DOCUMENT RESUME

ED 128 686 CG 010 791

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TITLE Confirmation of the Piagetian Logic of Exclusion and Combinations During Concrete and Formal Ope.ations.
PUB DATE Apr 76
NOTE 27p.; For related document, see CG 010 792; Paper presented at the Annual Meeting of the American Educational Research Association (San Francisco, California, April 19-23, 1976)

EDRS PRICE MF-$0.83 HC-$2.06 Plus Postage.
DESCRIPTORS *Cognitive Processes; Data Analysis; Deductive Methods; *Developmental Psychology; *Developmental Tasks; Educational Research; *Educational Theories; Junior High School Students; *Logical Thinking; Speeches; *Task Performance
IDENTIFIERS *Piaget (Jean)

ABSTRACT

A major premise of Piagetian theory relative to the periods of concrete and formal operations is that competence in specific logical operations is a necessary but not sufficient prerequisite to competence in other specific logical operations. The present study tested for the existence of specific concrete operations and specific formal operations that have been hypothesized to be developmental prerequisites to specific formal operations. The Multiple Hierarchical Analysis, a data analytic technique, was used to identify scales--or prerequisite sequences--that occur significantly more often than by chance. A sample of 622 junior high students were administered an assessment instrument consisting of two 16-item group administered written tests designed to measure various logical operations. The empirically-generated data of this study support the logically-generated Piagetian Theory. Also supported was the Piagetian postulation that each of the developmental periods is characterized by specific characteristics and a wholistic quality binding together the various characteristics. Combinatorial thought was found to be the quality that binds formal operations. (SJL)

# Confirmation of the Piagetian Logic of Exclusion and Combinations During Concrete and Formal Operations

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Paper presented at the annual meeting of the American Educational Research Association, San Francisco, April 1976.

# Confirmation of the Piagetian Logic of Exclusion and Combinations During Concrete and Formal Operations

A major premise of Piagetian theory relative to the periods of concrete and formal operations is that competence in specific logical operations is a necessary, but not sufficient, prerequisite to competence in other specific logical operations. This premise is sometimes referred to as the invariant sequence hypothesis and may be more specifically described according to the guidelines established by Guttman (1954) for defining the existence of unidimensional scales; that is, before there may be success in a task representing a specific logical operation, there must be success on tasks representing other logical operations that occur earlier in the scale. Evidence for this premise has been generated from two criterion sources. The first involves the difference between average age of success on tasks representing one logical and tasks representing another logical operation. The most frequent use of this criterion is in distinguishing between concrete operations and formal operations. Because tasks representing concrete operations are correctly solved at younger ages than tasks representing formal operations the inference is made that concrete operations are prerequisite to formal operations. Studies using this approach do not go beyond the ranking of tasks by levels of difficulty, whether it is the reporting of the difficulty sequence of the tasks used (Lee, 1971) or a determination of the significance of the difference in difficulty levels between some measure of the tasks (e.g., Lovell, 1961; Dale, 1970; Fischbein, Pampu, & Minzat, 1970; Brainerd, 1971; Mecke & Mecke, 1971; Jones, 1972; Somerville, 1974). In either approach, the existence of a scale or scales of prerequisites as defined by Guttman (1954) can not be determined. In those studies where an attempt was made to determine the existence of a scale via some variation of Guttman scalogram analysis, there is no way of determining whether the obtained scale(s) were a function of chance (e.g., Bart & Airasian, 1974).

The second criterion source is the use of a logical analysis of subject responses to specific Piagetian tasks and/or a determination of the logically necessary prerequisites for engaging in specific types of reasoning. A logical analysis of a subject's responses involves presenting a task and then identifying the logic used by the subject in attempting to solve the problem. For example, in studying how a child solves a problem that requires the formal operation of systematic exclusion of irrelevant variables, Inhelder and Piaget (1955/1958) used a simple pendulum and asked each subject to determine what changed the pendulum's oscillation frequency. Their results included examples of subject responses and an analysis of those responses. From the analysis of those responses, they inferred that the logical operation of seriation is prerequisite to the logical operation of making an inverse correspondence between an increasing series and a decreasing series, which is prerequisite to making a correct implication, which, in turn, is prerequisite to denying incorrect implications (Inhelder & Piaget, 1955/1958, pp. 70-79). This analysis supposedly describes what operations are prerequisite to other operations in the development of the formal operation of exclusion.

Similar analyses were performed for the tasks representing the formal operations of combinatorial thought, proportional reasoning, etc. Although the arguments presented are intuitively compelling, the important thing about them is that the prerequisites were <u>inferred</u>, and not directly tested by using tasks representative of the various logical operations hypothesized to be the developmental precursors of the various formal operations. Unfortunately, this approach of not directly testing for concrete operations but inferring their existence from lack of success on formal operational tasks tends to be quite frequent (e.g., Lovell, 1961; Dale, 1970; Fischbein, Pampu, & Minzat, 1970; Bart, 1971; Mecke & Mecke, 1971; Jones, 1972; Somerville, 1974; Kuhn & Angelev, Note 1).

An analysis of the logically necessary prerequisites for engaging in specific types of reasoning is also best illustrated in Inhelder and Piaget's (1955/1958) discussion of the formal operation of exclusion via the pendulum. In discussing the reasoning of subjects who were successful with the pendulum problem, they state the following:

> Thus we see that the exclusion of the three inoperant factors (which at first seemed so simple) as well as the reciprocal implications of the length and the result actually presuppose a complicated combinatorial operation which the subject cannot master except by ordering seriately the factors which are to be varied one-by-one, each time holding the others constant. . . . In comparing the correct inferences found at substage III-B with the earlier false ones, we see that the choice is again dictated by the presence of one or two conclusive combinations. Once more they presuppose a degree of mastery of the system of all possible combinations. (Inhelder & Piaget, 1955/1958, pp. 77-78)

The conclusion that formal operational combinatorial thought is a prerequisite to the formal operation of exclusion of irrelevant variables is clearly an inference because it is based on a logical argument and not on empirically generated data.

Thus, both lines of evidence have serious drawbacks: the first in that sequence of prerequisites are being formulated on the basis of some ordinal ranking of tasks and that, even when scalogram analysis is used, it does not ordinarily consider the role of chance in the normative determination of the obtained sequences; the second in that the existence of the prerequisite operations are logically generated and not concretely tested.

In contrast to the traditional types of evidence, the present study (a) tested for the existence of specific concrete operations and specific formal operations that have been hypothesized (Inhelder & Piaget, 1955/1958; Piaget & Inhelder, 1951/1975) to be developmental prerequisites to specific formal operations, and (b) used Multiple Hierarchical Analysis (Hofmann, Note 2), a data analytic technique that can identify scales, or prerequisite sequences, that are significantly greater than chance.

## Method of Investigation

### Hypotheses

1. Items representing the specific logical operations hypothesized to be the prerequisites of the formal operation of exclusion (Inhelder & Piaget, 1955/1958) will form a Guttman scale that is significantly greater than chance.

2. Items representing the specific logical operations hypothesized to be the prerequisites of the formal operation of combination (Inhelder & Piaget, 1955/1958; Piaget & Inhelder, 1951/1975) will form a Guttman scale that is significantly greater than chance.

3. Items representing the specific logical operations hypothesized to be the prerequisites of the formal operation of permutation (Piaget & Inhelder, 1951/1975) will form a Guttman scale that is significantly greater than chance.

### Sample

The original sample was composed of 622 junior high students participating in the Human Sciences Program© curriculum designed by Biological Sciences Curriculum Study. Five hundred seventy-eight were retained, as the remainder did not take both forms of the assessment instrument. Table 1 is a sex X age breakdown of the retained sample. No data on SES or ethnic background were available.

Insert Table 1 about here

Instrument

The assessment instrument consisted of two fifteen-item group administered written tests that included items designed to measure various logical operations hypothesized (Inhelder & Piaget, 1955/1958; Piaget & Inhelder, 1951/1975) to be prerequisites to the formal operations of exclusion, combination, and permutation. Three items from Form A (2, 8, 13) and three items from Form B (9, 10, 12) were eliminated from the analysis because they measured skills unrelated to Piagetian operations or were not reliably scorable. Table 2 lists the Piagetian operation,

Insert Table 2 about here

specific logical operation, and an illustration of each specific logical operation measured. Four concrete operational groupings were measured via six types of specific logical operations while two formal operational characteristics were assessed by four types of problems.

Concrete operational Grouping III (Bi-Univocal Multiplication of Classes) items (A11, B6, B11) required that each element of one set be placed in a one-to-or or-respondence with each element of a second set. Grouping IV (Co-Univocal Multiplication of Classes) items (A10, B4, B5) were similar to Grouping III items with the exception that the first set contained one entity, and that entity had to be placed in a one-to-one correspondence with each element in the second set.

Items A1, A4, B1, and B2 measured Addition of Asymmetrical Relations (Grouping V). Although these items used either an increasing series (A1, A4) or a decreasing series (B1, B2) the format was the same. The relationship between each of the adjacent elements in the series was presented and the subject had to determine the correctness of a statement about the extreme elements in the series.

Bi-Univocal Multiplication of Relations (Grouping VII) was measured by three types of items. One item measured conservation of continuous quantity by means of drawing and accompanying text indicating that two identical bottles of "coke" were poured into two different shaped glasses. Glass one was taller (↑) and thinner (←) than glass two. Thus, subjects had to determine that glass one was taller and thinner than glass two, as well as glass two being shorter and wider than glass one.

The other two types of multiplication of relation items had identical formats with slightly different content. One item (A6) presented two increasing series that were placed into one-to-one correspondence with the smallest entity in one series corresponding to the smallest entity in the second series, the second smallest entity of each series corresponding to each other, etc. The remaining items had one decreasing series and one increasing series. In both variations, one element of a series was given and the corresponding element in the other series had to be identified.

The formal operational characteristic of hypothetical-deductive thinking was measured by two types of items in which there were three or four sentences each with several clauses that were stated in the affirmative or in the negative.
An affirmative clause might say "John likes Mary" while its negation would be "John does not like Mary". Subjects were then given a specific clause and asked which of the other clauses consistently co-occured with the indicated clause. In three items (A3, A5, B8) there was a clause that co-occured with the identified clause; this type of logical operation is identified by Inhelder and Piaget (1955/1958) as making a correct implication. The item (B14) that did not have any clause consistenly co-occuring with the indicated clause was labeled as denying incorrect implications (Inhelder and Piaget (1955/1958).

Combinatorial thought was measured by two types of items, the first type involved making all non-redundant two-entity pairs from a set of entities presented in the item; the second, generating all-possible permutations of four entities. In both types of items, the key words "combinations" or "permutations" were not mentioned.

Table 3 is the distribution of items to the hypothesized scales according to Inhelder and Piaget (1955/1958, pp. 70-79, 111-122) and Piaget and Inhelder's (1951/1975, pp. 161-194) description of the logical operations represented by the various items. The concrete operational exclusion items are not separated into beginning or consolidated items as Inhelder and Piaget (1955/1958, pp. 70-73) do not adequately separate the subperiods according to specific logical operations, although one would intuitively expect the seriation items (A1, A4, B1, B2) to be prerequisite to the correspondence items (A14, A6, A7, B3, B7). The same lack of adequately distinguishing different subperiods for the formal operation of combination is evident. Note also, that the formal combination items are predicted to be in all three scales.

Seven items included a drawing for concrete reference and were multiple-choice, while the remainder included no drawing. Table 4 describes the item distribution by question type and use of a drawing.

Insert Tables 3 and 4 about here

## Procedure

The different forms were administered on two consecutive school days with Form A always given first. Administration was by classroom teachers trained by BSCS in the Human Sciences Program © Curriculum which includes formative testing by BSCS to assess the adequacy of the curriculum. Consequently, subjects were accustomed to being presented various assessment instruments from BSCS.

Subjects were informed that their scores would not count toward any grade and that the information was of benefit only to BSCS. On each test there were formal operational example items that were read aloud by the teacher and silently by the students. For each example question, the correct answer was provided with no indication of why the answer was correct. After answering the example items, subjects were instructed to answer all fifteen items on the form they were taking. The teacher read the items aloud while the subjects read silently. Total testing time for each form was approximately one hour.

Data Analysis

After subjects had responded to the individual questions, they were scored by graduate students at the University of Northern Colorado according to previously established objective criteria. After the responses had been scored the items were converted into a binary (1,0)

The binary data were then analyzed using a new multivariate procedure referred to as Multiple Hierarchical Analysis (MHA). The MHA model (Hofmann, Note 2) is a multidimensional scaling procedure developed for use with binary data. In particular, MHA identifies latent Guttman scales in a set of binary data. These latent scales when coupled with the original data are hierarchical in nature and are approximations to prefect non-chance Guttman scales. The resulting Guttman scales are defined descriptively by a $KR_{20}$, a hierarchical reproducibility index, and a Guttman reproducibility index. Although MHA does not identify all latent scales, it is a blind analytic procedure that identifies scales with large $KR_{20}$ indices and large reproducibilities.

In coupling the final latent scales with the original data, the items are ordered from easiest to hardest within scales and exact item level reproducibilities are determined. The scale reproducibility is just the average of the item level reproducibilities. If an item does not belong to a particular scale, its reproducibility on the scale is assumed to be zero.

If one takes the number of correct responses an individual obtains on a scale called j, defined by a group of items, it is assumed in the sense of Guttman (Goodenough, 1944) that the individual responded correctly to the j easiest items, in a normative sense. To the extent that this assumption is in error the observed scale will deviate from a perfect Guttman scale. The Guttman reproducibility for a total scale is the proportion of responses correctly predicted for all individuals when their summated composite score is used to make the prediction. For example, assume a 7 item scale with the items ordered from easy to hard. An individual with a score of 3 would show a response pattern of +++---- if the scale were a perfect Guttman scale. If, however, the response pattern appeared as +-++--- then 2 errors would be made in the prediction process: the second item would have been predicted as being correct but the individual missed it and the fourth item would have been predicted as being incorrect but the individual responded correctly to it. An item level reproducibility is nothing more than the proportion of responses correctly predicted, as being either correct or incorrect, across all respondents for the item.

Whereas the Guttman reproducibility is predicated upon the process of predicting a total response pattern from a composite score, a hierarchical reproducibility is based upon a hierarchical prediction process. The items of a scale are first ordered from easiest to hardest. It is assumed that an easier item is an empirical prerequisite to a harder item. Thus, if an individual responds incorrectly to an easy item, it is assumed that he will respond incorrectly to a more difficult item; if a correct response is given to an easy item, either a correct or an incorrect response may be given to a more difficult item. Two errors of prediction will occur when an individual responds incorrectly to an easy item and correctly to a difficult item. A hierarchical reproducibility for an item is in actuality a <u>link</u> between two items. It represents the proportion of correct predictions for a two-item sequence where the two items are adjacent to each other in a hierarchically ordered difficulty scale. In a very special sense, a hierarchical reproducibility is a Guttman reproducibility for a two-item scale. The hierarchical reproducibility for a total scale is the proportion of correct responses that are predicted for the items in a scale when the response to each item is predicted on the response given to the previous or prerequisite item. Thus the last item in a scale <u>will not</u> have a hierarchical reproducibility.

## Results[1]

Table 5 is a scale matrix. The columns correspond to the non-chance hierarchical scales in the twenty-four item data. The entries by row represent

Insert Table 5 about here

the item level reproducibilities for each item on each scale: A Guttman, reproducibility is the main entry and a hierarchical reproducibility is in parentheses. The items have been ordered easy to hard from top to bottom. With the exception of item A10, scale II is a subscale of scale I, while scales III, IV, V, and VI all share items from scale I as well as items from each other.

In Table 6 the intercorrelations of the observed scales are reported. For each scale, each individual had a scale score computed and the scale scores were the raw data for the Pearsonian correlations reported. Clearly, the scales are not independent of each other.

Insert Table 6 about here

Summary statistics for each scale are presented in Table 7. All six scales

Insert Table 7 about here

have significant hierarchical reproducibilities ($p < .001$), substantial reliabilities, and considerable practical significance, although it is clear that scales I and III are the "best" considering their difficulty range, average difficulty, and number of items. Practical significance is determined independently of statistical significance and is indicative of the percent improvement over minimum reproducibility made by a scale. The statistical significance of the reproducibilities is based upon a poisson distribution (see Hofmann, Note 2, Note 3).

Hypothesis 1

Hypothesis 1 was concerned with the prerequisites of the formal operation of exclusion. Scale III appears to represent the prerequisites for the development of formal operational exclusion. Within formal operations the items lined up exactly as predicted in Table 3; that is, items A3, A5, and B8 all were prerequisite to B14. Between concrete operations and formal operations the prediction was perfect for those concrete operational items (A1, B7) that were included in the scale. The remainder of the concrete items predicted as belonging with A1 and B7 were distributed across scales II (A14), IV (A14), and VI (A1, A4) with A6 and A7 not being represented in any scale. Consequently, the items representing the concrete operational prerequisites to formal operational exclusion, provided only partial support for the hypothesis.

Part of hypothesis 1 was the implied prediction that formal operational combinations are prerequisite to formal operational exclusion (Inhelder & Piaget, 1955/1958). Whether formal operational exclusion is considered as formal I - beginning formal operations - or formal II - consolidated formal operations - the data provide support for the hypothesis. In scale I items A9, A12, and B13 all were prerequisite to A5 and B14; while in scale III, A12 - the only formal combination item that was a member of the scale - was a prerequisite to A3, A5, B8, and B14. Thus, the Piagetian theoretical position that combinatorial thought is prerequisite to making correct implications, which is prerequisite to denying incorrect implications, was verified.

Hypothesis II

Hypothesis II focused on the prerequisites for formal operational combinations. Scale I provides almost perfect support for this hypothesis. Beginning concrete operational items (B4, B5) are prerequisite to consolidated concrete operational items (B6, B11), which are prerequisite to formal combination items (B13, A12, A9). The two concrete items not conforming to the prediction were A10 and A11. Item A10 is contained in scale II which is a subscale of scale I, while A11 is exactly the same as B11 except that it was multiple-choice and contained a drawing for reference. Apparently, the multiple-choice format made it more difficult than its constructed response version. It appears then that Piagetian theory is correct in its prediction of the prerequisites for the formal operation of combination.

Hypothesis III

The final hypothesis predicted that formal operational combinations would be prerequisite to formal operational permutations and scale I provides data supportive of the hypothesis. All formal operational combination items (B13, A12, A9) were prerequisite to the formal operational permutation items (B15, A15). It is interesting to note that denying incorrect implications (B14) as a consolidated formal exclusion operation is prerequisite to the formal operation of permutation. This contradicts the sequence implied by the age of success for consolidated formal exclusion (Inhelder & Piaget, 1955/1958, p. 75) versus the age of success for formal permutation (Piaget & Inhelder, 1951/1975, p. 191), and provides support for the reasoning presented earlier that inferring sequences from average age of success is fraught with problems.

Table 6 provides strong support for the Piagetian concept of structure d' ensemble. That is, although the items representing different prerequisite sequences, as evidenced by scales I and III, form different scales, they are related to each other. This is most evident in the magnitude of the correlation (.699) between the scales (I, III) representing the two sets of prerequisites studied.

## Discussion

The confirmation of the hypotheses provided twofold support for Piagetian theory. First, as Kohlberg (1971) has argued, there are two ways of providing support for stage oriented theories: logical analysis and empirical data. Piagetian theory (Inhelder & Piaget, 1955/1958; Piaget & Inhelder, 1951/1975) has provided detailed logical analyses of the various logical operations that are prerequisite to other logical operations. The present study provides a compliment to the logical analysis as its empirically generated data supports the logically generated theory. The results are important, not only because of their empirical base, but also because of the medium in which the results were obtained The original Piagetian studies and their various replications have been conducted in a one-to-one "clinical" interview situation where the subject was required to answer questions about and/or manipulate some physical situation. This interview paradigm has been criticized as being biased toward providing data automatically congruent with the theory (Brainerd, 1973). In contrast, the present study used group-administered written tests which had clear objective scoring criteria and provided results consonant with the theoretical predictions. There is a clear indication that Piagetian theory can be generalized beyond the traditional individual testing situation and it can be used to assess written items for their specific logical operation (Gray, Note 4).

The second type of theoretical support centers on the concept of structure d'ensemble which is the Piagetian postulation that each of the developmental periods is characterized by specific characteristics and a wholistic quality that binds together the various characteristics. Within the period of formal operations, there are a variety of logical operations that characterize the period. However, each of these characteristics has its ultimate base in the combinatorial system that is illustrated by all sixteen possible combinations of two assertions, *p*, *q*, and their respective negations $\bar{p}$, $\bar{q}$, (Inhelder & Piaget, 1955/1958). Thus, the formal operations of exclusion and permutation are based upon the combination of various propositions or various indicators of position. The clear results that formal combinations are empirically prerequisite to exclusion and permutations imply that combinatorial thought is the basic characteristic of formal operations and all of the other characteristics may be derived from it. That is, combinatorial thought is the glue that binds formal operations together.

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Footnote

[1]Data provided by the Human Sciences Program, with permission of the Biological Sciences Curriculum Study, Boulder, CO.

Table 1

Distribution of Sample

| Sex | Age | | | | | | | Total |
|---|---|---|---|---|---|---|---|---|
| | 9 | 10 | 11 | 12 | 13 | 14 | Missing | |
| Male | | | | | | | | |
| $\bar{X}$ | 9.50 | 10.48 | 11.70 | 12.33 | 13.22 | 14.06 | | 12.11 |
| s | 0.00 | .29 | .16 | .25 | .18 | .01 | | .57 |
| n | 1 | 4 | 113 | 135 | 19 | 3 | 11 | 275[a] |
| Female | | | | | | | | |
| $\bar{X}$ | | 10.54 | 11.70 | 12.27 | 13.25 | | | 12.07 |
| s | | .08 | .16 | .20 | .24 | | | .48 |
| n | | 4 | 121 | 142 | 18 | | 5 | 285[a] |
| Missing | | | | | | | | |
| $\bar{X}$ | | | 11.67 | | 13.08 | | | |
| s | | | 0.00 | | 0.00 | | | |
| n | | | 1 | | 1 | | | |
| Total | | | | | | | | |
| $\bar{X}$ | 9.50 | 10.51 | 11.70 | 12.30 | 13.23 | 14.06 | | 12.09 |
| s | 0.00 | .20 | .16 | .22 | .21 | .10 | | .53 |
| n | 1 | 8 | 235 | 277 | 38 | 3 | 16 | 562[a] |

*Note*. Total n = 578.

[a]Does not include subjects who did not report age.

Table 2

Item Distribution for Specific Logical Operations

| Piagetian Operation[a] | Specific Logical Operation | Example of Specific Logical Operation | Item |
|---|---|---|---|
| Concrete Operations Grouping III: Bi-Univocal Multiplication of Classes | 1-to-1 multiplication of each element in 1 group with each element in a second group | $\begin{array}{cccc} & L & M & N \\ A & AL & AM & AN \\ B & BL & BM & BN \\ C & CL & CM & CN \end{array}$ | A11 B6<br>B11 |
| Grouping IV: Co-Univocal Multiplication of Classes | 1-to-many correspondence | $\begin{array}{ccccc} & P_1 & P_2 & P_3 & P_4 \\ B & BP_1 & BP_2 & BP_3 & BP_4 \end{array}$ | A10 B4<br>B5 |
| Grouping V: Addition of Asymmetrical Relations | Increasing Seriation | $M < A < S < K$<br>$\therefore M < K$ | A1 A4 |
| | Decreasing Seriation | $P > K > J > R$<br>$\therefore P > R$ | B1 B2 |
| Grouping VII: Bi-Univocal Multiplication of Relations | Conservation of Continuous Quantity | $(G_1 \uparrow G_2)\ X\ (G_1 \leftarrow G_2) = (G_1 \uparrow\leftarrow G_2)$ | A14 |
| | 1-to-1 Correspondence of 2 Increasing Series | $\begin{array}{ccccccc} D & < & J & < & G & < & R \\ \updownarrow & & \updownarrow & & \updownarrow & & \updownarrow \\ F_1 & < & F_2 & < & F_3 & < & F_4 \end{array}$<br>$\therefore \begin{array}{c} J^b \\ \updownarrow \\ F_2 \end{array}$ | A6 |

Table 2 (continued)

| Piagetian Operation[a] | Specific Logical Operation | Example of Specific Logical Operation | Item |
|---|---|---|---|
| | 1-to-1 correspondence of a Decreasing and Increasing Series | $B > T > F > J$ <br> $\updownarrow \quad \updownarrow \quad \updownarrow \quad \updownarrow$ <br> $C_1 < C_2 < C_3 < C_4$ <br><br> $B^c \updownarrow C_1$ | A7 B3 B7 |
| Formal Operations Hypothetical-Deductive Thinking | Make Correct Implication[d] | $p \cdot q \cdot \bar{r} \cdot x$ <br> $\bar{p} \cdot q \cdot r \cdot x$ <br> $p \cdot \bar{q} \cdot \bar{r} \cdot \bar{x}$ <br><br> $\therefore q \left[ p \vee r \right] \leftrightarrow x$ | A3 A5 B8 |
| | Deny Incorrect Implications[d] | $p \cdot \bar{q} \cdot r \cdot x$ <br> $\bar{p} \cdot q \cdot \bar{r} \cdot \bar{x}$ <br> $\bar{p} \cdot q \cdot \bar{r} \cdot x$ <br> $p \cdot \bar{q} \cdot r \cdot \bar{x}$ <br><br> $\therefore \left[ p \vee q \vee r \right] * x$ | B14 |

Table 2 (continued)

| Piagetian Operation[a] | Specific Logical Operation | Example of Specific Logical Operation | | | | | | | Item |
|---|---|---|---|---|---|---|---|---|---|
| Combinatorial | Pair-Wise Non- | | A | D | L | M | N | S | A9 A12 |
| Thinking | Redundant | A | | AD | AL | AM | AN | AS | B13 |
| | Combination | D | | | DL | DM | DN | DS | |
| | | L | | | | LM | LN | LS | |
| | | M | | | | | MN | MS | |
| | | N | | | | | | NS | |
| | | S | | | | | | | |
| | Permutation | FGIP | GFIP | IGFP | PGIF | | | | A15 B15 |
| | | FGPI | GFPI | IGPF | PGFI | | | | |
| | | FPGI | GPFI | IPGF | PFGI | | | | |
| | | FPIG | GPIF | IPFG | PFIG | | | | |
| | | FIPG | GIPF | IFPG | PIFG | | | | |
| | | FIGP | GIFP | IFGP | PIGF | | | | |

[a] Grouping numbers follow Flavell (1963)

[b] Question is "Does J go with $F_2$"?

[c] Question is "Does B go with $C_1$"?

[d] Inhelder and Piaget's (1955/1958) term

Table 3

Item Distribution by Hypothesized Scales

| Period | Scale Defined by Formal Operation | | |
|---|---|---|---|
| | Exclusion | Combination | Permutation |
| Beginning Concrete | | A10 B4 B5 | |
| | A1 A4 B1 B2 | | |
| | A14 A6 A7 B3 B7 | | |
| Consolidated Concrete | | A11 B6 B11 | |
| Beginning Formal | A3 A5 B8 | A9 A12 B13 | A9 A12 B13 |
| Consolidated Formal | B14 | | A15 B15 |

Table 4

Item Distribution by Question Type and Concrete Reference

| Question Type | Reference: Drawing | Reference: No Drawing |
|---|---|---|
| Multiple - Choice | | |
| 4 alternatives | A14 | |
| 5 alternatives | A1 A4 A6 A7 A10 A11 | A9 |
| Constructed Response | | A3 A5 A12 A15 B1 B2 B3 B4 B5 B6 B7 B8 B11 B13 B14 B15 |

Table 5

Scale Matrix Defining Guttman and Hierarchical Level Reproducibility

| Item | Diffi-culty | Observed Scales Coupled to Latent Scales | | | | | |
|---|---|---|---|---|---|---|---|
| | | I | II | III | IV | V | VI |
| B4 | .94 | .95(.97) | | | | | .95(.96) |
| A10 | .93 | | .95(.95) | | | | |
| B5 | .92 | .93(.95) | | | | .93(.94) | |
| A6 | .89 | | | | | | |
| A14 | .87 | .89(.91) | .95(1.00) | | | | |
| B6 | .85 | .91(.95) | | | | | |
| A7 | .85 | | | | | | |
| A4 | .76 | | | | | | .90(.89) |
| B11 | .75 | .88(.95) | | | | | |
| A1 | .75 | | | .79(.82) | | | .86(1.00) |
| B7 | .74 | | | .80(.91) | | .84(.85) | |
| B3 | .74 | | | | .86(.88) | .83(.94) | |
| B2 | .71 | | | | .95(.95) | | |
| B1 | .68 | | | | .84(.99) | | |
| B13 | .56 | .86(.95) | | | | | |
| A12 | .47 | .84(.87) | | .75(.80) | | | |
| B8 | .45 | | | .82(.97) | | | |
| A11 | .39 | .76(.82) | | | | | |
| A9 | .36 | .77(.86) | | | | .88(*) | |
| A5 | .27 | .80(.98) | | .92(.96) | | | |
| A3 | .24 | | | .88(.97) | | | |
| B14 | .08 | .93(.96) | .96(.96) | .95(*) | | | |
| B15 | .05 | .93(.99) | .97(.99) | | .99(*) | | 1.00(*) |
| A15 | .03 | .97(*) | .98(*) | | | | |

Note. Table entries are Guttman reproducibilities. Entries in parentheses are hierarchical reproducibilities. (*) indicates that the last item of a scale has no hierarchical reproducibility.

Table 6

Intercorrelations Among Observed Guttman Scales

| Scale | I | II | III | IV | V | VI |
|---|---|---|---|---|---|---|
| I | 1.000 | | | | | |
| II | .564 | 1.000 | | | | |
| III | .699 | .424 | 1.000 | | | |
| IV | .380 | .296 | .325 | 1.000 | | |
| V | .564 | .224 | .457 | .536 | 1.000 | |
| VI | .446 | .366 | .492 | .286 | .251 | 1.000 |

Table 7

Total Scale Descriptive Indices

| Index | Scale I | II | III | IV | V | VI |
|---|---|---|---|---|---|---|
| Guttman Reproducibility | .880 | .959 | .843 | .911 | .868 | .926 |
| Hierarchical Reproducibility | .929* | .974* | .904* | .940* | .911* | .950* |
| Practical Significance | .721 | .897 | .735 | .850 | .776 | .853 |
| $KR_{20}$ | .891 | .664 | .886 | .915 | .651 | .725 |
| Number of Items | 13 | 5 | 7 | 4 | 4 | 4 |
| Difficulty Range | .94-.03 | .93-.03 | .75-.08 | .74-.05 | .92-.36 | .94-.05 |
| $\bar{X}$ Difficulty | .50 | .39 | .43 | .55 | .69 | .63 |

Note. $n = 578$

* $p < .001$