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ABSTRACT

A survey questionnaire designed to gather information concerning collegiate requirements for the mathematical training of secondary school mathematics teachers was sent to the 749 members of the American Association of Colleges for Teacher Education. This paper summarizes the responses on the 448 returns received. The survey was designed to determine the number of semester hours of mathematics courses required of future mathematics teachers, the extent to which institutions offer different programs for junior and senior high school teachers, and the nature and extent of curricular change in the mathematical preparation of teachers since 1960. In addition to questions concerning specific course requirements, respondents were asked to indicate whether certain topics are included in courses. They were queried concerning curricular changes and the major influences motivating these changes; three questions concerned outstanding features and needs of respondents' programs. (SD)

A SUMMARY

OF

AN ANALYSIS OF THE REQUIRED MATHEMATICAL PREPARATION FOR SECONDARY SCHOOL MATHEMATICS TEACHERS IN THE UNITED STATES

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STATEMENT OF THE PROBLEM

The primary objective of this study (5) was to examine the status of the preparation of preservice secondary mathematics teachers in institutions of higher learning in the United States. This examination was conducted in terms of recent programs and factors influencing change, to determine to what extent these practices are in agreement with the recommendations set forth by the Cambridge Conference on School Mathematics (CCSM) and the Committee on the Undergraduate Program in Mathematics (CUPM).

In particular, this study examined the following:

1. The changes in mathematics curricular offerings since 1960 in institutions of higher learning in the United States and the reasons for these changes.

2. The extent to which institutions of higher learning in the United States offer courses designed specifically for the preparation of junior high school mathematics teachers.

3. The differences in trends among institutions of higher learning in the United States of various degree levels regarding the mathematics course content units offered to preservice secondary school mathematics teachers.

4. The extent to which various organizations are perceived to have influenced changes in preservice programs for secondary school mathematics teachers since 1960.

5. The outstanding features and needs of current preservice secondary mathematics programs in institutions of higher learning in the United States.

RESEARCH DESIGN AND PROCEDURES

In order to answer the research questions stated above, the design called for the development of an instrument consisting of four sections. The instrument developed was similar to the one used by Pitts (6) in his study of the mathematics preparation of elementary teachers.

Section I was designed to obtain information about the following: (1) the number of semester hours of credit required for a major in secondary mathematics; (2) the mathematics content courses required of or frequently taken by preservice secondary mathematics teachers; (3) the approaches used in teaching geometry courses; (4) the number of secondary mathematics majors expected to graduate in the 1973-74 school year; (5) the extent of endorsement of the recommendations of the CUPM and the CCSM; and (6) the availability of courses designed specifically for junior high school mathematics teachers.

Questions in Sections I (5) and I (6) were taken from Pitts' questionnaire. The remaining five parts in Section I were developed by the writer with the assistance of Dr. Jackson Byars. The titles of the courses in Section I (2) were taken from a sampling of recent catalogs of institutions of higher learning. In Section I (2), each participant was asked to give the number of hours of the content courses, the approximate size of the 1973-74 classes of these courses, and whether or not the courses were required for junior high school mathematics teachers and/or senior high school mathematics teachers.

In Section II, curriculum development, each participant was asked to describe significant changes in content requirements for preservice secondary mathematics teachers since 1960 and to indicate the organizations influencing these changes. The two questions in this section were taken from Pitts' questionnaire, with only minor changes.

Section III focused on the content units recommended by the CUPM and the CCSM for preservice preparation of secondary mathematics teachers. These content units were taken from the CUPM publication entitled Commentary on a General Curriculum in Mathematics for Colleges (3) and the Cambridge Conference publication entitled Goals for School Mathematics (4). The writer divided the content units into four broad areas: algebra, probability and statistics, geometry, and analysis. A comprehensive compilation was made from the list of topics included in the above-mentioned books. Content units were listed and participants were asked to circle one of the following: (a) presented; (b) not presented, but appropriate; or (c) not presented and not appropriate. Some of the more "advanced" topics in analysis were excluded because of the questionnaire's length and because some of the content units seemed inappropriate for beginning teachers.

Section IV obtained information regarding the outstanding features of each participating institution's mathematics content program for undergraduate secondary mathematics teachers; and what mathematics courses utilized the computer. One question in this section, IV (3), was devised by the writer and the other two were taken from the questionnaire used by Pitts, again with minor changes.

Five graduate students, all of whom were mathematics teachers and four of whom had taught in college, were the first group to critique the questionnaire. The questionnaire was also presented to the mathematics faculty at Southeast Missouri State University for suggestions regarding improvements on the structure and wording of questions. A third group to offer suggestions for improvement was the writer's doctoral committee. Suggestions for improvements were made by all three groups and these were incorporated into the instrument.

DATA SOURCE AND COLLECTION

The source of the data for this study was 749 institutions of higher learning in the United States which held membership in The American Association of Colleges for Teacher Education Directory (1). Each institution also had to be listed as a four-year college or university in 1959, according to the listing in the ninth edition of The College Blue Book (2). This condition was placed on the sample since the study was limited to the changes in curriculum from 1960-1974.

On March 30, 1974, questionnaires (with the first page being a cover letter) were mailed to the chairperson of the mathematics department in each of the 749 institutions of higher learning sampled in this study. After the questionnaires numbered 1 through 749 were mailed, the institutions of higher learning were divided into three groups. Group I included 489 institutions of higher learning that offered the bachelor's degree as the highest degree in mathematics, as recorded in the fourteenth edition of The College Blue Book: U. S. Colleges: Tabular Data (8).

Group II consisted of 157 institutions of higher learning who offered the master's degree as the highest degree in mathematics, and Group III was comprised of 103 institutions of higher learning who offered the doctorate as the highest degree in mathematics.

On May 13, 1974, the first follow-up letter was mailed. At that time 227 responses (29 percent) had been received. A second follow-up letter was mailed on August 15, 1974. By that date 376 responses (50 percent) had been received. The last response used in the study was received October 7, 1974, which brought the total number of responses to 448 (60 percent). This included institutions of higher learning from 48 states and the District of Columbia. Of the 448 responses, 418 (56 percent) were used in the analysis. Thirty questionnaires were not used for reasons such as the following: (1) the institution prepared only elementary teachers; (2) institution offered mathematics courses only to graduate students; (3) questionnaire was inappropriate to their particular institution; (4) respondent said he had mailed questionnaire to writer but it was never received; and (5) institution had closed.

A self-addressed, stamped envelope and a questionnaire were enclosed in all three mailings and each participant was offered a summary of the results of the survey. Three hundred twenty-four, or 78 percent, desired a copy of the results.

Since the department chairperson was invited to ask colleagues to assist with the completion of the questionnaire, no direct effort was made to validate the data. A space was provided for the signature of participants, and of the 418 usable questionnaires, 395 were signed. Included in the 418 usable questionnaires were 249 (51 percent) responses from institutions of higher learning who offered the bachelor's degree as the highest degree in mathematics, and 64 (62 percent) responses from institutions of higher learning who offered the doctorate as the highest degree in mathematics.

SEARCH FOR SYSTEMATIC BIAS

Since 40 percent of the sample did not respond, the writer made an attempt to see if bias existed in the completed questionnaires that were received. The questionnaires were divided into three groups--according to when they were received. In addition, each group was divided by degree levels and the following items were examined: the number of semester hours required for teachers of junior and senior high school mathematics, and the listing of one or more outstanding features.

The number of semester hours required for teachers of junior high and senior high school mathematics remained rather stable throughout the study--with respect to various institutions' degree levels and time of receiving the completed questionnaires.

As a further check for possible bias, the number of responses from various geographic locations were considered. A response of 50 percent or greater was received from institutions in 38 states. Institutions in three of the 12 remaining states, Alabama, New York, and Texas, returned 42 percent or more of the questionnaires. Non-respondents from the other nine states were from only 31 different institutions. Eight questionnaires were mailed to the District of Columbia and three replies were received. Alaska and Wyoming were the only two states from which responses were not received.

In light of the above analysis the writer concluded that there was not sufficient evidence to support a claim of systematic bias based on the degree level of the institutions, the time of response, or the number of semester hours required for a mathematics major.

There did appear to be a trend in the frequency with which respondents reported outstanding features of their programs. Of those responding to the first, second, and third mailings respectively, 85 percent, 70 percent, and 52 percent reported such features. This might be used to infer that the institutions which responded earlier either felt better about their programs or took more care in their responses. If the former possibility were the real case, then one might assume that the non-respondents felt even less strongly that their programs had outstanding features than did the respondents.

Of the 448 questionnaires returned, 30 were from institutions that in one way or another were not appropriate for inclusion in the sample. It might be assumed that there were more such institutions which did not respond if non-response is considered to be a more likely option for a school which receives a questionnaire inappropriate to its offerings. Thus the 56 percent usable responses reported actually represents 58 percent or more of the schools which should be considered in the study.

If the data of the study are biased due to non-respondents, then the bias is most likely to be in the direction of showing the programs to be somewhat stronger than is the actual case.

METHOD OF ANALYSIS

The data from Sections I, II (2), and III were coded, placed on coding sheets, keypunched, verified, and double-checked for accuracy. Each questionnaire required three IBM cards. If Sections I (1) or I (2) were answered in terms of quarter hours, then they were converted to semester hours by multiplying by two-thirds and then rounded to the nearest integer. In addition, if the number of hours required for a major in mathematics, Section I (1), or the number of hours of the content courses, Section I (2), were not given on the returned questionnaires, then the writer obtained this information from the appropriate college catalog. Approximately 40 college catalogs were consulted. With these exceptions, the data for the study were obtained from the questionnaires.

Three programs from The Funstat Package in Fortran IV (7) were used in the analysis. They were Simple Tabulation for total, Bivariate Frequency Distribution by degree level, and Chi-Square test of independence.

The data from Sections II (1) and IV were tabulated by the writer's taking the information from the questionnaires and recording it by hand onto sheets of paper. The data were then analyzed and placed in appropriate tables.

Tables were constructed for each item in all sections of the questionnaire, giving the distribution and percentage of frequency of the responses--with the only exceptions being in Section I (4), where the responses were not satisfactory enough for consideration, and Sections II (2) and IV (2), where ranks were calculated. The questions asked in Section I (4) were: How many (a) secondary mathematics majors and (b) mathematics majors do you expect to graduate from your institution or branch in the 1973-74 school year? The number of secondary mathematics majors in part (a) should have been a subset of the number of mathematics majors given in part (b). However, this was not true for several of the responses. Since the questions in Section I (4) were misinterpreted by several people, the writer did not include them in the analysis.

A comparison of the opinions of institutions of higher learning of different degree levels regarding the presentation and appropriateness ratings of content units was another analysis performed in this study. Respondents were asked to indicate whether they felt that a given content unit was (a) presented; (b) not presented, but appropriate; or (c) not presented and not appropriate. Institutions were categorized according to the highest degree granted by the department of mathematics. A chi-square test of independence was performed to determine if there were statistically significant differences in the distributions of the ratings by institutions of higher learning of different degree levels.

A summary of the results of this study which was mailed to the respondents follows.

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3. Committee on the Undergraduate Program in Mathematics. Commentary on a General Curriculum in Mathematics for Colleges. Berkeley, California: Mathematical Association of America, 1972.

4. Goals for School Mathematics: The Report of the Cambridge Conference on School Mathematics. Boston: Houghton Mifflin Company, 1963.

5. Johnson, Carl S. "An Analysis of the Required Mathematical Preparation for Secondary School Mathematics Teachers in the United States." Unpublished Doctor's Dissertation, Kansas State University, 1975.

6. Pitts, Carl T. "A Study of the Required Mathematics Content Courses for Undergraduate Elementary Majors in the United States." Unpublished Doctor's Dissertation, Indiana State University, 1973.

7. Roscoe, John T. The Funstat Package in Fortran IV. New York: Holt, Rinehart and Winston, Inc., 1972.

8. Taylor, Allan T., and William E. Burgess, eds. The College Blue Book: U. S. Colleges: Tabular Data. Fourteenth edition, New York: CCM Information Corporation, 1972.

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DEPARTMENT OF CURRICULUM AND INSTRUCTION

November 25, 1975

A SUMMARY

OF

AN ANALYSIS OF THE REQUIRED MATHEMATICAL PREPARATION FOR SECONDARY SCHOOL MATHEMATICS TEACHERS IN THE UNITED STATES

Dear Respondent:

Your help in making this survey possible last year is greatly appreciated. I am sorry about the delay in getting you a copy of the summary of the results of the study. The data from this survey were used in my dissertation at Kansas State University, and it took a while to get everything compiled.

If you desire more information about this study you may contact me at Marshall University, Department of Elementary Education, Huntington, West Virginia 25701.

Thank you again for your help.

Sincerely yours,

Carl S. Johnson

Carl S. Johnson
Assistant Professor of
Curriculum and Foundations

SECTION I. General Information

1. In your institution or branch, what is the number of hours required for a major in:

   a. Junior high mathematics? (mean was 31.42 semester hours)

   b. Senior high mathematics? (mean was 33.28 semester hours)

2. Please provide the following information for the content courses in mathematics required of or frequently taken by prospective secondary teachers in your program:

Title of Course	Mean Number of Hours Required	Number of Schools Indicating a Requirement	
		For Jr. High Teachers	For Sr. High Teachers
1. Elementary Calculus	10.82	292	413
2. Abstract Algebra	2.72	200	320
3. Linear Algebra	2.05	173	260
4. Geometry	2.45	203	301
5. Probability & Statistics	1.56	117	183
6. Computer Science	0.67	67	92
7. History of Mathematics	0.23	28	36
8. Foundations of Mathematics	0.85	38	50
9. Set Theory & Logic	0.38	41	53
10. Introduction to Analysis	0.72	52	82
11. Number Theory	0.23	20	33
12. Topology	0.10	5	14
13. Differential Equations	0.47	38	61
14. The Real Number System	0.15	26	22
15. Advanced Calculus	0.79	51	82
16. Numerical Analysis	0.79	3	7
17. Applied Mathematics	0.05	4	5
18. Real Variables	0.14	11	16
19. Complex Variables	0.04	5	5
20. Other Required	0.60	41	60

3. Please circle the approach(es) used in teaching your geometry courses.

   a. The classical approach of Felix Klein (Circled by 238 respondents)
   b. The transformation approach (Circled by 157 respondents)
   c. The vector space approach (Circled by 72 respondents)
   d. Others (Circled by 73 respondents)

4. Do you endorse the Level II-J and Level III recommendations for undergraduate secondary teachers made by the CUPM?

   YES 291 NO 49 NOT FAMILIAR WITH RECOMMENDATIONS 66

5. Do you believe that The 1963 Cambridge Conference Goals for School Mathematics are realistic for secondary teachers?

   YES 69 NO 196 NOT FAMILIAR WITH RECOMMENDATIONS 147

6. Do you have courses designed specifically for junior high school mathematics teachers?

   YES 39 NO 379 If your answer is yes, please list the courses along with your comments.

SECTION II. Curriculum Development

1. What significant changes have been made in the content of your program for undergraduate secondary teachers in mathematics since 1960; and in what year were the changes made?

   The five changes listed most often were:

   (1) Computer science required or more computer science added.
   (2) Linear algebra required or more linear algebra added.
   (3) Geometry required or more geometry added.
   (4) Abstract algebra required or more abstract algebra added.
   (5) Probability and statistics required or more probability and statistics added.

2. Which of the organizations listed below have directly influenced changes (since 1960) in your undergraduate program for secondary teachers in mathematics content? Please rank in order of importance and omit those without influence. (1 is the most importance influence.)

   10* (a) State Department of Education

   4* (b) The College (or Department) of Education in Your Institution

   1* (c) State Mathematics Organizations

   52* (d) Committee on Undergraduate Preparation in Mathematics (CUPM)

   0* (e) The Cambridge Conference Goals for School Mathematics (CCSM)

   1* (f) National Council of Teachers of Mathematics

   10* (g) Your Institution's Curriculum Revision Committee

   14* (h) Others (please specify) ____________________

*Percent of time ranked first.

Note -- Eight percent of the respondents checked some of the organizations which had directly influenced curriculum change but failed to rank them.

SECTION III. Content

Listed below are content units recommended by CUPM and CCSM for undergraduate preparation of secondary mathematics teachers. Please indicate how each of these units fits in your present undergraduate secondary mathematics program by circling the appropriate letter. PLEASE THINK OF THESE AS TOPICS RATHER THAN COURSES REGARDLESS OF WHERE THEY APPEAR. You may want to consult some of your colleagues on this section.

a. presented
b. not presented, but appropriate
c. not presented and not appropriate

A. Algebra

a	b	c	x*		
91	3	2	4	1.	Review of properties of real and complex numbers
86	6	4	4	2.	Linear and quadratic equations and inequalities
85	8	2	5	3.	Rational forms and functions
80	9	6	5	4.	Modular arithmetic
94	1	1	4	5.	Systems of linear equations
85	8	2	5	6.	Euclidean algorithm
42	37	12	9	7.	Diophantine equations
37	30	23	10	8.	Complex numbers as residue classes of polynomials mod $x^2 + 1$
92	3	1	4	9.	Mathematical induction
86	8	2	4	10.	Fundamental theorem of algebra
79	11	3	7	11.	Archimedean property
94	1	1	4	12.	Groups, rings, fields, vector spaces over fields
58	22	10	10	13.	Cayley-Hamilton theorem
74	14	5	7	14.	Inner products and orthogonal transformations
90	5	1	4	15.	Vector spaces and subspaces
90	4	2	4	16.	Linear dependence, bases, dimension
94	2	0	4	17.	Matrices, determinants
88	7	1	4	18.	Equivalences of matrices, matrices of a transformation
79	12	3	6	19.	Triangular form of matrices, diagonal form of symmetric matrices
91	3	1	5	20.	Matrix inversion
47	32	12	9	21.	Estimation of characteristic roots
50	28	13	9	22.	Invariant subspaces
81	9	4	6	23.	Linear mappings
67	18	8	7	24.	Eigenvalues

B. Probability and Statistics

a	b	c	x*		
91	3	0	6	1.	Sample spaces, events as subsets, probability axioms
91	4	0	5	2.	Sampling from a finite population

*Did not complete

a	b	c	x*		
74	13	4	9	3.	Unordered sampling
79	10	2	9	4.	Ordered sampling with and without replacement
91	3	0	6	5.	Binomial coefficients and counting techniques applied to probability problems
90	4	0	6	6.	Conditional probability, independent events, Bayes' formula
91	3	1	5	7.	Mean, variance and expectation
72	16	5	7	8.	Chebychev's inequality
80	10	3	7	9.	Central Limit Theorem, Law of Large Numbers, statistical application
86	7	1	6	10.	Random variables (discrete and continuous) and their distributions--binomial, Poisson, uniform, exponential, normal
69	16	7	8	11.	Joint distribution of random variables and independent variables
72	13	6	9	12.	Probability for countable sample spaces
75	10	7	8	13.	Density and distribution functions
44	28	17	11	14.	Sequences of random variables
26	44	18	12	15.	Markov chains
36	33	19	12	16.	Stochastic processes
77	10	4	9	17.	Poisson and normal approximation to the binomial
54	28	8	10	18.	Analysis of variance
37	40	12	11	19.	Design of experiments
74	13	4	9	20.	Statistical estimation, sampling, hypotheses testing
53	26	10	11	21.	Nonparametric methods, power of a test, regression, point and interval estimates

C. Geometry

83	8	1	8	1.	The real numbers and geometry
75	13	3	9	2.	Logic of open statements and quantifiers
82	8	2	8	3.	Logic of formal proofs
82	7	3	8	4.	Axiomatic development of Euclidean geometry
84	6	1	9	5.	Incidence and order properties
77	10	2	11	6.	Congruence of triangles and inequalities in triangles
56	26	6	12	7.	Mappings by elementary functions, stereographic projection
54	29	7	10	8.	Projective geometry
79	11	1	9	9.	Non-Euclidean geometries
80	7	4	9	10.	Vectors, lines and planes in space, polar coordinates, parametric equations
72	14	4	10	11.	Conics and quadrics
63	21	5	11	12.	Constructions with ruler and compass
61	20	5	14	13.	Intuitive and synthetic geometry of the Pythagorean theorem
84	6	2	8	14.	Cartesian plane and space, lines, planes, circles and spheres
62	21	6	11	15.	Motions in Euclidean space, groups of motions

a	b	c	x*		
77	10	4	9	16.	Linear transformations and transformation laws
72	14	4	10	17.	Rotations in the plane and in space
17	26	42	15	18.	Dual spaces and tensor products
16	30	40	14	19.	Topology of the complex plane
17	27	42	14	20.	Differential geometry of curves in space

D. Analysis

95	1	1	3	1.	Review of the ideas of function, graph, slope of line, etc.
93	2	1	4	2.	Set terminology
78	14	3	5	3.	Cardinality
94	1	1	4	4.	Logarithmic, exponential and hyperbolic functions
96	0	0	4	5.	Chain rule--include derivatives of functions defined implicitly, inverse function and its derivative
96	0	0	4	6.	Limits, continuity
96	0	0	4	7.	Maxima and minima, curve sketching
96	0	0	4	8.	Differentiation of rational functions, trigonometric functions
96	0	0	4	9.	Definite integral, area, volume of solid of revolution
96	0	0	4	10.	The Mean Value Theorem, Fundamental Theorem of Calculus
95	1	0	4	11.	Sequences, series, power series
90	4	1	5	12.	Absolute and unconditional convergence
95	1	0	4	13.	Techniques of integration and applications
84	9	2	5	14.	Indeterminate forms, interpolation, difference methods
89	5	1	5	15.	Iterated and multiple integrals
94	1	1	4	16.	Indefinite integrals
80	9	3	8	17.	Multidimensional differential and integral calculus
84	10	2	4	18.	Calculus for functions of several variables
89	5	1	5	19.	Definite integral, its existence for continuous functions
86	7	3	4	20.	Theory of curves--parametric representation of curves, tangent, normal, arc length, curvature
91	4	1	4	21.	Partial derivatives
82	10	3	5	22.	Differential equations
71	18	4	7	23.	Numerical integration
63	22	7	8	24.	Uniform continuity
46	28	19	7	25.	Riemann-Stieltjes integral
43	28	20	9	26.	Analytic functions
36	33	23	8	27.	Topology and limits in metric spaces
34	30	27	9	28.	Continuous images of compact sets

SECTION IV. Outstanding Features and Needs of Your Program

In the space provided below, would you please identify and discuss the outstanding features and needs of your undergraduate content preparation program for teachers of secondary mathematics.

1. What do you consider to be the outstanding features of your program?

   Some of the most outstanding features, listed in order of more occurrence to less occurrence were:

   (1) Provides a solid mathematical background
   (2) Small classes
   (3) Breadth of offering
   (4) Flexibility of offering
   (5) Breadth and depth of topics covered
   (6) Dedicated faculty

2. What changes are needed to significantly improve your mathematics content program for prospective secondary teachers? (Please list them in order of importance.)

   Five of the most frequently listed needs were:

   (1) Greater utilization of the computer in mathematics courses
   (2) More geometry
   (3) More applications
   (4) Course on history of mathematics
   (5) Course on teaching junior high school mathematics

3. What mathematics courses do you offer that utilize the computer as part of the course and how is the computer used?

   The five mathematics courses utilizing the computer most often were:

   (1) Numerical Analysis
   (2) Computer Science
   (3) Calculus
   (4) Statistics
   (5) Differential Equations