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IDENTIFIERS *PLATO; Programmed Logic for Automatic Teaching Operations

ABSTRACT

This volume presents a partial description of the lessons in the preliminary version of the PLATO fractions curriculum. Each lesson has three parts: review, new material, and a student-selected option. Students receive immediate feedback from the computer as they progress through each lesson. Five groups of lessons are described: meaning of fractions, mixed numbers, equivalent fractions, addition and subtraction (like denominators), and addition and subtraction (unlike denominators). An outline and flowchart is presented for each group. For each lesson within a group, a statement of purpose, a brief description, and sample computer displays are provided. (SD)

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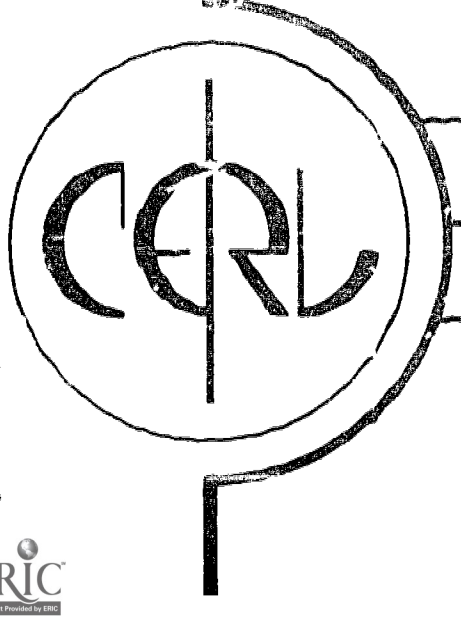
Share Jumping Beans (touch)

THE FRACTIONS CURRICULUM

PLATO ELEMENTARY SCHOOL MATHEMATICS PROJECT

SHARON DUGDALE
DAVID KIBBEY

March, 1975



Computer-based Education Research Laboratory

University of Illinois Urbana Illinois

FRACTIONS CURRICULUM

OF THE

PLATO ELEMENTARY SCHOOL MATHEMATICS PROJECT

Sharon Dugdale
David Kibbey

Computer-based Education Research Laboratory
University of Illinois at Urbana-Champaign

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Addition and Subtraction (unlike denominators)	E1 - E23

*Chapters on multiplication and decimals will be added when they are finished.

PREFACE

This book is a partial description of the PLATO elementary school fractions curriculum. Chapters on multiplication and decimals will be added at a later date. For each lesson described here, we have included pictures of a few selected screen displays which seem representative of the material in the lesson. The reader should bear in mind that the sense of a display often depends on the order and manner in which various parts of the display appear, so that interpreting a particular display as a whole is usually more meaningful than trying to read it from top to bottom. Of course, the few pictures with each lesson cannot show the entire content of the lesson. Anyone interested in more than a brief overview should try the actual lesson at a PLATO terminal. Many of our lessons require that the terminal be equipped with a touch panel. In this book each such lesson is identified by the word "touch" after the title on its description page.

The curriculum presented here is a preliminary version which has not yet had extensive testing in elementary school classrooms. This testing should be possible during the next several months. We anticipate the need for many changes and revisions based on information gathered during these classroom trials.

THE STUDENT SESSION

Each day's session has three parts called "slots."

Review Slot

Review slot is the "warm-up" period. It includes practice of already-learned skills, reinforcement of recent main slot material, and, occasionally, readiness for a lesson soon to occur in main slot. Each day's review slot has several short (often one page) lessons. A particular lesson may appear every day, or every few days, for several weeks.

Main Slot

Main slot is the heart of the session. It includes the new material. A student may see more than one lesson in each day's main slot. Many lessons, especially the long ones, are done a little at a time and will continue to appear for several days. For most of these lessons, the student is required to do a short segment, usually a specific number of problems, then is allowed to continue in the lesson for as long as he or she wants. When the student chooses to move on to some other available lesson (by pressing SHIFT-NEXT), the remaining part of the lesson is saved for future sessions.

Choice Slot

Choice slot allows the student to choose lessons from a list. Many of the lessons in the list are games that reinforce recently learned mathematical skills. Other lessons may encourage exploration of new and different ideas and strategies or provide practice of previously learned skills. The student is free to leave one lesson and choose another at any time.

STUDENT PERFORMANCE DATA

Evaluation of a student's performance in a lesson is complicated by two factors:

1) The student receives immediate feedback for each response. Error feedback may be a short hint or one of a variety of help sequences.

2) Many of these lessons continually adjust the difficulty of the task, based on the student's performance.

Because of this individualization, a percentage score or a count of correct responses for a lesson is, by itself, a poor measure of performance. Instead, we keep a "status." A student's status for a lesson reflects the amount of help the student required, the difficulty of the tasks performed, and other factors which depend on the particular lesson. This status and the specific criteria used to determine it are available to the teacher. For check-ups and some other lessons extra diagnostic data is also available.

The statuses used in the fractions curriculum are described below. If the student has finished a lesson, his or her status for that lesson is one of these:

Very good
OK
Poor
Help now

If the student has started a lesson but has not yet finished it, his or her status for that lesson is one of these:

Continued (abbreviated "contd")
Help-continued (abbreviated "helpctd")

If the student has not started a lesson, his or her status for that lesson is:

Not seen

The meaning of each status is as follows:

Very good	The student completed this lesson with little or no difficulty and is ready for any further work which depends on this lesson.
OK	The student completed this lesson with some difficulty. The student is probably ready for further work which depends on this lesson, but may have some trouble. Sometimes this category covers a broad range of performance.

Poor	This status indicates one of two things: 1) The student did very poorly at the beginning of this lesson and was dropped out of this lesson as not yet ready for it. OR 2) The student gave mostly wrong responses and either was stepped through this lesson with much help or was not allowed to complete it. The student is not ready for further work which depends on this lesson. If the student does poorly on the first try in a lesson, he or she usually is allowed to try it again later.
Help now*	(This is used mostly for check-ups.) There were enough wrong responses to indicate that the student is not ready for further work which depends on the material for this lesson. This lesson will be tried again later (how much later depends on the specific case). In most cases, the student will not see any new material until this lesson is passed.
Continued	The student has started this lesson but has not yet finished it.
Help-continued*	(This is used for key instructional lessons.) The student is doing poorly but will continue the lesson. He or she is not likely to finish this lesson without help.

*Help now and Help-continued are reserved for key lessons in which success is necessary in order to continue in the fractions curriculum.

ORGANIZATION OF THE FRACTIONS CURRICULUM

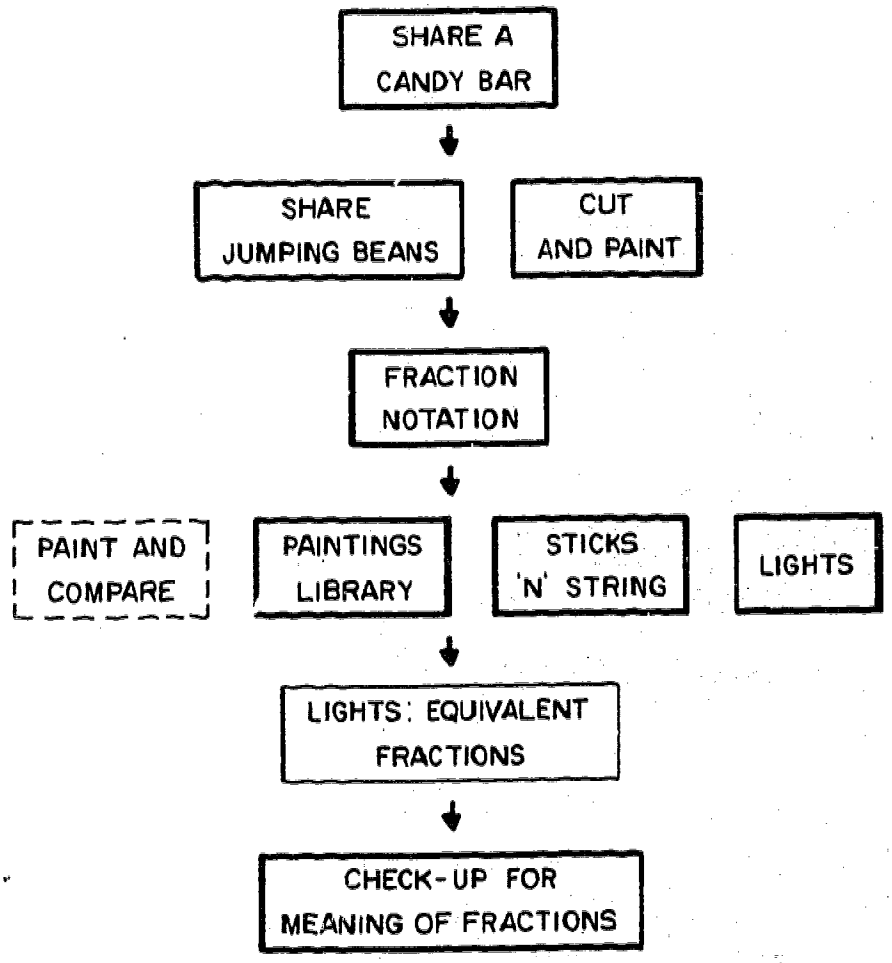
The curriculum is divided into chapters. Each chapter has a topic of major emphasis, but also includes lessons which provide readiness for later material, review of previous material, and general experience and enrichment.

The order in which these lessons are listed is approximately the order in which they appear for the student. However, the curriculum is not at all a linear sequence of lessons. Many lessons will take several days for the student to complete, while others will appear only once. In each session the student may see several lessons -- some new and some continued from previous sessions. The sequencing of the lessons is affected by variations in the performance and preference of the individual student. For example, a student who chooses to move on to a different lesson at every opportunity will have more lessons in a partially-finished state (and therefore have a less linear curriculum) than a student who tends to stay with a lesson until it is finished.

MEANING OF FRACTIONS

Simplified Curriculum Chart	A2
Lesson Descriptions	
Share a Candy Bar	A3
Share Jumping Beans	A4
Cut and Paint	A5
Fraction Notation	A6
Paint and Compare	A7
Paintings Library	A8
Sticks 'n' String	A9
Lights	A10
Lights: Equivalent Fractions	A11
Check-up for Meaning of Fractions	A12

MEANING OF FRACTIONS
a simplified flow chart



——— BASIC

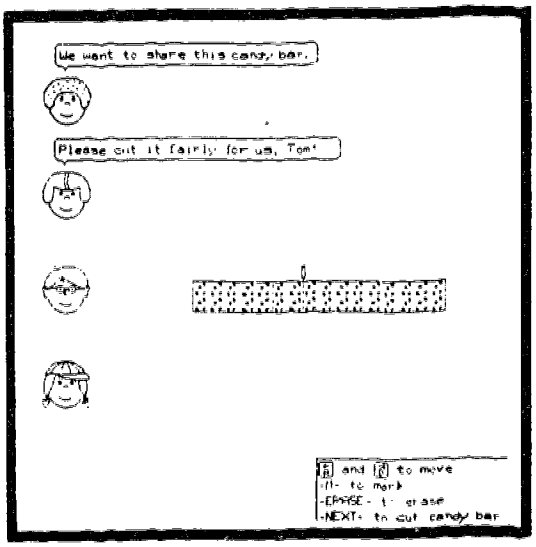
——— EXPERIENCE

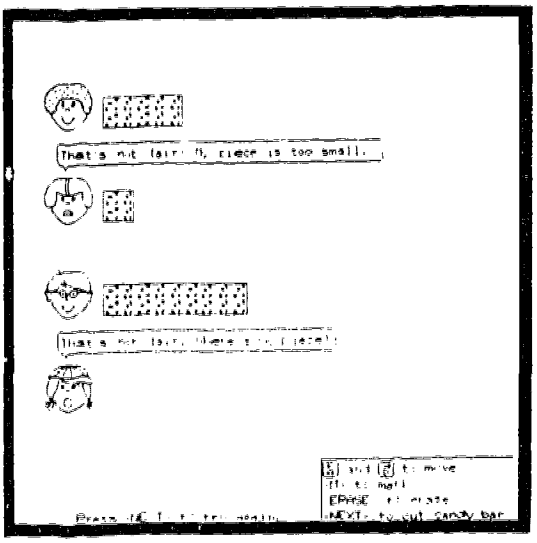
— — — PRACTICE

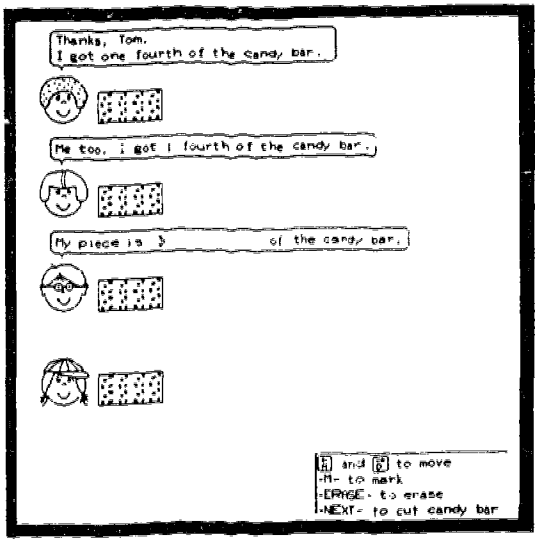
– – – – – ENRICHMENT

+– – – –+ REMEDIAL

Share a Candy Bar



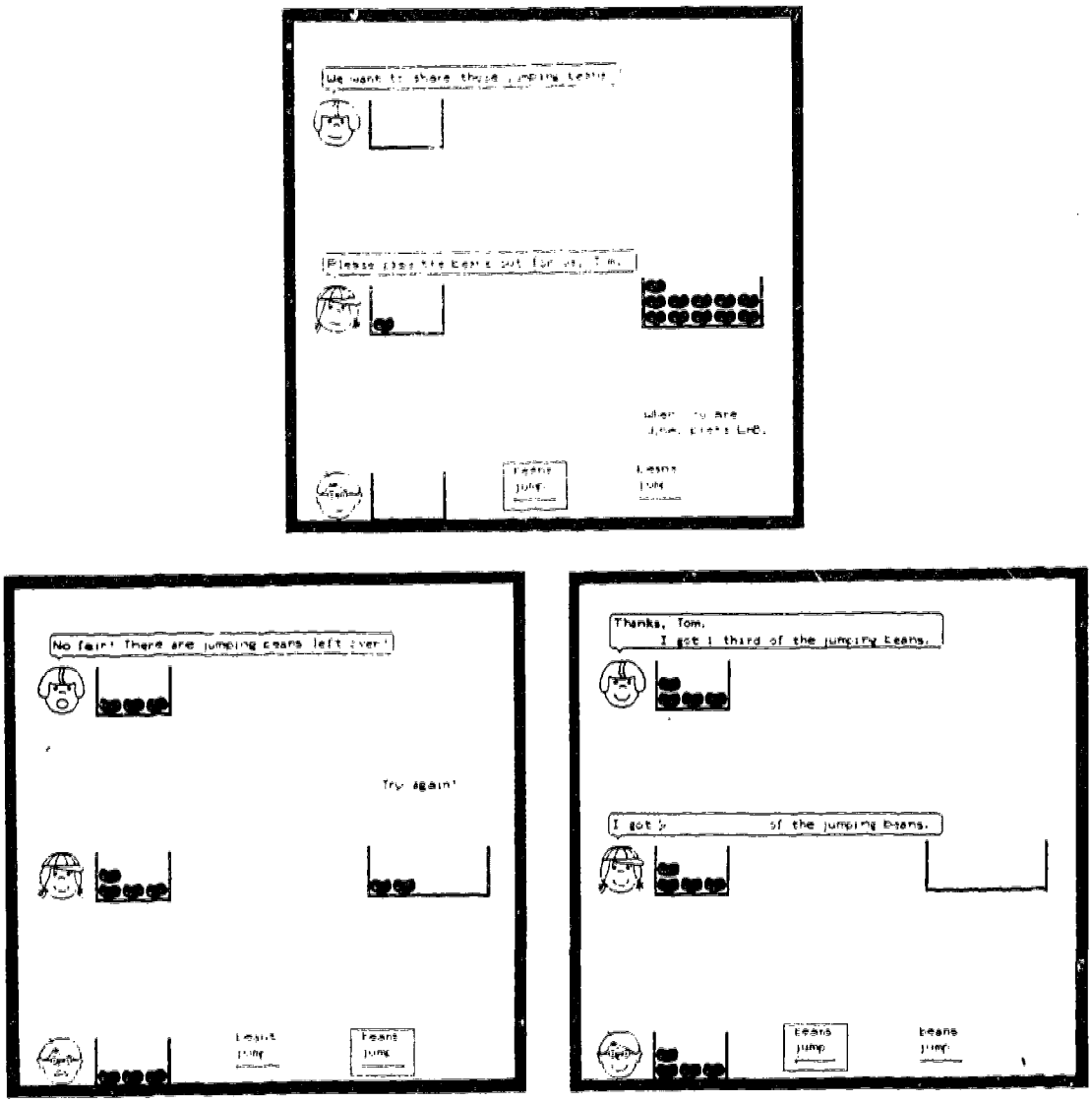




Purpose: Define 1 half, 1 third, 1 fourth, and 1 fifth of an object.

Description: There are four problems: the student shares a candy bar fairly for 2, 3, 4, and 5 kids on the screen. Emphasis is on equal pieces.

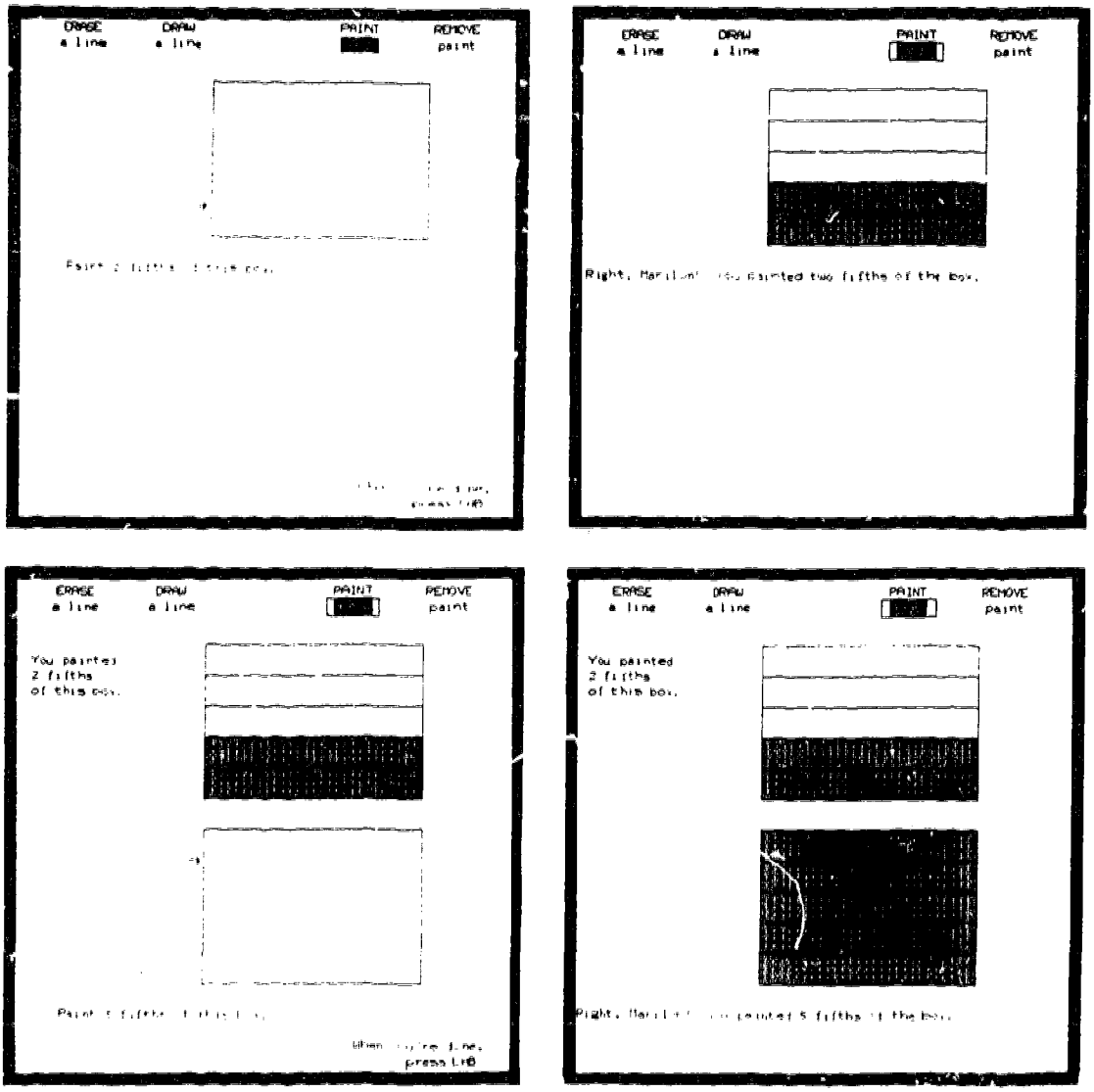
Share Jumping Beans (touch)



Purpose: Define 1 half, 1 third, 1 fourth, and 1 fifth of a set of objects.

Description: There are four problems: the student shares a box of jumping beans fairly for 2, 3, 4, and 5 kids on the screen.

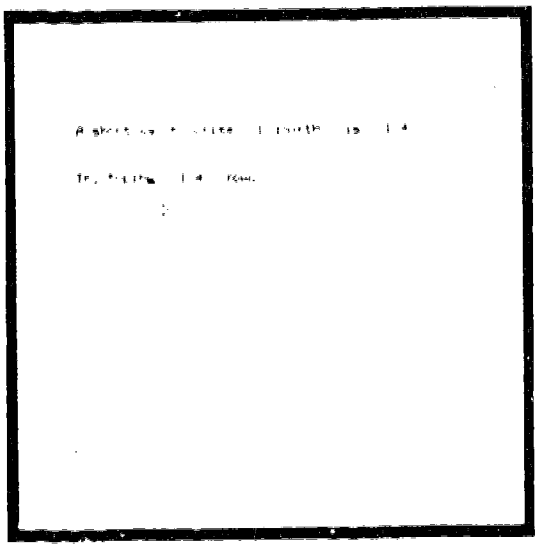
Cut and Paint (touch)

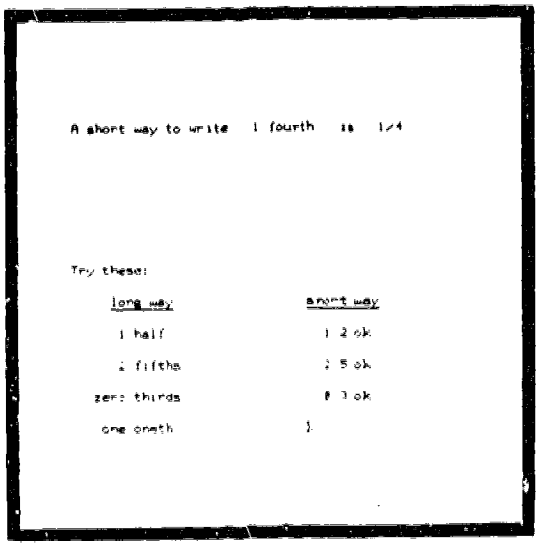


Purpose: 1) Let the student construct fractions of a region.
2) Bring in numerators greater than 1.

Description: The student paints fractions of rectangular boxes. The fractions include some equal to 0, and some equal to 1. Difficulty and length of the problem sequence adjust to the student's performance.

Fraction Notation

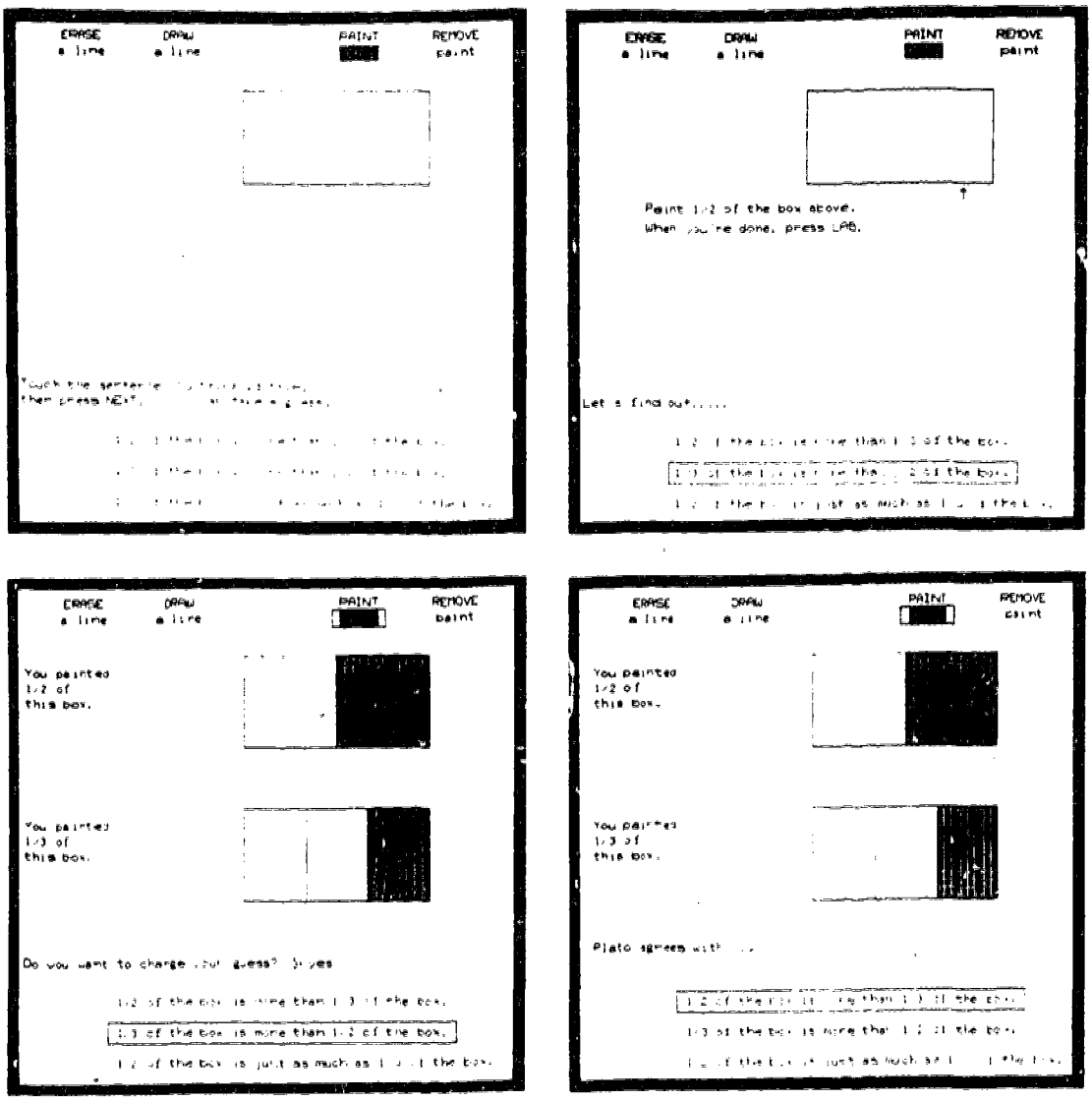




Purpose: 1) Introduce 1/4 as a short way to write 1 fourth.
2) Show the student how to write 1/4 on PLATO.

Description: PLATO explains that 1/4 is a short way to write 1 fourth. The student practices writing fractions using the new notation.

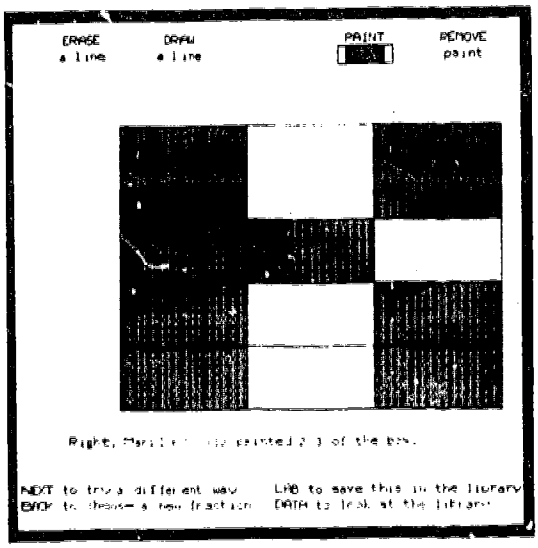
Paint and Compare (touch)

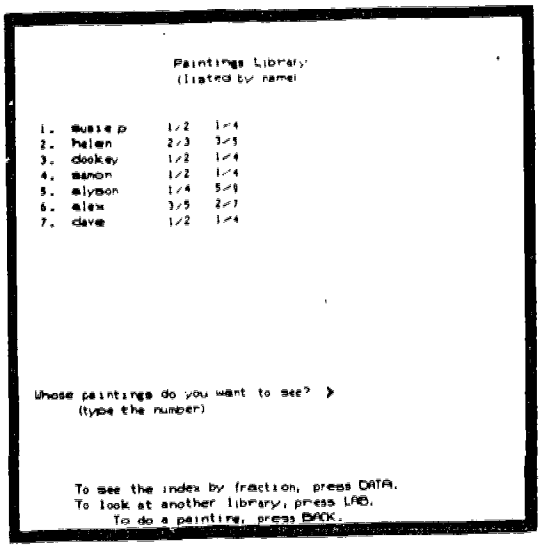


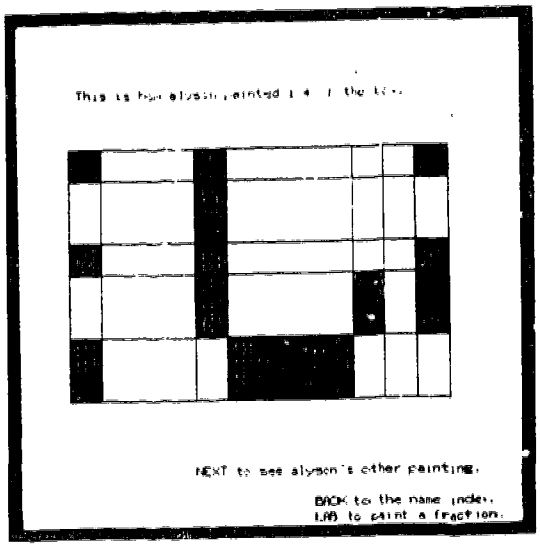
Purpose: Introduce a method of comparing two fractions.

Description: The student predicts which of two fractions is bigger, then paints two identical boxes (one for each fraction) to check the prediction. The prediction can be changed after painting. Some of the problems involve equivalent fractions.

Paintings Library (touch)



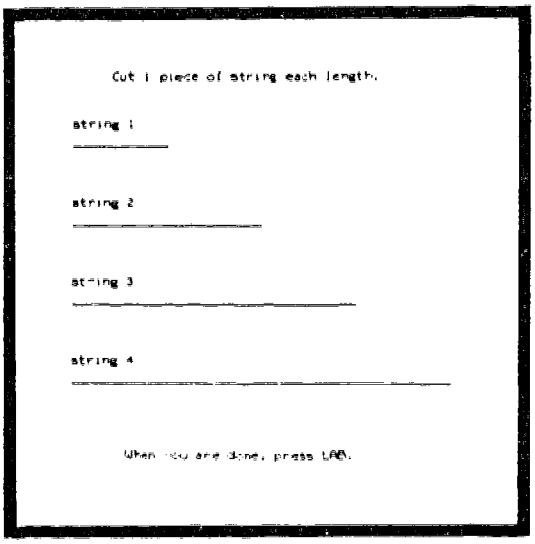


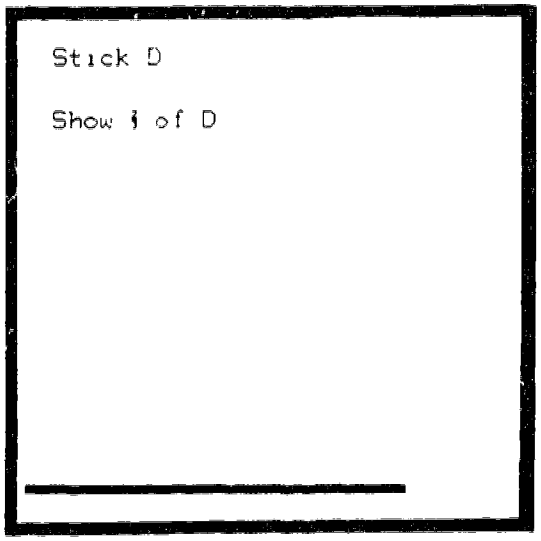


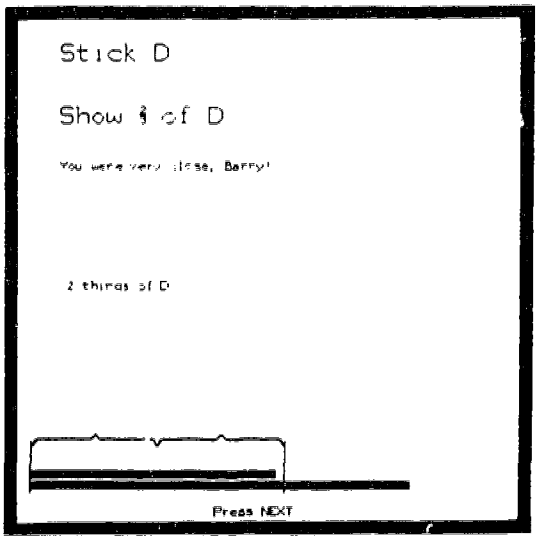
Purpose:
1) Encourage painting a fraction of a box different ways.
2) Initiate the idea of re-arranging the painted areas without changing the fraction painted.
3) Provide opportunities to notice equivalent fractions.
4) Let students share their work with others.

Description: The student can paint a box to keep in the library or look at paintings done by classmates. Due to space limitations, each student can keep only two paintings. A student can replace his or her paintings with new ones.

Sticks 'n' String



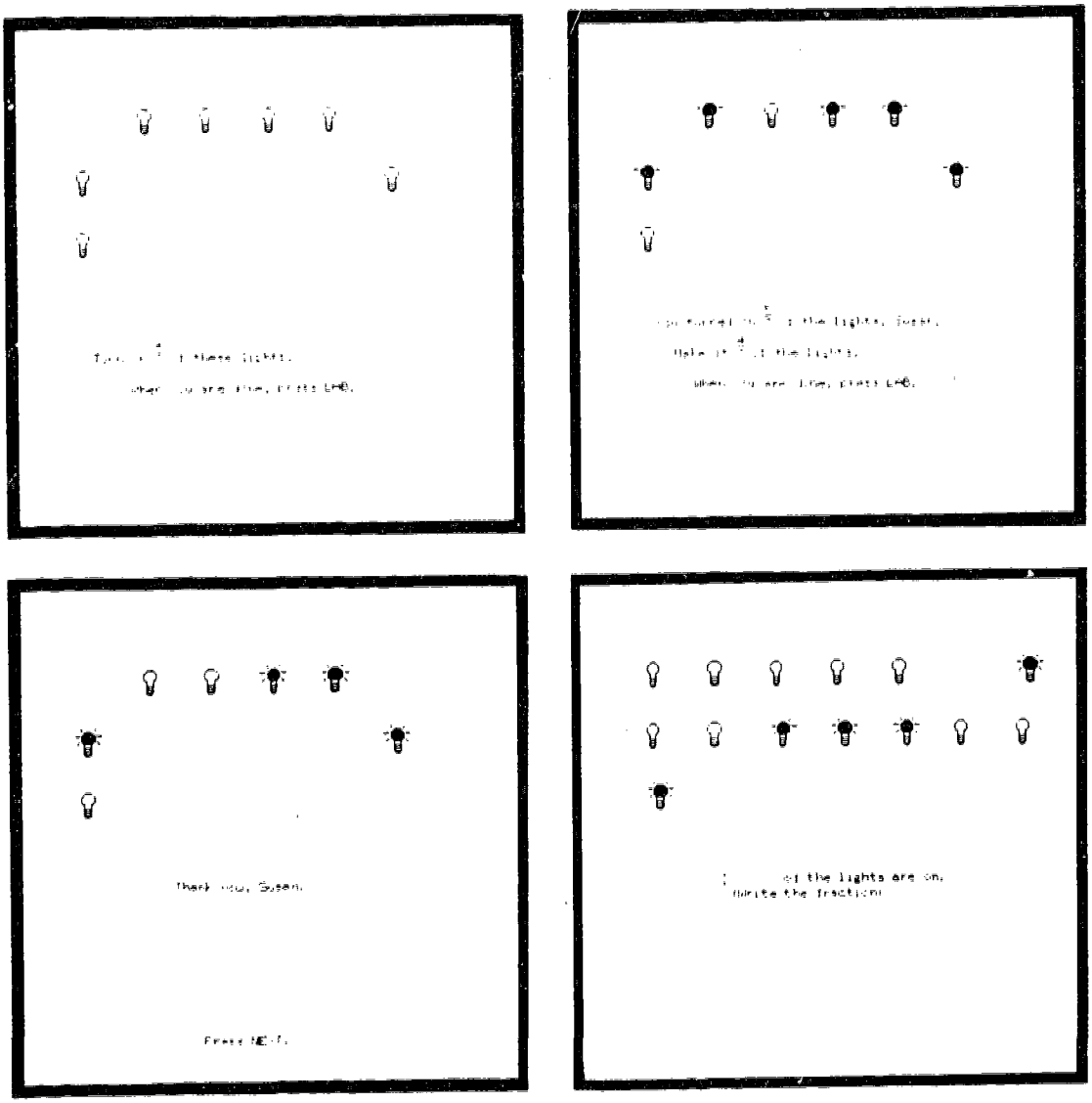




Purpose: 1) Let the student construct halves and thirds of various lengths.
2) Emphasize the importance of the unit length.

Description: The student constructs fractions of sticks of four different lengths. Using a string the same length as the stick, the student folds the string into halves or thirds and measures off the appropriate fraction. Students who have done well on previous lessons will do the standard problem sequence, which includes fractions < 1 and ≥ 1. Other students will do the remedial sequence, which includes only fractions < 1. Both problem sequences vary with the student's performance in the lesson.

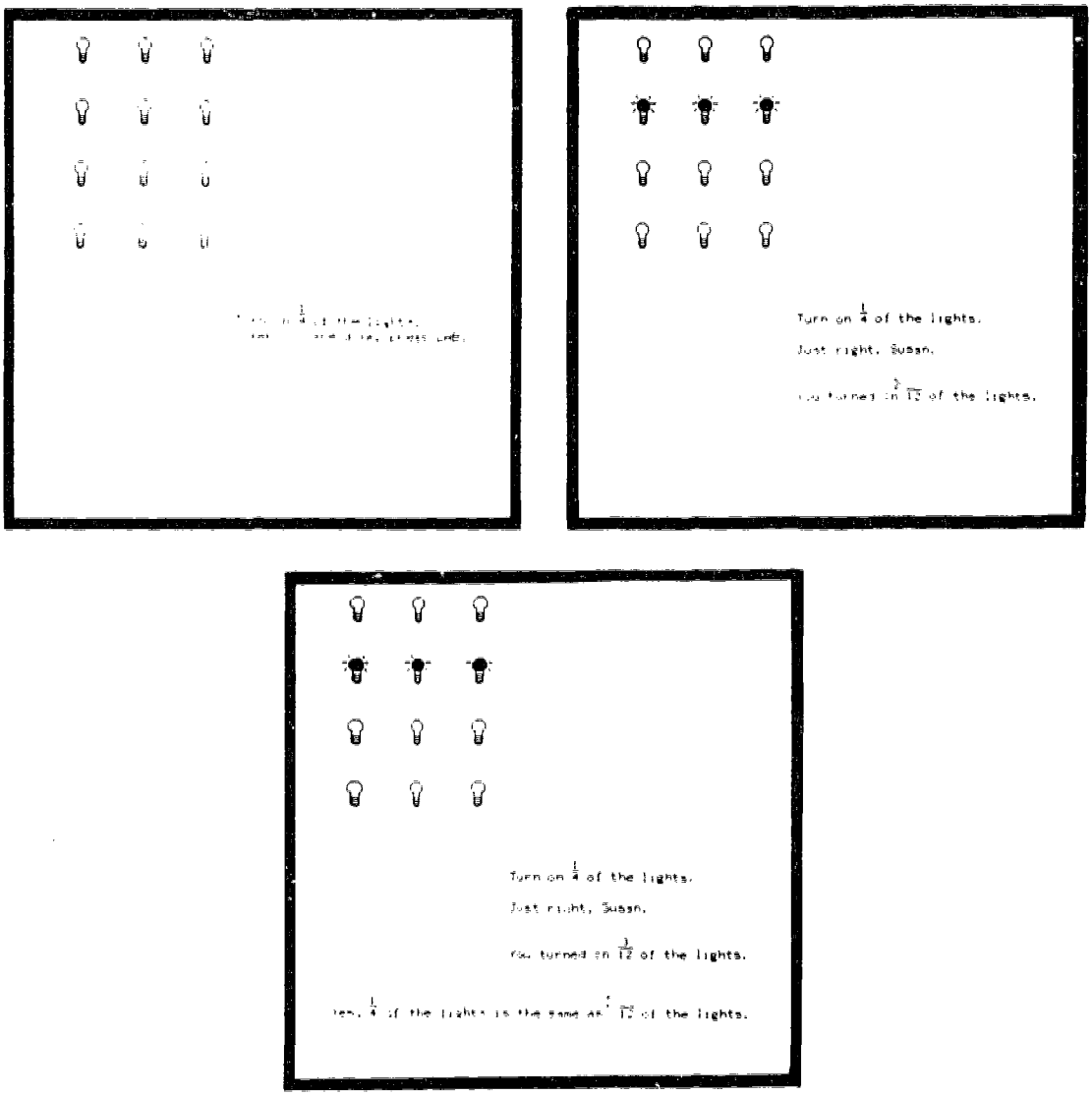
Lights (touch)



Purpose: Practice constructing and identifying fractions of a set of objects.

Description: The student turns on fractions of a set of light bulbs. After some success, PLATO turns on some lights and the student identifies the fraction represented. For this lesson the denominator of the fraction is always the number of lights in the set. The student is expected to move quickly to the next lesson ("Lights: Equivalent Fractions") where the denominator is different from the number of lights. Difficulty and length of the problem sequence adjust to the student's performance.

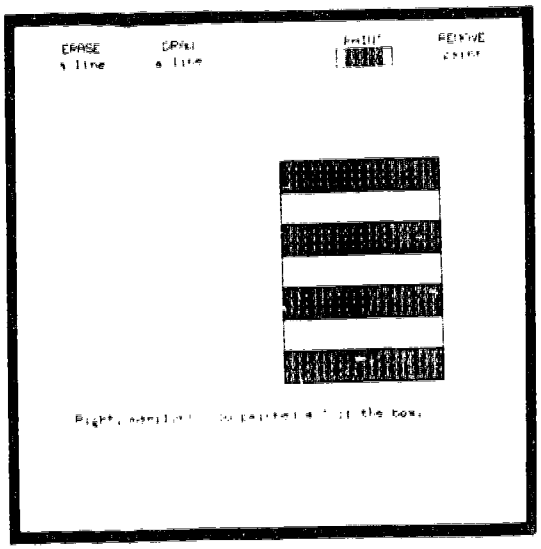
Lights: Equivalent Fractions (touch)

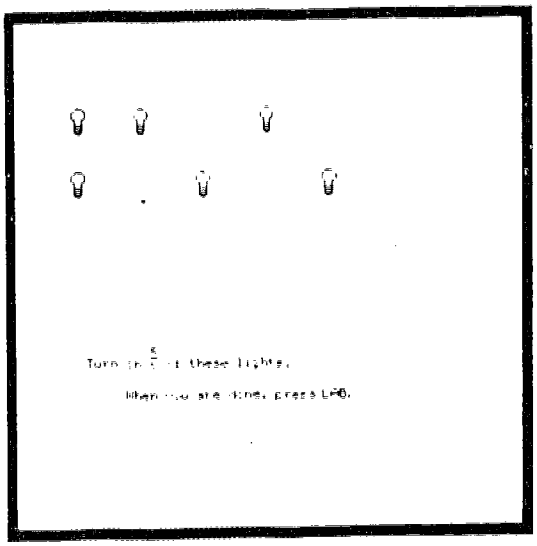


Purpose: Introduce equivalent fractions, using a set of objects.

Description: The lesson consists of problems like this: The student turns on 1/2 of 8 light bulbs, then notes that 4/8 of the lights are on, and concludes that 1/2 of the lights is the same as 4/8 of the lights.

Check-up for Meaning of Fractions (touch)





Purpose: 1) Review the meaning of fractions.
2) See if the student is ready for more advanced material.

Description: The check-up consists of a short sequence of problems from each of 2 lessons: "Cut and Paint," and "Lights." The sequence varies with the student's performance.

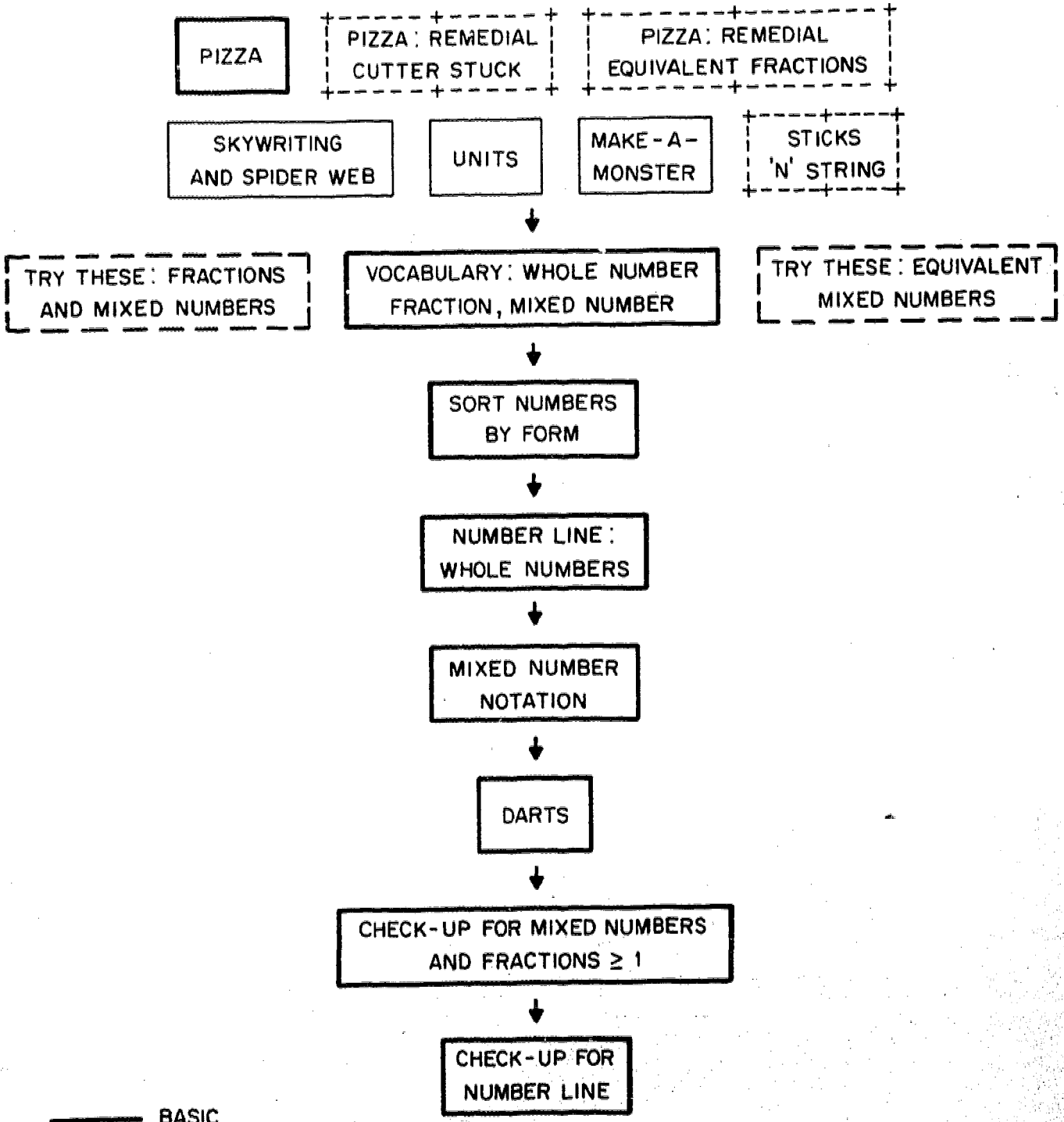
Note: If the student was dropped out of "Lights" as "not ready" for the lesson, he or she is allowed to re-try the lesson before going to the check-up. If performance is good on this second try, the student is allowed to try any of the enrichment lessons he or she seems ready for.

MIXED NUMBERS AND FRACTIONS ≥ 1

Simplified Curriculum Chart	B2
Lesson Descriptions	
Pizza	B3
Fractions	B4
Cutter Stuck	B5
2 Pizzas for 3 Kids	B6
2 Pizzas for 3 Kids--Generalized	B7
Mixed Numbers Introduction	B8
Mixed Numbers with Pizza	B9
Pizza: Remedial Cutter Stuck	B10
Pizza: Remedial Equivalent Fractions	B11
Skywriting and Spider Web	B12
Units	B13
Make-a-Monster	B14
Sticks 'n' String	B15
Try These: Fractions and Mixed Numbers	B16
Try These: Equivalent Mixed Numbers	B17
Vocabulary: Whole Number, Fraction, Mixed Number	B18
Sort Numbers by Form	B19
Number Line: Whole Numbers	B20
Mixed Number Notation	B21
Darts	B22
Check-up for Mixed Numbers and Fractions ≥ 1	B23
Check-up for Number Line	B24

MIXED NUMBERS and FRACTIONS ≥ 1

a simplified flow chart

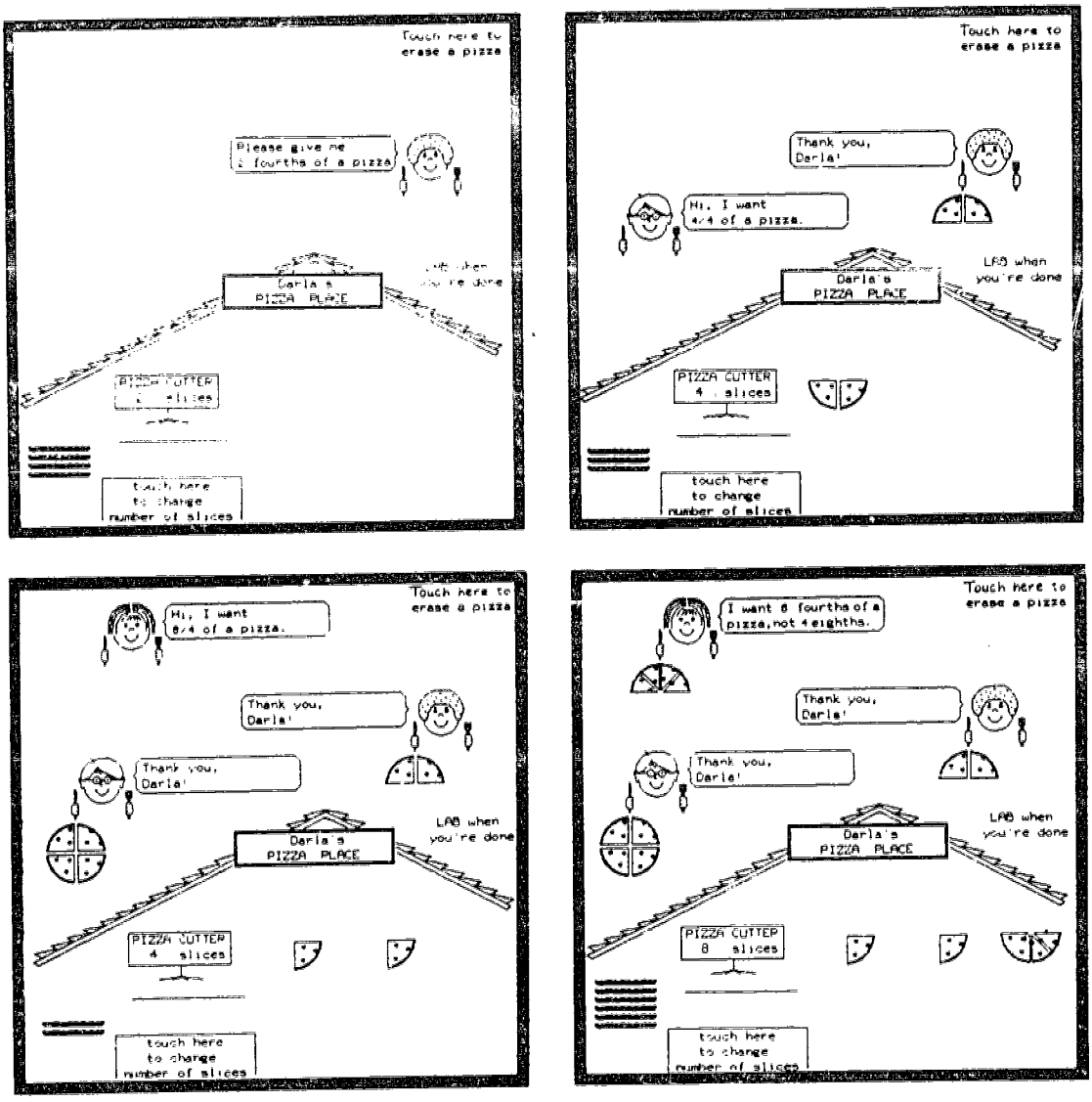


Pizza (touch)

Purpose: Work with fractions ≥ 1, mixed numbers, equivalent fractions, and fractions as division.

Description: This lesson is actually a composite of six lessons that have been mixed together to create a sequence that we hope will be more interesting and thought-provoking than each lesson done separately. The six individual lessons are described separately on the next six pages. These six lessons will be combined for the student into a sequence beginning with the most basic and mixing in other types of problems as the student seems ready. Student data will be kept separately for each of the six lessons. From the lesson index, the teacher may choose to see the combined sequence or any of the individual lessons.

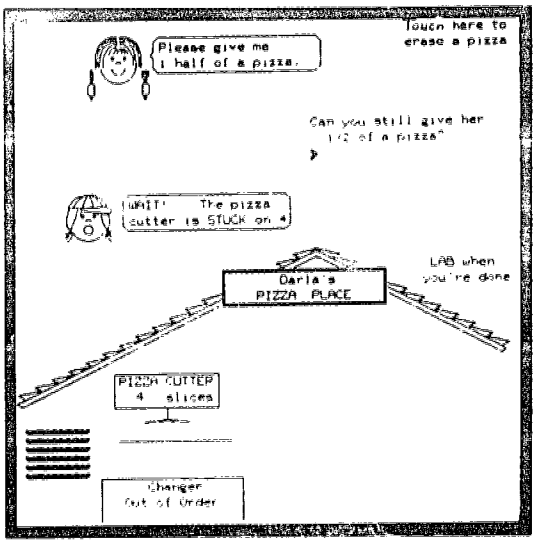
Pizza: Fractions (touch)



Purpose: 1) Re-emphasize with physical model that 3/4 is 3 of a fourth.
2) Extend this to fractions >1.

Description: The student operates a pizza place, serving kids who appear and ask for specific fractions of pizza. The problem sequence depends on the student's performance. It is designed to do fractions <1 until the student is familiar with the model, then lead into fractions equal to 1 and >1.

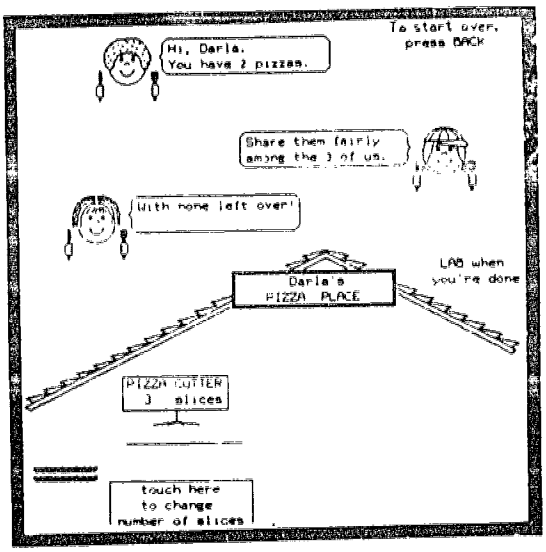
Pizza: Cutter Stuck (touch)

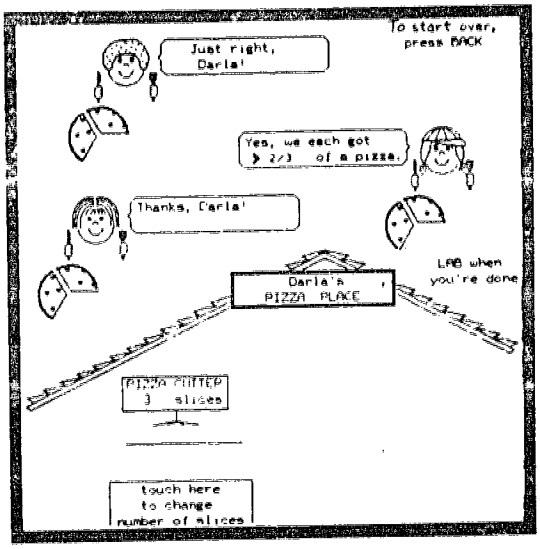


Purpose: Begin equivalent fractions.

Description: The student operates pizza place, serving kids who ask for specific fractions of pizza. The automatic pizza cutter is stuck on the "wrong" number of slices, so, for example, the student has to use eighths to make a half.

2 Pizzas for 3 Kids (touch)

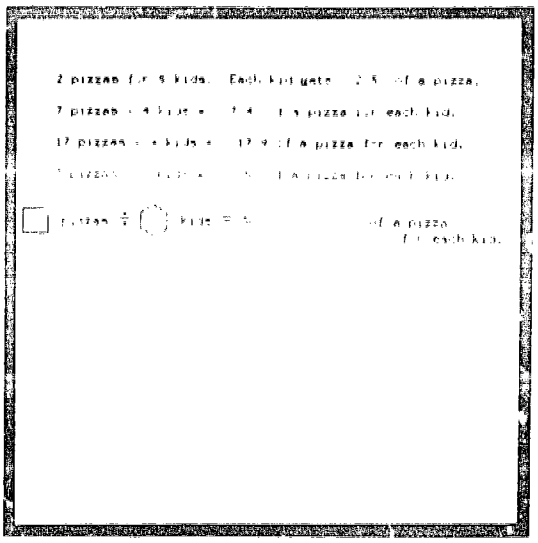




Purpose: Experience with $2 \div 3 = 2/3$

Description: Student has two pizzas which must be divided equally among 3 kids. Sequence of 4 or 5 problems using different numbers.

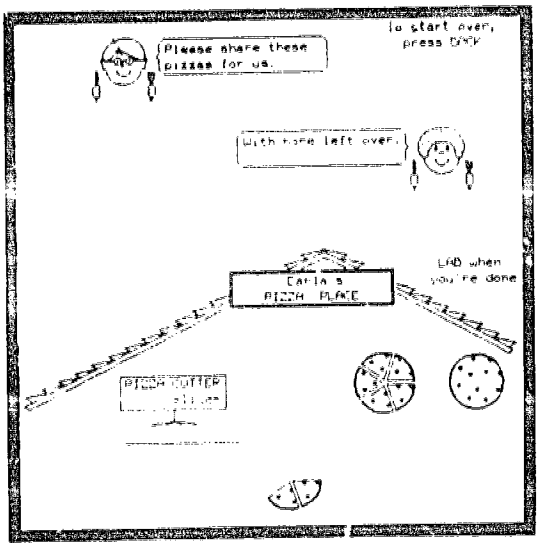
2 Pizzas for 3 Kids--Generalized

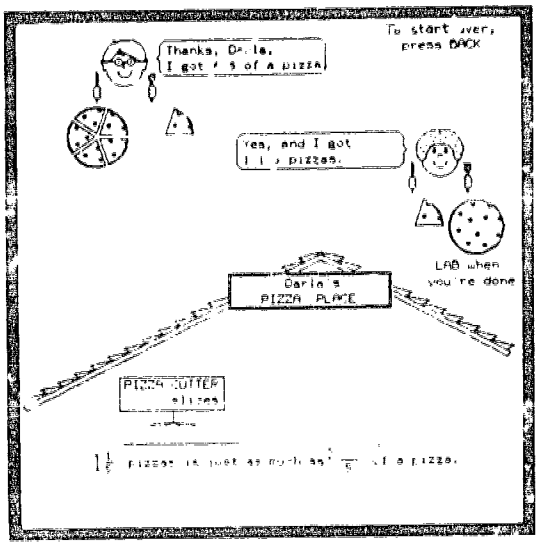


Purpose: Generalization of the 2 pizzas for 3 kids lesson.

Description: The student fills in "short stories" of the format: "2 pizzas for 3 kids. Each one gets ⟫ of a pizza." The lesson uses □ and ⬡ to generalize.

Pizza: Mixed Numbers Introduction (touch)

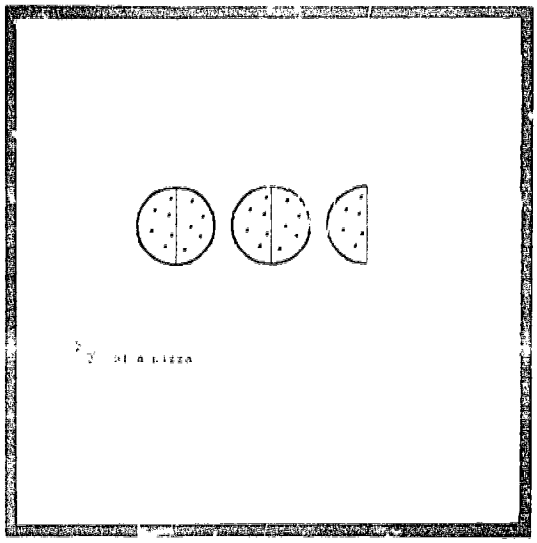


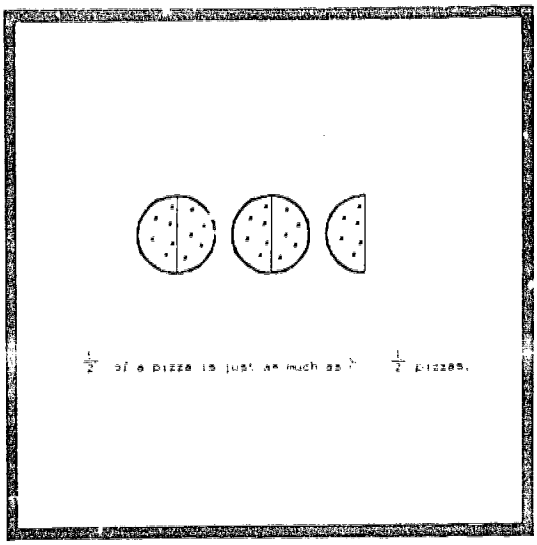


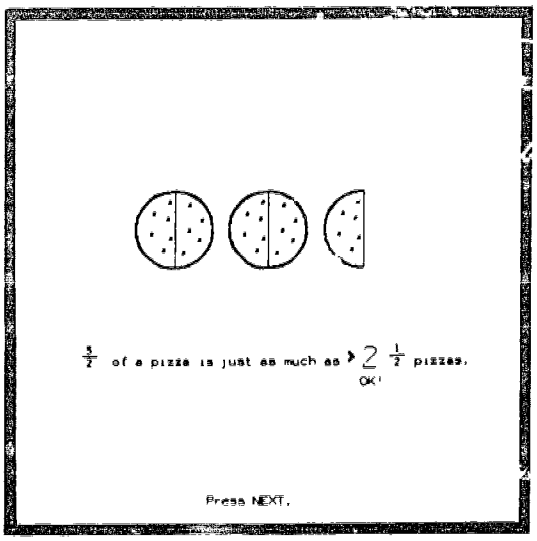
Purpose: Introduce the idea that a fraction and a mixed number can name the same number.

Description: The student shares 1 5/3 pizzas (and similar amounts) fairly between two kids. The kids get 1 1/3 pizzas and 4/3 of a pizza, respectively, so the conclusion is that 1 1/3 is equivalent to 4/3. The problem sequence adjusts to the student's performance.

Mixed Numbers with Pizza



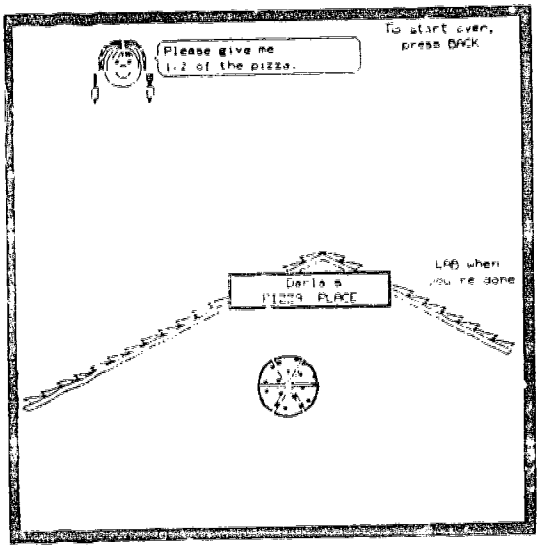




Purpose: Use a physical model to clarify the equivalence of a mixed number and a fraction greater than 1. This is basically a practice lesson which will appear about four problems at a time.

Description: The student fills in a fraction name and a mixed number name to describe a picture of pizzas. There is very specific step-through for errors. Difficulty adjusts to the student's performance. Higher level problems often have no picture as long as the student is doing well.

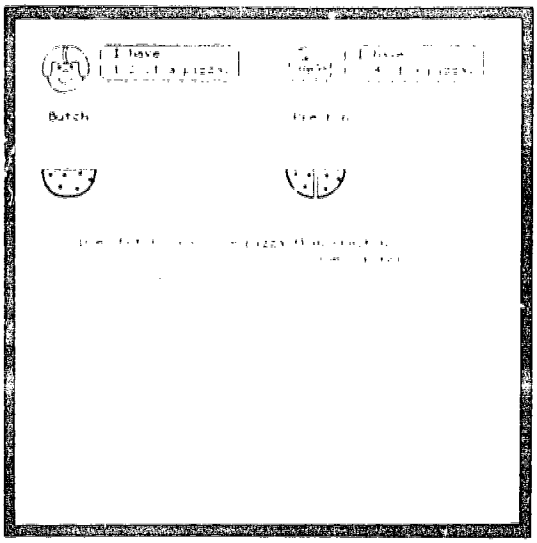
Pizza: Remedial Cutter Stuck (touch)

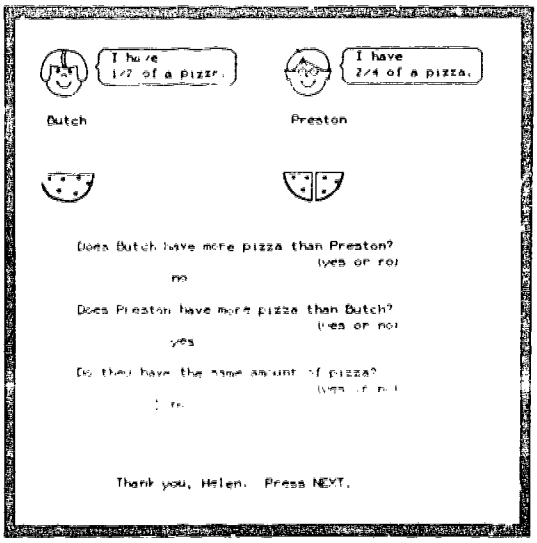


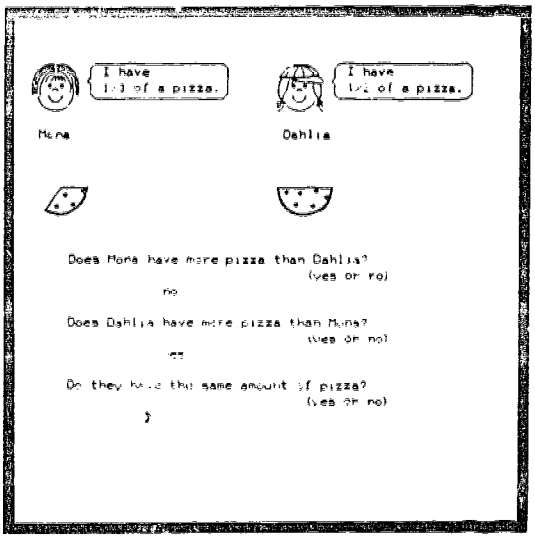
Purpose: Provide help for students who can't handle "Pizza: Cutter Stuck." See whether the trouble on "Pizza: Cutter Stuck" is mathematical or just difficulty with the complexity of the model.

Description: This is like "Pizza: Cutter Stuck," except that there is only one pizza, and it is already cut. The student has to use eighths to make 1/2, etc., but is not concerned with cutting the pizzas.

Pizza: Remedial Equivalent Fractions



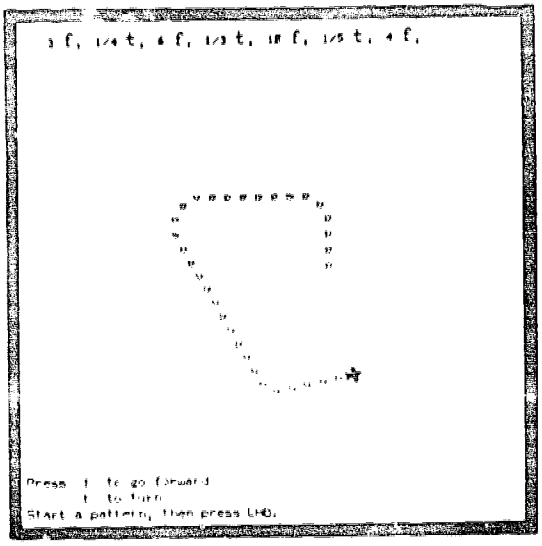


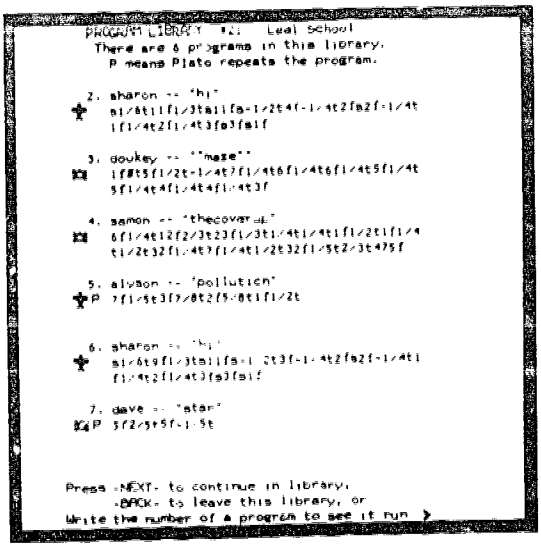


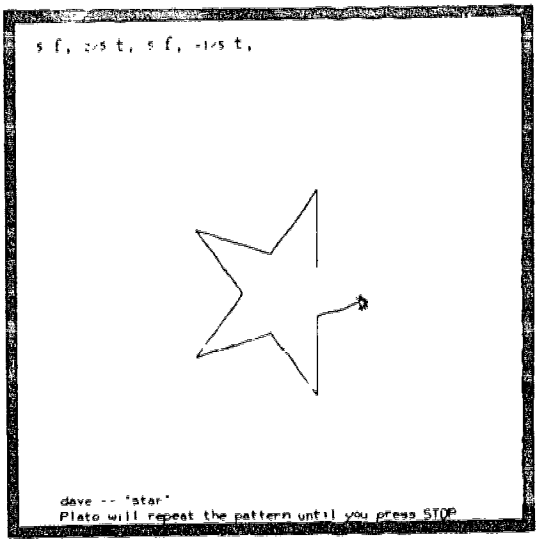
Purpose: Provide help for students having trouble with equivalent fractions.

Description: Two kids on the screen each have some pizza. The student is asked which kid has more and whether they have the same amounts. Answers are saved for diagnostic use by the teacher. There are five problems.

Skywriting and Spider Web



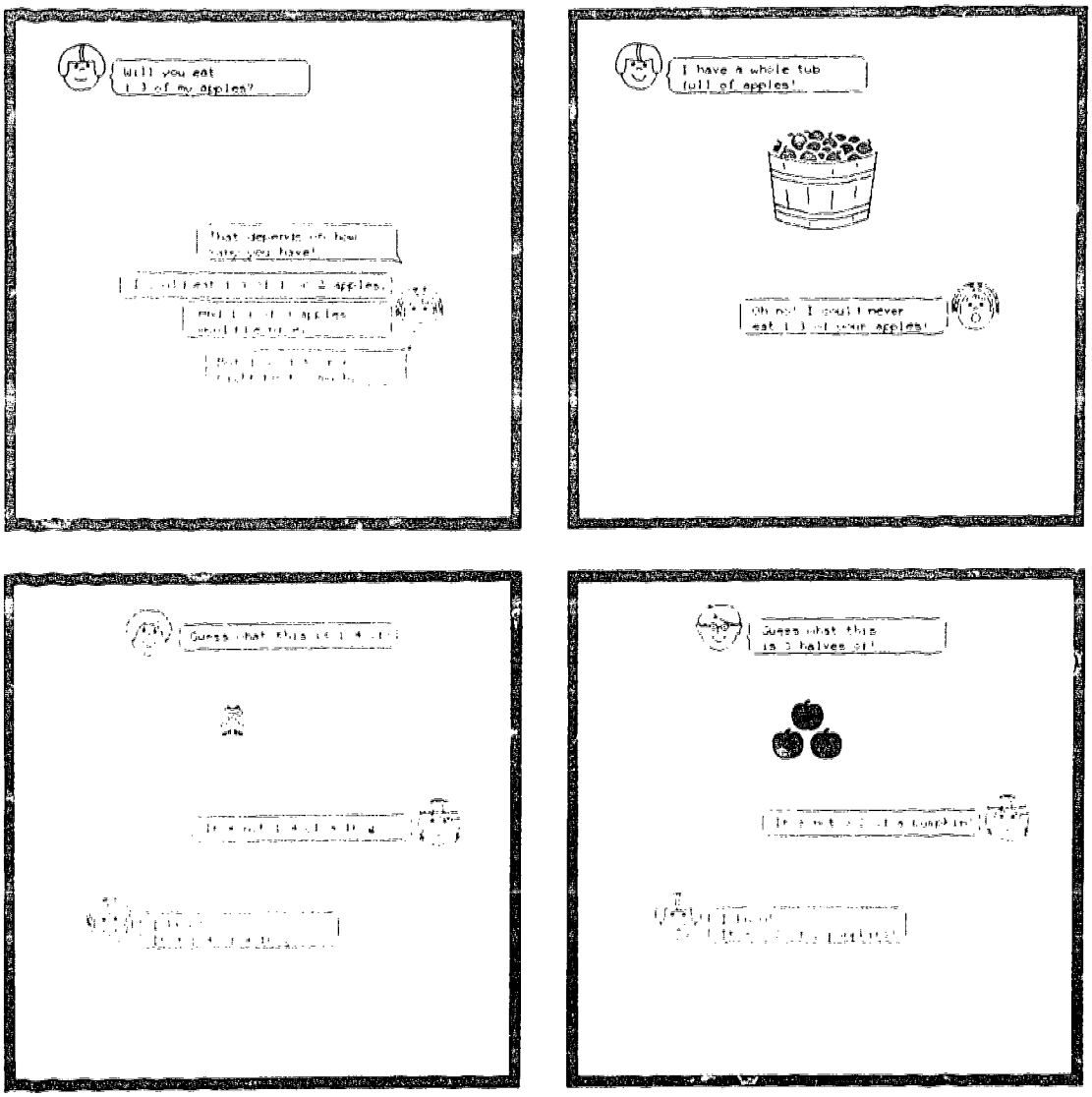




Purpose:
1) Provide experience with a new unit -- a turn.
2) Provide experience with very basic programming ideas.
3) Let students share their programs with other students through the program library.

Description: The student writes a program to move an airplane (or spider) around the screen, leaving a trail of smoke (or web). A program consists of a sequence of "forwards" and "turns." Each student can save one program in the library and replace it at any time with a new one.

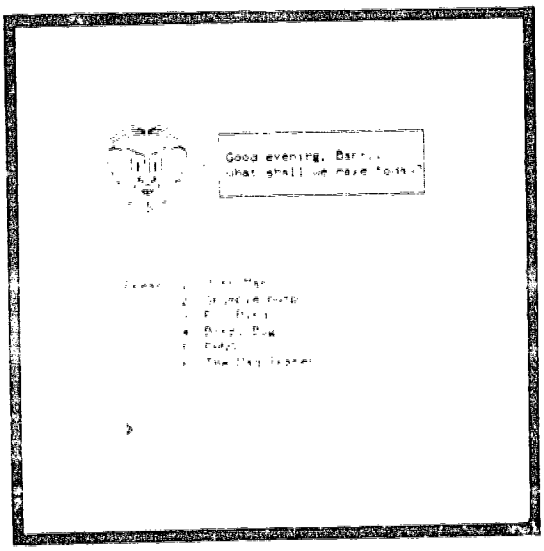
Units

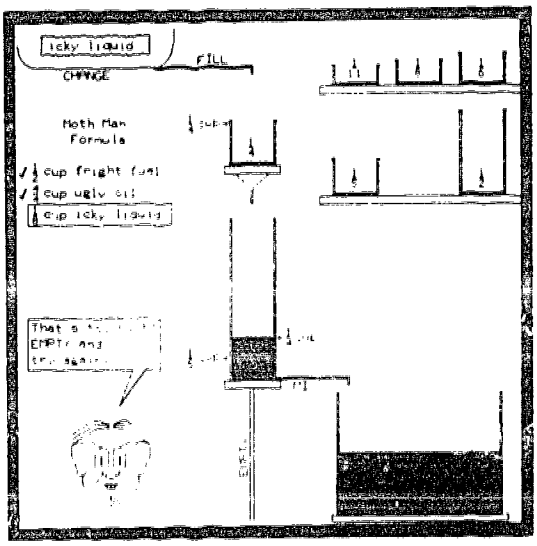


Purpose: Point out the importance of the unit. For example, 1/2 of one unit can be the same as 3/4 of another unit.

Description: There are ten independent pages in this lesson. On each page some kids on the screen discuss a situation involving units. This lesson will appear one page per session.

Make-a-Monster (touch)

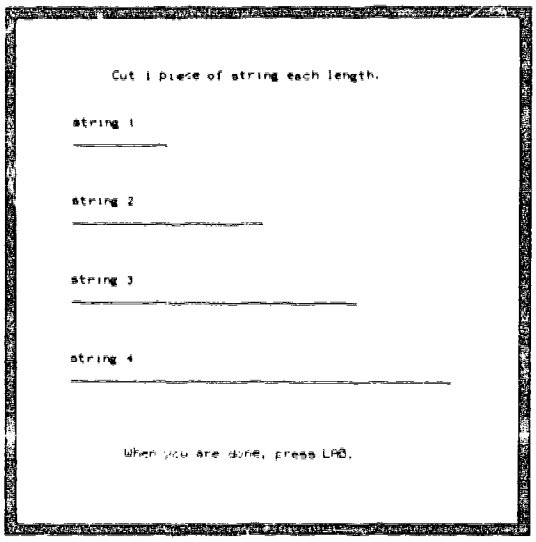


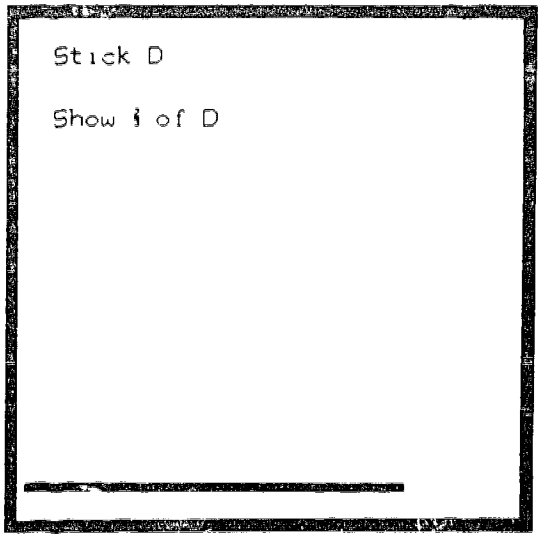


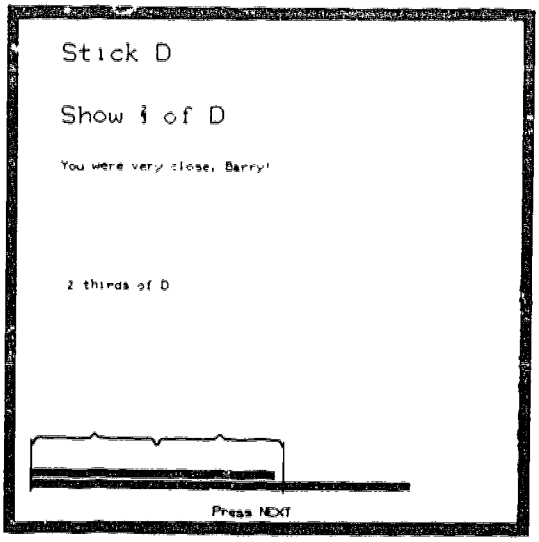
Purpose: 1) Re-emphasize that 3/4 is 3 of a fourth.
2) Provide experience with a new unit -- the cup.
3) Familiarize the student with a model that will be useful later for equivalent fractions and common denominators.

Description: The student uses measuring cups to mix monster formulas in an eccentric scientist's laboratory.

Sticks 'n' String



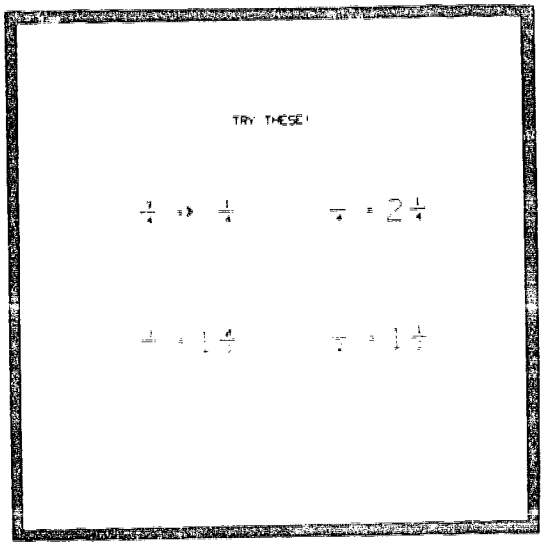


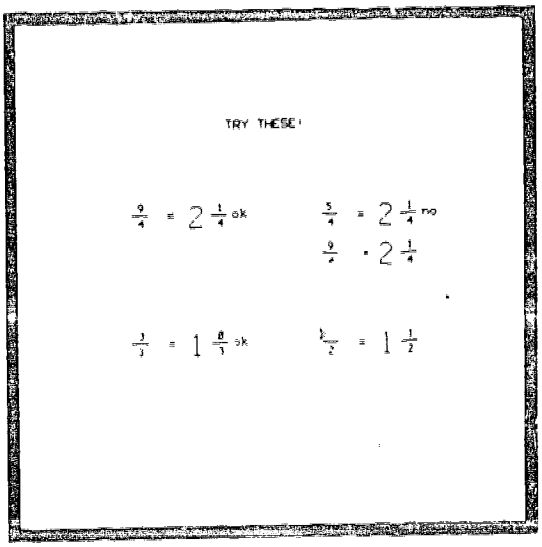


Purpose: Let the student construct 1/2, 2/2, 3/2, 1/3, 2/3, 3/3, and 4/3 of various lengths. This is for students who did the remedial "Sticks 'n' String" sequence in the "Meaning of Fractions" chapter (that sequence included only 1/2, 1/3, and 2/3). Students who did the standard sequence will not see this lesson.

Description: The student constructs fractions of sticks of four different lengths. Using a string the same length as the stick, the student folds the string into halves or thirds and measures off the appropriate fraction. This lesson is identical to the standard sequence of "Sticks 'n' String" in the "Meanings of Fractions" chapter.

Try These: Fractions and Mixed Numbers

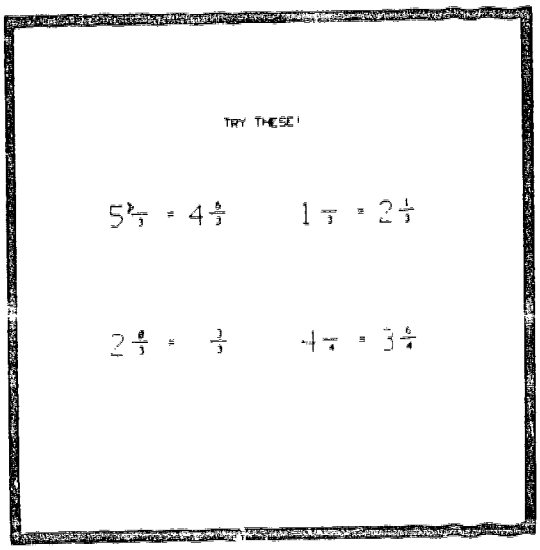


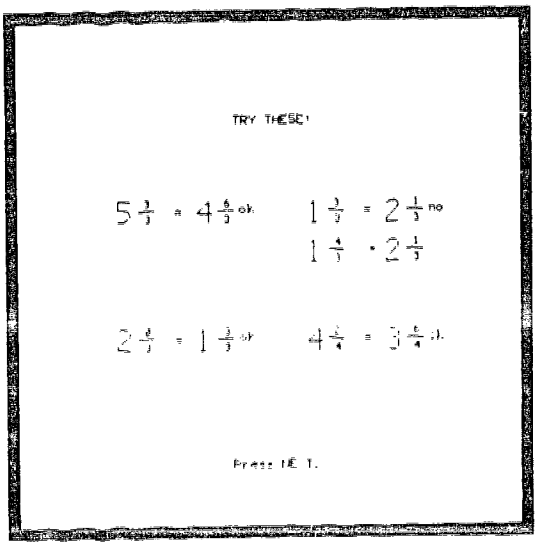


Purpose: Provide quick practice converting fractions to mixed numbers and vice versa. This is for students who already have a basic understanding, but need practice.

Description: This lesson occurs one page at a time for quick practice of an already learned skill. There are four problems per page, and no pictures or models. Problems are like 2 1/3 =》 /3. If the student is wrong twice on a problem, PLATO gives the answer to it. Difficulty adjusts to the student's performance.

Try These: Equivalent Mixed Numbers

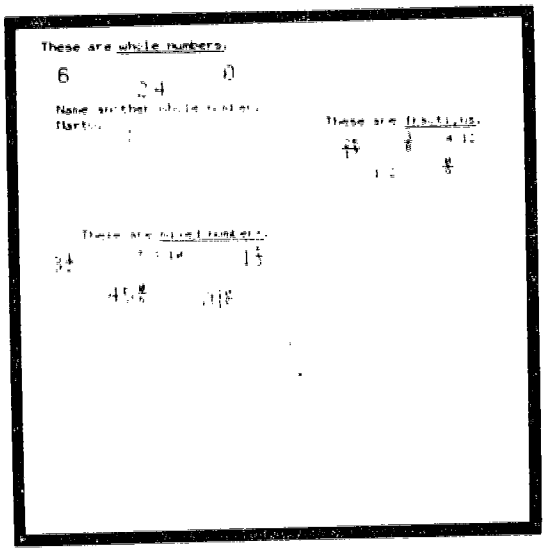


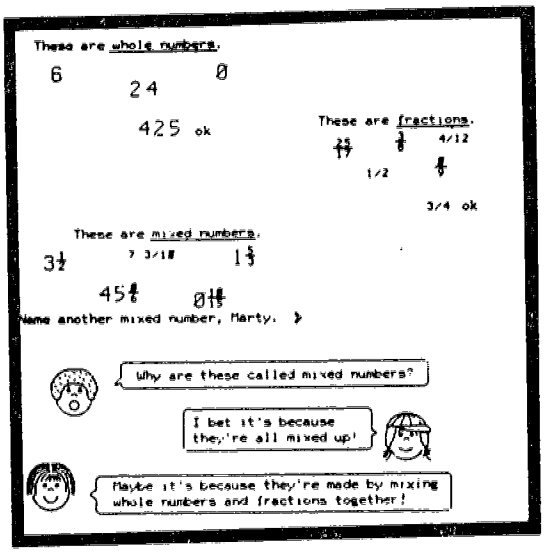


Purpose: Provide quick practice renaming mixed numbers as done in addition and subtraction of mixed numbers. This is for students who have a basic understanding, but need practice.

Description: This lesson occurs one page at a time for quick practice. There are four problems per page, and no pictures or models. Problems are like 4 1/2 = 3 ⟫ /2. If the student is wrong twice on a problem, PLATO gives the answer to it. Difficulty adjusts to the student's performance.

Vocabulary: Whole Number, Fraction, Mixed Number

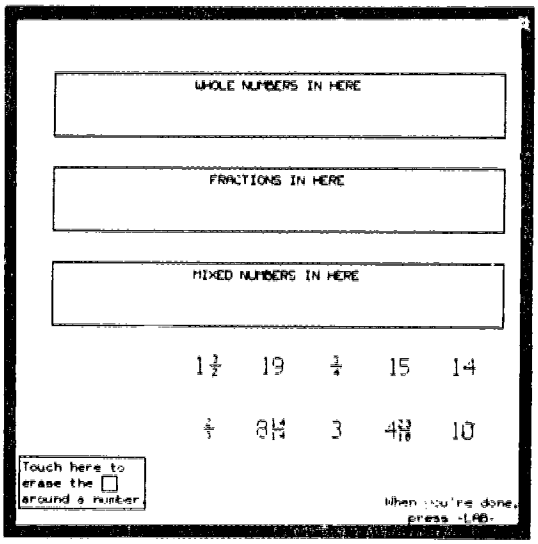


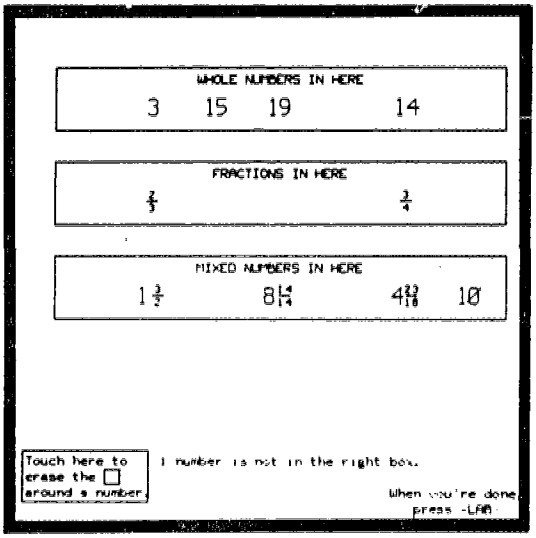


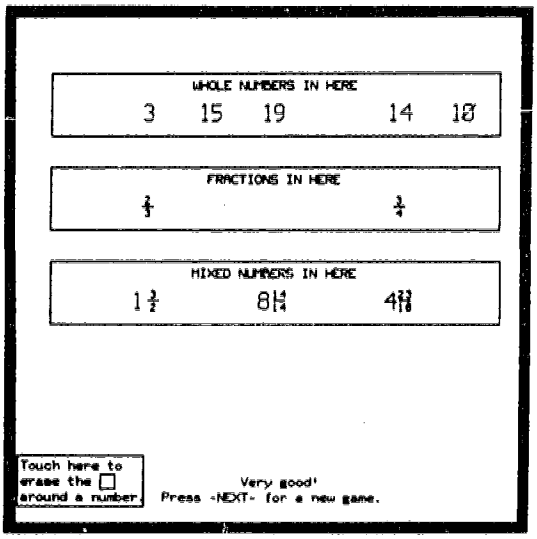
Purpose: Introduce vocabulary used in future lessons.

Description: The student is presented with examples of whole numbers, fractions, and mixed numbers, then asked to write one of each.

Sort Numbers by Form (touch)



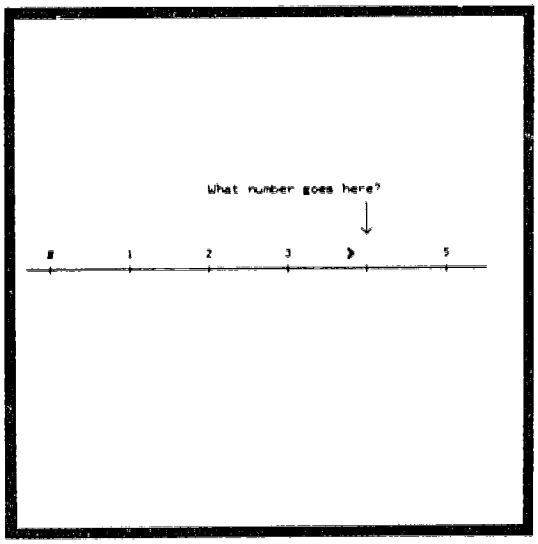


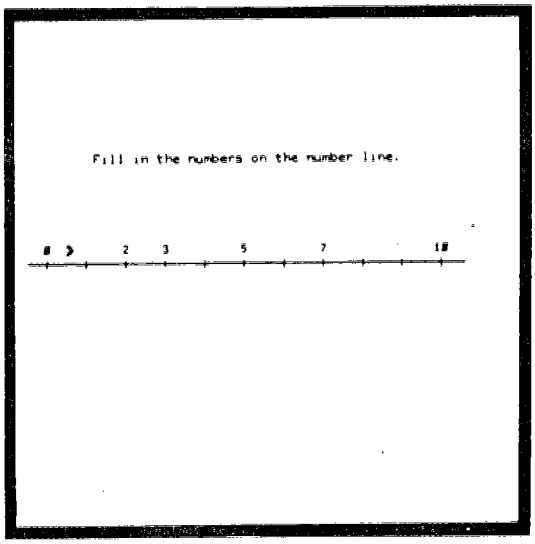


Purpose: Practice distinguishing "fractions," "mixed numbers," and "whole numbers." This is necessary for understanding the vocabulary used in future lessons.

Description: Several numbers (whole numbers, fractions, and mixed numbers) are scattered on the screen with three boxes to sort them into: one each for whole numbers, fractions, and mixed numbers. The student sorts the numbers into the appropriate boxes. This lesson will appear one page at a time.

Number Line: Whole Numbers





Purpose: Familiarize the student with whole numbers on the number line.

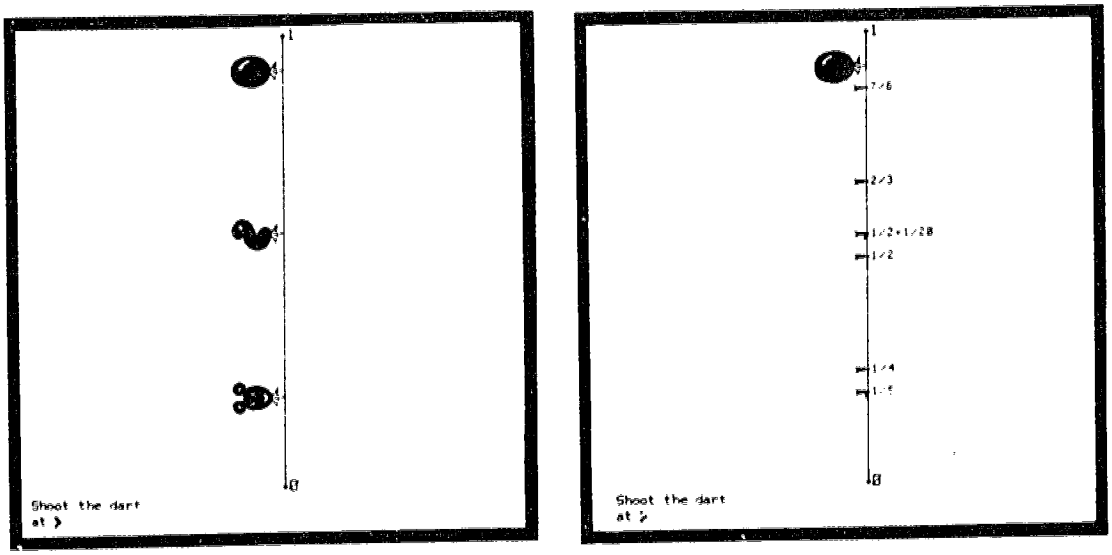
Description: The student fills in whole numbers on a number line. Difficulty adjusts to student's performance.

Mixed Number Notation

Purpose: Show the student how to write mixed numbers on PLATO.

Description: PLATO shows how to write 2 1/2. The student writes 1/2, 1 1/2, and 2 1/2 to mark points on a number line that already has the whole numbers labelled.

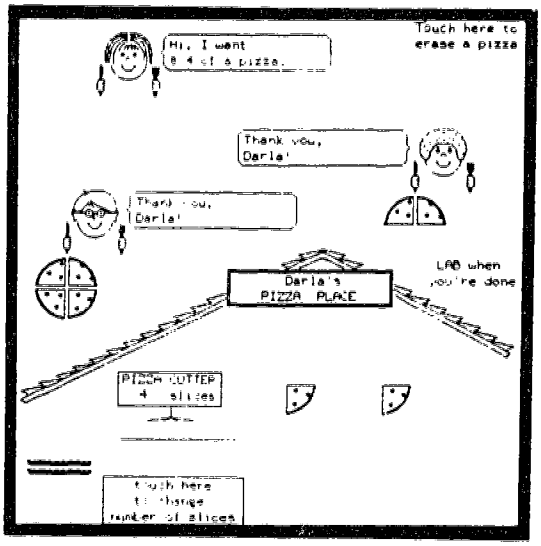
Darts

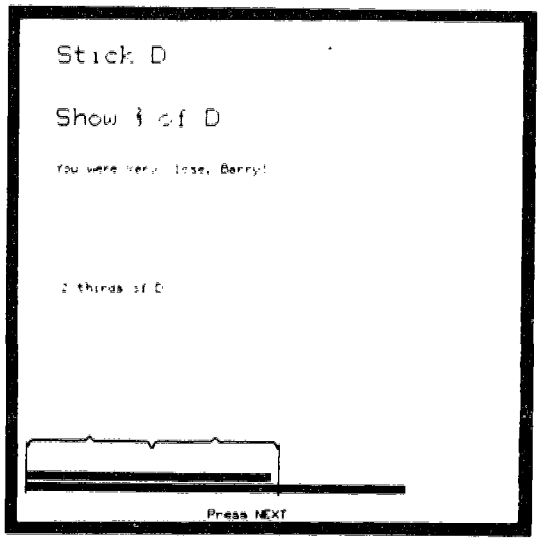


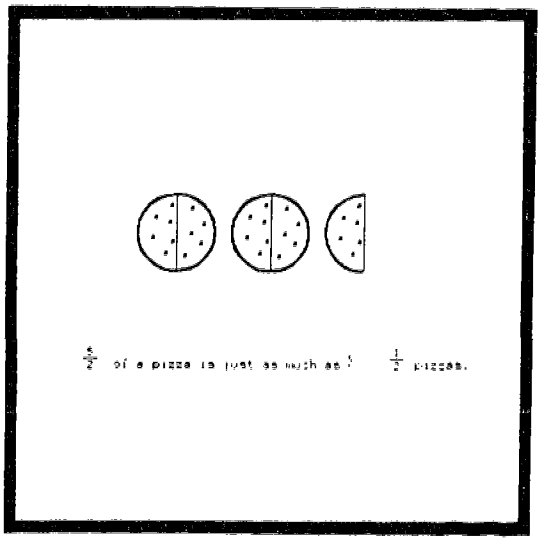
Purpose: Experience locating fractions and mixed numbers on the number line.

Description: The student writes numbers or expressions to shoot darts at balloons tied to a number line. Difficulty adjusts to the student's performance. Higher levels have fewer integers on the line and/or smaller balloons to shoot at.

Check-up for Mixed Numbers and Fractions ≥ 1 (touch)



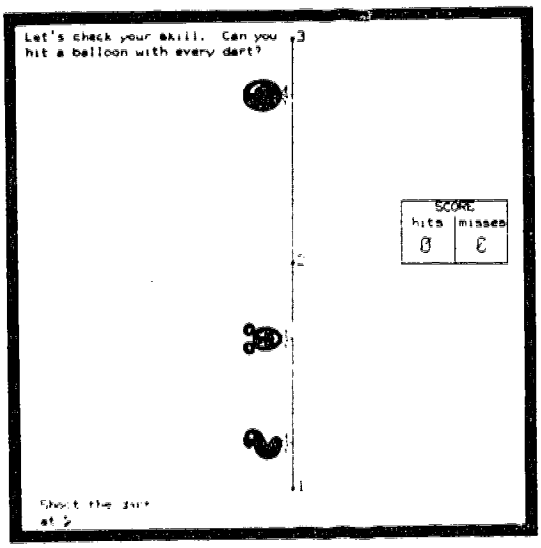


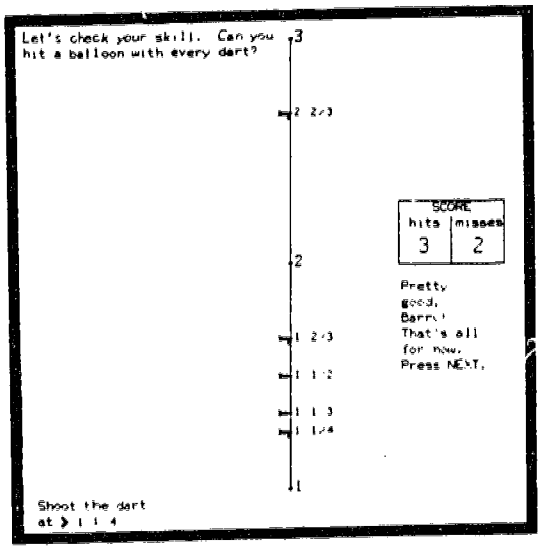

of a pizza is just as much as pizzas.

Purpose: 1) Review mixed numbers and fractions ≥ 1.
2) See if the student is ready for material that uses these concept

Description: The check-up includes a short sequence of problems from each of three lessons: "Pizza: Fractions," "Sticks 'n' String," and Mixed Numbers with Pizza." Most of the fractions used are ≥ 1.

Check-up for Number Line





Purpose: See if the student is ready for further number line work.

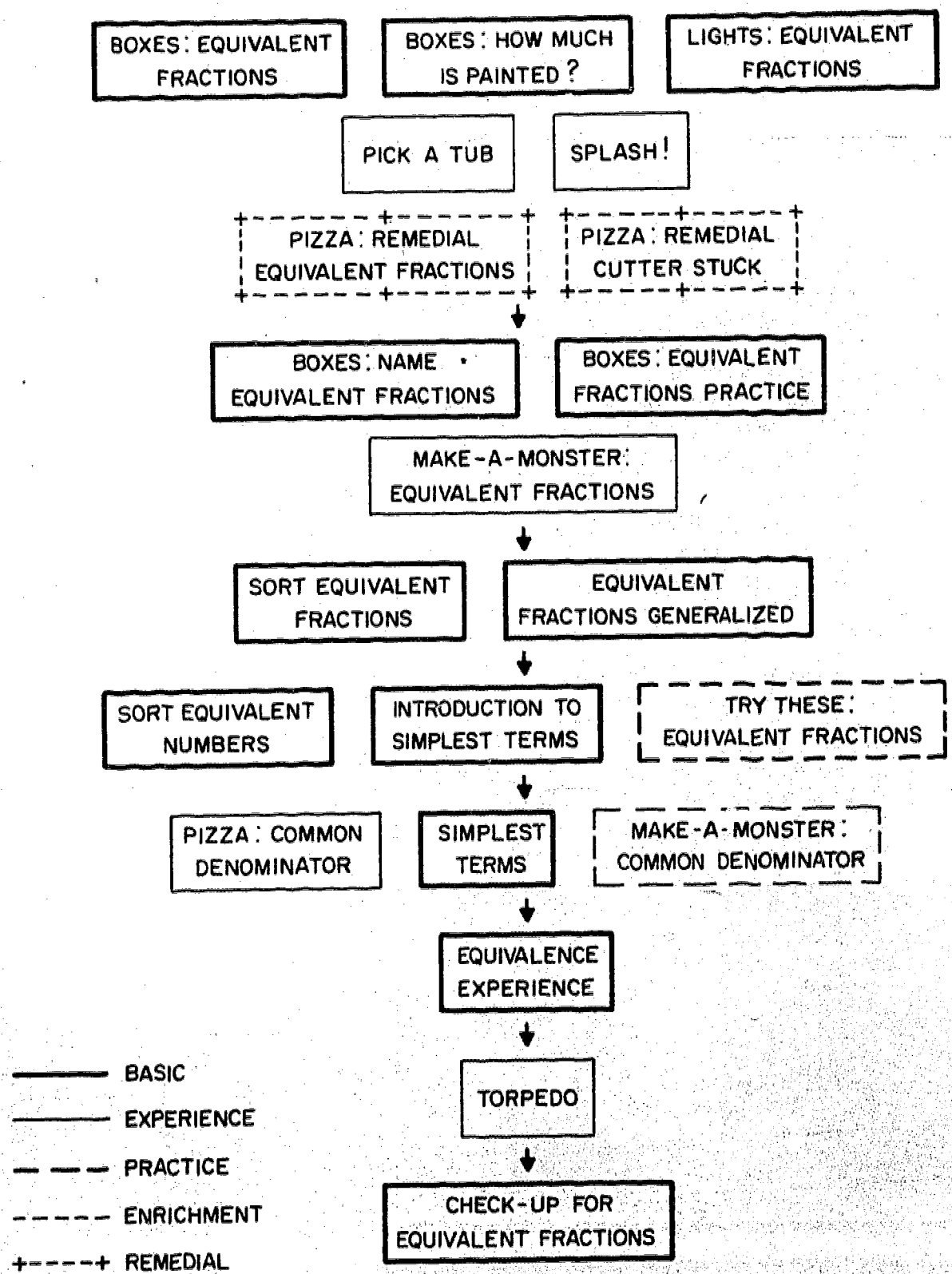
Description: The student plays two games of "Darts." Each game has three balloons to pop, all of which can be hit with denominators less than 7.

EQUIVALENT FRACTIONS

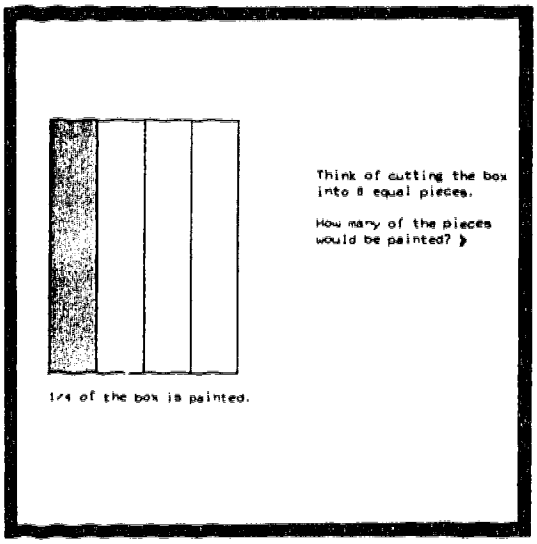
Simplified Curriculum Chart	C2
Lesson Descriptions	
Boxes: Equivalent Fractions	C3
Boxes: How Much Is Painted?	C4
Lights: Equivalent Fractions	C5
Pick a Tub	C6
Splash!	C7
Pizza: Remedial Equivalent Fractions	C8
Pizza: Remedial Cutter Stuck	C9
Boxes: Name Equivalent Fractions	C10
Boxes: Equivalent Fractions Practice	C11
Make-a-Monster: Equivalent Fractions	C12
Sort Equivalent Fractions	C13
Equivalent Fractions Generalized	C14
Sort Equivalent Numbers	C15
Try These: Equivalent Fractions	C16
Introduction to Simplest Terms	C17
Simplest Terms	C18
Pizza: Common Denominator	C19
Make-a-Monster: Common Denominator	C20
Equivalence Experience	C21
Torpedo	C22
Check-up for Equivalent Fractions	C23

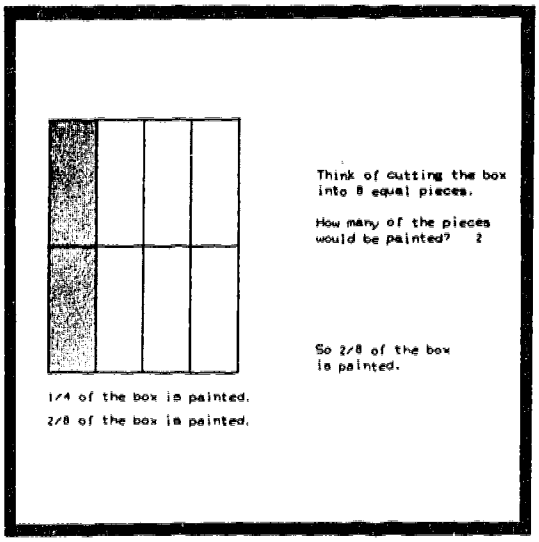
EQUIVALENT FRACTIONS

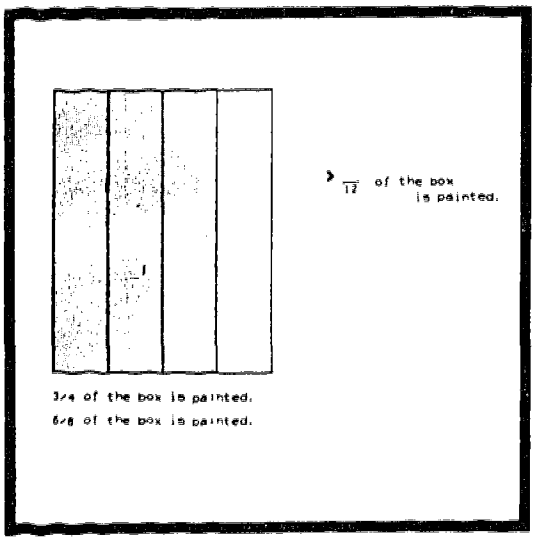
a simplified flow chart

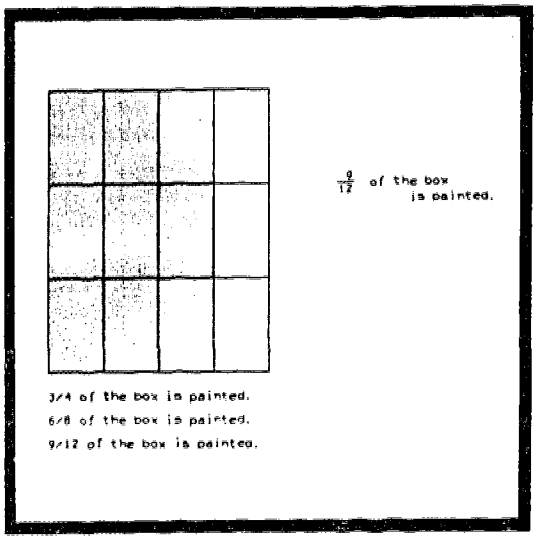


Boxes: Equivalent Fractions





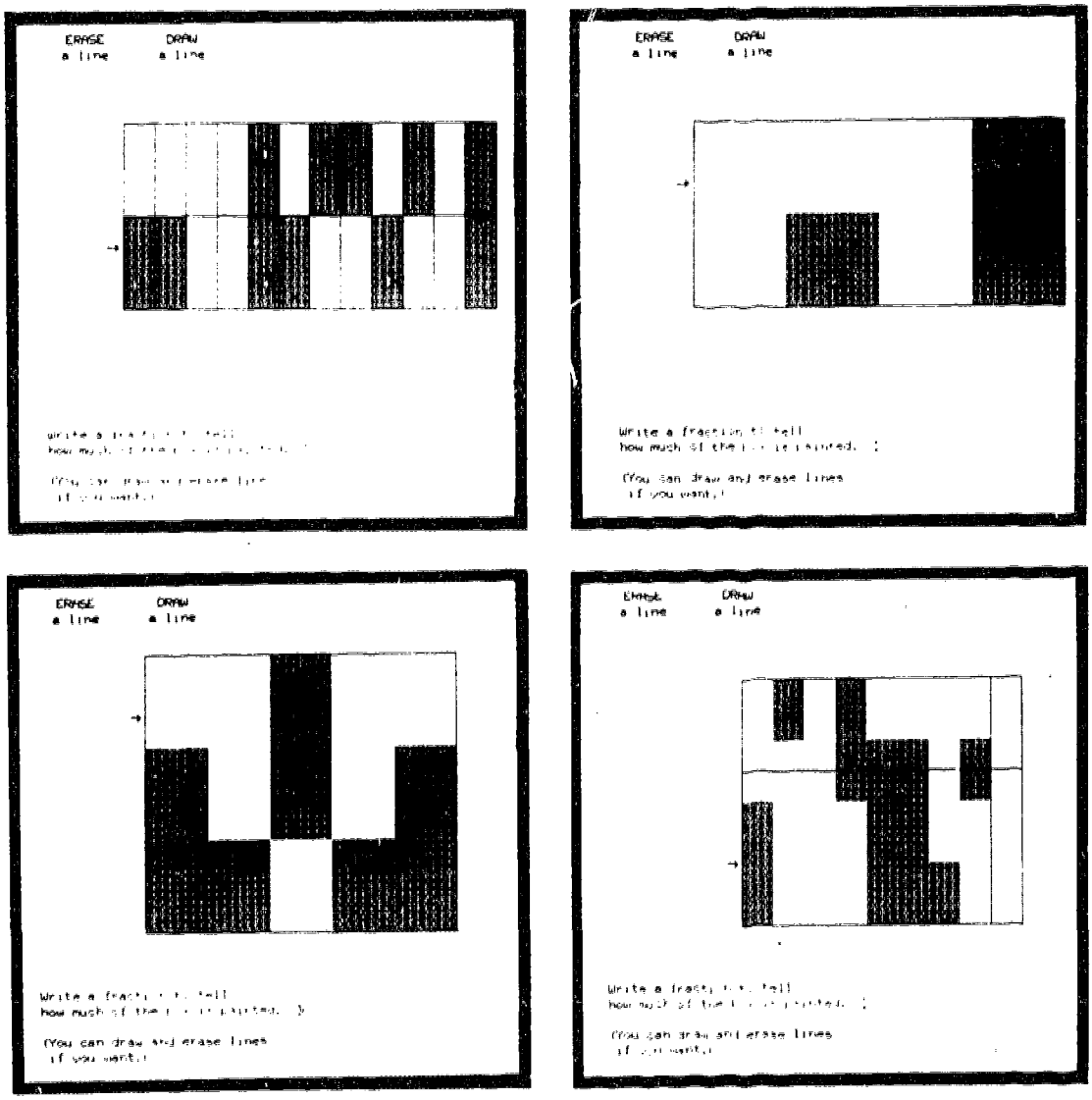




Purpose: Introduce a method for constructing fractions equivalent to a given fraction.

Description: PLATO paints a fraction of a box. The student is stepped through the process of cutting the box into more pieces to find another name for the fraction. Difficulty adjusts to the student's performance. The first problem gives a very specific step-through; later ones move faster and give less step-through.

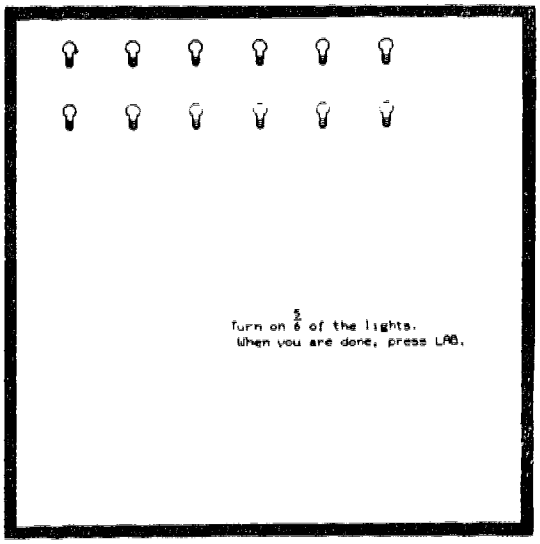
Boxes: How Much Is Painted? (touch)

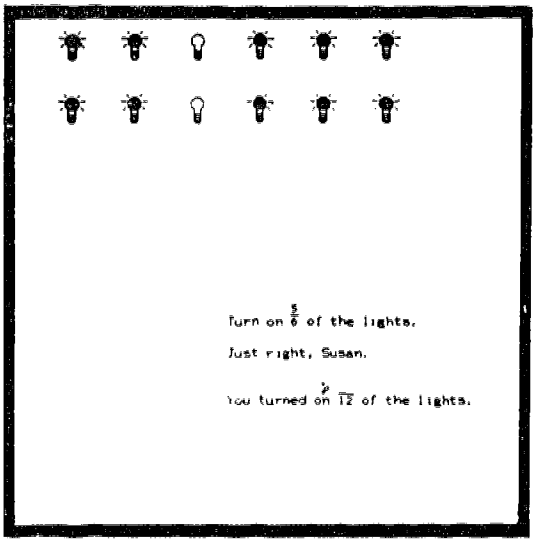


Purpose: Develop a strategy for figuring out what fraction of a box is painted. This skill will be useful later for addition of fractions using painted areas of a box.

Description: PLATO paints part of a box. The student can draw and erase lines to figure out what fraction of the box is painted. Difficulty adjusts to the student's performance.

Lights: Equivalent Fractions (touch)

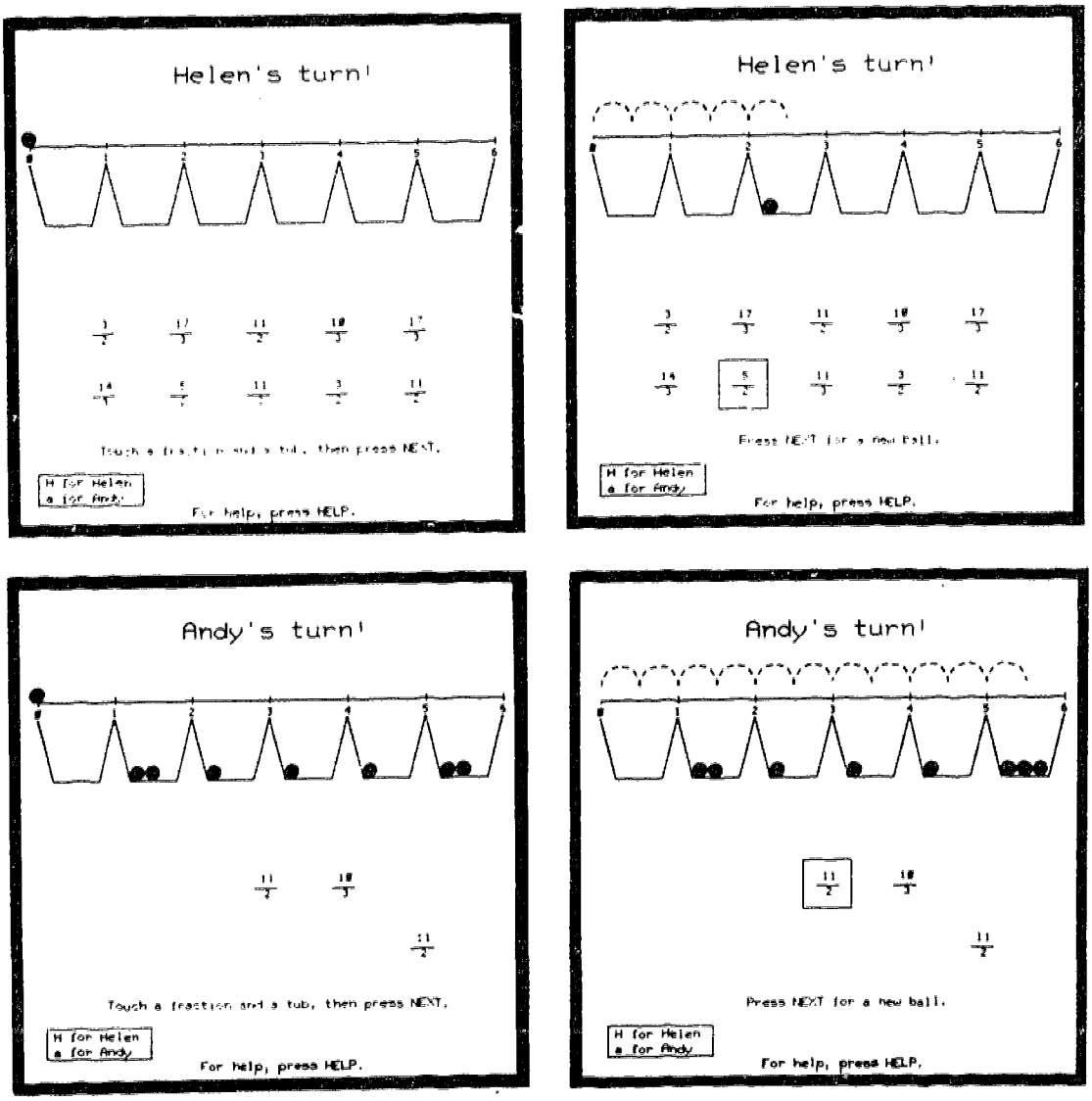




Purpose: Construct equivalent fractions, using a set of objects.

Description: This is a continuation of "Lights: Equivalent Fractions" which was started in the first chapter. The lesson consists of problems like this: the student turns on 5/6 of 12 light bulbs, then notes that 10/12 of the lights are on, and concludes that 5/6 of the lights is the same as 10/12 of the lights. The student continues the problem sequence where he or she left off in the first chapter.

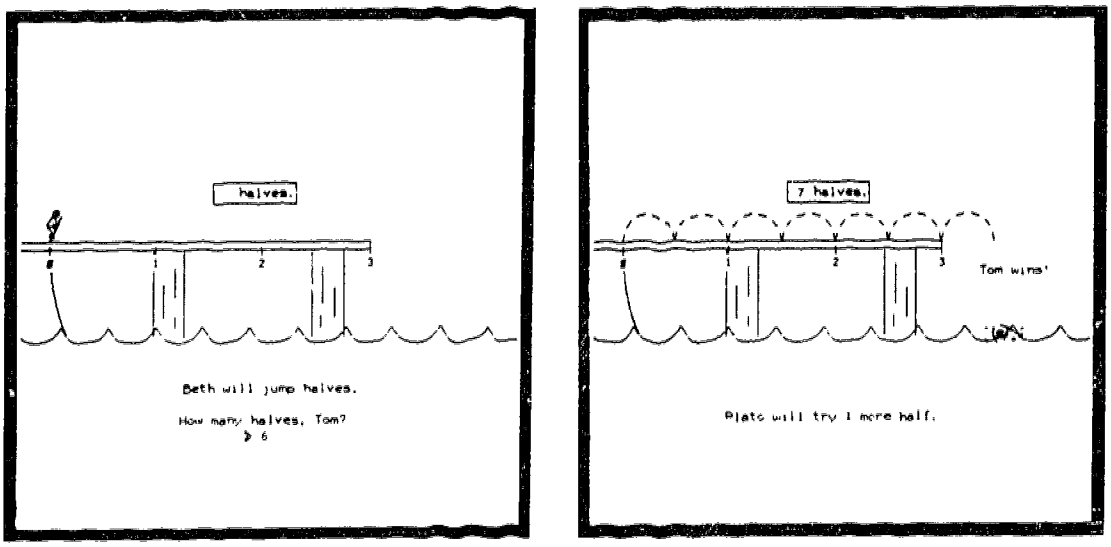
Pick a Tub (touch)



Purpose: Provide experience estimating positions of pure fractions on a number line.

Description: This is a game for one or two players. A ball is on a number line with tubs underneath, and ten fractions are given. Each turn consists of touching a fraction and a tub. The ball bounces the fraction distance and falls into a tub. If the student chose the right tub, the ball is marked with his or her initial. Difficulty adjusts to performance.

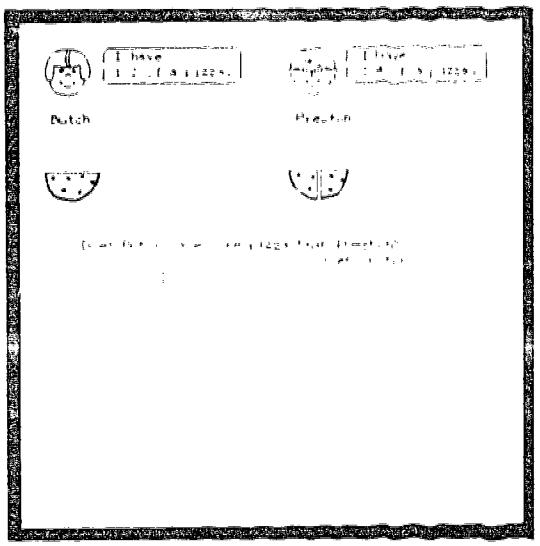
Splash!

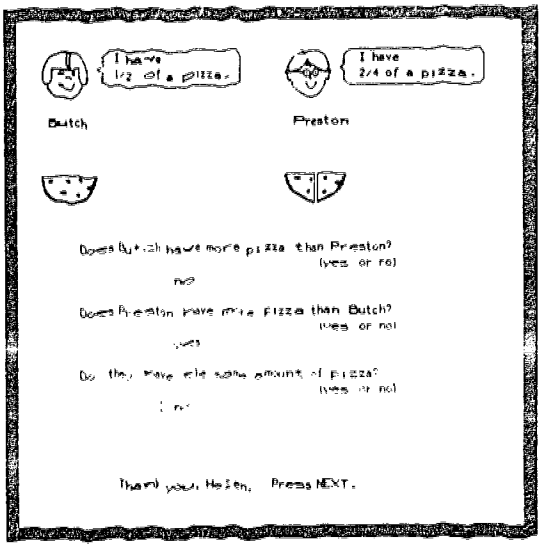


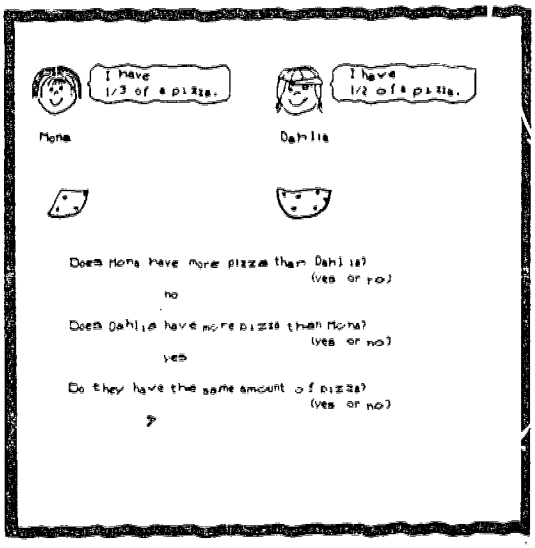
Purpose: Provide general experience relating and converting fractions to integers and mixed numbers.

Description: This is a game for one or two players. A kid on a pogo stick tries to hop to the end of a dock without falling into the water. The student tells the kid how far to go. Difficulty adjusts to the student's performance.

Pizza: Remedial Equivalent Fractions



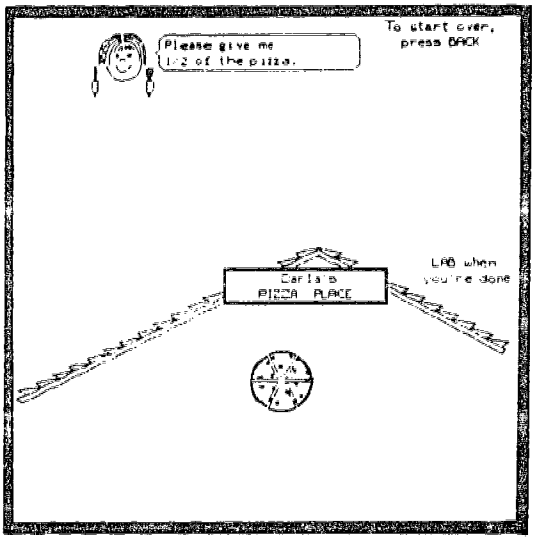




Purpose: Provide remedial and diagnostic help for students having trouble with equivalent fractions.

Description: Two kids on the screen each have some pizza. The student is asked which kid has more and whether they have the same amount. Answers are saved for diagnostic use by the teacher. There are five problems.

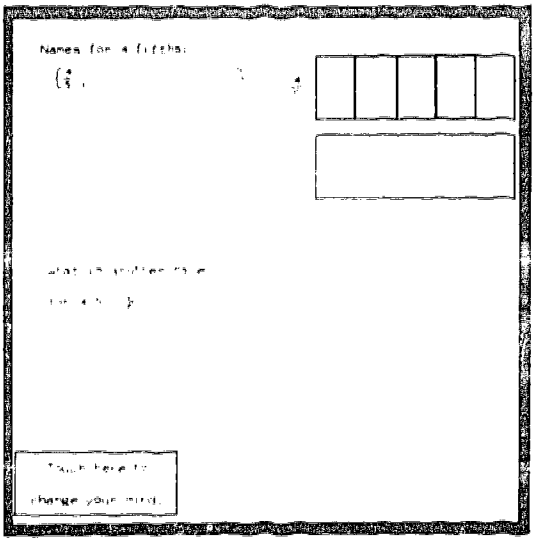
Pizza: Remedial Cutter Stuck (touch)

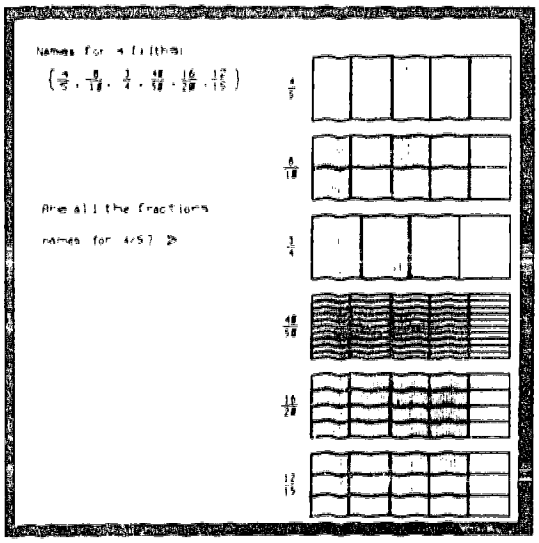


Purpose: Provide extra help for students who have trouble with equivalent fractions.

Description: The student has one pizza which is already cut into a number of equal pieces. A kid on the screen asks for a particular fraction of the pizza. Problems include using sixths to make 1/2, eighths to make 1/4, and similar problems.

Boxes: Name Equivalent Fractions (touch)

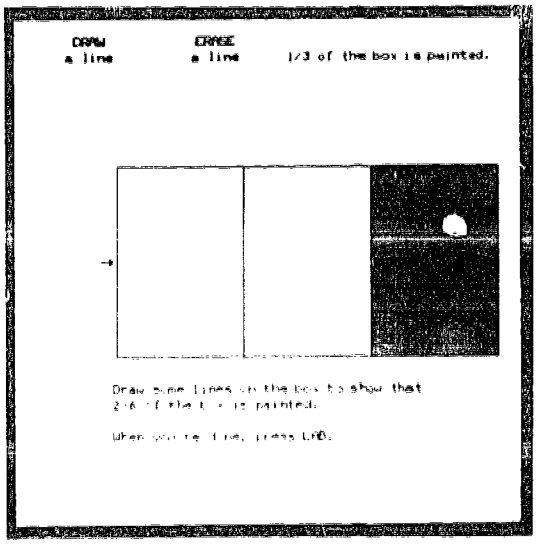


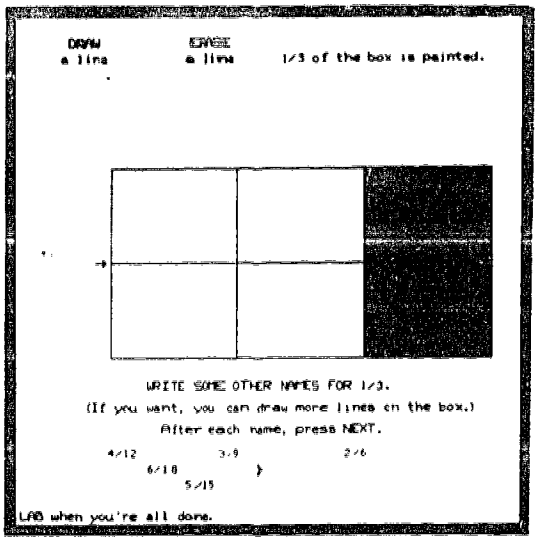


Purpose: Practice naming equivalent fractions with visual feedback comparing all of the named fractions.

Description: PLATO paints a fraction of a box. The student names fractions equivalent to the given one. Each named fraction is painted on a new box so that all of the fractions can be compared. The student removes any fractions that he or she decides are not equivalent to the given one. Difficulty adjusts to the student's performance.

Boxes: Equivalent Fractions Practice (touch)

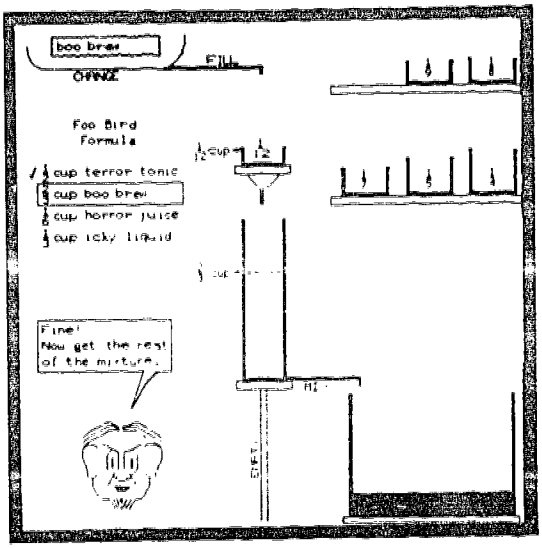




Purpose: Practice naming equivalent fractions with a familiar model available for help.

Description: PLATO paints a fraction of a box, and the student writes other names for that fraction. (The student can draw and erase lines on the box.) Some problems ask the student to draw lines on the box to show that two fractions are equivalent. Difficulty adjusts to the student's performance.

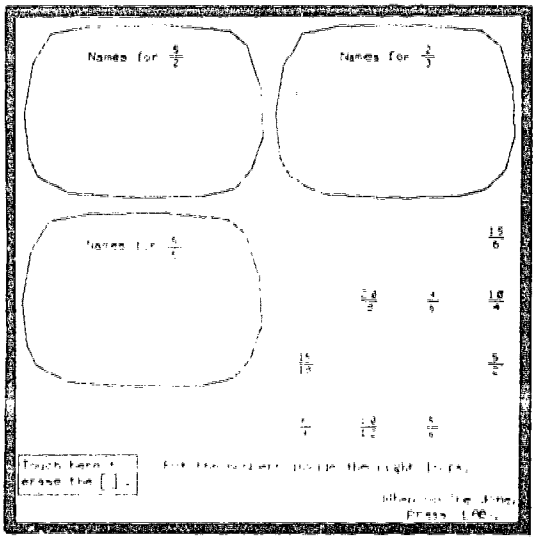
Make-a-Monster: Equivalent Fractions (touch)

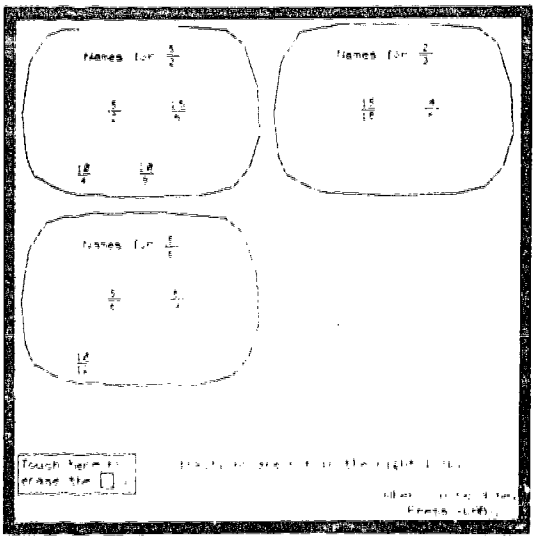


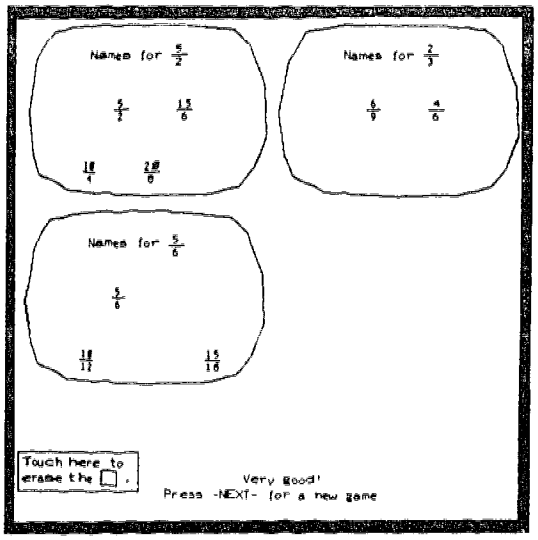
Purpose: Provide experience using equivalent fractions with a familiar model.

Description: This is like the earlier Make-a-Monster, but the student has to use equivalent fractions. When the formula includes 3/4 cup of something, the 1/4 cup is not available, but the 1/8 or 1/12 cups are. The lesson records which cups a student has used for each fraction, and when possible chooses available cups from those not yet used. If a student is doing particularly well, he or she will get "hard" formulas (e.g., a situation where he or she must measure 3/9 with a 1/6 cup).

Sort Equivalent Fractions (touch)



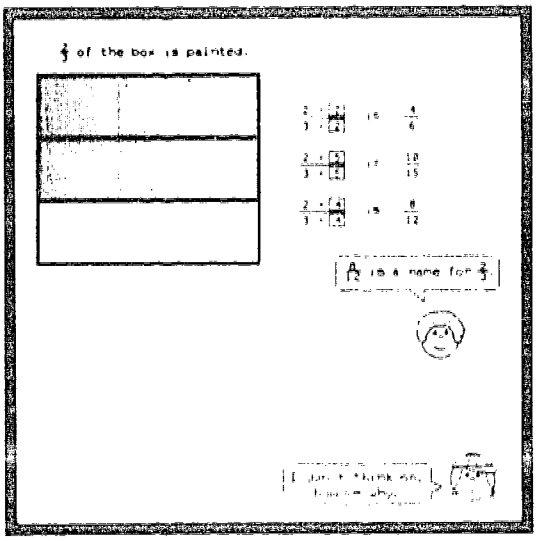


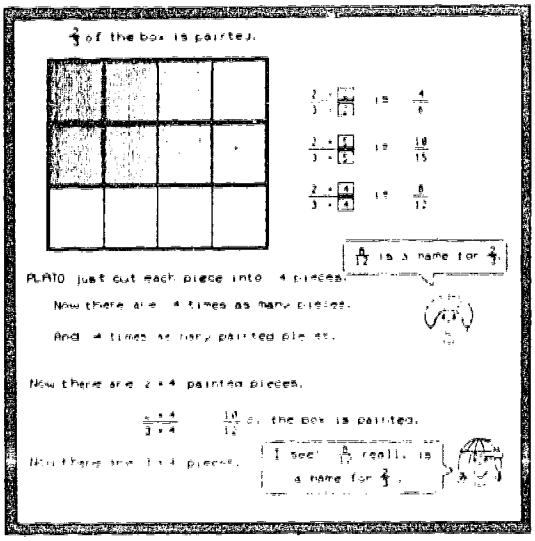


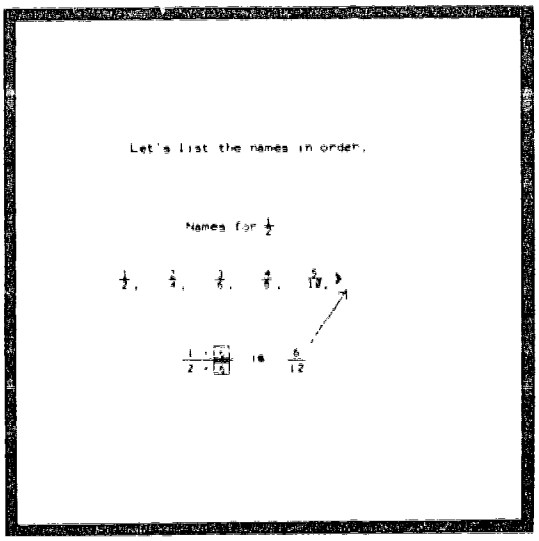
Purpose: Provide practice identifying equivalent fractions. This is for students who already have a basic understanding of equivalent fractions.

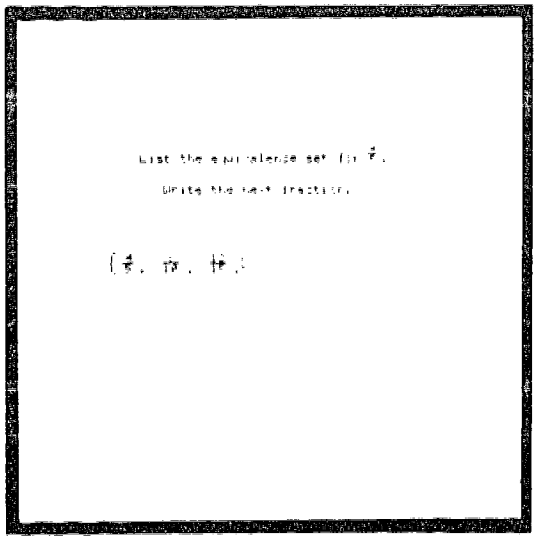
Description: Several fractions are scattered on the screen along with 2, 3, or 4 loops. The student sorts the fractions into the loops so that each loop contains an equivalence set. Difficulty adjusts to the student's performance. This is a practice lesson which will appear one page at a time.

Equivalent Fractions Generalized





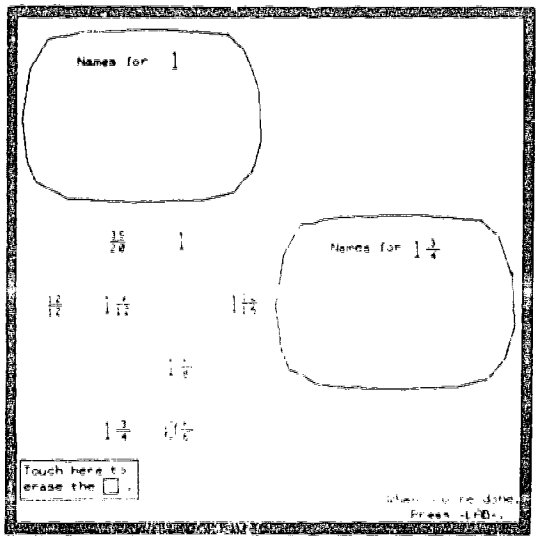




Purpose:
1. Present a systematic method for finding fractions equivalent to a given fraction.
2. Practice naming an equivalence set in order, starting with simplest terms. This skill will be useful later for systematically finding a common denominator for addition and subtraction of fractions.

Description: The first part of the lesson demonstrates that other names for a fraction can be found by multiplying the numerator and the denominator both by the same number. Kids on the screen question the method. PLATO justifies it for various cases by making more lines on a painted box and giving a short explanation. In the second part of the lesson, the student uses this method to list equivalence sets for given fractions. Difficulty adjusts to the student's performance. Lower difficulty levels offer much help. Higher levels encourage the student to work without PLATO's help.

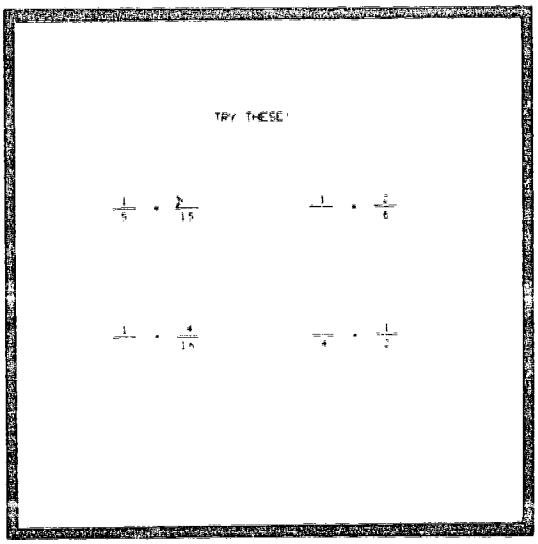
Sort Equivalent Numbers (touch)

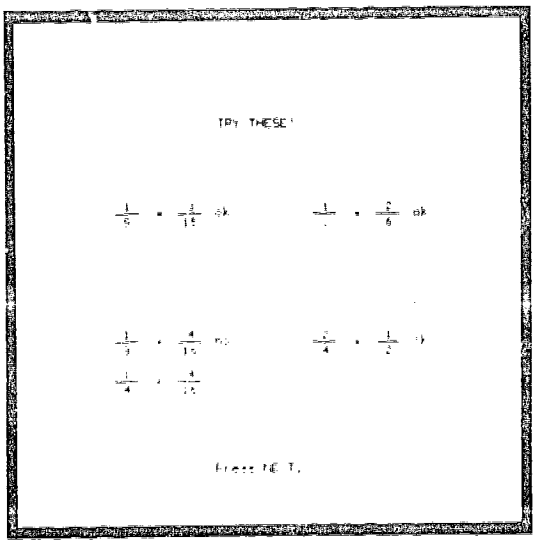


Purpose: Provide practice identifying equivalent whole numbers, fractions, and mixed numbers. This is for students who already have a basic understanding of equivalent fractions and mixed numbers.

Description: Several numbers (whole numbers, fractions, and mixed numbers) are scattered on the screen along with 2, 3, or 4 loops. The student sorts the numbers into the loops so that each loop contains an equivalence set. Difficulty adjusts to the student's performance. This is a practice lesson which will appear one page at a time.

Try These: Equivalent Fractions

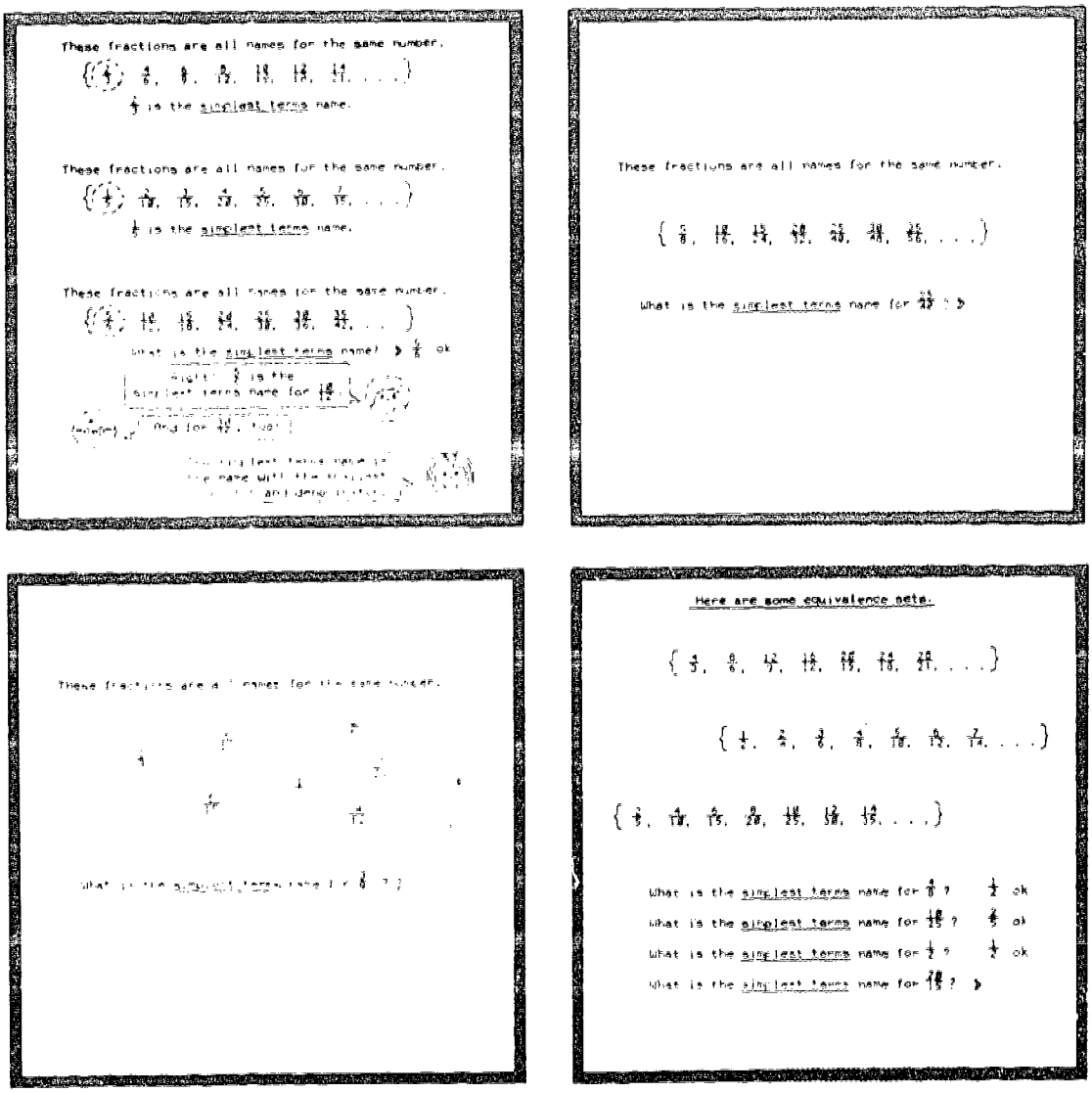




Purpose: Provide quick practice with equivalent fractions. This is for students who have a basic understanding of equivalent fractions, but need practice.

Description: This lesson occurs one page at a time for quick practice. There are four problems per page, and no pictures or models. Problems are like 1/2 = ⟫ /6. If the student is wrong twice on a problem, PLATO gives the answer to it. Difficulty adjusts to the student's performance.

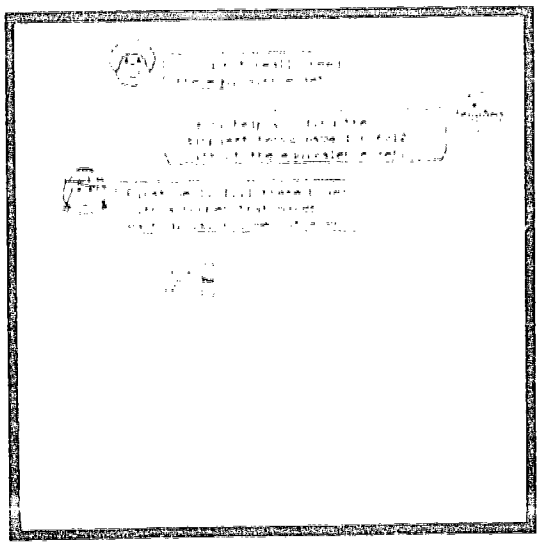
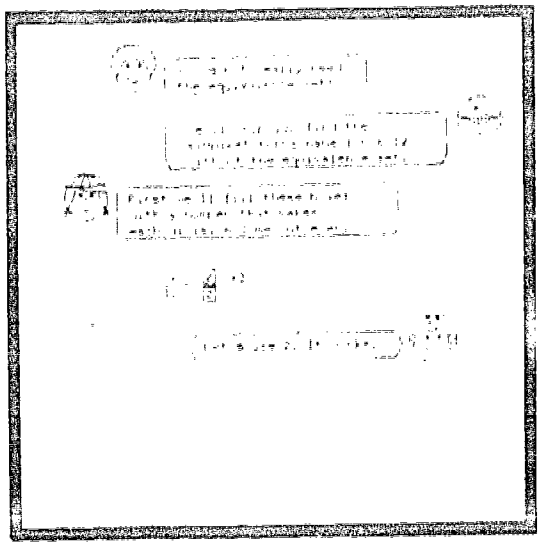
Introduction to Simplest Terms

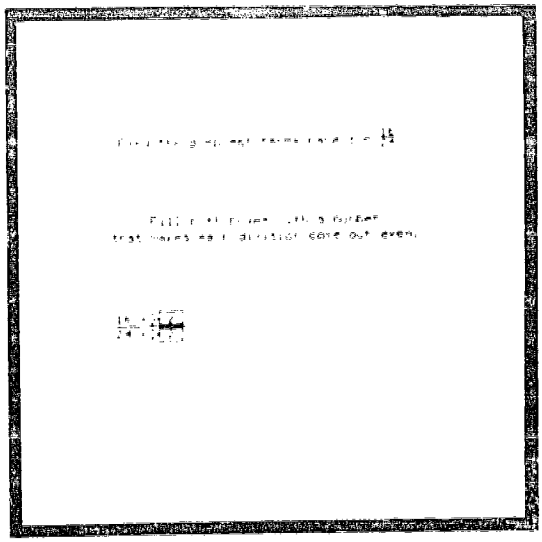


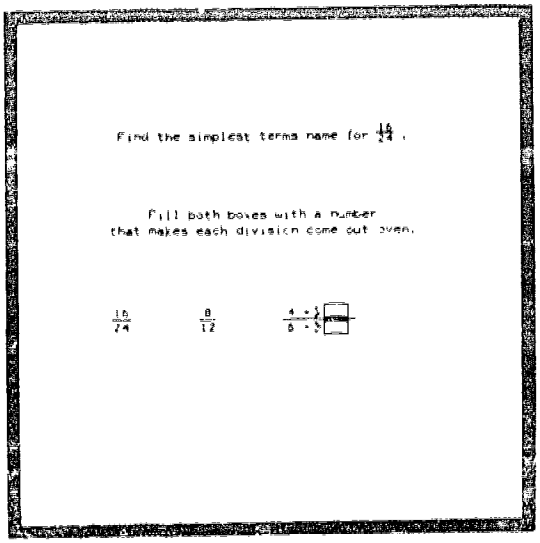
Purpose: Give the student a somewhat rough definition of the "simplest terms" name for a fraction.

Description: PLATO presents an ordered equivalence set for a fraction, with the simplest terms name listed first. PLATO points out the simplest terms name. After a few such examples, PLATO presents an equivalence set and asks the student to identify the simplest terms name. Kids on the screen make comments about "simplest terms." One kid points out that the simplest terms name is the name with the smallest numerator and denominator. Practice examples follow, some dealing with more than one equivalence set at a time and some having the names scattered in no particular order.

Simplest Terms



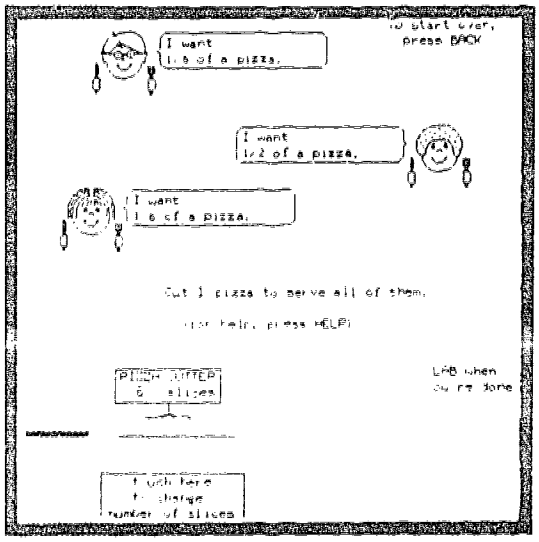




Purpose: Present and practice a systematic method for finding the simplest terms name for any fraction.

Description: Kids on the screen suggest that the equivalence set is not necessary for finding the simplest terms name for a fraction. For given fractions the kids help the student divide out common factors of the numerator and denominator to find the simplest terms name. Difficulty adjusts to the student's performance. PLATO's help is gradually reduced until the student is finding the simplest terms name independently.

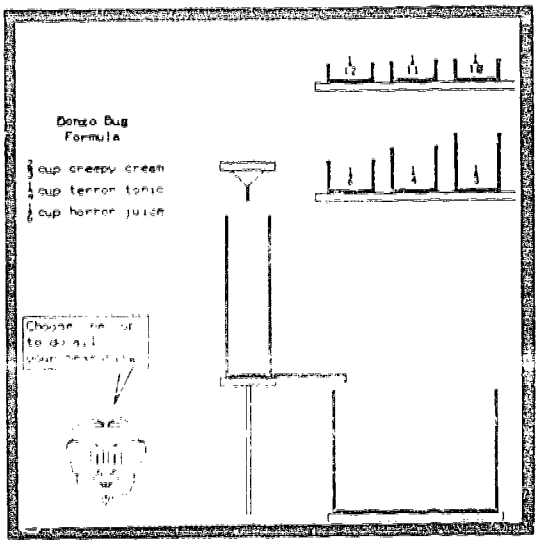
Pizza: Common Denominator (touch)



Purpose: Provide experience finding and using a common denominator with a familiar model.

Description: The student has one pizza with which to serve two or three kids on the screen who want different fractions of pizza. The pizza must be cut once to serve everyone. Difficulty of problems adjusts to the student's performance.

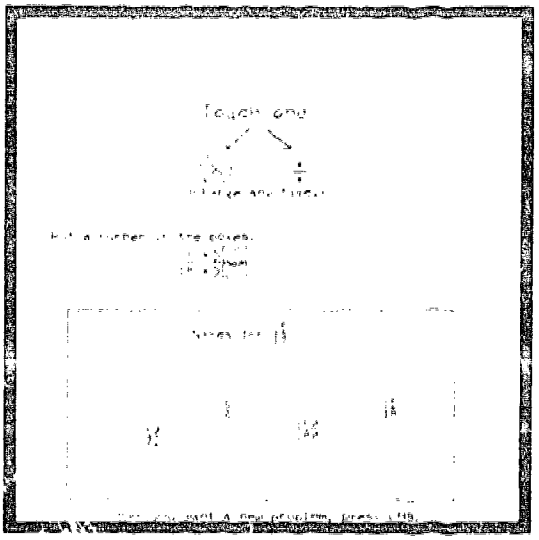
Make-a-Monster: Common Denominator (touch)

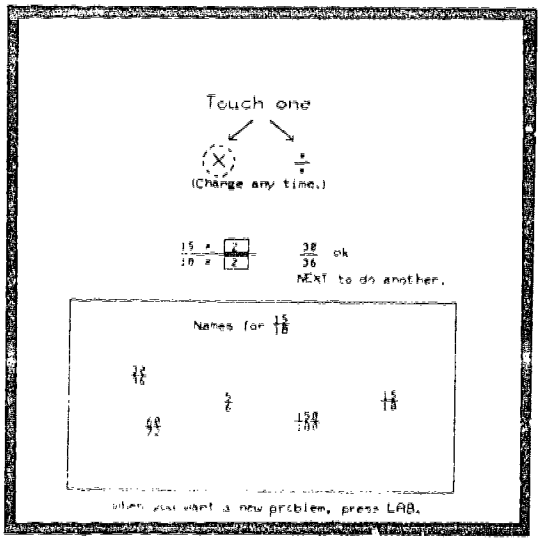


Purpose: Provide experience finding and using a common denominator with a familiar physical model.

Description: This is like the earlier Make-a-Monster, but the student must choose one measuring cup to do all the ingredients in the formula. For most students, this will be a straight forward common denominator exercise. But, for students who do particularly well in this program, there will be a few "hard" formulas. (A "hard" formula might include 6/9 cup, 2/4 cup, and 5/5 cup; the appropriate cup choice would be 1/6 or 1/12.)

Equivalence Experience (touch)

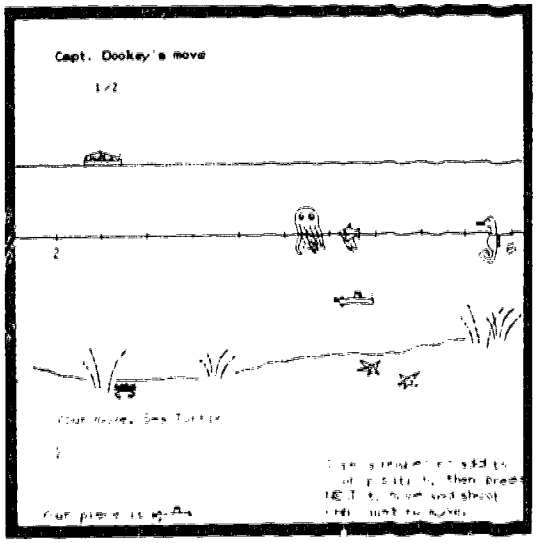


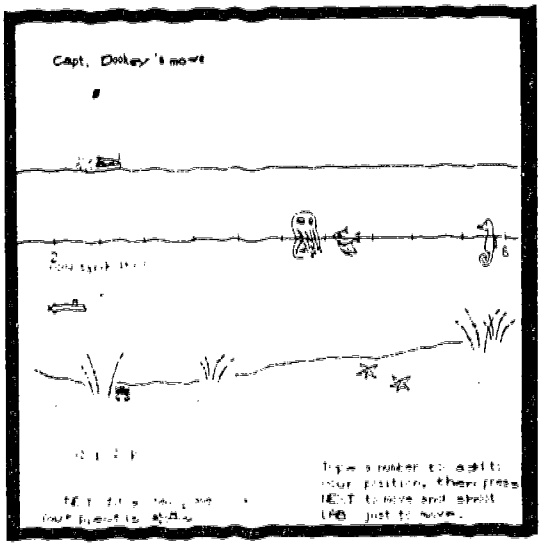


Purpose: 1) Encourage the student to think about why we can find the simplest terms name for a fraction by dividing out the common factors of the numerator and denominator.
2) Give the student a chance to experiment with factors and divisibility realted to equivalent fractions, using PLATO to do the computations.

Description: Some kids on the screen briefly explain that they can get another name for a fraction by dividing out common factors of the numerator and denominator. Then the student uses both multiplication and division to construct more names for given fractions. PLATO does the computation, so that the student is concerned only with exploring multiples, factors, divisibility, and any other patterns that he or she may find interesting.

Torpedo

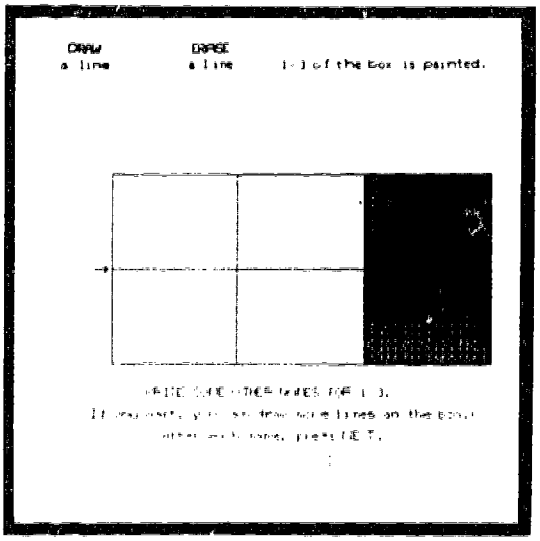


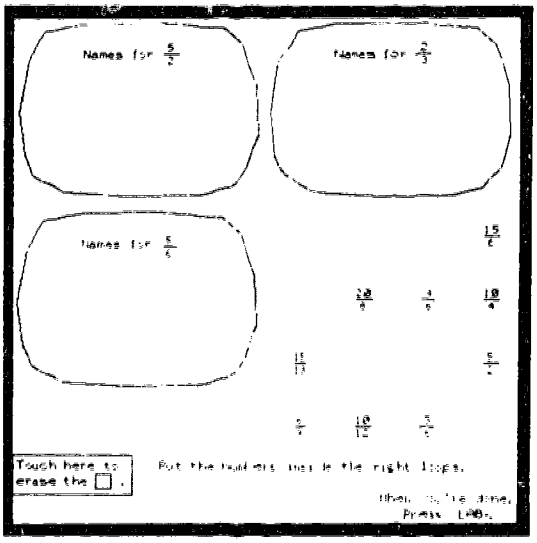


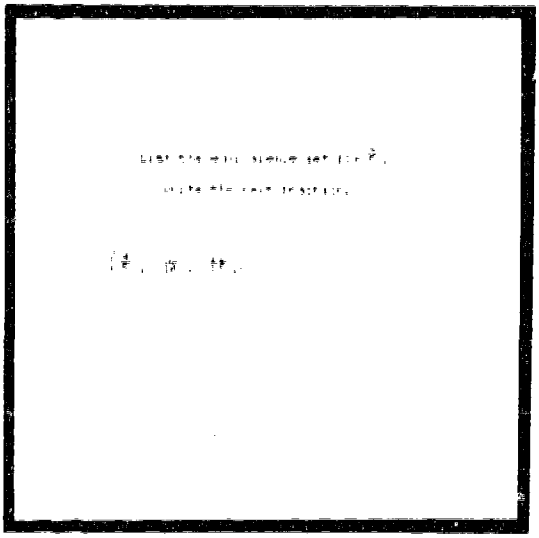
Purpose: Practice estimating fractional distances on a number line.

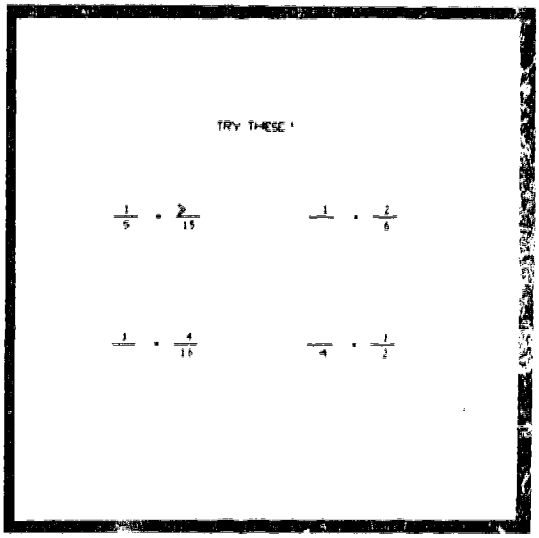
Description: This is an interterminal number line game. Two students compete. One controls a boat, the other a submarine. The object is to move to a point above (or below) the opponent's piece and shoot it. The number line varies from game to game.

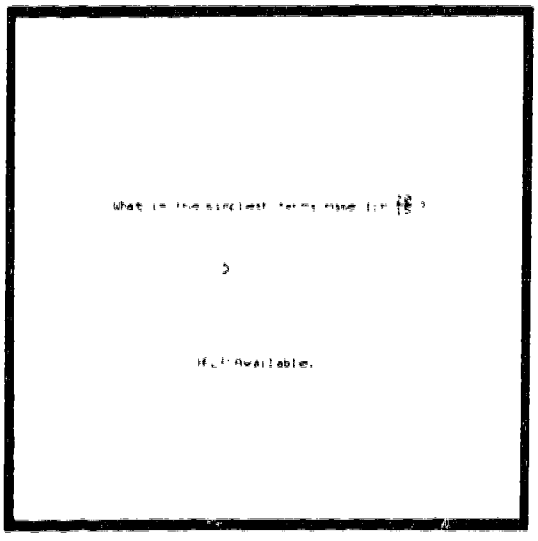
Check-up for Equivalent Fractions (touch)











Purpose: 1) Review equivalent fractions.
2) See if the student is ready for material that uses equivalent fractions.

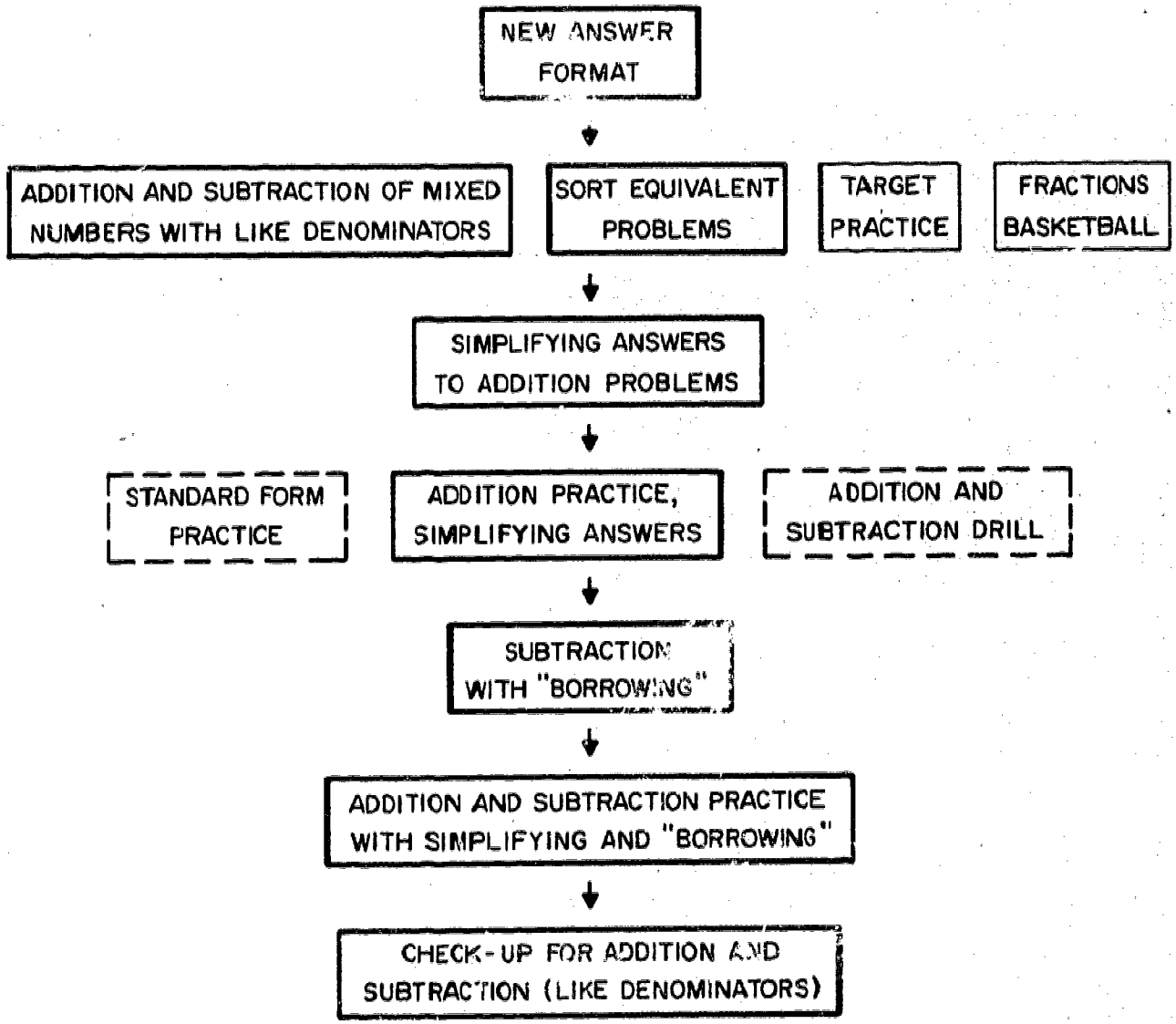
Description: The check-up includes from 1 to 3 selected problems from each of 5 lessons:
Boxes: Equivalent Fractions Practice
Sort Equivalent Fractions
Equivalent Fractions Generalized
Try These: Equivalent Fractions
Simplest Terms
The number of problems depends on the student's performance.

ADDITION AND SUBTRACTION (LIKE DENOMINATORS)

Simplified Curriculum Chart	D2
Lesson Descriptions	
New Answer Format	D3
Addition and Subtraction of Mixed Numbers with Like Denominators	D4
Sort Equivalent Problems	D5
Target Practice	D6
Fractions Basketball	D7
Simplifying Answers to Addition Problems	D8
Standard Form Practice	D9
Addition Practice, Simplifying Answers	D10
Addition and Subtraction Drill	D11
Subtraction with "Borrowing"	D12
Addition and Subtraction Practice with Simplifying and "Borrowing"	D13
Check-up for Addition and Subtraction (like denominators)	D14

ADD & SUBTRACT LIKE DENOMINATORS

a simplified flow chart



——— BASIC

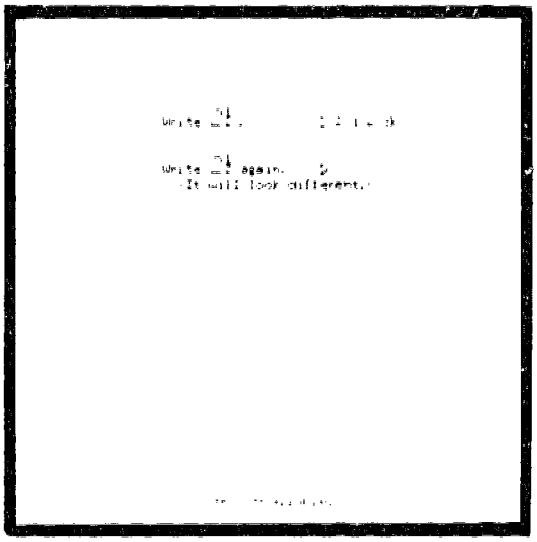
——— EXPERIENCE

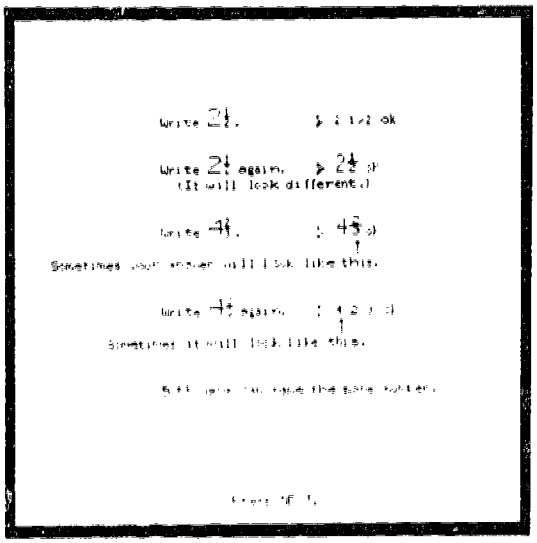
—'— — PRACTICE

– – – – – ENRICHMENT

+– – – – + REMEDIAL

New Answer Format



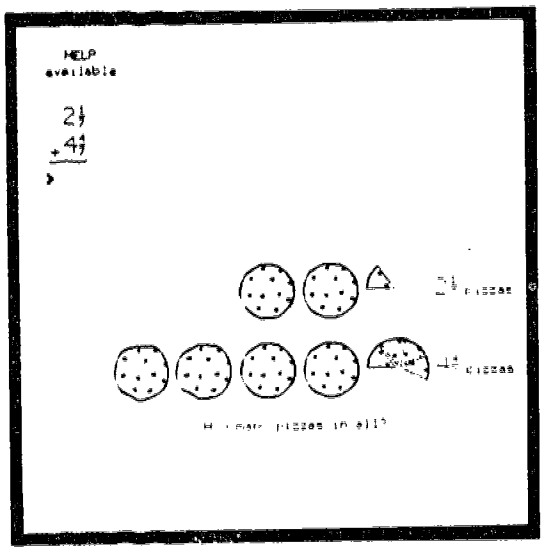


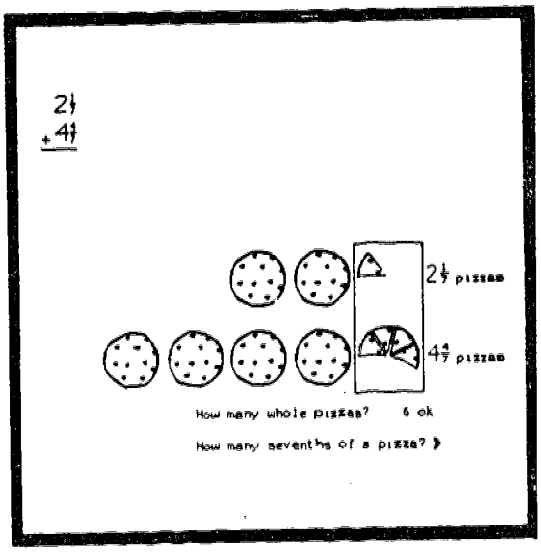
Purpose: Introduce the new way that a student's answer will look when he or she writes a fraction or mixed number on PLATO.

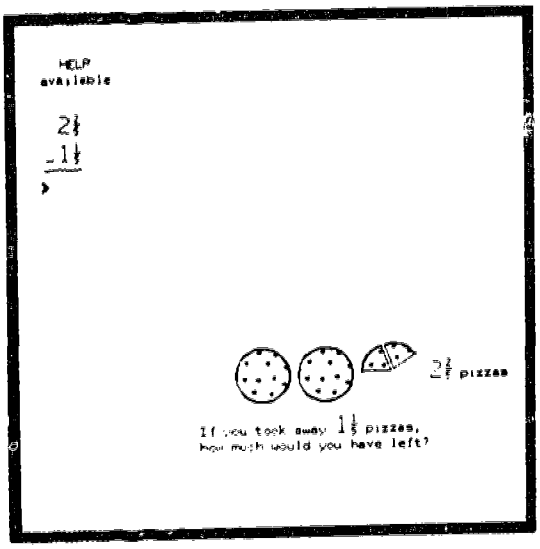
Description: The student writes several mixed numbers. Some of them appear as 3 1/2 (the "old" format); others appear as $3\frac{1}{2}$ (the "new" format).

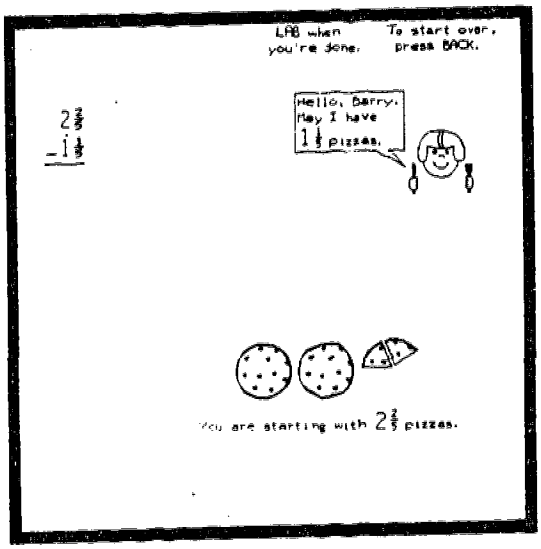
The new format will be used in the addition and subtraction lessons where the old format might look confusing. The new format involves no change in the keys pressed by the student, only in the way PLATO shows it. The old format is easier for PLATO to handle and allows many more options for the student, so it will continue to be used whenever the new format is not needed.

Addition and Subtraction of Mixed Numbers with Like Denominators (touch)





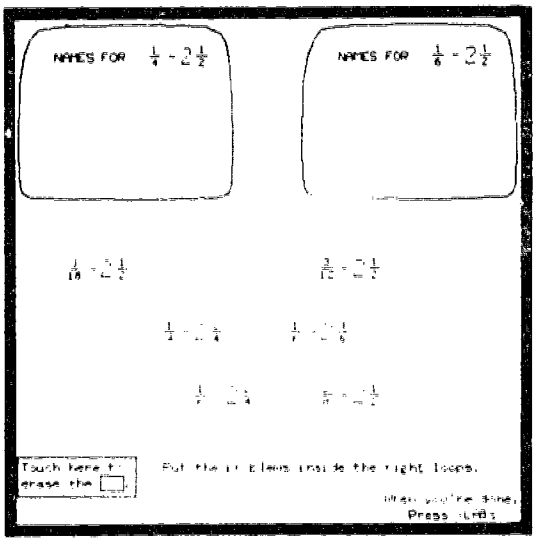


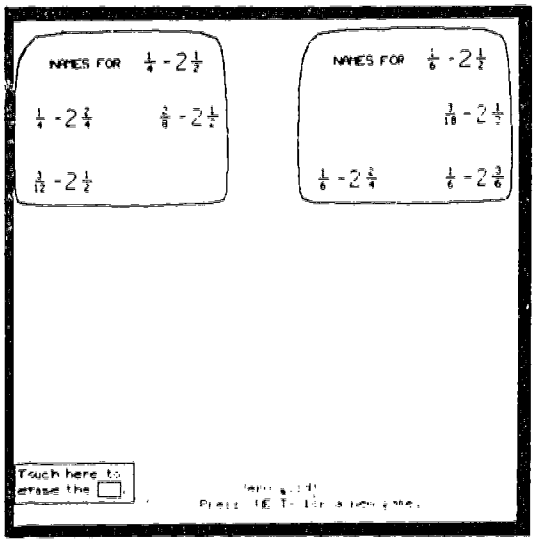


Purpose: Begin addition and subtraction of fractions and mixed numbers with like denominators.

Description: The student answers addition and subtraction problems. Pizza pictures and extra help are available. Difficulty adjusts to the student's performance. The student who is doing well gets some problems without pictures.

Sort Equivalent Problems (touch)

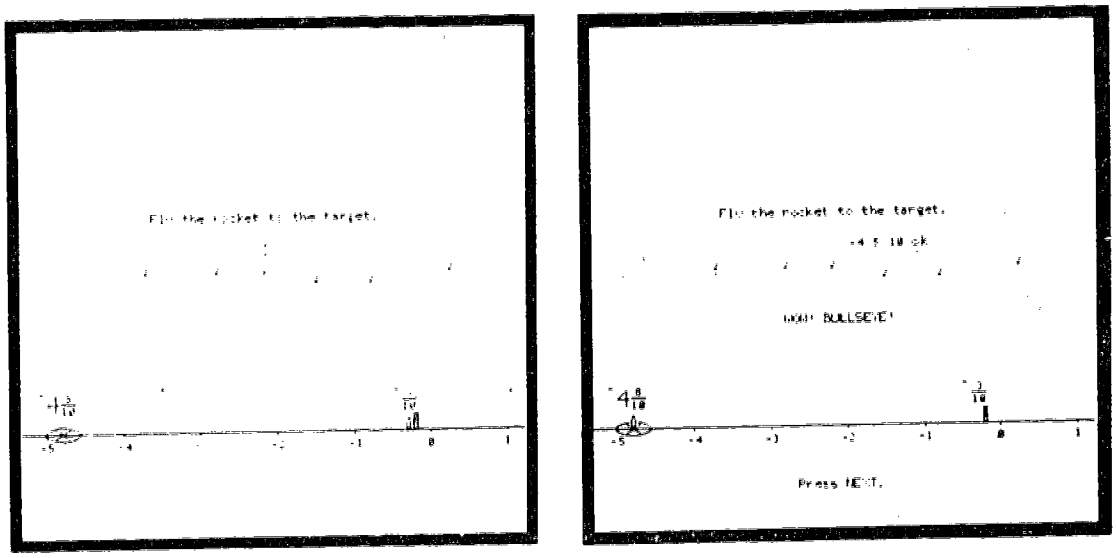




Purpose: Introduce the idea that renaming the numbers in an addition or subtraction problem makes another name for the same problem. This idea will be useful when we begin finding a common denominator for addition and subtraction.

Description: Several expressions involving addition and subtraction are scattered on the screen with two or three loops to sort them into. The student sorts the expressions into the loops so that each loop contains an equivalence set. Difficulty adjusts to the student's performance. This lesson will appear one page at a time.

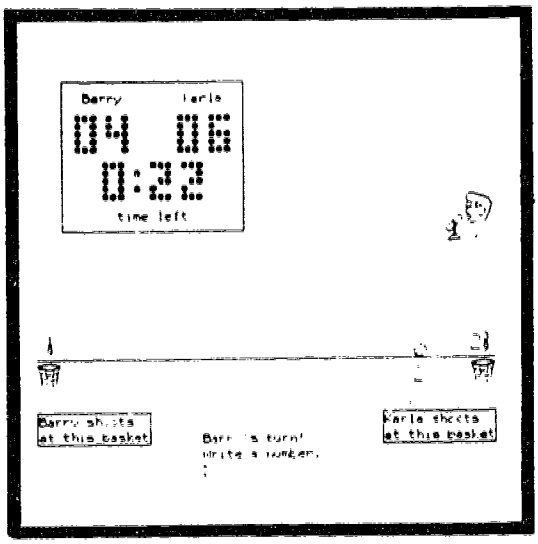
Target Practice

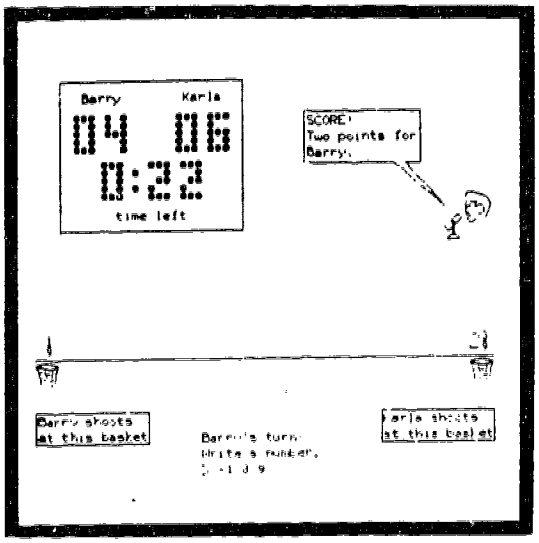


Purpose: Addition practice which may encourage checking the answer by estimation.

Description: The student fires a remote control rocket at a target. Both the rocket and the target are on a number line at known positions. An easy estimate will hit the target for a "SCORE," but only the exact answer gets a "BULLSEYE." Difficulty adjusts to the student's performance.

Fractions Basketball

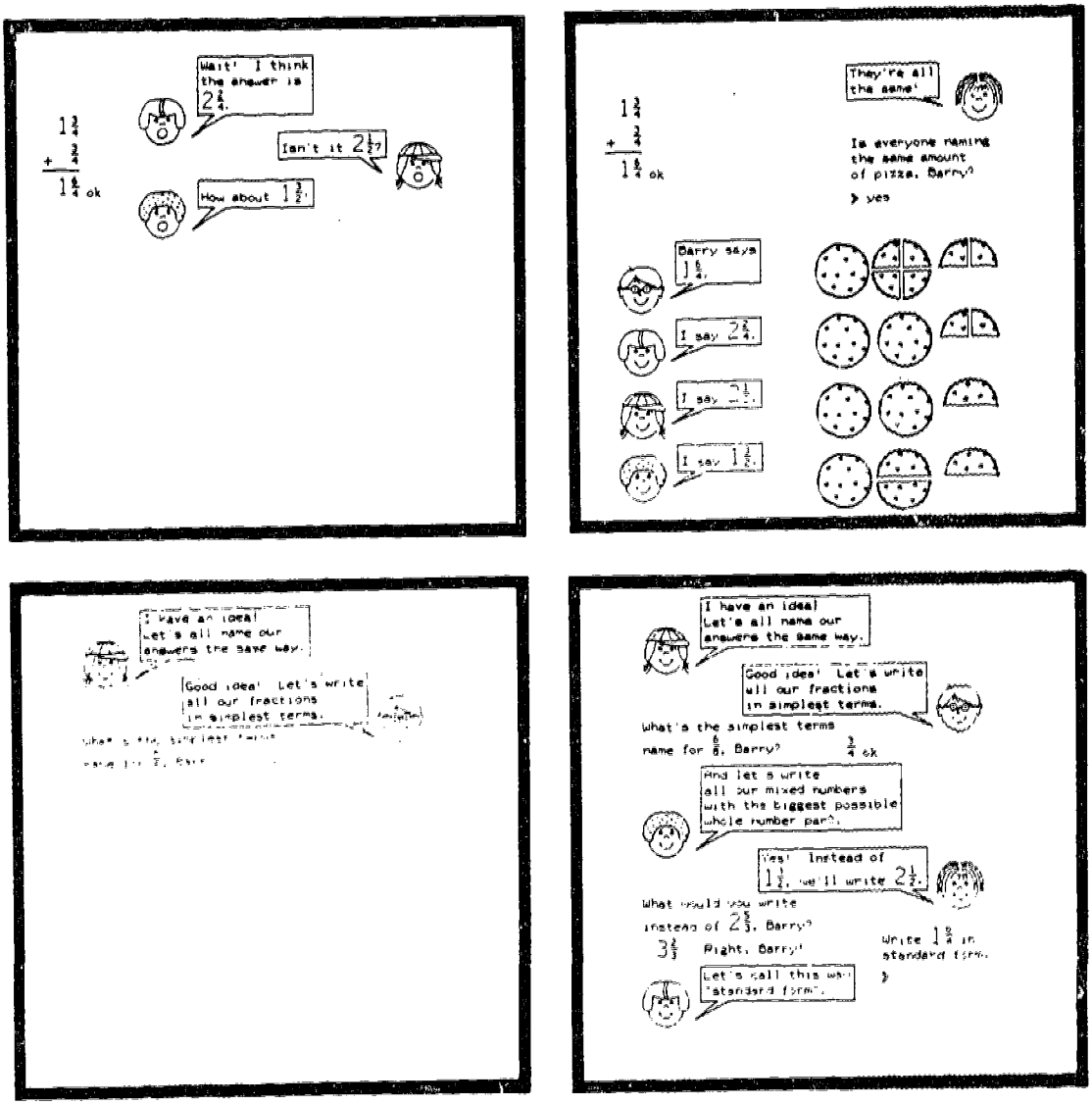




Purpose: Subtraction practice on the number line.

Description: This is a game for one or two students. Given ball and basket positions on the number line, the student bounces the ball some fraction or mixed number (e.g., 2 7/8 makes the ball do two bounces of length 1 then seven bounces of length 1/8) to make a basket. Difficulty adjusts to the student's performance.

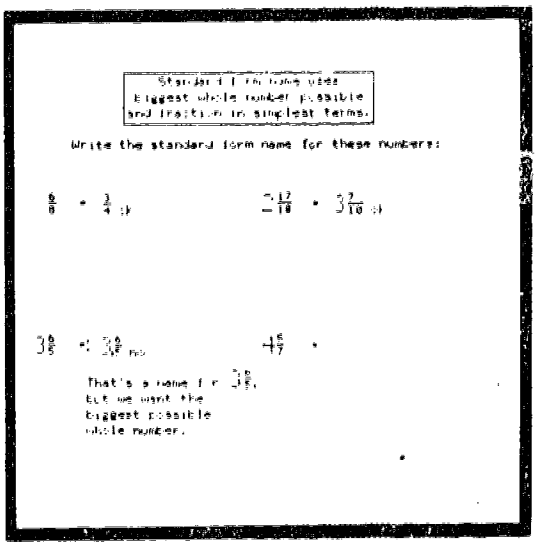
Simplifying Answers to Addition Problems



Purpose: Show a need for simplifying the answer to an addition problem.

Description: The student answers an addition problem. Several kids on the screen suggest different correct answers to the same problem. In order to avoid this confusion, they decide to always name their answers with the biggest possible whole number and with the fraction in simplest terms. They call this way "standard form." The student practices writing some numbers in standard form.

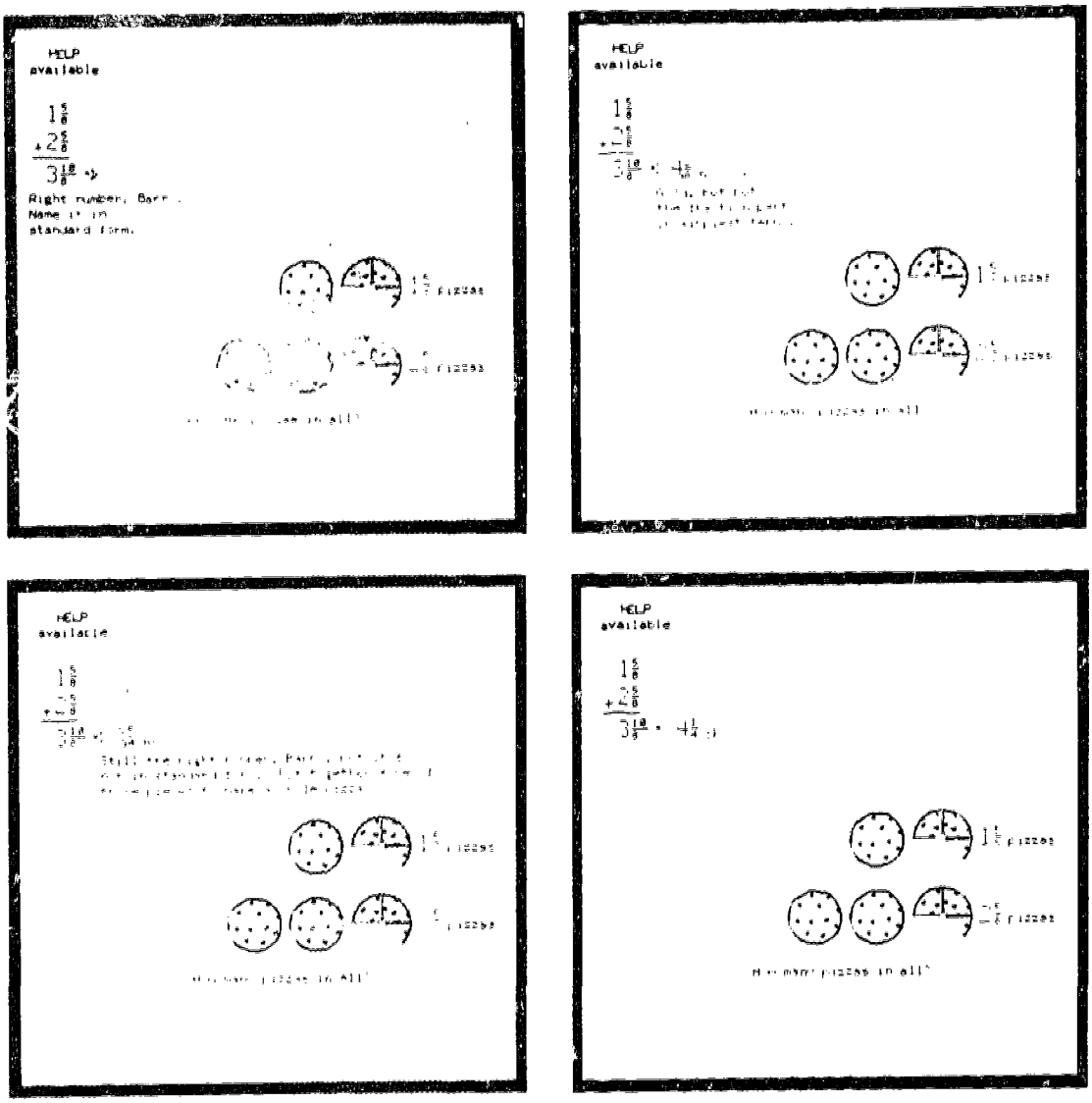
Standard Form Practice



Purpose: Practice writing fractions and mixed numbers in standard form.

Description: The student writes the standard form names for given fractions and mixed numbers. There are four problems per page. After the student's third wrong try on a problem, PLATO gives the answer. Difficulty adjusts to the student's performance. This lesson will appear one page at a time for most students. Students who have trouble with it will see more pages.

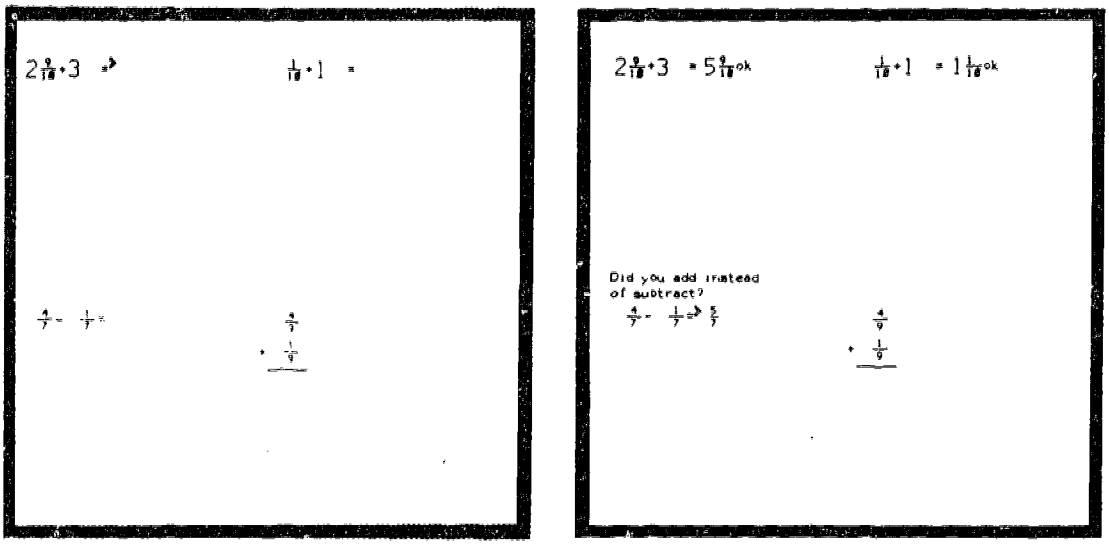
Addition Practice, Simplifying Answers



Purpose: Provide experience adding fractions and mixed numbers with like denominators, simplifying answers when necessary.

Description: The student answers addition problems, simplifying answers when necessary. Pizza pictures and extra help are available. Difficulty adjusts to the student's performance.

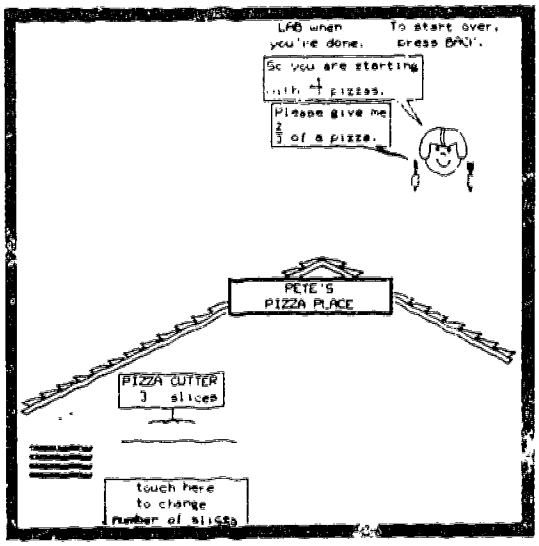
Addition and Subtraction Drill

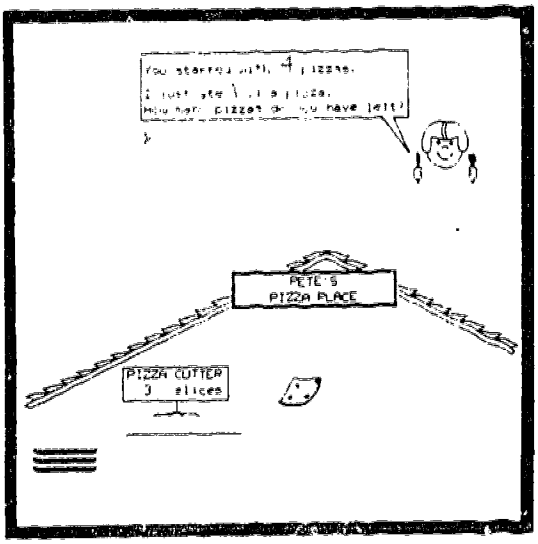


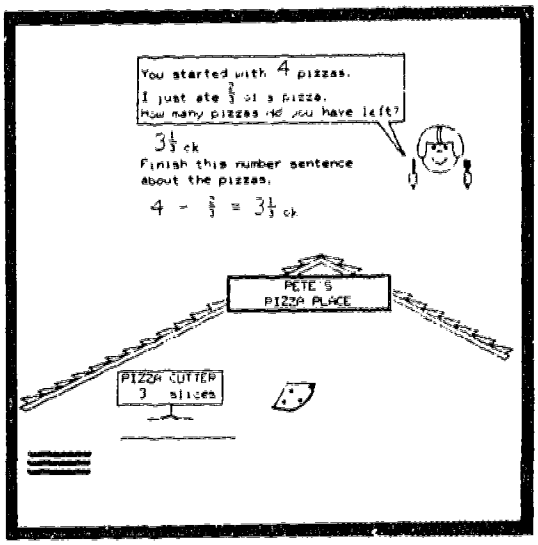
Purpose: Practice adding and subtracting fractions and mixed numbers with like denominators. This is for students who have a basic understanding but need practice.

Description: There are four problems per page and no pictures or models. There is help for students who have trouble "borrowing" or putting the answer in standard form. There are special messages for common errors like adding instead of subtracting. The student will see one page at a time. Difficulty adjusts to the student's performance. Problems which involve "borrowing" will appear in this lesson only after "borrowing" is introduced in other lessons.

Subtraction with "Borrowing" (touch)



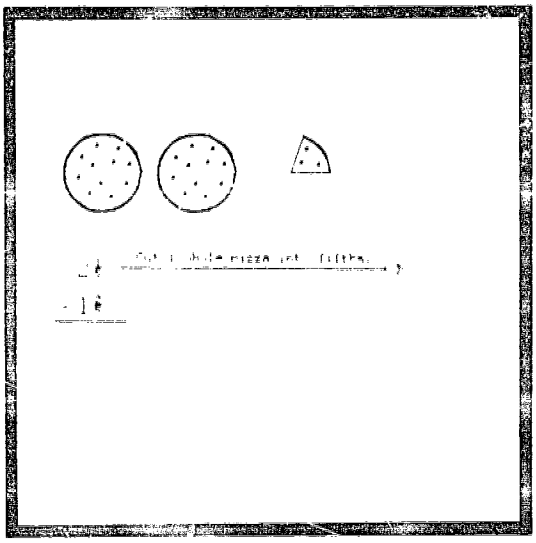


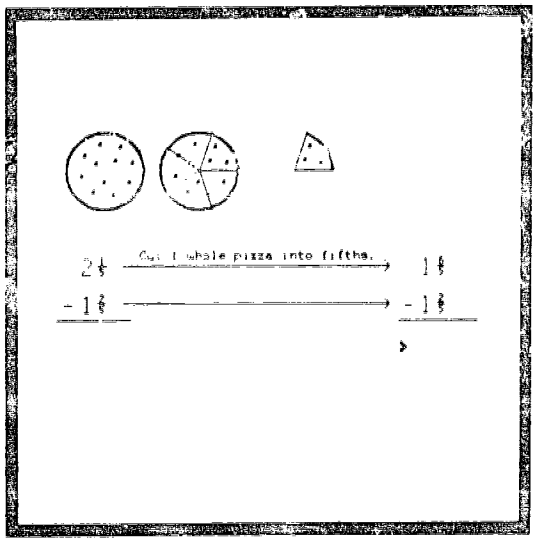


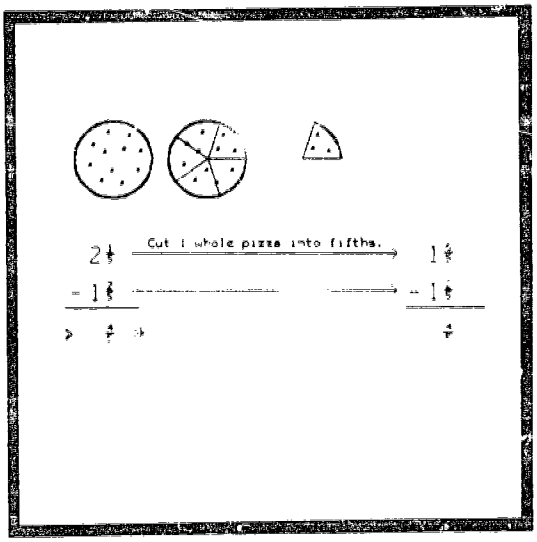
Purpose: Introduce a method for solving subtraction problems like $3\frac{1}{3} - \frac{2}{3}$.

Description: The student does "borrowing" problems while operating a pizza place. For example, the student starts with $3\frac{1}{3}$ pizzas, then gives away $\frac{2}{3}$ of a pizza by cutting one of the whole pizzas. The student writes a subtraction number sentence: $3\frac{1}{3} - \frac{2}{3} = 2\frac{2}{3}$. Difficulty adjusts to the student's performance. Harder problems have less specific step-through and ask the student to write the subtraction number sentence before cutting the pizza.

Addition and Subtraction Practice with Simplifying and "Borrowing"



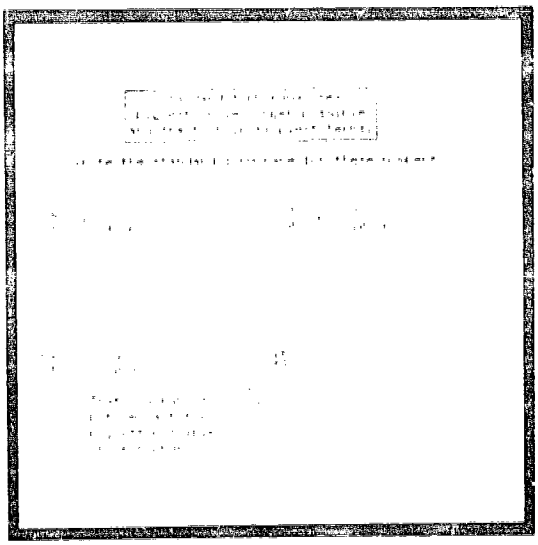


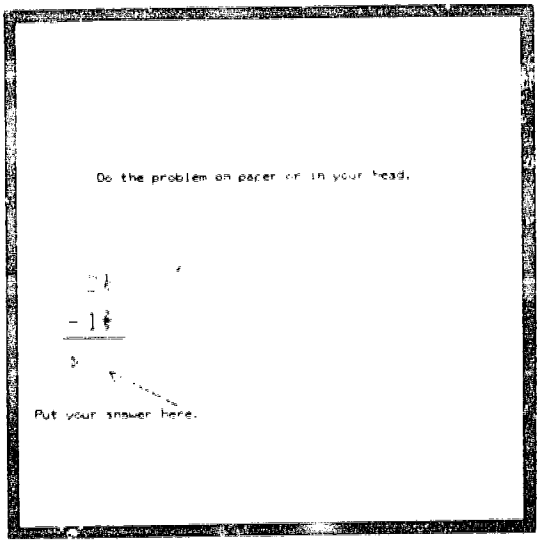


Purpose: Provide experie adding and subtracting fractions and mixed numbers, simplifying nswers and "borrowing" when necessary.

Description: The student is stepped through subtraction problems involving "borrowing." Difficulty adjusts to the student's performance. Higher levels of difficulty give less help and include addition problems as well as subtraction problems. Most, but not all, of the subtraction problems involve "borrowing."

Check-up for Addition and Subtraction (like denominators)





Purpose: 1) Review addition and subtraction of fractions and mixed numbers with like denominators.
2) See if the student is ready for further material.

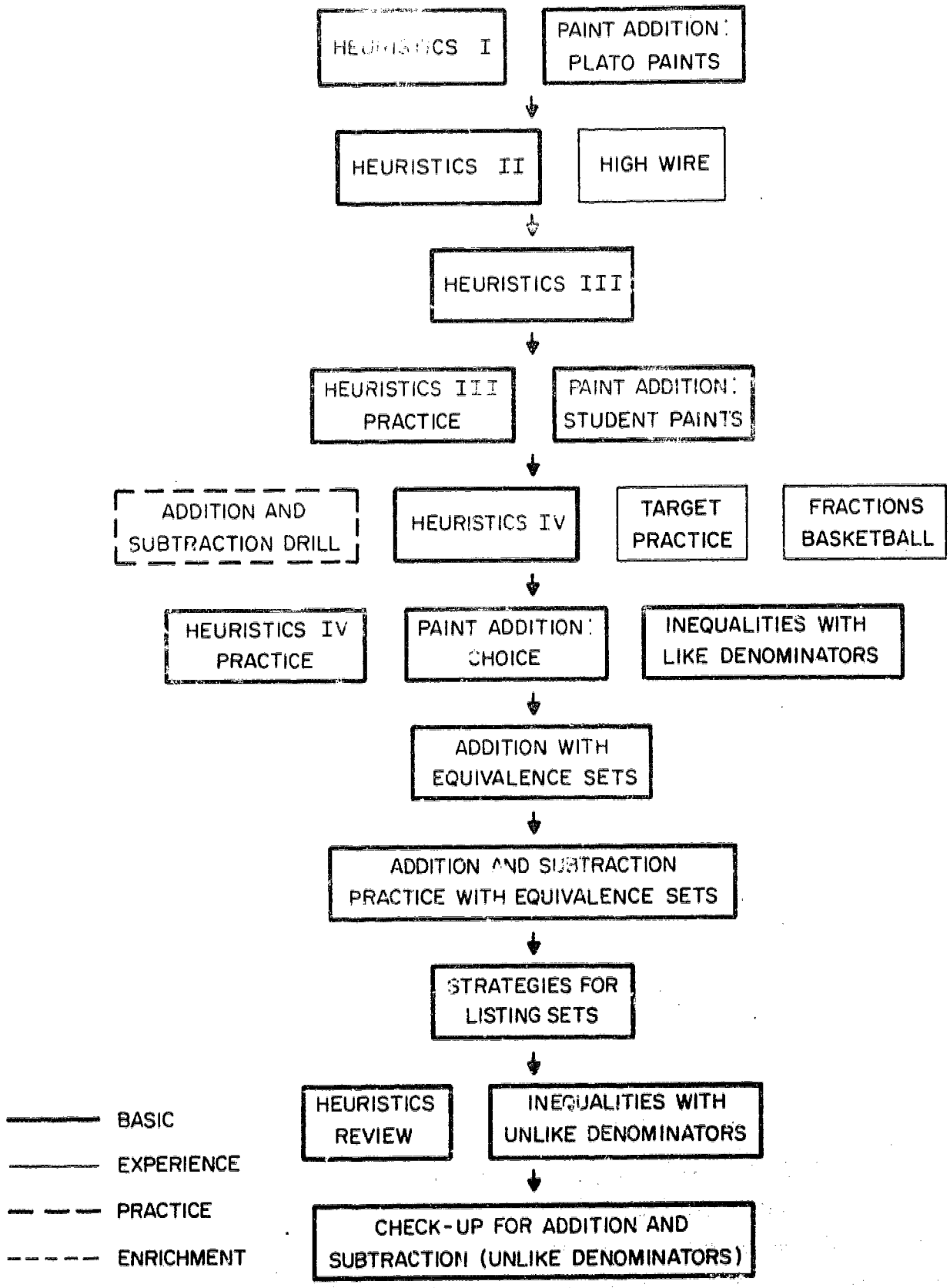
Description: The check-up consists of selected problems from two lessons: "Standard Form Practice" and "Addition and Subtraction Practice with Simplifying and Borrowing." The student tries each addition or subtraction problem [illegible] help. If he or she cannot do the problem, PLATO offers help

ADDITION AND SUBTRACTION (UNLIKE DENOMINATORS)

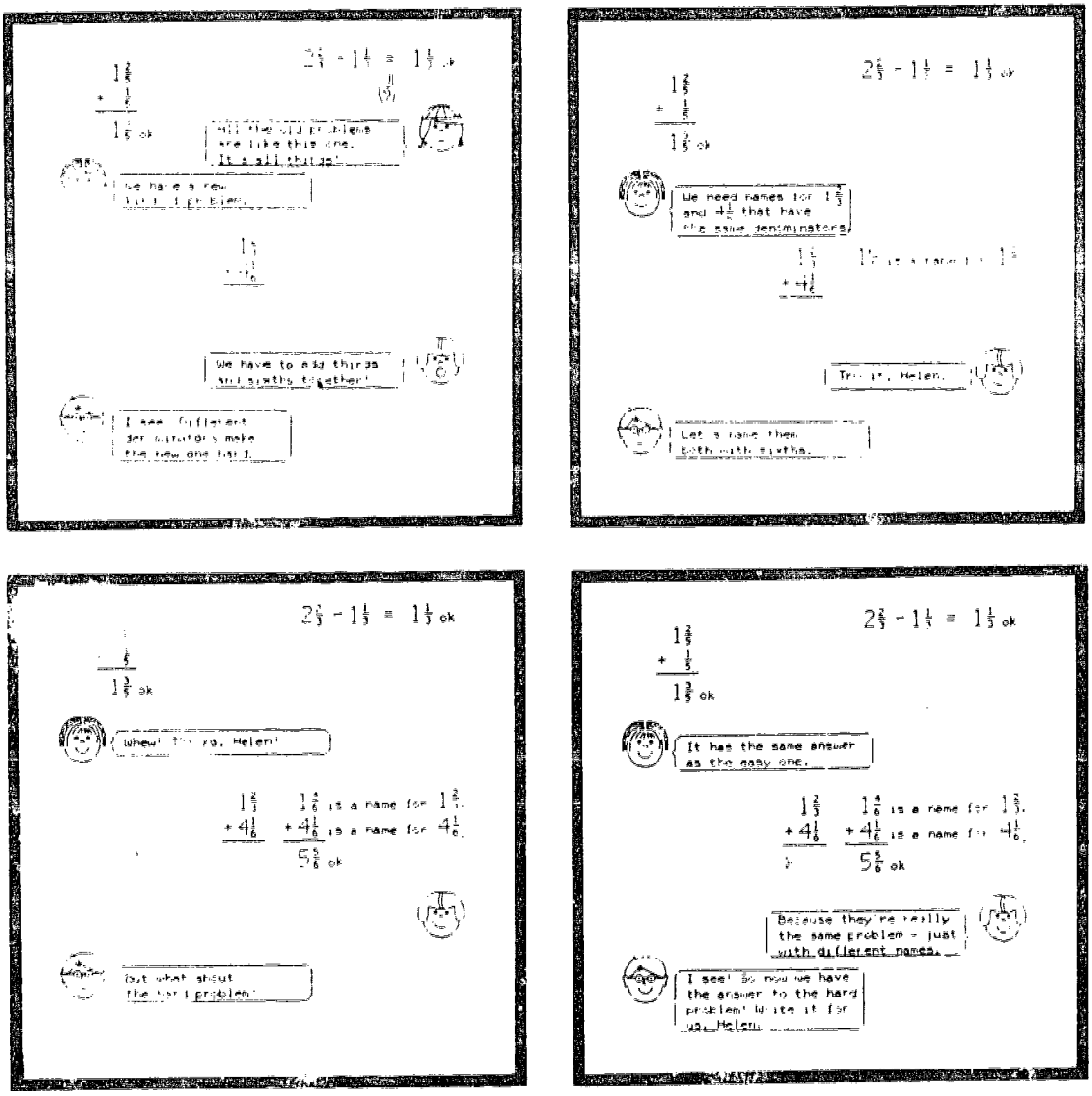
Simplified Curriculum Chart	E2
Lesson Descriptions	
Heuristics I	E3
Paint Addition: PLATO Paints	E4
Heuristics II	E5
High Wire	E6
Heuristics III	E7
Heuristics III Practice	E8
Paint Addition: Student Paints	E9
Addition and Subtraction Drill	E10
Target Practice	E11
Fractions Basketball	E12
Heuristics IV	E13
Heuristics IV Practice	E14
Paint Addition: Choice	E15
Inequalities with Like Denominators	E16
Addition with Equivalence Sets	E17
Addition and Subtraction Practice with Equivalence Sets	E18
Strategies for Listing Sets	E19
Heuristics Review	E21
Inequalities with Unlike Denominators	E22
Check-up for Addition and Subtraction (unlike denominators)	E23

ADD & SUBTRACT UNLIKE DENOMINATORS

a simplified flow chart



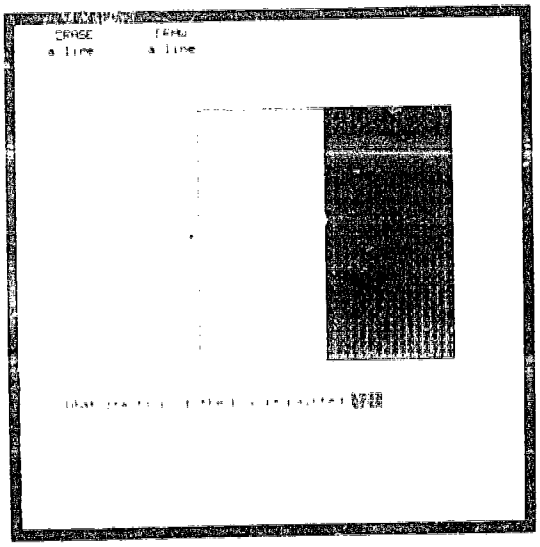
Heuristic I

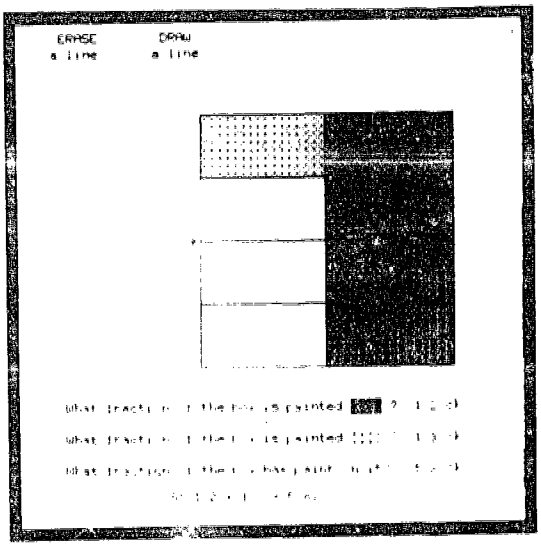


Purpose: 1) Introduce addition of fractions and mixed numbers with unlike denominators.
2) Introduce an approach to problem-solving.

Description: The student answers 2 problems with like denominators, then PLATO presents one with unlike denominators. Kids on the screen discuss what makes the new "hard" problem different from the similar "easy" ones. They decide that by renaming the fractions in the "hard" problem, they can find an "easy" problem that has the same answer as the "hard" one. The student fills in the numbers as the kids on the screen step through the process. This is a brief introduction and overview. Subesequent lessons develop these ideas more fully.

Paint Addition: PLATO Paints (touch)

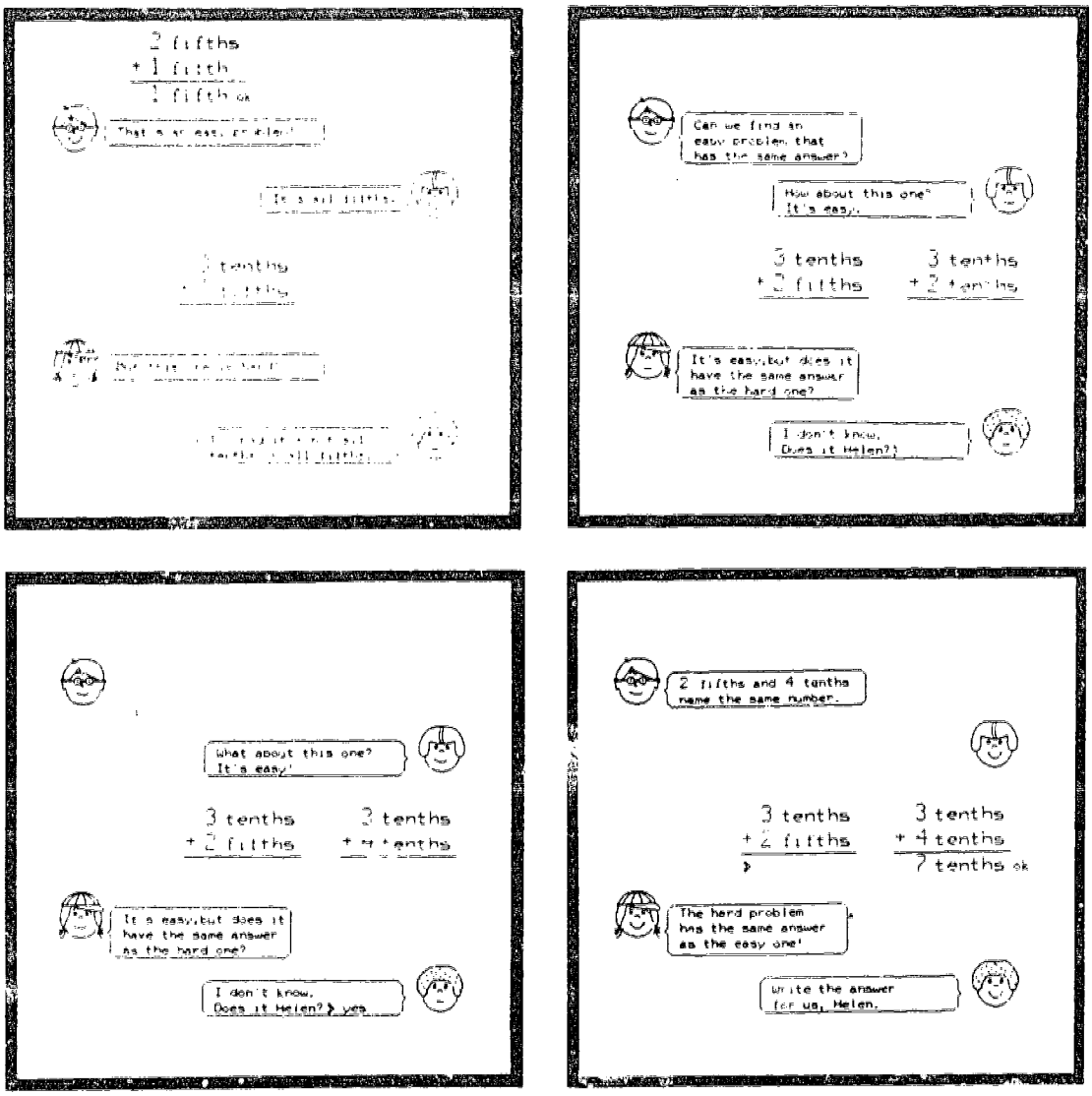




Purpose: Begin addition of fractions with unlike denominators, using a familiar model.

Description: PLATO uses 2 different kinds of paint to paint 2 fractions of the same box. The student tells what fraction of the box is painted with each kind of paint, then what fraction of the box is painted in all. The student can draw and erase lines on the box. This lesson relies on the student's experience cutting a box into sufficiently small equal pieces to tell how much is painted. Difficulty adjusts to the student's performance.

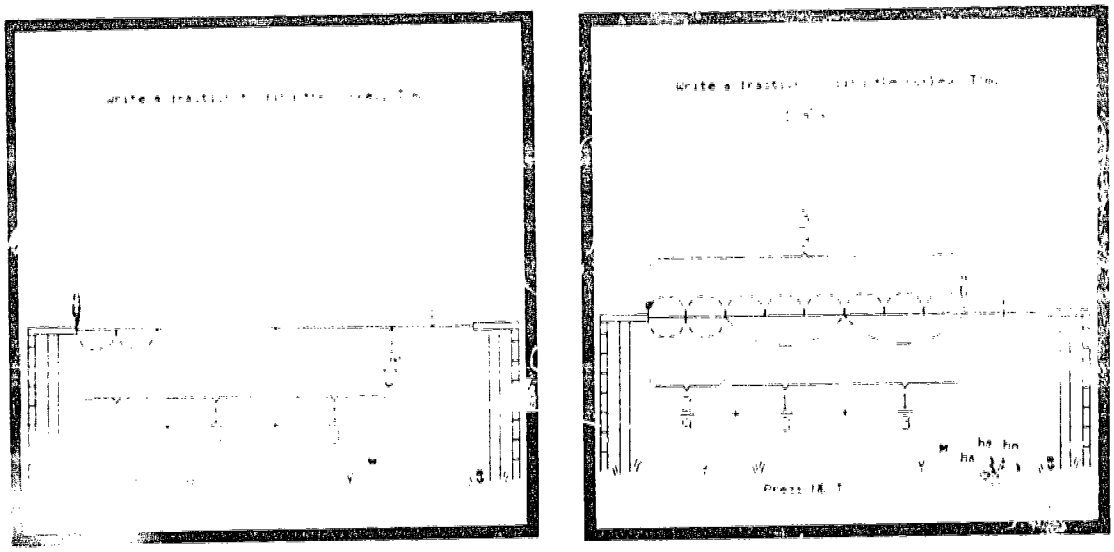
Heuristics II



Purpose: Give the student a more detailed look at the problem-solving approach to addition of fractions with unlike denominators.

Description: The student answers the problem 2 fifths + 1 fifth. PLATO presents a new problem, 3 tenths + 2 fifths. Kids on the screen discuss what makes the first problem "easy" and the second one "hard." They look for an "easy" problem that has the same answer as the "hard" one. They find the "easy" problem they need, but in the process they suggest 1 or 2 "easy" problems that won't do. Thus they emphasize that the appropriate "easy" problem must be another name for the "hard" one. The student answers questions and fills in numbers to help the kids on the screen.

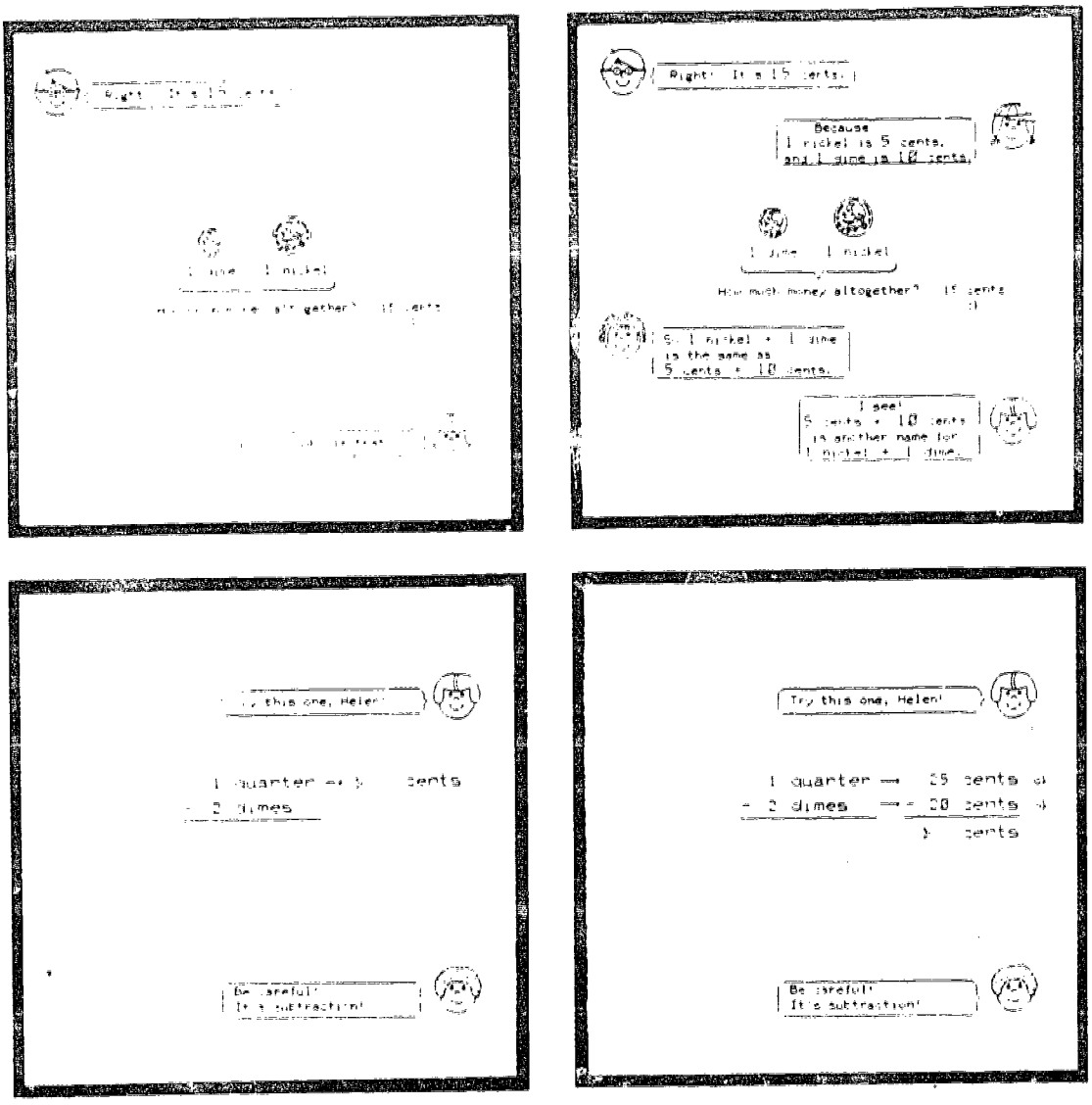
High Wire



Purpose: Show addition of fractions on the number line with visual feedback about common denominators.

Description: A monkey swings along under a high wire (number line) making an addition problem (e.g., 2/3 + 1/2 is 2 swings of length 1/3 and 1 swing of length 1/2). The student writes a fraction to hop a feather along above the number line. If the student is right, the feather lands where it can tickle the monkey; and the monkey falls off the line laughing. This is a number line illustration of common denominator; the monkey's swings and feather's hops match up to show equivalences. Difficulty adjusts to the student's performance.

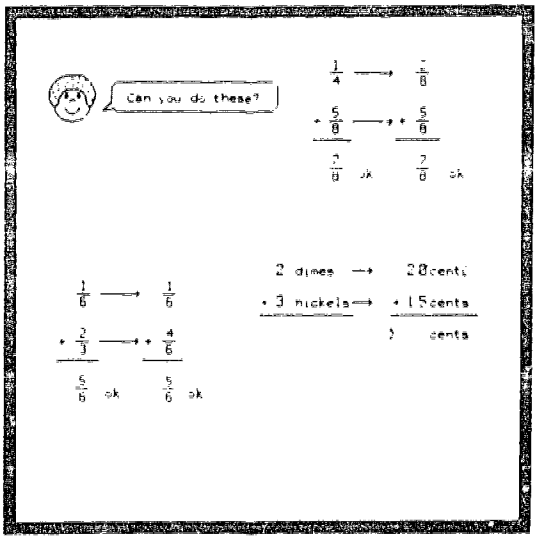
Heuristics III



Purpose: 1) Show addition of coins as a familiar process which is similar to addition of fractions with unlike denominators.
2) Reinforce the problem-solving approach to addition of fractions with unlike denominators.

Description: PLATO shows 1 nickel and 1 dime. The student tells how much money that is all together. Kids on the screen comment that "5 cents + 10 cents" is another name for "1 nickel + 1 dime." The student then does two or three coin addition problems by renaming each one in cents to make an "easy" problem that has the same answer as the "hard" one.

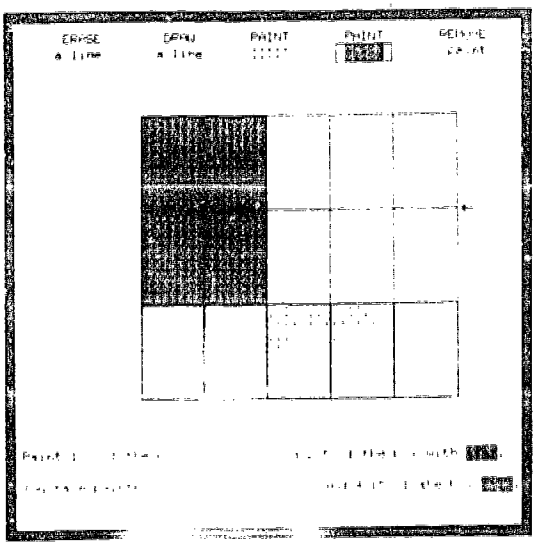
Heuristics III Practice

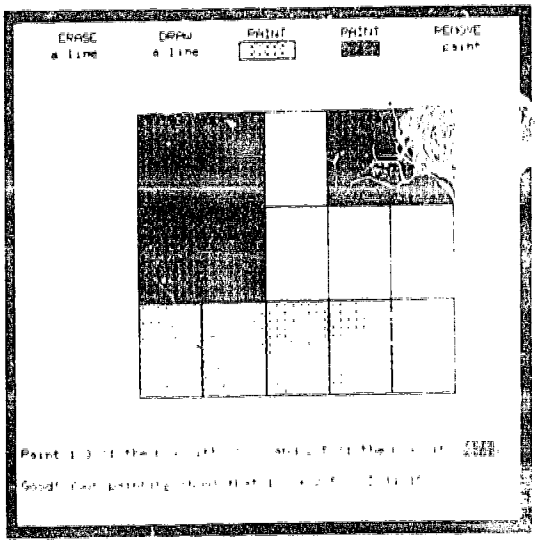


Purpose: Experience using the problem-solving approach to addition and subtraction of fractions with unlike denominators.

Description: The student solves addition and subtraction problems involving coins and fractions with unlike denominators. Difficulty adjusts to the student's performance. The lowest difficulty level is mostly coins problems. Higher levels have fewer coins problems, more fractions problems, and fractions written in fraction notation ("2/3" instead of "2 thirds"). The highest level has no coins problems and encourages the student to find a common denominator before PLATO suggests one. For fractions problems on all levels, one of the denominators in the problem can be used as the common denominator (i.e., one denominator always divides the other evenly).

Paint Addition: Student Paints (touch)

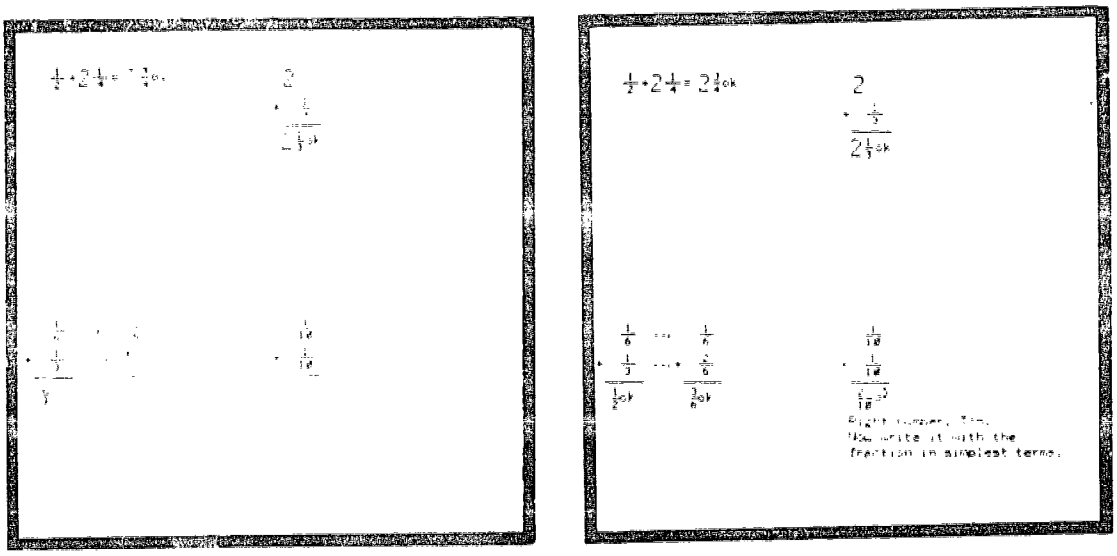




Purpose: Provide further experience with addition of fractions with unlike denominators, using a familiar model.

Description: The student is asked to paint a fraction of a box with one kind of paint, then paint another fraction of the box with a different kind of paint. The student then tells what fraction of the box has paint on it. Difficulty adjusts to the student's performance.

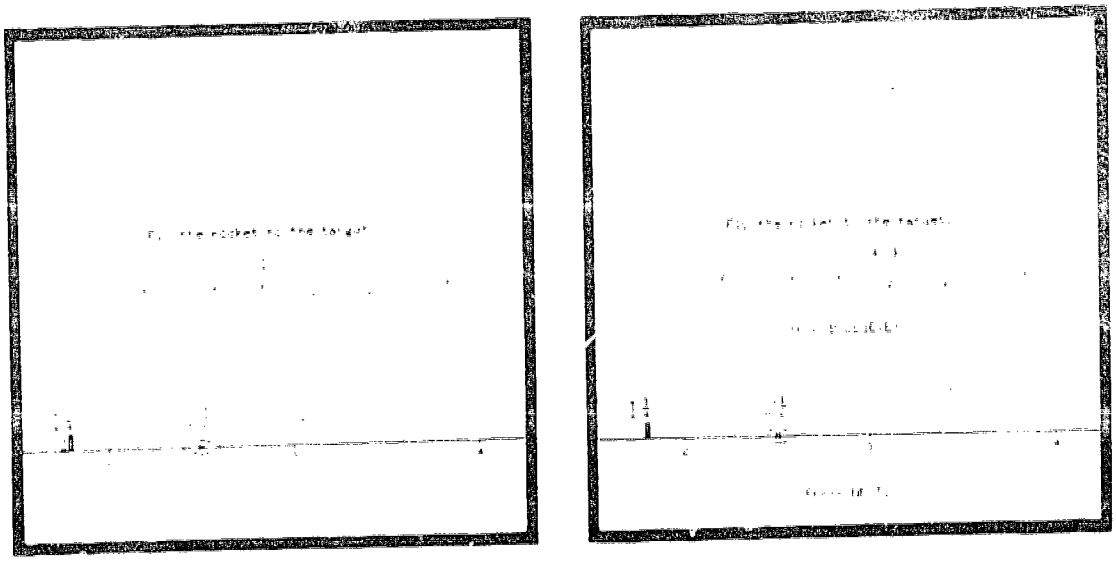
Addition and Subtraction Drill



Purpose: Practice adding and subtracting fractions and mixed numbers with unlike denominators. This is for students who have a basic understanding but need practice.

Description: There are 4 problems per page and no pictures or models. There is help for students who have trouble re-naming with a common denominator, "borrowing," or putting answers into standard form. There are special messages for common errors like adding instead of subtracting. The student will see one page at a time. Difficulty adjusts to the student's performance.

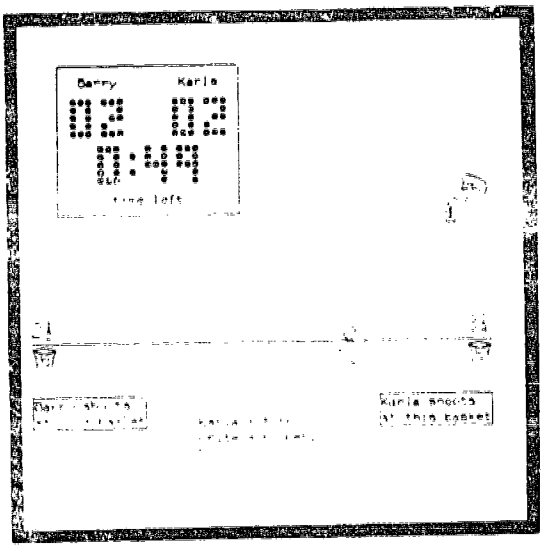
Target Practice

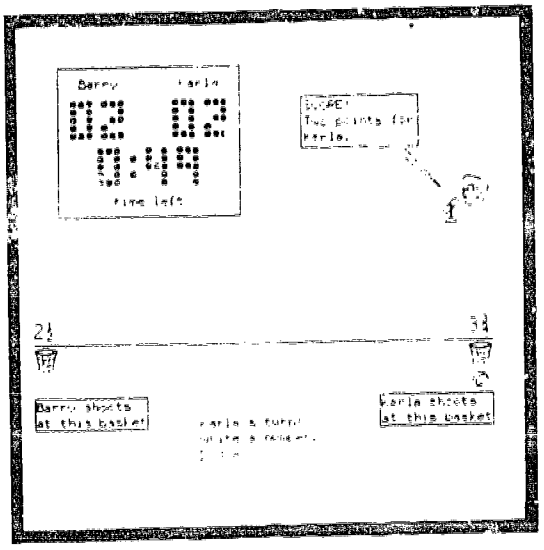


Purpose: Addition practice which may encourage checking the answer by estimation.

Description: The student fires a remote control rocket at a target. Both the rocket and the target are on a number line at known positions. An easy estimate will hit the target for a "SCORE," but only the exact answer gets a "BULLSEYE." Difficulty adjusts to the student's performance. This lesson appeared with like denominators in the previous chapter.

Fractions Basketball

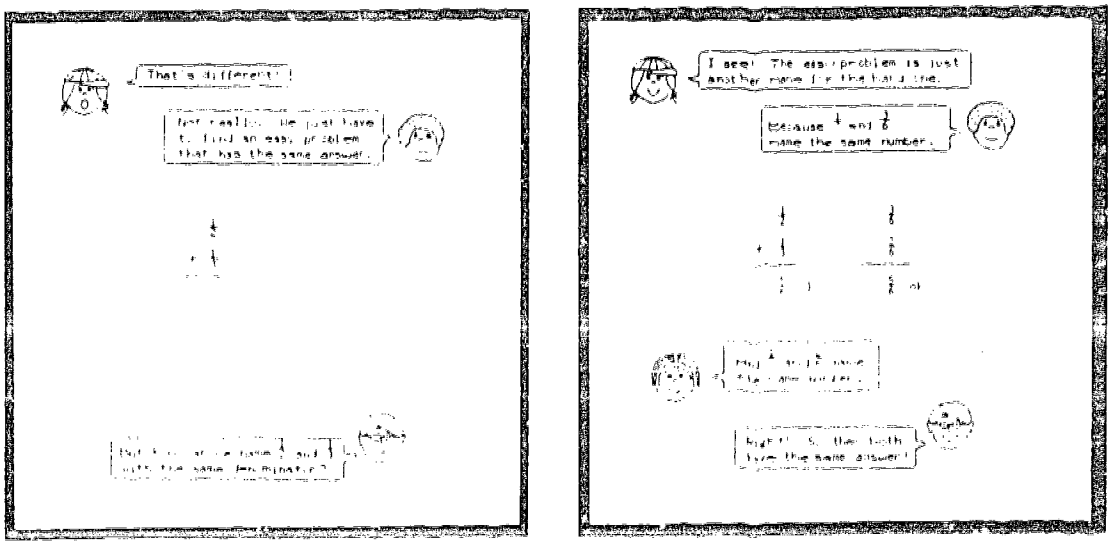




Purpose: Subtraction practice on the number line.

Description: This is a game for one or two students. Given ball and basket positions on the number line, the student bounces the ball some fraction or mixed number (e.g., 2 7/8 makes the ball do two bounces of length 1 then seven bounces of length 1/8) to make a basket. Difficulty adjusts to the student's performance. This lesson appeared with like denominators in the previous chapter.

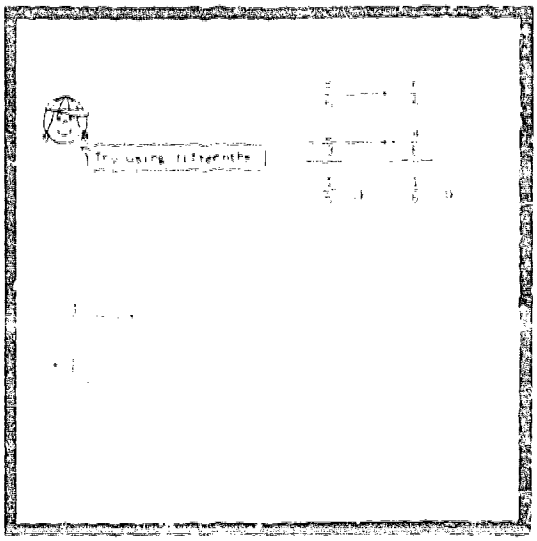
Heuristics IV



Purpose: 1) Review the problem-solving strategy for addition and subtraction of fractions with unlike denominators.
2) Apply this approach to problems in which neither of the given denominators can be used for the common denominators (i.e., neither denominator divides the other evenly).

Description: The student starts with a page from "Heuristics III Practice." If the student performs poorly on this page, he or she stays in the practice lesson. If the student does reasonably well, PLATO presents the problem, 1/2 + 1/3. Kids on the screen quickly review the problem-solving strategy and suggest renaming the problem in sixths. The student works the problem, then the kids on the screen review the idea that the "easy" problem is just another name for the "hard" one.

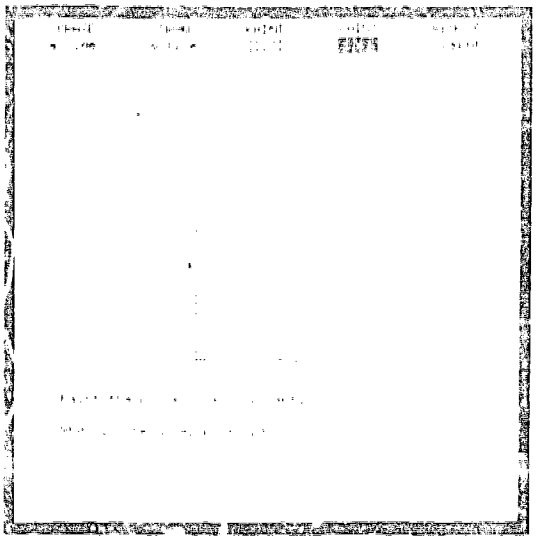
Heuristics IV Practice

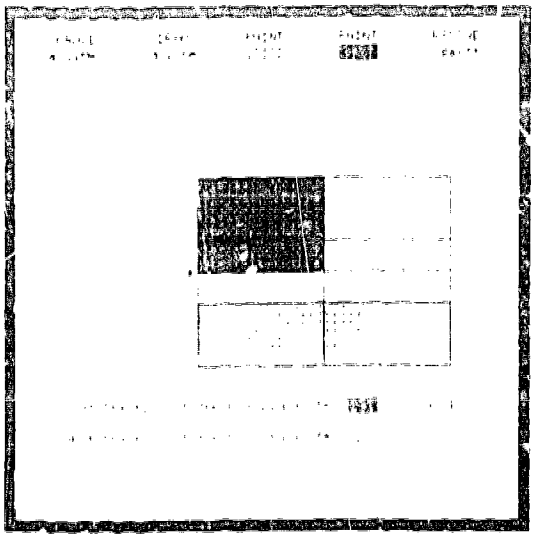


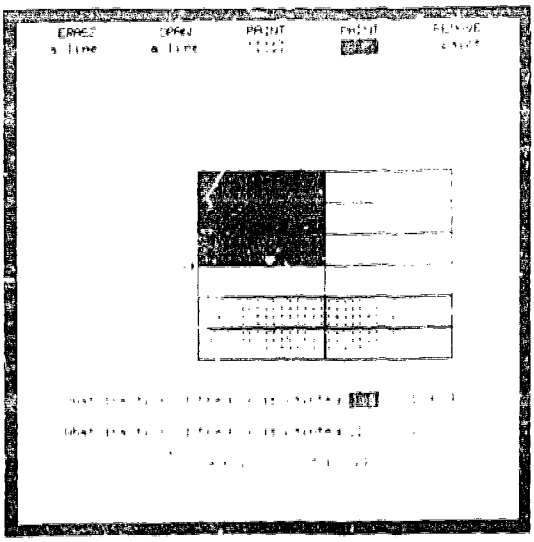
Purpose: Practice adding and subtracting fractions with unlike denominators. PLATO suggests a common denominator.

Description: The student solves addition and subtraction problems with unlike denominators. All problems use fraction notation ("2/3" instead of "2 thirds"). Difficulty adjusts to the student's performance. Higher levels have more problems in which neither of the denominators can be used as the common denominator (i.e., neither denominator divides the other evenly). The highest level encourages the student to find a common denominator before PLATO suggests one.

Paint Addition: Choice (touch)







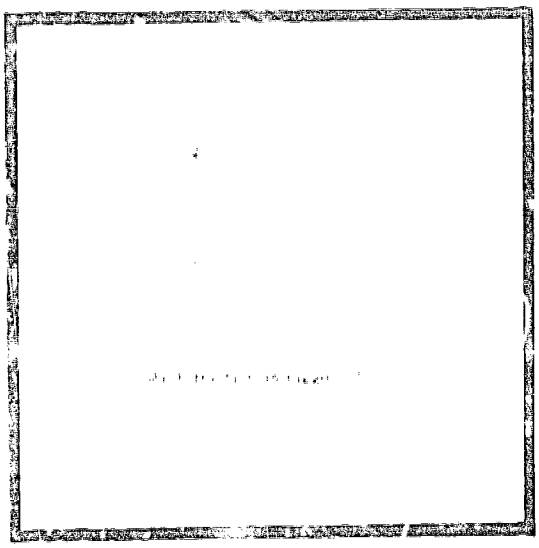
Purpose: Encourage the student to make up his or her own fraction addition problems and try painting problems in different ways.

Description: The student has a choice of three activities:

1) Write an addition number sentence for paintings done by PLATO (as in "Paint Addition: PLATO Paints")
2) Paint problems that PLATO chooses (as in "Paint Addition: Student Paints")
3) Paint a box, then write an addition number sentence to describe the painting.

The student may save his or her painting in a library for others to see.

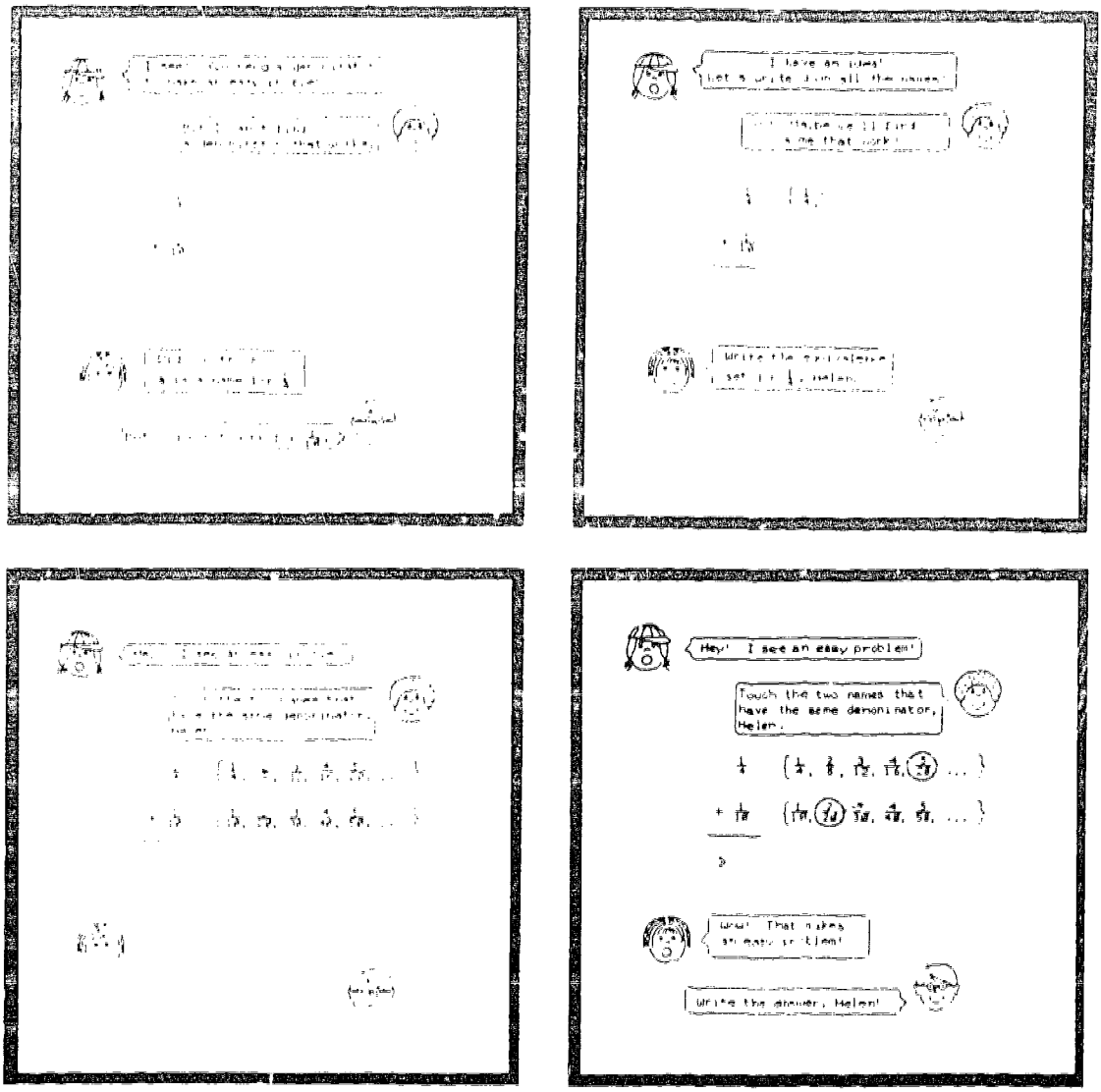
Inequalities With Like Denominators



Purpose: Provide readiness for comparing fractions with unlike denominators.

Description: The student tells which of 2 fractions with like denominators is bigger. If the student is not correct on the first try, PLATO displays the appropriate fractions of pizza for comparison.

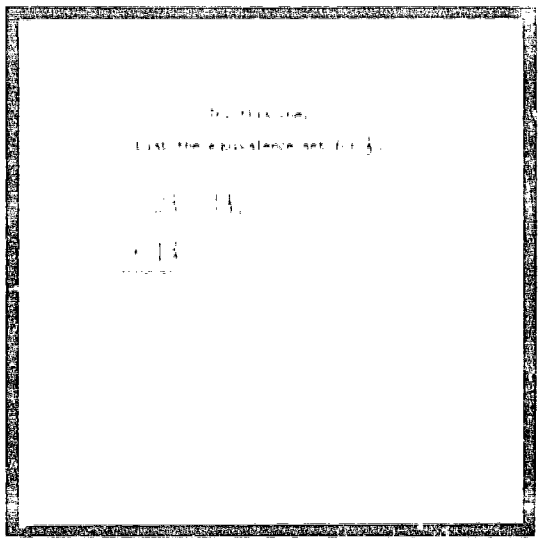
Addition With Equivalence Sets (touch)

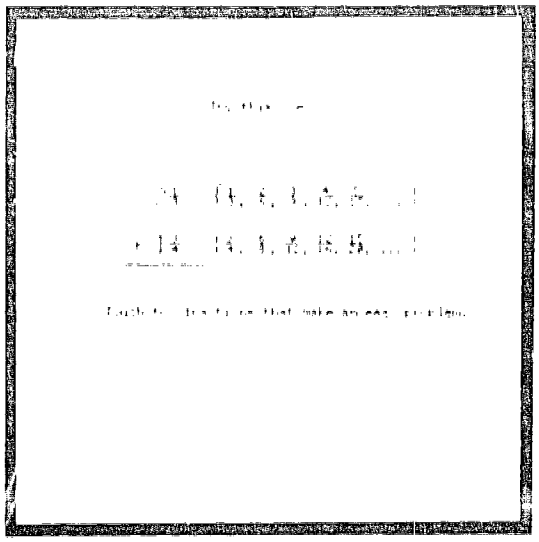


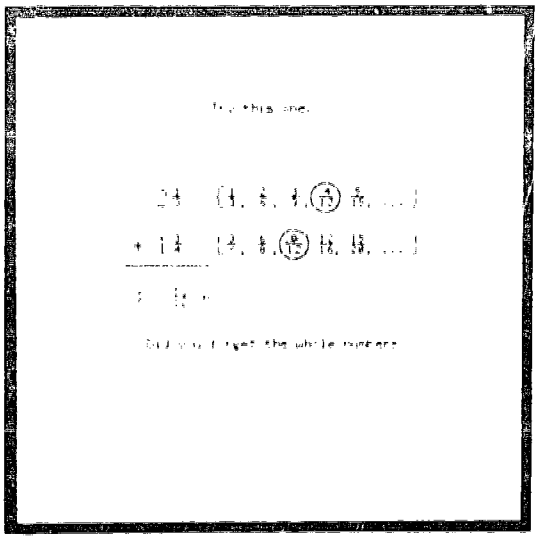
Purpose: Suggest a systematic method for finding a common denominator.

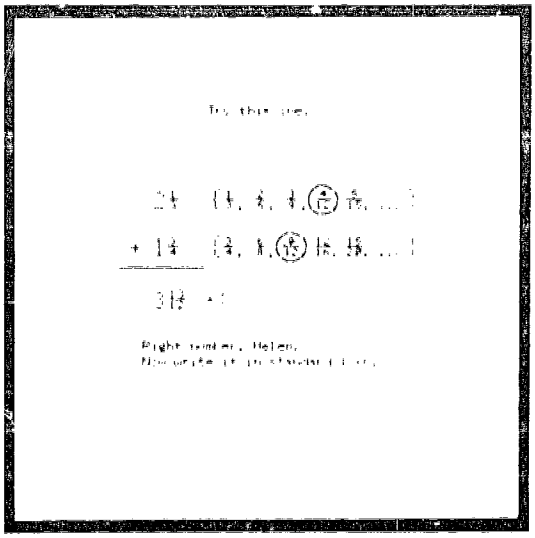
Description: Given the problem 1/4 + 1/10, the kids on the screen try to find an appropriate common denominator. After trying a few denominators that don't work, they decide to list the equivalence sets for 1/4 and 1/10. The student lists the sets, finds 2 names with a common denominator, and answers the problem.

Addition and Subtraction Practice With Equivalence Sets (touch)





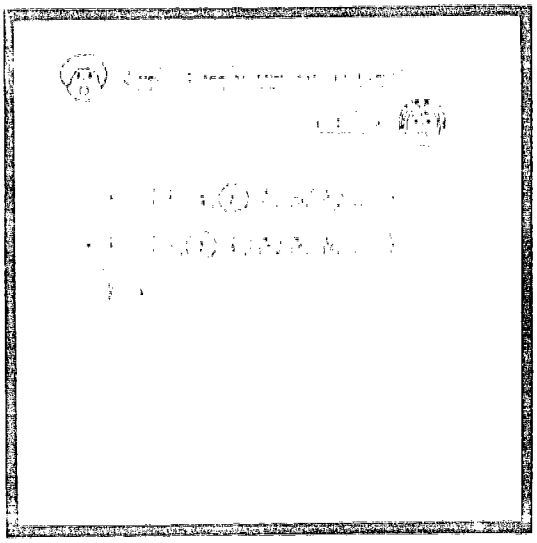


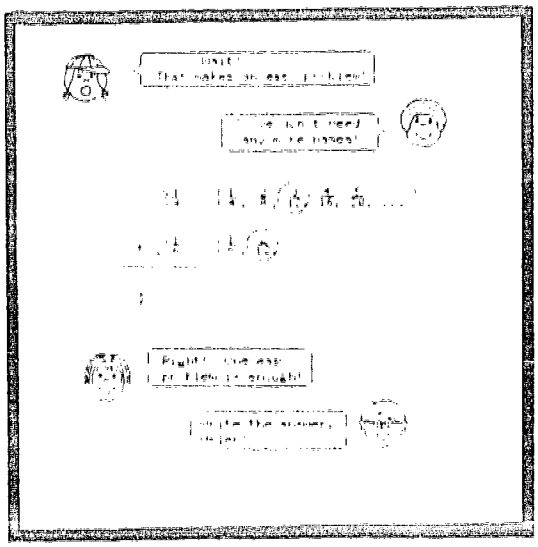


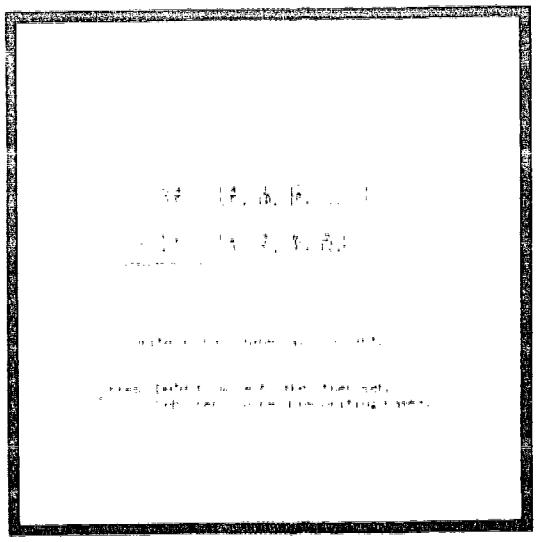
Purpose: Provide experience adding and subtracting fractions and mixed numbers, using a systematic method to find a common denominator.

Description: The student solves addition and subtraction problems by listing 5 elements in the equivalence set for each fraction, then picking out 2 names with the same denominator. Difficulty adjusts to the student's performance. This lesson involves no strategies for deciding how many names should be listed for each fraction. A student who does well can finish the lesson in as few as 3 problems and be ready for the lesson, "Strategies for Listing Sets."

Strategies for Listing Sets (touch)







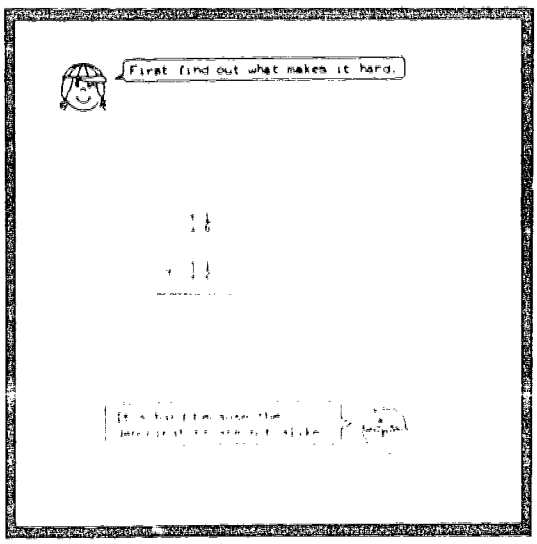
Purpose:
1) Provide practice adding and subtracting fractions and mixed numbers, using equivalence sets to find a common denominator.
2) Point out that more common denominators can be found by listing more names.
3) Develop strategies for deciding how many names to list to find a common denominator.
4) Enable the student to solve addition and subtraction problems without relying on guidance from PLATO.

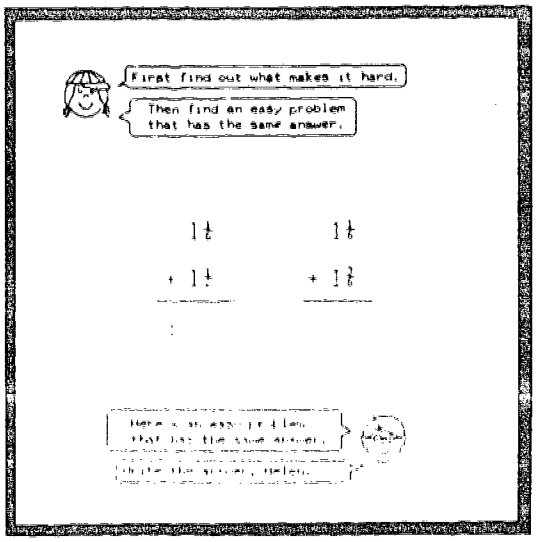
Description: There are 2 introductory problems followed by practice. When the student finishes the first problem, kids on the screen point out that there are 2 common denominators visible in the set listings. They see that either one can be used to find the correct answer to the problem.

They then list more names to find another common denominator and speculate about how many more common denominators could be found.

The student starts to solve the second problem by listing the equivalence sets. As soon as a common denominator shows up in the second set, the kids on the screen point out that there is no need to list more names. In the practice problems that follow, the student is free to list as many names for each fraction as he or she wants. Before writing the answer, the student must touch 2 names that have like denominators. For students who have trouble listing the sets, PLATO sometimes offers to calculate the equivalent fractions so that the student can view the overall addition process without getting bogged down in the individual calculations. Difficulty adjusts to the student's performance. For higher levels of difficulty, the student is not required to list sets or touch 2 names that have like denominators. The student can do whatever set-listing he or she needs.

Heuristics Review

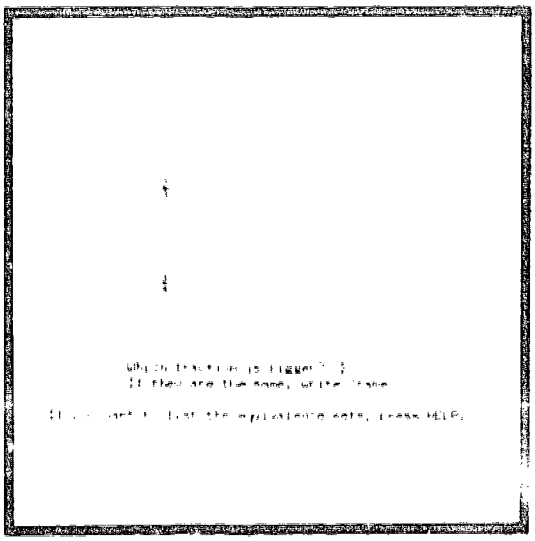


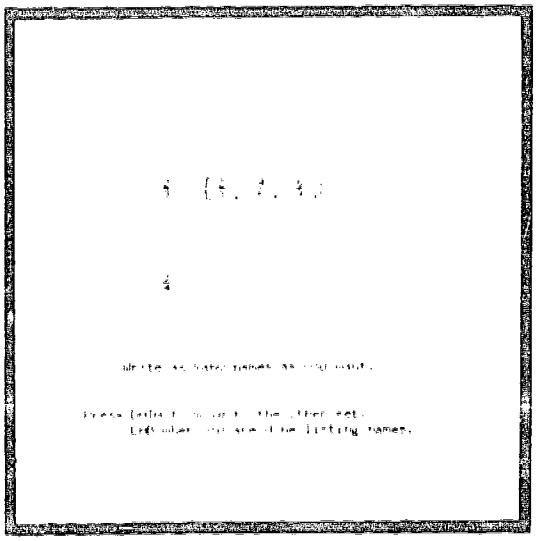


Purpose: Summarize and review the strategy for adding and subtracting fractions with unlike denominators.

Description: There are 6 problems involving addition and subtraction of fractions and mixed numbers with unlike denominators. For each problem, the kids on the screen review the problem-solving strategy and find an "easy" problem that has the same answer as the "hard" one. The student answers the problems. In a given session the student will see 1 or 2 of the problems.

Inequalities With Unlike Denominators (touch)

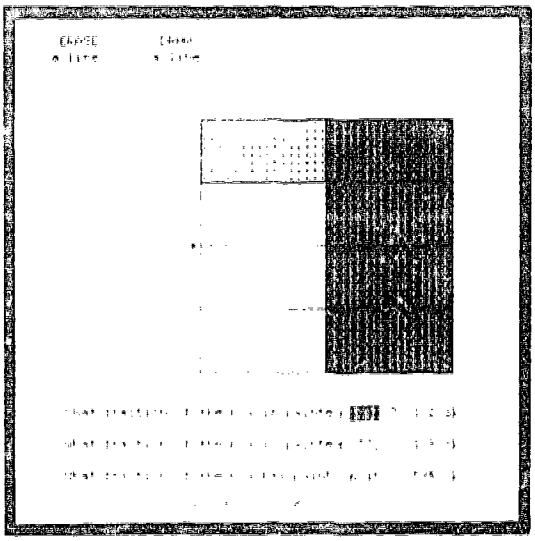


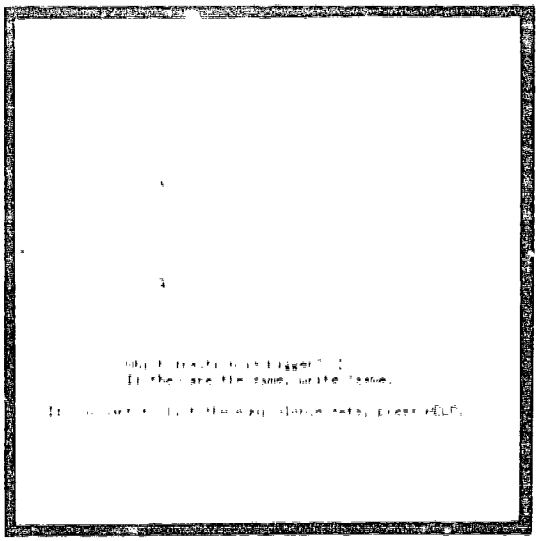


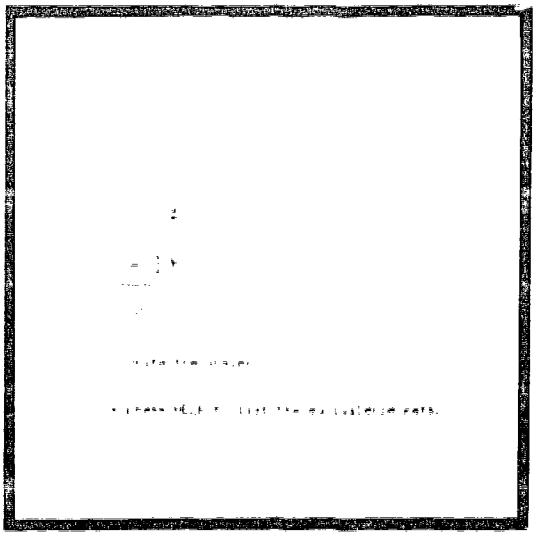
Purpose: Develop a systematic method for comparing the size of 2 fractions with unlike denominators.

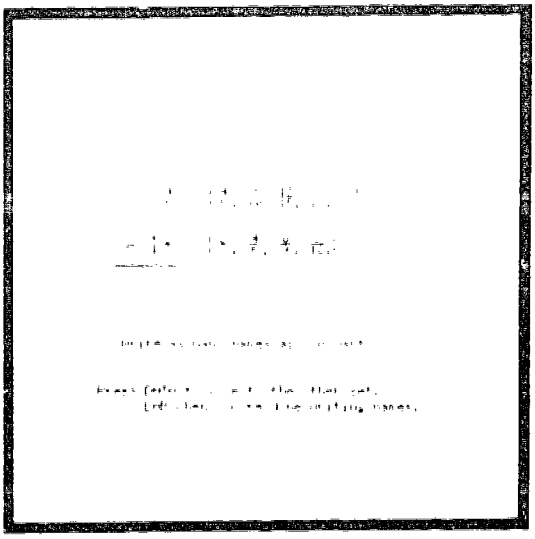
Description: Given 2 fractions with unlike denominators, the student tells which is bigger or that they're equivalent. Before responding, the student can choose to press HELP for a chance to list some names for either or both of the fractions. If the student's response is incorrect, he or she must list both equivalence sets and touch 2 names that have a common denominator before trying the question again.

Check-up for Addition and Subtraction (unlike denominators) (touch)









Purpose: 1) Review addition and subtraction of fractions and mixed numbers with unlike denominators.
2) See if the student is ready for material which depends on these concepts.

Description: The check-up consists of selected problems from each of three lessons: "Paint addition: PLATO Paints," "Strategies for Listing Sets," and "Inequalities with Unlike Denominators."