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ABSTRACT

The purpose of this paper is to propose a conception of the learning process and to suggest its mathematical form. In this framework, the characteristics of the schools children attend are emphasized. The objective is to suggest a framework for the evaluation of research on school effects and a point of departure for new research on the impact of schools' instructional resources and the environments they provide on the amount that students will learn. Some findings of an investigation using this framework are presented, but the inadequacy of existing data available for the analysis make these findings suggestive. The main emphasis is on the conceptual issues underlying the model and the methodological problems that the proposed model presents. The objective is to propose a model that can ascertain school effects, if there are any. It should focus on learning that takes place as a result of students' exposure to school activities, i.e., exposure to classrooms, teachers, and textbooks. Further, it should focus on learning that is relevant for a student's further educational and, perhaps, occupational career, for the question that ultimately generates the interest in school effects is whether it makes a difference to a child's future attainment which school he or she attends. (Author/JM)

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A RECONCEPTUALIZATION OF SCHOOL EFFECTS

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ABSTRACT

This paper presents a conceptual framework for the analysis of school effects on learning. A differential equation model for change in academic achievement is derived from the conceptual framework. Numerous implications for the evaluation of existing research is derived and preliminary analysis using the proposed model is presented.

A RECONCEPTUALIZATION OF SCHOOL EFFECTS

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Introduction

The freedom of parents to choose schools for their children has become an important political issue. This would not be so if it were believed that the school a child attends made little difference to his or her education. In fact, almost everyone concerned with schooling and educational policy seems to believe that school differences are important for the educational achievement of children. However, a considerable body of research accumulated over the last decade has failed to establish strong school effects.[1] Schools seem to make little difference in educational outcomes, when a child's ability and family background are adequately controlled. Particularly research on the effect of schools' instructional resources (facilities, curriculum and staff characteristics) has produced results of this nature, while research on school environmental variables -- measured by student body characteristics -- has been only slightly more successful.

Much of the research on school effects has been among the methodologically most sophisticated research in sociology, and it has been carried out on large and seemingly adequate samples. The main thrust of the research has been to show that apparent relations between school characteristics and the academic performances of children are in fact spurious when reasonable assumptions about the causal ordering of variables are made and adequate statistical methods are used. Of significance has been the use of regression methods in conjunction with linear models to test the relationships among variables. These are powerful methods that have gained general acceptance as a very efficient way of handling quantitative data.

If the research findings are valid, the widespread belief in the importance of schools for differences in educational outcomes is erroneous. An acceptance of these findings would have important consequences for educational policy. Existing inequalities in educational resources would be a much less salient issue, particularly if the injustice of these inequalities is argued to stem from the unequal educational outcomes they produce. These policy implications are somewhat depressing: if only the family background and the genetic endowment of children appear relevant for educational achievement, little can be done to remove inequalities in educational opportunities.

It is, however, not likely that a general acceptance of the research findings on school effects will come about very readily. A belief in the importance of schools is widespread and sustained by the interests of groups -- teachers and educational administrators -- who benefit from attempts at improving the instructional resources of schools. Characteristics of the pertinent research are also important, in our opinion. While the research on school effects is methodologically sophisticated and difficult to fault on technical grounds, it is not a line of research that has been much concerned with the conceptual issues involved in establishing school effects or the absence of them. The emphasis has been on establishing relations among variables, not on specifying the mechanisms that would produce such effects. Thus, the apparent lack of school effects is primarily an empirical finding for which a theoretical rationale is lacking. This property of school effects research hinders a widespread acceptance of the empirical findings, as the reasons for these findings remain unclear. Therefore, the controversy is likely to continue, resulting in new research and the reevaluation of existing findings.

The validity of an empirical finding regarding the relationships among two or more variables is dependent on the validity of the specification of the functional form of the relationship among the variables. Invariably, research on school effects has assumed a linear relationship. If this functional form is a misspecification, the validity of the findings is in doubt, regardless of how sophisticated the estimation procedures have been. The linear form has been chosen, it seems, primarily because of its convenience. This is not a peculiarity of research on school effects, but a common feature of sociological research. Still, the findings of this research would be much more convincing if the functional form could be argued on theoretical grounds to be the correct one. To make such an argument, it is necessary to specify the mechanisms that would produce school effects and give these mechanisms a mathematical form. This means specifying the mechanisms that produce learning as a function of the various relevant characteristics of students and their environments, including the teaching students are exposed to in schools. School effects, as well as any other effect on the academic achievements of children, are ultimately a question of influencing the learning process of children.[2]

The purpose of this paper is to specify the causal mechanisms that produce learning, i.e., to propose a conception of the learning process and to suggest its mathematical form. In this framework, the characteristics of the schools children attend will be emphasized. The objective is not necessarily to prove existing research on school effects wrong, but to suggest a framework for the evaluation of this research and a point of departure for new research on the impact of schools' instructional resources and the environments they provide on the amount that students will learn. Some findings of an investigation using this framework will be presented, but the inadequacy of existing data available for the analysis make these findings suggestive rather than conclusive. The main emphasis is on the conceptual issues underlying the model and the methodological problems the proposed model presents.

A Model for the Process of Learning

Learning is a process in time: the amount of learning achieved can be registered as change over some time interval in an individual's knowledge, skills or values. This section will propose a conception of this process and a mathematical model designed to implement that conception. There are two tasks involved -- one is to identify the relevant variables; the other is to formulate the mechanisms that link these variables. It appears useful to be quite elementary and to propose a simplified model that clearly expresses the basic properties desired. Further modifications are discussed in a subsequent section.

The objective is to propose a model that can ascertain school effects, if there are any. It should focus on learning that takes place as a result of students' exposure to school activities, i.e., exposure to classrooms, teachers and textbooks. Further, it should focus on learning that is relevant for a student's further educational and (perhaps) occupational career, for the question that ultimately generates the interest in school effects is whether it makes a difference to a child's future attainment which school he or she attends. These considerations imply that the focus should be on the learning of intellectual knowledge and skills that schools try to teach and test for in academic achievement tests, examinations and the like.

Schools may teach youngsters norms and values that are considered important to acquire by the community and may have a bearing on future attainments. However, these socializing activities of the school will not be considered here, as they also are usually excluded from the existing research on school effects.

Teaching is a communication process where knowledge and skills are transmitted to students who to a varying degree acquire the material taught. The variation among students in their learning is dependent on attributes of these students. In existing research, a host of characteristics of students have been assumed relevant for learning. Foremost of course is I.Q., but other cognitive attributes such as creativity and curiosity have also been suggested and assigned varying degrees of importance. A similar list of personality attributes -- such as anxiety, need for achievement, level of aspiration and attitudes toward learning -- has been suggested. These latter variables, in addition to ability variables, will be relevant for the amount a student learns. It appears that the introduction of these variables is primarily motivated by the need to account for variation in the degree of effort students will exhibit. More simply, we may say that variation in student learning is influenced by two broad sets of individual variables -- those determining ability and those determining effort.

Almost all research in educational psychology and sociology has focused upon individual determinants of learning and on the relevance of certain aspects of teacher behavior and teaching methods that would determine how effective teaching is. Implicitly it is assumed, it appears, that the amount of material communicated in the teaching process is a trivial variable for the amount of learning that takes place. But, however trivial this quantity is, it is nevertheless of crucial importance. No child will learn material he/she has not been exposed to regardless of how much ability and effort is displayed. Learning only takes place if there are opportunities for learning present. Variation in such opportunities will produce variation in learning independent of the abilities and efforts of children. Teacher behavior and teaching methods may be seen as ingredients in determining such opportunities. Other relevant features are curriculum organization and the amount of time spent in teaching as opposed to time spent in recreation and on keeping discipline.

We have identified three basic concepts -- ability, effort and opportunities for learning. These variables may in turn be linked to other variables like family background, characteristics of peer groups and school and teacher characteristics. First, however, we need to specify the interrelationship among these three concepts in producing learning.

Specification of the Model

We expect that ability, effort and opportunity produce variations in learning. One could carry out research where some measure of academic achievement is used as the dependent variable in an analysis where measures of the three main concepts are used as independent variables. The amount of variance explained by the three sets of variables would be focused upon. In fact, this is what most existing research on school effects has done, although a concept similar to "opportunities for learning" has not been employed.

However, an additive model for the dependence of learning on ability, effort and opportunities is not conceptually satisfactory. Such a model implies that the three sets of variables can compensate for each other, i.e., few opportunities for learning can be compensated for by high levels of ability and/or effort. But this seems unlikely. No one can learn material that he or she has not been exposed to. Hence, if opportunities for learning are nonexistent, no learning can take place. In general, the level of opportunities will determine an upper limit for how much learning can take place. Perhaps no one will reach that limit -- how much material a student acquires will depend on his/her ability and effort.

A more reasonable specification of the interrelationship among the three concepts would be some multiplicative form or a mixed additive and multiplicative form. The multiplicative formulation would capture the desired conception of the role of opportunities for learning. However, there are numerous alternative ways in which to capture this notion In this situation, one may proceed by trying out alternative formulations and choose the one that seems to fit the data best. Measurement problems and data limitations are likely to cause an indeterminate outcome of such a search. Furthermore, a form that happens to be well fitting need not necessarily be the most theoretically meaningful, and/or this form can be given different interpretations.[3] Instead, the most productive approach seems to be one where a specification of the fundamental mechanisms of the process is used to derive the functional form. This approach necessitates a clear specification of the fundamental assumptions so that theoretical adequacy as well as empirical adequacy can be a guideline.

To specify the fundamental mechanisms underlying change in academic achievement, that is learning, amounts to specifying a differential equation for change in achievement. Solving this differential equation will then give a functional form for the interrelationship among variables that can be used in empirical analysis.

Let us assume that achievement is measured as a continuous variable, and denote the level of achievement displayed after exposure to a learning process of length t by $y(t)$. The amount of learning in a small interval of time, dt, can now be represented by the change in $y(t)$, or $dy(t)$, that occurs in dt. The quantity $dy(t)/dt$ is the rate of learning to be explained by the opportunities for learning presented to the student and by his/her ability and effort.

Opportunities for learning are determined by another change variable. Knowledge and skills are communicated to the student in the instructional process. In some period t, a certain amount $v(t)$ has been communicated. It can be assumed that there will be a certain total amount v^* communicated in some subject (say, algebra, history, etc.), where this amount represents what teachers and schools deem an adequate coverage of the subject area.

The quantity $v^* - v(t)$ represents the amount of material not yet communicated by time t. The amount presented in a small time interval, dt, shall be denoted $dv(t)$. A student will learn a certain fraction of this material -- how much depends on his/her ability and effort. The amount learned in dt is $dy(t)$. The influence of ability and effort may be formulated as $dy(t)$ being proportional to $dv(t)$ with a constant of proportionality determined by ability and effort. Denote this constant s. Hence,

$$dy(t) = sdv(t) \tag{1}$$

This expression states that the amount learned is linearly dependent on the amount of new material presented. One may directly try to measure $v(t)$ and in this way test this simple expression. However, it seems difficult to operationalize $v(t)$, and it will usually only be possible to obtain information on $y(t)$ at different points in time. It is then necessary to specify the dependency of $y(t)$ on time. This means that $v(t)$'s dependency on time should be specified.

Teaching consists of both the repetition of old material and the presentation of new material. When a new subject is presented, most of the material will be new; toward the end of the period allotted to the subject, most material may be assumed to be repetition. If this is correct, then the amount of new material presented should depend on the quantity $v^* - v(t)$. This dependency can be formulated in the differential equation;

$$\frac{dv(t)}{dt} = b'[v^* - v(t)] \qquad b' > 0 . \tag{2}$$

The quantity b' is a parameter that determines how much will be taught per unit time. As such, it expresses the amount of effort displayed in the teaching process. It is reasonable to assume that b' is related to the total amount of material that has to be covered within the period allotted to the subject matter, that is b' should be determined by v^*. Without loss of generality, we may set $b' = \frac{1}{v^*}$.[4] This gives as a solution to (2):

$$v(t) = v^*(1 - e^{-\frac{1}{v^*}t})$$

$$= \frac{1}{b}(1 - e^{-bt}) , \qquad (3)$$

assuming $v(t) = 0$ for $t = 0$. This is the desired expression for the dependency of $v(t)$ on time.

Equation (1) has the solution $y(t) = sv(t)$, assuming $y(t) = 0$ at the start of the learning process. Inserting equation (3) in (1) and defining $b = -b'$ gives:

$$y(t) = \frac{s}{b}(e^{bt} - 1) \qquad b < 0 . \qquad (4)$$

This expression tells us how achievement depends on time, on the opportunities for learning, and on the ability and effort of the student.

It is useful for what follows to differentiate (4) with respect to time. This gives,

$$\frac{dy(t)}{dt} = s + by(t) . \qquad (5)$$

Hence the process formulated implies that the rate of change i academic achievement or the rate of learning will be a linear function of the ability and effort of the student, s, and the level of achievement obtained by time, t. The level of achievement will have a negative impact on growth in achievement since $b < 0$. Therefore, the quantity b constrains learning, and it expresses a negative feedback brought about by the opportunities for learning. The negative feedback will be greater the larger the absolute magnitude of b, that is, the smaller v^*, since $b = -\frac{1}{v^*}$

While b is a characteristic of the teaching that takes place in a school, s is a quantity that depends on individual students. Unless all students are identical, there will be variation in s that will have to be taken into account. More importantly, it is of major interest to specify how s depends on various characteristics of the students and their backgrounds. The simplest formulation of the dependency of s on other variables -- and here we follow the tradition of most research -- is to assume linear dependency,

$$s = a_0 + \sum_i a_i x_i \,, \tag{6}$$

where the x_i variables are individual characteristics such as the student's I.Q., need for achievement, family background, etc. The linear formulation assumes that these characteristics can compensate for each other. This seems more reasonable than to assume a linear dependency of achievement on school characteristics, since this amounts to assuming that learning can take place even in schools where no opportunities for learning exist.

Inserting (6) for s in (5) and solving the differential equation will give an expression similar to (4). This expression assumes that t is measured from the start of the teaching process. In empirical applications, it is more convenient to obtain an expression that relates achievement at two arbitrary points in time. In the sequel, we will therefore define t as $t = t_2 - t_1$, where t_2 and t_1 are two points in time, and further assume $y(0) \neq 0$. This gives as the solution,

$$y(t) = \frac{a_0}{b}(e^{bt} - 1) + e^{bt}y(0) + \frac{a_1}{b}(e^{bt} - 1)x_1 + \frac{a_2}{b}(e^{bt} - 1)x_2 \ldots +$$

$$\frac{a_n}{b}(e^{bt} - 1)x_n \quad (7)$$

Equation (7) may be used to obtain estimates of the parameters using the formulation,

$$y(t) = a_0^* + b^*y(0) + a_1^*x_1 + a_2^*x_2 \ldots a_n^*x_n \quad (8)$$

where $a_0^* = \frac{a_0}{b}(e^{bt} - 1)$ etc. Using least squares techniques to estimate a_0^*, b_1^*, a_1^*, . . . , a_n^*, the fundamental parameters may be obtained as (see Coleman, 1968),

$$b = \frac{\log b^*}{t}$$

$$a_0 = \frac{a_0^* \log b^*}{t(b^* - 1)}$$

$$a_1 = \frac{a_1^* \log b^*}{t(b^* - 1)}, \text{ etc.} \quad (9)$$

The model should be estimated separately for each school (or classroom) -- variations between schools in b then will provide the desired information on variations in opportunities for learning.

The model formulated in this section is a simple and, it seems, reasonable representation of the conception of the learning process proposed in the preceding section. In the next sections, the assumptions necessary for this formulation to hold will be described and some of its properties and implications will be outlined. The specification of the model presented here is clearly not the only possibility. In later sections of the paper, the model will be evaluated in relation to existing research and some preliminary results from an analysis will be presented.

Assumptions

Certain assumptions are necessary for the solution (7) to represent the relationship among opportunities for learning and the various measures of ability and effort. First, it is necessary to assume that the parameters are identical for all individuals and that they are constant over time. Second, it must be assumed that the independent variables, x_i, are constant over time. The assumption of no variation in parameters across individuals demands that all relevant variables measuring directly or indirectly a student's ability and effort are included in the list of x_i variables. Further, it must be the case that any member of a group of students for which the model is estimated has been exposed to the same set of opportunities for learning, that is, exposed to a process governed by the same value of b. This means that all members of a group should have been exposed to the same teaching process. Ideally then, analysis at the classroom level should be performed.

The last mentioned requirement implies that $y(t)$ should measure learning of material presented only in schools. Insofar as $y(t)$ measures achievement in material also available outside schools, variation in b's among students within a school (or classroom) will take place, and the model will not be empirically adequate. The most likely alternative learning agency to schools is the family, but its importance will vary with subject matter and perhaps also grade level. This problem introduces constraints on the choice of achievement test to be used as a measure of $y(t)$. Clearly, school effects cannot be found on learning that primarily takes place outside of the school.

A related problem stems from the fact that in investigations using the model presented here, students in different schools obviously should take the same test. Because there are variations among schools in curricula, there is a tendency in test construction to measure more general aptitudes rather than the learning of specific materials. However, such measures are not appropriate dependent variables for the model proposed here, as they confound actual learning with ability.

Variation in parameters over time would occur if the model was misspecified, i.e., if the mechanisms postulated above do not correctly represent the learning process. It would also occur if one or more of the independent variables change over time. The simplest situation is where one of the independent variables changes over time due to forces not related to the system specified by the model. In this instance, the change over time will have to be described by specifying the dependency of the variable on time. Procedures for doing so are described by Coleman (1968). A more complicated situation occurs if the change in the independent variable is endogenous to the system, i.e., if learning produces change in one of the independent variables. In this instance, a simultaneous differential equation model is needed so that the interdependency among variables can be modeled. Failure to use such models will introduce a bias in b -- the measure of opportunities for learning -- as we will show in a later section.

In the present context, the assumption of no endogenous change in independent variables implies that the ability and effort of students are constant over the period of observation; in particular, ability and effort are not influenced by learning. This seems a questionable assumption. Mos. will claim that ability may change as a result of learr.ing, and similarly it seems reasonable that effort should depend on past success, particularly if this success is rewarded with grades and encouragement from teachers.

The latter implications of the simple model (5), seem to call for revisions of the model. Such modifications will be discussed later in this paper. These complications may be avoided if observations are spaced closely enough for the assumption of no change in ability and effort to be reasonable, but such a solution may have other drawbacks.

Measurement error presents a serious problem in models that focus on change. What appears to be change may actually only be regression toward a mean due to measurement error. Such a phenomenon will bias the parameter b of equation (5) that is of such crucial importance for the argument presented here. Coleman (1968) has shown how it is possible to separate measurement error from true change in models such as the one proposed here. Observations at three or more points in time are needed. If this separation of change and error cannot be made, it is necessary to assume that errors are identically distributed for all subgroups among which comparisons of b 's are made.

All the various assumptions and requirements discussed will affect our ability to draw valid inferences from the variation among schools in opportunities for learning. Omitted variables measuring ability and effort will bias b due to the correlation among these variables and achievement and ability and effort. Variation in opportunities for learning within schools due to the operation of other learning agencies, will have similar results, as will the use of achievement measures that confound ability and learning. Finally, measurement error will also bias b , as just described.

Proper design of the research using the proposed framework will be essential to overcome the problems discussed here; in particular, the proper selection of measures of achievement and ability and effort should be emphasized. A fairly extneisve literature on lagged models of the type exemplified by equation (8) is available (see Grilliches, 1967, for a survey) for use in selecting proper estimation procedures.

These largely technical issues will not be discussed here, as they do not demand revisions of the basic model. In contrast, the presence of endogenous variables among the measures of ability and effort does demand a revision of the model. For the present purposes, we will accept the simple model (5) with the expansion of b given in (6) as a reasonable specification of the conception of the learning process advocated here, and discuss some important implications of this model.

Implications

The model mirrors a mechanism where growth in academic achievement is constrained by opportunities for learning. The latter is a characteristic of the school or more correctly of the teaching process the student is exposed to. The impact of schools through creation of opportunities for learning is an interactive one. This may be seen by finding the equilibrium level of achievement, i.e., the value of $y(t)$ for which $dy(t)/dt = 0$. This value can be obtained by taking t to infinity in equation (7):

$$y(e) = \frac{a_0}{b} - \frac{a_1}{b} x_1 - \frac{a_2}{b} x_2 - \ldots - \frac{a_n}{b} x_n \qquad (10)$$

Coefficients of the independent variables $d_i = -\frac{a_i}{b}$ may be obtained. These coefficients are those estimated in cross-sectional research as measures of the effect of independent variables on academic achievement. They will be positive in sign (since $b < 0$) and their relative magnitude will correspond to the a_i coefficients. However, in absolute magnit they will depend on b. The larger the absolute value of b -- that is, the fewer the opportunities for learning -- the smaller will be the effect of an independent variable on the level of achievement. In other words, the level of opportunities for learning will determine how much change (that is, learning) there is for the independent variable to act on, and thus determine how much of an effect these variables may have.

It further follows that the opportunities for learning will determine how much inequality in achievement will be generated by schooling. This is most easily seen if the existence of a comprehensive measure of ability and effort is assumed. The equilibrium value of achievement with such a measure will be $y(e) = -\frac{s}{b}$. The variance in $y(e)$, for a given variance in s, will then be greater the closer b is to zero, as $\text{var } y(e) = (-\frac{1}{b})^2 \text{ var. } s$. The opportunities for learning determine the extent to which schools reinforce inequality in achievement resulting from inequality in ability and effort. Family background is certainly an important cause of variation in ability and effort. If the relation between family background and achievement is assumed the same across schools, it follows from this and from the previous result, that good schools (i.e., those with many opportunities for learning) will increase inequality in achievement and increase the inequality among students due to family background. In other words, inequality of educational opportunity will be increased.

This is not a result usually looked for in research on school effects. Instead, school effects are most often conceived of as effects that modify the effect of family background on achievement and reduce inequality in academic achievement. Such effects are compensatory. They are clearly not produced by increasing opportunities for learning, but -- in the framework suggested here -- obtained by modifying a child's ability and effort. Characteristics of schools that have compensatory effects should be included alongside measures of individual attributes as independent variables adding to a person's ability and effort. They would not be characteristics that determine opportunities for learning. Since compensatory effects act on a child's ability and effort, they are probably effects of the interpersonal environments schools provide and not of the instrucational resources provided.

Thus, the conception of the learning process proposed here suggests two types of school effects -- effects of the opportunities for learning provided by schools that are interactive effects, and additive or compensatory effects produced by school environments as a result of their impact on students' ability and effort. The opportunity effect increases differences among students in learning and increases the absolute effect of students' background. Acting on variables that influence opportunities for learning will only produce increased equality in achievement by reducing opportunities for learning -- where no one can learn, the gifted will learn as little as the dull child. Acting on school environment variables may reduce inequality, other things equal, by reducing inequality in ability and effort.

A further implication of the model should be noted. Equation (10) only represents the interrelationship among achievement and measures of ability and effort in equilibrium. Cross-sectional studies using a linear model therefore implicitly assume equilibrium. In other words, it is assumed that no growth in achievement takes place at the point in time where the cross-sectional study is carried out. This is obviously a very dubious assumption. In the situation where change is still taking place, the coefficients that would be estimated using equation (10) would be functions of time. From equation (7) it follows that coefficients of the x_i variables, when the process is not in equilibrium, can be written:

$$d_i(t) = \frac{a_i}{b}(e^{bt} - 1) \qquad (11)$$

The coefficients will be increasing in magnitude in time until they reach their maximum value of $-\frac{a_i}{b}$ at equilibrium. In general, we should then expect that the effect of independent variables on achievement will increase by the amount of exposure to schooling.

The implications of the model presented here suggest certain patterns of results of empirical research, even though such research has not used the framework presented here. The next section provides a survey of existing research, which in indirect way, has a bearing on the plausibility of the model.

Evaluation of the Model

Equation (10) for the equilibrium level of academic achievement, is an additive static model in which the parameter b cannot be identified;

that is, a direct measure of the opportunities for learning cannot be obtained. Furthermore, equation (10) assumes that if academic achievement is in equilibrium, no further growth in achievement occurs at the time observations on the variables are obtained.

Nearly all research on school effects has nevertheless used a model that is formally identical to equation (10). In these applications, school characteristics are introduced as independent variables assumed to add to the effect of individual level measures of ability and effort. Since school characteristics are correlated with the individual level variables and consequently there is a substantial portion of shared variance, much attention has been directed to the problem of the order in which to introduce the variables. This problem has been particularly important in the controversy surrounding the massive study, Equality of Educational Opportunity by Coleman et al. (1966). In the Coleman study, the problem of shared variance was handled by introducing school characteristics last, after the individual level variables; in this way, only the variance not accounted for by individual level variables and not but shared by individual level variables and school characteristics is left for the school characteristics to act on. This gives a conservative estimate of the effect of school characteristics. Later reanalyses have used different orderings of the variables (e.g., Bowles, 1968), and found somewhat more substantial effects.

However, if the model suggested here is a reasonable approximation to the learning process, it is clear that the major problem in representing school effects is not one of the causal ordering of the variables.

Variables measuring school characteristics that determine opportunities for learning should not be included as independent variables in an additive specification at all. School characteristics that influence opportunities for learning determine the magnitude of b of the model and thus determine the effect of variables that are direct and indirect measures of ability and effort. Only those school characteristics that directly affect ability and effort should be used as independent variables. For those variables, the collinearity problem is relevant, and simultaneous equations may be useful to mirror their effect on ability and effort.

Thus, most of the research that reports no effect of schools on learning, in particular the research that reports no effect of a school's instructional resources (per pupil expenditure, library holdings, science labs, etc.), provides no evidence against the model proposed here. It is further consistent with the model that those school level variables for which an effect has been established are those that measure student body characteristics or provide other indicators of the interpersonal environments schools provide. Hence, schools' racial composition did exert some influence on achievement as a major result of the EEO study (Coleman et al., 1966). Studies that employ more direct measures of school environments -- for example, McDill, Rigsby and Meyers (1969) -- find significant effects on achievement of educational climates when controlling for a host of individual characteristics. In the framework we propose, this finding suggests that environments modify students' ability and effort and therefore will add to the variance ac ounted for by individual characteristics.

It is further consistent with the model that cross-sectional studies carried out at different grade levels find an increasing relation among achievement and family background and other independent variables as student age increases. The most massive documentation of this pattern is established in Mayeske et al.'s (1969) reanalysis of the EEO study. This result is to be expected from equation (11) because the dependency of $d_1(t)$ on time.

Even using the static formulation, it might be expected that significant differences in the size of parameters between schools should be found. Such an interaction has been tested for in a study of Wisconsin high schools by Hauser, Sewell and Alwin (1974). They found no between-school differences in the effects of sex, I.Q., SES, high school curriculum, peer and adult influences, college plans and occupational aspirations on educational attainment. Presumably this result casts doubt on the model proposed here. However, the dependent variable in the analysis carried out by Hauser et al., is educational attainment measured in years of schooling after high school. The model proposed here focuses on learning as measured by some form of academic achievement test. Academic achievement is of course relevant for attainment, but a host of other variables influence attainment. The evidence against the model from this research is therefore not very convincing.

Hauser et al. (1974) did use a measure of achievement in their model, and found no school interaction for that variable either.

However, the measure was rank in high school. i.e., the relative standing of a student among his/her peers. While this may be an appropriate dependent variables in testing for a differential compensatory effect of schools (i.e., a differential impact of schools on, say, the influence of family background), it is clearly not an appropriate measure of the dependent variable for the model proposed here.[5]

Research that directly focuses on change in achievement at least recognizes that learning is an ongoing process and is a more appropriate formulation of the model even though it might still be misspecified with respect to how variables measuring opportunities for learning are introduced. Such research is sparse, but it does provide some encouraging results for the framework proposed here.

Shaycroft (1967) analyzed achievement gain scores from grades 9 through 12 in the Project Talent Data. She reports significant between-school variance in achievement gains consistent with the argument presented here for the impact of opportunities for learning. However, her analysis focuses on the overall magnitudes of the gains and does not introduce explicit models for the impact of individual measures of ability and effort on the determination of the gains. The analysis is therefore vulnerable to criticism concerning the possible confounding of individual and school level variables.

Hanushek (1970) analyzed reading achievement gains of elementary school children in relation to teacher characteristics and found significant effects, where the magnitude of effect depends on the SES background of the students. Also relevant is the study by Summers and Wolfe (1974) on school children in Philadelphia. This study reports a number of significant effects on gains in achievement of school characteristics. All school characteristics are introduced as additive variables in the lagged models used, and all schools are pooled in the analysis. Hence, a direct test of the model proposed here is not possible. A number of interactions among school characteristics and individual level characteristics are however established, and some support for the model may be derived from this finding.

Overall direct support for the model proposed here is not available in the research on school effects, but some findings are consistent with what should be expected from the model, and nothing in the existing research contradicts it. However, no existing research is directly designed to implement the framework proposed here. This would demand estimating equation (8) on groups of students where each group was exposed to identical levels of opportunities for learning. Further, only measures of students' ability and effort -- or variables assumed to affect those attributes -- should be included as exogenous variables. Finally, the dependent variable should be a measure of achievement that taps what actually is taught in schools.

Not only does existing research not fulfill the requirements demanded by this model, but existing secondary data sources available to us suffer from a variety of limitations if a direct test of the model and its implications is to be carried out. In the next section, some findings from an analysis using the framework proposed here is presented. They are however, because of data limitations, preliminary.

Preliminary Findings

As a first step toward testing the proposed model, equation (8) was fitted to over-time data on academic achievement from the Project Talent study.[6] These data contain information on the achievements in various subject matters of a national sample of secondary school students. The data were collected in 1960 and again in 1963 when the students were in the 9th and 12th grades, respectively. In addition, information on the students' family background, I.Q., and sex is supplied. The latter variables provide measures of the x_i variables of equation (8). The achievement test variables provide measures of $y(t)$.

For the present analysis, we use a subset of the Project Talent sample consisting of all students whom we know were in the same school at both points in time: 2234 students from 63 schools meet this requirement. Not all of the students took the same battery of achievement tests. For any one test, the number of respondents is about 700.

The Project Talent Study tested for achievement in a number of areas (52 tests were given). Among these tests, two measures of achievement were chosen for the present analysis: (1) the score on a mathematics information test and (2) the total score in English achievement were obtained as a sum of scores on tests for spelling, effective expression, punctuation, etc. These tests cover material that very likely was presented to all respondents.

Ordinary least squares was used to estimate equation (8). Table 1 presents the results of such an estimation for all respondents who took the mathematics information test in both years.

Table 1

Estimation of $y(t) = a_0^* + b^*y(0) + \sum_i a_i^* x_i$

for Project Talent Data

using Mathematics Achievement as a Measure of $y(t)$

Independent Variables	Raw Regression Coefficients	Standardized	F
Mathematics Achievement	.634	.348	98.8
Sex[a]	-2.372	-.182	43.6
I.Q.	.054	.407	102.2
Socioeconomic Background[b]	.070	.107	12.9

a -- "Male" is coded 1; "Female" is coded 0.

b -- An index

The estimates presented are the quantities b_1^*, a_1^*, a_2^* . . . of equation (8). Since $b^* = e^{bt}$, one obtains that $b = \ln b^*$ setting $t = 1$. Inserting the empirical estimate for b^* gives a value of $b = -.425$. As expected, b is negative, indicating, according to the model, finite opportunities for learning. Using this estimate for b, the values of the remaining fundamental parameters can be obtained. Their absolute magnitude is of less interest here, but the standardized coefficients in the second column of Table 1 show that the largest relative effect is of I.Q. while sex and the socio-economic status of the family are of somewhat less relative importance.

The signs of the coefficients support the framework derived here. However it is a very weak support. The coefficient b would probably be negative with a variety of specifications of the model: measurement error alone would produce this result. As the available data only contain observations at two points in time, it is not possible to isolate the contribution of error to the results obtained in Table 1.

More firm support for the model would be obtained if indeed the parameters of the model, particularly b, showed meaningful variation across schools. Recall that b, according to the model, measures opportunities for learning. If this is so, then the size of b should vary with variables that may be assumed to measure or cause opportunities for learning. School resource measures available in the Project Talent Data are: percent teachers with M.A., number of library books, teacher experience, and expenditures per pupil.

It would be preferable to estimate equation (8) in each of the 63 schools and then demonstrate how the measures of school resources relate to the obtained estimates of b. However, the small number of students in each school who took the same test at both points in time make this procedure impractical. Instead, schools were divided into categories based on school characteristics, and b compared across categories. This is a less satisfactory procedure as it probably introduces some heterogeneity within categories.

The results of estimating b in the various categories of the school resource variables for the two tests are presented in Table 2.

Table 2

Values of b in $\frac{dY(t)}{dt} = a + bY(t) + \Sigma c_i X_i$ Across School Resource

Sample Size in Parenthesis

School Resource		Math: Info I	Total English
Percent Teachers with M.A.	Low (≤ 54%)	-.435 (254)	-.464 (241)
	Med. (55-84%)	-.327 (145)	-.962 (277)
	High (85-100%)	-.311 (264)	-.274 (245)
Number of Library Books	Low (< 2700)	-.697 (81)	-.327 (166)
	High (≥ 2700)	-.334 (547)	-.395 (538)
Teacher Experience	Low (3-8 yrs.)	-.246 (86)	a
	Med. (9-14 yrs.)	-.329 (233)	-.828 (453)
	High (≥ 15 yrs.)	-.493 (401)	-.168 (82)
Expenditure Per Pupil	Low ($50-266)	-.635 (396)	-.381 (277)
	Med. Low (275-398)	-.516 (105)	-1.343 (152)
	Med. High (418-573)	-.246 (180)	-.486 (147)
	High (696-1870)	+.122[b] (39)	-.155 (116)

a -- The absence of a b value indicates an insufficient number of cases in the category.

b -- The positive value of b may be due to measurement error stemming from the small number of cases in the category.

In Table 2, the sample size differs for the two tests in the various categories of the school resource variables because different schools are present for the analysis as a result of the way the tests were administered.

The expected pattern for the b 's is found for the mathematics information test for three of the school characteristics -- percent teachers with M.A., number of library books and per pupil expenditure. The fourth variable -- teacher experience -- also supports the model if the prediction is that teachers with less experience provide more opportunities for learning math because of their training and pedagogical techniques. The results for the Total English test are less clear-cut. The expected pattern is found for per pupil expenditure except for the low category. For teacher experience, the pattern found is the opposite of that found for the mathematics test. This may of course be given a post hoc explanation in terms of different teacher characteristics being appropriate in the two subject areas. The results for the two remaining school resource variables are also inconsistent with predictions.

The ambiguous results obtained, using the English achievement test, may be due to the nature of the material taught. It is, as shown earlier, a requirement for the model that achievement reflects learning of material presented only in schools; otherwise, between-student variation in opportunities for learning within schools will invalidate the model. This may be more likely to occur for English achievement than for mathematics. Background variables, such as encouragement to read by parents, may influence English achievement whereas schools may have more of a monopoly in teaching mathematics.

In conclusion, some evidence for the validity of the model has been found, but the evidence is not strong. Several limitations in the suitability of the Project Talent Data for this kind of analysis may be the reason for these results. Unfortunately, longitudinal data that include appropriate measures of achievement as well as individual level variables and measures of a number of school resources are not available. Such data are needed before an extensive test of the model is possible.

Measurement error and unmeasured variables have influenced our results, but the importance of error cannot be specified. Clearly, the model could also be wrong and misspecified. Assumptions about the homogeneity of parameters and the stability of exogenous variables over time may have been violated in the above analysis. Further, the variables used here as exogenous variables, though constant in themselves over time, may be indicators of unmeasured endogenous variables that change over time to the extent that ability and effort are affected by growth in achievement. Such phenomena demand revisions of the model. Some possible revisions and extensions are discussed in the next section.

Extensions and Modifications

The conception of the learning process proposed here has been shown to be consistent with the results of existing research, and the model that specifies this conception has received some support, although not unambiguous support, from a preliminary data analysis. No direct and stringent test of the model was possible, however, and the specification presented here therefore remains a first suggestion to be modified in subsequent research. This section will point to some modifications obtained at a price of having to deal with more complicated models, but possibly realizing a gain of a more valid specification.

The basic conception of learning as the outcome of an interplay between opportunities for learning and the ability and effort of students will be retained, but there are alternative ways of implementing this conception using less stringent assumptions than those made above.

Among the various assumptions that have to be made for equation (7) to be valid, one in particular seems in need of modification. This is the assumption that the x_i variables are exogenous variables not affected by change in achievement. This assumption means that a student's ability and effort is unaffected by his/her learning. However, learning is presumably rewarded in schools and those rewards likely affect a student's motivation, i.e., effort. Further, a student's ability to comprehend and learn new material is always ultimately dependent on what is already known. Both phenomena mean that ability and effort will depend on change in achievement. It may be argued that this problem will not occur if only background variables such as SES, sex and race are introduced as x_i variables in equation (7). However, these variables serve as indicators of the unmeasured quantities -- ability and effort. Hence, while a student's SES and other background characteristics remain unchanged, their relationship to ability and effort might change as a result of the learning process, and the model (7) remains misspecified.

The failure of other assumptions -- such as the assumption of identical parameters across individuals and over time -- obviously may also call for modifications in the model. However, insofar as the failure of these assumptions is not due to the interdependency of achievement and variables measuring ability and effort, the problems presented are of a more technical nature that may be overcome by using appropriate estimation procedures. In contrast, the problem of interdependency, or reciprocal causation among the main variables, demands a conceptually different model.

A simple solution to the problem of modeling the interdependency among ability, effort and learning would be to let the measure of ability and effort, s in equation (5), be a linear function of $y(t)$ -- the amount learned by time t . Let $s(t) = s(0) + ky(t)$; that is, the ability and effort at time t is assumed determined by variables other than learning (i.e., background factors) through $s(0)$ and by learning through $ky(t)$. Substituting in (5) gives,

$$\frac{dy(t)}{dt} = s(0) + ky(t) + by(t) , \qquad (12)$$

where k 0 . This equation has the solution,

$$y(t) = \frac{s(0)}{k + b} [e^{(k+b)t} - 1] + y(0)e^{(k+b)t}, \quad k > 0 . \qquad (13)$$

This expression is identical to the solution obtained earlier except that b is replaced by $k + b$. The quantity $s(0)$ could be written as a linear function of independent variables as before (cf., equation [6]).

In (12) it must be assumed that $k + b < 0$, i.e., $k < -b$; otherwise, achievement will increase forever. Ever increasing achievement is of course inconsistent with the conception proposed here, where it is assumed that there are finite opportunities for learning (which determine the size of b).

While the empirical analysis would take place using (13) in a manner similar to the analysis using (8), there is an important difference. The quantity k cannot be identified using (13) because only the sum $k + b$ can be estimated. It may, however, be argued that k should be assumed identical for everyone, in which case the relative magnitude of the school-specific b's still may be evaluated.

The formulation (13) shows that if there is some dependency of ability and effort on learning, the parameter b -- estimated from (13) -- is likely to be biased, that is, reflecting k as well as opportunities for learning. A more direct evaluation of that interdependency will, however, demand that simultaneous equations be used. Such a formulation will not only enable a direct identification of the magnitude of the dependency of ability and effort on learning, i.e., k, it will also permit a more flexible specification than the one suggested above.

It will be convenient to change the notation slightly. Let $y_2(t)$ denote s, i.e., ability and effort, and let $y_1(t)$ denote achievement. A simultaneous differential equation for the interdependency of $y_2(t)$ and $y_1(t)$ would be,

$$\frac{dy_1(t)}{dt} = a_1 + b_{11}y_1(t) + b_{12}y_2(t) \; ; \tag{14}$$

$$\frac{dy_2(t)}{dt} = a_2 + b_{21}y_1(t) + b_{22}y_2(t) \; . \tag{15}$$

Here b_{11} expresses the negative feedback of the level of achievement on learning; that is, b_{11} measures the opportunities for learning.

The quantity b_{12} expresses the contribution ability and effort make to learning. Similarly, b_{21} expresses the effect of achievement on ability and effort (in the same way as the parameter k above), while b_{22} expresses feedback of ability and effort on itself.

In matrix notation, equation (14) and (15) can be written:

$$\frac{dy(t)}{dt} = A + BY(t) \quad , \qquad (16)$$

where Y is a vector with elements y_1 and y_2, B is a matrix of coefficients, and A is a vector. The solution to this system of differential equations is:

$$Y(t) = (e^{Bt} - I)B^{-1}A + e^{Bt}Y(0) \quad , \qquad (17)$$

parallel to the expression for the single-equation model.

The system (17) will achieve an equilibrium only when the parameters fulfill certain restrictions. The restrictions are derived from the condition that a stable equilibrium will exist only if the real parts of the eigen values of the matrix B are negative. This parallels the conditions mentioned above that $b + k < 0$ in equation (14) for a stable value of achievement to obtain.

Direct use of equation (17) to estimate coefficients is possible, but assumes that a comprehensive measure of ability and effort exists. This is not likely to be the case. It is possible, however, that a direct measure of motivation may be obtained at two or more points in time. One may then restrict the attention to an interdependency between motivation and learning, and let the constant a_1 of equation (14) express ability and unmeasured components of effort assumed to be constant over time. This quantity could in turn be written as a linear function of background characteristics, I.Q., and other indicators of ability. In this interpretation, the constant a_2 would denote unmeasured contributions to a student's motivation, and this quantity could also be written as a function of independent variables (background characteristics, school environment variables, etc.).

Assuming ability unaffected by learning is only reasonable is if what is being learned does not depend on what already has been learned in the period over which change in achievement is established. This is a matter that can be controlled by choosing appropriate periods for observation in relation to a particular subject matter. Clearly, it is impossible to learn, say, trigonometry without knowing some elementary algebra: by learning algebra, one's ability to learn trigonometry therefore grows. However, over the period in which only trigonometry is presented, growth in achievement may be assumed to take place without corresponding change in "ability."

It should be noted that if $b_{22} = 0$ in equation (15) the system can be reduced to an expression identical to (12). In other words, the first suggestion for modeling the interdependency between ability and effort and learning assumes no feedback of change in ability and effort on itself. Such a feedback is, however, likely to take place, and the formulation that incorporates this term appears more reasonable.

The introduction of the simultaneous differential equation presents a number of complications with respect to estimation. It is therefore likely that the single equation model (5) will be preferred. If the mechanisms expressed in the simultaneous differential equation operate, the single-equation model therefore will be a reduced form of the system. It will then not be possible to separate the components of the estimated b 's due to opportunities for learning (b), and the component due to the dependency of ability and effort on learning (the parameters $b_{21} = k$ of equation [12]) and b_{22} if feedback is allowed). Still, inferences on the estimated variation in b 's -- due to opportunities for learning -- may be made if it is assumed that the interdependency between $y_2(t)$ and $y_1(t)$ operates with identical parameters across all schools. The situation is similar to the problem caused by measurement error, where -- if the error component cannot be estimated directly -- it is necessary to assume that error operates identically in all schools and therefore contributes the same amount to the estimated b's .

Conclusion

This paper has proposed a conception of the process of learning in terms of the opportunities for learning provided by schools, and the ability and effort of students. The conception has been modeled in a simple linear differential equation, where measures of ability and effort form the exogenous variables and change in a measure of achievement is the dependent variable. In this model, opportunities for learning determine the extent to which growth in achievement (that is, learning) is constrained by the level of achievement already obtained. This formulation was derived by assuming that the amount of new material communicated in a small interval of time depends on how much has already been presented. The total amount presented in turn is a quantity that represents the overall level of opportunities for learning.

In this framework, school effects due to variation in opportunities for learning can be measured by the coefficient of the endogenous variable -- achievement -- in a lagged equation that represents the solution to a differential equation model. The magnitude of this coefficient will determine the observed magnitudes of coefficients of exogenous variables that are measures of students' ability and effort. Thus, school effects due to opportunities for learning are interactive effects that determine the effect of ability and effort on learning. It has been shown further that schools that provide many opportunities will be schools where differences among students in academic achievement -- or inequality of achievement -- are large. In poor schools where no opportunities are provided, no one will learn, whether dull or gifted.

The school effects due to opportunities for learning should be distinguished from school effects due to the interpersonal environments schools provide. The school environment may affect student motivation and in this way influence learning. This is an additive effect of schools on achievement; the relevant school variables should be introduced alongside other measures of ability and effort as independent variables determining growth in achievement.

There are thus two types of school effects: (1) opportunity effects that determine the effect of a student's ability and effort on learning and that produce inequality in achievement resulting from differences among students in ability and effort, and (2) additive effects that directly influence ability and effort, and that may be compensatory, and reduce inequality. Only the latter effects can be identified directly from the linear additive models applied to cross-sectional data so common in research on school effects. Direct estimation of the level of opportunities for learning provided by schools is not possible using linear models on cross-sectional data.

No existing research has utilized the framework suggested here. However, it was shown that certain implications of the model are supported by existing research on school effects. It is thus consistent with the model that research has found largely no effects of schools' instructional resources on achievement, but has established effects of student body characteristics and other indicators of schools' interpersonal environments. Only the latter effects should be registered in additive models according to the argument presented here. Also, the model's prediction regarding the change over age in the parameters is supported by existing research.

The results of a preliminary investigation using Project Talent Data gave some support to the model, particularly for one of the test used -- a mathematics achievement test. The results using an English achievement test were more dubious. This may be explained by the lack of monopoly schools have in teaching language achievement;

but, the secondary data available for analysis, in general, suffer from a number of limitations for the kind of analysis suggested here. Therefore, no conclusive test of the model can be made at this time. Only new data collection can establish more conclusively the validity of the conception of the learning process and its mathematical specification presented in this paper. As it stands, however, the model does provide a number of qualitative implications that cast doubt on the prevailing belief among sociologists of education regarding the importance of schools.

FOOTNOTES

1. For a review of the literature, see Spady (1973) and for a survey of methodological issues, see Herriot and Muse (1973).

2. Harnischfeger and Wiley argue this point forcefully. They conceptualize the learning process as one in which the amount a child actually learns, and therefore his level of achievement, is constrained by institutional resources and structures, teacher characteristics and activities and student characteristics and pursuits.

3. A well-known problem of this kind is the problem of separating heterogeneity (i.e., differences in parameters among individuals) from nonstationarity (i.e., change in parameters over time) in work with stochastic process models. For a recent statement of the problem, see Taibleson (1974).

4. Setting $b' = \frac{1}{v^*}$ implies that the more that must be taught, that is, the larger v^*, the smaller is b' and the faster the teaching process.

5. Hauser et al. (1974) tested each interaction separately for computational reasons. This is not an entirely appropriate test for the model proposed here. A simultaneous test of between-school variation in all parameters would have been more appropriate.

6. The 1963 retest of the 1960 ninth graders that was part of the Project Talent Study forms the data base for the results to be presented. The data were obtained from the American Institute of Education, whose cooperation is appreciated.

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