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ABSTRACT

This volume is the twenty-eighth in a series of 29 coordinated MINNEMAST units in mathematics and science for kindergarten and the primary grades. Intended for use by third-grade teachers, this unit guide provides a summary and overview of the unit, a list of materials needed, and descriptions of four groups of lessons. The purposes and procedures for each activity are discussed. Examples of questions and discussion topics are given, and in several cases ditto masters, stories for reading aloud, and other instructional materials are included in the book. This unit begins by distinguishing between measurable and nonmeasurable properties of 2- and 3-dimensional objects. Topological transformations are then introduced using shadows, rubber sheets, and clay. Projective transformations are examined and used in the making of maps. (SD)

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MAPPING THE GLOBE

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UNIT 28

MINNESOTA MATHEMATICS AND SCIENCE TEACHING PROJECT

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	27.	NUMBERS AND THEIR PROPERTIES
	28.	MAPPING THE GLOBE
	29.	NATURAL SYSTEMS

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TRANSFORMATIONS

UNIT 28

MINNESOTA MATHEMATICS AND SCIENCE TEACHING PROJECT
720 Washington Avenue S. E., Minneapolis, Minnesota 55455

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MAPPING THE GLOBE

This unit was developed by MINNEMAST on the basis of the experiences of teachers who used an earlier trial version.

JEANNE BURSHEIM

ROGER S. JONES

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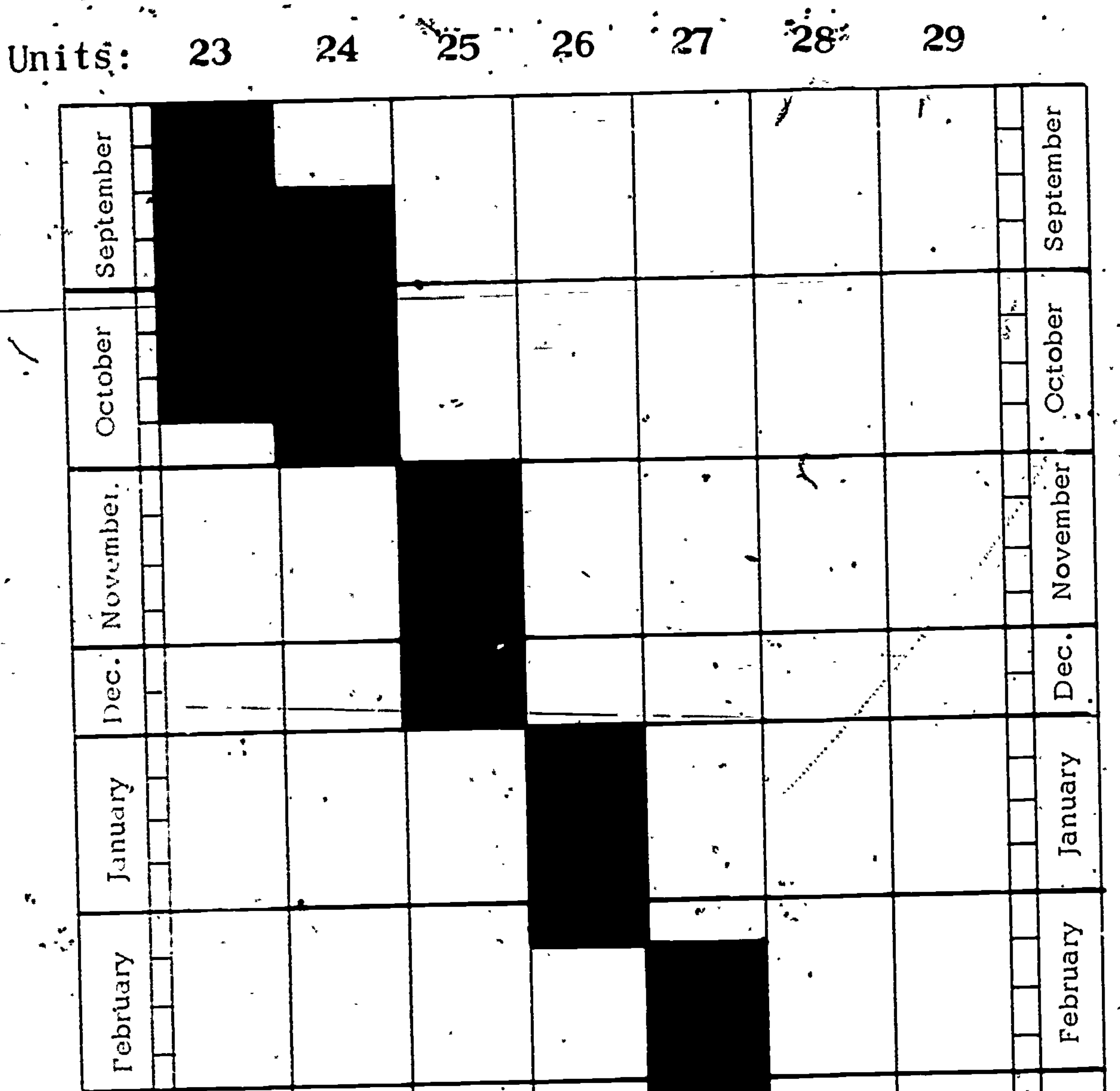
Suggested Teaching Schedule for MINNEMAST Third Grade Units



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Complete List of Materials for Unit 28

(Numbers based on class size of 30.)

total number required to teach unit	item	lessons in which item is used
30	** Student Manuals	
30	pairs of scissors	1,2,3,7,8, 10,17,18,19
30	** protractors (from Unit 26)	2,3,11,12
25-30	sheets of white paper	2
30	* rulers	2,3,7,10, 11,12,18,19
1	* ball of string	2,3,11,12
5	blank transparencies	2,12
1	overhead projector	2,12,18
	several rolls of transparent tape	3,19
1	measuring cup filled with sand	3
30	balls (assortment of different sizes, from golf balls to basketballs)	3,5
	crayons	4,5,7,8,14
1	* felt tip pen or grease pencil	5,18
20	* round balloons	5,6,10,17
30	large sheets of paper such as newsprint	6
1	* roll of masking tape	7
1	unopened flip-top can, such as soda pop comes in	7
	matches	7
	films: Monarch Butterfly Story, Encyclopedia Britannica film Frog's Life, Coronet film	7
	mealworm and beetle colony (optional)	7
1	candle	7
1	ice cube	7
	Tinkertoys (optional)	7

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30	* small plastic bags	8
1	commercial checkers game (optional)	8
	paste	8
2	* rubber bands	10
1	* red felt tip pen	10
1	* black felt tip pen	10
1	* long balloon	10
15	* sticks of clay	11
30	paper clips	11
	enough newspaper to cover desk tops	11
1	ruler or yardstick	12
	globes (1 for Lesson 14; 15 globes are preferable for Lesson 15, but 7 or 8 are acceptable; borrow from other teachers in the school)	14, 15, 16, 18
1	* hollow rubber ball	17
1	straight pin or other sharp instrument	17
	* projection equipment: transformer, plastic half-sphere, lamp, translucent plastic sheet	18, 19
	glue	19
	ESS Unit, Outdoor Mapping (optional; see page 138 for instructions on ordering)	19

* kit items as well as

** printed materials available from Minnemath Center, 720 Washington Avenue S.E., Minneapolis, Minnesota 55455

*** available from The Judy Company, 310 North Second Street, Minneapolis, Minnesota 55401

Some transformation activities —

Studying the globe.

Playing checkers.

Making a clay turtle.

Stretching a sheet of rubber.

INTRODUCTION

In this unit we introduce the mathematical idea of a topological transformation or deformation. Topology is the branch of mathematics concerned with the concepts of continuity and connectedness. In topology we study what happens to objects and surfaces when they are transformed by twisting, bending, pulling or stretching, but not by tearing or cutting. Under such deformations, some properties of objects and surfaces are not preserved. These are generally measurable properties and include area, length, mag of angle, shape, size, etc. Other properties are preserved and include, among others, connectedness, insideness-outsideness and continuity. Topology is commonly called "rubber sheet" geometry. For example, if you stretch a rubber sheet map of the United States, the areas of the states and the lengths of the rivers will change. However, the relative location of the states and rivers will not change, and Salt Lake, for example, will remain within the boundary of the United States.

One of the basic concepts of topology is that of the continuity of points in a set. The topologist, in his imagination, looks upon the objects and surfaces he deforms as continuous sets of points. We say that there is an infinite number of points on a line segment; see Units 10, 21, 22. Just as the points on a line segment form a continuous set, so also do the points on a surface or in a three-dimensional object. The continuity of points on a line is an essential characteristic of the real number system and of the number line we use to represent it geometrically. The continuity of points on a line or in a plane is also at the heart of the calculus and of all the basic theorems of analysis in mathematics. So, as the children study the deceptively simple properties of the surfaces and objects that they deform, they are introduced to the groundwork for much of higher mathematics. In this unit the children will often work with sets of discrete objects rather than with sets of continuous points. For example, the 48 states of the continental United States or the main features of a human body constitute the elements of some of the sets that they will study. They will observe transformations of these sets and study how measurable and non-measurable properties change under a transformation. Thus the children can transfer their ideas about sets of discrete objects to sets of continuous points.

Another concept that is basic to the idea of a topological transformation is that of one to one correspondence, with which the children in the MINNEMAST program are familiar. Consider a set of points or simple elements which is topologically transformed. The set, in its final form, will correspond point by point or element by element with the original form, in spite of changes in size or shape. The children are given simple examples of this concept. For instance, in Section 4, they will try various ways of flattening the globe in order to make a flat map. They will see that although angles, distances and shapes might change, the features of the flat map may be matched one to one with the features of the globe. They will realize that there is always a one to one correspondence between points (geographical features) on different versions of the same map. Thus the concept of one to one correspondence, which the children first worked with when they began counting and using natural numbers, is further developed and applied in this unit.

The concepts of continuity and one to one correspondence are two of the most fundamental ideas in set theory and, indeed, in mathematics. In their study of transformations in this unit, the children are exposed to these two concepts on a basic, intuitive and geometrical level.

NOTE: If possible, use the two films that are listed for Lesson 7. The films are:

Monarch Butterfly Story, Encyclopedia Britannica film, 11 minutes; color, 16 mm.
Frog's Life, Coronet film, 10 minutes, black and white, 16 mm.

Listed below are some books that you might find useful while teaching this unit.

Barr, S., Experiments in Topology, New York: Thomas Crowell Co., (Chapter 1).

Hilbert, D. and S. Cohn-Vossen, Geometry and the Imagination, New York: Chelsea House, (Chapter VI).

Kasner, Edward and James Newman, Mathematics and the Imagination, New York: Simon and Schuster, Inc., (Chapter 7).

Life Science Library, Mathematics, New York: Time, Inc., (Chapter 8).

This section reviews many of the ideas introduced in earlier MINNEMAST units, such as properties, classification, one to one correspondence and measurement.

In Lesson 1 the children review the idea that a property is a characteristic or attribute of an object or a set of objects, leading to the important idea that number is also a property of a set. By using one to one correspondence, they can compare the number property of two or more sets. They review classification ideas with sets of animal cards, using common properties such as the number of digits on a front foot, etc., as the basis for classification.

In Lessons 2 and 3 the children study measurable properties of two- and three-dimensional figures. A measurable property is one to which a number can be assigned, usually by measurement with some instrument (e.g., ruler, protractor). Examples of measurable properties are length, area and volume.

In Lessons 4 and 5, the children study non-measurable properties of two- and three-dimensional figures. In this unit the non-measurable properties are "boundedness" (pertaining to boundaries), order of points on a line or in a plane and "insideness-outsideness" (in-out), which has to do with the location of points in reference to a boundary.

This study of measurable and non-measurable properties provides a basis for investigating properties that change and properties that do not change under transformation.

Lesson 1: PROPERTIES OF OBJECTS AND SETS

The purposes of this lesson are:

- to review the usage of the word "property" when describing a characteristic or attribute of an object or a set of objects;
- to use one to one correspondence as a way of comparing the number property of two or more sets;
- to have the children classify sets of animal cards, using common properties of the animals as the basis of classification.

In this unit, as in Unit 26, <u>What Are Things Made Of?</u>, the word "property" refers to a characteristic or attribute of an object or a set of objects. Hardness, shape, color, thickness, texture, size, etc., are words that denote properties. When describing properties of sets, such words as "number" and "order" are also used.

MATERIALS

- Letter to the Student (provided in the Student Manuals)
- story, "The Education of Peter von Hahgg" (provided in this lesson and also in the Student Manuals)
- 30 pairs of scissors
- Worksheets 1 - 5

PROCEDURE

Activity A

Distribute the Student Manuals to the children and have them open to the student letter. Read it over with them. This letter introduces two story characters, Peter von Hahgg and Henry, who will appear from time to time to help introduce and review certain concepts. The first installment of the story follows the student letter. Have the children turn to the first installment and read through it with them. In this part of the story, Peter and Henry clarify a common misunderstanding children often have of the word "property," that is,

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they think of property as land. Peter and Henry also review one to one correspondence (matching) as a method of comparing the number of objects in two sets.

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Story by Elizabeth Ihrig

Illustrations by Judith Norman

THE EDUCATION OF PETER VON HAHGG

Peter von Hahgg, Pig-in-Chief of Hillendale Farm, was very proud of his highfalutin title and more than a little pompous, but he did have a very good heart, after all.

His friend, Henry, a clever but modest fellow, went to the Little Red School House five of the seven days out of the week. On the sixth day he took long walks with Peter, and on the seventh, he rested.

The two had been friends for quite some time now. When Henry came home from school, Peter would tell him the farm gossip and Henry, in turn, would explain much of what he had learned that day to Peter. Peter was a highly intelligent pig. He had taught himself the alphabet and could spell such words as pedestrian, antidisestablishmentarianism and reticule, which, you must admit, is pretty impressive. He even had a library card and checked books out of the local library so he could bone up on all sorts of subjects. All in all, Peter von Hahgg was one of your more well-informed and high-toned pigs. However, as you will soon see, there were certain topics about which Peter knew nothing, which is not to say that he wasn't perfectly willing to learn.

One fine Saturday afternoon in April, shortly before the beginning of daylight saving time, Henry and Peter went out for a stroll around Hillendale Farm. They were eager to look at the young lambs and chickens and ducks and to see how the gardens were growing.

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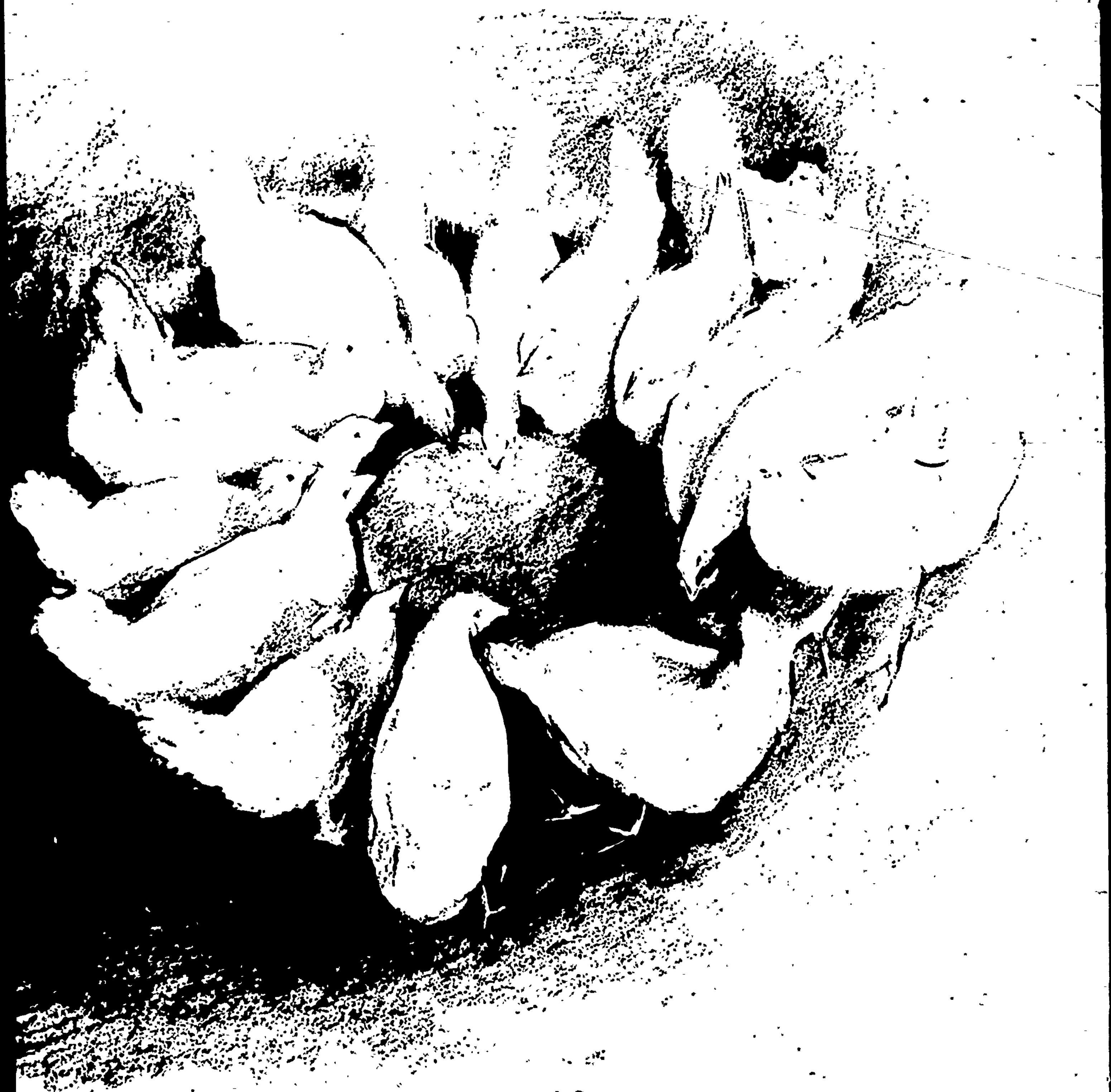
They were standing near the chicken coop looking at the chickens when Henry said to Peter, "Those chickens certainly have some interesting properties, don't they Peter?"



"Properties?" asked Peter. "Do they own property? Other than the ground they're scratching on, I mean?"

"Silly pig," replied Henry, "I mean — well, when I say property that way, I mean the things you can see and describe about the chickens that make them chickens — like their feathers. Let me see if I can explain it better. The properties of a chicken include its feathers, its wings, its ability to lay eggs, its two feet. You get the picture?"

"I'm not sure," answered Peter, curling up his tail as he thought about it, the way most people wrinkle their foreheads when they try hard to concentrate.

"Well, Peter, before you get your tail all bent out of shape thinking about it, I'm going to assemble a set of objects and let's see if you can describe some properties that all of them have. O.K.?" Henry asked.

"O.K.," replied Peter, but without much enthusiasm. He was a teeny bit afraid that he might not be able to answer Henry's questions and he hated to appear foolish in Henry's eyes.

And with that, Henry walked off to the barn and returned a quarter of an hour later, carrying two white baby rabbits, and leading a young lamb, a kitten and a puppy. He brought them over to the oak tree that Peter was standing under. As he sat the rabbits down on the ground, Henry said quietly to all of them, "Now, don't run off until I tell you you can go. You understand?" And turning to Peter, he said, "Very well, then, Peter, can you describe some properties of all the animals in this set?"

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"Hmm....let's see.....they all have four legs, they all are covered with fur, they are all of them living animals.....what I mean is, they're not plants or buildings or stars but they are animals," explained Peter.

"Verrrrry good," exclaimed Henry. "Anything else?"

"Well, the lamb and the two rabbits are white," Peter said.

"Yes, but the kitten and the puppy aren't white," Henry replied. "You see, Peter, I want you to tell me the properties that all the members of this set have in common. That means the properties that they all have — like they all have tails — or as you mentioned earlier on, they all have four legs."

"Oh, I see," Peter answered. "They all have two ears," he continued, "and two eyes and they all can move around. They all eat and they all sleep and they are warm when you curl up next to them."

"Now you have my idea," Henry said.

"Say, Henry, this is kind of fun," said Peter. "Why don't you — if it's not too much work, I mean — why don't you make another set of different kinds of objects and let me describe their common properties?"

"O.K." replied Henry, (who wanted to be a teacher when he grew up anyway) and so saying, he went to the tool shed and came back pushing a wheelbarrow that held a rake, a hoe, a shovel and a trowel. He lined up these things on the ground and asked Peter, "What are the common properties of the objects in this set?"

And quick as a bat, Peter answered, "They all have wooden parts and they all have parts made out of metal and they all have handles. And — oh yes — they are not animals. They are all used to work in a garden."

"Right!" exclaimed Henry. "Anything else?"

"No, not anything else that I can see," Peter answered. "But you know what, I just noticed something that's interesting."

"What's that?" Henry asked.

"Both sets — the set of living animals and the set of gardening tools — have five objects in them. Could you say that the <u>number</u> of objects in any set is a property of that set?"

"Not so fast," said Henry. "You mentioned two very important things. You're absolutely right when you say that the number is an important property. Whatever the number of objects in a set — 2, 49, 100 — it's an important property. You also noticed that in these two sets, both have the same number of objects. You must have <u>compared</u> them in some way. Can you show me just how you know that each set has the same number of objects in it?"

"Why, that's easy," replied Peter. "Look, you can match one object in this set to an object in that set and not have any left over."

And, wouldn't you know it, that's just what Peter von Hangg did! He matched the rake to the kitten, he matched the hoe to the puppy, the wheelbarrow to a rabbit, the shovel to the lamb and the trowel to the other rabbit.

"There you are, Henry," Peter exclaimed, stepping back to look at his work.

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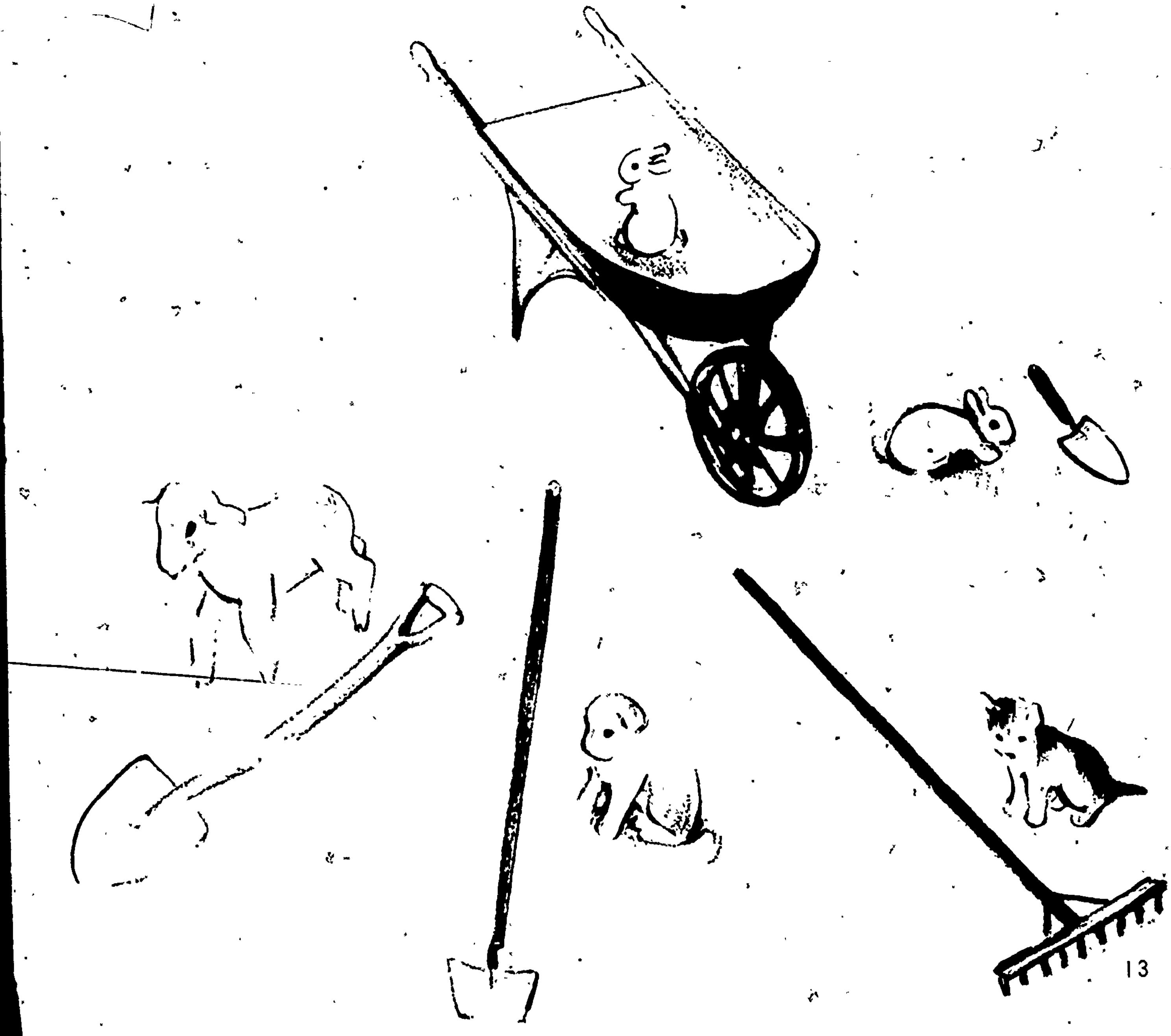
"That's very good," Henry said. "Do you know what you have just done? You, my friend, have matched one to one the objects in one set with the objects in another set. In other words, there is a one to one correspondence between these two sets."

"Well, dog my cats!" squealed Peter in excitement. (Peter had a hard time keeping cool when he got excited.) "So that's what I did. And now I understand what you mean when you say one to one matching of the objects in a set."



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Henry had been noticing that the shadows cast by the tree had been growing longer and longer, so he turned to Peter and said, "Well, old buddy, it's been a long afternoon and this has been a pretty deep discussion. What do you say we return to the house and have some lemonade before we clean up for supper?"

"Sounds like a good idea to me," said Peter, who was not one to turn down lemonade nor food nor even cleaning up for supper.

As the two friends walked down the lane to the farmhouse, Peter nudged Henry in the ribs and said, "Speaking of chickens, I have an important question for <u>you</u> to answer, Henry."

"So ask me," said Henry.

"Can you tell me," asked Peter, "why did the chicken cross the road?"

After the class has finished reading the story, ask the children if they would like to play a game in which they describe some properties of objects and sets of objects.

Game 1: Properties of Objects

First have the children describe some properties of an object, such as a book or an eraser. Then ask one student to leave the room for a few minutes and tell him that when he comes back into the classroom the other children will describe the properties of some object in the room. His job will be to figure out which object they described.

While the child is outside the room, the rest of the children decide which object they wish to describe. Caution them against looking at the object when they describe its properties. Also, they cannot tell the name of the object, i.e., they shouldn't say, "It's a clock." Call back the student who was sent out and proceed with the game.

Game 2: Properties of Sets of Objects

Have a set of four children stand at the front of the room. Ask the class to describe some properties of this set of children. Some possible properties are:

- there are four members in the set
- they all have two legs
- they all are wearing two shoes
- they are all girls
- they all have dark hair

Ask someone to divide the set into two subsets according to some property. (For example, three girls in one subset, one girl in another subset; one subset has three long-haired girls, the other subset has one short-haired girl in it.) The class should then try to guess by what property the child determined the subsets. (In this example, the property is hair length.)

Repeat the procedure, but this time choose a set of dissimilar objects, e.g., a child, a plant, a pair of scissors and a glass. Ask the children to describe some properties of this

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set of objects. Some possible properties are:

- they are all solid
- they are all taller than they are wide
- there are four members in the set

Have a child divide the set into subsets according to some property. For example, he may make a subset of the child and the plant and another subset of the scissors and the glass. The determining property in this example is whether the object is alive or not alive. Or perhaps he will divide these objects into a subset of the child and the scissors and another subset of the plant and the glass and say that the first subset has "legs" and the second subset does not. Or he may make one subset of the plant, the scissors and the glass and another subset of the child. In this case the determining property is whether the object is an animal or not.

The class should again try to guess by what property the child determined the subsets. The child who guesses the correct property gets to choose a new set of objects and also to determine the subsets.

Activity B

Ask the children to tear Worksheets 1 through 4 out of their manuals. They should cut each worksheet into four parts along the dotted lines. When they are done, they will have a set of sixteen animal cards. On one side of each card is a drawing of an animal. On the other side are smaller drawings of the front paws and the back paws of the animal.

Assign partners and have them sit next to each other. Ask each child to divide his set of animal cards into subsets, using any property he wishes. When they are finished, each child should determine what property his partner used to classify his set of animal cards. When this is done, ask them to reclassify their set of cards according to another property. Then they should check their partner's work as before.

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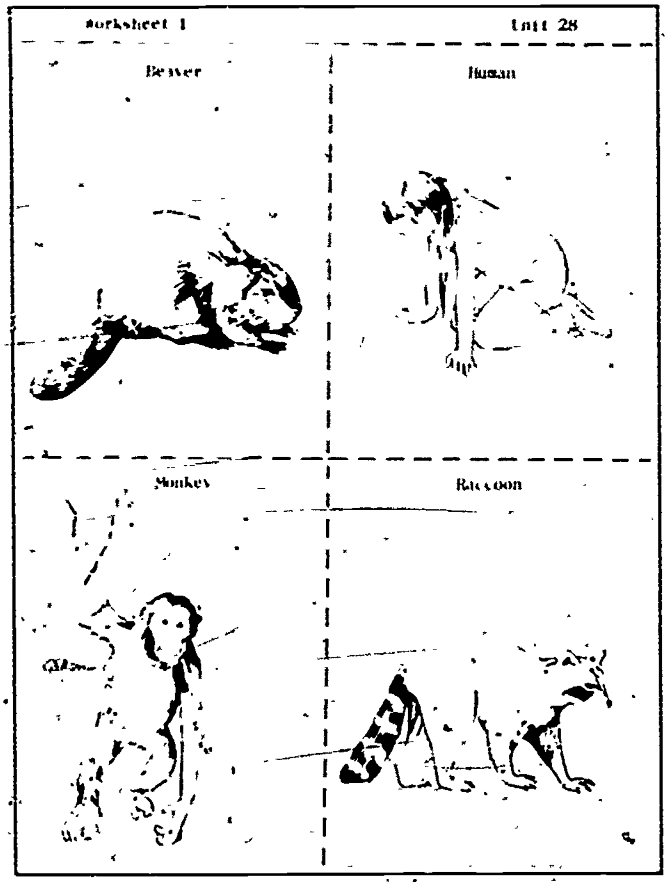
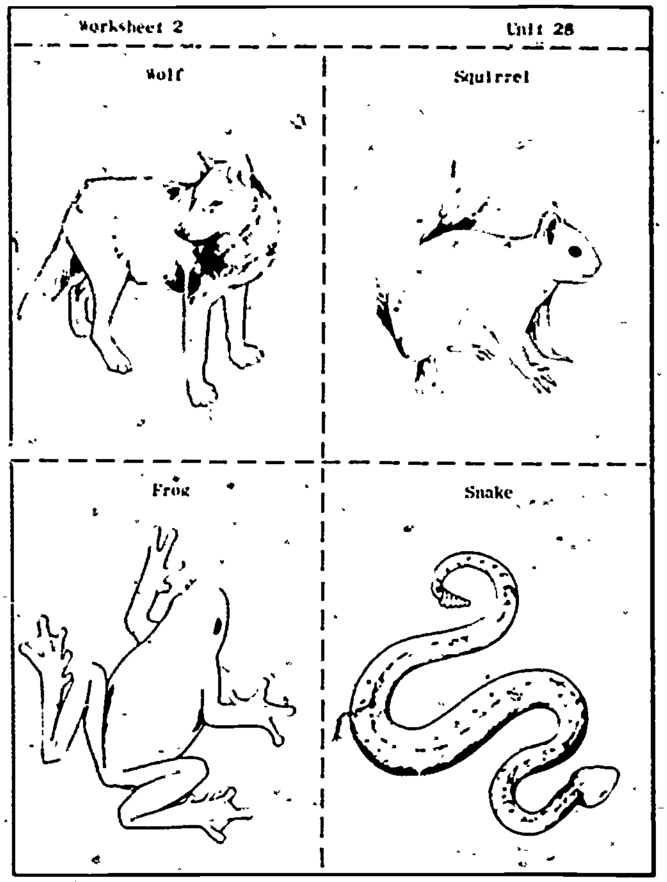
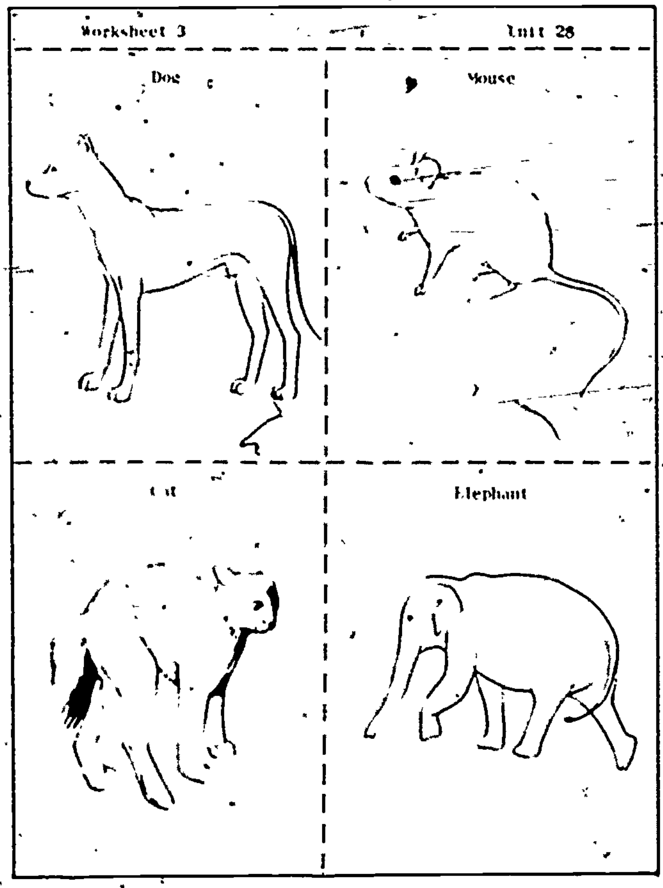
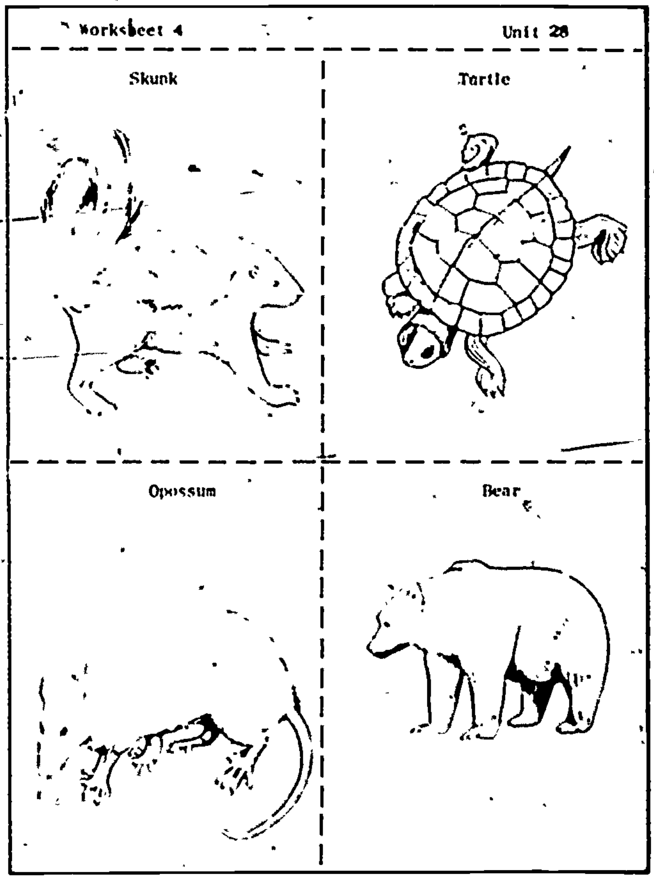
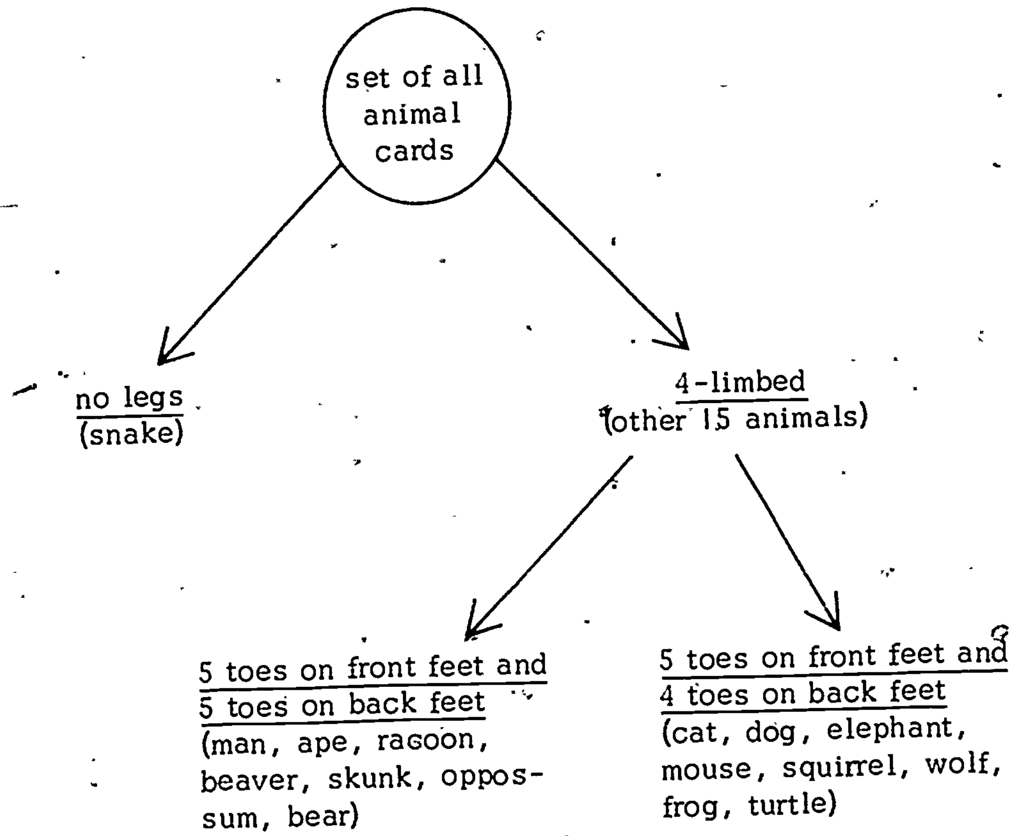

Worksheet 1
Unit 28
Beaver
Human
Monkey
Raccoon


Worksheet 2
Unit 28
Wolf
Squirrel
Frog
Snake

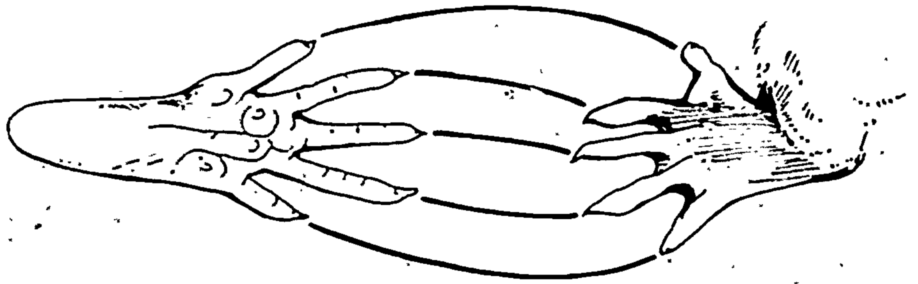

Worksheet 3
Unit 28
Dog
Mouse
Cat
Elephant


Worksheet 4
Unit 28
Skunk
Turtle
Opossum
Bear

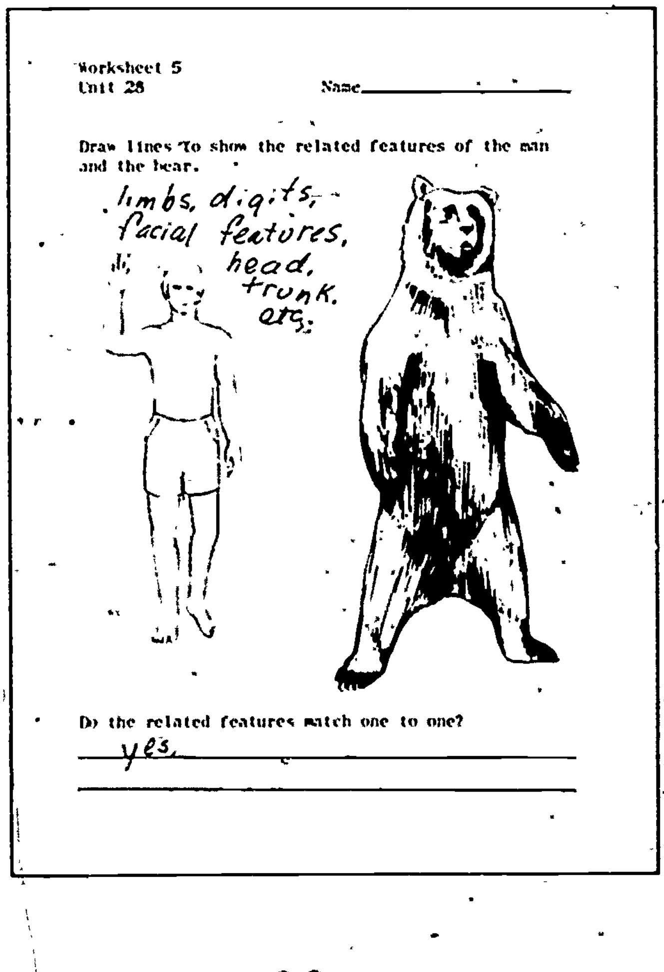
Extra motivation for this activity could be to have the children see which pair comes up with the greatest number of classification properties. When the children have had enough time to try several classification schemes, list on the chalkboard some of the properties that were used. The following diagram illustrates one possible classification scheme:



Discuss with the class the idea that the number of toes on an animal's hand or foot can be matched one to one with another animal's digits.



To further clarify the idea of one to one matching, have the children complete Worksheet 5. On this worksheet, all the main features match one to one on the man and the bear (2 eyes to 2 eyes, 5 fingers to 5 fingers, etc.), except for the bear's tail. Discuss the worksheet with the children after they have completed it.


Worksheet 5
Unit 28 Name_______

Draw lines to show the related features of the man and the bear.

limbs, digits, facial features, head, trunk, etc.

Do the related features match one to one?

yes.

Lesson 2: MEASURABLE PROPERTIES OF 2-DIMENSIONAL FIGURES

The purposes of this lesson are:

- to study such measurable properties of 2-dimensional figures as length, mag of angles and area;
- to provide the groundwork for investigating properties that change and properties that do not change under certain transformations.

Measurable properties are those to which a number can be assigned, usually by measurement with some measuring instrument (ruler, protractor, clock, weight scale, etc.). Examples of measurable properties are length, area, height, weight, time, volume, etc. Non-measurable properties are those that must be described without the use of numbers and measurements. For example, order of elements in a set, color, characteristic shape or form, open or closedness of a curve, etc., are non-measurable properties.

MATERIALS

- 30 protractors (from Unit 26)
- 30 rulers
- 30 pairs of scissors
- 30 pieces of $5\frac{1}{2}$" x 3" paper
- 30 pieces of string, each at least 12" long
- story, "The Same or Not the Same?" (provided in this lesson and also in the Student Manuals)
- transparency of Worksheet 6 (optional; printed original is included in the appendix of this manual).
- overhead projector (optional)
- Worksheets 6 - 11

PREPARATION

Before class, cut 30 pieces of $5\frac{1}{2}$" x 3" paper. Use any kind of paper.

Cut 30 pieces of string, each at least 12" long.

If you wish to make a transparency of Worksheet 6 for demonstration purposes, a printed copy of it is included in the appendix of this manual.

PROCEDURE

Activity A

Remind the children that in the previous lesson they talked about such properties of objects and sets of objects as color, thickness, hardness, etc. Today they are going to read a short story about two sisters who get into an argument about properties.

Have the children open their manuals to the story, "The Same or Not the Same?" It follows Worksheet 5. Read the story together with them. It introduces some of the fundamental ideas that will be investigated in this section and throughout the unit. That is, under certain transformations, certain properties change (measurable properties) and certain properties do not change (non-measurable-properties).

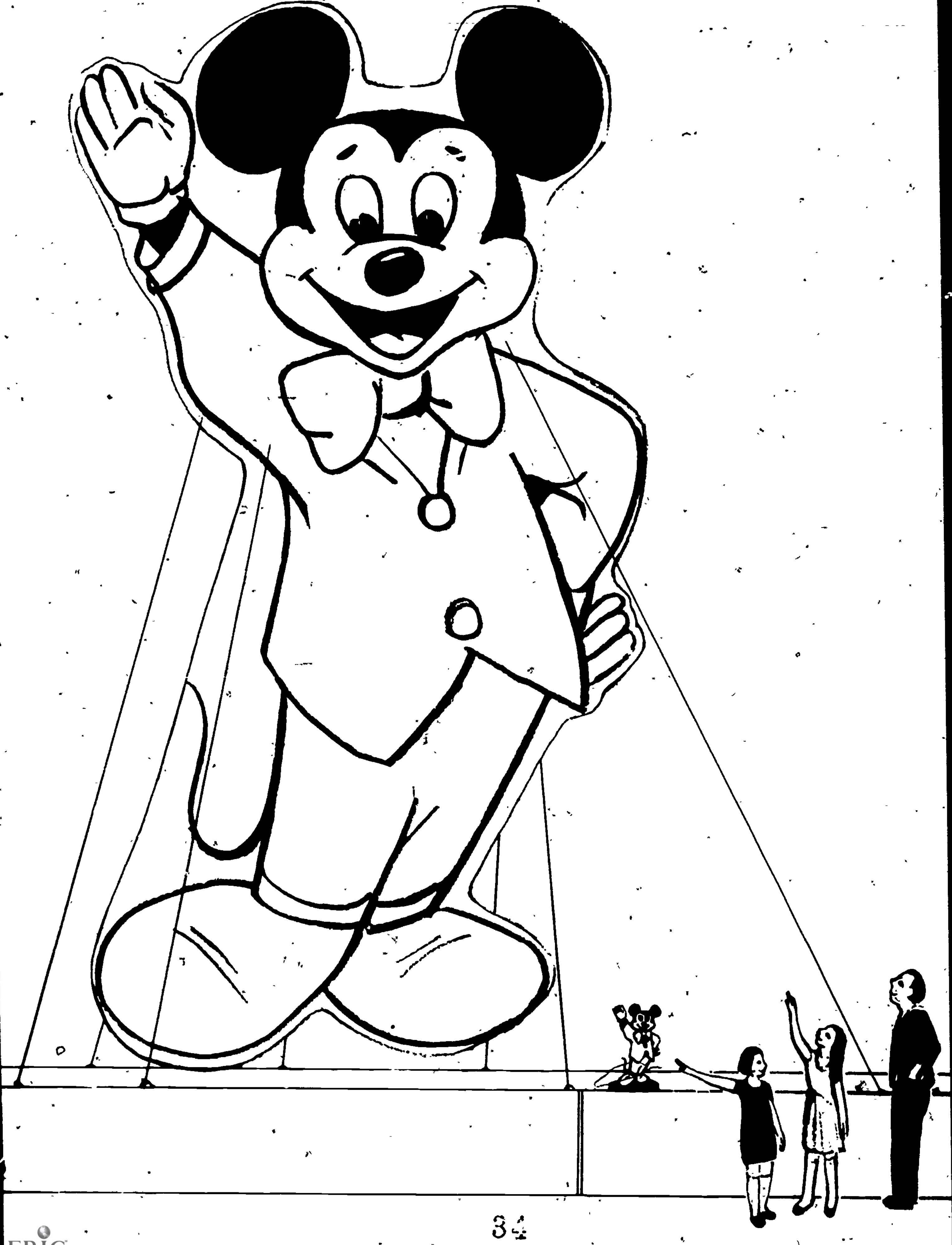
THE SAME OR NOT THE SAME?

Greta and Irene were very excited. They were visiting their grandfather in New York City and he was taking them downtown to visit a giant balloon factory. The girls were excited about the balloons because they were the ones used in the big Thanksgiving Day parade every year. Greta and Irene had often watched the parade on television in Minneapolis where they lived. They loved to see the drum majorettes, the clowns and jugglers, the beautiful fairyland floats and the live animals — and, of course, Santa Claus. But best of all were the giant balloons. There were balloons of many famous cartoon characters like Superman, Donald Duck, Mickey Mouse and Popeye. Some of the balloons reached to the very tops of the apartment buildings. They had seen all of this on television, and now they were going to see the real balloons at last!

"You girls are very lucky," the man at the factory said; "We usually keep the balloons flattened out and stored away in huge cartons all year. But Mickey Mouse is badly in need of repair and we've got him all blown up in our big warehouse."

The girls and their grandfather eagerly followed the balloon man into the warehouse. As they entered the huge room they were amazed to see a gigantic Mickey Mouse looming high above their heads and reaching almost to the top of the five-story roof. Greta and Irene both exclaimed "Wow!" They just couldn't believe how large the Mickey Mouse balloon really was.

"We're trying to find a couple of leaks," the man told them. "And we're also touching up some of the colors so he'll look just like our original Mickey Mouse statue over here." The girls looked at the table where the man had pointed. On the table was a small



statue of Mickey Mouse, about two feet high.

"That statue is funny," said Irene. "It's just like the big balloon."

"Oh no," argued Greta. "The Mickey Mouse balloon is completely different from this little statue. Can't you see that, silly?"

"Now wait just a minute," interrupted Grandfather. "I guess they're the same, but different too!"

Greta and Irene stopped arguing. They both looked puzzled.

☆ WHO WAS RIGHT? GRETA, IRENE OR GRANDFATHER?

☆ WHAT PROPERTIES WERE THE SAME ABOUT THE MICKEY MOUSE STATUE AND THE BALLOON?

☆ WHAT PROPERTIES WERE DIFFERENT?

After you have finished reading the story, discuss the questions at the end. Since these questions are meant to be open-ended, accept all suggestions that the children give. At the beginning of Section 3, you will refer to this story and give the children an opportunity to answer the questions in more detail.

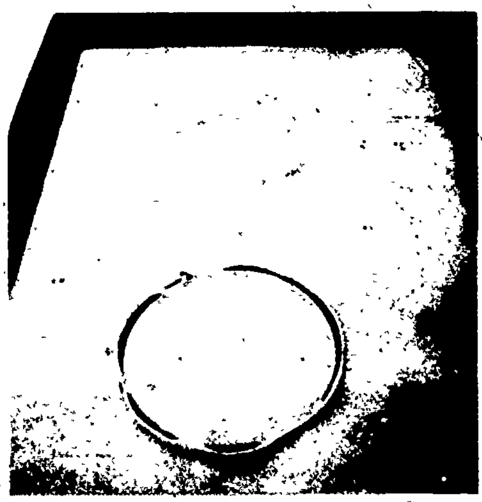
Tell the children that in this and the next few lessons, they will be investigating some of the types of properties the Grandfather in the story meant when he said that the statue of Mickey Mouse and the balloon of Mickey Mouse were the same, but also different.

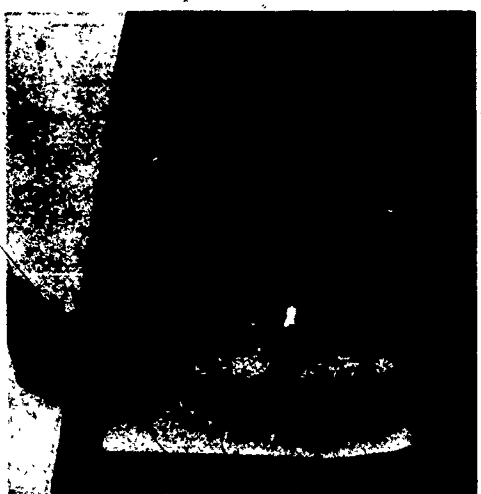
Activity B

Review the following measurement ideas. All of them have been used in previous MINNEMAST units, especially Units 12, 16, 21, 22, 24, 25 and 26.

1. The "appears to equal" sign
   When we measure lengths, widths, areas, weights, etc., we get only an approximate measurement, so instead of using the equal sign, we use the "appears to equal" sign. For example, "the length of the desk $\doteq$ 5 feet." Since it may actually measure 5.0000012 feet, it is incorrect to say that the length = 5 feet.

2. Length or distance measurements
   The children shouldn't have much trouble measuring line segments. However, they may not remember how to measure the distance around a circle. You may want to give the children time to think of a way to do it. One possible method is to lay a string along a circle, mark the circumference with a pencil and then lay the straightened string along a ruler.





3. Mag of angles

The children were introduced to the standard protractor in Unit 26, What Are Things Made Of?, pp. 172-178. The children will be using their protractors from Unit 26 throughout this unit. Review the ideas according to the needs of your class.

4. Area

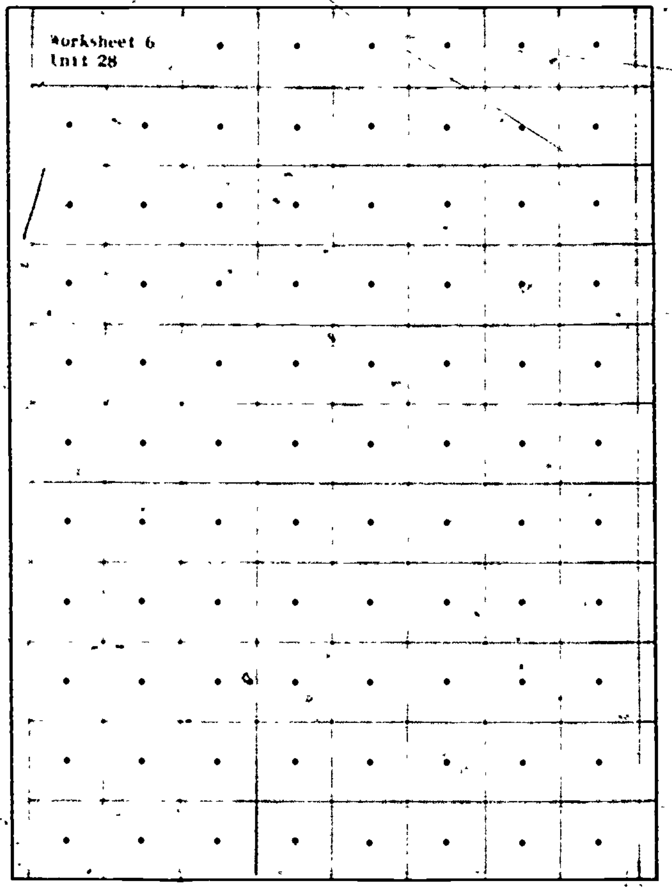
Children in the MINNEMAST program have not actually measured area since first grade in Unit 12, Measurement with Reference Units. Therefore, you may need to spend more time reviewing this notion than you spend on other measurement ideas.

Below is a possible sequence of ideas you may want to follow when discussing area with your class.

– Area was defined in Units 5 and 12 as the "size of a region on a surface bounded by a simple closed curve." For example, the size of the region bounded by the edges of this page is called the area of this page.

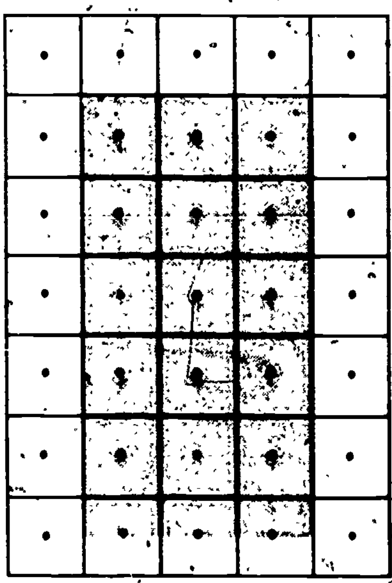
– Have the children look at the transparent grid on Worksheet 6. To find Worksheet 6, have them turn to the back of their manuals. Give each child a piece of paper that measures $5\frac{1}{2}$" x 3". Ask them how their grid could be used to measure the area of the piece of paper. (They could put the grid over the paper and count the number of grid squares it takes to cover the piece of paper.) Have them do this.

ERIC



They will soon discover that some of the grid squares fall only partly on the paper. Suggest that they use the following rule: "Count a grid square if its center falls in the region to be measured. The dots in the centers of the grid squares can help decide." Tell the children to try to get as many whole squares as possible onto the surface they are measuring.

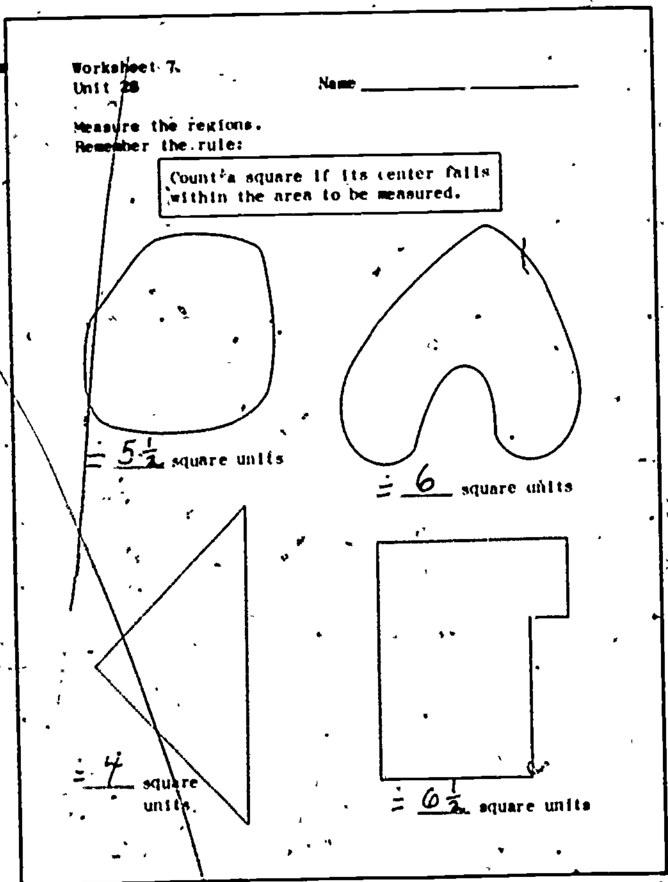
If the children have difficulty using their transparent grid sheets, make a transparency of Worksheet 6. Use the overhead projector and illustrate the measuring process for the children.



In this example, 15 of the dots fall completely in the region to be measured and 3 dots are on the boundary. In this case, the children should see that two half squares make one whole square, therefore, the area of this region is $16\frac{1}{2}$ square units.

– Rather than call the units "square units," ask the children what else they might be called. Have them measure the dimensions of the grid squares with their rulers. They should see that the squares measure 1" x 1". Suggest that the square units be called "square inches."

– You may want to give the children a chance to practice making measurements with their transparent grids. First, have them complete Worksheet 7 and then let them measure the surface area of several objects in the classroom. They can record their measurements on Worksheet 8. Suggest that they measure the area of some circular or "curved" regions, such as the cover of a jar, a closed curve drawn on the chalkboard, etc.


Worksheet 7
Unit 28
Name ____________

Measure the regions.
Remember the rule:

Count a square if its center falls within the area to be measured.

Worksheet 8
Unit 28
Name ____________

Area Record Sheet

The area of ____________ ≐ ________ square inches.
The area of ____________ ≐ ________ square inches.
The area of ____________ ≐ ________ square inches.
The area of ____________ ≐ ________ square inches.
The area of ____________ ≐ ________ square inches.
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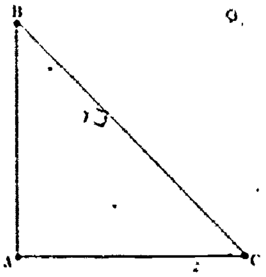
Worksheet 9
Unit 28

Name ______________

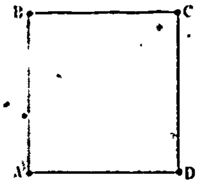


Find the measurable properties of this triangle.

* Length of sides
  - side AB ≐ 3 inches
  - side BC ≐ $4\frac{1}{4}$ inches
  - side AC ≐ 3 inches
* Meas of angles
  - angle A ≐ 90 degrees
  - angle B ≐ 45 degrees
  - angle C ≐ 45 degrees
* Area ≐ $4\frac{1}{2}$ square inches

Worksheet 10
Unit 28

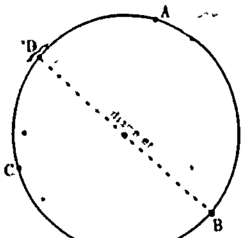
Name ______________



Length of each side ≐ 2 inches.

Meas of each angle ≐ 90 degrees.

Area of the square ≐ 4 square inches.



The distance around the circle (the circumference) ≐ 9 inches.

The distance across the circle through the middle (diameter) ≐ 3 inches.

The area of the circular region ≐ 7 square inches.

Worksheet 11
Unit 28

Name ______________

$\overline{AB}$ - height
$\overline{CD}$ - width

The height of the face ≐ 3 inches.

The width of the face ≐ $1\frac{3}{4}$ inches.

The area of the face ≐ 4 sq. inches

The distance around the face ≐ $7\frac{1}{2}$ inches.

Activity C

Have the children complete Worksheets 9, 10 and 11. They will need a standard protractor (from Unit 26), a ruler, a piece of string (at least 12" long) and the transparent grid (Worksheet 6). Discuss their answers when they have finished their worksheets.

The measurement ideas and devices reviewed in this lesson will be used extensively throughout the rest of the unit; therefore, have the children save their string, protractors, rulers and grids for later use. You may want them to keep their measuring devices together in a folder.

Lesson 3: MEASURABLE PROPERTIES OF 3-DIMENSIONAL OBJECTS

The purpose of this lesson is:

- to study some of the measurable properties of a cube and a sphere, such as surface area, circumference, mag of angles on a cube and volume.

MATERIALS

- 30 pairs of scissors
- transparent tape (as many rolls as possible, up to 30)
- 1 measuring cup, filled with sand
- measuring devices from Lesson 2 (protractor, ruler, string, and transparent grid)
- 30 balls (assortment of different sizes, from golf balls to basketballs), 1 per child
- Worksheets 12 - 15

PROCEDURE

Activity A

Briefly review some of the properties the children measured in Lesson 2, such as length (distance), area and mag. These were all properties of figures drawn on a flat surface. Tell the children that today they are going to investigate some of the measurable properties of objects that are not "flat" — objects that have length, width and height — like a box or a ball.

Each child will need scissors and a few pieces of transparent tape. Ask the children to tear out Worksheet 12 and follow the accompanying instructions to construct the cube. To find both pages of Worksheet 12, tell the children to turn to the back of their manuals.

When the children have constructed their cubes, have them take out their measuring devices from Lesson 2 (protractor, ruler, string and transparent grid). Then they should complete Worksheet 13. Discuss their answers with them. Some child may see that an easy way to find the total surface area of the

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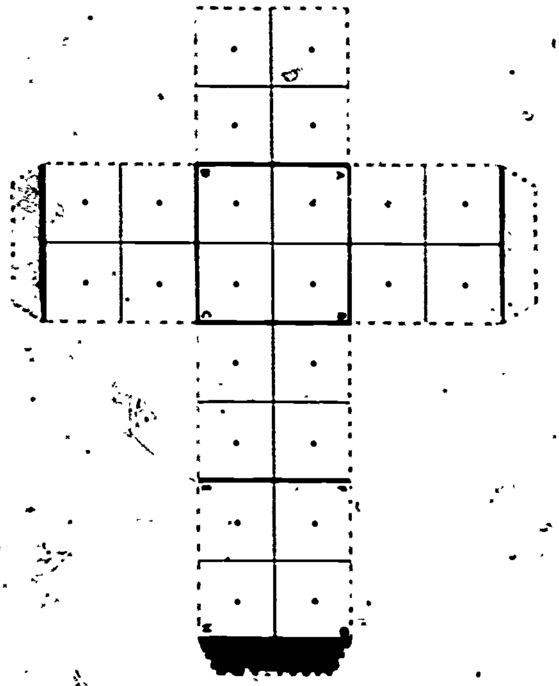
Worksheet 12
Unit 28



Worksheet 12, cont'd.
Unit 28

1. Cut out the figure along the dotted lines.

2. Fold along the darker lines.

3. Tuck in the tabs.

4. Tape the tabs shut.

5. You should have a box (cube) when you are finished.

cube is to find the area of one face and then multiply that by the number of faces ($2 \times 2 = 4$; $6 \times 4 = 24$ square inches).

In discussing the last question on Worksheet 13, "What other properties of the cube could you measure?" some possibilities are its volume (capacity) and its weight. One way to measure the volume of the cube is to tape completely the edges of one cube, except for a lid face. Then pour sand into the cube. Measure the amount of sand used by pouring the sand out of the cube into a measuring cup. Another method would be water displacement, but this would be more difficult.

Ask the children to save their cubes. They will be used again in Lesson 5.

Worksheet 13
Unit 28
Name ________________

Measurable Properties of a Cube

Look at the cube you made.
The surface area of the cube = 24 square inches.

Find corner A. Measure the distance from A to B to F to G to A. The total distance = 8 inches.

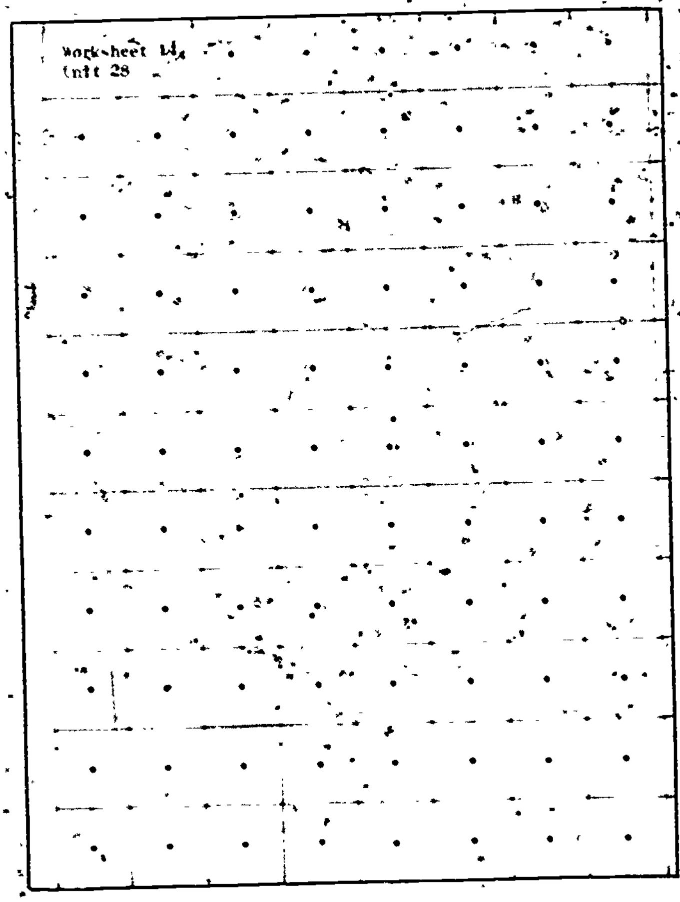
Measure the angles at the corners of your cube.
Each angle = 90 degrees.

What other properties of your cube could you measure?
volume, weight

Activity B

Review with the children some of the properties they measured of their cubes: surface area, distance around, mag of angles, volume and weight. Then hold up a ball. Ask the class what properties of the ball they could measure and how they would measure them. List their ideas on the chalkboard.

Then have them complete Worksheet 15. To complete this worksheet, they must actually measure the circumference, the surface area and the volume of a ball. On the reduced worksheet are sample answers for a child who has a softball. To answer the last question, the child must decide which kinds of balls are smaller and larger than his ball.



Worksheet 15
Unit 28
Name________

Measurable Properties of a Sphere

Kind of ball __softball (e.g.)__

The distance around (circumference) of my ball ≈ __12__ inches.

The surface area of my ball ≈ __25__ square inches.

The volume of my ball when it is compared to the other balls:

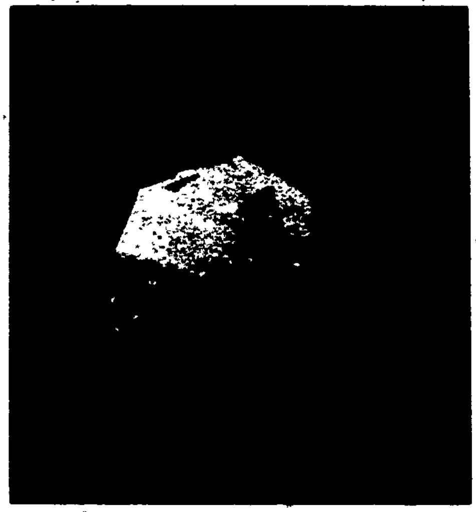
> volume of __tennis__ ball.

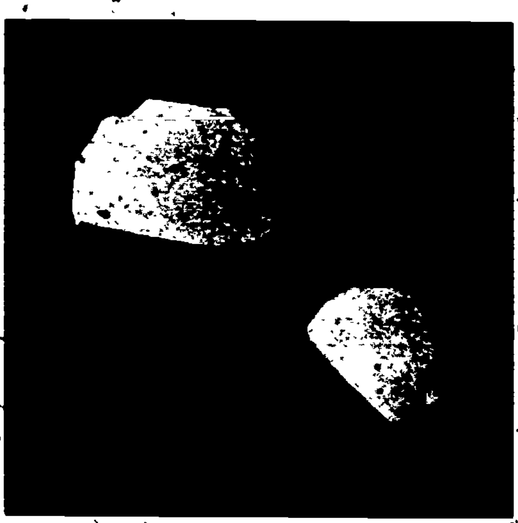
< volume of __volley__ ball.

Listed below are possible ways the children could make these measurements.

1. Circumference (distance around)
   The children could use a string to measure the circumference. Wrap the string around the ball at its "widest" part and then measure the length of the string with a ruler.

2. Surface area
   Worksheet 14 is a page of inch squares. These squares should be cut out and taped on half of each sphere. The total surface area of the ball can then be computed by multiplying by two the number of squares used.





When the children do this activity, they will soon see that the inch squares need to be "folded" or "pushed together" in order to conform to the spherical surface of the ball. (The smaller the ball is, the better.)

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This is an important experience for the children, since later in this unit they will encounter the reverse of this situation when they try to flatten the surface of a sphere. They will find that this cannot be done without "stretching" the spherical surface.

Children with similar balls may want to compare their surface area measurements. For some of the balls, area measurement may vary by as much as two square inches or more, depending on the amount of overlapping and gaps. The children should be able to make approximate measurements even with this rather crude measuring device.

3. Volume
   At this point, a comparative measurement of volume is all that is necessary. This could be done by lining up the balls on a demonstration table according to "size." The children could write on their data sheet, Worksheet 15, where their ball stands in relation to the other balls.

After the children have completed Worksheet 15, discuss their answers with them.

Finally, review some of the properties the class has measured in the last two lessons: length (or distance), area, mag, volume and perhaps weight. Remind the children of the different figures with which they worked — triangle, square, cube, circle, face, sphere. Then ask them to try to think of some properties of these figures that are not measurable properties. Tell them to think about this for the next lesson.

Save the cubes and spheres for Lesson 5.

Lesson 4: NON-MEASURABLE PROPERTIES OF 2-DIMENSIONAL FIGURES

The purpose of this lesson is:

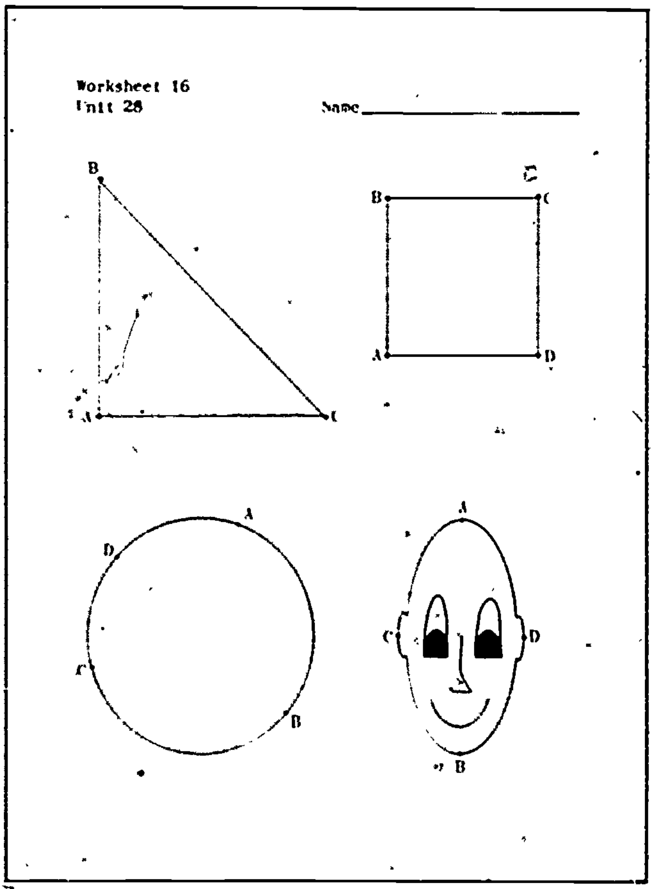
- to study two of the non-measurable properties of 2-dimensional figures. These two non-measurable properties are order and "boundedness."

MATERIALS

- crayons
- story, "The Education of Peter von Hahgg," part 2 (provided in this lesson and also in the Student Manuals)
- Worksheets 16 - 23

PROCEDURE

Activity A



Ask the children to turn to Worksheet 16. They will probably recognize these figures as the same ones they looked at in Lesson 2 (Worksheets 9, 10 and 11), when they were looking for measurable properties. Ask them if they can think of any properties these figures have other than such measurable properties as width, length (distance), area and mag of angles. Accept all reasonable responses and list them on the chalkboard. Although such non-measurable properties as order and boundedness are intuitively obvious, the children may have difficulty verbalizing them. This is to be expected.

After the children have had an opportunity to think about non-measurable properties of the figures on Worksheet 16, have them turn to the second installment of "The Education of Peter von Hahgg." It follows Worksheet 16 in the Student Manual. In the story, Peter and Henry have to think about the order of points on a curve as a non-measurable property. Another character, Ho Hum Bug, helps them and the children visualize the order of the points as being a constant; that is, the order of the points remains the same no matter where the observer stands.

ERIC

THE EDUCATION OF PETER VON HAHGG

Part 2

It was very, very late in the morning, but Peter von Hahgg was still snoring soundly in his corn crib. He probably would have slept the whole day through, except that the Saturday sunshine, flashing in through the slats, hit him in the eyes and awakened him. Rolling out of bed and rubbing his eyes, he exclaimed, "I'd better get moving if I want to get anything done today!"

He threw on his clothes and washed his face and it was while he was brushing his teeth that he remembered what it was he wanted to do that day. He wanted to go over to the library and read up on "things mathematical." Peter was a bit weary of hearing Henry tell about what he was studying at Little Red School. He had decided that with a little study here, a little more reading there, he could out-distance Henry and perhaps teach him a thing or three.

As he hung up his toothbrush, Peter thought to himself, "I just hope I don't run into Henry before I get a chance to go to the library." But, just as Peter came rushing out of the corn crib, he bumped smack into Henry.

"Oh, hi, Henry," gulped Peter, as his hopes went into a downward spiral.

"Hi, Peter," replied Henry. "I was just coming to get you up and ask you to help me with my homework. Well, actually, it's not so much homework. Our teacher, Mr. Picaresque, just told us to think about a few things over the weekend, and I need a little help with the thinking."

"Oh sure," answered Peter, "I'd be glad to help. What's up?"

"Yesterday afternoon in class we talked about measurable properties — you know what that means. Properties like width and length and area are measurable," explained Henry.

"Oh yeah, I know all about that stuff," replied Peter casually. (Actually, he didn't know all about it, but he knew enough to get by.)

"Most everything has measurable properties," continued Henry. "A piece of paper has a length and a width and an area and its angles have mags, and you can measure all those things."

"Yes, yes, I really do understand," said Peter impatiently. "So what are we supposed to think about anyway?"

"Non-measurable properties," answered Henry.

"NON-measurable properties?" asked Peter. "I'm not sure I know what you mean."

ERIC

"Neither do I. That's why we have to think about it," replied Henry.

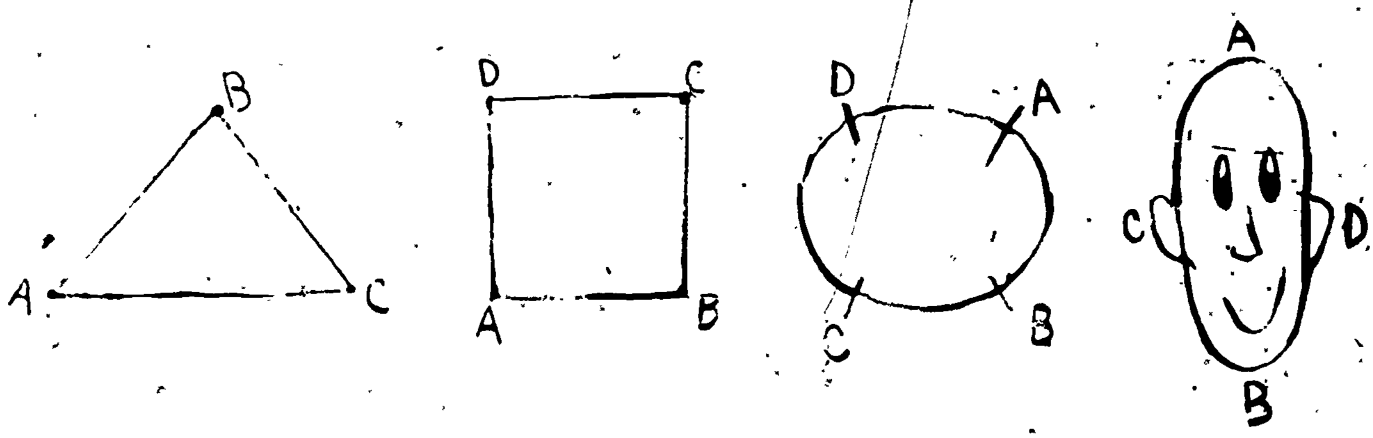
"Oh. Well." said Peter. That's all he could think of to say.

Then he said, "Maybe we should try to figure out what non-measurable means. I suppose it must mean a property that can't be measured with a ruler or a protractor or a clock or any device. Right?"

"That sounds pretty good to me," said Henry, wishing he had thought of it first.

"Do you have something we could study for its non-measurable properties?" asked Peter.

"Let me draw some of the figures we looked at yesterday in class," said Henry. "We looked at their measurable properties, but Mr. Picaresque told us that they have some non-measurable properties, too." Picking up a stick, he drew four figures in the dirt.



"What do all the letters stand for?" asked Peter.

"Beats me," answered Henry. "But they were on the pictures. They might have something to do with non-measurable properties."

Peter von Hahgg puckered up his snout and peered at the drawings, then he gazed at his front hooves as he always did when he thought deeply. After that he uncurled and recoiled his tail once or twice, but despite all these efforts, he was unable to see any NON-measurable properties.

At that moment, Peter glanced up and saw a most peculiar bug fluttering in his direction. The bug, in addition to the usual number of legs and wings and feelers that you find on any bug, had an extraordinarily intricate and colorful design which began on his head and covered his body, but faded into insignificance at his tail. The design swooped and curved and curled — except when it reached his tail, of course, where it stopped and left you somewhat dangling. The colors — azure, emerald green and ruby red, topaze and amber and deep, deep violet — shimmered and glowed in the late morning sun. The bug drifted lazily on the breeze, over to Henry and Peter, where he alighted on Henry's shoulder.

"Good morning, gents," said the bug.

"Good morning," replied Peter. "You're a very beautiful bug, most of you anyway. I'm curious to know why your design stops when it gets to your tail."

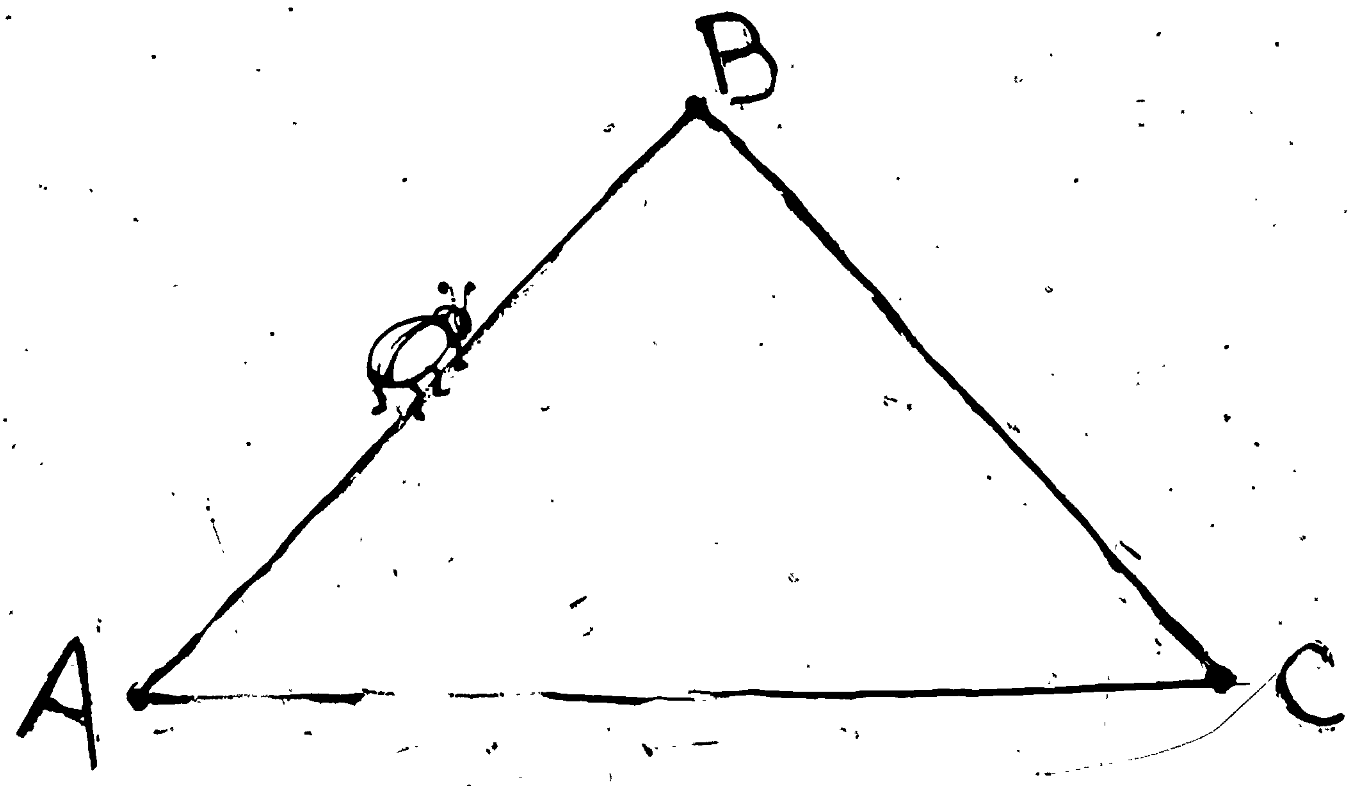
"Oh that," said the bug, yawning and stretching his legs. "Ho-hum....I just never had the energy to finish the design."

"I should like to know," said Henry cautiously, "what kind of a bug you are. I've never seen the likes of you in these parts before."

"Oh, my name is Ho Hum, yes Ho Hum Bug, at your service," answered the exotic, sleepy little bug.

"Well," said Peter, "so long as you are at our service, perhaps you can tell us what the non-measurable properties of these figures are." And he pointed to the figures that Henry had drawn in the dirt.

"Certainly," said Ho Hum Bug, shrugging his shoulders. "Watch me." And he fluttered down and began walking around the first figure, the triangle.



"You notice, fellows, that as I walk, I begin at Point A, then I move towards and through Point B and from there on to Point C. That's the order of my walk — moving from Point A to Point B to Point C. And as long as I keep moving in the same direction, the order of the points I go through is always the same — A, B, C. And the order of these points is a non-measurable property."

"Well," said Henry, to whom this explanation seemed awfully simple, "what about if I stand on the other side of you and the triangle? As I look down at you moving, you are going from my right to my left..."

"But as I look at him, he's going from my left to my right," interrupted Peter. "I don't see that that has anything to do with the order of the points on the triangle."

"But if I go stand on the other side and watch Ho Hum Bug move through the points, won't that change their order?" asked Henry.

"Try it," said Ho Hum Bug. "Try it and see."

So Henry walked around to the other side of the triangle, and Ho Hum kept crawling around the triangle and as he crawled, he still moved through the points in the order A, B and C.

"Why don't you try standing on your head, Henry?" suggested Peter. "Perhaps that will change the order of the points."

So Henry tried standing on his head, but even then, Ho Hum continued to move through the points in the order A, B and C.

"You see, Henry," explained Ho Hum, "it doesn't make any difference at all where or how you are standing. It doesn't make any difference at all if you pick up the triangle and me and move us to the other side of the earth. There will always be an order to the points on this triangle and what's important are those points and me. You've got to try to imagine what it's like to be me, moving through the points."

"OOohhh....I think I finally understand what you mean," said Henry. "For example, it's a property of Peter's face that his eyes are between his nose and his ears. But you can't measure that, it's a non-measurable property."

"Precisely," murmured Ho Hum Bug. "Now that you seem to see the light, I'll be off. I've got a long way to go yet today."

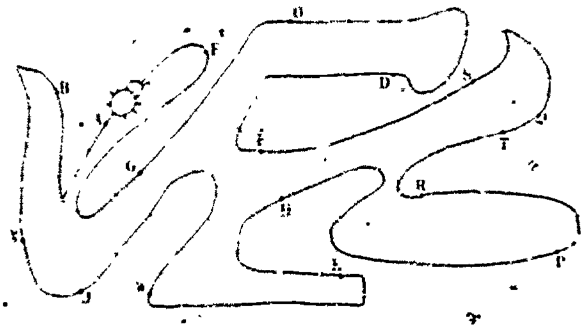
So saying, the exquisite little bug drifted away and Peter and Henry watched him until he was no more than an iridescent gleam in their eyes.

Then Peter turned to Henry and said, "That was very interesting, wasn't it, when Ho Hum said that we have to imagine what it's like to be him moving through the points."

"Yes, it was," answered Henry. "I've never before thought what it must be like to be in someone else's shoes."

Worksheet 17
Unit 28
Name ____________

Ho Hum Bug started crawling along the curve at point A. Next he came to point F. He kept crawling, in the same direction, and crawled all the way back to point A.



Write the order Ho Hum went through the points.

A	F	G	O
D	E	S	T
R	P	H	K
W	J	X	B

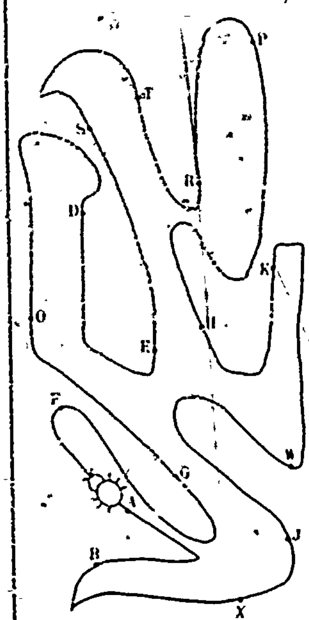
Worksheet 18
Unit 28
Name ____________

Ho Hum Bug started at point A.

Next he came to point F.

He kept crawling in the same direction.

Write the order Ho Hum went through the points.



A	F
G	O
D	E
S	T
R	P
H	K
W	J
X	B

Worksheet 19
Unit 28
Name ____________

Look at the curves on Worksheets 17 and 18.

Compare the curves to each other.

Are the curves the same or different? ____________

If you said the same, tell how they are the same.

order of points is the same

If you said different, tell how they are different.

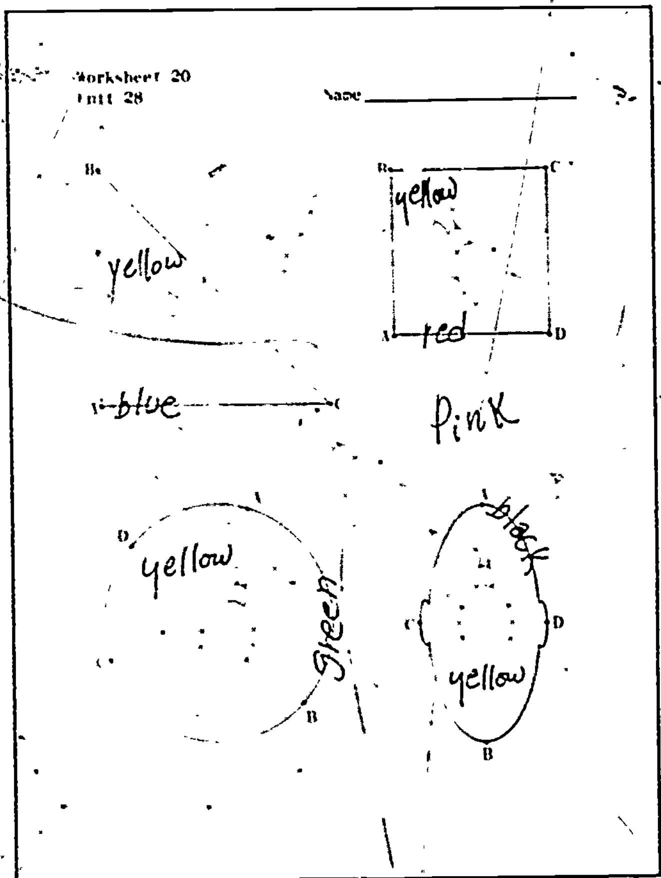
the curve has been rotated

When the children have finished reading the story, have them do Worksheets 17, 18 and 19. Then discuss order as a non-measurable property of the figures on Worksheet 16.

ERIC

Activity B

Have the children complete Worksheets 20 through 23. They will need their crayons for Worksheets 21 and 23. When they are finished, have them look again at Worksheet 20. Ask them if they can think of another non-measurable property these figures have. Eventually, the children should see that these figures have the property of separating an inside region from an outside region. This is called the property of "boundedness."



Worksheet 21
Unit 28 Name____________

Look at Worksheet 20.

Color red the boundary that separates Ho Hum from the other bugs.

Color blue the boundary that separates Waldo from the other bugs.

Color Oscar's boundary green.

Color Opal's boundary black.

Color all the regions that are inside boundaries yellow.

Color pink the region that is outside the boundaries.

How many regions are there? 5

Worksheet 22
Unit 28 Name ____________

Look at Worksheet 20

Without stepping over a boundary,

can Ho Hum visit Waldo? No

can Oscar visit Opal? No

can Ho Hum visit Opal? No

can Oscar visit Maggie? No

can Felix visit Maggie? Yes

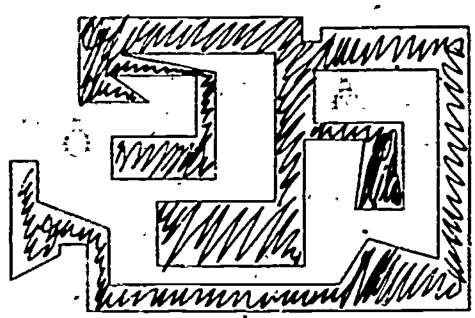
Which two bugs are in the same region? Felix + Maggie

Ho Hum's Rule

If a bug can go from one place in a region to another place, without stepping over a boundary, then those two places are in the same region.

Worksheet 23
Unit 28 Name ____________

Use Ho Hum's Rule to find out if Ho Hum and Maggie are in the same region. Are they? Yes



Color red the region inside the boundary.

Are Ho Hum and Maggie inside the boundary? No

The purpose of this lesson is:

- to study two non-measurable properties of a cube and a sphere. These two properties are order and "boundedness."

In studying both the cube and the sphere, the children discover what happens to the inside and outside spaces when the boundedness property is changed. These ideas will be important when they begin to study transformations in Section 2.

MATERIALS

- 30 cubes (from Lesson 3)
- crayons
- felt tip pen or grease pencil
- 1 ball (any size, preferably volleyball size)
- 1 balloon

PROCEDURE

Activity A

Review the two non-measurable properties that were introduced in Lesson 4 — order and boundedness. Then ask the children to take out the cubes they made in Lesson 3. Ask them if they think order is a property that their cubes might have and have them give reasons for their answers. To illustrate that order is a property of their cubes, have them color corner A red, corner B blue, corner F green, and corner G yellow. The children should see that no matter where they hold the cube (behind them, below them, etc.) corner A (red) is still located between corner B (blue) and corner G (yellow).

Some child might also suggest that the edges or the faces are in a particular order. You could label the faces 1 through 6, as on a die, and observe that opposite sides remain opposite, no matter how the cube is oriented or viewed. Give the children time to discover for themselves that the order remains the same no matter where the cube is held in space.

Ask the children if they think their cubes have the property of boundedness. They should see that the surface of the cube acts as a boundary, separating the space inside the cube from the space outside the cube (that is, if one ignores the openings between edges that are not taped securely).

Suggest that the children think of the classroom as a huge box with the walls, ceiling and floor of the room as a boundary separating the space outside the room from the space inside the room. Ask them what would happen to the space inside the classroom if someone opened the door. Remind the children of Ho Hum's Rule. To illustrate the rule, have one child stand in the hall and another stand at the front of the classroom.

According to Ho Hum's Rule (adapted to 3-dimensional space), if two bugs (or people) can get from one place in a region to another place, without going through a boundary, those two places are in the same region. With the door open, the child in the hall can reach the child in the room without going through a boundary. Therefore, they must both be in the same region when the door is open, even though it may seem that the two children are in separate regions.

The children should realize that by opening the door, the boundedness property of the classroom was changed. The space inside the room is no longer separated from the space outside the room.

Activity B

Gather the children around a demonstration table. With a felt tip pen or a grease pencil, draw a circumference around a ball. Label points on the circumference. Discuss with the class the order of the points as a property of the ball that does not change when the location of the ball is changed.

ERIC

Discuss the boundedness of the ball in terms of the surface of the ball as a boundary that separates the space inside the ball from the space outside the ball.

Next, blow up a balloon. Hold the neck of the balloon closed and discuss its boundedness property. Ask someone if he would like to change the boundedness property of the balloon. The child will probably open the neck passage or he might poke a hole in the balloon — either way, the space inside the balloon will no longer be separated from the space outside the balloon.

These ideas will be important later in the unit when you begin discussing which properties change under a transformation.

SECTION 2 TRANSFORMATIONS

In this section, the children are introduced to the idea of a transformation. A transformation can be thought of as an operation, real or imaginary, that changes a set from one form into another form. The initial form of the set, before the transformation, we shall call the "starting set" and the final form, after the transformation, we shall call the "ending set."

Lesson 6 introduces the words "starting set," "change" and "ending set." The children make their own examples of these things by casting shadows of their hands on paper and outlining them. They are asked to think of the hand as the starting set and its shadow as the ending set and to compare the properties of the two sets.

In Lesson 7, the children investigate several interesting transformations. These include, among others, making a Möbius strip, opening a flip top can, melting an ice cube, burning a candle, watching films of the metamorphoses of a caterpillar into a butterfly and a tadpole into a frog.

In Lesson 8, the children begin to develop some notion of what constitutes a "useful" ending set and which properties must be retained under transformation and which properties may change under transformation and still produce a "useful" ending set. In this lesson, they do this by studying several different checkerboards, actually playing checkers and making their own checkerboards. They see that for this situation, a "useful" checkerboard is one on which they are able to play checkers, no matter how odd the board may look at first glance. They also discover that to produce such a "useful" checkerboard, there must be one to one matching between the starting set (standard checkerboard) and the ending set (their special checkerboards) and that the non-measurable properties of boundedness, in-out and order must be retained, while the measurable properties may change.

In Lesson 9, these ideas are developed further. In this lesson, a "useful" ending set is defined as a "recognizable" one, and the children learn that the non-measurable properties must be retained under transformation if a "recognizable" ending set is to be produced.

You will see that as the children progress through this section, their idea of what a transformation is narrows down. They begin with the idea that there are many kinds of transformations and then focus their attention on a special kind of transformation that produces a "useful" or "recognizable" ending set. This kind of transformation occurs when the non-measurable properties are retained under transformation while the measurable properties may or may not change. This is called a topological transformation, although the children will not learn this term until later on in Section 3.

Lesson 6: TRANSFORMATIONS — HANDS AND SHADOWS

The purposes of this lesson are:

- to introduce the children to the terminology of "starting set" and "ending set";
- to study which properties change and which properties do not change when a "starting set" is transformed into an "ending set."

MATERIALS

- 1 balloon (optional)
- light source for casting shadows (if the sun is not shining, use several projectors such as a filmstrip projector, an overhead projector, a film projector, etc.)
- 30 large sheets of paper such as newsprint
- pencils
- Worksheets 24 - 29

PROCEDURE

Activity A

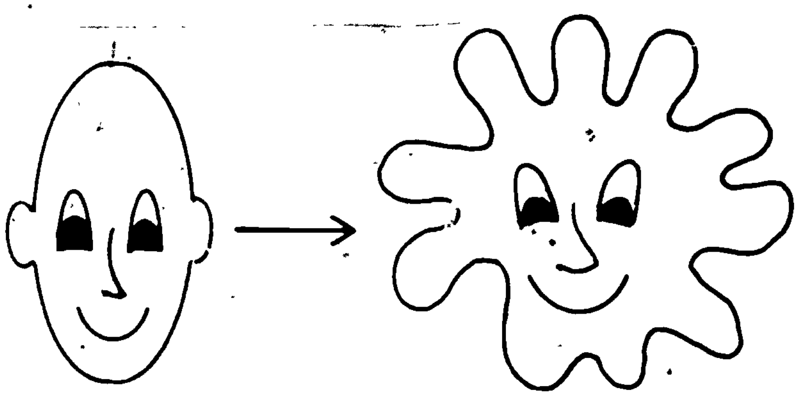
Briefly review some of the different properties discussed in earlier lessons, such as measurable properties (length, area, etc.) and non-measurable properties (order and boundedness). Tell the children that they will be using what they have learned about properties to discover some properties that change.

Review the balloon exercise from Lesson 5. Ask the children to think of the blown-up balloon as a "starting set." Explain that the "change" would be to poke a hole in the balloon, and that an "ending set" would be the deflated balloon. If the children have difficulty visualizing this, you may want to do this activity again with the balloon.

Ask the children what property of the "starting set" changed. They will probably say its boundedness changed. By poking a hole in the balloon you have changed the boundedness property because the rubber surface no longer bounds or holds the air in the inside space. The inside space is no longer separated from the outside space.

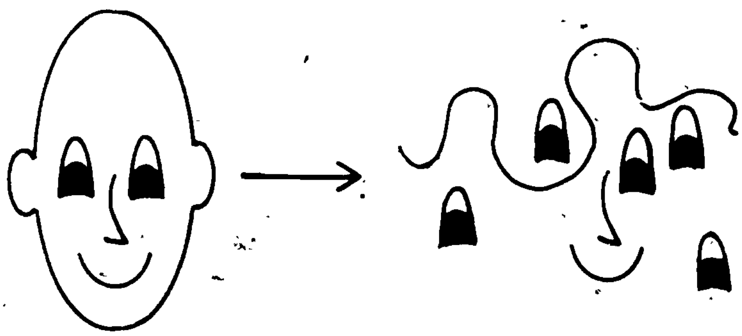
To further reinforce the ideas of "starting set," "change" and "ending set," draw the following sets on the chalkboard and label them as shown. In each case, discuss which properties changed from the "starting set" to the "ending set."

starting set — ending set

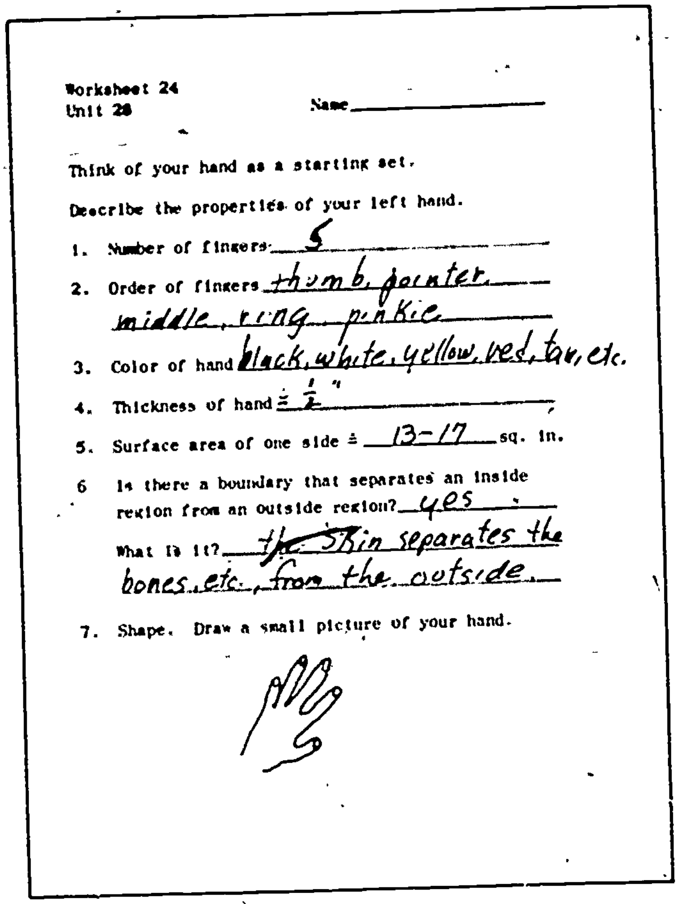


The properties that changed from the "starting set" to the "ending set" include shape, area and width.

starting set — ending set



In this example, the properties that changed from the "starting set" to the "ending set" include boundedness, one to one matching, order and shape, as well as such measurable properties as area and length.



Worksheet 24
Unit 28

Name________________

Think of your hand as a starting set.

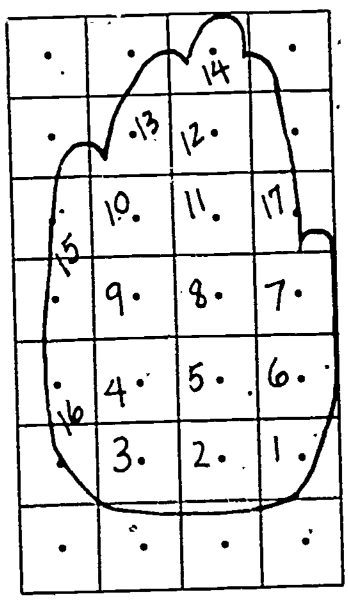
Describe the properties of your left hand.

1. Number of fingers: 5
2. Order of fingers thumb, pointer, middle, ring, pinkie
3. Color of hand black, white, yellow, red, tan, etc.
4. Thickness of hand ≐ 1/2"
5. Surface area of one side ≐ 13-17 sq. in.
6. Is there a boundary that separates an inside region from an outside region? yes

   What is it? the skin separates the bones, etc., from the outside.
7. Shape. Draw a small picture of your hand.

Activity B

Have the children open their manuals to Worksheet 24. Tell them that their left hand is going to be their "starting set" and on this worksheet they are to describe the properties of their "starting set." Go through each property with the children as they fill in the appropriate information. Question 5 asks for the surface area of one side of their hands. To make this measurement, have them tear out one of the area grids, Worksheets 27, 28 and 29. (Incidentally, reduced copies of these worksheets are not included in this lesson. They are exactly like Worksheet 14; see page 32.) They should lay a hand on the grid, with their fingers close together, and outline the hand. Remind them to include as many whole squares as possible. By counting the squares and parts of squares on the grid, the children should be able to get an approximate area measurement, probably from 13 to 17 square inches. The children should number the squares as they count them, as shown in the illustration below.



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When the children have completed Worksheet 24, tell them that they are going to use shadows of their hands for their "ending set." Then discuss the following procedure with them. You may want to list this procedure on the chalkboard.

1. Each person gets a partner.

2. Each pair gets a large sheet of construction paper or newsprint.

3. The class will go outside if the sun is shining. Otherwise, they will use projector equipment such as an overhead projector, filmstrip projector, etc., inside the classroom.

4. Each child will cast the shadow of his hand on the large piece of paper while his partner outlines the shadow with a felt tip pen or a pencil. Then they will trade jobs — the second child will cast the shadow of his hand and the first child will outline it. Tell the children that all of their shadows do not have to look alike. They should label each shadow-casting with the proper child's name.

5. When they are finished outlining their hand shadows, the children will return to the classroom and complete Worksheets 25 and 26.

Worksheet 25
Unit 28 Name__________

Answers will vary

Think of the shadow of your hand as an ending set.

Describe the properties of your hand's shadow.

1. Number of fingers__________
2. Order of fingers__________
3. Color of shadow__________
4. Thickness of shadow__________
5. Surface area of shadow ≐__________sq. in.
6. Is there a boundary that separates an inside region from an outside region? yes
   What is it? outline of the shadow separates inside region from outside region
7. Shape. Draw a small picture of your hand's shadow.

Worksheet 26
Unit 28 Name__________

Look at the properties of your starting set (your left hand) on Worksheet 24.

Look at the properties of your ending set (your hand's shadow) on Worksheet 25.

Which properties changed from the starting set to the ending set?

Answers will vary

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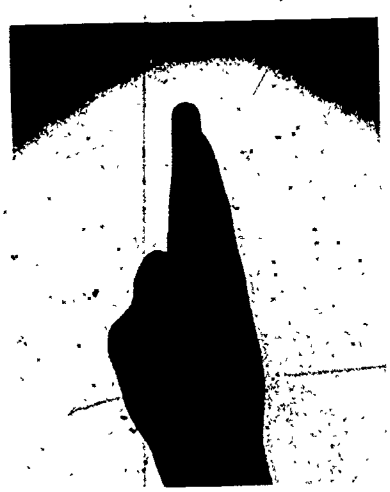
Ask the children whether they think the properties of their "ending set" will be the same as the properties of their "starting set." Then proceed with the activity.

Be sure the children understand that the actual shadow is their "ending set" and that the outline they make is only a record of the "ending set." Emphasize this point as they do this activity.

Question 5 on Worksheet 25 asks for the surface area of the shadow. To make this measurement, the children can use one of the surface grids provided (Worksheets 27, 28 and 29). When discussing the answers on Worksheet 25, you may want to have a projector set up for the children to use to illustrate their ideas. They would be especially helpful if someone claims that the number of fingers has changed. It is also helpful in showing how the order and shape of the hand have changed.



Other properties that will change are:

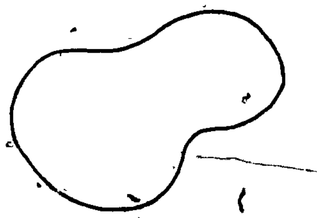
1. Boundedness
There will still be a boundary separating an inside region from an outside region, but the kind of boundary has changed. This is a result of going from a 3-dimensional

object to a 2-dimensional surface. In the "starting set," the skin is the boundary and in the "ending set," the shadow outline is the boundary.

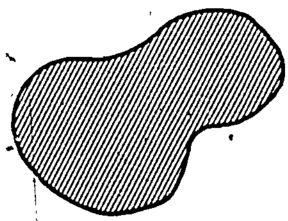
If the children have difficulty with these differences in kinds of boundaries, the following analogy may help to make it clearer.

> In the balloon example, the surface of the balloon separated the inside space from the outside space. If a hole is poked in the balloon, the air inside the balloon is no longer separated from the air outside the balloon. The skin of the hand separates the inside region from the outside region. If the skin is cut, the flesh, bones, etc. inside the hand are no longer separated from the outside.

Draw a boundary on the chalkboard.



Explain that this boundary separates the region inside (shade in the region inside the boundary) from the region outside.



The outline of the hand's shadow acts as a boundary to separate the inside from the outside. When we talk about 3-dimensional space, a surface serves as a boundary. When we talk about 2-dimensional space, a closed curve serves as a boundary.

2. Color
The "starting set" (hand) could be black, brown, white,

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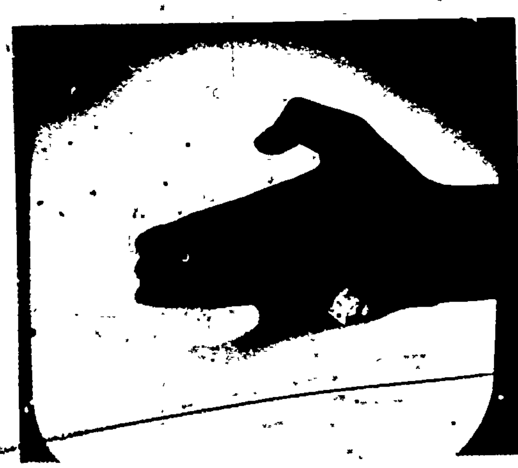
yellow or red. The "ending set" (shadow) is a darker shade of the color of the surface on which it is cast. In discussing this question indoors, the children may just look at their paper with the shadow outline on it and say that the "ending set" is the color of the paper. Remind them that this is only a record of the "ending set." Ask them what color the shadow was.

3. Thickness
The "starting set" (hand) is about ½" thick. The "ending set" (shadow) has no thickness.

4. Surface area
This will vary depending on the size of the shadow outline. However, the "ending set" could have the same area as the "starting set."



Lesson 7: TRANSFORMATIONS — MÖBIUS STRIP AND OTHERS

The purposes of this lesson are:

- to reinforce the notions of starting set and ending set;
- to provide more practice in describing which properties change and which do not change under a transformation.

MATERIALS

- 30 pieces of masking tape (approximately 1" long)
- 30 pairs of scissors
- crayons
- 30 rulers
- 1 unopened flip-top can, such as soda pop comes in
- a few sheets of paper
- matches
- films: Monarch Butterfly Story, Encyclopedia Britannica Film
  Frog's Life, Coronet Film
- mealworm and beetle colony (optional)
- 1 candle
- 1 ice cube
- Tinkertoys (optional)
- Worksheets 30 and 31

PROCEDURE

Activity A

In this activity, the children construct an ending set that is called a Möbius strip. When they investigate which properties change from the starting set to the ending set, they will be surprised to find that the number of surfaces and the number of edges changes unexpectedly.

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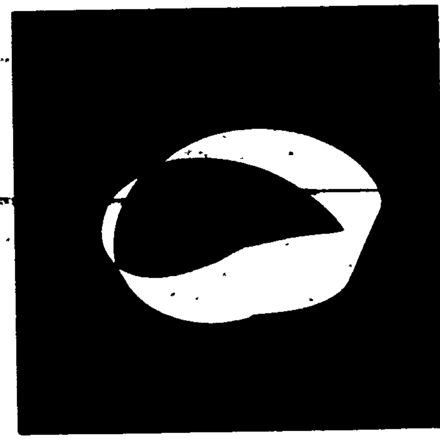
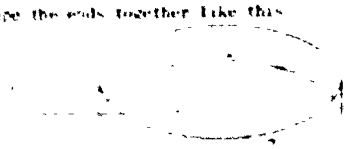
Each child will need his manual, a scissors, a 1" piece of tape, a ruler, and his crayons. Have the children turn to Worksheet 30. Go through the worksheet with them, giving assistance when necessary. After they have answered the questions at the bottom of Worksheet 30, have them follow the instructions on Worksheet 31. You may want to demonstrate for the children how to twist the strip before they try it.



Worksheet 30
Unit 28 Name ______________

Cut off the strip on the right side of this page along broken line.

Tape the ends together like this.



This is your starting set.

Color one surface red.
Color one surface blue.

* Properties of your starting set

1. Length 10 inches
2. Width 1 inches
3. Number of edges 2
4. Number of surfaces 2

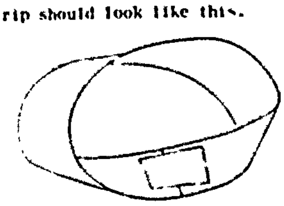
Worksheet 31
Unit 28 Name ______________

Untape the ends of your paper strip (starting set).

Twist the strip in a half turn.

Retape the ends.

Now your strip should look like this.



This is your ending set.

* Properties of your ending set.

1. Length = 10 inches.
2. Width = 1 inches.
3. Number of edges 1
4. Number of surfaces 1

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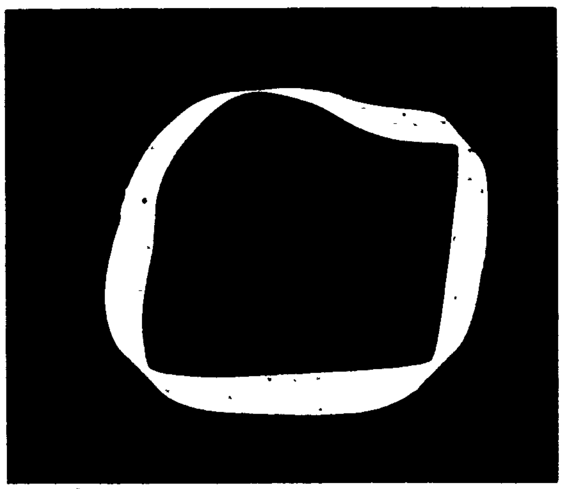
After the children have retaped the ends, ask them how many surfaces the strip has. They will probably say two. Then ask them to color one of the surfaces black. The children will discover that the strip now has only one surface. Ask them how many edges they think the strip has. (Answers may vary; some children will probably not be so sure of their instincts.) Tell them to make little tears in the edge, starting at any point on the edge.

They should continue making little tears along the edge in one direction until they get back to the starting point. The children will discover that the strip now has only one edge. Have them answer the questions at the bottom of Worksheet 31.

Discuss what happened to the strip; i.e., by twisting the strip, they changed the property of the number of surfaces on the strip and also the property of the number of edges.

Ask the children what they think would happen to the strip if they were to cut it down the middle. Most of the children will probably say that the strip will become two pieces. Have them cut down the middle of the strip.

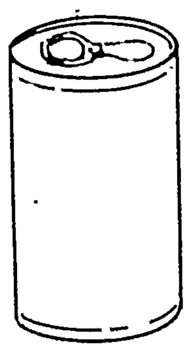
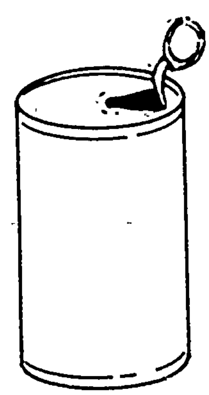


To their surprise, they will still have a taped strip, all in one piece. The children should check the number of surfaces and edges again. They will discover that now there are two surfaces and two edges. Ask them what properties of the starting set changed when it was transformed into the ending set. (Length and width.)

Activity B

In this activity, several other transformations are described. The examples embody the ideas about transformations that were introduced in this lesson and the previous one. You and the children will probably think of many more examples of transformations.

As much as possible, use the actual equipment described to perform the transformations. Then discuss what happened. Stress the terms "starting set" and "ending set," and use the words "change" and "transformation" interchangeably. Also discuss which properties change and which are retained under transformation. After each example, some of the properties that change or are retained have been listed. Be sure to do the first example described, because it reinforces the ideas about boundedness that were discussed in Lesson 6.

1.	starting set	transformation	ending set
		open the can	
	flip top can (closed)		flip top can (open)

Discuss the change in the boundedness property. Stress the idea that the soda pop inside the can is no longer separated from the outside region after the can has been opened.

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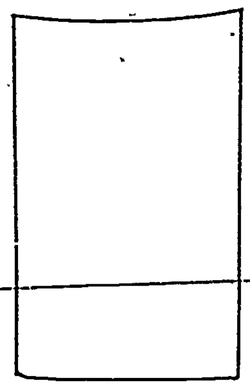
2. starting set | transformation | ending set

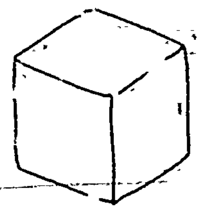


burn the paper

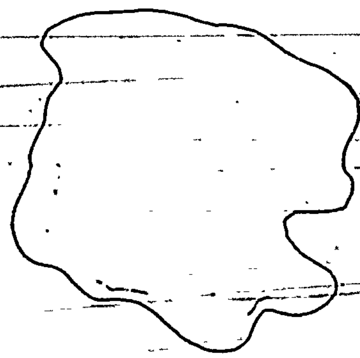
piece of paper | pile of charcoal

From the starting set to the ending set, the properties that changed include: shape, width, area, color, texture, order, boundedness, etc.

3. starting set | transformation | ending set



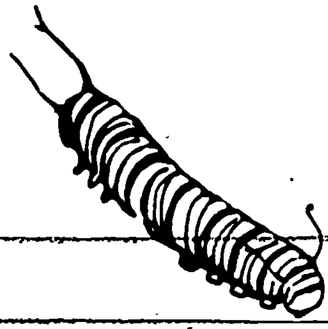
ice cube melts



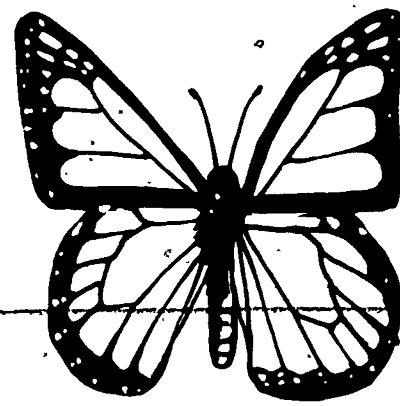
ice cube | puddle of water

From the starting set to the ending set, the properties that changed include: shape, boundedness, order, surface area, etc. Properties that were retained include: amount of material (volume).

4. <u>starting set</u> <u>transformation</u> <u>ending set</u>



metamorphosis



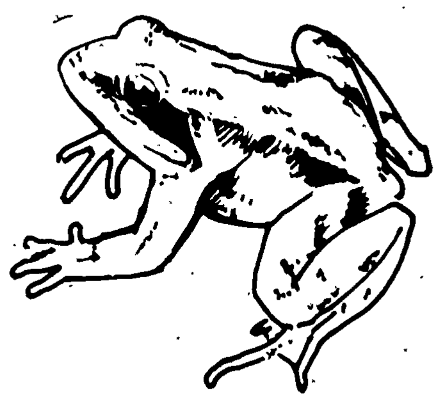
caterpillar

Monarch butterfly

If it is possible, use the Encyclopedia Britannica Film, <u>Monarch Butterfly Story</u>, which shows this transformation in slow motion.

5. <u>starting set</u> <u>transformation</u> <u>ending set</u>

metamorphosis

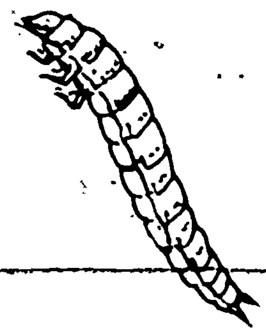


tadpole

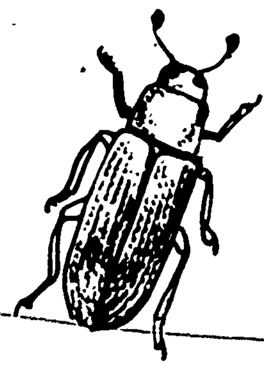
frog

Use the Coronet film, <u>Frog's Life</u>, to show this transformation.

6. | starting set | transformation | ending set



metamorphosis



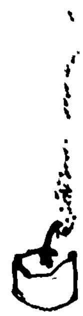
mealworm

beetle

Do this example if you have a mealworm and beetle colony in your classroom (or borrow a colony from another classroom).

7. | starting set | transformation | ending set

candle burns



candle

burned-out candle

From the starting set to the ending set, the properties that changed include: length, shape, width, one to one matching, order, etc. Properties that did not change include: color of wax.

8. starting set transformation ending set

From the starting set to the ending set, the properties that changed include order. Properties that were retained include: measurable properties of the sticks, their color; one to one matching may or may not change.

Lesson 8: TRANSFORMATIONS OF A CHECKERBOARD PATTERN

The purposes of this lesson are:

- to investigate transformations by transforming checkerboard patterns;

- to show the children that to produce a "useful" ending set of a checkerboard, the ending set must retain:

  1. a one to one correspondence with the elements of the starting set;
  2. the same boundedness property the starting set has;
  3. the original order of the distinguishing characteristics;
  4. the "in-out" property, that is, what was inside the boundary must remain inside and what was outside the boundary must remain outside. (The term "in-out" property is introduced in this lesson.)

In Activity A, the children become familiar with the game of checkers and the checkerboard. In Activities B and C, the children perform transformations upon the checkerboard patterns. Therefore, you may want to do Activity A one day, and Activities B and C the next.

MATERIALS

- 30 small plastic bags
- 1 commercial checkers game (optional)
- 30 pairs of scissors
- pencils
- crayons
- construction paper, at least 10 sheets
- paste
- Worksheets 32 - 42

PREPARATION

For Activity B, tear 30 pieces of paper that measure approximately 3" x 3". Do not cut these on the paper cutter.

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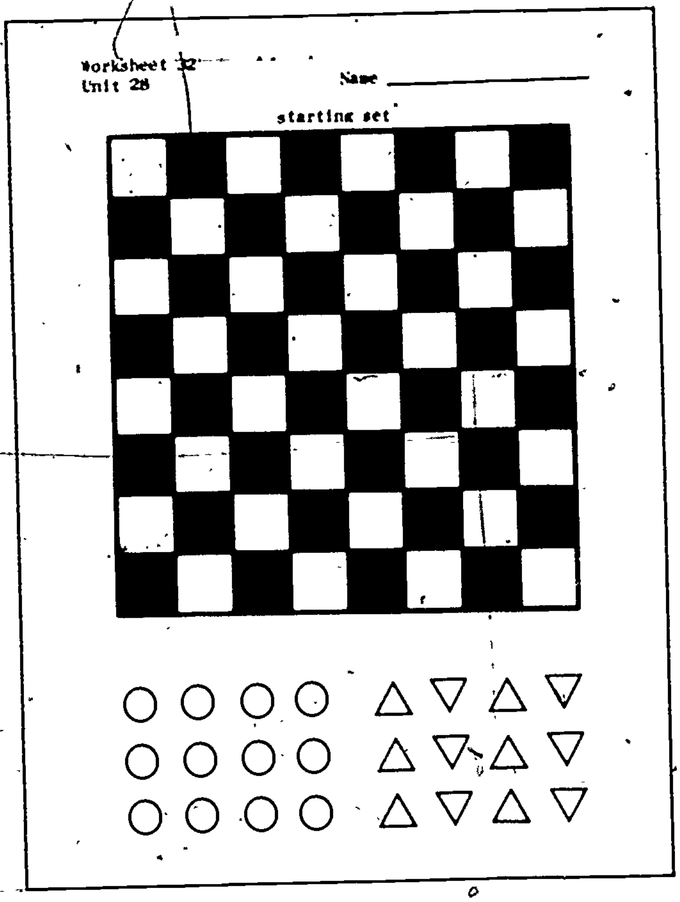


PROCEDURE

Activity A

Have the children tear out Worksheet 32 (standard checkerboard and checkers). To find Worksheet 32 tell them to turn to the back of their manuals. They should cut out the 24 checkers at the bottom of the worksheet and label them with their initials. Give each child a small plastic bag in which to keep his checkers and checkerboard.

Ask the children if they have ever played checkers. Discuss how their checkerboard and checkers are different from the checker sets with which they are familiar. (Their checkerboard has black and white regions; a regular checkerboard has black and red regions. Also, their checkers are white circles and white triangles, while regular checkers are red and black circles.) If you have a commercial checkerboard in the classroom, let the children compare it to their checkerboards.

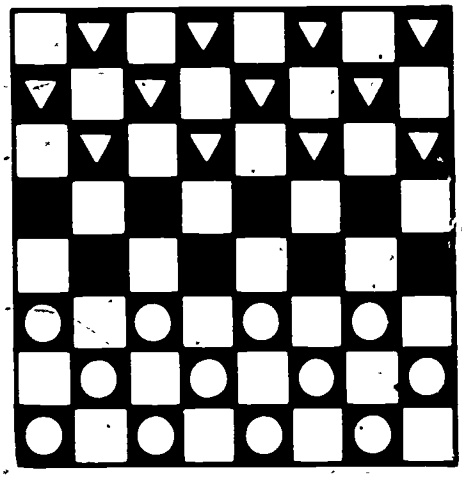
Tell the children that they will be doing some interesting activities with checkers and checkerboards, but first, they must become familiar with the rules of the game, which are given below. Go through the rules with the class and then have the children find partners with whom to play the game. Since each pair will need only one set of checkers and checkerboard, one set should be put away for future use. If you have several children who have never played checkers before, you may want to pair them with students who are familiar with the game.

Rules for playing Checkers

The main object of the game is to capture all of the opponent's checkers. Each player moves his checkers toward his opponent's beginning row.

1. The players sit opposite each other.

2. Each player has 12 pieces called "men" or "checkers." One player has 12 triangles, the other has 12 circles.



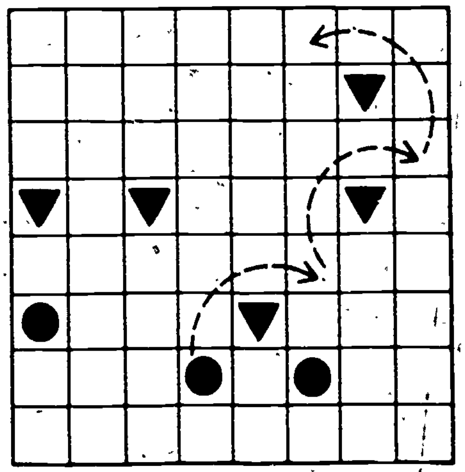
3. Each player arranges his checkers on the black squares of the first three rows.

4. Until it is crowned, a checker may only be moved forward diagonally.

5. The checkers may only be moved on the black squares.

6. When a checker reaches the beginning row on the opponent's side, it is crowned with a checker from those that the opponent has captured, and thus becomes a "king."

7. A king may be moved forward or backward diagonally.



8. A checker may jump and capture an opponent's checker if it is on an adjacent diagonal square and the square adjacent to the opponent's checker is empty. See diagram. (More than one checker can be captured at a time.)

Here the circle "man" jumps three triangles in one move. The circle becomes a king.

Tell the children to plan their moves carefully in order to put the other player in a defensive position.

Activity B

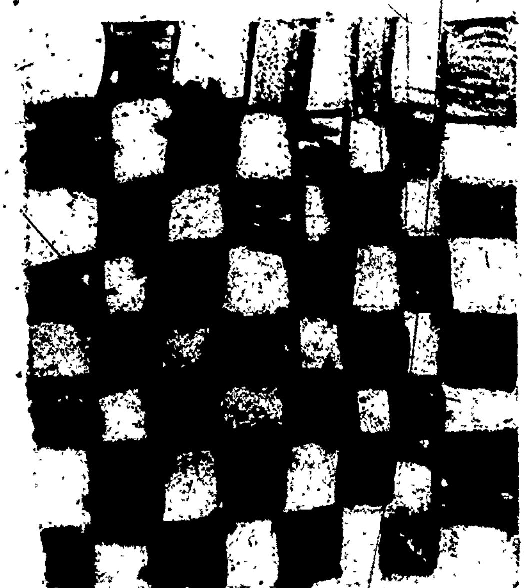
Have the children take their checkerboards out of the plastic bags. Give each child a piece of construction paper that measures approximately 3" x 3".

Ask the children to think of their checkerboard as a starting set. Tell them you want them to draw an ending set of their checkerboard on the piece of paper you gave them. They must follow these rules:

1. They cannot use a ruler or a straightedge to draw their ending sets.

2. It must be possible to play a game of checkers on the ending set, following the standard rules, with the regular number of checkers (assuming that the checkers would be small enough to fit on the ending set).

With these rules in mind, the children should begin. Some children may hesitate, while others will begin quickly to draw squares. In any case, have extra pieces of paper available, for the children will probably discover that to have a usable checkerboard, they need 8-regions x 8-regions, alternating black and white. That is, there must be one to one matching of each region of the starting set to each region of the ending set (the same number of black regions and white regions). Also, the property of order must be retained from the starting set to the ending set.



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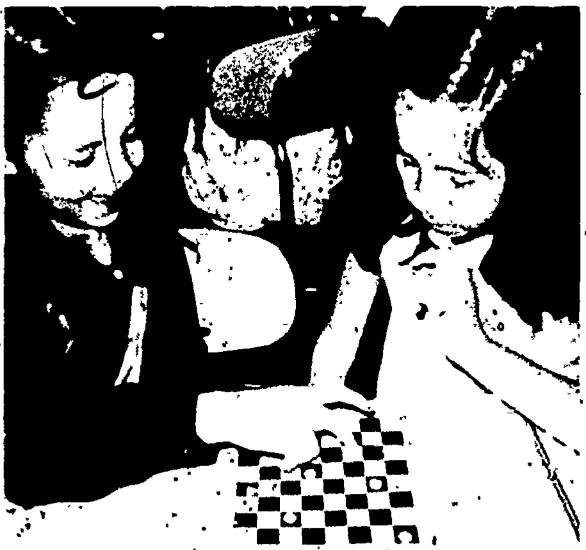
Ask the children to look at Worksheets 33 through 40. These worksheets show examples of different ending sets of a checkerboard. Ask the children if they can decide which ending sets would be useful as checkerboards. One way to decide would be to try to play checkers on each checkerboard (ending set). Suggest this if the children don't. Then let them find partners and play checkers. When they are finished playing checkers, they should classify the different checkerboards (ending sets), according to whether they are useful or non-useful ending sets. They can record their results on Worksheet 41.



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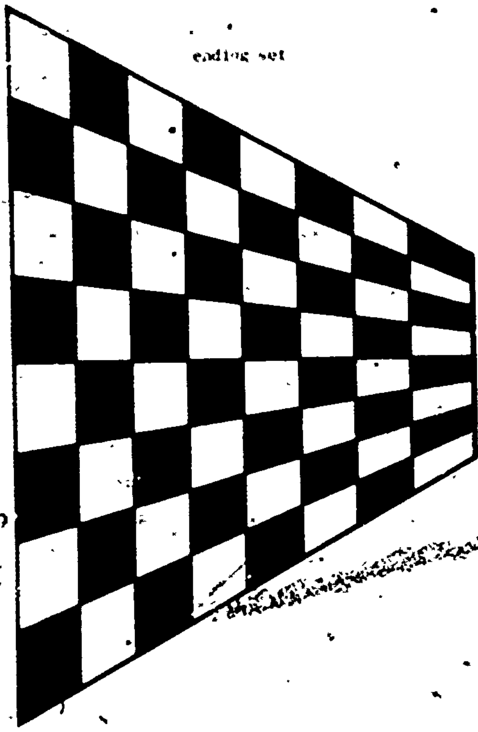
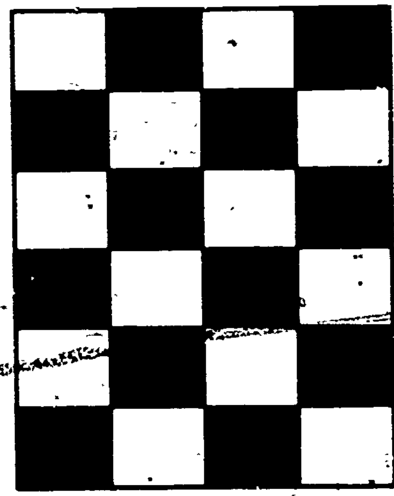
Worksheet 33
Unit 28 Name ____________

ending set

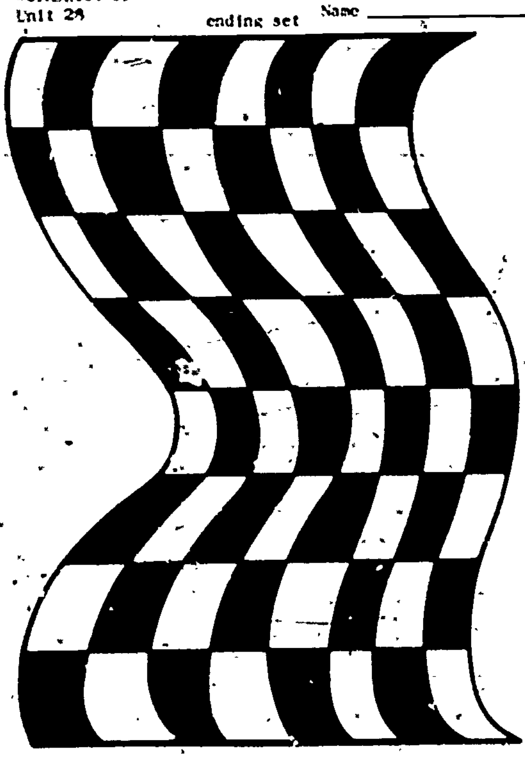


Worksheet 34
Unit 28 Name ____________

ending set

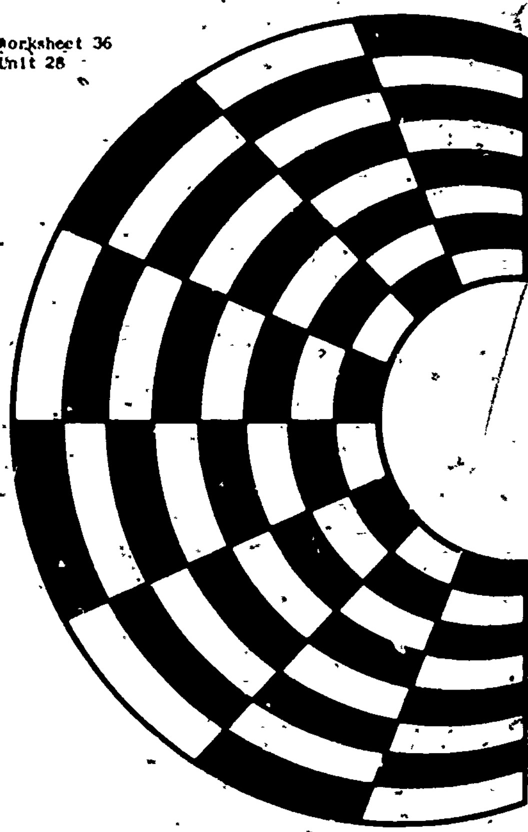


Worksheet 35
Unit 28 ending set Name ____________

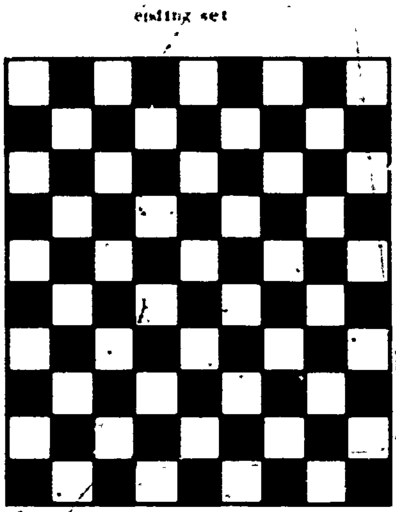


Worksheet 36
Unit 28

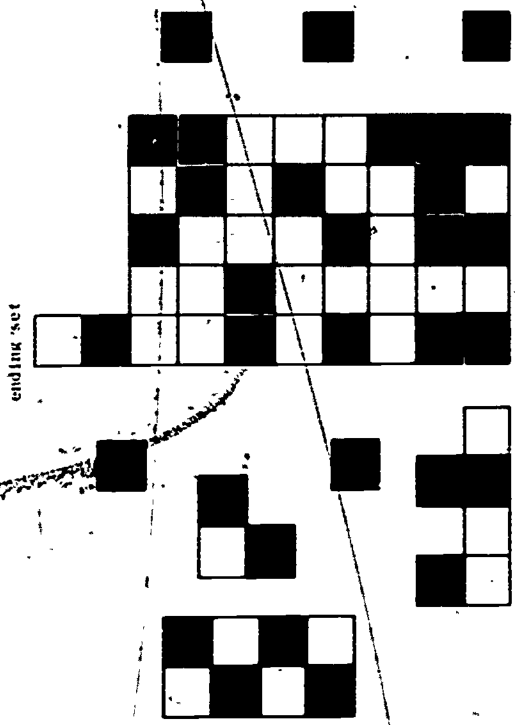
ending set



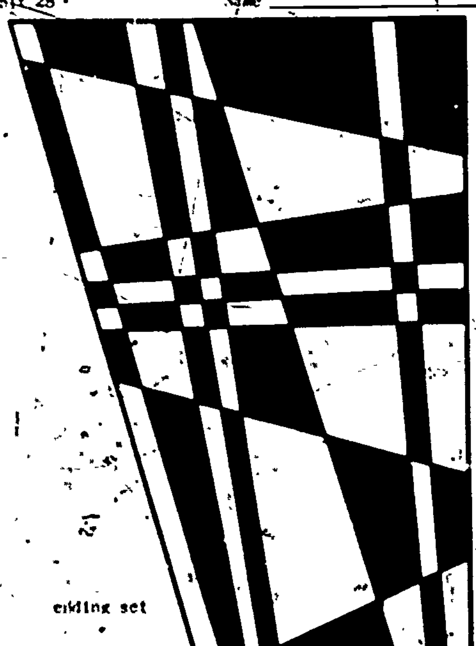
Worksheet 37
Unit 28 Name ____________



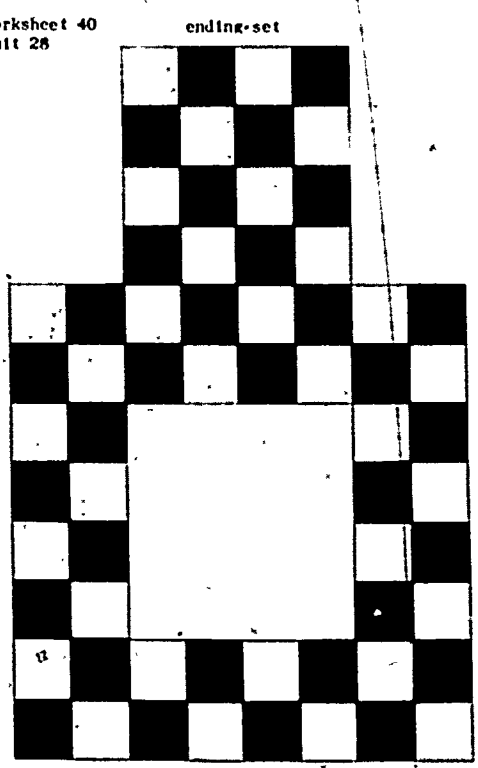
Worksheet 38
Unit 28 Name ____________



Worksheet 39
Unit 28 Name ____________



Worksheet 40
Unit 28



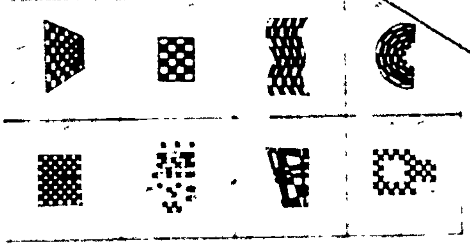
Worksheet 41
Unit 28 Name________

Record Sheet

Cut out the ending sets at the bottom of the page. Paste them in the correct column.

Useful ending sets	Non-useful ending sets
Worksheet 33	Worksheet 34
Worksheet 35	Worksheet 37
Worksheet 56	Worksheet 38
Worksheet 39	Worksheet 40

Ending Sets



Worksheet 42
Unit 28 Name________

Have your partner help you answer these questions.

★ What are the properties of the useful ending sets?

- They keep 1 to 1 matching with the starting set (same number of black and white regions).
- They all keep the same order as the starting set (alternating black and white regions).
- They are all distorted.
- The "boundedness" property didn't change.
- The "in-out" property didn't change.

★ What are the properties of the non-useful ending sets?

- 38 + 40. "Boundedness" property changed.
- 40. "In-out" property changed.
- 34 + 37. Do not keep 1 to 1 matching of regions with starting set (34 has too few + 37 has too many).
- 38. Order has changed (black + white regions don't alternate).
- 40. The region that was inside is no longer in the correct order.

When the children have finished Worksheet 41, they should try to figure out the properties of the useful sets and also the properties of the non-useful sets and record these properties on Worksheet 42. While they are completing Worksheet 42, remind them to look at their starting set (the standard checkerboard) so they can see which properties changed and which properties did not change.

The reduced copy of Worksheet 42 contains more answers and detail than is expected from the children. However, these answers should be helpful when you discuss Worksheets 33 through 42 with the class. You may want to write some of them on the chalkboard. On the reduced copy of Worksheet 42, the term "in-out" property is used for the first time. Use this term when you discuss Worksheet 40. Tell the children the in-out property refers to whether or not what is inside the boundary in the starting set remains inside the boundary in the ending set and to whether or not what is outside the boundary remains outside. On Worksheet 40, the in-out property

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has changed from the starting set, and because of this change, the checkerboard is not a useful ending set.

Activity C: Checkerboard Contest

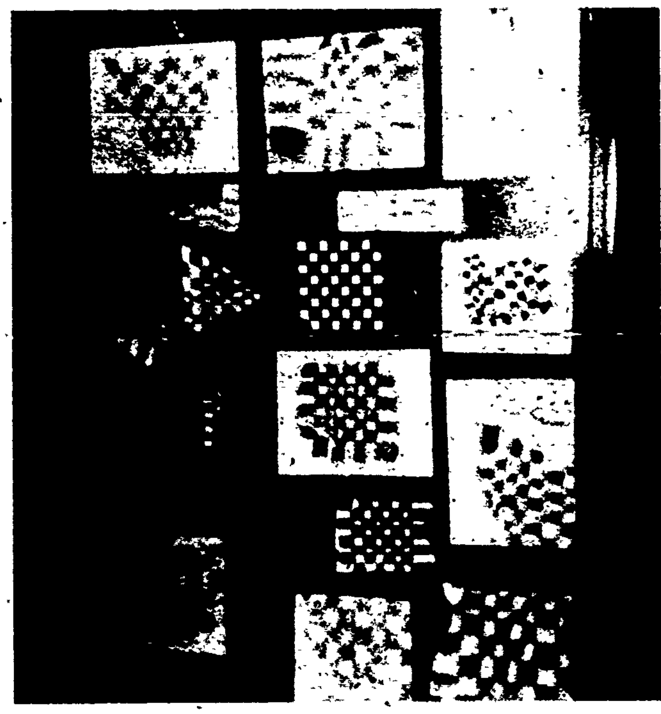
The purpose of this contest is to give the children an opportunity to construct their own transformations of their checkerboards. Review with them the following rules for a useful ending set.

1. There must be one to one matching of each region of the starting set to each region of the ending set. (There must be the same number of black regions and the same number of white regions.)

2. The original order of the regions must be retained in the ending set.

3. The original in-out property of the regions must be retained in the ending set.

The winner of the contest is the child who creates the most unusual transformation of his checkerboard, while following these rules.



Lesson 9: ONE TO ONE TRANSFORMATIONS

The purposes of this lesson are:

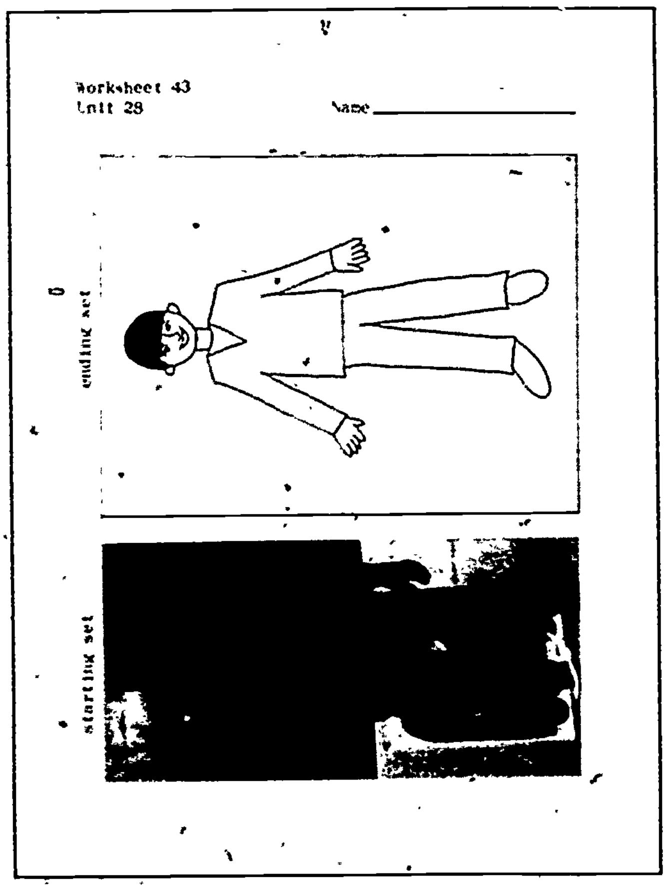
- to show that non-measurable properties of a set must be retained under a one to one transformation if the ending set is to be recognizable. These properties include order, boundedness and in-out;

- to show that even if the measurable properties of a set change, the ending set is still recognizable.

A one to one transformation is a transformation in which all the elements (objects) of the starting set are retained in the ending set. That is, the objects in the starting set can be matched one to one with the objects in the ending set.

MATERIALS

- pencils
- Worksheets 43 - 49



PROCEDURE

Have the children tear Worksheet 43 out of their manuals. Ask them what the starting set is. (A photograph of a boy.) Then ask them what the ending set is. (A drawing of the same boy.) Ask how they can tell that it is a drawing of a boy and not, for example, a drawing of an apple, or a truck, or a cat, etc. The children should see that the drawing has features similar to those of the starting set, such as two arms, two legs, two hands, five fingers on each hand and a body. These similar features are also in a particular order; the head and the legs are at opposite ends of the body, etc. The order of these features helps to make the drawing recognizable as that of a "boy."

Remind the children that by keeping certain properties, they were able, in the last lesson, to make ending sets (checkerboards) that were useful for playing checkers. Ask them what they think would happen if they drew ending sets of the boy that did not keep such properties as order, boundedness, in-out and one to one matching of the main features. Suggest that one way to find out is to try making just such drawings.

Have the children turn to Worksheet 44. Tell them that on this worksheet they should draw an ending set of the boy in which the features of the starting set cannot be matched one to one with the features of the ending set. You may have to review briefly what one to one matching of similar features means: 2 eyes in the starting set, 2 eyes in the ending set; 2 ears in the starting set, 2 ears in the ending set. Examples in which one to one matching is not retained are: 2 eyes in the starting set, 4 eyes in the ending set; 2 ears in the starting set, 0 ears in the ending set.

When the children have finished Worksheet 44, ask them if they think their ending set is recognizable as a drawing of a boy. (No, because boys don't have 3 eyes or 4 legs, etc.) Have them turn to Worksheet 45. On this worksheet, they should draw an ending set that retains one to one matching of all the main features of the starting set. But, they must change the order of the main features. Worksheets 46 and 47 should be completed in the same way, except that on Worksheet 46, the property of boundedness should be changed and on Worksheet 47, the in-out property should be changed. When the children do Worksheet 46, ask them to think of the outline of the boy as the boundary that separates the inside region from the outside region.

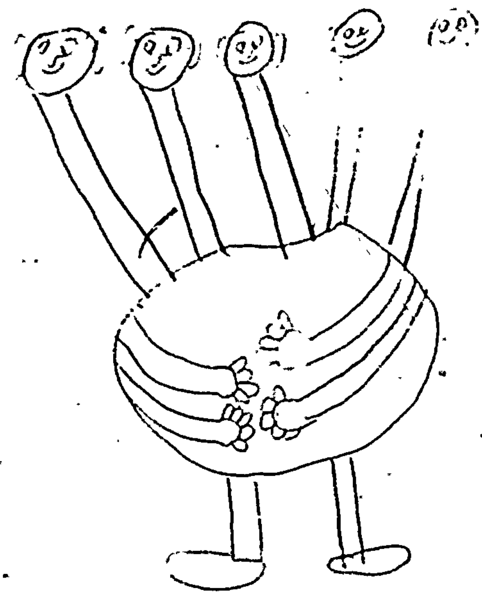
When the children have completed their drawings, have them answer the questions on Worksheet 48. Discuss what properties must be kept to produce a recognizable ending set of a boy. (It must be a one to one transformation. The properties of order, boundedness and in-out must be kept. These are all non-measurable properties.) Ask the children whether or not they think it is necessary to keep all the measurable properties

Worksheet 44
Unit 28

Name ____________

starting set

ending set



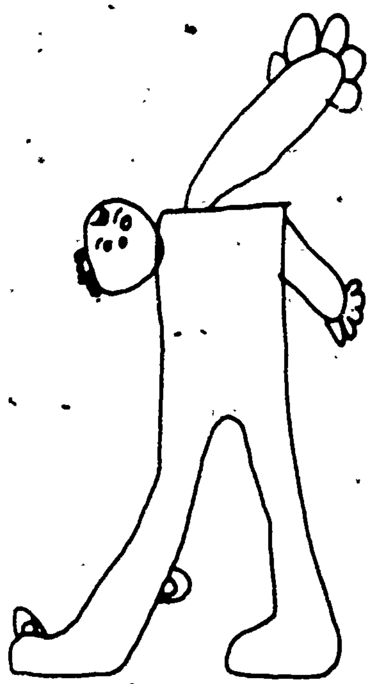
Change the one to one matching of the main features.

Worksheet 45
Unit 28

Name ____________

starting set

ending set



Change the order of the main features.

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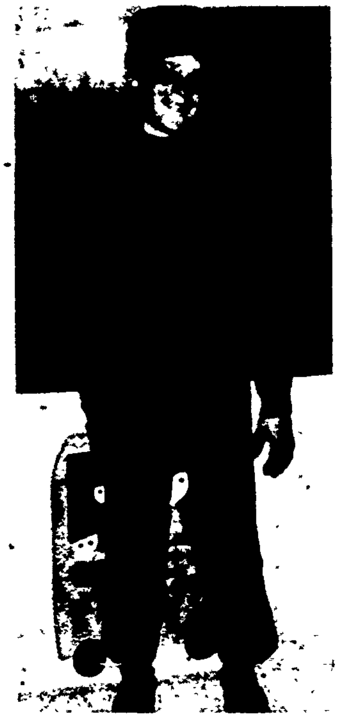
Worksheet 46
Unit 28

Name ______

starting set



ending set

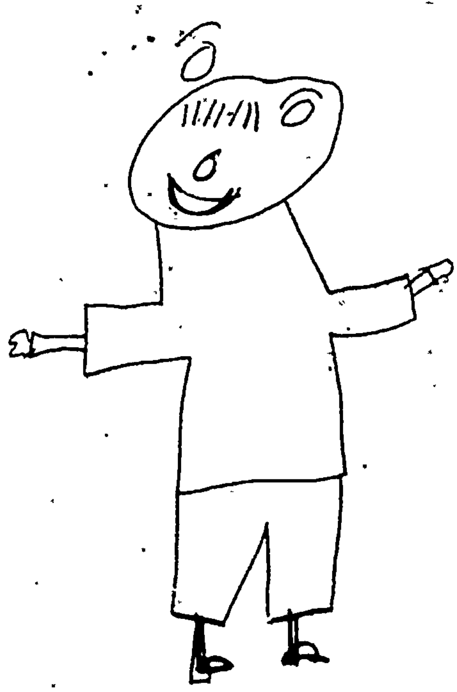
Change the boundedness property.

Worksheet 47
Unit 28

Name ______

starting set

ending set



Change the in-out property.

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Worksheet 48
Unit 28 Name ____________

Do any of the ending sets you drew look like a real boy? ____________

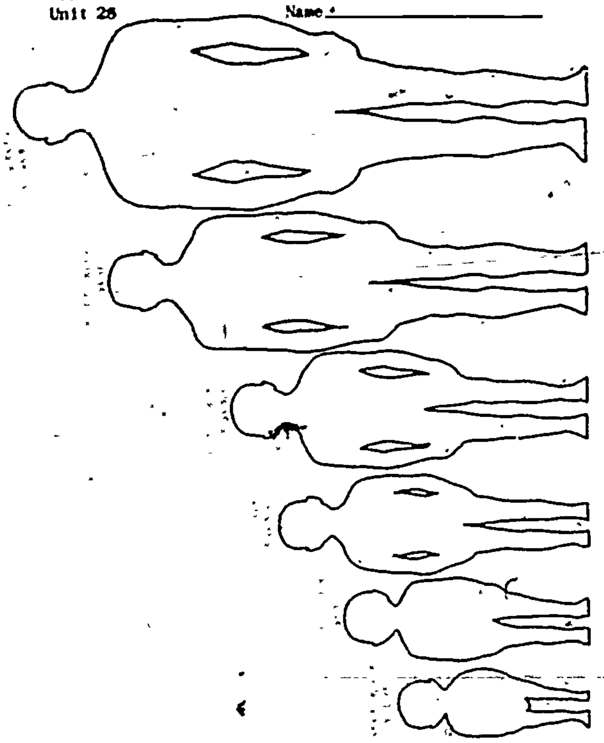
Why or why not? ____________

____

Would you like to look like any of your drawings?

____

Worksheet 49
Unit 28 Name ____________



to have a recognizable ending set. For example, does the height of the boy in the ending set have to be the same as the height of the boy in the starting set?

Have the children turn to Worksheet 49 and compare the properties of the starting set (the man at birth) to the properties of the various ending sets (man at other stages of development). Bring out the following ideas during the discussion:

1. Each ending set is a one to one transformation of the starting set. That is, each of the main features of the ending set can be matched one to one with its corresponding features in the starting set.

2. In the ending sets, measurable properties such as height, width and area have changed.

3. In the ending sets, non-measurable properties have been retained. These properties include order, boundedness and in-out.

Tell the children that in the next section, they will have a chance to construct some one to one transformations in which measurable properties change and non-measurable properties are retained.

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SECTION 3 TOPOLOGICAL TRANSFORMATIONS

In Section 3, the children study topological transformations ~~by working with a variety of materials. For Lesson 10, you~~ will arrange different "rubber sheet activity stations" at which the children get a chance to stretch, twist and bend sheets of rubber on which different starting sets have been drawn. This is one way to make a transformation in which the non-measurable properties are retained while the measurable properties change.

In Lesson 11, there are clay molding activities, where, without cutting, tearing or overlapping the clay, the children transform one figure into another figure. The first figure constitutes the starting set and the last figure, the ending set.

Lesson 12 introduces the term "topological transformation" for those transformations in which measurable properties change and non-measurable properties are retained. In this lesson, the children use projection equipment to make transformations.

In Lesson 13, the children see how a grid can be used to transform a starting set into an ending set. They also see that a grid reference system can be used to describe a location.

In Lesson 14, the children use a grid to transform one map of the United States into another form of that map, and see that when a map is thus topologically transformed, it still retains the properties of boundedness, in-out and order of points, which produces a "useful" ending set map.

Lesson 10: RUBBER SHEET TRANSFORMATIONS

The purposes of this lesson are:

- to review briefly the ideas introduced in Section 2. These include one to one transformations and the properties of boundedness, order and in-out;

- to discover that stretching a rubber sheet is one way to make a transformation in which non-measurable properties are retained and measurable properties are changed.

This lesson will probably require two or three class periods to complete. In Activity A, the children read the last installment of "The Education of Peter von Hahgg." In Activity B, they stretch, twist and bend different rubber sheets at ten different activity stations which you will set up in advance (see PREPARATION). They also observe which properties change and which are retained from starting set to ending set. In Activity C, the students analyze the information they gathered about the properties of the rubber sheet sets, and in Activity D, they use their newly acquired knowledge to answer more fully the questions from the story, "The Same or Not the Same?" in Lesson 2.

MATERIALS

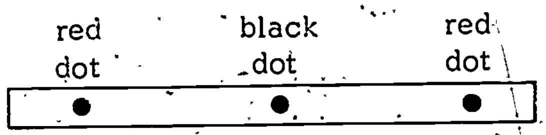
- 16 round balloons
- 2 rubber bands
- 1 long balloon
- 1 red felt tip pen
- 1 black felt tip pen
- 30 pairs of scissors
- 10 station labels (see PREPARATION)
- pencils
- 30 rulers
- story, "The Education of Peter von Hahgg," part 3 (provided in this lesson and also in the Student Manuals)

- rubber sheet activity booklets (in Student Manuals)
- Worksheet 50

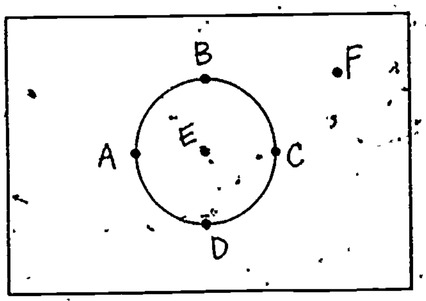
PREPARATION

Before class, prepare the following materials for the rubber sheet activity stations. (You will find that these preparations are very short and easy.)

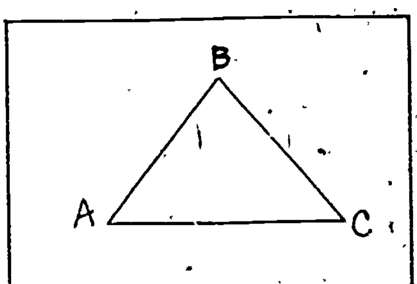
Station 1 Cut a rubber band once. With felt tip pens, put a red dot near each end of the strip and a black dot between them.



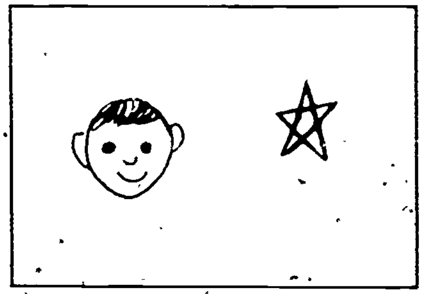
Station 2 Cut a rectangular rubber sheet out of a round balloon. Make it as large as possible. With a ball point pen or a fine tip felt pen, draw a circle and label points on the rubber sheet, as shown.



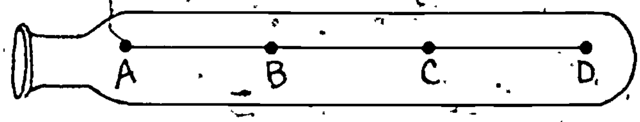
Station 3 Cut a rectangular rubber sheet from another round balloon. Draw the following pattern on it.



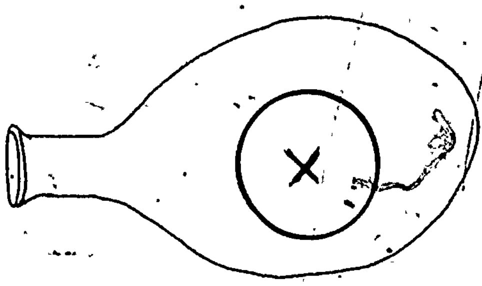
Station 4 Cut another rectangular rubber sheet from a round balloon. Draw the following design on it.



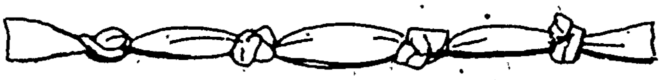
Station 5 Draw a line segment on a long rubber balloon. Label points A, B, C and D.



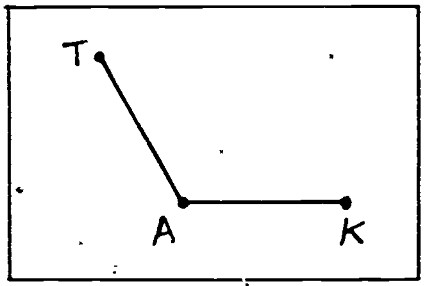
Station 6 Draw a circle on ten round balloons. Write X inside the circle.



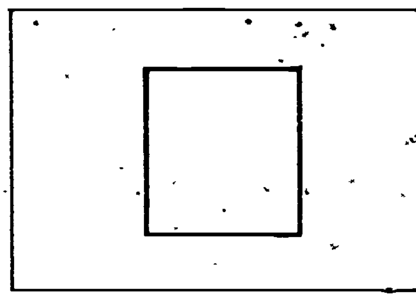
Station 7 Cut a rubber band once. Tie four knots in it.



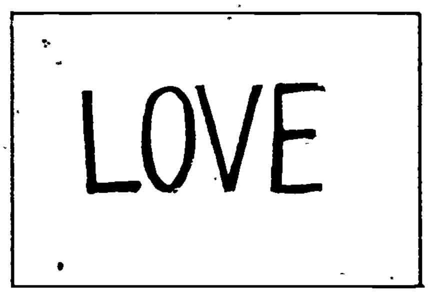
Station 8 Cut a rectangular rubber sheet from a round balloon. Draw angle TAK on it as shown.



Station 9 Cut a rectangular rubber sheet from another round balloon. Draw a square on it.



Station 10 Cut a rectangular rubber sheet from a round balloon. Write the word "LOVE" on it, as shown.



To make station labels, cut out ten 5" x 4" pieces of paper and write the numerals 1 through 10 on them. Place these station labels around the room and put the proper rubber sheet materials at the appropriate stations.

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PROCEDURE

Activity A

Have the children turn to the third and final installment of "The Education of Peter von Hahgg." It follows Worksheet 49 in their manuals. Read it together and give them an opportunity to answer the questions in the story.

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THE EDUCATION OF PETER VON HAHGG

Part 3

One evening, right after supper but long before sunset, Henry and Peter were ambling down the lane in search of adventure. Peter had just finished reading Treasure Island that afternoon, and he secretly hoped to stumble upon some buried treasure, or at least an old map pointing the way.

Just as they reached the end of the lane, a shaggy brown dog came loping up to Henry and said, "Hello there, Henry, how are you?"

"Why hello," said Henry, somewhat startled. "Do you know me?"

"Of course I do," said the dog. "Don't you remember me? Here, look," and he took out his wallet to show Henry a photograph. "I used to live here on Hillendale Farm, when I was a pup. My name is Old Brown, but you called me Brownie, when I lived here."

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Henry squinted at the photo of Old Brown as a pup. "Why yes, of course," he exclaimed, "I do recognize you now! It's great to see you Brownie!"

"And it's good to be back," replied Old Brown. "Who's your friend?"

"Oh, I beg your pardon. This is Peter von Hahgg, Pig-in-Chief of Hillendale Farm," said Henry, hastily making introductions. "And Peter, this is an old friend of mine, Brownie Dog, or as he says, Old Brown."

"Pleased to meet'cha," said Peter.

"The pleasure is mine," replied Old Brown.

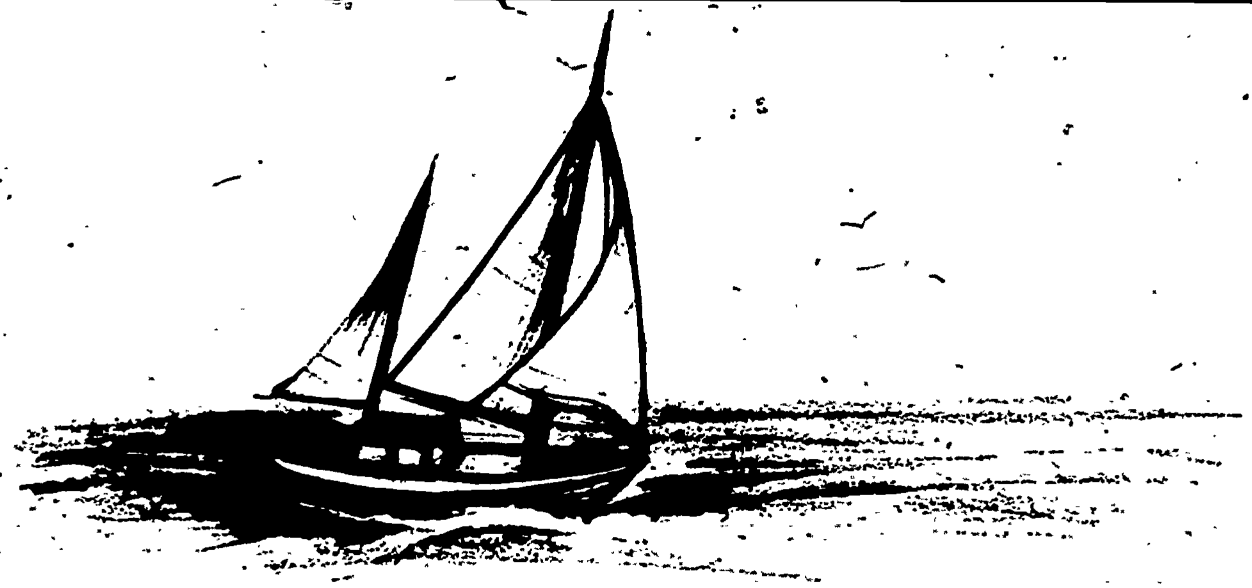
"Well, Old Brown, what have you been doing since I saw you last? Why did you leave Hillendale Farm?" asked Henry, who was extremely curious to find out what had become of his old friend.

"You remember I ran away from home as a young pup. I wanted to see the rest of the world, or at least as much of it as I could."

"And did you?" asked Peter.

"I saw quite a lot," Old Brown said. "First I traveled around this country and then I went to South America, all the way down to Tierra del Fuego, where I worked for a while, herding sheep. Then I met the captain of a ship going north, so I became a sea dog and eventually worked my way back here."

Peter said nothing. He had only read about the places Old Brown had been to and it boggled his mind to think of traveling that far alone.



"Wow!" exclaimed Henry. "I wish I would have run away with you! But maybe, someday....anyway, Old Brown, you must have some pretty exciting stories to tell."

"I do, at that," replied Old Brown.

"I can hardly believe it's really you," continued Henry. "If you didn't have that old photo with you, I wouldn't even recognize you, you've changed so much."

"We all do, Henry, m'boy, we all do," answered Old Brown.

"Yes," said Peter, who had finally found his voice. "It's as if that photo were a starting set and Old Brown here is the ending set."

"Now, that's very interesting, but what does it mean?" asked Old Brown.

"Well, you know — you started out one way, as the photo shows, and you ended up another way — as you are now. And in between, there were lots of changes," Peter explained.

"Yes," said Henry, picking up the thread of Peter's thought, "you are still recognizable as a dog, even after all these years and all your experiences."

"You don't say!" replied Old Brown, who was more than a little amused and even a bit intrigued by what Henry and Peter were saying.

"Sure," Peter said. "Your measurable properties have certainly changed. Look at that photo. You're a lot taller and longer and heavier now then you were when you were a pup."

"Well, yes, I've grown a lot since then, haven't I?" Old Brown remarked as he studied the photograph.

"But look, there are other properties that haven't changed at all," said Henry.

"Like what?" asked Old Brown.

☆ DO YOU KNOW WHICH PROPERTIES HAVE NOT CHANGED? ☆

"Well, the order of features is still the same, and there's one to one matching of the main features in the photo with the main features you have now," answered Henry. "Let me explain," he continued. "In the photo you have two ears, two eyes, one head, one body, one tail, four legs — and you still have the same number of features. That's one to one matching."

"I lost a couple of teeth in a dogfight once," said Old Brown.

"Well, that doesn't count," said Henry. "What's more, your features are still in the same order now as they are in the photo. Your ears and eyes are on your head and your legs are attached to your body and your tail is attached at the other end of your body from your head. Your front legs are still your front legs and your back legs are still your back legs. At least, I don't think they have switched places!"

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"No, I should say not," laughed Old Brown. "I don't reckon I could ever get used to a hind leg trading places with a front leg. Or if my head changed places with my tail!"

"Another non-measurable property that didn't change is the property of boundedness," said Peter. "And," he hastened to explain, "that's not as difficult to understand as it sounds. All it means is that your skin is still in one piece, as it was when you were a pup. It's still whole; there have been no cuts or breaks or tears in it."

"And along with that," added Henry, "your property of in-out hasn't changed from the photo to now. That means that what was inside you when you were a pup — things like your stomach and liver and heart and blood and bones and veins — are still inside you. And things outside you — like me — are still outside you."

"I think I understand what you're saying," said Old Brown. "It sounds like you two fellows have been learning a lot in school and reading a lot of books."

"Yes, we have," replied Henry. "But," he continued, with a far-away look in his eyes, "I'm sure there are other ways to learn that are just as good as school. I'll bet you know a lot of interesting things that Peter and I don't know."

"Yes, that's so," answered Old Brown. "And I'll tell you about my experiences sometime — there will be plenty of time for that. But right now, I'd like to have a good meal and then a good night's rest. I've traveled pretty far today."

"Of course," Henry said. "Let's head back to the farmhouse. We have the whole summer ahead of us, when we can tell stories and talk about the old days."

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The sun had already set and the birds were going to sleep as the three friends walked back to the farmhouse. Peter, excited by the mystery and adventure that Old Brown seemed to promise, recalled his secret hopes of finding buried treasure. "Say, Old Brown," he said, "did you ever find any buried treasures anywhere?"

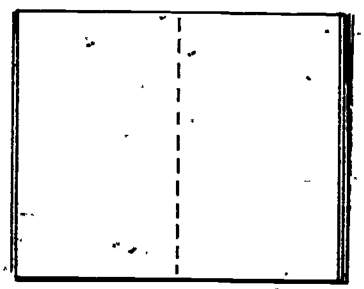
"Nope, I'm afraid not," laughed Old Brown. "But, there are places and there are friends that I'll never forget and that's a treasure of sorts, don't you think?"

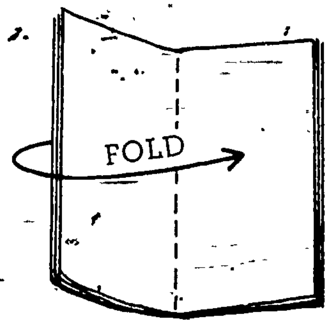
And Peter, thinking about it, nodded gravely.

Activity B

Ask the children if they can think of any method of changing a starting set into an ending set that would change the measurable properties while retaining the non-measurable properties. If no one suggests working with balloons or rubber sheets, show them a rubber sheet you prepared for one of the activity stations. Tell them that they will all have a chance to investigate some of the properties of rubber sheet "designs," before and after the rubber sheets are stretched. Also say that you want them to find out whether these transformations change or retain the non-measurable properties of the starting sets.

Divide the class into groups of three each. Have everyone tear the three rubber sheet data pages out of their manuals. (These follow the last installment of the story.) Holding these three sheets of paper together, they should fold the top half back so that a booklet is formed. Check to make sure that each child's data pages are in order and then staple them along the fold.





Explain that there are ten stations located around the classroom. Each group will experiment with the materials at each station. Call on one group to demonstrate the following procedure.

1. Each group uses only one data booklet while investigating the materials at the activity stations. When they are done with all of the stations, they can record the information in the other booklets. Each group will also need one pencil and one ruler.

2. Have the three children go to Station 2, for example. (The rest of the class should look at their data sheet for Station 2.)

Have one of the children hold up the rubber sheet at Station 2 and show it to the class. They should recognize it as the starting set pictured on their data sheet for Station 2.

3. Explain that two children in each group are to stretch, bend or twist this rubber sheet but they cannot break, cut or tear it. The third child should ask them to "freeze," holding the rubber sheet in its distorted state. He then makes a sketch of this ending set in the space provided on the data sheet for Station 2. When he is done, the rubber sheet should be put back at the labeled station for the next group to use.

NOTE: At Station 6, where the children are asked to blow up a balloon, each group should use a different balloon, for sanitary reasons.

4. Each group should answer the questions at the bottom of each data page. The questions for each station are different, so remind them to read carefully and answer the questions accurately. For some of the stations they are asked to measure the length of the ending set. One child should use his ruler to do this.

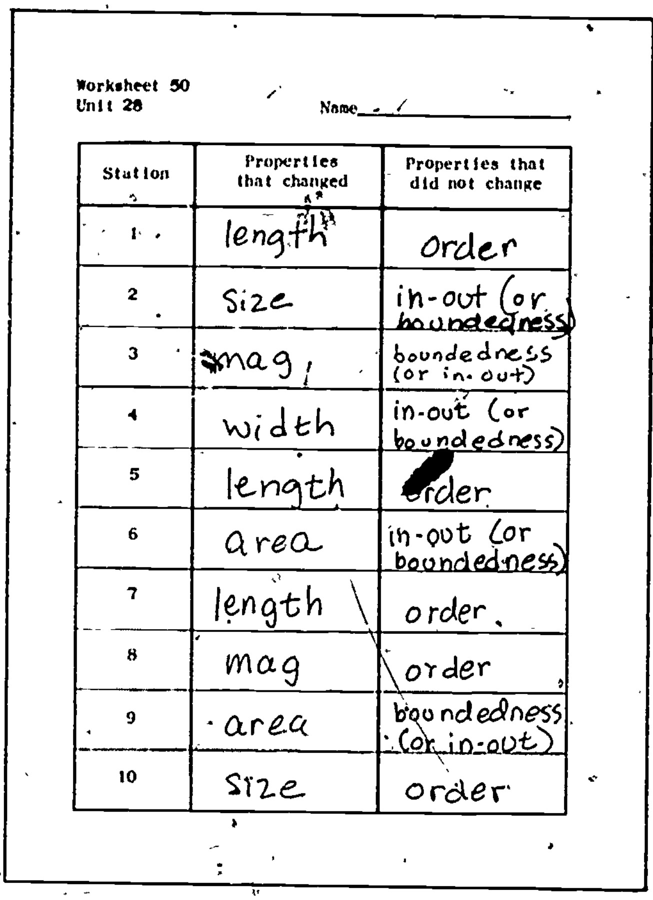
5. When they are done with each station, the group should look for a free station. They should go to the free station and repeat the procedure. It isn't necessary for them to do the stations in any particular order.

6. At the next station, the members within each group should switch jobs.

When you think the class understands these directions, assign a group to each station and let them begin.

Activity C

After all the children have transferred the rubber sheet information to their own data booklets, have them open their manuals to Worksheet 50. On Worksheet 50, they record which properties changed and which properties were retained in each rubber sheet activity. Do the first one or two exercises with the children. They should refer to their data booklets for the required information about each activity.



Worksheet 50
Unit 28

Name______

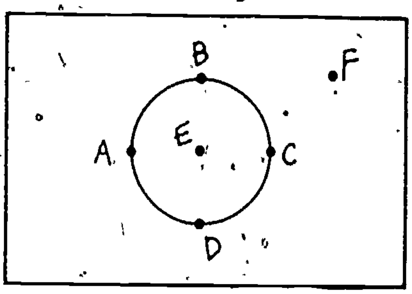
Station	Properties that changed	Properties that did not change
1	length	order
2	size	in-out (or boundedness)
3	mag.	boundedness (or in-out)
4	width	in-out (or boundedness)
5	length	order
6	area	in-out (or boundedness)
7	length	order
8	mag	order
9	area	boundedness (or in-out)
10	size	order

Since the properties of in-out and boundedness are so closely related, some children may have difficulty distinguishing between the two. This is the distinction the authors have made: When you talk about a boundary that <u>separates</u> an inner region from an outer region, you are talking about the boundedness property; when you talk about the <u>location</u> of some element as being inside or outside of some boundary, you are talking about the in-out property. However, if the children do not always make this distinction, don't press the point. Accept either "in-out" or "boundedness" as answers on Worksheet 50.

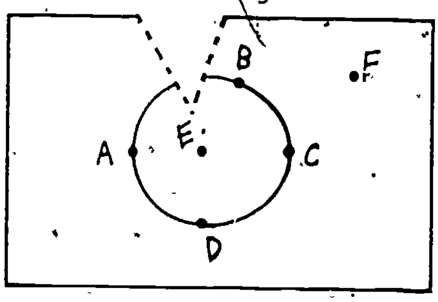
When all the children have completed Worksheet 50, discuss their answers with them. Ask the children if they can see any relationship between the properties that changed for each rubber sheet activity. Someone will probably notice that the properties that changed are all measurable properties and the properties that did not change are all non-measurable properties. Be sure to point out that by stretching, twisting or bending the rubber sheets, they were able to transform starting sets into ending sets that retained non-measurable properties and did not retain measurable properties.

Use one of the rubber sheets to demonstrate what would happen if it were cut or torn.

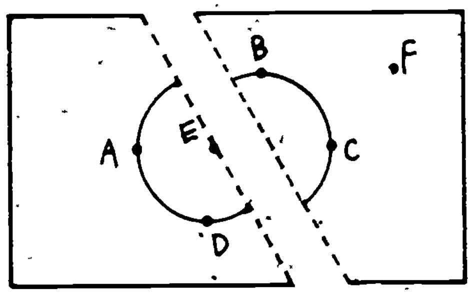
starting set



ending set



In the example on the previous page, the boundedness property has changed. The boundary is not connected the way it was in the starting set. Point F can be reached from point E without crossing the original boundary.



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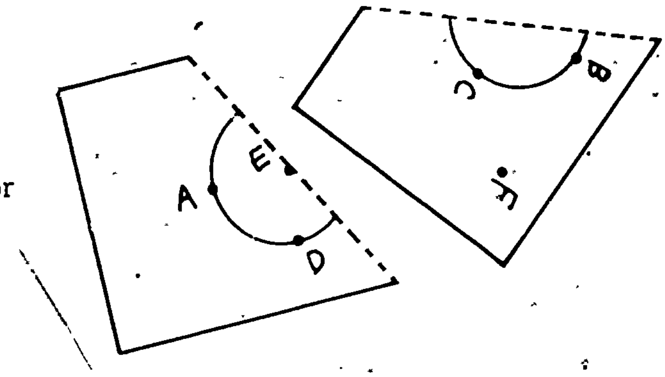

In this example, the properties of order, boundedness and in-out have changed.

In this discussion, stress that tearing, cutting or breaking would change the non-measurable properties as well as the measurable properties. These ideas will be developed in greater detail as the children progress through the rest of this unit; therefore, do not be too concerned if all the children do not understand them at this point.

Activity D

Ask the children if they remember the story they read earlier about the two girls named Greta and Irene and the argument they had. Read the story to the class again (p. 22) and discuss the answers to the questions, using the ideas the children have studied. The non-measurable properties of the ending set (the Mickey Mouse balloon) were not changed from the starting set (the Mickey Mouse statue). That is what is the same about the two figures. The measurable properties, however, changed from the starting set to the ending set and that is what is different about the two figures.

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Lesson 11: CLAY TRANSFORMATIONS

The purpose of this lesson is:

- to help the children discover that if they remold a clay figure (starting set) without cutting or overlapping the clay, the ending set keeps the non-measurable properties, while some of the measurable properties are changed.

MATERIALS

- one-half stick of clay for each child
- 30 paper clips
- enough newspaper to cover desk tops
- measurement devices from earlier lessons (ruler, protractor, string, inch grid)
- Worksheets 51 - 53

PROCEDURE

In this activity, the children mold clay into the figure of a turtle. This is their starting set. Then, without breaking, cutting, tearing or overlapping the clay of the turtle figure, they change it into a figure of a man. This is their ending set.

Distribute one-half stick of clay to each child. If your class seems interested in working with clay, you may want to let them play with their clay for a few minutes before giving any specific directions. When the class is ready, have them tear Worksheet 51 (both pages) out of their manuals. The activity described at the top of the first page should require very little direction from you. However, you may want to read through the instructions with the children before they start. When they have finished making their clay turtles, they should look at their turtles to answer the questions on the rest of the worksheet. They will also heed their measuring devices to answer the questions.

Have the children save both pages of Worksheet 51 when they have completed them. Then have them tear both pages of

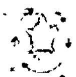
Worksheet 51
Unit 28 Name ______________

Put a paper clip inside your piece of clay.
You should not be able to see the clip.
Mold the clay into this starting set.

The main features of your clay turtle are:
4 limbs, 1 head and a body.

Non-Measurable Properties of Starting Set.

1. The paper clip is located inside your turtle.
   You are located in the region outside the turtle.



2. The order of the main features is:
   head, rt. front leg, rt. back leg,
   lt. back leg, lt. front leg, head

Worksheet 51, cont'd.

3. The surface of the turtle is a boundary that separates the inside region from the outside region.

Measurable Properties of Starting Set. Answers will vary.

1. Length of turtle from head to back leg ≐ ______ inches.

2. Width of turtle's body ≐ ______ inches.
3. Thickness of turtle ≐ ______ inches.
4. Surface area of top side ≐ ______ square inches.

5. Mag of this angle on the turtle ≐ ______ degrees.

Worksheet 52
Unit 28 Name ______________

Mold your clay turtle into this ending set.

Follow the rule: Do not cut, tear, break or overlap the clay.

Non-Measurable Properties of Ending Set.

1. The paper clip is located inside the man.
   You are located in the region outside the man.

2. The order of the main features is:
   head, right arm, right leg,
   left leg, left arm, head

Worksheet 52, cont'd.

3. The surface of the man is a boundary that separates the inside region from the outside region.

Measurable Properties of Ending Set. Answers will vary.

1. Height of man ≐ ______ inches.

2. Width of man's body ≐ ______ inches.
3. Thickness of man ≐ ______ inches.
4. Surface area of front (or back) ≐ ______ sq. in.

5. Mag of this angle on the man ≐ ______ degrees.

Worksheet 52 out of their manuals and follow the instructions at the top of the first page. When they have completed both pages of this worksheet, either collect the clay or have the children set it aside. They should refer to both Worksheets 51 and 52 to help answer the questions on Worksheet 53.

Worksheet 53
Unit 28 Name ________________

Use Worksheets 52 and 53 to answer these questions.

★ Which properties changed from the starting set to the ending set?

1. length
2. Width
3. thickness
4. surface area
5. mag of angles

These are the measurable properties.

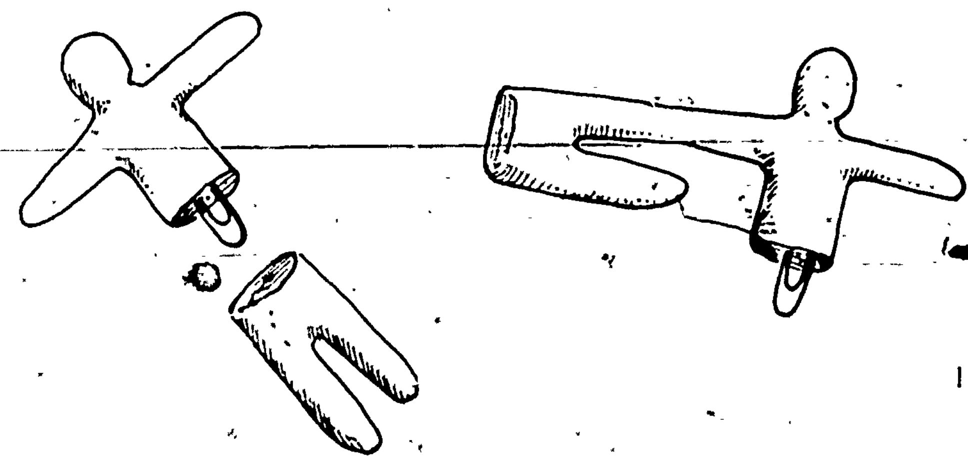
★ Which properties did not change?

1. in - out
2. order
3. boundedness

These are the non-measurable properties.

Discuss their answers on Worksheet 53. Bring out in the discussion the similarities of this clay transformation to the rubber sheet transformations they made in the last lesson. In both cases, the non-measurable properties were retained from the starting set to the ending set, while the measurable properties were changed. In both cases, they were not allowed to tear or overlap their materials. Ask the children if they can think of any reason why they weren't allowed to do this. Let them demonstrate their ideas using one of the ending sets (a clay man).

For example, if the figure of the man is broken in half, so that the paper clip is visible, the children will see that the in-out property has changed, the boundedness has changed, and if the pieces are moved in space and reconnected, the order could change. The children will undoubtedly think of many other examples. In such transformations, the non-measurable properties change; therefore, they were not allowed to cut, overlap or tear the clay.



Lesson 12: PROJECTION TRANSFORMATIONS

The purposes of this lesson are:

- to discover that when a starting set is transformed into an ending set by projection, the non-measurable properties are retained and the measurable properties are changed;

- to introduce the term "topological transformation" for types of transformations in which measurable properties change and non-measurable properties are retained.

The definition of "topological transformation" presented in this lesson is not mathematically precise. The condition that non-measurable properties be retained under transformation is not always sufficient to guarantee that all such transformations are topological. However, for the purposes of this unit, this definition is adequate.

MATERIALS

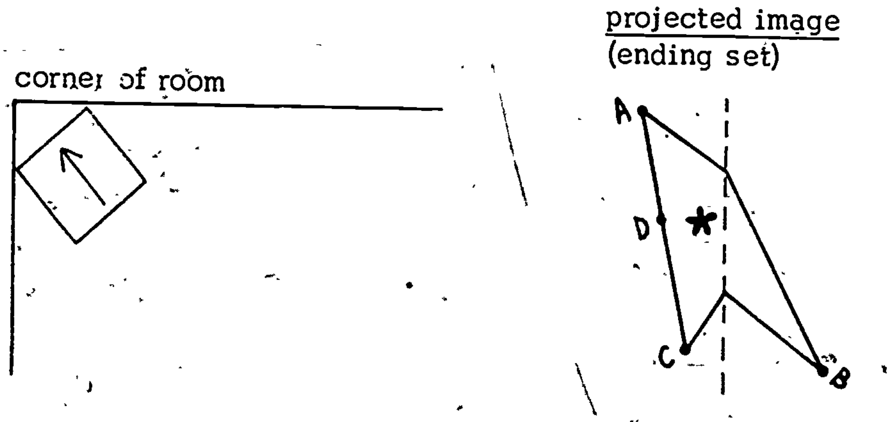
- 4 blank transparencies
- 3 printed originals for transparencies (included in the appendix of this manual)
- overhead projector
- projection booklets (in Student Manuals, following Worksheet 53)
- measuring devices (ruler, protractor, string, inch grid)
- pencils
- ruler or yardstick
- Worksheets 54 and 55

PREPARATION

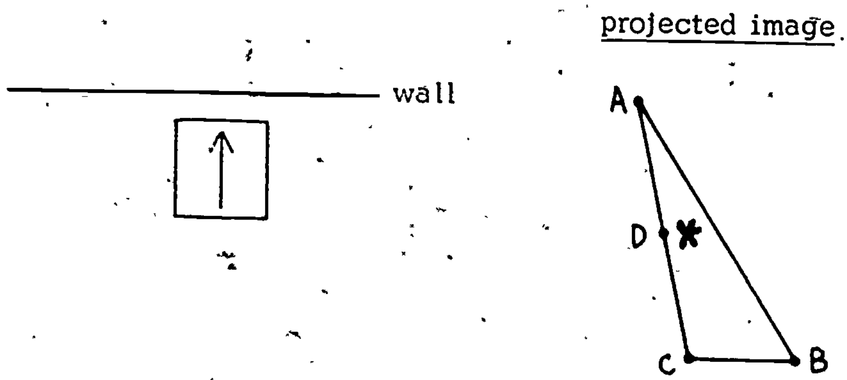
Make the three transparencies for this lesson, using the printed originals in the appendix. If you decide to draw the transparencies freehand, be sure the measurements correspond to those of the originals. On the first transparency, the line segment should measure 5", on the second, sides AC and CB of the triangle should each measure 3" and on the third transparency, each side of the square should measure 3".

These transparencies are identical to the starting sets in the children's projection booklets. Since you will use them to produce distorted ending sets, it would be a good idea to experiment with the transparencies and the overhead projector before starting the lesson. Suggested below are several ways to produce distorted ending sets. (The triangle transparency is used in each example.) In each case, the projector must be right next to the wall to get a noticeable distortion.

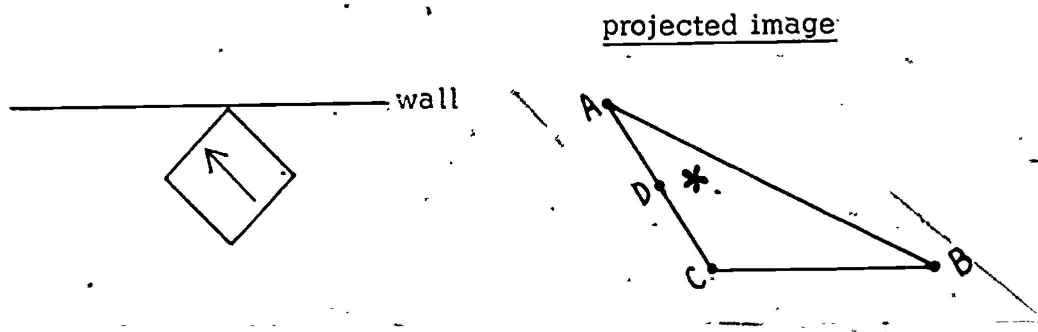
1. The projector faces into a corner of the room. Tilt it backward so the image is cast upward into the corner.



2. The projector faces the wall. Tilt the projector backward.



3. The projector is at an angle to the wall. Tilt it backward.



Other possibilities include projecting the image onto the seam where the wall and the ceiling meet or where two walls and the ceiling meet. During the activity, the children may think of other surfaces onto which the image may be projected.

PROCEDURE

Activity A

Briefly review with the children the two methods they have used so far to get transformations in which the measurable properties change and the non-measurable properties are retained. (Stretching rubber sheets and molding clay.) At this point, you might want to introduce a term for this type of transformation: topological transformation. (Sometimes it is also called a "rubber sheet" transformation. The children should be able to understand why these transformations are called "rubber sheet" transformations; figures are stretched out of shape and distorted, while certain non-measurable properties are retained.)

Tell the class that they are going to investigate another transformation of a starting set into an ending set. Say that you would like to have them decide if this transformation could be called a topological transformation.

Have the children tear the pages of the projection booklet out of their manuals. (These pages follow Worksheet 53.) They should fold the pages together as they did their rubber sheet booklets and staple on the fold. They will also need their

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measuring devices and a pencil. Have them open their projection booklets to Trial 1 and answer the questions about the starting set. When they have finished, show them the transparency you made of the starting set (line segment AF). Have someone show that it is identical to the starting set in the booklet by superimposing the transparency of the line segment over the line segment in the booklet.

Project the line segment onto several surfaces (see suggestions in PREPARATION section). You may have to tape the transparency down to keep it from sliding off the projector. Ask the children to pick one of the projected images as their ending set (for example, the image cast in the corner). Hold the projector steady (or have a student do so), and have the children draw a picture of this ending set in their booklets.

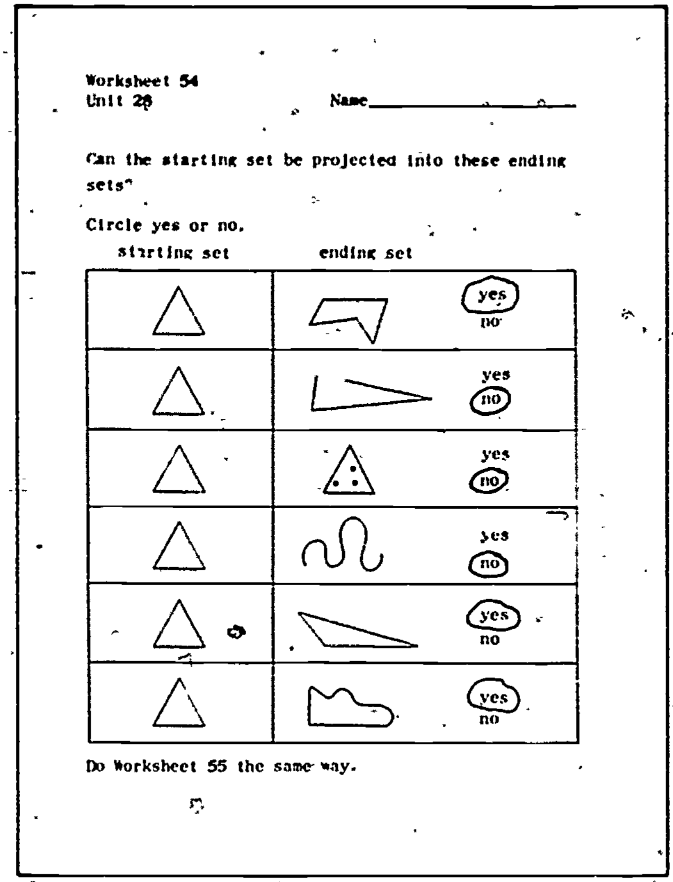
Have someone measure the length of the projected line segment with a yardstick or a ruler. The children should record the length in the appropriate blank beneath the sketch of the ending set.

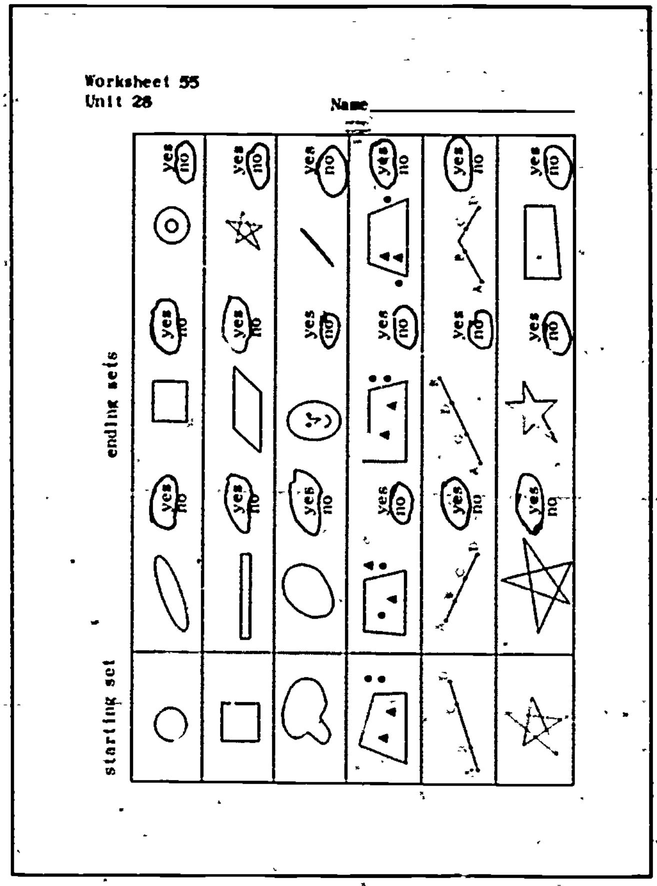
When they have finished the questions for Trial 1, follow the same procedure to complete Trials 2 and 3. Call on different children to make the required measurements of the ending sets. Several children may have to combine their inch grids to measure the area of the projected square (Trial 3). When all three trials have been completed, the children should use their information to complete the chart on the back page of the booklet. Discuss their answers with them.

Ask the children whether or not they think these projections were topological transformations. They should see that the measurable properties changed, while the non-measurable properties did not change; therefore, they are topological transformations.

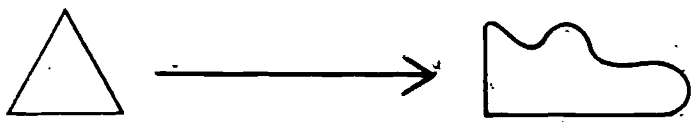
Activity B

Do Worksheet 54 with the children. You may want to have the overhead projector set up with a blank transparency on it. Draw the starting set on the transparency. Let the children check their answers by projecting the starting set onto various surfaces. The children should see that no matter where they project the image, the non-measurable properties (boundedness,

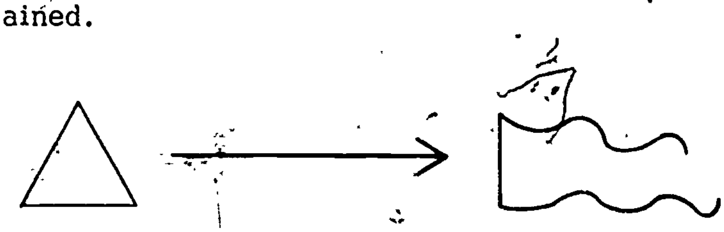




in-out, order) of the starting set remain unchanged in the ending set. Some children might be able to conceive that it is possible to project a triangle into an ending set like this.



This ending set is possible because the non-measurable properties are retained. However, the following is not a possible projection, because the boundedness property has not been retained.



Have the students complete Worksheet 55 on their own and then discuss the answers with them.

Lesson 13: USING A GRID TO MAKE TOPOLOGICAL TRANSFORMATIONS

The purposes of this lesson are:

- to review the properties of a topological transformation;

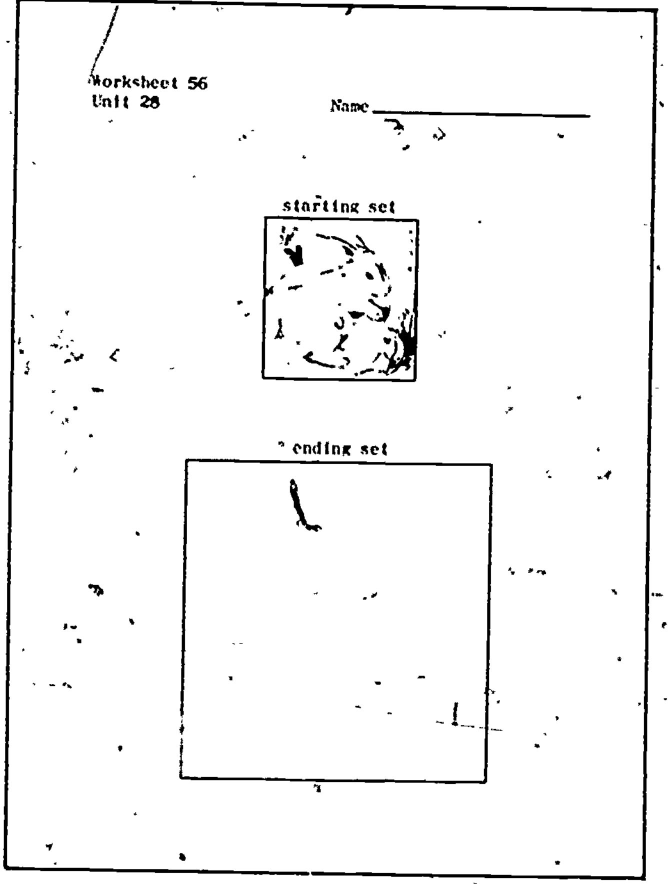
- to show how a grid can be used to transform a starting set into an ending set;

- to show how a grid reference system can be used to describe a location.

The children are introduced to a "tool" an artist might use if he wanted to reproduce a painting or a drawing; that is, superimposing a grid on the original drawing (starting set) and transferring what is in each section to corresponding sections on another grid. This produces the ending set.

MATERIALS

- pencils
- Worksheets 56 - 62

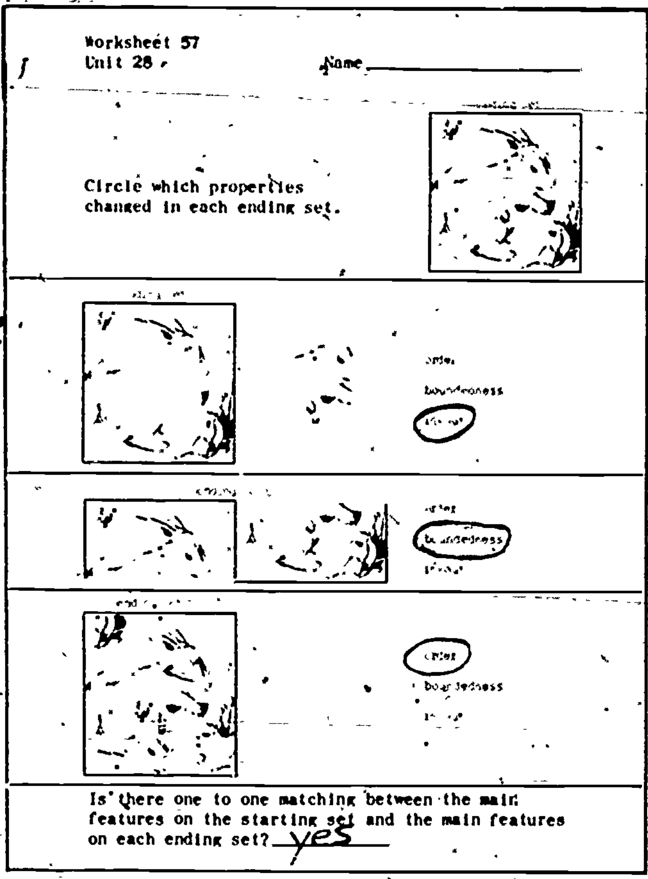


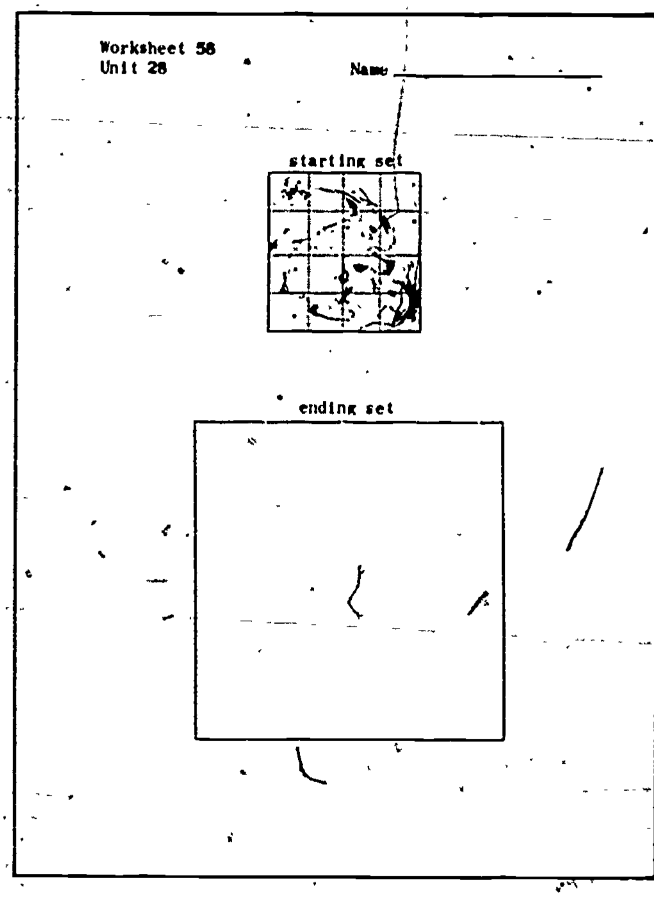
PROCEDURE

Activity A

Review with the children the three methods they have used to make topological transformations — stretching a rubber sheet, molding clay and projection. Tell them that today they are going to learn about a method that artists sometimes use when they want to transform a starting set (a picture) into an ending set (a picture).

Have the children turn to Worksheet 56. Tell them that perhaps an artist would want to draw an ending set in the larger square at the bottom of the worksheet and



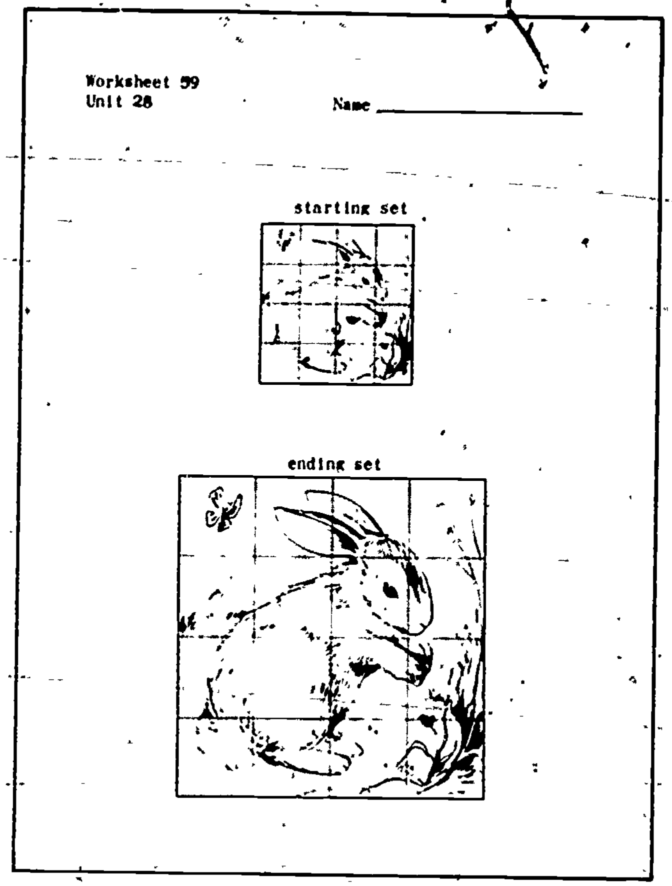


would want the result to look as much as possible like the original picture. Then ask:

WHAT PROPERTIES OF THE STARTING SET MUST BE KEPT IN THE ENDING SET IF IT IS TO BE RECOGNIZABLE?

Have the children look at Worksheet 57, which shows three ending sets of the picture that have not kept the properties of order, boundedness and in-out. Discuss with them why these are not very "useful" or "recognizable" ending sets. Then have the children look back at Worksheet 56. Ask them to think about the problem that confronts the artist who wants to draw an ending set that looks just like the starting set, except that it would fit into the larger square. The children might suggest using a projector, or if it were on a rubber sheet, stretching it so it would fit the larger square. Accept the children's ideas and then ask them to turn to Worksheet 58.

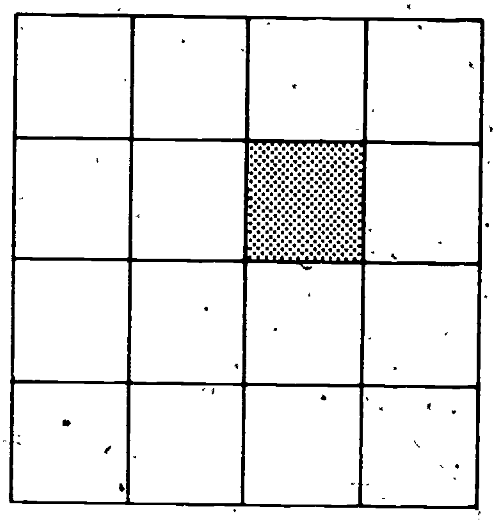
Ask the children what the artist has done to the starting set. (He has superimposed a grid over the picture.) Then ask:



HOW COULD HE USE THIS GRID TO MAKE HIS JOB EASIER?

Lead the children to see that if they were to construct a similar grid (four sections by four sections) on the square at the bottom of Worksheet 58, they could reproduce the drawing section by section. For example, the part of the drawing in the lower left-hand section of the starting set could be reproduced in its corresponding section of the ending set. (It isn't necessary to have the children actually transfer the drawing on Worksheet 58. Have them draw a grid on the larger square and let them transfer the picture in their free time or at home, if they want to.)

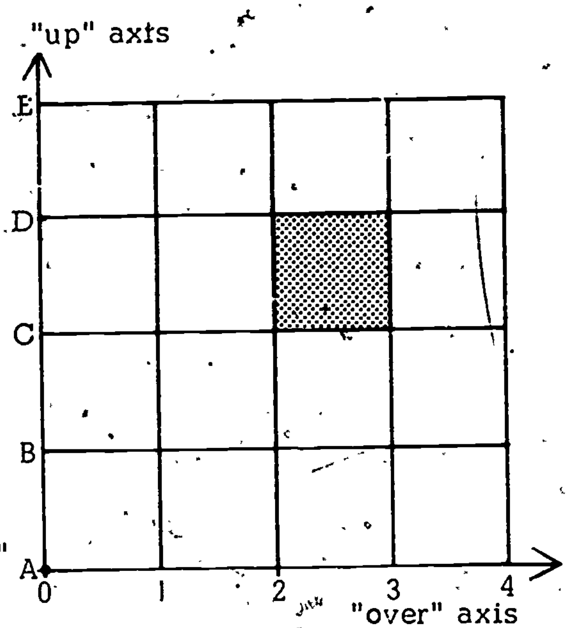
Have the children turn to Worksheet 59, which shows the drawing reproduced on the square by using a 4" x 4" grid. Have them compare the ending set drawing with the starting set drawing to be sure the one to one matching was retained. Also, check the properties of order, in-out and boundedness. During this discussion refer to certain sections for the students to check; the children should realize that it is difficult to refer to certain sections and that there is need for a better reference system. For example, it is awkward to say, "The square that is located in the second row from the top, the third to the right."



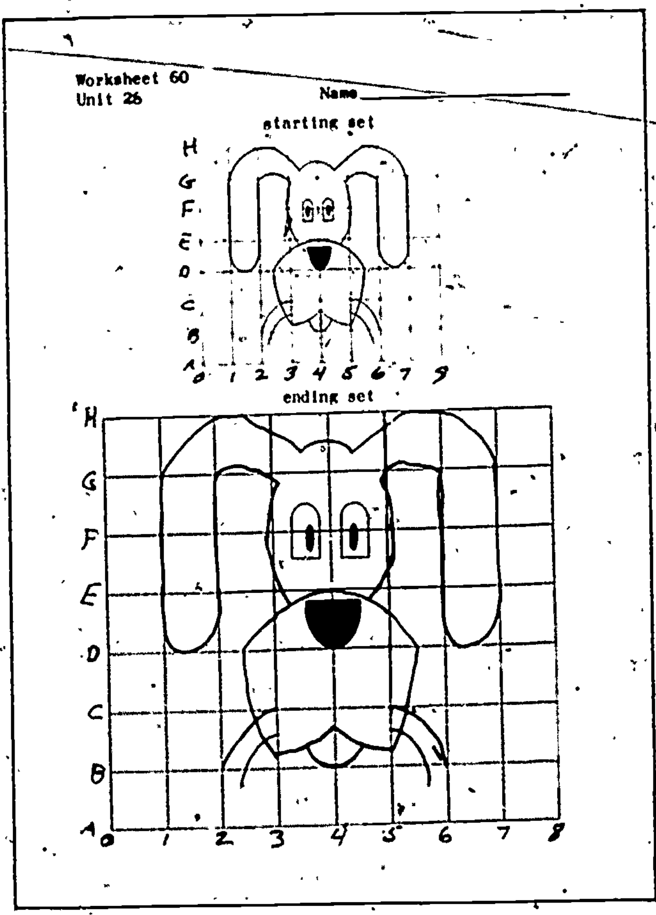
Ask the children to think of various reference systems and to discuss them. One simple reference system is shown on the next page.

1. Choose a starting point (0, A).

2. Label the grid lines (over-axis: 0, 1, 2, 3, 4; up-axis: A, B, C, D, E).

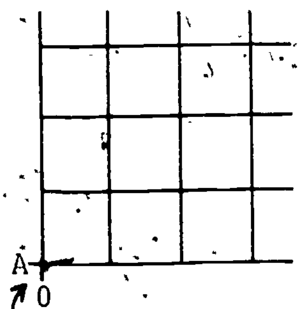


Now, the section that is shaded can be referred to as, "The region between lines 2 and 3, and C and D." Discuss the advantages this system has over other systems that make references to "right" or "left."



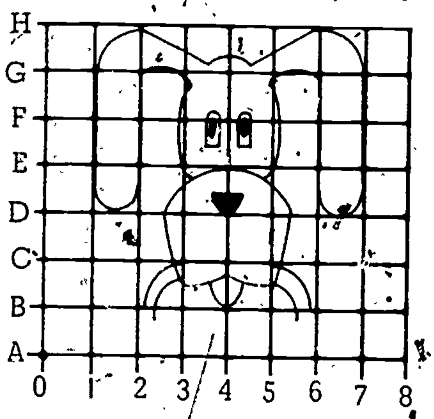
Activity B

Have the children open their manuals to Worksheet 60, which is a picture of a dog with a grid superimposed on it. Ask the children what they could do to the grid lines that would help them refer to certain sections of the grid. They will probably suggest that they label the grid lines. Have them do this. Remind them to label the starting point (0, A) first.



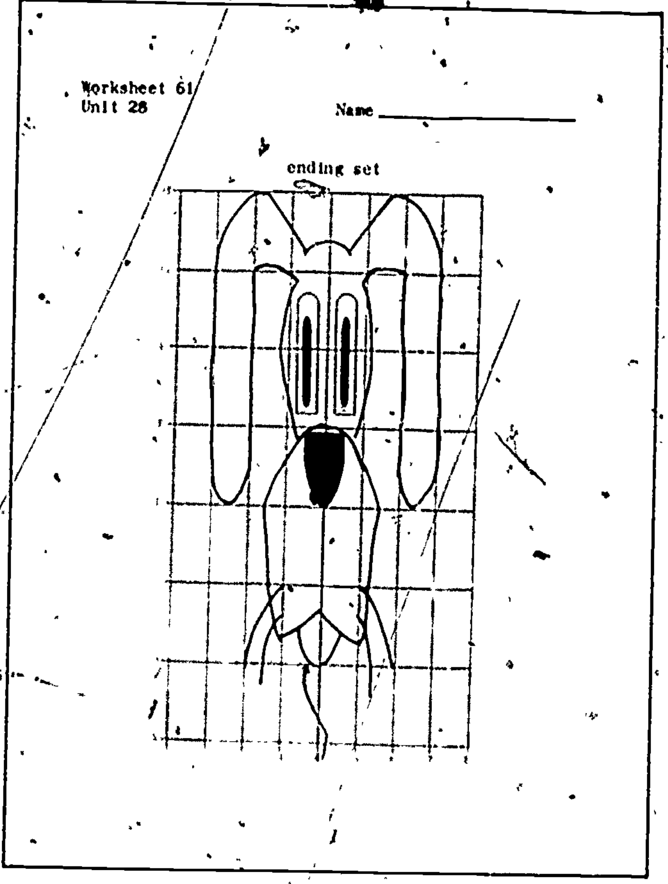
starting point (0, A)

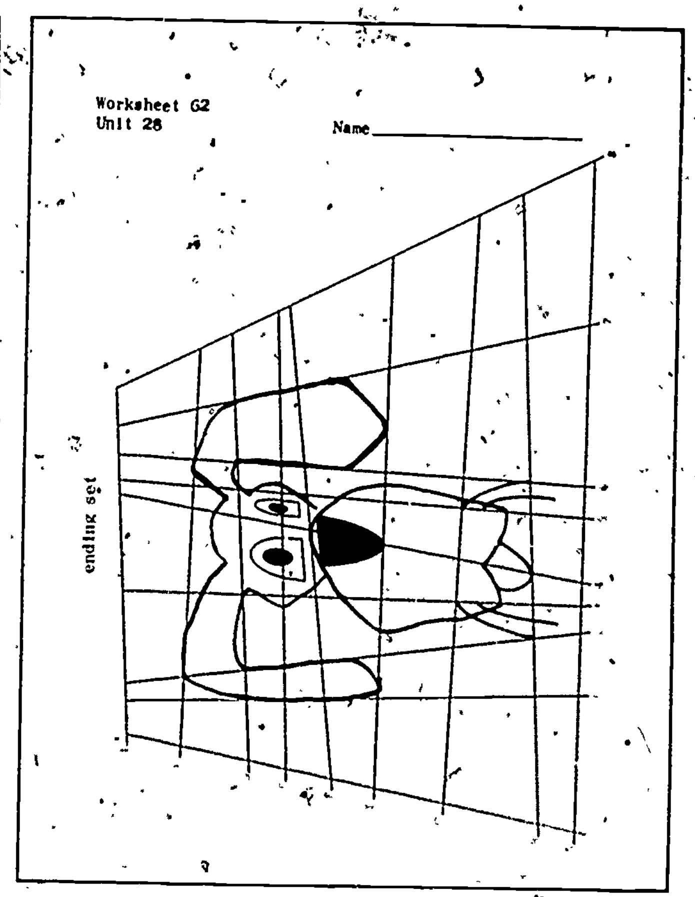
Starting set



This illustration shows how the starting set grid should be labeled. Have the children label the ending set grid the same way. Have them draw on the ending set the lines that are in the region between lines 3 and 4, G and H on the starting set. Quickly check their drawings and then let them finish. Check to make sure they get the appropriate lines in the correct regions.

When all the children have completed Worksheet 60, have them tear it and Worksheets 61 and 62 out of their manuals. They should use the starting set on Worksheet 60 as a guide for drawing the ending sets on Worksheets 61 and 62. When the children have completed these worksheets, discuss with them which properties changed and which properties were





retained in the ending sets on all three worksheets. List these properties on the chalkboard. Below is a sample list.

changed properties	properties retained
– area – height – width – shape is distorted (especially on Worksheet 6.2)	– order of main features (2 eyes, nose, tongue) – boundedness (the region inside the outline of the dog is still separated from the region outside the dog's outline) – in-out (the features inside are still inside)

The children should realize that the measurable properties changed and the non-measurable properties were retained. Therefore, these ending sets can be classified as topological transformations.

Lesson 14: TRANSFORMATIONS OF A MAP OF THE UNITED STATES

The purposes of this lesson are:

- to acquaint the children with some of the geographical features of the United States;

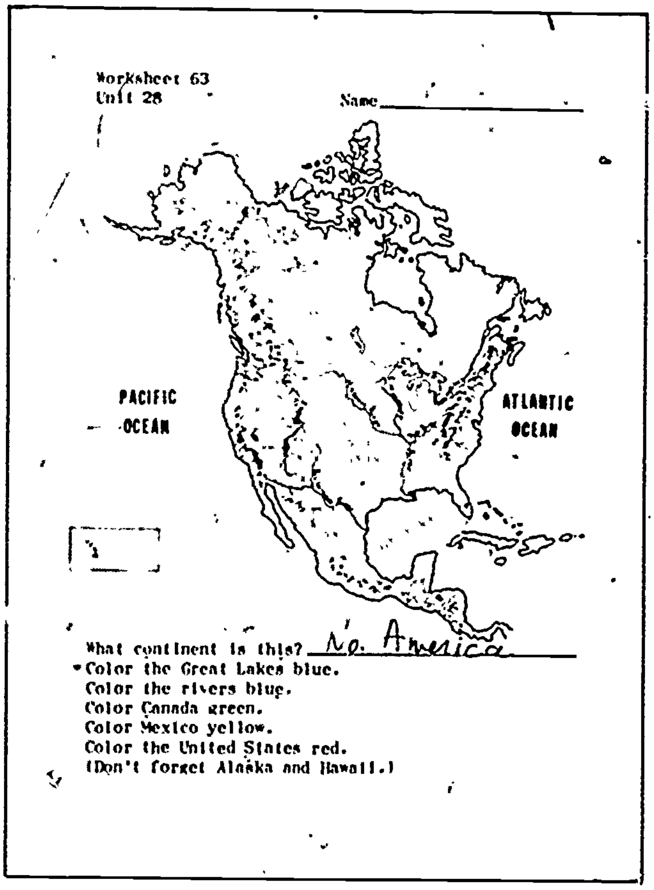
- to use a grid to transform one map into another map;

- to show that when a map is topologically transformed, it still retains certain properties (boundedness, in-out, order) which make the map "useful."

MATERIALS

- crayons
- globe
- Worksheets 63 - 66

PROCEDURE

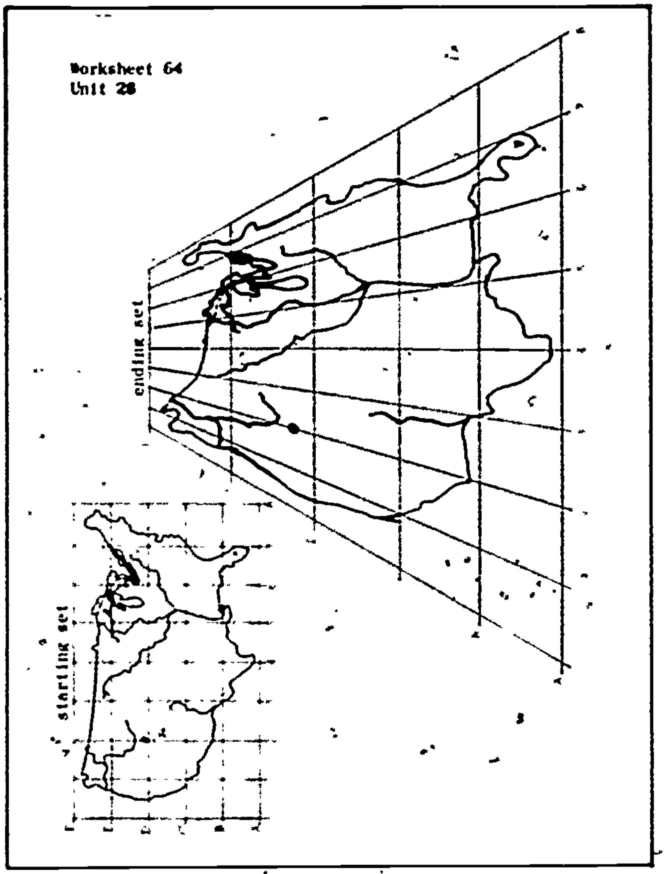


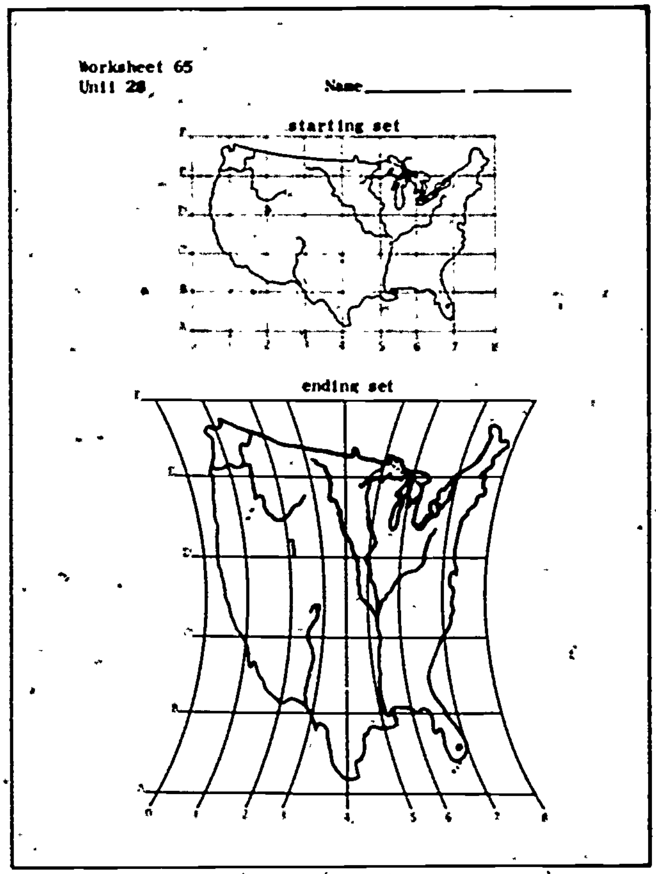
Activity A

Display a globe at the front of the classroom. Have the children open their manuals to Worksheet 63 and ask them to find on the globe the area shown on the worksheet. Ask:

> WHAT LARGE LAND AREA OR CONTINENT IS THIS? (North America.)
>
> CAN YOU FIND THE UNITED STATES?

Discuss briefly the geographical features (rivers, lakes, oceans, mountains). Then have them complete Worksheet 63. After everyone is finished, check the maps together, using the globe as a reference.





Activity B

Have the children tear Worksheets 64 and 65 out of their manuals. Tell them that they are going to transform the starting sets into ending sets, using the grid reference system as an aid. Discuss briefly the topological properties that should be retained in the transformations — order, in-out and boundedness. Then let them draw the ending sets.

After the children have completed the worksheets, discuss their maps with them. Ask such questions as the following.

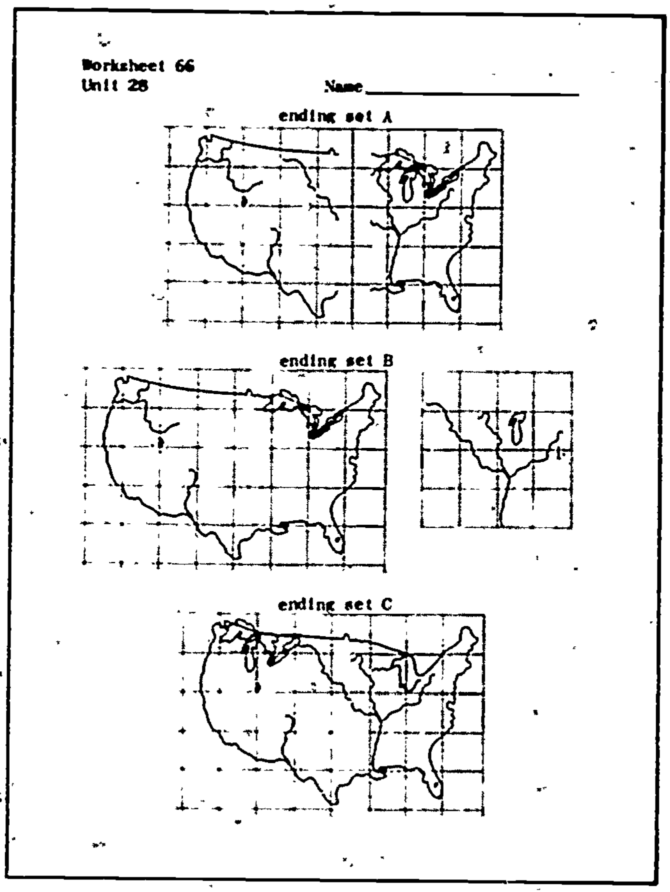
WHAT PROPERTIES HAVE CHANGED? (Area, width, height, shape.)

WHAT PROPERTIES HAVE BEEN RETAINED? (Order of main features, in-out, boundedness.)

DOES THE ENDING SET ON WORKSHEET 64 SHOW THE SAME REGIONS AS THE ENDING SET ON WORKSHEET 65? ARE THE SECTIONS IN CORRECT ORDER? ARE THE LAKES

AND RIVERS INSIDE OR OUTSIDE THE LAND AREA?

CAN YOU STILL RECOGNIZE THE ENDING SETS AS MAPS OF THE UNITED STATES?



Use Worksheet 66 as a discussion guide to help the children visualize how ending sets of the United States could look if the topological properties were not retained. Ask the children if the maps on Worksheet 66 are "useful" maps. For example, ending set C shows the Great Lakes located in the northwestern part of the United States. Ask the children if this map would be useful when traveling in this part of the United States. (No, the Great Lakes are not in Montana, Idaho and Washington. Ending set C is not a useful or faithful representation of this region.)

Discuss with them the usefulness of the maps on Worksheet 64 and 65 by comparing them to the maps on Worksheet 66. The children will probably see that even though the maps on

Worksheets 64 and 65 are distorted, they are still useful for showing relative locations of geographical features. (Check the location of the Great Lakes, for example.)

Tell the class that in the next section of this unit, they are going to use what they have been learning about topological transformations to make some very interesting maps.

In this section, the children work with the ideas they have learned about topological transformations to make flat maps of the globe. In this case, the globe is thought of as the starting set, and the flat maps as the ending sets.

Lesson 15 acquaints the children with the major geographical regions of the world and gets them to compare the properties of an area on the globe with the properties of that area on a flat map.

In Lesson 16, the children are asked to think about the problems that early mapmakers faced when they tried to make flat maps of the world that retained as much as possible the same topological properties as the globe.

In Lesson 17 they compare different methods of "flattening" the globe, including poking a hole in it, stretching it and cutting it. They learn that different methods are used to produce various kinds of flat maps and that different maps are useful for different purposes.

In Lesson 18, the children review the methods of making topological transformations that they learned about in Section 3. These were stretching rubber sheets, molding clay, using a grid reference system and projecting. They find that projecting is best for their purposes and so they use the projecting equipment to transform a spherical surface into a flat surface.

In Lesson 19, the children use the projecting equipment to project the northern hemisphere from a spherical surface onto a flat surface and then compare the properties of their starting set with the properties of their ending sets. They produce three different kinds of ending sets: the cylindrical projection, the conical projection and the gnomic projection. They also get a chance to predict which regions of the starting set (globe) will be most distorted by projection into an ending set (flat map) and to verify their predictions.

Lesson 20 is a summary of the most important ideas in the unit.

Lesson 15: SECTIONAL MAPS OF THE GLOBE

The purposes of this lesson are:

- to acquaint the children with the major geographical regions of the world;
- to compare the properties of sections on the globe (starting sets) to the properties of corresponding sections on flat maps (ending sets).

MATERIALS

- globes (see PREPARATION)
- pencils
- 30 transparent area grids (from Lesson 2)
- Maps A - E (in Student Manuals)
- Worksheets 67 - 72

PREPARATION

Try to get one globe for every two students; however, one for every four students is acceptable. Borrow the globes from other teachers in your school. They need not be identical globes. Plan to use them for at least two days.

PROCEDURE

Activity A

Many children have difficulty understanding the relationship among continents, countries, states and cities. To help clarify this, have the children help you develop the chart on the next page. Point out the different regions on a globe.

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The World

↓

Hemispheres (Northern and Southern, divided along the equator)

↓

Continents (Asia, Africa, North America, South America, Europe, Australia, Antarctica)

↓

Countries (within the North American continent, for example: Canada, Mexico, Greenland, United States, Cuba, Guatemala, etc.)

↓

States (within the country of the United States, for example: Texas, Minnesota, Hawaii, Alaska, etc.)

↓

Cities (within the state of California, for example: Los Angeles, San Francisco, San Diego, etc.)

Worksheet 67
Unit 28

Name_______________

STARTING SET — THE GLOBE

Find these regions on the globe.
If it is hard to find the boundaries, look at Map A.
Measure the area of each region.

1. The area of Greenland ≈ _____ square inches.
2. The combined area of the United States (remember Alaska), Canada and Mexico ≈ _____ square inches.
3. The area of Europe ≈ _____ square inches.

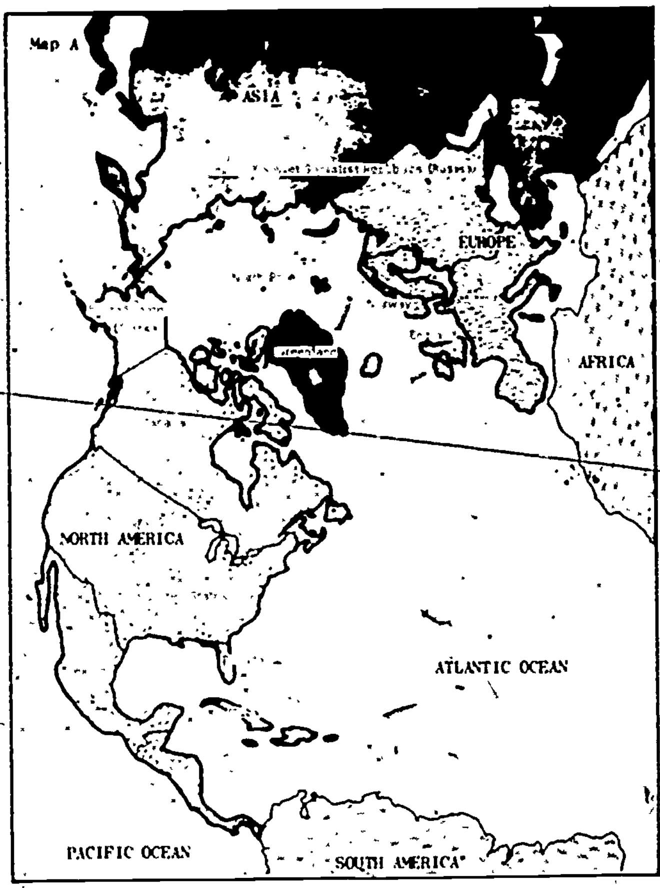
ENDING SET — MAP A

Find these regions on Map A.
Measure the area of each region.

1. The area of Greenland ≈ _____ square inches.
2. The combined area of the United States (remember Alaska), Canada and Mexico ≈ _____ square inches.
3. The area of Europe ≈ _____ square inches.

Activity B

Divide the class into groups. Each group should have one globe and all the children should have their transparent grids from Lesson 2. Have the children in each group complete Worksheets 67 and 68 together. They will probably have difficulty making the transparent area grids conform to the curved surface of the globe; however, they should try to get an approximate area measurement. Point out that on Map A the names of the continents are written in capital letters and that the rest of the maps in the manuals have been made this way.



Worksheet 68
Unit 28

Name____________________

Use your answers on Worksheet 67 to help you fill in the chart and answer the questions on this worksheet.

	starting set	ending set
Greenland	area ≈ _____ sq. in.	area ≈ _____ sq. in.
U.S., Canada and Mexico	area ≈ _____ sq. in.	area ≈ _____ sq. in.
Europe	area ≈ _____ sq. in.	area ≈ _____ sq. in.

1. Did the area of the regions in the starting set change or remain the same in the ending set?

____

2. Did the boundedness of the regions change or remain the same?

____

3. Did the order of the main features change or remain the same?

____

4. Did the in-out property change or remain the same?

____

When the groups have completed Worksheet 68, discuss their answers. The children will probably see that the area (a measurable property) changed from the globe (starting set) to Map A (ending set), while the non-measurable properties remained the same from starting set to ending set.

Let the children complete Worksheets 69 through 72 on their own or with partners. If you do not have several globes available, you may want to let the children complete the worksheets over a period of several days. After everyone has completed Worksheets 69 - 72, discuss the answers. Ask the children if there is one to one matching of the main features of the starting sets with the main features of the ending sets. They should see that on each map, the main features match one to one with the main features of that section on the globe. For example, on Map A there is just one North America, not zero or two or more North America's.

Discuss with the class the fact that each of the ending sets (Maps A - E) retained the properties of order, boundedness

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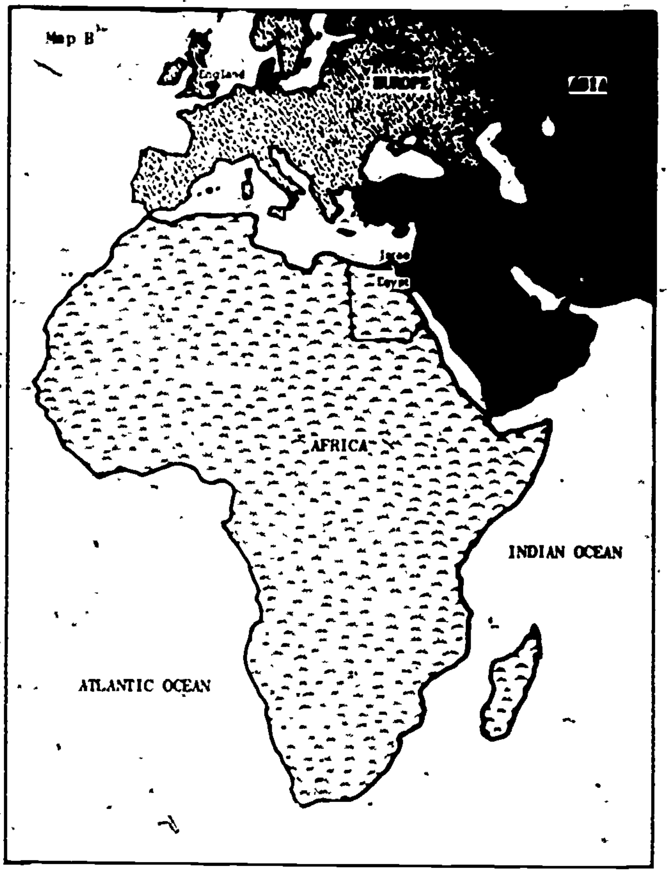


Worksheet 69
Unit 28 Name_______________

STARTING SET
Find Africa on the globe.
Measure the area.
The area of Africa ≐______square inches.

ENDING SET
Find Africa on Map B.
Measure the area.
The area of Africa ≐______square inches.

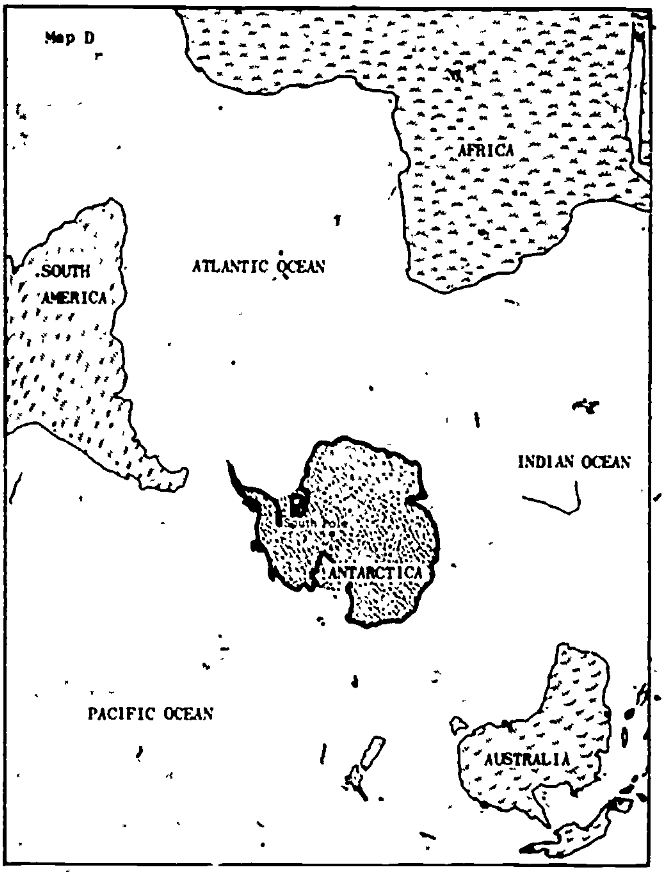
1. Did the area of Africa change or remain the same from the starting set to the ending set?
____
2. Did the boundedness of Africa change or remain the same?
____
3. Did the order of the main features change or remain the same?
____
4. Did the in-out property change or remain the same?
____

Worksheet 70
Unit 28 Name_______________

STARTING SET
Find South America on the globe.
Measure the area.
The area of South America ≐______square inches.

ENDING SET
Find South America on Map C.
Measure the area.
The area of South America ≐______square inches.

1. Did the area of South America change or remain the same from the starting set to the ending set?
____
2. Did the boundedness of South America change or remain the same?
____
3. Did the order of the main features change or remain the same?
____
4. Did the in-out property change or remain the same?
____



Worksheet 71
Unit 28
Name________________

STARTING SET

Find Antarctica on the globe.

Measure the area.

The area of Antarctica ≐_____square inches.

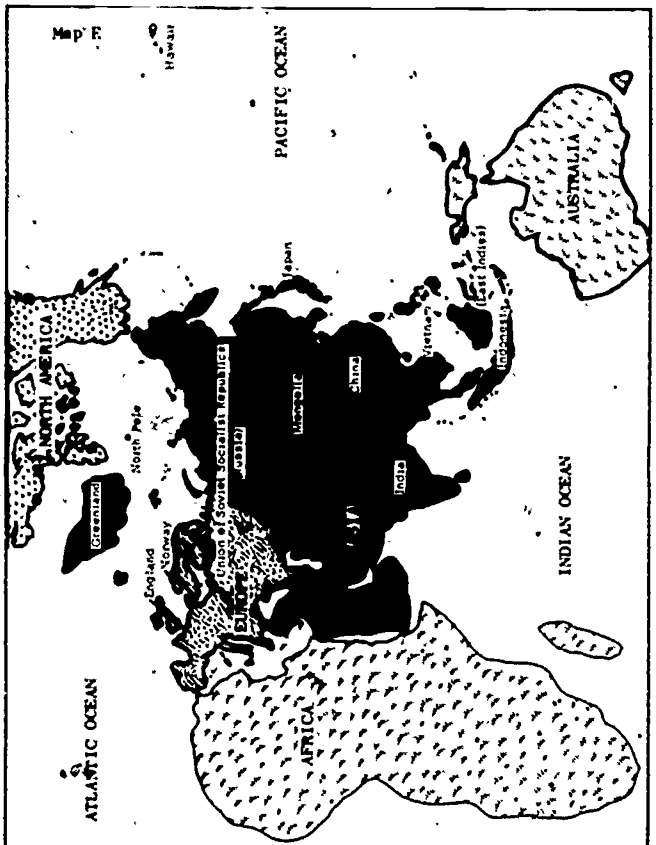
ENDING SET

Find Antarctica on Map D.

Measure the area.

The area of Antarctica ≐_____square inches.

1. Did the area of Antarctica change or remain the same from the starting set to the ending set?
   ____
2. Did the boundedness of Antarctica change or remain the same?
   ____
3. Did the order of the main features change or remain the same?
   ____
4. Did the in-out property change or remain the same?
   ____



Worksheet 72
Unit 28
Name________________

STARTING SET

Find Asia and Australia on the globe.

Measure the area.

The area of Asia ≐_____square inches.

The area of Australia ≐_____square inches.

ENDING SET

Find Asia and Australia on Map E.

Measure the area.

The area of Asia ≐_____square inches.

The area of Australia ≐_____square inches.

1. Did the area of Asia and Australia change or remain the same from the starting set to the ending set?
   ____
2. Did the boundedness of Asia and Australia change or remain the same?
   ____
3. Did the order of the main features change or remain the same?
   ____
4. Did the in-out property change or remain the same?
   ____

and in-out, while the measurable properties (area, etc.) changed. Ask if, instead of having several sectional maps of the globe, you could make a flat map of the whole world that would keep the non-measurable properties of in-out, order and boundedness. How would such a map look? Ask the children to think about this problem for the next few days and tell them that in the next lesson they will look at some early flat maps of the world that were devised by different men who thought about this same problem.

The geographical ideas introduced in this lesson could also be referred to in a social studies or current events class.

Keep the globes for use in the next lesson.

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Lesson 16: EARLY FLAT MAPS OF THE WORLD

The purposes of this lesson are:

- to study the topological properties of some early flat maps of the world;
- to determine, by comparing them to the globe, why these maps are not very useful ending sets today.

MATERIALS

- globes (from Lesson 15)
- pencils
- Maps F, G and H (in Student Manuals)
- Worksheets 73, 74 and 75

PROCEDURE

Have the children work in groups, using globes and Maps F, G and H to answer the questions on Worksheets 73, 74 and 75.

Worksheet 73
Unit 28 Name________________

Tear Map F out of your manual.
Use Map F and the globe to answer these questions.

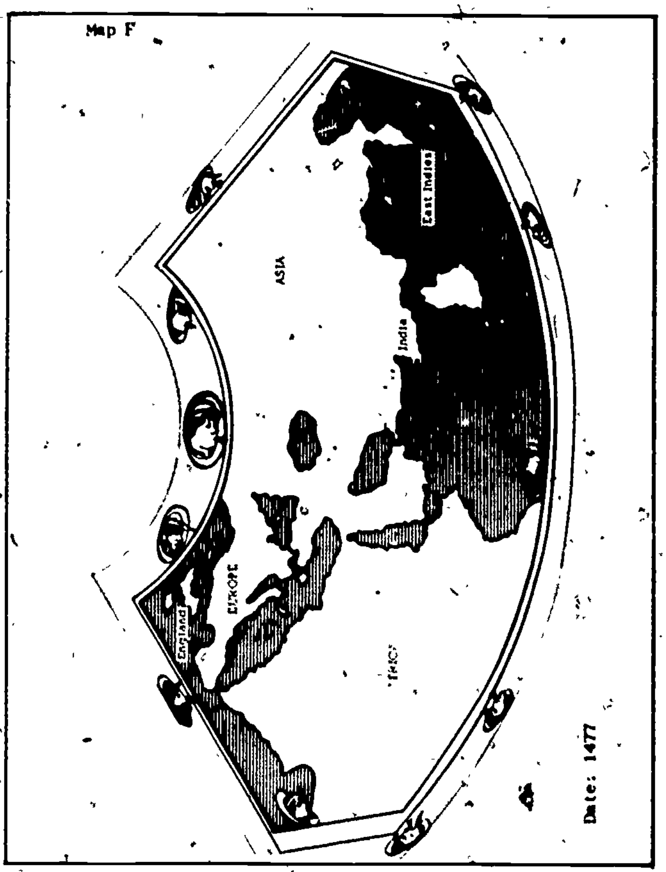
1. Does Africa look the same on both Map F and the globe?__________
If not, how is it different?__________
____

2. Can you find any large land areas that are on the globe that are not on Map F?__________
If so, list them below.
__________ __________ __________
__________ __________ __________
__________ __________ __________
__________ __________ __________

3. Is there one to one matching between the land areas on the globe and the land areas on Map F?
____

4. Write about any other differences you can find between the regions on the globe and the regions on Map F.__________
____
____

5. Is Map F a useful ending set of the globe?__________
Why or why not?__________
____



Worksheet 74
Unit 28 Name____________________

Tear Map G out of your manual.
Use Map G and the globe to answer these questions.

1. Does Greenland look the same on both Map G and the globe?____________
If not, how is it different?____________
____

2. What properties have been changed?____________
____

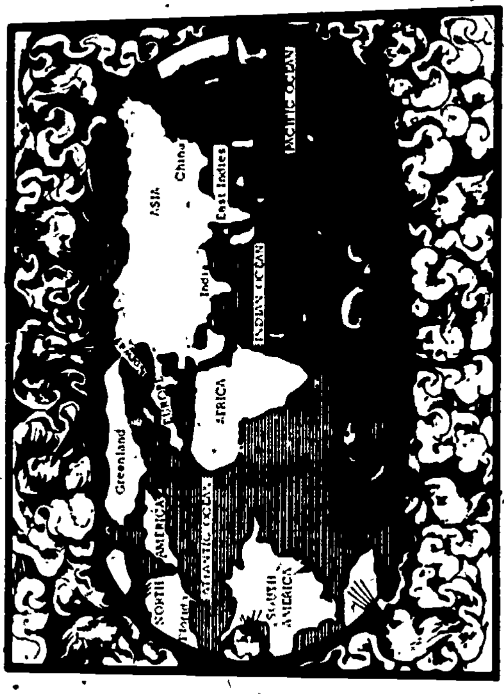
3. Find North America on the globe.
Find North America on Map G.
Does North America look the same on both?____________
If not, how is it different?____________
____

4. Are there any areas on the globe that are not on Map G?____________
If there are, list them below.
____________ ____________ ____________
____________ ____________ ____________
____________ ____________ ____________
____________ ____________ ____________

5. Is there one to one matching between land areas on the globe and land areas on Map G?____________

6. Is Map G a useful ending set of the globe?____________
Why or why not?____________
____
____

Map G



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Worksheet 75
Unit 28 Name____________________

Tear Map H out of your manual.
Use Map H and the globe to answer these questions.

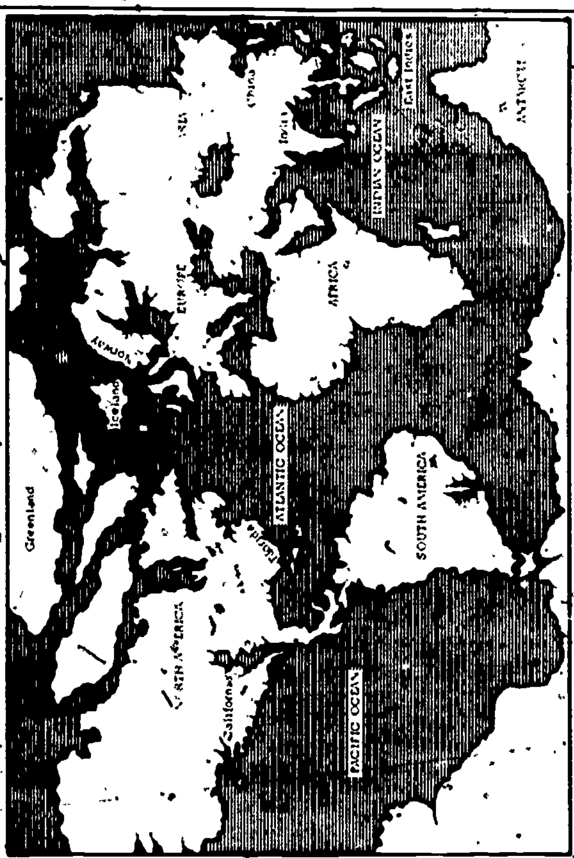
1. Does North America look the same on both Map H and the globe?____________
If not, how is it different?____________
____

2. Are there any other areas that are not the same on Map H and the globe?____________
Which area?____________
How is it different?
____
Which area?____________
How is it different?
____
Which area?____________
How is it different?
____

3. Is there one to one matching between land areas on the globe and land areas on Map H?____________

4. Is Map H a useful ending set of the globe?____________
Why or why not?____________
____
____

Map H



Date: 1675

(They should tear the maps out of their manuals in order to study them more easily.) When they have completed the worksheets, discuss their answers. Below are some ideas that should be covered in the discussion.

These maps are not one to one transformations of the globe; many regions have been left off the maps. Ask the children why they think this happened. Someone will probably notice the dates on the maps and remember that certain regions had not yet been discovered by explorers. Besides noticing the lack of one to one matching between the globes and these maps, the children will probably notice that non-measurable properties have changed. For example, on Map G, the boundary of Greenland is connected to Asia and Europe. Therefore, regions that are actually located outside the boundary of Greenland are not represented correctly; the in-out property has changed. Also, the inside region (the Greenland region on the globe) is no longer separated from the outside region; the boundedness property has changed. On the globe (starting set), the order is Greenland, Ocean, Norway. On Map G (ending set), the order is Ocean, Greenland, Norway; the order of the main features has changed.

Discuss why these old maps of the world are not very "useful" or "truthful" ending sets of the globe. (They don't keep one to one matching of the main features, nor do they keep non-measurable properties such as order, in-out and boundedness.)

Repeat the questions you asked at the end of Lesson 15.

CAN WE MAKE A FLAT MAP OF THE WORLD THAT RETAINS THE NON-MEASURABLE PROPERTIES SUCH AS ORDER, IN-OUT AND BOUNDEDNESS?

IF WE COULD, WHAT WOULD SUCH A MAP LOOK LIKE?

WOULD THE REGIONS LOOK LIKE THEY DO ON THE GLOBE?

Tell the children that in the next lesson, they will look for solutions to these questions.

Lesson 17: FLATTENING THE GLOBE

The purposes of this lesson are:

- to acquaint the children with the topological difficulties of "flattening" a sphere (the globe);
- to discuss the usefulness of different methods of flattening global maps;
- to compare the properties of a standard Mercator map with the properties of global maps;
- to show that different flat maps of the world are useful for different purposes and to let the children speculate about how these maps can be made with the fewest changes in the topological properties of the globe.

MATERIALS

- 1 hollow rubber ball (in third grade kit)
- 30 pairs of scissors
- 1 straight pin or other sharp instrument
- 1 balloon
- Worksheets 76 and 77

PROCEDURE

- Ask the children:

  CAN WE MAKE A FLAT MAP OF THE WORLD THAT KEEPS THE NON-MEASURABLE PROPERTIES OF ORDER, IN-OUT AND BOUNDEDNESS OF THE SPHERE?

Show the children the hollow ball. Ask them to think of this ball as the globe. Then ask if anyone can flatten it without changing its properties of order, in-out and boundedness. Let volunteers try out their ideas. No matter how they stretch it or step on it, they will not be able to make it lie flat.

Then ask:

WHAT DO WE HAVE TO DO TO THE BALL TO MAKE IT FLAT?

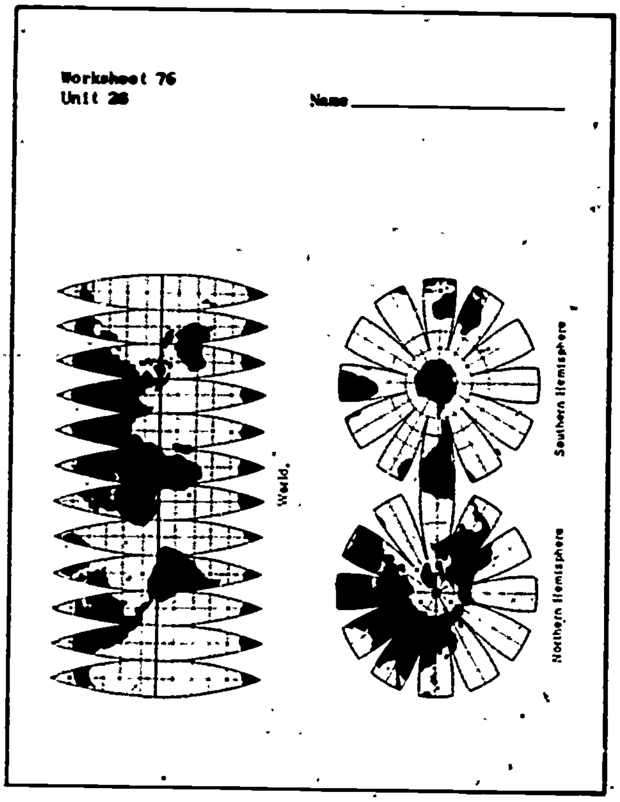
Someone will probably suggest poking a hole in it. Let him do this with a pin or some other sharp instrument. Squeeze the ball so that the air inside comes out. Ask the children what properties of the ball changed when the hole was poked in its surface. (The boundedness property and the in-out property have changed. Some children may recall the balloon activities they performed in Lessons 5 and 6, in which the properties of in-out and boundedness of a balloon were changed by poking a hole in it.)

Once a hole has been made in the ball and the ball is deflated, the main problem is to make the entire surface visible from one side. To illustrate this problem to the children, put an "X" on one side of the deflated ball. If you hold the ball with the X'd side toward you, the class will not be able to see the X, and vice versa.

Ask the children what can be done to the ball to make the entire surface visible from one side. Some child will probably suggest stretching the edges of the hole until the entire surface is flat. Do not do this with the ball; instead, try it with a balloon, which stretches out more easily.

The children should be able to visualize how the edges could be stretched out so the surface is flat and entirely visible from one side. Ask them what they think would happen to the land areas drawn on the surface of the globe if the globe were stretched this way. (They would become very distorted in some areas.)

Another method for flattening the ball is to make several cuts in it until it lies flat. Do this with the demonstration ball. Tell the children that cutting the ball is one way to make it lie flat. Have them look at the maps on Worksheet 76 (see reduction on the next page). These maps were made by "cutting" the globe. Discuss the properties of these maps and compare them with the properties of the globe's surface. Point out that in this case, not only have the topological properties of the sphere changed, but also the topological properties of the continents on the sphere's surface. For example, on the maps, the boundary of North America is disconnected in several places; this is not true of the boundary of North America on the globe.



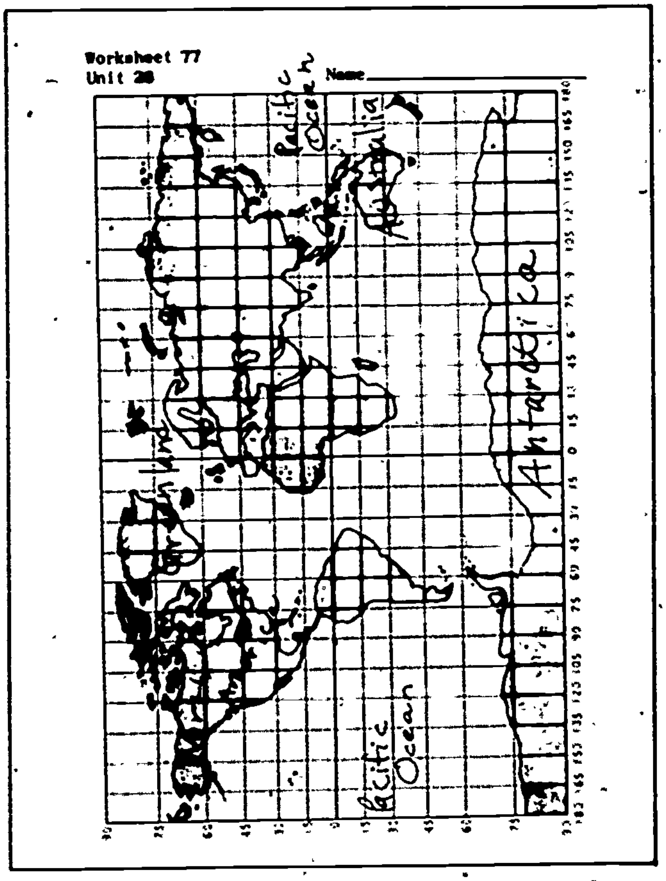
NOTE: Some children may become confused when they compare the properties of the globe's surface with the properties of the map's surface. They are not comparing the sphere (three dimensions) with the flat map (two dimensions). Rather, they are comparing the surface of the sphere (two dimensions) with the flat map (two dimensions). Perhaps the following examples will help to clarify this distinction.

1. The boundary of North America on the globe's surface can be compared to the boundary of North America on the flat map. In this example, a boundary on the 2-dimensional global surface is being compared to a boundary on the 2-dimensional flat map surface.

2. The boundary of the sphere, that is, the material (steel, rubber, etc.) that creates the boundary that separates the inside region from the outside region of the sphere, cannot be compared to the boundary of North America on the flat map.

3. The inside and outside regions of the sphere (i.e., the air inside the ball and the air outside the ball) cannot be compared to the inside and outside regions of North America on the flat map.

When discussing Worksheet 76, bring out the idea that these are not very useful ending sets of the globe because of the many cuts that were made that changed the non-measurable properties of the continents. Tell the class that although it is necessary to make some cuts, we want to make as few as possible when making a flat map of the globe. Ask the children to turn to Worksheet 77, which shows a standard Mercator map. Discuss the properties of this map (ending set) by comparing it to a globe (starting set).



Compare the lines of latitude and longitude on the globe to these lines on the Mercator map. The children will probably see that on the globe, the lines of longitude all meet at the North and South Poles (points, in both cases). However, on the Mercator map, the lines of longitude are parallel, and the points that were the North and South Poles on the globe have become "stretched out" to become lines the same length as the equator. Discuss what effect this "stretching-out" at the poles has on the land areas in these regions, especially Greenland and Antarctica. Have the children label these two land areas on Worksheet 77.

Have the children label Australia, the Pacific Ocean and North America on Worksheet 77. Ask them if they think this map would be very "useful" for the captain of a ship sailing from Australia through the Pacific Ocean to North America. (No. The Pacific Ocean is located on both the right and left sides of the map.) The cut that made the map lie flat

ERIC

was made along the 180° line of longitude. Discuss how the map could have been made so that the Pacific Ocean was left whole. (It could have been cut along a different longitude line, one that did not bisect the Pacific Ocean.)

Discuss the usefulness of this map for other purposes. For example, would a traveler going across Europe and Asia be able to use it? Or an airplane pilot flying the Polar Route between London and New York? Or an explorer in Antarctica? The children should see that the map on Worksheet 77 is adequate for some purposes but not for others. Stress the idea that for different purposes you need different maps, one for which the cuts that make it lie flat are made along different longitude lines..

Ask the children to think about how mapmakers construct flat maps that have only a few cuts in them and that keep the properties of order, in-out and boundedness as much as possible. Ask them to think especially about how the Mercator map (Worksheet 77) was made. Accept their speculations for now and leave the question open.

Lesson 18: PROJECTING IMAGES FROM A SPHERICAL SURFACE ONTO A FLAT SURFACE

The purposes of this lesson are:

- to review the methods used in earlier lessons to make topological transformations and to discuss whether these methods can be used to transform a global surface into a flat ending set;
- to investigate how a point light source and a transparent plastic half-sphere can be used to project designs from a curved surface onto a flat surface;
- to investigate the measurable and non-measurable properties of the ending sets that are formed by projection.

MATERIALS

- overhead projector
- transparency of designs 1 and 2 (included in the appendix of this manual)
- 1 globe
- projection equipment (in third grade kit): transformer, plastic half-sphere, lamp, translucent plastic sheet
- black grease pencil
- 30 rulers
- 30 transparent area grids (from Lesson 2)
- 30 pairs of scissors
- Worksheets 78 - 82

PREPARATION

Cut out the two design pieces on the transparency for this lesson. The transparency is in the appendix at the back of this manual. Set up the projection equipment on a demonstration table as shown on the next page. It would be helpful to study this equipment before conducting the lesson in order to become familiar with it.

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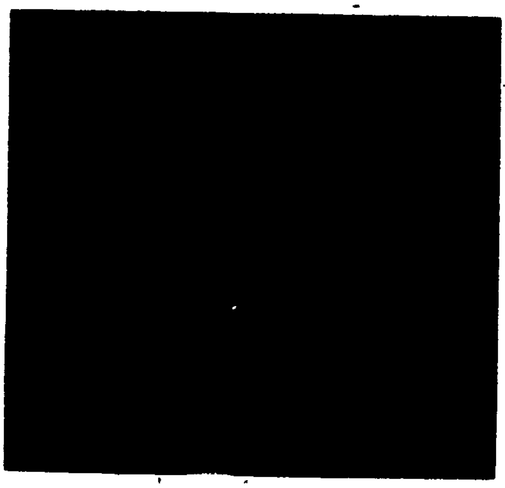


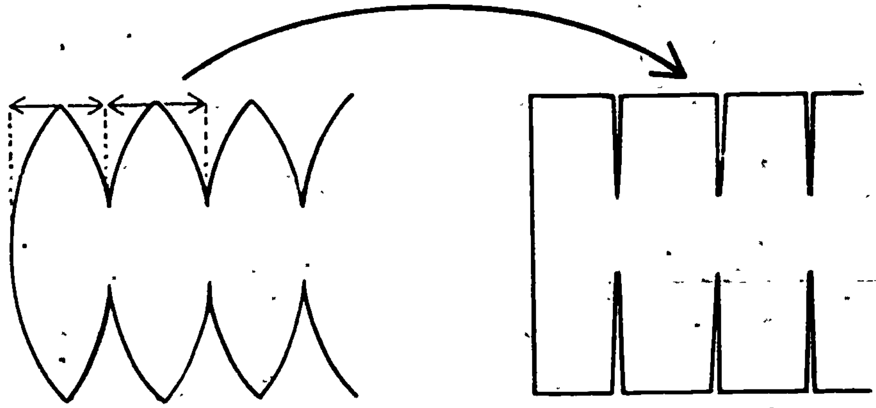
Set aside the projection equipment so it is not visible at the beginning of the lesson.

PROCEDURE

At the end of the previous lesson, the children were asked to think about how they could make different flat maps of the world that have the least amount of changes in the properties of order, in-out and boundedness of the continents. Discuss any ideas the children come up with. If they have difficulty thinking of ways to make such maps, ask them to recall some of the transformations they made earlier in this unit — transformations that kept the properties of boundedness, in-out and order. Ask them to think of how they could use these methods to make a flat ending set of the globe. Listed below are the different methods for making topological transformations that the children have studied and some suggestions for discussing them.

1. <u>Clay molding transformation</u>
   If they had a globe made out of clay, theoretically it could be molded into a flat map. However, this is not very practical.

2. <u>Rubber sheet transformation</u>
   If they had a globe made out of rubber, they could poke a hole in it and stretch the edges of the hole until the entire surface was flat. They could also cut the rubber globe and stretch out the areas by the polar regions.



Again, neither method is very practical.

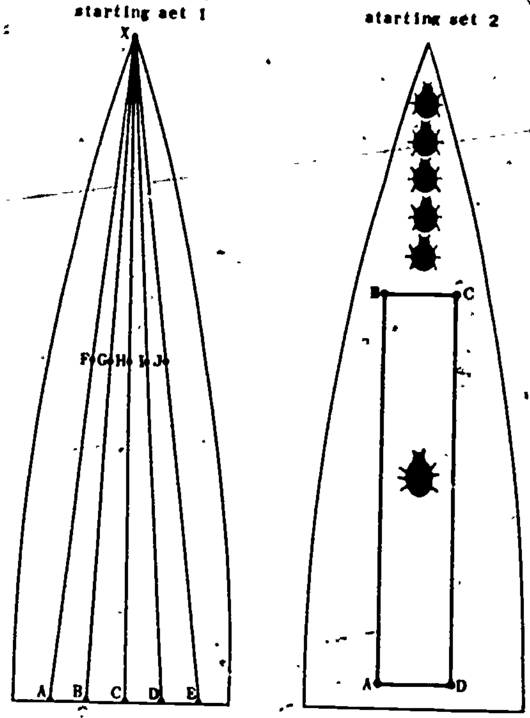
3. Grid transformation
   They could use the lines of longitude and latitude on the globe as their starting set. Each feature found in the grid sections could be transformed to a rectangular ending set grid. The ending set might look something like the Mercator map on Worksheet 77 from Lesson 17.

4. Projection transformation
   The children will probably remember the projection booklets they completed in Lesson 12 when they used the overhead to project figures onto the wall. Ask the children if they have any ideas about how this projection method could be used to project a global map onto other surfaces. Accept all reasonable ideas. Let several children test their ideas, using the overhead projector, globe and any other equipment they wish. For example, some child may place the globe on the overhead. He will soon discover that the spherical globe projects onto the screen as a black, circular region.

After the children have had a chance to test several of their ideas, show them the projection equipment you set up earlier. Ask them if they can think of some way this equipment could be used to project something from a curved, spherical surface onto a flat surface. Gather the children around the demonstration table on which the half-sphere equipment has been set up, so that they all can see it. Make a design with a grease pencil on the clear half-sphere. Ask the children for suggestions on how to project the design. Carry out their suggestions as closely as possible. Then show them the sheet of translucent plastic and suggest that it could be used as a possible "screen" on which to project the design. Darken the room and cast the design you drew on the half-sphere onto the ceiling and other surfaces (a large sheet of newsprint, etc.).

Show the children designs 1 and 2 that you cut out. Tell them that these two designs are going to be used as starting sets and the projected images of these starting sets will be the ending sets. However, before you begin projecting, ask the children to return to their desks and to tear Worksheet 78 out of their manuals. Worksheet 78 has copies of the transparency designs on it.

Worksheet 78
Unit 28
Name____________________



Worksheet 79
Unit 28
Name____________________

Properties of Starting Set 1

Measure the distance between the following points:

A and B ≐ $\frac{1}{2}$ inch
B and C ≐ $\frac{1}{2}$ inch
C and D ≐ $\frac{1}{2}$ inch
D and E ≐ $\frac{1}{2}$ inch
A and E ≐ 2 inches
F and G ≐ $\frac{1}{4}$ inch
G and H ≐ $\frac{1}{4}$ inch
H and I ≐ $\frac{1}{4}$ inch
I and J ≐ $\frac{1}{4}$ inch
F and ≐ 1 inch
X and C ≐ $8\frac{1}{2}$ inches

Starting at point A and going to point E, the order of the points is:

A, B, C, D, E

Starting at point F and going to point J, the order of the points is:

F, G, H, I, J

Worksheet 80
Unit 28
Name____________________

Properties of Starting Set 2

Measure the distance between the following points:

A and B ≐ 5 inches
B and C ≐ 1 inch
C and D ≐ 5 inches
D and A ≐ 1 inch

Find the area of rectangle ABCD without using your area grid.

area of rectangle ABCD ≐ 5 square inches

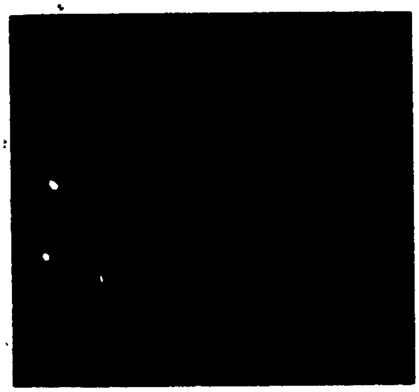
Check your answer. Measure the rectangle with your area grid.

Starting at point A and going in the direction of point D, the order of the points is:

A, D, C, B, A

No Hum is located inside the rectangle.
How many bugs are located outside the rectangle? 5

The students will need rulers, transparent area grids and Worksheet 78 to complete Worksheets 79 and 80, which are record sheets for listing the properties of the two starting sets (designs 1 and 2). When the children have completed Worksheets 79 and 80, tape design 1 (starting set 1) to the half-sphere, as shown.



Ask them to tear out Worksheets 81 and 82. These are record sheets for the ending sets. Project the starting set onto several surfaces, including the translucent sheet (screen). Hold

Worksheet 81
Unit 28 Name_______________

Properties of Ending Set 1

Measure the distance between the following points:

A and B ≐_____inch
B and C ≐_____inch
C and D ≐_____inch
D and E ≐_____inch
A and E ≐_____inch
F and G ≐_____inch
G and H ≐_____inch
H and I ≐_____inch
I and J ≐_____inch
F and J ≐_____inch
X and C ≐_____inch

Starting at point A and going to point E, the order of the points is:

A, B, C, D, E

Starting at point F and going to point J, the order of the points is:

F, G, H, I, J

Worksheet 82
Unit 28 Name_______________

Properties of Ending Set 2

Measure the distance between the following points:

A and B ≐_____inches
B and C ≐_____inches
C and D ≐_____inches
D and A ≐_____inches

Find the area of rectangle ABCD without using your area grid.

area of rectangle ABCD ≐_____square inches

Check your answer. Measure the rectangle with your area grid.

Starting at point A and going in the direction of point D, the order of the points is:

A, D, C, B, A

No bug is located inside the rectangle.
How many bugs are located outside the rectangle? 5

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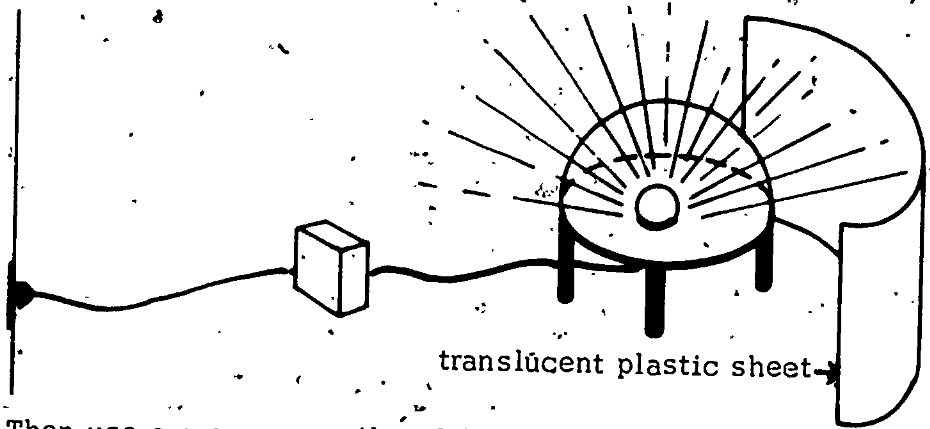
the screen in several positions. Ask the children to choose one position as the ending set position.



Then use a grease pencil or felt tip pen to outline the projected image on the screen. When this is done, lay the screen flat on the table and have someone measure the distances asked for on Worksheet 81. He should tell the rest of the class the measurements, so they can record them on their copies of Worksheet 81. Have some other student tell the class the other information asked for on the worksheet.

When everyone has filled in Worksheet 81, follow the same procedure for design 2 (starting set 2), recording the properties of the ending set on Worksheet 82. When the children have all completed Worksheet 82, ask them to compare the properties of each starting set to its ending set. Discuss which properties changed and which properties were retained.

The children should see that this method can be used to project a starting set on a spherical surface onto a screen that can be flattened, and in this way, the non-measurable properties are retained, while the measurable properties are changed.

End this lesson by asking the children to think about how this equipment could be used to make a flat ending set of the globe.

Save the equipment to use in the next lesson.

Lesson 19: MAPPING THE GLOBE

The purposes of this lesson are:

- to use the projection equipment from Lesson 18 to project the globe onto a flat surface;
- to compare the properties of a half-sphere starting set with the properties of three flat ending sets. These three ending sets are a cylindrical projection, a conical projection and a gnomic projection;
- to give the children a chance to predict which regions of the starting set will be the most distorted in each of the three ending sets, and to verify these predictions.

MATERIALS

- projection equipment (from Lesson 18)
- 4 transparencies of Northern Hemisphere pieces (included in the appendix at the back of this manual)
- transparent tape
- 30 pairs of scissors
- glue
- pencils
- 30 rulers
- 30 transparent area grids (from Lesson 2)
- ESS unit, Outdoor Mapping (optional); for information about obtaining this booklet, write to the Elementary Science Study of Education Development Center, Inc., 55 Chapel Street, Newton, Massachusetts 02160
- Worksheets 83 - 95

PREPARATION

Cut out the twelve map sections on the four transparencies for this lesson. The transparencies are in the appendix. Set them aside so they are not visible at the beginning of the lesson. Set up the half-sphere projection equipment on a demonstration table, as you did for Lesson 18.

PROCEDURE

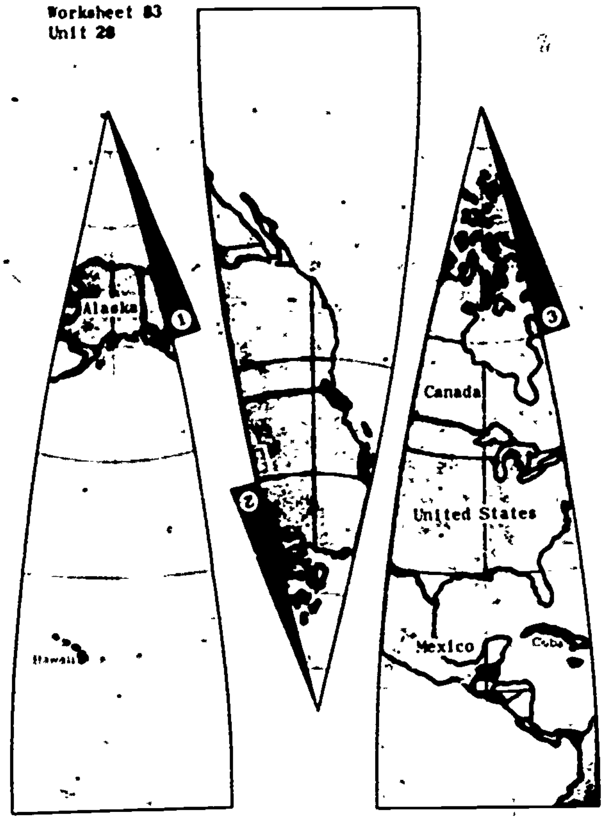
Activity A

Gather the children around the demonstration table on which the projection equipment is located. Ask them how they think the equipment could be used to make a flat map of the globe, and give them a chance to use the equipment to try out their ideas. They will probably discover that they need a transparent global map so that the light source can project the globe outward onto a screen. Show the children the transparent sections of the Northern Hemisphere that you cut out earlier. Ask them if they have any ideas about how they could use these. They will probably suggest taping the sections to the half-sphere.

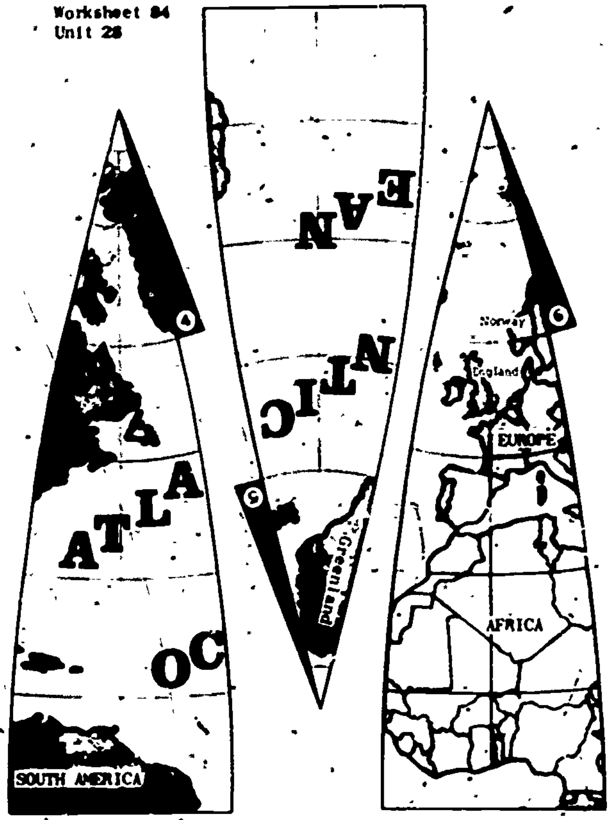
Tape the bottom of one transparent section to the bottom edge of the half-sphere and then tape the top down. Do this with the other eleven sections, making sure the latitude lines match. While you are taping on the sections, the children will probably notice that it is difficult to make the pieces conform completely to the curved surface of the sphere. However, for your purposes, these strips conform to the curved surface well enough.

Tell the children that this half-globe is the starting set and that they will project it onto different surfaces to make different ending sets. Then have them return to their desks. Tell them they will be working in pairs to investigate some of the properties of the starting set. Each pair will put together a half-globe similar to the transparent one you made. Worksheets 83 through 86 contain the twelve map sections.

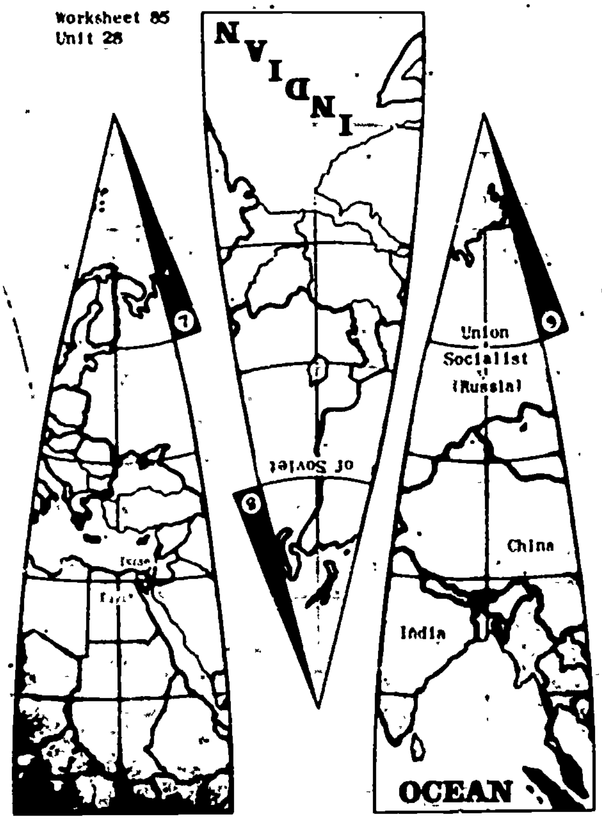
Worksheet 83
Unit 28



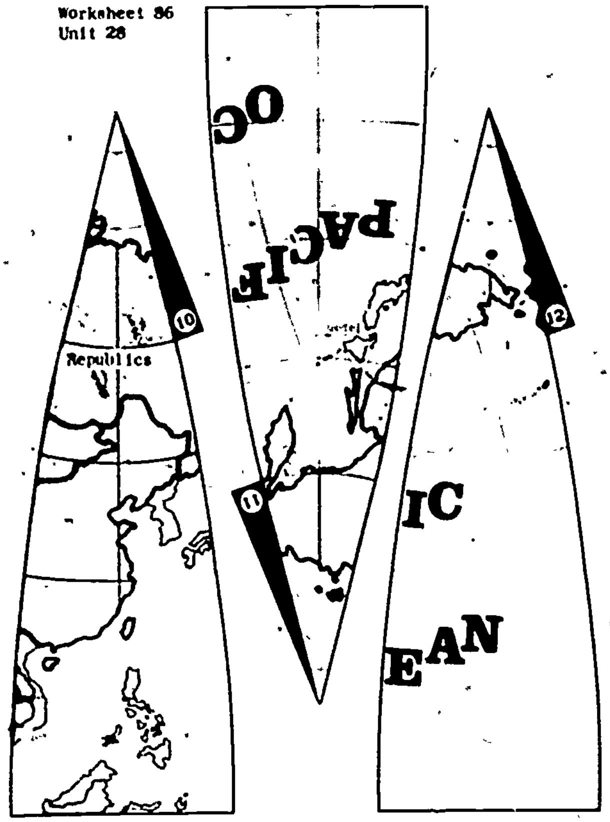
Worksheet 84
Unit 28



Worksheet 85
Unit 28



Worksheet 86
Unit 28



Worksheet 87
Unit 28 Name____________

Properties of Starting Set

Measure the area of these regions.

Greenland : ____ square inches
Canada : ____ square inches
Alaska : ____ square inches
United States : ____ square inches
Mexico : ____ square inches
U.S.S.R. : ____ square inches
Europe : ____ square inches
China : ____ square inches
India : ____ square inches

The height of the letters used to label the oceans : ____ inches.

Be ready to discuss the non-measurable properties of the starting set (the half-globe) with your classmates.

Discuss the following procedure with the children.

1. Each pair of children should use only one set of Worksheets 83 - 86.

2. One partner should carefully cut out the map sections and arrange them in the order of the numbered tabs.

3. The other partner should glue the sections together in order.

4. When the sections are glued together, the children should use their rulers and transparent area grids (from Lesson 2) to complete Worksheet 87.

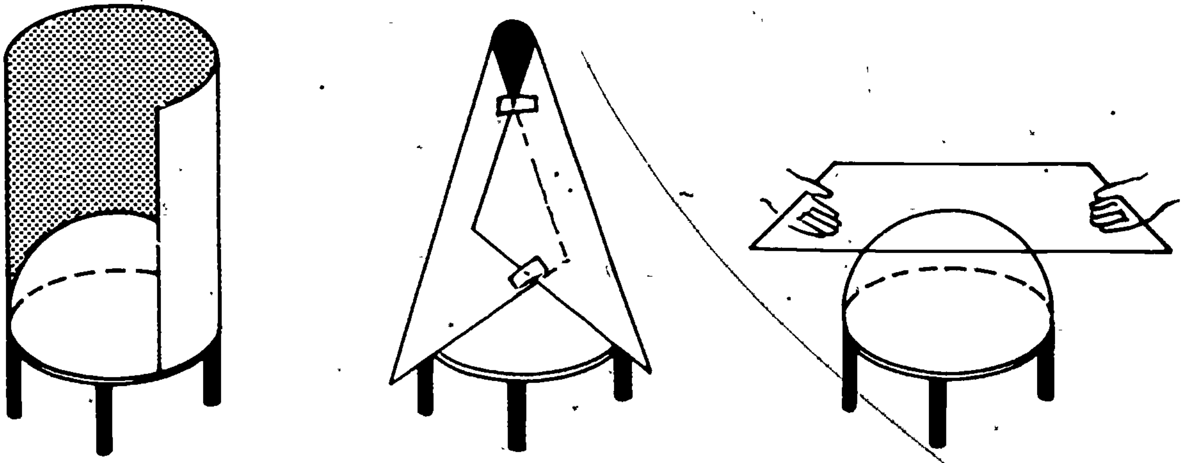
When the children measure the area of various regions on their half-globes, one partner should hold the sections together, while the other partner measures the area.

When they have completed Worksheet 87, gather the children around the demonstration table. Discuss the non-measurable properties (order, in-out, boundedness) of the starting set. For example, the boundary of Greenland separates the inside "land region" from the outside "ocean region." If a piece of Greenland were placed on the outside ocean region, the in-out property would be changed. Also discuss the order of the large land areas; for example, on the North American continent, the order of the main land areas is Alaska, Canada, U. S., Mexico. Ask the children to give an example of a change in the order of these land areas. (One example would be Alaska, Mexico, U. S., Canada.)

Activity B

Remind the children of the designs they projected in Lesson 18. Bring out in the discussion the idea that, depending on the position in which the screen was held in relation to the half-sphere, different ending sets were formed that had varying amounts of distortion.

Tell the children that they will project three ending sets of the half-globe starting set. The screen will be held in three different positions. Demonstrate the three positions as shown below.



cylindrical conical gnomic

Worksheet 88
Unit 28 Name____________

Prediction Sheet — Ending Set 1

If the screen is held like this:

★ Which region will be more distorted or stretched?

1. Greenland or Mexico

____

2. China or India

____

★ Which letters will be more distorted?

1. The "A" or the "C" in ATLANTIC

____

2. The "O" or the "N" in OCEAN

____

★ Will the properties of order, in-out and boundedness change?

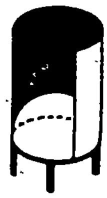
____

Worksheet 89
Unit 28 Name____________

Properties of Ending Set 1

Measure the area of these regions.

Greenland ±____square inches
Mexico ±____square inches
China ±____square inches
India ±____square inches



Measure the height of these letters.

The "A" in ATLANTIC ±____inches
The "C" in ATLANTIC ±____inches
The "O" in OCEAN ±____inches
The "N" in OCEAN ±____inches

Is the order of the main features the same or different? Same

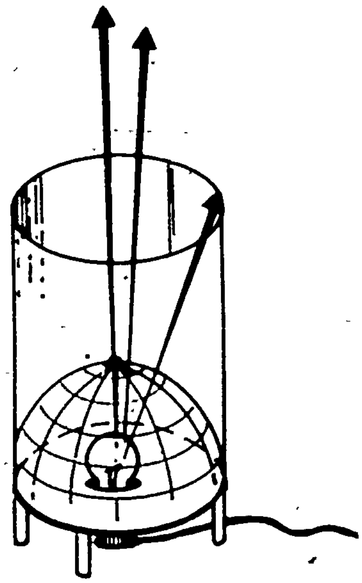
Has the boundedness property changed? No

Has the in-out property changed? No

Have the children turn to Worksheet 88. They should find Greenland, Mexico, China and India on their half-globes (starting sets). Ask them to recall their work with the design sheets and the different amounts of distortion that occurred. They should predict which regions will be more distorted when the screen is held in the position shown on the worksheet. Go through the rest of Worksheet 88 with the class. Each child should fill in his own predictions. When all the children have completed Worksheet 88, project the half-globe onto the cylindrically held screen. (Tape the bottom corners of the screen to the half-globe.)

Have the children turn to Worksheet 89. Have different children measure the area of the regions listed on the worksheet. (These measurements must be made on the screen, not on the half-globe.) They should repeat the measurements to the class so each student can record it on his copy of Worksheet 89.

Some children may notice that the Polar Region is not on the cylindrical projection. The diagram below may help you explain why this is so.



Notice that the North Pole is projected straight up and therefore cannot be seen on the cylindrical screen. The regions near the North Pole could be projected onto a cylindrical screen, if the screen were very tall and the lamp were much brighter than the one in the projection equipment.

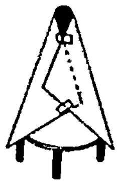
When all the children have finished Worksheet 89, have them compare it with Worksheet 88 and discuss which properties of the starting set changed and which were retained. Also discuss which regions were more distorted. Ask the children if this map (cylindrical projection) reminds them in any way of a flat map of the world they've seen before. Someone will probably notice that this projection is similar to the Mercator map; the North Pole becomes a line instead of a point, and the lines of longitude become parallel.

Do Worksheets 90 through 93 the same way as Worksheets 88 and 89. However, you may want to divide the class into small groups and let them experiment with the equipment over a period of several days, using Worksheets 90 through 93 as guides for their investigations. Whichever way the children do this activity, discuss the worksheets with them afterward.

Worksheet 90
Unit 28 Name____________

Prediction Sheet — Ending Set 2

If the screen is held like this:



* Which region will be more distorted or stretched?

1. Greenland or United States

____

2. Europe or the U.S.S.R.

____

* Which letters will be more distorted?

1. The "A" or the "C" in ATLANTIC

____

2. The "P" or the "F" in PACIFIC

____

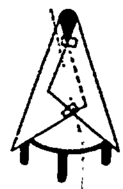
* Will the properties of order, in-out and boundedness change?

____

Worksheet 91
Unit 28 Name____________

Properties of Ending Set 2

Measure the area of these regions.



Greenland ≐____square inches
United States ≐____square inches
Europe ≐____square inches
U.S.S.R. ≐____square inches

Measure the height of these letters.

The "A" in ATLANTIC ≐____inches
The "C" in ATLANTIC ≐____inches
The "P" in PACIFIC ≐____inches
The "F" in PACIFIC ≐____inches

Is the order of the main features the same or different? Same

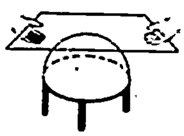
Has the boundedness property changed? No

Has the in-out property changed? No

Worksheet 92
Unit 28 Name____________

Prediction Sheet — Ending Set 3

If the screen is held like this:



* Which region will be more distorted?

1. Alaska or Mexico

____

2. The U.S.S.R. or India

____

* Which letters will be more distorted?

1. The "C" in ATLANTIC or the "O" in OCEAN

____

2. The "I" or the "A" in INDIAN?

____

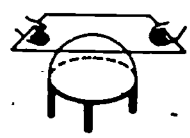
* Will the properties of order, in-out and boundedness change?

____

Worksheet 93
Unit 28 Name____________

Properties of Ending Set 3

Measure the area of these regions.



Alaska ≐____square inches
Mexico ≐____square inches
U.S.S.R. ≐____square inches
India ≐____square inches

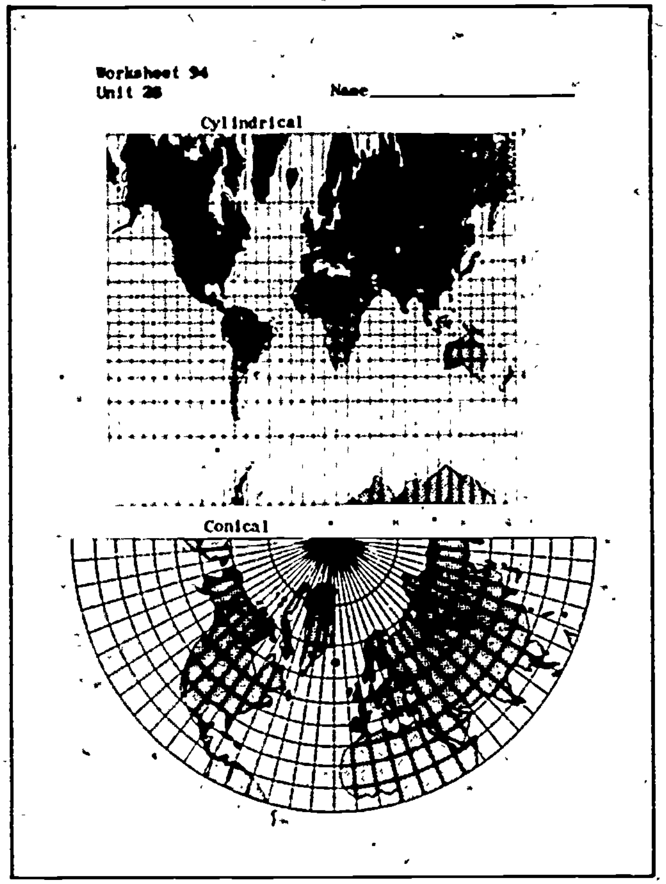
Measure the height of these letters.

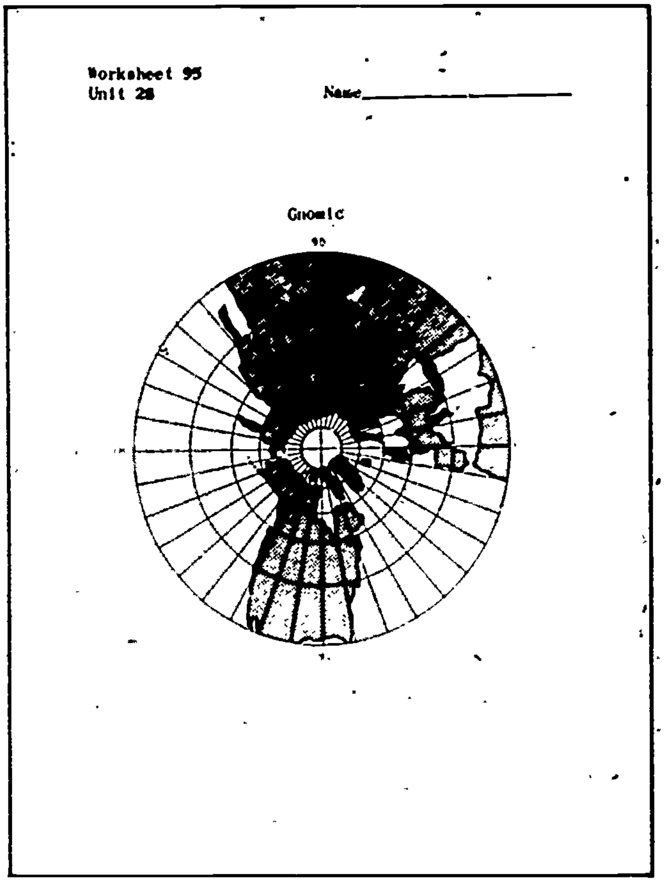
The "C" in ATLANTIC ≐____inches
The "O" in OCEAN ≐____inches
The "I" in INDIAN ≐____inches
The "A" in INDIAN ≐____inches

Is the order of the main features the same or different? Same

Has the boundedness property changed? No

Has the in-out property changed? No





Activity C

Have the children look at Worksheets 94 and 95. Using the information they have learned, ask them to speculate about how the screen was held to get the standard projections shown on these worksheets. Then gather the children around the demonstration table on which the projection equipment is set up. Have several children demonstrate how they think the screen should be held to get these projections. They should be able to support their answers by giving detailed descriptions of the identifying characteristics of each kind of projection. For example, on the cylindrical projection, the grid lines are parallel. Also, there is much distortion near the Polar Regions and little distortion near the equator. Some child may notice that it resembles the Mercator map.

After the children have discussed all three kinds of projection — cylindrical, conical and gnomic — discuss which properties have been retained in each projection (boundedness, order and in-out).

The children should see that even though the measurable properties changed, there is still one to one matching of the main features on the half-globe with the main features on each projection. For example, each major land area on the half-globe can be matched one to one with its projected image on the screen; there is just one North America on the screen, Cuba is not missing, etc.

Some children may notice the absence of the Southern Hemisphere on the conical and gnomic projections on Worksheets 94 and 95. Discuss with them the following suggestions on how projections of the Southern Hemisphere could be made.

1. <u>Conical</u>

   Form the screen into a cone shape and place it over a half-globe that has the Southern Hemisphere printed on it.

2. <u>Gnomic</u>

   Place the screen tangent to the South Pole on a half-globe that has the Southern Hemisphere printed on it.

Discuss these three standard projections in terms of their "usefulness" for different purposes. Here are some questions you might ask:

> WHICH PROJECTION WOULD AN ARCTIC EXPLORER USE? WHY?
>
> WHICH PROJECTION WOULD AN AIRPLANE PILOT FLYING FROM NEW YORK TO MOSCOW USE? WHY?
>
> WHICH PROJECTION WOULD A PERSON TRAVELING AROUND THE WORLD ON A SHIP USE?

The children should see that each projection is suitable for different purposes.

Ask the children which projection they would use if they wanted to make a map of North America. They will probably suggest the conical, since on it, North America is the least distorted.

The students should have an opportunity to experiment with some of their own ideas, using the projection equipment. You could keep the equipment set up in a corner of the room for a few days. The children should also be encouraged to follow their interests in mapping by investigating other types of maps, such as relief maps, contour maps, physical maps, etc. Some child might want to find out and report on how a globe is made, while others might enjoy making a clay contour map of the playground, as described in the ESS unit Outdoor Mapping. (See page 138 for instructions on ordering.)

Lesson 20: SUMMARY LESSON

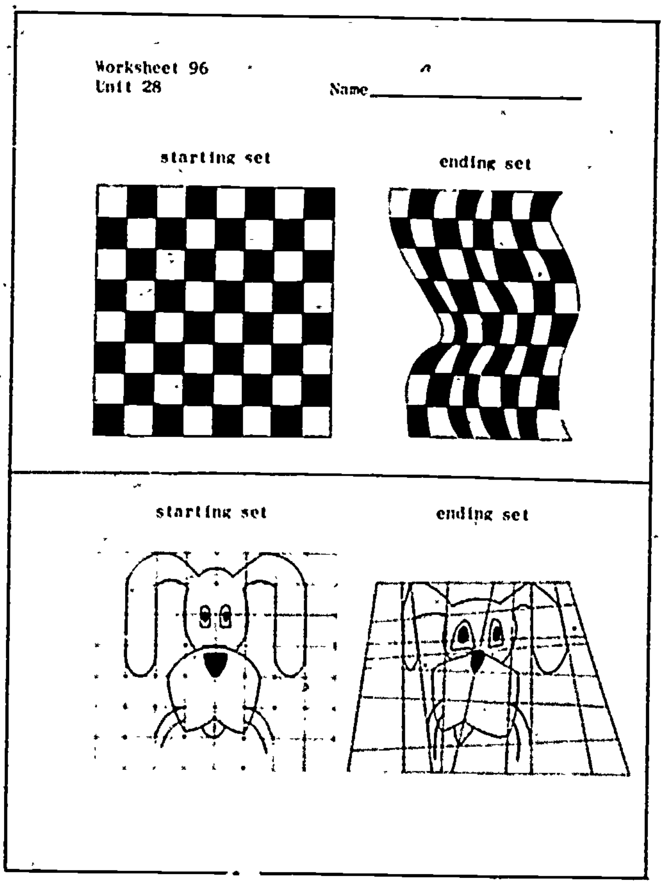
This lesson reviews and summarizes some of the main ideas introduced in this unit. Briefly, these include the notions that when a starting set is transformed into an ending set, both the measurable and the non-measurable properties may change. However, there is a large class of "useful" transformations that involve changing only the measurable properties. Examples of "useful" or topological transformations include checkerboard distortions, rubber sheet transformations, clay molding and grid transformations of maps and pictures.

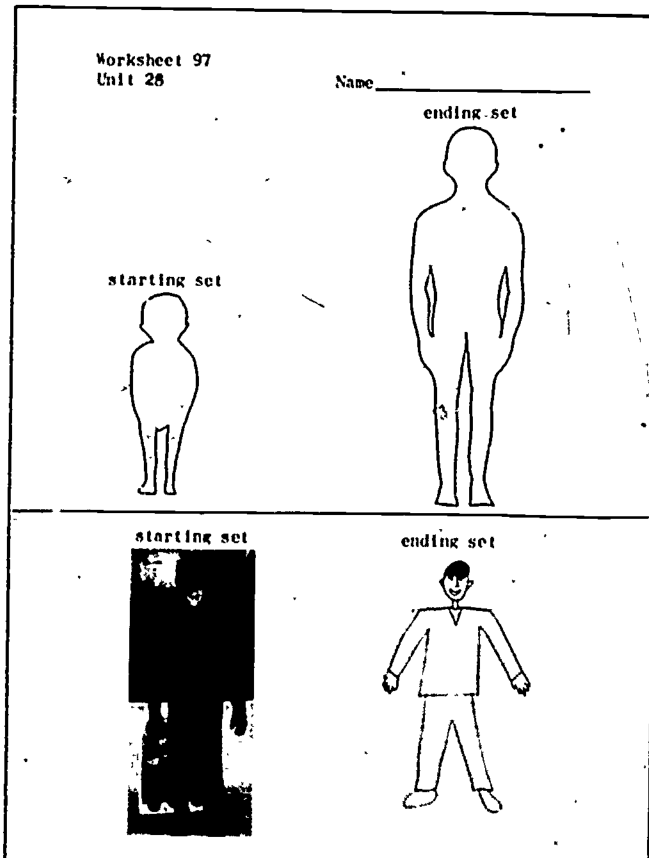
MATERIALS

- pencils
- Worksheets 96 - 99

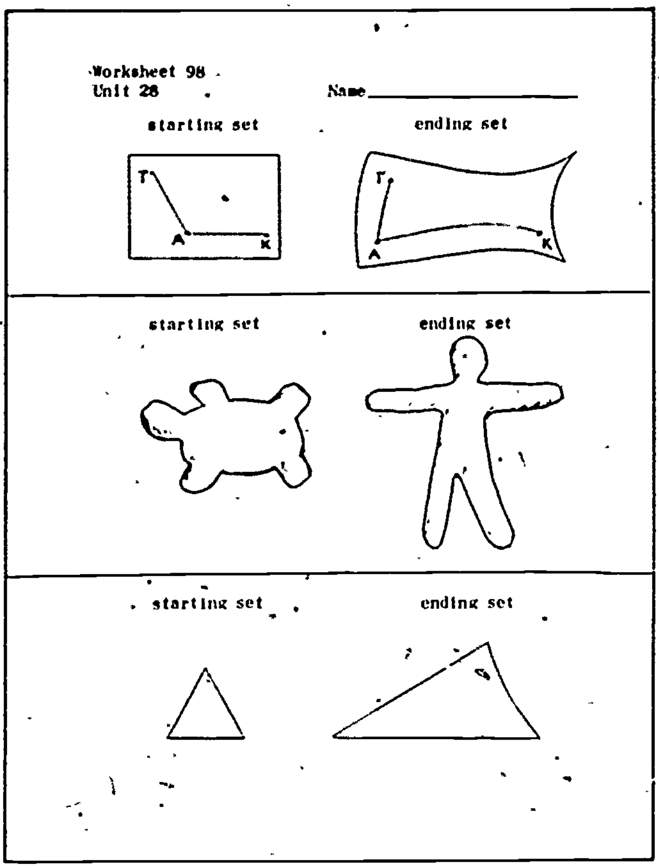
PROCEDURE

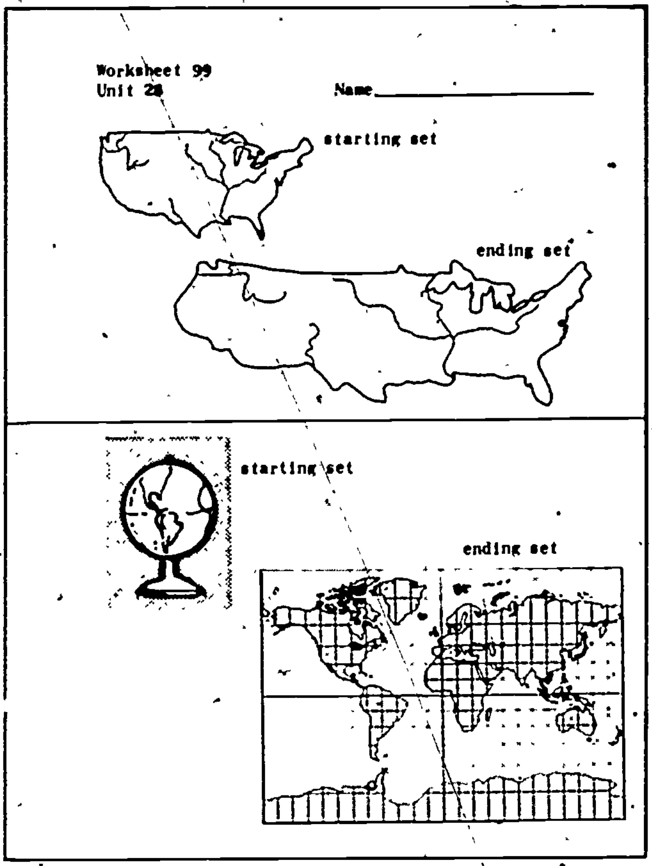
Use Worksheets 96 - 99 as a guide for reviewing and summarizing the main ideas in this unit. One possible procedure to follow would be to discuss which properties changed





ERIC





and which did not change from the starting sets to the ending sets pictured on each worksheet. (Each transformation that is pictured was actually studied in this unit.) Then you could ask the children to generalize about which kind of properties were retained to produce a "useful" ending set (non-measurable) and which kind of properties changed and still produced a "useful" ending set (measurable).

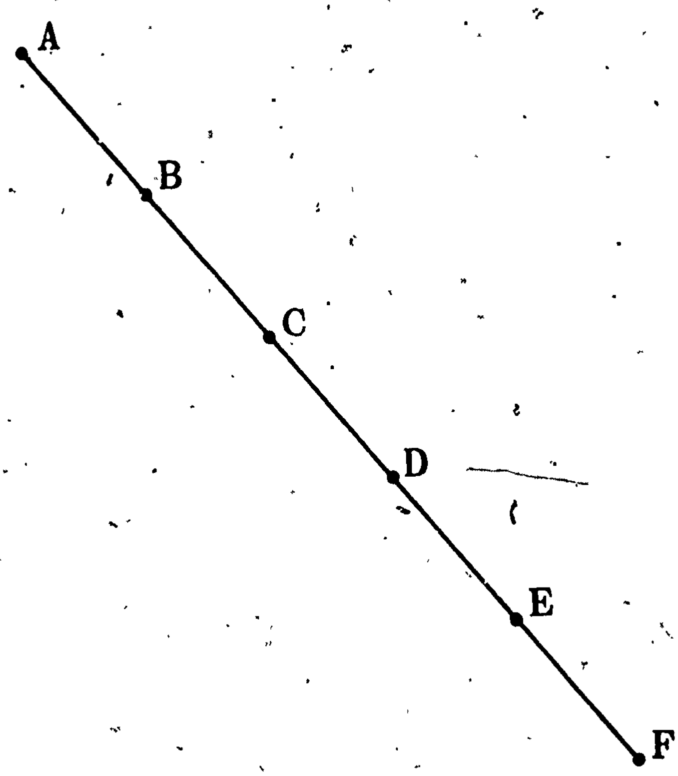
End the lesson by asking the children to think of other kinds of transformations in which the measurable properties change and the non-measurable properties do not change.

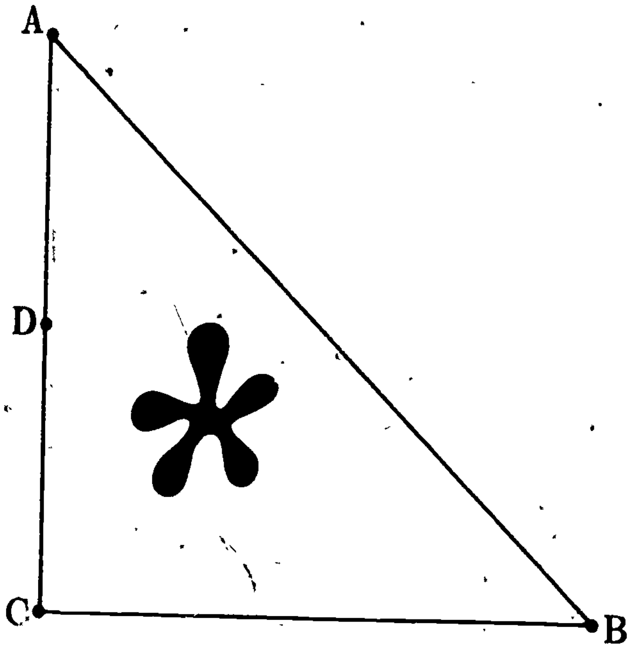
APPENDIX

ERIC

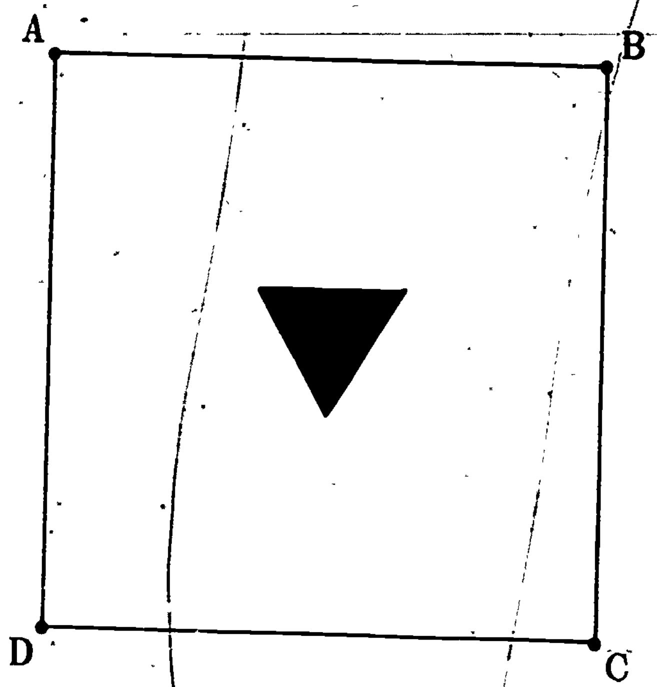
Lesson 2
Worksheet 6

Lesson 12
Trial 1




A
D
C
B

Lesson 12
Trial 3



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