DOCUMENT RESUME

ED 127 193 SE 021 230

AUTHOR Bray, Edmund C.; Redin, Paul
TITLE Multiplication and Motion: MINNEMAST Coordinated Mathematics - Science Series, Unit 25.
INSTITUTION Minnesota Univ., Minneapolis. Minnesota School Mathematics and Science Center.
SPONS AGENCY National Science Foundation, Washington, D.C.
PUB DATE 71
NOTE 163p.; For related documents, see SE021201-234; Photographs may not reproduce well
AVAILABLE FROM MINNEMAST, Minnemath Center, 720 Washington Ave., S.E., Minneapolis, MN 55414

EDRS PRICE MF-$0.83 HC-$8.69 Plus Postage.
DESCRIPTORS *Curriculum Guides; Elementary Education; *Elementary School Mathematics; *Elementary School Science; Experimental Curriculum; Graphs; *Interdisciplinary Approach; Learning Activities; *Mathematics Education; *Multiplication; Primary Grades; Process Education; Science Education; Units of Study (Subject Fields)
IDENTIFIERS *MINNEMAST; *Minnesota Mathematics and Science Teaching Project

ABSTRACT

This volume is the twenty-fifth in a series of 29 coordinated MINNEMAST units in mathematics and science for kindergarten and the primary grades. Intended for use by third-grade teachers, this unit guide provides a summary and overview of the unit, a list of materials needed, and descriptions of three groups of activities. The purposes and procedures for each activity are discussed. Examples of questions and discussion topics are given, and in several cases ditto masters, stories for reading aloud, and other instructional materials are included in the book. In this unit, data collected in foot racing and car racing activities are graphed on grids and properties of the graphs are examined. The idea of slope is used to motivate an introduction to multiplication. Multiplication as repeated addition is reviewed, as is the relationship between arrays and multiplication. A "multiplication machine" and cards for use in a multiplication game are provided. (SD)

MU

AND MOT

25

MINNEMAST

MINNEMAST **COORDINATED MATHEMATICS-SCIENCE SERIES**

KINDERGARTEN	1.	WATCHING AND WONDERING
	2.	CURVES AND SHAPES
	3.	DESCRIBING AND CLASSIFYING
	4.	USING OUR SENSES
	5.	INTRODUCING MEASUREMENT
	6.	NUMERATION
	7.	INTRODUCING SYMMETRY
FIRST GRADE	8.	OBSERVING PROPERTIES
	9.	NUMBERS AND COUNTING
	10.	DESCRIBING LOCATIONS
	11.	INTRODUCING ADDITION AND SUBTRACTION
	12.	MEASUREMENT WITH REFERENCE UNITS
	13.	INTERPRETATIONS OF ADDITION AND SUBTRACTION
	14.	EXPLORING SYMMETRICAL PATTERNS
SECOND GRADE	15.	INVESTIGATING SYSTEMS
	16.	NUMBERS AND MEASURING
	17.	INTRODUCING MULTIPLICATION AND DIVISION
	18.	SCALING AND REPRESENTATION
	19.	COMPARING CHANGES
	20.	USING LARGER NUMBERS
	21.	ANGLES AND SPACE
THIRD GRADE	22.	PARTS AND PIECES
	23.	CONDITIONS AFFECTING LIFE
	24.	CHANGE AND CALCULATIONS
	● 25.	MULTIPLICATION AND MOTION
	26.	WHAT ARE THINGS MADE OF?
	27.	NUMBERS AND THEIR PROPERTIES
	28.	MAPPING THE GLOBE
	29.	NATURAL SYSTEMS

OTHER MINNEMAST PUBLICATIONS

The 29 coordinated units and several other publications are available from MINNEMAST on order. Other publications include:

STUDENT MANUALS for Grades 1, 2 and 3, and
printed TEACHING AIDS for Kindergarten and Grade 1.

LIVING THINGS IN FIELD AND CLASSROOM
(MINNEMAST Handbook for all grades)

ADVENTURES IN SCIENCE AND MATH
(Historical stories for teacher or student)

QUESTIONS AND ANSWERS ABOUT MINNEMAST
Sent free with price list on request

OVERVIEW
(Description of content of each publication)

MINNEMAST RECOMMENDATIONS FOR SCIENCE AND MATH IN THE INTERMEDIATE GRADES
(Suggestions for programs to succeed the MINNEMAST Curriculum in Grades 4, 5 and 6)

MULTIPLICATION AND MOTION

UNIT 25

MINNEMAST

MINNESOTA MATHEMATICS AND SCIENCE TEACHING PROJECT
720 Washington Avenue S. E., Minneapolis, Minnesota 55455

MINNEMAST

DIRECTOR — JAMES H. WERNTZ, JR.
Professor of Physics
University of Minnesota

ASSOCIATE DIRECTOR FOR SCIENCE — ROGER S. JONES
Associate Professor of Physics
University of Minnesota

ASSOCIATE DIRECTOR FOR RESEARCH AND EVALUATION — WELLS HIVELY II
Associate Professor of Psychology
University of Minnesota

The Minnesota Mathematics and Science Teaching Project developed these materials under a grant from the National Science Foundation.

Third Printing
1971

MULTIPLICATION AND MOTION

This unit was developed by

EDMUND C. BRAY

PAUL REDIN

on the basis of experiences of the many teachers who taught earlier versions by

JEANNE BURSHEIM

ROY W. MUCKEY

ELIZABETH W. REED

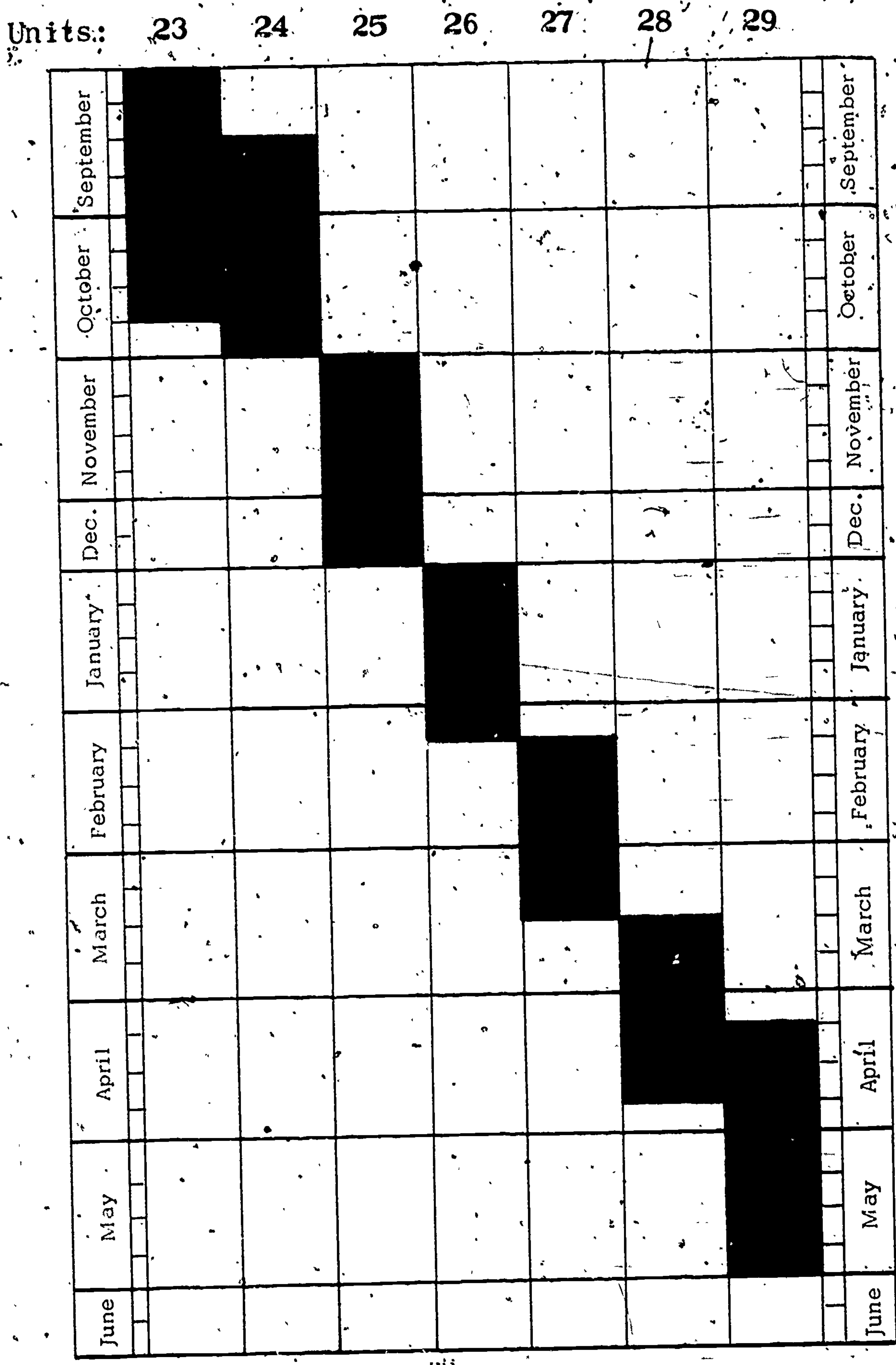
HOWARD T. STACKPOLE

LAURENCE VICTOR

EDMUND C. BRAY	Assistant to Director Curriculum Materials Development
PATRICIA A. ADAMS	Writer - Editor
LUANNE NYBERG	Writer - Editor
SONIA FORSETH	Art Director
CHARLES BJORKLUND	Illustrator
JUDITH L. NORMAN	Illustrator

CONTENTS

Suggested Teaching Schedule for MINNEMAST Third Grade Units



Complete List of Materials for Unit 25

(Numbers based on class size of 30.)

total number required to teach unit	item	lessons in which item is used
30	** Student Manuals	
1	* roll of masking tape	1, 2, 10, 14-24
60	* yards of string	1, 2, 8, 9, 10, 14
1	* red felt marker or pen	1
1	* blue or black felt marker or pen	1, 20
30	* 12 inch rulers	1-24
4	thumb tacks	1
4	* 1" paper clips	1
4	* weights (lead)	1
4	chairs	1, 2, 11-24
	+ space to run races	2
1	overhead projector (optional)	3-24
1	grid transparencies (optional)	3, 4, 7
30	crayons	3, 23
1	* battery operated toy cars	11, 14, 15, 18, 19
2	* batteries (AA, 1-1/2 volt)	11, 14, 15, 18, 19
2	* toothpicks	11, 14, 15, 18, 19
120 feet	* yarn, string or clothesline rope	11, 14, 15, 18, 19
2	* 2" paper clips	15, 18
	pictures of cars (optional)	15
1	potato	20

1	knife	20
1	tempera paint	20
3	scraps of paper	20
1	sheet of paper, 2 feet long	20
500	* counters (toothpicks, golf tees)	21
5	* rubber bands	22
1	envelope	22
2 ea.	* dice, numbered 0-5 and 4-9	24

* kit items as well as

** printed materials available from Minnemath Center, 720 Washington Avenue S.E., Minneapolis, Minn. 55455

+ In several lessons the children will need space for foot and car races. The best place for these races is the gymnasium, but some races can be held outdoors or in the classroom. The lessons are listed here so that you can make arrangements for the races and schedule the use of the gymnasium ahead of time.

Foot races: Lessons 2, 8, 9 and 10.
Car races: Lessons 11, 14, 15, 18 and 19.

x

INTRODUCTION

Unit 25 relates the graphs of motion at a constant speed to multiplication. The children begin to explore motion through a series of foot races. They time themselves running and hopping, and measure the distance covered at the end of five seconds. Through these races they become familiar with some of the problems involved in measuring speeds.

The data obtained from the foot races is analyzed by stating it in time-distance ordered pairs and graphing it on grids. When the graphed data is compared to the data as it was first collected, the children see the value of using graphs to represent and compare data.

Once the data has been graphed, the children find they can compare the relative speeds of two or more runners by comparing the steepness (slopes) of the lines representing their motion. The children discover that the steeper the slope of the line, the faster the runner.

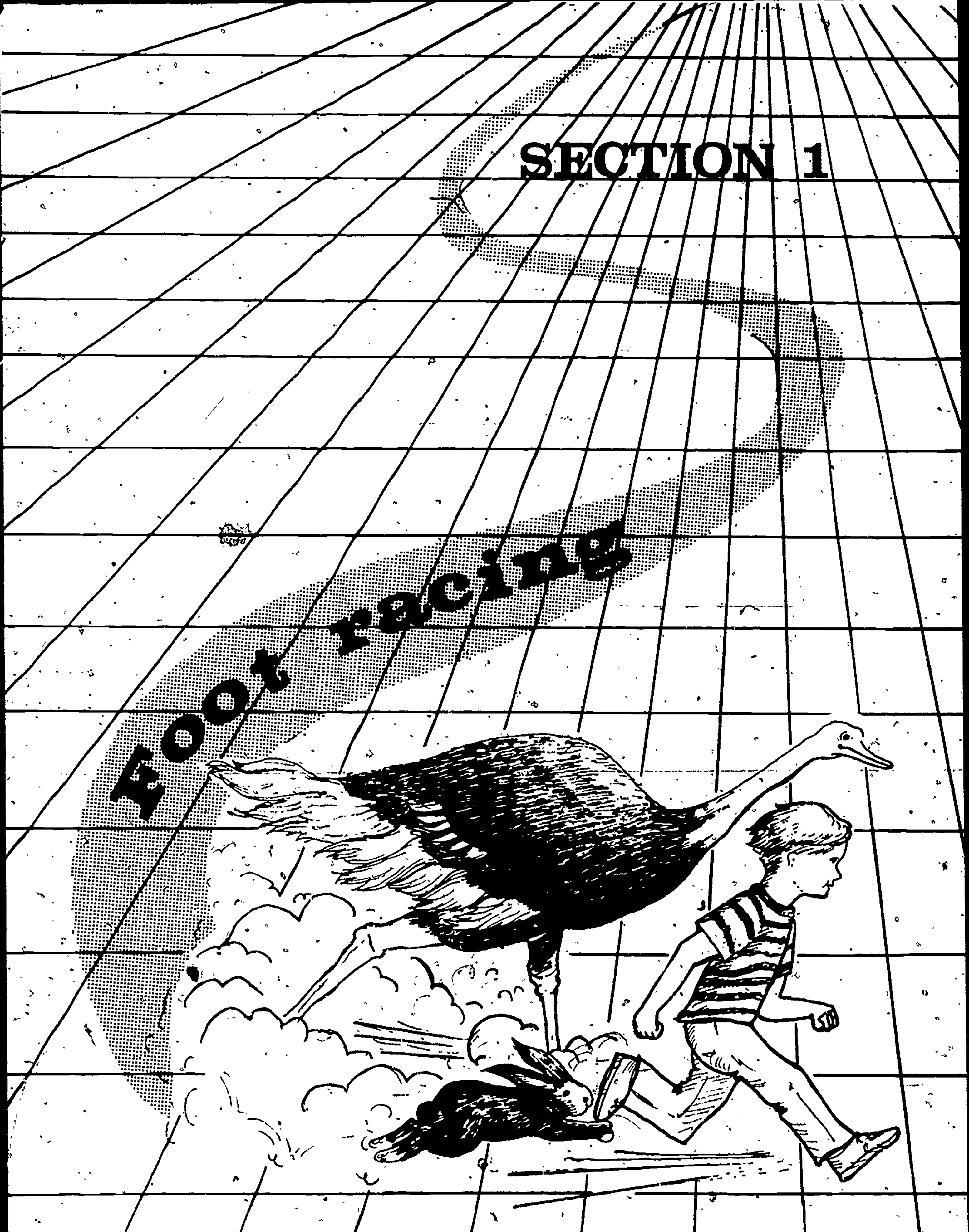
In Section 2 the class uses toy cars to study motion. They collect data on the cars when they are going at full speed and also when the cars have been slowed down. The motions of the cars are graphed and the children learn how to name the slopes of these lines by assigning each one a number. The slopes (or speeds) can now be compared by the size of their assigned numbers.

The relationship of multiplication to constant speed motion is made obvious when the children examine the increase in the slope of a line. The steepness of the line increases by the same number of grid units from one grid intersection that the line passes through to another. This increase is the same as the slope name. The children find that when the slope name is a whole number of units up per one unit over, they can multiply by this number using a line with that slope and numbers on the horizontal axis of a grid. The product is read from the vertical axis. From this relationship a chart with slopes one through nine is developed. The children use this chart to find the products of integers from zero through ten.

In the last section the children review some embodiments of multiplication other than slope. Their work with repeated addition and arrays serves both as a review and as preparation for Unit 27. They also explore the relation between multiplication and Cartesian products. Card games and dice games reinforce the relations among arrays, repeated addition sentences and multiplication sentences and also provide fun ways to practice the multiplication facts.

SECTION 1

Foot racing



SECTION 1 FOOT RACING AND GRAPHING

PURPOSE

- To develop an understanding of speed as a relationship between time and distance.

- To introduce the idea of slope as a way to represent and compare speeds.

COMMENTARY

The first section of this unit introduces the children to the study of motion. They plot time-distance data (speed) and interpret graphs of this data as showing faster and slower speeds. They are introduced to the idea of the slope of a graph and discover that the steepness of the slope indicates relative speed: the steeper the slope the greater the speed.

In Lesson 1 you and the children prepare the equipment they will use in the foot racing lessons. You introduce the racing procedures and the recording techniques. The children learn the jobs of team members.

In Lesson 2 the children practice the running and recording procedures. Each child runs at least once and his datum (distance covered in five seconds) is recorded. The children practice until you are sure they understand the procedure so that in subsequent lessons, when the children race again, they can concentrate on getting accurate time-distance data.

Lesson 3 reviews plotting points on a grid and writing ordered pairs for points already labeled on a grid. Since the children have done this in Unit 24, the lesson should not take much time. The lesson is included here because in following lessons the children must know how to graph their own time-distance data from foot races and, in Section 2, from car races.

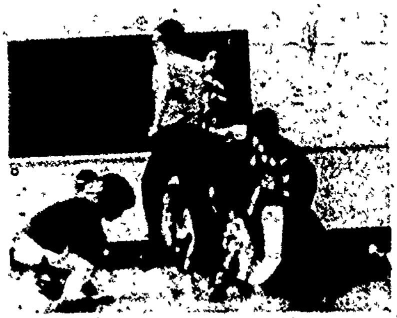
Lesson 4 includes two games that give practice in plotting points on a grid. The games can be introduced in class and then played by the children throughout the school year, and especially throughout the remainder of the unit.

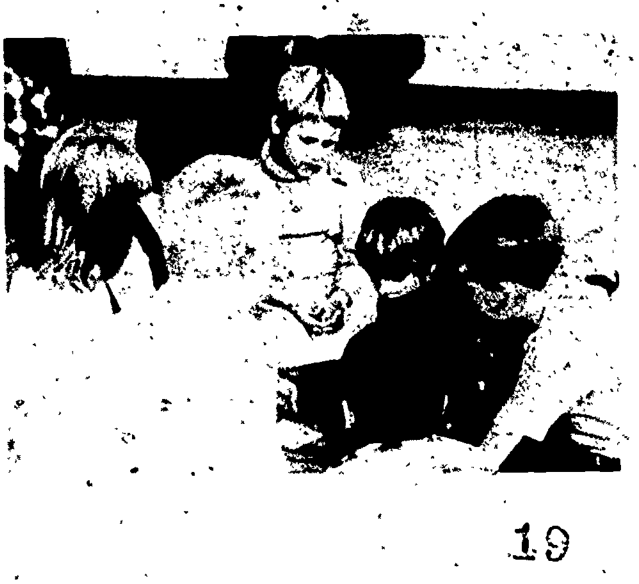
Lesson 5 deals with the value of graphing data, specifically height and weight data that is provided. The children discover that they can tell easily who is the tallest, heaviest, etc. when the data is plotted on a grid, whereas it is not so easy to answer such questions when the data is given in the form of ordered pairs.

Lesson 6 involves plotting time-distance data. The children discuss the meaning of phrases denoting speed, such as "four miles per hour." The class discussion and worksheets involve the interpretation of constant speed data.

In Lesson 7 the children again plot linear (constant speed) time-distance data. The idea of slope and its meaning are introduced. The children interpret a steeper slope on a graph as an indication of faster speed.

In Lesson 8 the children run races for a five-second time duration. Then each child plots the data for three of his team members: the fastest, the slowest and one other child. As a class the children discuss these graphs, answering such questions as "Who is the fastest runner? Who is the second fastest girl runner?"





The children run another race in Lesson 9, this time recording the distance of each runner at one, two, three, four and five seconds. When this data is plotted and the points connected, the children see that each graph has an approximately constant slope indicating that a child ran at about the same speed for the entire five seconds.

In Lesson 10 the children run races in which the speed is not constant. The procedure suggested involves having a child hop for three seconds and then run for three seconds. Again the time-distance data for each child is plotted, but this time the graph is not a straight line. The children see that the steepness, or slope, of the graph varies. The first three seconds show a less steep slope than the last three seconds, indicating that a child did not go as fast the first three seconds.

Lesson 1: ORGANIZATIONAL LESSON

In this lesson the class prepares for work in subsequent lessons by setting up the equipment and going over the procedures for the foot races which they will begin in Lesson 2.

The class will need four pendulums and four measuring strings for the car races in later lessons. Though only two pendulums and two strings are needed for the foot races, you will probably want the children to make four of each now, and set the extras aside for use in Section 2.

There will be four teams. Two teams work together to learn to run, time, judge and record data for the foot races. In Lesson 2, the class practices the races in the gymnasium. Since Lesson 2 should be done while the race procedures are still fresh in the children's minds, you may want to do Lesson 1 in the morning and Lesson 2 in the afternoon.

By developing, in this lesson, the skills needed in the races, the children will be able to concentrate more on the study of motion in the lessons that follow. They will begin to understand that in determining speeds, data about distances and times must be gathered.

MATERIALS

- roll of masking tape
- 4 measuring strings, each 42' long
- red marker and blue or black marker
- Worksheet 1

-- for pendulums --

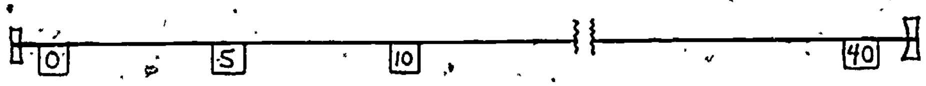
- 4 foot rulers
- 4 thumb tacks
- 4 paper clips, 1" long
- 4 pieces of string, 12" long
- 4 lead weights for the bob
- 4 chairs

PREPARATION

The procedures for making the pendulums and measuring strings are explained here. You should have one of each ready to show the class when you begin the lesson. You may wish to have volunteers help you make all of them at one time, or you may wish to assemble the equipment in class.

If you have not already done so, arrange for the use of the gymnasium or other large area for subsequent lessons. (See the materials list at the beginning of this unit.)

To make the measuring strings, use a piece of string slightly longer than 40 feet for each one. Mark off each string at six-inch intervals, using a red felt marker for the foot marks and a black or blue marker for the half-foot marks. Attach a piece of masking tape every five feet, labeled with the proper numeral: 0, 5, 10, 15, etc.



Worksheet 1 shows how to construct a pendulum. If you are doing this in class, have the children turn to the worksheet in their student manuals.

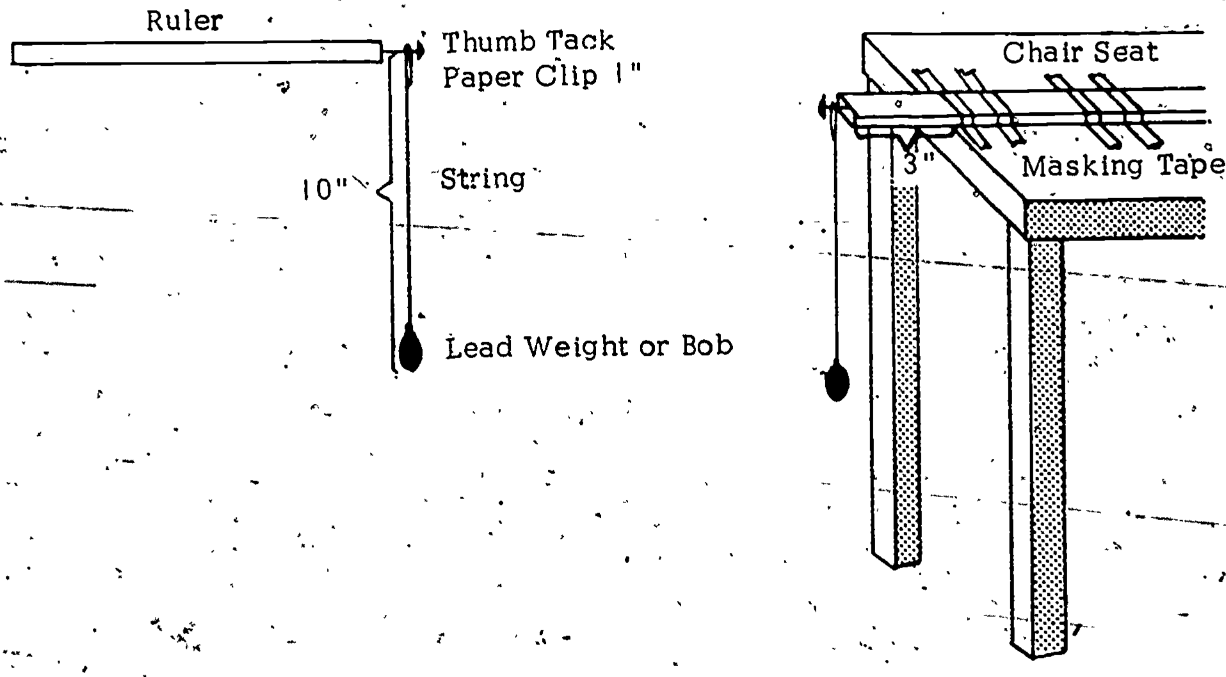
Attach a lead sinker, or other small weight such as a nut, to one end of a 12-inch string. Tie the other end of the string to a one-inch paper clip and adjust it so that the entire length of the pendulum, including the paper clip and the bob, is 10 inches. This will make the period (time for one complete swing) of the pendulum about one second.

22

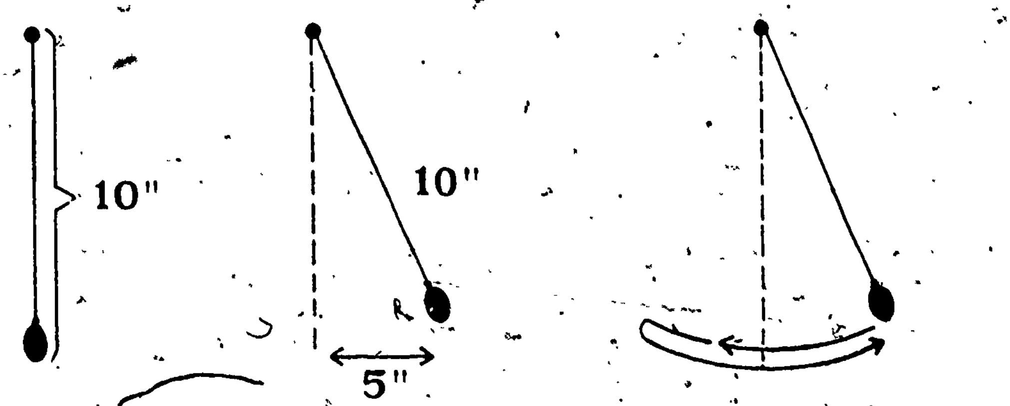
Worksheet 1
Unit 25

Name________________

Pendulum Construction

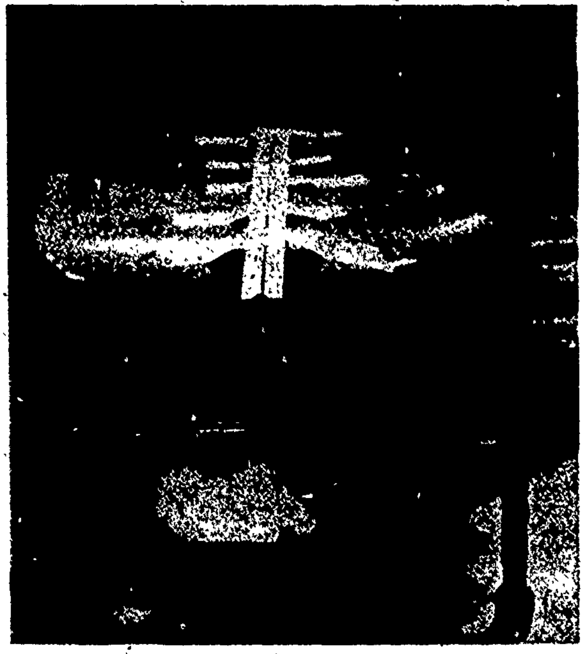


Procedure For Starting Pendulum



(one complete swing for each count)

Counting 2, 1, 0, 1, 2, 3, 4, 5, 6, 7, etc.



In order that the bob can swing freely without hitting anything and at the same time be portable, the supporting ruler should be extended from the seat of a chair with the end held down by masking tape. The ruler should not extend more than three inches beyond the chair seat, thus avoiding the tendency for it to vibrate as the pendulum swings.

PROCEDURE

Activity A

To make the children aware of the need for the measuring techniques they are going to practice in this lesson, use questions like those below to stimulate discussion. The children should begin to associate distance and time duration with speed. This activity should take about ten minutes.

Ask the class:

WHAT DO WE MEAN BY SPEED? (How fast something goes.)

CAN ANYONE TELL ABOUT A TIME WHEN THEY TALKED ABOUT SPEED OR WHEN THEY HEARD SOMEONE TALKING ABOUT SPEED? (When I rode in a car, during a space flight.)

HOW FAST DID THE CAR — SPACESHIP — GO? (60 miles per hour; 15,000 miles per hour.)

If the child does not include a phrase like miles per hour in his answer, say:

IT WENT SIXTY — SIXTY WHAT? (Sixty miles per hour.)

WHAT DOES SIXTY MILES PER HOUR MEAN? (It would go sixty miles in one hour.)

HOW FAR COULD A SPACESHIP GO IN ONE HOUR AT 15,000 MILES PER HOUR? (15,000 miles.)

WHEN WE TALK ABOUT SPEEDS WE SAY, "MILES PER HOUR." SO WHEN WE TALK ABOUT SPEEDS, WHAT MEASUREMENTS DO WE TALK ABOUT? (Miles and hours.)

WHAT DO WE MEASURE IN MILES? (Distances; length measurements.)

WHAT DO WE MEASURE IN HOURS? (Time durations; time measurements.)

THEN TO FIND OUT ABOUT SPEEDS, WHAT DO WE HAVE TO MEASURE? (Distances and time durations.)

CAN ANYONE THINK OF SOME OTHER UNITS BESIDES MILES WE COULD USE TO MEASURE DISTANCES? (Feet; yards; centimeters; etc.)

CAN ANYONE THINK OF SOME OTHER UNITS BESIDES HOURS WE COULD USE TO MEASURE TIME DURATIONS? (Minutes; seconds; weeks; years; etc.)

WE SAID BEFORE THAT IN ORDER TO FIND OUT ABOUT SPEEDS, WE MUST MEASURE DISTANCES AND TIME DURATIONS. COULD WE MEASURE SPEEDS IN FEET PER

SECOND OR YARDS PER WEEK INSTEAD OF MILES PER HOUR? (Let the children speculate. They should see that the important thing to remember is that they measure distances and time durations. It doesn't matter what units they use.)

Tell the class that in the next lesson they will begin experiments to study speeds. Before they begin the races, they will learn how to measure distances and times.

Activity B

If you have not yet made four pendulums and four measuring strings, show the class the demonstration string and pendulum and have the children make the others.

When the equipment is ready, organize the children into four teams by choosing four captains who can select team members in rotation or by picking the teams yourself.

Explain that when the races are run in following lessons, two teams will work together on each track. The children on one team will be the runners, while the children on the other team will be the officials. When all the members of one team have run, the teams will change jobs. Every child will be an official at some time, either in this lesson or future lessons.

Runners — There will be seven or eight children on each team, but only four will run at one time. While four children of Team A are running, the other Team A members sit quietly by the side of the track.

Demonstrate how to use the pendulum. To start the pendulum swinging, hold the bob between two fingers about five inches from its rest position and release it by simply opening the fingers. Using too large a swing or pushing the bob may alter the period of the pendulum. Have the children count the swings with you until the procedure is clear.

Timer — One child from Team B (the team that is not running) will be the timer. He starts his pendulum and counts its swings as if he were doing a count down for a blast-off. He will say "two" when he releases the pendulum, "one" when it first returns to the release point, "zero" the next time it returns to the release point, and then "one," "two," etc. to "seven" as it returns to the release point.

Let one child from each team use the pendulum and count its swings while the other members from the teams watch. Remind each timer that he should hold the bob no more than five inches from the bottom of the swing when he releases it and that he should not push the bob, but simply let it go.

Starter — One child from Team B will bring his arm down at the count of "zero" just as the starter of an auto race brings his flag down to send the cars on their way. (Later, when toy cars are used in experiments, the starter will lower the rear wheels of the car to the floor when the timer says "zero.")

Choose one child from each team to be a starter. Let each starter practice bringing his arm down at "zero" while the timer from each team counts the pendulum swings.

Judges — There will be one judge from Team B for each runner of Team A. Each judge will mark the location of the front toe of his runner (front of the car in later experiments) at the count of "five" by laying down a ruler at a right angle to the track. The judge will give the time interval and the distance run to the nearest half-foot (e.g., "5 seconds, 28½ feet").

When the children run the race in the next lesson, a few trial runs will be necessary to position the judges at the approximate spot where most runners will be at the count of "five."

Lay out part of the measuring string on the classroom floor and let one child serve as runner and one as judge. Let the runner walk a short distance and have the judge demonstrate how to mark the distance with the ruler. (See note on page 14.)

Recorder One child from Team B will keep track of the runners from Team A. He will record the location of each runner and the count when that location was marked.

Discuss the information for the recording sheets and ask the recorder from each team to develop a sheet to be used in the next lesson. This sheet can be similar to the one on the next page.

Team ____________ Date ____________

Recorder ____________________

Name	Distance from Start (feet)	Time (counts)

Note: Caution the judges not to trip the runners when they lay down the rulers. Each judge will have to watch carefully where the runner is at count "five" because the runner keeps running to the end of the track.

Feel free to use as much time as you consider necessary to develop these procedures before going on to the next lesson. You may want to have the children practice their jobs in the classroom before going to the racing area.

Lesson 2: PRACTICE RACES

In this lesson, the children will practice timing and marking a race for a time interval of five seconds. Each judge will mark the distance covered by one runner in that time interval. Some of the children may begin to realize that when runners move for the same time interval, the one who covers the greatest distance moves the fastest, though this will not be developed until later lessons.

MATERIALS

- space in the gymnasium or outdoors

-- equipment to take to the racing area --

- 2 pendulum assemblies
- recording sheets and pencils
- 2 measuring strings
- 8 rulers
- masking tape
- 2 chairs for pendulums

PROCEDURE

Before going to the gymnasium or outdoors, plan the practice races with your class. Keep in mind the following suggestions as the plans are made:

1. Two teams are involved with each race: one team provides the officials, and members of the other team are runners. When all members of one team have raced, the teams exchange roles. Have two tracks set up so that all four teams are involved.

2. To ensure more accurate results, only four children from a team should run at the same time. The other children can sit on a warm-up bench. Thus there will have to be more than one heat for each team.

3. All runners on the same team should start from the same starting line, start at the same time, and run in the same direction — parallel to the marking string which is taped to the floor.

4. The judges and runners should be numbered and arranged so that Judge 1 can mark Runner 1, etc. Be sure that each judge has a chance to mark at least one runner.

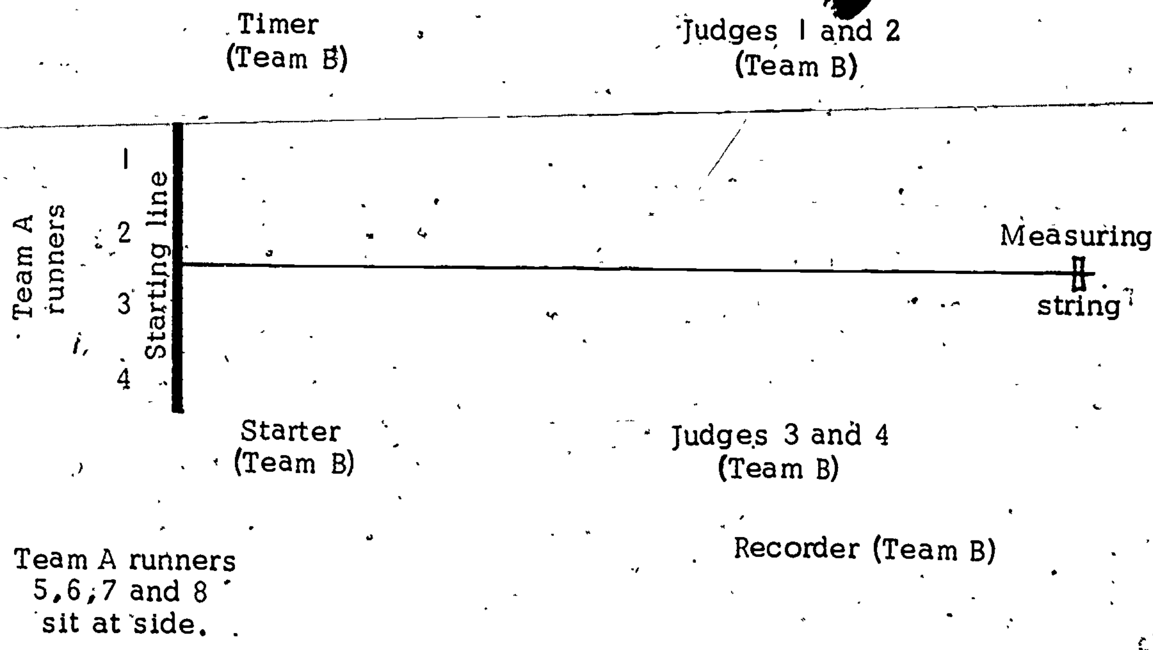
5. The runners should start when the starter drops his arm and keep running to the end of the track.

6. Their positions should be marked by the judges when the timer calls "five."

7. The distance run by each runner is recorded by the recorder who goes to each judge to get the information.

8. A diagram on the chalkboard (similar to the one on the next page) will help the children to visualize how the races are to be run.

When the children have the procedure clearly in mind, gather up two pendulum assemblies, two chairs, several recording sheets and pencils, two measuring strings, eight rulers, and a roll of masking tape.



In the gymnasium, the measuring string should be taped to the floor at both ends so that it is taut. A starting line, long enough so that all four runners can stand by it without being crowded, should be made of masking tape perpendicular to the measuring string.

When the timer has his pendulum in place and the judges have been given rulers and stationed properly, let the children practice a few races. Each team should get a chance to act both as officials and as runners. Be sure the children run enough times so that they are familiar with the running and recording procedures. Explain to the children that these procedures will be used again in Lesson 8 and similar procedures will be used in other lessons involving foot races and car races. They must understand how to run and record their data now so that the next time they race they can concentrate on the experiment involved rather than the set up and recording techniques.

Lesson 3: ORDERED PAIRS AND POINTS ON A GRID

This lesson reviews and gives practice in plotting points on a grid and assigning ordered pairs to points on a grid. The amount of time you should spend on this lesson will depend on how much your class remembers about ordered pairs and points on a grid from Unit 24.

MATERIALS

- overhead projector or chalkboard grid
- transparency of grid on pages 19–20 and transparency of Worksheet 5 in student manual
- ruler or other straightedge for each child
- Worksheets 2 through 8

PREPARATION

Make a transparency of Worksheet 5 from the student manual and of the grid on pages 19–20 of this manual. (The grid transparency will be used in several lessons, so you may want to make more than one.) Have the overhead projector ready. If you do not have a projector, you can draw the grid on the chalkboard.

PROCEDURE

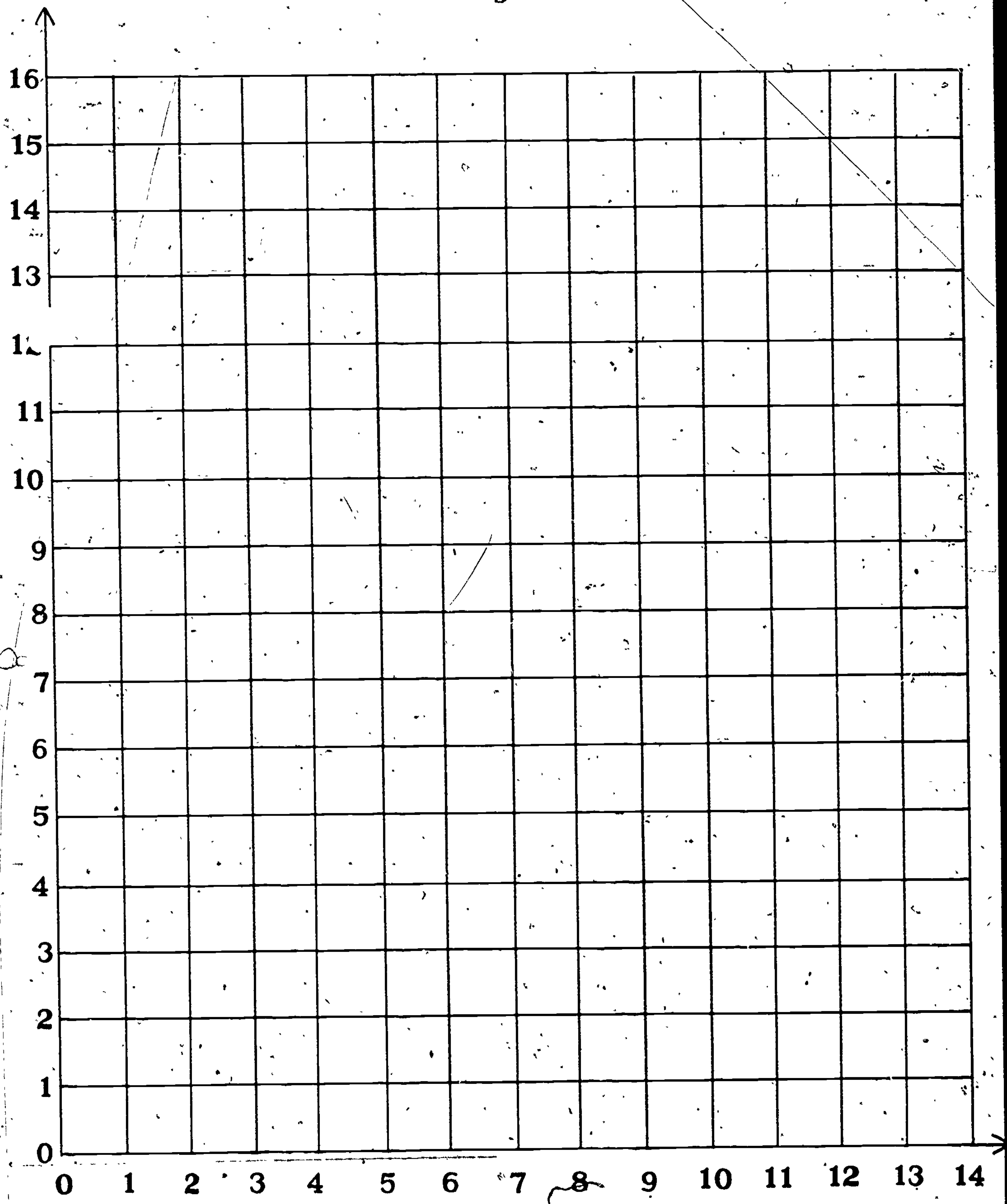
Activity A

Ask the children to recall what they know about ordered pairs and plotting ordered pairs on a grid. Emphasize that the order of the numbers in the pair is important. Illustrate this by plotting the ordered pairs (3,6) and (6,3). (3,6) means "over 3 and up 6." Locate this point on the grid. (6,3) means "over 6 and up 3." Locate this point on the grid. (3,6) and (6,3) name different points on the grid.

Have different children come to the overhead and locate the points that correspond to the following ordered pairs:

(3,2), (1,5), (5,1), (9,4), (8,9), (4,7), (7,4).

Have them write the ordered pair next to the point on the grid.



16
15
14
13
12
11
10
9
8
7
6
5
4
3
2
1
0
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14

Activity B

In this activity the children complete several worksheets that provide practice with ordered pairs and plotting points on a grid. Use only as many as you feel the children need.

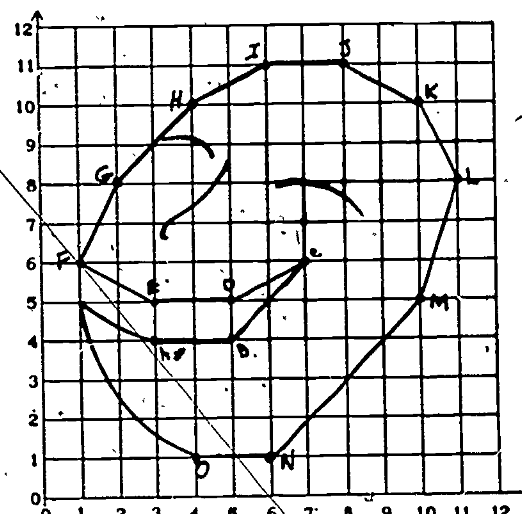
Worksheet 2

Have the children turn to Worksheet 2. Using the overhead projector and the grid transparency of page 19 locate and label points A and B using the ordered pairs from the worksheet. Emphasize that the ordered pair tells you "how many to go over and how many to go up" from the origin in order to locate the point. Point A is located by going over 3 and up 4. Point B is found by going over 5 and up 4.

Have different children come up to the overhead and locate points C-O as the other children locate these points on their worksheets. Circulate about the room to be sure that each child has located the points correctly. You may want to have the children exchange grids to check each other's work. After all the points have been located, have each child use a straight edge to connect the points in alphabetical order with line segments.

Worksheet 2
Unit 25

Name________________



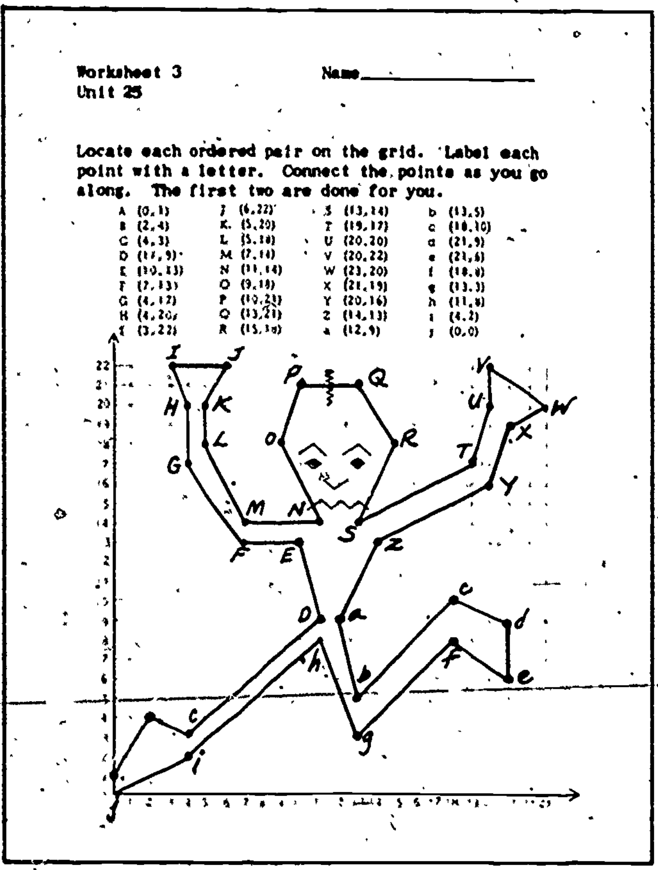
A (3,4)
B (5,4)
C (7,6)
D (5,5)
E (3,5)

F (1,6)
G (2,8)
H (4,10)
I (6,11)
J (8,11)

K (10,10)
L (11,8)
M (10,5)
N (6,1)
O (4,1)

Worksheets 3 and 4

Have the children work independently on Worksheets 3 and 4. You may want to use these worksheets only with those children who had difficulty with Worksheet 2. On these worksheets the children again locate and label points on the grid and then connect them using line segments to draw a picture.

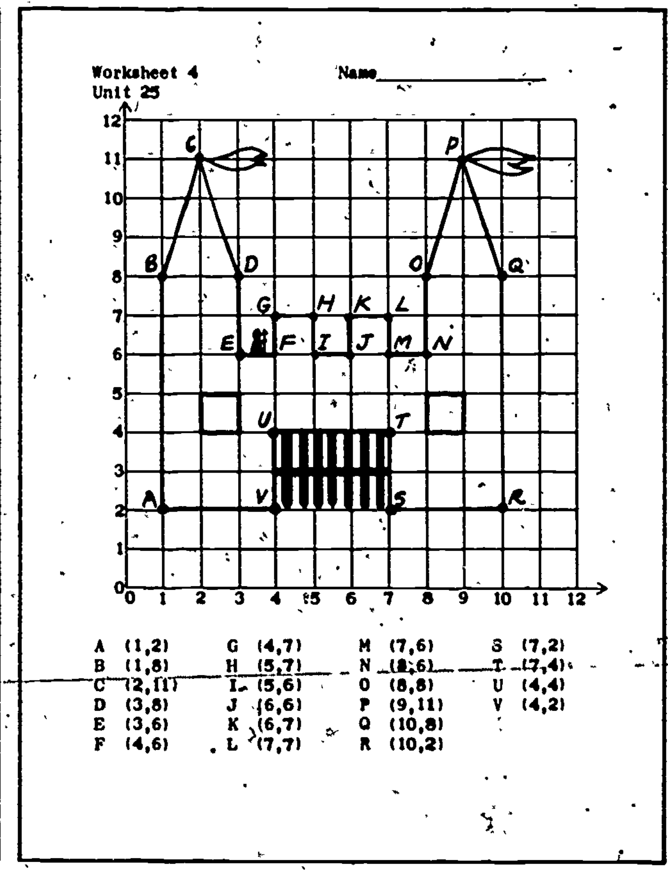


Worksheet 3
Unit 25

Name________________

Locate each ordered pair on the grid. Label each point with a letter. Connect the points as you go along. The first two are done for you.

A (0,1)	J (6,22)	S (13,14)	b (13,5)
B (2,4)	K (5,20)	T (19,17)	c (18,10)
C (4,3)	L (5,18)	U (20,20)	d (21,9)
D (11,9)	M (7,14)	V (20,22)	e (21,6)
E (10,13)	N (11,14)	W (23,20)	f (18,8)
F (7,13)	O (9,18)	X (21,19)	g (13,3)
G (4,17)	P (10,21)	Y (20,16)	h (11,8)
H (4,20)	Q (13,21)	Z (14,13)	i (4,2)
I (3,22)	R (15,18)	a (12,9)	j (0,0)



Worksheet 4
Unit 25

Name________________

A (1,2)	G (4,7)	M (7,6)	S (7,2)
B (1,8)	H (5,7)	N (8,6)	T (7,4)
C (2,11)	I (5,6)	O (8,8)	U (4,4)
D (3,8)	J (6,6)	P (9,11)	V (4,2)
E (3,6)	K (6,7)	Q (10,8)	
F (4,6)	L (7,7)	R (10,2)	

Worksheet 5

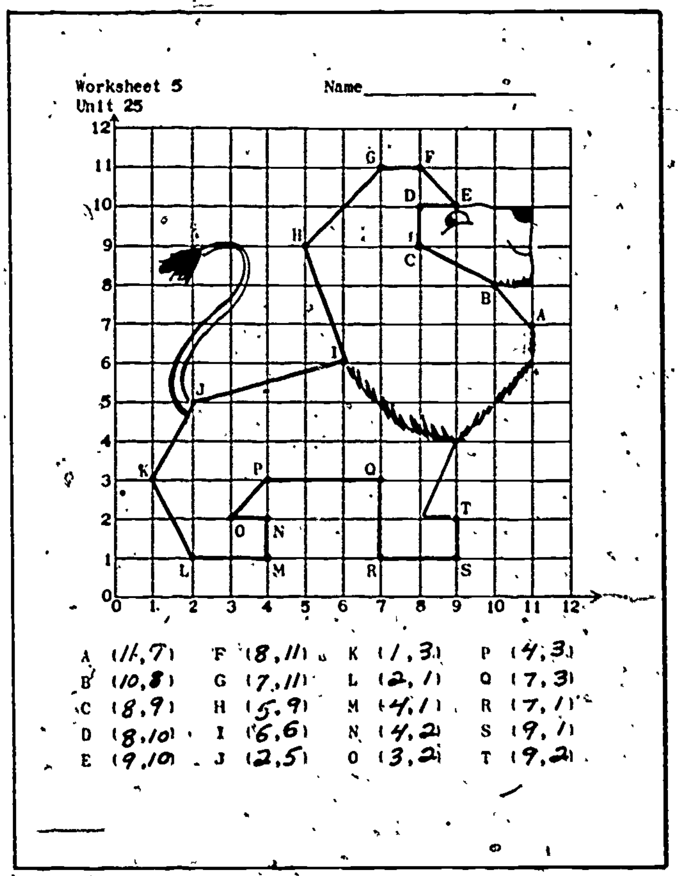
Have the children turn to Worksheet 5. Use a transparency on the projector.

Ask the children to find point A on their worksheets.

WHAT ORDERED PAIR WOULD CORRESPOND TO POINT A?

Remind the children that the first number of the ordered pair tells how many units to go over while the second number of the ordered pair tells how many to go up. Be sure all the children understand that point A corresponds to the ordered pair (11,7) before going on. Help the children find the ordered pairs for points B–E. The children should then complete the worksheet by themselves and connect the points to make a picture.

Circulate throughout the class to be sure each child is giving the ordered pairs correctly. You might post the correct answers several places throughout the room so that the children can check their own work when they are finished.

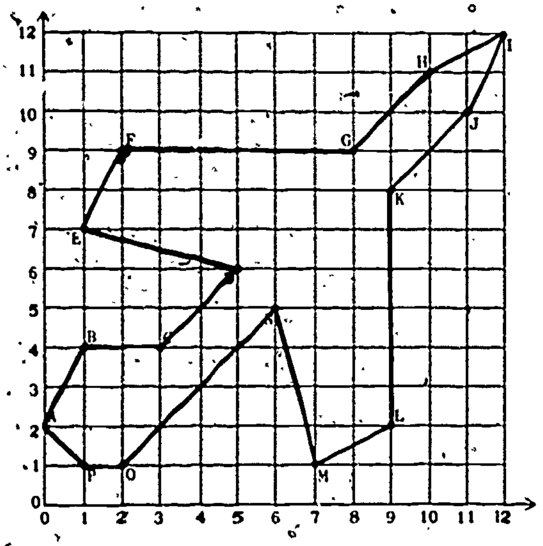


Worksheets 6 and 7

These worksheets reinforce skills of naming points on a grid for those children who need more practice. You may want to help children who had difficulty with Worksheet 5 complete these two. The other children can complete the worksheets on their own time if they wish. After the children write ordered pairs for the points they should connect the points with line segments from A to B, B to C, etc. They will discover a picture on each worksheet.

Worksheet 6
Unit 25

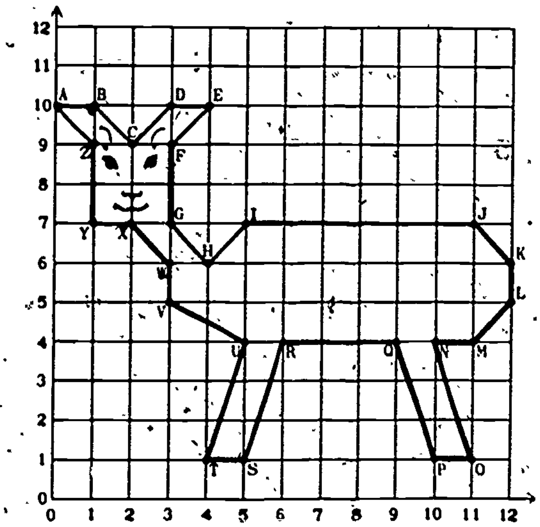
Name________________



A (0,2)	E (1,7)	I (12,12)	M (7,1)
B (1,4)	F (2,9)	J (11,10)	N (6,5)
C (3,4)	G (8,9)	K (9,8)	O (2,1)
D (5,6)	H (10,11)	L (9,2)	P (1,1)

Worksheet 7
Unit 25

Name________________



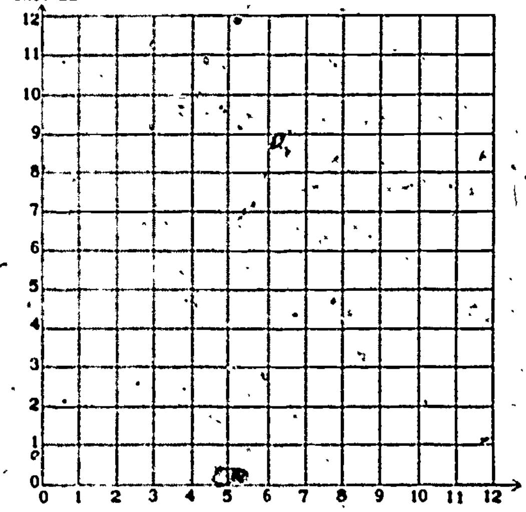
A(0,10)	F(3,9)	K(12,6)	P(10,1)	U(5,4)
B(1,10)	G(3,7)	L(12,5)	Q(9,4)	V(3,5)
C(2,9)	H(4,6)	M(11,4)	R(6,4)	W(3,6)
D(3,10)	I(5,7)	N(10,4)	S(5,1)	X(2,7)
E(4,10)	J(11,7)	O(11,1)	T(4,1)	Y(1,7)
				Z(1,9)

Worksheet 8

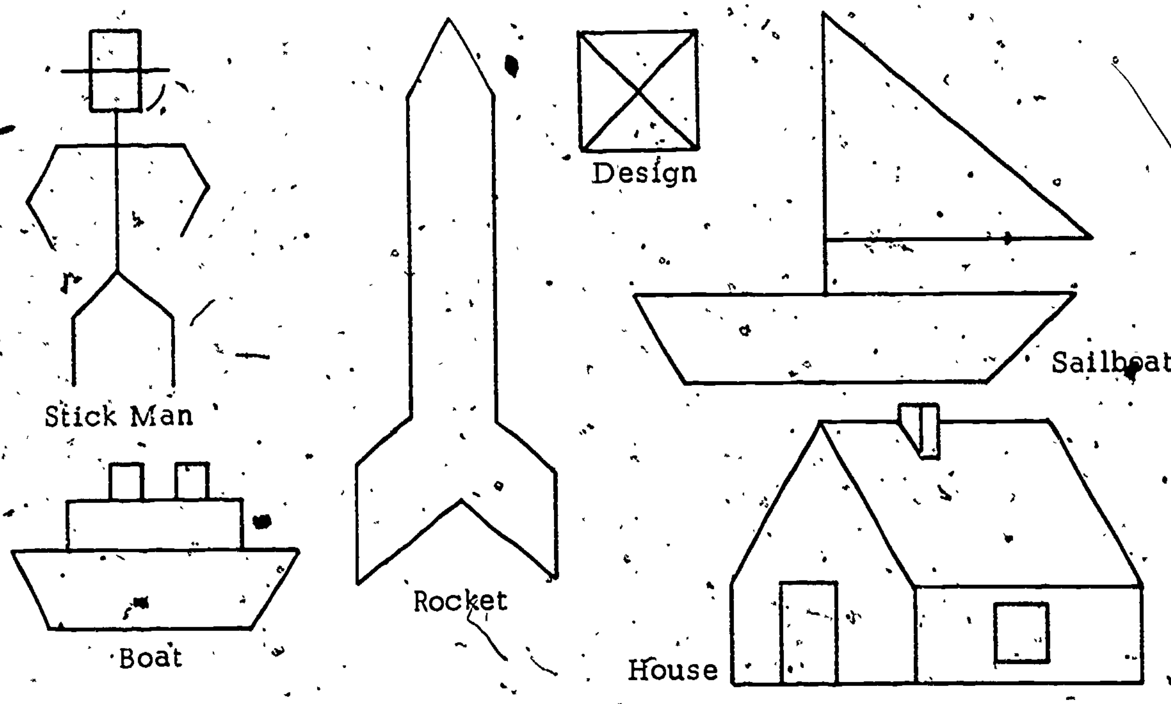
On this worksheet the children draw a line picture on the grid. The drawing can be as simple or complicated as they desire. In case the children need some ideas you might sketch the pictures on the following page on the chalkboard.

Worksheet 8
Unit 25

Name________________



Write ordered pairs to name the end points of your picture.



Encourage originality in the drawings. The only restrictions that need be put on the drawings is that they consist only of line segments and that the end points of the segments lie on the grid intersections. When a child has completed his drawing have him label the end points of each segment using the letters of the alphabet. He may want to label them in a specific order. At the bottom of the worksheet he should list the letter and then the ordered pair of the point it names.

For additional practice the children might want to exchange sets of ordered pairs and see if they can figure out what their neighbor's picture is. (Two copies of the worksheet are provided for this purpose.) Also the best pictures might be posted on the bulletin board.

NOTE: Save the transparencies you have made for later lessons.

40

Lesson 4: PLOTTING ORDERED PAIRS (GAMES)

This lesson gives practice in plotting points on a grid in game situations. Introduce the games in class and then let the children play during free time throughout the school year.

MATERIALS

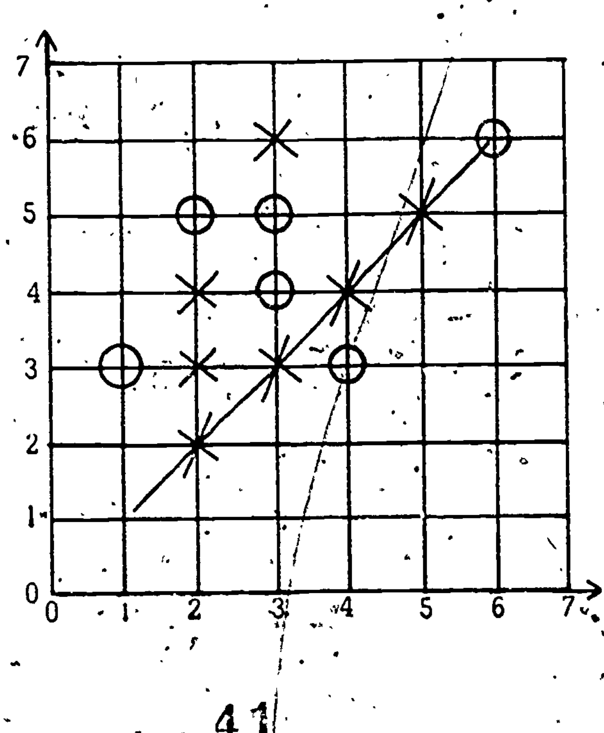
- overhead projector
- grid transparencies (from Lesson 3)
- crayons for each student
- Worksheets 9 and 10

PROCEDURE

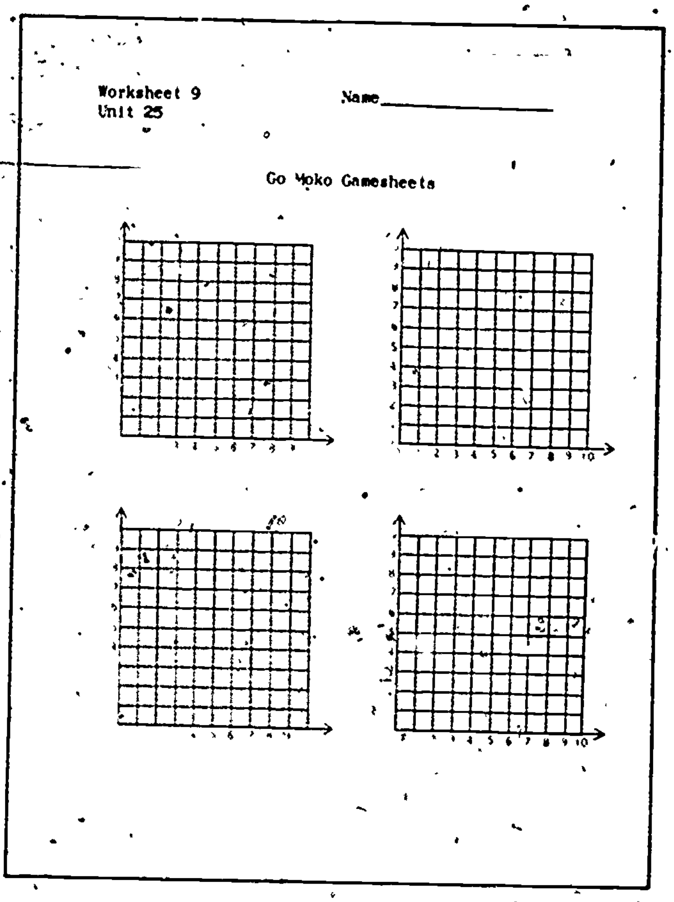
Activity A: Go Moko

The game "Go Moko" provides good practice in plotting ordered pairs on a grid.

Divide the class into two teams. Designate one team as the X's and the other as the O's. While explaining the game use a chalkboard grid or a grid transparency on the overhead projector. The object of the game is to gain control of any four adjacent intersections which lie in a straight line horizontally, vertically or diagonally. The first team to do so wins the game. You may want to copy the scoresheet below and explain to the children that the X team won because they have four X's in a row at points (2,2), (3,3), (4,4) and (5,5).



Now have children from each team take turns calling out ordered pairs to name points on the grid. They must give the order of the numbers correctly. (The first number tells how far to go over and the second tells how far to go up.) As each ordered pair is given, you or a student for each team mark that point with the appropriate X or O on the overhead grid.


Worksheet 9
Unit 25

Name______________

Go Moko Gamesheets

A team may lose its turn by:

(a) one member calling an intersection out of turn,
(b) one member conferring with another,
(c) calling an ordered pair that has already been called,
(d) marking an ordered pair incorrectly.

After playing "Go Moko" several times in class the children may want to play the game with partners or in teams of two or three during their free time. There are several grids provided for each child in his student manual on Worksheet 9.

Activity B: Claim Jumper

This game provides opportunity for additional practice in locating coordinates on a grid. Initially the game may be introduced by having the class play against you. They will try to find the treasure you buried. Later the game can be played by pairs of children at their desks. It will be easier for the children to start with only a 5 x 5 grid. These grids are on Worksheet 10. The children can use a larger grid after they have played a few times. There are two versions of the game; the easier version is explained first.

Two children play the game. In the first version, child 1 secretly marks the location of four bars of gold on a 5 x 5 grid. One comstock bar, covering four intersections in a row, two homestake bars, each covering two intersections in a row, and one nugget bar, covering one intersection. Child 2, who will be digging in different places to find the gold bars, also has a 5 x 5 grid where he records his hits and misses.

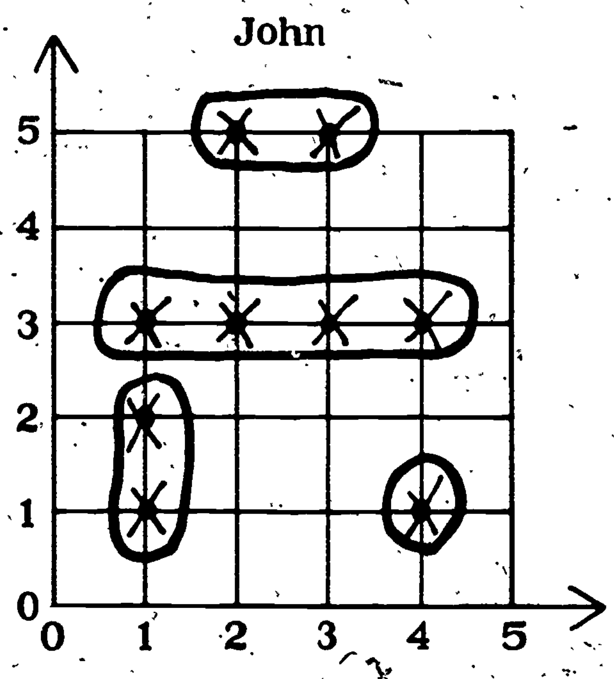
To draw these bars child 1 puts a dot on each of the intersections that his bars cover, and circles the area covered by each bar so that it is easy to see. He must keep his grid covered so his opponent cannot see it.

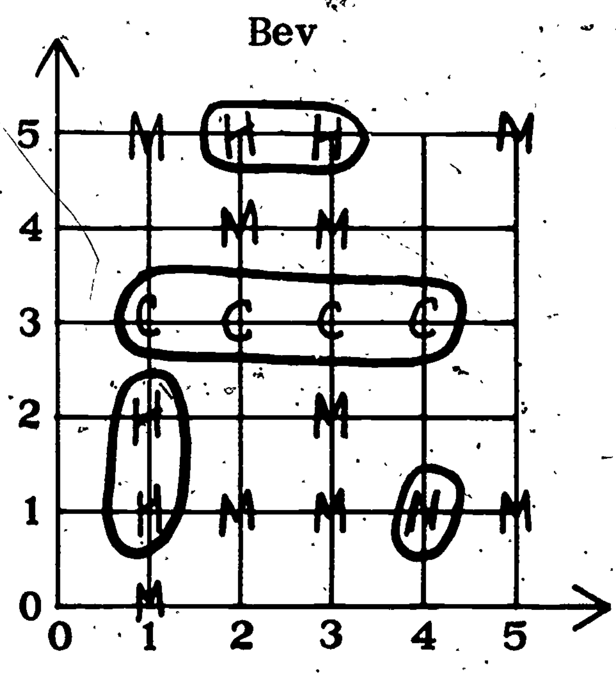
Child 2, with a blank grid, begins by calling out an ordered pair in an attempt to locate his opponent's gold bars. Since he is unaware of their positions, he must dig his first hole as an exploration. After each ordered pair is named, child 1, who buried the gold, tells whether the hole hit or missed the bars. If it hit, child 1 says "hit," and tells which bar was hit. Then he puts an X through that intersection on his grid to remind himself that his bar has been hit. Child 2, who is digging, now marks his blank grid with an M for miss at the intersection, or, if it is a hit, he writes C, H, or N for comstock, homestake, or nugget.

To claim a bar, all points (intersections) of that bar must be found. When a bar is claimed, the owner (child 1) must announce this. Child 2 circles the location of the claimed bar so that he will remember he has found that bar.

When all the bars have been claimed, child 2, who did the searching, counts up the number of holes it took to find and claim all four bars. This is his score. Then the players change roles. Each child has a new grid. The child who searched for gold now secretly draws four bars on his grid. They play the game again. The child who had fewer holes is the winner.

In the first round of the game below, John buried a nugget bar at (4,1), a homestake bar at (1,1) and (1,2), another homestake bar at (2,5) and (3,5), and the comstock bar at (1,3), (2,3), (3,3), and (4,3). He put a dot at each intersection and circled the area each bar covered. Bev started with a blank grid. She called out ordered pairs and marked the misses with an M and the hits with C, H, or N, showing which bar was hit. As John's bars were hit, he placed an X at each point. When a bar was claimed, Bev circled it on her grid. Bev dug 18 holes to claim John's four gold bars. In the next round, Bev would bury the gold bars and John, starting with a blank grid, would call out ordered pairs until he located the bars. Whichever player dug fewer holes to claim his opponent's bars would be the winner.





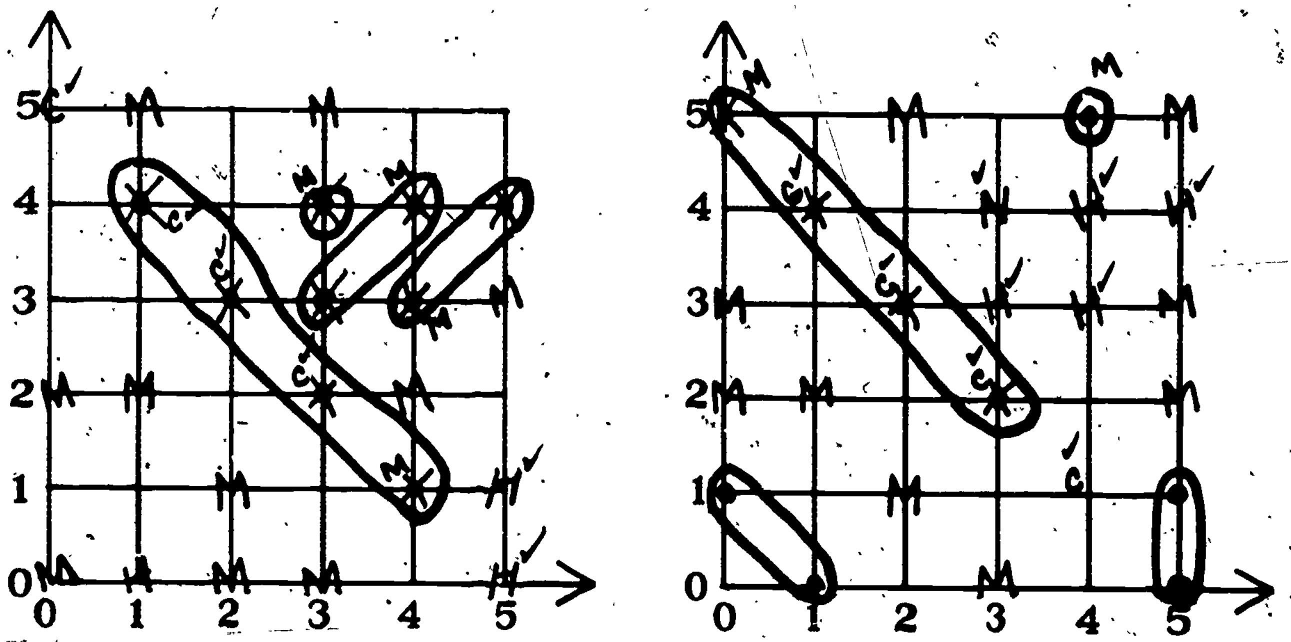
If some children would like a more complicated version, they can try this one. Two children play. Each secretly draws four bars on his grid by marking the intersections with dots and circling the area with a crayon to keep track of his own buried bars. This version is more difficult because each player must keep track of the hits and claims on his own bars, while at the same time keeping track of his hits and claims on his opponent's bars.

The children take turns alternately digging three holes at a time. When a player calls out an ordered pair, the opponent announces a miss or a hit of the comstock, homestake or nugget bar. If the player misses, he puts an M at that intersection so that he does not call that intersection again. If the player hits his opponent's bar, he writes a C, H, or N at the intersection to remind him which bar he hit and where it is. The player whose bar is hit, puts an X at that intersection just to remind him how his opponent is doing.

When a player has claimed one of his opponent's bars by finding all of its intersections, he checks off the letters, indicating that bar on his grid so that he knows it has been claimed. He does not circle the location of the bar as he did in the first version of the game because the grid would be too confusing with both players' bars circled.

Each child's grid will have many marks on it and there will be many things he has to remember. Therefore, this version is much more difficult than the first. The children will discover that they must study their grids closely to determine where to dig for the opponent's gold. Some intersections will have several marks, especially when both players put their gold bars in the same places. The children may prefer to use a larger grid so that the gold bars do not overlap.

The sample scoresheets below are for the second version. Mike is the winner because he located his opponent's four bars first. Mike's scoresheet shows 1) the locations of his bars (circled), 2) the locations of his opponent's bars (marked with letters indicating the type of bar and with check marks indicating the bar had been claimed), and, 3) his misses (each marked with an M indicating that his opponent did not have a bar at that location).

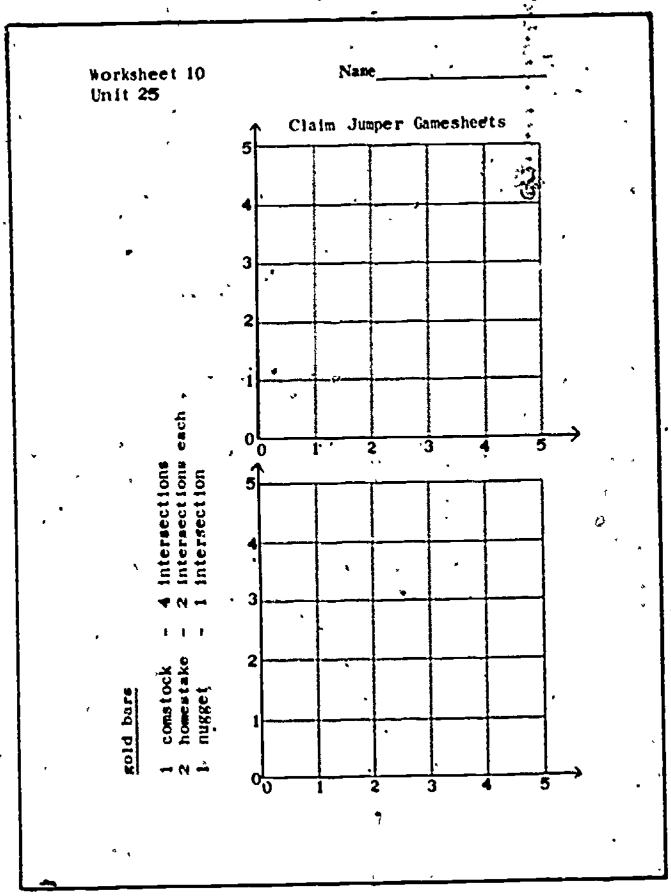


Pat Mike (winner)

40

After the children have played either version of the game a few times they may want to vary the size of the grid and the shape and size of the gold bars. For each game the players must agree on the shape and size of each gold bar.

Tell the children that they may use one or two of the four grids that follow Worksheet 57 in their student manuals for the game. The children might enjoy making up their own versions by varying the objects which are hidden. For example, they could hide animals, such as an octopus whose body covers four intersections and legs cover three adjacent intersections each. The objects to hide, size of objects and size of grids are limitless. However, two opponents must hide identical objects.



Lesson 5: THE VALUE OF GRAPHING DATA

In this lesson the children will see the value of graphing information on a grid so that it can be more easily analyzed. The children will plot and analyze weight and height data. This lesson is provided to give the children an opportunity to analyze data they are somewhat familiar with before they begin graphing time and distance data in the next lesson.

Comparisons among ordered pairs (height, weight) are very difficult or at least time-consuming to analyze. The value of plotting information is that the graph provides a visual representation of the data which shows interrelationships. Maximum and minimum values within the data can be found easily and relationships between one piece of data and the rest of the data can be seen. Questions, such as, "Who is the tallest?, Who is the lightest?, Who is the shortest and lightest?, Who is the tallest and heaviest?, How much variation in height is there in the class?" etc. can be answered rather easily once the data is represented by points on a grid. We would like to emphasize in this unit the analytical value of graphing data.

MATERIALS

- transparency of Worksheet 12 (made from student manual)
- overhead projector
- Worksheets 11 and 12

PROCEDURE

Activity A

Have the children turn to Worksheet 11. Explain to the children that we often need to store two pieces of information together. For example, a teacher needs to store a child's name and the grade he received on an exam. An employee in a store may need to store the name of an item and its price. Other information pairs might be the year a car was made and its make. The ordered pairs given on the worksheet are storing special information — the height and weight of the children in a third grade classroom.

Review the idea that the order of the numbers in the pair makes a difference. In these ordered pairs the first number gives the child's height in inches and the second gives the child's weight in pounds. For child A (52,61) the height is 52 inches and the weight is 61 pounds.

Ask the children to look at the ordered pairs and tell who is the tallest child. Quickly ask more questions, but do not spend much time on answers: "Who is the shortest child? Who weighs the least? Who weighs the most? Who is the biggest child (both the tallest and weighs the most)? Who is the smallest child (is the shortest and weighs the least)? What could you say about Sandy? Is she the tallest? shortest? heaviest? lightest? How does she rank within the class? How many children weigh more than 55 pounds? How many weigh less than 50 pounds?"

Worksheet 11
Unit 25

Name____________

		(height,weight)
Mary	A	(52,61)
Bob	B	(50,62)
Sally	C	(50,54)
Paul	D	(53,69)
Bob	E	(48,57)
Jim	F	(51,64)
Pat	G	(52,57)
Jane	H	(54,76)
Linda	I	(48,56)
John	J	(53,78)
Marilyn	K	(53,64)
Walt	L	(49,51)
Jean	M	(50,56)
Bill	N	(56,75)
Tom	O	(51,62)
Sandy	P	(51,59)
Chris	Q	(51,55)
Pam	R	(48,45)
Chuck	S	(51,53)
Edith	T	(52,62)
Kim	U	(47,46)
Ned	V	(52,60)

49

The answers to these questions are not immediately obvious by looking at the data when they are given in ordered pairs. In fact, some questions are not only very difficult to answer but raise other questions as well. For example, what is meant by "biggest" and what if there is no child that is both the tallest and heaviest?

Suggest to the children that since these are ordered pairs we might be able to tell more about them by plotting them on a grid. Have the children turn to Worksheet 12.

Emphasize that the horizontal (over) axis represents the height in inches and the vertical (up) axis represents the weight in pounds. Review once again the meaning of the ordered pairs and graph several with the children (use a transparency of Worksheet 12 and the overhead projector) and then let the children complete the worksheet on their own. The letters that correspond to each ordered pair should be used to name the point as it is plotted. The completed graph is on the next page.

After the children have completed the graph ask:

WHERE WOULD THE POINTS REPRESENTING THE TALLEST CHILDREN BE LOCATED ON THE GRID? (To the right since the numbers on the height scale increase as they go to the right.)

WHERE WOULD THE POINTS REPRESENTING THE SHORTEST CHILDREN BE LOCATED? (To the left.)

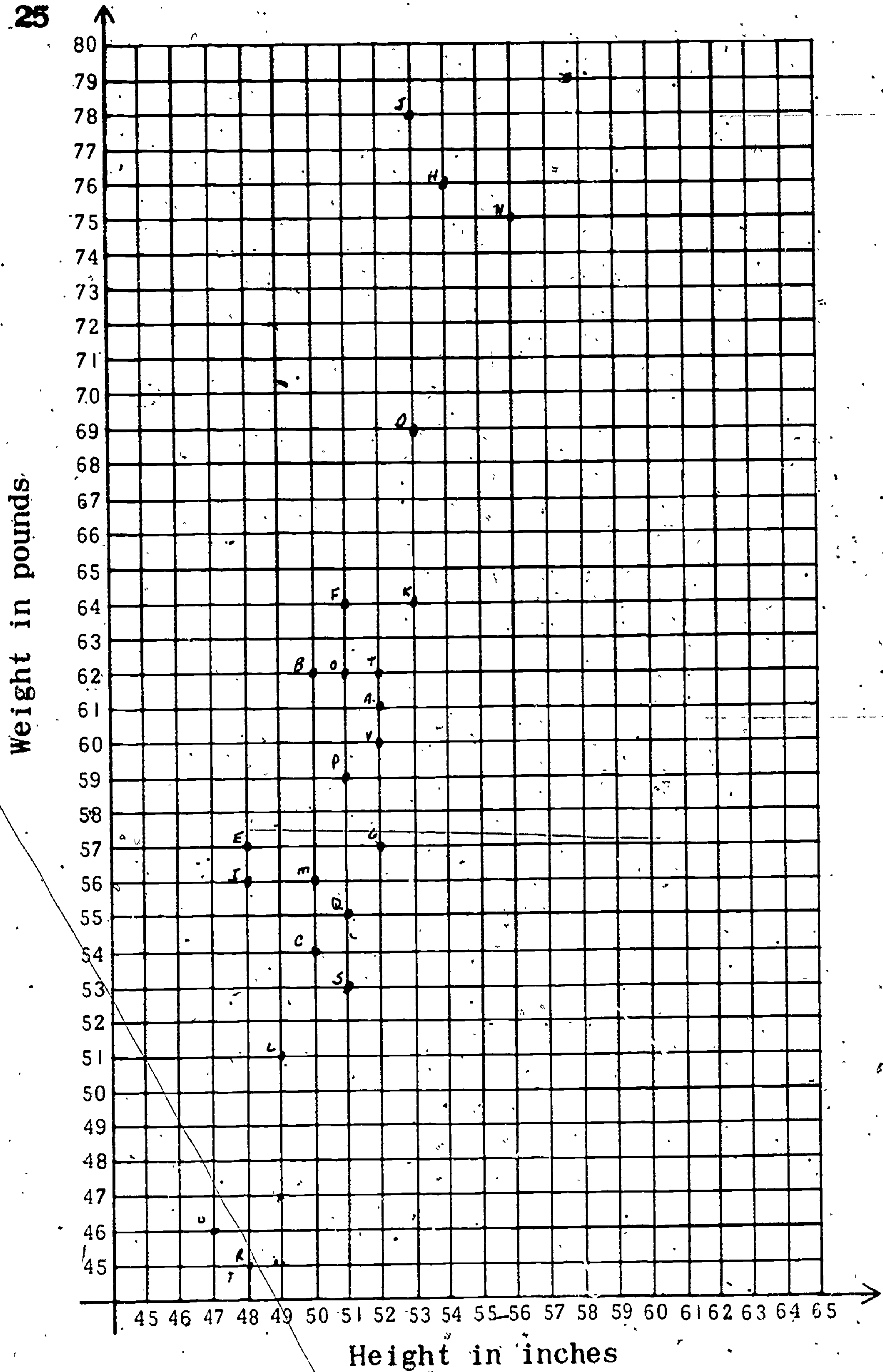
WHERE WOULD THE POINTS REPRESENTING THE HEAVIEST CHILDREN BE LOCATED? (Near the top since the numbers on ..e weight scale increase as they go up.)

WHERE WOULD THE POINTS REPRESENTING THE LIGHTEST CHILDREN BE LOCATED? (Near the bottom.)

Now ask the same questions that you asked when the children had only the ordered pairs to look at: Who is the tallest? etc.

Worksheet 12 (Answers) Name_______________

Unit 25



Ask the children to explain their answers. When you ask "Who is the biggest child?" they will see there is no child who is both tallest and heaviest. But the graph shows data (in the upper right hand corner) for three children who could be called the biggest children. Similarly those in the lower left corner would be the smallest children.

From the graph the children can see that most of the points fall rather close together in the middle. You can compare the height and weight of any child with that of any other child by locating their points on the graph. Emphasize the value of having a "picture" of the information and relationships.

Any child who wants to can plot his height and weight on the graph. The data come from an actual third grade class.

Lesson 6: PLOTTING TIME - DISTANCE DATA

The lesson introduces plotting of time-distance data on a coordinate system. Since time and distance are continuous measures these points are connected by a line. The line graph is used to find how far a runner went in a certain amount of time, or how long it took to run a certain number of feet.

MATERIALS

- overhead projector
- transparency of Worksheet 13 (made from student manual)
- Worksheets 13, 14 and 15
- ruler or other straightedge

PREPARATION

Make a transparency of Worksheet 13 from the student manual. A full size copy of the worksheet with answers appears in this manual for your use.

PROCEDURE

The children discussed the meaning of speed in Lesson 1, but you may wish to discuss the concept again as an introduction to plotting time-distance data.

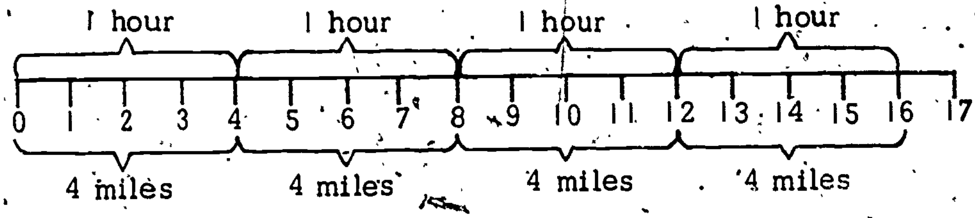
HOW MANY OF YOU HAVE BICYCLES? HOW FAST CAN YOU RIDE?

The children will probably describe speed only in terms of fast and slow, though they can probably ride their bicycles as slow as two miles per hour or as fast as 20 miles per hour. Ask about the speed of other means of transportation such as the horse (35 mi/hr), the car (60-70 mi/hr), racing cars (200 mi/hr); airplanes (small planes, 200 mi/hr; commercial jets, 600 mi/hr; air force jets, 2400 mi/hr; space craft, 25000 mi/hr). Help the children to order the vehicles they mention from slowest to fastest.

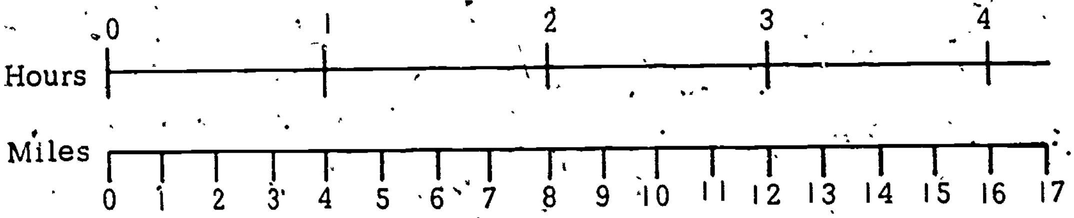
Walking is probably the slowest of any transportation. Walking speeds range from two to four miles per hour. Ask one of the children to demonstrate a brisk walk. Describe it as about four miles per hour. Ask the children what four miles per hour means. An acceptable answer is that if you continued to walk at that constant speed, in one hour you would travel a distance of four miles. Emphasize the time-distance relationship when you refer to motion. When the children have a feeling for "four miles per hour," ask,

HOW FAR WOULD YOU TRAVEL IN TWO HOURS AT THIS SPEED? (Eight miles.) IN THREE HOURS? (12 miles.) IN ONE HOUR? (Four miles.)

If the children seem to have difficulty answering these questions you might place the following sketch on the board to help them.



Parallel number lines can also be used.

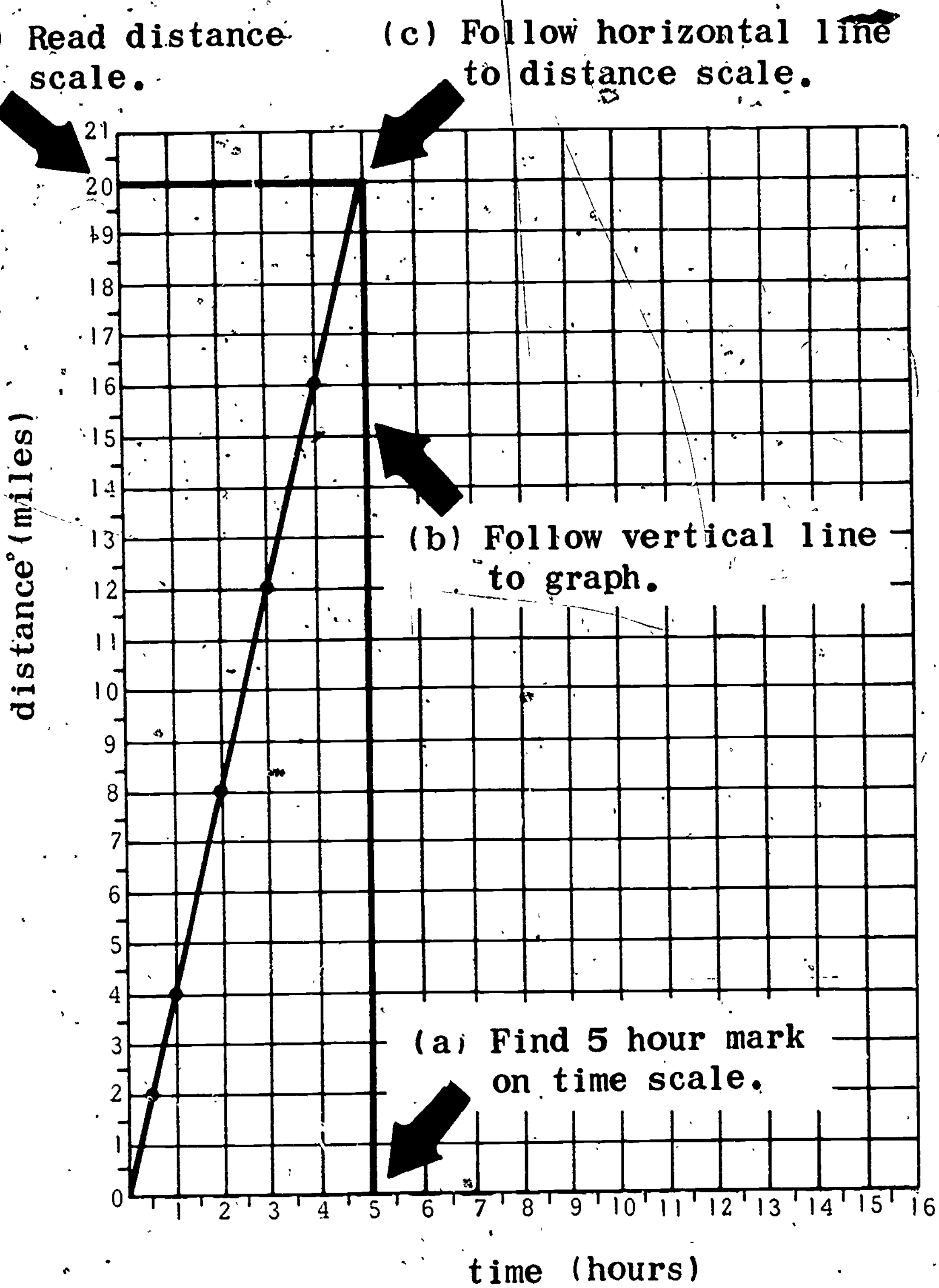


Suggest to the children that they record this information as ordered pairs. Review the idea that order makes a difference in an ordered pair and that we will use a time-distance ordered pair. The first number represents time and the second distance. The class would then have these ordered pairs: (1,4), (2,8), (3,12), (4,16).

Have the children turn to Worksheet 13 as you place a transparency of the worksheet on the overhead projector. Explain that since graphing gives a picture of relationships, we might be able to learn something about a motion of four miles per hour if we plot the points representing the motion.

Worksheet 13 (Answers) Name__________________

Unit 25



After the children have plotted the points for four miles per hour on their graphs (and you have the points plotted on the overhead transparency) ask for observations and comments. One of the observations should be that one line could be drawn that would contain all the points. This line would pass through the origin (0,0).

Have the children draw in the line that contains all of the points. This also provides a check to see if they plotted the ordered pairs correctly.

Explain that this line represents the time-distance relationship at four miles per hour. Since both time and distance are continuous measures there will be many ordered pairs (points) that represent four miles per hour. In fact, all the points on the line represent this relationship.

Have the children use the graph to help find more ordered pairs. Ask questions like the following.

HOW FAR WOULD YOU TRAVEL IN FIVE HOURS? (20 miles.) Steps a through d on your copy of Worksheet 13 explain how to determine the distance traveled in five hours.

HOW LONG WOULD IT TAKE TO GO TEN MILES AT THIS RATE? ($2\frac{1}{2}$ hours.) SIX MILES? ($1\frac{1}{2}$ hours.)

HOW FAR WOULD YOU TRAVEL IN THREE AND ONE-HALF HOURS? (14 miles.)

Have the children do Worksheets 14 and 15. These sheets present word problems and graphs involving speeds of two and three miles per hour. Worksheets with answers appear on the next page.

Worksheet 14 Name_______________
Unit 25

A puppy walks 2 miles per hour.

In 1 hour he walks 2 miles.
In 2 hours he walks 4 miles.
In 3 hours he walks 6 miles.

The ordered pair (4,8) means in 4 hours he walks 8 miles.

Complete these ordered pairs.			Give some ordered pairs of your own.
(1, 2)	(4, 8)	→	(6 ,12)
(2,4)	(5,10)		(7 ,14)
(3, 6)			(8 ,16)

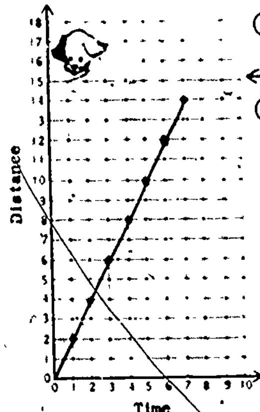
A boy walks 3 miles per hour.

In 1 hour he walks 3 miles.
In 2 hours he walks 6 miles.
In 3 hours he walks 9 miles.

The ordered pair (4,12) means in 4 hours he walks 12 miles.

Complete these ordered pairs.			Give some ordered pairs of your own.
(1,3)	(4,12)	→	(6 ,18)
(2,6)	(5,15)		(7 ,21)
(3, 9)			(8 ,24)

Worksheet 15 Name_______________
Unit 25

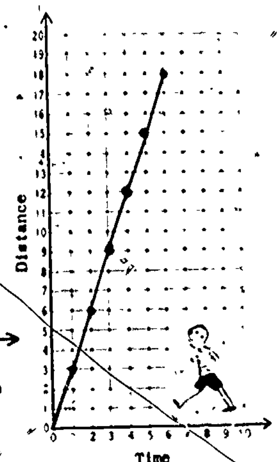


① Plot the ordered pairs for the puppy from Worksheet 14.

② Draw a straight line that contains all the points.

③ Plot the ordered pairs for the boy from Worksheet 14.

④ Draw a straight line that contains all the points.



Lesson 7: INTERPRETING THE SLOPE OF A GRAPH

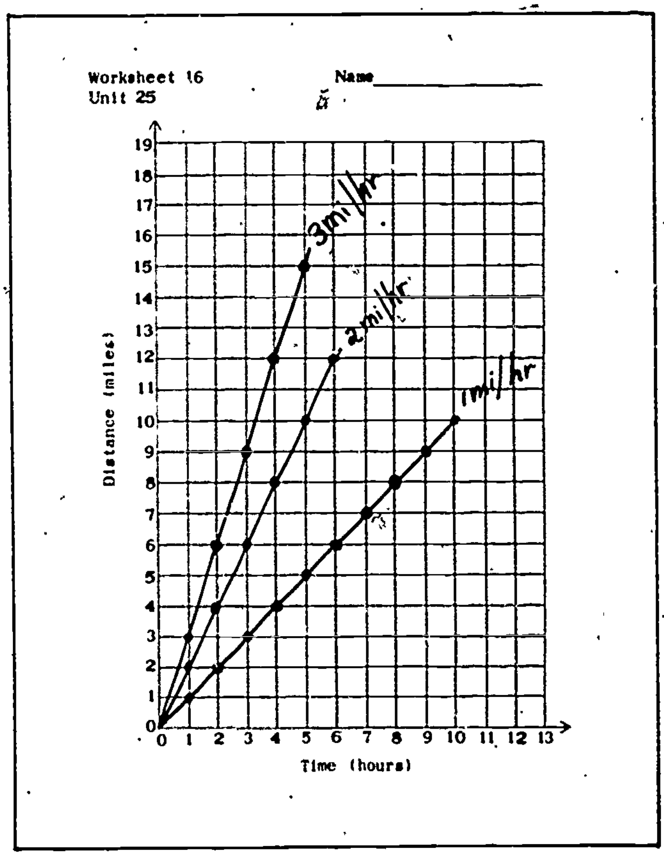
The children will plot linear data (constant speed data) and interpret graphs as indicating faster or slower speeds than other graphs. The word "slope" and its meaning are introduced.

MATERIALS

- grid transparency (from Lesson 3) and one blank transparency
- Worksheets 16 and 17

PROCEDURE

Using a grid transparency and the overhead projector, draw and label a two miles per hour line and a three miles per hour line. Have the children turn to Worksheet 16. Help them locate points for the two and three miles per hour lines. Remind them that two miles per hour means that in one hour you would travel two miles. The time-distance ordered pair (1, 2) is on the two miles per hour line. Discuss the meaning of other points on each line. When the children have plotted several points for each line, have them draw the lines starting from the origin (0, 0). Ask what (0, 0) means and why each line starts there. (It means that in zero hours, zero miles have been traveled. If no time has elapsed, then no distance has been traveled.)



Draw the one mile per hour line on your grid and ask the children what one mile per hour means. (In one hour you travel one mile.) Develop ordered pairs for this line and have the children explain what each means: (1, 1), (2, 2), (3, 3), etc. Have the children plot these on their grids and then draw in and label the one mile per hour line. Show them how to indicate the units (miles and hours). This is written miles/hour or mi/hr. and read "miles per hour" or "miles in an hour."

Ask the children to interpret these lines. What speed does each line represent? Which speed is faster -- one, two or three miles per hour? How does the graph represent which speed is faster? How is the line for the fastest speed different from the other lines?

HAVE YOU EVER HEARD THE WORD "SLOPE"? WHAT DOES IT MEAN? CAN YOU USE IT IN A SENTENCE?

CAN YOU THINK OF A SENTENCE USING BOTH THE WORDS "STEEP" AND "SLOPE"?

This discussion should lead the children to the idea that the steepness of a line (or a hill) is the slope of that line (or hill). The children should be able to explain that the slope of the line for a faster speed is steeper.

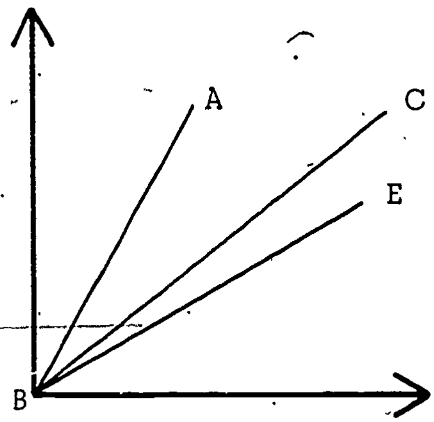
Ask the children to look at the one mile per hour line.

WHEN WE GO OVER ONE UNIT, HOW MANY UNITS UP DO WE GO TO RETURN TO THE LINE? (One.) Show the path of over one and up one on your transparency.

Ask the same with the two and three miles per hour lines. For each line have the children pick several points and go over and up until they return to the line. They will see that for the three miles per hour line they go over one and up three from any point to return to the line. They go over one and up two for the two miles per hour line.

(The children will begin naming slope in Section 2. The examination of slope as a certain number of units up per one unit over is meant only as an introduction to the idea of slope and to a concrete method of determining which slope is steeper.)

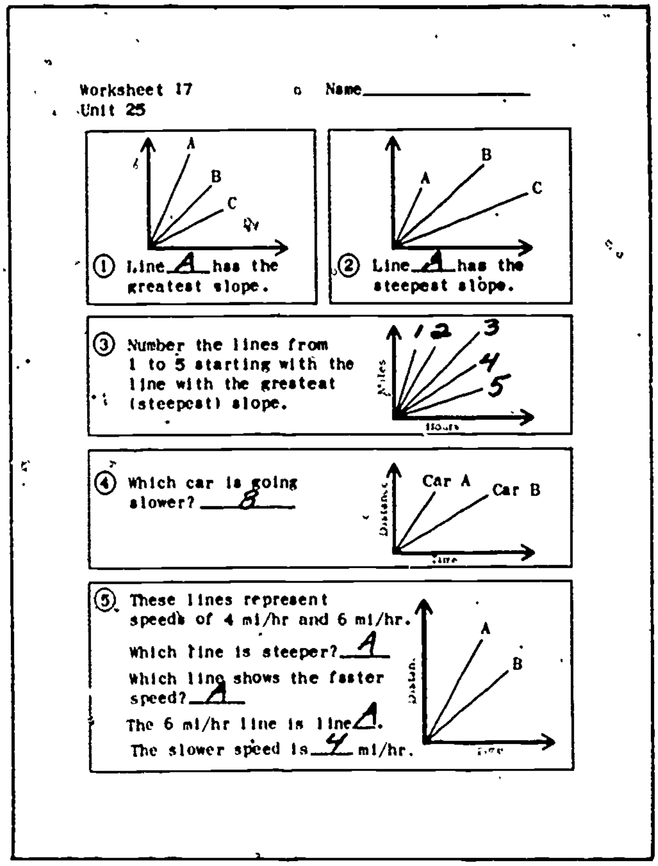
On a blank transparency (one with no grid lines) draw three different lines starting at the origin. Label the lines AB, CB and EB, but do not label the speeds.



Then ask:

> WHICH LINE REPRESENTS THE FASTEST SPEED? (AB.) THE SLOWEST? (EB.) HOW CAN YOU TELL?

Have the children complete Worksheet 17 by themselves. Check each child's work to see that no one has confused the length of a line with the slope or steepness of that line. If any children have made this mistake, draw some graphs on the chalkboard to show that any line can be extended or made shorter without affecting the slope of that graph or the speed the line represents.



Worksheet 17 Name________________
Unit 25

① Line A has the greatest slope.

② Line A has the steepest slope.

③ Number the lines from 1 to 5 starting with the line with the greatest (steepest) slope.

④ Which car is going slower? B

⑤ These lines represent speeds of 4 mi/hr and 6 mi/hr.
Which line is steeper? A
Which line shows the faster speed? A
The 6 mi/hr line is line A.
The slower speed is 4 mi/hr.

Lesson 8: RACES WITH DURATION TIMING

This lesson involves races for which data will be recorded and plotted. Each child runs and the distance he covers in five seconds is marked and recorded. Each team plots and compares the distance covered by the fastest, the slowest and a middle speed runner. By comparing the steepness of various team graphs, the class can determine who was the fastest runner. The children see that the slope of the graph is steepest for the fastest runner.

MATERIALS

- space in the gymnasium or outdoors
- 2 pendulum assemblies (from Lesson 2)
- 2 measuring strings (from Lesson 2)
- 8 marking objects (rulers)
- recording sheets
- Worksheet 18

PROCEDURE

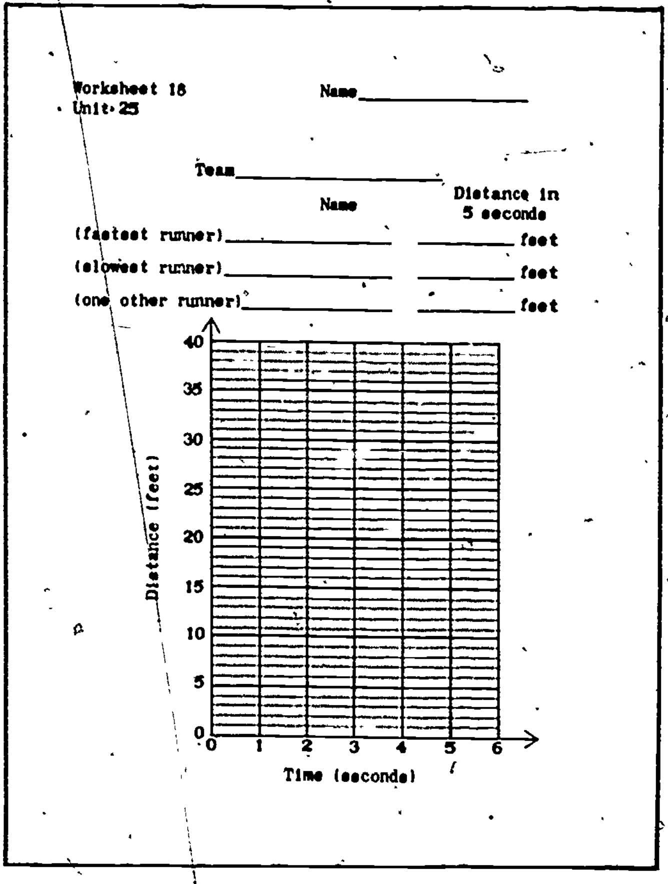
Activity A

Before going to the racing area, review the racing procedures from Lesson 2, going over the diagram to refresh the children's memories. Remind them that two teams work together, one team officiating while the other team runs. After all the members of the first team have raced, they serve as officials for the other team.

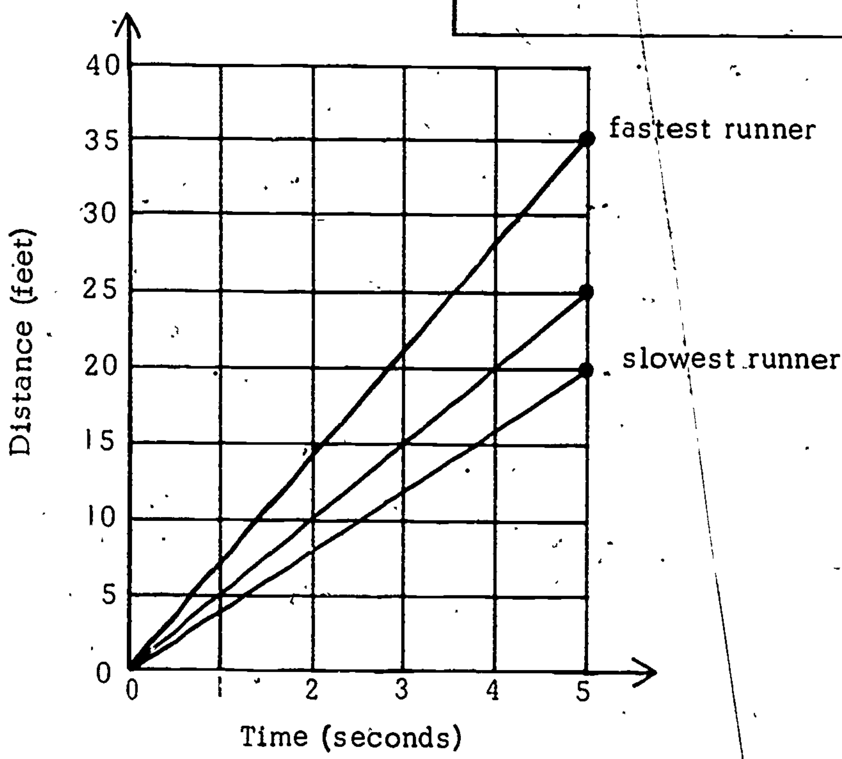
When the class is sure of the racing and recording procedures, take them to the racing area. Have the teams lay out the measuring strings, run the races and record each runner's distance traveled in five seconds. When the races are done, have the children pick up the equipment and return to the classroom.

Activity B

Ask each child on each team to prepare a graph on Worksheet 18 showing the distance traveled in five seconds by three members of his team: the fastest, the slowest and one other member. All members of the same team should agree on which third member to plot. Each child on a team plots the points on his own graph.



Have the children draw a line segment connecting the origin of the graph (0 seconds, 0 feet) with the plotted point for each runner. (See the sample graph below.)



Ask each team to study the graph of its three runners. You may want to copy one team's graph on the chalkboard. The line segment showing the time and distance of each runner is called a "graph;" the complete grid with times and distances for these runners is also called a "graph." You should make this distinction in the following discussion.

HOW ARE THE GRAPHS (THE LINE SEGMENT FOR EACH RUNNER'S TIME AND DISTANCE) THE SAME? (Each shows the distance a runner went in five seconds.)

HOW ARE THE GRAPHS DIFFERENT? (Some have a steeper slope showing a faster speed.)

Ask the children to consider the origin for each graph.

WHY DOES THE ORIGIN OF EACH GRAPH START AT 0,0? WHAT DOES 0,0 MEAN? (0,0 means zero seconds and zero feet. It means that when each runner was at the starting line, no time has passed and they had not run any feet.)

Now ask the children again how the graphs differ. (Some are steeper than others.)

DO THE SLOPES (STEEPNESS) OF THESE LINES TELL US ANYTHING ABOUT THE SPEEDS OF THE RUNNERS? (The steeper the slope, the faster the runner.)

IF WE SHOWED THIS GRAPH (the grid with three line segments) TO A STRANGER, COULD HE TELL WHO WAS THE FASTEST RUNNER? (Yes.) HOW COULD HE TELL? (The fastest runner is the one who ran farthest in five seconds. The graph of his speed has the steepest slope.)

Have the class discuss and compare the graph of each team (the four grids, each with the times and distances of three runners). The following questions may be helpful in guiding the discussion.

WHO IS THE FASTEST RUNNER IN THE WHOLE CLASS?

63

It is difficult to determine the fastest runner in the class when the data for the fastest of each team is plotted on a separate grid. Lead the children to suggest plotting the data for the fastest of each team on one grid if they cannot tell by looking at the separate graphs.

WHO IS THE SLOWEST RUNNER? Avoid this if you feel the slowest runner would be embarrassed.

WHO IS THE FASTEST BOY? THE FASTEST GIRL? THE SECOND FASTEST?

After this discussion the children should be able to read a graph and to interpret the slope of a graph as indicating a faster or slower speed than another graph.

Each child should plot the data for his time and distance on Worksheet 18 if he was not one of the children whose data were plotted for the class discussion.

Have the children save Worksheet 18 for comparison with data obtained in the next lesson.

Lesson 9: RACES WITH INTERVAL TIMING

The position of each runner at counts "1", "2", "3", "4", and "5" is marked, recorded and plotted. This graph should be an approximately straight line. Each child compares his new graph with the graph of his race in Lesson 8. The two graphs will be similar. The children should begin to realize that the graph of a constant speed is a line with a constant slope -- a straight line.

MATERIALS

- space in the gymnasium or outdoors
- 2 pendulums
- 2 measuring strings
- 10 marking objects (rulers)
- record sheets
- Worksheet 18 (from the previous lesson) and Worksheet 19

PROCEDURE

Activity A

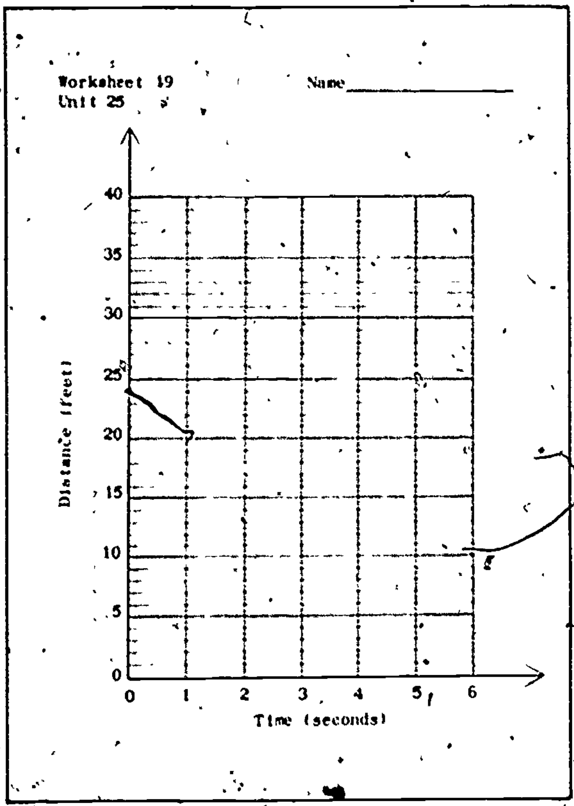
Before going to the racing area, discuss the procedures for the race. There will be five judges, one each to mark the runner's distance at one, two, three, four and five seconds. Only one child runs at a time. The children on Team A should line up behind the starting line. Five children from Team B will be positioned along the track as judges. The timer, starter and recorder will carry out their jobs as in previous races. When the timer calls "zero", the starter lowers his arm and the first runner starts. When the timer calls out "one", the first judge along the track marks the position of the toe of the runner's front foot. At "two", the second judge marks the runner and so on.

When the runner is done, the recorder records the position at each count. He will need a record sheet like the one on the next page.

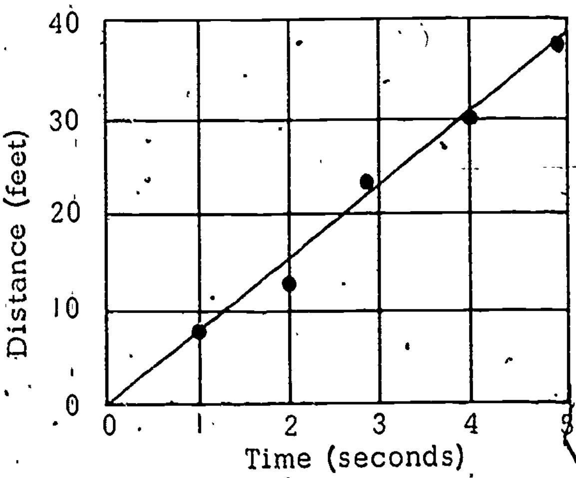
Team: A	Position at each count				
Name	1	2	3	4	5

Then the second child on Team A runs. When all the children on Team A have run, Team B runs and Team A officiates. When all the races have been completed, have the children return to the classroom.

Activity B



Have the recorder give each child his data for the race. Then each child plots the data on Worksheet 19. At the chalkboard, plot one runner's data and discuss with the class how a line segment can be drawn which approximates all the points plotted. (See the sample graph below.) Have each child draw such a line.



Have the children study the data on Worksheet 19. Most children's graphs will show a nearly constant speed. The points representing the distance at each second all fall close to the line segment. Ask the children why the points are near (or on) the line and what this means. (It means the child ran at about a constant speed. His speed did not change much.)

COULD A RUNNER KEEP UP THIS CONSTANT SPEED FOR AN HOUR? (No, he would get tired.) WHAT WOULD HAPPEN TO HIS SPEED? (It would get less because he would slow down.)

IF A RUNNER SLOWED DOWN, WHAT WOULD HAPPEN TO HIS GRAPH? WHAT WOULD IT LOOK LIKE? (The slope of the graph would change. It would become less steep. The graph would not be a straight line.)

Ask the children to compare the data they plotted for the first race (Worksheet 18) and that for the second race (Worksheet 19). For many children, the slope of each line was similar. For some children the data differed because they ran faster or slower in one race than in the other.

COULD WE PLOT THE POINTS FROM THE FIRST RACE (Worksheet 18) ONTO THE GRAPH OF THE SECOND RACE (Worksheet 19)?

Have each child transfer his distance at five seconds from Worksheet 18 onto Worksheet 19. He should draw a line from this point to the origin (0, 0). Have the children put away Worksheet 18 and look at Worksheet 19.

DID ANYONE RUN FASTER IN THE SECOND RACE THAN HE DID IN THE FIRST?

DID ANYONE COVER LESS DISTANCE IN FIVE SECONDS IN THE SECOND RACE THAN HE DID IN THE FIRST RACE?

ARE THERE ANY RUNNERS WHO RAN ABOUT THE SAME DISTANCE IN FIVE SECONDS IN THE SECOND RACE AS IN THE FIRST RACE? IF SO, WHAT DO YOU NOTICE ABOUT THE TWO LINES? (They are similar.)

HOW ARE THE DATA DIFFERENT FOR THESE RACES? (For the first race we plotted only two points -- the origin and the distance at five seconds. For the second race we plotted six points -- the origin and the data at each of five seconds.)

Lesson 10: NON-CONSTANT SPEEDS

In Lesson 9 a question was raised about the graph of a race in which the speed changed. In this lesson the children will plot and interpret data for runners who change speed. They will see that the plot of a race in which the speed changed significantly will not be a straight line and that an increase in speed is indicated by an increase in slope.

MATERIALS

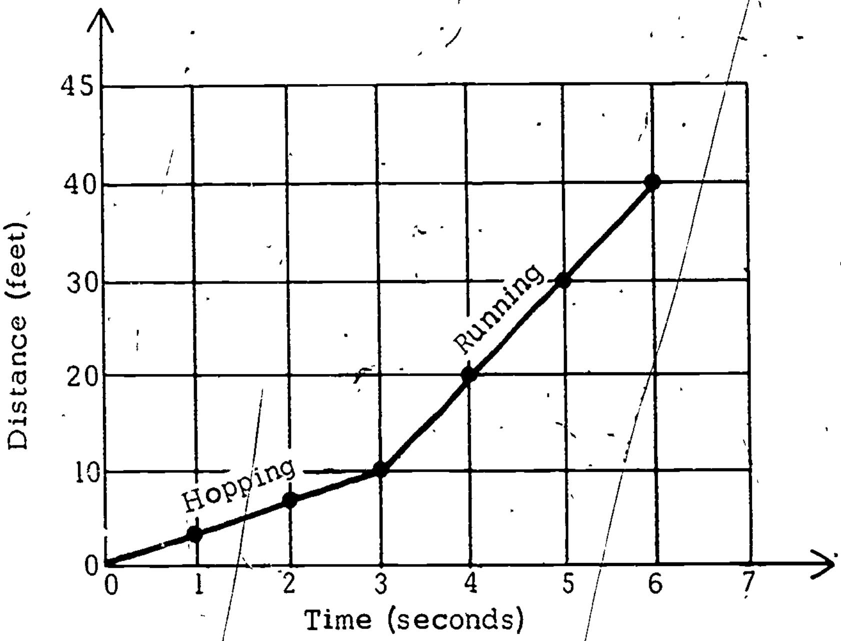
- space in gymnasium or outdoors
- 2 pendulums
- 2 measuring strings
- masking tape
- 12 rulers
- recording sheets
- pencils
- Worksheet 20

PROCEDURE

Review what the children found when they compared the graphs from their first two races. They were running at constant speeds, so the graph of the time-distance relation was always a straight line. This relation between constant speed and straight line graphs should be stressed.

Explain that the class is going to run races today in which they will change their speeds. Remind them of the discussion of the child who might get tired after running for an hour and slow his speed. How would his graph change? (The slope for the faster speed would be steeper than the slope for his slower speed.)

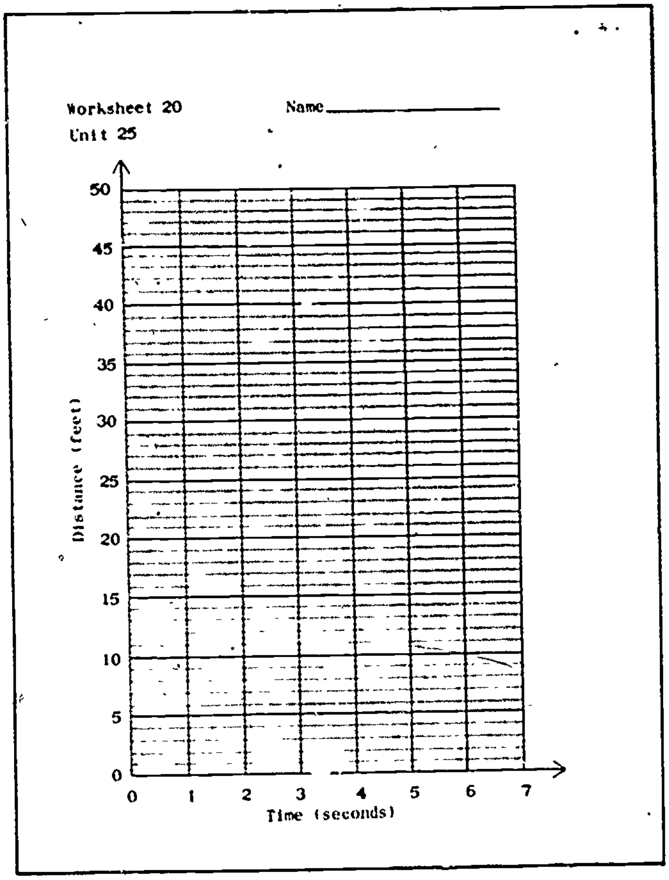
A simple way to change speed during a race is to hop for three counts and then run for the next three. Have a few children sketch their ideas of what a graph of this race would look like. A sample graph appears on the next page.



. The procedure for this race is a modification of the one used in Lesson 9. There are six judges, each marking the runner's position for one of the counts from one through six. The runner starts out hopping and changes to running when the timer calls "three." There is a second starter positioned near judge three who lowers his arm at "three" to remind the runner to run.

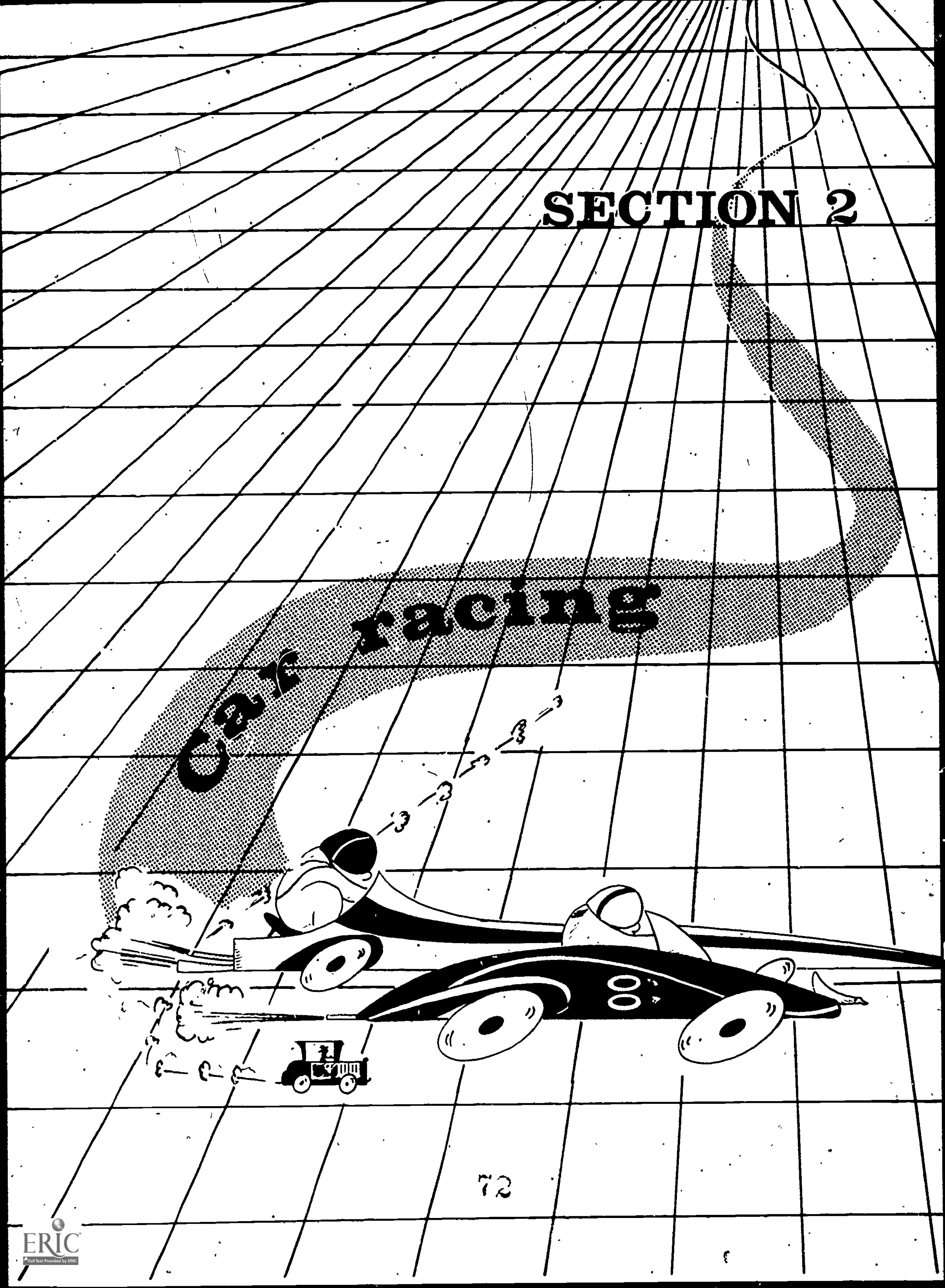
70

After the races, the recorder should read off each child's data. Each child should plot his own data on Worksheet 20. Then discuss the race, the graphs, and the relation between the change in speed and the change in slope on the graphs.



SECTION 2

car racing



SECTION 2 CAR RACING AND MULTIPLYING WITH SLOPE

PURPOSE

- To introduce the idea of naming slope.
- To develop an understanding of constant speed represented by a straight line graph.
- To develop the idea of slope as an embodiment of multiplication.
- To provide practice in multiplying using graphs of different slopes.

COMMENTARY

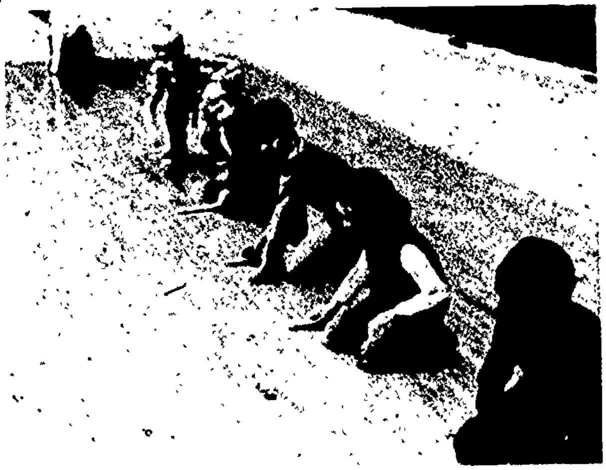
In Lesson 11 the children race toy cars and graph the data on grids. By examining the slope of each graph, they can answer questions such as: "Which car went fastest in trial 1?", "Did the same car go fastest in each trial?", and "Can we tell which car went the greatest distance in five seconds in any of the trials?"

Lesson 12 introduces the children to naming the slope of a line as a number of units up per a number of units over on a grid. The children draw lines that represent given slopes and also name the slopes of lines that are given on grids.

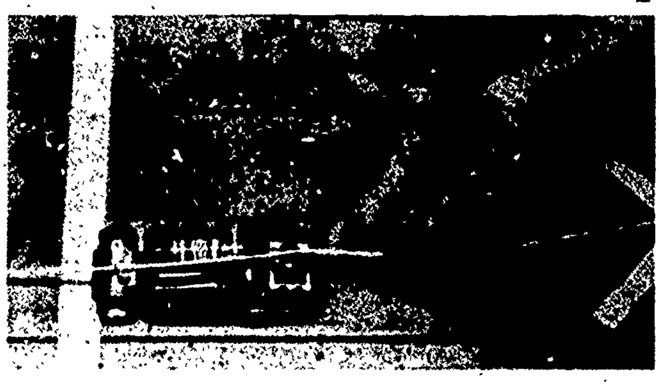
73

In Lesson 13 the children discover that any line on a grid can have infinitely many names for its slope. Games and worksheets give practice in finding many different names for the same slope. It is usually easiest to name slope as a whole number of units up per one unit over, although some slopes are more easily named as a whole number of units up per two or three units over.

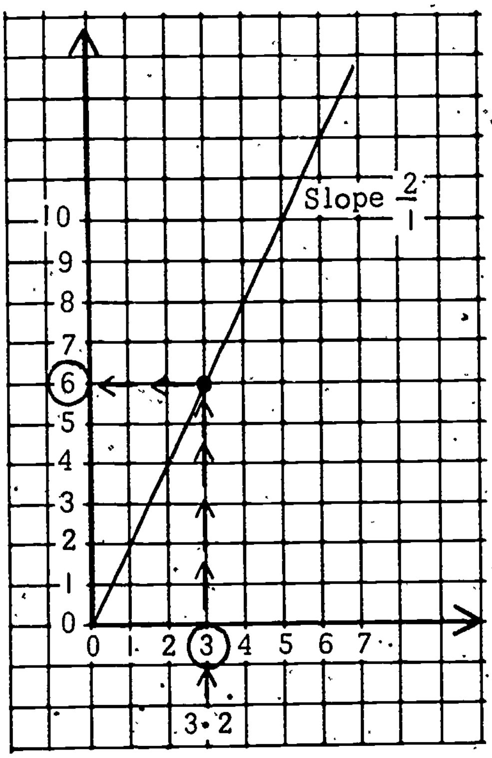
In Lesson 14, the children race their cars again, marking the position of the car at each of five seconds. They graph the data and name the slope. They discuss the fact that when a car goes the same distance during each second of a race, it is traveling at a constant speed. The children examine their graphs and see that a graph of a constant speed is a straight line.



In Lesson 15 the children modify their cars to slow them down and then compare the speeds of the modified and unmodified cars. The children find that in both races the car went at a constant speed (the graphs are straight lines) and that the unmodified car always went faster (its graph always has the steeper slope).



In Lesson 16 the children see that slope is related to multiplication. They find that they can use a line with slope two per one, for example, to multiply by two. The product of three times two can be found by starting at three on the horizontal axis, moving up to the line, and moving across to the vertical axis. The number there is the product, six.



The children get a "multiplication machine" in Lesson 17 which they can use to multiply integers up to ten times nine. This machine is a grid and lines with slopes ranging from one per one through nine per one. The children complete several worksheets using the machine. The lesson includes two optional activities using this machine. One optional activity involves multiplying mixed numbers and the other involves division of whole numbers.

In Lesson 18, the children race their cars on circular tracks. In previous lessons, to find which car went fastest, they determined which went the greatest distance in a fixed amount of time. Since the circular track is a fixed distance, the time durations will vary.

In order to compare the speed of different cars, the children must determine the amount of time it takes the car to complete one revolution. The children race their cars modified and unmodified, and observe the effects of turning the cars' front wheels in or out.

76

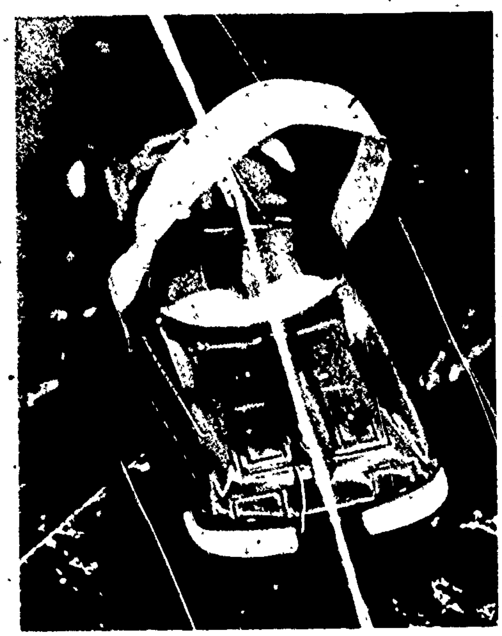
In Lesson 19 the children predict the amount of time it will take their cars to travel a fixed distance and test their predictions. Some children may realize that if their car went 20 feet in five seconds in previous races, it will take about six seconds to go 25 feet since it travels at four feet per second. By looking at the accuracy of the children's predictions and the discussions of the results, you should be able to evaluate somewhat how well the children understand the relation between time and distance and the concept of speed.

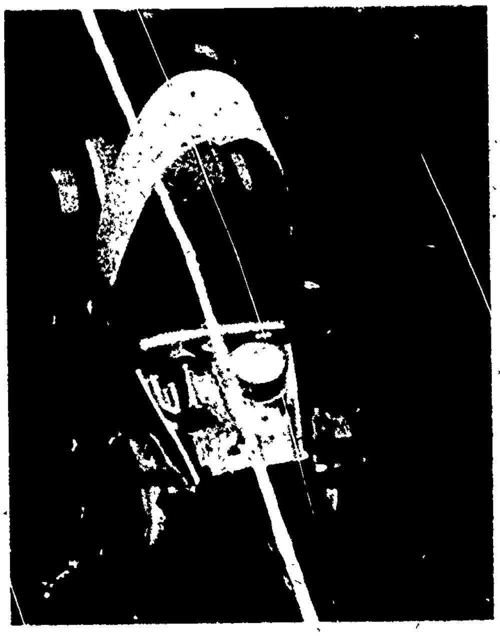
77

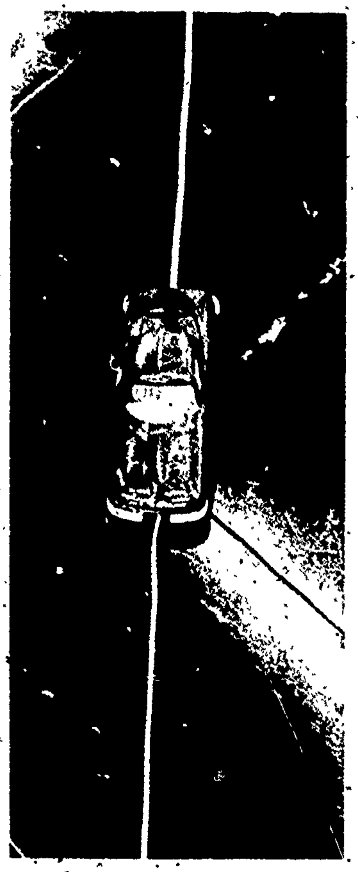
In Section 2 several lessons involve car races. The purposes of the car races are: 1) to teach children experimental procedures, 2) to generate data for lessons in which the children learn to draw and name slopes of lines, 3) to help children perfect their data-gathering methods. Cars are used to provide a fun way of gathering slope data. Therefore, the type or speed of the cars does not really matter. The instructions for racing procedures given in the lessons refer to one kind of car, but these can easily be modified to fit whatever car your class is using. For schools that do not get the OMSI kit, and for teachers who would like the children to work in smaller groups, we suggest you ask children to bring any battery operated car they may have.

The purpose of the guide string discussed in the racing procedures is to help the car run in a straight line. The car pictured in the lessons accomodates toothpicks. The string goes over the car, between the toothpicks. There are other ways of using the guide string:

1. Make a masking tape loop with two pieces of masking tape. One piece should be an inch longer than the other. Put sticky sides together, leaving a half-inch of the sticky side at each end so that it will stick to the car. The guide string can run over the car and through the loop.







2. Run the car over the guide string. If you have a large car, use yarn or clothes line rope for the guide string. Such a guide string will keep the car running in a straight line.

In the lessons, the cars are modified by replacing one of the batteries with a worn-down battery or a two-inch paper clip. If your car can not be adapted to these methods, you could tape weights to the car or tie something on that will drag behind. Before the children use their modified cars for the lesson, have them time the cars to be sure that they have been slowed down significantly.

If your racing space is limited, you can change the duration of the races or the distance that the cars run.

Lesson 11: CAR TIME TRIALS

In this lesson the children use the procedures they developed during the foot races to time cars. Each team runs a car for five seconds in three different races, and records the distances it travels. Then they graph the various races of all teams and compare the data, asking which car went slowest, fastest and so on.

MATERIALS

-- for each team --

- racing space
- car
- 2 size AA penlight batteries, $1\frac{1}{2}$ volt
- 2 toothpicks
- guide string (30 feet)
- measuring string (from previous lessons)
- pendulum (from previous lessons)
- object to mark position of car (eraser or ruler)
- ruler (to measure distance of car)
- record sheet
- chair

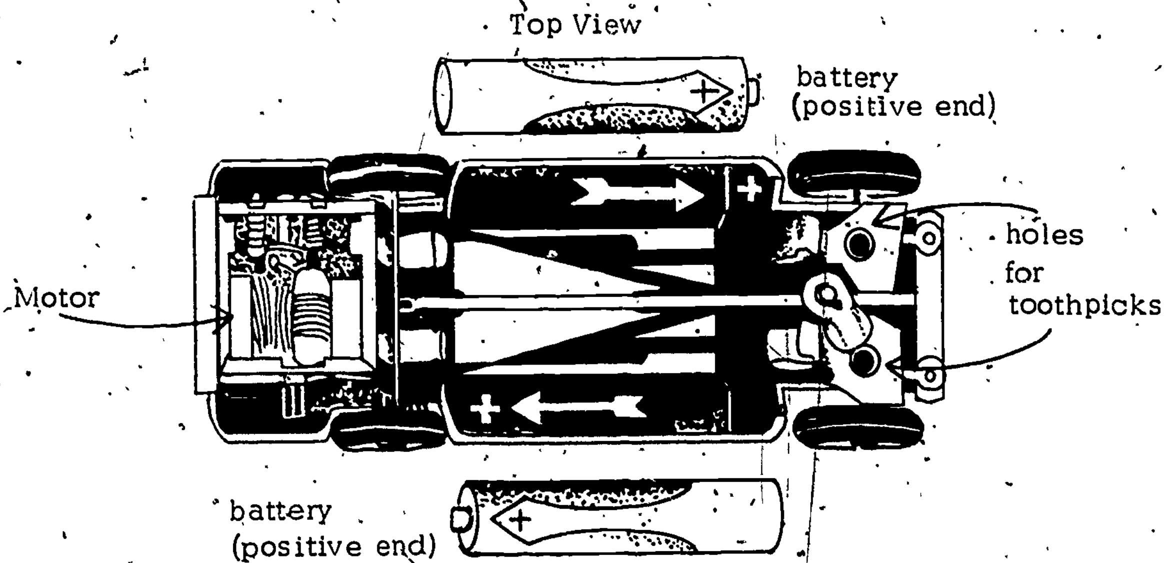
-- for each child --

- Worksheet 2.1

PREPARATION

Each team needs a space about five feet wide and thirty feet long. The best place to run the races is in the gymnasium, but if you can move the desks in your classroom, the races can be held there. The measuring string need not be taped down past the 30-foot mark. If your classroom is too short to allow the cars to run for five seconds, you may run the car races for only four seconds.

To put the batteries in the car, place them so that the positive ends (tops) are at the head of the arrows on the car floor. Tape them to the car chassis with the masking tape to make sure they stay in place. Toothpicks should be inserted in the two holes at the front of the car. The switch for the motor is underneath the car behind the front wheels. Be sure the wheels are not touching any surface when the motor goes on.



Warn the children not to push down on the car or hold the wheels when the motor is on or the motor will burn out. If the car motor stops working, you may be able to fix it by gently spinning the back wheels while the motor is on. Warn the children each time they use the cars that they must not push down on the car or hold the back wheels when it is in motion. The motor snaps into the back end of the car. If it should fall out when the children are using the cars, simply put it back in place.

Cut four guide strings, each about 30 feet long. Each car runs under a guide string, which is taped to the floor. The string runs over the car and between the toothpicks.

PROCEDURE

Activity A

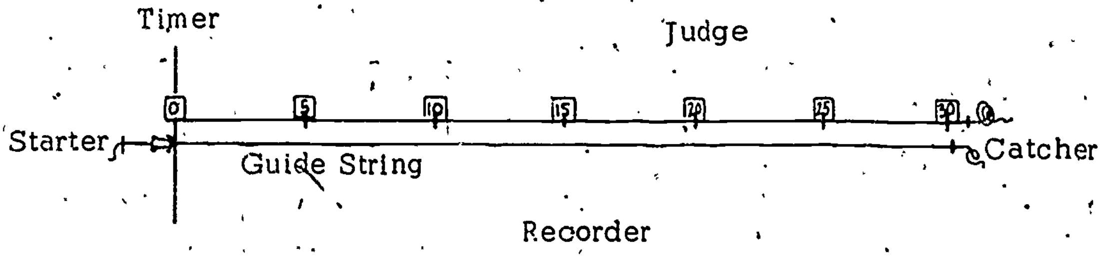
Explain the procedures before beginning the races.

1. Each team needs this equipment: one measuring string, one guide string, one pendulum assembly, one chair, masking tape, one car, two batteries, two toothpicks, one marking object, a ruler, a recording sheet and a pencil. Each team chooses a name and labels its car with an identifying mark. The same children must work together and with the same car in all car races.

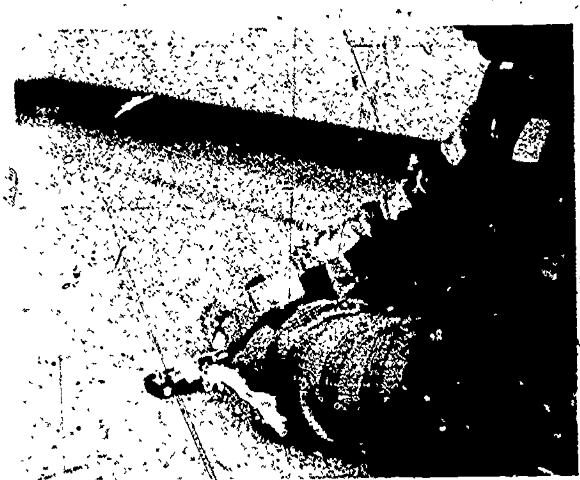
2. The children tape the measuring string at both ends to the floor. The guide string should be taped parallel to the measuring string and about six inches from it. Each end should be held down firmly with several pieces of tape. The guide string should extend at least a foot behind the starting line so that the car can be placed underneath it.

3. A judge with a marking object should be stationed along the measuring string at approximately the distance the car will go in five seconds. The children might want to make a trial run to determine where the judge should stand, or the judge can follow the car along the track so he will be with it after five seconds.

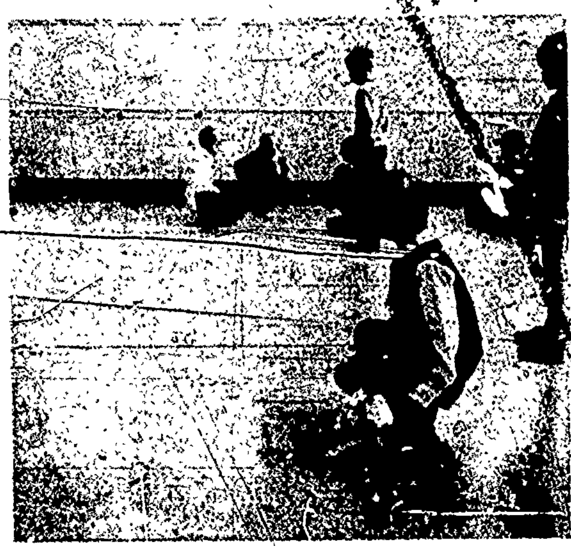
4. A car catcher should be stationed at the end of the guide string. He will lift the car off the floor when it runs to the end of the string, turn the motor off, and return the car to the starter. (Warn him not to push down on the car when the motor is running.)



5. To prepare the car for the race, have the starter check to see that the front wheels are straight. He turns on the motor. Then he gently lifts the guide string and puts the car under it so that the string runs over the car and between the toothpicks. The starter moves the car up the string until the front wheels are even with the zero mark of the measuring string. He lifts the back wheels so that they are not touching the floor.



6. The timer begins the countdown ("two, one, zero, one..."). When the starter hears "zero," he puts the rear wheels of the car on the floor and lets the car go. When the timer calls "five," the judge marks the front of the car with his marking object. The car runs to the end of the guide string where it will stop. Have the catcher return it to the starter. Remind him not to push down on the car when the motor is on. He should pick up the car, turn off the motor, and carry it back to the starter.



7. The recorder notes the position of the marking object to the nearest inch by finding the nearest half-foot on the measuring string, then measuring the intervening inches with a ruler. He records this datum in the correct place on a record sheet the class has developed. It might look like this:

Team ____________	Distance car traveled in 5 seconds
Trial 1	
Trial 2	
Trial 3	

Help the children set up their equipment. Have each team run three trials and record the datum for each. Remind the children to exchange jobs for each trial so that every child on a team can participate. When the trials are finished, gather up the equipment and return to the classroom for a discussion of the results.

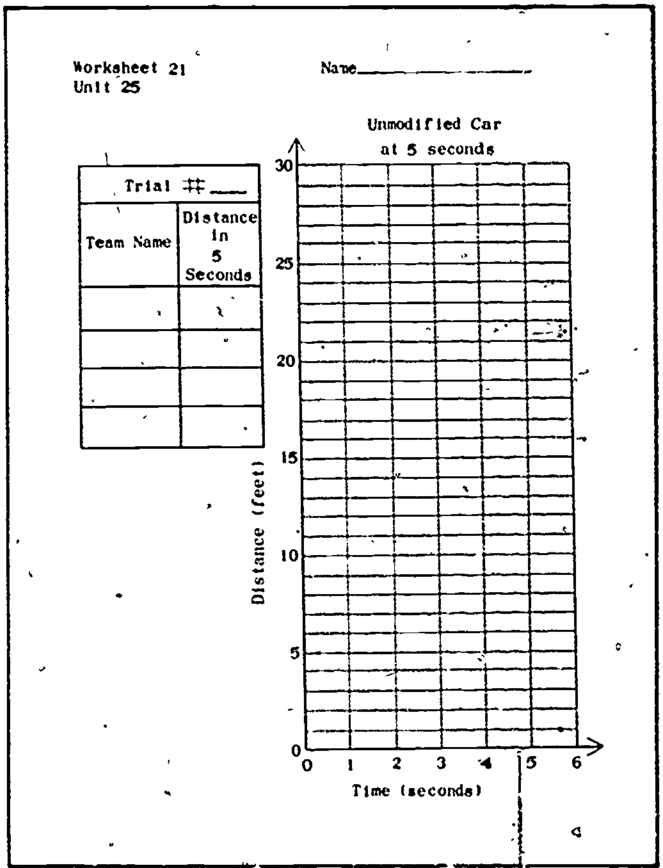
Activity B

Let the members of each team discuss their data and decide how far the car went in the fastest trial. (Have them turn ahead in their student manuals to Worksheet 32 and record this distance in the box marked "Unmodified." They will use this data in Lesson 15.) They should decide if any differences in the data for the three trials were caused by the movement of the car or were due to inaccuracies in measurement.

Have each team divide into three subgroups: 1, 2 and 3. Each subgroup plots the data for one of the time trials their team ran (1, 2 or 3 respectively) on Worksheet 21, drawing a line connecting the origin and the five-second point. Then each subgroup exchanges data with the corresponding subgroup

84

of every other team. This data should also be graphed on Worksheet 21. For example, trial #1 group will graph time trial #1 for each car.



Have the children in corresponding trial groups sit together while you lead a discussion of such questions as:

ARE ALL THE GRAPHS FOR TRIAL 1 THE SAME? (No.) Repeat the question for trials 2 and 3.

HOW CAN YOU TELL WHICH CAR WENT FASTEST IN TRIAL 1? IN TRIAL 2? TRIAL 3? (By looking at the distances on the chart, or at the slopes of the lines on the grid. The graph for the fastest car has the steepest slope.)

CAN WE LIST THE FOUR CARS IN ORDER BY THEIR SPEEDS FOR EACH TRIAL?

Have each trial group rank the cars 1 through 4 (fastest to slowest) on the chart on Worksheet 21.

Have the children from the appropriate trial group display their graphs as you ask the following questions.

WHICH CAR WENT SLOWEST IN EACH TRIAL? (In trial 1, the Red Baron car went slowest, etc.)

DID ANY TWO CARS GO AT ABOUT THE SAME SPEED IN TRIAL 1? 2? 3?

WHICH TEAM'S CAR WENT FASTEST IN THE FIRST TRIAL? IN THE SECOND? IN THE THIRD?

DID THE SAME CAR GO THE FASTEST IN EACH TRIAL?

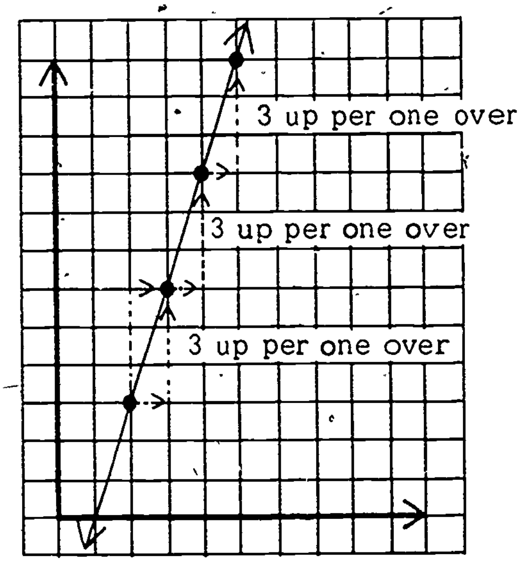
DO WE HAVE A CAR WHICH WE CAN CALL THE FASTEST? (Yes, if one car ran fastest in two out of three trials.)

CAN WE TELL WHICH CAR RAN THE GREATEST DISTANCE IN FIVE SECONDS IN ANY OF THE THREE TRIALS? (Yes, the one that went fastest covered the greatest distance.)

Lesson 12: INTRODUCTION TO THE NAMING OF SLOPE

In this lesson the children begin naming slopes of lines. Slope is named as the number of units up per the number of units over on the grid. The easiest way to determine the slope of a line is to start at any point on the line that is at an intersection of the grid, and move over one square and up as many squares as necessary to return to the line. Although slope names are read "three up per one over" or "three per one," they are written like fractions, $\frac{3\ \text{(up)}}{1\ \text{(over)}}$.

This particular slope name could be reduced to an integer (3), but we will retain the fractional form to emphasize the up per over meaning of the name. In the diagram below, the slope of the line is three up per one over. At any point on that line, the steepness increases by three units up per one unit over to return to the line.



MATERIALS

- overhead projector
- grid transparency
- Worksheets 22-27

PROCEDURE

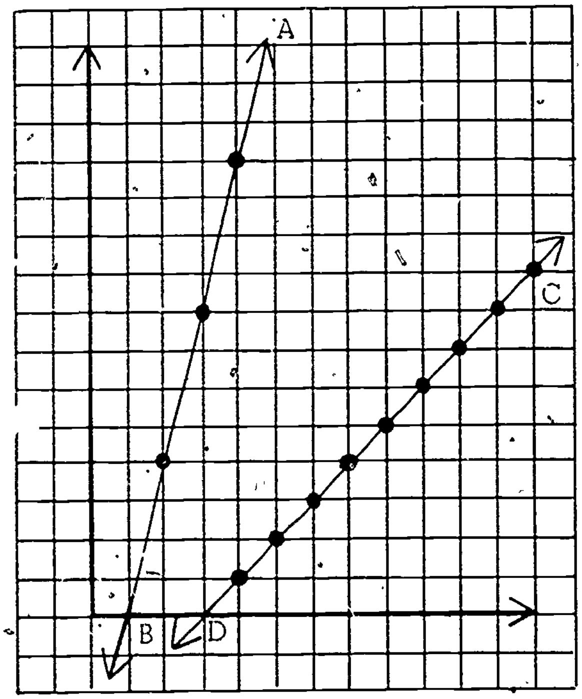
Activity A

In previous lessons the children talked about their idea of the meaning of slope and steepness, but they may need to review these ideas again. A discussion of the children's practical experience with slope may help develop the mathematical idea of slope and the naming of slope.

Begin by asking the children if they have ever run or ridden a bicycle up a hill and how they felt once they reached the top. Are some ~~hills ha~~rder to go up than others? How do you know they are harder? Do you get more tired? Why do you get more tired going up some hills than others?

These questions should lead into a discussion of steepness and slope. Encourage the children to mention things other than hills that have slope: roofs, playground slides, etc.

On a chalkboard grid or grid transparency on the overhead projector, draw two lines, AB (slope four per one) and CD (slope one per one). Do not label the slopes, but mark the grid intersections that each line passes through. Ask the children which line has the steeper slope.



The children will probably say that line AB is steeper than line CD. Ask them if they can assign a number to the steepness of each line. Encourage suggestions. If no one has any ideas, have a child start at a grid intersection that line CD passes through, move one step (one unit) to the right, and then take one step back to the line. Have him do this starting at another grid intersection that line CD passes through. Then have him move one unit to the right from different points on line AB, each time moving up as many units as necessary to return to the line. Ask what the children notice about these walks. After you have elicited the observation that it takes more steps up to get back to line AB, you may spell out the details: "If you take one step to the right of line AB, the line with the steeper slope, it takes four steps up to get back to the line. After you have gone one step to the right of line CD, the line whose slope is less steep, it takes just one step up to get back to the line." As you say this, trace the appropriate paths with a pointer.

Ask the children whether or not they can use these relationships to assign a number to the steepness of the lines AB and CD. Develop the idea that the steeper the slope of a line, the greater the number of units (or steps) one must go up to return to the line for each step taken to the right. The children should see that they can compare the slopes of different lines by determining the number of steps needed to return to the line for each step taken to the right.

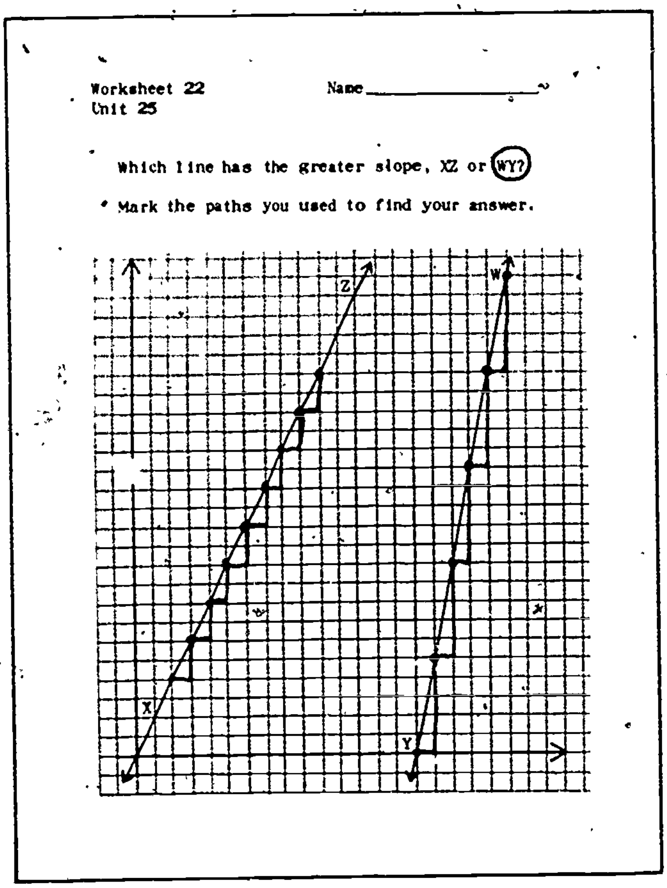
Explain that we can name the slope of a line by the following method:

$$\text{slope} = \frac{\text{number of units to get back up to the line}}{\text{number of units over to the right}} \text{ or } \frac{\text{up}}{\text{over}}.$$

For example, on line AB, if you take one step to the right you must take four steps up to get back to the line. So the slope is $\frac{4}{1}$. This should be read as "four up per one over."

For line CD, if you take one step to the right you must take one step up to get back to the line. So the slope is $\frac{1}{1}$ (one up per one over).

Start at several places on each line to show that the slopes will be the same no matter where you start your walks. Remind the children that it is easiest to determine slope if they start at a grid intersection the line passes through.



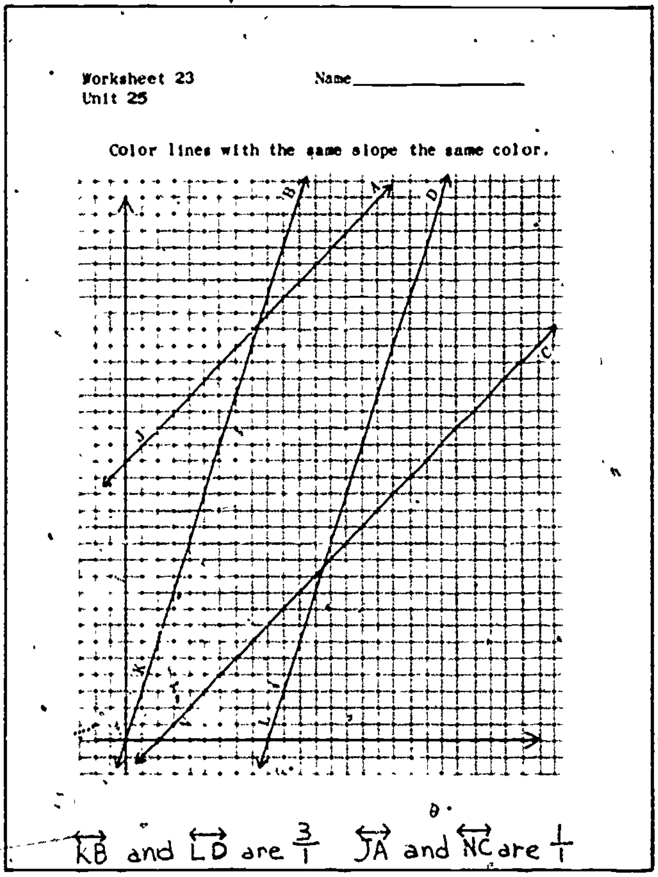
Have the children complete Worksheet 22. Ask them to describe the slope of line XZ. Have a child demonstrate on the chalkboard how he determined his answer. The easiest way is to start at a point on a line that falls on a grid intersection and count over and up. Tell the children they can start at any grid intersection the line passes through. Mark another point, and ask if someone could find the slope from that point. Ask if the slope would be the same from any point on the line and have the pupils measure the slope from several points on line XZ on their papers. The slope is two up per one over.

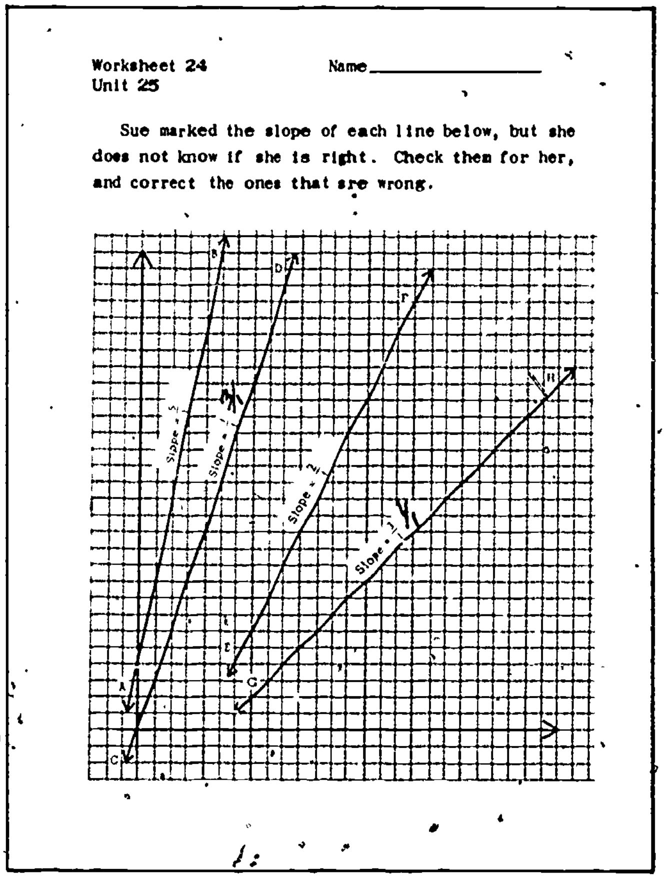
Then ask how many units they needed to go up to return to line YW on their worksheets when they had taken one step to the right. Tell them we would say that this line has a slope of five up per one over, because we have to go five units up to return to the line after having gone one unit to the right from the line.

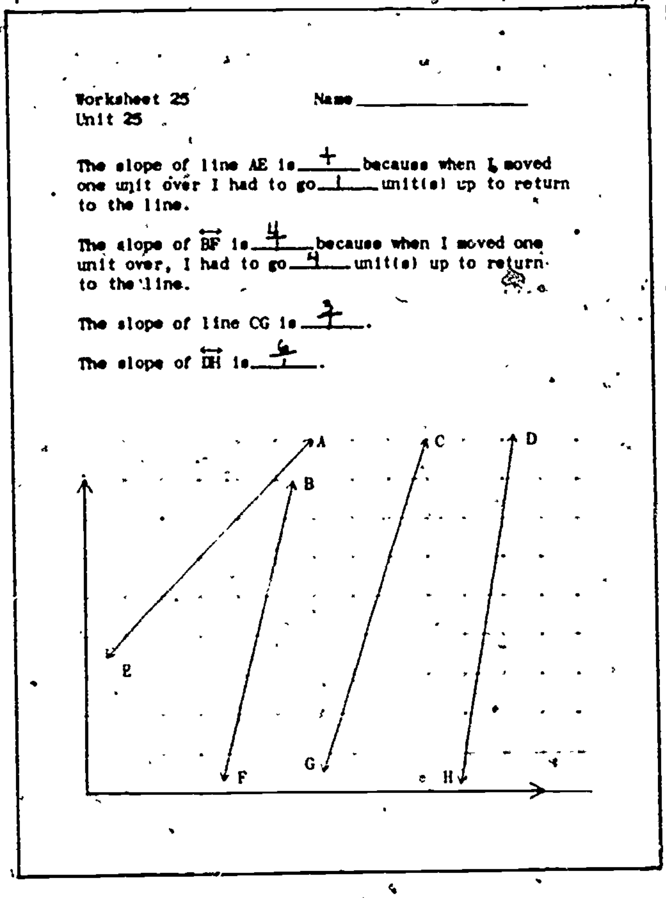
Have the children complete Worksheets 23 through 26 as a class or individually. If the symbol for "line" (↔) is unfamiliar to the children, explain it using Worksheet 25.

NOTE: Watch to see if any children are confusing the idea of an ordered pair with the idea of naming slope. An ordered pair, such as (1,3) indicates a location — one point on a graph. A name for a slope, such as three per one, describes the steepness of a line. At any point on that line, the name of the slope is three per one. The children know that the first numeral of an ordered pair refers to the over axis and the second to the up axis. For a slope name, the top numeral tells how many units to go up and the bottom numeral tells how many to go over. You should make explicit the distinction between an ordered pair and a slope name only if the children are confused.

Be sure the children do not read $\frac{3}{1}$, for example, as three over per one up. Emphasize that the name is always read: "_____ up per _____ over."







Worksheet 25 Name_______________

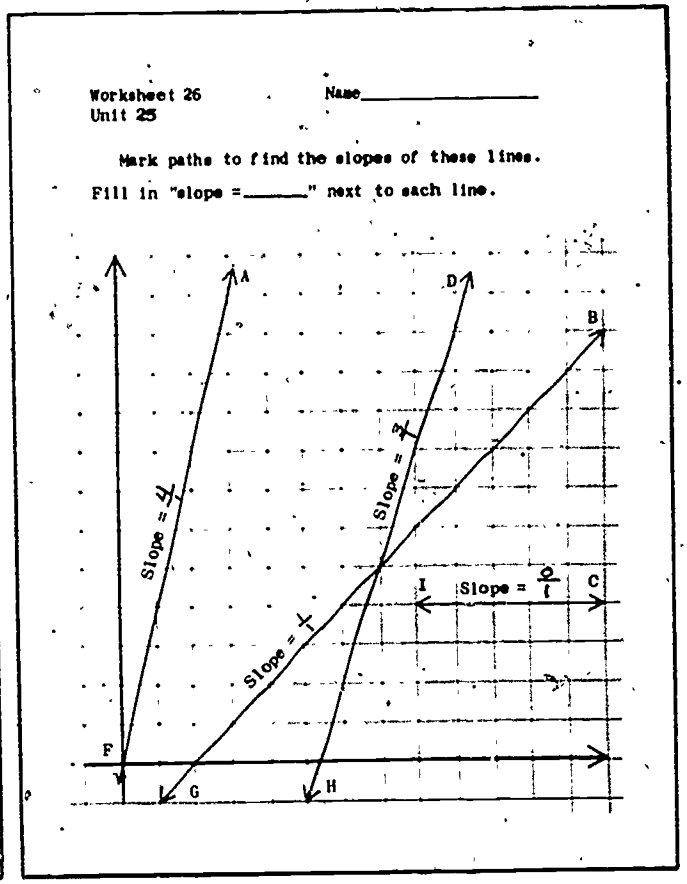
Unit 25

The slope of line AE is $\frac{1}{1}$ because when I moved one unit over I had to go 1 unit(s) up to return to the line.

The slope of $\overleftrightarrow{BF}$ is $\frac{4}{1}$ because when I moved one unit over, I had to go 4 unit(s) up to return to the line.

The slope of line CG is $\frac{3}{1}$.

The slope of $\overleftrightarrow{DH}$ is $\frac{6}{1}$.



Worksheet 26 Name_______________

Unit 25

Mark paths to find the slopes of these lines.

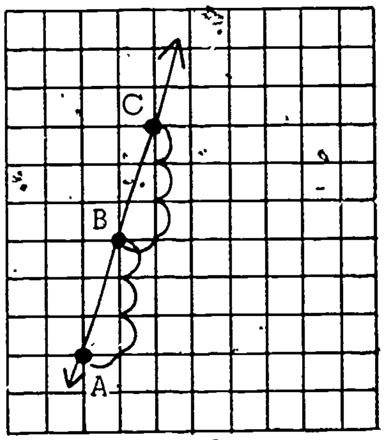
Fill in "slope = ______" next to each line.

Activity B

In the previous activity the children named the slopes of lines. In this activity they draw lines of given slopes.

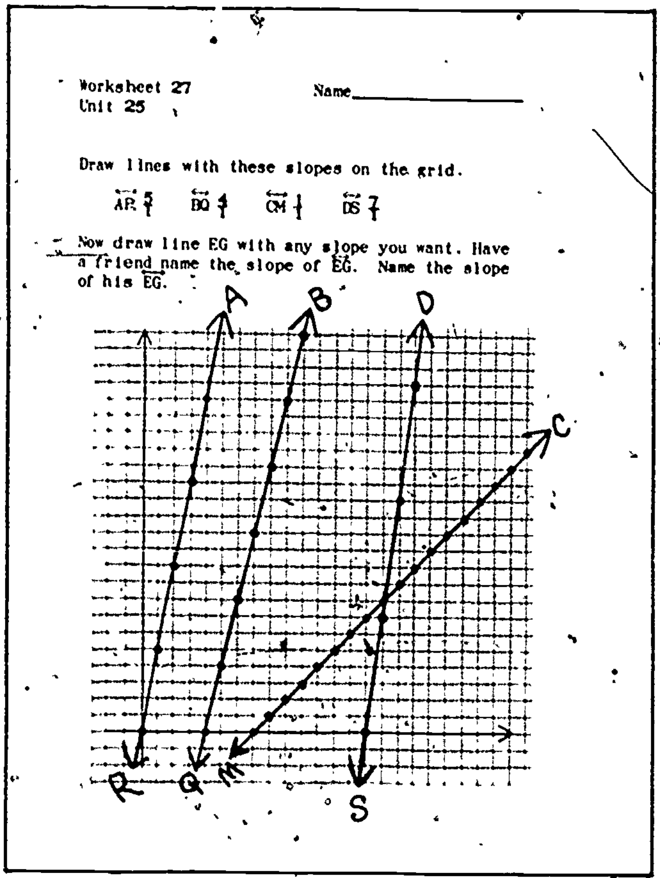
Demonstrate for the children how to draw a line with a given slope, for example, $\frac{3}{1}$:.

1. Choose any grid intersection (point A).
2. Count over one space (the bottom numeral of the slope name).
3. Go up three spaces (the top numeral of the slope name).
4. Repeat these steps from point B to arrive at point C.
5. Draw a line connecting the plotted points.



On the chalkboard grid or a grid transparency, have several children draw lines of given slopes. Have these volunteers explain the steps in drawing a line of a given slope through a given point. Let enough children draw lines of given slopes until the class understands how to do it.

Have the students complete Worksheet 27 and as many other examples as needed until all children can 1) name the slope of a given line, and 2) draw a line of a given slope. If the children need more practice have them draw the lines on one of the blank grids in the back of the student manual.



Worksheet 27 Name____________
Unit 25

Draw lines with these slopes on the grid.

$\overleftrightarrow{AR}$ $\frac{5}{1}$ $\overleftrightarrow{BQ}$ $\frac{4}{1}$ $\overleftrightarrow{CM}$ [illegible] $\overleftrightarrow{DS}$ $\frac{7}{1}$

Now draw line EG with any slope you want. Have a friend name the slope of $\overleftrightarrow{EG}$. Name the slope of his $\overleftrightarrow{EG}$.

Lesson 13: MORE WORK WITH NAMING SLOPE

In the previous lesson the children named slope as a whole number of units up per one unit over. Now they will learn that slope can be named as a whole number of units up per any whole number of units over, and that there are an infinite number of names for the same slope. Six per two, nine per three, and twelve per four are other names for a slope of three per one.

There are other slopes, such as five per two, that when reduced to units up per one unit over result in a fractional number of units up (two and one-half up per one over, in this case). The children are given examples of this type, but are not asked to reduce them to units up per one over.

MATERIALS

- overhead projector
- grid transparency
- Worksheets 28-30

PROCEDURE

Activity A

Using a chalkboard grid or grid transparency show the children how to name several lines as a certain number of units up per two units over. Also ask several children to draw lines whose slopes are named as so many units up per two over. Emphasize that slope is not always named as a certain number up per one over and that the same procedures are followed no matter how many units over we go. For example, three per one could be named six per two.

Activity B

Erase the lines drawn on the grid. Put a point on the grid anywhere except at 0, 0 and explain that a line on a grid can start anywhere. Tell the children that they are going to draw lines that must pass through this point.

Ask a child to draw a line with a slope of three per one through the point. The other children should watch carefully to see that he has done it correctly. Leave this line on the grid. Ask another child to draw a line with a slope six per two that also passes through the point. He will discover that his line is the same as the line the first child drew. Have him draw his line anyhow, on top of the first line. Ask other volunteers to draw lines with the slopes nine per three, twelve per four, and fifteen per five, all passing through the point. Each child will see that his line is the same as the other children's line.

Ask the class why all these lines fell in the same place. Lead them to see that there are many names for the same slope. For each name, the slope of the line does not change. Stress the fact that the children could use many different names for the same slope, but that it is usually easiest to use a certain name, such as three per one in this example.

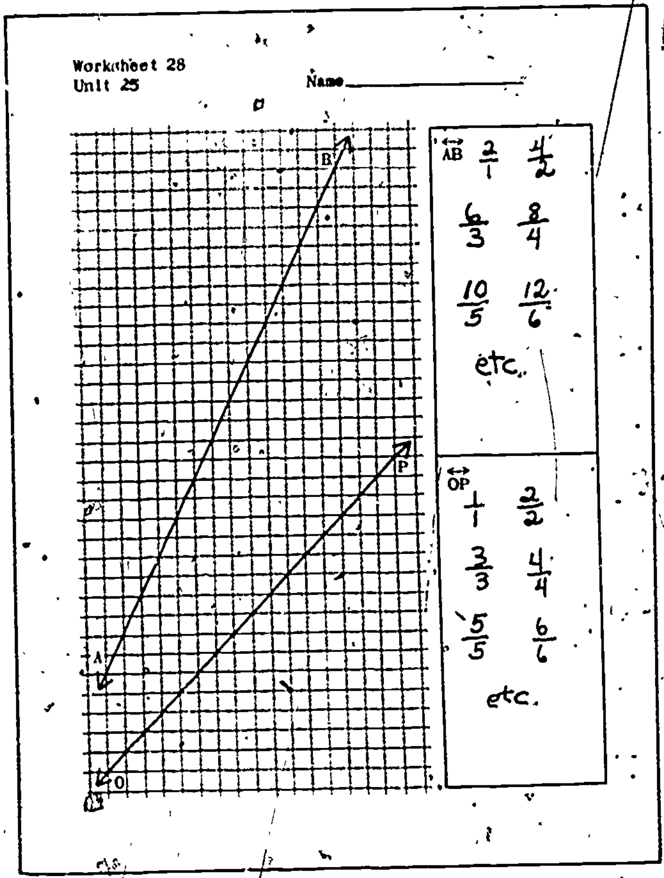
On a fresh grid, ask a child to draw a line with the slope two per one through a given point. Ask other children to draw lines with the slope four per two, six per three, and eight per four through this same point. The children will see that these are different names for the same line.

Activity C: Name Game

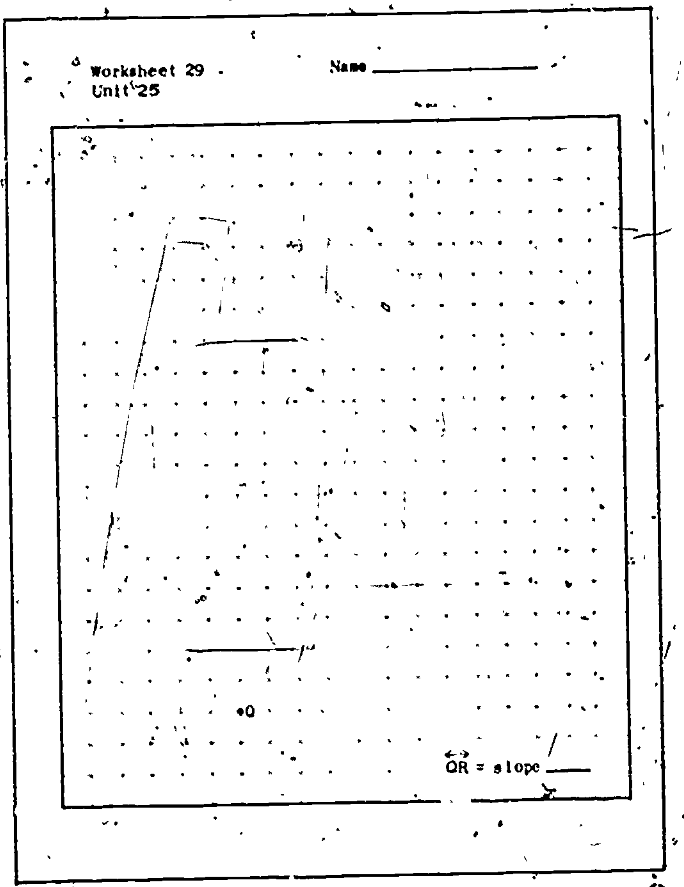
Have the children turn to Worksheet 28. The graph on this page shows line AB with a slope of two per one, and line OP with a slope of one per one. Do not tell the children these slopes. Explain that the class is going to play a game, giving different names for the slope of line AB on the worksheet.

Divide the children into two teams. Choose a scorekeeper who will write down each correct response in the column of the team that gave it. Have him make a scoresheet. One child on team 1 begins by giving a slope name for line AB. He might say "two per one." The children on team 2 decide whether or not this is correct. If the name is correct, the scorekeeper writes the name on the scoresheet under the team 1 column. The children on both teams should indicate on their worksheets that this name has been chosen. Then a child on team 2 gives a name. He might say "four per two." The scorekeeper records the response.

The teams alternate naming the slope. If a child gives an incorrect name, the other team must challenge it, and say why it is wrong. The team with the wrong response gets no credit for that response and loses its turn. The team that gets the most correct answers wins.



Not every child on each team will get a chance to give a slope name unless there are many incorrect responses. Have the teams play again, using line OP, beginning with the children who have not yet had a turn.



Activity D

Have the children form teams of three or four members each and turn to Worksheet 29. Choose a slope (____ up per one over) and write on the board three or four names for that slope. For example: $\frac{5}{1}$, $\frac{10}{2}$, $\frac{15}{3}$, $\frac{20}{4}$.

Each child on a team chooses a different slope name and records that name in the space at the bottom of his worksheet. Then he draws a line QR with the slope name he has chosen; his line must pass through point Q.

After each child on a team has drawn his line, have the children tear out their worksheets. Then the children on a team compare the lines they have drawn by holding up two worksheets at a time, one on top of the other, to a light. Tell the children to be sure the dark border of one worksheet is lined up with the border of the other worksheet; also point Q should be lined up on both worksheets.

The children should discover that each member of the team drew a line with the same slope, only the names are different. If some child's line is not the same as the others, he should check his work and re-draw the line.

Activity E

Commentary

The slopes the children have been working with so far are all reducible to a whole number of units up per one unit over (e.g., three up per one over) and can be expressed as integers (e.g., three). But there are some slopes that are named by a fraction or mixed number of units up when reduced to "___up per one over" and cannot be expressed as integers. For example, five per two is reduced to two and one-half up per one over.

It is sometimes more convenient to eliminate the fraction or mixed number by naming the slope as a certain number of units up per two (or three or four, etc.) units over. In this activity the children will work with slopes, such as five per two, but will not be asked to reduce them to, say, two and one-half up, per one over.

When the children encounter a fractional slope, they will find that if they go over one unit, they must go up a mixed number of units to return to the line. There is a way of finding another name for the slope — a name that uses whole numbers only.

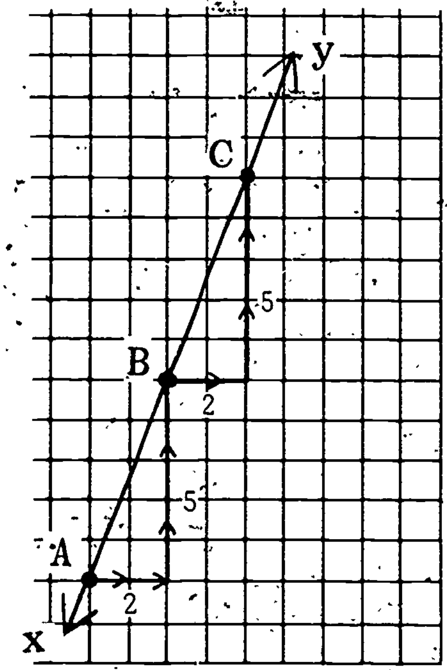
The technique is:

1. Choose a point (A) that lies on a grid intersection which line XY passes through.

2. Then find the next grid intersection that line XY passes through (point B).

3. Count the number of units over and up that it takes to get from A to B. The slope name is five up per two over.

4. Check this answer by counting the units over and up from point B to the next point that lies on a grid intersection (C). Again it is five up per two over to return to the line.



Fractional slopes (those that are not reducible to an integer) are included in this lesson so that the children will learn to name the slope of any given line and to draw a line of any given slope. No matter what slope or what name is given, the procedure of naming or drawing that slope is the same: the number of units up per the number of units over.

Class demonstration

Put a line MW with slope five per two on the overhead grid and ask the children to name the slope. Some child may say "two and one-half up per one over." This is a very good answer, but tell the children they can name this slope without using a mixed number of units up. If no child thinks of how to do it, remind them of the different names they gave for the slope of lines in the previous lesson. (Remind them that slope six per two names the same line as slope three per one.) Ask the children if this idea could be used to name the slope of line MW. (Yes, it can.)

Elicit from the children the name five per two for this line by demonstrating the procedure mentioned on the previous page. Ask the children for other names that can be expressed in whole numbers, such as ten per four and fifteen per six.

Remind the children that there are many names for the slope of a line. All these names mean the same thing; the name changes, but the slope of the line remains the same. Also remind the children that the procedure of naming slope is always the same. Slope is named as the number of units up per the number of units over.

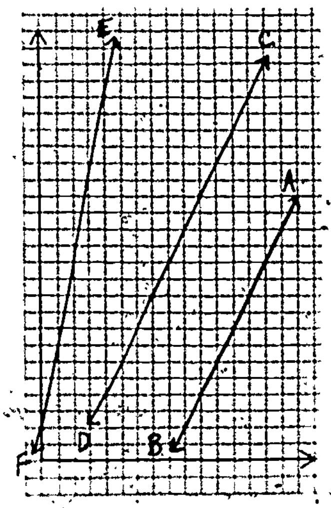
Have the children name the slope of lines such as three per two and seven per three. Encourage them to use the technique of moving from one grid intersection to the next to find a name that uses whole numbers only.

Have the children complete Worksheet 30. Ask the children what they notice about the slopes of line AB and CD. (They are the same.)

Worksheet 30
Unit 25 Name______________

Draw the slopes. Choose any starting point.

$\overleftrightarrow{AB}$ $\frac{2}{1}$ $\overleftrightarrow{CD}$ $\frac{6}{3}$ $\overleftrightarrow{EF}$ $\frac{10}{2}$



Give other names for $\overleftrightarrow{CD}$

2 per 1
4 per 2
8 per 4
10 per 5
12 per 6

Lesson 14: CAR RACES WITH MULTIPLE TIMING

The children race their cars and each team marks the position of the car at one second intervals for five seconds. The data are graphed and the children see that a constant speed produces a constant slope on the graph. A constant slope is a straight line, although all the points on the graph may not fall exactly on the line. The children compare the graphs of different teams for the same race and discover that they can tell which line has the steepest slope by examining the slope names.

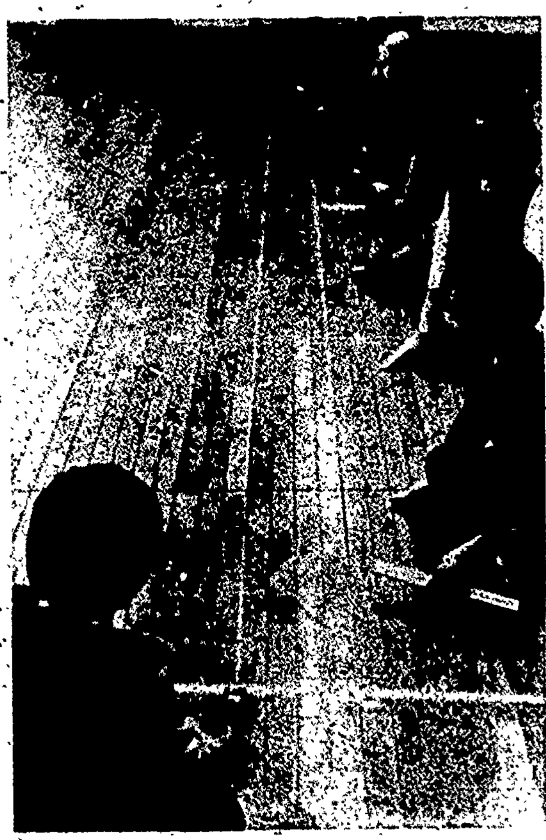
MATERIALS

-- for each team --

- racing space
- car
- 2 batteries
- 2 toothpicks
- measuring string
- guide string
- pendulum
- chair
- masking tape
- 5 marking objects
- ruler
- recording sheet

-- for each child --

- Worksheet 31



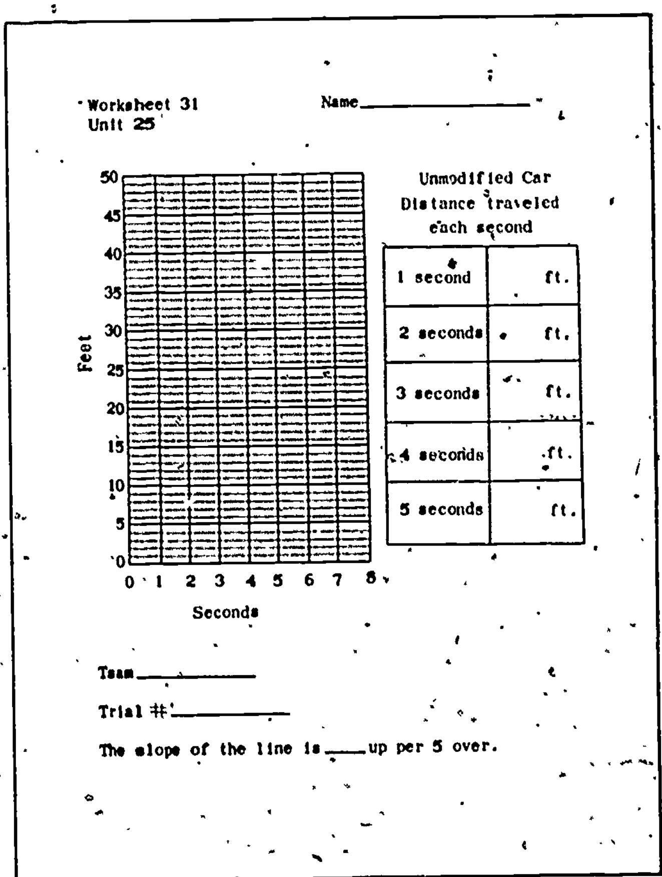
PROCEDURE

Activity A

Review the procedure that the class used in Section I when they ran foot races and marked the runner each second for five seconds. The same setup will be used to race and time the cars. Remind the children that there will now be five judges. Each marks the position of the front of the car at the appropriate count. One of the judges may have to act also as catcher to retrieve the car and carry it back to the starter. The recorder should make a recording sheet so he can write down the distance of the car at each second. When the children are familiar with the procedure, have each team set up the equipment and run three races, marking the car each second for five seconds. Be sure the recorder is measuring the distance in inches using the measuring string and a ruler. If the batteries are becoming weak, replace them but save the old ones for later lessons. Remind the children again that they must not press down on the car when the motor is on.

Activity B

Have each team divide into three groups. Have them turn to Worksheet 31. Trial #1 group plots the data from trial one, trial #2 group plots trial two, and trial #3 group plots trial three. When the five points are plotted, the child draws a straight line which goes as close as possible to all the points. Then each child determines the slope of his graph (__ up per five over) and records it on the worksheet.



Choose a graph (one that shows a constant speed) of one team and put this line on a chalkboard grid or an overhead grid. Ask the class if they can tell whether the car went the same speed each second by looking at the graph. (Yes.)

HOW FAR DID THE CAR GO DURING THE FIRST SECOND? DURING THE SECOND SECOND? DURING THE THIRD? FOURTH? FIFTH?

Have the children determine how far the car went during each second. You may have to help them do this. (If the car was at the 12-foot mark at two seconds and the 18-foot mark at three seconds, it went six feet during the third second.)

DID THE CAR COVER ABOUT THE SAME NUMBER OF FEET DURING EACH SECOND? WHAT CAN WE SAY ABOUT THE SPEED OF THE CAR? (It traveled at about the same speed.) WHAT WAS THE SPEED OF THIS CAR IN TERMS OF FEET PER SECOND?

Tell the children that when the car goes the same distance each second, we can say that it is going at a constant speed. (If the speed varied, ask the children to suggest reasons.)

WHAT IS THE CONSTANT SPEED FOR THIS RACE? IF THE CAR KEPT GOING AT THIS CONSTANT SPEED, HOW FAR WOULD IT GO AFTER SIX SECONDS?

Remind the children of the race in which they hopped for three seconds, then ran for three seconds. When they graphed this race, the slope of the line changed because the speed changed. Ask the class whether they think the graph of a car race would be a straight line if the speed of the car changed. Draw them to the conclusion that when the speed of a car is constant the graph of that race will be a straight line.

Activity C

Ask one child who plotted trial one from each team to come to the front of the room with his graph. These four children should hold up their graphs so that the children in the classroom can see them, but they should cover the slope name that is written on the worksheet. Ask the class to decide which car went fastest and which went slowest just by looking at the steepness of the graphs. Unless there was a great difference in speed among the cars, the children will discover that they cannot tell easily which car went slowest and fastest because the graphs are not all on the same grid.

Ask the children for suggestions on how to compare the cars for trial one. They may propose plotting data from trial one of all four teams on the same grid. This is a good suggestion, but there is an easier way. Someone should suggest comparing the slope name of each graph. Each child with trial one data has already determined the slope of his graph as so many units up per five units over. The slope with the largest number up per five over is the steepest slope and indicates fastest speed because that car went farthest in five seconds.

The children who plotted trial two may want to get together and determine which car went fastest in trial two. Trial three children may want to do the same. Children who plotted trial one could determine which car went second fastest.

Ask the children if they can think of ways to slow down their cars. Let them give a few suggestions, and tell them that they will be doing this in the next lesson.

Have the children turn ahead to Worksheet 44 in their student manuals. Each child should copy the data from the chart on Worksheet 31 (distance traveled each second) onto the chart on Worksheet 44. They should also indicate what trial the data is for. They will use this data in Lesson 19.

Lesson 15: MODIFIED CAR RACES

In this lesson the children develop methods to slow down their cars. They predict how far the modified cars will go in five seconds and then test their predictions. The class compares the modified speeds with the original speeds of the cars. They see that the graph of the faster speed always has a steeper slope, and that as long as the speed is constant, the graph will be a straight line.

MATERIALS

-- for each team --

- racing space
- car
- object for modifying car (2" paper clip, old battery)
- 2 penlight batteries
- 2 toothpicks
- guide string
- measuring string
- pendulum
- chair
- masking tape
- marking object
- recording sheet
- ruler

-- for the class --

- overhead projector (optional)
- eight auto cutouts
- Worksheet 32

10

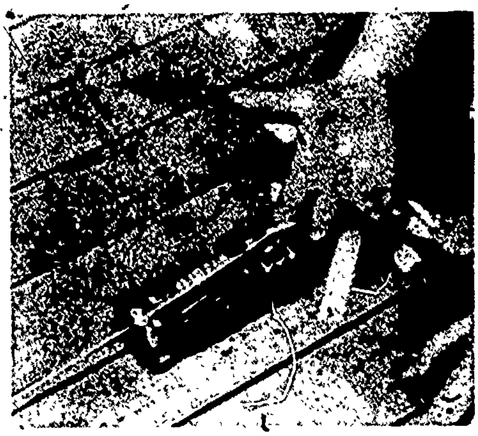
PREPARATION

Have some volunteers cut eight photos of autos from magazines. They could paste these on heavy paper so the cutouts are easier to handle. These will be used in Activity B.

PROCEDURE

Activity A

Remind the children of the problem posed in Lesson 14: Can we slow down the cars somehow? Let students suggest some methods. These might include taking one battery out of the car, using old batteries, taping weights to the car, or taping something to the car that will drag behind it. The team that takes a battery out will have to replace it with a paper clip to close the circuit. If the weights on the car stop it completely, some should be taken off immediately or the motor will burn out. If children have difficulty modifying their cars, ask questions which will help them solve the problem themselves.



Ask the class to predict how far they think the modified cars will go in five seconds. You may want to remind them of the speeds from the unmodified car races in previous lessons. Record a few of the predictions for later discussion.

Take the children to the racing area. Have the same teams work with the same cars as in the previous lessons. They should use the setup and procedures from Lesson 11. Each team runs its modified car in three trials, and each time records in feet and inches the position of the car at the five-second count. The recorder will have to use a ruler, measuring from the nearest mark on the measuring string. Remind the children to change jobs after each trial. When the races are done, gather up the equipment and return to the classroom to discuss the results. (Tell the recorders to keep the data to use in Activities B and C.)

Activity B

Draw four horizontal number lines on the chalkboard to represent the tracks of the four teams. Mark each number line in five-foot intervals. Put one team's name beside each of the lines. Have the children turn to Worksheet 32 where they have recorded the greatest distance their car went in the unmodified five-second trials. One of the members of one team should tape a car cutout to their number line so that the front of the car is at the distance the car traveled in the unmodified race. Have this team tape a second car at the farthest distance their car went in the modified trials. This datum is provided by the recorder. Have the other teams repeat this process on their number lines. When the record is complete, the children can compare their predictions for the modified cars with the results. (Have the children save Worksheet 32 for the next activity.)

Refer to the number lines on the chalkboard while the class discusses the following questions:

LOOK AT THE FASTEST CAR. HOW MUCH FARTHER DID IT GO IN THE FIRST RACE THAN IN THE MODIFIED RACE?

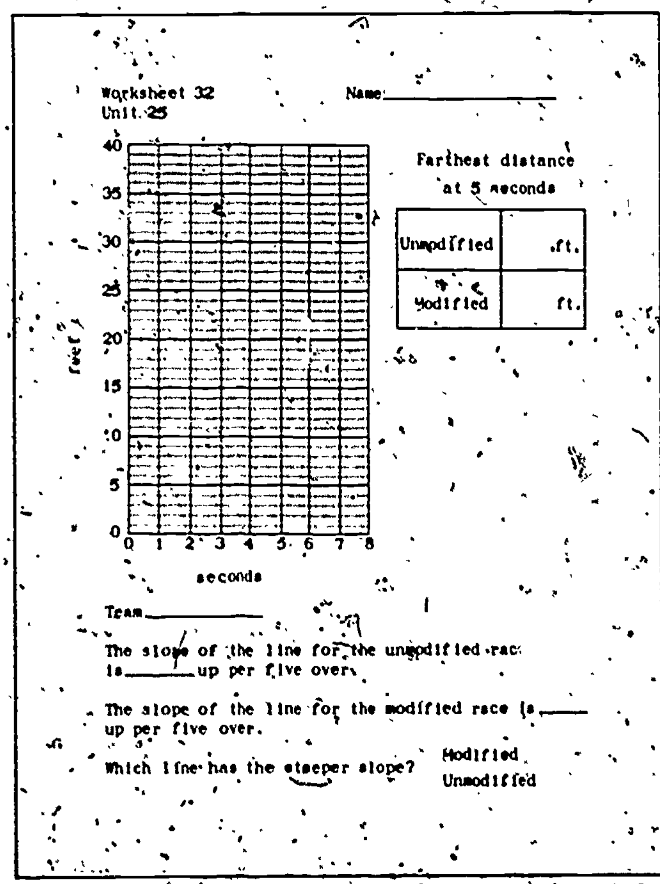
WILL THIS DIFFERENCE BE THE SAME FOR EACH OF THE

OTHER CARS? (No.) WHY? (The methods of slowing the cars were different. Not all the cars went the same speed in the first race. Also, batteries wore down and marking techniques differed.)

Let's compare the speed of one team's car in the unmodified race with the speed of that car in a modified race. Remember, each car ran for five seconds. In which race did it go farther? (The unmodified race.) In which race did it go faster? (The unmodified race.) If we graphed an unmodified race and a modified race for each of the cars, which graph would have a steeper slope? (The unmodified. If the children do not know which would be steeper, do not tell them. They graph the data in the next activity.)

Activity C

Have each team get together and turn to Worksheet 32. They will see that they have already recorded the farthest distance that their car traveled in the unmodified five-second races (in Lesson 11). Now have each team record the farthest distance their car traveled in the modified races. The recorder tells his team this information. Have the children graph the two races they have recorded.


Worksheet 32 Unit 25 Name:

Farthest distance at 5 seconds	
Unmodified	ft.
Modified	ft.

Team

The slope of the line for the unmodified race is ____ up per five over.

The slope of the line for the modified race is ____ up per five over.

Which line has the steeper slope? Modified Unmodified

Ask each team to report which graph has the steeper slope, that of the modified car or the unmodified car? The children will see that both graphs are approximately straight lines and that the unmodified race has the line with the steeper slope.

Review ideas, such as: the car that travels the farthest distance in five seconds is the faster car (the unmodified car went faster); the graph of the faster car has a steeper slope; the steeper the slope, the faster the speed; a graph of a constant speed is a straight line. Save any batteries that have worn out for Unit 26.

Lesson 16: THE MULTIPLICATION RELATION EVIDENT IN SLOPE

In this lesson the children see that slope is related to multiplication. They find that on a line with a slope that is an integer (such as three per one), for one move over, the number of moves up from the origin increases by exactly the same number of squares as the slope name (three in this example).

Since the multiplication relationship exists between the slope of a line and coordinates of points on this line, a graph of a straight line can be used as a kind of multiplication machine.

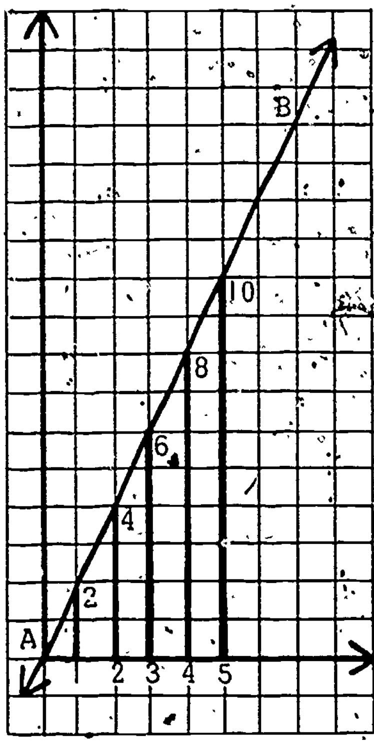
MATERIALS

- overhead projector (optional)
- Worksheet 33

PROCEDURE

Activity A

Using a chalkboard grid or the overhead projector, draw line AB with a slope of two per one through the origin (0, 0). Ask the children to name the slope of the line. Then ask someone how many units we would go up to get back to the line if we move one unit over (right) from the origin. (Two.) You or a child should sketch in these moves on the grid and label them with the number of units moved in each direction.



Now ask how many units we must go up if we move two units to the right from the origin. (Four.) Trace these moves. Continue moving one more over (three over, four over, etc.) and the necessary number of units up until your grid looks like the diagram on the previous page.

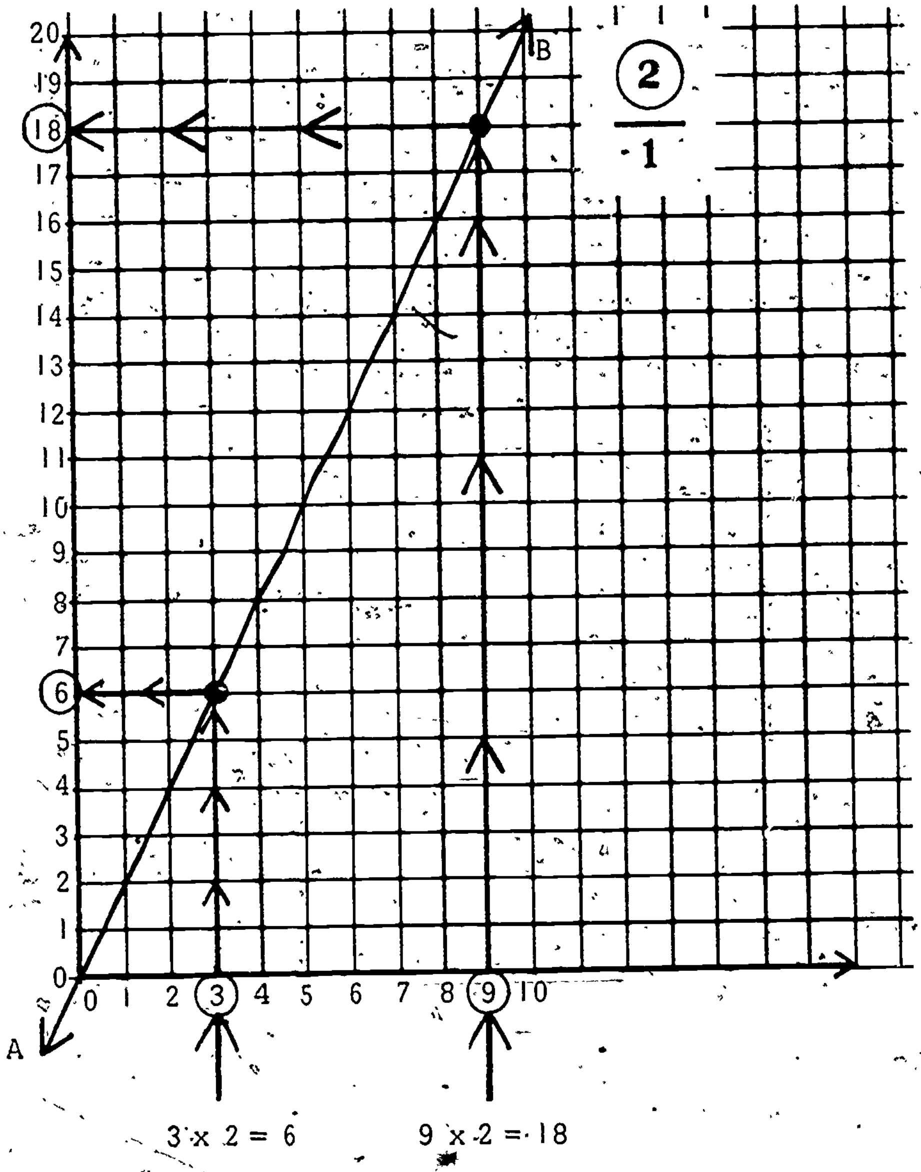
Ask the children if they see a pattern in these moves. If they do not, trace the moves as you say: "One over and two up. Two over and four up. Three over and six up. Four over and eight up, etc." Then ask the children if they see a pattern in the number of moves up for each successive move over. (Each number of units up is two larger than the previous number. Each number of moves over is one larger than the previous number.) This pattern is similar to the technique the children used in finding different names for the same slope: two per one, four per two, six per three, etc.

Repeat the procedure using a line with the slope three per one. With this slope, the pattern is an increase of three units up for each increase of one unit over: three up per one over, six up per two over, nine up per three over, etc. The increase in units up is the same as the slope name, three (per one).

Activity B

Using a grid transparency or a chalkboard grid draw a line AB passing through the origin (0, 0) with a slope of two per one. Ask the children to name the slope of the line you drew. Someone should say the slope is two up per one over. Review what this slope means. It means for every unit over, we go up two units to return to the line. In Activity A the children demonstrated graphically how, for example, a line with slope two per one increases by two units up for an increase of one unit over. Tell the children they can use a graph as a multiplication machine to help them find the product of multiplication problems. Ask if anyone knows how to use a graph to multiply.

Explain that a graph with the slope two per one can help us multiply by two. (See diagram on next page.) Circle the 2 of the slope name. If the children ask, you may want to explain the meaning of the fraction two per one. This fraction can be interpreted as two ones, which for our purposes, is the same as two.



Write 3 x 2 on the board. Ask the children if they know the product of 3 x 2. Find the 3 on the horizontal scale and demonstrate how to follow the grid line for 3 up to line AB. When you have located this point on the line, demonstrate how to follow the grid line from this point over (left) to the vertical axis, which will produce the product, 6.

Ask a child to come to the grid and multiply 9 x 2 using the graph with slope two per one.

110

Ask other children to do more examples until all children understand how to use the graph as a multiplication device. Show them how this multiplying technique is similar to what they were doing in Activity A. In the previous activity, the children saw that with a graph with slope two per one, the increase in number of units up is two when the increase in units over is one: over one, up two; over two, up four; over three, up six; etc. is similar to multiplying by two: 1 x 2 = 2, 2 x 2 = 4, 3 x 2 = 6, etc.

(In the next lesson the children will complete several worksheets of multiplication problems using graphs of slopes one per one through nine per one.)

Draw line PQ with slope three per one. Ask the children how they can use this line to multiply. (They can use it to multiply by three.) Have the children turn to Worksheet 33 and complete the problems with your help and using the line PQ you drew. Ask a child to come to the overhead projector and demonstrate how to multiply 1 x 3. Remind him to find the 1 on the horizontal scale, which will be multiplied by 3, the slope name. Have the other children enter the numerals 1 and 3 in the first two blanks on the top row of the worksheet. Have the children read this problem: "1 (over) times 3 (slope name)." The children should know the answer to this problem without using the graph, but have the child at the overhead projector demonstrate how to find the answer with the graph anyhow.

Now ask:

> IF I GO OVER TWO UNITS, HOW MANY WOULD I GO UP TO RETURN TO THE LINE? THIS IS THE SAME AS MULTIPLYING 2 X 3.

Now have the children enter 2 in the over line and 3 as the slope name on the worksheet. Ask for the answer and have a child demonstrate how to find the answer using the graph.

Go through the same procedure for 3 over (3 x 3), 4 over (4 x 3), 5 over (5 x 3), and 6 over (6 x 3).

Do other examples with lines of different slopes until the children understand how to use the graph to multiply. In the next lesson the children will complete several worksheets of multiplication problems using graphs.

Worksheet 33
Unit 25

Name____________

Slope of PQ = $\frac{3}{1}$

over		top numeral of slope name		up
1	x	3	=	3
2	x	3	=	6
3	x	3	=	9
4	x	3	=	12
5	x	3	=	15
6	x	3	=	18

Lesson 17: MULTIPLYING WITH SLOPE

In this lesson the children use their multiplication machines to find the answers to multiplication problems on several worksheets.

MATERIALS

- Multiplication machine (in student manual)
- Worksheets 34-41

PROCEDURE

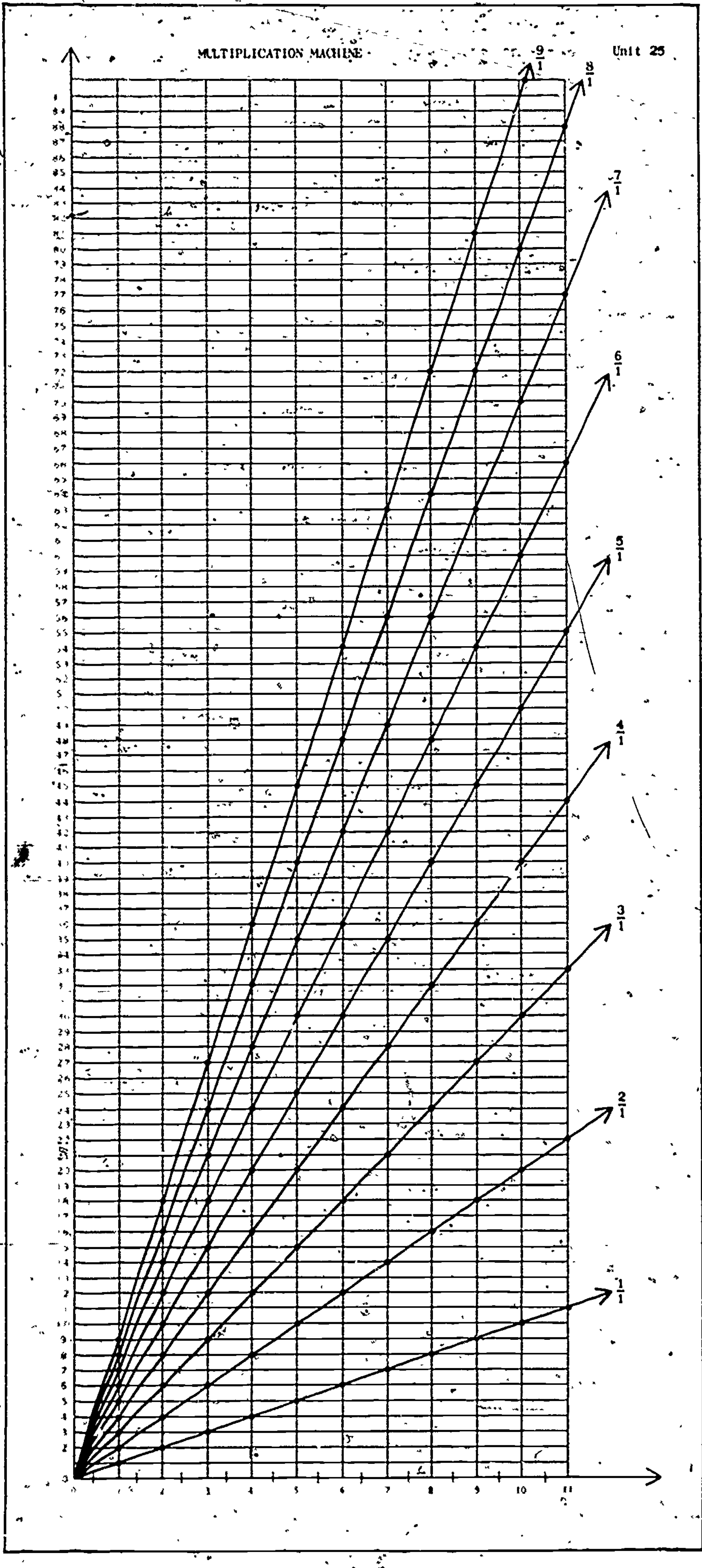
Activity A

In the back of this manual and the student manual there is a two-page foldout "multiplication machine." This machine is a graph of lines with slopes one per one through nine per one. With this chart the children can multiply any combination of numbers up to 9 x 10.

The grid units on this chart are rectangles rather than squares. For the purpose of multiplying it makes no difference what size or shape the grid units are. If the children notice this and wonder about it, explain that the shape is not important here. You should say "units" and not "squares."

Review the procedure for using this chart. If a child wants to multiply 9 x 7, he uses the slope seven (per one) line. He finds 9 on the horizontal axis, follows the grid line up to the slope seven (per one) line and then follows the vertical grid line over (left) to the vertical axis. The number there will give him the product, 63.

Have the children complete Worksheets 34 through 40 using this chart. You may want to work one problem from each worksheet to get the children started, but they should complete the worksheets independently during class. Check the children's work from time to time to make sure they are using the multiplication machine correctly. (Worksheet 41 is optional and goes with Activity B.)



100

Worksheet 34
Unit 25

Name______________

Look at your multiplication machine.

Using a line with slope three per one, you can multiply by 3.

Use your multiplication machine to work these problems:

$9 \times 3 =$ 27 $\quad 6 \times 3 =$ 18

$8 \times 3 =$ 24 $\quad 7 \times 3 =$ 21

Using a line with slope $\frac{2}{1}$, you can multiply by 2.

$7 \times 2 =$ 14 $\quad 8 \times 2 =$ 16

$9 \times 2 =$ 18 $\quad 10 \times 2 =$ 20

$4 \times 2 =$ 8 $\quad 0 \times 2 =$ 0

Worksheet 35
Unit 25

Name______________

Look at these problems: 4×6,
7×6,
9×6,
3×6.

To find the products of these problems you should use the line with the slope six per one. This will help you multiply by six.

Use your multiplication machine to do the problems below.

$4 \times 6 =$ 24 $\quad 1 \times 6 =$ 6

$7 \times 6 =$ 42 $\quad 5 \times 6 =$ 30

$9 \times 6 =$ 54 $\quad 2 \times 6 =$ 12

$3 \times 6 =$ 18 $\quad 8 \times 6 =$ 48

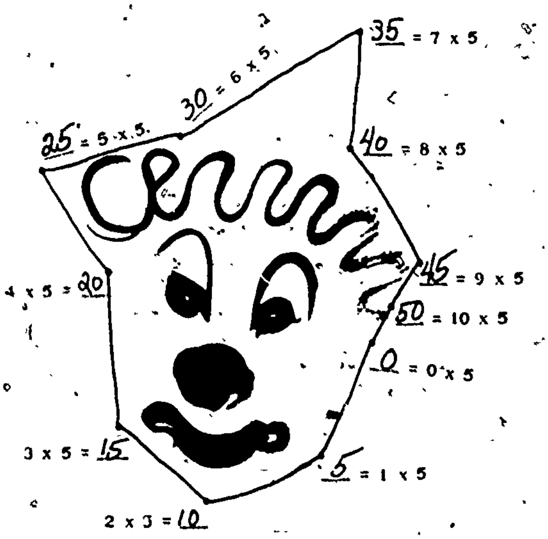
Worksheet 36
Unit 25

Name______________

Write the answer for each problem.

Each answer names a dot.

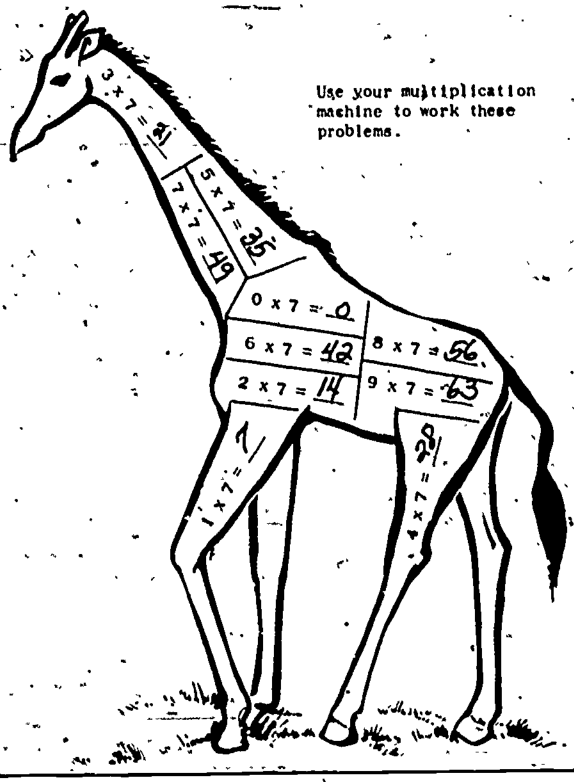
Begin at 0 and connect the dots in numerical order.



Worksheet 37
Unit 25

Name______________

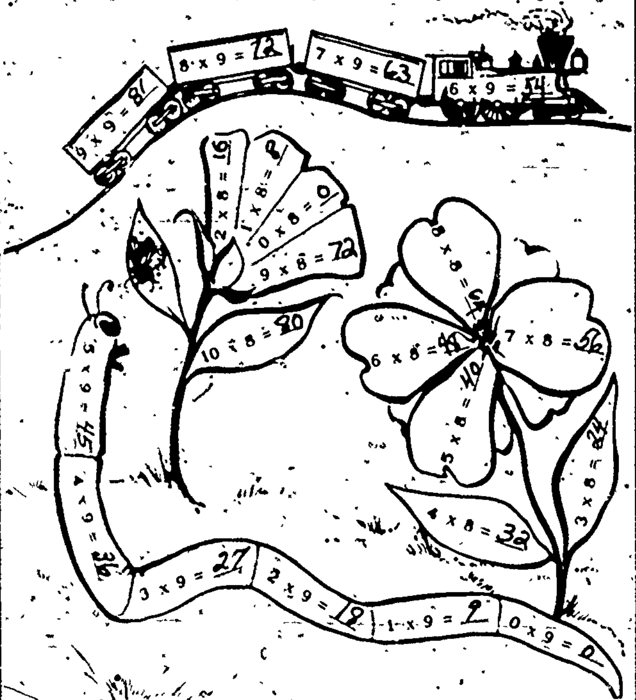
Use your multiplication machine to work these problems.



Worksheet 38
Unit 25

Name________________

Use your multiplication machine to work these problems.



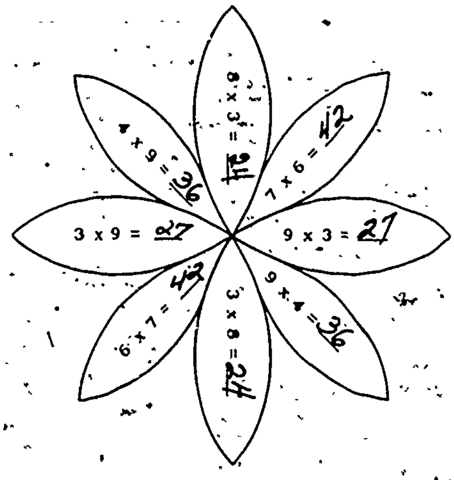
Worksheet 39
Unit 25

Name________________

Work the problems.

Color the pattern using this code:

42 = blue 36 = green
27 = yellow 24 = red



Worksheet 40
Unit 25

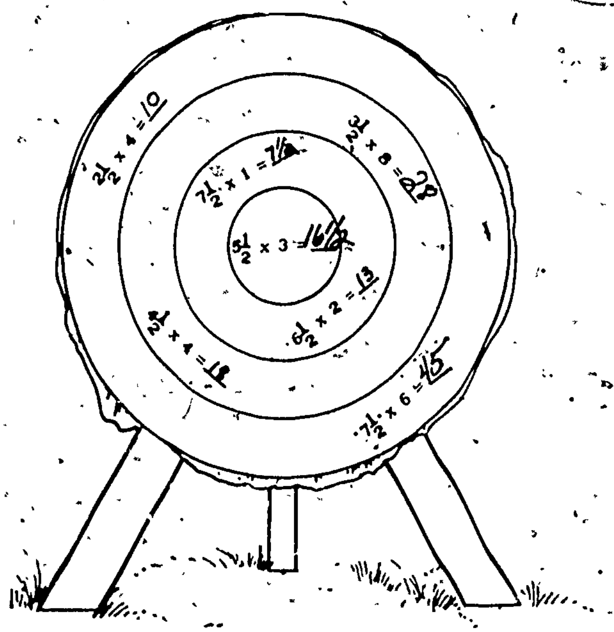
Name ________________

10 x 3 = 30	6 x 6 = 36
7 x 5 = 35	3 x 4 = 12
8 x 8 = 64	8 x 9 = 72
9 x 9 = 81	7 x 7 = 49
3 x 9 = 27	7 x 6 = 42
5 x 4 = 20	3 x 8 = 24
0 x 5 = 0	1 x 7 = 7
5 x 9 = 45	4 x 7 = 28

Worksheet 41
Unit 25

Name________________

Use your multiplication machine to solve these problems.



Activity B (Optional)

In this activity you demonstrate how to multiply by mixed numbers using the multiplication machine. Multiplication involving fractions is usually not introduced in third grade, but is provided here for those children who show special interest and ability in mathematics.

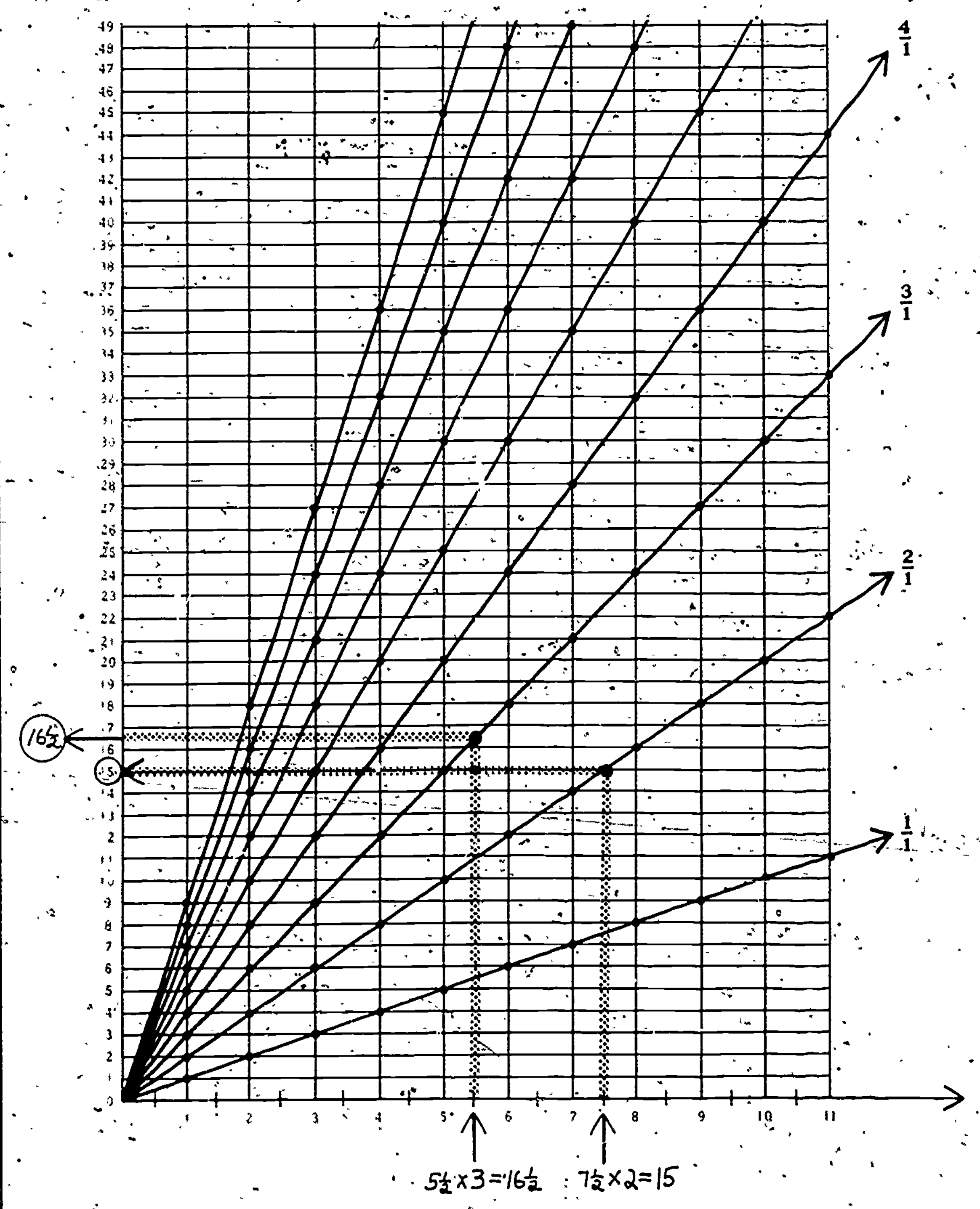
The multiplication machine can be used to multiply a whole number by a mixed number in the same way it is used to multiply two whole numbers. (See the diagram on the next page.)

For a problem such as $7\frac{1}{2} \times 2$, you use the line with slope two per one. Find $7\frac{1}{2}$ on the horizontal axis of the grid. Follow the $7\frac{1}{2}$ position up to the slope two per one line. Then follow the grid line over (left) to the vertical axis to find the product, 15.

Multiplying mixed numbers is a little more difficult with the machine because often there are no grid lines to follow for the halves. Show the children how to use a ruler to find the position of, say, $5\frac{1}{2}$ on the slope three per one line.

Some multiplication problems involving mixed numbers result in a product which is also a mixed number. An example is $5\frac{1}{2} \times 3$. The procedure for finding the product is the same as for multiplying whole numbers. Locate $5\frac{1}{2}$ on the horizontal scale. Follow the $5\frac{1}{2}$ position up to the slope three per one line. (This point is between two grid lines.) Follow this point over (left) to the horizontal axis. The position on the horizontal axis is between 16 and 17; the product is $16\frac{1}{2}$.

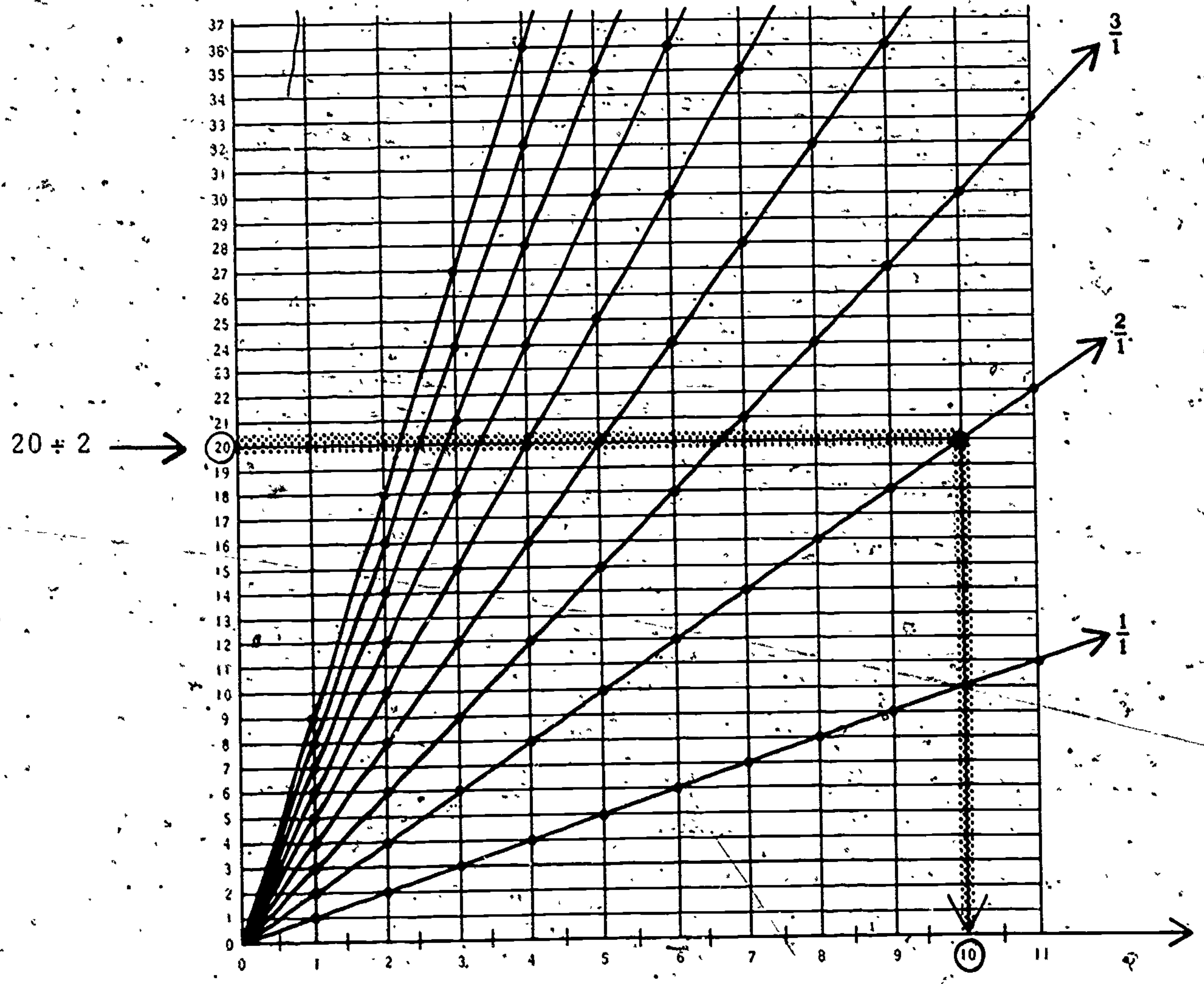
Worksheet 41 is an optional worksheet for those children who would benefit from an introduction to multiplying mixed numbers. It includes only two problems whose products are not whole numbers.


4/1
3/1
2/1
1/1
16½
15
5½ x 3 = 16½
7½ x 2 = 15

Activity C (Optional)

In this activity the children learn how to use the "multiplication machine" to work division problems. You may want to try a few division problems with the whole class or with just those children who are ready for division. You may choose to use this activity some time later in the school year. Encourage the children to save the multiplication charts to use all year long.

Using the lines of different slopes, the children can divide any number on the vertical axis by any slope name and find the answer on the horizontal axis. The procedure is just the opposite of that used for multiplication. Choose problems with answers that are whole numbers or mixed numbers involving one-half. The diagram below illustrates how to find the quotient of 20 ÷ 2.



Lesson 18: CIRCULAR MOTION

The children race their cars on a circular track for one revolution. They are asked to determine whether the car goes faster when it is modified or unmodified. Since one revolution is a fixed distance, the children must record the time duration of one revolution to determine in which trial the car went faster. The time is the variable in this race in contrast to previous lessons in which time was constant (five seconds) and the distance varied.

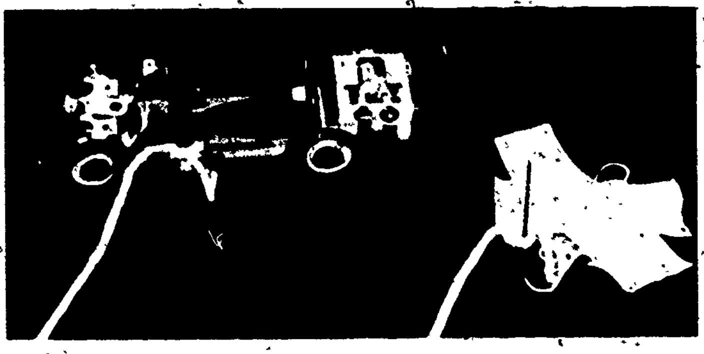
MATERIALS

-- for each group --

- racing space
- car (and equipment to modify car)
- 2 two-inch paper clips
- 28-inch string
- masking tape
- pendulum
- Worksheets 42 and 43

PREPARATION

Bend one end of a two-inch paper clip so that it is perpendicular to the rest of the clip. Tie a loop in one end of the string. Tie or tape the other end of the string to the side of the car as shown and tape it toward the front of the car when you tape down the batteries. Tape the paper clip firmly to the floor and slip the loop end of the string over the extended end of the paper clip. The string must be 24 inches from the car to the clip.

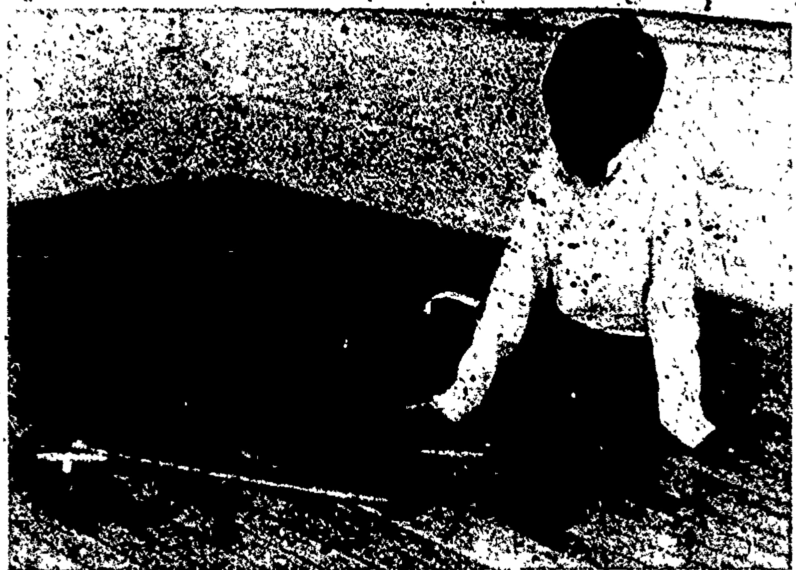


PROCEDURE

Tell the children that they are going to experiment with circular motion. Explain what is meant by a circular motion race. The other races have been linear — the cars and the children ran in a straight line, starting at one place and ending at another. In this race the car will travel in a circle, and will end up in the same place it started. Explain that the teams will run their races on a circular track.

Show the children the new setup, explaining how to tape the paper clip to the floor and loop the string over the clip. In this race there will be a timer, starter; recorder and one judge. (The children will have to take turns doing the jobs.)

Emphasize how this race differs from previous races. The car will run for a fixed distance — one revolution of the track (12½ feet). The judge will record the number of counts (seconds) it takes the car to travel one revolution. The judge must listen closely to the timer and note at what count the front of the car returns to the starting line. He tells the recorder this datum.



Worksheet 42
Unit 25

Name ____________

Circular Motion Record Sheet

Unmodified	Wheels Straight
Trial 1	seconds
Trial 2	seconds

Unmodified	Wheels Turned In or Out
Trial 1	seconds
Trial 2	seconds

Modified	Wheels Straight
Trial 1	seconds
Trial 2	seconds

Worksheet 42 is the record sheet. The recorder should bring the worksheet to the racing area. Each team runs three races, two trials of each race. The first race is with an unmodified car with its wheels straight. The second race is the unmodified car with the wheels turned either in or out. Have two teams do it one way and two teams do it the other. Make sure each recorder indicates on the record sheet whether the race was done with the wheels in or out. The third race is with a modified car with straight wheels. This information is on the worksheet. Before going to the racing area show the children how to turn the wheels on the car. If the wheels of your car cannot be turned, omit this race.

Before the children run the races you should discuss the first two points below. When the children return from the races, again discuss the first two points and also the third.

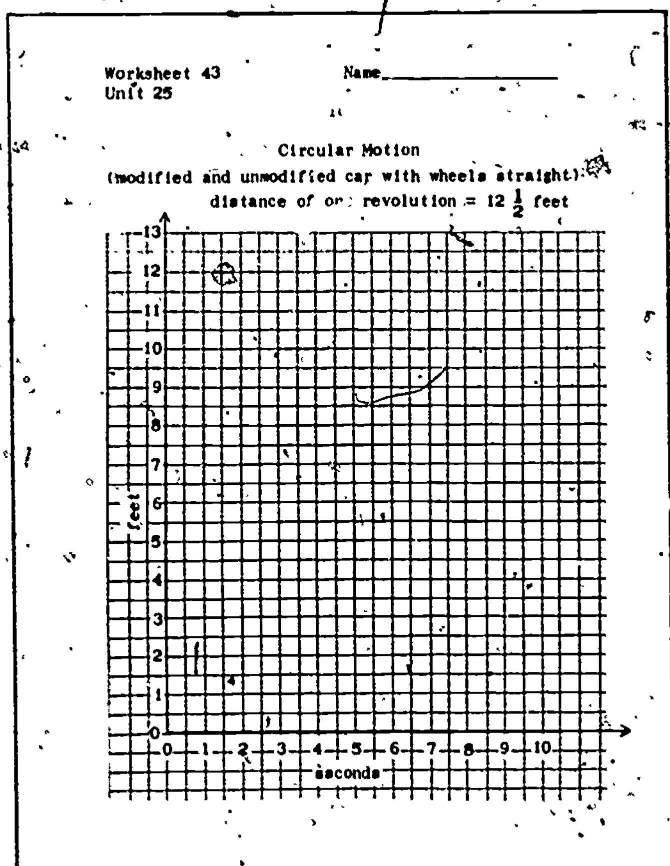
1. How long will it (did it) take for the unmodified car to go around the track once? When the car is modified how long will it (did it) take? In order to determine the speed of the cars, the children will have to find a way to determine how long it takes the car to make one revolution. Lead them to see that this is done by constructing a starting line, starting the car at that point, and noting at what count the front of the car reaches the starting line after going one revolution.

2. What happens to the car when the wheels are turned in toward the center? What happens when they are turned out away from the center? When they are straight? Does the speed change? What reasons can you think of

for what you have observed? If the wheels on your cars do not turn you can eliminate this discussion or let the children speculate about what might have happened.

3. If we plot the data for the two races with the wheels straight, which graph would have a steeper slope, the one for the modified race or for the unmodified race? The car went the same distance (one revolution equals $12\frac{1}{2}$ feet) each time.

Have the children test their predictions by graphing the data for the races (straight wheels) on Worksheet 43.


Worksheet 43 Name________________
Unit 25

Circular Motion
(modified and unmodified car with wheels straight)
distance of one revolution = $12\frac{1}{2}$ feet

Lesson 19: PREDICTING RACE RESULTS

In this lesson the children use their racing experiences to help them solve prediction problems. In previous linear races the children measured how many feet they or the car went in a fixed amount of time. Now they will be asked to examine the data of previous races (especially the data of the unmodified car with multiple timing), and predict how long it will take a car to travel a fixed distance, 25 feet. Some children might realize that if their car went 15 feet in five seconds, it should go 25 feet in about eight seconds since it traveled at a rate of three feet per second. From the children's predictions and the discussion of results, you should be able to evaluate what the children have learned about the relation between distance, time and speed.

MATERIALS

-- for each team --

- racing space
- car
- measuring string
- guide string
- masking tape
- pendulum
- recording sheet
- Worksheet 44

PROCEDURE

Discuss with the children the idea that one way of determining the speed of the car is to race it for a fixed amount of time as they have done in previous lessons. Ask them how they knew which car went fastest. Answers include: the one that went the farthest; the one that had the graph with the steepest slope.

Now ask the children how they would determine the fastest car if they raced each car for 25 feet — a fixed distance rather than a fixed time. Give the example of runners racing to a

12

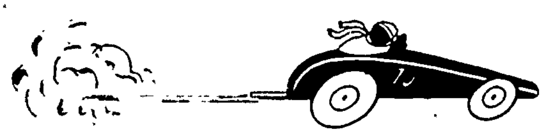
finishing line (a fixed distance). The children should know from experience that whoever got to the line first (in the shortest time duration) ran the fastest.

Worksheet 44
Unit 25

Name ________________

Team:	distance traveled				
multiple timed race with unmodified car	1 sec	2 sec	3 sec	4 sec	5 sec
trial #					

I predict that the car will take ☐ seconds to go 25 feet.



Have each team sit together. The children should turn to Worksheet 44 in their student manuals. Ask them to look at the data for the multiple-timed unmodified race which is recorded there. Can they use this data to predict how long it would take their car to travel 25 feet? (If any car went 25 feet in five seconds in previous lessons, you may want to change the distance.) Do not tell the children how to predict, but allow them plenty of time to discuss the problem. Have each child record his prediction on the worksheet.

Review the jobs of team members. In this race, the judge will sit by the 25-foot mark and take note of the count when the car reaches that mark. The recorder enters the number of seconds for each trial on his record

	Time duration for a 25-foot distance
Trial 1	seconds
Trial 2	seconds
Trial 3	seconds

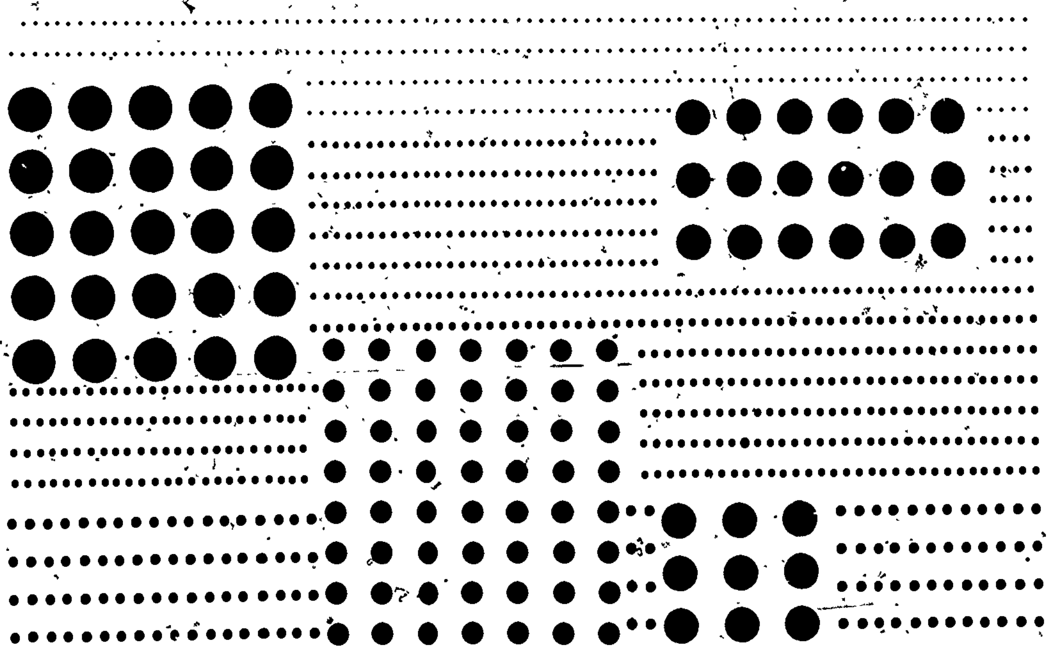
sheet. Have each recorder make a record sheet like the one on the previous page. Have the teams set up the equipment and run three 25-foot trials.

When the races are finished, have the children compare their predictions with the results. Ask those children who predicted most accurately to tell the class how they went about making their predictions. Lead the children to see the relationship between how far a car traveled in five seconds and how long it took a car to go 25 feet. If some team correctly predicted a time duration according to the data, but the results did not bear out the prediction, ask them what might have happened. (The timing, starting or judging procedures might not have been accurate; the car batteries may have worn down since the previous race, etc.) Encourage the children to make other predictions such as how long the car would take to go 30, 40 or 50 feet.

SECTION 3

Multiplication

SECTION 3 MULTIPLICATION



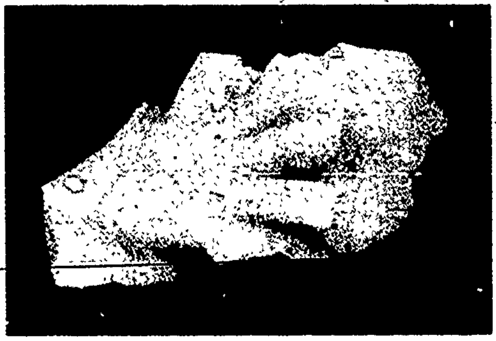
PURPOSE

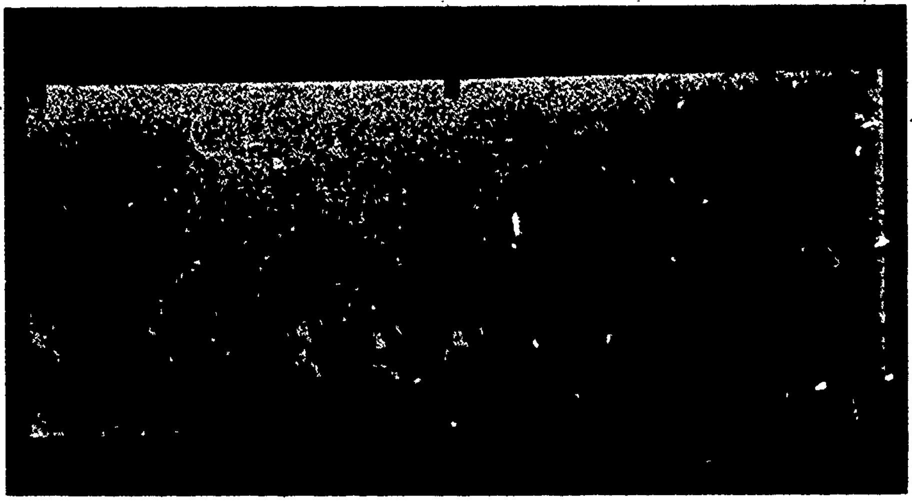
- To review repeated addition and arrays as embodiments of multiplication.
- To introduce the relationship of Cartesian products to multiplication.
- To provide practice with the multiplication facts in game situations.

COMMENTARY

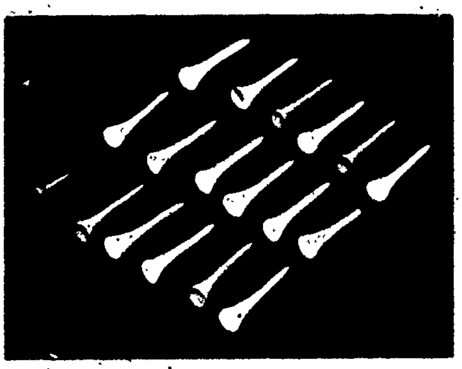
The last section of this unit reviews and introduces embodiments of multiplication other than slope. In Sections 1 and 2 the children studied motion, and then the multiplication relation evident in a constant speed represented by a straight line on a graph. In this section the children will study multiplication relations evident in repeated addition, arrays and Cartesian products. Games are provided to give practice with the multiplication facts.

In Lesson 20 the children review repeated addition as it relates to multiplication. The children print "trees" with potatoes and tempera paint. Each tree has five branches, illustrating how repeated addition of fives is similar to multiplication by five. Worksheets and folding number lines illustrate the similarity of repeated addition and multiplication.





Lesson 21 reviews arrays and how repeated addition can be illustrated by an array of a certain number of sets each with the same number of members. For example, 3 · 6 can be found by adding 6 + 6 + 6 and can be illustrated by drawing an array with three rows and six objects in each row.



In Lesson 22 the children learn some card games that review repeated addition, use arrays and also give practice with multiplication facts. The children should play these games as much as possible.

Lesson 23 introduces the Cartesian product as the set of all ordered pairs that can be made by pairing every member of one set with every member of another set. Cartesian products are related to multiplication in that the total number of members in the Cartesian product of two sets is the same as the product of the number of members in the first set and the number of members in the second set. Worksheets illustrate situations where finding a Cartesian product is desirable. The children will work more extensively with arrays and Cartesian products in Unit 27.

Lesson 24 introduces dice games that provide more practice with multiplication facts. The dice games (and the card games) should be played often because they provide a fun way for the children to learn the multiplication facts.

This section of the unit is intended to show the children embodiments of multiplication other than slope: repeated addition, arrays and Cartesian products. The children should see how these embodiments are similar to the multiplication relation evident in slope, and that there are many methods of finding the product of a multiplication sentence: a line with a certain slope on a graph, repeated addition, drawing arrays, etc. The children will realize that it is much easier to memorize the basic multiplication facts rather than drawing an array or adding repeatedly, but they should be aware of these devices and their value in finding the product of a multiplication fact that has been forgotten. The games provide an excellent opportunity for the children to memorize the single-digit multiplication facts.

Lesson 20: REPEATED ADDITION

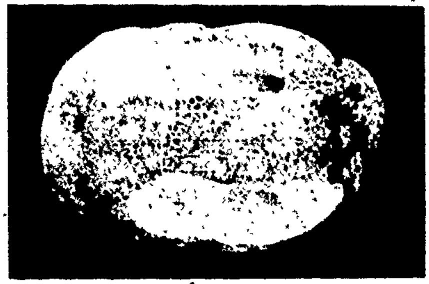
This lesson reviews the idea that multiplication is much like repeated addition. The class prints trees with five branches and fills out a repeated addition-multiplication chart, making a new entry as each new tree is printed. Worksheets reinforce the relation between repeated addition sentences and multiplication sentences. Then the children use the printed trees to fill out a parallel number line chart.

MATERIALS

- potato
- knife
- thick tempera paint
- scrap paper
- sheet of paper at least 2 feet long
- masking tape
- felt tip pen
- Worksheets 45–47

PREPARATION

Cut a potato in half. Cut a "tree" with five branches in one half of the potato as shown. The potato will be easy to hold if you cut two grooves in the back for a handle.



Tape a strip of paper about two feet long to the chalkboard, and next to it draw a chart like the one on the next page.

Number of Branches on Each Tree		
Number of trees	Total number of branches	Multiplication sentence
0		
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		

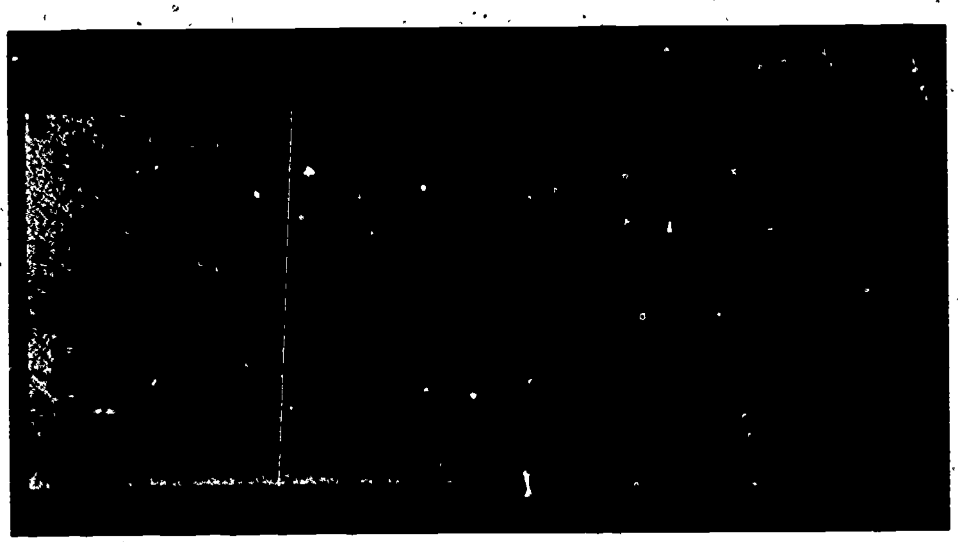
PROCEDURE

Activity A

Gather the children in front of the chalkboard. Using a piece of scrap paper taped to the chalkboard, show them how to print with the five-branch potato by dipping it in the paint, placing it on the paper, pressing hard, and carefully lifting it off.

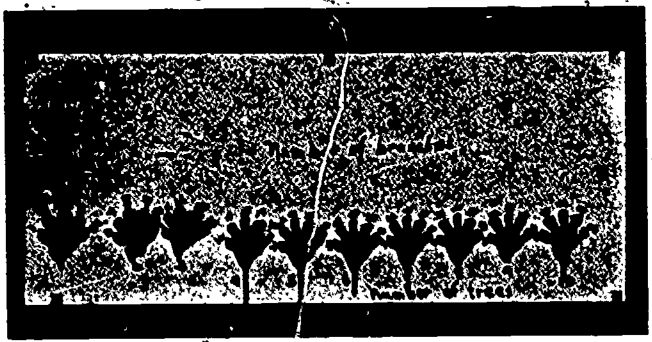
Explain that the class is going to print five-branch trees, and write in "5" at the top of the chart. Pointing to the zero in the number of trees column, ask the children how many branches there are on zero trees on the strip of paper. Put a zero in the number of branches column on the chart. In the column for the multiplication sentence, write the numerals in the appropriate boxes: 0 x 5 = 0. Discuss the meaning of the multiplication sentence.

Now have a child print the five-branch potato at the left end of the paper strip. Another child should number each branch and number the tree using a felt tip pen.



Have the children help you fill in the row on the chart for one tree. They can look at the potato print for information.

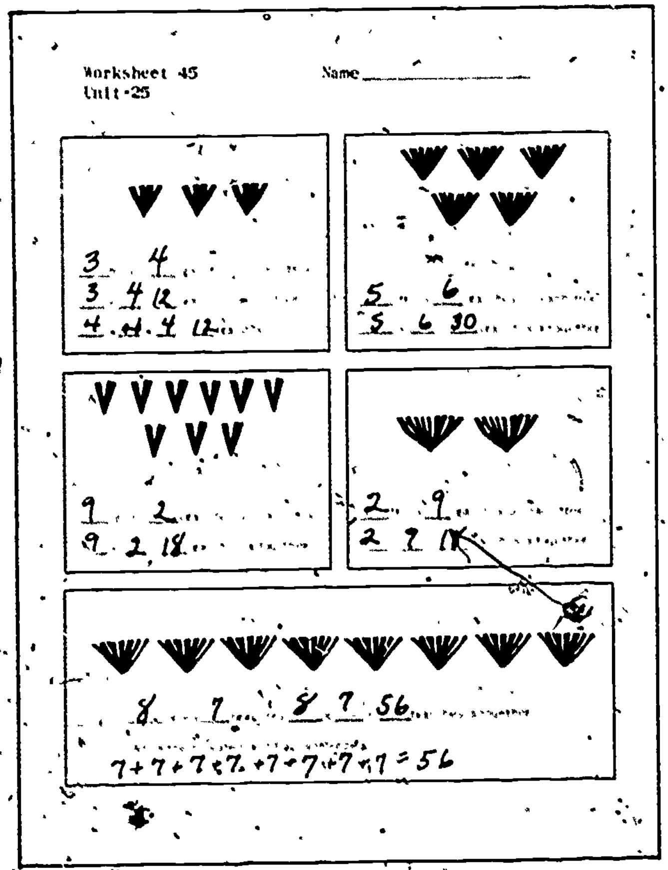
Ask a child to print a second five-branch tree next to the first tree. A child should number these branches consecutively, and number the tree. Fill in the chart for two trees. Remind the class that 5 + 5 = 10 is a repeated addition sentence. Follow this procedure discussing the addition and multiplication sentences each time, until the children have printed 10 trees. Label the top of the tree strip "total number of branches," and the bottom "number of trees."

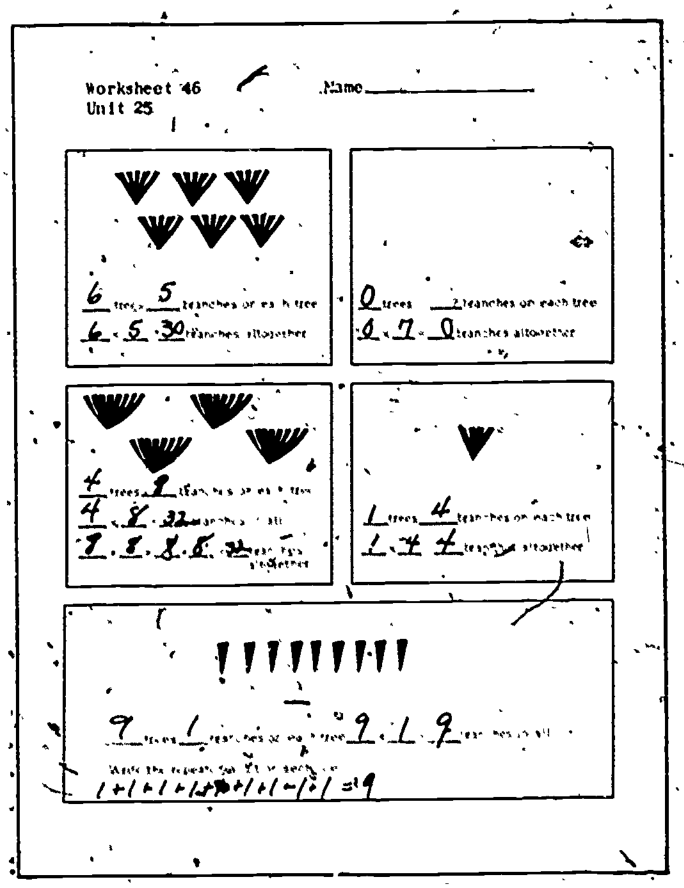


The completed chart will look like this.

Number of Trees	Number of Branches on Each Tree 5 Total number of branches	Multiplication Sentence
0	0	0x5=0
1	5	1x5=5
2	5+5	2x5=10
3	5+5+5	3x5=15
4	5+5+5+5	4x5=20
5	5+5+5+5+5	5x5=25
6	5+5+5+5+5+5	6x5=30
7	5+5+5+5+5+5+5	7x5=35
8	5+5+5+5+5+5+5+5	8x5=40
9	5+5+5+5+5+5+5+5+5	9x5=45
10	5+5+5+5+5+5+5+5+5+5	10x5=50

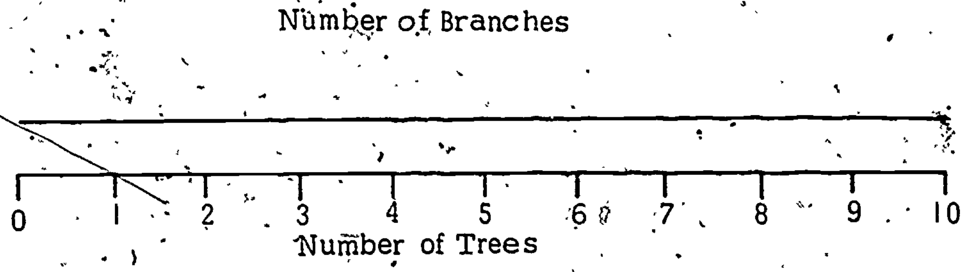
Have the children turn to Worksheet 45. This worksheet presents sets of trees with a different number of branches on the trees in each set. The children are asked to give the number of branches on each tree, the number of trees, and the multiplication sentence that the trees represent. Work the first problem with the children; then let them complete the worksheet individually. They can then go on to Worksheet 46. The second problem on this page has no trees in the box. This is an instance of multiplying by zero. If the children are puzzled, the class may want to discuss this problem.





Activity B

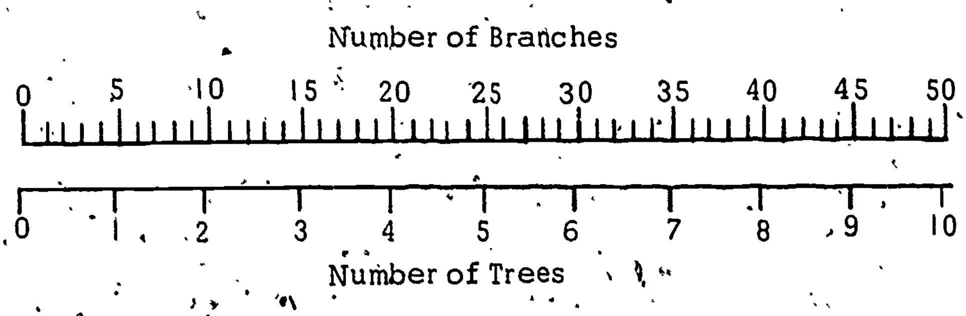
Put the chart of printed five-branch trees in a place where the children can see it. Draw parallel number lines on the chalkboard and label the bottom number line "Number of trees." This line should be labeled 0-10 at rather wide intervals. The top line should be labeled "Number of branches."



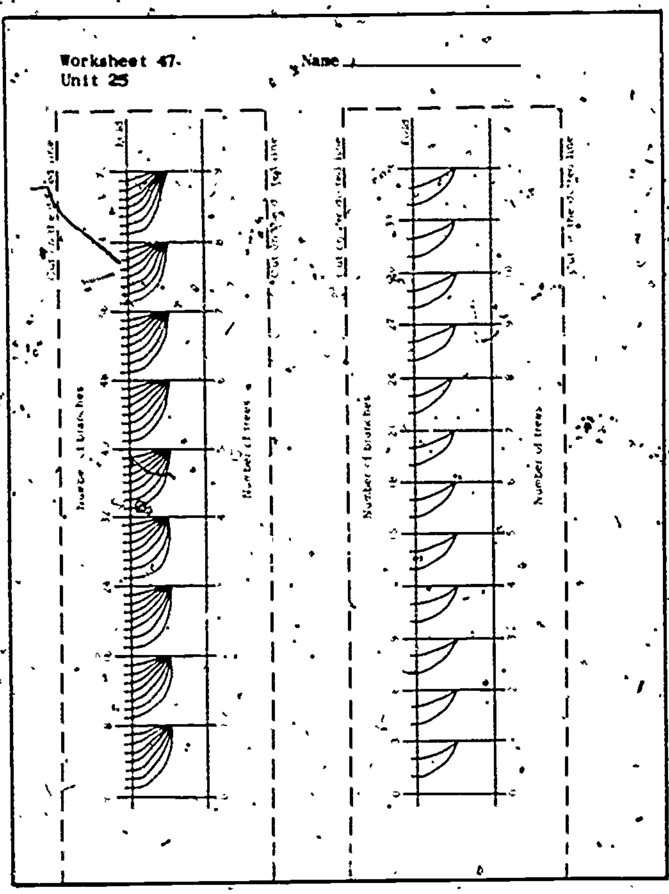
Ask the children to help you fill in the top number line. For zero trees, there are zero branches. For one tree, there are five branches. Put a five on the top number line above the one on the bottom number line. You may want to draw slashes for each of the branches between zero and five. Continue filling in the top number line with the children. When the tenth set of branches has been filled in, ask the children if the chart could be extended.

> IF THERE WERE ELEVEN TREES, HOW MANY BRANCHES WOULD THERE BE? (55.)

The completed number lines will look like this:

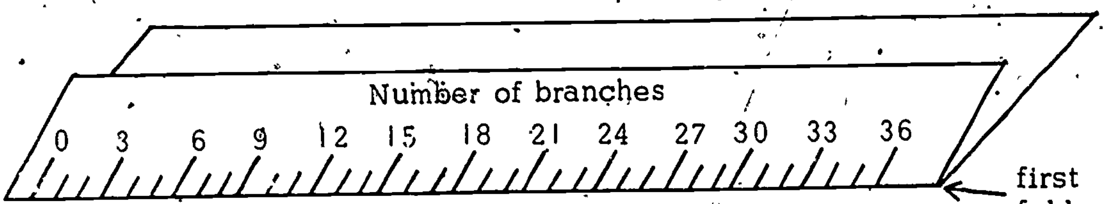


Discuss with the children the fact that we can use repeated addition to multiply, and that this can be shown with parallel number lines. When comparing this method to using the multiplication chart, the children should find that repeated addition is rather slow.

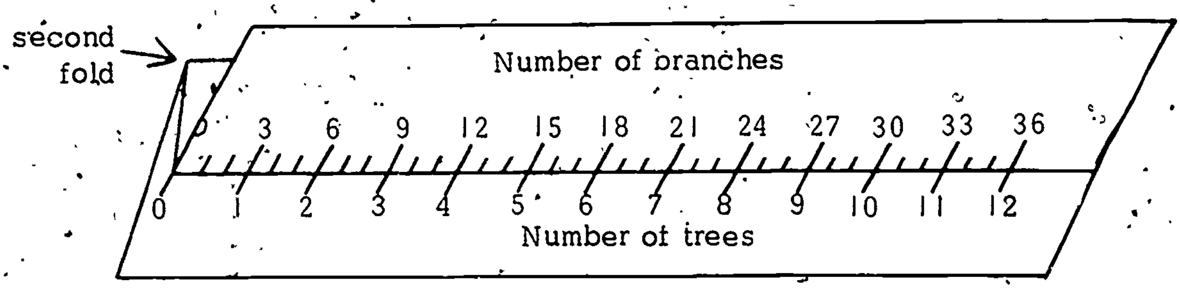


Activity C

Have the children turn to Worksheet 47 and cut out the tree diagrams along the dotted lines. Ask them to look at the diagram with three branches on each tree. The trunks are numbered consecutively along the bottom number line, and the branches are numbered consecutively along the top number line. Show the children how to fold back the paper along the top number line as shown.



Then show them how to bring the top number line down to the bottom number line. Have them crease the paper to make it easier to handle.



Now only the tips of the branches and the bottoms of the trunks show along one number line. It is easy to see that on three trees there are nine branches, on ten trees there are 30 branches, etc.

Have the children cut and fold the eight-branch tree diagram. Discuss it with the class, and let them use it to find the number of branches on various numbers of trees.

Lesson 21: REVIEWING ARRAYS

One method of finding the product of two real numbers is by interpreting multiplication as repeated addition. For example, $3 \cdot 4$ can be found by adding $4 + 4 + 4$. Repeated addition can be thought of as the combination of equivalent sets (in this example, combining three sets with four members each). When multiplication is thought of in this way, it can be given a simple physical representation by building an array.

The children were introduced to arrays in Unit 17, Introducing Multiplication and Division. This lesson reviews the arrays as one multiplication algorithm. This review will prepare the children for the extensive use of arrays in Unit 27, Numbers and Their Properties.

MATERIALS

- 20 counters for each child (toothpicks, bits of paper, golf tees, etc.)
- Worksheets 48-52

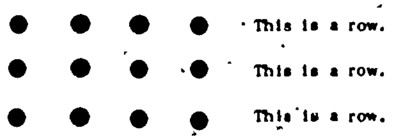
PROCEDURE

Activity A

Have the children turn to Worksheet 48. Read the worksheet with them and have each child build an array with his counters so that he gets the physical experience of building an array. Discuss any of the questions that the children have trouble with. Stress the fact that in an array, each row has the same number of objects. When the children have finished the worksheet, put a simple array on the chalkboard. Have the children determine the number of dots in each row, and the number of dots in the whole array. Then have them do Worksheet 49.

Worksheet 48 Name________________
Unit 25

This is an array. Arrays have rows.



Rows go across.

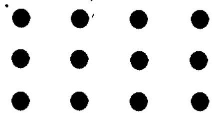
In this array, there are 3 rows.

Make a different array with your counters.

In my array, there are ______ rows.

Worksheet 48 (Cont.)
Unit 25

In this array, each row has some objects in it.



Row 1 has 4 dots.
Row 2 has 4 dots.
Row 3 has 4 dots.

Does each row have the same number of objects in it?
Yes No

In the array above, there are 3 rows with 4 dots in each row.

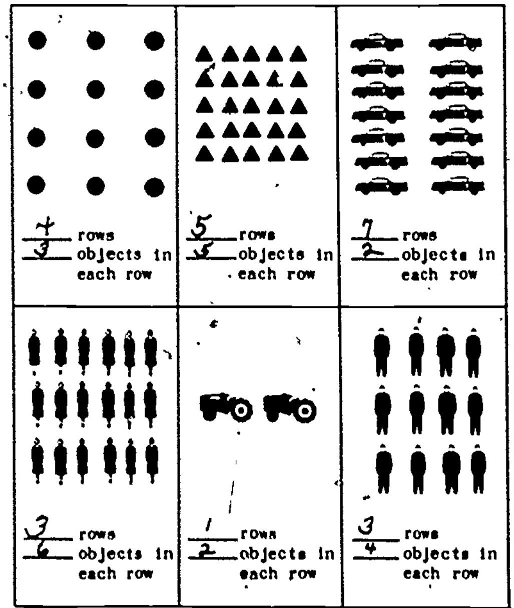
There are 12 dots in all.

In my array, there are ______ rows with ______ objects in each row.

There are ______ objects in all.

Worksheet 49 Name________________
Unit 25

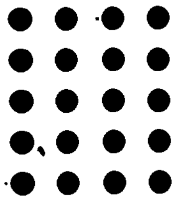
These are arrays. Fill in the blanks.



Activity B

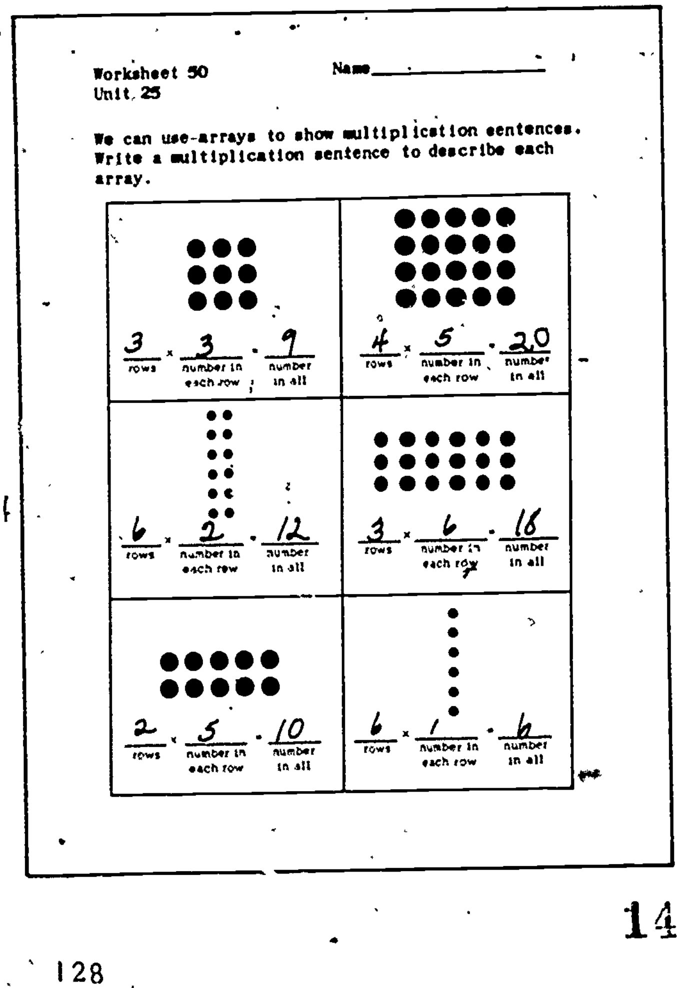
Put the multiplication problem 5 x 4 on the board. Ask the children if they could make an array to find the product of these two numbers. Explain that the first number of the problem represents the number of rows in the array and that the second number of the problem represents the number of objects in each row. As children suggest arrays, put them on the chalkboard and ask a child to find the number of objects in all. He may do this by counting each dot. Suggest that there is a faster way. Since there are the same number of objects in each row, we could use repeated addition to find the product.

Write the repeated addition sentence next to the correct array. Your work should look like this:



$4 + 4 + 4 + 4 + 4 = 20$

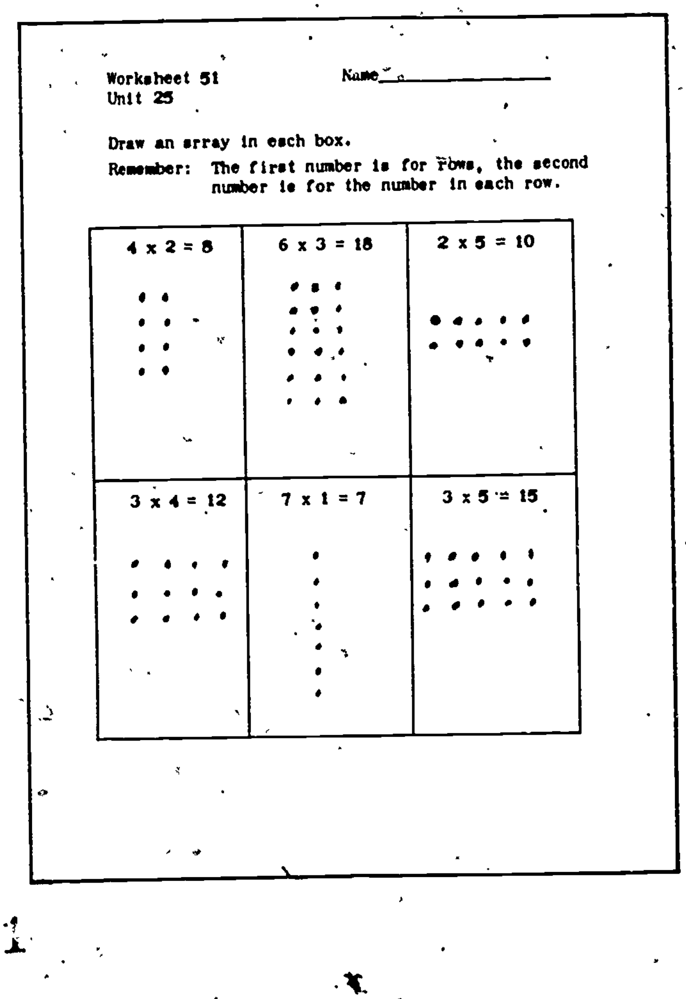
Put a new array on the chalkboard and have the class give you the multiplication sentence that describes it. Remind them that the first number in the sentence describes the number of rows and the second number describes the number of objects in each row. Have the children complete Worksheets 50 and 51. You may want to help them with the first problem on each worksheet.



Worksheet 50 Name__________
Unit 25

We can use arrays to show multiplication sentences. Write a multiplication sentence to describe each array.

3 (rows) × 3 (number in each row) = 9 (number in all)	4 (rows) × 5 (number in each row) = 20 (number in all)
6 (rows) × 2 (number in each row) = 12 (number in all)	3 (rows) × 6 (number in each row) = 18 (number in all)
2 (rows) × 5 (number in each row) = 10 (number in all)	6 (rows) × 1 (number in each row) = 6 (number in all)



Worksheet 51 Name__________
Unit 25

Draw an array in each box.

Remember: The first number is for rows, the second number is for the number in each row.

4 x 2 = 8	6 x 3 = 18	2 x 5 = 10
3 x 4 = 12	7 x 1 = 7	3 x 5 = 15

141

Activity C

Put a simple multiplication sentence, for example, 3 x 2, on the board. Explain to the children that instead of using the x for a times sign, it is often better to use a dot. Erase the x from your problem and replace it with a dot, $3 \cdot 2$. Tell them that from now on, they will be using a dot instead of an x in multiplication sentences.

Read through Worksheet 52 with the class. The children should discover that they can find the product of the two numbers by drawing arrays, though they will find that this is a slower method than using their multiplication machines.

Worksheet 52 Name________________
Unit 25

Multiply. Draw arrays only if you need to.

$7 \cdot 4 =$ 28	$9 \cdot 3 =$ 27
$6 \cdot 8 =$ 48	$7 \cdot 5 =$ 35
$4 \cdot 6 =$ 24	$8 \cdot 3 =$ 24

Lesson 22: MULTIPLICATION CARD GAMES

In this lesson the children will learn several card games that provide practice with repeated addition, arrays and multiplication. The card games are played with special cards that are included in the back of the student manual and teacher manual. Each child will have his own five decks in case he wants to take some home. Each deck has a distinguishing mark in the lower right corner so that the decks can be sorted if they get mixed up. Each deck consists of 30 cards with repeated addition sentences, multiplication sentences and arrays that match. The children will learn the games in this lesson, and then should play as often as possible to provide practice with multiplication facts.

MATERIALS

-- for each team --

- 1 deck of cards
- pencil and paper
- space to play (table, floor, or desks)

-- for each child --

- 5 decks of cards (provided in student manual)
- 5 rubber bands
- envelope

PREPARATION

Before class begins cut up the five decks of cards in this manual and the five decks from one student manual. It will save time to cut these on the paper cutter. Ten decks of cards will be enough for this lesson. The other children can cut up their cards during free time or at home. The children should put a rubber band around each deck and keep the five decks in an envelope. The children may want to put their initials on the back of their own cards. Your five decks and at least five other decks should remain in the classroom. Each child may want to take a couple of his own decks home.

PROCEDURE

These games involve matching multiplication cards with the correct repeated addition and array cards. For example, in the deck marked with triangles, there is a $5 \cdot 6$ multiplication card. The matching repeated addition card has on it: $6 + 6 + 6 + 6 + 6$. The matching array card has an array of dots with five rows and six dots in each row. In each deck there is a bonus card that matches a multiplication card in place of the repeated addition card. These bonus cards are for multiplication cards that involve one times some number. For example, there is no repeated addition sentence for $1 \cdot 7$. Therefore the bonus card repeats the multiplication card and is to be used in place of the repeated addition card. The mathematical reason for this is that there are no repeated addition sentences that illustrate multiplication by zero or one.

Introduce the games in any way you wish. You may want to teach one game at a time to the entire class, or you may want to start different teams on different games and then let the children teach each other.

Multiplication Rummy

Two, three or four children play this game with one deck of cards. Each player is dealt five cards. The rest of the deck is turned face down and used as a draw pile. The dealer turns the top card face up to start the discard pile. The object of the game is to be the first player to get rid of all his cards by laying down matching pairs and triples (of multiplication, repeated addition and array cards).

The player to the left of the dealer begins the game by drawing a card from either the draw pile or the discard pile. He lays down (face up in front of him) any pairs or triples he can make, and discards one card. The next player draws a card from either pile, lays down any pairs or triples, and discards one card. If any player has the third card to a pair some other player has put down, the player with the third card puts it down (face up) by his own cards. He can do this only during his turn.

If any player thinks another player has not matched the cards properly, he should challenge that player at the time the pair or triple is placed on the table. The child who matched the cards must explain why he thinks he is right. If he is wrong, he picks them up and loses his turn.

When the draw pile is gone, the dealer shuffles the discard pile and makes it a draw pile. He turns over one card to start the new discard pile. The first child to lay down all his cards is the winner. It does not matter how many cards the winner has matched, but only that he got rid of all his cards first.

Multiplication Match

Three children play this game. The dealer deals out all cards from one deck. The children arrange their cards face up on the table in front of them so that all players can see all cards. The object of the game is to be the first player to match up (in pairs or triples) all his cards.

To begin the game, each child finds all the pairs or triples in the cards he has been dealt. He stacks each pair or triple in front of him face up so that one card from each match shows. Each child should keep his matched cards separated from his unmatched cards, but all cards should be kept in view. When the children have made as many pairs and triples as they can find, the dealer begins playing by drawing one unmatched card from the player on his left and putting it with his own cards. Since the object of the game is to match up all unmatched cards, he should pick a card that will either make a pair with one of his unmatched cards or a triple with one of his matched pairs. If he finds a matching card, he stacks the pair face up or, if it is the third card, puts it face up with the matched pair.

The children continue drawing one card, always from the player on the left, until one child has matched all his cards in pairs or triples. Since each child must draw a card each turn, he should always try to draw one that can either be paired with one he has or matched as the third card to a pair he already has. The children will be able to see every player's cards and will know which ones they want. Since they can only draw from the person on the left, sometimes they may have to wait several rounds to get the card they need. The first child to match all his cards in pairs or triples is the winner. It does not matter how many pairs or triples he matched, only that he has no unmatched cards left.

Keep the Card

Three, five or six children can play this game. All cards are face down in one pile in the middle of the table. The object of this game is to get the most cards.

One child begins by drawing the top card from the pile. He examines the card, shows it to the other players and then gives either 1) the product if he drew a multiplication card, 2) the sum if he drew a repeated addition card, or 3) the multiplication sentence for an array if he drew an array card. If necessary, the children can use paper and pencil, counters, multiplication machine, etc. to help figure out the answers, but the children may want to set a time limit on how long each player gets to figure out his answer.

The players can challenge any answer they think is wrong. The challenger must say, "I think you are wrong. The correct answer is ______." When an answer is challenged both the player and challenger must try to prove his own answer by drawing an array, writing a repeated addition sentence, or using the multiplication machine. If the player's answer was correct, he keeps the card. If the challenger was right, he gets the card. If both the challenger and player were wrong, the card goes back into the deck on the bottom of the pile.

The next player draws a card, examines it, shows it to the other players and gives the correct answer. Again, his answer can be challenged. If he gave the correct response, he keeps the card. Then it is the next player's turn. The game continues in this way until there are no more cards left. Then each player counts up his cards. The winner is the player with the most cards, indicating he gave the most correct responses.

In case of a tie, each player involved in the tie tries to match his cards into pairs and triples. Each player gets two points for each pair and three points for each triple. The child with the most points, wins the tie. This method of breaking a tie depends mostly on luck rather than skill, so you may just want to have a game end as a tie.

Lesson 23: CARTESIAN PRODUCTS

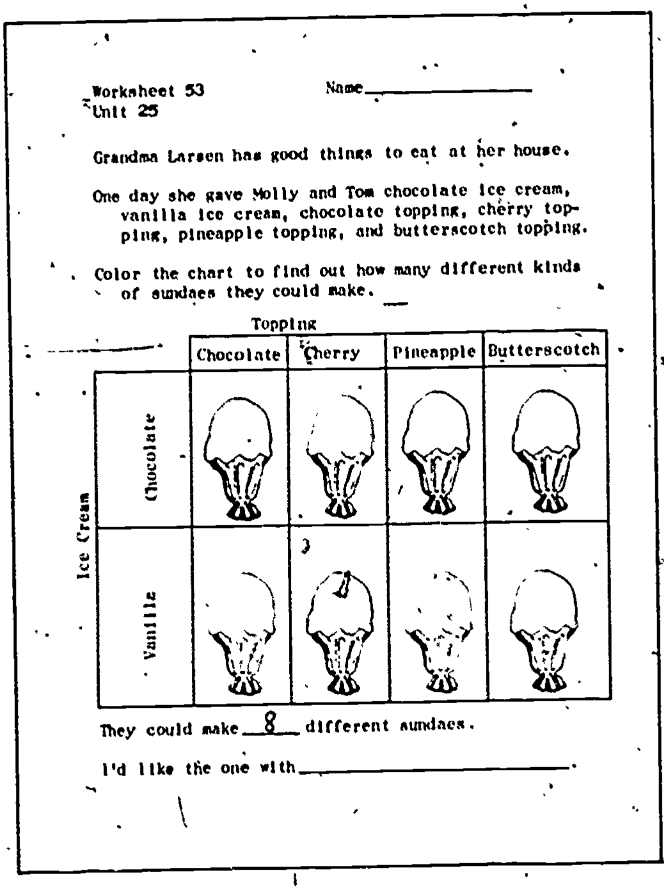
A Cartesian product is the set of all ordered pairs that can be made by combining every member of one set with every member of another set. Cartesian products are related to multiplication in that the total number of members in the Cartesian product of two sets is the same as the number of members in the first set times the number of members in the second set.

In this lesson the children work with two simple situations in which finding the Cartesian product is desirable. Though this process is discussed, you need not introduce the term at this time. This lesson prepares the children for work with Cartesian products in Unit 27.

MATERIALS

- crayons
- Worksheets 53-55

PROCEDURE



Worksheet 53 Name____________
Unit 25

Grandma Larsen has good things to eat at her house.

One day she gave Molly and Tom chocolate ice cream, vanilla ice cream, chocolate topping, cherry topping, pineapple topping, and butterscotch topping.

Color the chart to find out how many different kinds of sundaes they could make. ___

	Topping: Chocolate	Cherry	Pineapple	Butterscotch
Ice Cream: Chocolate				
Ice Cream: Vanilla				

They could make 8 different sundaes.

I'd like the one with____________.

Have the children turn to Worksheet 53. Read through the top with them and let the children complete it individually. When they have finished, ask them if the chart on the worksheet reminds them of an array.

> HOW IS IT LIKE AN ARRAY? (It has rows, and each row has the same number of objects as the other rows.) HOW MANY ROWS ARE THERE? (Two.) HOW MANY THINGS IN EACH ROW? (Four.)

Have the class determine the multiplication sentence that describes this array.

Read Worksheet 54 with the class. Have the children suggest a way to show the Cartesian product of the two sets. Draw the chart on the chalkboard and have the children suggest and demonstrate how to fill it in. The Cartesian product here is the number of members in the set of plants (two) times the number of members in the set of conditions (three). The product tells how many tests would have to be made to test every condition on each plant.

During the discussion of how to fill in the chart, you should bring out the fact that with this chart, the class can tell whether or not every set of variables has been tested. In order for a scientist to be confident of his results, he has to experiment with every possible combination of variables. When the children have completed the chart so that every combination has been made, each child can copy it onto his worksheet. Then read through the top of Worksheet 55 with the class and have the children complete it individually or in pairs.

Worksheet 54 Name ____________
Unit 25

In Unit 23, you tested two kinds of plants to see how some conditions affected them.

The set of plants had two members. These were:

1 member grass

1 member radish

2 members

You used three different conditions.

The members of this set were:

1 member moisture

1 member temperature

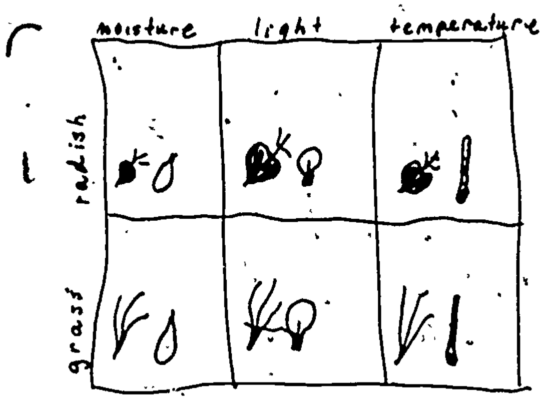
1 member light

3 members

Worksheet 54 (continued)

Draw a chart to show how many tests you did in all.

(Hint: It could be like the one on Worksheet 53.)



We made 6 tests in all.

Worksheet 55 Name______________

Unit 25

After you tested the plants, you tested three different kinds of animals to see how some conditions affected them.

This set of animals had three members. These were:

1 member	mealworm beetle	
1 member	sowbug	
+ 1 member	earthworm	
3 members		

The set of conditions you tested were the same as before. The members of this set were:

1 member	moisture	
1 member	temperature	
+ 1 member	light	
3 members		

Worksheet 55 (continued)

Draw a chart to show how many tests you made in all.

	beetle	sowbug	earthworm
light	BL	SL	eL
temperature	BT	ST	eT
moisture	Bm	BS	BE

We made 9 tests in all.

Lesson 24: MULTIPLICATION DICE GAMES

The games in this lesson provide more practice with multiplication facts. The children should be encouraged to play these dice games often. Teachers who do not have the dice from the third-grade kit, or children who want to play at home, can modify these games and use regular dice.

MATERIALS

-- for each team --

- two dice (one numbered zero through five and one numbered four through nine)
- pencils and paper

-- for each child --

- Worksheets 56 and 57

PROCEDURE

Game I Math Madness

Divide the children into groups of three, four or five. Each child should have a pencil and paper. Give each team a low-numbered die. The object of the game is to gain the most points before an agreed upon time has elapsed or to be the first to reach a predetermined score. You or the children can decide which way to play.

To begin the game, one child calls out a number from one through nine. The child on his left rolls the die, multiplies the number on the die times the number the first child called out, and tells the group the product. If his answer is correct he gets a point. One child can be scorekeeper or each child can keep his own score. The children can check the product using a multiplication machine, arrays, counters, pencil and paper, etc. If any player thinks an answer is wrong, he challenges it and shows the correct answer. If the challenger is correct, he gets the point.

151

Now the child who rolled the die calls out a number. The player to his left rolls the die and multiplies the number on it by the number the previous child called out. He tells the group the product and gets one point if his answer is correct. The game continues in this way. The winner is the child who gets the most points before an agreed upon time has elapsed or who reaches a predetermined score.

Variations

1. When the children are able to call out the products rapidly and accurately, they can use the second die (four through nine) which will generate larger numbers.

2. The children can use both dice and multiply the number on one times the number on the other. Each child writes down the product he generated on his first turn. After his second turn, he writes down the product and adds it to the first product. After each turn he adds the product to his previous total. If a child does not multiply correctly (and is challenged by an opponent), he does not get to add anything to his running total. The child who reaches a score of, say, 200 first, is the winner.

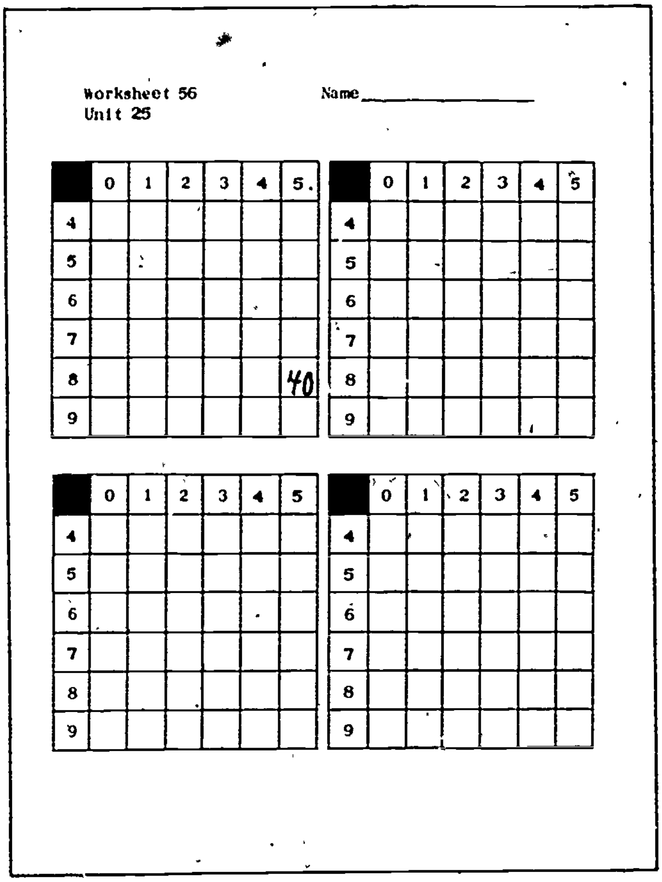
3. The children can use both dice. The first child calls out "add" or "multiply." The second child rolls both dice and does what the first child told him to do. One point is given for each correct answer.

4. The game can be played with two-digit numbers generated by multiplying the roll of one die times the roll of the other die. That product is recorded on a piece of paper. Then one die is rolled again, and the first product is multiplied by this number. A child gets one point if he correctly multiplies the first two numbers and one more point if he multiplies correctly the second time.

Game 2 Dice Boxes

For this game the children use two dice and the scorecards on Worksheet 56. Each child can use one scorecard or several children can play on one card, initialling the boxes they fill in.

To begin the game the first child chooses a number along either the horizontal or vertical axis of the scorecard. If he chooses a number from zero through five along the horizontal axis, he rolls the high-numbered die (and vice versa). For example, he may choose 5 along the horizontal axis, and then roll an 8 on the high-numbered die. He multiplies $5 \cdot 8$ and fills in the product, 40, in the correct box of his scorecard. (If several children are playing on one card, he initials the box.)



Worksheet 56
Unit 25

Name________________

	0	1	2	3	4	5
4						
5						
6						
7						
8						40
9						

	0	1	2	3	4	5
4						
5						
6						
7						
8						
9						

	0	1	2	3	4	5
4						
5						
6						
7						
8						
9						

	0	1	2	3	4	5
4						
5						
6						
7						
8						
9						

Then the next child chooses a number along one axis. (If several children are playing on the same card, he cannot choose a number that has already been taken.) Then he rolls the correct die, multiplies and fills in the product.

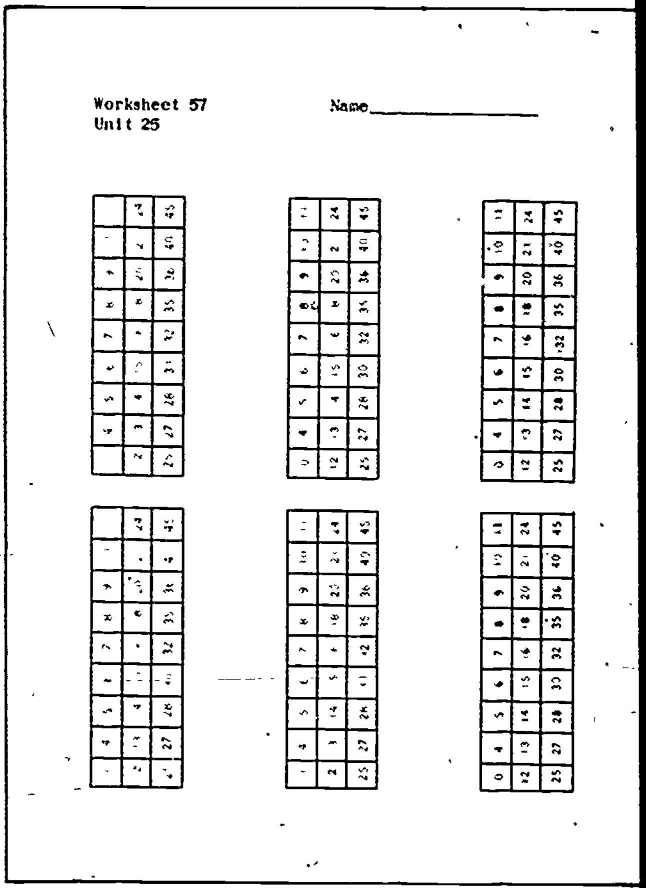
The game continues in this way until one player has filled in his whole card. (If several children are using one card, the game continues until it is filled in. The winner is the child who filled in the most boxes.)

A child could play this game by himself, keeping track of the number of turns it takes him to fill in the whole card.

Game 3 Dice Choice

For this game the children use both dice and the scorecards on Worksheet 57. Any number of children can play, each using his own scorecard.

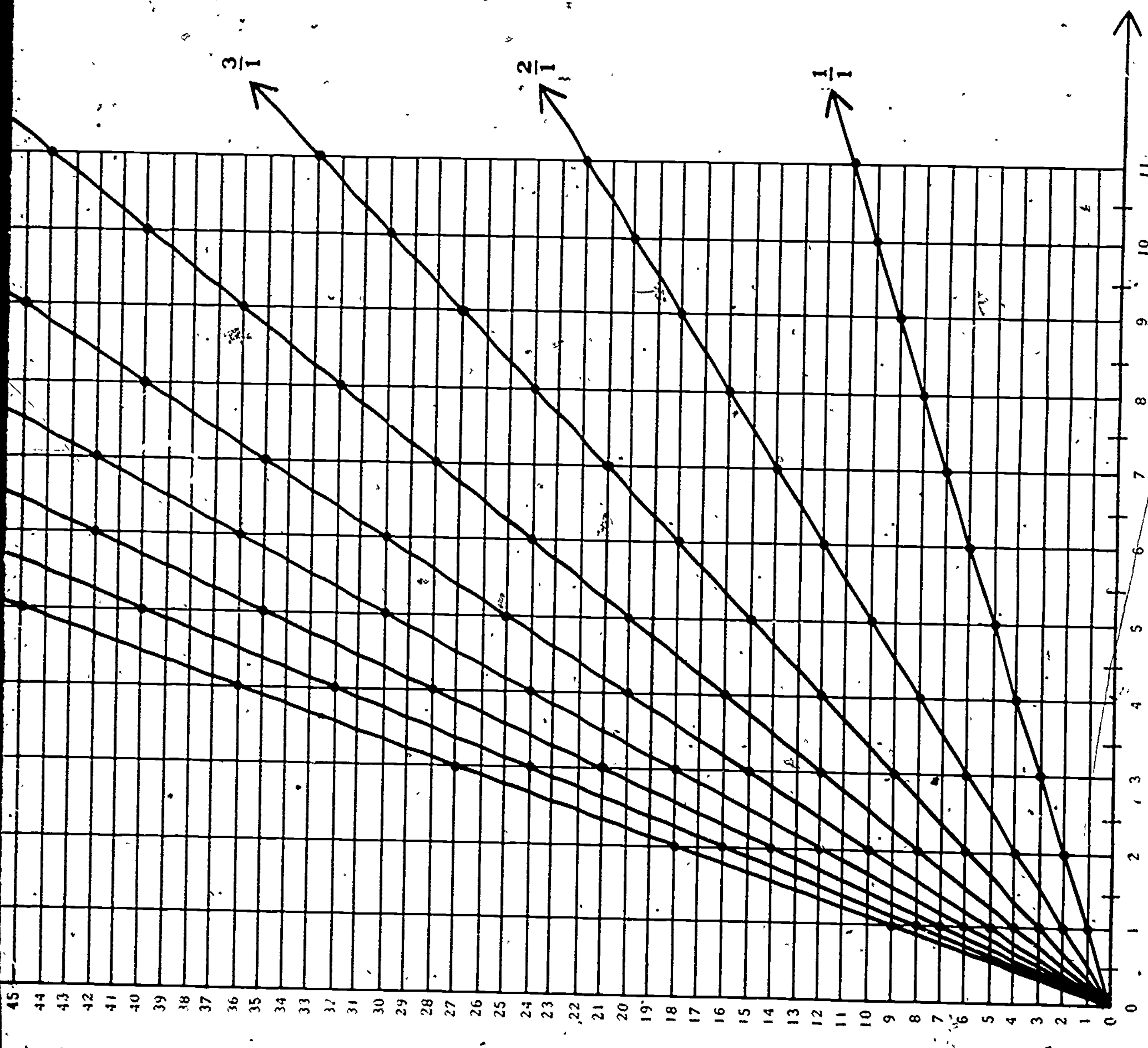
One child rolls both dice. He can either add or multiply the two numbers to generate a product or sum shown on the scorecard. The object of the game is to be the first player to cross out all numbers on the scorecard. If a player rolls a 4 and a 5, he can either add them and cross out the 9 or he can multiply them and cross out the 20.



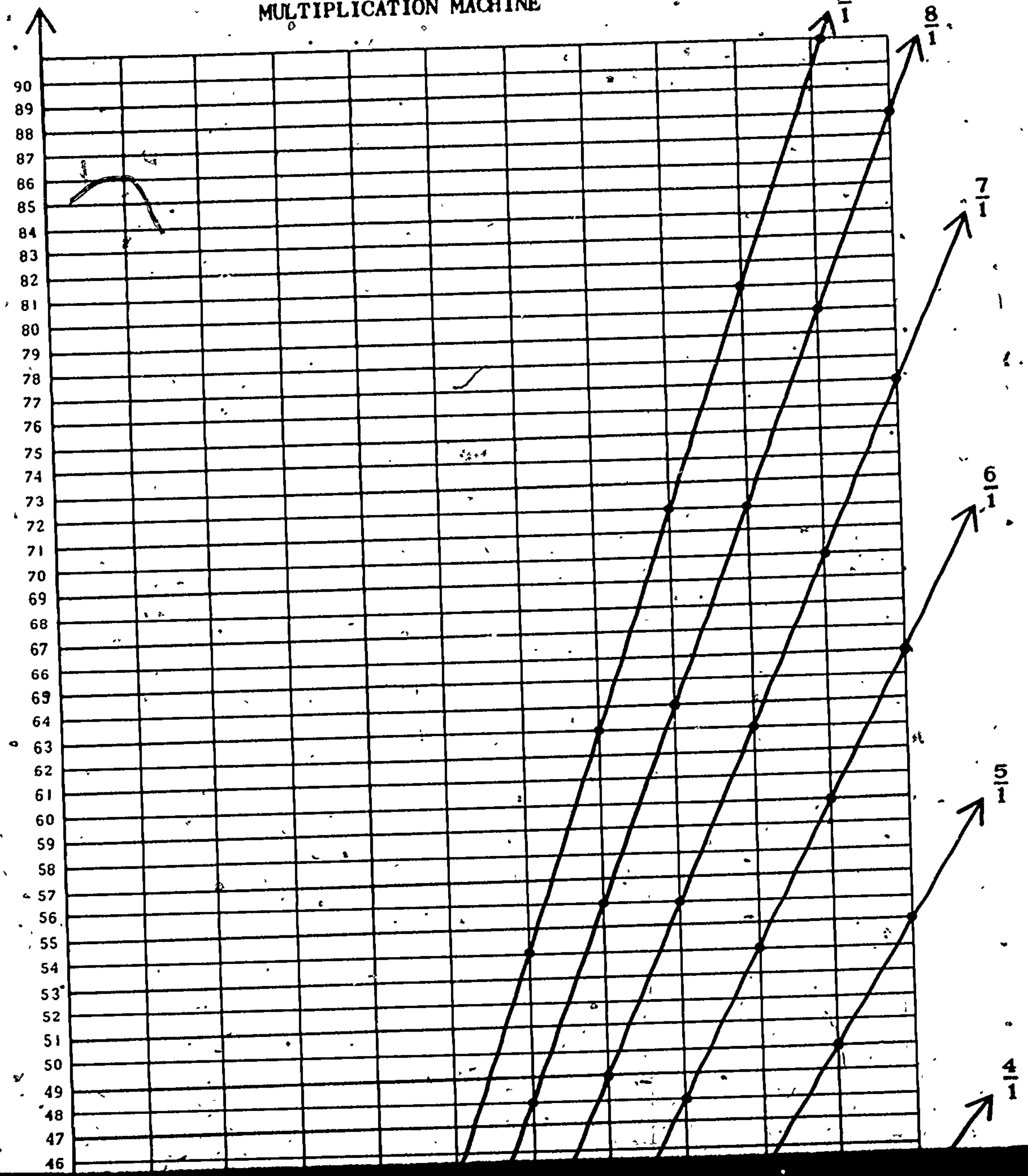
The game continues in this way until one player has crossed out all the numbers on the scorecard. The children will realize that on many turns they will not be able to generate a number shown on a scorecard or they will generate a number that has already been crossed out.

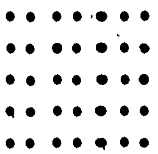
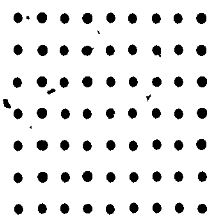
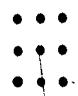
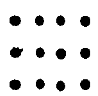
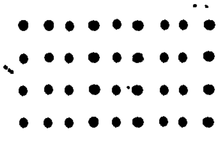
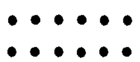
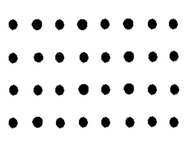
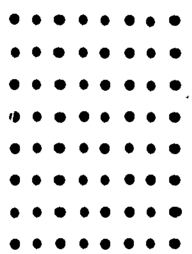
For variation, an opponent could call out "add" or "multiply" before the player rolls the dice. Then the player must add or multiply the numbers according to what the opponent said. A child could play this game by himself, keeping track of how many turns it takes him to cross out all the numbers.

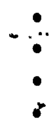
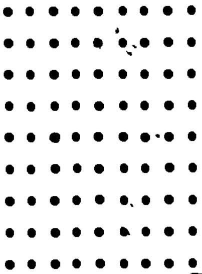
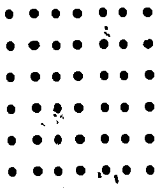
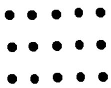
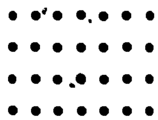
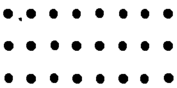
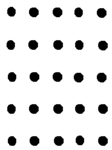
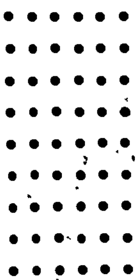
In the back of this manual and in each student manual is a letter to the parents concerning Unit 26, What Are Things Made Of? Please have each child tear out the letter and take it home a few days before you plan to start Unit 26.

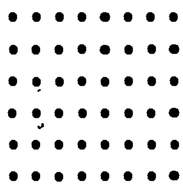
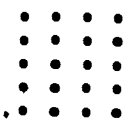
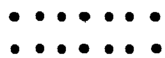
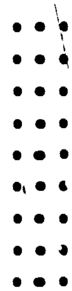
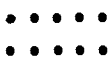
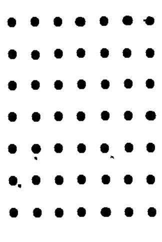
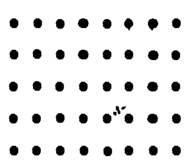
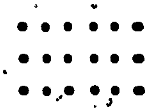

3/1
2/1
1/1
0 1 2 3 4 5 6 7 8 9 10 11
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45

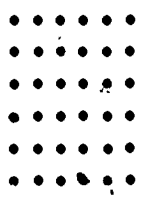
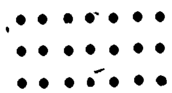
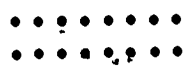
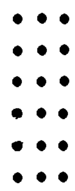
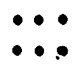
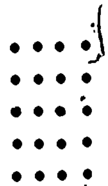
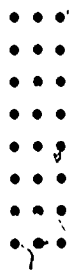
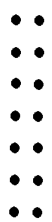
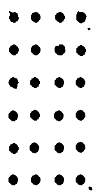
MULTIPLICATION MACHINE

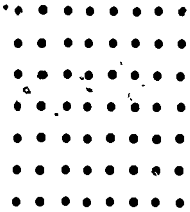
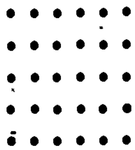
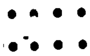
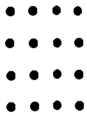
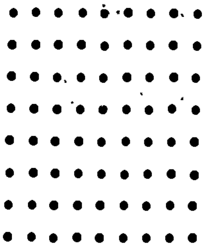
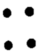
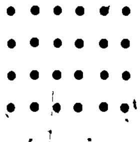
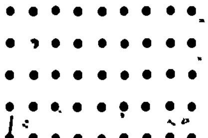


5 · 7	2 · 1	7 · 9	3 · 3	3 · 4
4 · 9	2 · 6	4 · 8	1 · 5	8 · 8
7 + 7 + 7 +7 + 7	1 + 1	9 + 9 + 9 +9 + 9 + 9 +9	3 + 3 + 3	4 + 4 + 4
9 + 9 + 9 +9	6 + 6	8 + 8 + 8 +8	Bonus  1 · 5 No repeated addition card for 1 · 5.	8 + 8 + 8 +8 + 8 + 8 +8 + 8
				
				

4 · 1	9 · 9	6 · 7	3 · 5	4 · 7
3 · 8	9 · 2	1 · 6	5 · 5	9 · 6
1 + 1 + 1 +1	9 + 9 + 9 +9 + 9 + 9 +9 + 9 + 9	7 + 7 + 7 +7 + 7 + 7	5 + 5 + 5	7 + 7 + 7 +7
8 + 8 + 8	9 + 9	Bonus  1 · 6 No repeated addition card for 1 · 6.	5 + 5 + 5 +5 + 5	9 + 9 + 9 +9 + 9 + 9
				
				

1 • 1	6 • 8	4 • 5	2 • 7	9 • 3
2 • 5	7 • 7	5 • 8	3 • 6	9 • 1
Bonus  1 • 1 No repeated addition card for 1 • 1.	8 + 8 + 8 +8 + 8 + 8	5 + 5 + 5 +5	7 + 7	3 + 3 + 3 +3 + 3 + 3 +3 + 3 + 3
5 + 5	7 + 7 + 7 +7 + 7 + 7 +7	8 + 8 + 8 +8 + 8	6 + 6 + 6	1 + 1 + 1 +1 + 1 + 1 +1 + 1 + 1
				
				

$6 \cdot 6$	$3 \cdot 7$	$2 \cdot 8$	$6 \cdot 3$	$2 \cdot 3$
$1 \cdot 8$	$5 \cdot 4$	$8 \cdot 3$	$7 \cdot 2$	$6 \cdot 4$
$6 + 6 + 6$ $+6 + 6 + 6$	$7 + 7 + 7$	$8 + 8$	$3 + 3 + 3$ $+3 + 3 + 3$	$3 + 3$
Bonus  $1 \cdot 8$ No repeated addition card for $1 \cdot 8$.	$4 + 4 + 4$ $+4 + 4$	$3 + 3 + 3$ $+3 + 3 + 3$ $+3 + 3$	$2 + 2 + 2$ $+2 + 2 + 2$ $+2$	$4 + 4 + 4$ $+4 + 4 + 4$
				
				

7 · 8	5 · 6	2 · 4	1 · 7	4 · 4
3 · 1	8 · 9	2 · 2	4 · 6	5 · 9
8 + 8 + 8 +8 + 8 + 8 +8	6 + 6 + 6 +6 + 6	4 + 4	Bonus  1 · 7 No repeated addition card for 1 · 7.	4 + 4 + 4 +4
1 + 1 + 1	9 + 9 + 9 +9 + 9 + 9 +9 + 9	2 + 2	6 + 6 + 6 +6	9 + 9 + 9 +9 + 9
				
				

UNIVERSITY OF *Minnesota*

MINNESOTA SCHOOL MATHEMATICS AND SCIENCE CENTER
720 WASHINGTON AVENUE S.E. • MINNEAPOLIS, MINNESOTA 55414

Office of the Director

Dear Parent:

Your child will soon begin studying a new MINNEMAST unit, What Are Things Made Of? As the name implies, it is concerned with the great variety of materials of which objects are composed. At the beginning of the unit, the children are going to take apart discarded useless objects so that they can identify and sort each material. It would be most helpful if your child could bring a few broken toys or useless articles to school within the next few days.

The most interesting objects are those made of more than one material, such as a toy car made from metal, plastic and rubber. Old utensils are also useful. Some suggestions include wornout metal pans with wooden handles, broken clocks and discarded paint brushes.

If your child could also bring a hammer, pliers or file, this would be helpful. Care will be taken to mark any tool so that it can be returned.

Thank you for your cooperation. I am sure your child is going to enjoy this unit and will tell you about the different tests for distinguishing one material from another.

Cordially,

Teacher