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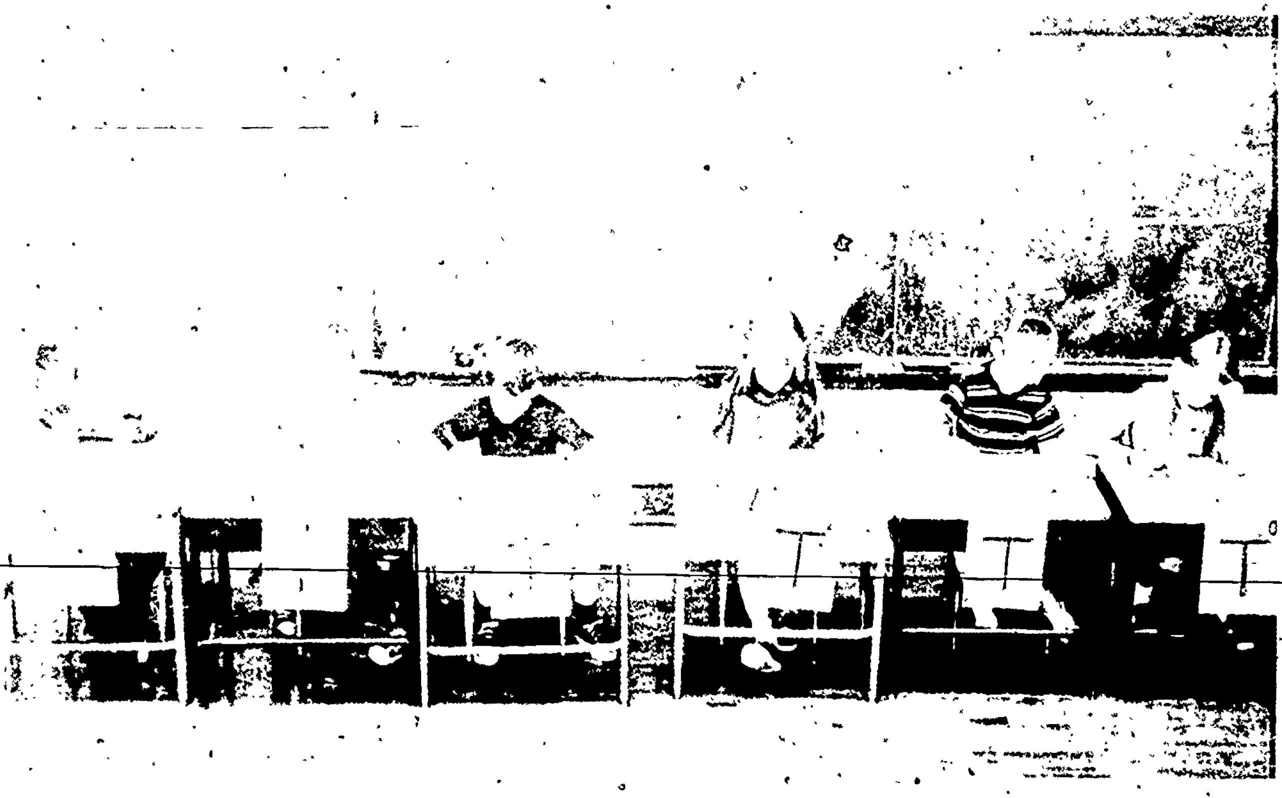
ABSTRACT

This volume is the twenty-fourth in a series of 29 coordinated MINNEMAST units in mathematics and science for kindergarten and the primary grades. Intended for use by third-grade teachers, this unit guide provides a summary and overview of the unit, a list of materials needed, and descriptions of four groups of lessons and activities. The purposes and procedures for each activity are discussed. Examples of questions and discussion topics are given, and in several cases ditto masters, stories for reading aloud, and other instructional materials are included in the book. This unit covers both computational and measurement ideas. In the two sets of lessons on computation, the class simulates a computer in activities designed to promote understanding of addition and subtraction in a place value system. Measurement activities are related to liquid volume, length and time. Two job booklets, one on pouring and the other on balancing as methods of measurement, provide activities for independent work by students. (SD)

CHANGE AND CALCULATIONS



UNIT 24

MINNEMAST COORDINATED MATHEMATICS-SCIENCE SERIES

KINDERGARTEN	1.	WATCHING AND WONDERING
	2.	CURVES AND SHAPES
	3.	DESCRIBING AND CLASSIFYING
	4.	USING OUR SENSES
	5.	INTRODUCING MEASUREMENT
	6.	NUMERATION
	7.	INTRODUCING SYMMETRY
FIRST GRADE	8.	OBSERVING PROPERTIES
	9.	NUMBERS AND COUNTING
	10.	DESCRIBING LOCATIONS
	11.	INTRODUCING ADDITION AND SUBTRACTION
	12.	MEASUREMENT WITH REFERENCE UNITS
	13.	INTERPRETATIONS OF ADDITION AND SUBTRACTION
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SECOND GRADE	15.	INVESTIGATING SYSTEMS
	16.	NUMBERS AND MEASURING
	17.	INTRODUCING MULTIPLICATION AND DIVISION
	18.	SCALING AND REPRESENTATION
	19.	COMPARING CHANGES
	20.	USING LARGER NUMBERS
	21.	ANGLES AND SPACE
	22.	PARTS AND PIECES
THIRD GRADE	23.	CONDITIONS AFFECTING LIFE
	● 24.	CHANGE AND CALCULATIONS
	25.	MULTIPLICATION AND MOTION
	26.	WHAT ARE THINGS MADE OF?
	27.	NUMBERS AND THEIR PROPERTIES
	28.	MAPPING THE GLOBE
	29.	NATURAL SYSTEMS

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CHANGE AND CALCULATIONS

UNIT 24

MINNEMAST

MINNESOTA MATHEMATICS AND SCIENCE TEACHING PROJECT
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Third Printing
1971

CHANGE AND CALCULATIONS

This unit was developed by MINNEMAST on the basis of experiences of the many teachers who taught an earlier version in their classrooms.

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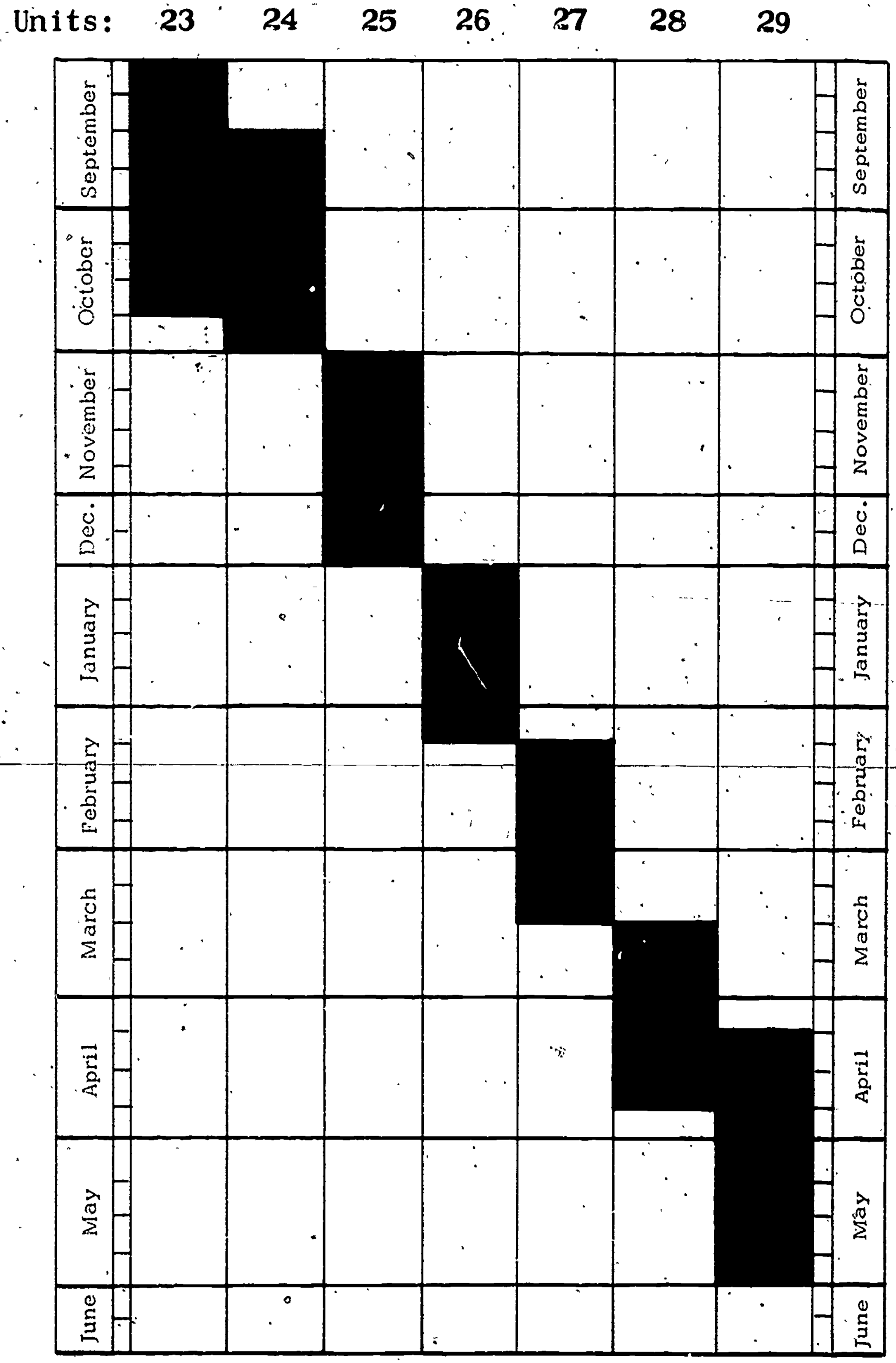
Suggested Teaching Schedule for MINNEMAST Third Grade Units



CONTENTS

Complete List of Materials for Unit 24

(Numbers based on class size of 30.)

total number required to teach unit	item	lessons in which item is used
30	**Student Manuals	
1	*marking pen	1
1	*masking tape	1,3,7,9,12, 14, Section 4
30	**Sheets A and B	1
1	abacus (optional)	1
30	scissors	2
30	*small plastic bags	2-14
15 pr.	*dice labeled 0-5 and 4-9	6
1	*ruler or straightedge	7
19	*half-pint containers	12
9	*pint containers	12
4	*quart containers	12
2	*half-gallon containers	12
1	gallon container	12
1	*water dipper of cup (measuring)	12, Section 4
1	*yarn or string	12
1	large pail (optional)	12, Section 4
1	sponge (optional)	12
15	*yard and meter sticks	13
1	clock with second hand	14
2	*plastic tubes	Section 4
30 lengths	*centimeter tape	Section 4
1	*butcher tray	Section 4
30 ea. color	*red, black and silver paper clips	Section 4
1	*beam support block	Section 4

*kit items as well as

**printed materials available from Minnemath Center,
720 Washington Avenue S.E., Minneapolis, Minn. 55455

INTRODUCTION

In the first two sections of the unit, the children set up a computer to work addition and subtraction problems. These problems can be written in several ways:

1.	horizontally	$25 + 71$
2.	vertically	$\begin{array}{r} 25 \\ \underline{+71} \end{array}$
3.	in T notation	$\begin{array}{r} 2T^1 + 5T^0 \\ \underline{+ 7T^1 + 1T^0} \end{array}$

Regardless of the form of the problem, the same rules are followed. We add or subtract the numbers in each place (5 + 1 and 2 + 7) to get the sum or difference. Though most people prefer to write addition and subtraction problems vertically and to work from right to left, this need not be done. No matter how the problem is set up, we still follow the rule that the numbers in each place must be added or subtracted to find the answer.

In Section 1 algorithms (rules) for addition are developed with heavy emphasis on the place value system. The children review T notation for expressing place value. In T notation the number 385 would be rewritten as $3T^2 + 8T^1 + 5T^0$. Since T stands for 10, the ten's place is called T^1 because the elements have been grouped by ten just once. The one's place is called T^0 because no groups of ten have been made in this place, and the hundred's place is called T^2 because we have grouped by tens twice. Each place is ten times as great as the one to its right ($T^0 = 1$, $T^1 = 10T^0$ or 10, $T^2 = 10T^1$ or 100). First the children do addition using a T-piece computer. Then they add using numerals in their computer. Finally the vertical form for addition is developed from an addition chart which represents the computer.

Section 2 develops the subtraction algorithm, again with emphasis on place value. The children set up a computer and work subtraction problems with T notation. The progression of subtracting with a computer, then a chart and finally in the standard vertical form is similar to the format and procedures for addition in Section 1.

Section 3 involves measurement systems of length, volume and time duration. These systems will contrast the place value of the base-ten numeration system the children worked with in Sections 1 and 2. The children will see that measurement units for length and time duration are not regular. That is, there are 12 inches in one foot but only three feet in one yard and there are 60 minutes in one hour and 24 hours in one day. For volume measures the children set up a base-two computer and find that these units are regular: two pints equal one quart, two quarts equal one half-gallon, etc. Besides learning different measurement units, the children should compare the different place values of measurement systems to the base-ten numeration system.

Section 4 is made up of two job booklets which the children will complete independently. One of the booklets is a pouring system and the other presents a balance beam system. By pouring water from one measured tube into another or balancing a meter stick, the children generate ordered pairs. They graph these pairs and use them to predict new states of their systems. The booklets are designed to be self-instructional with a minimal amount of help from you.

Each booklet should be completed in one week.

NOTES ON TEACHING THIS UNIT

This unit is ~~to be taught~~ concurrently with Unit 23, Conditions Affecting Life, starting one week after Unit 23. Sections 1 through 3 should be taught for approximately 30 minutes each day for four weeks. The Job Booklets of Section 4 will take two weeks to complete. For the first week, half of the class completes the Pouring System Booklet while the other half completes the Balance Beam System Booklet. During the second week each group of children completes the other booklet.

For Lesson 12 of Section 3, you or the children must provide one half-gallon container and one gallon container. Milk cartons or plastic pitchers work well.

SUGGESTED SCHEDULE FOR TEACHING UNITS 23 AND 24 CONCURRENTLY

	UNIT 23 This unit generally should be taught three days a week for periods of 40 to 50 minutes.					UNIT 24 ** This unit generally should be taught every day for periods of approximately 30 minutes.				
	Mon.	Tues.	Wed.	Thurs.	Fri.	Mon.	Tues.	Wed.	Thurs.	Fri.
1st week	Lesson* 1	Lesson* 2	Lesson* 3							
2nd week			Lesson 4	Lesson 5		Lesson 1		Lesson 2		Lesson 3
3rd week	Lesson 6		Lesson 7	Lesson 8		Lesson 3	Lesson 4		Lesson 5	Lesson 6
4th week	Lesson 9		Lesson 10			Lesson 7	Lesson 8	Lesson 9	Lesson 10	Lesson 11
5th week	Lesson 11	Lesson 12	Lesson 13		Lesson 14	Lesson 12		Lesson 13		Lesson 14
6th week	Lesson 15	Lesson 16	Lesson 17	Lesson 18		Pouring** System	Pouring System	Pouring System	Pouring System	Pouring System
7th week	Lesson 19					Balance** Beam System	Balance Beam System	Balance Beam System	Balance Beam System	Balance Beam System

* It is important to teach Lessons 1, 2 and 3 on the days specified.

** Half of the class does one system one week, then the other system the following week.

SECTION I · ADDITION

PURPOSE

- To develop an understanding of and facility in using place value (T notation).
- To introduce and provide practice with the addition algorithm.
- To provide game situations for enjoyable practice with basic addition facts and the addition algorithm.

COMMENTARY

In Lessons 1 and 2 a classroom computer is set up using children as components. Each desk represents one place in the place value system. The children develop rules which tell the computer how to add using T pieces. It does so by physically combining pieces which represent the number of elements in each place. The rule of combining the elements in each place is always followed, though the form for writing addition problems changes as the section progresses.

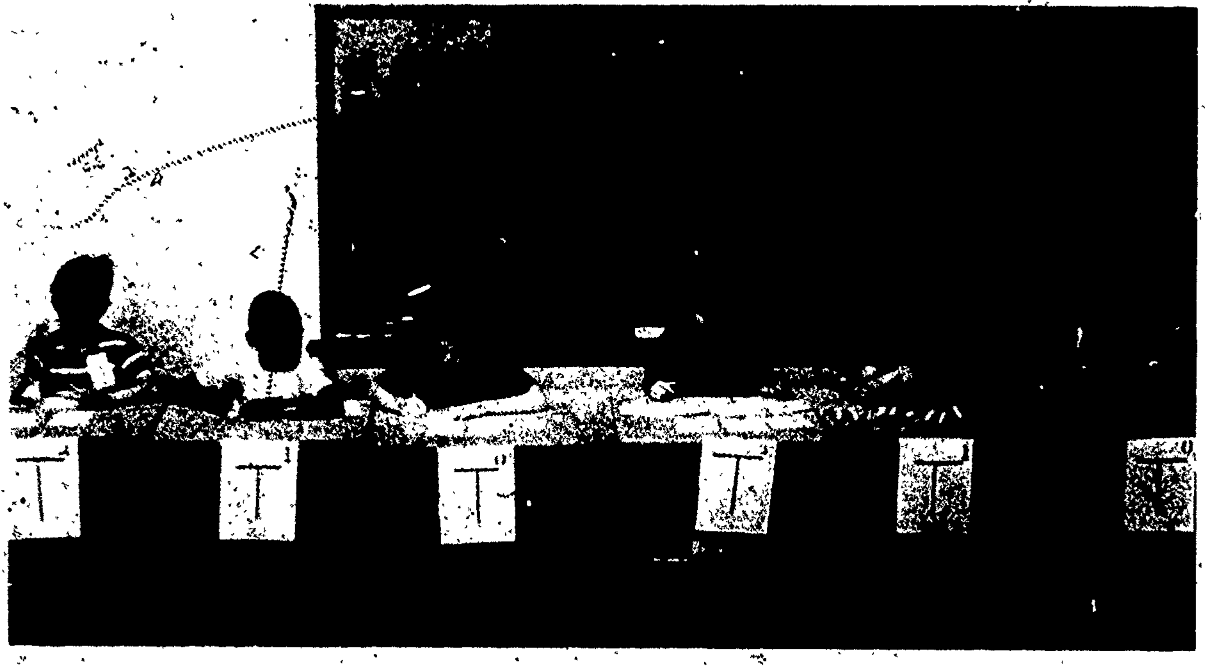
An algorithm for regrouping is developed in Lesson 2. The children learn that they must trade 10 pieces for 1 of the next larger pieces. For example, 10 T^0 pieces are traded for 1 T^1 piece. When the computer adds a problem such as 9 + 8, the T^0 place will receive first 9 T^0 pieces and then 8 T^0 pieces. The child at that place trades 10 of his 17 T^0 pieces for 1 T^1 piece. He then gives the 1 T^1 piece to the child at the next desk, the T^1 place, and keeps the 7 T^0 pieces at the T^0 place.

In Lesson 3 the children explore the place value system further by making representations of the T^3 and T^4 places. They tape together 10 T^2 pieces to make a T^3 piece and then 10 T^3 pieces to make a T^4 piece. The idea that the place value system goes on and on is developed.

In Lesson 4 the rules for the T-pieces computer are modified so that numerals written on slips of paper are accepted by the computer and added mentally by the children who are computer components. When the problem 9 + 8 is added with the numeral computer, the child at the T^0 place will receive a slip of paper marked $9T^0$ and another marked $8T^0$. He adds these mentally and fills out one slip for each place value represented in the sum. That is, the sum 17 is rewritten as $1T^1 + 7T^0$. He passes the $1T^1$ slip to the T^1 place. Though the form of regrouping has changed, first using T pieces and then using numeral slips, the rule that 10 elements are grouped and passed to the next larger place remains.

When the children are familiar with the addition process using the numeral computer, the vertical form for adding is developed. In Lesson 4 a chart is drawn on the chalkboard which represents each place in the computer. The children add a problem using the computer while you add it using the chart. You stress the fact that the place value addition is the same using both methods. In Lesson 5, lines are removed from the chart until only the vertical form for writing addition problems remains.

Games in Lessons 3 and 6 provide an enjoyable way for the children to practice the basic addition facts. These games can be modified to include more difficult problems as the children become proficient. Since most of the games require only a few minutes to play, one or more of them should be used almost every day throughout the year.



Lesson 1: THE T-PIECES COMPUTER

This lesson begins a review of addition and place value using the T notation approach that the children worked with in Unit 20. You may find that they need to regain familiarity with this system. For this reason material from second grade has been included in the activities. Those children who do not recall T notation will have considerable experience with it in the first few lessons. Be sure to use T notation language yourself but do not demand that the children use it exclusively.

The children make their own classroom computer in which they serve as components. A set of rules is devised that tells the computer how to add numbers and respond with the sum. The operation of the computer stresses place value and provides practice using the addition facts.

The time allowed for this lesson is two days. If the children remember T notation from the second grade, Activity A should go very quickly and you could start Activity B on the same day. If many children in your class do not understand T notation, spend the first day on Activity A only. The rate of progress through these first three lessons will depend on the abilities of the children. Feel free to speed up the introduction to the computer in Activity B if you think the children understand the procedure.

MATERIALS

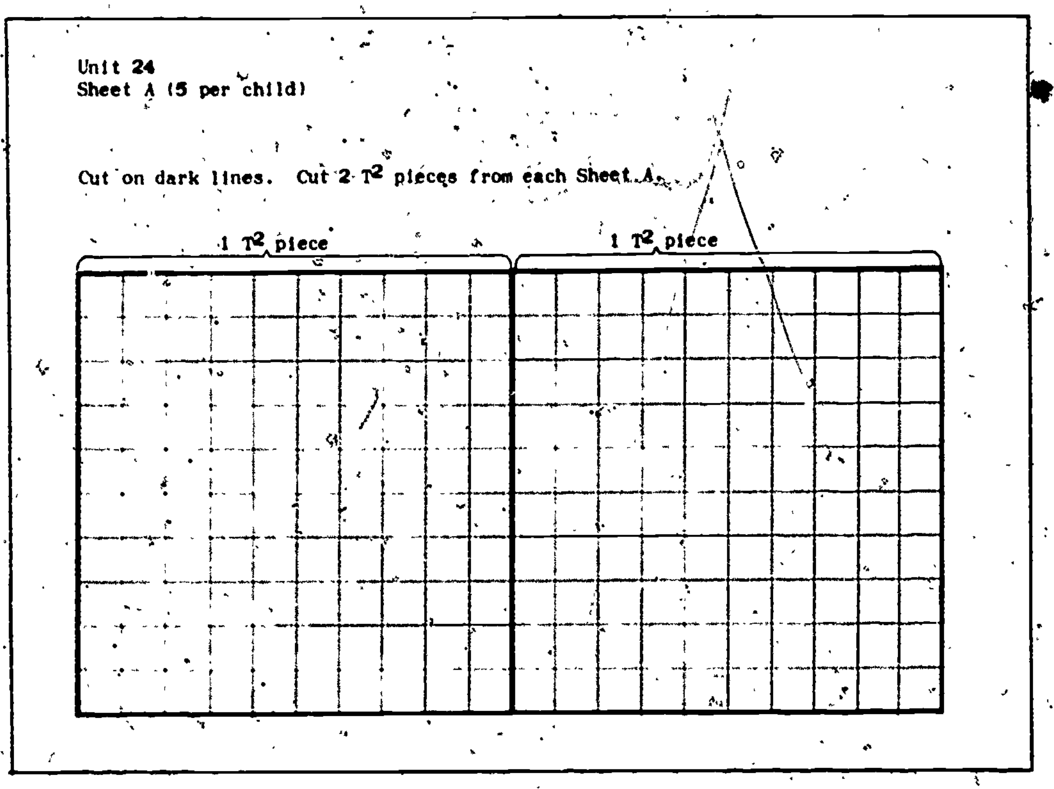
- 3 large sheets of paper, one each labeled T^0, T^1, T^2
- marking pen
- masking tape
- set of T pieces made from 5 copies of Sheet A and 1 copy of Sheet B (available from MINNEMAST)
- abacus (optional)

-- for each child --

- sheet of paper
- Worksheets 1 - 3

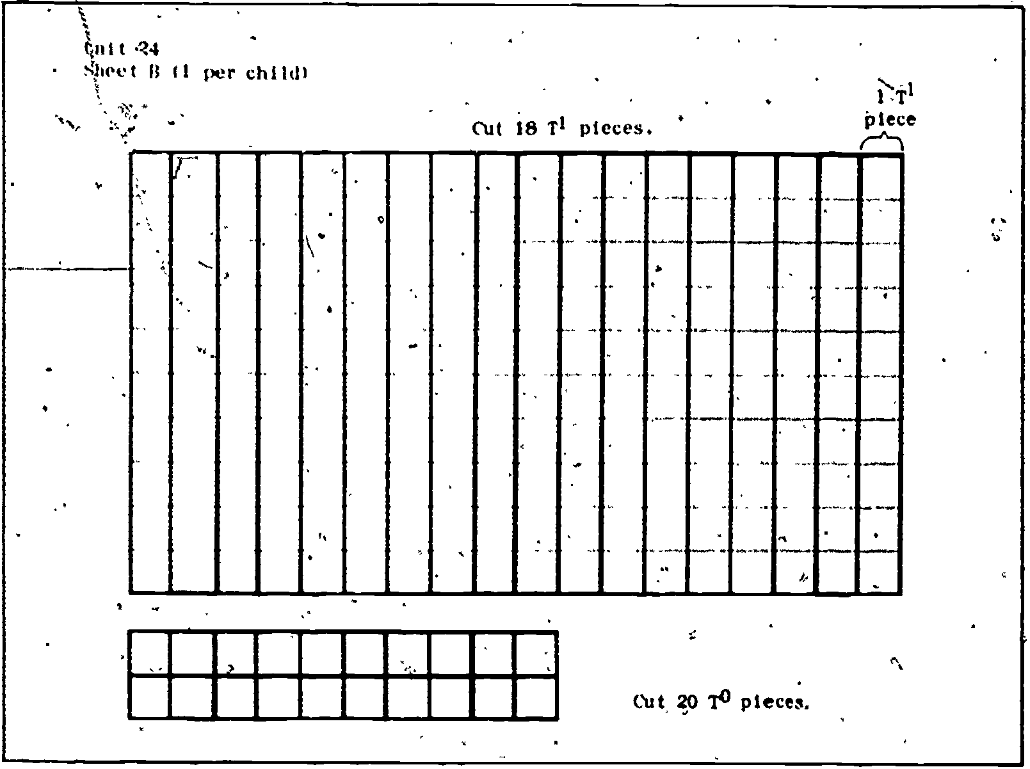
PREPARATION

Make a set of T pieces by cutting up five copies of Sheet A and one copy of Sheet B. You should now have one set of T pieces, containing 10 T^2 pieces, 18 T^1 pieces and 20 T^0 pieces. (Full-size copies of Sheets A and B can be found in the Appendix.) Label three large sheets of paper T^0, T^1, and T^2.



Unit 24
Sheet A (5 per child)

Cut on dark lines. Cut 2 T^2 pieces from each Sheet A.

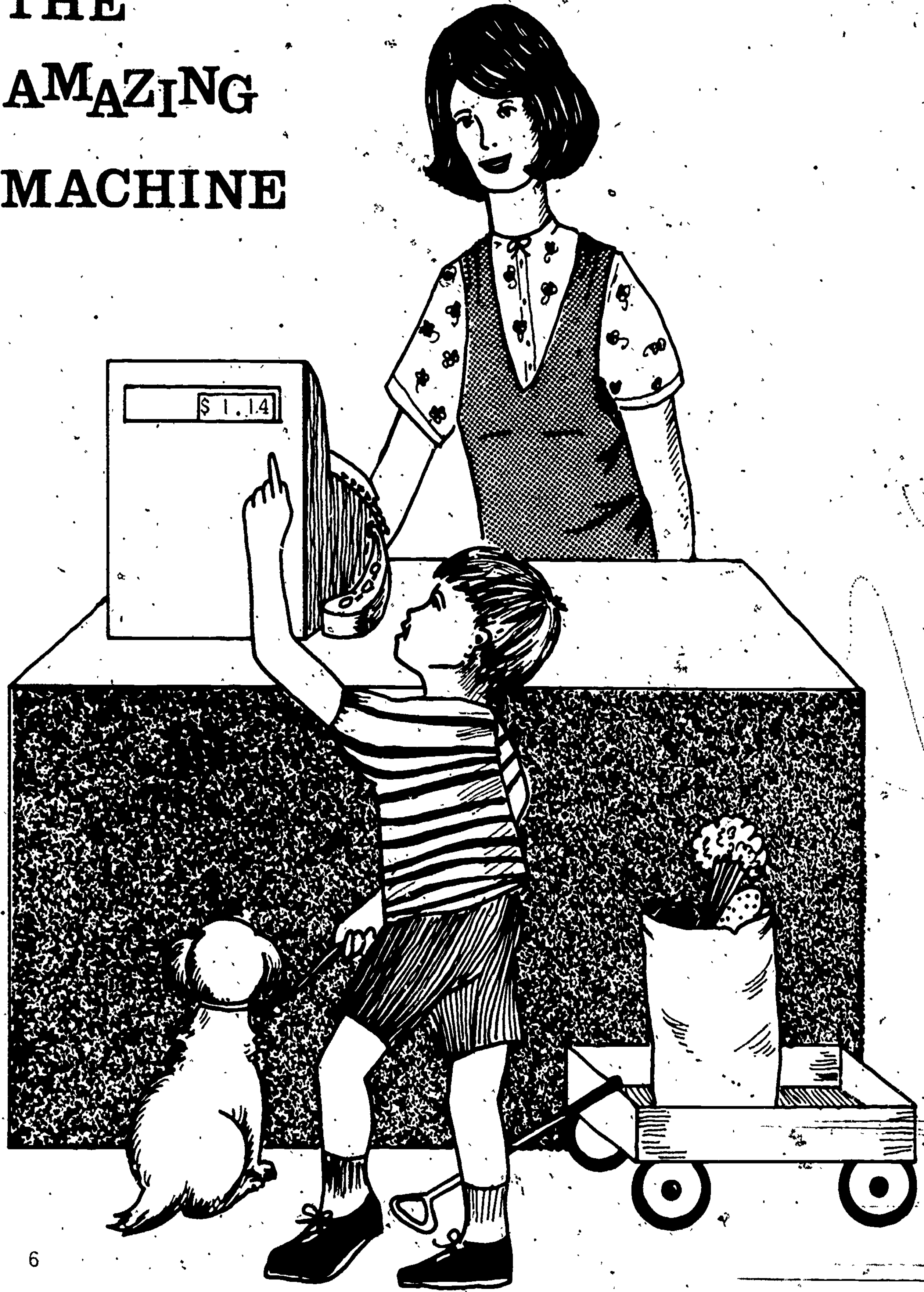


PROCEDURE

Activity A

Read the story, "The Amazing Machine" to the class. The story is on page 7.

THE AMAZING MACHINE



THE AMAZING MACHINE

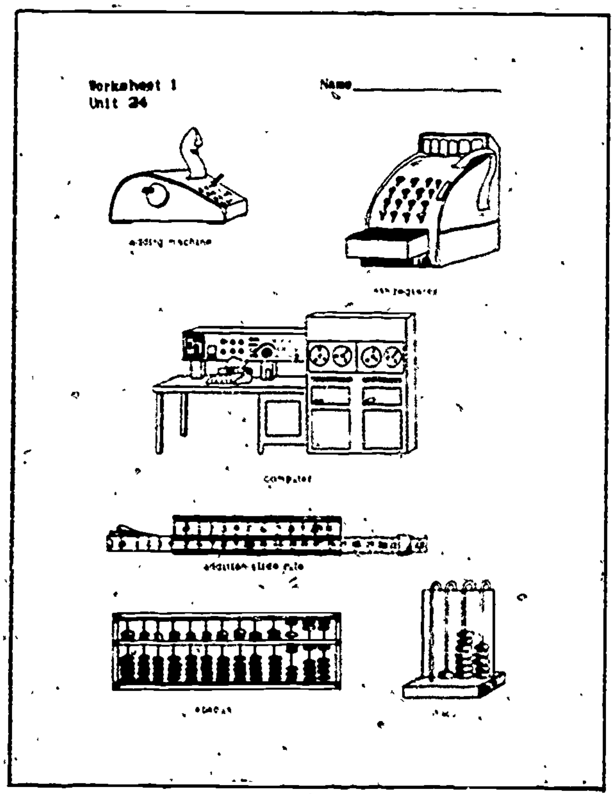
Freddy was shopping at the supermarket. He found all the things on his mother's list and wheeled his cart up to the check-out counter. It was fun to watch the clerk add up his purchases on the cash register. The machine showed a total of $3.86. Freddy gave the clerk the five-dollar bill his mother had given him. With a clatter and a clang the machine registered the amount of change Freddy was to receive -- $1.14. Fourteen cents came sliding down a little chute for him to pick up, and the clerk gave him a dollar bill. He put all the money in his pocket and took his bag of groceries out through the door that opened by itself when he came to it.

All the way home Freddy thought about the machine. It gave the right answer every time. How did it do it? How did it know when to add and when to subtract? Freddy wondered why it never multiplied the numbers when the clerk pushed the buttons. And why did it always send just the right coins down the chute? Freddy couldn't wait to ask someone how machines like the cash register work.

Ask the children if they have ever watched a machine that computed for us, like the cash register, and wondered how it worked.

> WHAT OTHER MACHINES CAN YOU THINK OF THAT COMPUTE FOR US AS CASH REGISTERS DO? (Adding machines, odometers, pedometers, abaci, computers. Do not expect the children to use these words.)

You might mention that the addition slide rule used in Unit 13 is a kind of computer.



Worksheet 1
Unit 24

Name________________

Have the children turn to Worksheet 1. Discuss the machines with them and explain that numbers are put in each machine and answers come out. Clarify the fact that machines do only what they are instructed to do. They can only follow the rules someone has made for them. Tell the class that you have a plan to make a computer right in the classroom but you will need everyone's help to do it.

With masking tape, tape the T signs to the chalkboard in this order:

$$\boxed{T^2} \quad \boxed{T^1} \quad \boxed{T^0}$$

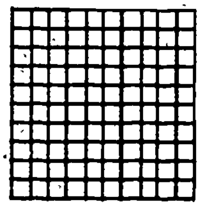
(Save these labels for Activity B.)

Ask the children what T^0, T^1, and T^2 represent. If your class does not remember that these are names for the ones, tens and hundreds places, you may want to use the following review. If the review is not necessary, go on to Activity B.

Hold up a T^0 piece □ and a T^1 piece  .

HOW MANY OF THESE PIECES (T^0) DOES IT TAKE TO MAKE THIS PIECE (T^1)? (10.)

Holding the T^1 piece, count with the class the 10 T^0 sections marked on it. Then hold up the T^1 and T^2 pieces and ask:



HOW MANY OF THESE PIECES (T^1) DOES IT TAKE TO MAKE THIS PIECE (T^2)? (10.)

Count the 10 T^1 strips with the class. Explain that because it takes 10 small pieces to make a long piece and it takes 10 of those strips to make the next sized piece, we say we are "grouping by tens."

Hold up the appropriate pieces again during the following discussion.

WHEN WE GROUP 10 SMALL PIECES WE GET A LONG PIECE. WE CALL THIS PIECE T^1. THE T TELLS US THAT WE ARE GROUPING BY TENS AND THE 1 TELLS US THAT WE HAVE GROUPED BY TEN JUST ONCE.

NOW SUPPOSE WE GROUP 10 T^1 PIECES -- WHAT CAN WE CALL THIS NEW PIECE? (T^2.)

THE T TELLS US THAT WE ARE GROUPING BY TENS AND THE 2 TELLS US THAT THE PROCESS OF GROUPING BY TENS HAS BEEN DONE TWICE. FIRST WE GROUPED 10 OF THESE PIECES (show a T^0 piece) AND GOT ONE OF THESE PIECES (show a T^1 piece). THEN WE GROUPED 10 OF THESE

PIECES (show a T^1 piece) AND GOT ONE OF THESE PIECES (show a T^2 piece).

Now hold up a T^0 piece and ask:

WHEN WE ARE GROUPING THIS WAY, WHAT CAN WE CALL THIS PIECE? FIRST LET'S WRITE DOWN T TO TELL US THAT WE ARE GROUPING BY TENS. NOW HOW MANY TIMES HAVE WE GROUPED BY TENS? (None; zero times.) WHAT CAN WE WRITE TO TELL US THAT? (T^0.)

Explain to the children how to read T notation:

T^0 is "T to the zero power."

T^1 is "T to the first power."

T^2 is "T to the second power," etc.

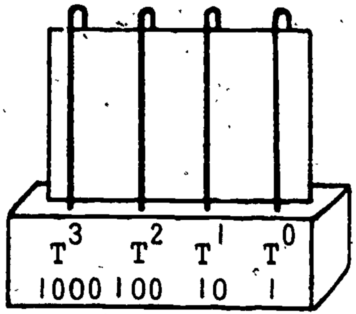
Tell them that we can also say it a shorter way:

T^0 is "T to the zero."

T^1 is "T to the one," or "T to the first."

T^2 is "T to the two," or "T to the second."

Label the places on an abacus T^0, T^1, T^2, and higher if you desire. Also label the places 1, 10, 100, etc.



Show a number on the abacus and have each child write the numeral in the following form on a piece of paper.

$$345 = 300 + 40 + 5$$

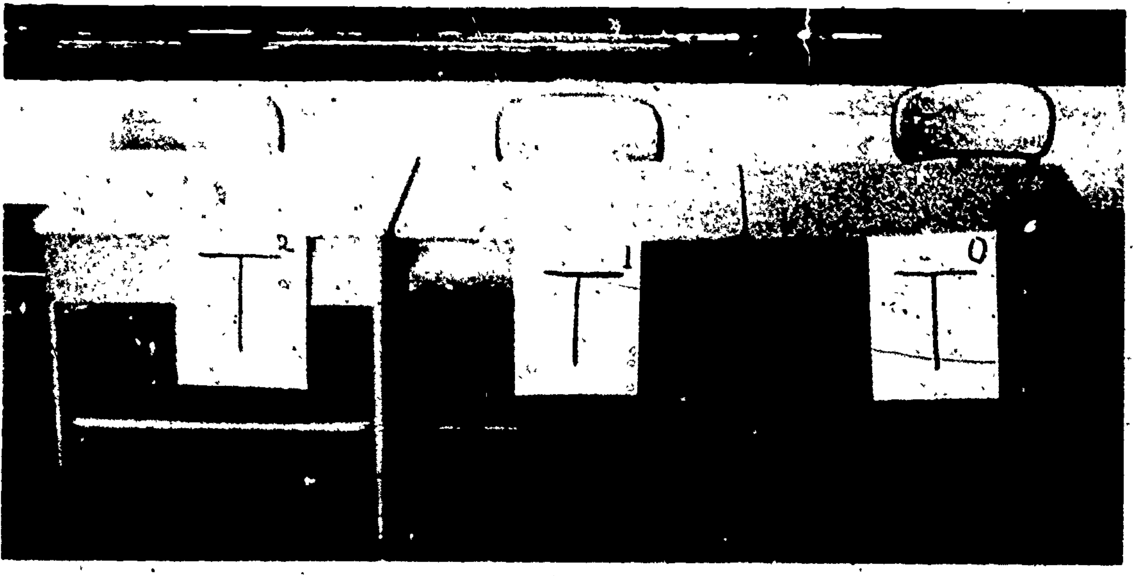
$$345 = 3T^2 + 4T^1 + 5T^0$$

The relationship between T notation and the expanded base-ten notation should be shown and discussed even if an abacus is not available.

Have each child write a 3-digit number on a small piece of paper and pass it to the child behind him. Each student writes this number in expanded base-ten notation (549 = 500 + 40 + 9) and in T notation (549 = $5T^2 + 4T^1 + 9T^0$). The student then hands the paper back to the child who wrote it. This child checks to decide if the notations have been written correctly. Go around the room and quickly check those the children think are wrong.

Activity B

Tell the children they are now going to build the computer you discussed in Activity A. Remove the T signs from the chalkboard. Place three desks at the front of the room and label them as shown in the photograph below. Explain to the children that the desks are part of the computer. (Save these labels for subsequent lessons.) You may want to draw an imaginary start button on the chalkboard next to the computer desks.



Ask one student to sit at each of the three desks. These children are part of the computer. Explain that computers will do only what they are instructed to do. Tell the children that you have made up some rules for the computer to follow. Then write these rules on the board in your own words:

Rule 1. The computer is to add the numbers fed into it.

Rule 2. Each desk can take only one kind of T piece. The T^0 place can take only T^0 pieces, the T^1 place can take only T^1 pieces and the T^2 place can take only T^2 pieces.

Rule 3. Each place must tell its contents when the Programmer calls for it.

Choose three more children to help operate the computer by acting as Programmer, Banker, and Recorder. The Recorder should use the chalkboard and the Banker should be given all the T^0, T^1, and T^2 pieces. The jobs of the Programmer, Banker, and Recorder are explained in the example below. The rest of the children should act as Repairmen. Tell the Repairmen to watch carefully to see if the Programmer, Banker, or Recorder make any mistakes. Repairmen should be allowed to correct any errors they find. (The first problem will probably go quite slowly until the children learn the procedure.)

LET'S SEE HOW OUR COMPUTER WORKS. LET'S HAVE IT ADD 21 AND 32.

Recorder: The Recorder writes the problem on the chalkboard. (The box shows what is written at each step and need not be drawn on the chalkboard.)

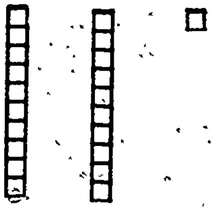
$$\boxed{\begin{array}{r} 21 \\ +32 \\ \hline \end{array}}$$

Ask: HOW DO WE WRITE 21 IN T NOTATION?
$(21 = 2T^1 + 1T^0)$

The Recorder writes this on the chalkboard. He should have written:

$$\begin{array}{r} 21 = \boxed{2T^1 + 1T^0} \\ +\ 32 \qquad\qquad\quad \\ \hline \end{array}$$

HOW CAN WE SHOW THIS NUMBER WITH THE T^0, T^1 AND T^2 PIECES?



Banker:	The Banker holds up the proper pieces for the T^0 place (in this case for the T^0 notation of 21).
Programmer:	The Programmer takes the pieces, checks to be sure they are the correct number and value, and feeds them to the computer (places them on the T^0 desk).
Computer T^0:	The child sitting at this place checks the pieces he is given and nods if he thinks they are the correct ones.
Banker:	While the Programmer and the T^0 place child are checking the T^0 pieces, the Banker picks up the T^1 pieces and passes them to the Programmer.
Programmer:	He checks the T^1 pieces and feeds them into the computer.
Computer T^1:	This child checks the pieces and nods if he thinks they are correct.
Note:	In this problem there are no T^2 pieces, but if there were the Banker would hand them to the Programmer, who would check them and hand them to the T^2 child who would check them and nod if they were correct.

Now ask: HOW DO WE WRITE 32 IN T NOTATION? ($32 = 3T^1 + 2T^0$).

Recorder: The recorder writes this on the chalkboard.

$$\begin{array}{r} 21 = 2T^1 + 1T^0 \\ \underline{+32 = \boxed{3T^1 + 2T^0}} \end{array}$$

The children go through the same procedure to feed the T pieces for 32 into the computer.

Programmer: When the T pieces representing 21 and 32 have been fed into the computer the Programmer pushes an imaginary start button.

Computer: The students at each place again count their pieces and then, one at a time starting with the T^0 place, they tell the total number of pieces they have. (3 pieces for the T^0 place, 5 for the T^1 and 0 for the T^2.)

Recorder: When the T^0 place child calls out the total number of pieces he has, the Recorder writes it in the proper place on the chalkboard. He does the same for the T^1 place. T^2 has no pieces so the Recorder does not write anything.

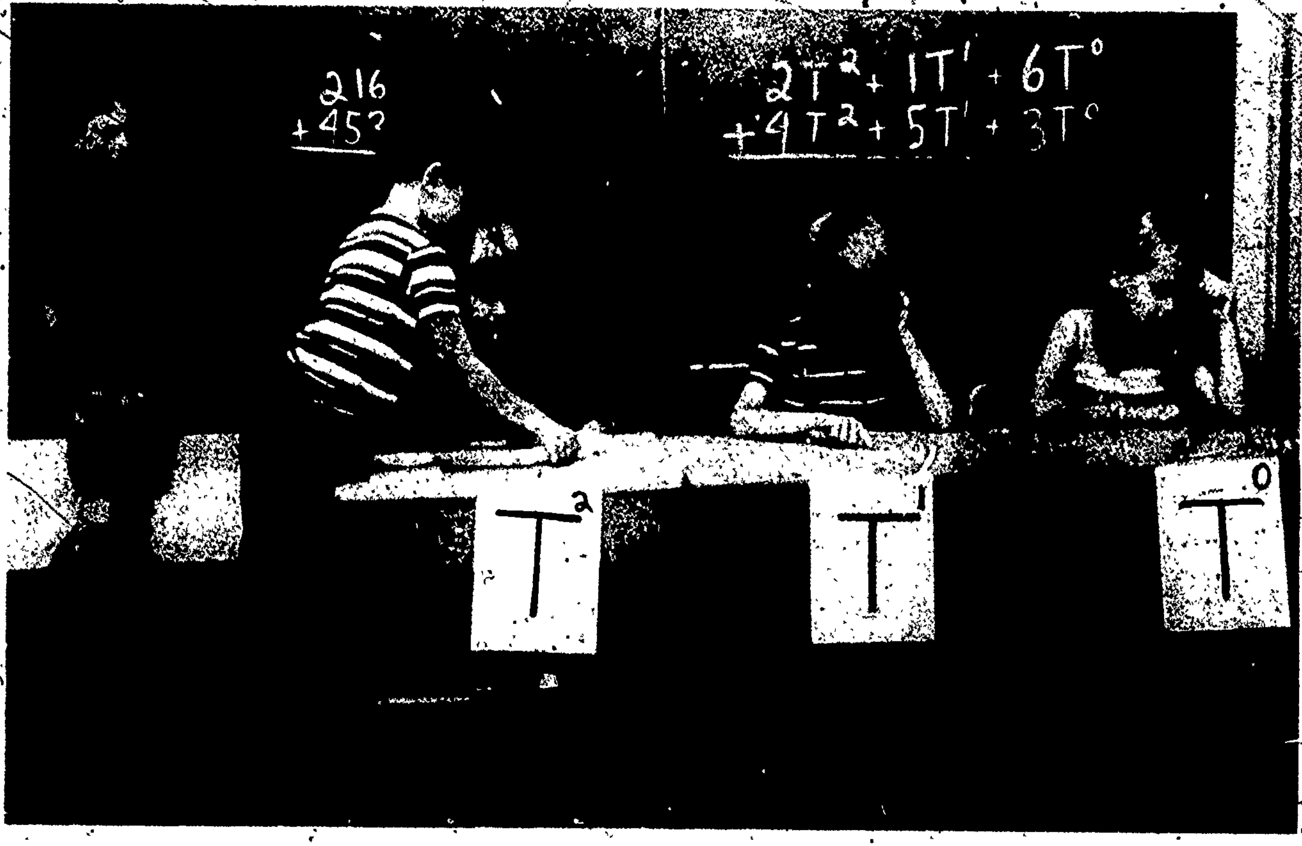
$$\begin{array}{r} 21 = 2T^1 + 1T^0 \\ \underline{+32 = 3T^1 + 2T^0} \\ \boxed{5T^1 + 3T^0} \end{array}$$

Now the Recorder converts the T notation answer to the standard notation (53) and writes it under the original problem. The completed problem should look like this:

$$\begin{array}{r} 21 = 2T^1 + 1T^0 \\ \underline{+32 = 3T^1 + 2T^0} \\ 53 = 5T^1 + 3T^0 \end{array}$$

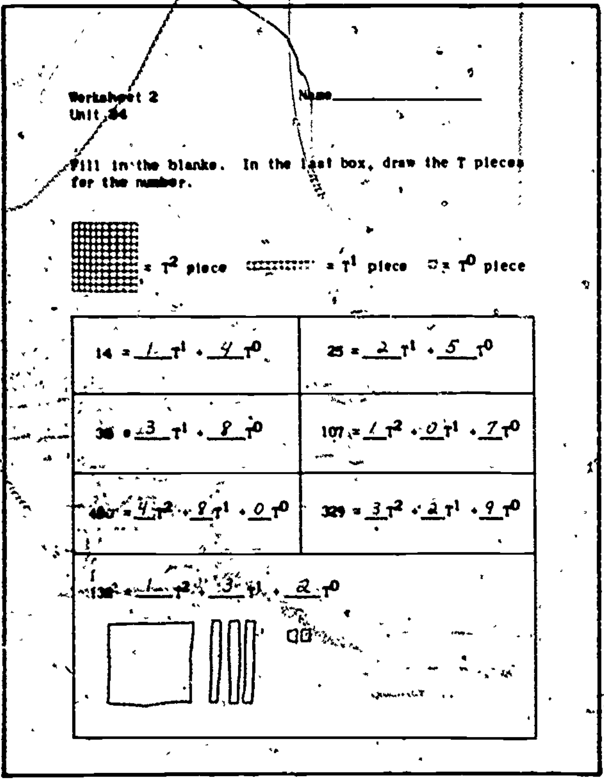
Review what the computer has done. It has added the pieces representing the first number (21) to the pieces of the second number (32) to give the sum of the two numbers (53).

Choose three different children to sit at the desks and three more to act as Programmer, Banker and Recorder. Let them choose another problem that does not involve regrouping and help them go through the addition procedure with the computer. The procedure should go faster this time. Work enough problems, such as 34 + 462, 291 + 4 and 128 + 40, so that each child has a chance to be a part of the computer set up. You may want to work a few more problems with those children who have difficulty the first time. All the children should become proficient at adding with the computer. Remind the Repairmen to report any mistakes they notice.



Save the T place labels and T pieces for future use.

Have the children complete Worksheets 2 and 3. These worksheets provide review of the concept of T notation and give practice in converting numerals to T notation. Have all the children do the bonus problems on Worksheet 3, with your help if necessary. You may want to provide a few more problems of this type so that the children understand that the T^2 place, for example, stands for the hundred's place even when T^1 and T^0 are not written out. If the children do not read the problem's carefully they may answer $8T^2 = 8$ instead of 800. Occasionally you should refer to the T places as the "hundred's place," "ten's place" and "one's place."



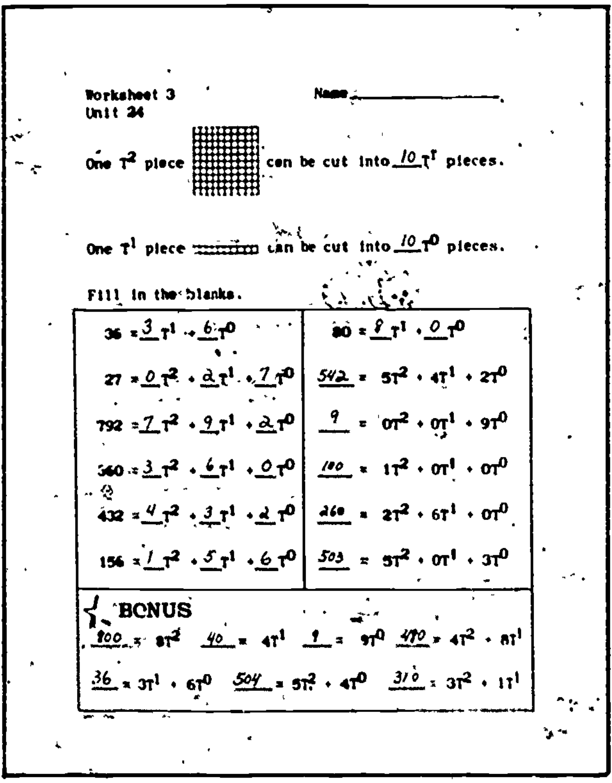
Worksheet 2
Unit 24

Name ____________

Fill in the blanks. In the last box, draw the T pieces for the number.

▦ = T^2 piece ▭ = T^1 piece □ = T^0 piece

$14 = \underline{1}\,T^1 + \underline{4}\,T^0$	$25 = \underline{2}\,T^1 + \underline{5}\,T^0$
$38 = \underline{3}\,T^1 + \underline{8}\,T^0$	$107 = \underline{1}\,T^2 + \underline{0}\,T^1 + \underline{7}\,T^0$
$480 = \underline{4}\,T^2 + \underline{8}\,T^1 + \underline{0}\,T^0$	$329 = \underline{3}\,T^2 + \underline{2}\,T^1 + \underline{9}\,T^0$
$132 = \underline{1}\,T^2 + \underline{3}\,T^1 + \underline{2}\,T^0$	



Worksheet 3
Unit 24

Name ____________

One T^2 piece ▦ can be cut into $\underline{10}$ T^1 pieces.

One T^1 piece ▭ can be cut into $\underline{10}$ T^0 pieces.

Fill in the blanks.

$36 = \underline{3}\,T^1 + \underline{6}\,T^0$	$80 = \underline{8}\,T^1 + \underline{0}\,T^0$
$27 = \underline{0}\,T^2 + \underline{2}\,T^1 + \underline{7}\,T^0$	$\underline{542} = 5T^2 + 4T^1 + 2T^0$
$792 = \underline{7}\,T^2 + \underline{9}\,T^1 + \underline{2}\,T^0$	$\underline{9} = 0T^2 + 0T^1 + 9T^0$
$360 = \underline{3}\,T^2 + \underline{6}\,T^1 + \underline{0}\,T^0$	$\underline{100} = 1T^2 + 0T^1 + 0T^0$
$432 = \underline{4}\,T^2 + \underline{3}\,T^1 + \underline{2}\,T^0$	$\underline{260} = 2T^2 + 6T^1 + 0T^0$
$156 = \underline{1}\,T^2 + \underline{5}\,T^1 + \underline{6}\,T^0$	$\underline{503} = 5T^2 + 0T^1 + 3T^0$

BONUS

$\underline{800} = 8T^2$ $\underline{40} = 4T^1$ $\underline{9} = 9T^0$ $\underline{480} = 4T^2 + 8T^1$

$\underline{36} = 3T^1 + 6T^0$ $\underline{504} = 5T^2 + 4T^0$ $\underline{310} = 3T^2 + 1T^1$

Lesson 2: ADDING AND REGROUPING WITH THE COMPUTER

In this lesson the children work in computer teams to solve addition problems, some of which involve regrouping. This lesson will take two class periods, one for each activity.

MATERIALS

-- for each child --

- set of T pieces cut from 5 copies of Sheet A and 1 copy of Sheet B
- scissors
- small plastic bag
- Worksheets 4 - 7

-- for each group --

- several sheets of paper

PROCEDURE

Activity A

Give each child five copies of Sheet A and one copy of Sheet B. The T pieces should be cut apart along the heavy lines. Each child should have 10 T^2 pieces, 18 T^1 pieces and 20 T^0 pieces in all. His T pieces should be stored in a plastic bag.

Divide the class into teams of three. Each team will set up its own computation machine consisting of a Computer, a Programmer and a Recorder. Ask the Programmer to turn to Worksheet 4 in his Student Manual and tear out the desk computer.

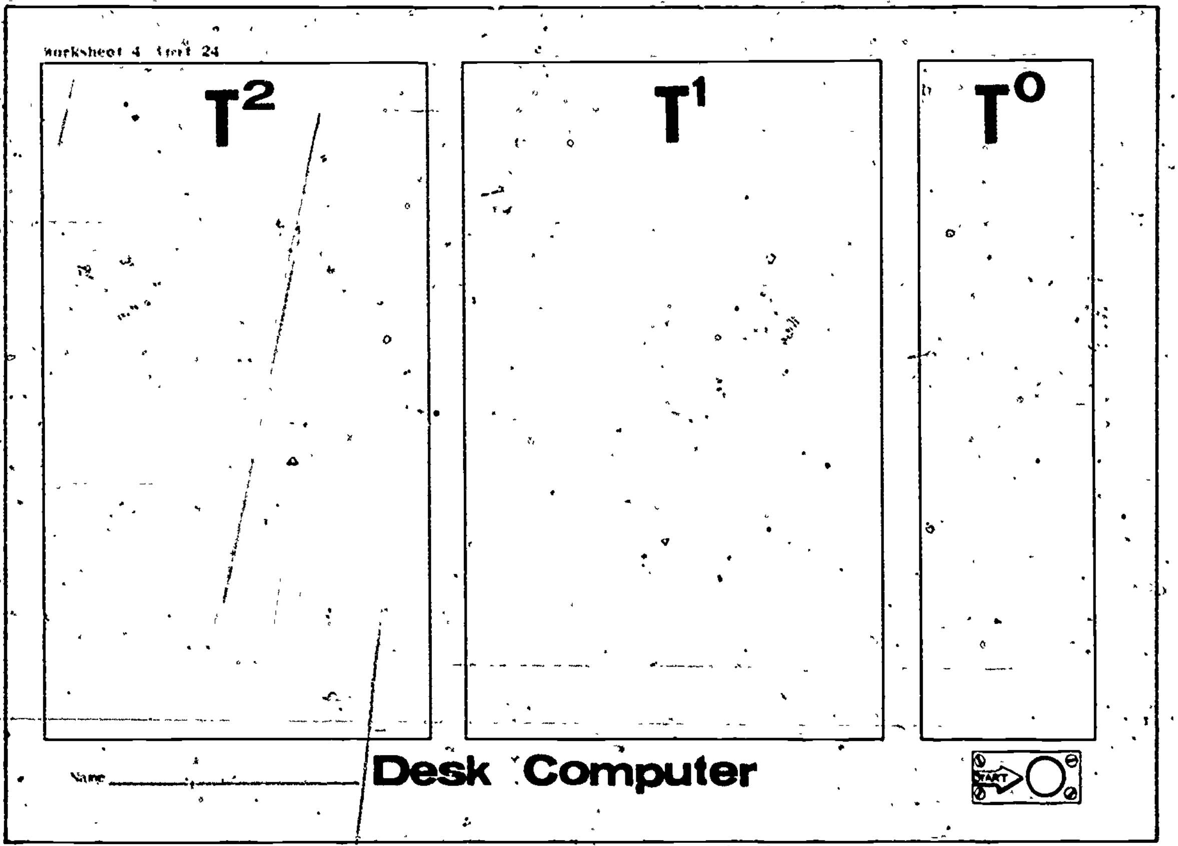
Explain the responsibilities of each assignment:

R: The Recorder writes the problem in both standard and T notation on a sheet of paper.

P: The Programmer puts the pieces representing the addends of the problem into the desk computer and presses the start button.

C: The Computer computes (counts the pieces if necessary) and tells the Recorder the number of T pieces he now has in each place.

R: The Recorder writes the answer, first in T notation and then in standard notation.



To demonstrate the proper procedure and recording techniques to the class, write this problem on the chalkboard in vertical form:

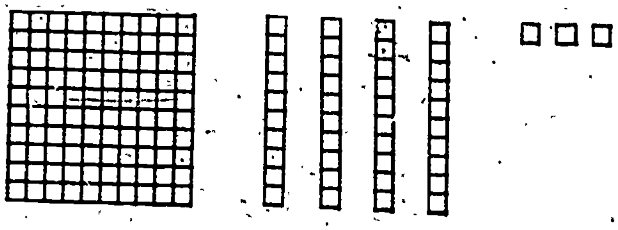
$$\begin{array}{r} 143 \\ \underline{+245} \end{array}$$

Choose one team to help with the demonstration while the other teams go through the problem at their desks. The children at the desks should check their work against the work of the demonstration team at each step. Allow time for discussing and correcting errors.

Step 1: The Recorder writes the problem and converts it to T notation. The Recorder on the demonstration team should write it on the chalkboard while the other Recorders write it on their paper.

$$\begin{array}{r} 143 = 1T^2 + 4T^1 + 3T^0 \\ +245 = 2T^2 + 4T^1 + 5T^0 \\ \hline \end{array}$$

Step 2: The Programmer gets T pieces from his plastic bag (the bank) to represent the first number, 143.



He feeds the pieces into their proper places in the desk computer. Then he gets the pieces that represent 245 and feeds them into the computer.

Step 3: The child who has the job of Computer counts the total number of pieces in each place and tells the Recorder how many he has of each kind of piece. For this problem it should be 8 T^0 pieces, 8 T^1 pieces and 3 T^2 pieces.

Step 4: The Recorder writes $3T^2 + 8T^1 + 8T^0$ under the T notation problem and puts 388 under the standard notation problem.

Give the teams two more problems that do not require regrouping. The team members should change jobs so that each child has a chance to do each job. Move about the room while the children are working and check their methods, work and answers. At the conclusion of this activity, select a Recorder to read his group's results to the class so that all can verify their answers. If some groups finish early, you may want to give them additional problems.

Activity B

Have each group work the problem, 246 + 325, and give you their answers. The answer they are likely to get is:

$$5T^2 + 6T^1 + 11T^0$$

If any team regrouped to get $5T^2 + 7T^1 + 1T^0$, ask the rest of the children to try to figure out how they got such an answer. If no team got the correct answer, ask someone to come up and write $5T^2 + 6T^1 + 11T^0$ in standard notation. He will probably write 5611. Explain that this answer is much too large and suggest that the children check their work.

Go over the pieces in each place of one team's computer with the class while the other teams check their computers. Someone should discover that 10 of the T^0 pieces can be exchanged for one T^1 piece with one T^0 piece left over. Put 10 of the T^0 pieces together to show this. Rewrite $5T^2 + 6T^1 + 11T^0$ as $5T^2 + 6T^1 + 1T^1 + 1T^0$. Insert parentheses around the $6T^1 + 1T^1$. Then add to get $5T^2 + 7T^1 + 1T^0$ or 571.

> WOULDN'T IT BE EASIER IF THE MACHINE WOULD DO THIS REGROUPING FOR US?

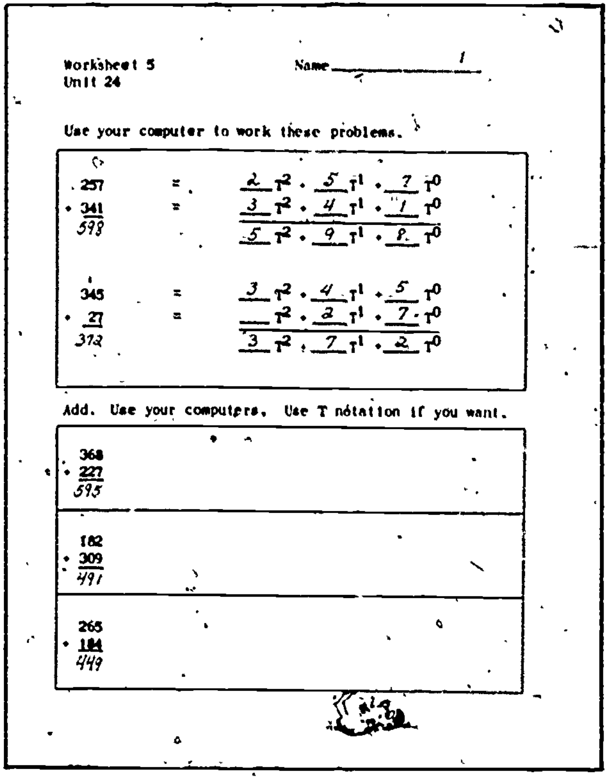
Remind the children that the machine will do what we tell it to do. Ask them to develop a rule for regrouping. It should be similar to the following rule:

> There may be no more than nine pieces in each place. If there are 10 or more pieces in a place, 10 pieces must be exchanged for one of the next larger pieces. The new piece must be entered into the proper place.

Have each team try adding 246 + 325 again and help them to regroup.

Let each team apply the regrouping rule to several problems such as 46 + 38, 132 + 49, 269 + 150. The children should change jobs for each problem. When the children are ready, let them try a problem requiring regrouping in both the T^0 and the T^1 places, for example, 49 + 76 or 98 + 23.

Review the rules that our computers must follow and have the teams complete Worksheets 5 and 6 using their desk computers. Each child should serve as Recorder to fill in his own worksheets. Worksheet 7 should be completed individually. It requires the children to build on their familiarity with the place value system. Some of the children may not be able to complete it. Do not press them. This material will be covered in Lesson 3.



Worksheet 5
Unit 24

Name 1

Use your computer to work these problems.

$$\begin{array}{rcl} 257 & = & \underline{2}\,T^2 + \underline{5}\,T^1 + \underline{7}\,T^0 \\ +\ \underline{341} & = & \underline{3}\,T^2 + \underline{4}\,T^1 + \underline{1}\,T^0 \\ 598 & & \underline{5}\,T^2 + \underline{9}\,T^1 + \underline{8}\,T^0 \end{array}$$

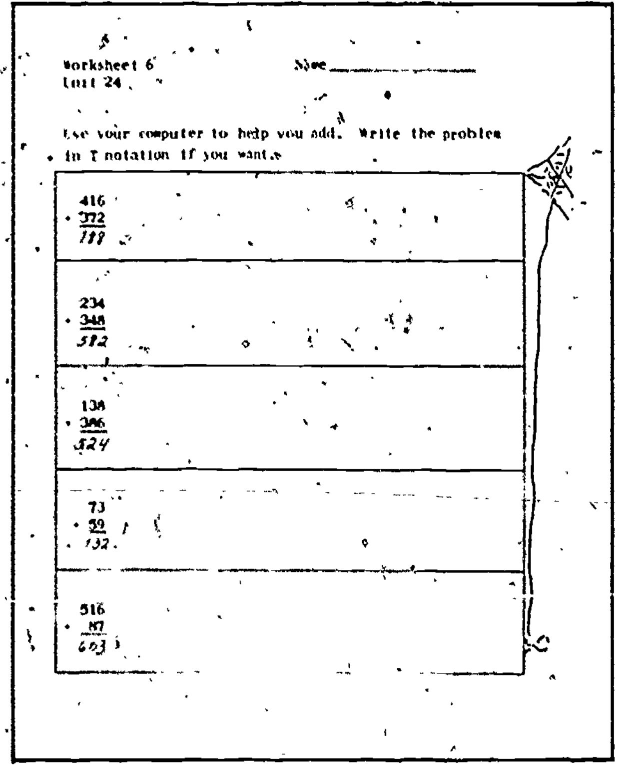
$$\begin{array}{rcl} 345 & = & \underline{3}\,T^2 + \underline{4}\,T^1 + \underline{5}\,T^0 \\ +\ \underline{27} & = & \underline{\quad}\,T^2 + \underline{2}\,T^1 + \underline{7}\,T^0 \\ 372 & & \underline{3}\,T^2 + \underline{7}\,T^1 + \underline{2}\,T^0 \end{array}$$

Add. Use your computers. Use T notation if you want.

368
+ 227
595

182
+ 309
491

265
+ 184
449



Worksheet 6
Unit 24

Name

Use your computer to help you add. Write the problem in T notation if you want.

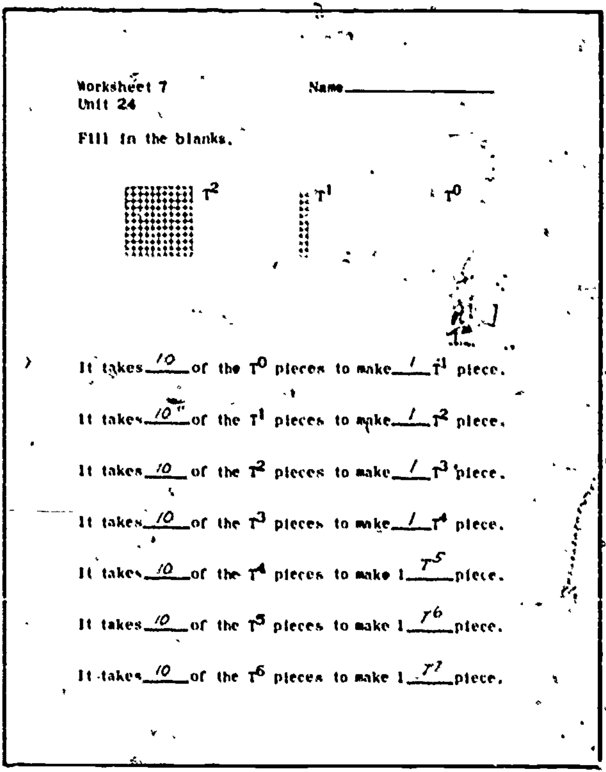
416
+ 372
788

234
+ 348
582

138
+ 386
524

73
+ 59
132

516
+ 87
603



Worksheet 7
Unit 24

Name

Fill in the blanks.

It takes 10 of the T^0 pieces to make 1 T^1 piece.

It takes 10 of the T^1 pieces to make 1 T^2 piece.

It takes 10 of the T^2 pieces to make 1 T^3 piece.

It takes 10 of the T^3 pieces to make 1 T^4 piece.

It takes 10 of the T^4 pieces to make 1 T^5 piece.

It takes 10 of the T^5 pieces to make 1 T^6 piece.

It takes 10 of the T^6 pieces to make 1 T^7 piece.

Lesson 3: PLACE VALUE

This lesson develops the idea of place value as a repeating pattern generated by the rule that each place is ten times larger than the place to its right. In the first activity the children regroup the T^0, T^1 and T^2 places and see a need for pieces to the left of T^2. In Activity B the children tape together T pieces to produce values to the left of T^2. Then these pieces are taped to the wall, providing a visual representation of the relationships among T places. The last three activities include worksheets and games that provide practice with simple addition facts. Spend two days on this lesson. Activities A and B should be done on the same day.

MATERIALS

- labels marked T^0, T^1, T^2 (from Lesson 1) and T^3, T^4, and T^5
- masking tape
- wall or chalkboard space about 2 yards wide

-- for each child --

- plastic bag with set of T pieces
- Worksheets 8 - 12

PROCEDURE

Activity A

Set up a three-desk computer. Assign a child to each desk plus a Programmer and a Recorder to operate it. Give them the problem, 562 + 659. The Recorder writes the problem in T notation and the Programmer feeds the pieces into the computer. The children playing the role of computer must add up the number of pieces in each place and exchange ten of their pieces for the next larger piece from the Programmer.

In this problem the child in the T^0 place will have 11 pieces. He exchanges 10 of these for a T^1 piece and then gives the T^1 piece to the child at the T^1 place. The child at the T^1 place exchanges ten of his pieces and passes the T^2 piece to the T^2 place. When the student in the T^2 place finds that he must exchange ten of his pieces for the next larger piece, he will be faced with the question of what to use for the next larger piece.

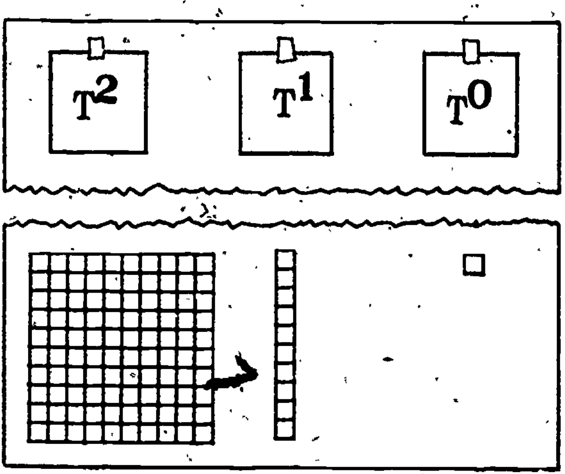
(If the children at the T^0 and T^1 places had trouble regrouping, go through a few more problems before explaining what the child at the T^2 place should do. The children should not try to regroup the T^2 pieces until they can do it for T^0 and T^1.)

Ask the class to help the child in the T^2 place with his regrouping. The children should realize that they have no T^3 piece. Raise the question of what this piece should look like and ask the children if they would like to make one. (They will do this in the next activity. Save the desk labels for Activity B.)

Activity B

You will need masking tape, place value labels for T^0 through T^5, and wall or chalkboard space about two-yards wide. Each child should have his bag with a set of T pieces.

Tape a T^0 piece to the wall or chalkboard and ask how this place should be labeled. Tape the T^0 sign above the piece. Ask the children what piece belongs in the next place, and tape a T^1 piece to the left of the T^0 piece. Label the T^1 place. Have the children suggest which piece goes in the next place and tape a T^2 piece there. Tape the T^2 sign above it.



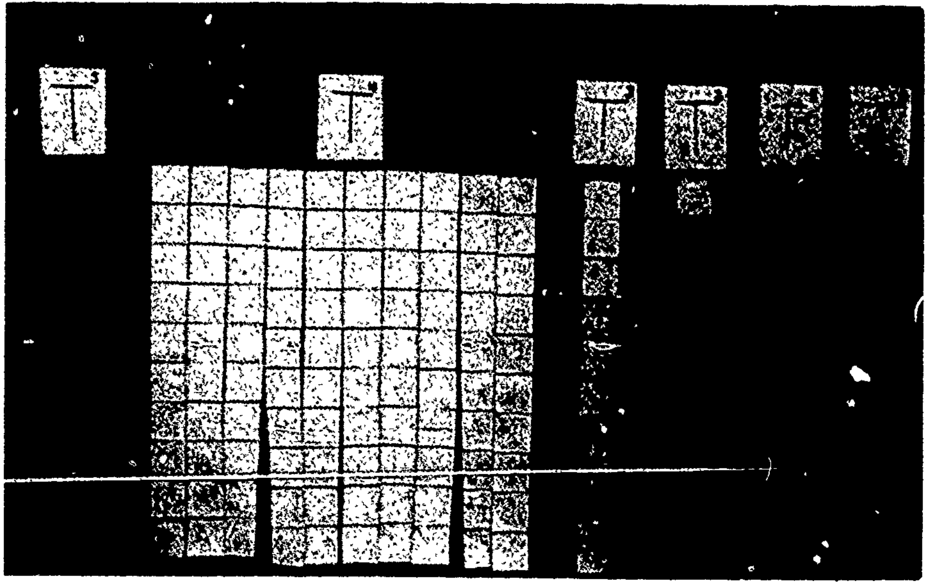
WHAT WOULD A PIECE FOR THE NEXT PLACE LOOK LIKE? HOW MANY T^0 PIECES DID IT TAKE TO MAKE A T^1 PIECE? (10.) HOW MANY T^1 PIECES DID IT TAKE TO MAKE A T^2 PIECE? (10.) SO HOW MANY T^2 PIECES WILL IT TAKE TO MAKE A PIECE FOR THE NEXT PLACE? (10.)

Tape together 10 T^2 pieces to make a T^3 piece and put it up to the left of the T^2 piece. Ask the class how this should be labeled, and tape the label above it.

> WHAT WOULD WE CALL THE NEXT PLACE? (T^4.) WHAT WOULD A T^4 PIECE LOOK LIKE? LET'S TRY TO MAKE ONE.

Have the children work on the floor in groups of four or five. Each child should bring his T pieces. Pass around a roll of masking tape and scissors so that each group can cut off nine two-inch pieces. Have the children take their T^2 pieces out of the bag, lay 10 of them upside down on the floor and tape them together to make a T^3 piece. As soon as a group finishes a T^3 piece, have them bring it to you so that you can tape it to the wall. Several groups may have to make two T^3 pieces, so that there will be 10 T^3 pieces in all, enough to make a T^4 piece.

When the T^4 piece is completed, tape the T^4 label above it. Your T^4 piece should be 50 inches high and 50 inches wide.



HOW WOULD THE NEXT PIECE BE MADE? (By taping 10 T^4 pieces in a row.) WHAT SHOULD THIS NEXT PLACE BE LABELED? (Tape the T^5 label to the left of the T^4 label.)

Ask the children if they could continue this pattern and whether the place pattern could go on in the other direction (to the right of T^0). Let them speculate about what one of these pieces might look like. If some of the children are interested, let them explore this question, with your help, during free time. They should find that there is no limit to the place values in either direction. If one T^0 piece were cut into 10 equal strips, each of those strips would represent a T^{-1} piece. In later units or grades the children may learn that $T^{-1} = 10^{-1} = \frac{1}{10}$ and $T^{-3} = 10^{-3} = \frac{1}{1000}$.

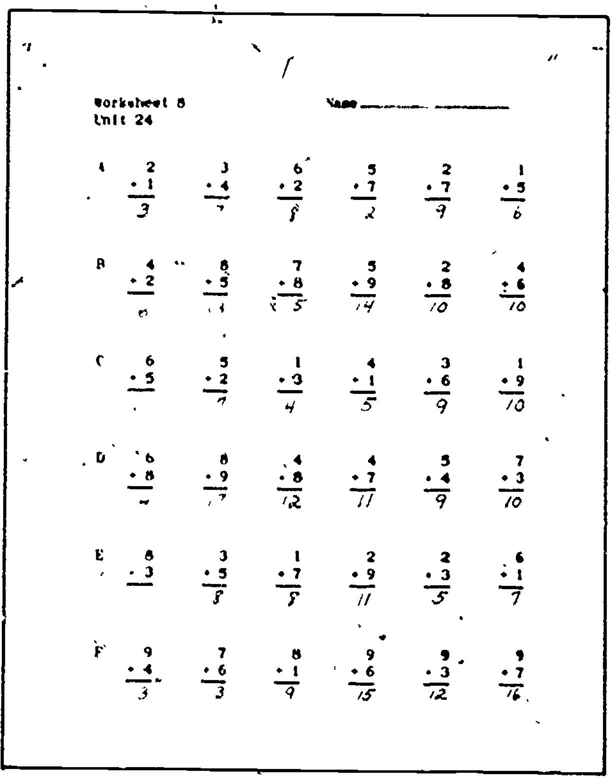
Any children who did not complete Worksheet 7 in the previous lesson should do so now.

Activity C

Tell the children that when a real computer works it does more than our desk models. It stores information in its memory bank. This memory bank is, in some ways, like our brains. We can store facts in our brains and remember them when we need to. We are going to test our own computers, our brains, to see how many addition facts they have stored.

Have the children complete Worksheet 8 without the computer if they can. Although the worksheet includes most of the addition facts, you may want to make supplementary worksheets which reverse the order of the addends and cover problems like 4 + 4, that are not included.

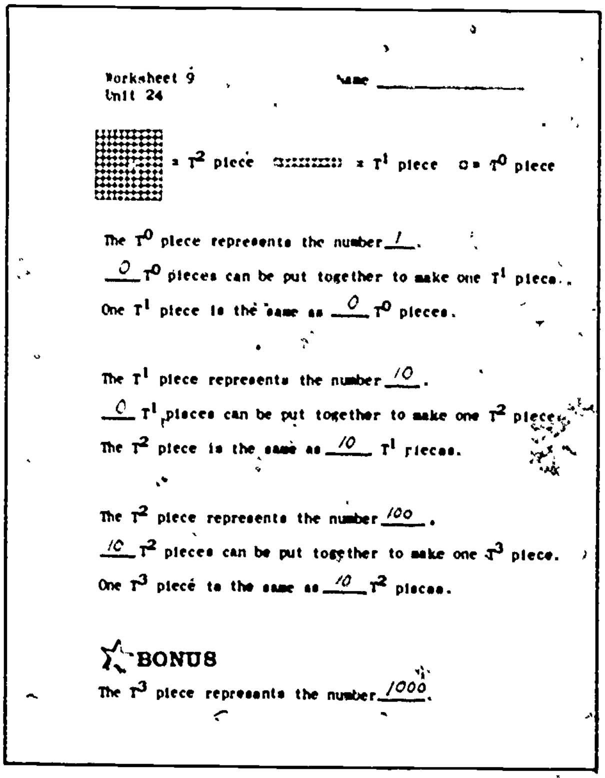
Worksheets 9, 10, 11 and 12 involve word problems illustrating regrouping. Assign these according to the reading and mathematics skills of your class. You may want to set aside time to help children who have reading difficulty.



Worksheet 8
Unit 24

Name ____________

A	$\begin{array}{r} 2 \\ +1 \\ \hline 3 \end{array}$	$\begin{array}{r} 3 \\ +4 \\ \hline 7 \end{array}$	$\begin{array}{r} 6 \\ +2 \\ \hline 8 \end{array}$	$\begin{array}{r} 5 \\ +7 \\ \hline 2 \end{array}$	$\begin{array}{r} 2 \\ +7 \\ \hline 9 \end{array}$	$\begin{array}{r} 1 \\ +5 \\ \hline 6 \end{array}$
B	$\begin{array}{r} 4 \\ +2 \\ \hline 6 \end{array}$	$\begin{array}{r} 8 \\ +5 \\ \hline 13 \end{array}$	$\begin{array}{r} 7 \\ +8 \\ \hline 15 \end{array}$	$\begin{array}{r} 5 \\ +9 \\ \hline 14 \end{array}$	$\begin{array}{r} 2 \\ +8 \\ \hline 10 \end{array}$	$\begin{array}{r} 4 \\ +6 \\ \hline 10 \end{array}$
C	$\begin{array}{r} 6 \\ +5 \\ \hline \end{array}$	$\begin{array}{r} 5 \\ +2 \\ \hline 7 \end{array}$	$\begin{array}{r} 1 \\ +3 \\ \hline 4 \end{array}$	$\begin{array}{r} 4 \\ +1 \\ \hline 5 \end{array}$	$\begin{array}{r} 3 \\ +6 \\ \hline 9 \end{array}$	$\begin{array}{r} 1 \\ +9 \\ \hline 10 \end{array}$
D	$\begin{array}{r} 6 \\ +8 \\ \hline \end{array}$	$\begin{array}{r} 8 \\ +9 \\ \hline 17 \end{array}$	$\begin{array}{r} 4 \\ +8 \\ \hline 12 \end{array}$	$\begin{array}{r} 4 \\ +7 \\ \hline 11 \end{array}$	$\begin{array}{r} 5 \\ +4 \\ \hline 9 \end{array}$	$\begin{array}{r} 7 \\ +3 \\ \hline 10 \end{array}$
E	$\begin{array}{r} 8 \\ +3 \\ \hline \end{array}$	$\begin{array}{r} 3 \\ +5 \\ \hline 8 \end{array}$	$\begin{array}{r} 1 \\ +7 \\ \hline 8 \end{array}$	$\begin{array}{r} 2 \\ +9 \\ \hline 11 \end{array}$	$\begin{array}{r} 2 \\ +3 \\ \hline 5 \end{array}$	$\begin{array}{r} 6 \\ +1 \\ \hline 7 \end{array}$
F	$\begin{array}{r} 9 \\ +4 \\ \hline 3 \end{array}$	$\begin{array}{r} 7 \\ +6 \\ \hline 3 \end{array}$	$\begin{array}{r} 8 \\ +1 \\ \hline 9 \end{array}$	$\begin{array}{r} 9 \\ +6 \\ \hline 15 \end{array}$	$\begin{array}{r} 9 \\ +3 \\ \hline 12 \end{array}$	$\begin{array}{r} 9 \\ +7 \\ \hline 16 \end{array}$



Worksheet 9
Unit 24

Name ____________

▦ = T^2 piece ▭ = T^1 piece ○ = T^0 piece

The T^0 piece represents the number 1.

0 T^0 pieces can be put together to make one T^1 piece.

One T^1 piece is the same as 0 T^0 pieces.

The T^1 piece represents the number 10.

0 T^1 pieces can be put together to make one T^2 piece.

The T^2 piece is the same as 10 T^1 pieces.

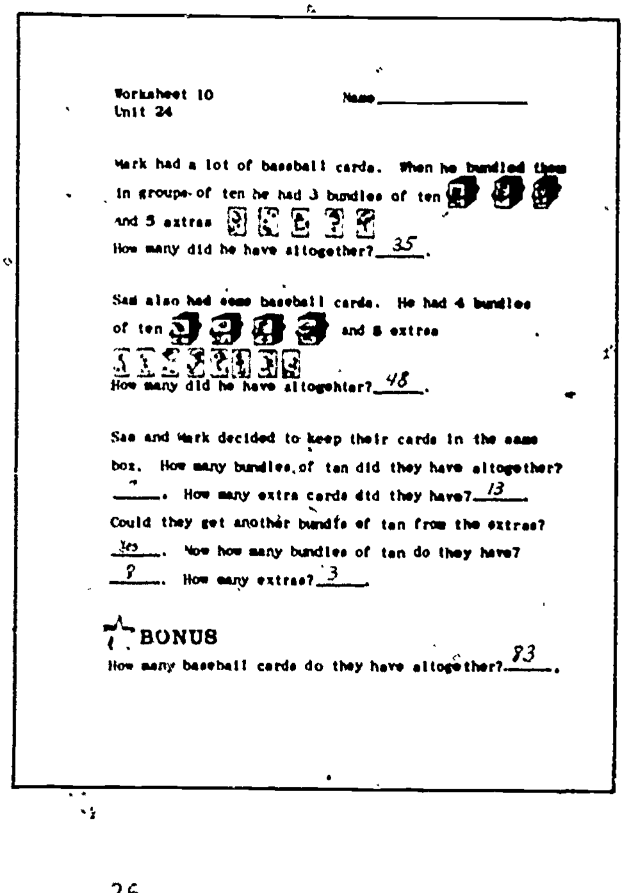
The T^2 piece represents the number 100.

10 T^2 pieces can be put together to make one T^3 piece.

One T^3 piece is the same as 10 T^2 pieces.

☆ **BONUS**

The T^3 piece represents the number 1000.



Worksheet 10
Unit 24

Name ____________

Mark had a lot of baseball cards. When he bundled them in groups of ten he had 3 bundles of ten and 5 extras

How many did he have altogether? 35.

Sam also had some baseball cards. He had 4 bundles of ten and 8 extras

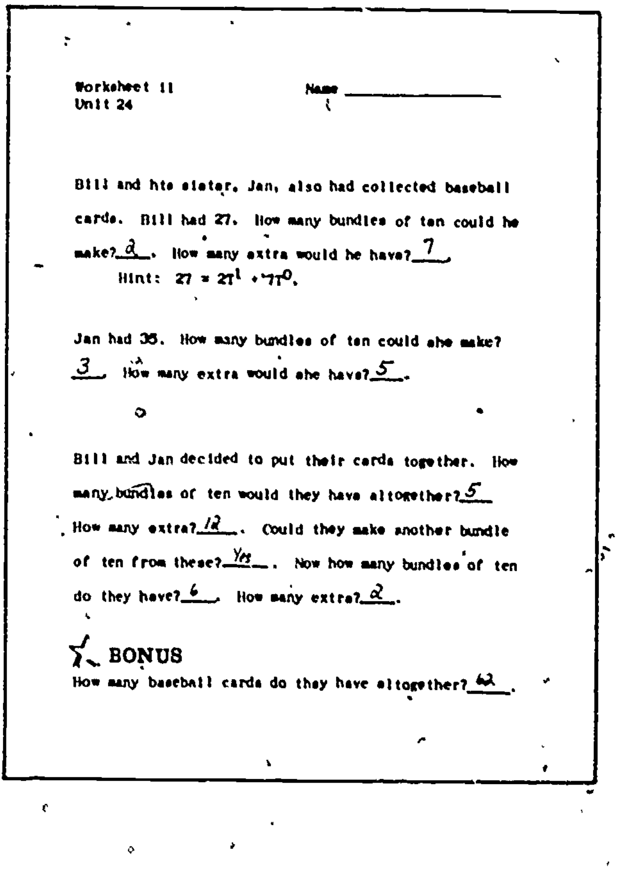
How many did he have altogehter? 48.

Sam and Mark decided to keep their cards in the same box. How many bundles of ten did they have altogether? ____. How many extra cards did they have? 13.

Could they get another bundle of ten from the extras? Yes. Now how many bundles of ten do they have? 8. How many extras? 3.

☆ **BONUS**

How many baseball cards do they have altogether? 83.



Worksheet 11
Unit 24

Name ____________

Bill and his sister, Jan, also had collected baseball cards. Bill had 27. How many bundles of ten could he make? 2. How many extra would he have? 7.

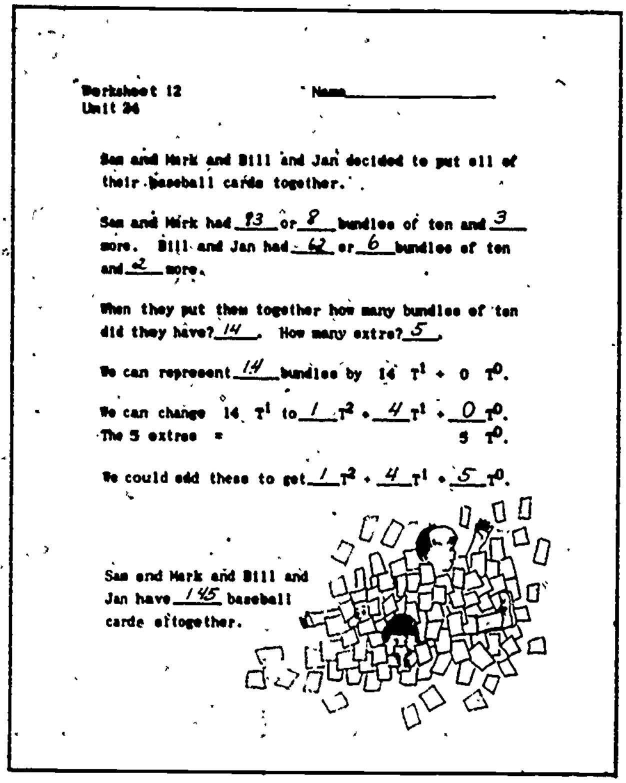
Hint: $27 = 2T^1 + 7T^0$.

Jan had 35. How many bundles of ten could she make? 3. How many extra would she have? 5.

Bill and Jan decided to put their cards together. How many bundles of ten would they have altogether? 5. How many extra? 12. Could they make another bundle of ten from these? Yes. Now how many bundles of ten do they have? 6. How many extra? 2.

☆ **BONUS**

How many baseball cards do they have altogether? 62.



Worksheet 12
Unit 24

Name________________

Sam and Mark and Bill and Jan decided to put all of their baseball cards together.

Sam and Mark had 83 or 8 bundles of ten and 3 more. Bill and Jan had 62 or 6 bundles of ten and 2 more.

When they put them together how many bundles of ten did they have? 14. How many extra? 5.

We can represent 14 bundles by 14 T^1 + 0 T^0.

We can change 14 T^1 to 1 T^2 + 4 T^1 + 0 T^0.
The 5 extras = 5 T^0.

We could add these to get 1 T^2 + 4 T^1 + 5 T^0.

Sam and Mark and Bill and Jan have 145 baseball cards altogether.

Activity D: Relay Game

This game is a favorite with the children and should be played all year. It can be varied to accomodate more difficult addition problems as well as subtraction, multiplication and division facts.

Have the children sit at their desks. The child in the first desk of a vertical column of desks stands beside the child in the second desk of that column. You present a simple problem (such as 5 + 5) to these two children. The child of this pair who calls out the answer first is the winner and moves to stand beside the child in the third desk. You present a simple problem to this pair of children. The one who answers first gets to compete with the child in the fourth desk and so on. The object of the game is to see if someone can move all the way around the room and back to his own desk. To do so, he would always have to answer every problem faster than the child he is competing with.

Activity E: Mental Arithmetic

Many teachers have found this review technique to be highly successful. Do the activity for one or two minutes each day.

Tell the class that you are going to give them problems to compute with their own computers, their brains. Start out with simple problems such as 4 + 7, and work up to problems such as 12 + 8 or 3 + 5 + 7. Ask the children to raise their hands when they have the answer, and call on one child to give the answer. Look for speed and accuracy.

Lesson 4: ADDITION WITH A NUMERAL COMPUTER

In this lesson a classroom computer is designed which adds numerals rather than T pieces. The children devise rules by which the new computer can perform regrouping operations. Then the steps in computer addition are recorded in chart form. This activity provides a transition between adding with T pieces and adding in vertical form.

MATERIALS

- 2 sets of T^2, T^1, T^0 desk labels
- demonstration T pieces
- several hundred pieces of paper (about 2" x 2")
- Worksheets 13 - 17

PREPARATION

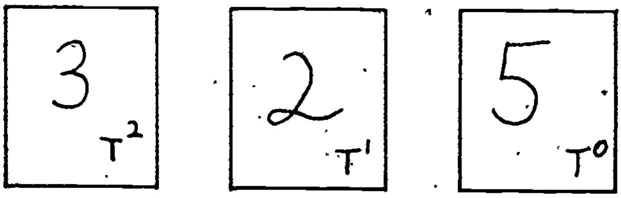
Before the lesson begins, prepare several hundred slips of paper by cutting up larger sheets or providing scratch pads.

PROCEDURE

Activity A

Set up and label three desks for a computer as you did in Lesson 3. Assign a Recorder, a Programmer and three children to sit at the desks. Have the computer work an addition problem such as 432 + 516 using T pieces. After the problem is completed, discuss with the class the value of a more efficient computer that doesn't need to use T pieces. Suggest trying to make one that uses just numerals.

Arrange another three-desk computer next to the first one. Label the desks T^0, T^1, and T^2, put several pieces of paper on each desk and assign a child to sit at each desk. Write the following problem in vertical form on the chalkboard: 325 + 462. Have the Programmer put 325 into the first computer with T pieces. Ask the children how 325 might be put into the numeral computer. If no child suggests it, propose that a Programmer write the numerals 3, 2, and 5 on slips of paper. Choose another Programmer to come to the numeral computer and fill out the slips. Each slip should be labeled with the proper place notation. The Programmer then feeds these slips into the numeral computer at the proper places.



Have the T computer Programmer enter 462 into his computer with T pieces while the numeral computer Programmer feeds in numeral slips for 462.

Without going through the procedure, review what the person in each place of the T computer does with his pieces:

1. He combines his pieces,
2. Checks to see if he has more than nine pieces,
3. Exchanges 10 for a T piece of the next larger size if necessary, and
4. Tells how many pieces he has when the Recorder calls for the answer.

Ask the children to suggest how the new computer might combine the numbers in each place. (Each child can combine his numbers mentally, using the addition facts, and write the answer on a slip of paper.) For example, the child at the T^0 place would add 5 and 2, and mark a slip of paper with the total (7) and the place value (T^0). He would hand it to the Programmer when it is called for. The Programmer takes each slip to the Recorder who then writes each numeral on the chalkboard. (Choose a Recorder for the numeral computer.) Similarly, the children at the T^1 and T^2 positions complete slips for the Programmer and the Recorder writes these numerals on the board. Let the children at both computers complete the problem.

Now tell the children that the computers are going to have a race. Hand each Recorder a slip of paper with this problem written on it: 487 + 512. Both Recorders start at the same time. The Recorder for the T computer writes the problem in T notation on the chalkboard while the other Recorder writes it in standard vertical form.

The Programmer for each computer starts his job as soon as he sees the problem. The T computer Programmer counts out the pieces and feeds them into the correct places. The numeral computer Programmer fills out the slips and hands them to the students at the computer desks. Each child at the T computer desks counts his total number of pieces and tells the Programmer this number. The Programmer tells the Recorder who must write it in T notation and then convert it to standard notation. At the same time the children at the numeral computer add the numbers on their slips, fill out new slips showing the sum and the place value. They hand these slips to the Programmer, who then carries them to the Recorder. The Recorder writes the total in standard notation. The other children in the classroom act as Repairmen to catch mistakes and also watch to see which computer finishes first. The work of each Recorder should look like this:

$$\begin{array}{r} 487 \\ +512 \\ \hline 999 \end{array} \qquad \begin{array}{r} 4T^2 + 8T^1 + 7T^0 \\ +5T^2 + 1T^1 + 2T^0 \\ \hline 9T^2 + 9T^1 + 9T^0 \end{array} = 999$$

The class should compare the efficiency of the two machines and do another problem, if necessary, to confirm the speed of the numeral computer. Ask why the numeral computer is faster (because it is making use of the addition facts that each child has stored in his brain).

Have the children summarize the rules that each place of the numeral computer is following when it adds. These should include: each place accepts the proper numerals, adds them, writes the sum on a slip of paper, and feeds it out on demand.

Write on the chalkboard a problem that involves regrouping: 274 + 119. Ask the children to develop a rule for the numeral computer to deal with this situation. This rule might be:

> No place may feed out a numeral larger than 9. If the sum of its numerals is more than 9, two slips must be filled out, one for each of the place values in the sum. The larger place value slip must be given to the next place to the left.

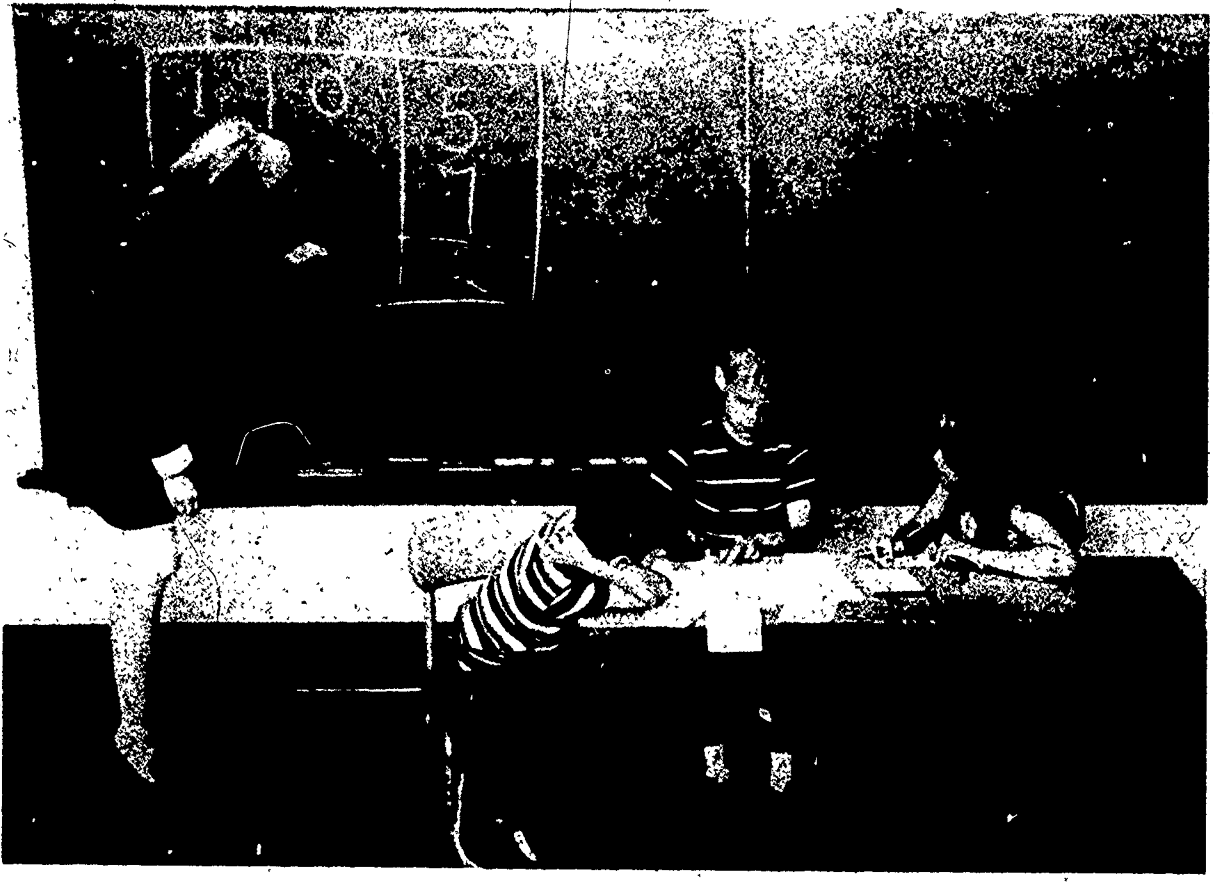
Tell the children at the T notation computer that they will now become parts of a numeral computer. Have the children in each numeral computer work the problem you wrote on the board (274 + 119). They will find that the sum, $13T^0$, must be written as $1T^1 + 3T^0$. For each computer the $1T^1$ slip is passed to the T^1 place. The T^1 place must add this slip to its other two ($7T^1$ and $1T^1$) and fill out a slip for the proper sum ($9T^1$).

Choose new children to sit at each computer. Give both computers the same problem, one which requires regrouping in the T^0 and T^1 places. Remind the children to write and label a slip for each of the places in the sum, and to hand the proper slips to the place to their left.

If you feel the children are ready to work on their own, they can now form groups of three. Give each group 40 or 50 slips of paper. The groups are to solve addition problems using their desk computer and numeral slips. Write a few sample problems such as $\begin{array}{r}208,\\ +\underline{463}\end{array}$ $\begin{array}{r}29,\\ +\underline{92}\end{array}$ $\begin{array}{r}560\\ +\underline{377}\end{array}$ on the chalkboard and circulate to check that the numeral slips are being marked and regrouped properly.

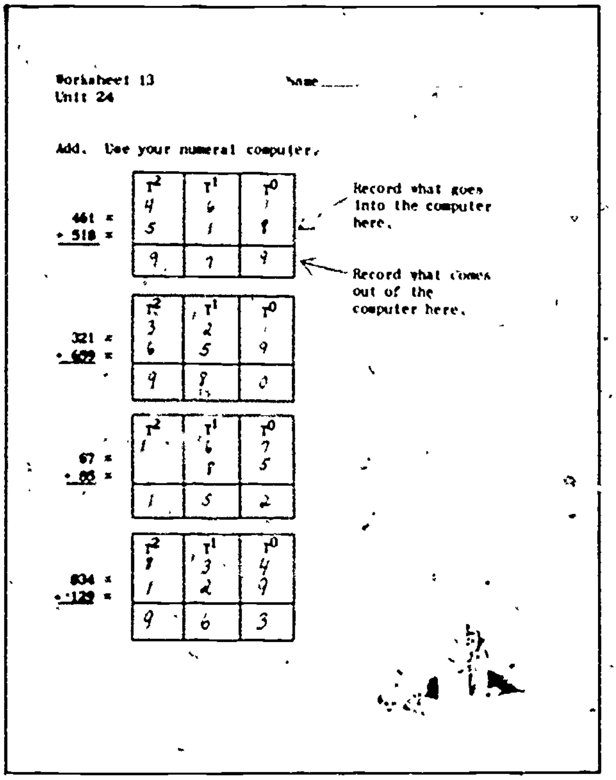
Activity B

Draw a chart on the chalkboard to introduce a system for recording the steps that the computer goes through when it adds. Write a problem beside it.



Organize the children into groups of three and have one child in each group take out his desk computer. You will act as Recorder for all groups. One child in each group fills out slips for the first addend and another child fills out slips for the second addend. These slips are then put in the correct places of the desk computer. Each child acts as one place of the computer, adding the numerals in his place. Starting at the T^0 place, call on a different group for each place to tell you its answer. As the children tell you the sum for each place, write these on the chart. When a number is regrouped and a slip is passed to the next place, enter a 1 in the next place on the chart. Your chart should finally look like this.

	T^2	T^1	T^0
		1	
105	1	0	5
+237	2	3	7
	3	4	2



Worksheet 13
Unit 24

Name____

Add. Use your numeral computer.

461 =
+ 518 =

T^2	T^1	T^0
4	6	1
5	1	8
9	7	9

Record what goes into the computer here.

Record what comes out of the computer here.

321 =
+ 659 =

T^2	T^1	T^0
3	2	1
6	5	9
9	8	0

67 =
+ 85 =

T^2	T^1	T^0
1	6	7
	8	5
1	5	2

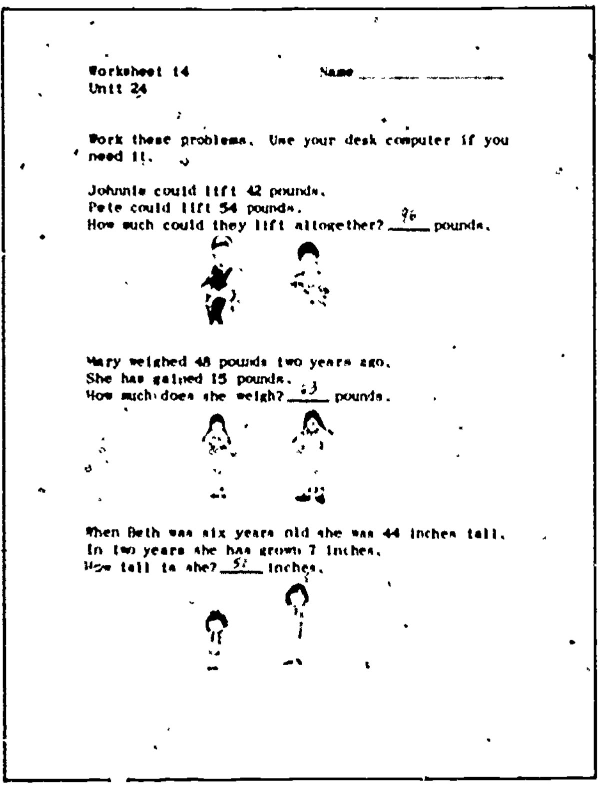
834 =
+ 129 =

T^2	T^1	T^0
8	3	4
1	2	9
9	6	3

Do as many problems with the numeral computer and the chart as you feel are necessary for the children to understand the chart. Choose new children to run the computer each time.

Have the children turn to Worksheet 13. If they need help let them work a few problems with friends. They should complete most of the worksheet by themselves. The children should do Worksheets 14 - 17 individually on their own time. Some children may need help in reading some of the problems.

Note: Save the desk computers for Lesson 7.



Worksheet 14
Unit 24

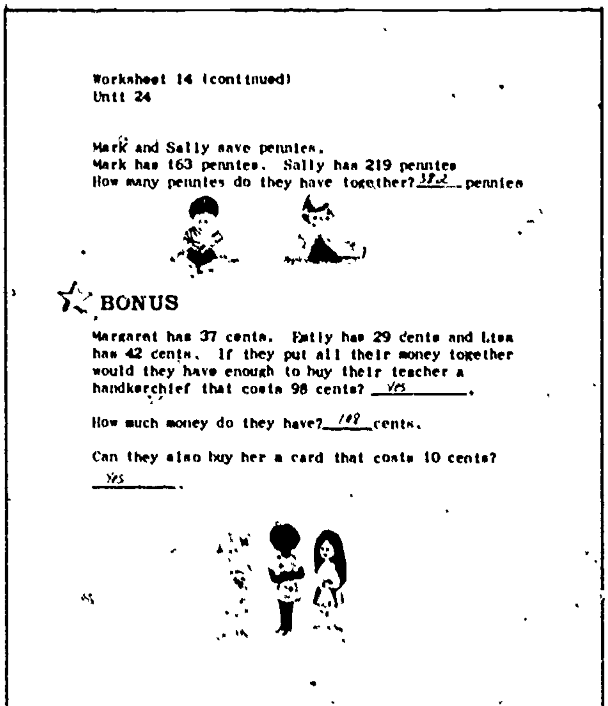
Name ________

Work these problems. Use your desk computer if you need it.

Johnnie could lift 42 pounds.
Pete could lift 54 pounds.
How much could they lift altogether? 96 pounds.

Mary weighed 48 pounds two years ago.
She has gained 15 pounds.
How much does she weigh? 63 pounds.

When Beth was six years old she was 44 inches tall.
In two years she has grown 7 inches.
How tall is she? 51 inches.



Worksheet 14 (continued)
Unit 24

Mark and Sally save pennies.
Mark has 163 pennies. Sally has 219 pennies
How many pennies do they have together? 382 pennies

☆ BONUS

Margaret has 37 cents. Emily has 29 cents and Lisa has 42 cents. If they put all their money together would they have enough to buy their teacher a handkerchief that costs 98 cents? Yes.

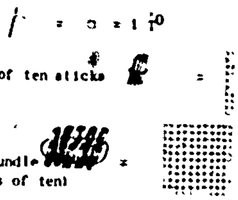
How much money do they have? 108 cents.

Can they also buy her a card that costs 10 cents?
Yes.

Worksheet 15 Name______________
Unit 24

The kids at school decided to see how many popsicle sticks they could collect. Bill's and Mary's third grade class collected 475 sticks. That would be __4__ giant bundles (T^2 bundles), ______ T^1 bundles and __5__ extras.

Let's compare our sticks with the pieces we used in our computer.



one stick = 1 T^0

one bundle of ten sticks = 1 T^1

one giant bundle (ten bundles of ten) = 1 T^2

We could represent 4 giant bundles by 4 T^2.

We could represent 7 bundles of ten by 7 T^1.

We could represent 5 extra sticks by 5 T^0.

Worksheet 16 Name______________
Unit 24

The class across the hall collected 319 sticks. This would make __3__ giant bundles or __3__ T^2, __1__ bundle(s) of ten or __1__ T^1, and __9__ extras or __9__ T^0.

One class had $4T^2 + 7T^1 + 5T^0$.

The other had $3T^2 + 1T^1 + 9T^0$.

That equals __7__ T^2 bundles, __8__ T^1 bundles and __14__ T^0 extras.

Will the 14 extra sticks make another bundle of ten? __Yes__ Then we will have altogether __7__ giant bundles (T^2), __9__ bundles of ten (T^1) and __4__ extras (T^0). In T pieces this would be __7__ T^2 + __9__ T^1 + __4__ T^0 or __794__ altogether.

Worksheet 17 Name______________
Unit 24

For experts only!

Mr. Walters, the popcicle man said that if the third grade classes could collect enough popcicle sticks to make a super bundle, he would give each third grader a popcicle.

Everyone wanted to know how big a super bundle is.

A small bundle (T^1) is __0__ sticks.

A giant bundle (T^2) is __0__ small bundles or __100__ sticks.

A super bundle (T^3) is __0__ giant bundles or __100__ small bundles or ______ sticks.

A small bundle (10 sticks) = __T^1__

A giant bundle (100 sticks) = __T^2__

A super bundle (1000 sticks) = __T^3__

Worksheet 17 (continued)
Unit 24

Two of the classes together had almost 800 sticks or __8__ giant bundles (T^2 bundles).

The other third grade class had 317 sticks. This would be __3__ T^2 (giant bundles), __1__ T^1 (bundles of ten) and __7__ T^0 (extras).

Did the three classes together have at least 10 giant bundles? __Yes__.

Did they have enough sticks to make 1 super bundle? __Yes__.

The first two classes had exactly 794 sticks.
The third class had 317 sticks.
How many did they have altogether? __1111__.

Lesson 5: DEVELOPING THE VERTICAL FORM FOR ADDITION

In this lesson the numeral computer technique for addition is converted to the vertical form for addition.

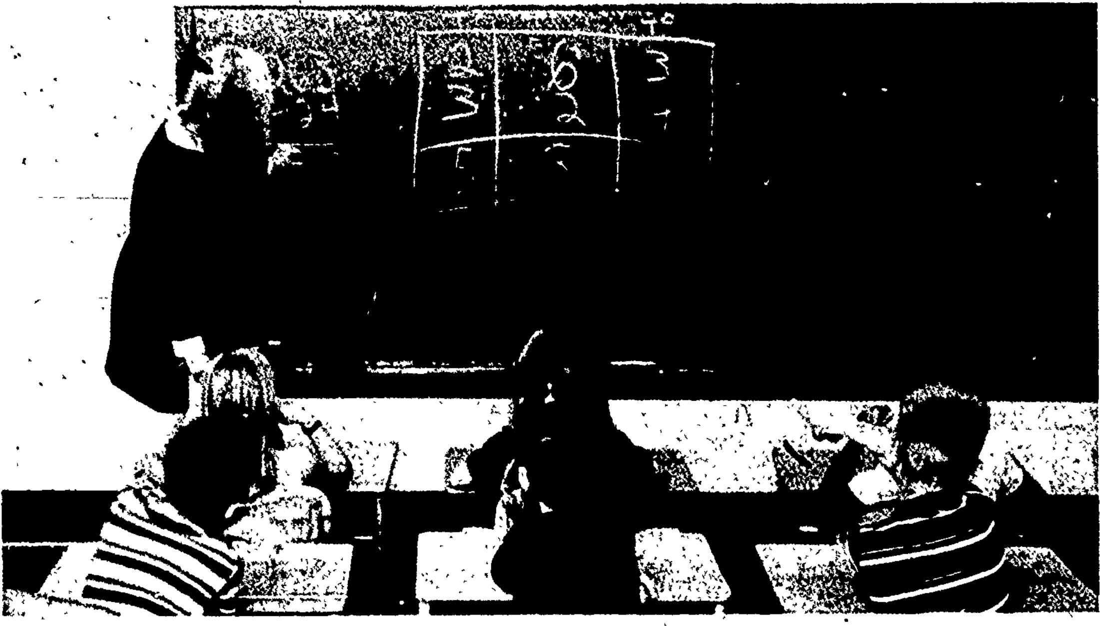
MATERIALS

- Worksheets 18-20

PROCEDURE

Discuss with the class the fact that we often need to do addition problems but we do not always have access to a computer. Develop the idea that each child can be his own computer by using his own memory device (his brain) and by following certain rules. Put the problem 263 + 324 in vertical form on the board and draw a computer chart next to it.

Enter the numerals in the chart as you review their place values in the standard algorithm. Then add the numerals in each place of both problems, stressing their place value again. The completed record should look like the one in the photograph below.



Work as many problems using the standard algorithm and the chart as you feel the children need to understand that the numerals of each addend on the chart are in the same place and same relation as they are when they are written in the usual vertical form.

CAN WE SHOW WHAT WE DO WHEN WE ADD WITHOUT USING THE CHART? LET'S TRY.

Put this problem on the chalkboard:

$$\begin{array}{r} 265 \\ +722 \\ \hline \end{array}$$

WHAT NUMERAL WOULD GO INTO THE T^0 PLACE IN OUR FINAL ANSWER? (7.) WHY? (5 + 2 = 7.)

WHAT NUMERAL WOULD GO INTO THE T^1 PLACE IN OUR FINAL ANSWER? (8.) WHY? (6 + 2 = 8.)

WHAT DOES THIS ADDITION SENTENCE MEAN IN TERMS OF PLACE VALUE? ($6T^1 + 2T^1 = 8T^1$ or 60 + 20 = 80.)

WHAT NUMERAL WOULD GO INTO THE T^2 PLACE IN OUR FINAL ANSWER? (9.) WHY? (2 + 7 = 9.)

WHAT DOES THIS MEAN IN TERMS OF PLACE VALUE? ($2T^2 + 7T^2 = 9T^2$ or 200 + 700 = 900.)

WHAT IS OUR ANSWER? (987.)

Notice that the order in which we add the columns does not matter when we do not have to regroup. But, when regrouping is necessary, we should add the T^0 column first, then the T^1 column, etc.

Now put this problem, which involves regrouping, on the chalkboard in vertical form: 438 + 249. Use the standard algorithm which begins by adding the first place on the right (the T^0 place).

WHAT NUMERAL WOULD GO INTO THE T^0 PLACE IN OUR FINAL ANSWER? ($8 + 9 = 17$. We regroup 17 to $1T^1 + 7T^0$. Be sure the children understand this transition of 17 into T notation. The 7 is recorded in the T^0 place and the 1 is entered at the top of the T^1 column. The children should see that the 1 in this case means $1T^1$ or 10.)

WHAT NUMERAL WOULD GO INTO THE T^1 PLACE IN OUR FINAL ANSWER? (8.) WHY? ($1T^1 + 3T^1 + 4T^1 = 8T^1$. The 8 is recorded in the T^1 place. Be sure the children understand that the 8 means $8T^1$ or 80.)

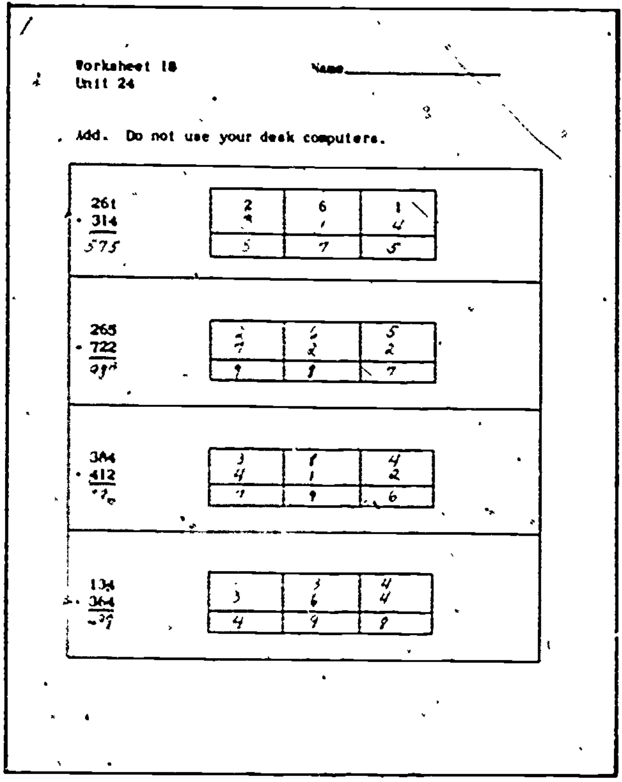
WHAT NUMERAL WOULD GO INTO THE T^2 PLACE IN OUR FINAL ANSWER? (6.) WHY? ($4T^2 + 2T^2 = 6T^2$. The 6 is recorded in the T^2 place because in this example the 6 stands for 600.)

Your final work should look like this:

$$\begin{array}{r} {}^{1} \\ 438 \\ +\,249 \\ \hline 687 \end{array}$$

Worksheets 18 and 19 should be done individually during free time. Each problem on Worksheet 18 is to be added using both the addition chart and the vertical form.

All the problems on Worksheet 19 involve regrouping. You may want to go through the first problem with the class to see that they understand how to regroup using the standard algorithm. If the children are still having difficulty regrouping after they have completed Worksheet 19, you may want to use the problems on Worksheet 20 for class demonstration. Otherwise, the children can complete Worksheet 20 on their own.



Worksheet 18
Unit 24

Name________________

Add. Do not use your desk computers.

261
+ 314
575

2 3	6 1	1 4
5	7	5

265
+ 722
987

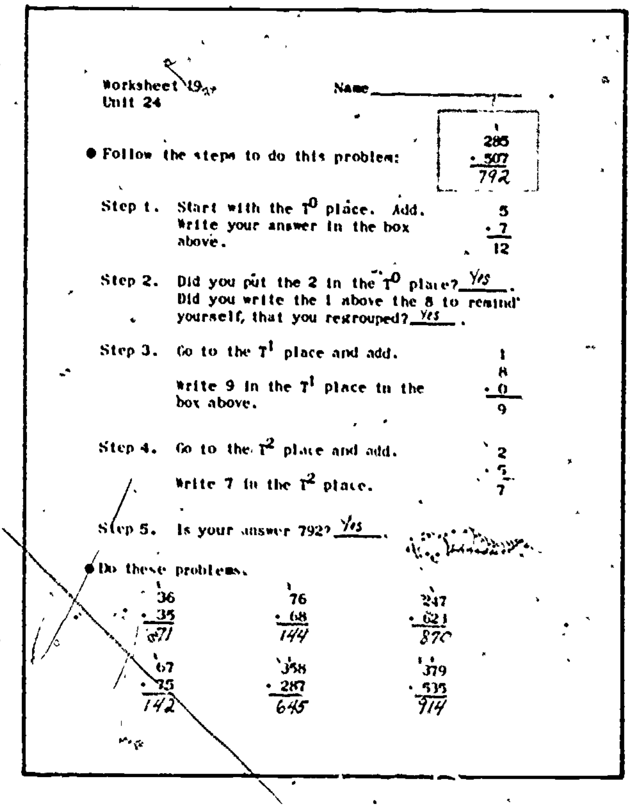
2 7	6 2	5 2
9	8	7

364
+ 412
[illegible]

3 4	6 1	4 2
7	9	6

134
+ 364
[illegible]

 3	3 6	4 4
4	9	8



Worksheet 19
Unit 24

Name________________

- Follow the steps to do this problem:

1
285
+ 507
792

Step 1. Start with the T^0 place. Add. Write your answer in the box above.

5
+ 7
12

Step 2. Did you put the 2 in the T^0 place? Yes.
Did you write the 1 above the 8 to remind yourself, that you regrouped? Yes.

Step 3. Go to the T^1 place and add.

Write 9 in the T^1 place in the box above.

1
8
+ 0
9

Step 4. Go to the T^2 place and add.

Write 7 in the T^2 place.

2
+ 5
7

Step 5. Is your answer 792? Yes.

- Do these problems.

1
36
+ 35
71

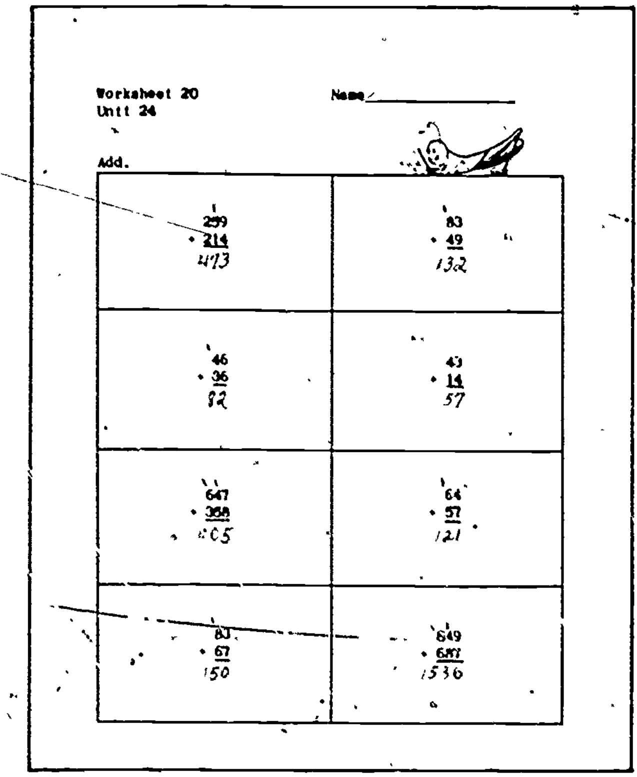
1
76
+ 68
144

1
247
+ 623
870

1
67
+ 75
142

1 1
358
+ 287
645

1 1
379
+ 535
914



Worksheet 20
Unit 24

Name________________

Add.

1 299 + 214 473	1 83 + 49 132
1 46 + 36 82	43 + 14 57
1 1 647 + 358 1005	1 64 + 57 121
1 83 + 67 150	1 1 849 + 687 1536

Lesson 6: ADDITION PRACTICE GAMES

This lesson provides practice in addition that should be pleasant for the children. Spend this class period introducing the games, and encourage the children to play them in free time throughout the year.

MATERIALS

- 15 pairs of dice (one of each pair labeled 0-5, the other labeled 4-9)
- pencils and paper
- Worksheet 21

PROCEDURE

Activity A

Begin by reviewing the addition algorithm. Have a child go to the chalkboard and work a problem that requires regrouping while the other children do it at their seats. Do as many more problems as you think the children need. Then play the following game.

Let each column of children be a team. Write a number between 1 and 20 on the chalkboard in front of each team. The first child on each team goes to the chalkboard, writes another number under the first and adds the two. Then the second child goes to the board, writes a number under the first answer and adds these two. This continues until each person in the team has had a chance to add a pair of numbers. The team whose final total is closest to 100 is the winner.

For example:

13	(the number you put on the chalkboard)
+20	(the number the first pupil puts on the board)
33	(the answer the first pupil gets)
+ 5	(the number the second pupil puts on the board)
38	(the answer the second pupil computes)
+17	(the number the third pupil adds)
55	(the answer for the third pupil, etc.)

One minute should be long enough for each pupil, but this is not a race against time, so allow more time if your class needs it. Have each team of children check its work at the chalkboard. Allow them to tell the person at the board if his work is wrong, but not what the mistake is.

This game provides an opportunity for you to determine which children in the class are having trouble regrouping. You can provide more practice and individual attention for those children who need it. When you play the game a second time, start from the back of each column so that every child gets a chance to regroup with larger numbers.

Activity B

Let the children choose partners. Give each pair of children a die with the numerals 0, 1, 2, 3, 4, 5 on the face. The first player throws the 0 - 5 die twice. The first throw is the ten's place digit, the second throw is the one's place digit. The player records this number on his paper. Then the second player generates a number the same way. Both players repeat this procedure and record their numbers in the column form.

Sample:	1st Player	2nd Player
	35 + 24	41 + 04

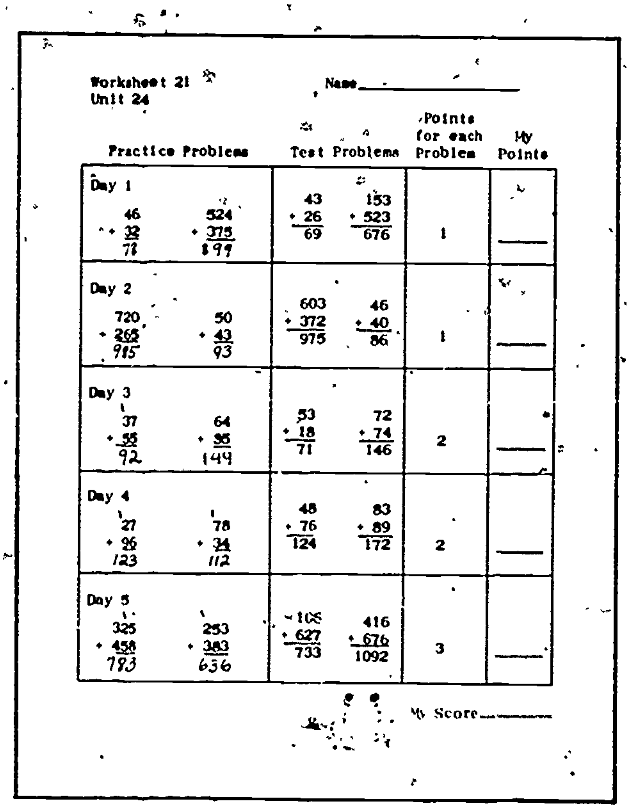
Each player adds his numbers. The player whose sum is larger scores one point. If a player makes an error and his opponent notices the error, the opponent wins a point. The first player with 5 points wins the game. The next game should be played the same way, but with the die marked 4 - 9. You can vary the game by using the die marked 0 - 5 for the first set of numbers and the die marked 4 - 9 for the second set of numbers and vice versa. Three and four-digit numbers should also be used. After they have played the game once or twice, let the children decide how many digits to use for each number. These variations will give the students practice with all of the addition facts and many opportunities to regroup.

Activity C

Computational skills are developed by frequent practice over an extended period of time. This practice need not be tedious, but should be provided often and with emphasis on accuracy.

Worksheet 21 provides practice in computational skills. It will be used each day for one week. Explain to the children that the class is going to have a contest in computation for one week. Have them turn to Worksheet 21 and look at the left-hand column, which contains the practice problems. These problems are similar to each day's test problems. Set up a certain time to give the test each day and tell the children to complete each day's practice problems on their own before the test. Put the answers to the practice problems on the chalkboard or bulletin board each day (under a cover piece of paper) so that the children can check their own answers.

When all the children have done the practice problems for the day, write the test problems on the chalkboard. The children copy these on their worksheets and solve them. Correct the problems together and have the children record their points as indicated on the worksheet. You may wish to devise a reward system for those scoring the most points during the week, or for those scoring above a certain number of points. You or the children can make up similar worksheets throughout the year, varying the difficulty as is appropriate for different students.



Worksheet 21
Unit 24

Name________________

Practice Problems		Test Problems		Points for each Problem	My Points
Day 1 46 + 32 78	524 + 375 899	43 + 26 69	153 + 523 676	1	______
Day 2 720 + 265 985	50 + 43 93	603 + 372 975	46 + 40 86	1	______
Day 3 37 + 55 92	64 + 85 149	53 + 18 71	72 + 74 146	2	______
Day 4 27 + 96 123	78 + 34 112	48 + 76 124	83 + 89 172	2	______
Day 5 325 + 458 783	253 + 383 636	105 + 627 733	416 + 676 1092	3	______

My Score________

SECTION 2 SUBTRACTION

PURPOSE

- To introduce and provide practice with the subtraction algorithm.
- To provide game situations for enjoyable practice with basic subtraction facts.

COMMENTARY

In this section the subtraction algorithm is developed and then applied to problems written in the vertical form. In Lesson 7, the children program the T-pieces computer to do subtraction. They divide each place of the computer in half horizontally. The minuend is in the top or "start" level and the subtrahend in the bottom or "subtract" level. Subtraction is done in the computer by a comparison process. Each piece in the bottom level is matched with one in the top level. To find the difference, the computer counts the number of unmatched pieces in the start level.

In Lesson 8, the rule for regrouping is developed. One piece from the next larger place than the place lacking pieces is exchanged for ten of the needed pieces. The new pieces are entered in the start level of their place. Then the pieces are paired and the difference is noted.

The physical process of subtraction represented by the T-pieces computer is replaced by a mental process when the computer is converted to accept numerals in Lesson 9. The child at each place does the subtraction in his head and writes the difference on a slip of paper.

When the children are familiar with the subtracting and regrouping algorithms, the vertical form for subtraction is developed. In Lesson 9 a representation of the numeral computer is drawn on the chalkboard. The minus sign replaces the "start" and "subtract" labels. Place value is stressed while problems are worked using this chart.

In Lesson 10 lines are removed from the chart until only the vertical form remains. Then the class develops a notation for regrouping and works practice problems using the vertical form.

Games in Lesson 11 provide enjoyable practice of addition and subtraction facts.

Lesson 7: COMPUTER SUBTRACTION

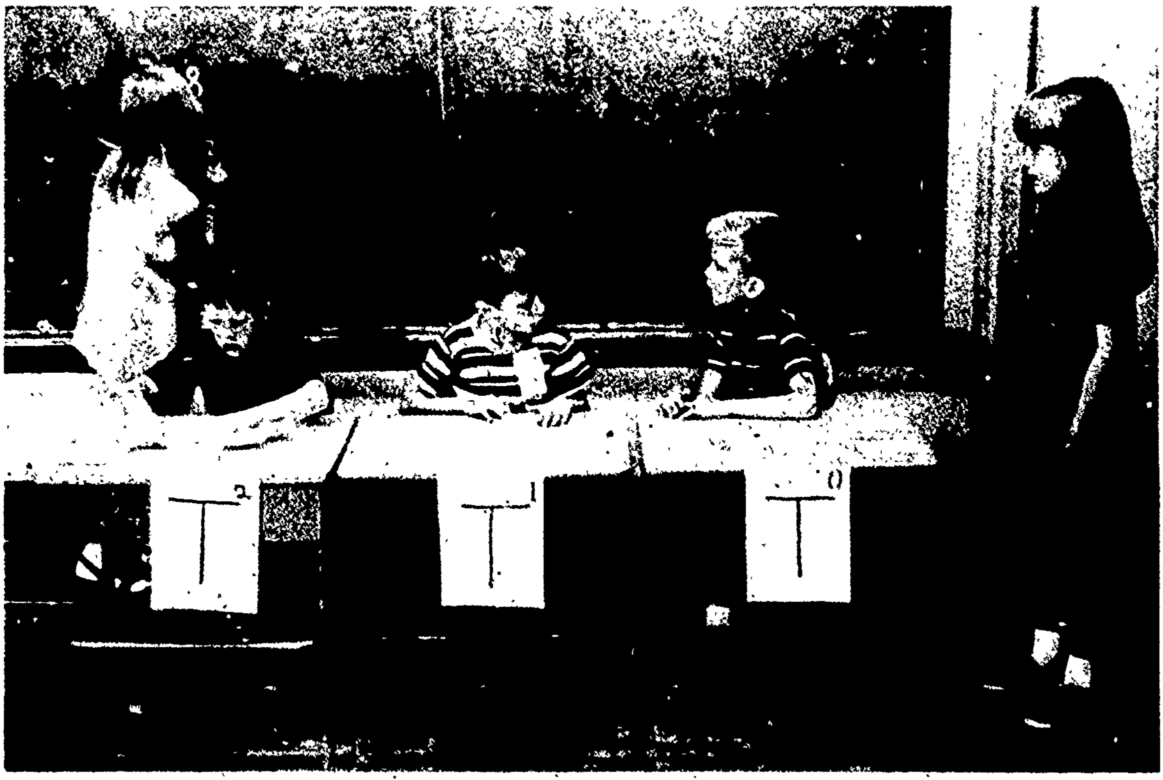
This lesson extends the computer rules so that problems involving subtraction can be solved. It is the first of four lessons that will develop the procedure for subtraction first with the T pieces computer and finally in vertical form.

MATERIALS

- computer desk labels T^2, T^1 and T^0
- 6 pieces of 3" x 5" paper (3 labeled "start" and 3 labeled "subtract")
- demonstration T pieces
- masking tape

-- for each child --

- desk computer (Worksheet 4)
- ruler or other straightedge
- 2 pieces of 3" x 5" paper
- bag with set of T pieces
- Worksheets 22 and 23



PROCEDURE

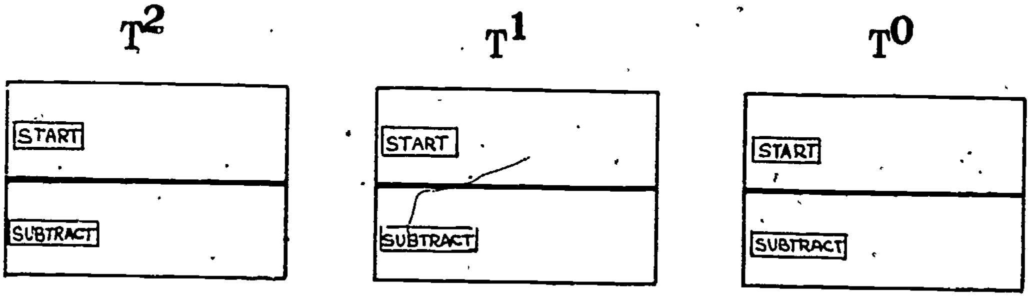
Activity A

Discuss with the class how the T pieces computer worked for addition. (It would take only certain pieces in each place, combine the pieces, regroup if necessary and produce the answer.) Then ask the class:

WILL THIS COMPUTER LET US SOLVE A PROBLEM SUCH AS, 5 - 3? (No.)

WHAT IS DIFFERENT ABOUT THIS PROBLEM FROM THE OTHERS WE DID WITH OUR COMPUTER? (It involves subtraction.)

Place three desks at the front of the room, labeling them T^2, T^1 and T^0, as in previous lessons. With masking tape, divide each desk top into two sections and label as shown. (Save these labels for later lessons.) Also draw this three-desk computer on the chalkboard.



Use the following procedure to develop the subtraction process.

THE COMPUTER MUST SOLVE THE PROBLEM, 5 - 3. HOW DO WE WRITE THIS IN T NOTATION? Write both equations on the chalkboard:

$$\begin{array}{rcl} 5 & = & 5T^0 \\ \underline{-3} & = & \underline{-3T^0} \end{array}$$

Choose five children to work the computer (a Recorder, a Programmer and three Computers). Tell the children they will work at the desk computer doing the same thing that you do at your computer drawn on the chalkboard. The Recorder copies the problem in vertical and T notation forms on the chalkboard.

> WHAT NUMBER WOULD THE COMPUTER LOOK AT FIRST IN OUR PROBLEM, 5 - 3? THAT IS, WHAT NUMBER WOULD IT START WITH? (5 or $5T^0$.)

Enter the pieces representing 5 (the first number or minuend) in the start level of the T^0 place in your computer on the chalkboard. You can draw in the pieces, but it will be better to tape the paper T pieces in place. (Remind the other children in the classroom to watch closely so that they will be able to work in groups later.) The Programmer counts out the T^0 pieces and feeds them into the T^0 place of the desk computer.

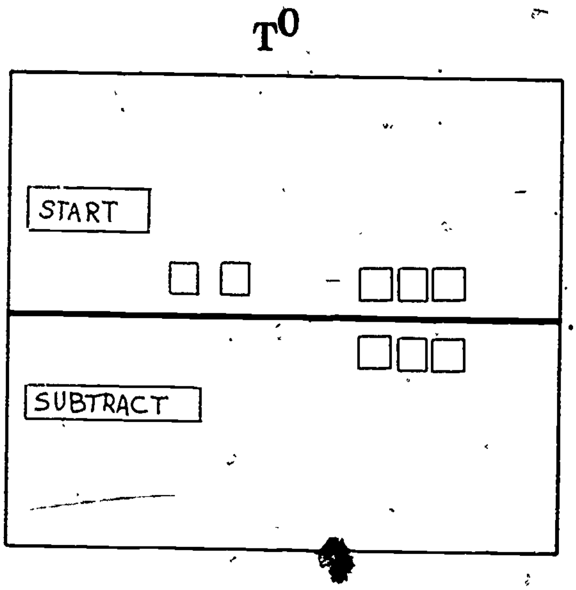
> WE HAVE NOW TOLD THE COMPUTER TO START WITH 5. WHAT IS THE NEXT THING WE MUST TELL IT TO DO? (Subtract 3.)

Enter the pieces representing 3 (the number to be subtracted or the subtrahend) in the subtract level of your computer. The Programmer does the same at the T^0 desk.

> WE MUST NOW PROVIDE THE COMPUTER WITH RULES IT CAN USE ALL THE TIME TO SOLVE SUBTRACTION PROBLEMS.

Suggest a rule similar to this and write it on the chalkboard:

> One by one, take the pieces in the start level and pair them with pieces in the subtract level.



Draw lines pairing the pieces in your computer. Or, if you taped T pieces to the board, rearrange them so that they are paired up. Instruct the child at the T^0 place of the desk computer to pair his pieces. Tell him not to remove the pieces from the start level, but only slide them together so there is an obvious physical pairing of the pieces. Each piece in the subtract level must be paired with a piece in the start level.

When you have performed these operations ask the class:

> HOW MANY UNPAIRED PIECES DO WE HAVE LEFT IN THE START LEVEL OF THE T^0 PLACE OF THE COMPUTER? (2 T^0 pieces.)
>
> WHAT IS THE SOLUTION TO THE PROBLEM, 5 - 3? (2.)

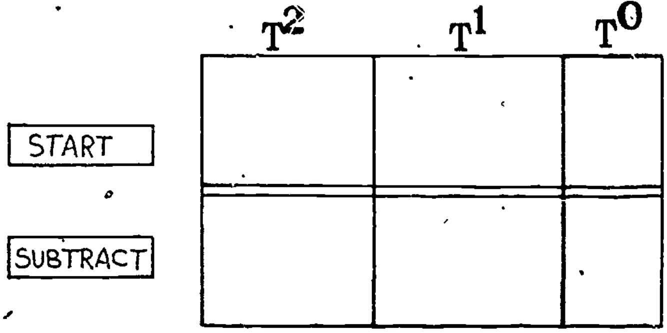
Ask the child at the T^0 place of the desk computer how many unpaired pieces he has left. He tells the Recorder this number and the Recorder writes the answer on the chalkboard in T notation and then converts it to standard decimal notation.

Mention that when working in groups the Computer tells the Recorder the number of unpaired pieces remaining in the start level at each place value. Select five new children to operate the computer. You may want to do a problem on the chalkboard while the children do it at the computer so that all the children in the classroom can see what is being done. Feel free to go through this procedure faster if the children catch on quickly. Write this problem on the chalkboard near your computer:

$$\begin{array}{rl} 348 = & 3T^2 + 4T^1 + 8T^0 \\ \underline{-235 =} & \underline{-2T^2 + 3T^1 + 5T^0} \end{array}$$

The Recorder also writes the problem on the board. As you feed the pieces for 348 (the minuend) into the start level of your computer, the programmer feeds the pieces into the correct places of the desk computer. Then you feed the pieces for 235 (the subtrahand) into the subtract level of your computer while the Programmer does the same with the desk computer. Ask the children in the classroom what to do next. They should tell you to pair the pieces in the subtract level at each place with pieces in the start level of the same place. The child at each desk pairs his pieces and tells the Recorder the number of unpaired pieces remaining in the start level at each place. The Recorder writes each number on the chalkboard under the problem. While he is doing this, you pair the pieces in your chalkboard computer. The children in the classroom should act as Repairmen and check your work to see if it is correct. The children at the desk computer check their work against yours to see if they are correct. After pairing the pieces in each place these pieces remain: $1T^2 + 1T^1 + 3T^0$. The children should have little difficulty transposing this into base ten notation, 113.

Have each child take out his desk computer and draw a double line through each place compartment of the computer to separate the start level from the subtract level. Use 3" x 5" pieces of paper to label the divisions "start" and "subtract." These labels can be placed next to the computer. The new desk computer should look like this:

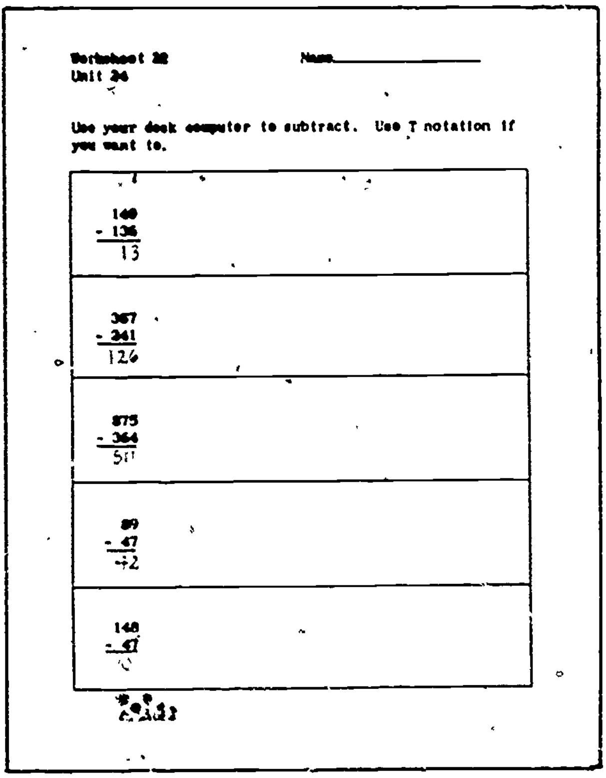


Divide your class into groups of three. Put at least one child who participated in the demonstration into each group to help the others. Let the children choose jobs and work several problems such as 74 - 32 and 629 - 512. Walk around the room checking to see that each group is using its subtraction computer correctly.

Point out to the children that in the T^0 place of their desk computers, they will have to match groups of pieces instead of pairing individual pieces since only three pieces can be put flush with the dividing line.

Activity B

Each group of three children should now complete Worksheet 22 together. The members of the group take turns being Programmer and Computer. Each child acts as his own Recorder and writes the computations on his worksheet. The children should complete Worksheet 23 independently.



Worksheet 22
Unit 24

Name________________

Use your desk computer to subtract. Use T notation if you want to.

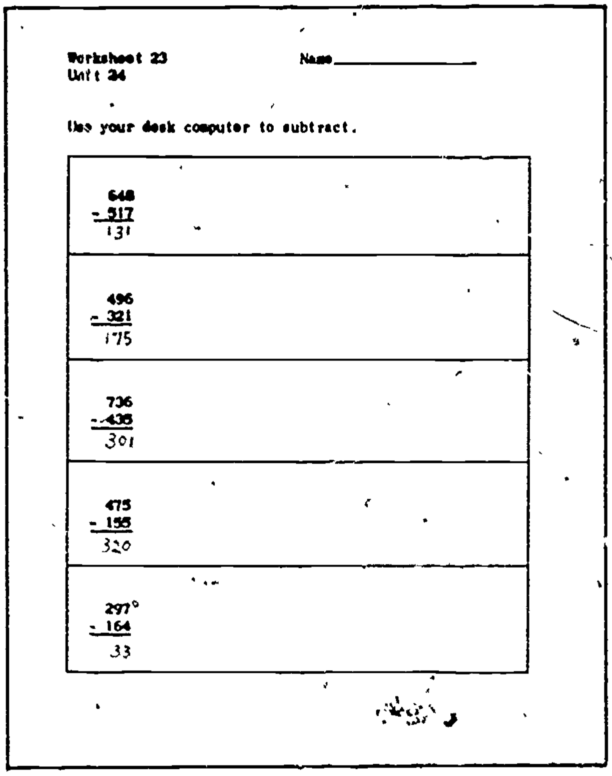
149
- 136
13

367
- 241
126

875
- 364
511

89
- 47
42

148
- 47
[illegible]



Worksheet 23
Unit 24

Name________________

Use your desk computer to subtract.

648
- 517
131

496
- 321
175

736
- 435
301

475
- 155
320

297
- 164
33

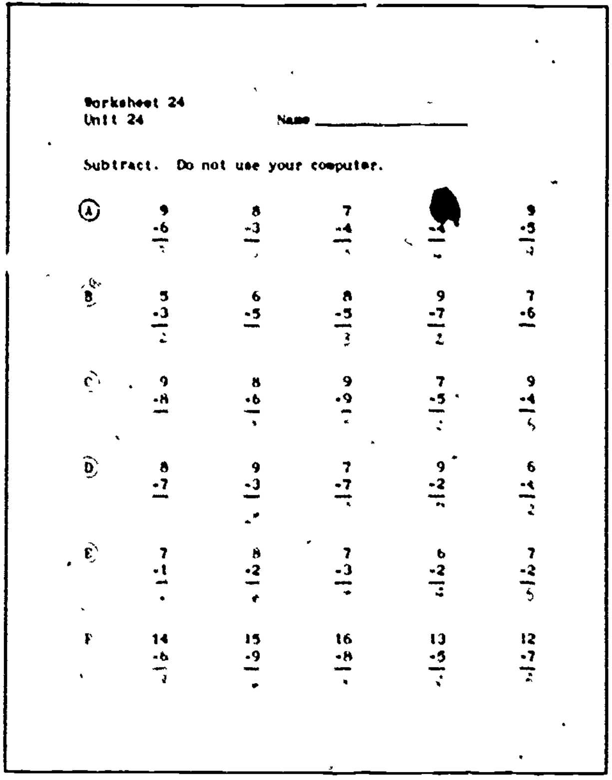
Lesson 8: COMPUTER SUBTRACTION WITH REGROUPING

In this lesson a rule is devised which tells the computer how to subtract when regrouping is necessary. In Activity A the children complete a worksheet of subtraction problems using the facts they have stored in their own memories. In Activity B the children use their computers to regroup in subtraction problems.

MATERIALS

-- for each child --

- desk computer
- set of T pieces
- Worksheets 24 and 25



Worksheet 24
Unit 24 Name ______________

Subtract. Do not use your computer.

Ⓐ	9 -6	8 -3	7 -4	[illegible]	9 -5
B	5 -3	6 -5	8 -5	9 -7	7 -6
C	9 -8	8 -6	9 -9	7 -5	9 -4
D	8 -7	9 -3	7 -7	9 -2	6 -4
E	7 -1	8 -2	7 -3	6 -2	7 -2
F	14 -6	15 -9	16 -8	13 -5	12 -7

PROCEDURE

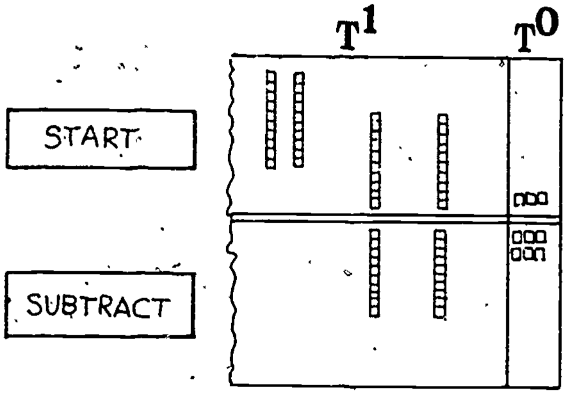
Activity A

To check the children's knowledge of subtraction facts, ask them to do Worksheet 24. Introduce this test as you did with the addition test by mentioning that the children's brains are like computers, and can be tested to find out what information they have stored.

Activity B

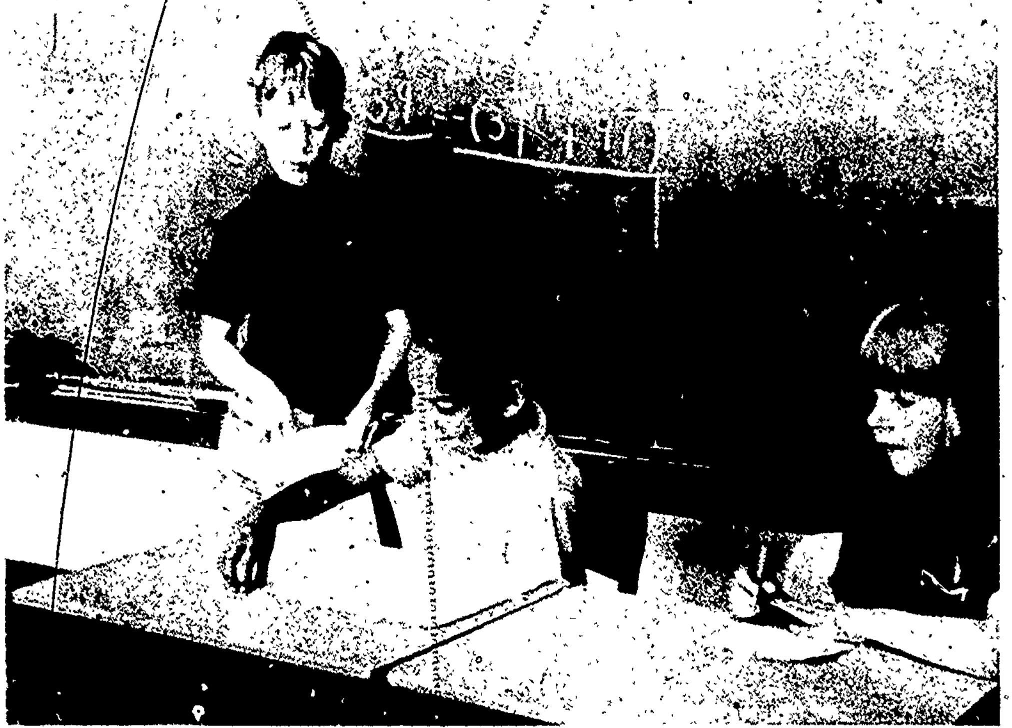
Ask the children to form groups of three and see that each child has his desk computer and T pieces. Have each child in the group work this problem with his desk computer: 43 - 26. The children in each group can consult each other about this problem.

As they proceed, the children will find that they do not have enough pieces in the start level of the T^0-place to pair with each piece in the subtract level of that place.

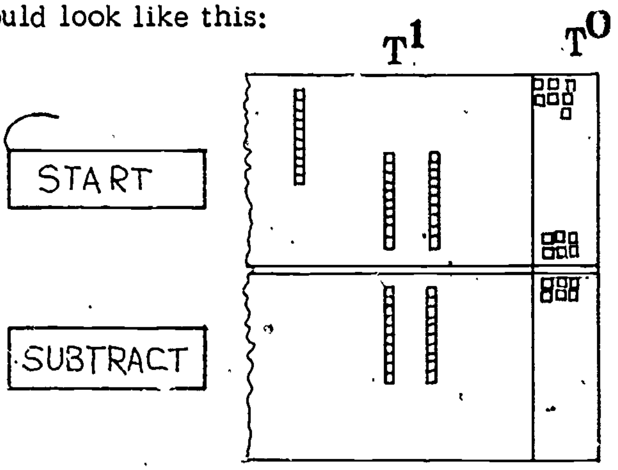


Remind the children of the rule for subtracting with the computer: each piece in the subtract level must have a mate in the start level. Ask the children for suggestions on how to pair the pieces in the two levels of the T^0 place. If no one suggests exchanging a T^1 piece for T^0 pieces, ask them what they did with the extra T^0 pieces when they were adding. Remind them that they exchanged 10 T^0 pieces for 1 T^1 piece. Ask the children how they could reverse this process to get some more T^0 pieces to solve the problem. When they have discovered what to do, formulate a rule that tells the computer how to regroup for subtraction problems:

> To regroup, the computer must know which T place needs more pieces. Then the computer removes one piece from the start level of the place to the left of the one that needs more pieces and exchanges it for ten of the next smaller pieces. These ten smaller pieces are entered into the start level of the T place that needs the extra pieces.



Now have the children again try the problem, 43 – 26, using the new rule for subtracting with the computer. Each computer should look like this:

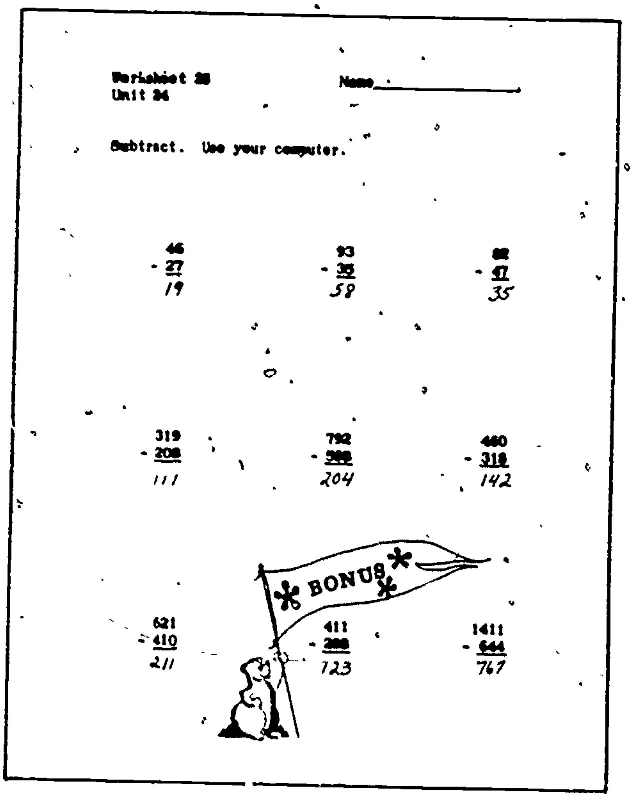


All the pieces in the subtract level of both places can now be paired with pieces in the start level of both places. The children should count the unpaired pieces in each place to determine the answer. They will see that 1 T^1 and 7 T^0 pieces remain unpaired and that $1T^1 + 7T^0 = 17$.

Two children in each group can put away their desk computers and T pieces, so that only one computer is out for each group. Remind the children to rotate jobs of Programmer, Recorder and Computer. Then have the groups work as many problems as you feel necessary for the class to master the regrouping procedure. Go around the room checking the work and helping those groups that do not understand the procedure. Suggest problems such as:

$$\begin{array}{r} 64 \\ -39 \\ \hline \end{array} \qquad \begin{array}{r} 482 \\ -257 \\ \hline \end{array} \qquad \begin{array}{r} 328 \\ -236 \\ \hline \end{array} \qquad \begin{array}{r} 452 \\ -376 \\ \hline \end{array}$$

After you are assured that the children know how to subtract with regrouping, assign Worksheet 25 to be done individually during free time with the desk computers. If there is time, you may want to let the children start the worksheet during class time to determine whether any children are having trouble regrouping.



Worksheet 25
Unit 24

Name________

Subtract. Use your computer.

$$\begin{array}{r} 46 \\ -27 \\ \hline 19 \end{array} \qquad \begin{array}{r} 93 \\ -35 \\ \hline 58 \end{array} \qquad \begin{array}{r} 82 \\ -47 \\ \hline 35 \end{array}$$

$$\begin{array}{r} 319 \\ -208 \\ \hline 111 \end{array} \qquad \begin{array}{r} 792 \\ -588 \\ \hline 204 \end{array} \qquad \begin{array}{r} 460 \\ -318 \\ \hline 142 \end{array}$$

$$\begin{array}{r} 621 \\ -410 \\ \hline 211 \end{array} \qquad \begin{array}{r} 411 \\ -288 \\ \hline 123 \end{array} \qquad \begin{array}{r} 1411 \\ -644 \\ \hline 767 \end{array}$$

Lesson 9: DEVELOPING THE VERTICAL FORM FOR SUBTRACTION

In this lesson the rules for the subtraction computer are changed so that it will handle numerals rather than T pieces. The arrangement of levels and columns in the desk computer used in Lesson 8 provides a smooth transition to the development and use of the algorithm for subtraction in vertical form.

MATERIALS

- labels marked T^2, T^1, T^0
- masking tape
- 3 "start" and 3 "subtract" labels (from Lesson 7)
- 30 to 40 slips of paper
- Worksheet 26

PREPARATION

Before the lesson, prepare 30 or 40 small slips of paper from larger sheets or provide a scratch pad for your demonstration.

PROCEDURE

Activity A

Discuss the inadequacies of the T pieces computer for subtraction (it is slow, we will not always have one available). Ask the children to think of a way we might change the computer so that it accepts numerals instead of T pieces. Remind them that they used an addition computer that accepted numerals in Lesson 4.

Set up the three-desk computer used in Lesson 7 by dividing each desk into sections with masking tape, and labeling each section "start" or "subtract." Select a Programmer, a Recorder and three Computers.

Present this problem for the computer to solve: 469 - 327. Establish the procedure for putting numerals into the computer. The Recorder writes the problem on the chalkboard and rewrites it in T notation. His record should look like this:

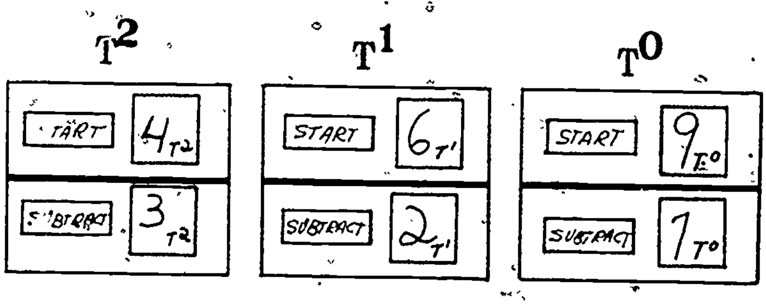
$$\begin{array}{rcl} 469 = & & 4T^2 + 6T^1 + 9T^0 \\ \underline{-327} = & - & \underline{3T^2 + 2T^1 + 7T^0} \end{array}$$

Then the Recorder transfers the numeral and place value notation for each place onto separate slips of paper.

$$\boxed{4_{T2}} \qquad \boxed{6_{T1}} \qquad \boxed{9_{T0}}$$

$$\boxed{3_{T2}} \qquad \boxed{2_{T1}} \qquad \boxed{7_{T0}}$$

The Programmer enters the first number (the minuend) by putting the appropriate slip of paper in the start level of each place. The number to be subtracted (the subtrahend) is entered in the subtract level of the computer.

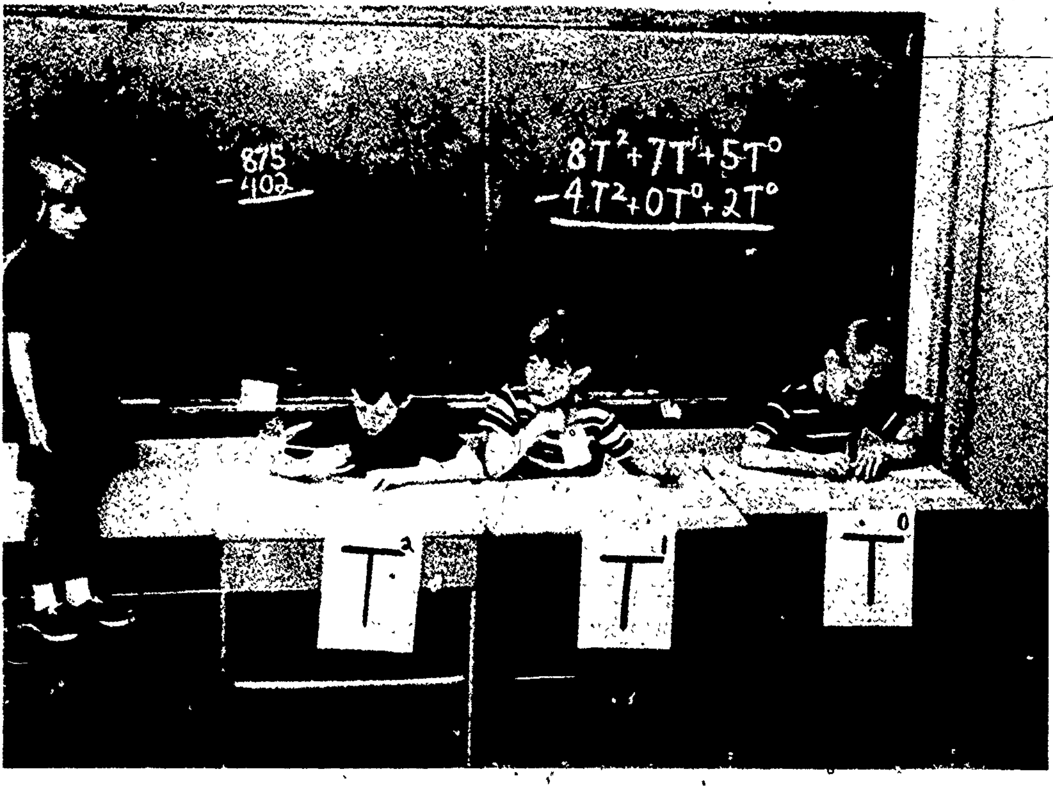


After the Programmer has pushed the start button, the Computer at each desk mentally subtracts and then writes the answer on a slip of paper, labeling it with the proper place value. Each Computer hands his slip to the Programmer when it is called for. The output for the sample problem is:

$$\boxed{1_{T2}} \qquad \boxed{4_{T1}} \qquad \boxed{2_{T0}}$$

The Recorder writes the answer in T notation under the T notation problem on the chalkboard, and records the difference in vertical notation (142) under the original problem.

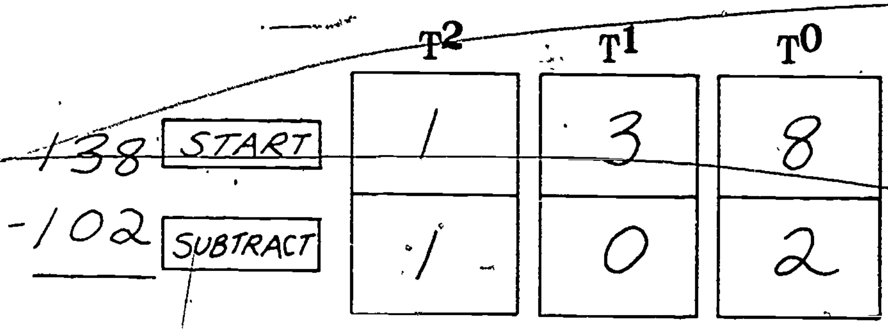
Give the children as many examples of subtracting with the numeral computer as you feel are necessary for them to understand the process. Suggest various types of problems, but do not include any that involve regrouping. Give as many children as possible a chance to be part of the computer.



Activity B

Discuss with the children the fact that we do not always have a computer available to help us work subtraction problems. Tell them that you have an idea for making a chart that can be used to help subtract.

Put a sample problem on the chalkboard. Next to it make a sketch of each place and level of the subtraction computer. As the children tell you where to enter each numeral, write it in.



Point to the T^0 place in the computer and ask what we do with the numerals there. (We subtract 2 from 8.)

Write in the result below the bottom line of the computer in the T^0 column. Enter the differences in the T^1 and T^2 places in the same way. Ask the children to compare the computer with the original problem to see if they notice anything interesting about the placement of the numerals. (The numerals in the computer are in the same places as the numerals in the problem.)

Erase the lines that separate the start and subtract levels of the computer and ask:

- HAVE I CHANGED THE PROBLEM BY REMOVING THESE LINES? (No.)

Erase the start label and ask:

IF I TAKE AWAY THIS LABEL, CAN I STILL TELL WHICH NUMBER I STARTED WITH? THAT IS, CAN I TELL WHICH NUMBER I WANT TO SUBTRACT FROM? (Yes, it is always on top.)

Discuss the original problem:

IN THIS EXAMPLE, WHAT TELLS US THAT WE ARE GOING TO SUBTRACT THE SECOND NUMBER FROM THE FIRST? HOW DO WE KNOW THAT WE SHOULDN'T ADD THESE NUMBERS? (The minus sign tells us to subtract.) CAN WE REPLACE THE WORD SUBTRACT IN OUR COMPUTER WITH THIS SIGN? (Yes.)

Erase the subtract label and replace it with a minus sign. Draw a line beneath the lower numerals of the problem separating them from the space for the answer. Make the necessary changes in your chalkboard computer to make it look like this:

	T^2	T^1	T^0
	1	3	8
−	1	0	2
		3	6

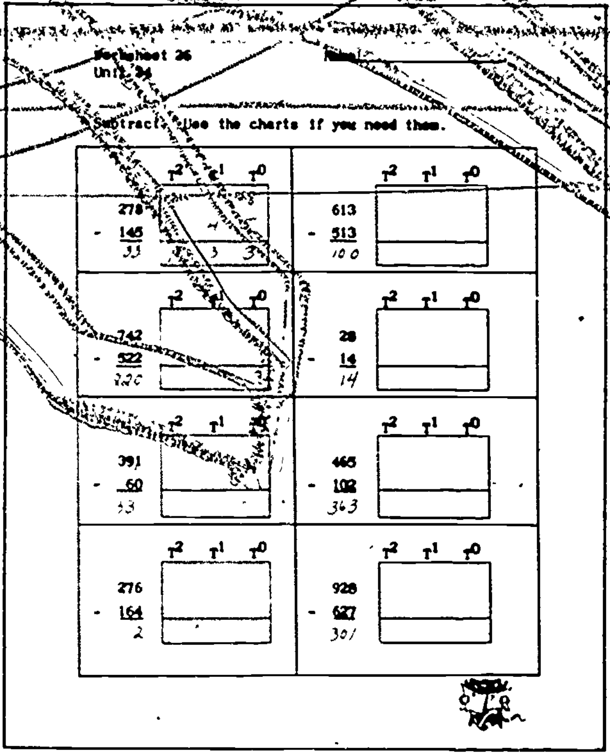
Explain that nothing has been changed by altering the computer. Verify this by again subtracting the numerals in each T place. Have several children do problems at the board using the simplified chart.

When you are sure the children understand the use of the chart, ask:

CAN WE SHOW WHAT WE DO WHEN WE SUBTRACT WITHOUT USING THE CHART? LET'S TRY.

Write a problem on the chalkboard in vertical form, for example, 633 - 502, and subtract directly. Remind the children of the place value subtraction: that is, 6 - 5 = 1 is really $6T^2 - 5T^2 = 1T^2$ or 600 - 500 = 100. 6 - 5 = 1 is a shortcut.

Have the children do Worksheet 26 using either the subtraction chart or the vertical form.



Worksheet 26
Unit 34

Name ____________

Subtract. Use the charts if you need them.

T^2 T^1 T^0 278 - 145 33	T^2 T^1 T^0 613 - 513 100
T^2 T^1 T^0 742 - 522 220	T^2 T^1 T^0 28 - 14 14
T^2 T^1 T^0 391 - 60 33	T^2 T^1 T^0 465 - 102 363
T^2 T^1 T^0 276 - 164 2	T^2 T^1 T^0 928 - 627 301

Lesson 10: SUBTRACTION WITH REGROUPING IN VERTICAL FORM

In this lesson the students use the subtraction chart to help them with problems involving regrouping. Then they learn to use the vertical form to solve subtraction regrouping problems.

MATERIALS

- Worksheets 27, 28 and 29
- sheet of 8½" x 11" paper for each child

PROCEDURE

Activity A

Draw a subtraction chart on the chalkboard and pose a problem which involves regrouping, for example, 482 - 265. Ask the children to copy the problem and solve it using a similar chart they can draw on a sheet of paper.

$$\begin{array}{r} 482 \\ -265 \\ \hline \end{array}$$

	T^2	T^1	T^0
	4	8	2
−	2	6	5

The question of how to subtract 5 from 2 will arise. Review subtraction regrouping with T pieces. One of the next larger pieces was removed and exchanged for 10 of the needed pieces. Explain that since we cannot subtract 5 from 2, we must take a T^1 piece away and exchange it for 10 T^0 pieces. Cross out the 8 in the T^1 place and put in a 7 so that everyone will know we have taken 1 T^1 away. Emphasize that you take the T^1 piece from the top number, the one you are subtracting from, and not from the bottom number. Then, on the chart, add the 10 T^0 to the 2 T^0 already there, making 12 T^0. Now we can take 5T^0 away, leaving 7. Subtract in the other places, reminding the children to subtract 6T^1 from 7T^1, not from the 8T^1 that is crossed out. Your completed chart should look like this:

$$\begin{array}{r} 482 \\ -265 \\ \hline \end{array}$$

	T^2	T^1	T^0
			10+2=12
	4	$\overset{7}{\cancel{8}}$	$\cancel{2}$
−	2	6	5
	2	1	7

Activity B

Explain to the children that we do not need the chart to do subtraction that involves regrouping. Do a sample problem with the class using vertical form, but stressing the place value of each operation.

As you work the problem, point out the place that needs more numerals (the T^0 place in this example).

$$\begin{array}{r} 3\,2\,6 \\ -2\,1\,7 \\ \hline \end{array}$$

Cross out the numeral you will take 10 from (the 2 in the T^1 place). Write in the remaining numeral (1) to remind yourself that you have taken away 1 T^1.

$$\begin{array}{r} 3\,\overset{1}{\cancel{2}}\,6 \\ -2\,1\,7 \\ \hline \end{array}$$

Remind the children that the 1 T^1 or 10 must be added to the T^0 place. Show them how you wrote this on the chart: cross out the 6 and write above it, 10 + 6 = 16. Now the children know that they can subtract 7 from 16.

$$\begin{array}{rll} & 1 & 10+6=16 \\ 3 & \cancel{2} & \cancel{6} \\ -2 & 1 & 7 \\ \hline \end{array}$$

Tell the children there is a shorter way to indicate regrouping in the T^0 place. We can do the addition in our heads (10 + 6 = 16) and only put down the sum (16).

$$\begin{array}{rll} & 1 & 16 \\ 3 & \cancel{2} & \cancel{6} \\ -2 & 1 & 7 \\ \hline \end{array}$$

Now ask:

> CAN ANYONE THINK OF AN EVEN SHORTER WAY TO SHOW THAT WE HAVE REGROUPED TO THE T^0 PLACE?

If no one thinks of it, show the children that they can write a 1 next to the 6 to indicate 16 rather than cross out the 6 and rewriting 16. The children should learn to do the problems this way.

$$\begin{array}{rll} & 1 & \\ 3 & \cancel{2} & {}^{1}6 \\ -2 & 1 & 7 \\ \hline \end{array}$$

Be sure the children understand that when they regroup a T^1 to the T^0 place, they are adding 10 to the number already in the T^0 place.

Complete the problem. Now have the children work several problems, preferably without the chart. If they need the chart let them use it for one or two problems. Then each child should work a few problems without the chart before going on to the next part of this activity. Write problems like these on the chalkboard in vertical form: 47 - 39, 627 - 243, 250 - 135. Go around the room checking work or let a few children come to the board to demonstrate the subtraction regrouping procedure.

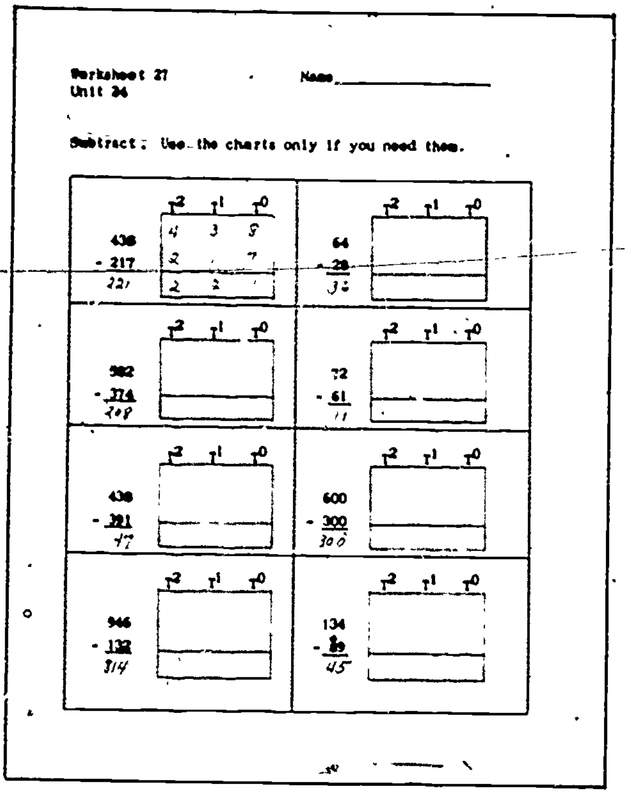
When the children feel confident about regrouping, pose a problem on the chalkboard that involves regrouping in more than one place, for example, 322 - 158. Work the problem with the class, letting them tell you how to proceed.

If some children are confused about subtracting in the T^1 place remind them that they must regroup from the next larger place. They must take 1 T^2 from the T^2 place. As T^2 equals 10 T^1, so they add 10T^1 to the number already in the T^1 place.

Activity C

Have the children turn to Worksheet 27 and work the first problem or two with them. They should be able to complete the rest of the worksheet themselves. Worksheet 28 provides more practice with subtraction. Encourage the children to become efficient in using the subtraction algorithm with problems written in vertical form rather than depending on the subtraction chart.

The practice and test problems on Worksheet 29 should be used for one week. Each day the children do the practice problems and then you give them the test problems.

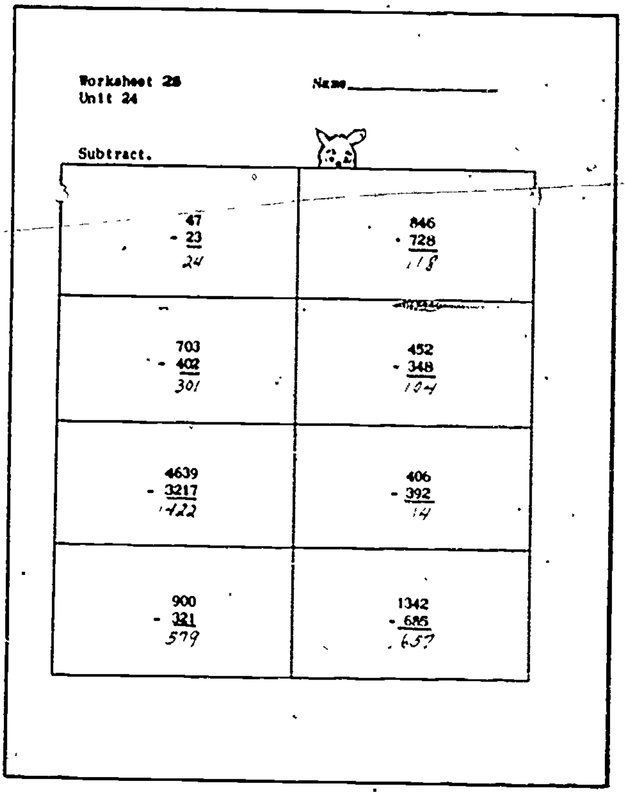


Worksheet 27
Unit 24

Name________________

Subtract. Use the charts only if you need them.

438 - 217 221 T^2 T^1 T^0 4 3 8 2 7 2 2	64 - 28 36 T^2 T^1 T^0
582 - 374 208 T^2 T^1 T^0	72 - 61 11 T^2 T^1 T^0
438 - 391 47 T^2 T^1 T^0	600 - 300 300 T^2 T^1 T^0
946 - 132 814 T^2 T^1 T^0	134 - 89 45 T^2 T^1 T^0

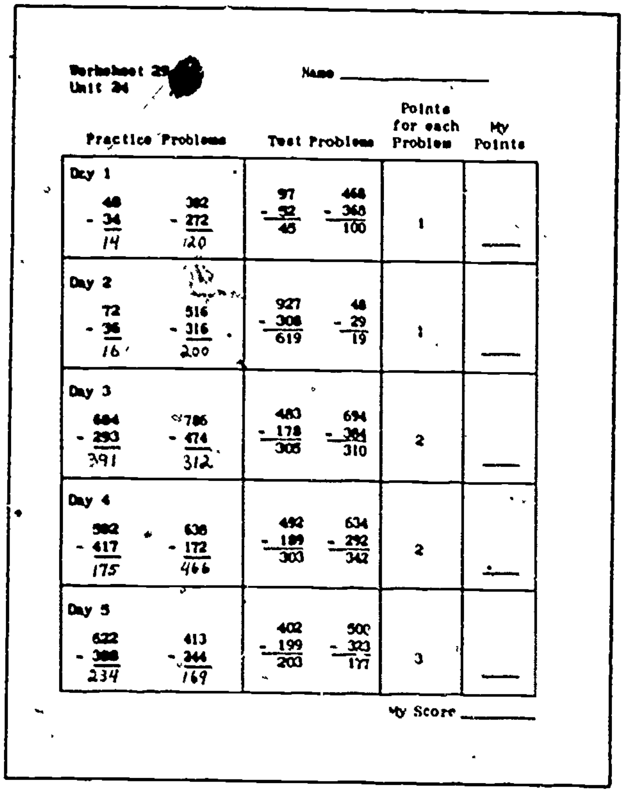


Worksheet 28
Unit 24

Name________________

Subtract.

47 - 23 24	846 - 728 118
703 - 402 301	452 - 348 104
4639 - 3217 1422	406 - 392 14
900 - 321 579	1342 - 685 657



Worksheet 29
Unit 24

Name ________________

Practice Problems		Test Problems		Points for each Problem	My Points
Day 1 48 - 34 14	392 - 272 120	97 - 52 45	468 - 368 100	1	____
Day 2 72 - 36 16	516 - 316 200	927 - 308 619	48 - 29 19	1	____
Day 3 684 - 293 391	786 - 474 312	483 - 178 305	694 - 384 310	2	____
Day 4 592 - 417 175	638 - 172 466	492 - 189 303	634 - 292 342	2	____
Day 5 622 - 388 234	413 - 244 169	402 - 199 203	500 - 323 177	3	____

My Score ________

Lesson 11: ADDITION AND SUBTRACTION GAMES

Games and a worksheet give the children practice with addition and subtraction problems. The games "Fast Math" and "Mental Arithmetic" can be varied and played as often as you wish.

MATERIALS

- Worksheet 30

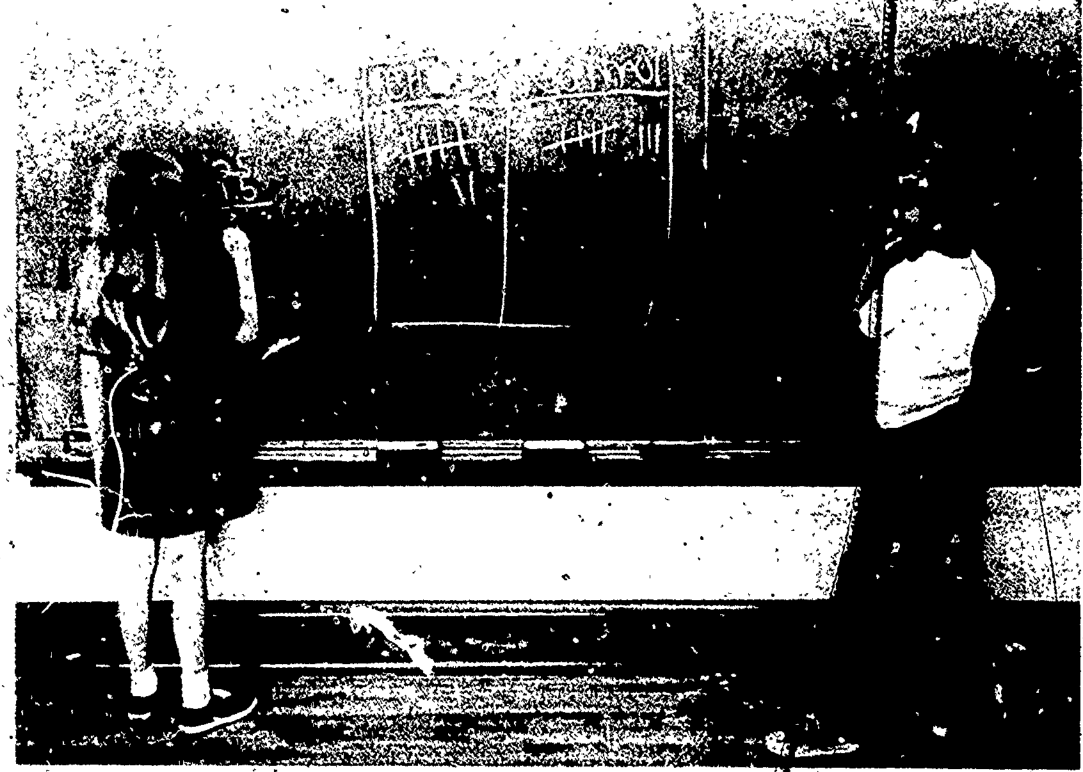
PROCEDURE

Activity A: Fast Math

Divide the class into two teams. Each team may select a name, for example, "Control" and "Chaos." If you have an odd number of children, ask one to serve as Recorder. Seat each team so that the first person on Control is matched with the first person on Chaos, etc. Since the pairs will be competing, match children of equal ability.

The first pair go to the chalkboard, standing on either side of a scoreboard drawn there. Give the pair a problem. Both are to write the problem, add it, and record the answer. The first to write the correct answer wins a team point. The point is recorded on the scoreboard under the team name. When they have finished, the first pair return to their desks and the second pair go through the same procedure. This continues until everyone has done a problem. Play the game again, this time using subtraction problems.

You may want to stop during the game to discuss any problems that arise. Such discussions can be a good learning experience. Children who are slow writers will be at a disadvantage, but may find they can still win by doing the computation rapidly. Both their writing speed and computational skills may benefit from the competition.



This game can be used in several ways. Children who finish their work early can get together in small groups to play a round or two. You can divide the children into teams according to abilities and needs, and assign the problems accordingly. Children who need remedial work should be especially encouraged to play the game on their own. For another variation, alternate addition and subtraction problems during the same round.

Activity B: Mental Arithmetic

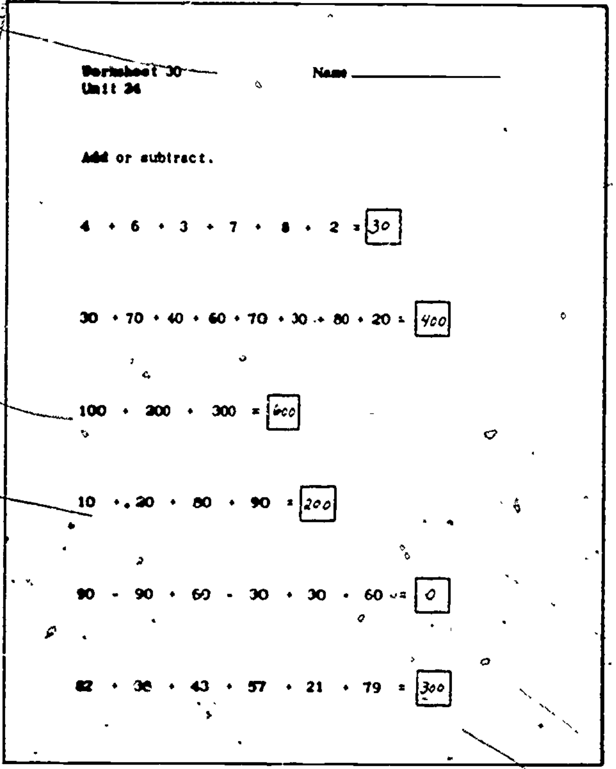
The Mental Arithmetic game from Lesson 3 can now be expanded to include subtraction. Pose problems such as, 7 - 2, 8 - 5, 11 - 9. When the children become proficient at adding and subtracting mentally, you can vary the game by asking problems that involve two operations, such as 10 - 6 + 3 and 8 + 10 - 8. You can use this activity during unstructured time, for example, when the children are lining up to leave the room or when they are getting out new materials. Spend no more than two or three minutes each time you play.

Activity C

Worksheet 30 gives the children practice in recognizing addition and subtraction combinations which yield 10's or 100's. As the children complete the worksheet ask them if they can find a pattern that helps them finish the problems faster. The pattern in the first problem is:

$$\underbrace{4 + 6}_{10} \quad + \quad \underbrace{3 + 7}_{10} \quad + \quad \underbrace{8 + 2}_{10} \quad = \quad 30$$

To provide more practice with these types of combinations, use similar problems during the Fast Math game.



Worksheet 30
Unit 24

Name ____________

Add or subtract.

4 + 6 + 3 + 7 + 8 + 2 = $\boxed{30}$

30 + 70 + 40 + 60 + 70 + 30 + 80 + 20 = $\boxed{400}$

100 + 200 + 300 = $\boxed{600}$

10 + 20 + 80 + 90 = $\boxed{200}$

90 - 90 + 60 - 30 + 30 - 60 = $\boxed{0}$

82 + 38 + 43 + 57 + 21 + 79 = $\boxed{300}$

SECTION 3 MEASUREMENT

PURPOSE

- To review units of measure and notation for these units.
- To observe the relationships between units in each system of measure and compare them with other systems of relationships (place value systems, etc.).
- To establish a feeling for relative size among some common units of measure.

COMMENTARY

In Section 3 the students investigate several systems of measurement. Physical representations of these measures are used to establish a feeling for the size of each measure and for the relationships between the measures. The children should be encouraged to play and experiment with the materials during free time.

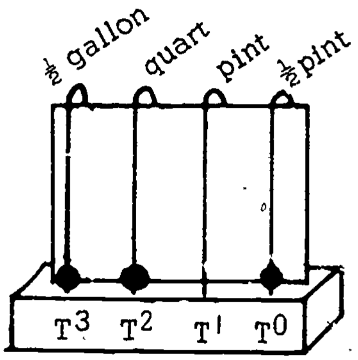
The study of measures of volume, length and time duration is included here to emphasize and contrast the relationships among the units in each of these systems with those of our base-ten numeration system. The relationships between measures of volume are regular, as are the relationships between place values in the base-ten numeration system. The relationships between measures of length and time are not regular. These similarities and differences should be emphasized.

Lesson 12 deals with the measure of liquid volume. The units that are studied include the half-pint, pint, quart, half-gallon and gallon. The children set up a base-two computer to help them in the addition of the units of liquid volume measure. The students draw concept tree diagrams to illustrate the relationships among these units. Lesson 12 will take two class periods. You must supply one half-gallon container and one gallon container.

In Lesson 13 the children study the British system for measuring length. They compare the British system of inches, feet and yards with the metric system and find that the relationships among the metric units are regular: each unit is ten times as large as the next smaller unit. The children make a concept tree illustrating the irregular relationships among the British units. The children do not set up computers in this lesson, but feel free to encourage them to do so on their own. As a class the children should discuss how these computers would work even if they do not actually set them up. This lesson will take two class periods.

Lesson 14 is an investigation of the measurement of time duration. The children make physical representations of seconds, minutes and hours to illustrate the relationships among these units. Encourage the children to discuss how a computer for time duration would add amounts of time. They should be encouraged to set up such a computer, or at least make up the rules for it, even though they do not actually set up the computer in this lesson. The lesson will take one class period.

The abacus may be used to provide a representation of the relationships that exist among the units of measure in these measurement systems. For example:



Section 3 attempts to emphasize the relationships among units of measure within any given measurement system. In all of these investigations, the students should be encouraged to finish the following statement: ______ units of this size are equal in value to one unit of the next larger size.

Lesson 12: LIQUID VOLUME MEASURE

Though most of the world uses metric units to measure liquid volume, the United States uses the half-pint, pint, quart, half-gallon and gallon. In this lesson the children investigate the relationships among these common units of liquid measure. The lesson should take two class periods.

In Activity A the children study the relationships among the half-pint, pint, quart, half-gallon and gallon containers. In Activity B they build a base-two computer that adds volume measures. These two activities should be done the same day.

Activity C is optional, depending on whether or not your class needs more practice in adding with a base-two computer as they did in Activity B. You may wish to use the activity for remedial work with some children or as a ten-minute review at the beginning of the class period on the second day.

In Activity D the children make a diagram called a concept tree which provides a visual representation of the relationships among the different sized containers.

A time schedule for this lesson could be:

Day 1:	Activity A	10 minutes
	Activity B	20 - 30 minutes
Day 2:	Activity C	(Optional or 10 minutes if used for review)
	Activity D	30 minutes

MATERIALS

- desk labels marked "Gallon," "$\frac{1}{2}$-gallon," "Quart," "Pint," "$\frac{1}{2}$-pint" and "Bank"
- containers (with sizes labeled): 19 half-pint, 9 pint, 4 quart, 2 half-gallon and 1 gallon

- masking tape
- water dipper or cup
- yarn or string
- water
- 10 - 20 slips of paper
- large bucket or pail and a sponge (optional)
- Worksheets 31 - 35

PREPARATION

For this lesson you or the children should bring in one half-gallon container and one gallon container. Before the lesson, make one each of the desk labels mentioned in the Materials List. Using masking tape, also label the size of each container. Place the containers on the table you will be using for the bank. The children will need a supply of water for this lesson. If you do not have a sink, have a bucket of water in a place that is convenient for the children. Since there is bound to be some spilling, have a sponge ready.

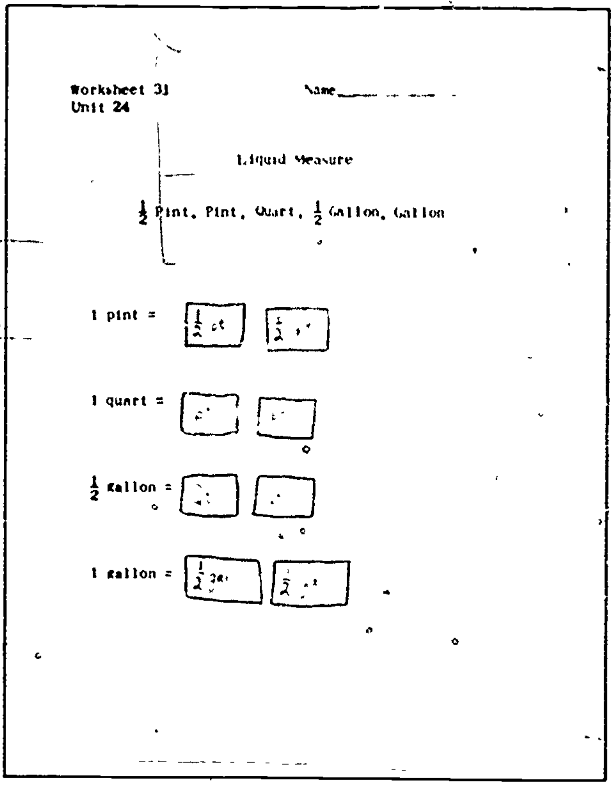
PROCEDURE

Activity A

This activity illustrates relationships among the various liquid measures. The children will have an opportunity to handle the containers and talk about the relationships they discover. The activity serves as an introduction to units of volume measurement and will provide an opportunity to compare a base-two system with the base-ten numeration system.

Place one labeled container of each size or a table. The water supply should be nearby. Have two students come to the table and assist you. One student will be the Experimenter and one will serve as Recorder. Write the title, "Liquid Measure," on the chalkboard. In a column to the left on the board write "1 pint," "1 quart," "½-gallon," and "1 gallon."

Have the children turn to Worksheet 31 which is a copy of the liquid measure chart you have drawn on the chalkboard. The children at their desks will fill in the worksheet as the Experimenter and Recorder demonstrate at the front of the room. Explain that the Recorder will be responsible for entering information on the chalkboard chart.



Worksheet 31
Unit 24

Name________

Liquid Measure

½ Pint, Pint, Quart, ½ Gallon, Gallon

1 pint = ½ pt ½ pt

1 quart =

½ gallon =

1 gallon = ½ gal ½ gal

Give the Experimenter the half-pint container. Then ask the class:

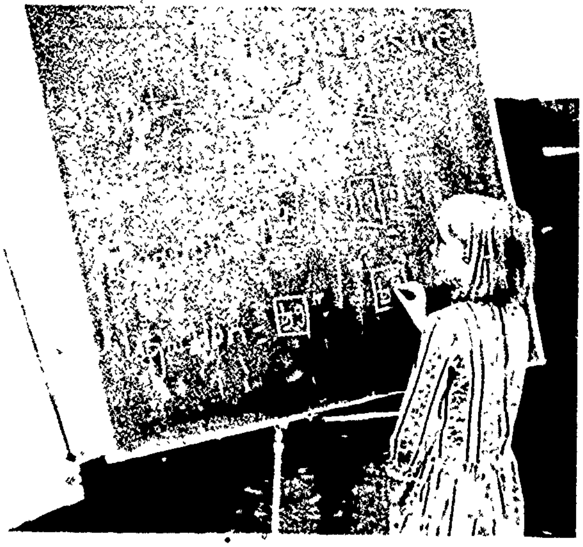
> HOW MANY TIMES WILL THE EXPERIMENTER HAVE TO EMPTY THE HALF-PINT CONTAINER INTO THE PINT CONTAINER IN ORDER TO FILL THE PINT CONTAINER?

Guesses may vary. Have the Experimenter fill the half-pint container with water and pour it into the pint container. The other children in the class keep track by counting each half-pint that is poured into the pint. Each child draws a representation of each half-pint on his worksheet. The Recorder draws and labels a representation of the half-pint container on the chalkboard to the right of the one-pint measure. Each time the Experimenter fills and empties the half-pint container, the Recorder and children at their desks should sketch and label a representation of the container. When the pint container is full ask:

> HOW MANY HALF-PINTS DID IT TAKE TO FILL THE PINT CONTAINER? (2.)

The Recorder should repeat this as he checks his diagram. (Reading from left to right, "one pint equals two half-pints.")

Now use the pint container and ask the class to guess how many times the pint will have to be filled and emptied into the quart container. Follow the same procedure using the quart to fill the half-gallon and the half-gallon to fill the gallon. The Recorder and the children at their desks draw and label representations of the containers. The chart should look like this:



Ask the children to imagine a computer set up to add volume measures.

> WHAT WOULD SUCH A COMPUTER LOOK LIKE? (There would be five desks labeled "½-pint," "pint," "quart," "½-gallon" and "gallon.")

(You should erase the liquid measure chart at this time.)

Draw a chart representing this computer on the chalkboard. Tell the children that last week you bought a half-gallon of milk, a quart of orange juice and a half-pint of whipping cream. Write these amounts in the top line of the chart. Have the children tell you where each numeral goes. Then say that yesterday you bought a quart of milk and a half-pint of coffee cream. Put these figures in the chart. Then ask the children the total quantity of liquid you bought, but do not write down their answers.

	G	$\frac{1}{2}$G	Q	P	$\frac{1}{2}$P
	0	1	1	0	1
+	0	0	1	0	1

OUR COMPUTER NEEDS SOME RULES TO TELL IT HOW TO ADD LIQUID VOLUME MEASURES. LET'S SET UP A COMPUTER AND MAKE UP THE RULES SO THAT IT CAN ADD THE QUANTITIES IN THE CHART.

Leave the chart on the board and go on to Activity B.

Activity B

Briefly review how the T piece computers were set up. (Desks were labeled to represent place values and children were selected from the class to act as Recorders, Programmers or Computers.) Recall that the computer did only what it was told to do. Emphasize that it was necessary to make certain rules for the computer to follow. The rules were:

Rule 1. Each place could accept only one kind of piece.

Rule 2. Each place could not have more than nine pieces. (If there were more than nine, ten pieces had to be traded in for the next larger piece.)

Rule 3. Each place must tell its contents to the Programmer when he calls for it.

Have five children push their desks together in front of the room. They will act as Computers. Bring out the desk labels and masking tape. By studying the column headings the children should be able to tell you where each label is to be placed. Tape the labels to the desk as the children locate the proper positions. Give each child a few small blank slips of paper.

Select one student to serve as the Banker. Have him come to the bank and group the containers according to size with the smallest containers on the right side of the table. (The containers should already be labeled.) Ask the children if they agree with the grouping and ordering the Banker has done.

ARE THE CONTAINERS GROUPED IN ORDER ACCORDING TO SIZE? (Yes.)

Ask the Banker to read to the class the label on the smallest container. Point to the title in the right-hand column of the chart on the chalkboard. The Banker should continue to read the labels on the containers in each size category as you point to its place on the chart. Then ask:

> SHOULD WE FEED T PIECES INTO OUR COMPUTER? (No, we should use half-pints, pints, quarts, half-gallons and gallons.)

Choose a Recorder and two Programmers. Have the Recorder read the quantities in the first row of the chart (one half-pint, one quart and one half-gallon) while one Programmer gets the container corresponding to each measure from the bank. These containers should be filled with water and put in the proper places in the computer.

The other Programmer should then go to the bank and get the containers that illustrate the amounts for the second row as the Recorder reads these quantities. The Programmer fills these with water and puts them in the computer. Now the computer must find the total. Before it can do so, it must have a set of rules to follow. Keeping in mind the rules that the addition and subtraction computers followed, ask the children to suggest rules for this computer. These should include:

Rule 1. Each place in the computer can accept only one kind of unit.

Rule 2. The computer counts the number of containers in each place.

Rule 3. The computer feeds out the number of units in each place.

When the computer follows these rules it will feed out these totals. (Write them on the chart.)

	G	$\frac{1}{2}$G	Q	P	$\frac{1}{2}$P
	0	1	1	0	1
+	0	0	1	0	1
	0	1	2	0	2

Suggest a rule for the computer to regroup:

Rule 4. If any place in the computer contains enough of one kind of unit to form the next larger unit, it must exchange the correct number of smaller units for one unit of the next size and put that unit in the proper place.

Remind the children that in the T pieces computer, 10 pieces of one kind were exchanged for 1 of the next larger pieces. But this computer is different. Lead the children to suggest filling the smaller containers with water and then pouring the water into the larger container. In this way they can determine for sure how many smaller containers equal a larger one.

There are two half-pints in the computer, so the child at that place asks the bank for the next larger unit, one pint. He pours one of his half-pints into the pint container and discovers that the pint container is only about half full. If he thinks another half-pint will fit into the pint, let him pour it in. He will see that two half-pints equal one pint. Then he puts the filled pint container into the proper place in the computer and returns the half-pints to the bank. Emphasize that two half-pint containers hold exactly the same amount of water as one pint container, that is, these two measures are equivalent. (If the two half-pints did not completely fill the pint container, have the children speculate as to why they did not. Perhaps the two half-pint containers were not completely full or some water may have spilled while the child was pouring.)

The last rule for this computer should now be revised to say:

Rule 4. If two containers appear in any one place at the computer these two must be exchanged for one of the next larger containers. An exception is gallon's place which can hold more than two containers.

The child at the quart place of the computer exchanges the two quart containers for one half-gallon container. Again emphasize that these are equivalent in volume. When the new half-gallon container is put in the proper place, two half-gallon containers can be exchanged for a gallon container.

The children at the computer, starting with the half-pint place, feed out the numerals 0, 1, 0, 0, 1 by writing them on slips of paper. Each slip should also be labeled with the place value. The Recorder fills in the chart on the chalkboard.

Choose other children to operate the computer and work a few more problems. The children should learn to discriminate visually among the sizes of the containers, so remove the labels as soon as possible.

While the children are using the computer to do problems, discuss with them its similarity to the addition and subtraction computers they used that were base-ten computers. The rules by which those computers and this one operate are the same except that this computer must regroup when it has two or more containers in a place. The base-ten computer did not regroup until it had ten or more pieces in a place. Let the children discuss the idea that this computer could be used to add and subtract numerals using any base, with only slight changes in the rules to allow for regrouping in the proper base.

Activity C

In this activity the children do more addition problems with volume containers and the computer. If the children in your class did well on Activity B, you can omit this activity, or perhaps go through the procedure once as review before starting Activity D.

Place all the containers listed in the beginning of this lesson on a table. Designate five desks to be the computer and label them with the place value names. The children at these desks can serve as Computers. Select a child to be Recorder and have him draw a diagram of the computer on the chalkboard. The children who do not have a role should act as Repairmen, watching carefully to see that the containers are fed into the proper places. Randomly distribute one container to each Repairman. Call on three students to "feed" the containers they have into the computer one at a time at the proper place. It is important that no two children you call on have the same size container.

The Recorder should write on his chart in the proper place the number of containers that have been entered or fed into the computer. For example:

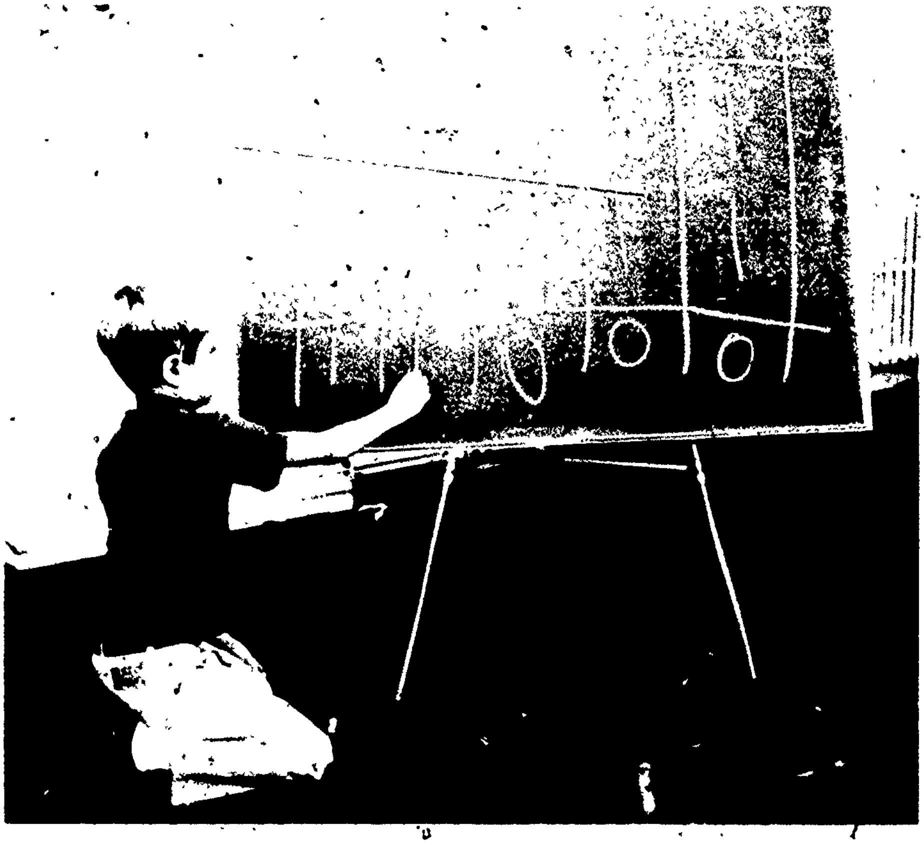
	G	$\frac{1}{2}$G	Q	P	$\frac{1}{2}$P
	0	1	1	0	1
+					

Call on two to four more children to feed their containers one at a time into the computer. The Recorder should write in the second part of the problem on his diagram.

	G	$\frac{1}{2}$G	Q	P	$\frac{1}{2}$P
	0	1	1	0	1
+	0	1	0	1	1

Now that the computer has been programmed, have each place add the number of containers it has. Remind the Computers that each place must trade in its containers if it has two or more. If a place needs to exchange two of its containers for the next larger, that child should find the proper container on someone's desk and exchange it for his two. He should take the larger container back to the computer and put it in the next larger place. While the computer is regrouping, the Repairmen should watch for errors. You may want to allow the person finding an error to replace the person who made the mistake.

7. When the computer has finished its computation, the Recorder calls on each place to tell its answer. The solution to the sample problem is:

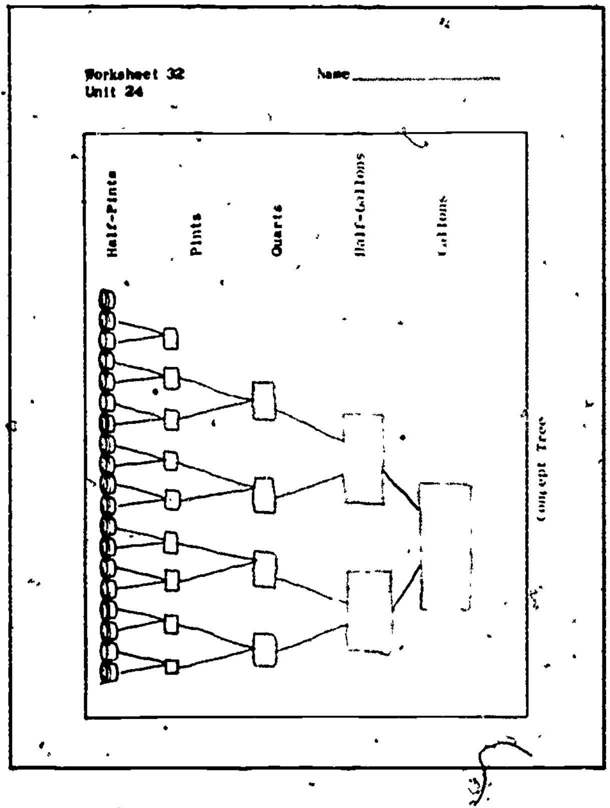


In some problems the gallon's place may end up with more than two containers. Remind the children that the number of gallons can go over two without regrouping. (A barrel is usually a 31½-gallon container and a hogshead is a 63-gallon container, but the children do not need to know this.)

Have the class do several problems following this procedure. Let different children operate the computer each time so that everyone in the class gets a chance.

Activity D

In Units 2, 8 and 21 the children used concept trees to classify curves and sort sets of objects into subsets. The relations among the various liquid measures can be described by building a concept tree.



Place 19 half-pint containers in a row on a table or on the floor. Have the children turn to Worksheet 32 which they will complete as you demonstrate. Then ask:

> HOW MANY HALF-PINT CONTAINERS DOES IT TAKE TO MAKE A PINT CONTAINER?

(It may be necessary to recall the regrouping the children did in Activity B.) Then place one empty pint container in front of two half-pint containers in your display. Use yarn or string to connect the two half-pints to the pint. The children should draw a pint container on their worksheet that depicts the same relationship. They should draw lines to the pint container.

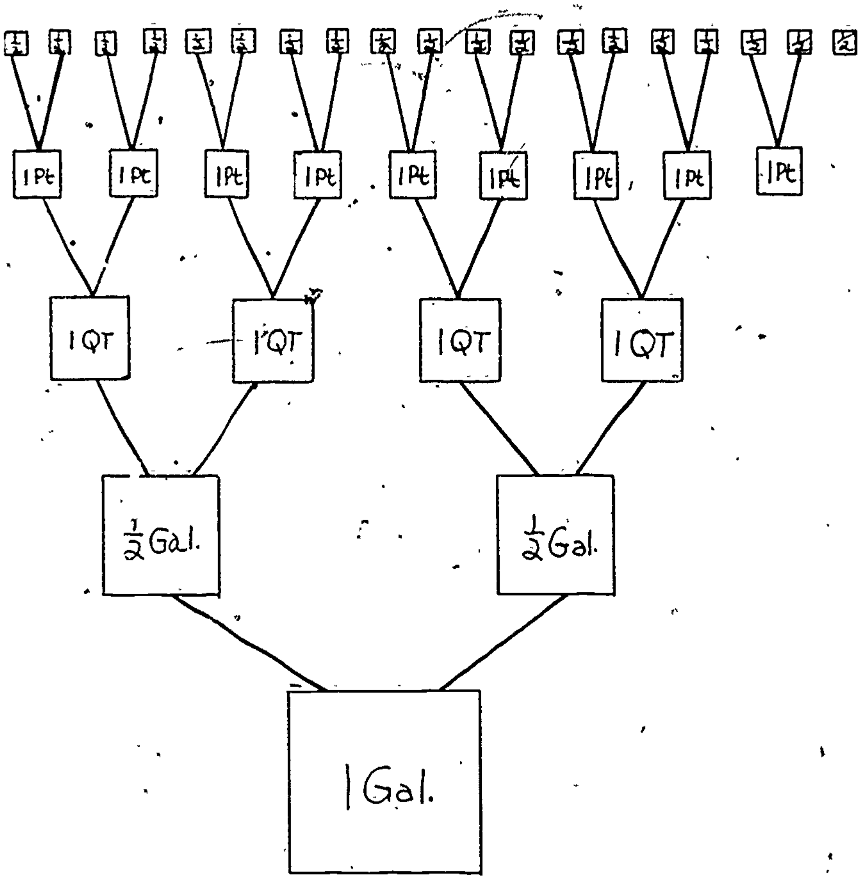
> HOW MANY MORE TIMES CAN WE SHOW THAT TWO HALF-PINTS MAKE A PINT? (8.)

Have a child come up and put 8 pint containers in the proper relationship to 16 half-pint containers in the display. Again he can use yarn to connect the containers. Similarly have the children draw eight more pint containers each in relation to two half-pint containers on the worksheet and connect them with lines.

Note that one half-pint container is left over and cannot be paired. Ask the class:

HOW MANY PINTS DOES IT TAKE TO MAKE ONE QUART? (2.)

Select another child to come to the display and depict this relationship. He should place one quart container in front of each pair of pint containers and connect them with yarn. The other children represent the relationship on their worksheets by drawing four quart containers and connecting each quart to the two pint containers it equals. Again, one pint cannot be paired. Follow this procedure and combine quarts to make half-gallons and half-gallons to make a gallon. Have the children complete Worksheet 32 as you complete the display. The completed display should look like the diagram below.



Review the tree diagram and what it shows.

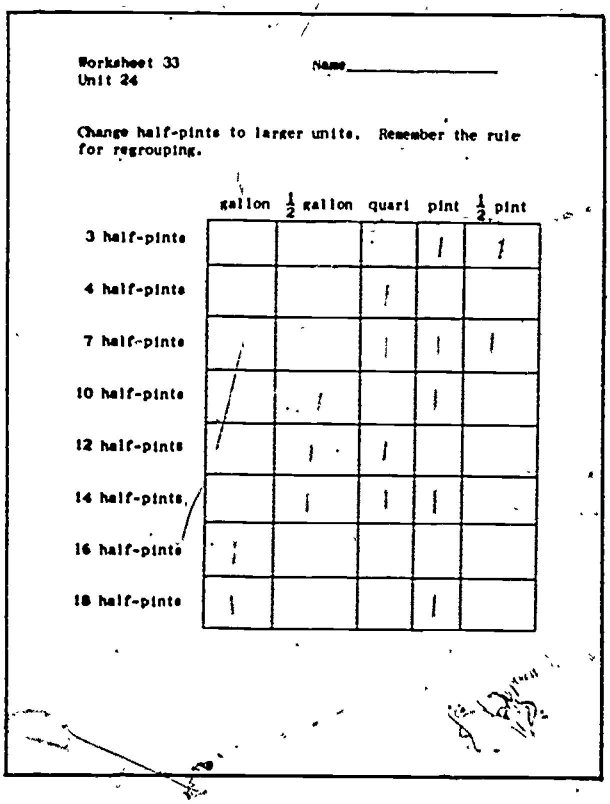
WE BEGAN WITH 19 HALF-PINTS. WHAT DO THEY EQUAL?

Answers may be given in terms of pints, quarts, half-gallons or gallons. Accept all answers but be sure that the children do not forget the pint and half-pint that could not be paired.

IF THERE WERE MORE HALF-PINTS COULD WE BUILD ANOTHER BRANCH FOR OUR TREE? (Yes.)

Continue asking the children what each level of the tree equals in terms of the next larger level. Summarize the tree diagram:

WE HAVE FOUND THAT 19 HALF-PINTS EQUAL ONE GALLON, ONE PINT AND ONE HALF-PINT.



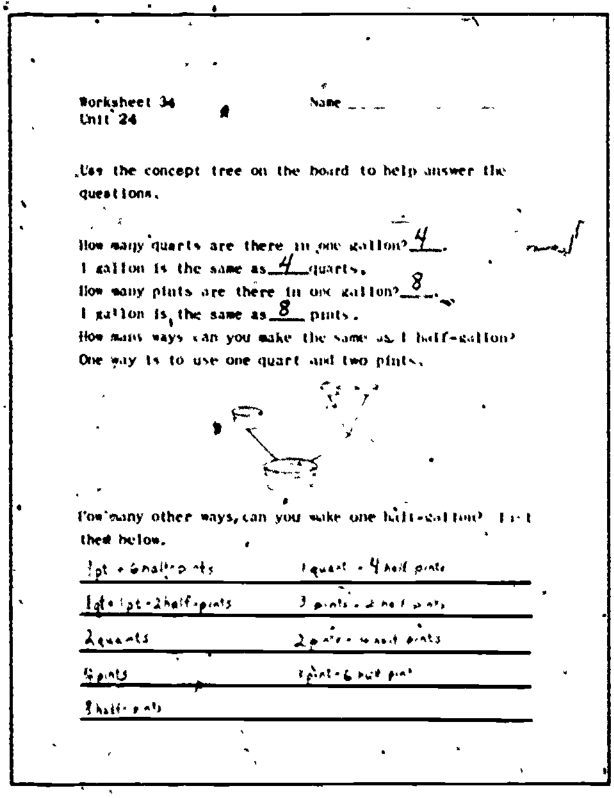
Worksheet 33
Unit 24

Name________________

Change half-pints to larger units. Remember the rule for regrouping.

	gallon	$\frac{1}{2}$ gallon	quart	pint	$\frac{1}{2}$ pint
3 half-pints				I	I
4 half-pints			I		
7 half-pints			I	I	I
10 half-pints		I		I	
12 half-pints		I	I		
14 half-pints		I	I	I	
16 half-pints	I				
18 half-pints	I			I	

Help the children start Worksheets 33 and 34. Remind them (if they forget) that they must regroup when there are two or more containers in one place. Be sure the children understand how to use the concept tree to find the relations among the various units of liquid measure. Have the class complete these two worksheets and Worksheet 35 by themselves.



Worksheet 34 Name ______
Unit 24

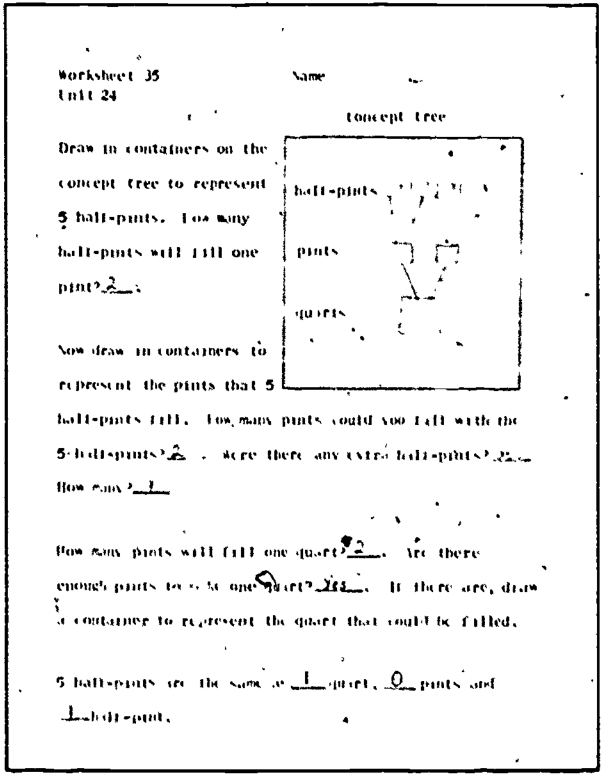
Use the concept tree on the board to help answer the questions.

How many quarts are there in one gallon? 4 .
1 gallon is the same as 4 quarts.
How many pints are there in one gallon? 8 .
1 gallon is the same as 8 pints.
How many ways can you make the same as 1 half-gallon?
One way is to use one quart and two pints.

How many other ways can you make one half-gallon? List them below.

1pt + 6 half-pints
1qt + 1pt + 2 half-pints
2 quarts
4 pints
8 half-pints

1 quart + 4 half pints
3 pints + 2 half pints
2 pints + 4 half pints
1 pint + 6 half pint



Worksheet 35 Name
Unit 24

Concept tree

Draw in containers on the concept tree to represent 5 half-pints. How many half-pints will fill one pint? 2 .

Now draw in containers to represent the pints that 5 half-pints fill. How many pints could you fill with the 5 half-pints? 2 . Were there any extra half-pints? yes
How many? 1

How many pints will fill one quart? 2 . Are there enough pints to fill one quart? Yes . If there are, draw a container to represent the quart that could be filled.

5 half-pints are the same as 1 quart, 0 pints and 1 half-pint.

Lesson 13: LENGTH

This lesson begins with an activity that presents a brief history of measurement. It demonstrates the necessity of standardized systems of measurement and motivates the children to explore the two systems that are in use today. The students compare the British and metric systems, and are introduced to the units of measurement used in each. The first two activities should take one class period.

In Activities C and E the children investigate the relationships among the units of measurement in the British system. Activity D is an optional activity that encourages further investigation of the metric system and the relationships among its units.

MATERIALS

- foot ruler for each child
- yardstick for every pair of children
- meterstick
- paper
- Worksheet 36

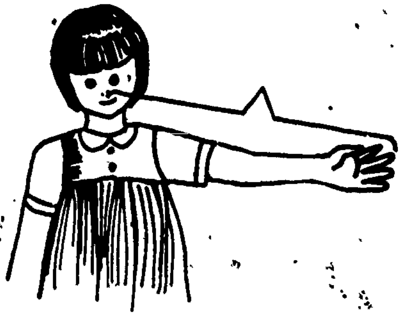
PROCEDURE

Activity A

Before standardized measures were introduced, the lengths of various parts of the body were used as approximate measures. In this activity, the children will use their feet, thumbs, hands and arms to measure things in the classroom.

Explain to the class that a long time ago these measures were used:

yard = the distance from the tip of the nose to the end of the middle finger on the hand of an outstretched arm.



foot = the length of any available foot. 

palm = the width of four fingers when the fingers are closed. 

inch = the width of the thumb. 

Have the children choose partners. For each measurement the children make, one child in each pair should lay down the appropriate appendage while the other marks where the measure falls. Have the pairs keep records of their measures on paper.

Ask each pair to measure the width of a student's desk in palms and inches and record their results. Then have several different pairs measure the width of the door in palms, the length of the chalkboard in yards, the length of the radiator or other object on the floor in feet, and the length of a textbook in inches. Choose children who vary in size to measure the same object.

Have all the pairs that measured the same object read their results. There should be some variety among the answers. Ask why these discrepancies occurred (people are different sizes). Lead the children to the idea that a more accurate system of measurement is needed for precise measuring.

Activity B

Tell the children that a long time ago people discovered what the class has just discovered: that a more precise system of measurement was needed. Write the words "British" and "Metric" on the chalkboard and explain that these are names of two systems of measurement. Show the children a yardstick and write "yard" under the proper heading. Explain that this is one of the measures in the British system. Then show the children a meterstick, compare it with the yardstick and write "meter" under "Metric." Explain that this is one of the units in the metric system.

Write the other units of measure used in each system beneath the proper heading. Ask the children to examine these closely and tell you how many of each kind of unit it takes to make the next larger unit. As you enter the measure, draw lines on the board to show the lengths of those measurements that will fit on the chalkboard. The completed chart is shown below.

British			Metric		
12 inches	=	1 foot	10 millimeters	=	1 centimeter
3 feet	=	1 yard	10 centimeters	=	1 decimeter
5280 feet	=	1 mile	10 decimeters	=	1 meter
			10 meters	=	1 dekameter
			10 dekameters	=	1 hectometer
			10 hectometers	=	1 kilometer

Point out to the children that the relationships among the metric units are regular. Each unit is 10 times larger than the next smaller unit. Ask the children what a computer using the metric system would look like. Ask how many metric units could be in each place of the computer before they would have to regroup. (9. If there were 10 units, the computer could exchange them for 1 of the next larger units.)

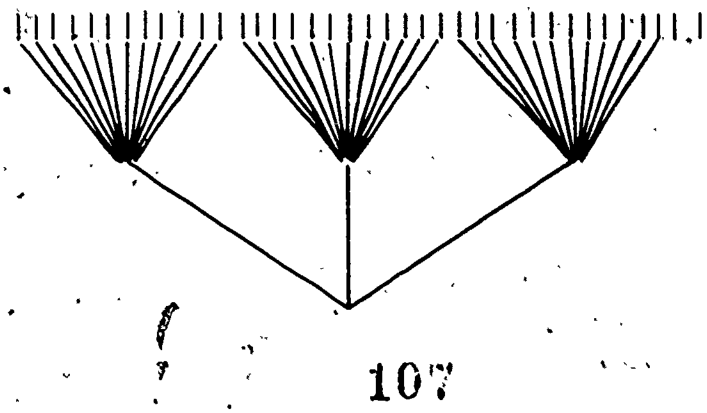
In contrast, point out that the British units are not regular. For example, three feet make one yard, but 12 inches make one foot. Ask the children to imagine a computer using the British system. Lead them to see that they would have to have a different rule for regrouping at each place.

Activity C

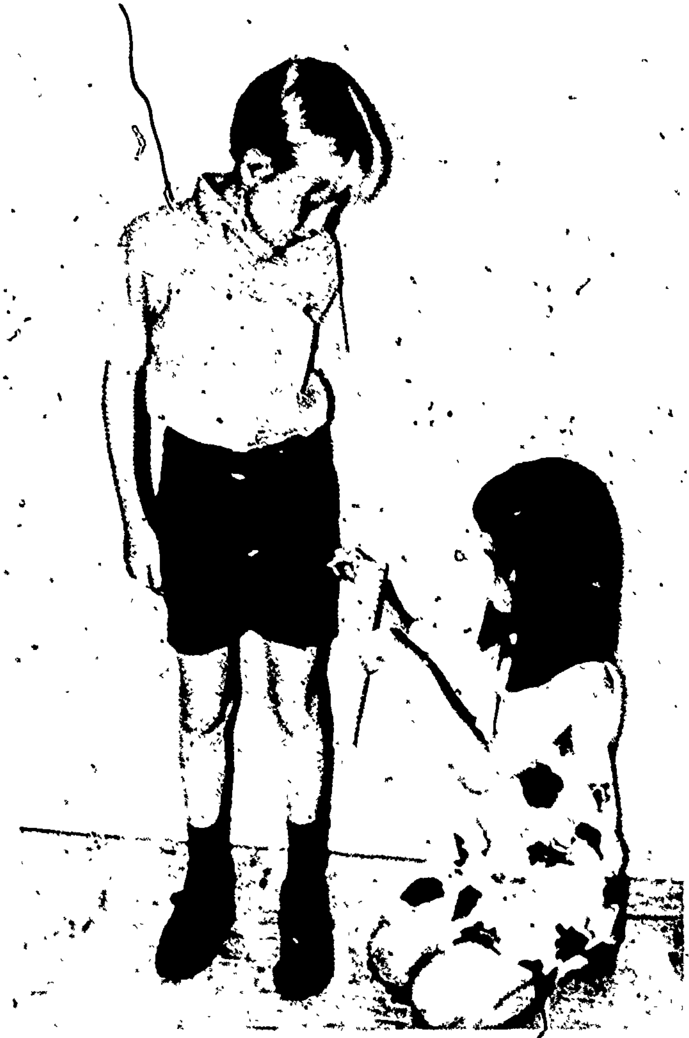
Choose an object in the room, for instance your desk top, and have a child measure its length in inches with a yardstick. Have him write the length of the object in inches on the chalkboard.

Then have each student take a blank sheet of paper and make the same number of dots as inches in the object along one edge of the paper. You should represent the inches with dots on the chalkboard. Then ask how we could tell the number of feet in the object. (Connect every 12 inches.) Draw lines to make a tree diagram which illustrates that every twelve inches are equivalent to one foot, and have the children do the same on their papers. When they have finished, ask them to count the number of feet and remaining inches on their charts.

In the tree diagram below, there are 38 inches or 3 feet, 2 inches, or one yard, 2 inches.



Have the children choose two other objects to measure. Ask them to make diagrams to represent the lengths they find in yards, feet and inches. The children may choose to measure and compare their own heights.



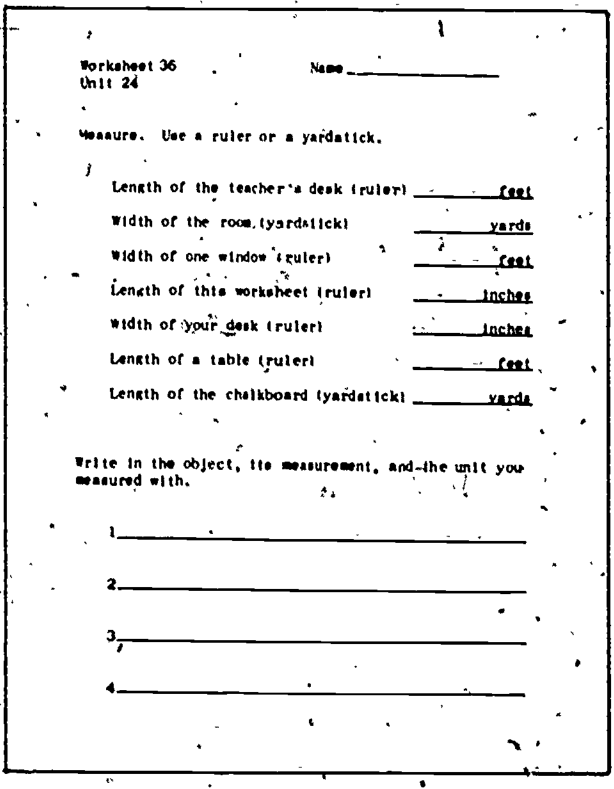
Activity D

This activity is optional. Repeat Activity C using the meterstick for measuring lengths. Have the children draw a concept tree to illustrate the length of an object they measured. The completed tree will show the relationships among the units of measurement in that system. Compare these with the relationships among the British system of measure.

Activity E

In this activity the children have the opportunity to practice their measuring skills. Worksheet 36 provides a list of things to be measured. The children should measure each item to the nearest units indicated at the right of the worksheet. For example, the width of a window is to be measured to the nearest foot while the length of the chalkboard is to be measured to the nearest yard. Have the children work individually or in pairs.

The last four lines on the worksheet are blank so that you or the children can choose some items to measure. Ask each child or pair of children to decide which unit of length to use to measure each object. After all the measurements have been taken, children can check with each other to see if they found the same measurement for each object. Have them justify the appropriateness of the units they choose for the last objects.



Worksheet 36
Unit 24

Name ____________

Measure. Use a ruler or a yardstick.

Length of the teacher's desk (ruler) ______ feet
Width of the room (yardstick) ______ yards
Width of one window (ruler) ______ feet
Length of this worksheet (ruler) ______ inches
Width of your desk (ruler) ______ inches
Length of a table (ruler) ______ feet
Length of the chalkboard (yardstick) ______ yards

Write in the object, its measurement, and the unit you measured with.

1. ____________________
2. ____________________
3. ____________________
4. ____________________

Lesson 14: TIME

Clocks measure time in seconds, minutes and hours. We also use day, week, month, year and century to describe units of time. In this lesson, the children investigate relationships among some of the units of time measurement.

MATERIALS

- 2 or 3 sheets of paper per child
- clock with second hand
- masking tape
- Worksheets 37 and 38

PROCEDURE

Activity A

Discuss with the children the relations among some of the units of time measure. As various units are mentioned, list them on the chalkboard. Your list might include:

60 seconds = 1 minute
60 minutes = 1 hour
24 hours = 1 day

365 days = 1 year
366 days = 1 leap year
100 years = 1 century

We are accustomed to watching a clock to determine the passage of a certain amount of time. It is interesting to try to judge some amount of time with our eyes closed. Tell the children that they will try to judge when 60 seconds have passed.

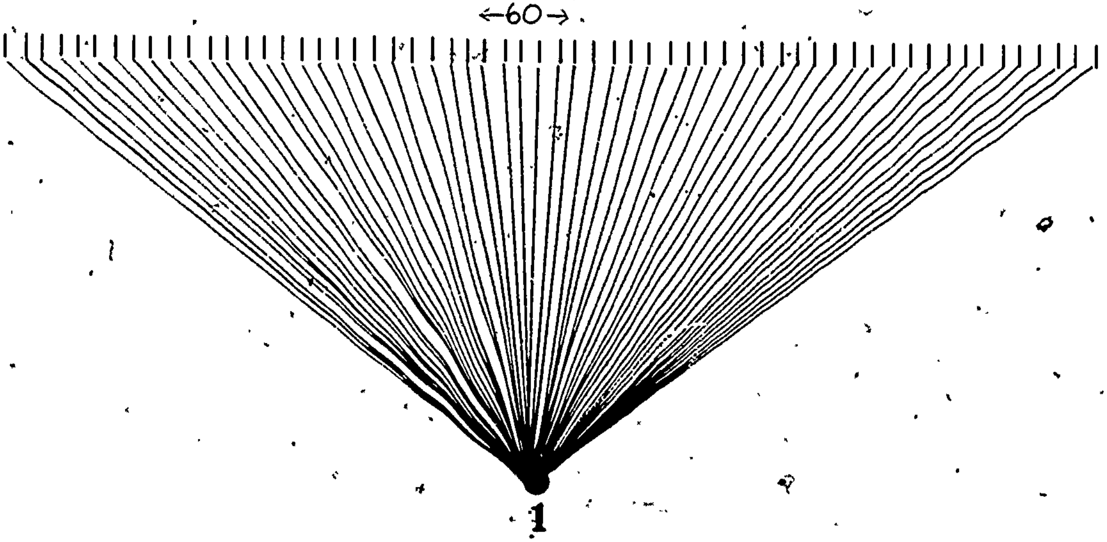
Ask the children to close their eyes and keep them closed for 60 seconds starting when you say "go." Tell them to open their eyes when they think the time has passed. When they have their eyes closed you will be writing numerals on the chalkboard to indicate the number of seconds that have lapsed. Start writing 5-second intervals after 15 or 20 seconds have gone by. As each child opens his eyes he notes the numeral you have just written to discover how long he actually had his eyes closed. Caution the children not to talk after they open their eyes so they do not give away the time. There will probably be much variation in the amount of time the children keep their eyes closed.

Ask the children if they think they could come closer to judging 60 seconds if given another chance. Repeat the procedure three or four times to allow each child to modify his techniques for estimating the length of a minute's duration. After a few trials you may want to ask those children who came close to 60 seconds what methods they used to make judgments, but do not spend too much time on this.

Activity B

Give each student two or three sheets of paper. Appoint one child to be the clock watcher. He is to call out each second in one minute. When you say "go," the clock watcher is to start his count and each child is to make one tally per second for 60 seconds on the edge of one sheet of paper. Some of the children will need to use a second sheet of paper while others will crowd all their marks into a small space along the edge. Point out that these differences do not matter; each child's marks represent one minute.

The children can illustrate that it takes 60 seconds to make one minute by drawing lines to show that all the second tallies produce one minute-tally.



Ask:

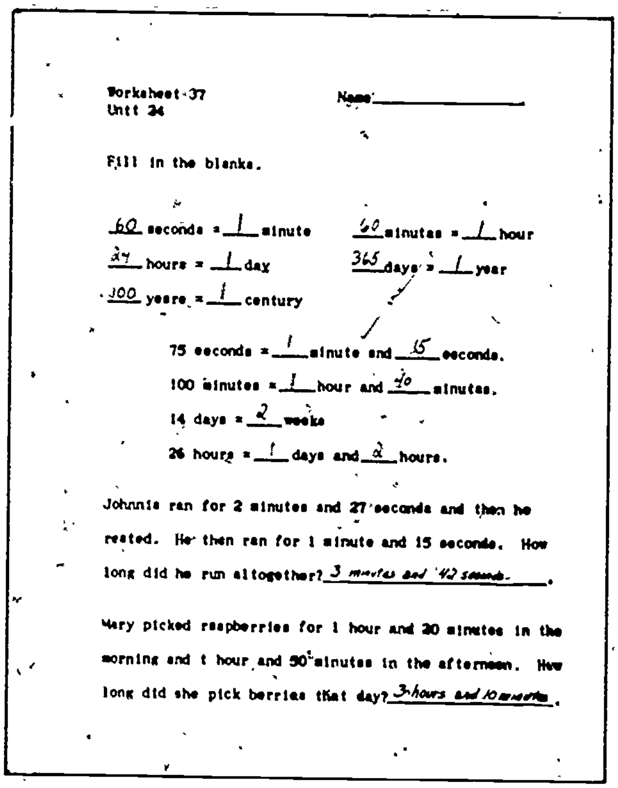
> HOW MANY MINUTES DOES IT TAKE TO MAKE ONE HOUR? (60.)
>
> HOW CAN WE USE THE MINUTE REPRESENTATIONS WE HAVE MADE TO ILLUSTRATE ONE HOUR?

Allow the children to give their ideas, and then suggest that an hour could be illustrated by taping 60 minute representations to the chalkboard. Have the children bring their minutes to you. Since there will not be enough to represent an hour, have the class repeat the procedure to generate more minute representations.

When 60 minutes have been made, the representation of an hour can be constructed by taping them to the chalkboard and connecting each one to a central point with chalk lines. This procedure may seem to be busy work, but do not hasten to drop the activity. Although students have occasion to discuss the relationships that exist between units of time measure, they seldom see these relationships other than on a clock face.

Ask the children how this system of time measurement would work in a computer. Point out that there are 60 seconds in one minute and 60 minutes in one hour, but there are 24 hours in one day. The children will see that a computer would need one rule when regrouping the seconds and minutes, but it would need different rules for regrouping hours into days, days into weeks, weeks into months, and months into years. Ask the class to state these rules. If you have time, set up a time measurement computer and have the children do some problems that involve regrouping.

To review the relationships among time units, complete the first part of Worksheet 37 with the children. Worksheet 38 reviews material covered in Section 2.



Worksheet 37
Unit 24

Name________________

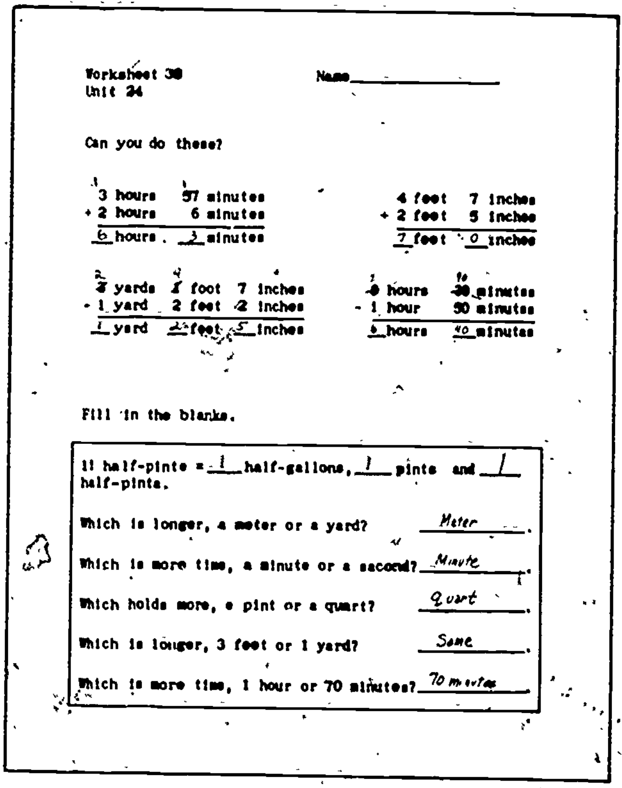
Fill in the blanks.

60 seconds = 1 minute
60 minutes = 1 hour
24 hours = 1 day
365 days = 1 year
100 years = 1 century

75 seconds = 1 minute and 15 seconds.
100 minutes = 1 hour and 40 minutes.
14 days = 2 weeks
26 hours = 1 days and 2 hours.

Johnnie ran for 2 minutes and 27 seconds and then he rested. He then ran for 1 minute and 15 seconds. How long did he run altogether? 3 minutes and 42 seconds.

Mary picked raspberries for 1 hour and 20 minutes in the morning and 1 hour and 50 minutes in the afternoon. How long did she pick berries that day? 3 hours and 10 minutes.



Worksheet 38
Unit 24

Name________________

Can you do these?

$$\begin{array}{rlrl} & \overset{1}{3}\text{ hours} & \overset{1}{57}\text{ minutes} \\ + & 2\text{ hours} & 6\text{ minutes} \\ \hline & \underline{6}\text{ hours .} & \underline{3}\text{ minutes} \end{array}$$

$$\begin{array}{rlrl} & 4\text{ feet} & 7\text{ inches} \\ + & 2\text{ feet} & 5\text{ inches} \\ \hline & \underline{7}\text{ feet} & \underline{0}\text{ inches} \end{array}$$

$$\begin{array}{rlll} & \overset{2}{\cancel{3}}\text{ yards} & \overset{4}{\cancel{1}}\text{ foot} & 7\text{ inches} \\ - & 1\text{ yard} & 2\text{ feet} & 2\text{ inches} \\ \hline & \underline{1}\text{ yard} & \underline{2}\text{ feet} & \underline{5}\text{ inches} \end{array}$$

$$\begin{array}{rll} & \overset{7}{\cancel{8}}\text{ hours} & \overset{90}{\cancel{30}}\text{ minutes} \\ - & 1\text{ hour} & 50\text{ minutes} \\ \hline & \underline{6}\text{ hours} & \underline{40}\text{ minutes} \end{array}$$

Fill in the blanks.

11 half-pints = 1 half-gallons, 1 pints and 1 half-pints.

Which is longer, a meter or a yard? Meter.

Which is more time, a minute or a second? Minute.

Which holds more, a pint or a quart? Quart.

Which is longer, 3 feet or 1 yard? Same.

Which is more time, 1 hour or 70 minutes? 70 minutes

SECTION 4 THE POURING SYSTEM AND THE BALANCE BEAM SYSTEM

PURPOSE

- To introduce the children to self-instructional materials.
- To provide the children opportunity to analyze and compare two different systems.
- To provide practice in using graphing to analyze systems and to store data.

COMMENTARY

This section consists of two booklets in the Student Manual, both of which are designed to be read and completed by the students with a minimum of help from you.

1. You will want to read through the copy of the booklets with answers, but you should also complete both booklets on your own to familiarize yourself with the content before introducing them to the class. This will help you anticipate problems the children might encounter.

2. The necessary materials for the booklets should be placed in a convenient spot in the classroom. The students work in pairs. Each child fills in his own answers in the booklet in his manual.

3. Half the class begins with the Balance Beam System while the other half begins on the Pouring System because there is not enough equipment for all children to do the same booklet at the same time. As each pair finishes one booklet, they go on to the other one. Each book should take no more than one week. Even if some children do not finish in that time, they must start on the other booklet. They can complete the booklets in their own time if they wish. Those children who finish both booklets quickly can spend the remaining time playing the mathematical games from Lessons 3, 6 and 11.

4. A letter to the students appears just before the booklets in the Student Manual. The whole class should read the letter together. Each pair of students work together with the pouring and balancing, but each child should decide his own answers. Then the two compare answers and go over the work, if necessary, until they agree.

5. The students should work swiftly but carefully through their booklets. You should check each pair's work daily to be sure that they are not making errors such as working with an unbalanced beam, spilling water from a measured tube, or writing ordered pairs incorrectly. You may want to walk around as the students experiment, checking their procedures and written work.

Though the children may not have worked with self-instructional materials before, they will be working this way often in third grade units. Encourage them to work out difficulties they may have by themselves.

Because the children will have to become accustomed to working on their own, the first job booklet might go more slowly than the second.

Students with reading problems might be grouped together so that you can help them get started on each job. At no time should one student in any pair be allowed to dominate the work to the extent that the other student doesn't get a chance to generate his own answers. If this occurs, have the children change partners.

As the students pour water and balance the beam, they are asked to find and describe many different states of the systems in which:

a. The depth in either tube is a whole unit.

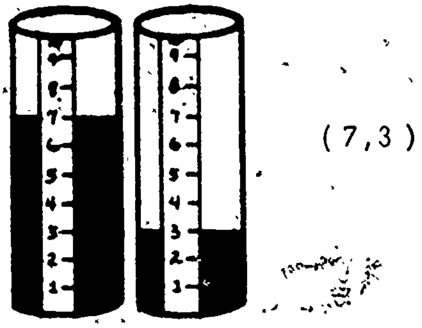
b. The depth in either tube is not a whole unit.

c. The beam is balanced.

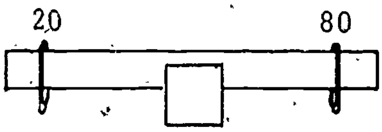
d. The beam is not balanced.

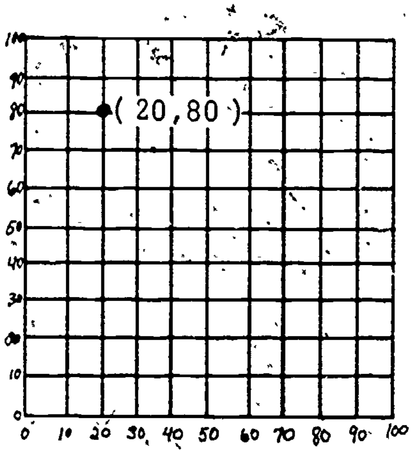
Once they have generated these states, the children describe them by one of these two techniques:

1. Ordered pairs. The ordered pair that describes the state of this system (the depth of the water in each tube) is (7,3). For this example the children would be asked to find and describe all the states of this system of 10 units of water. Other states would be (8,2), (9,1), (10,0), etc.



2. Points on a grid. The children are asked to find many different combinations of places to hang two paper clips so that the beam balances. They then graph these combinations on a grid. One point that describes a balanced state is at (20, 80). Later an extra clip is added and the children are asked to find and graph balanced states of this new system.





In the Pouring System booklet the children should discover that when they change the state of a system, the total amount of water does not change. If the children are asked to write four ordered pairs for a system containing nine units of water, they should discover that the sum of each ordered pair will equal nine.

With the Balance Beam System booklet the children learn that to balance the beam with two paper clips, the ordered pair must total 100. For example, if they are told that the red clip is at 70, they can predict that the black clip must be at 30.

Another way the children can predict the position of the black clip when given the position of the red clip is to determine how far from the center the red clip is. If it is at 70, they see that it is 20 units from the center. Then they will discover that to balance the beam, the other clip must also be 20 units from the center, at the 30 mark.

During later jobs in the Balance Beam Booklet the children use three paper clips: one red, one black, and one stationary silver clip. When the silver clip is in position, say at 60, the children must determine where to put the other two clips. They will see that the distance of one clip from the center of the beam must equal the distance of the other clip plus the distance of the silver clip. In other words, the distance of the clip on one side must equal the combined distances of the two clips on the other side. The ordered pair (10, 80) balances the beam when the silver clip is at 60. The silver clip is 10 units from the center and the clip at the 80 mark is 30 units from the center. These two clips are a total distance of 40 units from the center. The third clip at 10 is also 40 units from the center, so both sides balance.

40 distance of red clip from center of beam	=	30 distance of black clip from center	+	10 distance of silver clip from center

In some jobs the children are asked to write several ordered pairs that will balance the beam when the silver clip is at a certain point, say 60. After writing several such pairs, the children are asked to add the numbers in each pair. They discover that the sum of each pair is the same when the beam is balanced and the silver clip is stationary.

MATERIALS

-- for each pair of children --

Pouring System:

- 2 plastic tubes, $1\frac{1}{4}$ inches in diameter and 4 inches long
- measuring cup
- butcher tray
- masking tape

Balance Beam System:

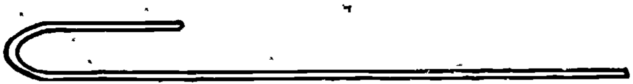
- meter stick
- wooden block
- 6 paper clips, (2 red, 2 silver, 2 black)
- straightened paper clip
- masking tape

-- for the class --

- pail (optional)

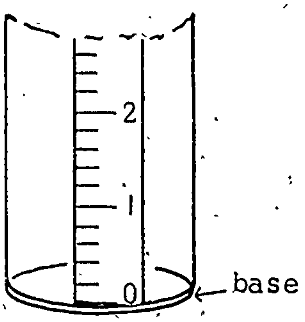
PREPARATION

Straighten one paper clip for each pair of children who will be working on the Balance Beam System. The clip will support the meter stick which acts as a balance arm.



A completed beam balance appears on page 126.

Each child working on the Pouring System will need two plastic tubes, 1¼ inches in diameter and 4 inches long. For every tube, cut off a ten-unit length of centimeter tape. Tape it to the outside of the tube, making sure that the zero mark rests where the tube cavity begins rather than at the base of the tube. For a completed tube, see page 108.



UNIVERSITY OF *Minnesota*

MINNESOTA SCHOOL MATHEMATICS AND SCIENCE CENTER
720 WASHINGTON AVENUE S.E. • MINNEAPOLIS, MINNESOTA 55414

Office of the Director

Dear Student,

Your teacher has told you to turn to either the Pouring System booklet or the Balance Beam System booklet in your Student Manual. You and your partner should work together in your booklets. Read each page very carefully and try to fill in every blank and to answer all the questions. Be sure to discuss all your answers with your partner. If you and your partner disagree, the two of you should work the problem again to find the correct answer.

Sometimes it may be hard to find the answer to a question. When this happens, read the question again. Try working the problem a different way. Don't go to your teacher unless you and your partner just can't find the answer. For some questions, there may be more than one correct answer.

From time to time, the teacher will look at the work you've done. She will be checking to see that you have done the jobs correctly and that you are working as fast as you can. Each booklet should take one week. If you don't get done with one of the booklets in time, you can finish it in your spare time.

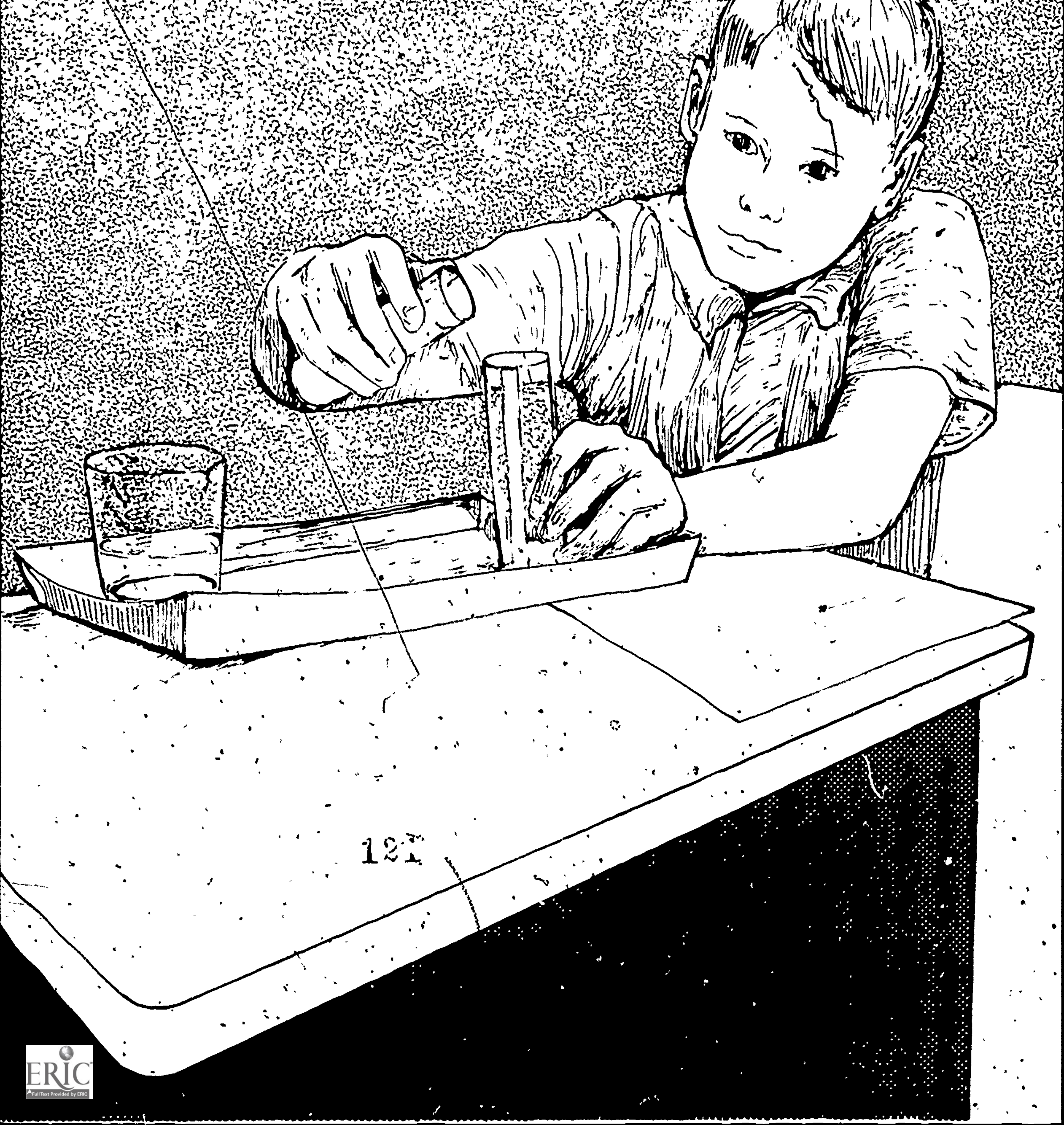
Have fun.

Sincerely,

The Authors

JOB BOOKLET

POURING SYSTEM



Job 1

Describe each picture.

(2 boys , 3 girls)

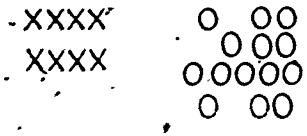
(2 , 3)

(3 oranges , 5 apple)

(3 , 5)

(2 rockets , 4 airplanes)

(2 , 4)



(8 x's , 14 o's)

(8 , 14)

(6 units in A , 3 units in B)

(6 , 3)

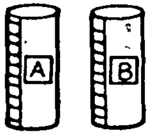
(7 units in A , 3 units in B)

(7 , 3)

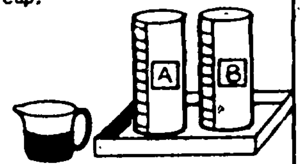
Job 2

Get these materials from your teacher.

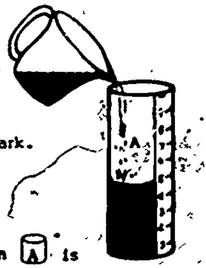
Use the two pieces of tape.
Letter the tubes A and B.



Put some water in the pouring cup.

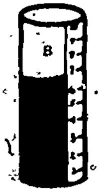


Fill A to the 5 mark.



The water in A is 5 units deep.

Fill B to the 7 mark.



The depth of the water in B is 7 units.

Job 3

Mary and I are partners.

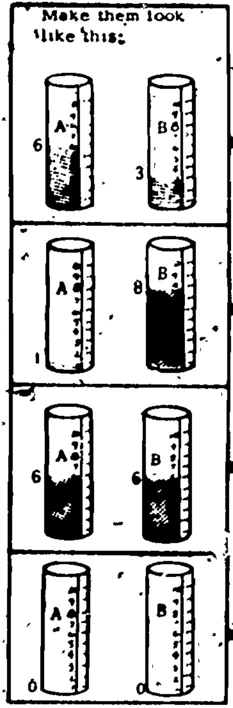
① We both try to do all the jobs. [✓ True] False

② We compare all our answers. [✓ True] False

③ If we have different answers, we discuss them to see who is correct. [✓ True] False

Pour water into A and B.

Make them look like this:



Describe A and B

Depth of water in A is 6 units.
Depth of water in B is 3 units.

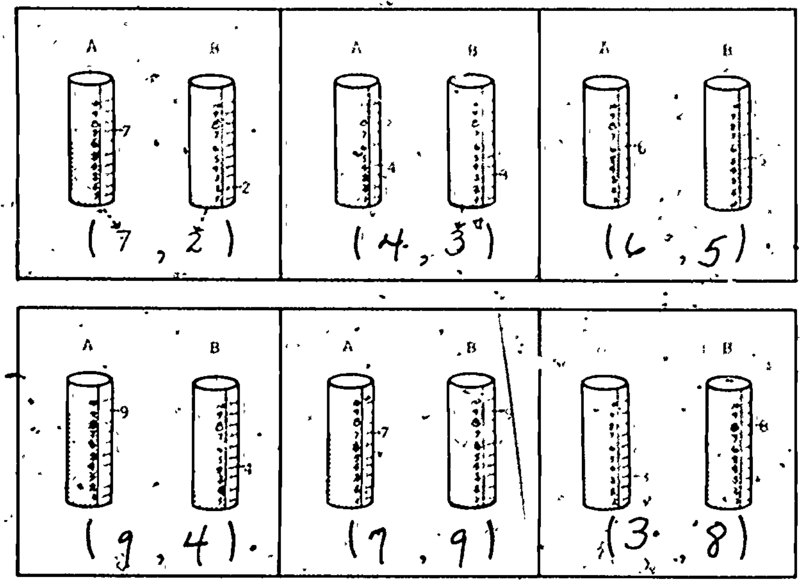
Depth of water in A is 1 units.
Depth of water in B is 8 units.

Depth of water in A is 6 units.
Depth of water in B is 6 units.

Depth of water in A is 0 units.
Depth of water in B is 0 units.

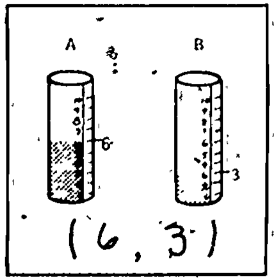
Job 4

Describe the state of each system.



(7, 2) (4, 3) (6, 5)

(9, 4). (7, 9) (3, 8)



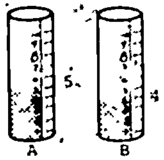
(6, 3)

(6,3) is an ordered pair.

6 means that there are 6 units of water in tube A.

3 means that there are 3 units of water in tube B.

Job 5



The ordered pair (5,4) describes the state of my system.

The ordered pair (5,4) is an (A,B) ordered pair.

The first member of (5,4) is 5.

The second member of (5,4) is 4.

The first member describes the state of tube A.

The second member describes the state of tube B.

So (5,4) is called an (A,B) type of ordered pair.

The ordered pair (4,5) describes the state of my system.

The ordered pair (4,5) is an (B,A) ordered pair.

The first member of (4,5) is 4.

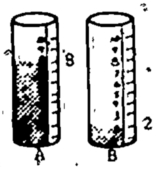
The second member of (4,5) is 5.

The first member describes the state of tube B.

The second member describes the state of tube A.

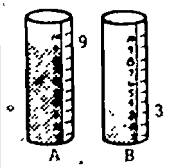
So (4,5) is called a (B,A) type of ordered pair.

Write (A,B) and (B,A) pairs to describe the systems.



A B
(8, 2)

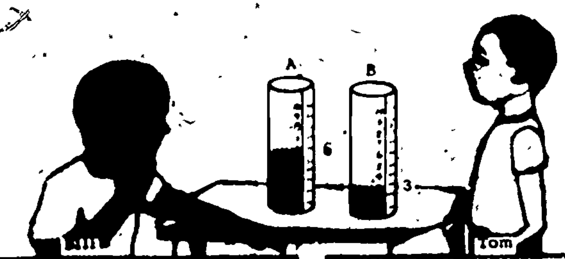
B A
(2, 8)



A B
(9, 3)

B A
(3, 9)

Job 6



① Bill wrote (6,3) to describe the state of the system.

② (6,3) is called an ordered pair.

③ The 6 means there are 6 units of water in tube A.

④ The 3 means there are 3 units of water in tube B.

① Tom wrote (3,6) to describe the state of the system.

② (3,6) is an ordered pair.

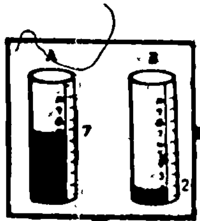
③ The 3 means there are 3 units of water in tube B.

④ The 6 means there are 6 units of water in tube A.

Bill wrote (6,3). Was he correct? [✓ Yes] No

Tom wrote (3,6). Was he correct? [✓ Yes] No

Job 7



Bill wrote (7 , 2).

There are 7 units of water in tube A

There are 2 units of water in tube B.

Tom wrote (2 ,7).

There are 2 units of water in tube B.

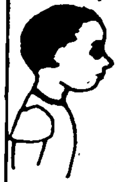
There are 7 units of water in tube A.

I wrote (7, 2).

(7, 2) is an ordered pair.

The first member of the ordered pair describes the state of tube A.

The second member describes the state of tube B.



I wrote (2 , 7).

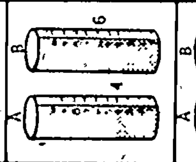
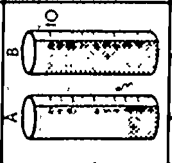
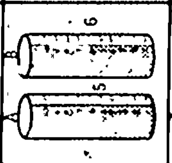
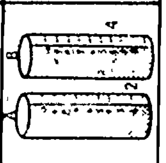
(2, 7) is an ordered pair.

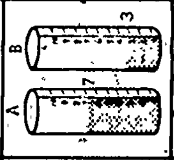
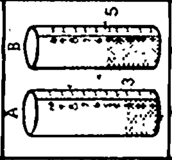
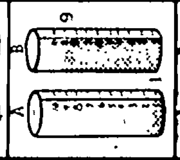
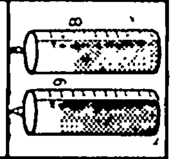
The first member of the ordered pair describes the state of tube B.

The second member describes the state of tube A.

Job 8

Fill in the charts.

Ordered pair (B, A)	6 , 4	10 , 3	6 , 5	4 , 2
State of the system				
Ordered pair (A, B)	4 , 6	3 , 10	5 , 6	2 , 4

Ordered pair (B, A)	3 , 7	5 , 3	9 , 1	8 , 9
State of the system				
Ordered pair (A, B)	7 , 3	3 , 5	1 , 9	9 , 8

Job 9

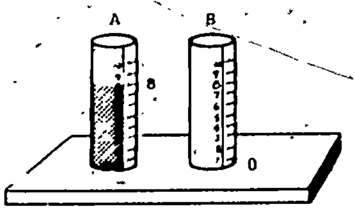
Make your systems look like this.

Your system contains

tube A

and

tube B.



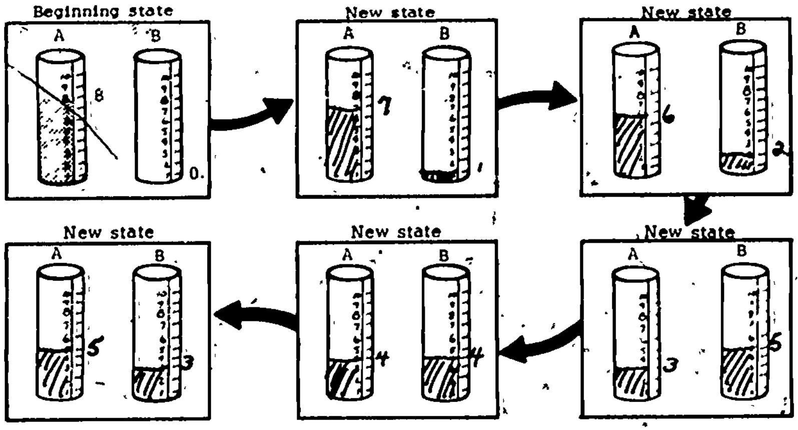
There are 8 units of water in this system

No water can be added to or removed from your system.

Use only the water in tubes A and B.

Pour the water back and forth between [A] and [B].

Draw a picture of each new state of your system.



Job 10

1 is a whole number.

2 is a whole number.

$\frac{1}{2}$ is not a whole number.

$1\frac{2}{3}$ is not a whole number.

5 is a whole number.

$3\frac{1}{2}$ is not a whole number.

Circle all the whole numbers.

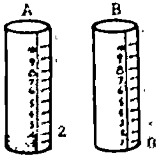
$\frac{1}{4}$ $3\frac{1}{2}$ [✓ 16] [✓ 2]

[✓ 5] $7\frac{1}{2}$ [✓ 6] [✓ 3]

[✓ 8] [✓ 0] $4\frac{1}{2}$ [✓ 13]

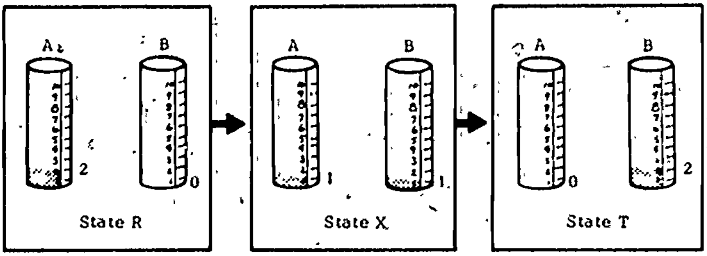
[✓ 0] [✓ 6] [✓ 9] $\frac{1}{4}$

Put your system in the state $(\overset{A}{2}, \overset{B}{0})$.



Pour water back and forth between A and B.

Can you get the same states as shown below?



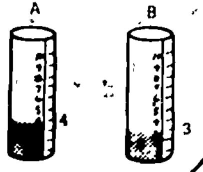
Write the ordered pairs that describe each state. Fill in the blanks.

STATE R	(2, 0)	2 is a whole number. 0 is a whole number
STATE X	(1, 1)	1 is a whole number. 1 is a whole number.
STATE T	(0, 2)	0 is a whole number. 2 is a whole number.

Job 11

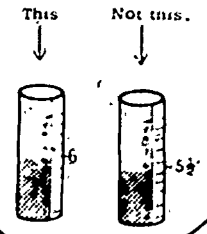
1. Put your system in the state $\overset{A}{4}, \overset{B}{3}$ (4, 3).

2. Your system should look like this:



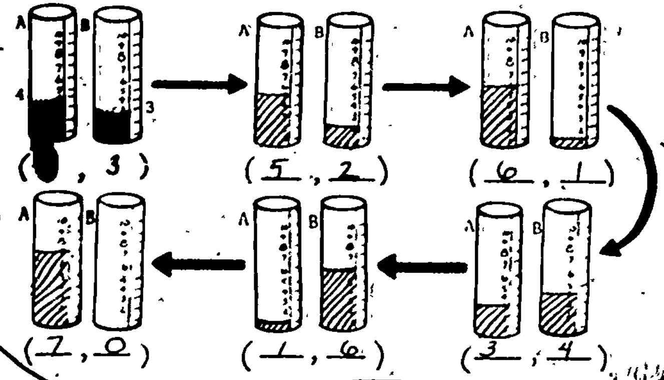
3. Pour the water back and forth between A and B.

The water level in either tube should be at a whole number mark.



4. Draw a picture of each new state of your system.

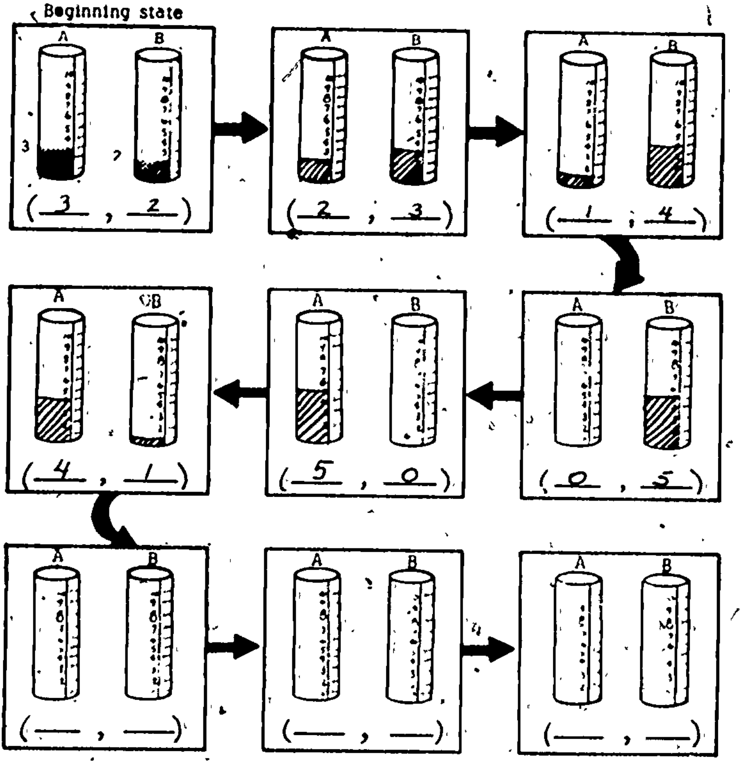
Write an ordered pair to describe each state.



Job 12

- Put your system in the state (3, 2), A over 3 and B over 2. Your system should have 3 units of water in tube A and 2 units of water in tube B.
- No water can be added or removed from your system.
- Pour water back and forth between A and B.

Record each of the states in which the water level in one tube is a whole unit.



Job 13

addend + addend = sum

5 = 4 + 1 sum = addend + addend
7 = 2 + 5 sum = addend + addend
3 + 2 = 5 addend + addend = sum
3 + 6 = 9 addend + addend = sum

Use the numbers 0,1,2,3,4,5, as addends.
Add any two of the addends to get the sum of 5.
Write addition sentences to show the different ways.

4 + 1 = 5	5+0=5
3 +2 =5	
2 +3 =5	
1 +4 =5	
0+5 =5	

I found six ways to add the addends to get the sum of 5. [✓ True] False

Job 14

1. Pour five units of water into A and no units into B.
The system contains a total of 5 units of water.

2. No water can be added or removed from the system.
There must always be a total of 5 units of water.

3. Pour the water back and forth between A and B.
After each pouring, the water level in at least one tube should be at the whole unit mark.

4. Record all the possible states of your system.
Count only those states where the water level is at a unit mark.

A	B	Ordered Pair	Addition Sentence
Beginning state 5	0	5, 0	5+0 =5
4	1	4,1	4+1=5
3	2	3,2	3+2=5
2	3	2,3	2+3=5
1	4	1,4	1+4=5
0	5	0,5	0+5=5

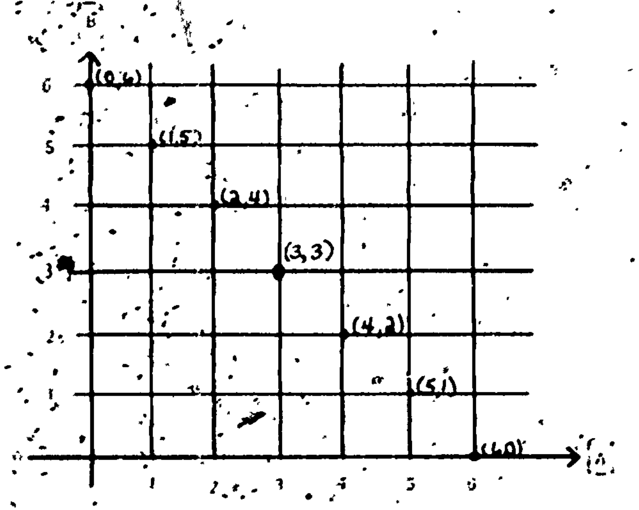
5. There should be 6 possible states. Did you find them all? [✓ Yes] No
If you did not find 6 states, go back and find the others.

Job 27

① Put your system in the state $(\overset{A}{3},\overset{B}{3})$.

② Do not pour water back and forth between A and B. Just think about doing it.

③ Think only of states in which the water level is in whole units. Record these states as ordered pairs and as points on a grid.

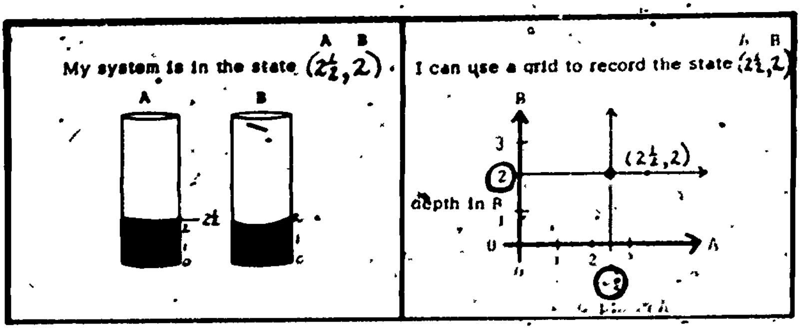


④ Now pour water back and forth between A and B.
I found these states.

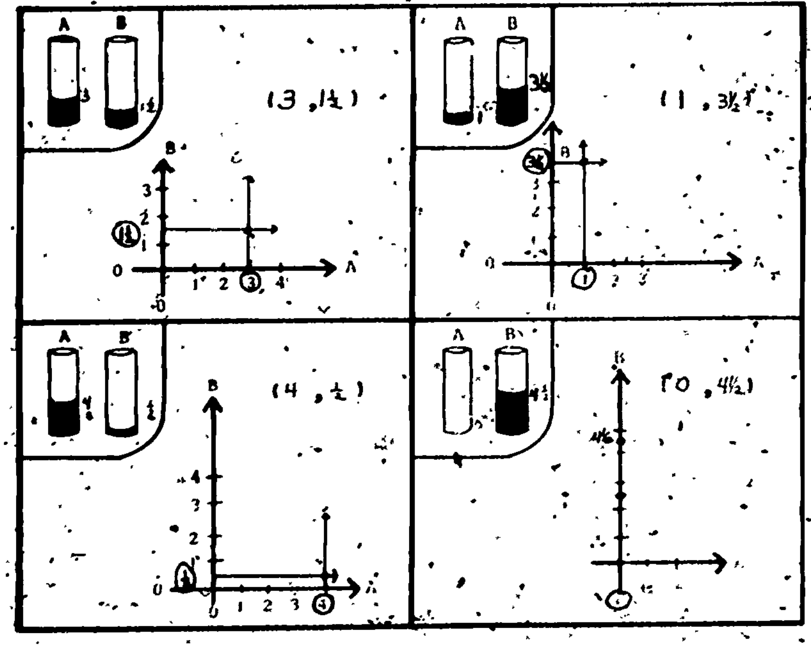
0,6	1,5	2,4	3,3	4,2	5,1	6,0

⑤ These states are the same as the ones I recorded on the grid. [✓ Yes] No
The grid points fall in a straight line. [✓ Yes] No

Job 28



Locate the grid points that can be used to describe the states of the system.
Write ordered pairs to describe the states.

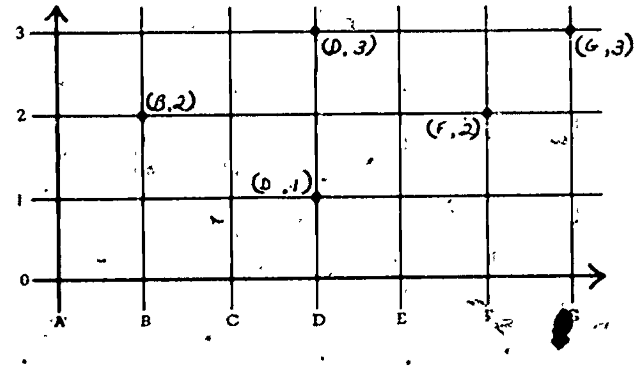


Job 17

A,3 The first member is a letter.
The second member is a numeral.
A,3 is a (letter, numeral) pair.

7,R The first member is a numeral.
The second member is a letter.
7,R is a (numeral, letter) pair.

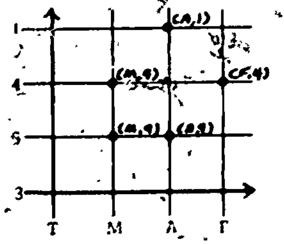
Write a (letter, numeral) ordered pair next to each • on the grid.



Did you write: (B, 2) (D, 1) (F, 2) (D, 3) (G, 3) [✓ Yes] No

Use some of the ordered pairs below to name the • on the grid.

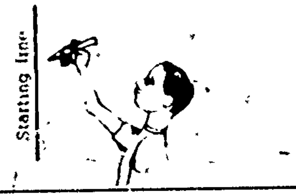
(Z, 5) (A, 9) (M, 3)
(F, 4) (M, 9) (M, 4)
(K, 7) (L, 4) (A, 4)



Job 18

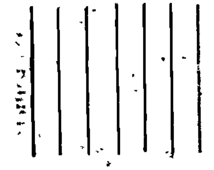
1

I draw a line segment and call it my starting line.



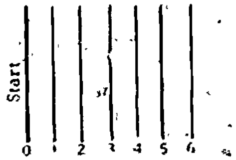
2

Now I'll draw other line segments next to the starting line.



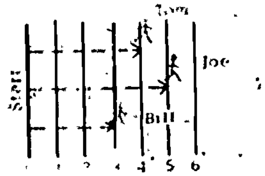
3

I call the starting line zero. The other lines are named 1, 2, 3, 4.



4

Three boys had a race. The arrows show their paths.

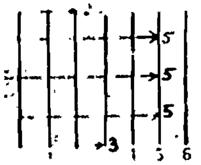


5

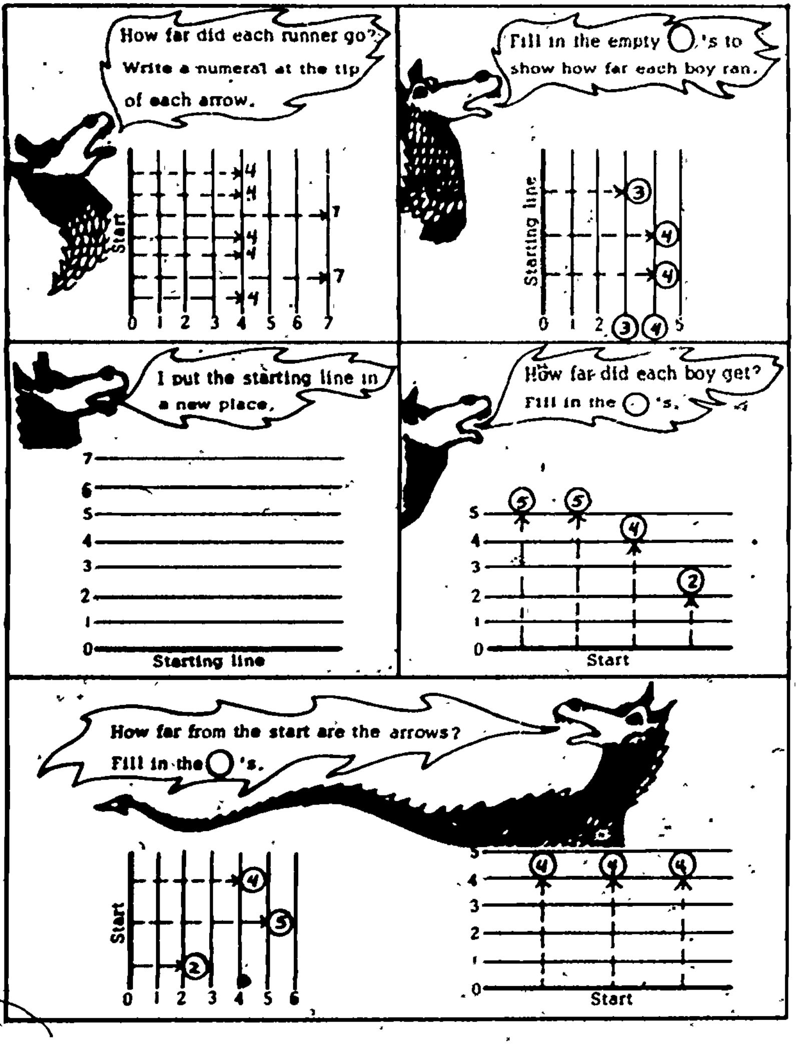
In the race in picture 4,
Bill has run 3 units.
Joe has run 5 units.
Tom has run 4 units.

6

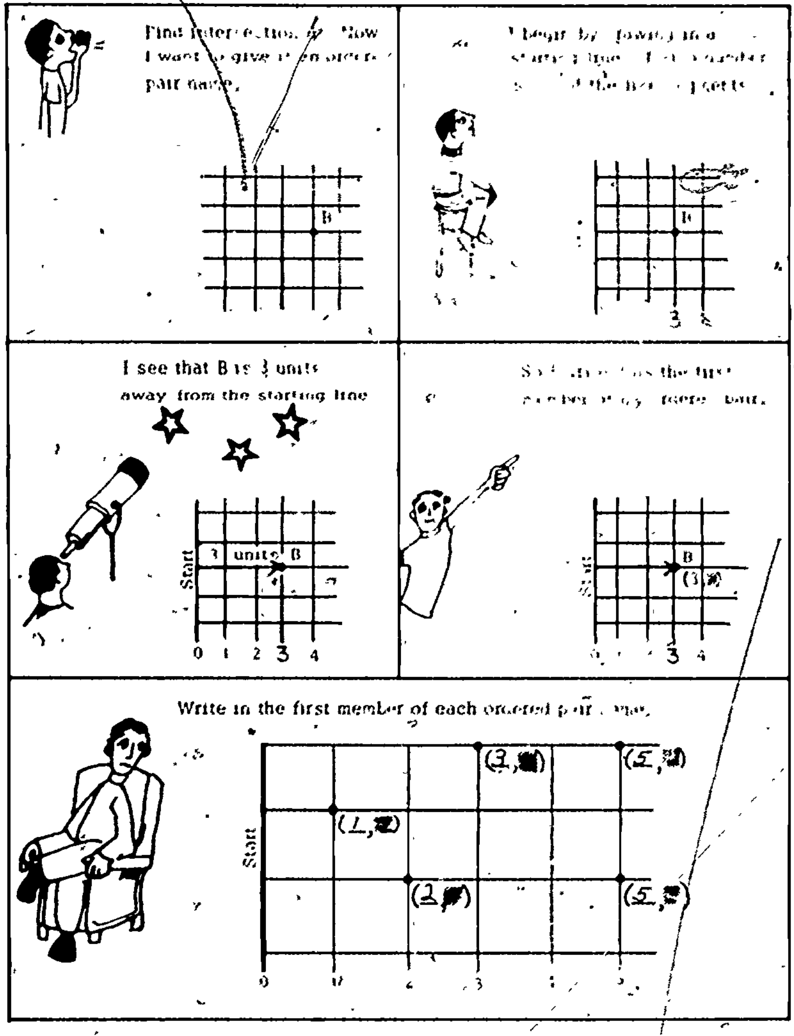
In the race below, show how far each runner ran. Write a numeral at the tip of each arrow.



Job19

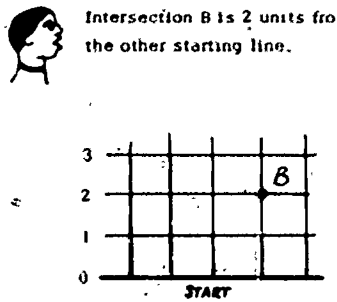

How far did each runner go? Write a numeral at the tip of each arrow.
Start
4
4
7
4
4
7
4
0 1 2 3 4 5 6 7
Fill in the empty ○'s to show how far each boy ran.
Starting line
3
4
4
0 1 2 3 4 5
I put the starting line in a new place.
7
6
5
4
3
2
1
0
Starting line
How far did each boy get? Fill in the ○'s.
5 5 4 2
5
4
3
2
1
0
Start
How far from the start are the arrows? Fill in the ○'s.
Start
4
5
2
0 1 2 3 4 5 6
4 4 4
5
4
3
2
1
0
Start

Job20

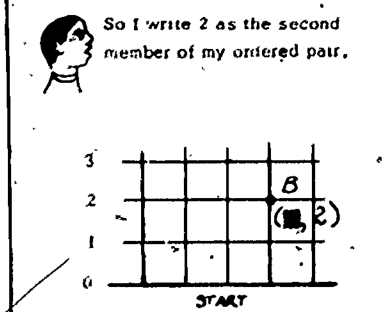

Find intersection B. Now I want to give it an ordered pair name.
B
I begin by [illegible] starting line [illegible]
B
3 4
I see that B is 3 units away from the starting line
Start
3 units B
0 1 2 3 4
So 3 is the first member of my ordered pair.
Start
B
(3,■)
3 4
Write in the first member of each ordered pair name.
Start
(3,■)
(5,■)
(1,■)
(2,■)
(5,■)
0

Job21

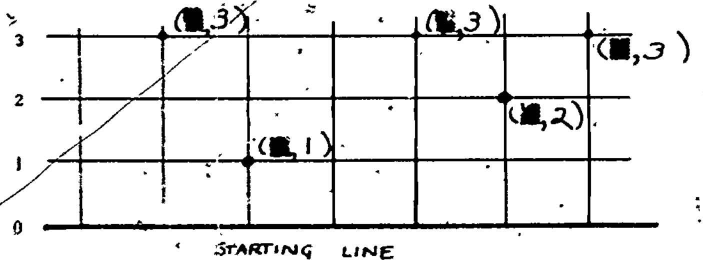
Intersection B is 2 units from the other starting line.



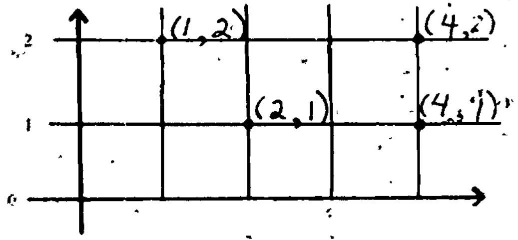
So I write 2 as the second member of my ordered pair.



Write the second member of each ordered pair name.

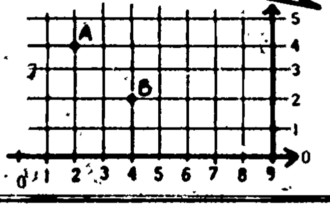


Write both members of the ordered pairs.

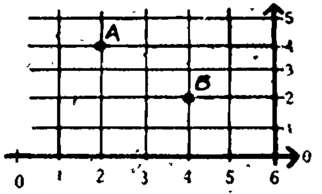


Job22

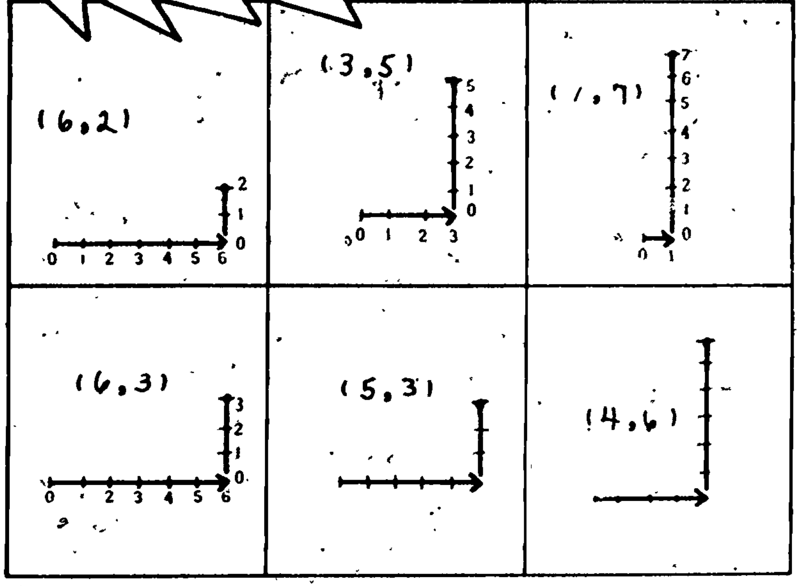
The address of my house is (4,2).
Do I live at Point A or Point B?
Write your answer here. B



My house is at Point B.
The address at Point B is (4, 2).
The address at Point A is (2,4).

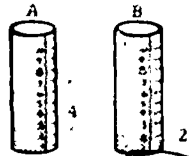


Study the arrows. Write ordered pair names for each point.



Job23

Put your system in this state.



There are 4 units of water in tube A.
There are 2 units of water in tube B.

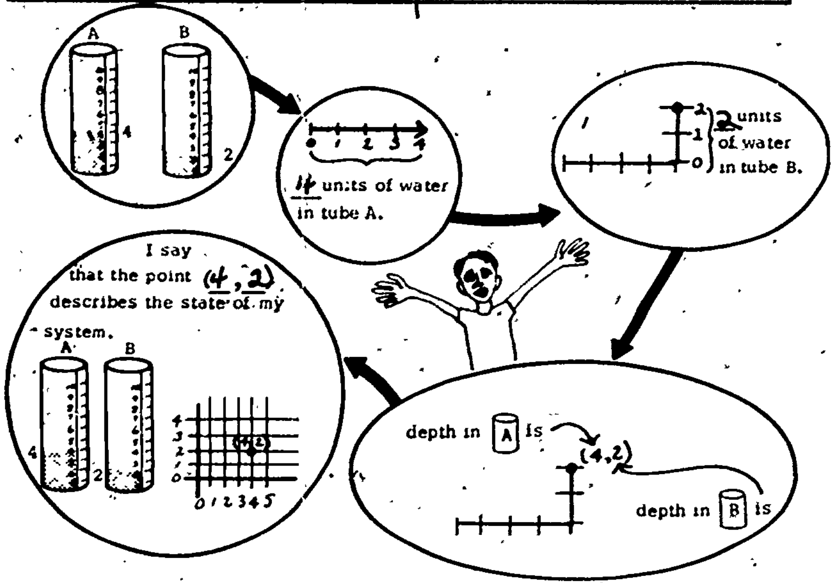
The ordered pair (4,2) describes the state of my system.

The first member of the ordered pair is 4.

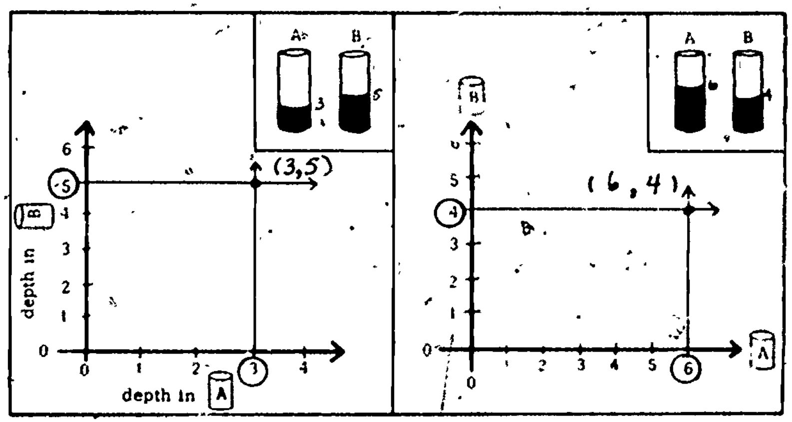
The second member of the ordered pair is 2.

In an (A, B) ordered pair, the first member always describes tube A.
the second member always describes tube B.

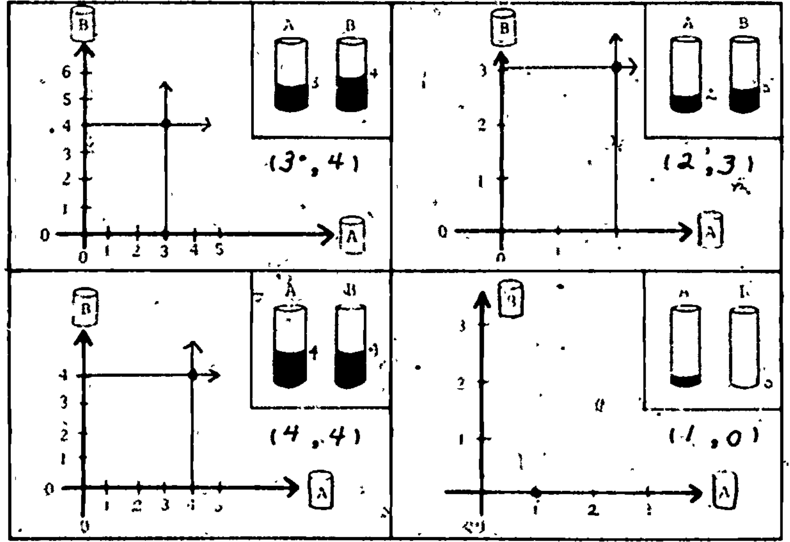
In an (B, A) ordered pair, the first member always describes tube B.
the second member always describes tube A.



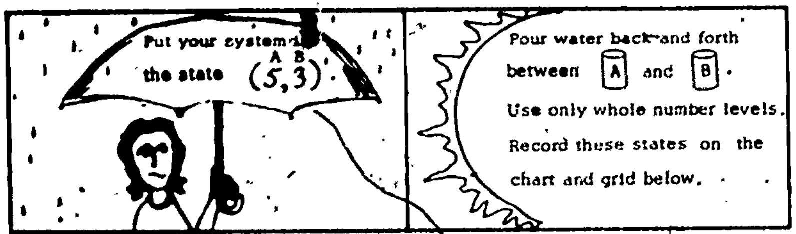
Job24



Draw arrows to find the point which represents each system.



Job25



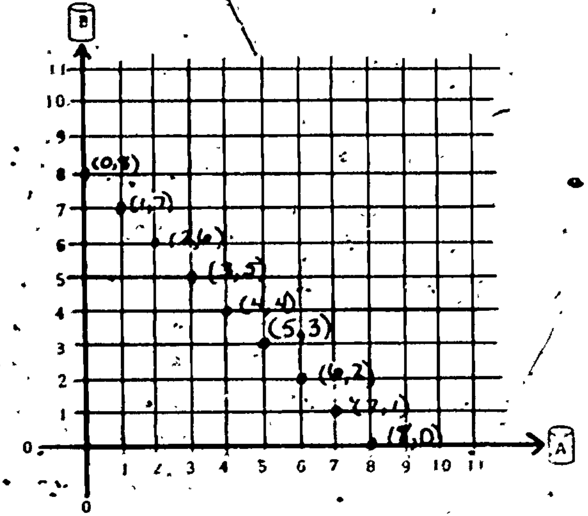
Pour water back and forth between A and B.

Use only whole number levels.

Record these states on the chart and grid below.

Whole Number Levels

A, B
5,3
4,4
3,5
2,6
1,7
0,8
8,0
7,1
6,2

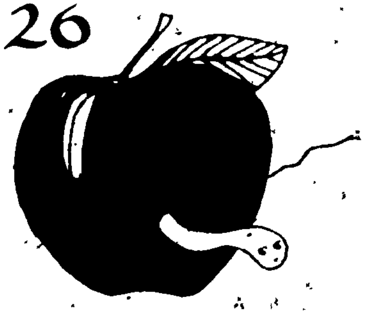


Look at the points you marked on the grid.

Do they lie on a curved line? Yes [✓ No]

Do they lie on a straight line? [✓ Yes] No

Job26



① Put your system in the state (6,4).

② Use only this water to form new states.

③ Find only states that have the water level at a whole unit.

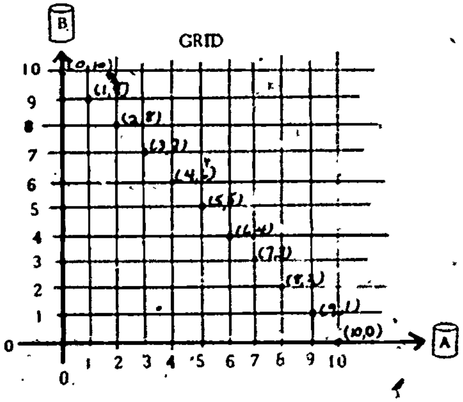
④ Record these states on the table. The point (6,4) describes one state of your system.

⑤ On the grid below, locate and name the other states you recorded on your table.

TABLE

A	B	Ordered Pair
6	4	6.4
5	5	5;5
4	6	4,6
3	7	3,7
2	8	2,8
1	9	1,9
0	10	0,10
10	0	10,0
9	1	9,1
8	2	8,2
7	3	7,3

GRID



These are the states you should have listed.

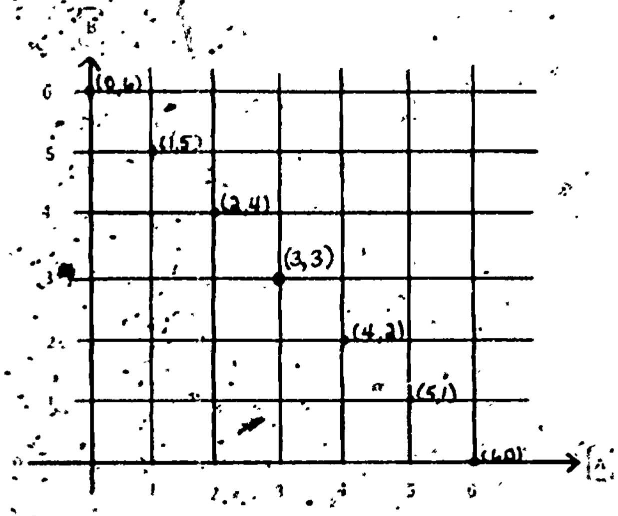
(6,4) (5.5) (4.6) (3.7)
(2,8) (1,9) (0,10) (10,0)
(9.1) (8,2) (7,3)

Job 27

① Put your system in the state $(\overset{A}{3},\overset{B}{3})$.

② Do not pour water back and forth between A and B. Just think about doing it.

③ Think only of states in which the water level is in whole units. Record those states as ordered pairs and as points on the grid.



④ Now pour water back and forth between A and B.
I found these states.

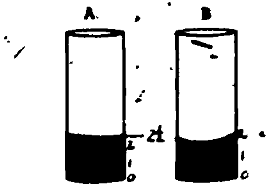
0,6	1,5	2,4	3,3	4,2	5,1	6,0

⑤ These states are the same as the ones I recorded on the grid. [✓ Yes] No

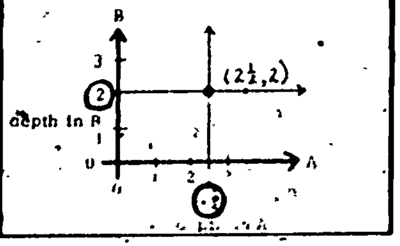
The grid points fall in a straight line. [✓ Yes] No

Job 28

My system is in the state $(\overset{A}{2\frac{1}{2}},\overset{B}{2})$.

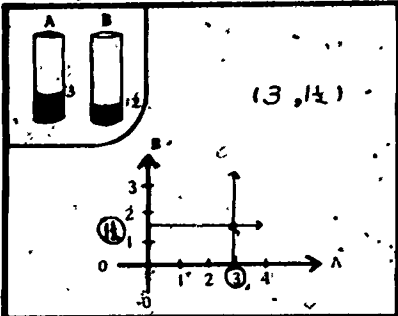


I can use a grid to record the state $(\overset{A}{2\frac{1}{2}},\overset{B}{2})$.

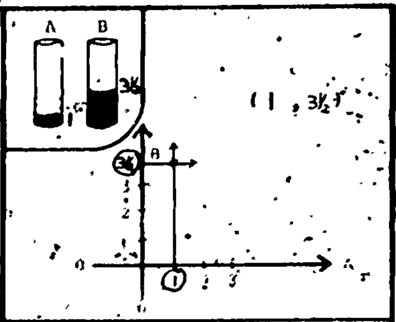


Locate the grid points that can be used to describe the state of the system.

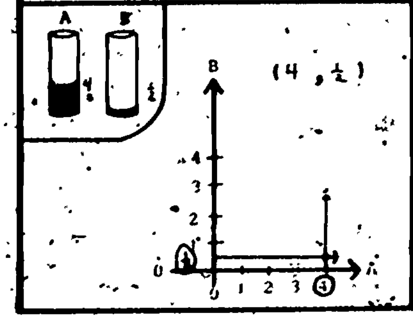
Write ordered pairs to describe the states.



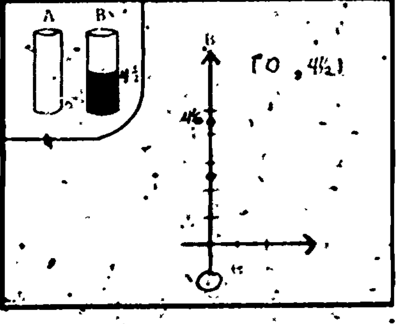
(3 , 1½)



(1 , 3½)

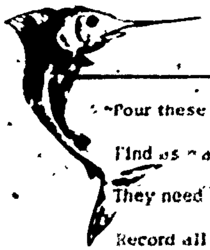


(4 , ½)



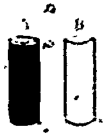
(0 , 4½)

Job29



Fill tube A to the ten unit mark.

Put no water in tube B.

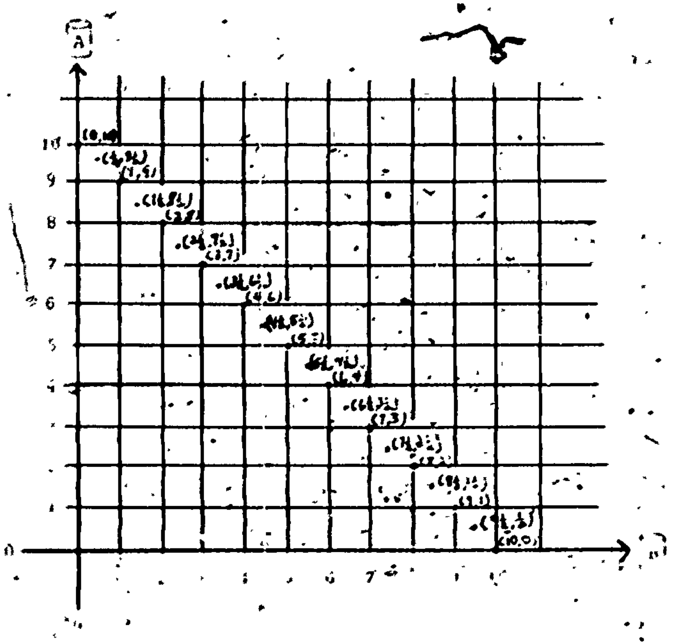


Pour these ten units of water back and forth between the tubes.

Find as many different states as possible.

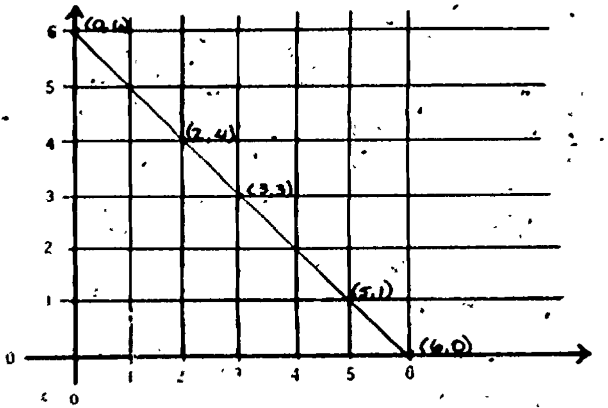
They need not be whole number states. ($3\frac{1}{2}$, $6\frac{1}{2}$) is okay.

Record all your states on the grid below.



Job30

Put your system in state $\overset{A}{(}6, \overset{B}{0})$.	Pour all the water from A into B.
There are <u>6</u> units of water in tube A.	Now there are <u>0</u> units of water in tube A.
There are <u>0</u> units of water in tube B.	Now there are <u>6</u> units of water in tube B.
Find that point on the grid below.	Find the ordered pair $(\overset{A}{0}, \overset{B}{6})$ on the chart below.



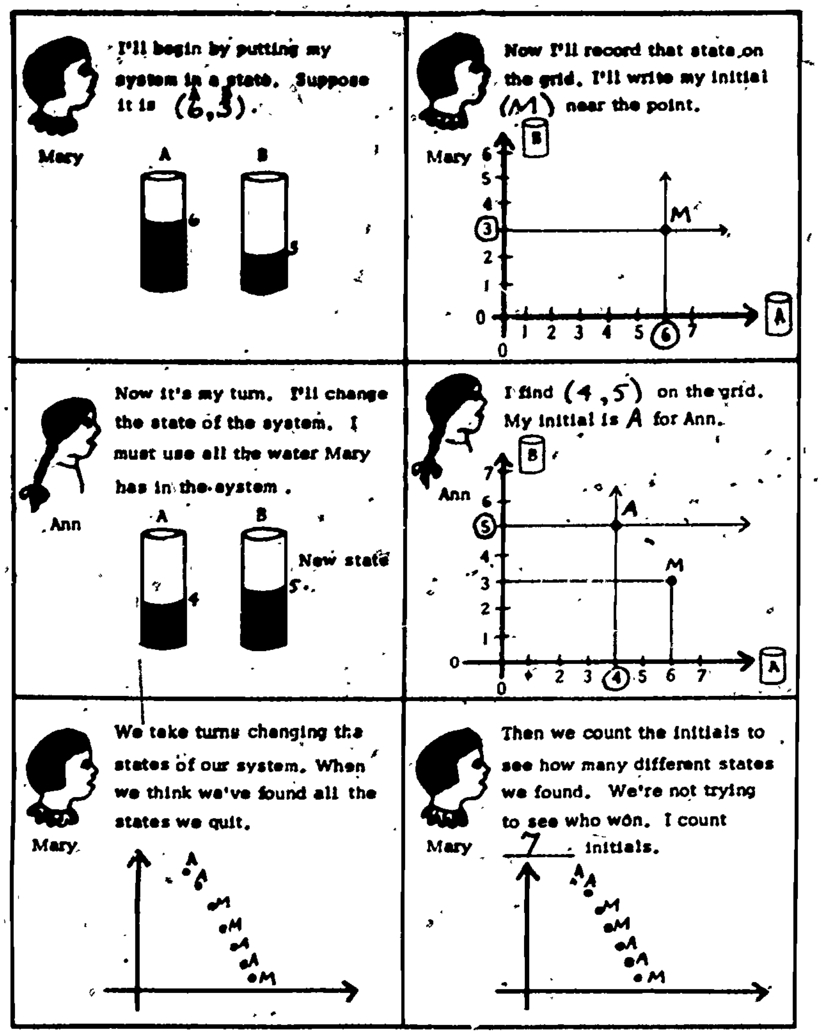
Draw a straight line through the two points (0,6) and (6,0).

Pour the water back and forth between A and B.

Record several different states on the grid.

Do all the points you named fall on the line segment? [✓ Yes] No

Job 31



Job 32

① Put your system in the state $(\overset{A}{7},\overset{B}{3})$.

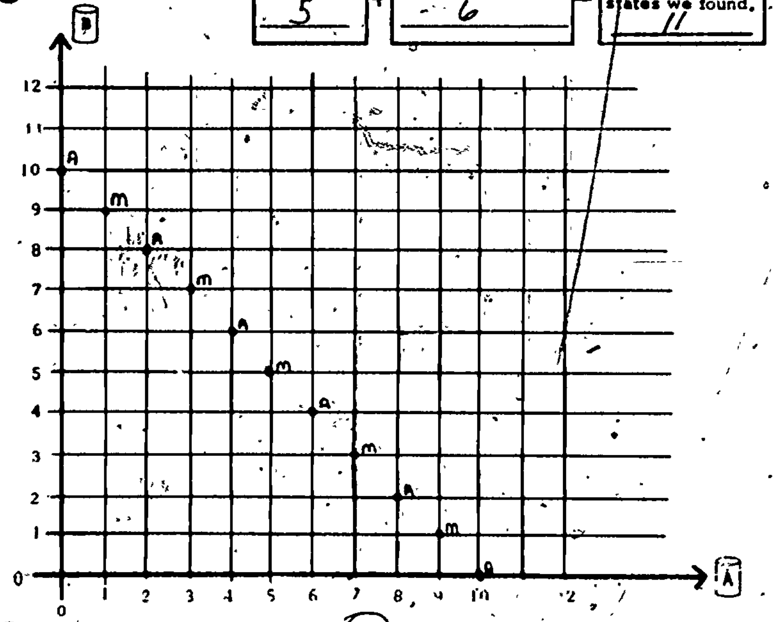
② One partner should record this state on the grid. Write your initial beside the point.

③ The other person should change the state of the system. No water can be added to or removed from the system.

④ Find the point that describes this new state. Write your initial next to it.

⑤ Take turns changing states and locating points.

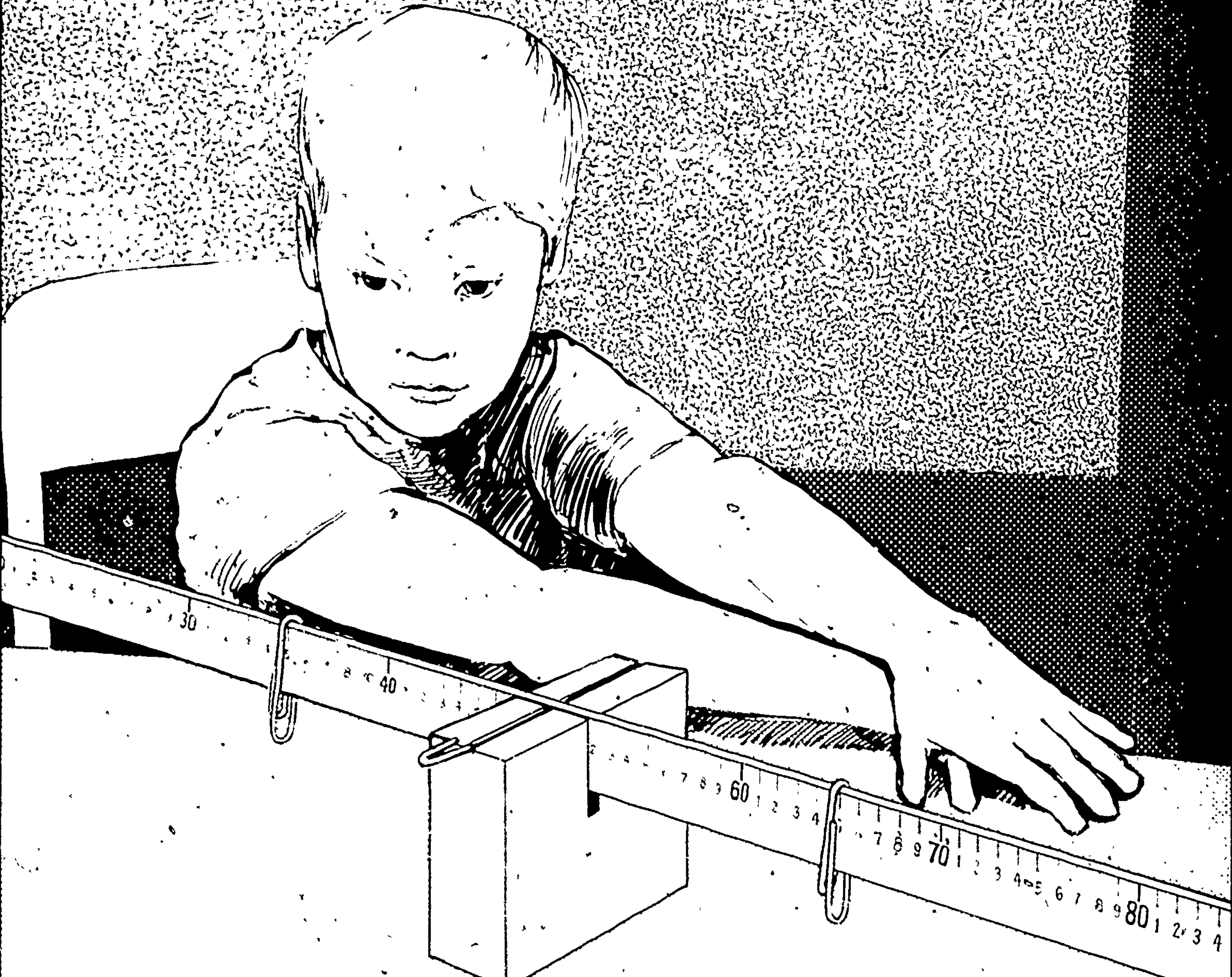
⑥ Finish this sentence: My states 5 + my partner's states 6 = total number of states we found. 11



⑦ Did you find at least 11 states? [✓ Yes] No

JOB BOOKLET

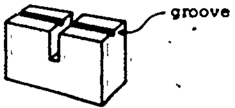
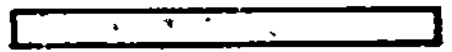
BALANCE BEAM SYSTEM



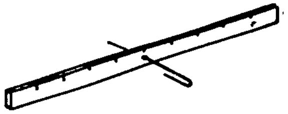
Job I

① Write your partner's name here. → Helen

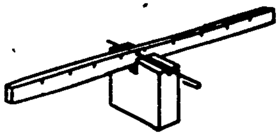
② Get these materials from your teacher.
One set of materials is enough for both of you.

meter stick (called beam)	
wooden block	
6 paper clips	
1 straightened paper clip	
masking tape	

③ Put your balance beam together like this:



Paper clip goes through hole in beam.

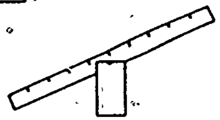


Paper clip rests in groove in block.

● Job I continued

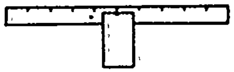
④ Picture 2 describes our balance beam.

1



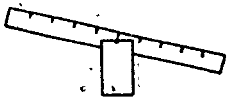
not balanced

2



balanced

3



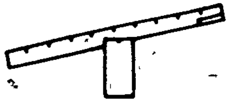
not balanced.

⑤ The balance beam we made is balanced. [✓ True] False

⑥ If your beam is *balanced* go on to Job 2.
If your beam is *not balanced* do what the pictures below show you.

1

Lightly stick some masking tape on the arm that is up.



Then let go to see if it balances.

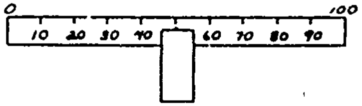
2

If the beam balances, go on to Job 2. If it doesn't balance, move the tape back and forth until it balances.

Be sure to stick the masking tape on tightly once the beam is balanced.

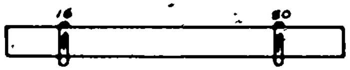
Job2

① Be sure your balance beam is balanced.
Have the 0-100 side facing you.



② The paper clip will slide over the ends of the balance beam.

Hang a red paper clip at the 16 mark.
Hang a black paper clip at the 80 mark.
Is the beam balanced? Yes [✓ No]
In the chart below, record the positions of the clips.



③ Change the positions of the black and red clips.
Record the results.

Many possible answers

Position of red clip	Position of black clip	Describe the Beam B = balanced NB = not balanced
16	80	NB

● Job 2 continued

④ Hang one red clip and one black clip from the beam.
Change the positions of the clips.
Record the different states in which the beam is balanced.

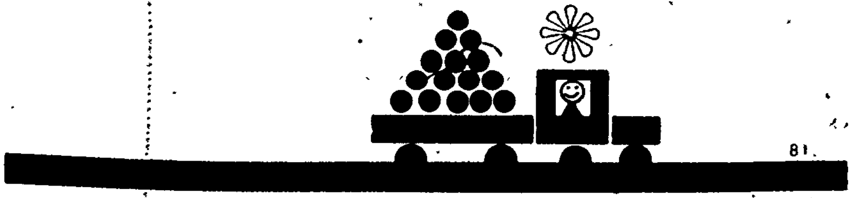
Any ordered pair that totals 100 is correct

Position of Red Clip	Position of Black Clip

Position of Red Clip	Position of Black Clip

Game—

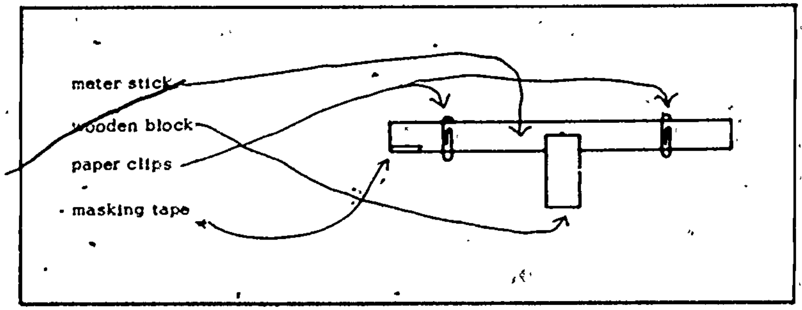
1. One of you begins by choosing a numeral where the red clip will be hung.
2. Then the other person writes the numeral where the black clip must be hung to balance the beam.
3. Hang the red clip and the black clip at the numerals you wrote. How did you do? Did the beam balance?
4. Take turns writing numerals and hanging the red clip and the black clip.



Job 3

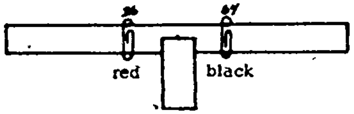
We call the balance beam a system.

Match the names with the parts of the system.



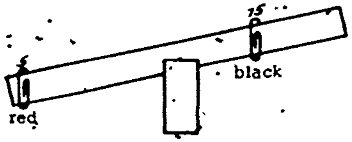
The state of a system is the condition of the system.

Describe the states of the system below:



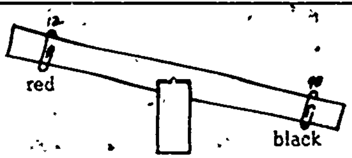
red clip at 36

black clip at 64



red clip at 5

black clip at 75



red clip at 12.

black clip at 90.

● Job 3 continued

From now on, try to find only states in which the beam is balanced.

①	Place the red clip at 39. Place the black clip so that the beam is balanced.	red clip at 39 black at 61 39, 61
②	Place the black clip at 72. Place the red clip to balance the beam.	red clip at 28 black at 72 28, 72

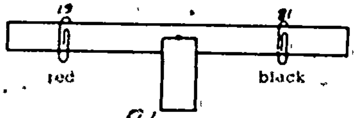
Find only balanced states of your system.

Describe each balanced state. *Any pair that totals 100 is correct.*

red clip at ____ black at ____ ○, ○	red at ____ black at ____ ○, ○
red at ____ black at ____ ○, ○	red at ____ black at ____ ○, ○
red at ____ black at ____ ○, ○	red at ____ black at ____ ○, ○

Job 4

- Make your system look like this:
- Describe the state of your system.



red clip at 19 black clip at 81

19 , 81

① Did you write (19,81) to describe the state of your system? Yes No

Two numbers like 19 and 81 are called a pair of numbers.

The 19 in (19, 81) describes the position of the red clip.

The 81 in (19, 81) describes the position of the black clip.

② Suppose we agree to write our pairs of numbers in a special order.

The first number describes the position of the red clip.

The second number describes the position of the black clip.

(40,60)

The first member of this pair is 40.
The second member of this pair is 60.
The 40 describes the position of the red clip.
The 60 describes the position of the black clip.
(40, 60) is called an ordered pair of numbers.

③ Put your balance beam in the states $\overset{R}{(40}, \overset{B}{60)}$, $\overset{R}{(75}, \overset{B}{25)}$, $\overset{R}{(18}, \overset{B}{82)}$.

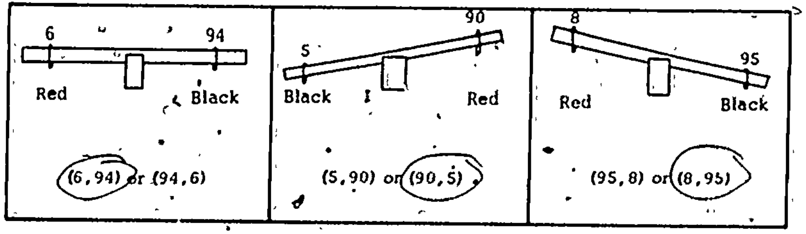
Job 5

①

Remember our order for describing the state of our system.

The first member of the ordered pair describes the position of the red clip.
The second member of the ordered pair describes the position of the black clip.

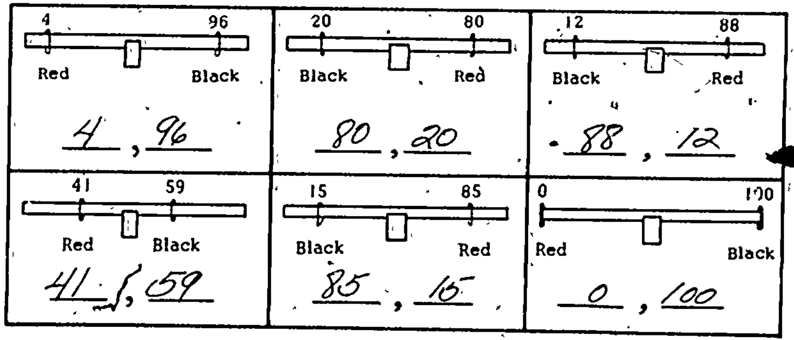
② Pick the ordered pair of numbers that describes each state.



(6,94) or (94,6)	(5,90) or (90,5)	(95,8) or (8,95)

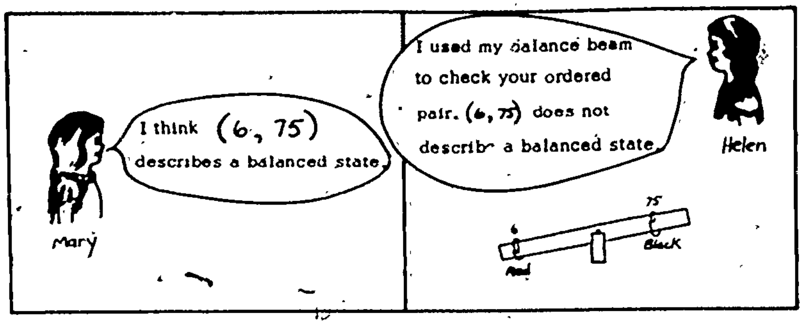
③ Your answers should be (6,94) (90,5) (8,95).

④ Write ordered pairs to describe the states below.



4 , 96	80 , 20	88 , 12
41 , 59	85 , 15	0 , 100

Job 6



① Take turns writing ordered pairs that describe balanced states.

Check each ordered pair with your balance beam.

any pair that totals 100 is correct

State of a System	
Red, Black	B = Balanced NB = Not Balanced
6, 75	NB

State of a System	
Red, Black	B = Balanced NB = Not Balanced

Job 7

① Put your beam balance in four different states that are balanced.

Write the ordered pair for these states. any pair that totals 100 is correct

(—,—) (—,—) (—,—) (—,—)

② Look carefully at the ordered pairs you wrote.

Can you decide ahead of time where to put the clips so that the beam is balanced?

If you have a way, try it a few times to be sure it works.

③ Try to think of another way to decide ahead of time.

Talk it over with your partner.

④ Fill in the chart.

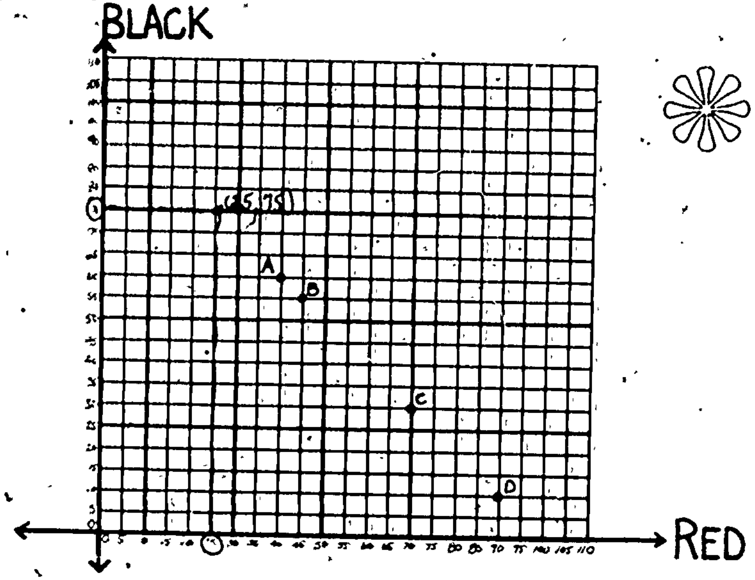
Balanced States

Red Black	Addition Sentence
10, 90	10 + 90 = $\boxed{100}$
any pair that to	als 100 is correct

Not Balanced States

Red Black	Addition Sentence
20, 90	20 + 90 = $\boxed{110}$
any pair that does not total 100 is correct.	

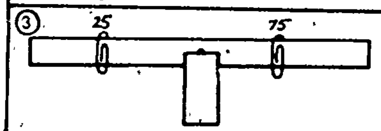
Job 8



① Use a red crayon to color the line labeled RED.

② Use a black crayon for the black line.

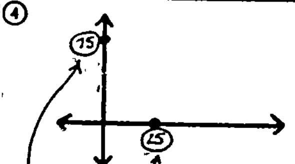
③



Ordered pair is (25, 75).

System is ([✓ Balanced] Not balanced).

④

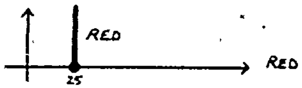


Circle 25 on the RED line with your red crayon.

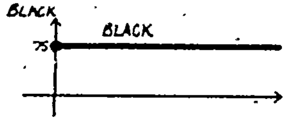
Circle 75 on the BLACK line with your black crayon.

● Job 8 continued

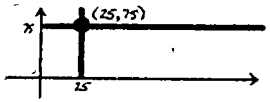
⑤ Find 25 on the RED line. Draw a RED line segment to the top of the grid.



⑥ Find 75 on the BLACK line. Draw a BLACK line segment to the right side of the grid.



⑦ Find the intersection of the two line segments you drew. Label the intersection (25, 75).



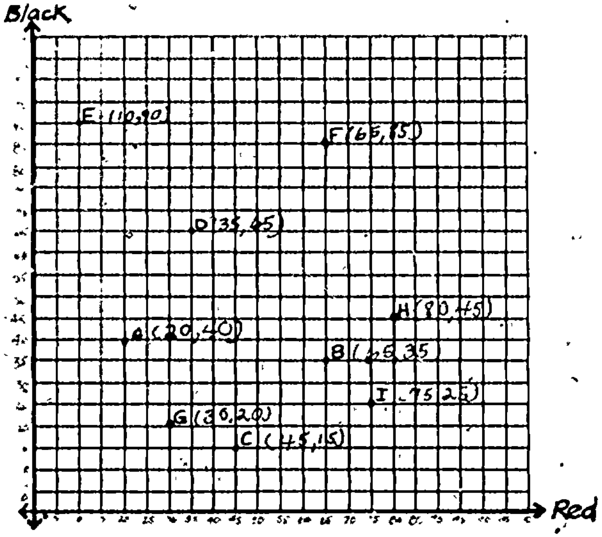
⑧ Look at the grid on the previous page. Put your system into states A, B, C, and D.

STATE	ORDERED PAIR	BALANCED or NOT BALANCED B NB
A	40, 60	B
B	45, 55	B
C	70, 30	B
D	90, 10	B

Job 9

① Several points are marked on the grid below. Assign an ordered pair of numbers to each point. Then fill in the chart next to the grid.

A = 20, 40
B = 65, 35
C = 45, 15
D = 35, 25
E = 10, 90
F = 65, 85
G = 30, 20
H = 80, 45
I = 75, 25



② Put your system in the states represented by points A, B, C, D, E, F, G, H, I. Is the system balanced or unbalanced?

	B or NB
A	NB
B	B
C	NB

	B or NB
D	B
E	B
F	NB

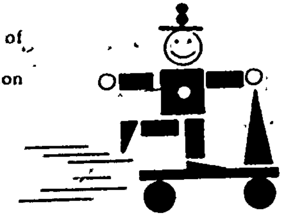
	B or NB
G	NB
H	NB
I	B

Job 10

① Be sure your beam balances when there are no clips on it.

② When a pair of numbers have a special order, we call them an ordered pair.
The first member of the pair describes the position of the red clip.
The second member of the pair describes the position of the black clip.

③ Use one red clip and one black clip.
Balance the beam for 8 different states.
Record these states on the chart.

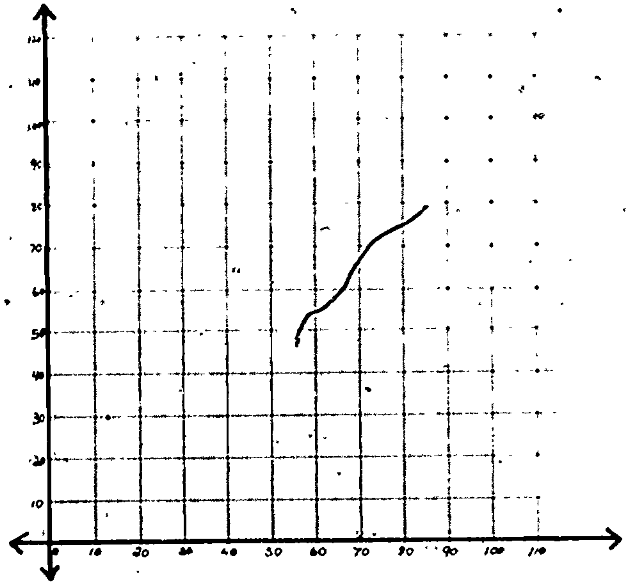


Red	Black	Ordered Pair	Addition Sentence	Letter Name
		(,)	___ + ___ = ___	A
			Any pair that totals 100	B
			is correct.	C
				D
				E
				F
				G
				H

● Job 10 continued

④ Locate the 8 points that describe the states of your system on the grid below.

Write ordered pair names and letter names.



⑤ Draw a line connecting the points you marked on the grid.

What do you notice? Do the points lie on a [✓ straight line] or a curved line?

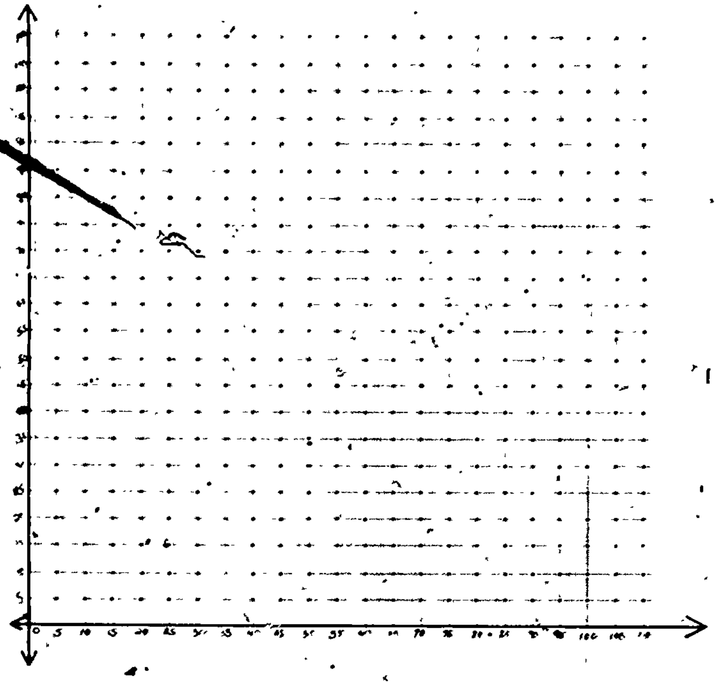
(3, 4) is called an ordered pair because we agree that:

- The first member of the pair describes the position of the red clip.
- The second member of the pair describes the position of the black clip.

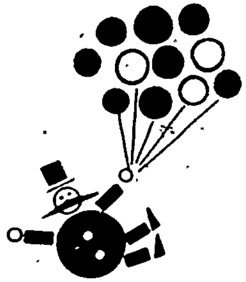
Job 11

① Use one red clip and one black clip. Any pair that totals
Find 4 states where the beam is balanced. 100 is correct

Letter	Red	Black	Ordered Pair	Addition Sentence
A			(,)	__ + __ = __
B			(,)	__ + __ = __
C			(,)	__ + __ = __
D			(,)	__ + __ = __



● Job 11 continued



② Use a green crayon.
Mark points (A, B, C, D) on the grid.
These points are on a straight line. [✓ Yes] No

③ Use one red clip and one black clip.
Find 4 states in which your system is not balanced.

any pair that does not
total 100 is correct.

Letter Name	Red	Black	Ordered Pair	Addition Sentence
R			(,)	__ + __ = __
S			(,)	__ + __ = __
T			(,)	__ + __ = __
U			(,)	__ + __ = __

④ Use an orange color to mark points (R, S, T, U) on the grid.
These points are on a straight line. Yes [✓ No]

⑤ Points which represent a balanced system fall on a [✓ straight line.] curved line.

Points which represent an unbalanced system do [✓ do not] fall on a straight line.

⑥ Try some more states where your system is balanced or unbalanced.
Locate more points on the grid.

Job 12

① Use one red clip and one black clip.
Write ordered pairs to represent the states of the system.
This chart shows ordered pairs for balanced and unbalanced states we found.

Balanced

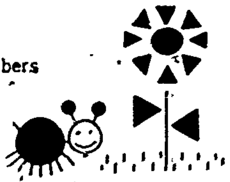
Red , Black	Addition Sentence
60, 40	60 + 40 = 100
	any pair that totals 100 is correct

Unbalanced

Red , Black	Addition Sentence
30, 80	30 + 80 = 110
	any pair that does not total 100 is correct.

② When the beam is balanced, the sum of the two members of the ordered pair is 100.

③ When the beam is unbalanced, the sum of the two members of the ordered pair cannot be 100.



Find some more states.

Balanced

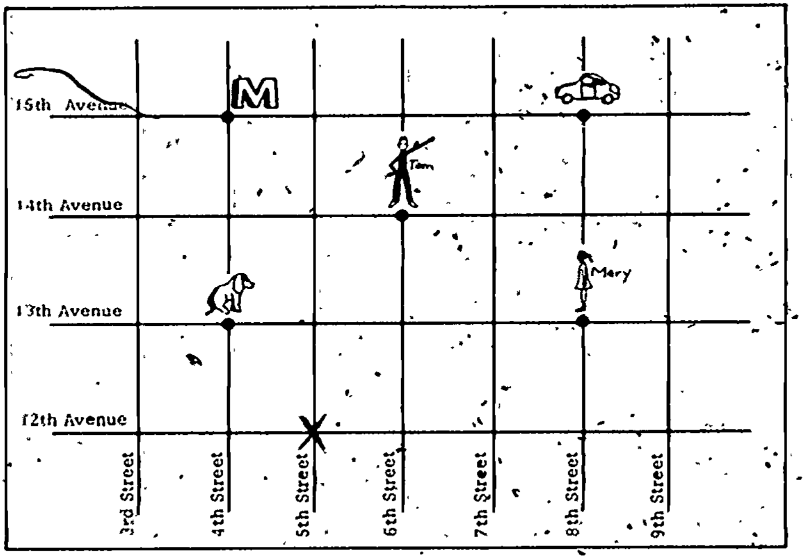
(,)	___ + ___ = ___

Unbalanced

(,)	___ + ___ = ___

● Job 12 continued

Map of Fairville



① Tom is standing on the corner of 6 Street and 14 Avenue.

② Mary is standing on the corner of 8 Street and 13 Avenue.

③ The dog is resting at the corner of 4 Street and 13 Avenue.

④ The car is stopped at the corner of 8 Street and 15 Avenue.

⑤ Place an X on the corner of (5th Street, 12th Avenue).

⑥ The address of point M is (4 Street, 15 Avenue).

Job 13

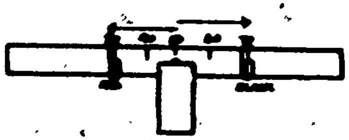
Fill in your own chart.
Use your balance beam to check your answers.

Any pair that does not total 100 is correct

Balanced	
(40, 60)	40 + 60 = 100
(65, 35)	65 + 35 = 100
(82, 18)	82 + 18 = 100
(28, 72)	28 + 72 = 100
(55, 45)	65 + 45 = 100

Not Balanced	
(35,)	___ + ___ = ___
(43,)	___ + ___ = ___
(, 12)	___ + ___ = ___
(, 85)	___ + ___ = ___
(,)	___ + ___ = ___

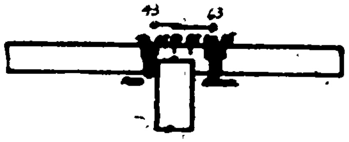
The state is (30, 70).
Is the beam balanced? [✓ Yes] No



The red clip is 20 units from 50.

The black clip is 20 units from 50.

The state is (43, 63).
Is the beam balanced? Yes [✓ No]



The red clip is 7 units from 50.

The black clip is 13 units from 50.

● Job 13 continued

There are 3 apples in this picture.
There are 4 oranges in this picture.

① We will write an (apple, orange) ordered pair to describe the picture.
The ordered pair we write is (3, 4).

② We can write an (orange, apple) ordered pair to describe the picture.
The ordered pair we write is (4, 3).

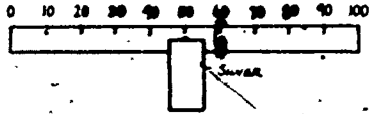
Write ordered pairs to describe each picture.

	
(truck, airplane) ordered pair 2 , 3	(boy, girl) ordered pair 4 , 2
(airplane, truck) ordered pair 3 , 2	(girl, boy) ordered pair 2 , 4

Job 14

Place a silver clip at 60 on your beam.

DO NOT MOVE THIS CLIP FOR THE REST OF THIS PAGE.



Find six ordered pairs that give balanced states of the beam with the silver clip at 60.
Any pair that totals 90 is correct when the silver clip is at 60

Red Black	Addition Sentence
(,)	___+___=___
(,)	___+___=___
(,)	___+___=___

Red Black	Addition Sentence
(,)	___+___=___
(,)	___+___=___
(,)	___+___=___

The silver clip is at 60.

Put the red clip at 70.

I predict that the black clip must be at 20 if I want the beam to balance.

Check your prediction.

The beam is balanced if the red clip is at 70 and the black clip is at 20.

(70 , 20)
red black

The silver clip is at 60.

Put the black clip at 90.

I predict that the red clip must be at 0 if the beam is to balance.

Check your prediction.

The beam is balanced if the red clip is at 0 and the black clip is at 90.

(0 , 90)
red black

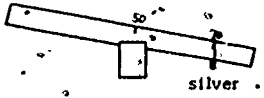
● Job 14 continued

GAME PREDICTION

① Take the silver clip off the beam.

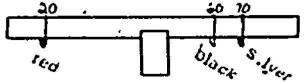
Check the beam to see that it is balanced without clips.

② Place a silver clip at point 70. Keep the silver clip on the beam for the rest of this game.



③


④

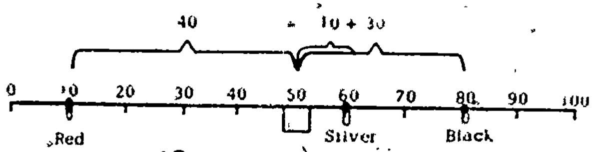
Write your prediction for states in which the system is balanced. Test each prediction. Any pair that totals 70 is correct when the silver clip is at 70.

red black	TEST B or NB
(,)	
(,)	
(,)	
(,)	
(,)	

red black	TEST B or NB
(,)	
(,)	
(,)	
(,)	
(,)	

Job 15

1. Put the silver clip at 60. Put your system in the state (10, 80)
red, black



40	-	10	+	30
(distance of red clip from center)		(distance of silver clip from center)		(distance of black clip from center)

2. The distance of the clip on one side must equal the distance of the two clips on the other side.

3. The red clip is 40 units from the center of the stick.

4. The silver clip is 10 units from the center of the stick.

5. The black clip is 30 units from the center of the stick.

6. When the silver clip is at 60 and the system is in the state (10, 80) is the beam balanced? [✓ Yes] No

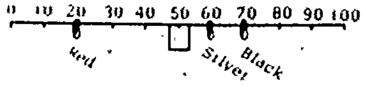
7. Place the silver clip at 60. Place the red clip at 25. Where should the black clip go? 45

8. HINT: the distance of the clip (from the center) on one side. = the distances of the two clips (from the center) on the other side.

$$\frac{25}{(\text{Red})} = \frac{10}{(\text{Silver})} + \frac{15}{(\text{Black})}$$

9. Should the black clip be put at 65 to balance the beam? [✓ Yes] No

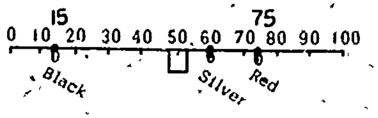
● Job 15 (continued)



The state of this system is (20, 70). The silver clip is at 60.

The red clip is 20 units from the center.	=	The black clip is 20 units from the center.	+	The silver clip is 10 units from the center.

Is the beam balanced? [✓ Yes] No



The state of the system is (75, 15). The silver clip is at 60.
Is the beam balanced? [✓ Yes] No

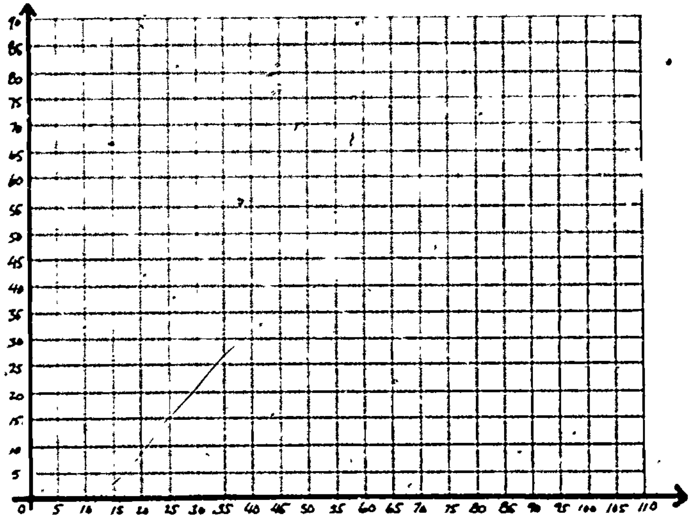
The black clip is 35 units from the center	=	The red clip is 25 units from the center	+	The silver clip is 10 units from the center

Job 16

① Place the silver clip at 70. Use one red clip and one black clip.

② Find 4 states in which the beam is balanced.

	Distance from center		
A (20, 60)	30 red	= 10 black	+ 20 silver
B (,)	___ red	= ___ black	+ ___ silver
C (,)	___ red	= ___ black	+ ___ silver
D (,)	___ red	= ___ black	+ ___ silver



● Job 16 continued

③ Use a green crayon to locate points (A, B, C, D) on the grid.

Do these points fall on a straight line? [✓ Yes] No.

④ Find 4 states in which the beam is unbalanced.

Many possible answers.

		Distance from Center red	black	silver
E	(,)		=	+
F	(,)		=	+
G	(,)		=	+
H	(,)		=	+

⑤ Use an orange crayon to locate points (E, F, G, H) on the grid.

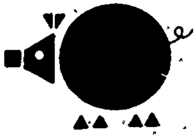
These points fall on a straight line. True [✓ False]

⑥ Fill in the blanks.

The beam is balanced. The silver clip is at 70. The red clip is at 25. The black clip must be at 55.	The beam is balanced. The silver clip is at 70. The black clip is at 75. The red clip must be at 5.
The beam is <u>not</u> balanced. The silver clip is at 70. The red clip is at 15. The black clip must <u>not</u> be at 65.	The beam is unbalanced. The silver clip is at 70. The black clip is at 69. The red clip must not be at 11

Job17

① Move the silver clip to a new position on the beam.
The position of the silver clip is____.



The silver clip stays at____for the rest of this job.

② Find some states where the beam is balanced and some where it is not balanced.

Balanced Many possible answers. Unbalanced

A	(,)	__+__=__
B	(,)	__+__=__
C	(,)	__+__=__
D	(,)	__+__=__
E	(,)	__+__=__
F	(,)	__+__=__

M	(,)	__+__=__
N	(,)	__+__=__
O	(,)	__+__=__
P	(,)	__+__=__
Q	(,)	__+__=__
R	(,)	__+__=__

③ Use a green crayon to locate points (A, B, C, D, E, F) on the grid on the next page.

If this system is balanced, the points lie on a straight line. [✓ True] False

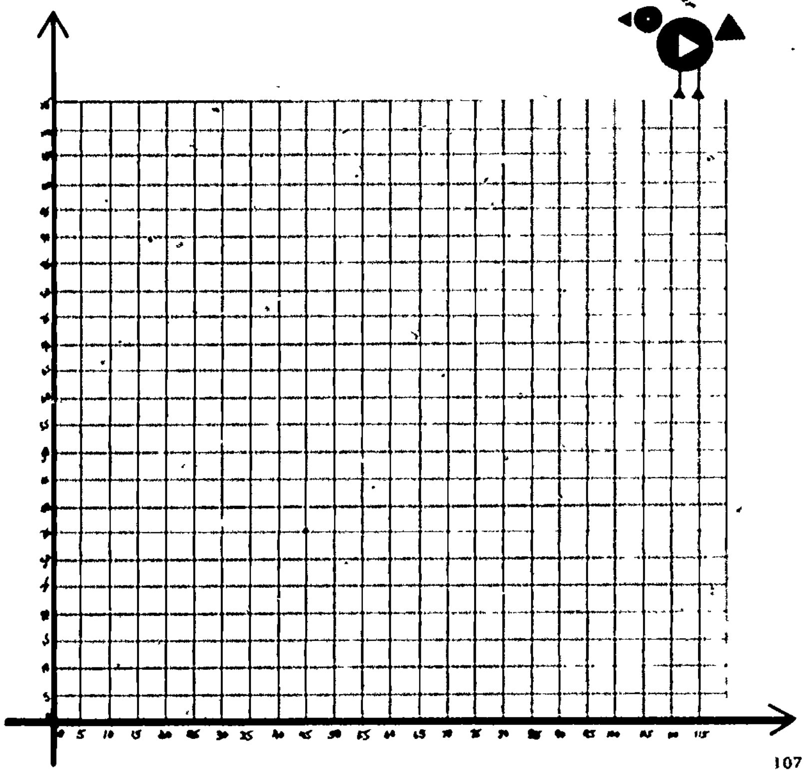
When the beam is balanced, the sum of the two members of the ordered pair is______.

● Job 17 continued

④ Use an orange color to locate points (M, N, O, P, Q, R).

If this system is unbalanced, the points lie on a straight line. True [✓ False]

When the beam is unbalanced, the sum of the two members of the ordered pair is not ______



Job 18

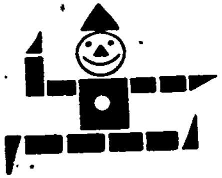
Put the silver clip at 40. Do not move the clip.

- The tables show the position of either the red clip or the black clip.
- Predict where the other clip must be placed to balance the beam.
- Check your prediction. If your prediction is not correct, write in the correct position.

Balanced System with Silver Clip at 40

my prediction		test
Red	Black	Correct Position
30	80	(30 , 80)
20	90	(20 , 90)
35	75	(35 , 75)
85	25	(85 , 25)
69	41	(69 , 41)
28	82	(28 , 82)
72	38	(72 , 38)

my prediction		test
Red	Black	Correct Position
80	30	(80 , 30)
40	70	(40 , 70)
90	20	(90 , 20)
39	71	(39 , 71)
25	85	(25 , 85)
73	37	(73 , 37)
98	12	(98 , 12)



Job 19

① Remove all clips from the balance beam.

② The center of the balance beam is at point 50.

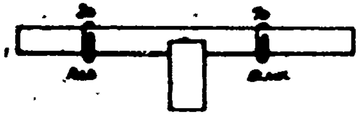
③ Use only the red clip and a black clip.

Find several states in which the beam is balanced.

Fill in the table. any pair that totals 100 is correct.

Red Black (,)	The red clip is ____ units from 50. The black clip is ____ units from 50.
(,)	The red clip is ____ units from 50. The black clip is ____ units from 50.
(,)	The red clip is ____ units from 50. The black clip is ____ units from 50.
(,)	The red clip is ____ units from 50. The black clip is ____ units from 50.
(,)	The red clip is ____ units from 50. The black clip is ____ units from 50.
(,)	The red clip is ____ units from 50. The black clip is ____ units from 50.

Job20



① The ordered pair(30 , 70)can be used to describe the state of this system.

② The first member of the ordered pair is 30 .
30 is the position of the red clip.

③ The second member of the ordered pair is 70 .
70 is the position of the black clip.

④ Why is (30,70) an ordered pair?
Because we agree to write the pair in the order of (red position, black position).

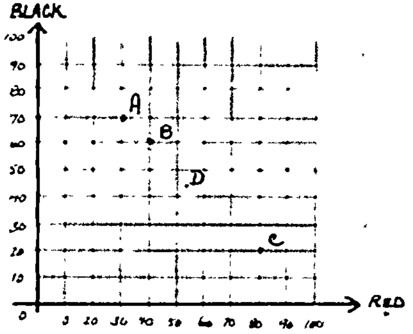
Locate these points which describe states of a balanced beam system.

A = (30 , 70)

B = (40 , 60)

C = (80 , 20)

D = (56 , 44)



Job21

This is the last job of this booklet.

What did you learn from this booklet?

Do this job to help you find out.

① Do not use the beam to check your answers in this job.
Just talk things over with your partner.

② We write pairs of numbers to describe states of our system.
These pairs must follow a special order.

The order is (red clip position, black clip position)

③ Think about using one black clip and one red clip.
Fill in the following chart.

any pair that totals 100 correct

	Beam Balanced
A	(___ , ___)
B	(___ , ___)
C	(___ , ___)
D	(___ , ___)
E	(___ , ___)

any pair that does not total 100 is correct.

	Beam Not Balanced
M	(___ , ___)
N	(___ , ___)
O	(___ , ___)
P	(___ , ___)
Q	(___ , ___)

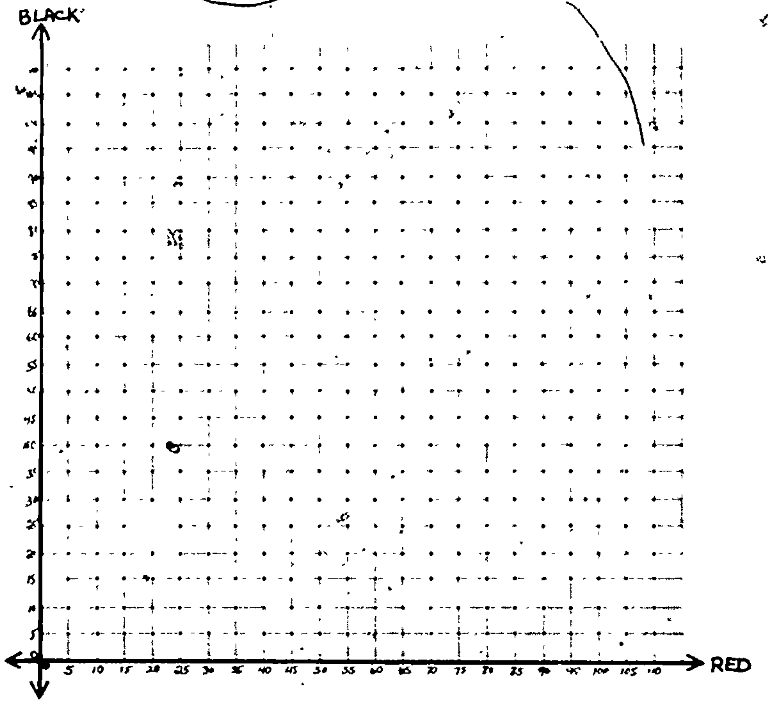
- Job 21 continued

④ Use a blue crayon to mark points (A, B, C, D, E) on the grid below.

These points ([✓ fall], do not fall) on a straight line.

⑤ Use a green crayon to mark points (M, N, O, P, Q) on the grid below.

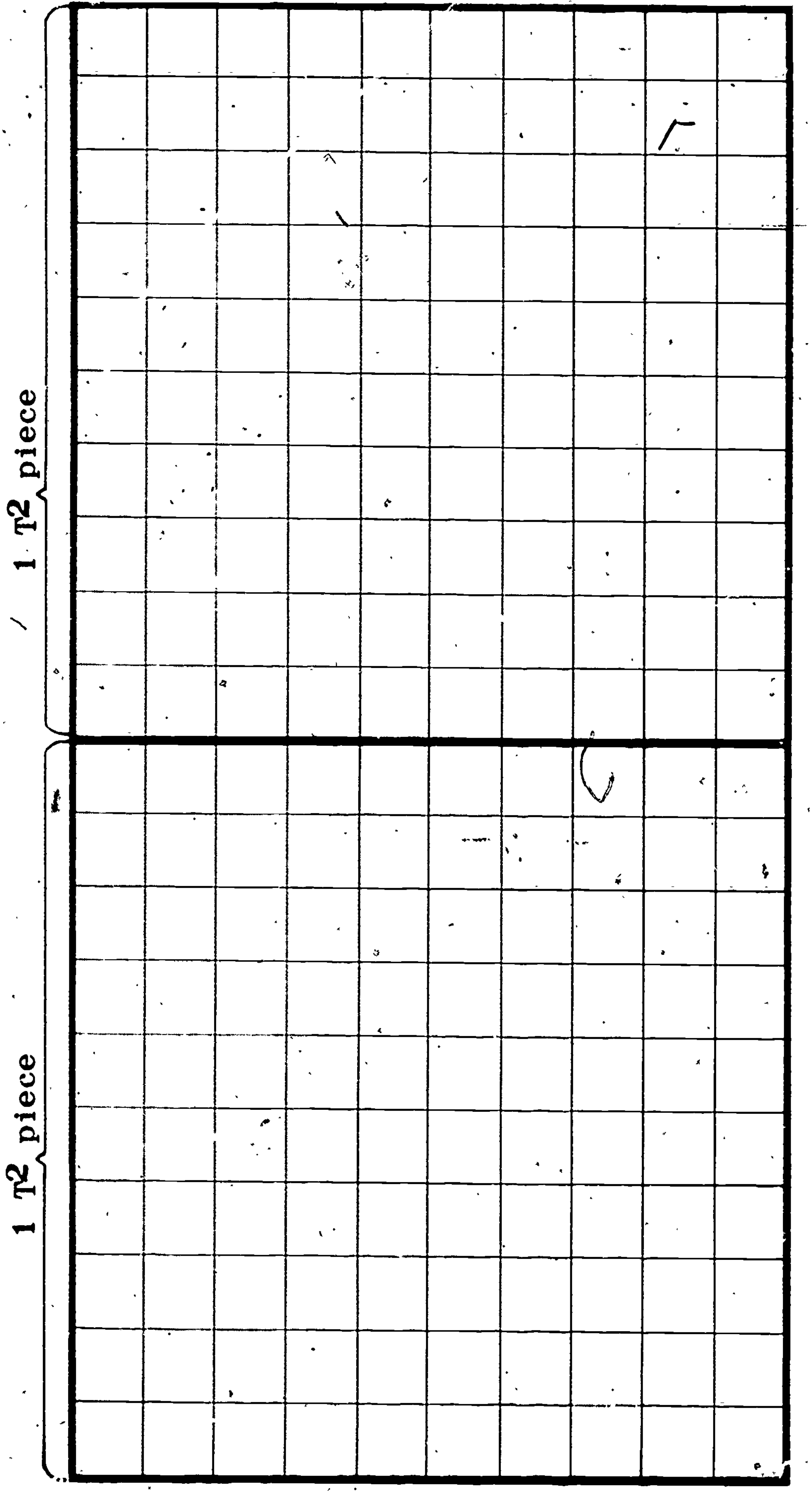
These points (fall, [✓ do not fall]) on a straight line.



⑥ Show this job to your teacher. She wants to know how you did.

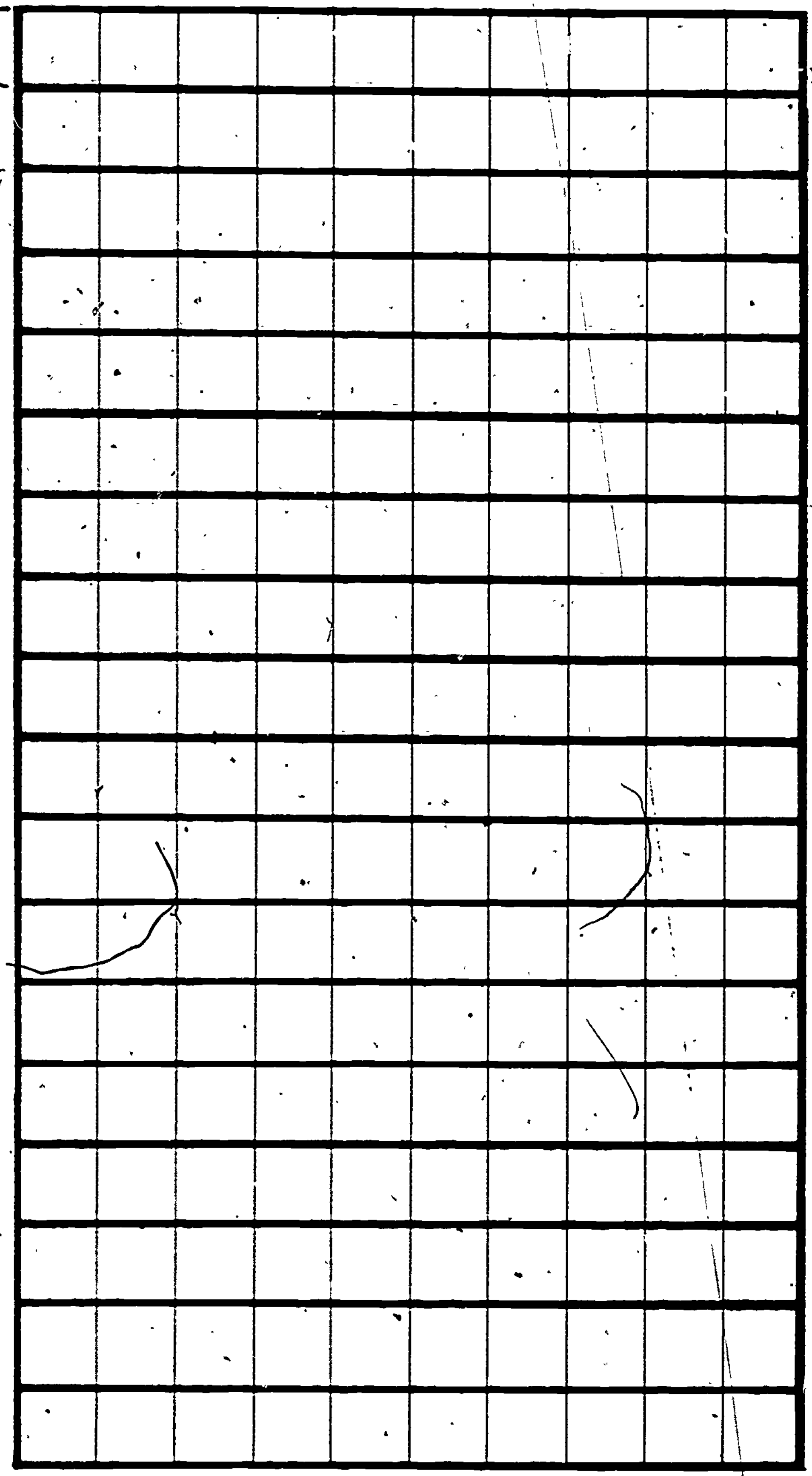
Sheet A (5 per child)

t on dark lines. Cut 2 T^2 pieces from each Sheet A.



Sheet B (1 per child)

1 T^1 piece



Cut 18 T^1 pieces.

Cut 20 T^0 pieces.