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ABSTRACT

This volume is the twenty-first in a series of 29 coordinated MINNEMAST units in mathematics and science for kindergarten and the primary grades. Intended for use by second-grade teachers, this unit guide provides a summary and overview of the unit, a list of materials needed, and descriptions of three groups of lessons. The purposes and procedures for each activity are discussed. Examples of questions and discussion topics are given, and in several cases ditto masters, stories for reading aloud, and other instructional materials are included in the book. The first section of this unit is concerned with angles and their measurement. The unit of measurement used is called a Mag and angles are measured with a special circular protractor. The other sections deal with polygons and polyhedra. (SD)

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	7.	INTRODUCING SYMMETRY
FIRST GRADE	8.	OBSERVING PROPERTIES
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	10.	DESCRIBING LOCATIONS
	11.	INTRODUCING ADDITION AND SUBTRACTION
	12.	MEASUREMENT WITH REFERENCE UNITS.
	13.	INTERPRETATIONS OF ADDITION AND SUBTRACTION
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	● 21.	ANGLES AND SPACE
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	25.	MULTIPLICATION AND MOTION
	26.	WHAT ARE THINGS MADE OF?
	27.	NUMBERS AND THEIR PROPERTIES
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ANGLES AND SPACE

UNIT 21

MINNEMAST

MINNESOTA MATHEMATICS AND SCIENCE TEACHING PROJECT
720 Washington Avenue S. E., Minneapolis, Minnesota 55455

MINNEMAST

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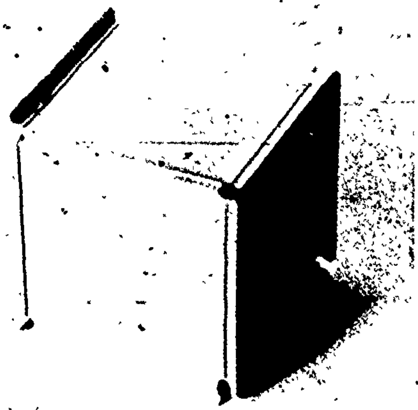
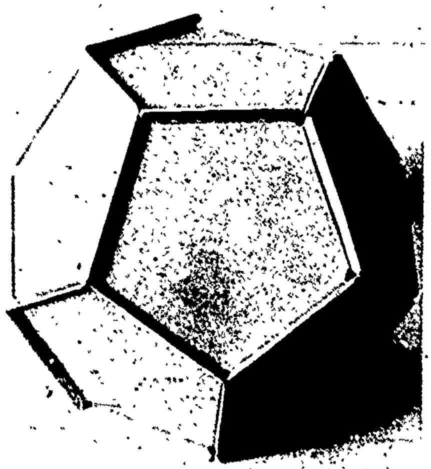


ANGLES AND SPACE



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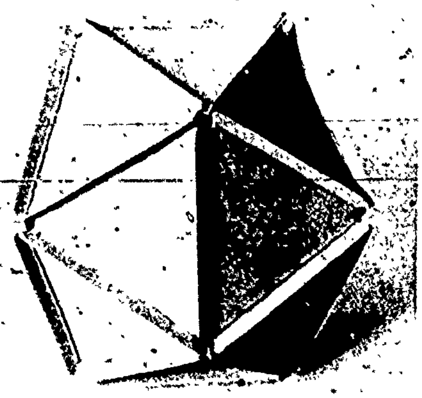

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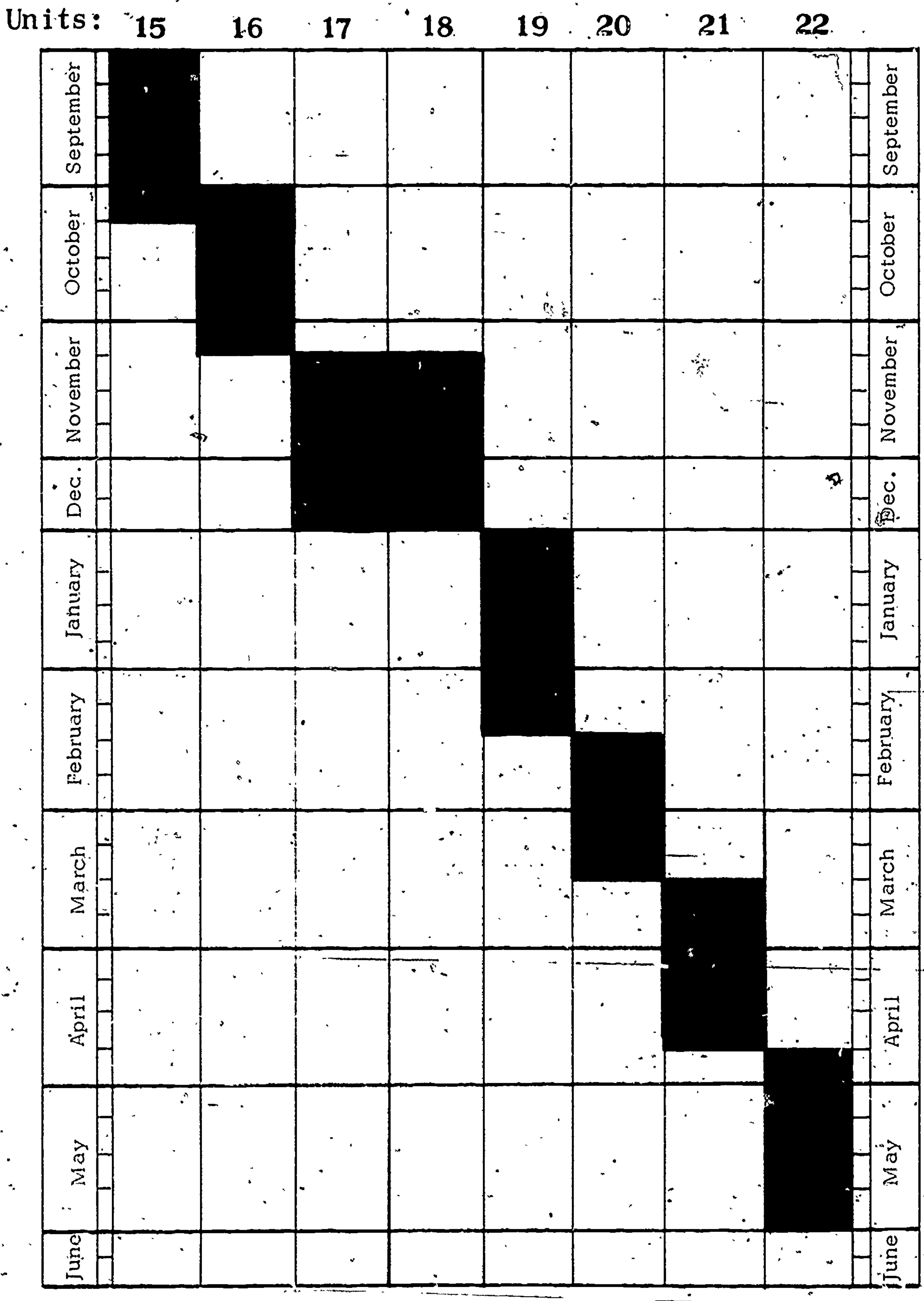
Suggested Teaching Schedule for MINNEMAST Second Grade Units



CONTENTS

Complete List of Materials for Unit 21

(Numbers based on class size of 30.)

total number required to teach unit	item	lessons in which item is used
30	**Student Manuals	
30	straight pins	1
30	*magnifying glasses	1
30	boxes of crayons	1, 14, 16
30	*rulers	1, 3, 5, 6, 8, 9, 11, 12
9 or 10	blank transparencies	1, 5, 10, 12
1	overhead projector	1, 3, 5, 10, 11, 12, 13, 14
1	box of colored pencils	1, 6, 8
	yarn and needles (optional)	1
1	plastic shoe box (optional)	1
	soil or sand (optional)	1
1	package of white radish seeds (optional)	1
1	roll of plastic wrap (optional)	1
1	*ball of string or yarn	2, 5, 7, 11, 12
100	*pipe cleaners	3, 5
1	large pair of scissors, 10" to 12" long	3
1	small pair of scissors	3, 8
1	Elmer's glue (optional)	3, 14
1	ream of 8-1/2" x 11" construction paper	3, 9, 14
5	boxes of toothpicks (optional)	3
1	**transparency of clock protractor	5
1	ray cutout	5
30	**clear plastic clock protractors	5, 6, 8, 11, 12

15	angle-finders	6,7,8
1	filmstrip projector or slide projector	7
90	sheets of black construction paper	7,14
2 or 3	dusty chalkboard erasers	7
1	roll of masking tape	7,8
30	*mirrors on blocks	7,8,11
5 or 6	flashlights	7,8
4 or 5	rubber playground balls	7
4 or 5	plastic bowling pins	7
2	*small mirrors (2" x 4-1/4")	8
1 quart	quart milk carton	8
30	scissors	9,12,13,14,16
30 ea.	*colored pencils: red, blue, green	9,12
1	box of paper clips	10,12,16
1	*package of think sticks and connectors	10,16
1	**Transparency A	11,13
1	sheet of tagboard	12
70-75	sheets of 12" x 18" construction paper	13,15
1	paper cutter	14
1	globe	14
150	baggies (small plastic bags)	14
180	*square counters	15
15	*rolls of transparent tape	5,10,13,15,16
1 set	*Flexagons	15,16

*kit items as well as

**printed materials available from
Minnemath Center, 720 Washington Ave. S.E., Mpls., Minn. 55455

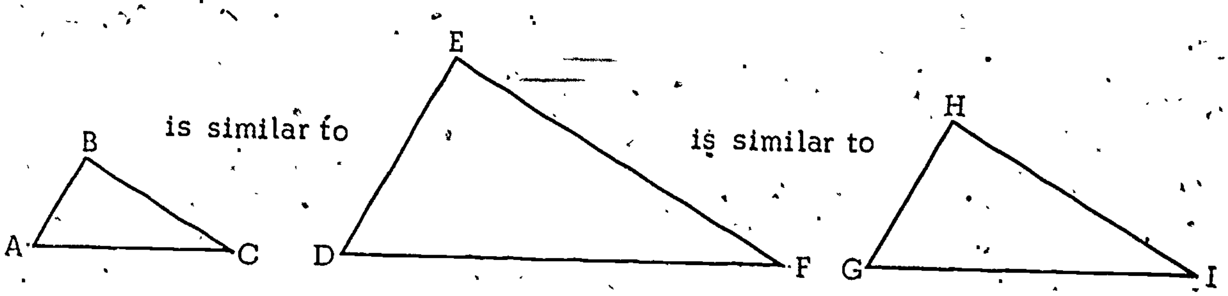
***available from The Judy Company,
310 North Second Street, Minneapolis, Minnesota 55401

INTRODUCTION

Unit 21, Angles and Space, concentrates on geometry concepts. There are numerous reasons for including the study of geometry in the elementary curriculum.

1. Geometry is the study of spatial relations and concepts. These are important in the application of science, engineering and technology, in art and architecture, in sewing and picture hanging, in baseball and billiards. Our sense of spatial relations is exercised every day in countless ways. It is therefore obviously valuable to strengthen the children's grasp of these concepts.

2. Geometry is an important discipline for developing logical thinking. For example, the children will work with similar and congruent triangles. This should help them begin to realize that if triangle ABC is similar to triangle DEF, and triangle DEF is similar to triangle GHI, then logically, triangle ABC can be expected to be similar to triangle GHI.



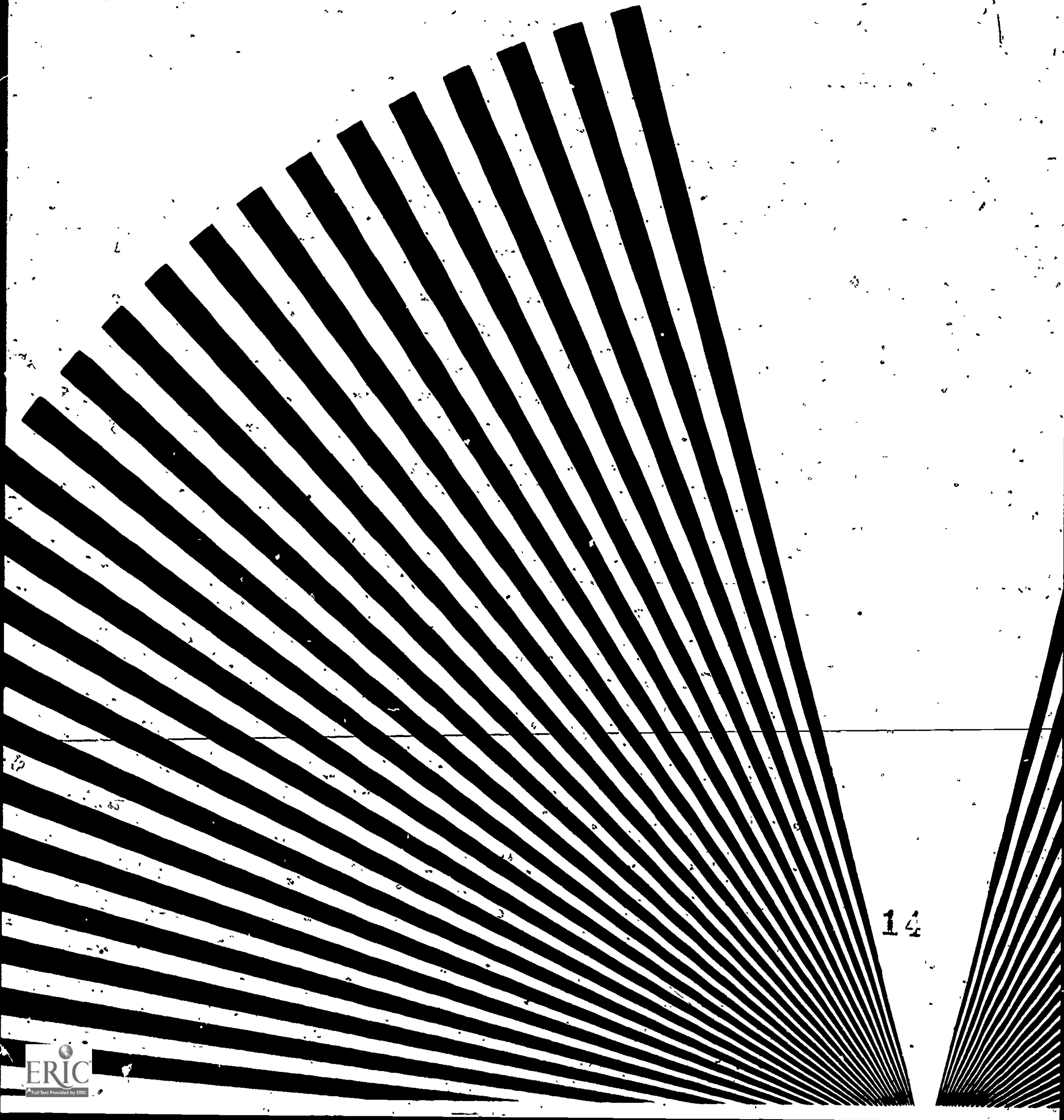
Another simple exercise in logic is that if two shapes have the same number of sides, and if those sides are the same length, and if all the corresponding angles have the same measurement, then the two shapes are congruent. This kind of logical thinking carries over in countless situations.

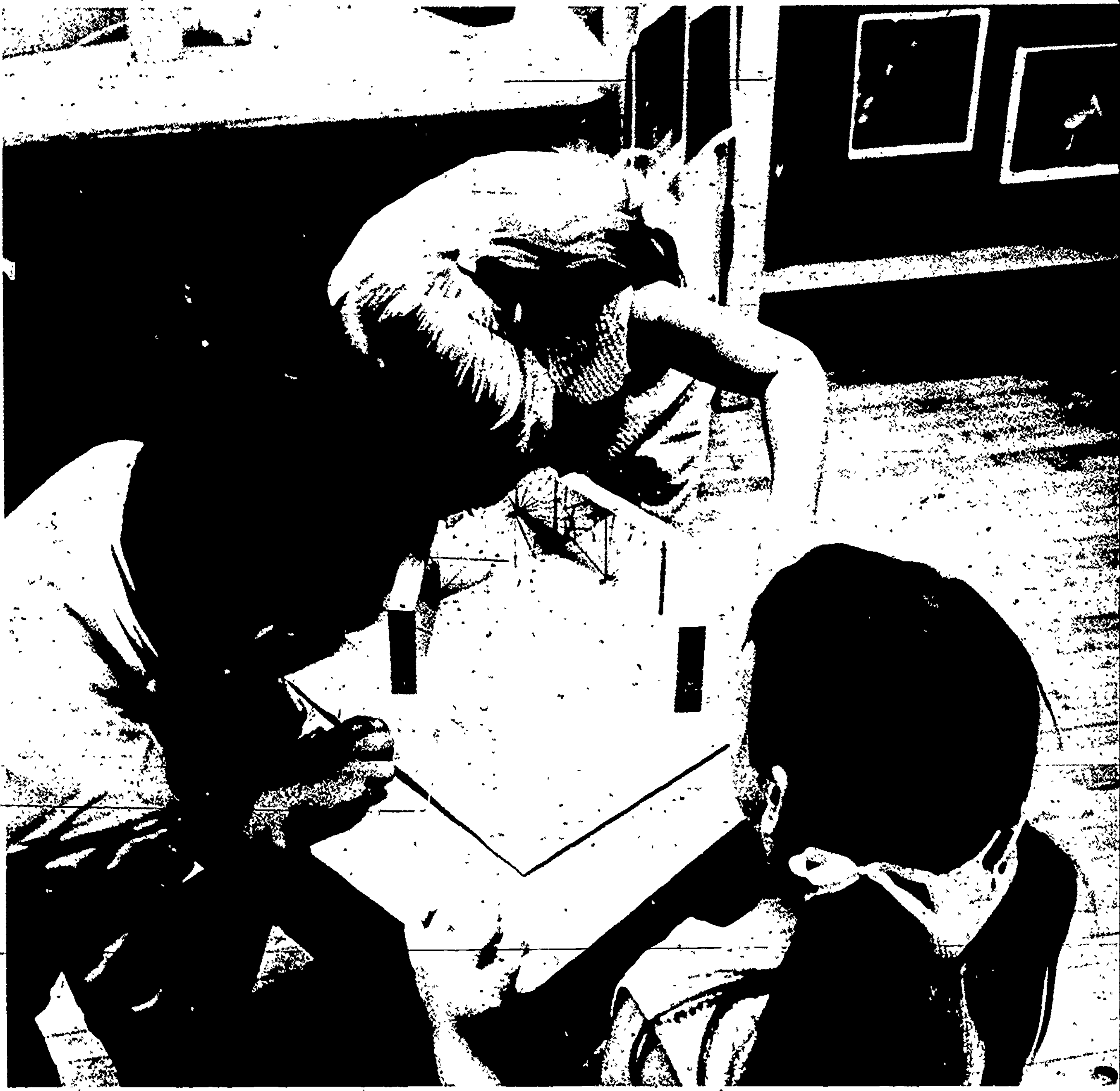
3. In most practical uses of spatial relations and concepts, measurement is an essential element. Therefore the measurement aspect of geometry is extended in this unit. When measurement is applied to spatial ideas, the children's use of numbers is reinforced. Consider the study of a line: when you measure a segment of a line and apply numbers to it, you are able to define it not only in terms of its location and direction, but also in terms of how far it reaches from here to there. You can look at an angle in quantitative terms also. When you apply measurement to an angle, you find its "mag." (This is the word the authors have coined to mean "the measure of an angle" just as "length" means "the measure of a line segment.")

4. The understanding of dimension is basic to an appreciation of the structure of our three-dimensional universe. Dimension underlies our concepts of maps, surface, structure, volume, design and architecture. In the development of these ideas, the children will work in one dimension (with lines), in two dimensions (with plane figures) and in three dimensions (with solid figures).

As you read through the unit before teaching it, you will no doubt think of games and equipment you can use that are already in your classroom. You may want to integrate some of them into the lessons, or have them available for use during free time. Mosaic tile sets, parquetry blocks and building blocks are the most obvious examples of materials that could be used. You will surely find many ways in which the substance of this unit can be extended into other classroom activities, since the concepts are so clearly reflected in everyday life.

SECTION ONE : ANGLES





SECTION 1 ANGLES AND MEASUREMENT

Section 1 begins with an extended review of the concepts of point, line and line segment, and introduces the concept of ray. Then intuitive notions of angle are explored and developed. The children are expected to be able to discover a simple definition of angle (two rays with a common origin) and to learn how to name angles.

In this section, the children add the measurement of angles to the other measurement skills they have already developed. First, they compare angles indirectly by using various simple devices and then, directly, by superposition. This leads them to see that a more practical and precise method of measurement would be useful. For this, they are introduced to the clock face protractor. It should be easier for the children to read the clock face, rather than the traditional 180° protractor markings. (Degree measure will be introduced in Unit 26.) The children are also introduced to a new word, "mag." This word means "the measurement of the angle." We coined this word to help the children avoid the confusion that often occurs when discussing the concept of angle and the measurement of angle. You can talk about the mag of an angle just as you talk about the length of a line segment or the weight of a mass.

Having acquired these geometry concepts and the vocabulary for expressing them, and having acquired a tool and the skill for measuring the mag of angles, the children are given a good deal of experience measuring angles they find in the world around them. For example, they find and measure angles in photographs and drawings of trees, leaves and animals. They are given another tool, the angle finder. They can use this tool, along with their clock face protractors, to discover and measure angles they can form with parts of their own bodies.

In the later lessons of this section, light beams are used as physical embodiments of geometric rays. This leads to the study of angles of incidence and reflection, as in optics, and illustrates the application of angles and geometry to the physical sciences. In Lesson 8, the last lesson in this section, we describe a light reflection activity center, where the children can experiment freely to discover interesting facts about light and its reflection.

Lesson 1: POINTS, LINES AND LINE SEGMENTS

The concepts of points, lines and line segments were introduced in Unit 10. This lesson is an extended review of those concepts. The amount of time you spend on the activities will depend on whether or not the children have studied Unit 10 and how much they remember of it.

The children review the ideas that a line is made up of points and that the shortest distance between two points is measured along a straight line. They also develop methods of naming points, lines and line segments.

MATERIALS

- 30 straight pins
- 30 magnifying glasses
- 30 boxes of crayons
- 30 rulers
- transparency of Worksheet 4
- colored pencils (optional)
- yarn and needles (optional)
- transparency of "Curve of Pursuit" (optional)
- Worksheets 1 - 5

PREPARATION

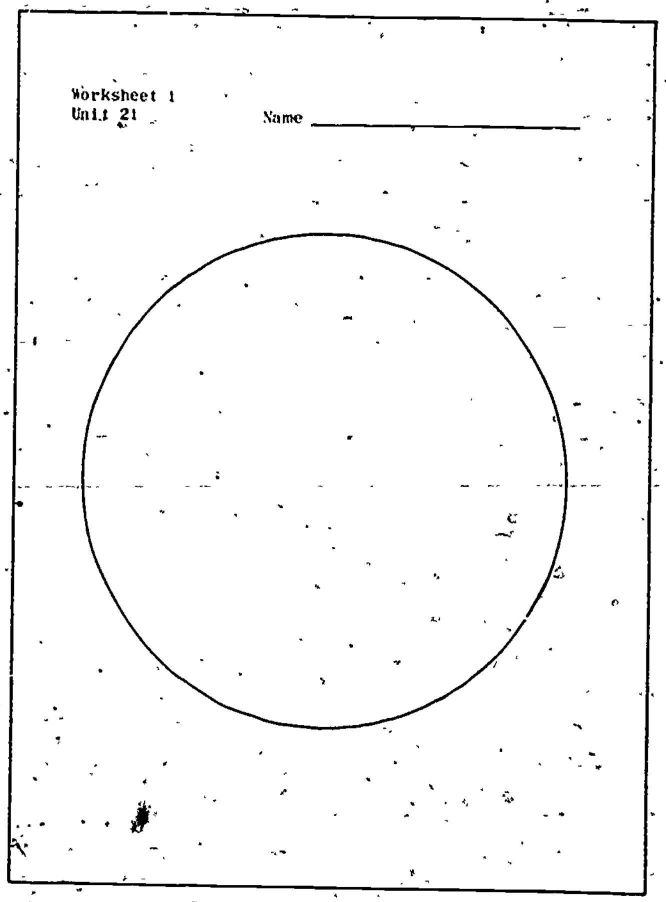
Make a transparency of Worksheet 4 and of the "Curve of Pursuit." Use a copy of Worksheet 4 from the Student Manual; the printed original of the curve of pursuit is included in the Appendix at the back of this manual.

PROCEDURE

Activity A

The purpose of this activity is for the children to realize that a geometric point is so small that we can't see it. They should realize that to refer to a point, they need something to represent that point, i.e., a dot.

Have the children make a dot with a crayon near the center of



the circle on Worksheet 1. Ask them to use a pencil to make a smaller dot in the center of the crayon dot. Then ask the children if they could make an even smaller dot inside the pencil dot. Ask:

> WHAT COULD WE USE TO MAKE THIS SMALLER DOT? (Let them speculate.)

Give each child a pin and a magnifying glass. They should then make a pin hole in the center of the pencil dot and look at it with their magnifying glasses. Ask if it would be possible to make an even smaller dot in the pin hole. They might say that they haven't got anything with which to make a smaller dot, but it appears that it would be possible. Someone may suggest that a microscope would have to be used to see such a small dot. Ask what would happen to the "smallest" dot if they kept making their dots smaller and smaller. (It would become invisible or it would disappear.)

Tell the children:

> WE HAVE A NAME FOR THIS "SMALLEST" DOT THAT WE CAN'T SEE. IT IS CALLED A POINT.
>
> IF WE WANT TO TALK ABOUT A POINT, LIKE A POINT NEAR THE CENTER OF THE CHALKBOARD, HOW COULD I SHOW YOU WHERE IT IS LOCATED?

Let the children discuss different possibilities. Lead them to the idea that we need something like a dot to represent a point.

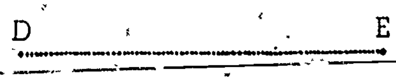
Have someone draw two dots anywhere on the chalkboard. Tell the class that these two dots represent two different points. Say that you want to talk about just one of these points. How will they know which point you are talking about? Have someone describe its location. Then ask if there is an easier way to show which point you want to talk about. Lead the children to name the point in some way, e.g. point "Harry." Tell them that mathematicians use an even shorter method of naming a point. They use a capital letter which is like a person's initials. Have a child label the two points on the chalkboard with any capital letters.

Draw several more dots on the chalkboard and have other children label them, using different letters. Then ask someone to locate specific points by naming them.

Activity B

Put two dots on the chalkboard, about six feet apart. Name them D and E. Tell the class that D and E are names for the points represented by the two dots.

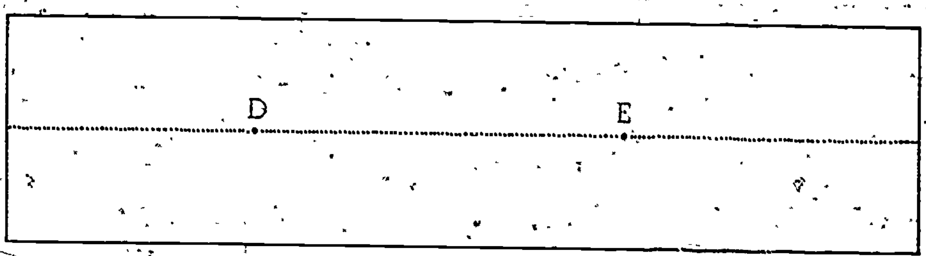
Ask the class how many dots they think could be placed between points D and E. Write a few of their guesses on the chalkboard. Then ask two children to put as many dots as possible between points D and E. Remind the class that these dots represent points so small that we can't see them.



When the two children are satisfied that they have made as many dots as possible between points D and E, ask if some other children think they could place more dots between D and E. Let them try. Keep insisting that they can place more dots between points D and E. Someone will probably start erasing larger dots and replacing them with many smaller dots; encourage this. The children should eventually realize that if they continued this process of making smaller and smaller dots, they could place "billions and billions" of dots between these two points.

Then ask them to imagine how many points there could be between these two points. However large a number the children come up with there is always a larger one. There is actually an infinite number of points between any two points; however, at this level we do not expect the children to comprehend the the concept of infinity. We only expect them to consider the concept of infinity as being "fantastically large," or "going on and on forever" or "larger than anything we can imagine."

Now ask a group of children to place as many dots as possible on the straight line extensions past D and past E to the ends of the chalkboard.



When the series of dots resembles a line, ask the children to return to their desks. Ask them how far they could make dots if they didn't have to stop at the edges of the chalkboard but could go on through the school wall, the city, country, sky, outer space, etc. The children should reply something like: "on and on forever." Then ask the children what their series of dots looks like. (A line.)

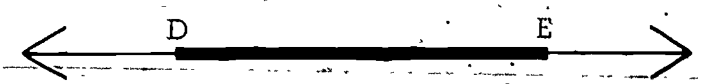
Ask the children how they could show that the line doesn't have to end at the chalkboard edges but could go on and on. Lead them to place arrowheads on the ends of this representation.

Ask the class how they made this line. (By placing a set of dots on the chalkboard.) Then ask:

WHAT DO THE DOTS REPRESENT OR STAND FOR? (Points.)

The children should realize that since their chalkboard line is made of a set of dots, and that since the dots represent points, the line can be thought of as a set of points that go on and on forever.

Use colored chalk to outline the section of the line between points D and E.



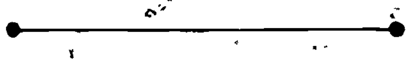
Ask the class if they can remember the special name for part of a line, such as the colored part between points D and E. Remind them of the name "line segment." You may want to tell the class what the word "segment" means by giving examples of how it can be used in various contexts. Then ask:

WHAT ARE THE POINTS AT THE ENDS OF THIS LINE SEGMENT? (Points D and E.)

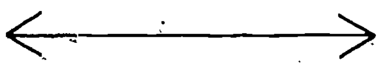
DOES A LINE HAVE END POINTS? (No, because a line goes on and on. A line segment has two end points.)

Review the following ideas with the class.

Draw a figure like the one below on the chalkboard:

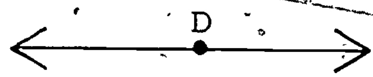


Ask the class what this figure represents. (A line segment.) Then ask how they know that it is a line segment instead of a line. (It has two end points.) Ask them what they would have to do to show that the figure represents a line. (Put arrowheads at the ends to show that it extends endlessly in both directions.)



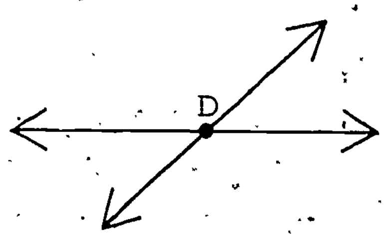
Activity C

Review with the class the procedure used to name points. Then ask them if they can think of a way to name a line. (Someone may remember from the work in Unit 10 that two points are needed to name a line. If so, ask him why we can't always name a line by using just one of its points.) Someone may suggest naming it using one of the points on the line, e.g., line D.



IS THIS THE ONLY LINE THAT CAN PASS THROUGH POINT D? (No.)

Draw another line through point D.

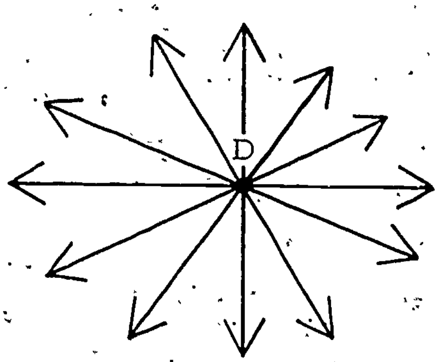


WHICH LINE IS LINE D? (The children will probably see that both lines could be called line D.)

Then ask:

HOW MANY STRAIGHT LINES CAN WE DRAW THROUGH POINT D?

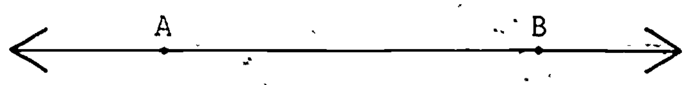
Let various children draw lines through point D, using a straightedge.



The children should realize that it is possible to draw many straight lines through point D. Then ask:

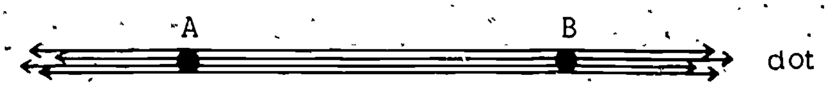
> HOW DO WE KNOW WHICH LINE IS LINE D? THEY COULD ALL BE CALLED LINE D. THIS DOESN'T SEEM TO BE A VERY ACCURATE WAY TO NAME JUST ONE LINE. CAN ANYONE THINK OF SOME OTHER WAY TO NAME ONE LINE? (Name two points on that line.)

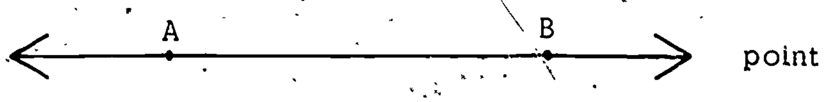
Draw another line on the chalkboard and label two of its points.



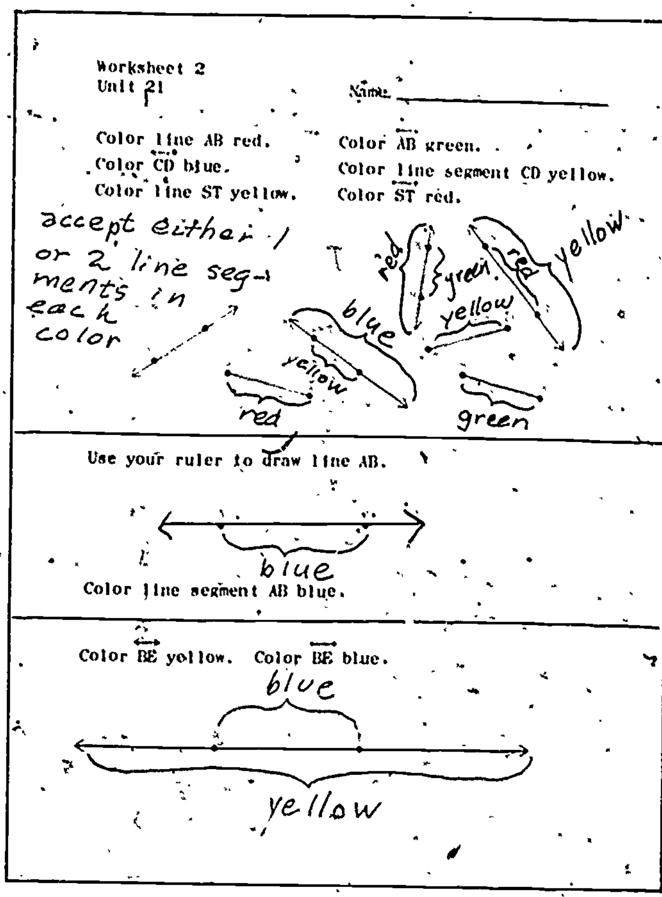
This is called "line AB" or in mathematical notation "$\overleftrightarrow{AB}$". Write line AB and $\overleftrightarrow{AB}$ on the chalkboard and explain the notation to the children. Also explain that if we are talking about line segment AB, we would write either line segment AB or $\overline{AB}$. Draw several lines and line segments on the chalkboard, label two points on each, and have the children write the correct notation on the chalkboard.

Erase all lines and line segments except line AB, and ask the class if naming two points on the line is enough to determine one straight line. Let several children try to draw another straight line through points A and B using a straightedge. If the dots are drawn too large, they will be able to draw more than one line through the two dots. Remind the children that dots A and B represent points A and B which are so small (they have no dimension) that only one line can extend through them.





Have the children complete Worksheets 2 and 3.


Worksheet 2
Unit 21

Name ____________

Color line AB red.
Color $\overleftrightarrow{CD}$ blue.
Color line ST yellow.

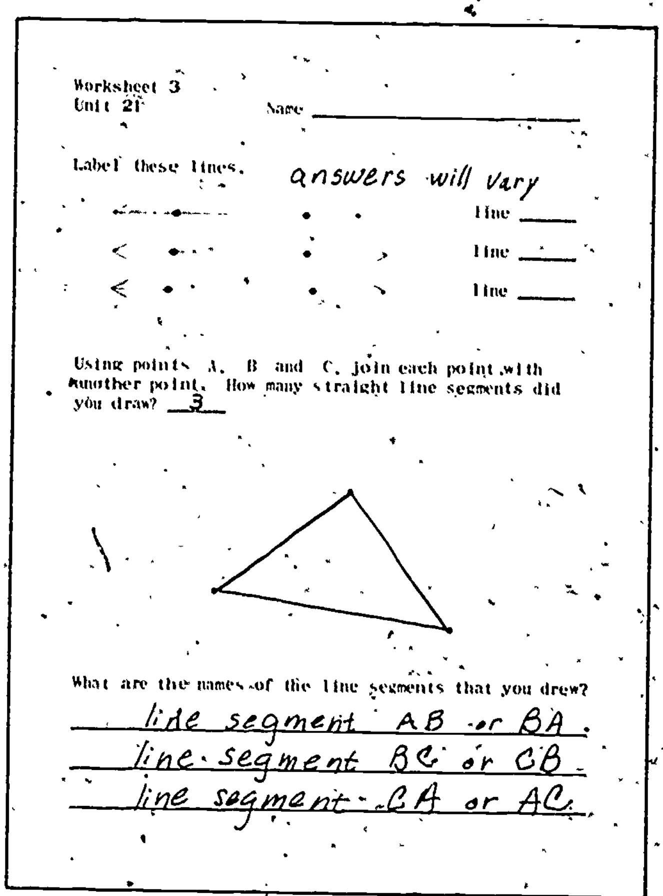
Color $\overline{AB}$ green.
Color line segment CD yellow.
Color $\overline{ST}$ red.

accept either 1 or 2 line segments in each color

Use your ruler to draw line AB.

Color line segment AB blue.

Color $\overleftrightarrow{BE}$ yellow. Color $\overline{BE}$ blue.


Worksheet 3
Unit 21

Name ____________

Label these lines.

answers will vary

line ____
line ____
line ____

Using points A, B and C, join each point with another point. How many straight line segments did you draw? 3

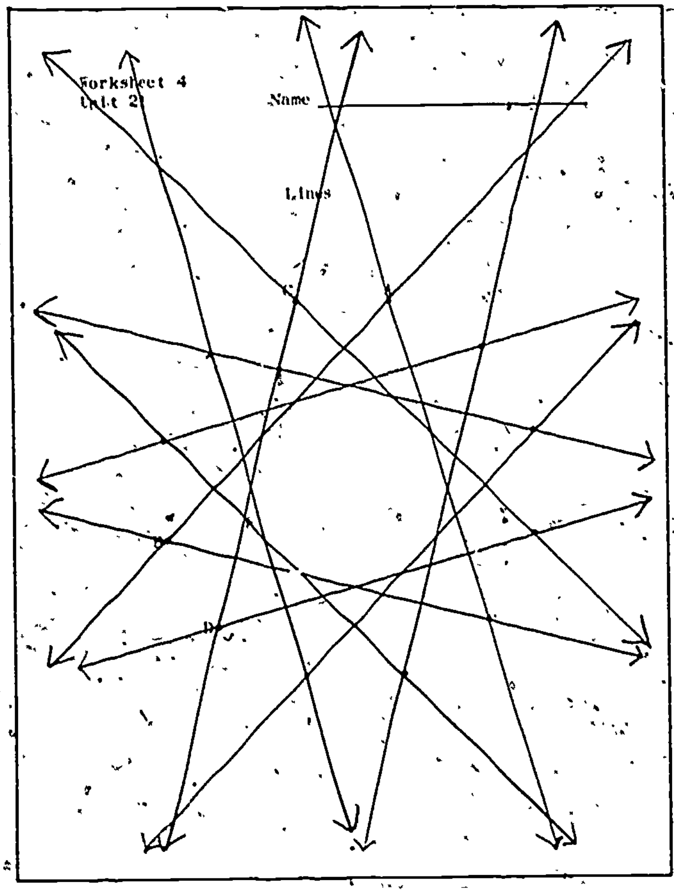
What are the names of the line segments that you drew?

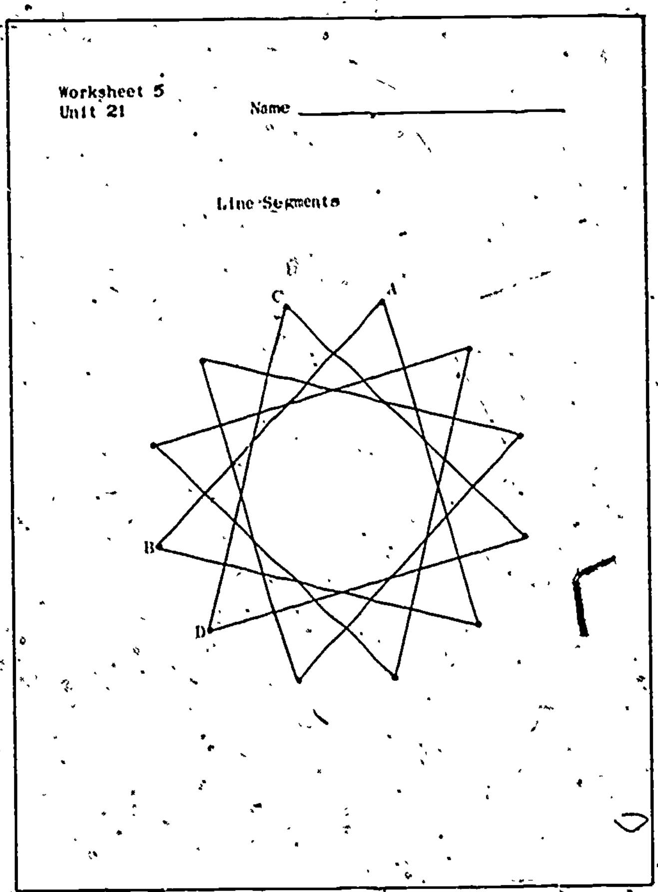
line segment AB or BA
line segment BC or CB
line segment CA or AC

Activity D

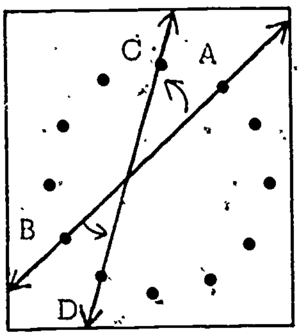
Each child will need a pencil and a ruler to do Worksheets 4 and 5. These worksheets show the difference between lines and line segments. They also give the children experience in recognizing patterns showing rotational symmetry (repeated patterns). The children could start working on them during math class and complete them as an art activity.

On Worksheet 4, the children are to draw straight lines using a ruler and a pencil. Copy the worksheet on the chalkboard or project a transparency of it and demonstrate how to join the points. First draw line AB to the edges of the worksheet. Then draw line CD all the way to the edges of the worksheet.

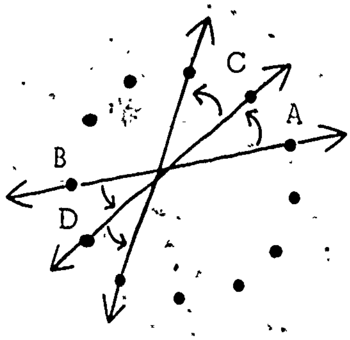




Point out that after you drew line AB, you moved over one space from points A and B to draw line CD.



To draw the next line, move over one space from points C and D to the next pair of points.



Ask someone to draw the line through the next pair of points, etc. When you feel the children understand the procedure, have them do Worksheet 4. As the children complete it, quickly check their work and then instruct them to go on to Worksheet 5.

Worksheet 5 is the same as Worksheet 4, except that the children draw line segments between the points instead of lines through the points.

Activity E (Optional Art Activities)

Coloring Patterns

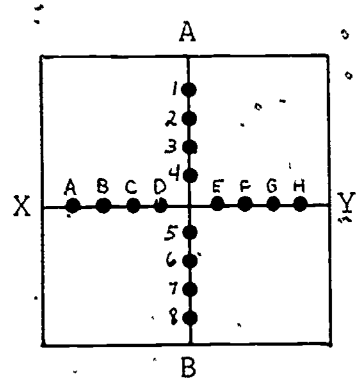
Have the children use colored pencils to color the repeated patterns on Worksheets 4 and 5. They could then carefully outline the lines and line segments with a black crayon. These worksheets would make a very attractive bulletin board display.

Curve Stitching

If the children are interested, you may want to do some curve-stitching, using yarn and needles to stitch patterns drawn on tagboard.

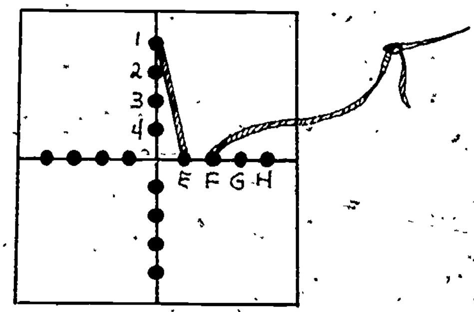
1. Mark a piece of 10" x 10" tagboard into quarters.

2. Mark points along the line segments at 1-inch intervals. Label the points on line segment XY with letters and the points on line segment AB with numbers.

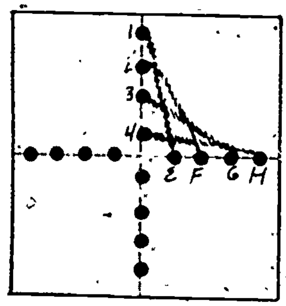


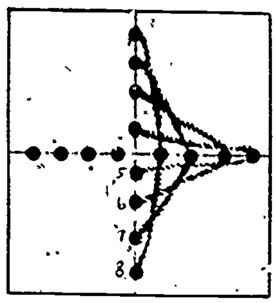
3. Before starting the stitching, the children should poke a hole through the tagboard with the yarn needle at each marked point.

4. Begin stitching from the back of the tagboard. Knot yarn, then pull needle and yarn through 1. Insert in E and come up through F. Demonstrate the stitching to the children and have them follow you step by step.

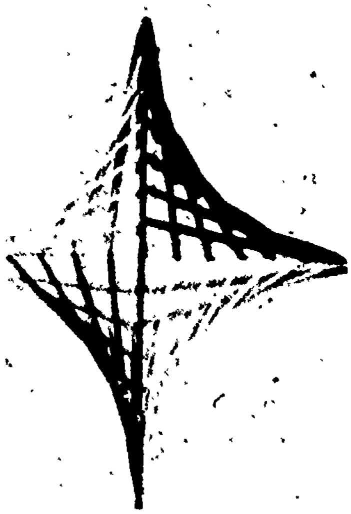


5. Continue sewing curve by stitching from F, over to 2, up through 3, and over to G, up through H, and over to 4.





6. Those children who did not have trouble with this may want to do another section. Number along the lines, as shown. Then stitch next curve as follows: H to 5, 6 to G, F to 7, 8 to E.



7. Sew each quarter until pattern is made.

Activity F: Curve of Pursuit (Optional)

Project the transparency of the curve of pursuit. Tell the following story while completing the curve.

Abbreviated version of story

The rabbit comes in through the hole in the fence and goes to the upper left hand corner of the yard where the carrot garden is located. He starts eating the carrots. Suddenly the sleeping dog wakes up and sees the rabbit.

(On the transparency, this is shown by the line from the dog's eye to the first fence post where the rabbit is located.)

The rabbit sees the dog and hops one space (one fence post) toward the opening in the fence. The dog moves one space up the line toward the rabbit.

(To mark the dog's new location, measure up $\frac{1}{2}$ inch from the dog's eye on the given line and make a dot.)

The dog sees the rabbit at the second fence post.

(Draw a line from the dog's location -- the dot you just finished marking -- to the second fence post.)

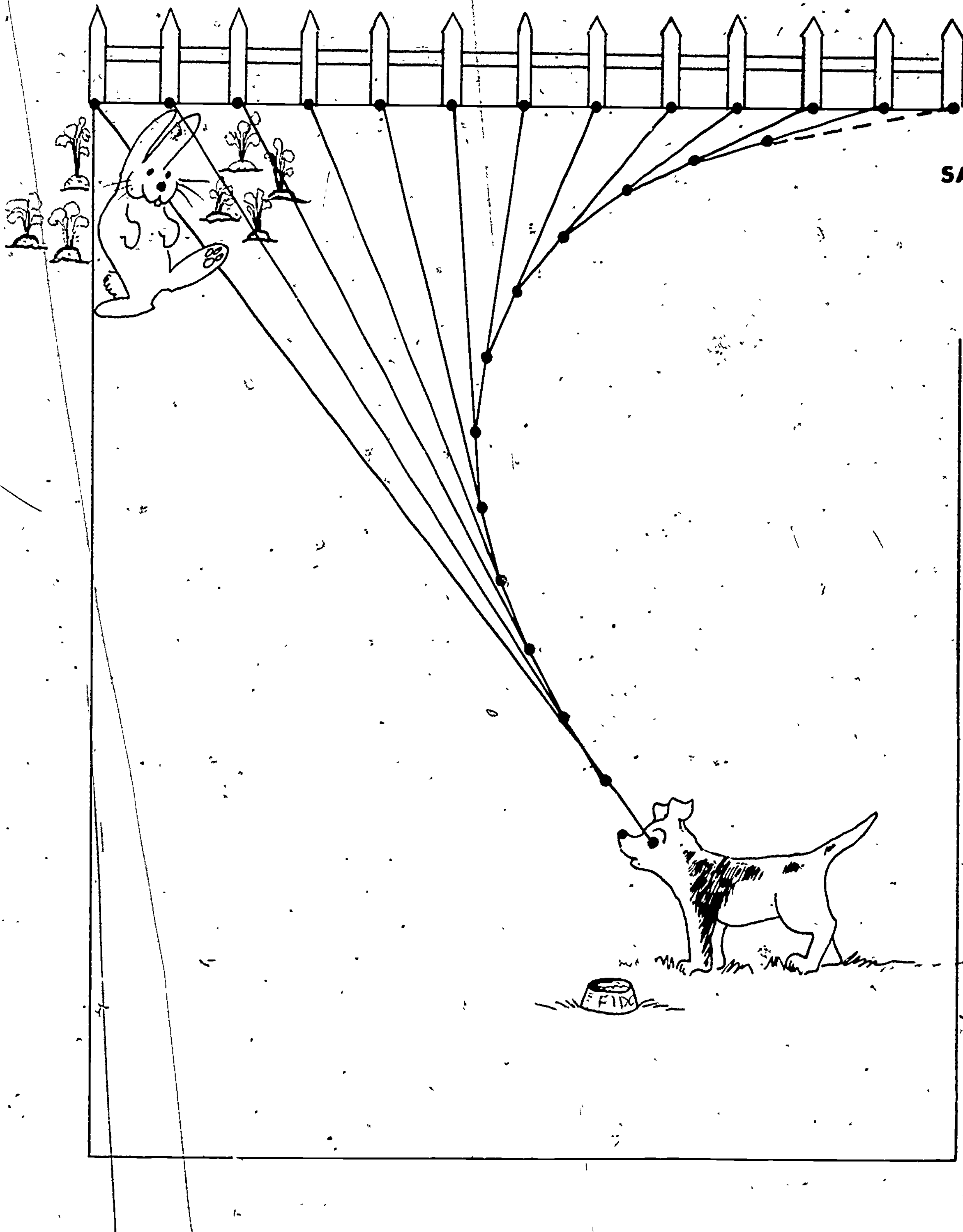
Each time the rabbit moves one fence post to the right, the dog moves one space closer.

(To mark the next location in the dog's path, measure up $\frac{1}{2}$ inch on the line you just drew and make another dot. Draw a line from this dot to the third fence post, etc.)

Ask:

- WILL THE DOG GET TO THE RABBIT BEFORE THE RABBIT CAN GET TO SAFETY?

(Complete the curve of pursuit by marking dots and drawing lines from the dots to the fence posts, as you did earlier. See diagram of completed curve on the next page.)



SAFETY

OPTIONAL PREPARATION

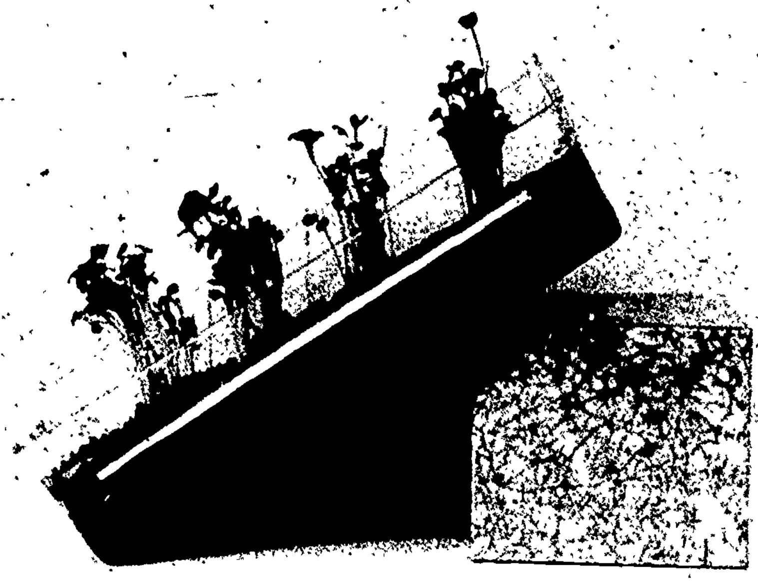
In Lesson 6, "Finding and Measuring Angles," an optional activity is suggested that requires radish seedlings. If you plan to do this activity, the radish seeds should be planted now. This will allow approximately one week for them to grow.

MATERIALS

- 1 plastic shoe box
- soil or sand
- white radish seeds
- plastic wrap or baggies
- water

PROCEDURE

1. Fill the shoebox approximately half full of soil or sand.
2. Level the surface.
3. Plant the seeds in rows, about $\frac{1}{4}$ inch below the surface.
4. Water the seeds. Sprinkle so as not to wash the seeds away.
5. Prop up the shoebox at an angle, using a box or a stack of books.
6. Cover the box with plastic wrap until the seeds sprout.
7. Sprinkle with water daily, especially the higher area.



Lesson 2: RAYS

The children are introduced to the concept of a ray and learn what properties rays have in common with lines and line segments.

MATERIALS

- ball of string or yarn
- Worksheets 6, 7 and 8

PROCEDURE

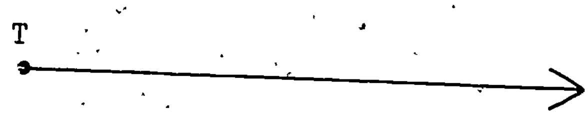
Activity A

Select one student (Tom) to represent a point. Have him stand at the front of the room. Draw a dot on the chalkboard to represent Tom and label it "T." Give Tom the end of a ball of string and select another child (Mary) to walk away from Tom, unwinding the ball of string as she walks. The string should be kept as taut as possible. Have Mary walk away from Tom as far as she can go in a straight path — you may want her to walk down the hall and out the door.

Tell the class:

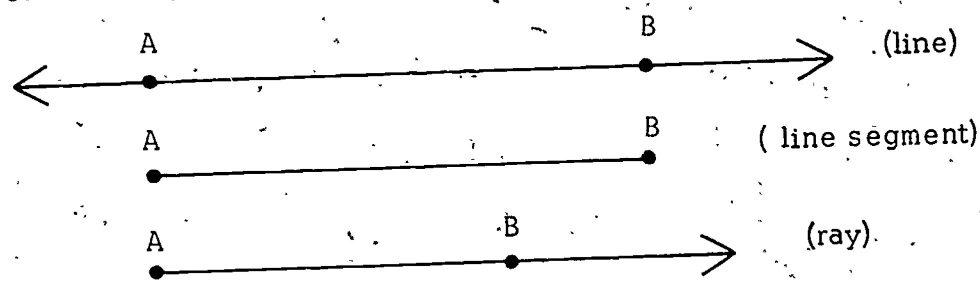
IMAGINE THAT MARY NEVER RAN OUT OF STRING AND SHE COULD CONTINUE GOING AWAY FROM TOM FOREVER IN A STRAIGHT PATH. HOW COULD WE SHOW THIS ON THE CHALKBOARD USING POINT T TO REPRESENT TOM?

Use T as the starting point and draw a "line" in one direction away from it. Place an arrowhead at the end of the "line" to show that it could go on and on forever.



Ask the class if this representation reminds them of something they've seen before. The class should see that it resembles both a line and a line segment in certain ways. Tell the class that there is a name for this new representation. It is called a "ray." We call its starting point the "origin" of the ray.

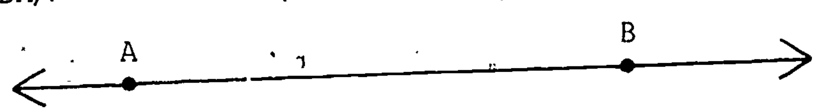
Draw a representation of a line and a line segment on the chalkboard, above the ray.



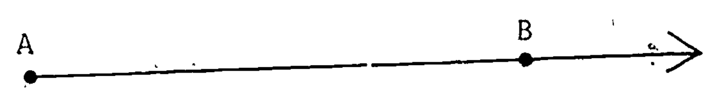
Discuss the following possible comparisons with your class.

1. All three representations may be named by and are determined by two points.
2. Each one may be thought of as a set of points.
3. The line extends endlessly in both directions, and the new representation (ray) extends endlessly in one direction.
4. The line segment has two endpoints and the new representation has only one endpoint.
5. The line segment can be measured (it has a certain length); the other two representations cannot be measured.

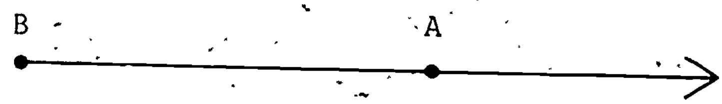
Discuss the fact that a ray can be named, just as the line and line segment are, but there is one important difference. Remind the children that line AB ($\overleftrightarrow{AB}$) can also be called line BA ($\overleftrightarrow{BA}$).



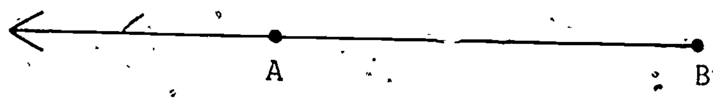
It doesn't make any difference in what order we say the letters. The same is true for line segment AB. This is not true for a ray.



When we say ray AB ($\overleftrightarrow{AB}$), the A comes first and the B second. This shows that A is the origin of the ray. Reading from left to right, the first letter is always the origin of the ray. If we say ray BA ($\overleftrightarrow{BA}$), we are referring to rays similar to the following.



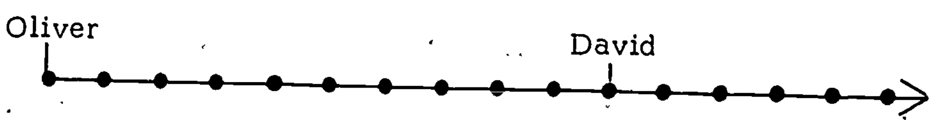
— or —



Lead the class to realize that it is less confusing to name a ray by saying the origin first, and then another point on the ray. You may want to draw several rays on the board and have various children label and name them, making sure they always name the origin first. Use the notation $\overleftrightarrow{AB}$, etc.

Activity B

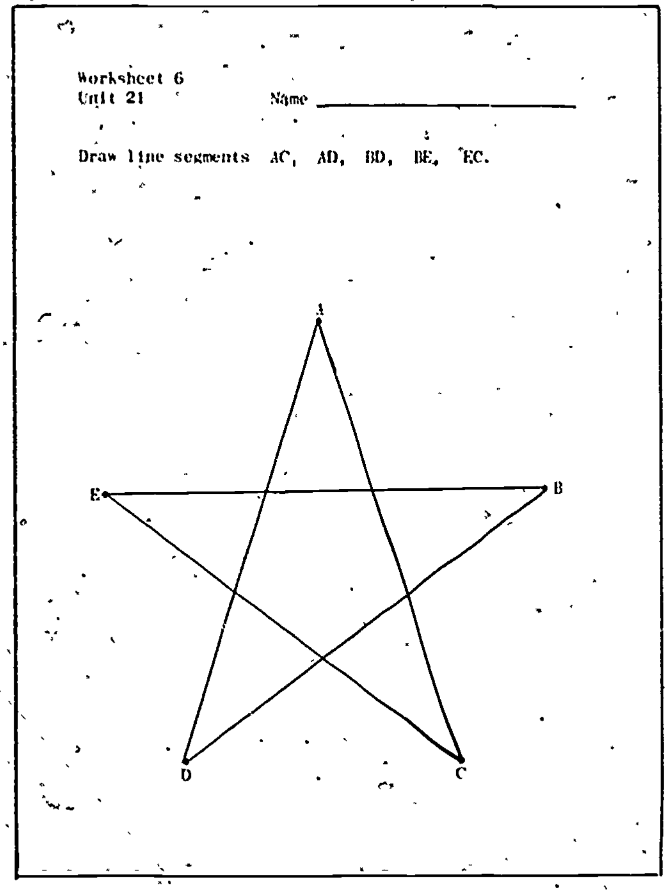
This activity may be done in the classroom or outdoors. Choose a child (Oliver) to be the origin of a ray. Choose another child (David) to be the direction marker. David may stand anywhere. Ask the other children to imagine the ray which originates, starts, or has its origin at Oliver's feet and extends in the direction from Oliver to David and beyond. They should stand on this ray. Most children will probably stand in a roughly straight line between Oliver and David. Suggest that they are rather crowded. Where else could some of them stand, and still be on the imaginary ray which starts at Oliver's feet and goes through David's? (Beyond David.) You may decide to have someone sketch the ray on the floor with chalk, with an arrowhead to indicate the direction in which it continues.



Ask children to suggest possible names for the ray (ray OD or $\overleftrightarrow{OD}$). Make sure the first letter is the origin.

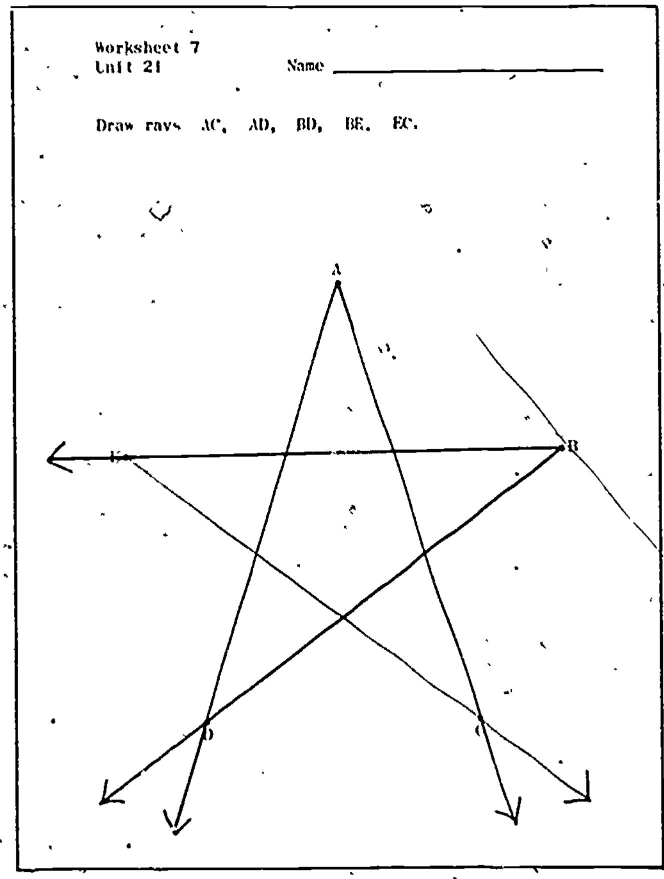
Have the children stand around in a haphazard arrangement. Call out the names of two children for origin and direction marker of an imagined ray — these two are not to move. The other children should hurry to stand somewhere on the imaginary ray. Be sure you always say which child is the origin and and which the direction marker, i.e. "This ray has Opal as origin and it goes toward Dianne and beyond her." Repeat with other children who are standing in different locations. Be alert for children who stand on a line through Opal's and Dianne's feet, on the other side of Opal and Dianne, but not on the ray which starts at Opal and extends only in the direction toward and beyond Dianne.

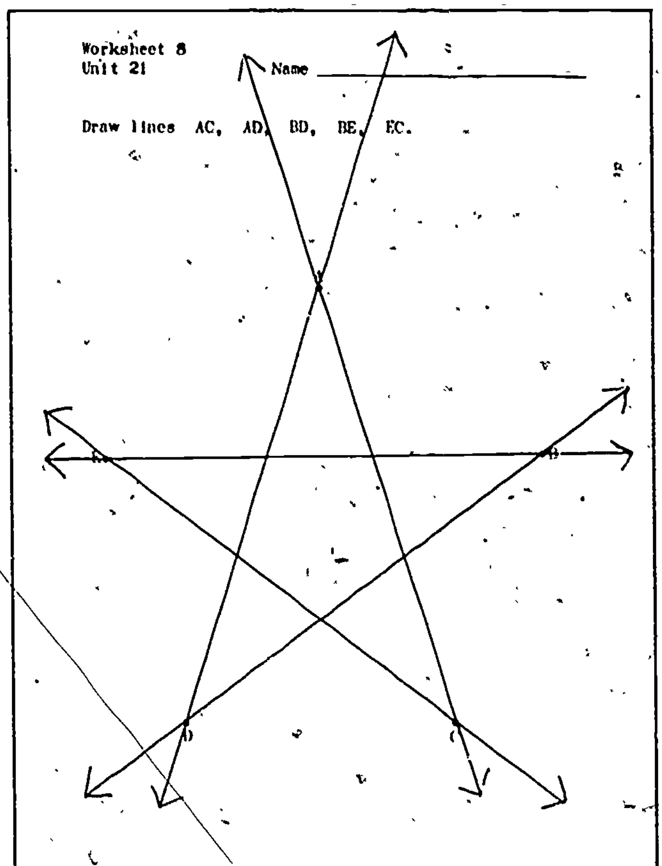
For variation, you could use objects — trees, bushes, playground equipment, or chairs, desks, books and erasers placed on the classroom floor — as origin and direction markers. Rays could be marked with string, chalk, or a stick drawn through sand. You could also make this a timed game, calling out the ray's origin and direction marker, and counting quickly to ten. All who are not standing on the ray when you reach "ten" are out. You could also call out lines and line segments.



Activity C

Have the children complete Worksheets 6, 7 and 8. These worksheets are printed on tracing paper so that when they are laid on top of each other the children can make comparisons of an enclosed figure made by lines (Worksheet 8), line segments (Worksheet 6) or rays (Worksheet 7). This way, the children should see that a ray and a line segment are always part of some line, and that a line segment may be extended into a ray or a line.





Lesson 3: INTRODUCING ANGLES

In the next few lessons, the children will be studying angles. In this lesson they are introduced to the concept of an angle; they learn that angles can be different and that the length of the sides of an angle has nothing to do with the measure of the angle. The children should get an intuitive feeling for an angle. They also develop the definition of angle and find examples of angles in the classroom.

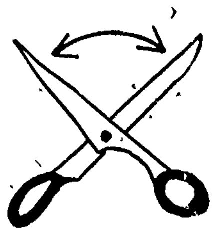
MATERIALS

- 100 pipe cleaners
- 1 large pair of scissors, 10" to 12" long
- 1 small pair of scissors
- rulers
- Elmer's glue (optional)
- overhead projector (optional)
- construction paper
- 5 boxes of toothpicks (optional)

PROCEDURE

Activity A

Hold up a large pair of open scissors. Trace the angle formed by the cutting edges of the scissors with your finger and ask the children if they know what we call this figure. Make angles of various magnitudes, i.e., acute angle, right angle, obtuse angle, by moving the blades.



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If no one thinks of the word angle, say it. Then have the children point out examples of angles, such as those formed by the corner of a desk, book, property block, etc. They can also trace with their finger the angles generated when they move different parts of their bodies.

Ask the class:

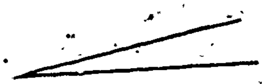
WHAT WOULD A PICTURE OF AN ANGLE LOOK LIKE? (Discuss various answers for a few minutes.)

Then ask:

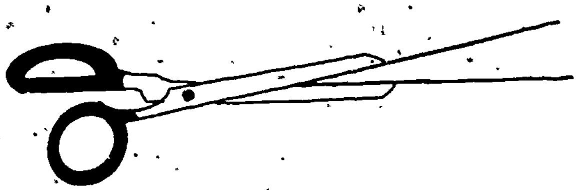
WOULD SOMEONE LIKE TO TRY TO DRAW A PICTURE OF AN ANGLE?

Let different children come to the chalkboard and try to draw an angle. Have chalk and straightedge on the chalk tray for the children to use. After they have experimented for a while, help them draw an angle.

Your chalkboard diagram may look like this.



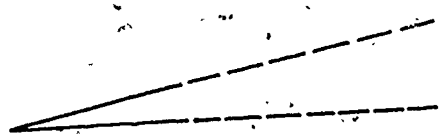
Place the scissors alongside the chalkboard diagram so that the angle formed by the blades of the scissors will coincide with the diagram.



DOES THE CHALKBOARD DIAGRAM REPRESENT THE ANGLE I HAVE FORMED WITH THIS SCISSORS? (Yes.)

Place a much smaller scissors alongside the diagram and ask the same question. (The chalkboard diagram represents the angle formed with the smaller scissors, too.) Point out the different blade lengths of the two scissors. Mention that the same chalkboard diagram represents the angles formed by the different scissors, even though the lengths of the sides of the scissors were not the same.

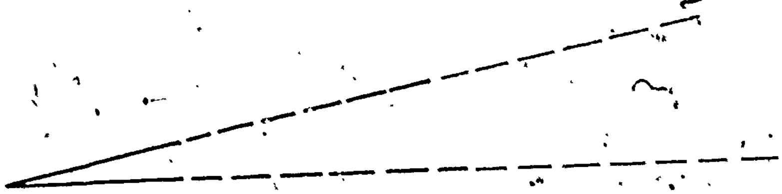
Using a straightedge, extend the sides of the chalkboard angle.



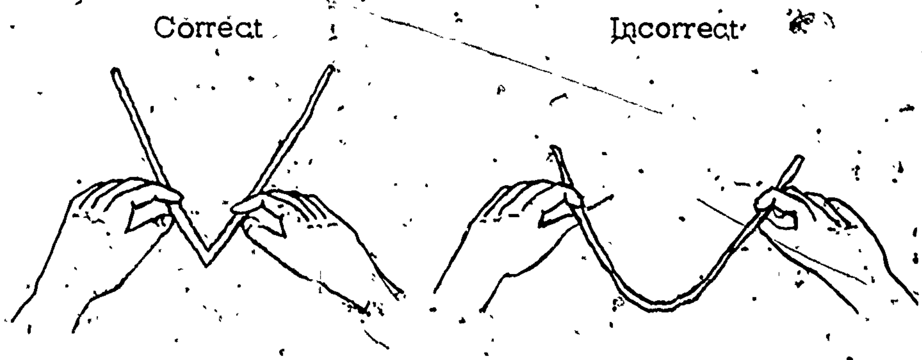
IS THIS THE SAME ANGLE I HAD BEFORE? (Let the children speculate.)

WHAT HAS CHANGED IN THIS DIAGRAM? (The length of the sides.)

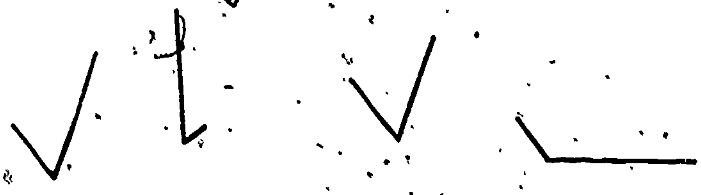
Extend the sides of the diagram even more and ask the same questions.



Tell the class that they are going to make representations of angles from pipe cleaners. Give each child three pipe cleaners. Demonstrate how to make the representation. The correct way is to put your fingers close to the spot where you want to bend the pipe cleaner. The wrong technique is to hold the pipe cleaner at each end. Demonstrate both the right and wrong techniques.



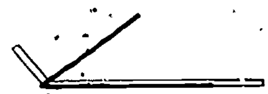
Encourage them to bend the pipe cleaner in different places, not just in the middle.



After everyone has made his pipe cleaner angles, divide the class into groups of four. Each group should put all its angles into one collection. Ask each group to try to arrange its angles in order from the largest to the smallest. Each group should be encouraged to work together to try to figure out a method of deciding which angles are larger or smaller. Some children may focus their attention on the length of the sides. When this happens, remind them of the scissors activity.

This activity should give the children an opportunity to investigate and experiment with angles before they develop a definition of what an angle is and how it is measured. Accept and encourage any reasonable arrangement so long as the children have devised a definite scheme for ordering and can explain it to you.

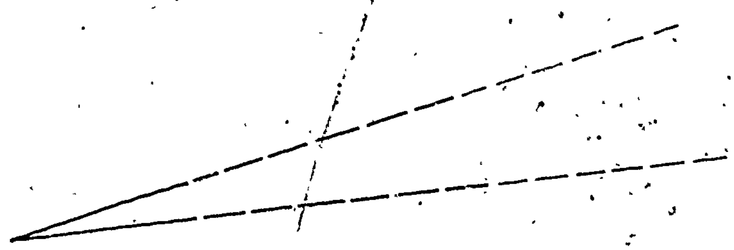
After 5 or 10 minutes, check each group's arrangement. If their angles are not in correct order, place one angle on top of another and ask the group which angle is larger. Lead the group to superpose one angle on another with one side and the vertex superposed.



Caution the children to hold just one side of each angle together; some may want to hold both sides together, bending a pipe cleaner in the process. Lead the children to the conclusion that the angle whose side extends farther to the left, when the second sides are superposed, is the larger angle. (This will be explained in more detail in Lesson 5.)

For added motivation, you may want to make this activity a contest; the group that correctly arranges its angles first is the winner. When one group has completed the activity, and the other groups have nearly finished it, ask the children in the winning group to demonstrate to the class the method they used to order their angles.

Have the children look back at your chalkboard diagram of an angle with extended sides.



Show the children two pipe cleaners that will both fit exactly on the chalkboard angle but are bent in different places.

Superpose each of them over the chalkboard diagram and ask:

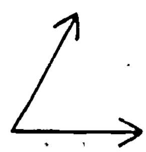
DO THE TWO PIPE CLEANER ANGLES REPRESENT THE SAME ANGLE AS THE CHALKBOARD DIAGRAM OR DIFFERENT ANGLES? (The same.) WHY? (If they were larger or smaller, one of the sides would protrude.)

ARE THE LENGTHS OF THE SIDES THE SAME? (No.)

Lead the children to the conclusion that the length of the sides has nothing to do with the size of the angle. Discuss the chalkboard diagram in more detail. Ask:

HOW CAN WE SHOW ON THE CHALKBOARD THAT THE LENGTH OF THE SIDES DOES NOT MAKE ANY DIFFERENCE WHEN WE MEASURE THE SIZE OF THE ANGLE?

Lead them to draw arrowheads on the endpoints of the line segments to show that they could go on and on, without changing the size of the angle.



The class should now be able to tell you in their own words what an angle is. (An angle is two rays with a common origin.)

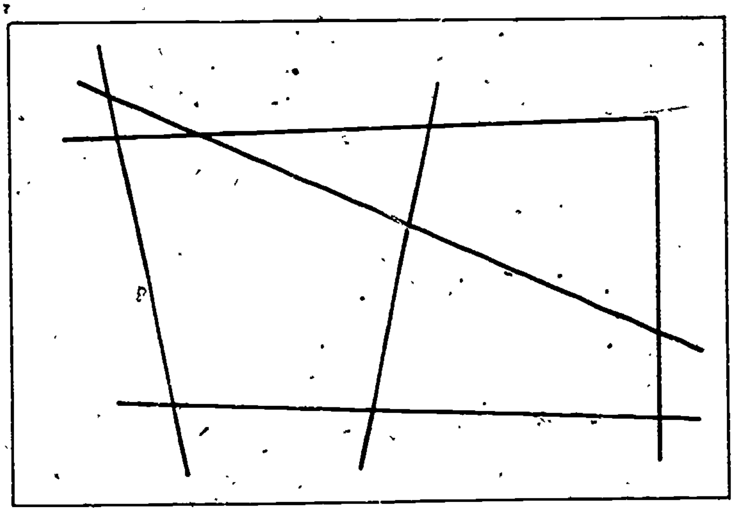
Save the pipe cleaners for Lesson 5.

Activity B

This activity has two versions. Version II is optional.

Version I

Show the children a diagram, similar to the one below, made up of line segments. (Use the overhead projector if possible.)



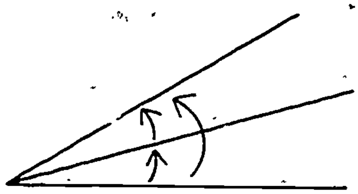
Ask:

CAN ANYONE SEE ANY ANGLES REPRESENTED ON THIS DIAGRAM?

Let different children come to the projector and point out different angles. Emphasize that the line segments only represent angles -- that angles are rays and that rays go on and on.

Let each child draw a picture of something using only straight line segments. Time this activity for about 5 or 10 minutes. As soon as the time period is over, each child trades his picture with a friend, who counts the number of angles he can find in two minutes. He writes this number in a corner of the picture and then returns the picture to its owner. Then time the children for two more minutes, while each child counts the number of angles in his own picture. He should write in a corner of the picture the number of angles he found. The pair of children now compare their findings. They can work together counting the angles and resolving disagreements.

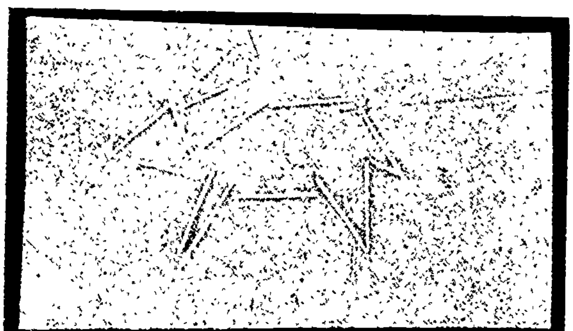
Some child may say this figure determines two different angles, while his partner may see three different angles. (The angles are indicated by the arrows.)

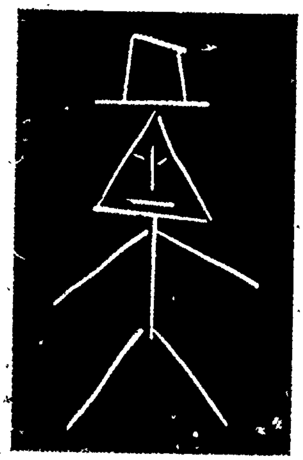


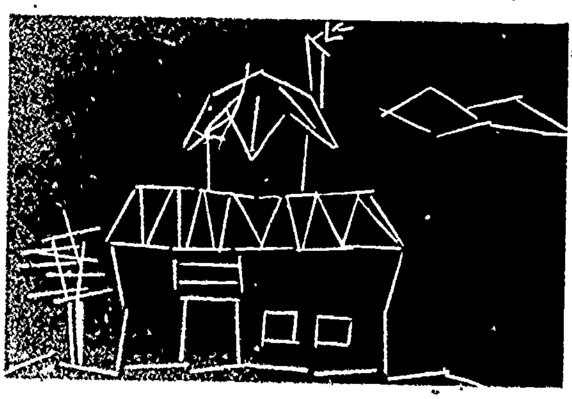
When using a diagram such as the one above, you may want to explain that even though angles are rays, we often leave off the little arrows (shortcut notation).

Version II (Optional)

Have the children make two dimensional figures or three dimensional sculptures with toothpicks and Elmer's glue. Then they should count the angles as they did in Version I.







Lesson 4: NAMING ANGLES

In this lesson the children continue to work with angles and develop a method for naming angles.

MATERIALS

- Worksheets 9 and 10

PROCEDURE

Activity A

Begin this activity with a discussion like the following.

HOW DID WE NAME A LINE? (We named two points on it. $\overleftrightarrow{AB}$ Line AB.)

HOW DID WE NAME A LINE SEGMENT? (We named its end points. $\overline{AB}$ Line segment AB.)

HOW DID WE NAME A RAY? (We named its end point first and then any other point. $\overrightarrow{AB}$ Ray AB.)

You may find it necessary to quickly review the procedure used for naming lines, line segments and rays. Emphasize that the origin of a ray is named first; the order is important.

COULD WE USE TWO POINTS TO NAME AN ANGLE? (Let the children speculate, and then try it.)

Example 1

In Example 1, naming two points, or even one point, does not present any problems; we can tell which angle is which.

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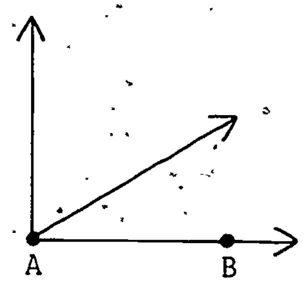
Example 2

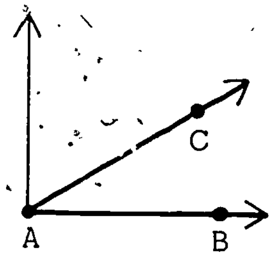


WHICH ANGLE IS ANGLE AB?

Outline two of the three angles with colored chalk. Then ask:

> BOTH ANGLES HAVE RAY AB AS ONE OF THEIR SIDES. HOW CAN WE SHOW WE ARE TALKING ONLY ABOUT THIS ANGLE? (Point to the smaller of the two angles you outlined.)

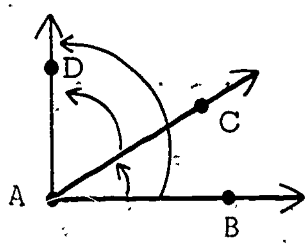
Lead the children to the realization that they need to name three points instead of two.



> NOW WE CAN CALL OUR ANGLE, "ANGLE CAB." OR SHOULD IT BE ABC, OR BAC, OR BCA?

Tell the class that mathematicians had this same problem at one time; so, they agreed always to have as the middle letter of the name of the angle the letter at the vertex of the angle. Thus, this angle can be called Angle CAB or Angle BAC; either way is correct, just so the middle letter is always the letter at the vertex of the angle. Introduce the word "vertex" to the children at this point.

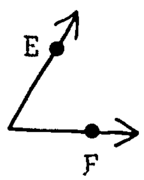
Ask the class how they can name the larger angle you outlined earlier. They will have to name another point on the other ray, e.g., point D.



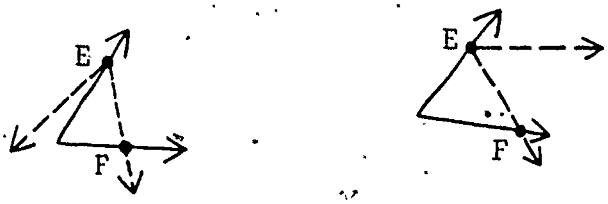
Now have someone name all three angles as you outline them with chalk or your finger.

Angle DAC or CAD
Angle CAB or BAC
Angle DAB or BAD

You may want to discuss whether or not two points would be adequate to name an angle if they were located as follows:



If another angle can be drawn through these two points, then confusion would arise as to which one is angle EF. Have the children try to draw another angle through points E and F.

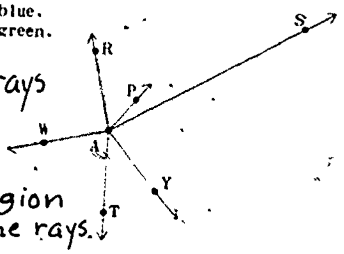


Have the children complete Worksheets 9 and 10. Remind them that an angle is made up of the rays only.

Worksheet 9
Unit 21

Name ____________

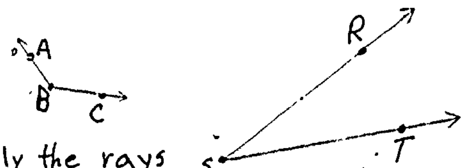
Color angle RAS red.
Color angle PAT blue.
Color angle WAY green.



only the rays should be colored, not the region between the rays.

Color the larger angle red and label it so its name is angle ABC.

Color the smaller angle blue and label it so its name is angle RST.

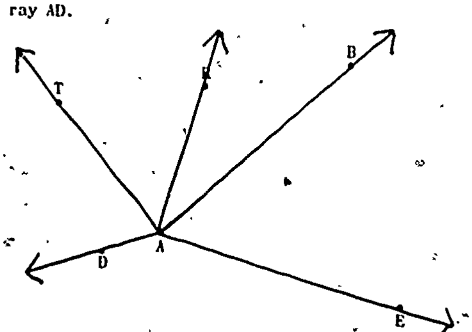


Only the rays should be colored, not the region between the rays.

Worksheet 10
Unit 21

Name ____________

Draw ray AB.
Draw $\overrightarrow{AE}$.
Draw ray AD.
Draw $\overrightarrow{AR}$.
Draw $\overrightarrow{AT}$.



Color angle DAT blue.
Color angle EAR green.
Color angle BAE yellow.

Only the rays should be colored, not the region between the rays.

What is the name of the ray that you colored that is part of two angles? ray AE

What is the vertex of all of these angles? point A

Lesson 5: FINDING THE MAG OF AN ANGLE

The word "mag" is introduced for the measure of an angle. We coined this word for the MINNEMAST curriculum because there is no single adequate term in mathematics to distinguish the concept of an angle (a union of two rays having a common endpoint) from the measure or size of an angle. This deficiency has caused endless confusion, even in teaching high school geometry, and we hope that the introduction of an explicit word will help clear up the ambiguity. You need only recall that an angle (a special set of points) is distinct from its measure or "mag" just as a line segment (a different special set of points) is distinct from its measure or length. The children come to see the need for a way to compare angles other than by superposition and they are introduced to the "clock" protractor as a device for finding the mag of an angle.

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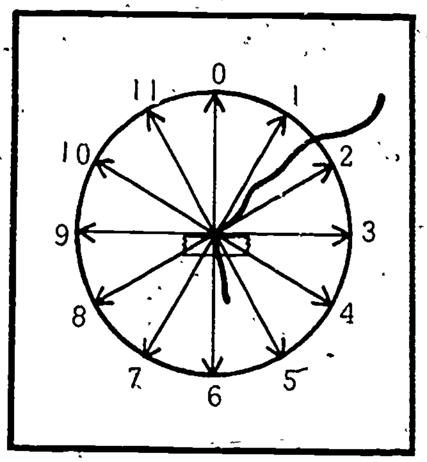
MATERIALS

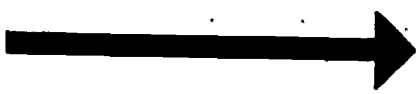
- pipe cleaners
- ruler or yardstick
- overhead projector
- transparency of "clock" protractor (provided with printed materials; printed original also included in Appendix)
- 6" piece of yarn
- transparent tape
- ray cutout
- clear plastic "clock" protractors, 1 per child (provided with printed materials; printed original also included in Appendix)
- 1 or 2 blank transparent sheets
- Worksheets 11 and 12

PREPARATION

Before teaching Activity B, assemble the demonstration clock transparency. Tape the 6" piece of yarn to the middle of the clock protractor so that it can swing freely all the way around the clock face.



To make a ray cutout, cut a ray shape from a sheet of paper. Make it 8" to 10" long and $\frac{1}{4}$" to $\frac{1}{2}$" wide, with an arrowhead at one end.



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PROCEDURE

Activity A

Discuss the following questions with the children:

WHEN YOU STUDIED MEASUREMENT BEFORE, WHAT DID YOU MEASURE WHEN YOU MEASURED A LINE SEGMENT? (Its length.)

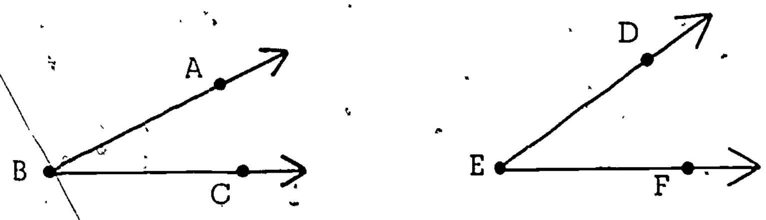
WHEN YOU STAND ON A SCALE, WHAT ARE YOU MEASURING? (Your weight.)

WHEN YOU MEASURE THE SIZE OF A REGION, YOU FIND ITS AREA.

NOW, IF WE WANT TO MEASURE AN ANGLE, WHAT WOULD WE MEASURE? ... LENGTH? ... WEIGHT? ... AREA? WE NEED A WORD FOR THE MEASURE OF AN ANGLE. THIS NEW WORD IS "MAG."

WHEN WE TALK ABOUT LINE SEGMENTS, WE SAY WE ARE MEASURING THEIR LENGTH. NOW, WHEN WE TALK ABOUT ANGLES, WE'LL SAY WE ARE MEASURING THEIR "MAG." WHEN WE WORKED WITH THE PIPE CLEANER ANGLES THE ONE WHICH EXTENDED FARTHER TO THE LEFT HAD THE GREATER MAG.

Draw two angles on the chalkboard that are nearly the same in measure.



Then ask:

WHICH ONE OF THESE ANGLES HAS THE GREATER MAG?

There will probably be disagreement. Have the children vote for the angle they think is the larger. Write their choice on the board.

Ask:

> CAN ANYONE THINK OF A WAY HE CAN SHOW THE CLASS WHICH ANGLE HAS THE GREATER MAG? (Let them speculate.)

If necessary, remind the children of the method they used when they put the pipe cleaner angles in order. (Superposing one angle on another.) Tell them we can't cut out one chalkboard angle and place it on top of the other. At this point someone will probably suggest bending a pipe cleaner to match one of the angles, and then placing the pipe cleaner angle over the other angle. Do this. Tell the class this is one method for comparing the mags of these angles. But what would they do if they didn't have any pipe cleaners? Suggest that there must be another method of comparing the mags of these angles.

> WE NEED SOMETHING TO MEASURE THE MAG OF AN ANGLE. WHAT DO WE USE TO MEASURE THE WEIGHT OF SOMETHING? (A scale.) WHAT DO WE USE TO MEASURE TIME INTERVALS? (A pendulum, water clock, watch, clock.)
>
> WHAT DO WE USE TO MEASURE THE LENGTH OF A LINE SEGMENT? (A yardstick, ruler, etc.)
>
> COULD WE USE A RULER TO MEASURE THE MAG OF AN ANGLE? (Answers will vary.)

Let someone come up and try to measure the mag of the angles you have drawn on the chalkboard. The direction your class takes at this point should be interesting. Let different children try their ideas using a ruler. The following are some possibilities you should watch for.

1. A child may try to measure the length of the sides. Someone will probably remind him that the sides go

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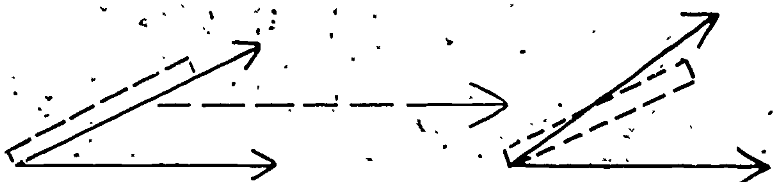
on and on, so the length of the sides has nothing to do with the mag of the angle.

2. He may try to place the ruler on one ray of an angle and move it over to a corresponding ray on the other angle.



If one ray of one angle is on the same line as a ray of the other angle, as in the diagram, and if the child is able to hold the ruler at a set angle while moving it over, he will be able to find out which angle is bigger, but he will still not be able to actually measure the angles.

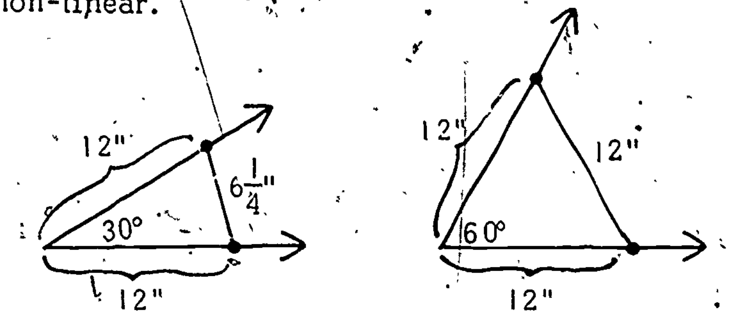
3. Some very alert child may come up with the following method, which does happen to be a method of measuring the mag of an angle. Unfortunately, problems soon develop. But first, the method he may use.

He might measure off the same length on all the rays.

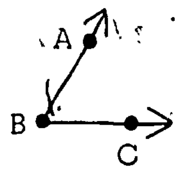
Then he might draw straight line segments joining the two indicated points on both angles.

If he measures these two line segments, the angle with the longer line segment will have the greater mag.

The main reason we don't use this method for measuring angles is that it is not linear. For example, if we used this method for a 30 degree angle, we would expect the length of the line segment for a 60 degree angle to double. It does not. The relationship is non-linear.

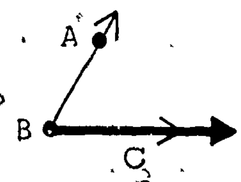


After discussing the ruler as a device for measuring the mag. of an angle, tell the class you have an idea you want to try. Draw an angle (ABC) on the chalkboard.

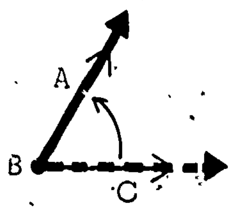


Pick up the ray cutout and say:

IF I PLACED THIS RAY CUTOUT ON TOP OF RAY BC (do so),

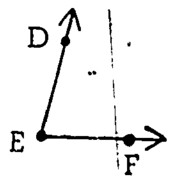


AND ROTATED IT TO RAY BA (do so),

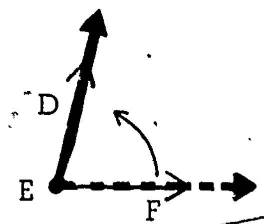


I COULD TALK ABOUT HOW MUCH OR HOW FAR I ROTATED (TURNED) IT.

Draw a larger angle (DEF) on the board.



Using the ray cutout, rotate ray EF into ray ED.



Then ask:

IN WHICH ANGLE DID I ROTATE MORE? ANGLE ABC OR ANGLE DEF? (If necessary, rotate angle ABC again.)

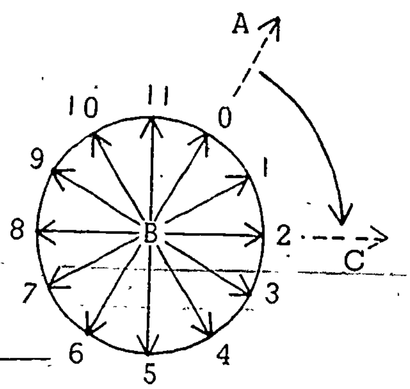
WE CAN SAY THAT I ROTATED MORE IN ANGLE DEF THAN IN ANGLE ABC. BUT CAN WE TELL HOW MUCH MORE?

DO YOU SEE ANYTHING IN THIS ROOM THAT HAS SOMETHING THAT ROTATES AND ALSO HAS NUMERALS? (The children should readily notice the classroom clock.)

Activity B

Show your class the demonstration clock you prepared earlier. Ask them how it is different from the classroom clock. (Zero instead of 12, only one hand, rays drawn out to all the numerals, it's transparent, etc.)

Draw a two hour angle (ABC) on a transparent sheet. Then ask the children if anyone can think of a way to use the demonstration clock to measure the mag of this angle. Let various children try their ideas. One method is to put the demonstration clock transparency over the transparent sheet with angle ABC so the zero ray is over one of the rays of the angle (ray BA). Put the "hour hand" (the yarn) at zero and rotate it into the other ray (ray BC).



Ask:

HOW FAR DID I ROTATE? (2 hours or 2 spaces.)

WHAT COULD WE SAY THE MAG OF THIS ANGLE IS? (2 hours or 2 spaces.)

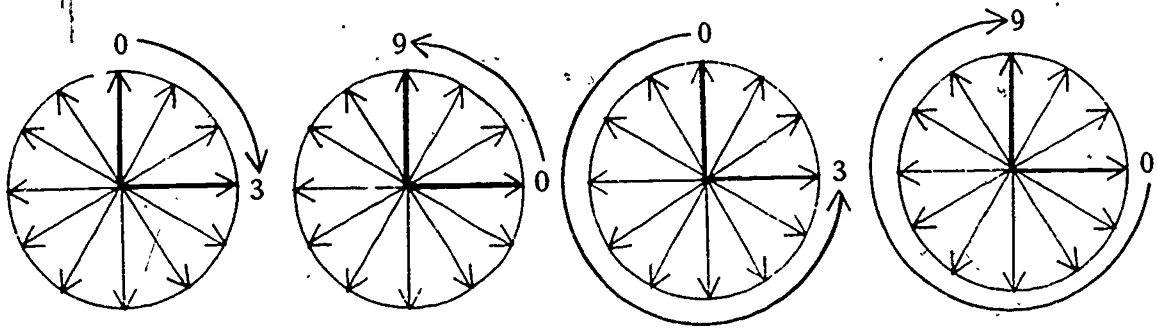
Draw other angles on the transparent sheet and have different children use this procedure to measure them. If the measurement comes between two of the hour marks, you could call the angle a $2\frac{1}{2}$ hour angle, a 2:30 angle, or say the mag of the angle is between 2 and 3.

If no one asks, raise the following questions:

DOES IT MAKE ANY DIFFERENCE ON WHICH RAY OF THE ANGLE WE START? DOES IT MAKE ANY DIFFERENCE IN WHICH DIRECTION WE ROTATE? (Let the children speculate.)

Draw a 3 o'clock angle on a transparent sheet. Place the zero on either ray, and rotate in different directions.

Examples



OUR ANGLE MEASURED AS A 3 HOUR ANGLE OR 9 HOUR ANGLE. WHICH MEASUREMENT IS CORRECT OR ARE THEY BOTH CORRECT? (Let the children speculate.)

Then ask:

DID WE CHANGE THE ANGLE ITSELF IN ANY WAY? (No.)

You may want to change the position of the transparency and measure the same angle again.

Ask the class if they always have to start at zero to measure this angle. Have someone measure the angle with the clock starting at 2 or 4 on one ray. They should count the "hours" they rotated to get to the other ray of the angle. Again, they will get 3 hours or 9 hours as their measurement.

NOTE: At this point you can see the advantage of separating the measure (mag) of an angle from the concept of an angle (union of two rays). The old confusion about which is the angle, or whether we use "interior" or "exterior" angles, now disappears. An angle is unique and is simply the set of all points on two rays from a common origin. Confusion arises only when we attempt to assign a size of measure to the set of points we call an angle. So there is no confusion about the definition of an angle, but there is some choice in how we measure it. We simply choose the most convenient method to suit our purposes.

Discuss with the class that there are many measures of an angle's mag, and that these are two of them (3 hour and 9 hour). Tell them that in order to avoid confusion, we must decide which measure to use. Tell the children that an angle can be measured either way, but unless there is a special reason to do otherwise, we usually use the smaller reading.

IF WE ALWAYS USE THE SMALLER MEASUREMENT OF AN ANGLE, WE WILL ONLY HAVE TO USE THE HALF OF THE CLOCK LABELED 0 THROUGH 6.

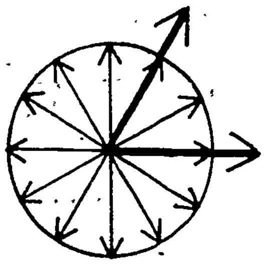
CAN ANYONE MAKE AN ANGLE THAT WE COULD NOT MEASURE WITH JUST HALF OUR CLOCK? THE ANGLE WOULD HAVE TO BE GREATER THAN A 6 O'CLOCK ANGLE.

At this point many children will believe they can. Let them try, using pipe cleaners, etc. Each time, measure their angles with the demonstration clock. A 6 o'clock angle is the largest angle they will be able to make. As soon as they bend a pipe cleaner it becomes less than a 6 o'clock angle. When you feel the class is convinced that they can measure any angle using only half of the clock, ask them if they would like to have some small clocks of their own with which to measure the mag of some angles.

Summary

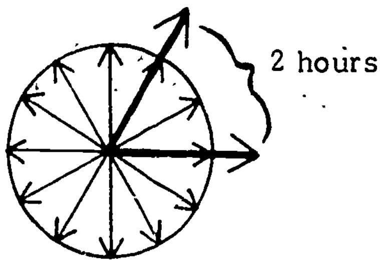
You may want to discuss and demonstrate this simplified method of measuring the mag of an angle with your class before they do Worksheets 11 and 12.

Step 1. Place the center of the clock on the vertex of the angle.

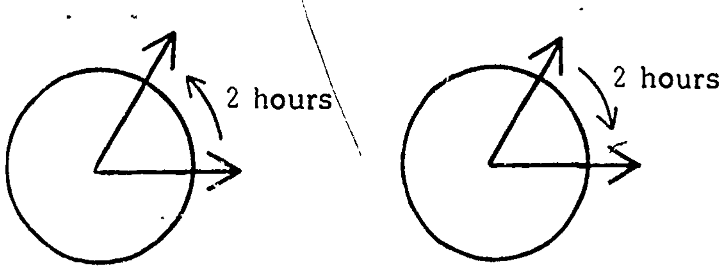


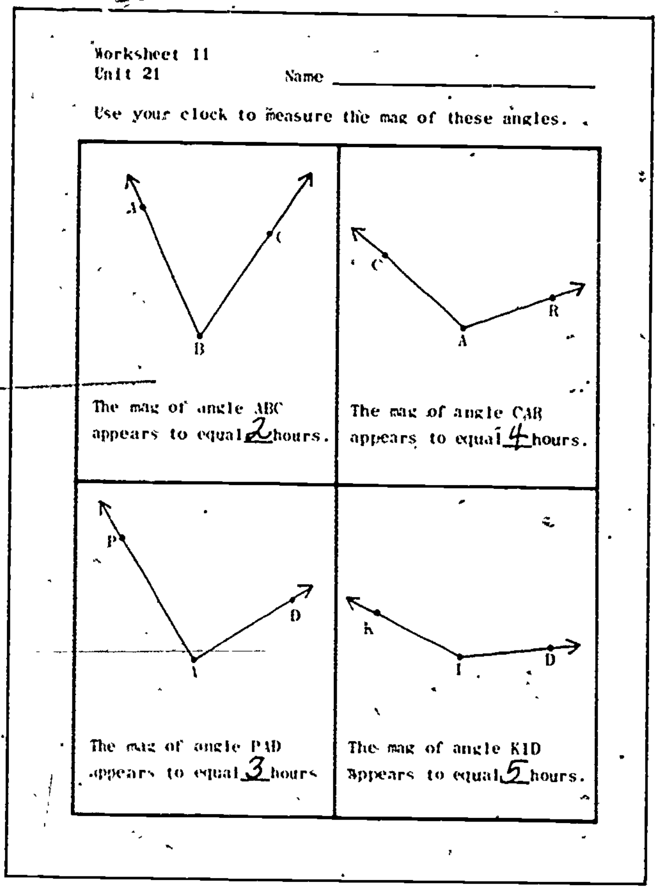
Step 2. Line up one of the clock's rays with one of the rays of the angle.

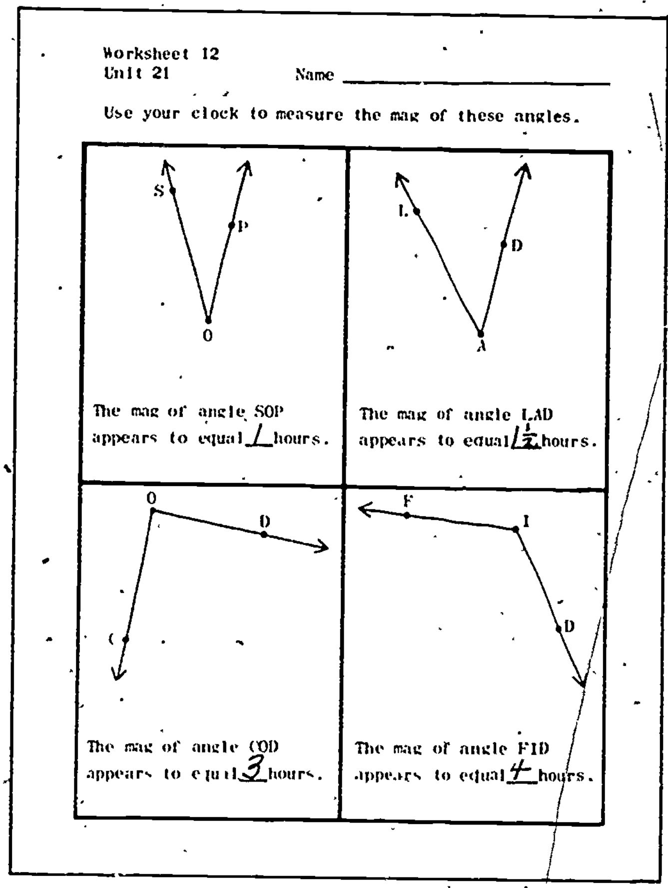
Step 3. Count the hours on the clock between the angle's two rays.



Show the children that the 2 hour mag measurement found in this example could be counted in either direction, clockwise or counterclockwise.





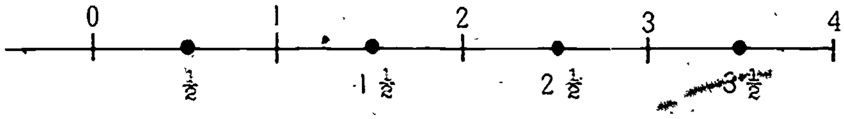


Activity C

Give each child a clear plastic clock protractor. Worksheets 11 and 12 give the children practice in measuring the mags of angles with their clock protractors. We have given only the small measure of the angles on your copy of the worksheets; however, if the children give the larger measure, it is also correct.

You will notice that on Worksheets 11 and 12, the words "appears to equal ___ hours" are used. Remind the children that this language is used because measurements cannot be made with complete accuracy, even with equipment more precise than their clock protractors.

After the children have completed the two worksheets, discuss their answers. Angle LAD on Worksheet 12 should be of particular interest. Let the children figure out this measurement on the chalkboard and then discuss the names of the points halfway between one and two, two and three, etc.



You could also discuss the relation of $1\frac{1}{2}$ to 1:30 on a regular clock.

Have the children save their clock protractors. They will be used in the following lessons.

Lesson 6: FINDING AND MEASURING ANGLES

In this lesson the children use their clock protractors to measure angles that they find in nature. Sets of worksheets showing different natural objects are provided for Activity A. The first set, Worksheets 13 and 14, shows the angles formed by trees and buildings and the slope of hills. The second set, Worksheets 15, 16 and 17, focuses on the angles formed by the joining of the branches of trees to the trunks of the trees. The children should also notice the angles formed by the vein structures of the leaves. The third set, Worksheets 18 through 21, shows the angles formed by the skeletal structures of animals. The children will be asked to draw line segments on pictures of a dinosaur, a frog, and a grasshopper to show the angles that are formed by the joints. This set shows animals in a static position, but since angles formed by joints change as the animal moves, Worksheet 21 is a sequence of pictures showing a dog running. The children should notice how the angles formed by the dog's legs change.

In Activities B and C, the children are given an opportunity to visualize the changing angles formed by parts of their own bodies. An "angle-finder" is used to find angles almost anywhere -- on the playground, in the classroom, on a field trip, at home, etc.

MATERIALS

- 15 angle-finders
- clock protractors, 1 per child
- rulers, 1 per child
- 1 set of colored pencils
- radish seedling set-up (optional; see page 20)
- Worksheets 13-22

PROCEDURE

Activity A

Discuss with the children the idea of being able to see angles in the classroom. Have the children trace angles they can find -- desk corner, chalkboard corner, etc. Ask the class if they think they could find angles anywhere else, such as on the playground or on the way home from school.

Tell the children that today they are going to practice finding angles by looking at some pictures in their workbooks. If they want to compare the mag of two angles they find, how will they do it? (They should remember their "clocks" from the last lesson.)

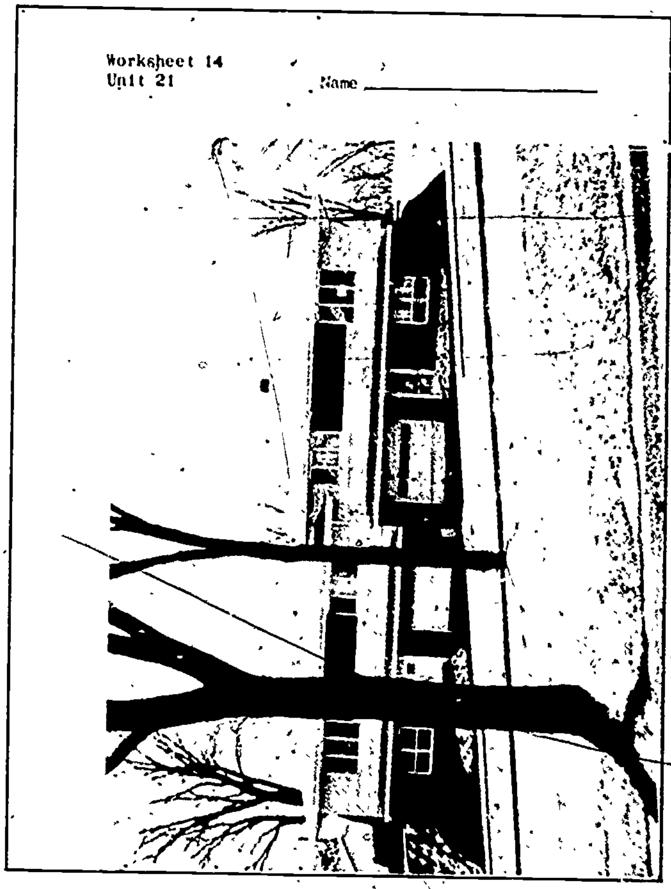
Have the children turn to Worksheet 13. Tell them that some of the angles in this picture have already been found for them and have been marked on the overlay. Have them look only at the trees that have their angles drawn on the overlay and ask:

> DO ALL THESE TREES FORM THE SAME ANGLE WITH THE HILL, OR DIFFERENT ANGLES? HOW CAN WE FIND OUT? (Measure the angles with our "clocks.")

Have them write the measurement by each angle on the overlay. All the angles should measure approximately the same mag.

Challenge the children to find angles in the picture other than those already marked. Have them use a ruler and colored pencils to draw in the angles, either on the overlay or directly on the picture. This activity continues on Worksheet 14.

After the children have had a chance to find angles, you



may want to discuss with them questions like those below.

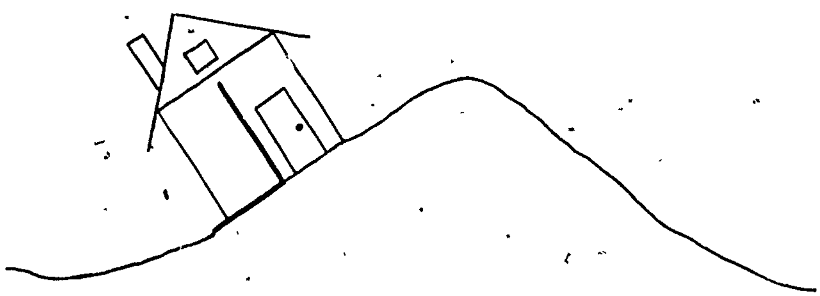
WHAT WAS THE MAG OF THE SMALLEST ANGLE YOU FOUND ON WORKSHEET 13? ON WORKSHEET 14?

DID YOU FIND ANY 3 HOUR ANGLES ON WORKSHEET 13? ON WORKSHEET 14? WE CALL A 3 HOUR ANGLE A RIGHT ANGLE.

ON WORKSHEET 14, DID THE HOUSE AND THE TREE FORM THE SAME ANGLE WITH THE HILL, OR DIFFERENT ANGLES?

DID THE HOUSE AND THE HILL FORM A RIGHT (OR 3 HOUR) ANGLE?

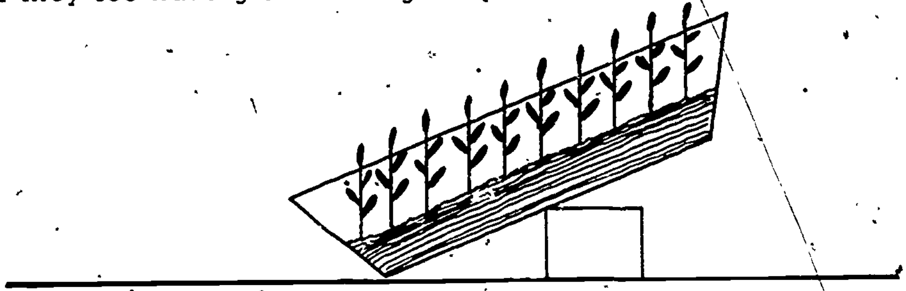
Draw on the chalkboard a sketch of a building that is at a right angle to the hill on which it is built.



Ask the children if they can think of any problems they might have if they lived in a building that looked like this. (Furniture would slide downhill, dishes wouldn't stay on the table, etc.)

The class should notice that on Worksheets 13 and 14, both the trees and the building go straight up from the hill.

At this time display and discuss the radish seedlings that have been grown in a tilted box. The children should see that they too have grown straight up.



When the children were finding angles on Worksheets 13 and 14, many of them probably found angles formed by the branches of the trees. Have the children turn to Worksheets 15, 16 and 17, which show pictures of six trees. Discuss with them the following questions:

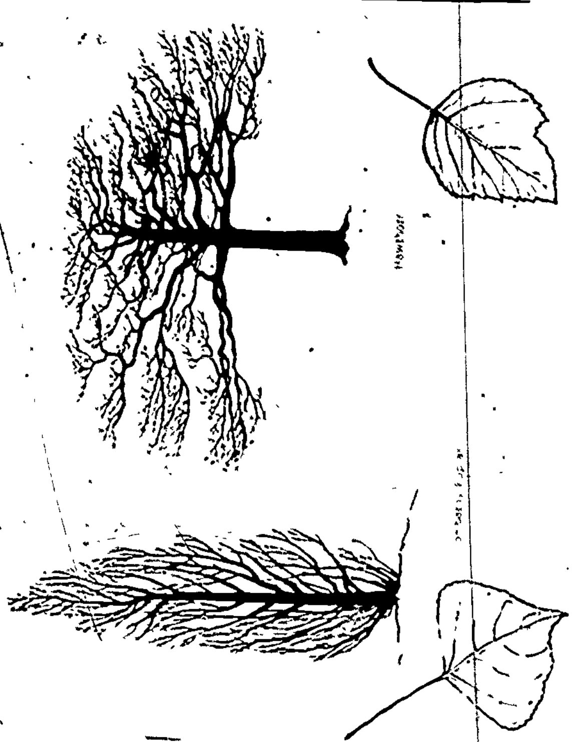
ARE ALL TREES THE SAME? (No. Have them name a few common trees.)

HOW CAN WE TELL ONE KIND OF TREE FROM ANOTHER KIND OF TREE? (Look at its leaves, its bark, its size, etc.)

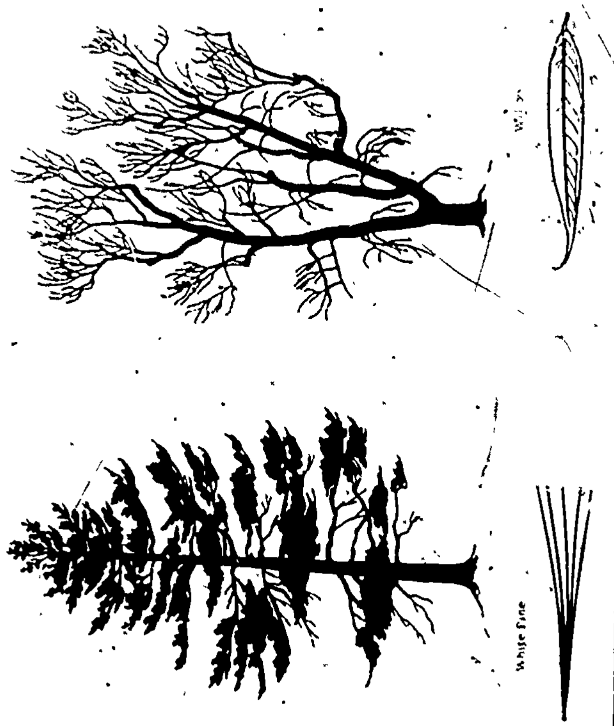
Have the children tear out Worksheets 15-17. Ask if they can think of other ways to tell one tree from another. If no one suggests looking at the angles formed by the branches and the trees, lead to this idea. Discuss how some of the trees are alike and how they are different. However, do not spend too much time on the discussion or on these worksheets. Some children may want to measure the angles formed by the branches to the trunks of the trees, finding the trees that have the greatest and smallest angles. They might also notice the angles formed by the vein structures of the leaves.

Worksheet 18 shows a picture of a daddy longlegs and Worksheet 19 shows a picture of a bat skeleton. The children should notice the angles formed by the joints and make comparisons of the various angles they find.

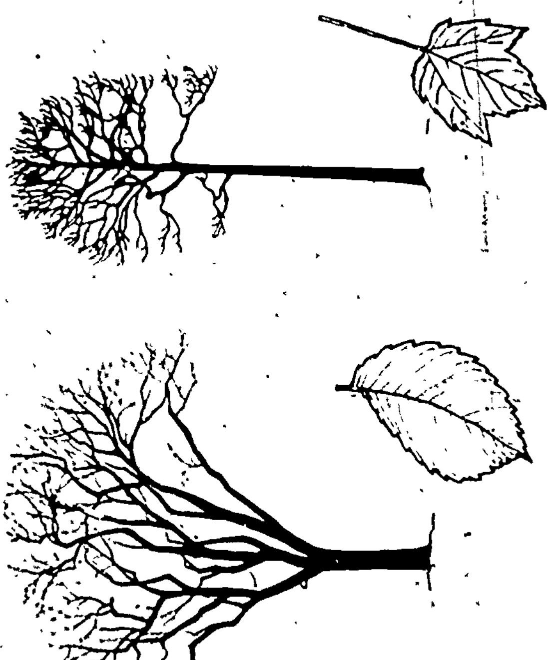
Worksheet 15
Unit 21 Name ____________________



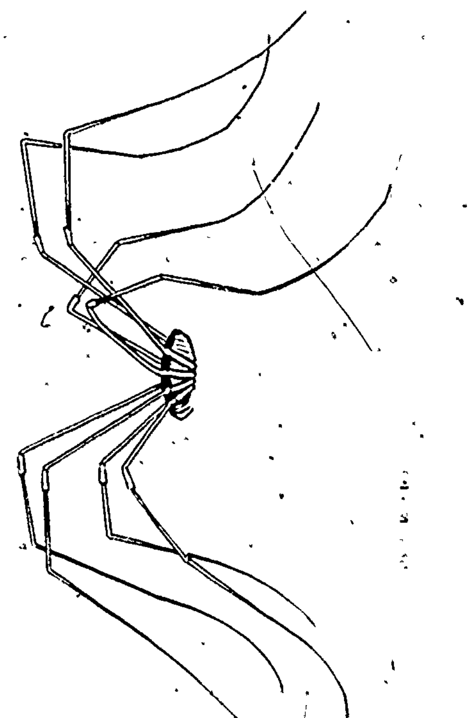
Worksheet 16
Unit 21 Name ____________________

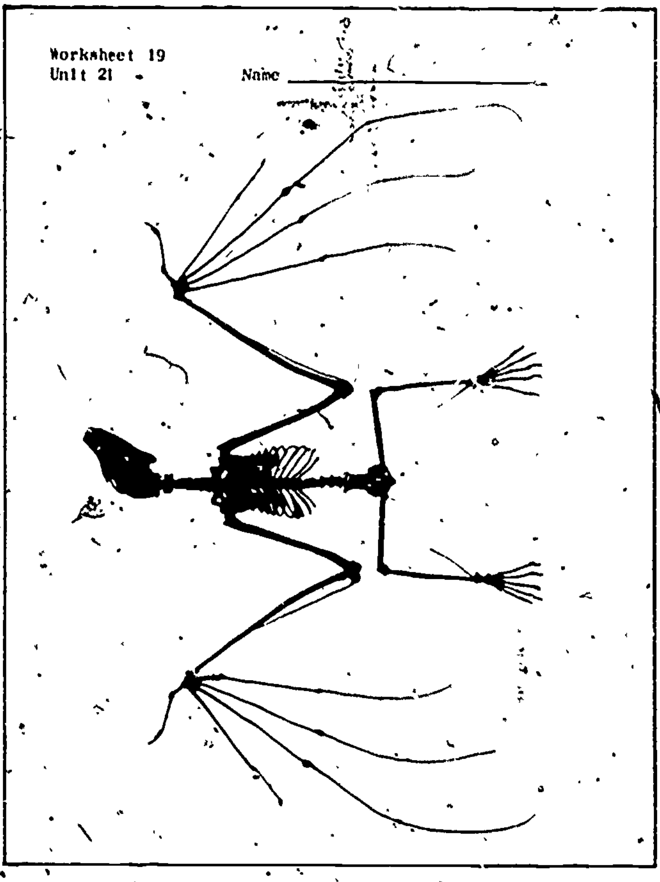


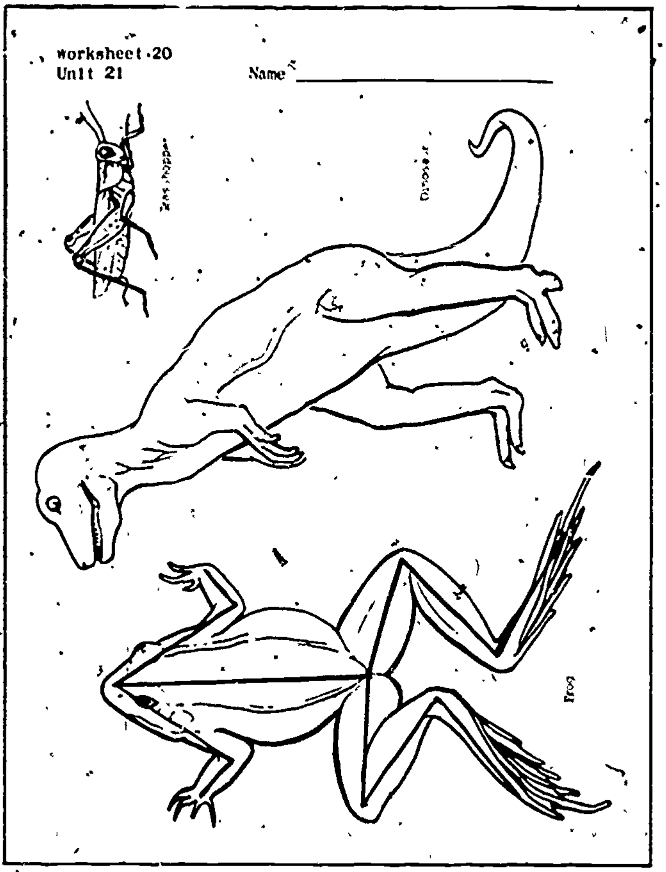
Worksheet 17
Unit 21 Name ____________________



Worksheet 18
Unit 21 Name ____________________







On Worksheet 20, which shows drawings of a frog, a dinosaur and a grasshopper, the angles formed by the joints are not as evident as the angles on Worksheets 18 and 19. Discuss with the children how they could better show the angles formed by the joints. One possible solution is to draw straight line segments on the figures to represent the skeletal structure. Have the children do this with rulers and colored pencils. (For an example, see frog on reduced copy of Worksheet 20.)

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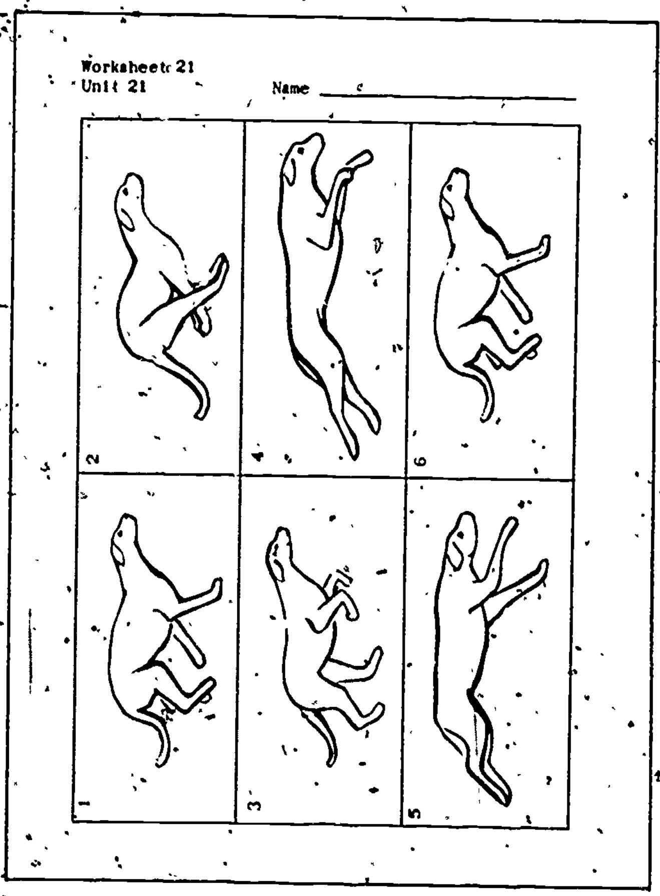
After the children have completed Worksheet 20, ask them:

DO THE FROG'S JOINTS ALWAYS FORM THE SAME ANGLES, OR DO THE ANGLES CHANGE?



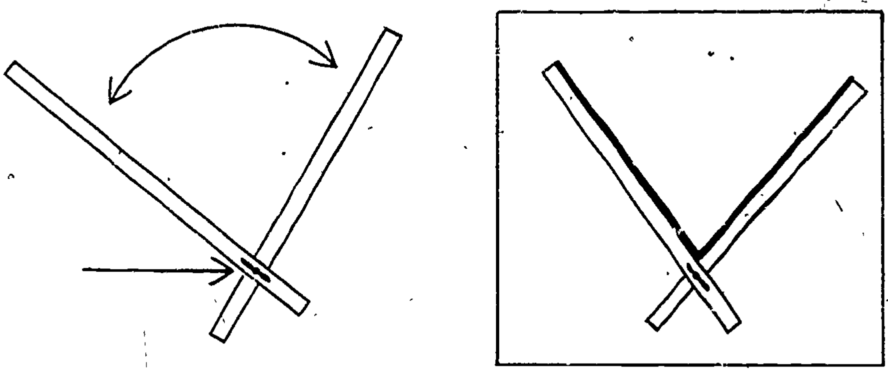
The children should realize that this drawing was made of the frog in a certain position; if the frog moved, the drawing would be different and the angles would change.

Tell the children that Worksheet 21 shows examples of changing angles formed when a dog runs. Discuss the sequence of the drawings and ask the children to point out which drawings are alike. (1 and 6.) Ask them to predict what drawing 7 would look like and why. (Drawing 2. The sequence repeats itself.)

Ask the children to compare the angles formed by the dog's back legs as he runs. What are the smallest and largest angles the joints make? They could draw line segments to show the bone structure, and then measure the angles with their "clocks."

Activity B

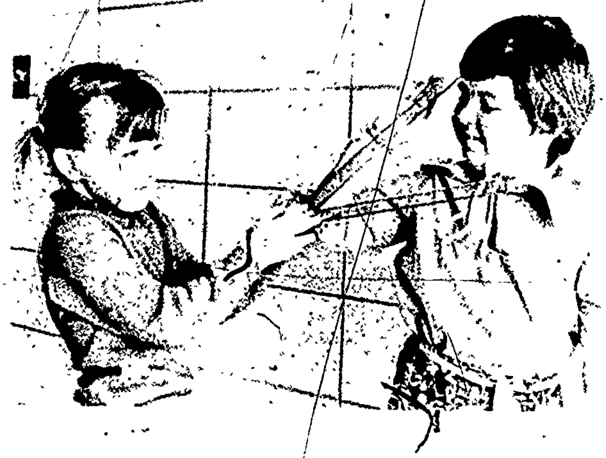
Ask the children if they would like to find out what are the smallest and largest angles that can be formed by their bodies' joints. Discuss with the children how they could do this since they don't have drawings of themselves running or moving. After the children have had a chance to make suggestions and have tried out their methods, show them the "angle-finder." Demonstrate how the angle-finder works. Unscrew the wing-bolt, change the angle, tighten the bolt, and trace the angle formed on a piece of paper or the chalk-board.



Ask the children if they can think of a way this angle-finder could help them find the smallest and largest angles formed by their bodies' joints. Have various children demonstrate their ideas to the class, using the angle-finder. One possible method is as follows:

A child holds his arm straight. Open the angle-finder wide enough so that its angle matches the angle formed by the child's elbow.

Now have the child "close" his arm as much as he can. Again, place the angle-finder over the elbow, matching the angle formed by the child's elbow. Trace the angle and measure it.



Worksheet 22
Unit 21

Name ____________

Answers will vary

	Mag of Smallest Angle	Mag of Largest Angle
Elbow	$1\frac{1}{2}$ hrs.	$5\frac{1}{2}$ hrs.
Knee	$2\frac{1}{2}$ hrs.	6 hrs
Ankle	$2\frac{1}{2}$ hrs	5 hrs.
Wrist	3 hrs	6 hrs

Ask the children if they think everyone in the class will form the same angle with their elbows. How could they find out? Measure everyone's elbow "angles" and then compare the findings.

Divide the class into pairs and give each pair an angle-finder. Each child should tear out Worksheet 22 from his workbook. They are to work together, finding the smallest and largest angles formed by each child's elbow, and record their findings on the worksheet. When they are through finding the mag of the angle formed by their elbow joints, they can find the smallest and largest angles formed by their knees, ankles and wrists.

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If a group of children finishes early, they may want to find angles formed by other joints of their bodies not listed on the worksheet.

After the children have finished finding the mag of their bodies' angles, discuss their findings. You could make a classroom chart showing the range they found for each joint. Then they could underline or circle with one color the most common measure found for each joint, and the least common measure with another color.

The children should feel free to use the angle-finder to find angles anywhere. Some may want to take an angle-finder home to find more angles. If their interest is high, you may want to take them on a nature hike to find angles around the school.

Activity C

If the children are interested in observing changing angles, they could use their angle-finders and clock protractors to measure the angles formed by parts of their bodies when they are sitting down and when they are standing. Other simple movements they could perform include hopping, walking up or down some stairs, and bowing. They could also play "Statue" and measure the different angles that are formed.

Lesson 7: LIGHT BEAMS

In this lesson, light beams are used as physical embodiments of geometric rays. Beams of light are like geometric rays in some ways -- they have their origin at the light source and they extend away from their origin in a certain direction. (Even though light from stars travels trillions of miles through space, light beams do not extend endlessly in one direction, as geometric rays do. Also, geometric rays are abstractions that have no "thickness." Light beams do have "thickness.") Angles are generated using light beams and the children discover interesting relations between geometry and light.

MATERIALS

- filmstrip projector or slide projector
- 1 sheet of black construction paper
- 14-foot piece of yarn
- dusty chalkboard erasers
- masking tape
- 30 mirrors on blocks
- 5 or 6 flashlights (bring these from home, or borrow from other teachers or the custodian)
- 1 or 2 angle-finders (from Lesson 6)

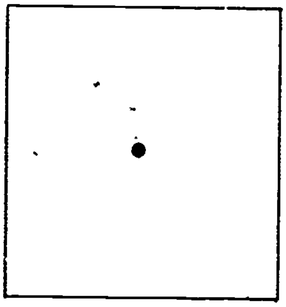
-- for each group of 6 or 8 children --

- rubber playground ball
- plastic bowling pin
- sheet of 6" x 9" paper

PREPARATION

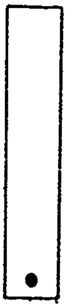
For this lesson you will be using a slide projector or a filmstrip projector as your main source of light. In order to have a concentrated beam of light, use one of the following devices in your projector.

1. Slide projector:



This slide is made from a 2" x 2" piece of black paper. Poke a small hole in the middle of the paper with a pin. Put this slide in your projector. When the projector is turned on and pointed toward the wall, only a small spot of light should be seen. Begin Activity A.

2. Filmstrip projector:

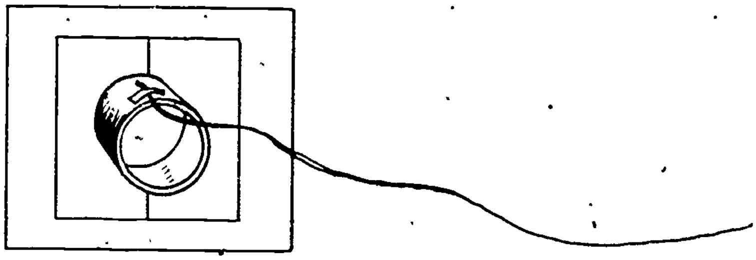


This strip is made from a 1 $\frac{1}{8}$" x 6" piece of black paper. Poke a small hole in the center, about 1" from the bottom. Put this strip of paper into the projector as if it were a filmstrip. The small hole should be positioned so that light will show through it onto the wall. Begin Activity A.

PROCEDURE

Activity A

Tape one end of the 14-foot piece of yarn to the front lens of the projector.



Darken the room. Turn on the projector and point it toward someone (Tom) in the room. Ask another child to describe what he sees happening. (Light from the projector is shining on Tom.) Ask:

DOES THE LIGHT SHINE JUST HERE (point to lens of projector) AND ON TOM?

Have someone hold his hand in front of the lens of the projector and then move slowly toward Tom, keeping his hand in the beam of light. The class should see that there is a continuous beam of light from the projector to Tom. To illustrate this further, have a child clap two dusty erasers together while following the beam of light.

Have Tom take the other end of the yarn that is taped to the projector and hold it taut. Tell the class that the yarn represents the beam of light. You may want to point the projector at a few other children, letting them hold the yarn, and have someone else clap dusty erasers to make the light beam visible.

Turn on the classroom lights and direct the children's attention to the chalkboard. Tell the class you are going to draw a representation of the light beam on the chalkboard. Ask them to raise their hands when they notice something familiar about your drawing. Draw the following representation on the chalkboard.

Use chalk of a different color to represent the light "ray."

The class should be able to see the similarity between a geometric ray and the beam of light. Discuss with the class how they are alike and how they are different. (Both have an origin and extend away from it in one direction in a straight path; however, a beam of light has dimension and

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a geometric ray does not. A beam of light does not extend endlessly; a geometric ray does.) You should also discuss the relationships among the drawing on the board, the piece of yarn, and the beam of light. (They are all representations of a ray.)

Ask the class:

IS A "RAY" OF LIGHT ALWAYS STRAIGHT? (Let the children speculate.)

Give each child a mirror. Turn off the classroom lights, cover the windows and turn on the projector. It should be the only source of light in the room. This should be a time for free experimentation by the children. Have them hold their mirrors so that the light from the projector will shine on the mirrors. Keep changing the location of the projector. The children should reflect the light beam throughout the room with their mirrors. After a few minutes of experimentation, turn on the classroom lights and collect the mirrors.

Then ask:

IS A "RAY" OF LIGHT ALWAYS STRAIGHT? (No. It can be "bounced off" or reflected off mirrors.)

Discuss with the children whether they have ever seen other things that reflected rays of light. (Bodies of water, reflections on a lake, windows or glass surfaces, etc.)

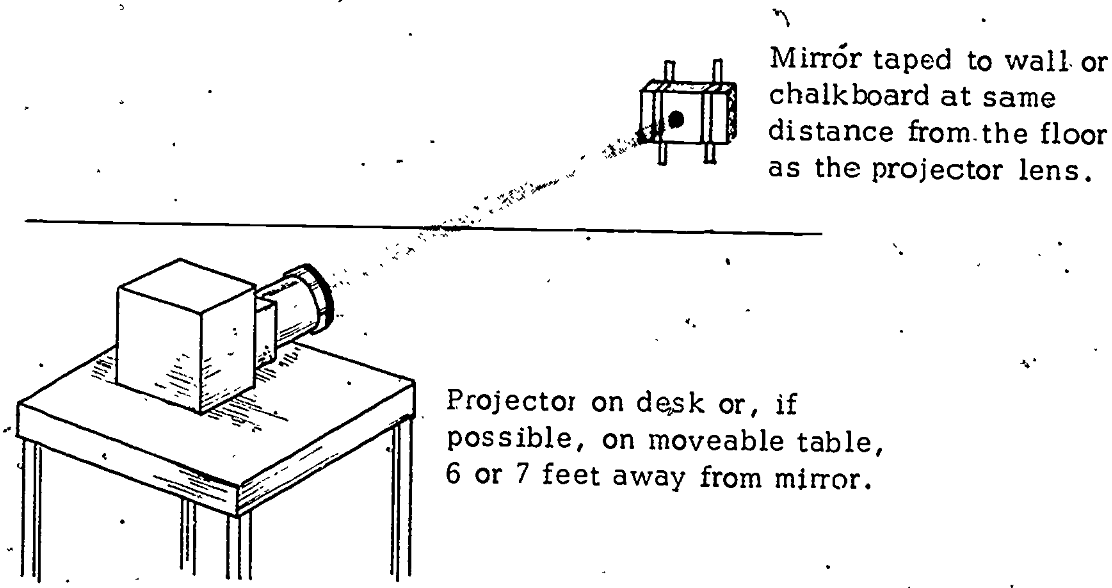
Bring out five or six flashlights. (The children could bring these from home or you could borrow some from the custodian, or from other teachers.) Ask the class if they would like to experiment some more with their mirrors and rays of light. Divide the class into groups, so that each group has a light source. One group could use the projector.

Seat each group on the floor somewhere in the room. Tell them to work as a team, using their mirrors. Each group will have a chance to report back to the class whatever interesting discoveries they make about reflected rays. Have each group turn on its flashlight, and then turn off the classroom lights.

The amount of time spent on this activity will depend on your class. When you feel they are ready, collect the mirrors and flashlights. Let each group come up to the front of the room and demonstrate something they discovered to the class. Have a demonstration set of equipment available for them to use.

Activity B

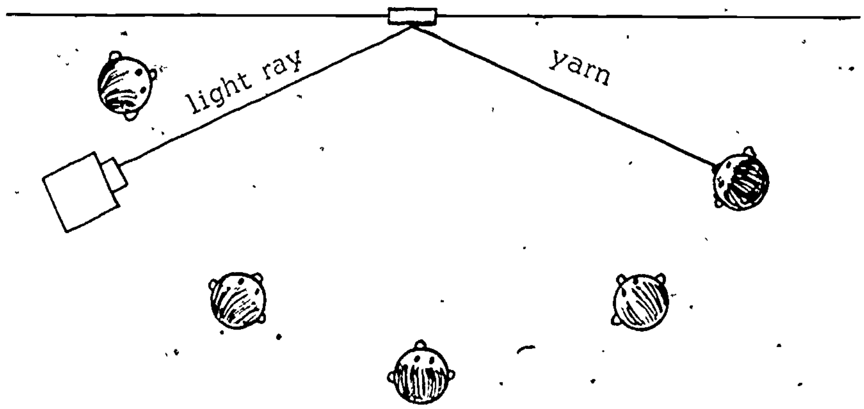
Set up a mirror and the projector in the following manner:



Before you begin, be sure to focus the spot of light on the mirror. Then turn off the classroom lights. Ask someone to describe the set-up to the class. (Projector on table, mirror taped to wall or chalkboard, spot of light on mirror.) Ask someone to clap dusty erasers to make the light ray visible. Ask the class what they could use to represent the light ray. They should remember the piece of yarn from Activity A. Tape the yarn to the projector again, extending it to the mirror. Holding the yarn taut, tape it to the mirror with transparent tape.

Turn the projector off and have five children come up and stand in a semicircle in front of the mirror. Make sure no one stands between the projector and the mirror.

Have the class predict who they think will get "hit" by the reflected beam of light. Then turn on the projector. The one who is "hit" by the light ray should hold the other end of the yarn, keeping it taut.



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Move the projector around the semicircle of children so the reflected light ray hits another person. That person should hold the yarn. The children should see that when the projector is moved, the reflected light ray also moves and hits someone else. Turn off the projector and move it to a different location. Ask the class who they think will be hit by the light now. Let them vote for their choice. Then turn on the projector, revealing the "victim." Do this a few more times.

Have the five children return to their desks. Ask the class if they would like to play a game called "Hit Me," using the light ray and mirror. Tell them only three people can play the game at one time, but the rest of the class should pay close attention and try to figure out what the trick is. The first person to figure out the trick will be the winner.

GAME: HIT ME

Use the same set-up that you used in Activity B. Be sure that the light spot is focused on the mirror. The object of the game is to "hit" someone in the stomach with a reflected light ray. The three players are:

#1 - Projector mover
#2 - Light ray duster
#3 - Person to be hit

Player 3 stands anywhere within the semicircle in front of the mirror, four to five feet away from the mirror.

Player 1 moves the projector to a position within 5 or 6 feet of the mirror where he thinks he will be able to reflect the light ray off the mirror and hit Player 3. Then he turns on the projector, making sure the light ray strikes the mirror.

Player 2 hands the other end of the yarn to Player 3. Then he claps the dusty erasers to make the beam of light visible.

The class should be able to see if Player 3 was hit by the light beam or, if he was missed, by how much.

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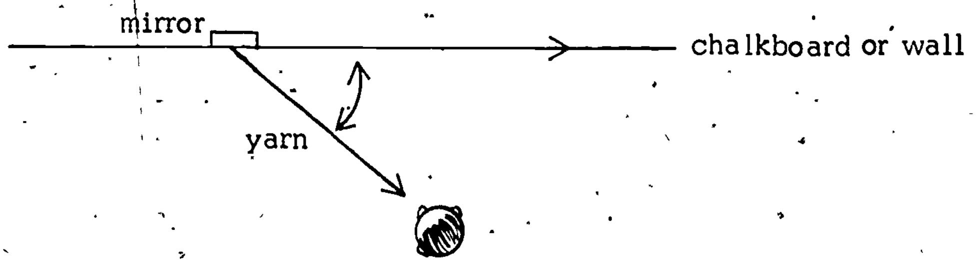
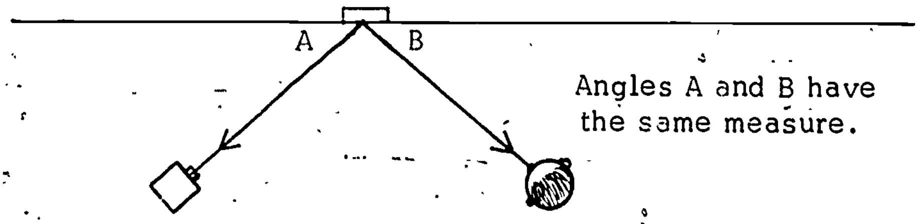
Let three other children play the game. Continue until someone in the room is able to figure out the strategy for hitting Player 3 every time. Have him demonstrate his strategy to the class. The strategy is as follows:

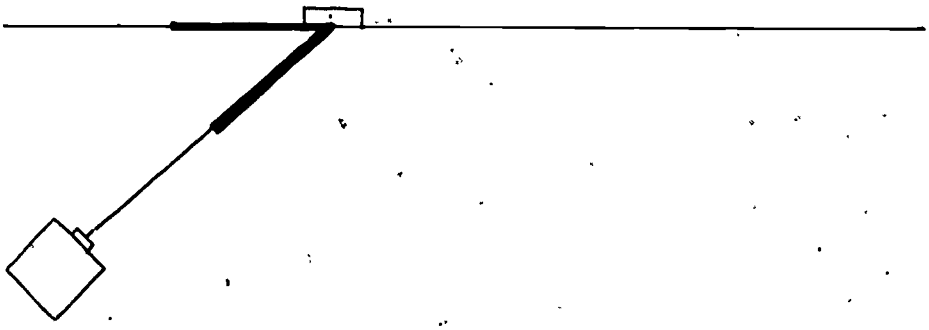
A person who holds the yarn and stands to the right of the mirror forms an angle. The chalkboard or wall and the yarn are the sides of the angle. The mirror is the vertex.



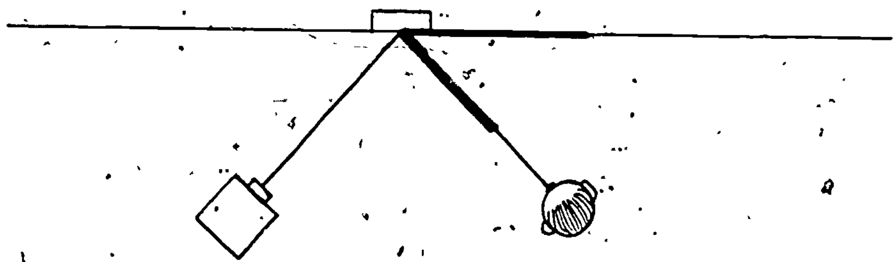
The projector must be placed so that the angle formed by the chalkboard and the light beam equals the angle formed by the chalkboard and the yarn.



To test this, turn off the projector and use an angle-finder to determine the projector-chalkboard angle.



Then move the angle-finder to the other side of the mirror. Have Player 3 stand so the yarn and the chalkboard form an angle equal to the projector-chalkboard angle.



Turn on the projector. The ray of light should hit Player 3. Test the procedure again with different angles. You may want to draw a diagram on the chalkboard showing the relationship of the two angles.

Activity C

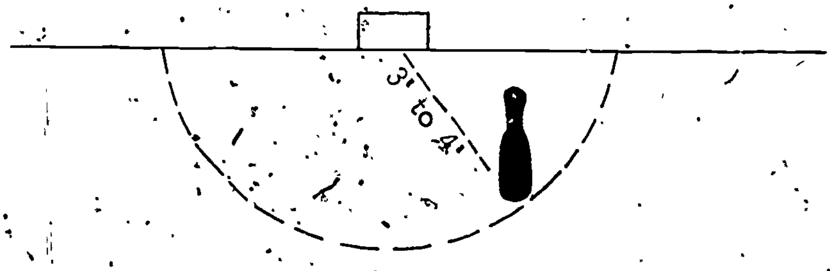
This activity is best done in the gymnasium. The children apply the knowledge of angles they gained in the light-reflection activities to a new situation. Instead of a light beam being reflected off a mirror, a rolling ball will be reflected off a wall.

Divide the class into groups of six to eight children. Each group will need a rubber playground ball or volleyball, a plastic bowling pin (or some object that is easily knocked over) and a sheet of 6" x 9" paper. Tape each group's sheet of paper to the wall in the gym, at floor level. Each group should be allowed approximately ten feet of wall space.

Explain the following procedure to the class and have one group demonstrate it.

Version I

Place a bowling pin on the floor within a three to four foot radius of the paper.



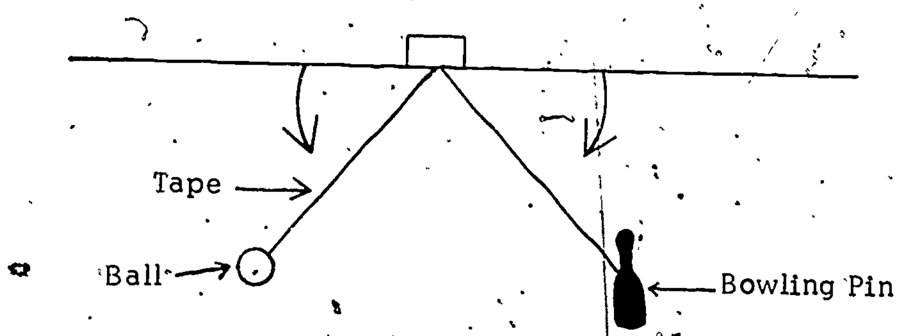
The children are to take turns rolling the ball so that it hits the paper, bounces off and hits the bowling pin. They get ten points each time they knock over the bowling pin.

The children may stand wherever they wish to roll the ball. If a child misses the paper, he gets another chance. If he knocks over the pin without hitting the paper, he doesn't receive any points. Caution the children against spinning the ball as they roll it. (The angle of reflection will not be regular if they do so.)

Each team repeats this procedure three times, changing the position of the bowling pin each time. They should keep track of their total points. The team with the greatest number of points wins.

Tell the teams to try the game again, but to try to think of a strategy that will help them knock over the pin more often. When the teams have completed this game, call them together in front of one of the team areas. Ask the winning team if they figured out a strategy which helped them knock over the pin more times.

You may want to use masking tape to show the class the path of the ball. This will help them visualize the angles formed by the ball's paths and the wall.



Set the bowling pin on one of the tapes and have a few children roll the ball along the other tape. Most of them should be able to hit the bowling pin. Change the position of the tapes, forming two nearly equal angles with the wall. Again, place the pin on one of the tapes and have some children roll the ball along the other tape.

Emphasize the similarity between the reflected ball and reflected light beams. (They both formed equal angles with the wall.)

Version II

Instead of using a bowling pin, have a child stand still on a given spot. The other children should try to make the ball hit the paper on the wall and then the child.

Version III

Challenge a student to put the bowling pin (or another child) in a position where the rest of his team will not be able to hit it by reflection from the wall. Remind them that the point they select must remain on the three-four foot radius line. (The closer to the wall the pin is, the harder it should be to hit. Also, if it is set on a line almost perpendicular to the paper, it will be more difficult to hit.)

Listed below are some books the children might enjoy reading at this time.

Farquhar, Margaret C. Book to Begin on Lights. New York: Holt, Rinehart and Winston, 1960.

Kohn, Bernice. Light You Cannot See. Englewood Cliffs, N.J.: Prentice-Hall Inc., 1965.

Pine, Tillie S. and Joseph Levine. Light All Around. New York: Whittlesey House, a division of McGraw-Hill Book Co. Inc., 196[illegible].

Lesson 8: LIGHT REFLECTION ACTIVITY CENTER

If the children are interested in light and how it is reflected, you will find it well worth your time to set up an activity center where they can explore many interesting combinations of the materials that are available. After a few days you could have the children report and demonstrate their discoveries to the class.

Prepare each display as shown in the photographs. We have given a detailed description of one possible use of each display. This should help you give suggestions to the children. However, the materials are not meant to be used in only these ways; there is an unlimited number of combinations that can be discovered.

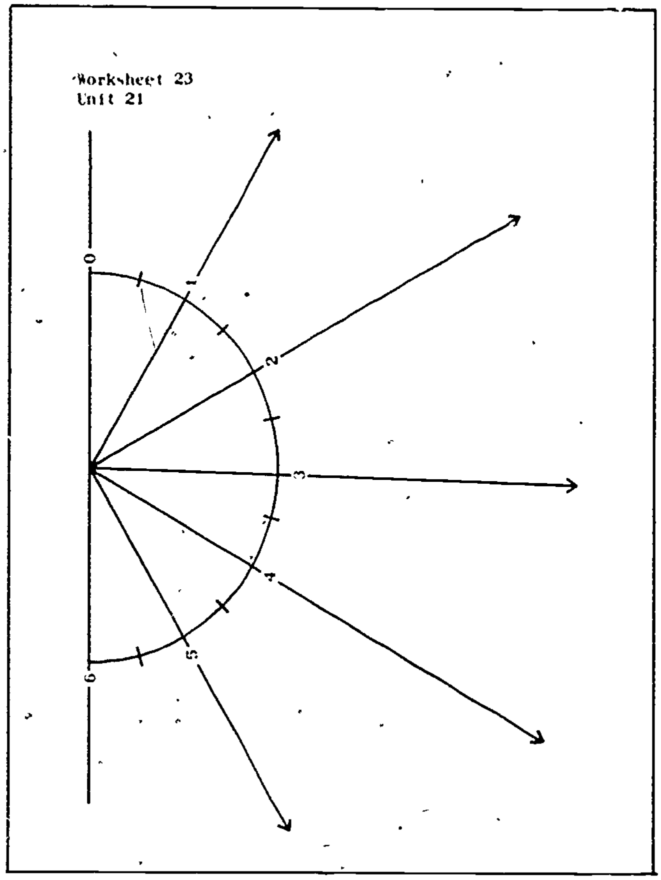
MATERIALS

- 2 small mirrors (2" x $4\frac{1}{4}$")
- 12 - 15 mirrors on blocks
- 1 set of colored pencils
- pencils
- 2 rulers
- blank sheets of paper
- 1 qua : milk carton
- masking tape
- striped transparency (printed original in Appendix)
- 5 angle-finders (from Lesson 6)
- 5 clock protractors (from Lesson 5)
- 2 flashlights
- 1 scissors
- Worksheet 23

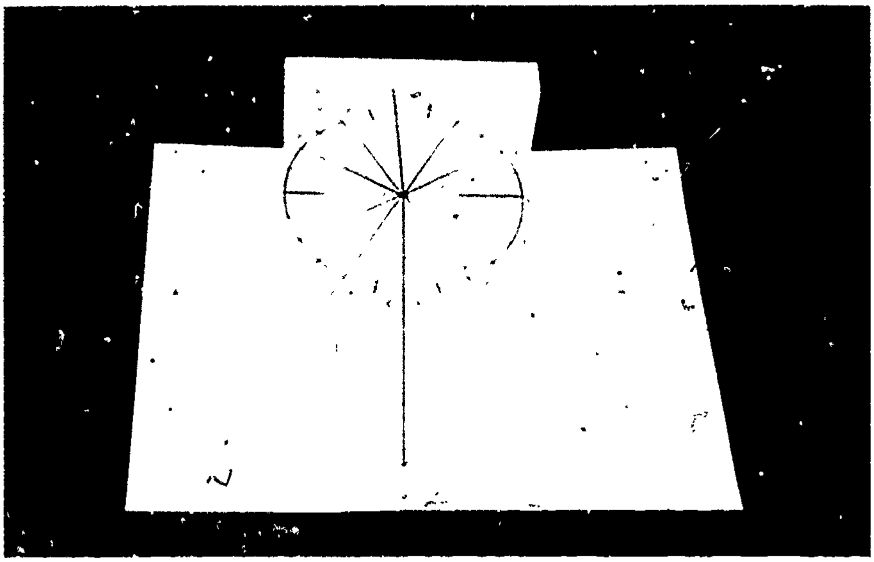
PREPARATION

For Display 5 make the striped transparency, using the printed original in the Appendix. For this display, you can also use combinations of colored paper or paper with designs on it.

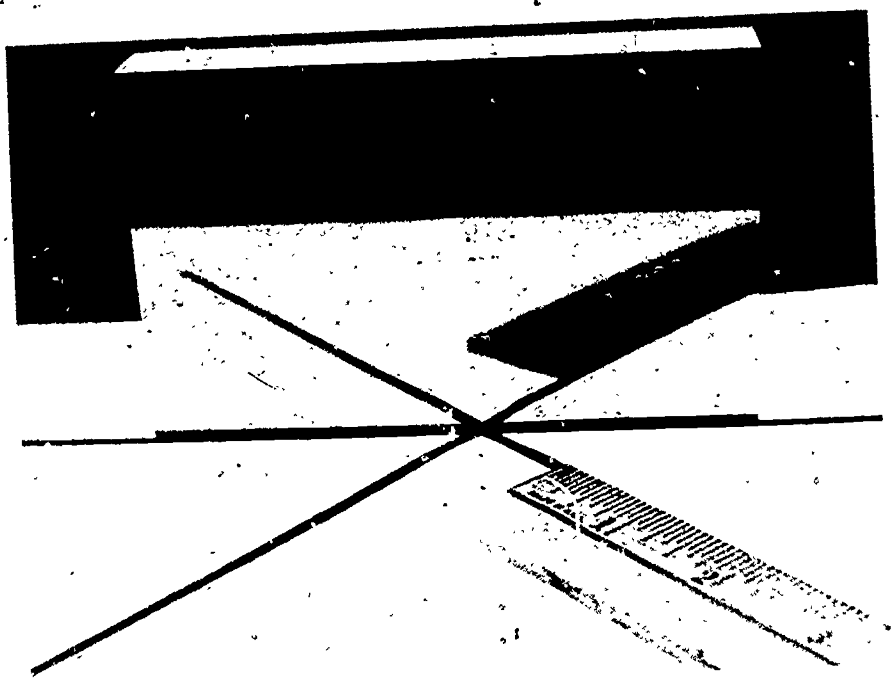
Display I



Look along ray 5 toward the middle of the mirror. Looking into the mirror, you can see that the mirror image of ray I is the mirror extension of ray 5. Color ray 5 and ray I the same color. Measure the angles formed by these rays and the mirror's edge. They are equal in mag. Do the same thing with other rays and the angles they form.

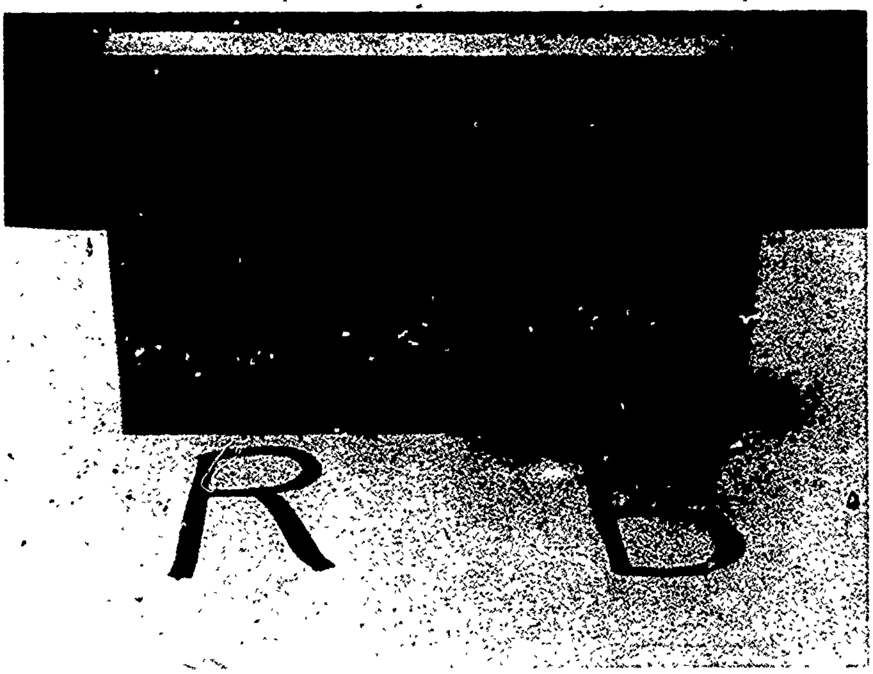


Display 2



Draw a straight line segment on a blank sheet of paper. Place a mirror so that the edge of the mirror and the line segment form an angle. Place one edge of a ruler so that its mirror image is the mirror extension of the line segment. Draw a line along the edge of the ruler up to the mirror. Also draw a line along the edge of the mirror. Use a clock protractor to measure the mag of the two angles you drew.

Display 3



Write the letter R on a blank sheet of paper in front of the mirror. Try to copy its mirror image next to the original letter. Now look at the mirror image of the second letter. Try to think of a figure whose mirror image is the same as the actual figure, e.g., the letter I.

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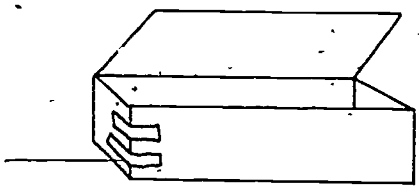
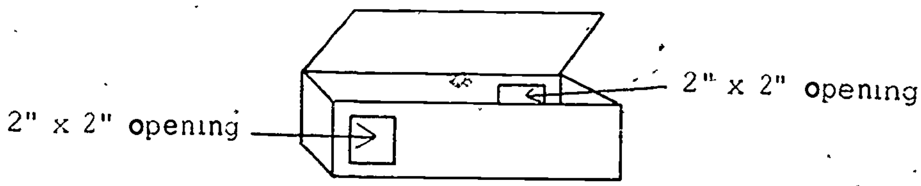
Display 4: A Periscope

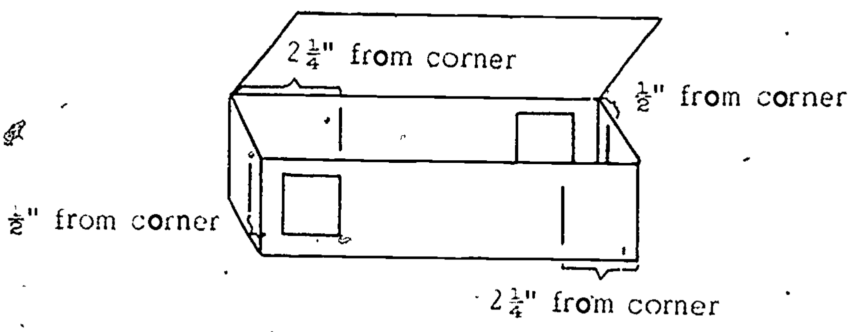
Straighten the top of a quart milk carton by opening it. Cover the open end with a piece of cardboard and masking tape. Then cut along one long edge of the carton and at both ends. This forms a "lid."



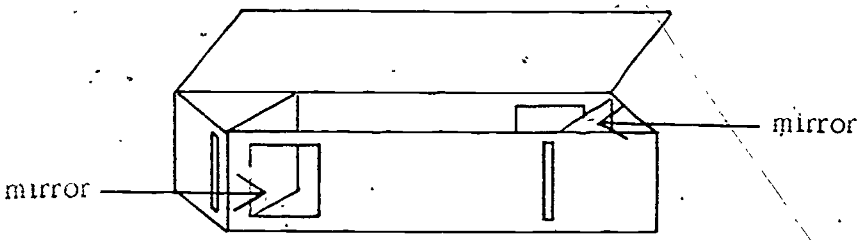
Cut 2" x 2" openings at opposite ends of the two long sides.



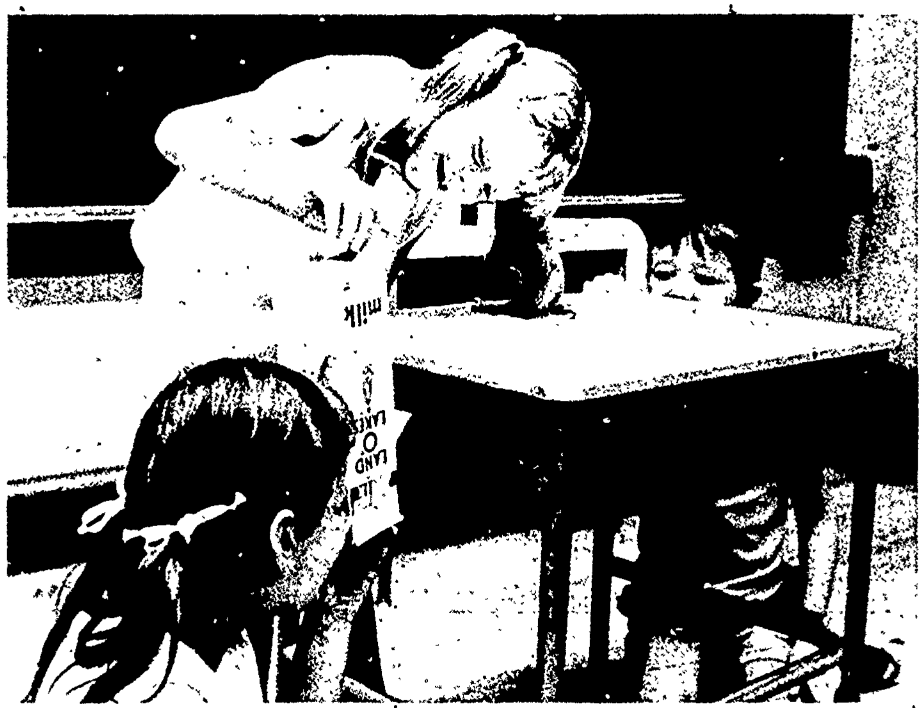
Make a $2\frac{1}{4}$" slit on each side as shown in the diagram below. The slits should be approximately $2\frac{1}{4}$" from the corners on the long sides and $\frac{1}{2}$" from the corners on the short sides.



Small mirrors can be placed in the corners opposite each opening by fitting them into the slits.



Prepare the milk carton with the openings and the slits, but do not insert the mirrors. Tell the children that these materials can be put together to make a periscope and let them try to construct one themselves.



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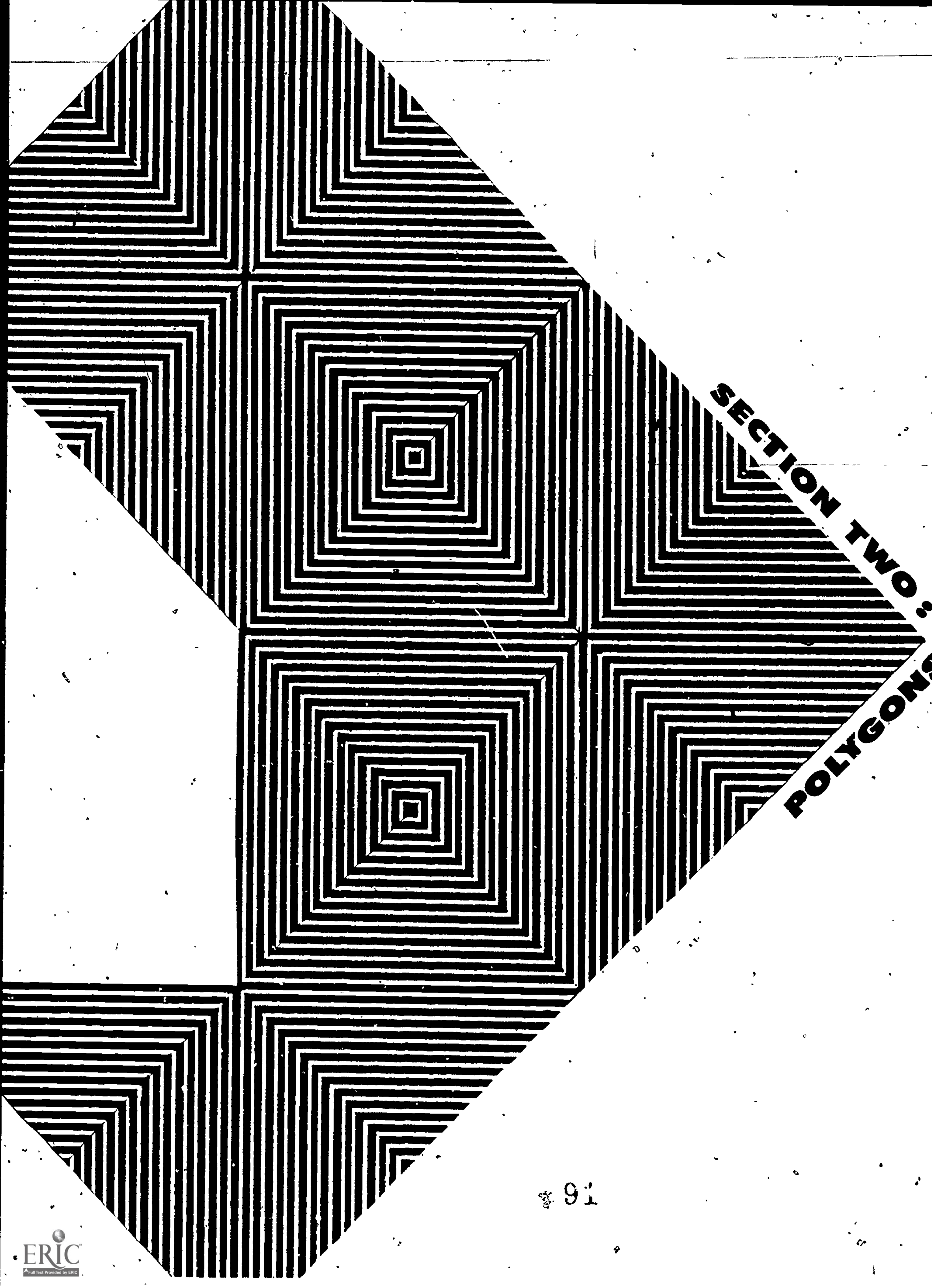
Display 5: A Kaleidoscope

Assemble three mirrors as shown so that the shiny surfaces are facing. Wrap masking tape around them to secure them. Hold combinations of striped transparencies over one end. Move the transparencies while looking through the other end.



SECTION TWO: POLYGONS

SECTION 2 POLYGONS

In Section 2, the children study polygons. They also review many of the set and classification concepts that were introduced in earlier MINNEMAST units. First the children classify a set of flower cards. The organizing principle is to look at the polygon shapes formed when the tips of the petals are connected with line segments. Then they classify sets of cards with drawings of different polygons on them. In classifying these cards, the children consider such properties as number of sides and whether the polygon is concave or convex, regular or non-regular. Working with think sticks to make different polygons, the children also consider the property of rigidity of shape. (A rigid polygon is one whose angles cannot be changed by pushing on the sides.) They are led to see that the triangle is the only rigid polygon.

There are several interesting activities in which mirrors are used to reflect lines, forming polygons with infinite numbers of sides.

The children also study similar triangles and congruent figures. Verbalization of these concepts is not expected at this time. The notion of scaling as it was introduced in Unit 18 is incorporated into the discussion of similar triangles.

Finally, the children play with paper polygons to see what relationships they can discover among shapes and what repeating patterns they can find in designs consisting of polygons. These investigations should be left open-ended.

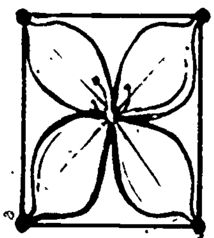
Throughout this section, there is continued emphasis on the concept of angle and the measurement of mag as developed in Section 1.

Lesson 9: FLOWER POLYGONS

The children are introduced to the concept of polygons. They classify flowers according to the polygons formed when line segments are drawn to connect the tips of the petals.

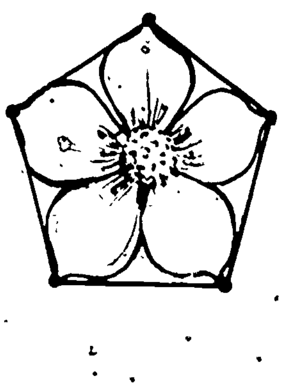
Flowering plants differ in the numbers and arrangement of the parts of the flowers. These differences are useful in plant classification. The children can use the arrangements of some flowers parts to derive polygons. At the same time, they can classify the specimens by the geometric figures derived from the flowers and incidentally discover that there are many kinds of flowers.

Worksheets 24, 32, 36 and 38 are pictures of flowers which have four parts. Line segments connecting the tips of the petals of these flowers will yield four-sided polygons.



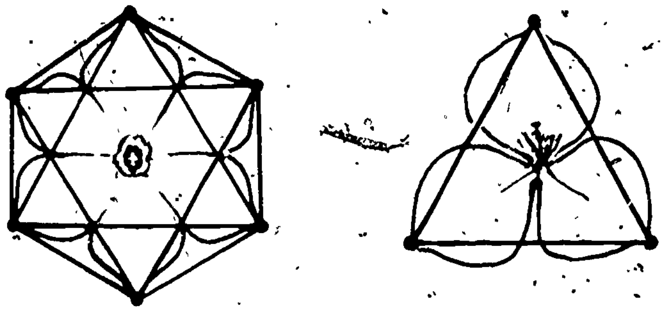
All these worksheets show flowers belonging to the cabbage or crucifer family, so named because the four petals form a cross. The flowers of many of our common vegetables belong to this group, including cabbage, radish, turnip, brussels sprouts, broccoli and cauliflower.

Worksheets 25, 29, 30, 31, 35 and 37 are pictures of flowers with five parts which yield pentagons with sides of equal or unequal length.



These flowers all belong to the rose family. The flowers of many common fruits belong to this family, including pears, plums, peaches, apples, strawberries and raspberries.

Worksheets 26, 27, 28, 33, 34, 39 and 40 are pictures of flowers which have three or six parts. The polygons which can be derived are either a single triangle, one triangle superimposed on another triangle, or a hexagon.



All the flowers shown are members of the lily family, except for the spiderwort, which is closely related to them. Most flowering bulbs belong to this family.

Worksheet 41 shows a picture of the water lily, which represents a fairly common flower type -- that with many parts. The magnolias and buttercups have flowers like this. This worksheet also shows a picture of the sunflower, which represents the large family of sunflowers that includes asters, daisies, dahlias, chrysanthemums and dandelions. The single head which appears to be a single flower is really an aggregation of tiny flowers.

It would be very interesting for the children to see and dissect real specimens of each type of flower. Almost any common spring flowering bulb will supply examples of three and six parted flowers. You can use the flowers of paper-white narcissus or hyacinth, crocus, scilla, any lily or amaryllis, or tulip. Many florists will give over-age flowers to classes. The easiest four-parted flowers to find are those on a stalk of fresh broccoli. Many buds will open if the stalk is kept in water in a warm room.

Specimens of five-parted flowers can be found on flowering crab, bridal wreath, firethorn, hawthorn, juneberry, quince, pear, apple, mountain ash and wild plum.

An optional activity for interested children could be the collection and classification of flower pictures from catalogues, advertisements or household magazines. Naturally, this same activity can be continued outdoors when the weather permits.

A word of caution should be added. The correct classification of plants depends on many structures; not all plants with four-parted flowers are in the cabbage family. There are some plants with five-petaled flowers that do not belong to the rose family because of other structures in the flowers. The worksheets are designed to limit the examples to family. Wild flower books will supply helpful information about other plant families.

MATERIALS

-- for each child --

- 1 scissors
- ruler
- colored pencils: 1 red, 1 blue and 1 green
- 4 sheets of $8\frac{1}{2}$" x 11" construction paper
- Worksheets 24 through 41

PROCEDURE

Activity A

While the children are working on their flower worksheets, be sure to introduce the pronunciation of the following words: polygon (pol'i-gon'); quadrilateral (kwad'ra-lat'er-al), pentagon (pen'ta-gon'), hexagon (hek'sa-gon') and triangle. Use these words correctly but do not insist that the children learn them. You may wish to point out that a polygon is a closed curve made up of line segments.

Have the children complete Worksheets 24 through 27. When everyone is finished, discuss the different types of polygons they have constructed.

Tell the children they are to complete Worksheets 28 through 41. They can continue working on the worksheets whenever they have some extra time. These should be completed by the next MINNEMAST class period. Then the children should remove their flower worksheets from their Student Manuals and cut them on the center lines making 19 flower cards. Ask them to think of a way they could organize or classify their sets of flower cards into subsets.

Activity B

Direct the children to classify their sets of flower cards into subsets, using whatever properties they wish as a basis for classification. When everyone has finished this work, have various children report to the class the properties they used to classify their cards. The children may use various properties; whichever they choose is acceptable so long as they can explain why they classified this way. Hopefully, someone will classify his cards according to the polygon determined by each flower's petals. In any case, borrow the five following cards from a student and display them, classified as shown. Label each subset. Discuss each flower's properties.

Quadrilaterals	Pentagons	Hexagons or Triangles	Unknowns
(4-sided polygons)	(5-sided polygons)	(6 and 3-sided polygons)	
Wild Turnip	Wild Rose	Narcissus Daffodil	Waterlily

Give each child four sheets of 8½" x 11" construction paper. Each sheet should be folded in half and used as a folder to hold each subset of flower cards. Have the children label each folder with the name of the subset: Quadrilateral (4-sided polygon), Pentagon (5-sided polygon), Hexagon or Triangle (6-sided or 3-sided polygon), and Unknown. Each child then completes the classification of his flower cards, including the student whose five cards you borrowed for display. (These cards should be left on display, however.)

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When all the children have finished classifying their cards, call on one student to come up and tape all the cards in his quadrilateral folder on the board under the proper label. Have three other students do the same with the cards in the other folders.

A completed set would be classified like this:

Quadrilaterals (4-sided)	Pentagons (5-sided)	Hexagons or Triangles (6 and 3-sided)	Unknowns
Wild Turnip	Wild Rose	Narcissus	Waterlily
Toothwort	Strawberry	Daffodil	Sunflower
Radish	Blackberry	Day Lily	
Rocket	Apple	Hyacinth	
	African Violet	Trillium	
		Spiderwort	
	Raspberry	Amaryllis	

Tell the children to check their classification with that on the board. Discuss this classification system with them. Then return all the cards on the board to the proper owners. Have the children save their classification folders and cards for Lesson 11.

Activity C (Optional)

The children might enjoy making up games to play with their flower cards. For example, they could combine sets of cards and play a game similar to "Old Maid." Encourage them to make up their own games and rules.

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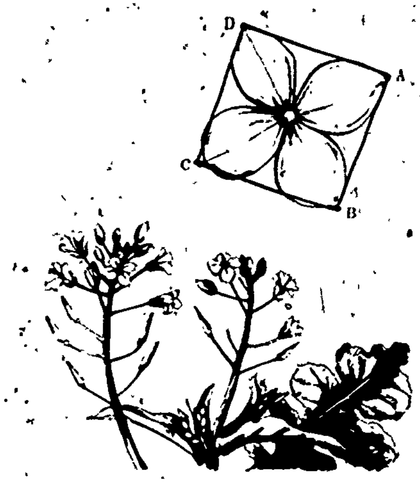
Worksheet 24
Unit 21



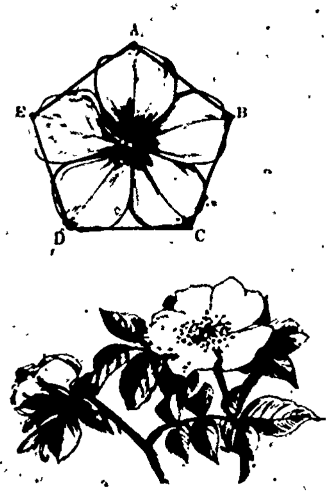
Wild Turnip

Draw line segments AB, BC, CD, and DA.

How many sides does the figure you drew have? 4

This figure is called a 4 sided polygon, or a quadrilateral.

Worksheet 25
Unit 21



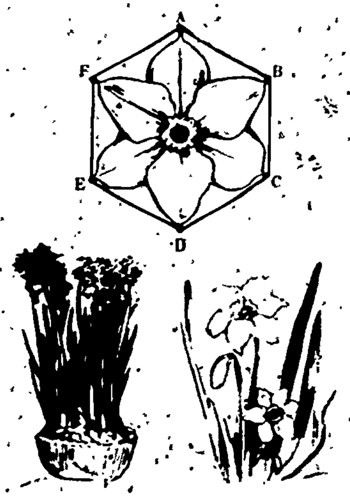
Wild Rose

Draw line segments AB, BC, CD, DE, and EA.

How many sides does your figure have? 5

This figure is called a 5 sided polygon, or a pentagon.

Worksheet 26
Unit 21



Narcissus

Draw line segments AB, BC, CD, DE, EF, and FA.

How many sides does your figure have? 6

This figure is called a 6 sided polygon, or a hexagon.

Worksheet 27
Unit 21

Daffodil

Draw line segments AC, CE, and EA.

How many sides does your figure have? 3

This figure is called a 3 sided polygon, or a triangle.

Draw line segments BD, DF, and FB.

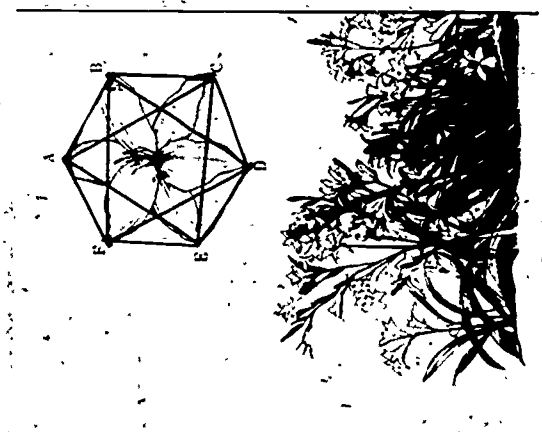
What figure did you draw? a triangle

Worksheet 28
Unit 21

Use a red colored pencil to draw line segments AB, BC, CD, DE, EF, and FA.

Use a blue colored pencil to draw line segments AC, CE, and EA.

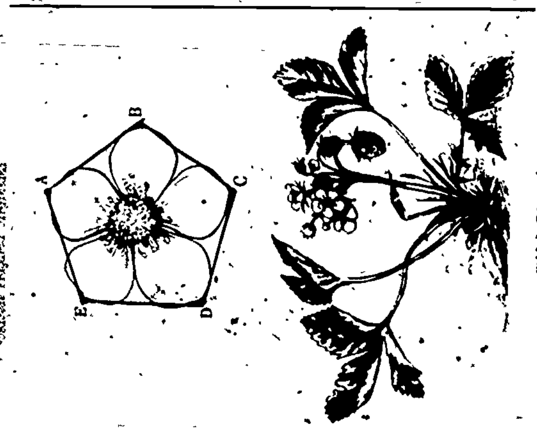
Use a green colored pencil to draw line segments FB, BD, and DF.



Day Lily

Worksheet 29
Unit 21

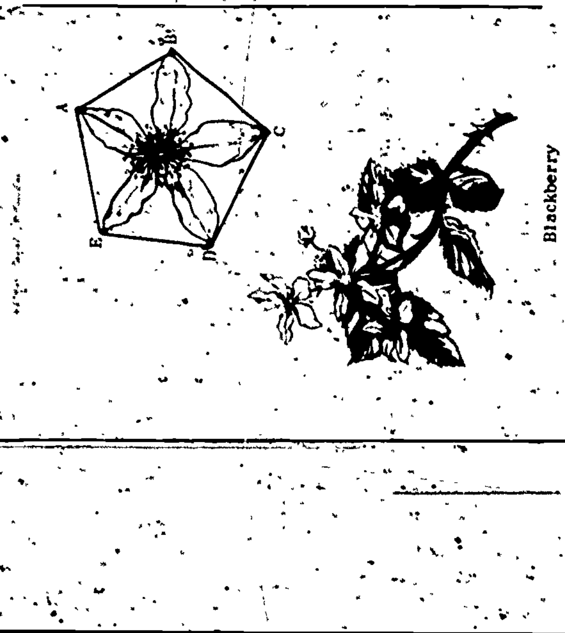
Draw line segments AB, BC, CD, DE, and EA.



Wild Strawberry

Worksheet 30
Unit 21

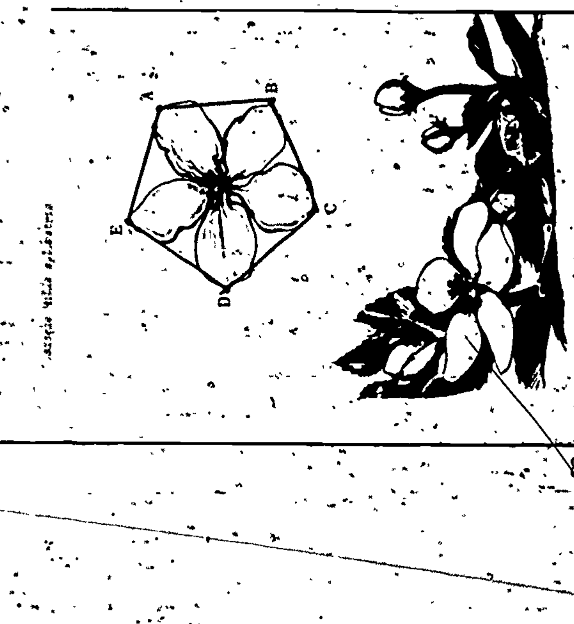
Draw line segments AB, BC, CD, DE, and EA.



Blackberry

Worksheet 31
Unit 21

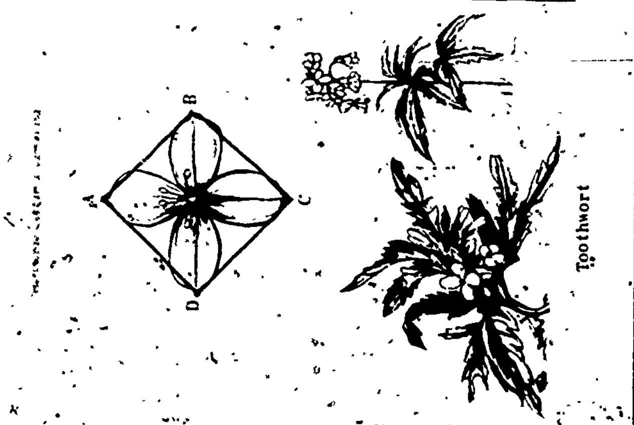
Draw line segments AB, BC, CD, DE, and EA.



Apple

Worksheet 32
Unit 21

Draw line segments AB, BC, CD, and DA.



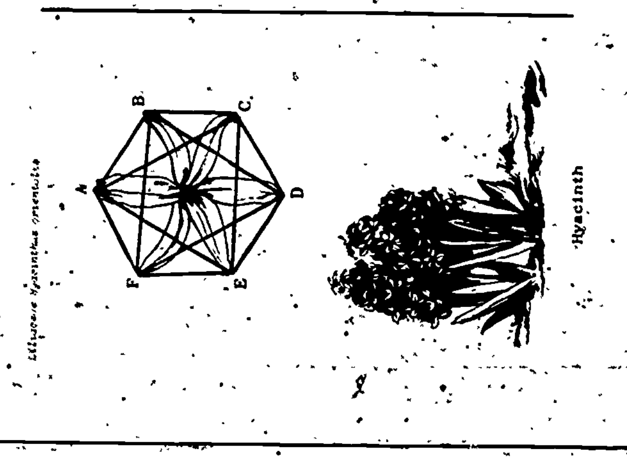
Toothwort

Worksheet 33
Unit 21

Use a red colored pencil to draw line segments AB, BC, CD, DE, EF, and FA.

Use a blue colored pencil to draw line segments AC, CE, and EA.

Use a green colored pencil to draw line segments FB, BD, and DF.



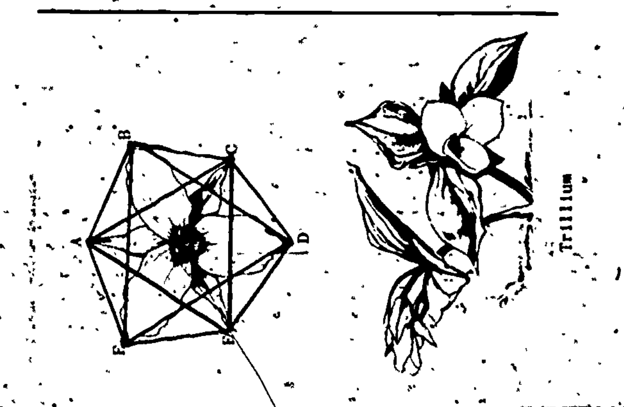
Hyacinth

Worksheet 34
Unit 21

Use a red colored pencil to draw line segments AB, BC, CD, DE, EF, and FA.

Use a blue colored pencil to draw line segments AC, CE, and EA.

Use a green colored pencil to draw line segments FB, BD, and DF.



Trillium

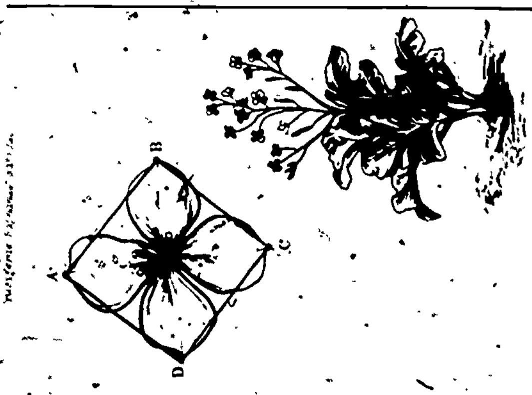
Worksheet 35
Unit 21

Draw line segments AB, BC, CD, DE, and EA.

African Violet

Worksheet 36
Unit 21

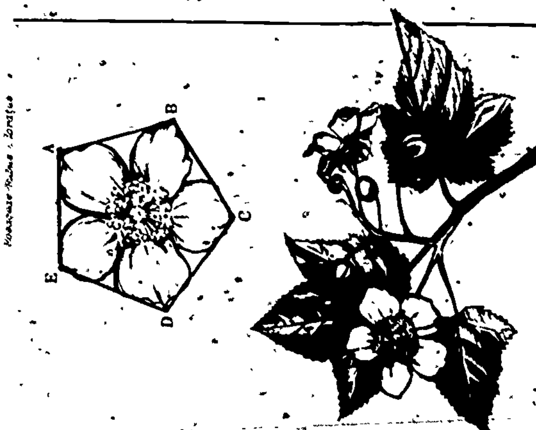
Draw line segments AB, BC, CD, and DA.



Radish

Worksheet 37
Unit 21

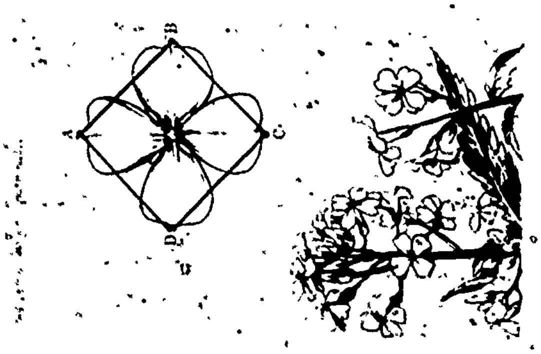
Draw line segments AB, BC, CD, DE, and EA.



Raspberry

Worksheet 38
Unit 21

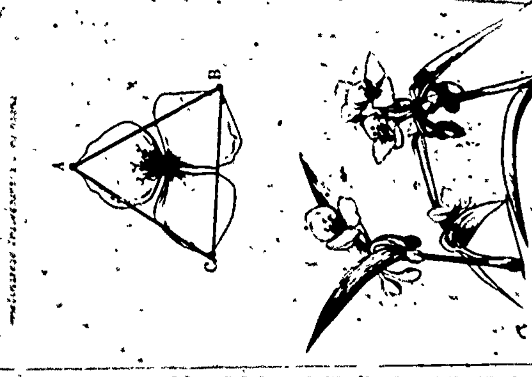
Draw line segments AB, BC, CD, and DA.



Rocket

Worksheet 39
Unit 21

Draw line segments AB, BC, and CA.



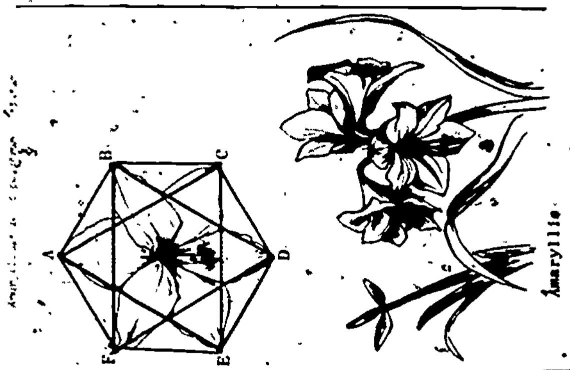
Spiderwort

Worksheet 40
Unit 21

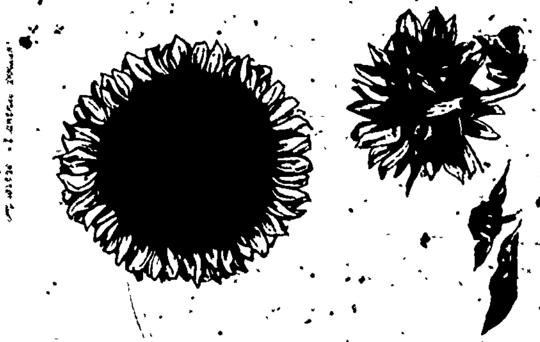
Use a red colored pencil to draw line segments AB, BC, CD, DE, EF, and FA.

Use a blue colored pencil to draw line segments AC, CE, and EA.

Use a green colored pencil to draw line segments FB, BD, and DF.



Worksheet 41
Unit 21



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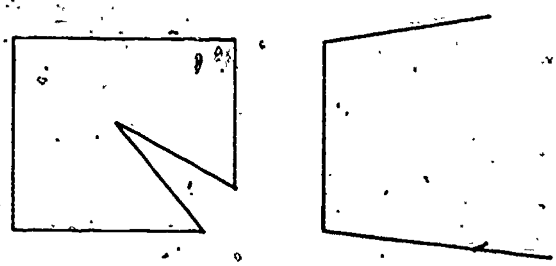
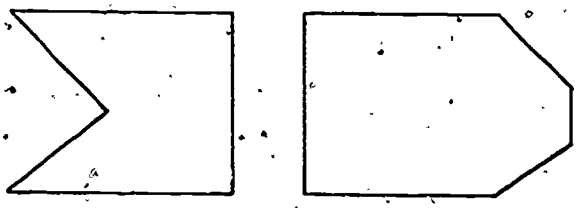
Lesson 10: CLASSIFYING POLYGONS

In this lesson the children classify a set of cards showing polygons and non-polygons into various subsets. The first subsets they will use are polygons and non-polygons, the polygons being closed curves made up of line segments, and the non-polygons being open curves.



Polygon Non-Polygon

Then they classify the set of polygons into subsets of polygons with 3, 4, 5, 6, or 8 sides. Each one of these subsets can be classified into subsets of "caved" (concaved) or "non-caved" (convex) polygons.



Caved Non-Caved

As the children do this classifying, they will discover that the triangles do not have a subset of "caved" figures; in other words, the set of "caved" triangles is an empty set. They use think sticks to show that the triangle is the only rigid polygon (it is the only polygon whose angles cannot be changed by pushing on the sides). As they form each subset, the children tape their cards to a large chalkboard tree diagram. This tree diagram will be completed in Lesson 11.

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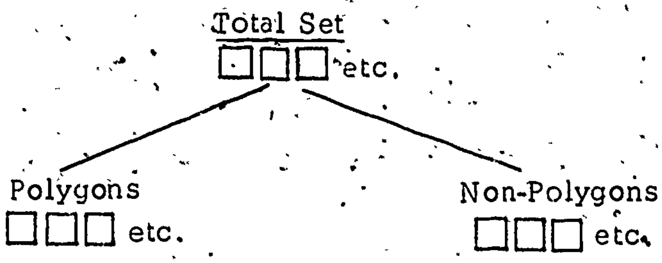
WHICH ONE OF THESE CURVES IS A POLYGON? THE CLOSED CURVE OR THE OPEN CURVE?

Lead the children to the conclusion that a polygon is a closed curve made up of line segments.

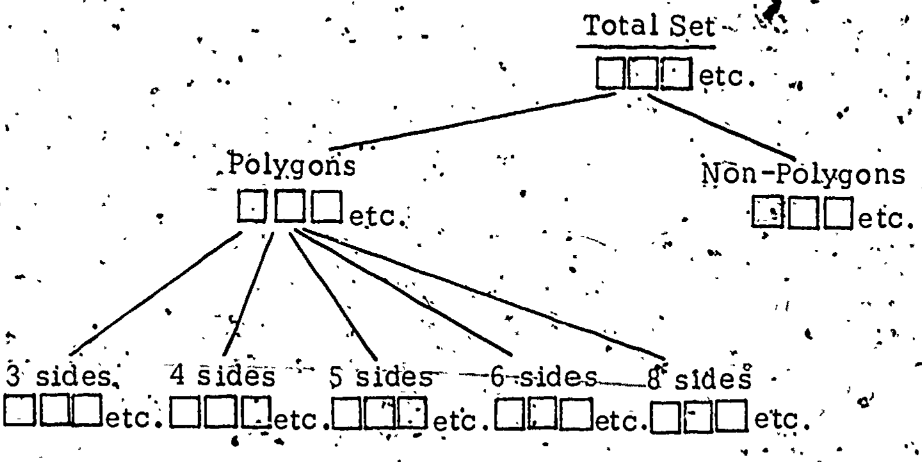
Have the children remove the sheets of polygon and non-polygon cards from their workbooks. You should cut each sheet into quarters on the paper cutter. Give each child a large paper clip to keep his set of cards together. Then collect the sets of cards from the children. Divide the class into groups of three each. (Keep a list of the children in each group. You will need this information for Lesson 11.)

Give each group of three a set of cards. Ask them to classify their set of cards into two subsets, polygons and non-polygons. While the children are doing this, tape one set of cards to the chalkboard, wall or bulletin board and label it "Total Set". (See diagram below.) When all the groups have finished their classification, have one group tell the class why they placed certain cards in either subset. Tape their set of cards on the chalkboard, wall or bulletin board, labeling the two subsets polygons and non-polygons.



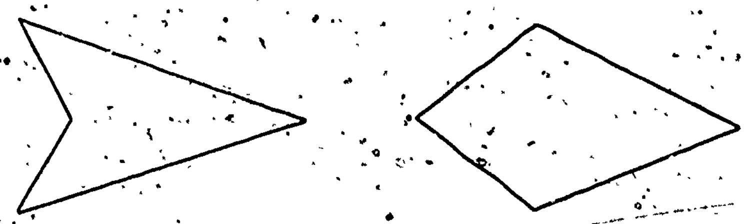
Give this group of children another set of cards to work with.

Ask the children to put aside their sets of non-polygons. (It might be convenient to give each group a paper clip to keep its set of non-polygons together.) Ask the children to classify their sets of polygons into subsets, using whatever properties of the polygons they want as a basis for classification. When the groups have finished classifying their polygon cards, discuss their methods of classification. Most groups will probably classify their polygons according to the number of sides. Tape an extra set of polygon cards on the board using this method of classification. Label each subset.

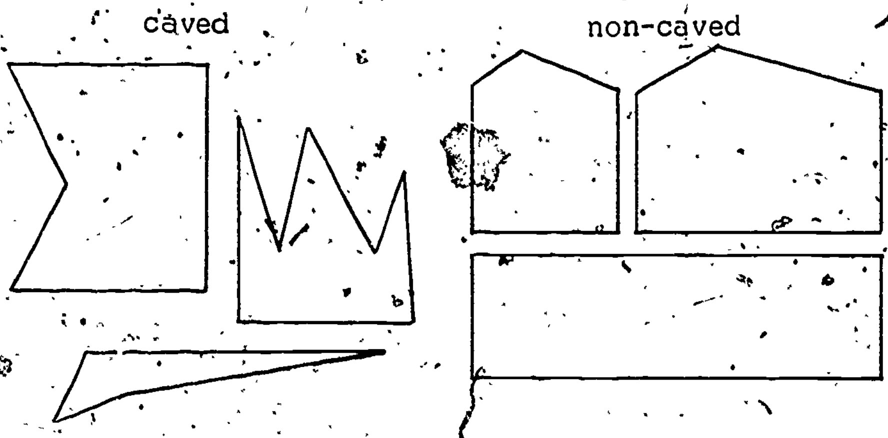


Give each group five paper clips to keep its subsets of polygons together. Ask a child to collect all the sets of 3-sided polygons; ask another child to collect all the sets of 4-sided polygons, etc. When all the polygon sets have been collected, divide the class into six groups. Give one group two sets of 3-sided polygons; give another group two sets of 4-sided polygons, another group two sets of 5-sided polygons, etc.

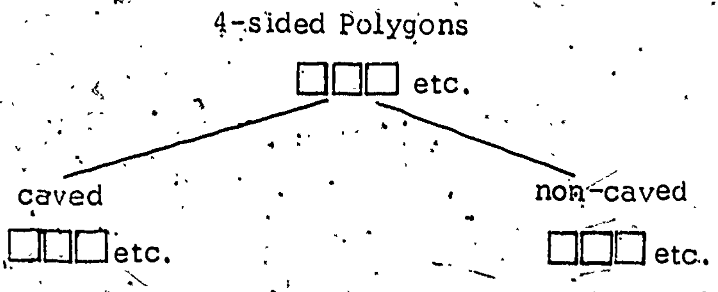
Draw these two polygons on the chalkboard or the overhead projector:



Ask the children to tell you how these two figures are different. Lead them to the realization that one figure has "caved-in" or "bent-in" sides, and the other figure does not. Explain that the first polygon is called a "caved" polygon and the second ond is called a "non-caved" polygon. Draw other examples of polygons with "caved" sides and polygons with "non-caved" sides.

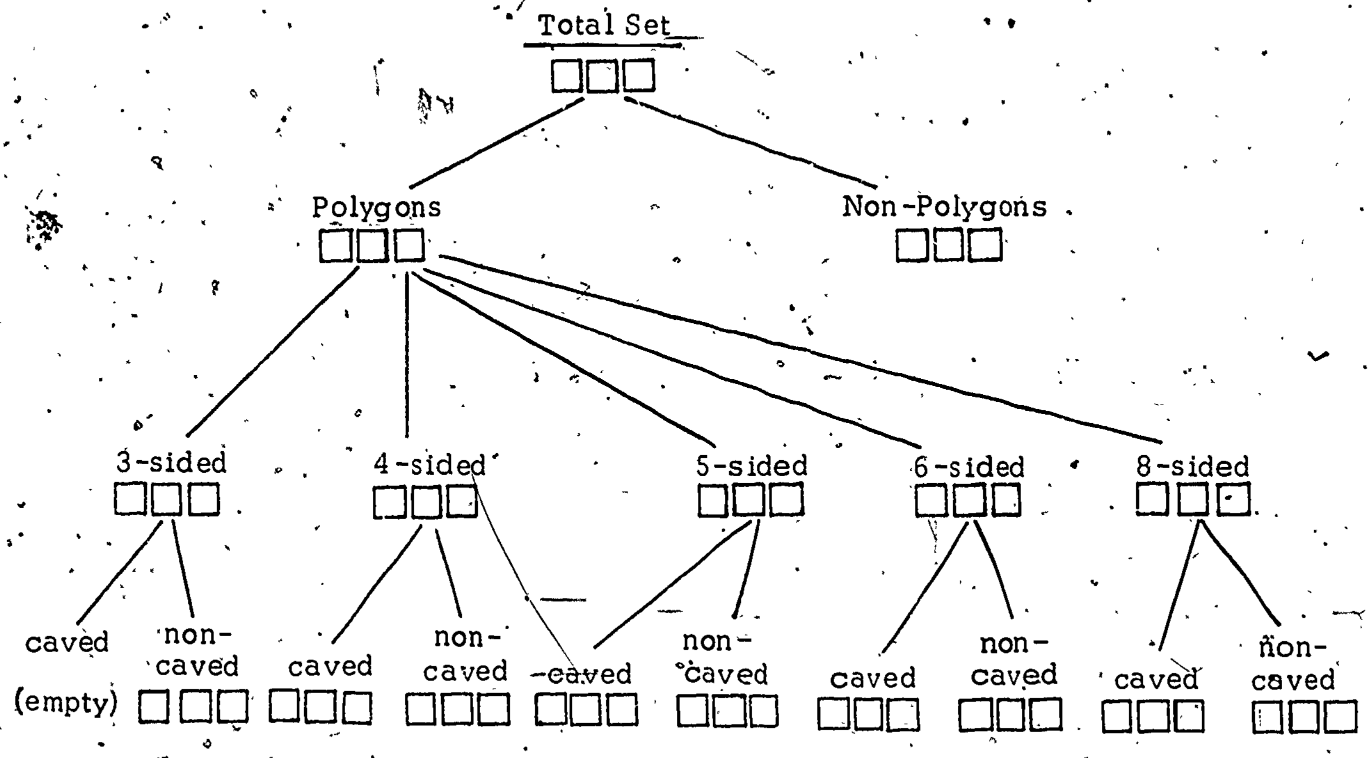


Ask each group to classify one of its sets of polygons into "caved" and "non-caved" polygons. When they are finished, two children in each group should tape their polygons on the chalkboard in the correct place. The tree diagram for the group with 4-sided polygons would look like this:



While some of the children are taping their polygon cards to the chalkboard, the other children in their group should classify the other set of polygons into caved and non-caved polygons.

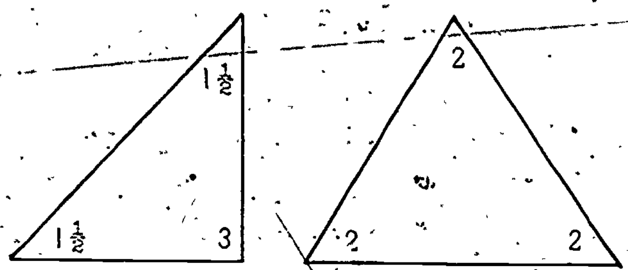
Give the children paper clips to secure their subsets. Then have them set aside their subsets. The chalkboard tree diagram should now look like the diagram on the next page.



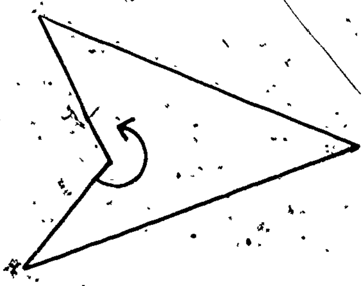
Discuss the tree diagram with the class. Someone should notice that the group working with 3-sided polygons did not classify any of their polygons as caved. Have the class check the non-caved 3-sided polygons to see if any of them are caved.

Challenge ~~the class~~ to try to construct a 3-sided caved polygon. Remind them that they can only use three straight line segments. They should discover that they cannot construct a caved 3-sided polygon using three straight line segments.

Note: The sum of the mags of the angles of a triangle is always equal to 180° or a six hour angle.

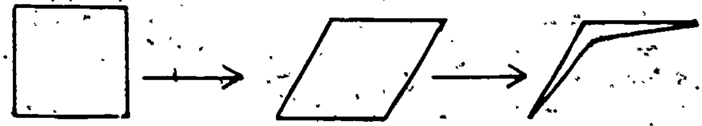


In a polygon with caved sides, the two sides that are caved form an angle whose interior mag is greater than 180° or 6 hours.



Since the total mag of all the interior angles of a triangle equals 180°, no one of the three angles could measure 180° or more.

You need not try to show this to the children; instead, they will use think sticks to construct 3 to 8 sided polygons. They should discover that the triangle is the only rigid figure -- its angles cannot be changed. The other polygons are not rigid, that is, angles can be changed by pushing the sides in different directions.



They can construct caved 4 to 8 sided polygons by pushing in two sides.

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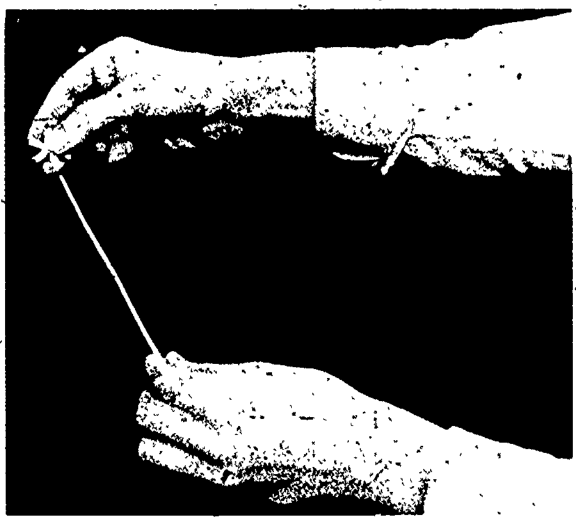
Demonstrate to the class how to use the think sticks. Tell them the sticks will break if used incorrectly; demonstrate both the right and wrong ways of putting the sticks and connectors together.

Correct

Incorrect



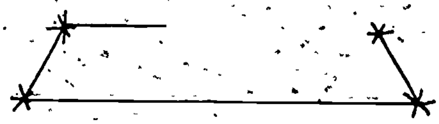
They should use this same method when taking the sticks and connectors apart.

Separate the children into the original groups of three and divide the package of think sticks and connectors among the groups, making sure each group gets a variety of lengths. Distribute the extra connectors so that each group has a total of at least eight connectors.

Ask each group to make two different 4-sided polygons with its think sticks. When this has been completed, have each group report to the class some properties of its 4-sided think stick polygons. List these properties on the chalkboard. On the next page is an example of a possible list.

4-sided Polygons

1. Flexible, floppy.
2. Angles can be changed.
3. Sides can be caved in.
4. If one side is longer than the sum of the other three sides, you cannot make a polygon.



Now ask each group to make a triangle using their think sticks. Have the groups report some properties of their think stick triangles. List the properties on the chalkboard and compare them to the properties of the 4-sided figures.

3-sided Polygons

1. Rigid.
2. Angles cannot be changed.
3. Sides cannot be caved in.
4. If one side is longer than the sum of the other two sides, you cannot make a triangle.

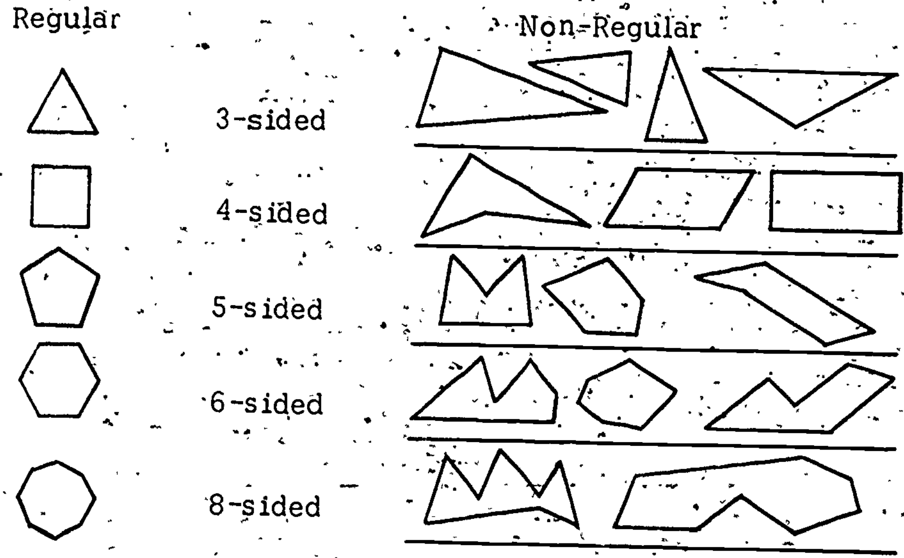
Challenge the class to make another polygon, other than the triangle, that is rigid. They should soon discover that the other polygons will have the same basic properties as the 4-sided polygon. The triangle is the only rigid figure.

Collect the think sticks from each group. You may want to leave the sticks out on an activity table for the children to experiment with in their spare time. Ask the groups to keep their extra sets of caved and non-caved polygons for the next lesson. You should also save the chalkboard tree diagram for Lesson 11.

Lesson 11: REGULAR POLYGONS

In this lesson the children continue to classify polygons, the final subset being regular and non-regular polygons. A regular polygon is a polygon whose sides are all equal in length and whose angles are all equal in mag.

Examples:



The children also classify their flower card polygons as regular and non-regular. In the last activity they use mirrors to generate various polygons.

MATERIALS

- Transparency A (included in printed materials; the printed original is also included in the Appendix)
- Transparency B (½" grid; printed original in Appendix)
- two 17" pieces of yarn or string
- overhead projector
- polygon cards (from Lesson 10)
- flower cards (from Lesson 9)
- mirrors on blocks, 1 per child
- blank sheets of paper
- pencils

- rulers, 1 per child
- student clock protractors, 1 per child
- Worksheets 42, 43 and 44.

PREPARATION

Carefully cut out the polygons and labels from the top half of Transparency A (colored red). Save the bottom half of Transparency A for use in Lesson 13. Make Transparency B, using the printed original in the Appendix.

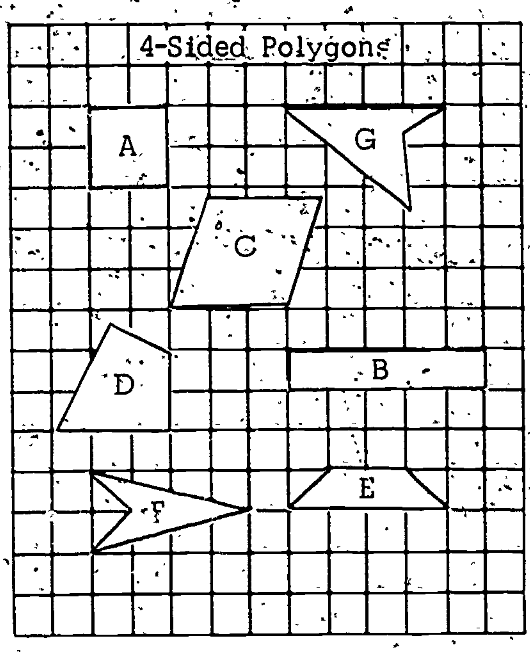
PROCEDURE

Activity A

Use the overhead projector. Place the red polygons on Transparency B ($\frac{1}{2}$"grid). Ask:

WHAT PROPERTY DO ALL THESE POLYGONS HAVE IN COMMON? (They are all 4-sided polygons.)

Write this on Transparency B.



Then ask:

WHAT TWO SUBSETS CAN WE MAKE FROM THIS SET OF 4-SIDED POLYGONS? ("Caved" and "non-caved.")

Divide the polygons into these two subsets and arrange them on Transparency B. Be sure to write "non-caved" and "caved" over the two subsets.

Ask the class to look at Figure A (the square).

Ask:

WHAT DO YOU NOTICE ABOUT THE LENGTH OF FIGURE A's SIDES? (They appear to be equal in length.)

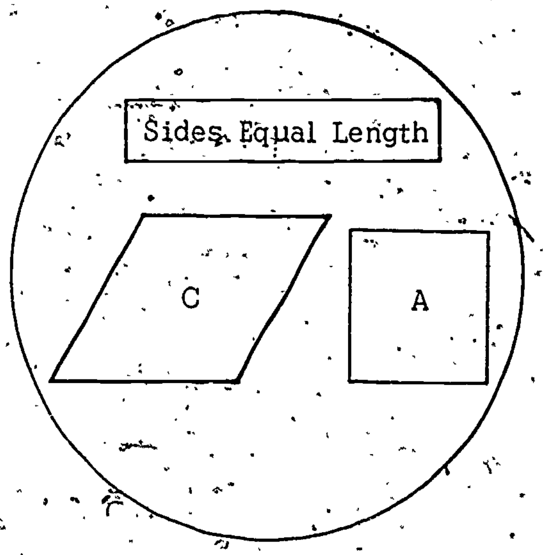
HOW CAN WE FIND OUT IF THE SIDES ARE EQUAL IN LENGTH? (Measure them.)

Have someone line up the sides of the square with the grid. They should see that each of the four sides is two units long.

CAN ANYONE FIND ANOTHER FIGURE THAT HAS SIDES OF EQUAL LENGTH? (Figure C.)

Someone should measure the length of Figure C's sides by lining up each side with a 3-unit segment of the grid.

Tell the class that we can separate Figures A and C from the other non-caved figures by putting a closed curve around them. Do this with one piece of yarn. Place the label "Sides Equal Length" inside the curve with Figures A and C.



Then ask the class:

WHAT CAN YOU TELL ME ABOUT THE MAGS OF FIGURE A's ANGLES? (The mags are all equal; all the angles are right angles.)

Someone can show this either by measuring the angles with his clock protractor or by superimposing each angle on a right angle of the grid.

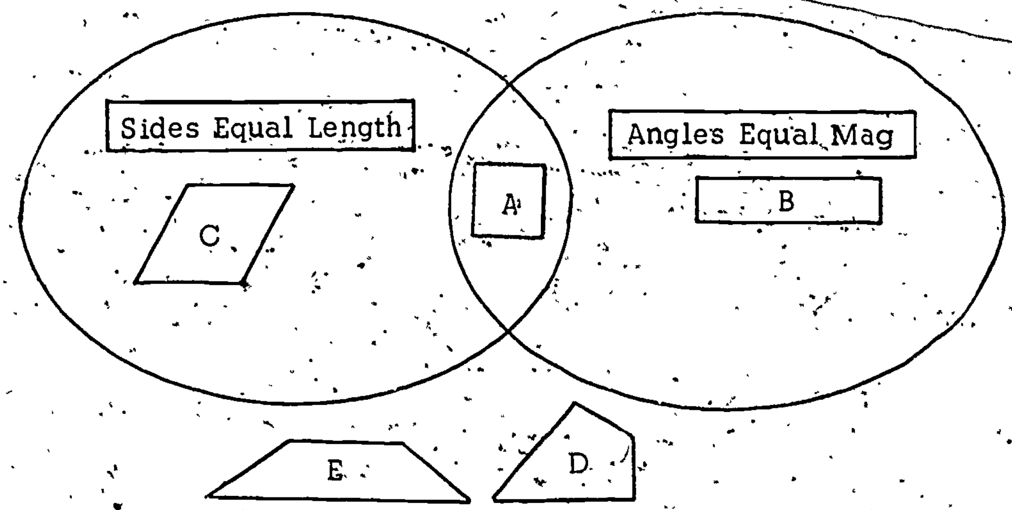
CAN YOU FIND ANOTHER FIGURE WHOSE ANGLES ARE ALL EQUAL IN MAG? (Figure B.)

Make a closed curve around those figures that have angles of equal mag. Put a curve around Figure B and the label "Angles Equal Mag."

Ask the children:

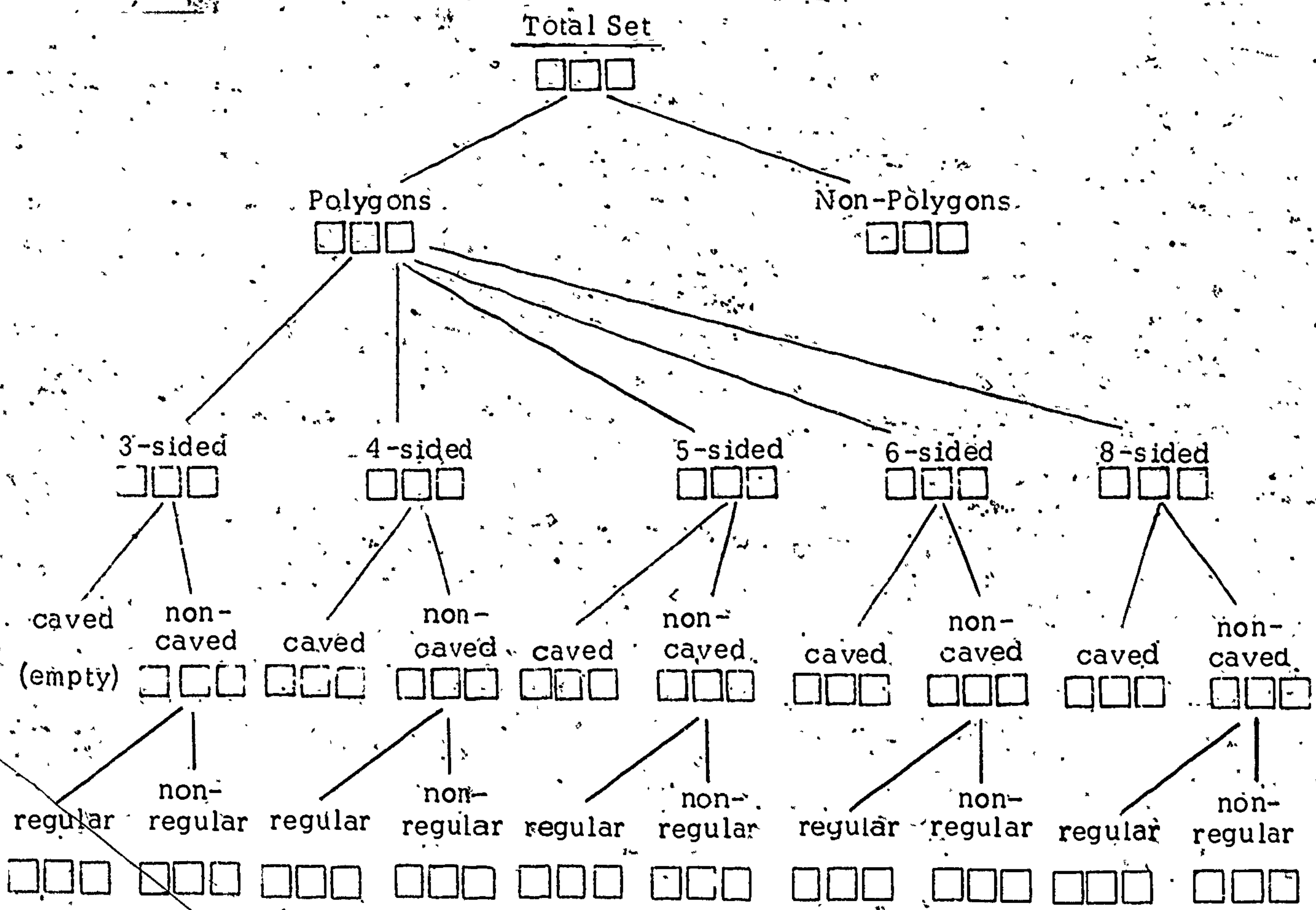
IN WHICH CLOSED CURVE SHOULD FIGURE A BE PLACED?

Lead the children to the realization that Figure A belongs in both closed curves. Have someone arrange the yarn and Figure A so that it is inside the intersection of the two closed curves.



Tell the class that there is a special name for polygons that have both the properties of the sides being equal in length and the angles being equal in mag: they are called "regular" polygons. The square is a regular 4-sided polygon.

Divide the class into the same groups of three each as you did for Activity A, Lesson 10. Direct the children's attention to the chalkboard tree diagram they made in Lesson 10. Ask the groups to take out their extra set of non-caved polygon cards. Ask them to separate their non-caved polygons into two subsets -- one subset should contain the regular polygons and the other subset the "non-regular" polygons. (The common name for "non-regular" polygons is "irregular." You may want to use both words interchangeably.) When they have completed this task, two people should tape their group's subsets in the appropriate place on the chalkboard tree diagram. The completed tree diagram should look like the diagram below.



The class should examine and discuss each group's placement of the regular and non-regular polygons.

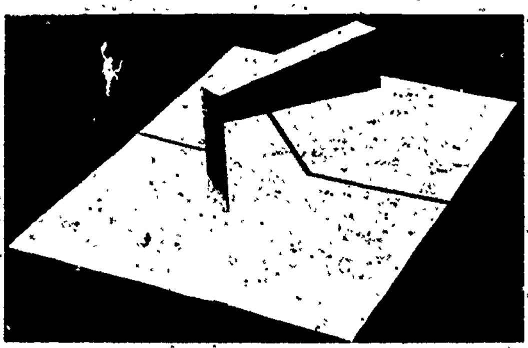
Activity B

Have the children take out their flower card folders from Lesson 9. Using what they have learned about regular polygons, have the children roughly classify (by eye) their sets of flower polygons as regular and non-regular (irregular).

Some children may want to make a tree diagram of their classified flower polygons. They could list the names of the flowers under each category.

Activity C

For this activity, have the children work in pairs. Each pair will need two mirrors on blocks, a blank sheet of paper, a pencil and a ruler. Ask the children to draw a straight line segment across the middle of their paper. Have them place one mirror upright at some angle to the line segment.



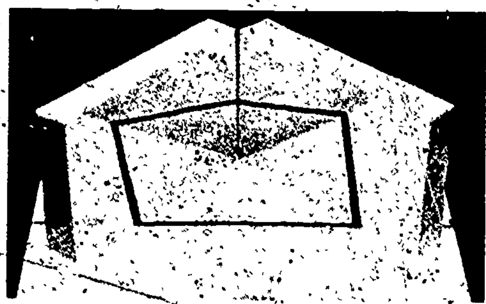
Ask them to look in the mirror and find out how many angles they can see. (One.)

Say:

IF WE PUT ANOTHER MIRROR SO THAT ONE OF ITS EDGES TOUCHES THE EDGE OF THIS MIRROR, (demonstrate by holding two mirrors together but off the paper)

AND THE SECOND MIRROR ALSO FORMS AN ANGLE WITH THE LINE SEGMENT, HOW MANY ANGLES DO YOU THINK YOU WILL SEE? (Let the children predict.)

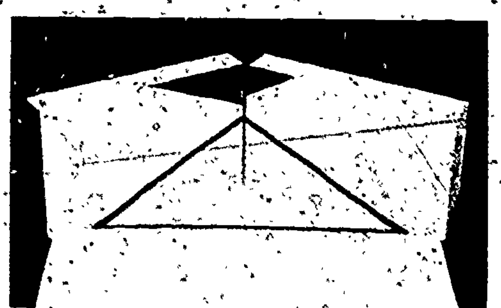
Then have the children put their second mirror on their paper according to your instructions.

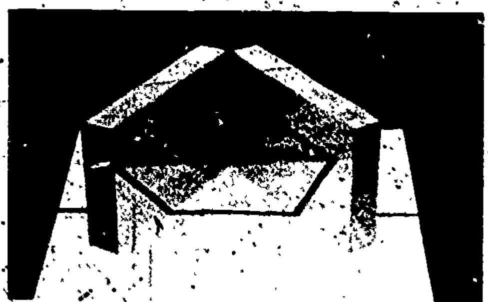


HOW MANY ANGLES DO YOU SEE NOW WHEN YOU LOOK INTO THE MIRRORS?

Call on several children. Their answers will vary, depending on the mag of the angle formed by the two intersecting mirrors.

Ask the children to change the angle between the two mirrors until they are able to see a triangle. When they have done this, ask them to look into a neighbor's mirrors and see if his triangle is just like their own. Let them experiment with their mirrors, making triangles that have sides of different lengths.





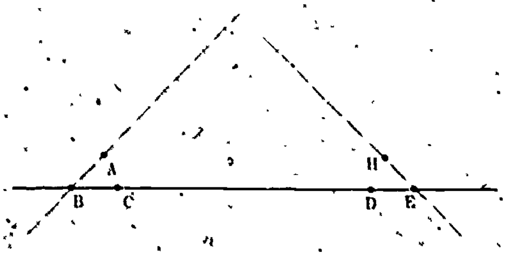
Ask the children to move both mirrors together slowly, making the angle formed by the two intersecting mirrors smaller and smaller.

Ask:

WHAT HAPPENS TO THE NUMBER OF SIDES AND ANGLES AS YOU MOVE THE MIRRORS CLOSER TOGETHER? (The number of angles and the number of sides increase.)

Have the children turn to Worksheets 42 and 43. They will need their two mirrors and their clock protractors. When they have completed Worksheet 43, discuss their answers with them.

Worksheet 42
Unit 21 Name ______

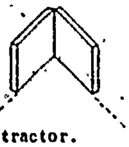


Use your clock protractor.

1. What is the mag of angle ABC? $1\frac{1}{2}$ hours
2. What is the mag of angle HED? $1\frac{1}{2}$ hours

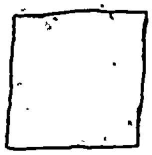
Worksheet 43
Unit 21 Name ______

Use Worksheet 42 to help you answer these questions.
Put 2 mirrors on the dotted lines like this:



Use your clock protractor.

1. If the mag of angle ABC is $1\frac{1}{2}$ hrs, what do you guess the mag is of angle CBC? 3 hours
2. If the mag of angle HED is $1\frac{1}{2}$ hrs, what do you guess the mag is of angle DED? 3 hours
3. Carefully draw the shape you see with the mirrors.



4. Did you guess correctly the mag of angles CBC and DED? yes

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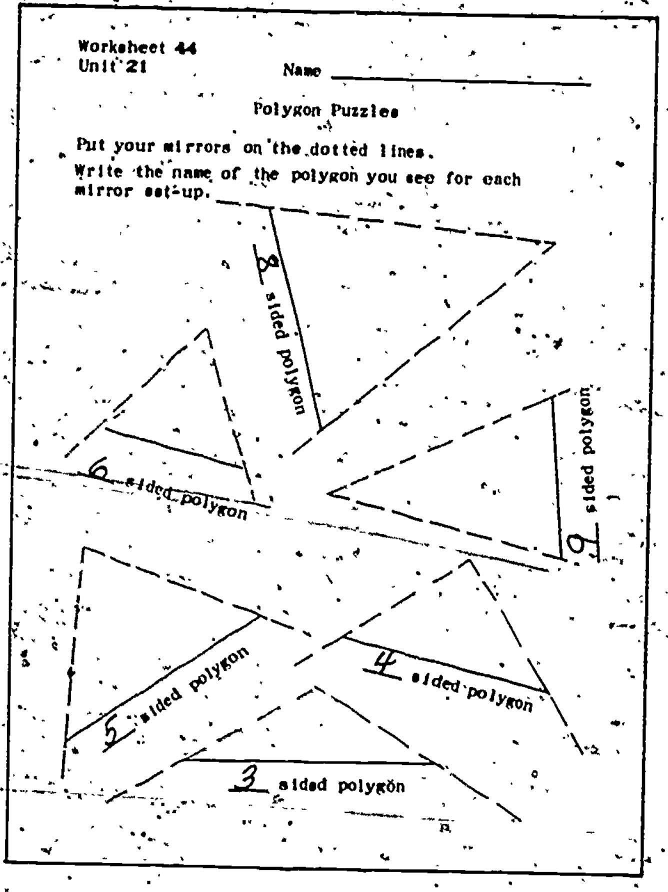
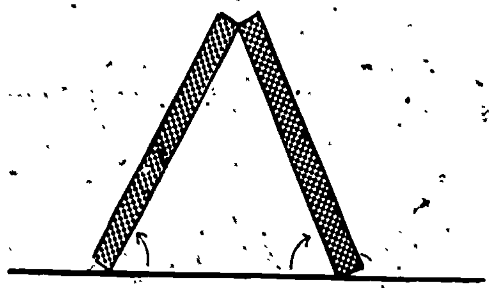
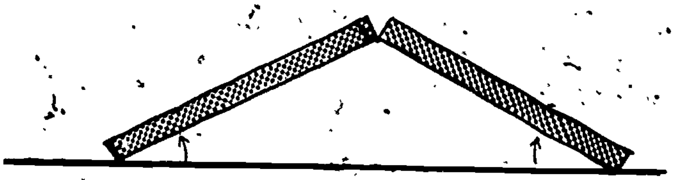


Worksheet 44, "Polygon Puzzles," should be done now by the children. When they are finished, ask them if they noticed any relationship between the size of the angles and the number of sides when they looked in their mirrors.



Greater mag of angles yields polygons with greater number of sides.



Smaller mag of angles yields polygons with lesser number of sides.

Lesson 12: SIMILAR TRIANGLES

The purpose of this lesson is to introduce the concept of similar triangles and to tie together similarity and scaling. The children compare corresponding sides and angles of similar triangles.

Two triangles are similar when their sides are proportional and the mag of their angles is equal; that is, if two triangles are similar we can always find a proportional relationship between the sides of one triangle and the sides of the other triangle, arranged in some arbitrary order.

MATERIALS

- several paper clips
- overhead projector
- yarn (optional)
- tagboard, 1 sheet
- blank transparency

-- for each child --

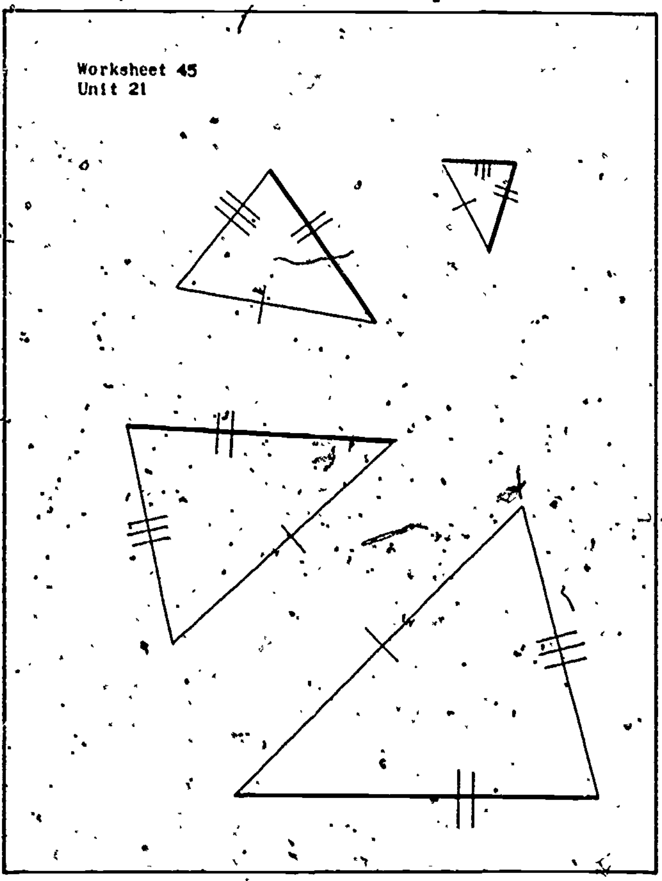
- 4 paper clips
- clock protractor (from Lesson 5)
- scissors
- pencil
- colored pencils: 1 red, 1 blue and 1 green.
- Worksheets 45 - 48 (sets of similar triangles)
- Worksheet 49
- ruler

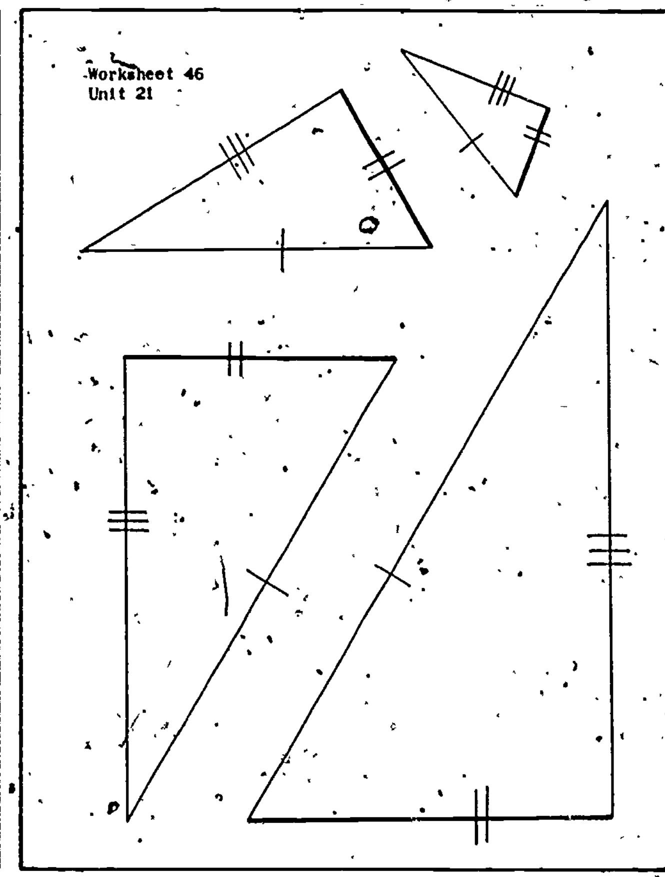
PROCEDURE

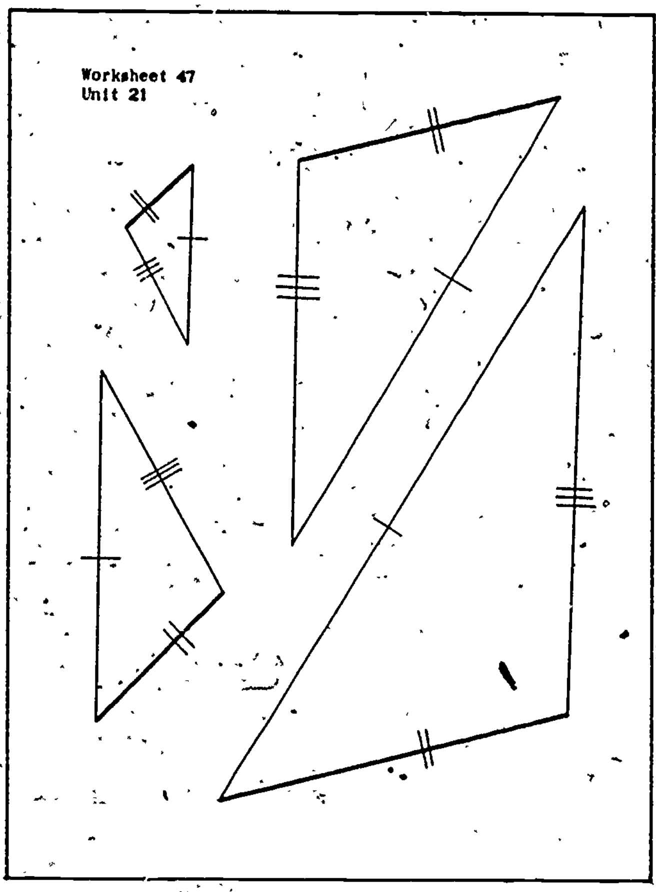
Activity A

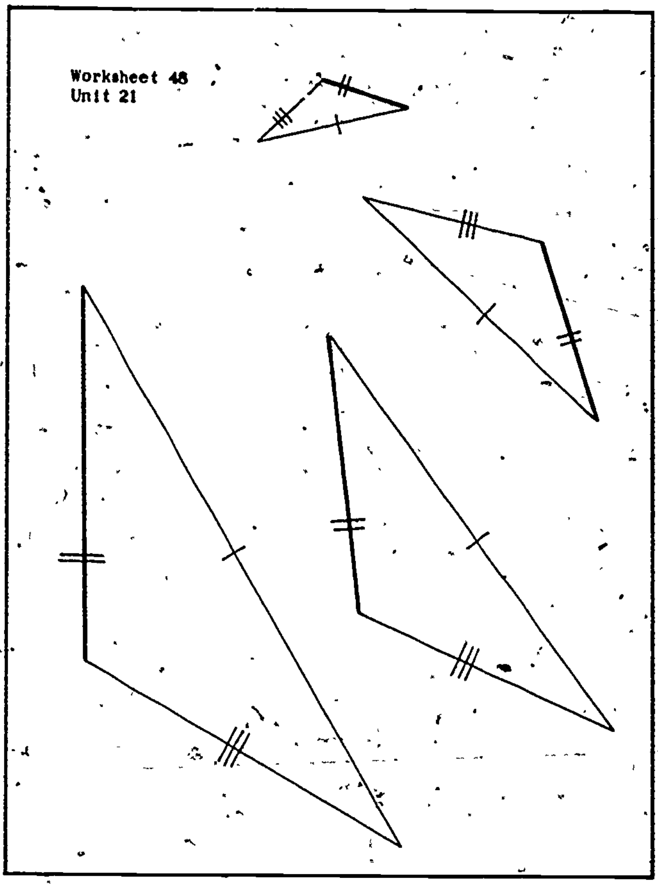
The purpose of this activity is for the children to discover the relationships in a set of similar triangles. They should see that the corresponding angles of similar triangles are equal in mag, and that the corresponding sides of similar triangles are in proportion. The concept of proportion presented in this lesson is related to the concept of scaling presented in Unit 18, Scaling and Representation.

Each child will need a pair of scissors, a pencil, a red colored pencil, a blue colored pencil, a green colored pencil and his Student Manual. Have the children tear out Worksheets 45 - 48.

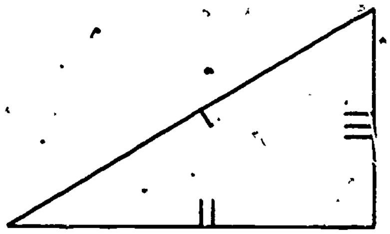




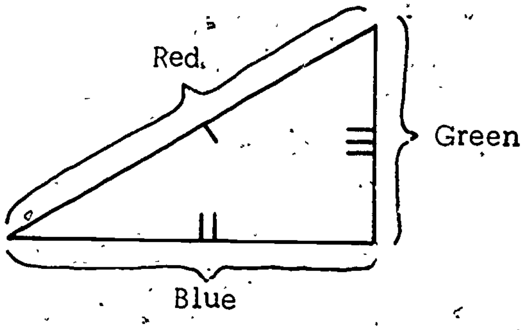




Ask someone to describe the set of triangles on Worksheet 45 to the class. He should note the different sizes of the triangles and the "hash" marks on the sides of the triangles; some sides have one "hash" mark, some have two "hash" marks, and others have three "hash" marks. On the chalkboard draw a large triangle with 1, 2, and 3 "hash" marks on the respective sides.

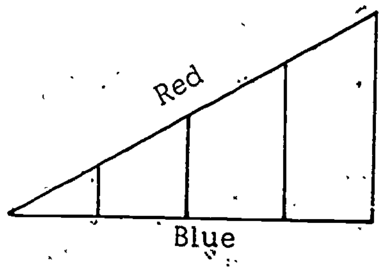


With red chalk, outline the side with one "hash" mark. Tell the class to outline with their red colored pencil the sides of all the triangles on Worksheets 45 - 48 that have one "hash" mark. When everyone has completed this, color the side with two "hash" marks blue and the side with three "hash" marks green. Tell the children to do the same with the triangles on Worksheets 45 - 48.



Now tell the children to cut out the set of triangles on Worksheet 45 very carefully. When they are finished, each child should stack his triangles in order, with the largest on the bottom of the stack. Ask them to place their triangles in such a way that the blue sides are on top of each other, and the red-blue angles are also on top of each other.

They should cut out the sets of triangles on Worksheets 46, 47 and 48 and stack them the same way.



Ask them what they notice about the mag of the red-blue angles. (They appear to be equal in mag — they all fit on top of each other evenly.) Tell the class that their sets of triangles have certain relationships that are very interesting. Say that if they compare the other angles and the lengths of the sides they should be able to discover what these interesting relationships are. Ask them to raise their hands when they think they have discovered something. Allow time for free experimenting and discussion.

You may want to walk among the children drawing attention to certain methods of comparison being used. Some children may want to use their clock protractors for comparing the mag of the angles, others may use the superposition method.

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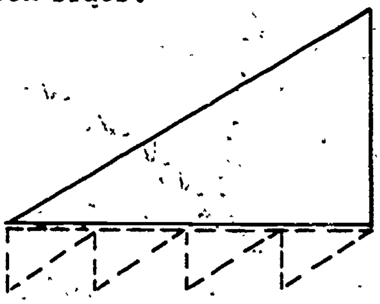
There are various methods that the children can use to compare corresponding sides. They could use the smallest triangles as their standard unit of measure, marking off how many small blue sides there are on the blue sides of each of the other triangles. This method can also be used to compare the red and green sides.



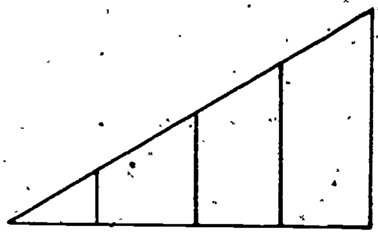
Some children may want to use a ruler to compare the lengths of corresponding sides; however, many of the measurements will not come out to an even number of centimeters or inches.

After you feel the children have had enough time for experimentation, have different children show and discuss the relationships they discovered. The following is a list of a few concepts that could be discussed:

1. The corresponding angles (the angles with the same colors) are equal in mag. You may want to introduce the word "corresponding" at this time, giving examples of things that can go together — left and right hands have corresponding points, etc. In this case, the red sides are corresponding sides, as are the green sides and the blue sides.

2. The corresponding sides are in proportion. You may not want to use the word "proportion," but discuss the relationships of the length of the sides, i.e., the red sides of the smallest triangle and the largest triangle have a $1 \rightarrow 4$ relation, etc.

3. Some children may notice that if they stack the triangles in order from largest to smallest and line up one corresponding side and angle, the sides opposite the angle form parallel line segments.



Tell the children we have a special name for these triangles: they are called "similar triangles."

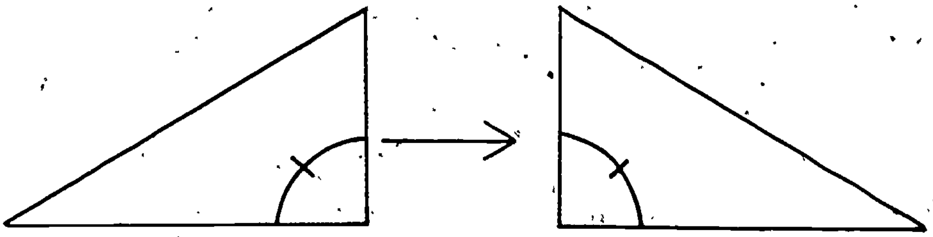
Give each child a paper clip to keep his triangles together. They will be used in Activity B.

Activity B

Divide the class into groups of four and ask the children in each group to move their desks close together. They should remove the paper clips from their sets of similar triangles and then each group should thoroughly mix its sets of triangles, turning some over in the process.

Using what they have learned about similar triangles, the children should now be able to sort their combined sets of triangles into sets of similar triangles again.

On the overhead projector place a set of similar triangles right side up. Flip over one of the triangles. Ask the children if this triangle is still similar to the other triangles.



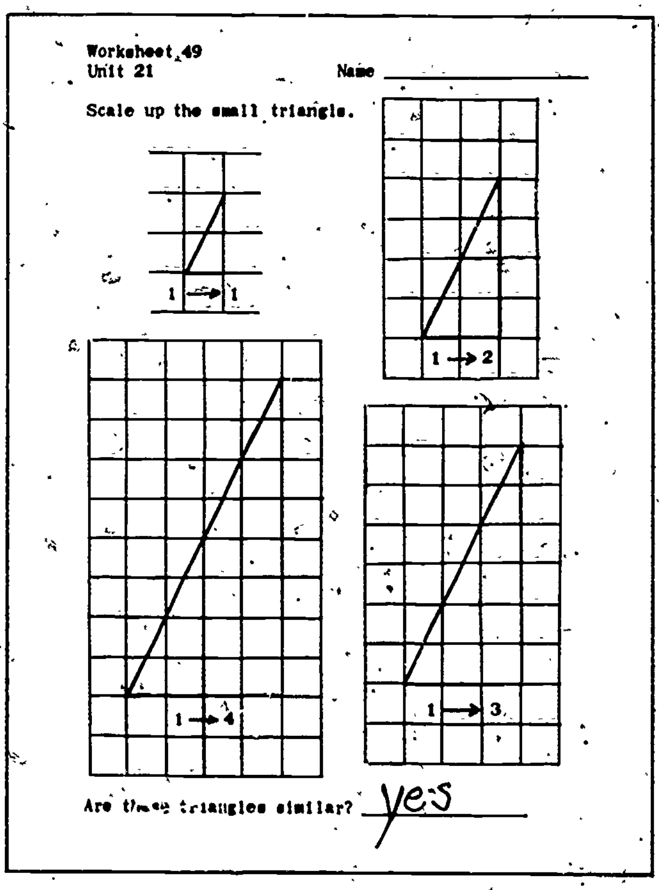
The following definition of similar triangles is provided for your information and the children should not be expected to repeat it. "Two triangles are similar if their corresponding angles have the same mag or their corresponding sides are in the same ratio." Therefore, since flipping a figure over does not change the figure, the two triangles are still similar.

Discuss this idea with the children, in terms that they can understand.

You may want to make a bulletin board display using the children's sets of similar triangles. You could use yarn to make a closed curve around each set of triangles, or arrange each set on a large sheet of different colored construction paper.

Activity C

In this activity the children use what they learned about scaling in Unit 18, Scaling and Representation, to generate a set of similar triangles. You may find it necessary to quickly review the notation used in Unit 18, i.e., $1 \rightarrow 3$, $1 \rightarrow 2$, $1 \rightarrow 1$, etc. and the term "scale up." Have the children complete Worksheet 49.


Worksheet 49
Unit 21
Name

Scale up the small triangle.

Are these triangles similar? yes

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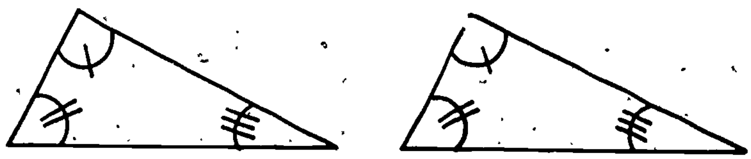
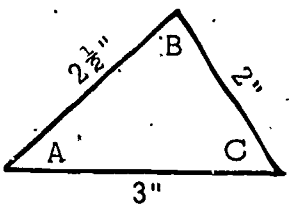
Activity D

So far, the children have worked with similar triangles whose sides are in integral proportions, i.e., $1 \rightarrow 2$, $1 \rightarrow 3$, $1 \rightarrow 4$, etc. In this activity the class works together constructing a set of similar triangles whose sides are not in integral proportion. While these similar triangles are being constructed the children should also notice that once two equal corresponding angles are constructed,

the third corresponding angles will also be equal.

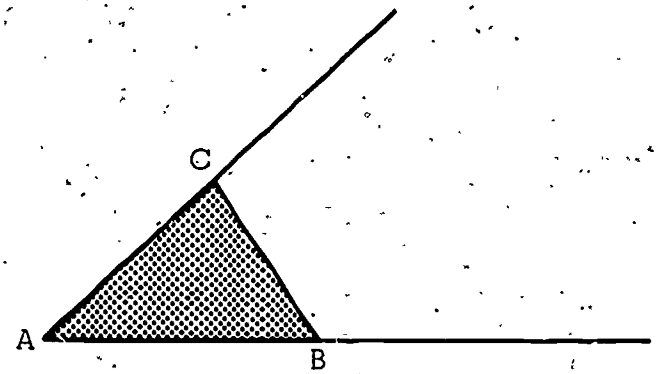


Cut a triangle like the following out of a piece of tagboard.

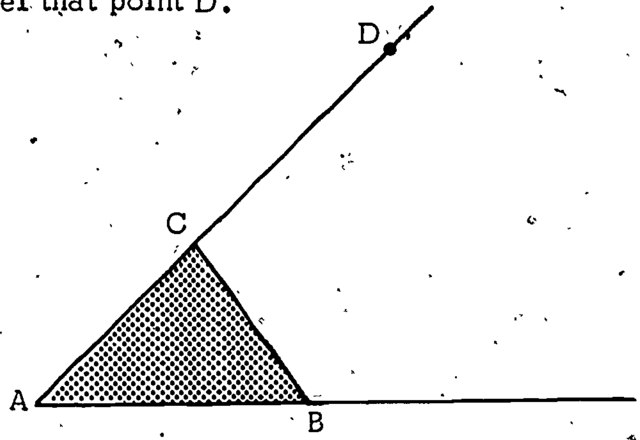


Outline your triangle cutout on a blank transparency. Label the three vertices. Project it on the overhead projector.

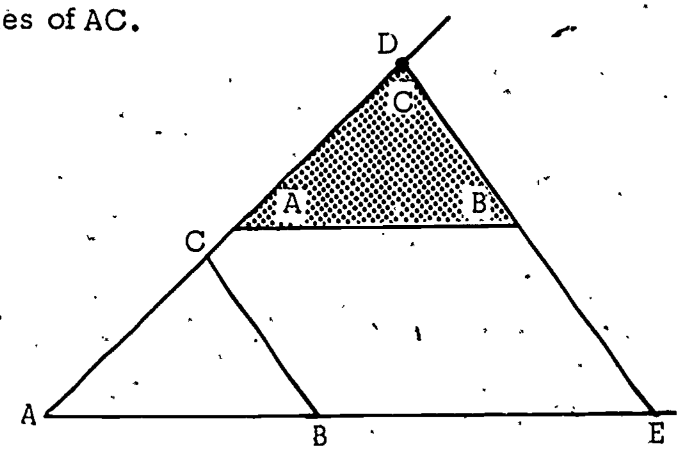
Use a ruler to extend line segments AC and AB for approximately 3 or 4 inches.



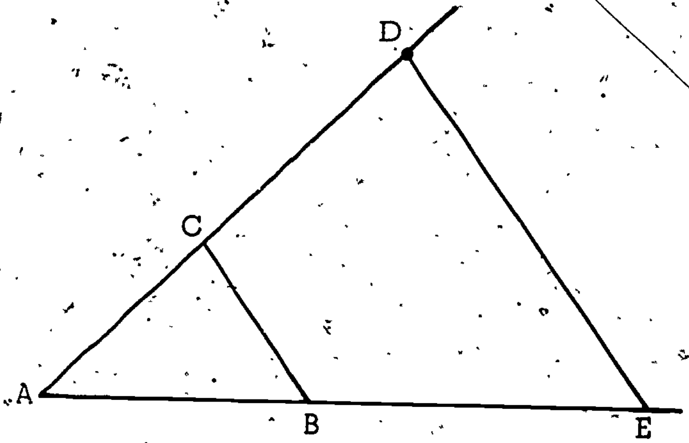
Ask a child to come up and mark any point on line segment AC. Label that point D.



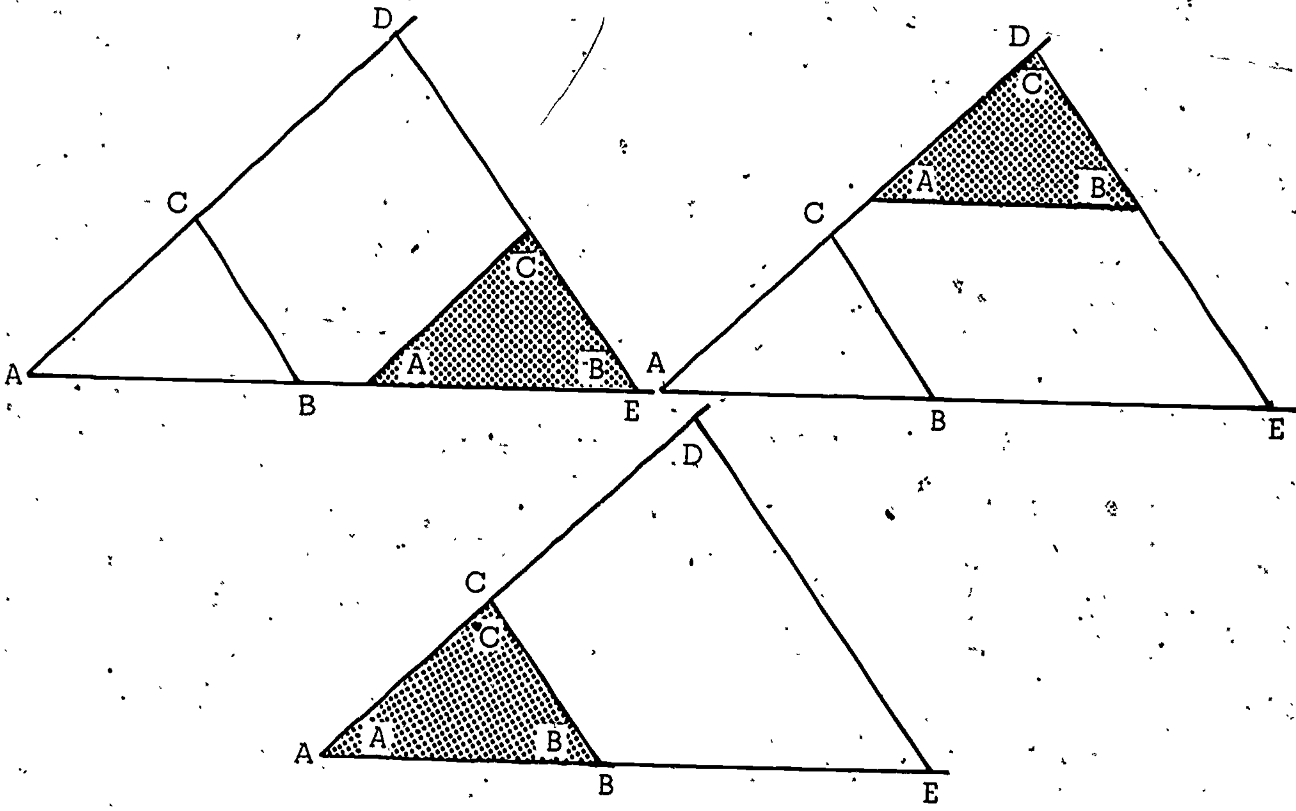
Place point C of your cutout triangle on point D and line up the sides of AC.



Outline side CB of the cutout triangle. With your ruler, extend this line segment until it intersects line segment AB. Label this intersection E.

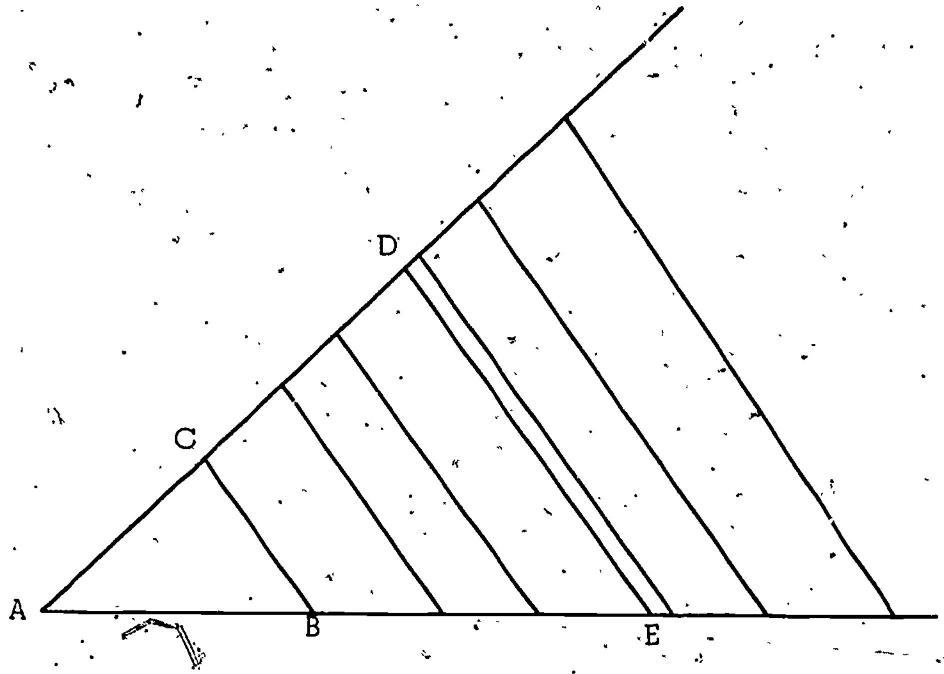


Outline triangle ADE in red. Ask the children if triangle ADE is similar to triangle ACB. Have someone show the class that they are similar by moving the cutout triangle ABC so that B coincides with E to show that those two angles are equal. Then move the cutout triangle so that C coincides with D to show that those two angles are equal. Finally, move the cutout triangle so that A coincides with A to show that those two angles are equal.



Ask the children if they notice any particular property of line segments CB and DE. (They are parallel, but the children might not know or use this word.)

Using cutout triangle ABC, construct several other similar triangles using this same procedure. The finished transparency might look like this.



Lesson 13: CONGRUENT FIGURES

In this lesson we introduce an intuitive approach to congruence. A congruent figure is a special kind of similar figure. Two geometric figures are congruent if they have exactly the same size and shape — when superimposed on one another they will fit exactly. The children discover that the congruency of two figures does not change when one of the figures is flipped over.

The concept of congruency can also be applied to symmetrical patterns.

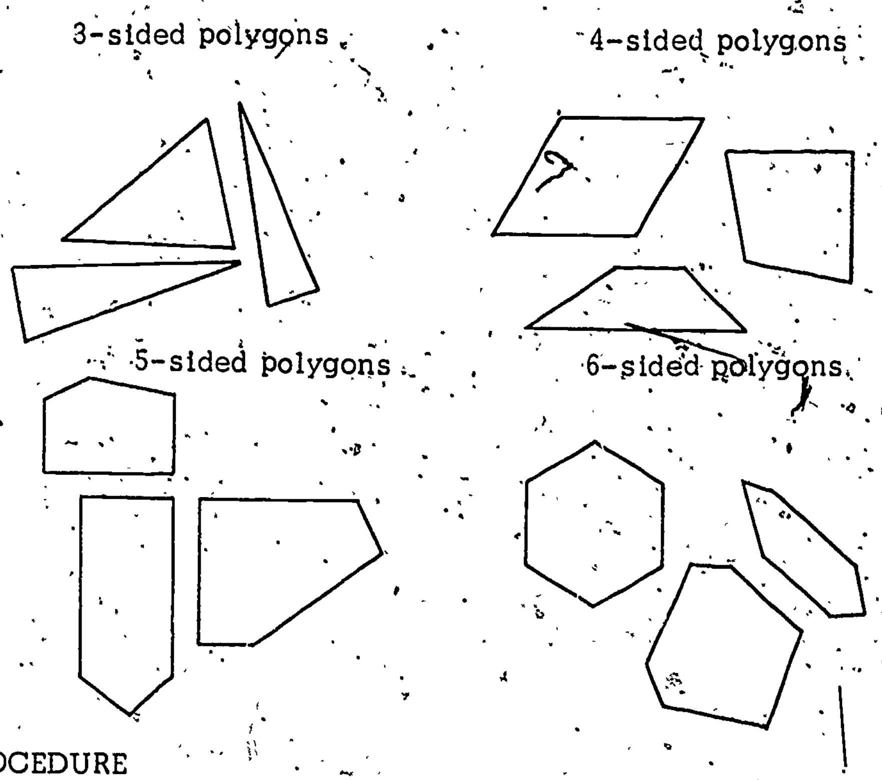
MATERIALS

- 10-12 sheets of 12" x 18" construction paper
- scissors
- Transparency B (from Lesson 11)
- bottom half of Transparency A (from Lesson 11)
- overhead projector
- transparent tape
- Worksheet 50

PREPARATION

You will need cutout congruent polygons of three, four, five and six sides. You can prepare these cutouts easily and quickly by putting two or three sheets of construction paper on top of each other and cutting out the figures freehand.

Each polygon should be approximately 4 or 5 inches across. You should be able to cut out 4 to 6 polygons from a large sheet of construction paper (12" x 18"). (See examples on the next page.)



PROCEDURE

Activity A

Gather the class into a group and put the set of cutouts in the center of the group. Choose any one of the cutouts and hold it up. Ask the class if anyone can find another polygon cutout just like the one you're holding. Let the children suggest some methods for finding the matching polygon. Call on a volunteer to find a matching polygon. He will probably rummage through the set of cutouts looking for one that matches. When a few figures have been tried, suggest that this method takes too much time. Ask:

IS THERE SOMETHING WE COULD DO TO MAKE IT EASIER TO FIND A POLYGON THAT GOES WITH THIS ONE?

Lead the children to suggest that they should sort the polygons into subsets of polygons that have the same number of sides. Call on four children to sort the set of cutouts into subsets of three, four, five, and six-sided polygons.

When the subsets are assembled, divide the class into four groups and assign each group to a set of cutouts. Tell them that each polygon in the set has at least one other matching

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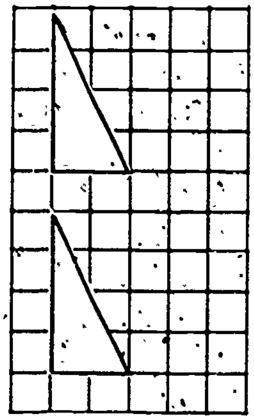
polygon, and that they should sort their set into subsets of polygons that are alike. Let them devise their own method of sorting.

When the children have finished sorting their cutouts, ask a child from each group to demonstrate the method his group used to find the matching cutouts. Some children will report that they found the matching polygons when the sides were the same, and show two superposed figures. Elicit the idea that all corresponding sides have to be the same length in order for the cutouts to be the same. If no one has mentioned it, direct the children's attention to the angles. They should also see that the corresponding angles appear to be equal in mag.

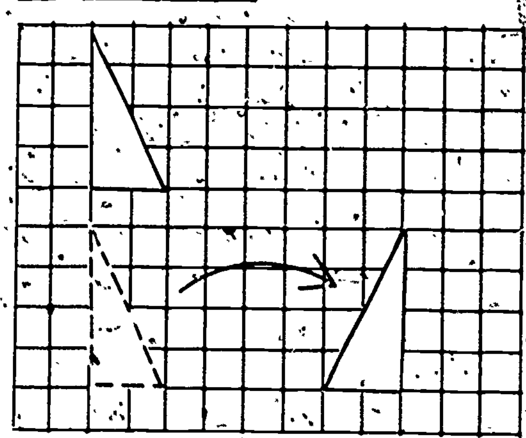
Tell the children we have a special name for figures that will fit on top of each other or figures that "match"; they are called "congruent." Write the word "congruent" on the chalkboard.

Activity B



Cut out the two right triangles from the bottom half of Transparency A (red acetate). You will also need Transparency B (clear grid transparency).

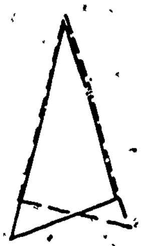
Place the red, cutout triangles on the grid transparency.



Flip over one of the triangles.

The sides of the right angles should be lined up with the grid lines. You may want to tape the two triangles to the grid transparency.

Ask the class if they think these two triangles are congruent. Have various children give reasons for their answers. The final verification should be to superimpose one triangle on the other. Do this without flipping the one triangle back over. You will not be able to place one neatly on top of the other.



Superpose corresponding angles on each other. The children should see that all the corresponding angles are equal. They should also see that the corresponding sides are equal (either by measuring with the grid, or lining up each pair of sides). Ask:

ARE THESE TWO TRIANGLES CONGRUENT?

The children will probably disagree — the corresponding sides and angles are equal, but they can't seem to be able to fit one on top of the other so they fit exactly.

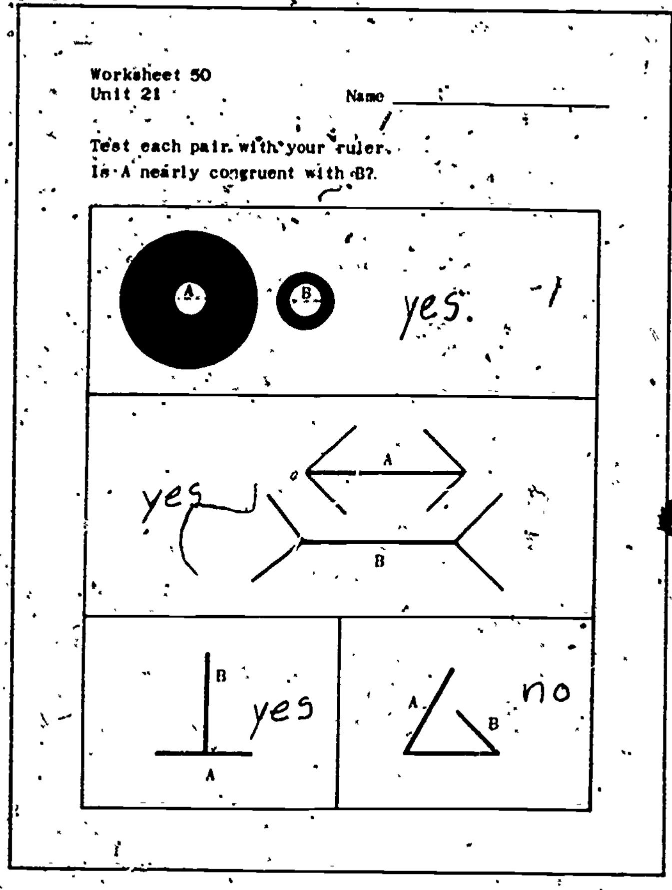
Then ask:

IS THERE ANYONE WHO CAN DO SOMETHING SO THAT THESE TWO TRIANGLES WILL FIT EXACTLY ON TOP OF EACH OTHER?

From their experiences in Activity A, someone should be able to suggest that one of the triangles be flipped over. When this is done, the triangles will fit exactly on top of each other. The triangles are congruent.

So far, we have limited ourselves to congruent polygons; however, the concept of congruency can be applied to many geometric figures. Line segments are congruent if they have the same length. Angles are congruent if they have the same mag. Regions are congruent if they have the same area and shape.

13_

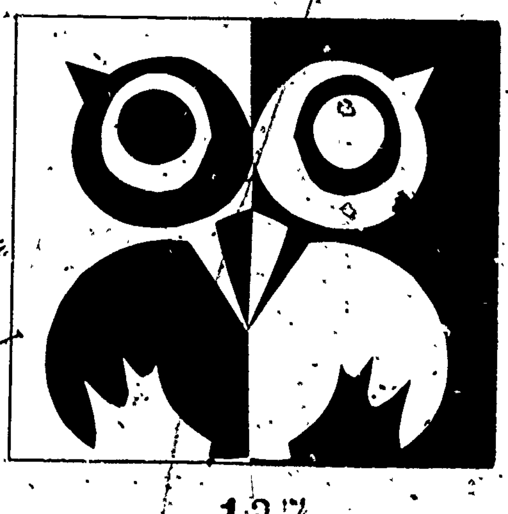


Discuss these ideas with the children and then have them do Worksheet 50.

Activity C (Optional)

Units 7 and 14 are MINNEMAST symmetry units. Many of the art activities in these units could be used at this time as examples of congruency.

Example. page 86, Unit 14, Exploring Symmetrical Patterns.



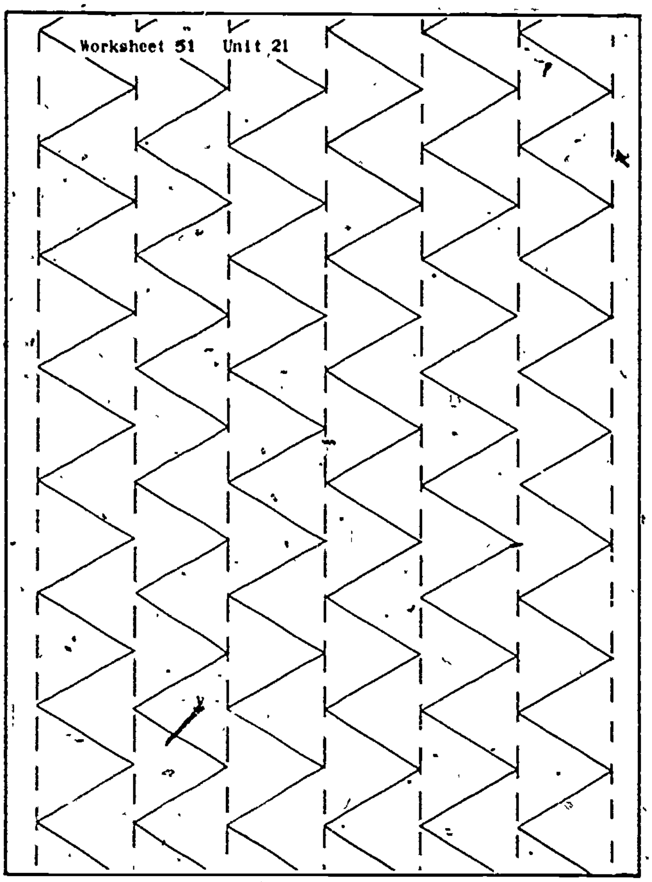
Lesson 14: PATTERNS WITH POLYGONS; TESSELATIONS

Throughout this lesson, the children are given many different problems to look at, think about and solve. You should involve the children in discussion of the questions and manipulation of the materials, get them to think about the problems and try to solve them for themselves. In some cases you may want to leave drawings and questions on the board to give the children time to think about them and to arrive at their own conclusions. The major emphasis in the problems is on finding patterns in designs and shapes. Doing this kind of work develops skill in looking for relationships among elements. The ability to see such relationships is fundamental in understanding mathematics and science.

The word "tesselation" means any arrangement of polygons that fit together to cover a plane surface completely, with none of the polygons overlapping. The polygons can be regular or irregular, concave or convex and may form a symmetrical or an asymmetrical tesselation when they are fitted together. In this lesson, the work is limited to those tesselations which exhibit repeating symmetry. Although the children will be working with and forming tesselations, you need not teach the word "tesselation" to them.

MATERIALS

- scissors, 1 per child
- paper cutter
- 9" x 12" black construction paper, 2 or 3 sheets per child
- overhead projector
- globe
- baggies (small plastic bags), 5 per child
- crayons
- construction paper and paste (optional)
- Worksheets 51-61



PROCEDURE

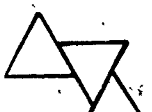
Activity A

Have the children cut off two strips of triangles from Worksheet 51. Then they should carefully cut out the triangles in each strip. Discuss with the class the properties of the equilateral triangles. (All sides are of equal length, all angles are of equal mag.)

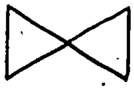
After the triangles have been cut out, give each child a sheet of 9" x 12" black construction paper. The triangles will be easier to manipulate on the coarse construction paper than on a smooth desk top. Ask the children to make different shapes by placing their triangles together. Allow time for free experimenting. Some children may want to show the class some of the shapes they make. You could use the overhead projector for this. The following are some shapes they might make.

1. Sides together, so that they coincide.

2. Sides together, but without coincidence.

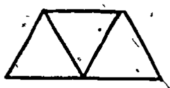
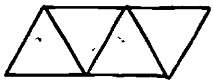
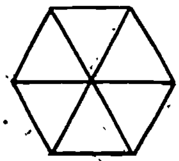


3. Vertices together.

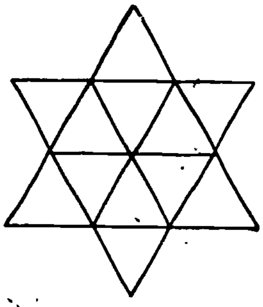


4. Triangles on top of other triangles.

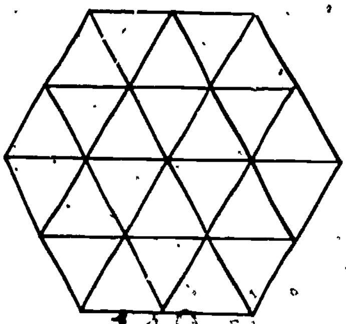
Tell the children that they should use a rule that they can only make shapes that have the sides of the triangles together with the endpoints of the triangles matching (see #1 on page 129). After the children have had time to experiment, discuss several basic shapes they can make this way.

rhombus	trapezoid	parallelogram	hexagon
			

Suggest to the children that they try to discover more shapes by combining other shapes. Some may make a star.



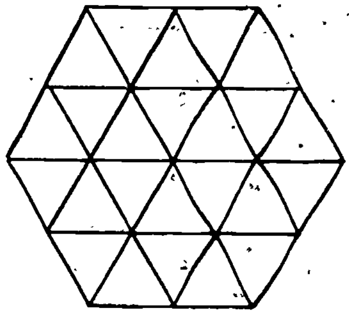
Show them how a larger hexagon is formed if more triangles are added to the star.



Activity B

Let the children investigate the following problems. Put the drawings on the board. You may also want to write the questions on the board and leave them up for several days. Go on to other activities in this lesson and then come back to this one later and discuss the children's observations with them.

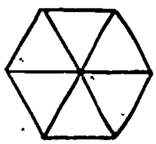
1.



HOW MANY SMALLER HEXAGONS CAN YOU FIND WITHIN THIS LARGER HEXAGON? THE TRIANGLES OF THE SMALLER HEXAGONS CAN OVERLAP. (7 smaller, overlapping hexagons.)

WHAT HAPPENS IF WE BUILD ONTO THIS FIGURE WITH TRIANGLES? (We can get bigger and bigger hexagons with other shapes inside of them.)

2.



WHAT DIFFERENT SHAPES CAN YOU FIND WITHIN THIS HEXAGON? THEY CAN OVERLAP OR NOT OVERLAP. (There are six rhombuses that overlap. There are three rhombuses that do not overlap. If someone notices this, ask him how many different ways this hexagon can be separated into three rhombuses. There are also six trapezoids that overlap.)

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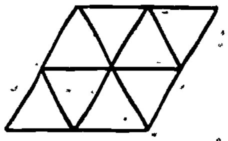
3.



HOW MANY SMALLER RHOMBUSES ARE THERE WITHIN THIS LARGER RHOMBUS? WHAT HAPPENS IF WE BUILD ONTO IT WITH TRIANGLES?

Follow the same procedure with the trapezoid and the parallelogram as the starting shapes.

Eventually the children should see that as they continue to build on with triangles, they will form larger and larger versions of the starting shapes.

Activity C

Have the children work in pairs. They should use their supply of equilateral triangles to try to tile a piece of black construction paper. Have them cut extra strips of triangles from Worksheet 51 as they need them. Ask them if they can cover the paper completely without leaving any spaces and without overlapping. This is possible only if they cut the triangles when they get to the edges of the paper. Ask them to try covering one of the corners of the paper with triangles. They should see that they would have to cut some triangles to cover the corner neatly, too. Then ask:

COULD WE COVER THE WHOLE CLASSROOM FLOOR WITHOUT LEAVING ANY SPACES?

Let the children speculate. Some children may want to try to cover part of the floor with their triangles. Eventually someone may suggest that he would have trouble with the corners and edges. Ask:

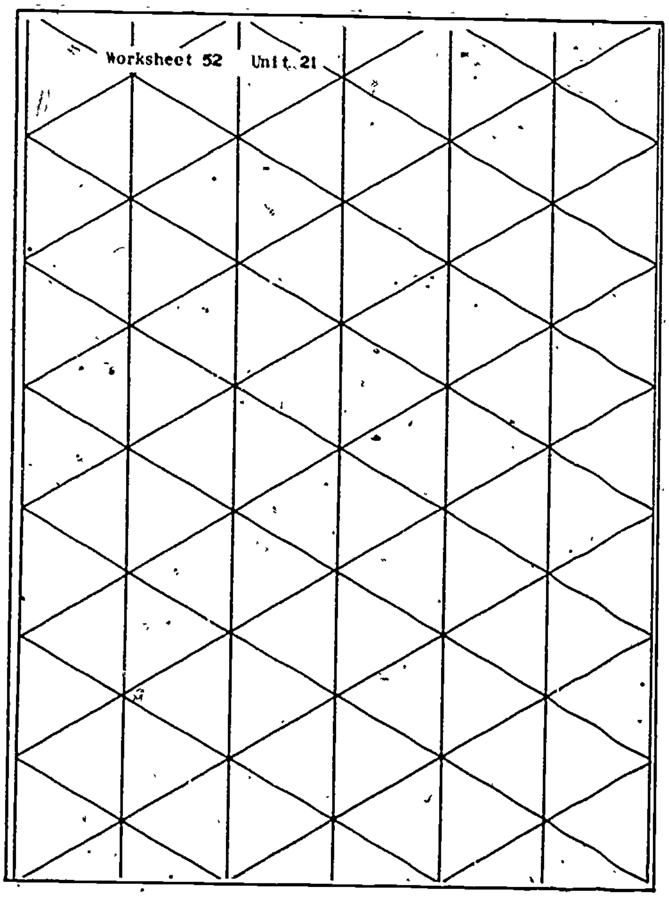
COULD WE COVER THE WHOLE PLAYGROUND? (Yes, but we might have difficulty at the corners and edges.)

WHAT SHAPE WOULD THE PLAYGROUND HAVE TO BE SO THAT THERE WOULD BE NO PROBLEM AT THE EDGES? (Triangle, rhombus, trapezoid, parallelogram, hexagon, etc.)

COULD WE GO ON COVERING THE GROUND WITH TRIANGLES AND LEAVE NO OPEN SPACES?

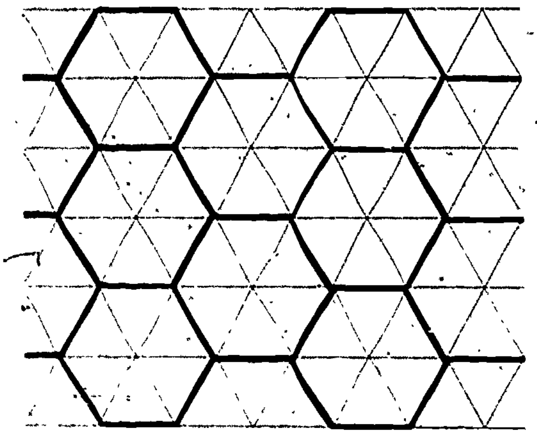
Some may say yes, if the ground was flat. Someone else may say you would have trouble eventually, because the earth is not flat, it is round. Have a child try to cover a section of the globe with his triangles. The children should see that they have to bend or disfigure the triangles in order to cover an area of the globe.

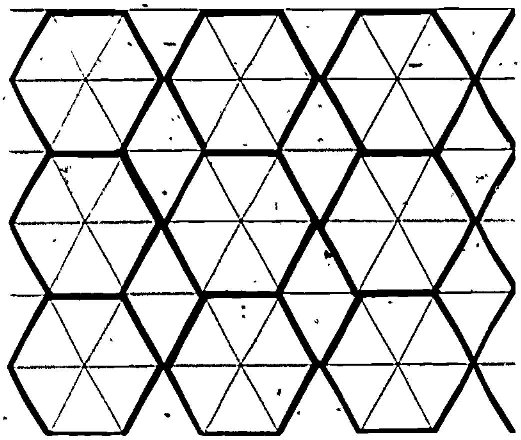
Give each child a baggie in which to keep his cut out triangles. They will be used again in Lesson 15.



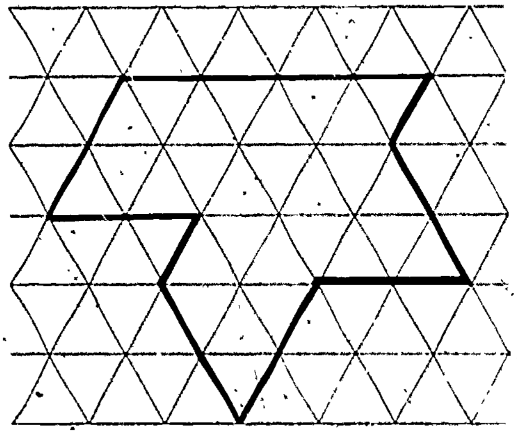
Activity D

Have the children look at Worksheet 52. Discuss the repeating symmetry of the equilateral triangles. Then ask the children if they can see any shapes other than triangles on the page. This should get them to look for combinations of triangles that make other polygons, as they did in Activity A when they fit cutout triangles together in different combinations. When a child finds a shape, ask him to outline it. Then ask if he can find another shape like the first one. They should go on outlining the same shape wherever they find it on the page. Some children may pick out a shape and repeat it in such a way that the page is tesselated. Others may make a design that is not a tesselation. Shown below are two examples of a design in which the hexagon is the single element. The first example is a tesselation; the second is not, because the surface is not completely covered. (If, however, we think of the second design as being made up to two elements -- the hexagon and the diamond shape -- it is a tesselation.)





Some children may pick out an initial shape of which they cannot find another example.



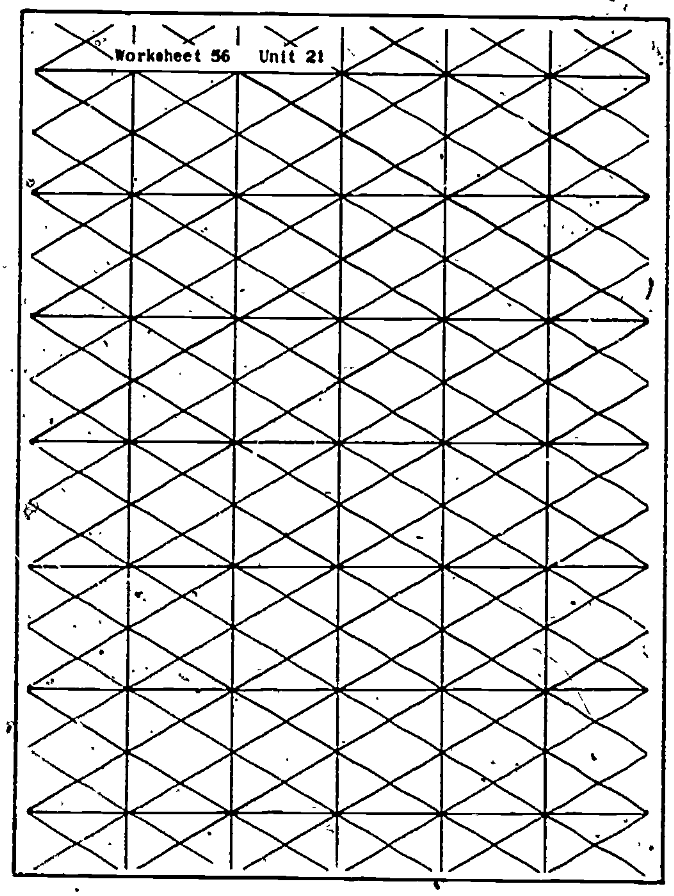
Don't emphasize the idea of making a tesselation, but rather, have the children compare and contrast their designs with each other. Hold a discussion about the different kinds of designs.

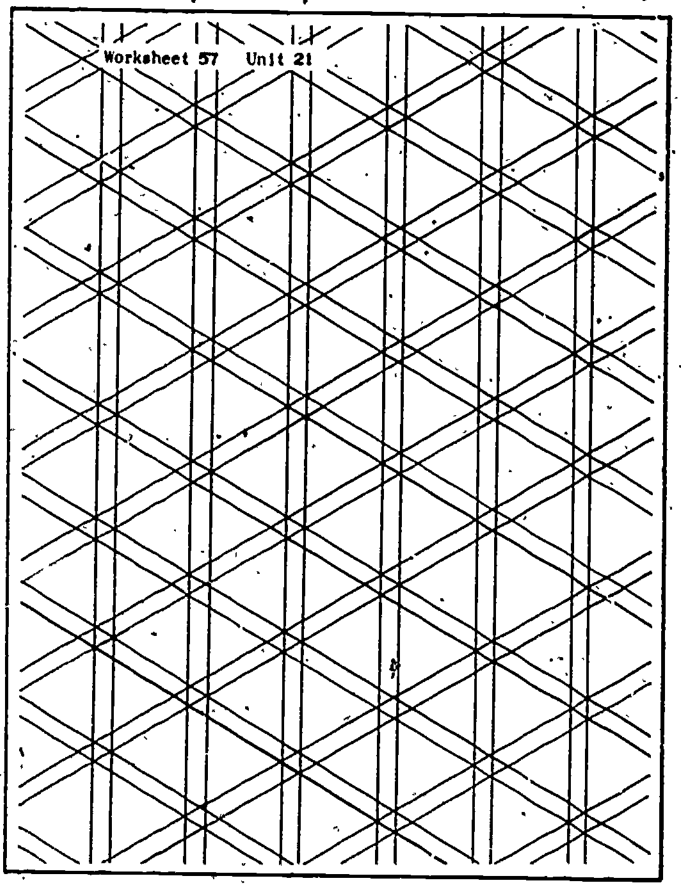
This activity continues on Worksheets 53-55. These worksheets are the same as Worksheet 52, except for the worksheet number.

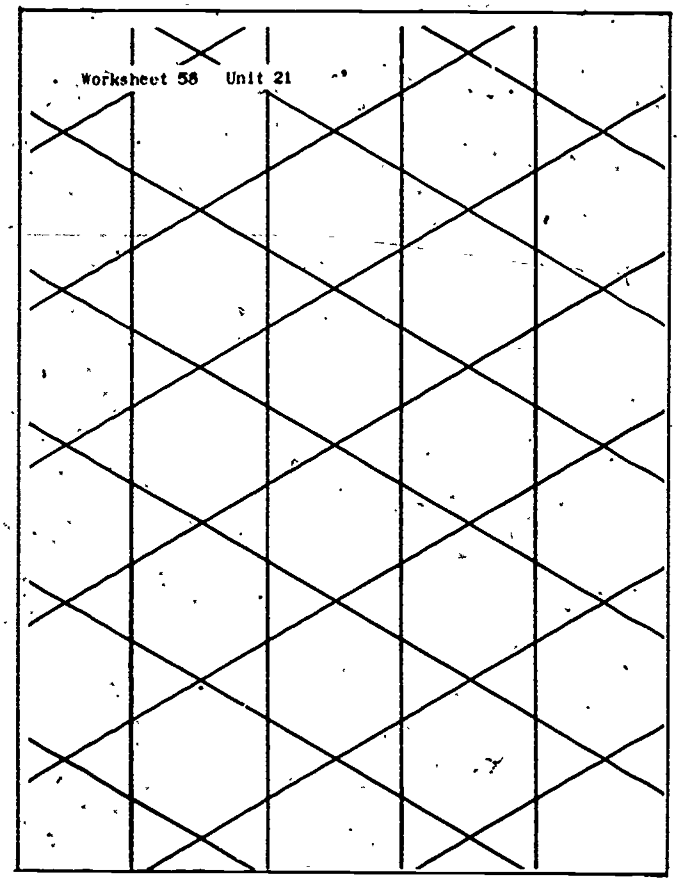
Activity E

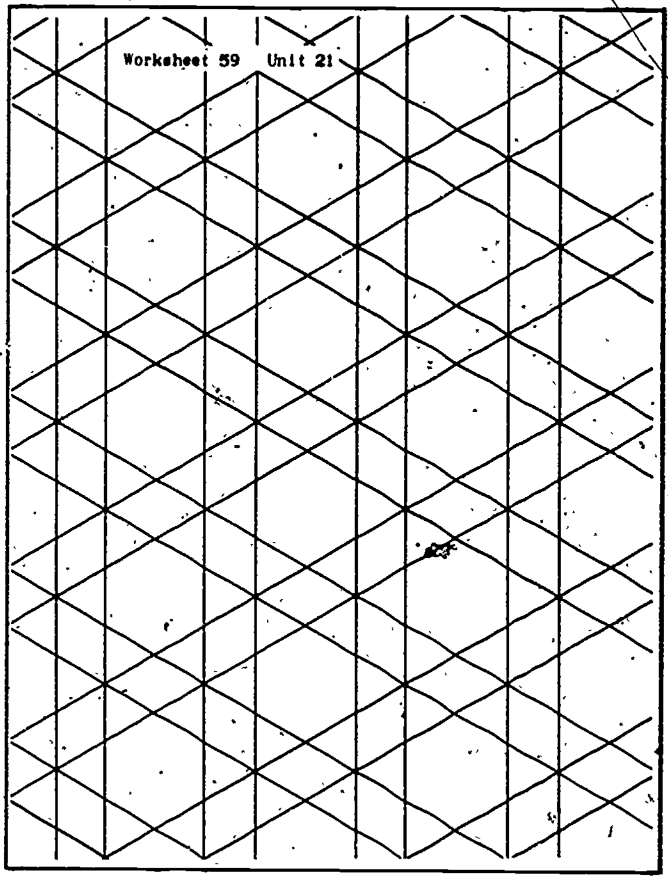
On Worksheets 56-59 are more complex patterns than that on Worksheet 52. The children should do the same thing on these worksheets that they did on Worksheets 52-55: outline a shape or shapes, try to find that shape again (i.e., repeat the pattern), and then color the pattern. Suggest that they try to maintain repeating symmetry in their coloring.

Encourage the children to design their own patterns and to color them. Discuss where the patterns they have made might be used or found. (Bathroom floor tiles, kitchen floor patterns, wallpaper designs, carpet designs, brick wall designs or garden landscaping.)



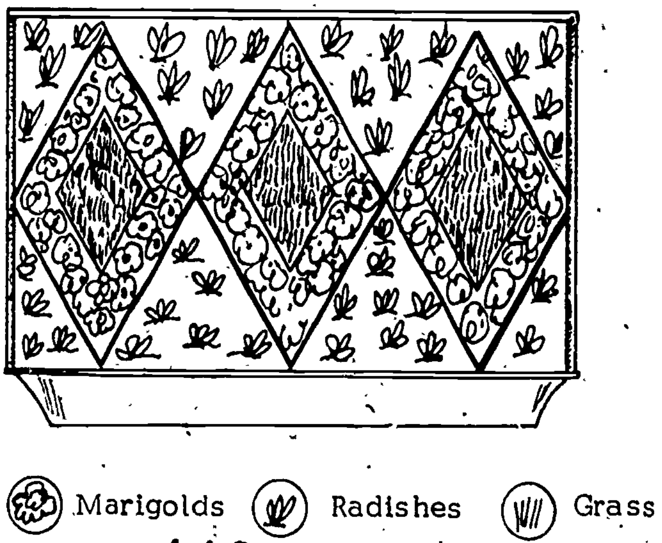


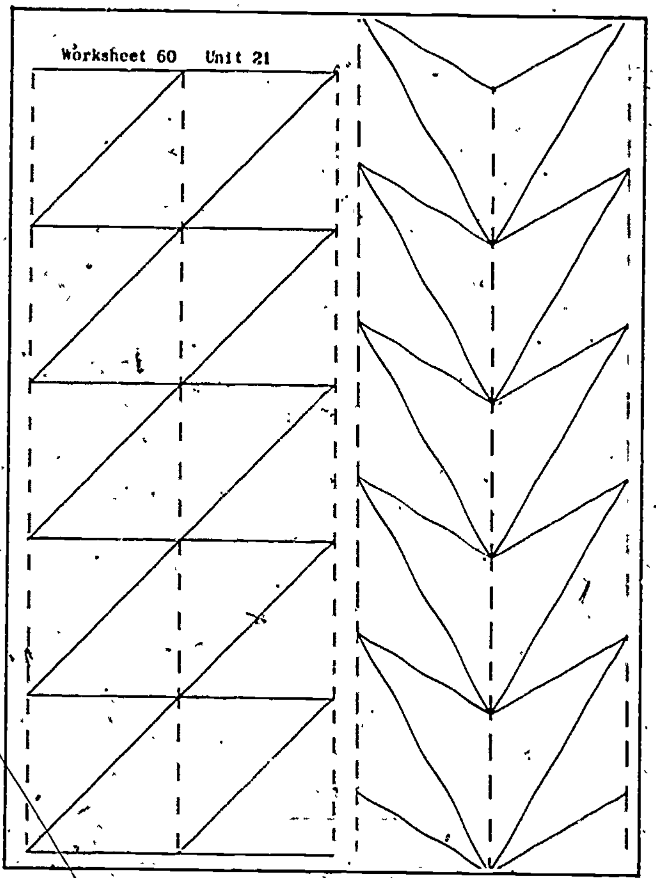


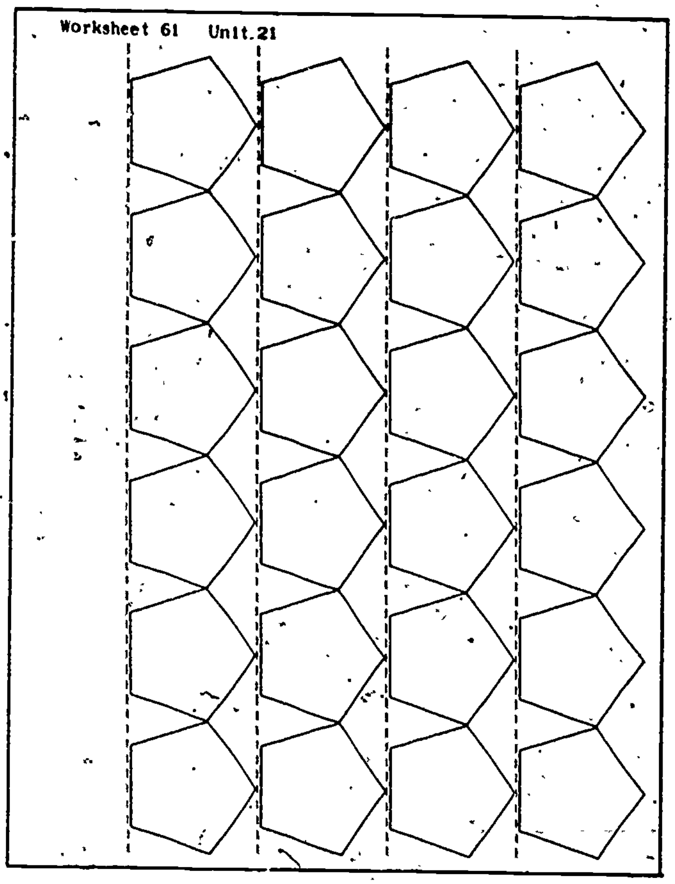


Activity F (Optional)

The children might want to design a pattern that can be used for a tile floor or for landscaping. They could also design a tesselated flower and vegetable garden and plant seeds in large trays, following their pattern. Below is an example of such a garden.







Activity G

Have the children remove Worksheets 60 and 61 from their Student Manuals and cut them into strips along the dotted lines. Then they should cut out the individual triangles and pentagons.

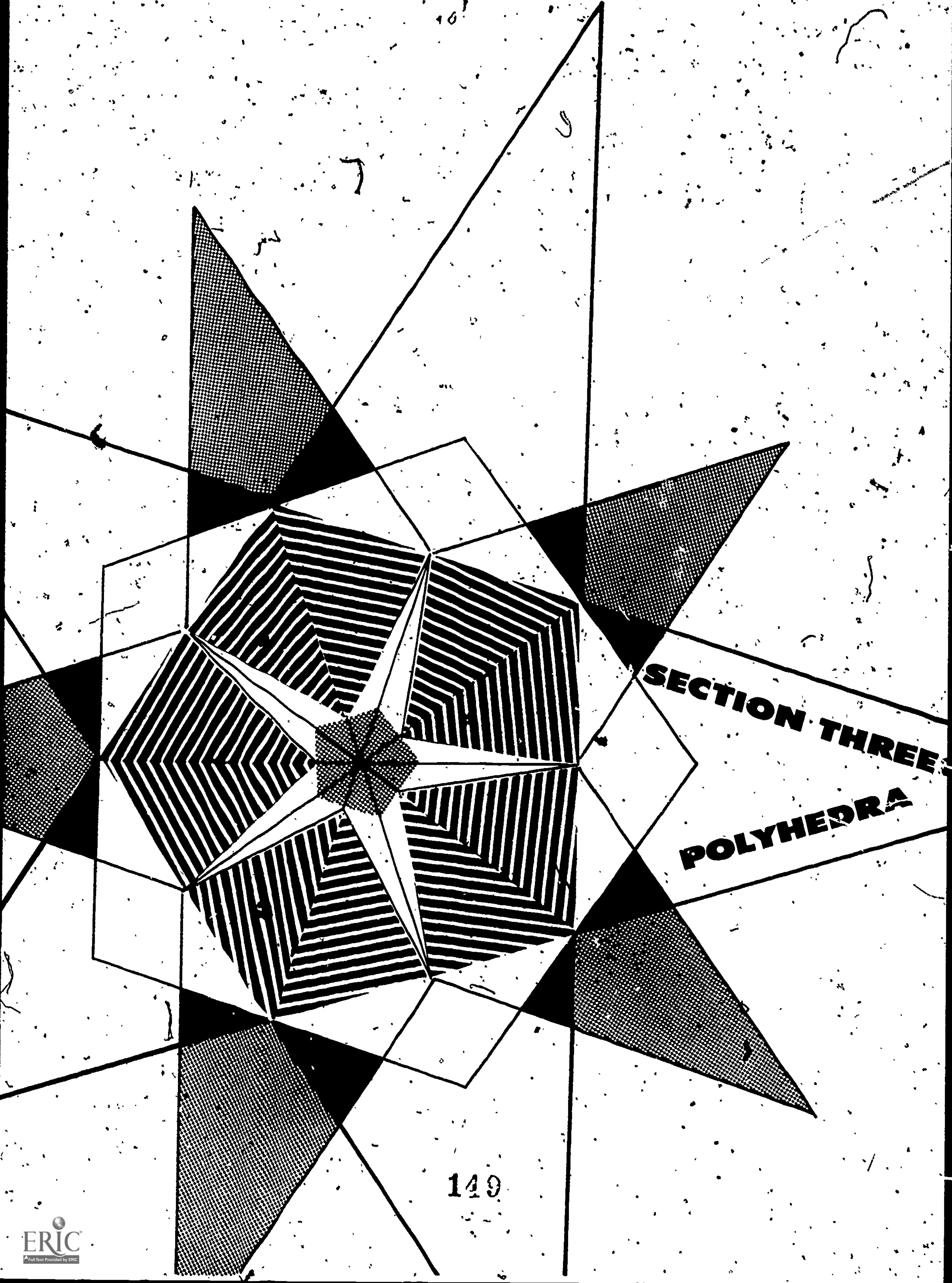
Give them three baggies apiece in which to keep their cut-out polygons.

Working with their triangles and pentagons, the children should experiment to see what different shapes they can make by fitting their cutout polygons together. Then suggest that they try to cover an area such as a piece of construction paper with their cutout polygons, without leaving any empty spaces and without overlapping. When they work with the pentagons, they should discover that they cannot cover a surface completely.

The children may want to make designs that show bilateral, repeating or rotational symmetry. Some may want to paste their cutout polygons on a sheet of construction paper, making a permanent record of what they have discovered. You could make a bulletin board display of these. Use a different example from each child in the class.

You may want to allow time for a show-and-tell situation during which the children report to the class some of the discoveries they have made.

SECTION THREE: POLYHEDRA

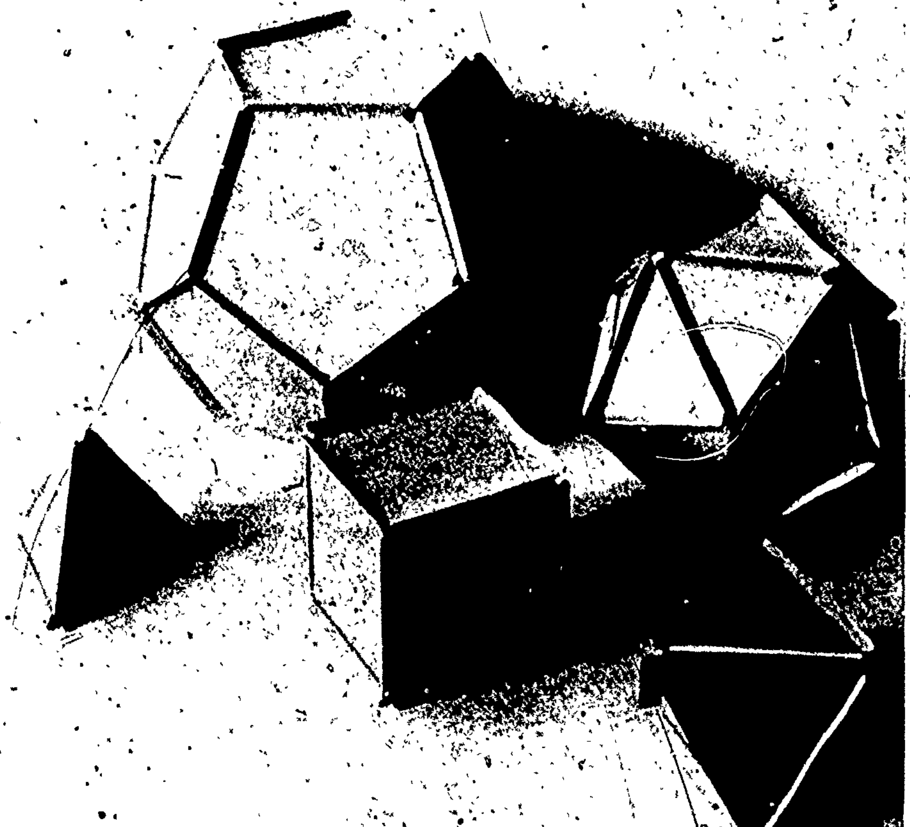


SECTION 3 POLYHEDRA

In Section 3, there are numerous open-ended investigations with plane and solid figures. There are two lessons in this section. In the first one, the children make a transition from two-dimensional to three-dimensional shapes. They work with flat shapes (for example, squares) and put them together with tape to form a three-dimensional shape (for example, the cube). An important part of this study is free experimentation with Flexagons, both during class time and during free time, if possible.

In the second lesson, the children construct additional different three-dimensional solids from two-dimensional patterns.

There are many big words in these lessons, such as tetrahedron, icosahedron, dodecahedron, etc. It is not necessary for the children to memorize or learn how to spell these words. However, they might enjoy hearing the technical names for the different shapes as they work with them.



Lesson 15: FROM 2-DIMENSIONAL SHAPES TO 3-DIMENSIONAL SHAPES

The children visualize congruent or similar shapes that are in different orientations. They also make the transition from two-dimensional shapes to three-dimensional shapes.

MATERIALS

- square counters, 6 per child
- 12" x 18" construction paper, 1 or 2 sheets per child
- transparent tape, 15 rolls
- equilateral triangles (from Lesson 14)
- Flexagons

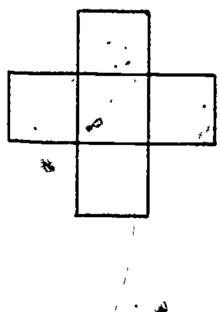
PROCEDURE

Give each child six square counters and a piece of construction paper. Tell them that today they are going to solve a problem. Ask:

HOW MANY DIFFERENT PATTERNS CAN WE MAKE USING FIVE SQUARES? (Let the children make several guesses.)

Ask them to use the same rule they used in Lesson 14 -- the edges must fit together evenly.

The children can use the construction paper to keep a record of the different patterns they make. They can use their sixth square as a guide for drawing the patterns. Demonstrate the procedure by placing five squares in the following pattern.



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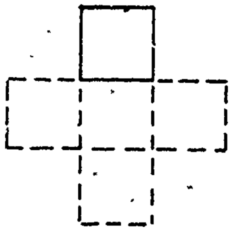
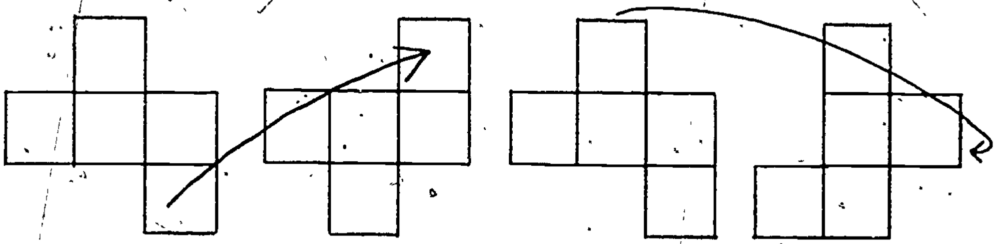
Then demonstrate how to use the sixth square to draw the pattern.

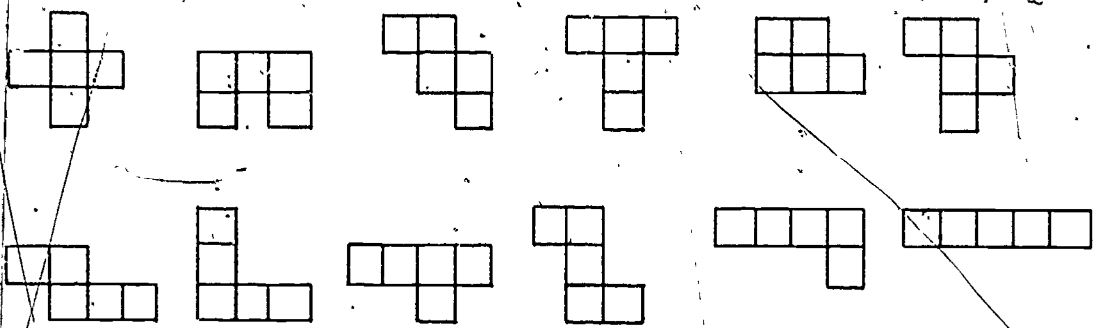


When the children understand what to do, have them proceed. Walk among them and give suggestions. Many children may have trouble deciding if two patterns are the same if one is flipped or if one is at a different orientation than the other, e.g.,

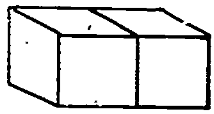


flipped   different orientation

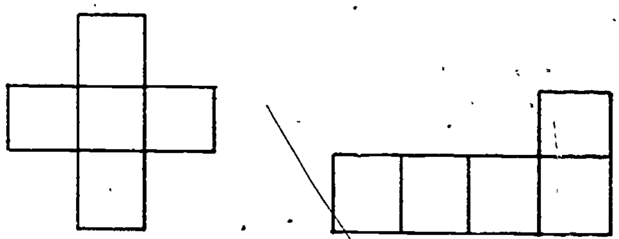
There is a total of twelve different patterns:



Pass out a roll of transparent tape to every two children. Ask the children to see what shapes they can make by taping the squares together. Tell them that the squares do not have to lie flat on the desk. Some children should discover a cube. If two children put their pieces together, they could make a rectangular box.

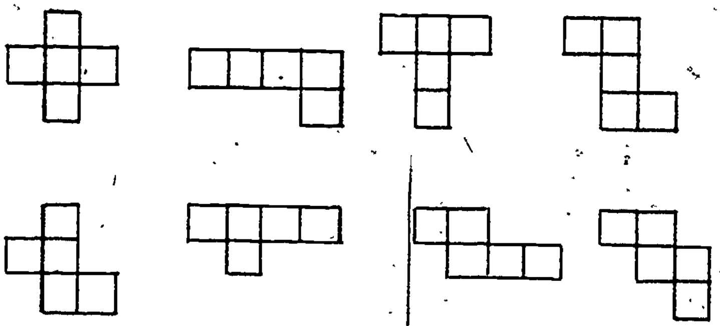


Have everyone make a cube without a top. Then ask them to try to visualize what the box would look like if it were flattened out and to draw a picture of what they think it would look like. These pictures should be quite interesting. If the children count the faces (5 squares), their drawings mig... resemble some of the five-square patterns they made earlier, for example:



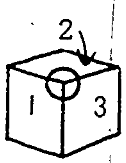
If someone notices this, have the children speculate which patterns their flattened-out boxes would look like. They should check their hypotheses by cutting the tape holding their boxes together and trying to flatten their boxes so they look like whatever patterns they have chosen.

The following eight patterns (out of the twelve original patterns shown on page 142) are the possible patterns that the flattened-out boxes will look like.

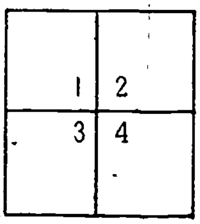


Some children may want to tape their boxes together again and try flattening them to form another pattern.

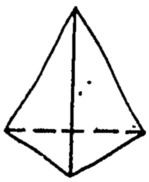
Have the children count the number of faces at a corner. (3.)



Ask what would happen if there were four faces meeting at a corner. They should try this. Someone should see that there will be a flat surface when there are four faces at a corner.



Have the children take out the equilateral triangles they used in Lesson 14. Ask them to see what shapes they can make by taping their triangles together. Again, the triangles do not have to lie flat. Someone should discover the tetrahedron.



Ask:

HOW MANY TRIANGLES DO WE HAVE AT EACH CORNER? (3.)

Bring out the Flexagons and show the children how to put them together with rubber bands. They can use these during their free time to make solid figures like the cube and tetrahedron. Ask them to find out if they can have more than three triangles meeting at a corner and still make a solid. They might also try to find out if they can make solids using pentagons, hexagons and squares.

If there are other second grade classes in your school, you may want to borrow their sets of Flexagons for your class to use for a few days.

Lesson 16: POLYHEDRA

In this lesson, the children learn how to construct three-dimensional solids from two-dimensional patterns, how to construct rigid and non-rigid solids, and how to construct congruent irregular tetrahedrons.

A polyhedron (pol-ee-HE-drun) is a solid figure in three dimensions, all of whose surfaces or faces are flat or plane polygons. Cubes, boxes, pyramids, tents, and buildings are all examples of polyhedra. Polyhedra are more difficult to describe than polygons, because each face may be a different kind of polygon. For example, in Figure 1, we see a 7-sided polyhedron whose end faces are pentagons and whose other five faces are rectangles.

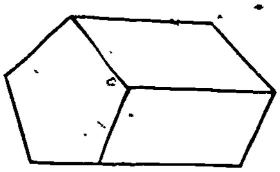


Figure 1

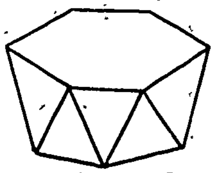
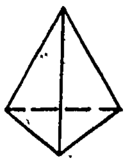


Figure 2

In Figure 2, we see a 14-sided drum-shaped polyhedron constructed from triangles and hexagons.

A regular solid or polyhedron is one in which all of the faces are regular polygons, the edges are all alike, and the vertices are all alike. (We mean that there is a symmetry operation that will transform any face, edge, or vertex into any other face, edge or vertex.) As amazing as it may seem, there are only five regular polyhedra, a fact discovered over 2,000 years ago by the Greeks. The five regular polyhedra are:

1. The four-sided pyramid or tetrahedron. (4 faces, all equilateral triangles.)



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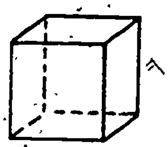
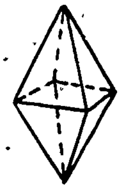
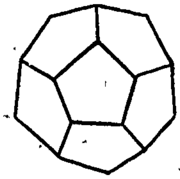
2. The cube or hexahedron. (6 faces, all squares.)



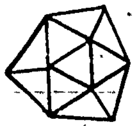
3. The octahedron. (8 faces, all equilateral triangles.)



4. The dodecahedron. (12 faces, all regular pentagons.)



5. The icosahedron. (20 faces, all equilateral triangles.)



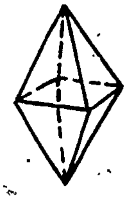
MATERIALS

- transparent tape, 15 rolls (from Lesson 15)
- Flexagons (from Lesson 15)
- think sticks and connectors (from Lesson 10)
- scissors
- paper clips, 6 per child
- crayons
- Worksheets 62-66 (regular polyhedra construction sheets)
- Worksheet 67 (irregular tetrahedron puzzle construction sheet)

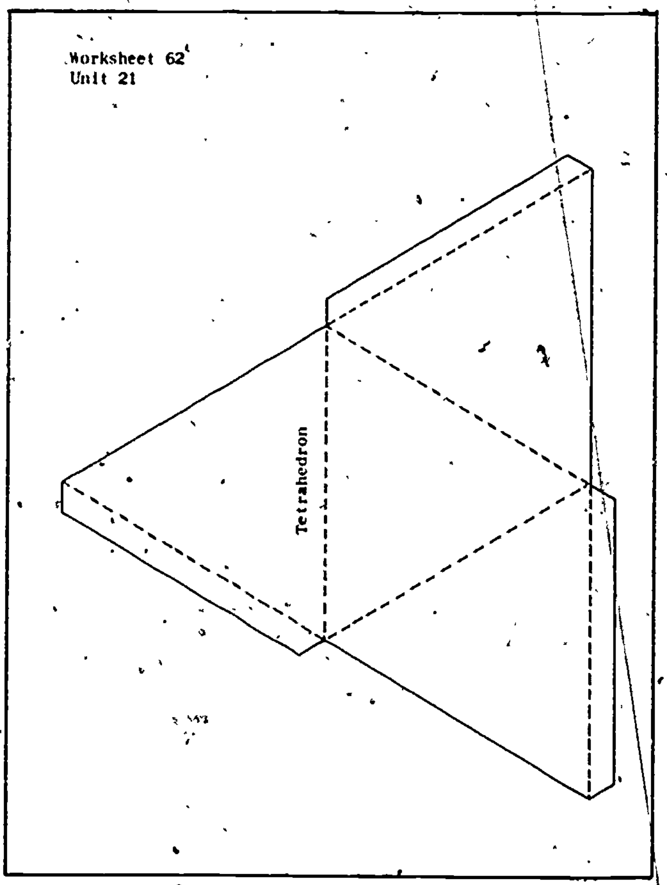
PROCEDURE

Activity A

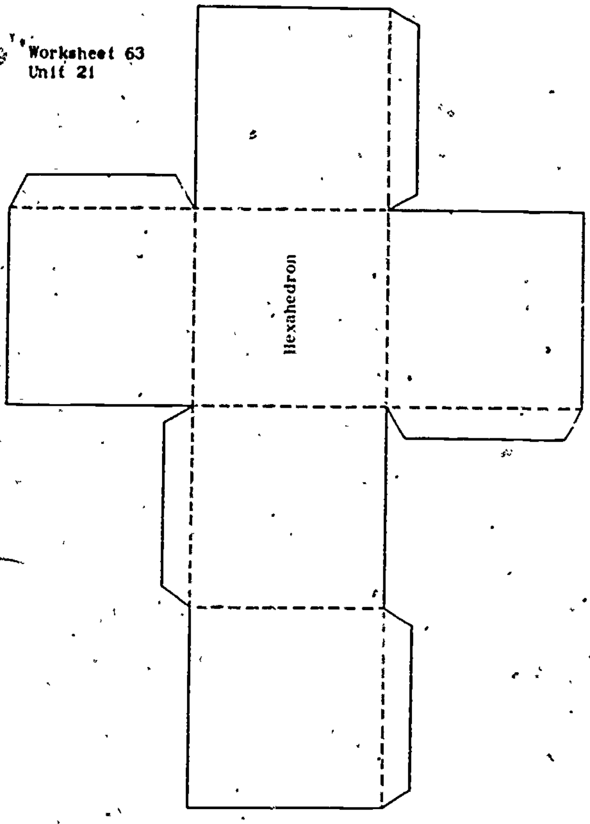
Review the procedure used in the previous lesson to make the cube and tetrahedron. If the children have had time to experiment with the Flexagons, some may want to report their findings to the class. Ask them if anyone was able to construct a figure using more than three triangles at a corner. Someone may have been able to make the octahedron.



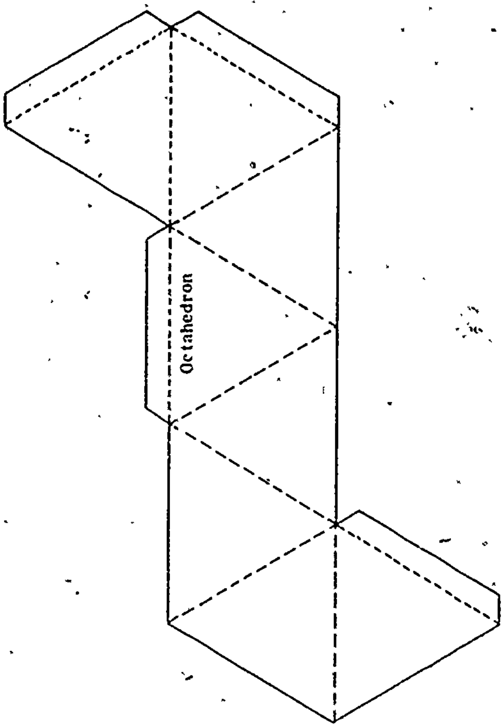
Have the children remove Worksheets 62-66 (regular polyhedra construction sheets) from their Student Manuals. Pass out the rolls of transparent tape. Ask the children to look at their polyhedra construction sheets. Tell them that after these figures have been cut out, they can be folded to make solid shapes.



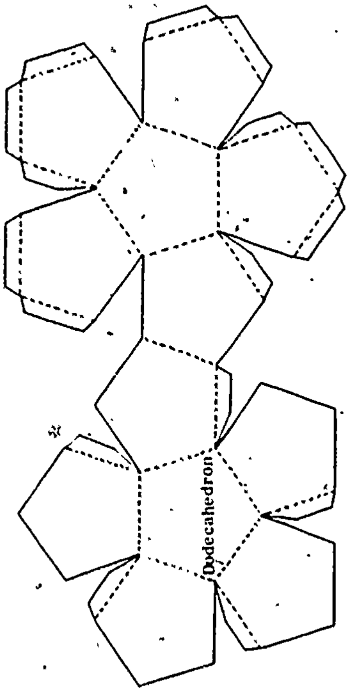
Worksheet 63
Unit 21



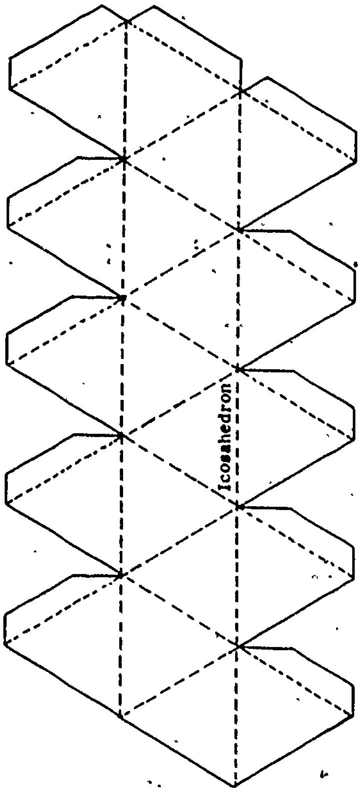
Worksheet 64
Unit 21



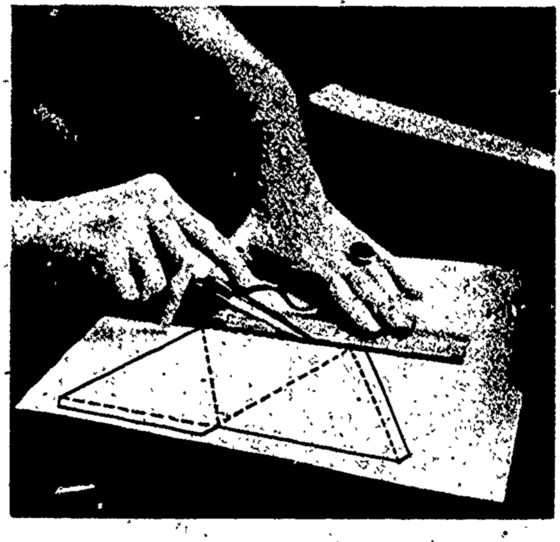
Worksheet 65
Unit 21



Worksheet 66
Unit 21



The figures will be easier to fold if the edges are scored. With some practice the children should have little difficulty with scoring. The procedure for scoring is the same as for drawing a straight line segment using a ruler, except that the open pointed end of a scissors is used instead of a pencil.



All the dotted lines should be scored.

Divide the class into groups of five. One child should put together the cube, one child the tetrahedron, one the octahedron, one the dodecahedron and one the icosahedron. (They should construct their extra solids when they have free time, or they could take them home.)

Within each group, the children should examine each others' solids. When all the groups have finished making a solid of each type, discuss their properties: number of faces, number of corners, polygons used to make the solids, etc. Discuss why these are called regular solids.

Activity B

Pass out think sticks and connectors to each group of five children. Give each group at least six sticks the same length, twelve more sticks of another length and twelve connectors. Ask the children in each group to make a tetrahedron (4-sided polyhedron) using their think sticks. Ask them how many edges and how many corners their tetrahedron has. (It has the same number of edges as the number of sticks used and the same number of corners as the number of connectors used.) Now ask each group to make a cube. They should notice the number of sticks needed and the number of connectors needed -- twice as many as for the tetrahedron. (The tetrahedron has six edges and four corners; the cube has twelve edges and eight corners.) Ask them if they can notice another difference between the cube and the tetrahedron. Someone should mention that the cube is "floppy" and the tetrahedron is "rigid." Some children may remember when they used the think sticks to make polygons in Lesson 10. At that time they found the triangle to be the only rigid figure.

Some children may want to experiment with the think sticks and try making mixed "floppy-rigid" figures.

Have the class compare a think stick cube to a tagboard cube.

> WHY ISN'T THE TAGBOARD CUBE FLOPPY? (The faces act as bracing for the sides.)

Some children may want to try cross-bracing their cubes to make them rigid.

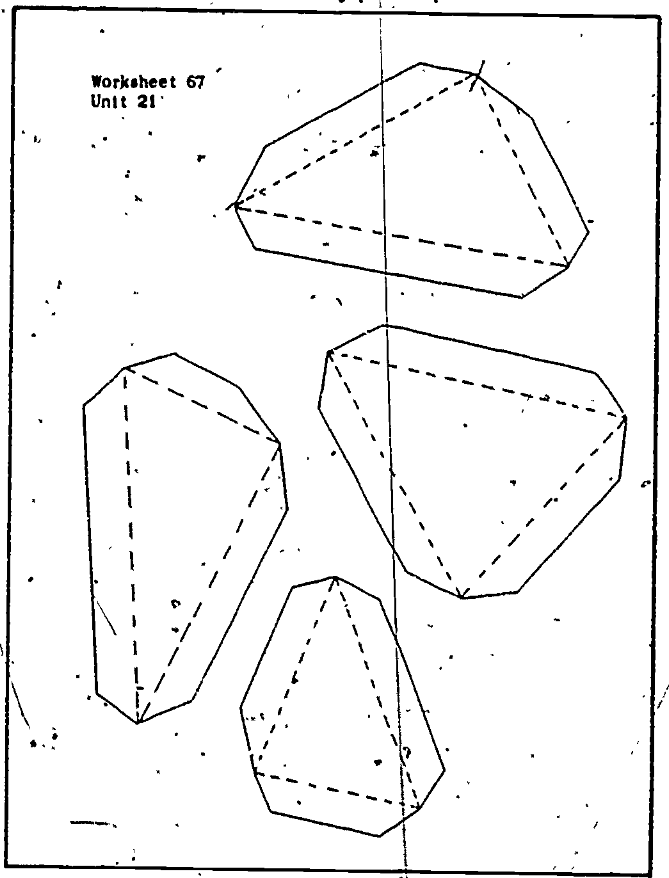
For further investigation, the following question could be raised:

> If the triangle is the only rigid polygon and the tetrahedron is made of triangles and is also rigid, does that mean that all solids that are made up of triangles will be rigid?

As a special project, some children may want to try constructing the octahedron and icosahedron using think sticks.

Activity C

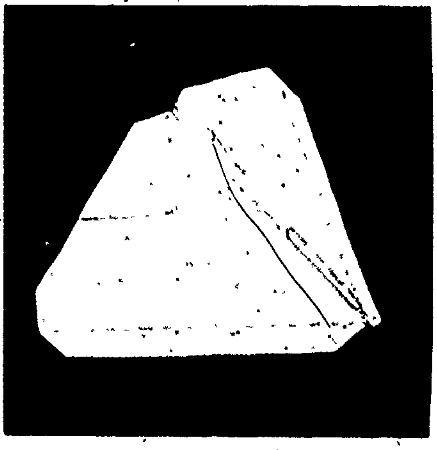
Have the children remove Worksheet 67 (irregular tetrahedron puzzle sheet) from their Student Manuals. Give each child six small paper clips. The children should color one side of their worksheets. Ask them to cut out carefully the four



puzzle pieces. Tell the children that the tabs can be folded in either direction and that these pieces can be fitted together to make a tetrahedron. Ask if it will be a regular tetrahedron. They should be able to conclude that since the triangles are not regular the tetrahedron will not be regular. Ask half the class to make a tetrahedron so that the white side is on the outside and the other half of the class to make a tetrahedron so that the colored side is on the outside.

Show the children how to use the paper clips to secure two flaps.

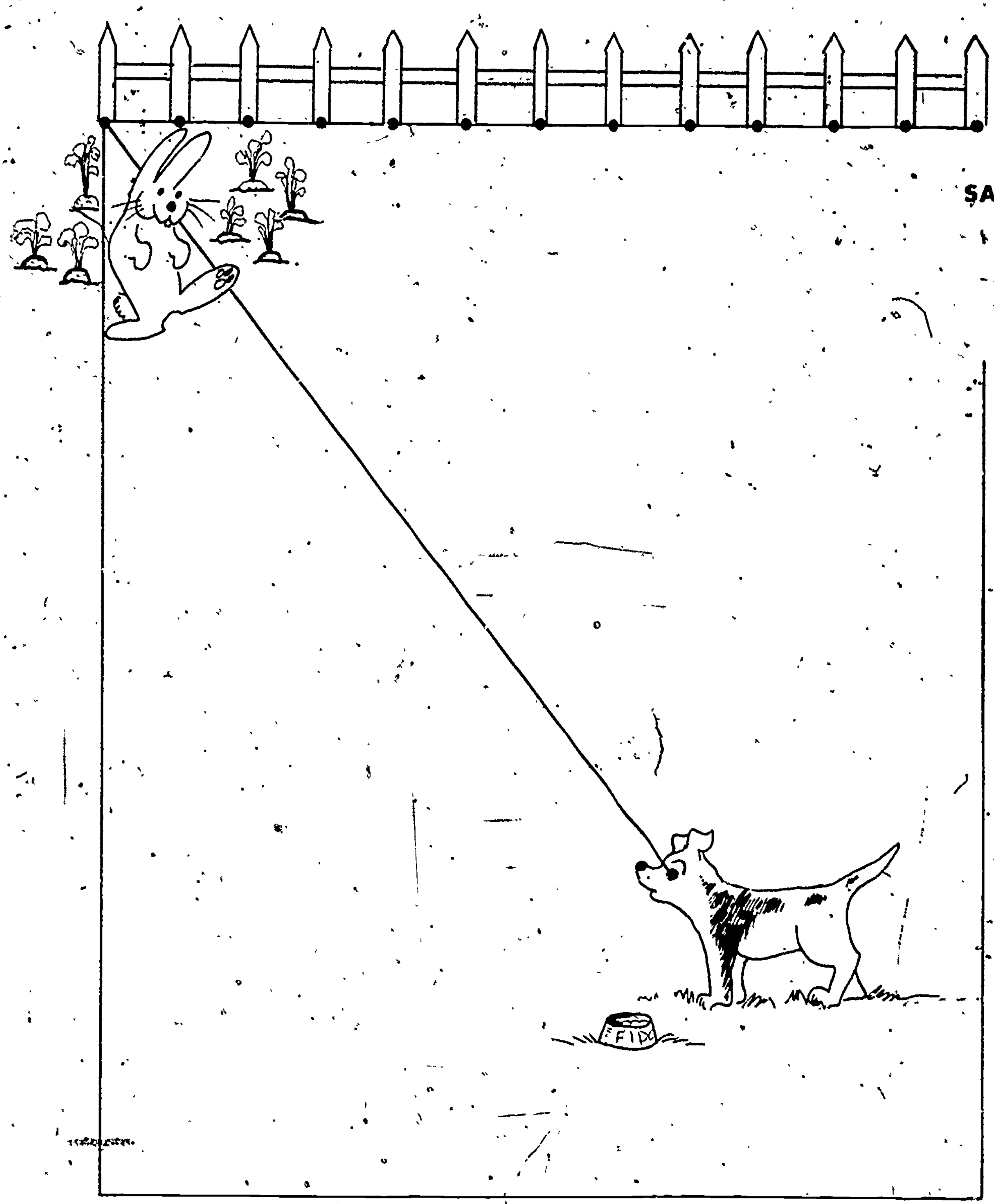
When the children have completed their tetrahedrons, have them compare the white and colored tetrahedrons. Ask them if their tetrahedrons are the same. (They are the mirror images of each other -- like your left and right hands.)



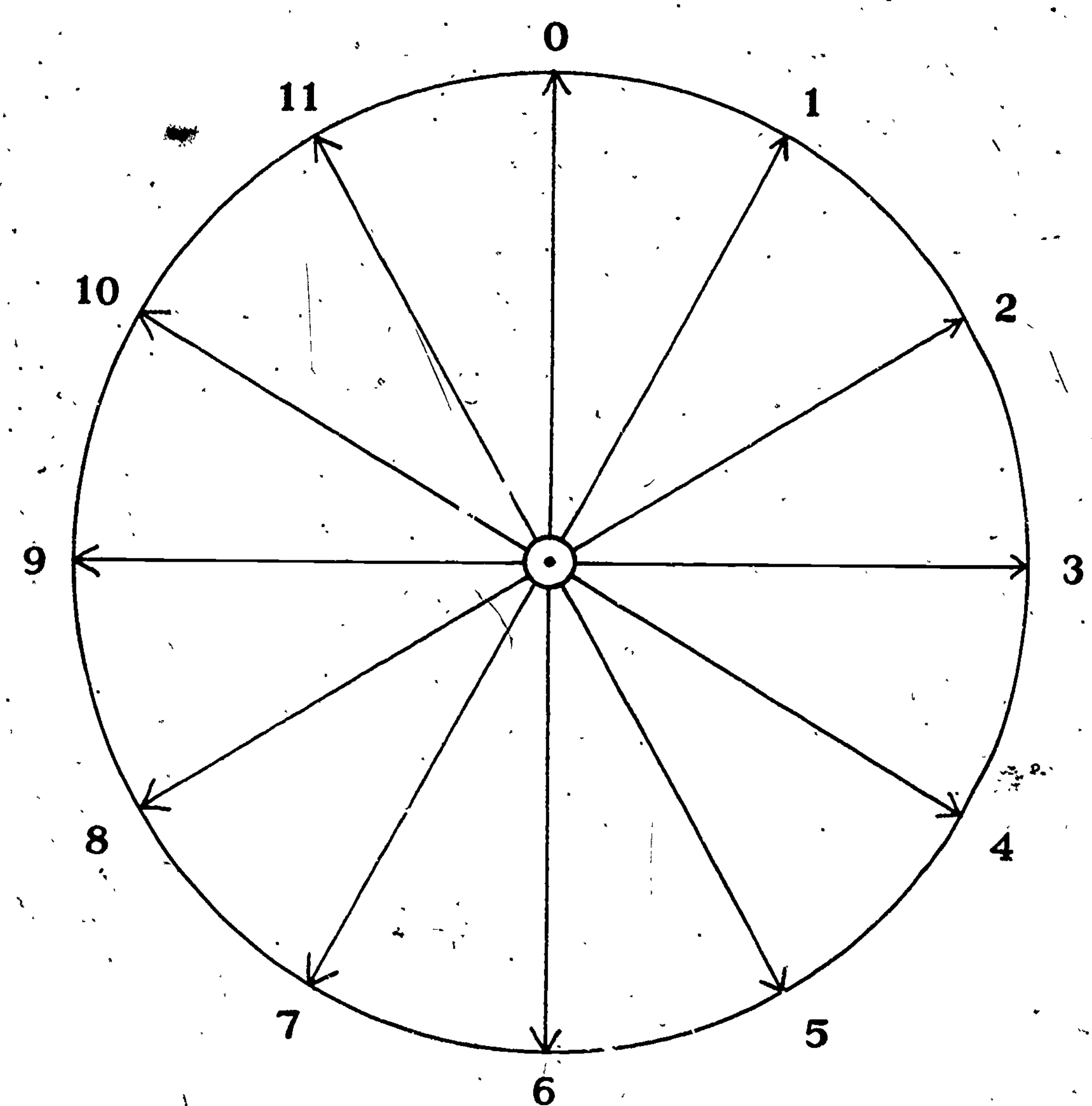
Some children may want to investigate the possibility of making a tetrahedron with both colored and white faces. How many ways of arranging the two colors are there? How are the colors arranged on the inside of these tetrahedrons? These questions can be left open-ended.

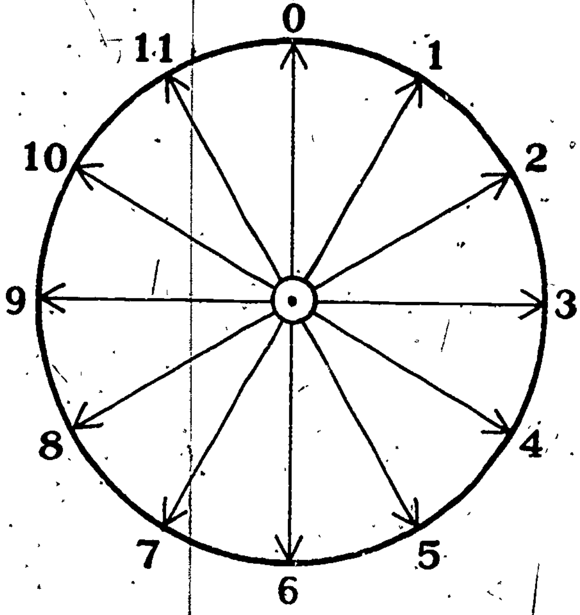
APPENDIX

SAFETY



Demonstration Clock Protractor

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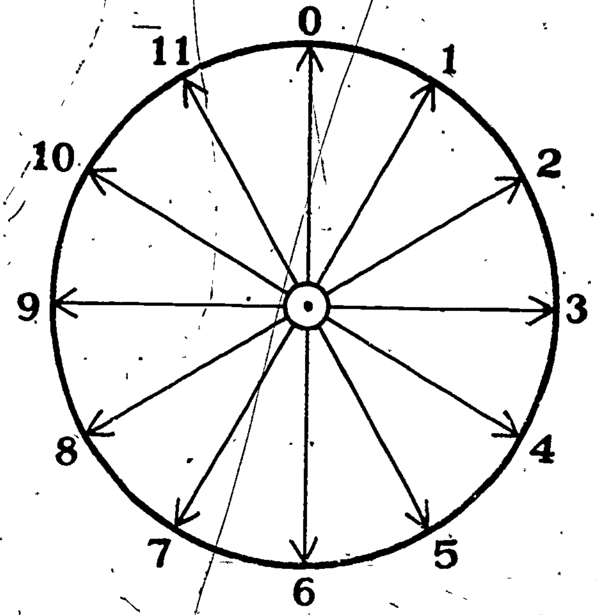
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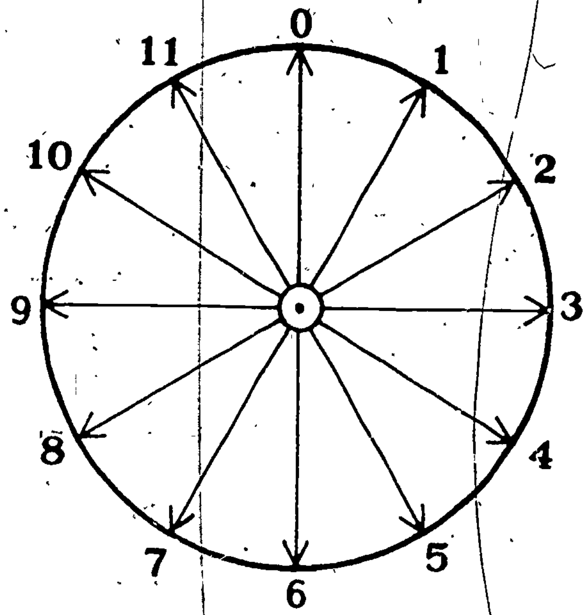
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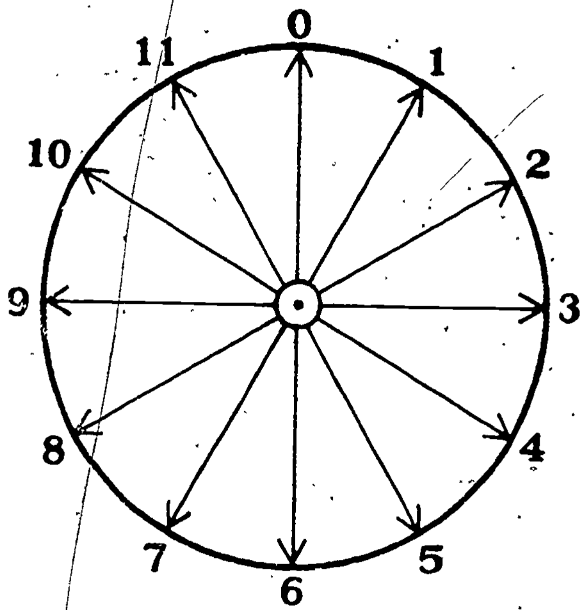
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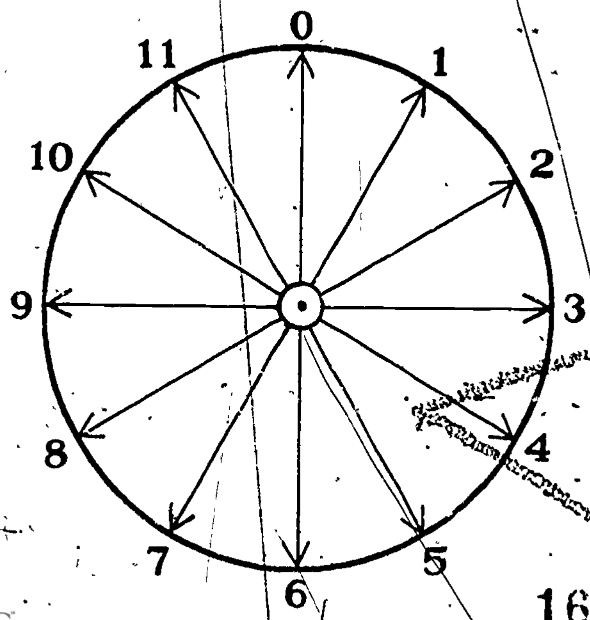
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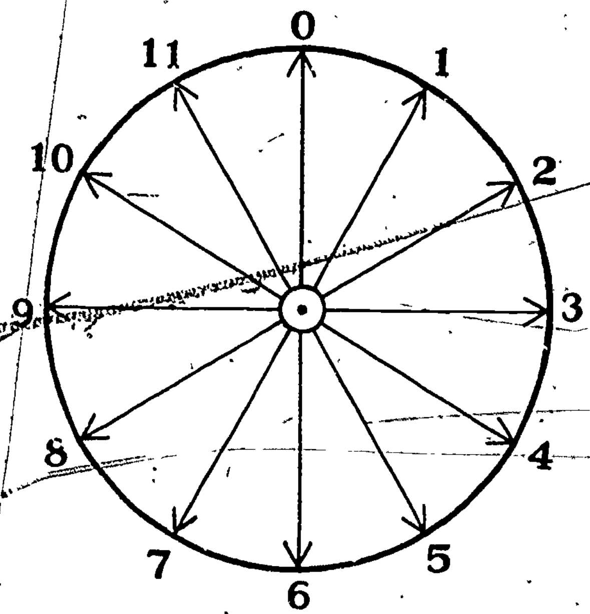
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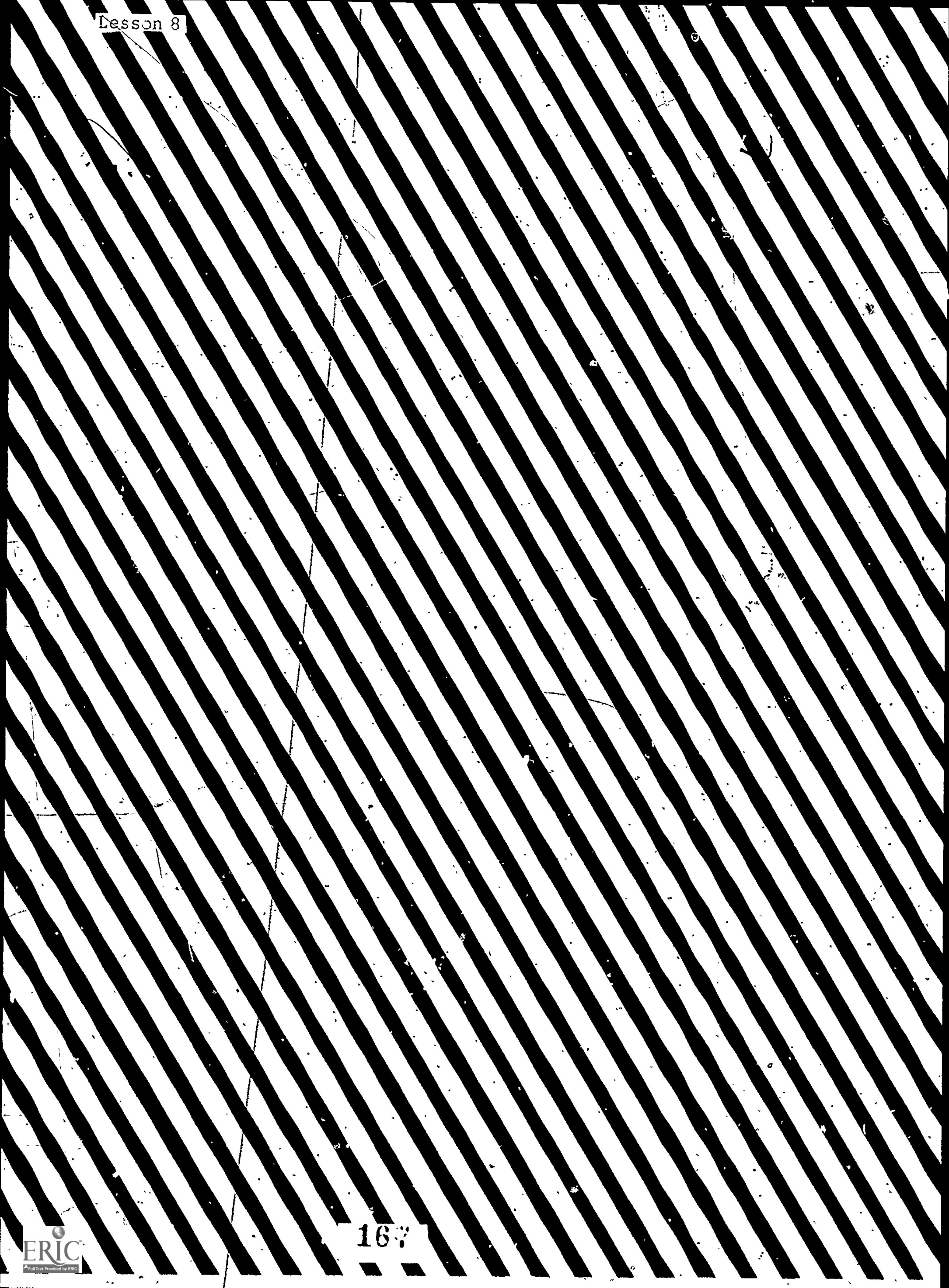
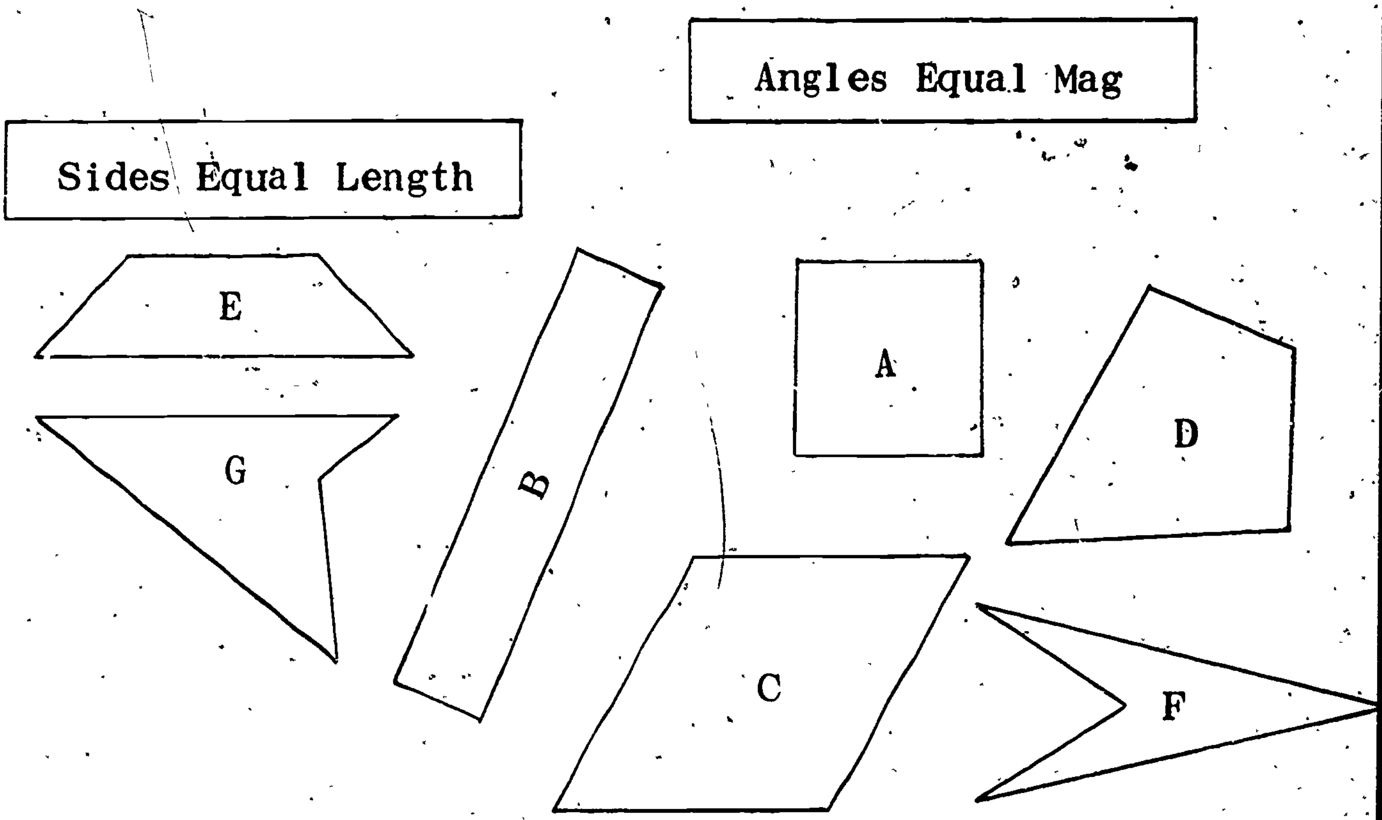


Unit 21
Lesson 11
Transparency A



Lesson 13