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ABSTRACT

This volume is the eighteenth in a series of 29 coordinated MINNEMAST units in mathematics and science for kindergarten and the primary grades. Intended for use by second-grade teachers, this unit guide provides a summary and overview of the unit, a list of materials needed, and descriptions of 12 lessons. The purposes and procedures for each activity are discussed. Examples of questions and discussion topics are given, and in several cases ditto masters, stories for reading aloud, and other instructional materials are included in the book. This unit concerns the representation of objects, the use and construction of scale drawings and three-dimensional models, and the use of instruments. (SD)

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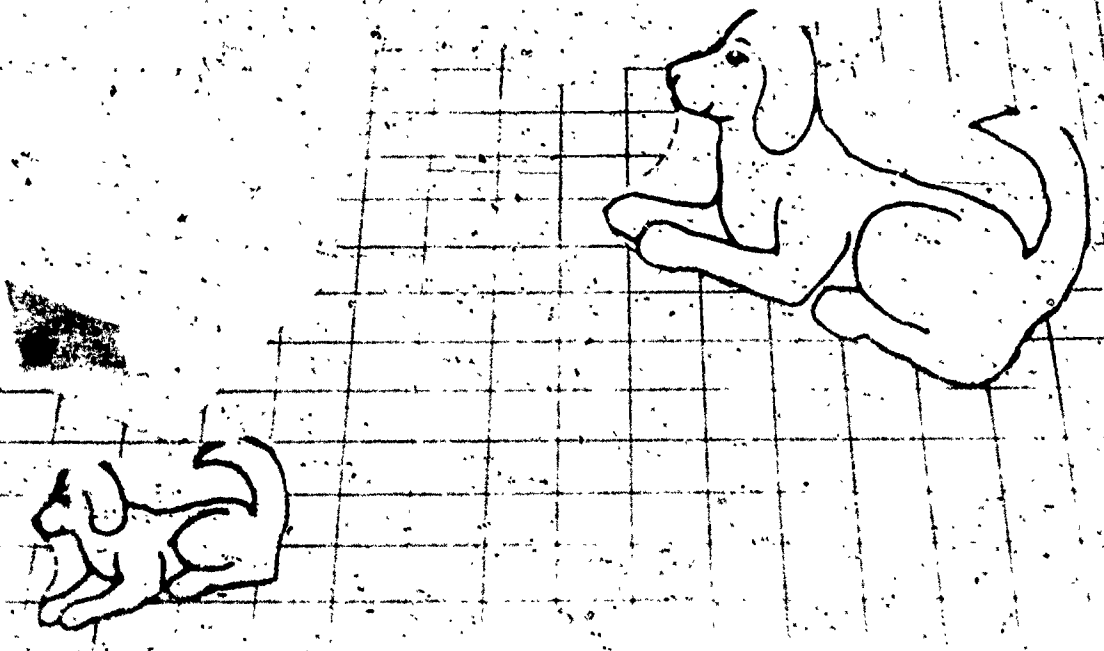

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UNIT

MINNESOTA MATHEMATICS AND SCIENCE TEACHING PROJECT

MINNEMAST COORDINATED MATHEMATICS–SCIENCE SERIES

KINDERGARTEN

1. WATCHING AND WONDERING
2. CURVES AND SHAPES
3. DESCRIBING AND CLASSIFYING
4. USING OUR SENSES
5. INTRODUCING MEASUREMENT
6. NUMERATION
7. INTRODUCING SYMMETRY
8. OBSERVING PROPERTIES

FIRST GRADE

9. NUMBERS AND COUNTING
10. DESCRIBING LOCATIONS
11. INTRODUCING ADDITION AND SUBTRACTION
12. MEASUREMENT WITH REFERENCE UNITS
13. INTERPRETATIONS OF ADDITION AND SUBTRACTION
14. EXPLORING SYMMETRICAL PATTERNS
15. INVESTIGATING SYSTEMS
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17. INTRODUCING MULTIPLICATION AND DIVISION
18. SCALING AND REPRESENTATION
19. COMPARING CHANGES
20. USING LARGER NUMBERS
21. ANGLES AND SPACE
22. PARTS AND PIECES

THIRD GRADE

23. CONDITIONS AFFECTING LIFE
24. CHANGE AND CALCULATIONS
25. MULTIPLICATION AND MOTION
26. WHAT ARE THINGS MADE OF?
27. NUMBERS AND THEIR PROPERTIES
28. MAPPING THE GLOBE
29. NATURAL SYSTEMS

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SCALING AND REPRESENTATION

UNIT 18

MINNESOTA MATHEMATICS AND SCIENCE TEACHING PROJECT

720 Washington Avenue S.E., Minneapolis, Minnesota 55455

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Third Printing, 1971

SCALING AND REPRESENTATION

This unit was developed by MINNEMAST on the basis of experiences of the many teachers who taught an earlier version in their classrooms.

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CONTENTS

Complete List of Materials for this Unit

(Numbers based on class size of 30.)

total number required to teach unit	item	lessons in which item is used
30	large, square property blocks (*** or borrow from Kindergarten teacher)	1
10-15	assorted natural objects	2
1	overhead projector	2,3,4,5
1 each	** transparencies of Worksheets 5,7,9,11, and 13	2,3,4
30	* concave lenses ("reducers")	2
30	* convex lenses ("magnifiers")	2
30	* 4-oz. plastic cups	2
30	* rulers	2,9,11,12
1 each	** silhouettes of man, house, rabbit and 1-inch square	3,4
1	** piece of 1-inch grid paper	3,4
4-5	assorted geometric shapes, e.g., triangles	5
30	scissors	6,12
	paste	6
34	sheets of construction paper	6,12
30	*** sets of Minnebars	7,8
3-4	unsharpened pencils	9
3	* yardsticks	9,11
300	* jumbo paper clips ("clips")	9
300	* #1 paper clips ("clips")	9
1	film: Maps--Where Am I?	10
1	book: A Map is a Picture, by Barbara Rinkoff (Crowell, 1965)	10
30	* 24-inch lengths of yarn (60 feet)	10,12

1	story: "Goldilocks and the Three Bears"	11
3	chairs: 1 kindergartener's, 1 second grader's, 1 teacher's	11
2	pairs of sunglasses: 1 child size, 1 adult size	12
2	footballs: 1 small, 1 large	12
2	sweaters: 1 child's, 1 adult's	12
2	carriages: 1 for small dolls, 1 for large dolls	12
2	bicycles: 1 child's, 1 adult's	12
2	shoes: 1 child's, 1 man's	12
1	infant's shoe, 3½ inches long or less	12
2	toy trucks, 6 inches and 2½ inches long	12
1	book: Gulliver's Travels, by Jonathan Swift	12

*kit items as well as

**printed materials available from
Minnemath Center, 720 Washington Ave. S.E., Mpls., Minn. 55455

***available from The Judy Company,
310 North Second Street, Minneapolis, Minnesota 55401

INTRODUCTION

"Scaling" and "representation" describe familiar techniques that are used every day. For example:

1. A map is a representation of a geographic area. The map does not show every detail within the area, such as traffic lights on streets. Symbols are used to represent boundaries, cities, rivers, railroads, highways, in a simplified way. A road map may be drawn to a scale of 10 miles to one inch, so that it can be printed on a piece of paper of manageable size. Sometimes a state map will include an insert of a city map drawn to a scale of about 1/10 of a mile to one inch so that more details of the city can be represented. The scale and amount of detail in the representation are chosen for convenience and purpose.

2. When a dressmaker picks a size 14 pattern she is choosing a representation of the dress scaled to the size of the person it is intended for. The tissue paper shapes are representations of the pieces of fabric needed to make the dress.

3. Architects draw scaled representations of buildings. Construction workers follow the plans that tell them the size and arrangement of rooms and precise details of construction. Plans of this type are usually drawn to several different scales according to need. For example, the architect makes a small drawing of the finished building, a medium-sized floor plan, and perhaps a larger-than-life detail drawing of an unusual type of construction.

The importance of scaled representations in mathematics and science can hardly be overestimated. Objects and ideas can be represented in pictures with only the essentials shown. When these pictures are drawn to scale, their measurements are meaningful for quantitative work. Representations are used therefore not only as illustrations but also as simplifications and as accurate indications of size.

These concepts are introduced in Unit 18 in their simplest forms. To begin with, the children trace outlines of their hands on paper. They study the differences between the outline and the hand, and learn that the outline is a representation of the hand. From it they can determine size, shape, number of fingers, etc. But it is easy for them to see that while a representation records some properties of an object, it also omits many properties. The children go on to note that for some objects only a few properties must be represented for identification, while for other objects more properties are necessary.

Changing Sizes

The next idea to be introduced is that a representation of an object may be the same size, larger, or smaller than the object. The children are provided with a number of different experiences involving the expansion and reduction of the sizes of patterns. They use magnifying and reducing lenses and draw representations of what they see through the lenses. This activity produces a group of representations of the same object in assorted sizes.

Another example of magnification and reduction is achieved by moving a projector further from or closer to a screen. The same image is made larger or smaller in this manner. When the image is projected on a grid it is easy to assign numbers to the dimensions of the image. A figure that measures two grid units in length is enlarged to measure four grid units in length. Its dimensions have been increased by a scale factor of 2. In this way the relation between scaling and multiplication is established.

Linking Mathematics and Science

The relation between scaling and multiplication demonstrates one of the links between mathematics and science. The child observes a change in size. Then he measures the change and assigns a number to it. Once a number has been assigned,

he can use mathematical operations to predict further change without manipulating the actual materials. The children have many experiences in this unit with this kind of moving back and forth between actual objects and mathematical operations. The assignment of numbers to qualitative observations is precisely what enables us to use mathematics in discussing natural phenomena.

First the children scale up a figure by using two different grid sizes. They copy a simple figure, with each small grid unit represented by a larger grid unit. This gives them a larger figure that is the same shape as the smaller one, but they do not know exactly how much larger it is. Then they repeat the activity with the assignment of numbers to the dimensions of the figure. This time the two drawings are made on grids of the same size. The scale factor tells how many grid units on the second drawing are needed to represent each unit of length of the original figure. The children write the mathematical equation for this increase in the dimensions. The scale factor times the old length equals the new length.

Inverse Operations

The children are given many experiences with expansion and reduction as inverse operations. They learn that one is undone by the other. They learn that expansion or scaling up can be expressed by a multiplication equation. We use the term "division" to describe what happens to the dimensions of a figure that is scaled down, but we do not introduce division equations. This mathematical procedure will be left for later. But we do establish the groundwork now for the fact that multiplication and division are inverse operations.

Area and Volume

Note that we always refer to scaling up or down the length of the side of a figure by a given factor. The area and volume change by different factors than does the length of the side, but these are probably too difficult for most children at this level.

Area and volume are introduced briefly in Unit 17, Lesson 10, and in more detail here. The children get substantial practice with multiplication of two dimensions to find area and three dimensions to find volume (that is, how many squares or cubes are in the representation). But this concept is secondary to the concept of scaling. If children seem to have difficulty with either area or volume, some of those exercises may be omitted without affecting the main purposes of the unit.

Scaling Techniques

Much of the work with scaling is done on grid paper. Minnebars are introduced at first as an aid in visualizing two-dimensional scaling, and then as a three-dimensional representation.

The architects ruler is a device that helps in the actual process of scaling and also helps illustrate the multiplicative relation of two scales by way of parallel number lines. The children make and use their own rulers with scales ranging from 1 → 1 to 1 → 6. They also work with arbitrary scale units such as 1 small paper clip → 1 large paper clip or 1 line segment → 1 handspan.

Applications for Second Graders

The usefulness of scaling is illustrated in several ways. One lesson is devoted to comparing the size of a child with the size of a chair that is comfortable for him. In another, the children study maps of different scales and plan a visit to the zoo. The lesson in map reading is greatly facilitated by a film Maps—Where Am I? which shows very clearly how a geographic area is represented on a map.

The children also build scale models—dog houses to fit dogs of different sizes, garages to fit trucks, shoe boxes to fit shoes. Thus their work with scaling and representation has direct application in second grade terms.

NOTES ON TEACHING THIS UNIT

Unit 18 is designed to be taught with Unit 17, Introducing Multiplication and Division. They may be started at the same time, or Unit 18 may be started a week or two after Unit 17 is begun. The unit should require one class period a day for about five weeks.

Be sure to order the film Maps—Where Am I? from your usual sources well ahead of time. If you have difficulty obtaining it, write directly to the producer, Charles Cahill and Associates, Inc., 5764 Sunset Boulevard, Hollywood 28, California.

Please note that though the magnifiers supplied by MINNEMAST are marked 3X and 6X, the actual image through the larger lens is closer to 2X, as called for in Lesson 2.

You will note that sometimes lengthy directions are included in the worksheets. We do not overestimate the early second grader's reading ability. The directions are always to be read to the children, and those who are able to can read along with you. Our purpose in including them is to get the children accustomed to the idea of following written directions.

Those of you who do not have the MINNEMAST printed materials kit will have to prepare your own worksheet transparencies for those activities that call for use of the overhead projector.

Lesson 1: MAKING REPRESENTATIONS OF OBJECTS

In this lesson the children draw outlines around familiar objects including a hand, objects from their desks; and property blocks. They discuss ways in which the representations differ from the objects themselves and note that the size and shape of the objects' sides are the only properties represented by the drawings.

The children make outline drawings of objects that are easily recognized by the distinctive shape of one side. Then they are given large, square property blocks of different thicknesses and asked to represent them. They find that they must draw outlines around more than one surface of a block to give enough information for someone else to identify the representation. (Not all objects can be identified in this way. For example, the same set of outlines might represent several different things such as a box, a book and a block. Size and shape do not provide sufficient information for distinguishing an object from other objects of the same dimensions.)

MATERIALS

-- for each child --

- large square property block, either thick or thin (borrow from Kindergarten)
- Worksheets 1, 2, and 3

PROCEDURE

Have the class turn to Worksheet 1, which is a blank page. Each child is to place one hand on the page, spread his fingers apart, and trace around his hand with a pencil. Ask the children to look carefully at their drawings and to compare them with their hands.

WHAT IS SIMILAR ABOUT YOUR DRAWING AND YOUR HAND? (Approximate length, width, shape of fingers, etc.)

WHAT IS DIFFERENT? (Color, thickness, material they are made of; the real hand has fingernails, skin, knuckles, the ability to feel things.)

Ask the children to suggest a name for what they have drawn. Ask if they think they could call it a hand. If they say "Yes," ask them to tell again what the differences are between the real hand and their drawings.

The children may suggest calling the representation a picture, but a picture would show the fingernails and other details that are not in these drawings. "Outline" or "representation" would be more appropriate. Introduce the word "representation" at this time, but do not require the children to use it. Ask the children if they can think of any uses for these representations they have made. (Shopping for mittens without the child.)

Let the children select some things from their desks (pencil, book, ruler, scissors). Have them place one or more such objects on a piece of paper and draw outlines of them. Then discuss the differences between the real objects and the representations of the objects.

WHICH PROPERTIES OF THE OBJECTS ARE REPRESENTED BY YOUR DRAWINGS? (Size and shape of one side.)

ARE THESE PROPERTIES SUFFICIENT TO IDENTIFY THE OBJECTS?

Discuss the properties that are not represented by the outlines.

Have the children turn to Worksheet 2, a $\frac{1}{2}$-inch grid. Give each child a large, square property block. (Some will be thick and some will be thin.) Tell the children to make an outline representation of the block. (Suggest that they line up the block along the lines on the grid paper.) Most children will draw around the square side of the block. Ask if someone else could tell exactly what size and shape the block is from such a drawing.

WOULD YOU SAY THAT THE OUTLINE YOU MADE SHOWS ENOUGH ABOUT THE SHAPE OF THE PROPERTY BLOCK SO

THAT OTHERS COULD TELL EXACTLY WHICH SHAPE BLOCK YOU HAVE? (No.)

WHICH OTHER BLOCK COULD YOURS BE MISTAKEN FOR? (A large square block of a different thickness.)

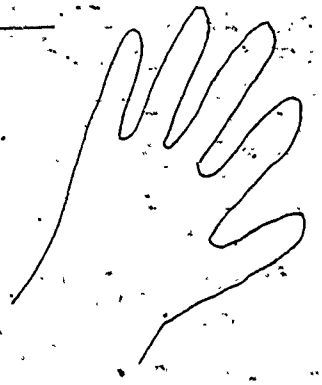
WHAT OTHER DRAWING COULD YOU MAKE TO SHOW MORE ABOUT THE BLOCK'S SHAPE? (An outline of the side. Note that although the block has six sides, they are not all differently shaped. Four sides are of one shape and two sides are of another. Therefore the two outlines are sufficient to represent the view from all six sides.

Have them make an outline of the side.

HOW IS THIS DRAWING MORE COMPLETE THAN THE DRAWING YOU MADE OF YOUR HAND? (The representation of the hand shows the outline only from the top or the bottom. The side view of the hand is not represented.)

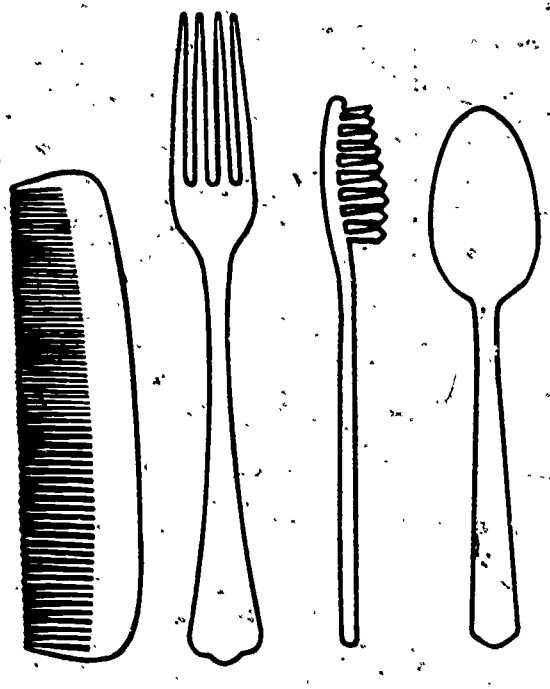
Ask the children to look at Worksheet 3 and to identify the objects represented.

Worksheet 1
Unit 18 Name_____



Worksheet 2
Unit 18 Name_____

Worksheet 3
Unit 18 Name_____

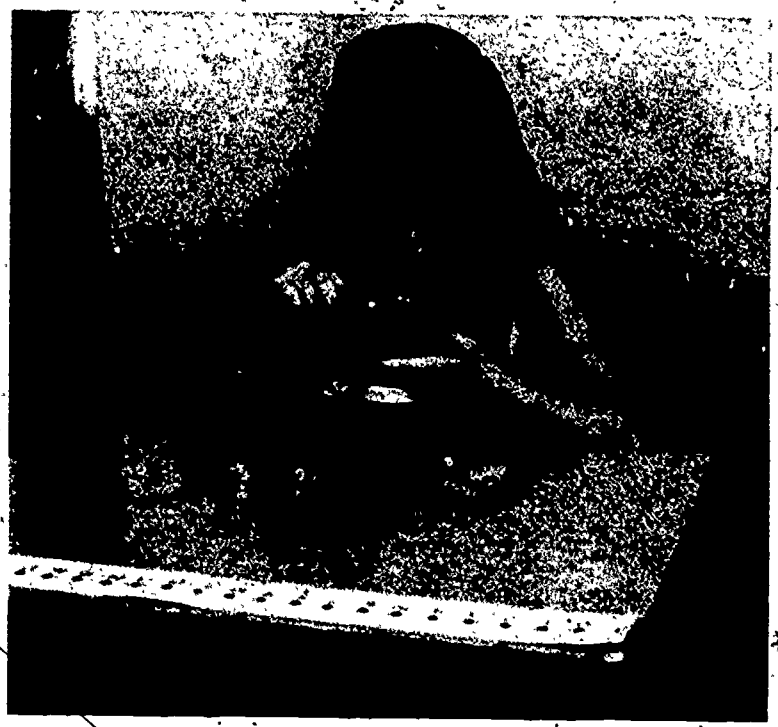


Lesson 2: SCALING WITH LENSES

This lesson introduces the idea that representations of objects can be made that are smaller or larger than the objects themselves.

First the children observe natural objects and pictures of natural objects through magnifying and reducing lenses. The children have an opportunity to observe that the appearance of a shape enlarges or decreases in all its dimensions when seen through these lenses. They also see that details are either more or less discernible when the objects are magnified or reduced.

Then they are asked to make 1 → 1 scale drawings of letters and shapes as they are seen through a lens. That is, a picture of an object seen through the magnifying lens should be about twice as long as the object itself, and a picture of an object viewed through the reducing lens should be about half as long as the object. This operation scales the object up or down by a factor of 2. Introduce this terminology to the children as you describe the procedure.



MATERIALS

- natural objects such as leaves, hair, feathers, insects
- overhead projector
- transparency of Worksheet 5

-- for each child --

- 2 lenses, one concave and one convex
- 4-ounce plastic cup
- ruler
- Worksheets 4, 5 and 6

PROCEDURE

Activity A

Give each child a sheet of plain paper and ask him to draw an outline of his thumb. After the drawing is completed give each child two lenses, one concave and one convex. Have them look through each of these lenses, first at their thumbs and then at the representations of them. (Show them how to move the lenses up and down until the image is sharp.)

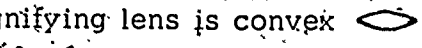
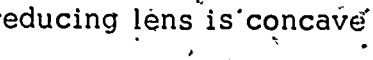
Ask what happens to the appearance of their thumbs when they look at them through the magnifying lens. What happens to the appearance of the representations of their thumbs? They should observe that both the real thumb and the representation appear larger through the lens.

Now have them look at their thumbs and the representations of them through the other lens. They should observe that this lens makes both the thumb and its representation appear smaller.

To call attention to the effect of magnification and reduction, have the children look at their thumbs carefully and then look at them again through the magnifier.

WHAT DETAILS DO YOU SEE THAT YOU DIDN'T SEE WITHOUT THE MAGNIFIER?

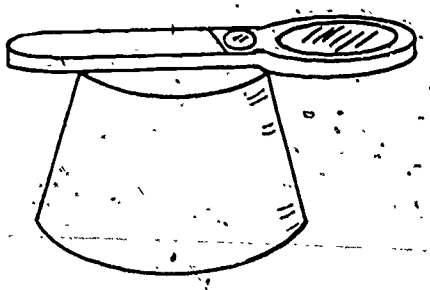
WHAT DO YOU SEE IF YOU LOOK AT YOUR THUMB THROUGH THE REDUCING LENS? (The children should note that magnification reveals greater detail and that after reduction some details are no longer visible.)

Call attention to the shape of the lenses. Have the children examine and feel them. The magnifying lens is convex  and the reducing lens is concave  .

Activity B

Make a collection of natural objects for the children to take turns observing through the lenses. They should observe many things, such as leaves, butterflies from a collection, human or animal hair, and feathers. Also have the children look at various objects from their desks through the lenses.

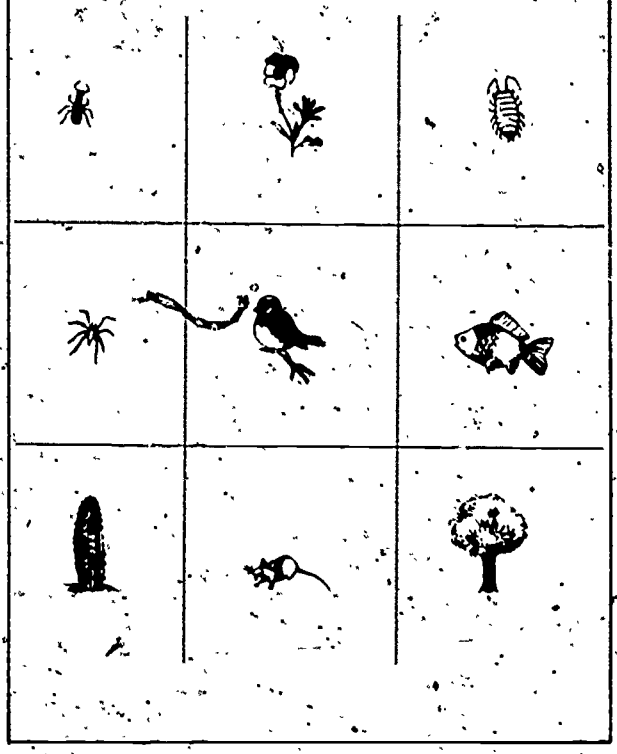
Give each child a 4-ounce plastic cup which he can invert to use as a support for the handle of his lens.



Worksheet 4 provides a number of pictures for the children to examine with their magnifying and reducing lenses.

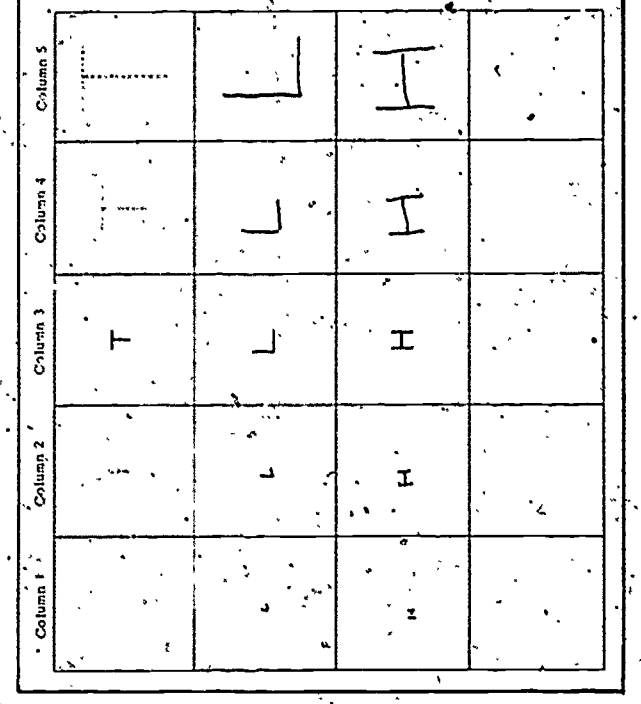
Worksheet 4
Unit 18

Name_____



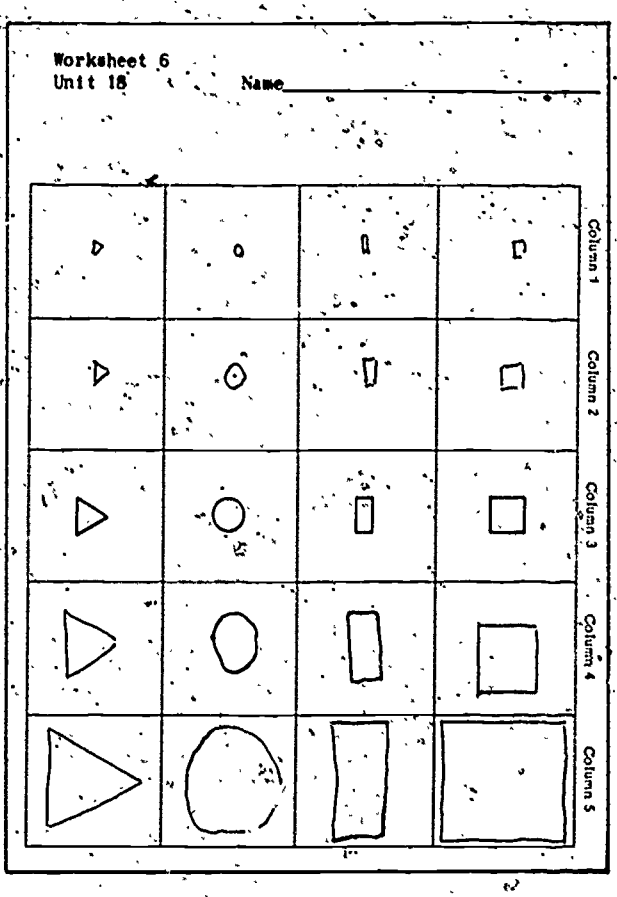
Worksheet 5
Unit 18

Name_____



Worksheet 6
Unit 18

Name_____



After they have examined several natural objects and pictures of natural objects, ask the children how much larger they think the magnifying lens makes things look and how much smaller the reducing lens makes them appear to be.

ARE ALL OF THE THINGS WE LOOK AT ENLARGED OR REDUCED EQUALLY WHEN SEEN THROUGH THE SAME LENS?

Tell the children that by looking at letters and shapes and drawing some of them, they may be able to find out how much the appearance of the object is enlarged or reduced through each of the lenses.

Have the children turn to Worksheet 5.

Call attention to the letters in the center column of the worksheet. Tell the children that they will use their lenses to look at the letters, and then copy them in the sizes they appear to be. It may be easier for them to work if they rest the handle of the magnifier on the bottom of the inverted 4-ounce plastic cup. Those children who have trouble working to the left of the lens may turn their papers upside down.

Work together on the first row. Have the children look at the T in Column 3 through the magnifying lens. When it is sharply in focus they should notice that it appears to be about the same size as the dotted T in Column 4. Have them look through the lens carefully and trace over the dotted T in Column 4 to make a magnified letter that looks like the original one as seen through the magnifier.

Now have them look at the T in Column 3 through the reducing lens. They should notice that what they see is about the same size as the T in Column 2. Have them trace over the dotted T in Column 2 to make a reduced T that looks like the original one as seen through the lens.

Suggest that they make an even smaller T by looking through the reducing lens at the one they just drew and drawing in Column 1 what they now see. They may make a larger T than the one in Column 4 by looking at it through a magnifying lens and drawing what they see in Column 5.

Tell the children to complete the worksheet and to compare the letters they have made in the different columns. (Each letter to the right should be almost twice as high as the letter in the column before it. Each letter to the left should be about half as high.) The last row will be used later.

DO THE LETTERS APPEAR TO BE JUST TALLER WHEN YOU LOOK AT THEM THROUGH THE MAGNIFIER, OR DO THEY LOOK WIDER ALSO?

DO JUST SOME PARTS OF THE LETTERS BECOME LARGER OR DO ALL OF THE PARTS APPEAR TO INCREASE IN SIZE?

HOW CAN YOU CHECK THE LENGTH AND WIDTH OF THE LETTERS AS SEEN THROUGH THE LENSES? (By measuring the drawings.)

Activity C

On Worksheet 6 the children will look at shapes through the lenses and scale them up and down in the same way that they did the letters on Worksheet 5. Then they will study the shapes to find out how the lengths and the widths change. Have the children complete Row 1. Ask them to compare the dimensions of the shapes they have drawn in the five columns. At this time the children should only make the general observation that each shape seems to be half or twice as high as the one next to it. Review what the children have done so far, and then ask them to finish the worksheet.

Have the children compare the enlargements and reductions in each row, using their rulers for measuring. The dimensions will not be in whole units. Have them use the nearest unit of measurement.

IS EACH SHAPE IN COLUMN 2 ABOUT TWICE AS WIDE AND TWICE AS HIGH AS EACH SHAPE IN COLUMN 1? (Yes.)

IS EACH SHAPE IN COLUMN 3 ABOUT TWICE AS WIDE AND TWICE AS HIGH AS EACH SHAPE IN COLUMN 2? (Yes.)

IS EACH SHAPE IN COLUMN 4 HALF AS WIDE AND HALF AS HIGH AS EACH SHAPE IN COLUMN 5? (Yes.)

DO YOU THINK THAT OTHER THINGS SEEN THROUGH THE SAME MAGNIFYING LENS WOULD ALSO APPEAR TO BE TWICE AS HIGH? (Yes.)

DO YOU THINK THAT OTHER THINGS SEEN THROUGH THE SAME REDUCING LENS WOULD ALSO APPEAR ABOUT HALF AS HIGH? (Yes.)

Have the children go back to Worksheet 5 and measure to see if the T in Column 3 is about twice as high as the T in Column 2.

CAN YOU ADD ANOTHER COLUMN TO THE WORKSHEETS BY USING THE SAME MAGNIFIER? (No.)

WHY NOT? (The magnifier isn't large enough. The shape or letter extends beyond the edge of the lens. You could do it if you had a magnifier with a greater diameter.)

Activity D

Have the children look at a shape in Column 5 through the reducing lens. Ask if they can find another shape on the worksheet that looks like what they see through the lens. (They should find it in Column 4.) Have them recall how they drew the shape in Column 5. (They copied what they saw in Column 4 through the magnifier.) Repeat with several other shapes in Columns 3, 4 and 5, to provide experience with the effect of the reducing lens undoing the effect of the magnifying lens.

Go to Columns 1 and 2 on the worksheet and follow the same procedure, using the magnifying lens to get back to the size from which the reductions were made.

The children may draw a letter or shape of their own choosing in Column 3 of the last row of Worksheet 5, and enlarge and reduce it as they did before.

Activity E

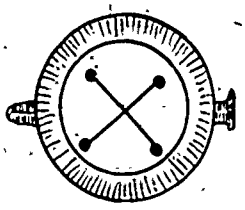
In presenting the following sequence of pictures, fold your manual back so that you can show just one page at a time.

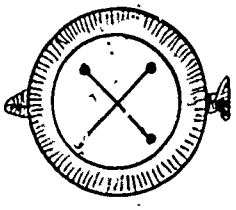
Show the title page and then page one of "Through the Magic Window" to the children. Ask the children what the picture is. (Three buttons.) If they do not guess correctly, don't tell them. They can find out if they were right when they look at the following pictures. Ask the children what they notice about the pictures. (Each one is twice as long or half as long as the one before it.)

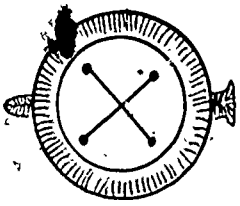
Then have the children make a similar picture story using a subject of their choice. Some possible subjects are a bug, a butterfly, a fresh flower, a plant leaf (especially a hairy one), carrot or celery tops. The most interesting subjects are those with additional details to be revealed under the magnifier, and those that come from environments the children can find out about and include in their scaled-down drawings.

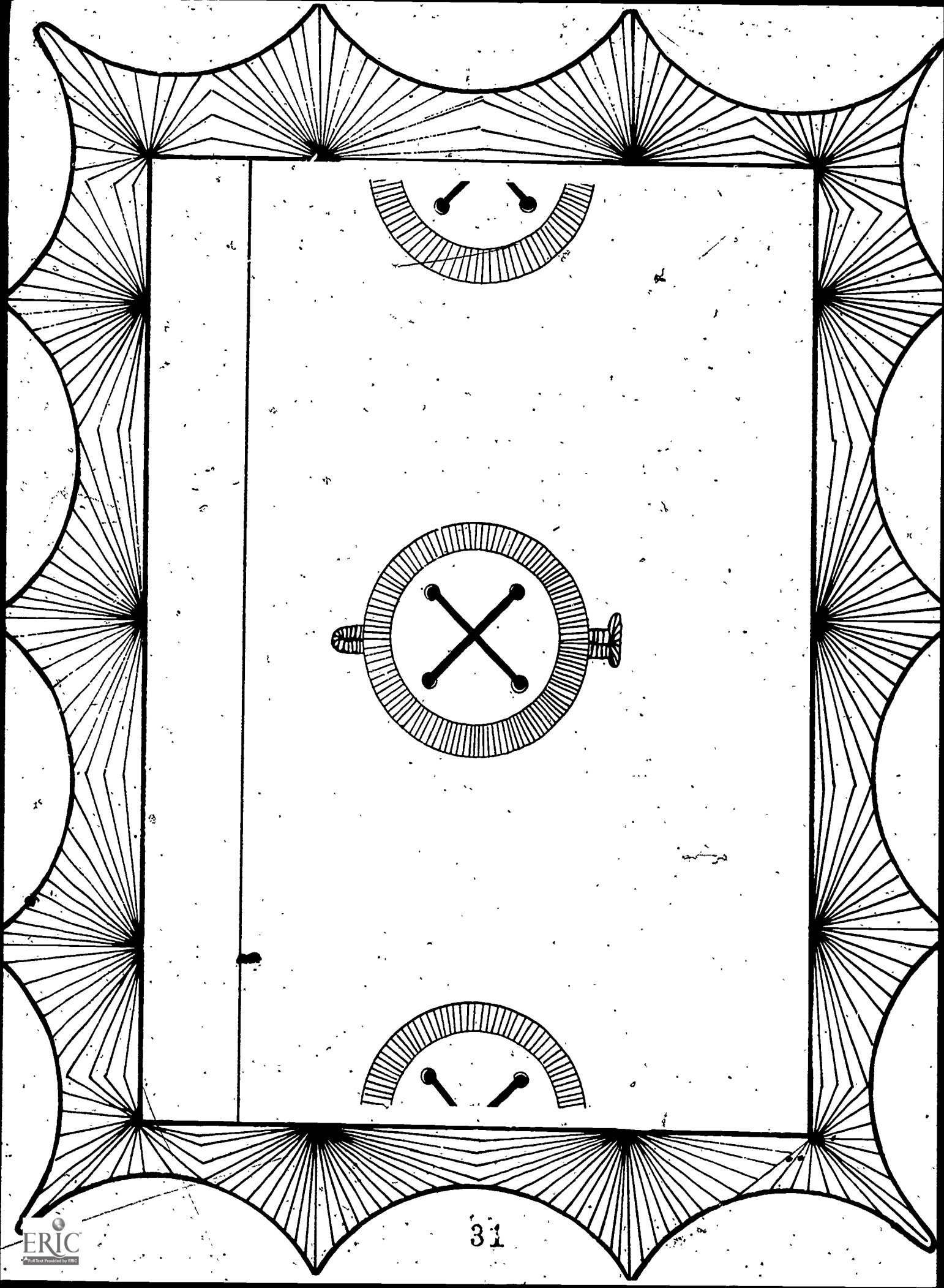
Blank "Magic Window" frames appear in the Student Manuals after Worksheet 6. Have the children draw a middle-sized picture in window frame number 3 and then make two-step enlargements in frames 2 and 1, and two-step reductions in frames 4 and 5.

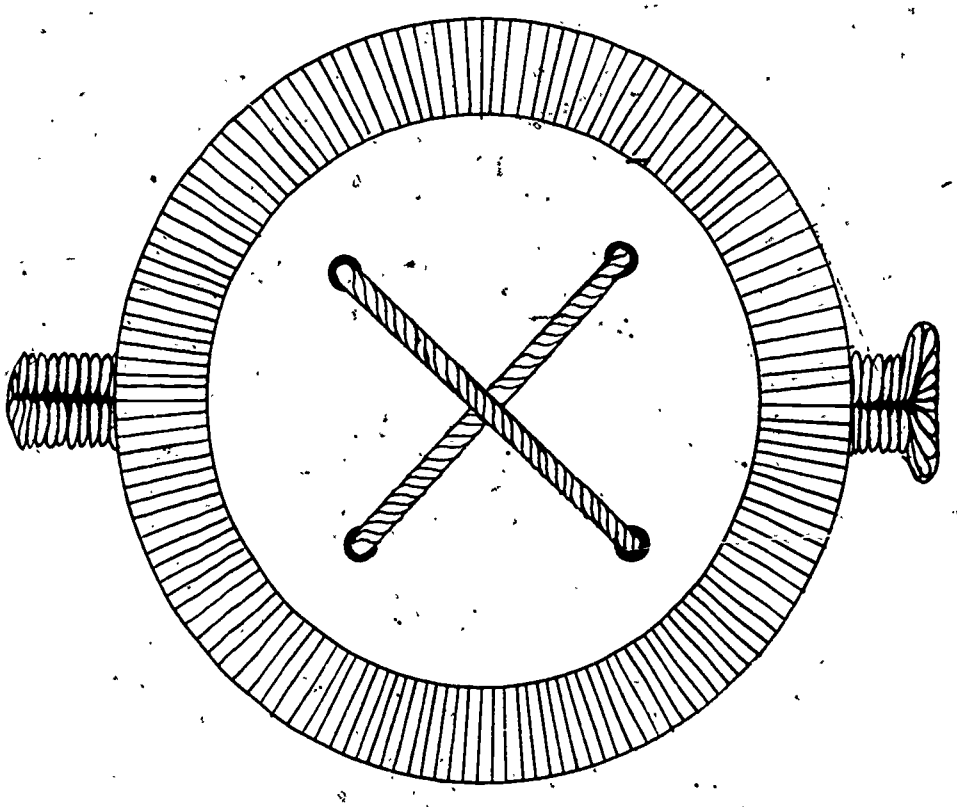
THROUGH THE MAGIC WINDOW

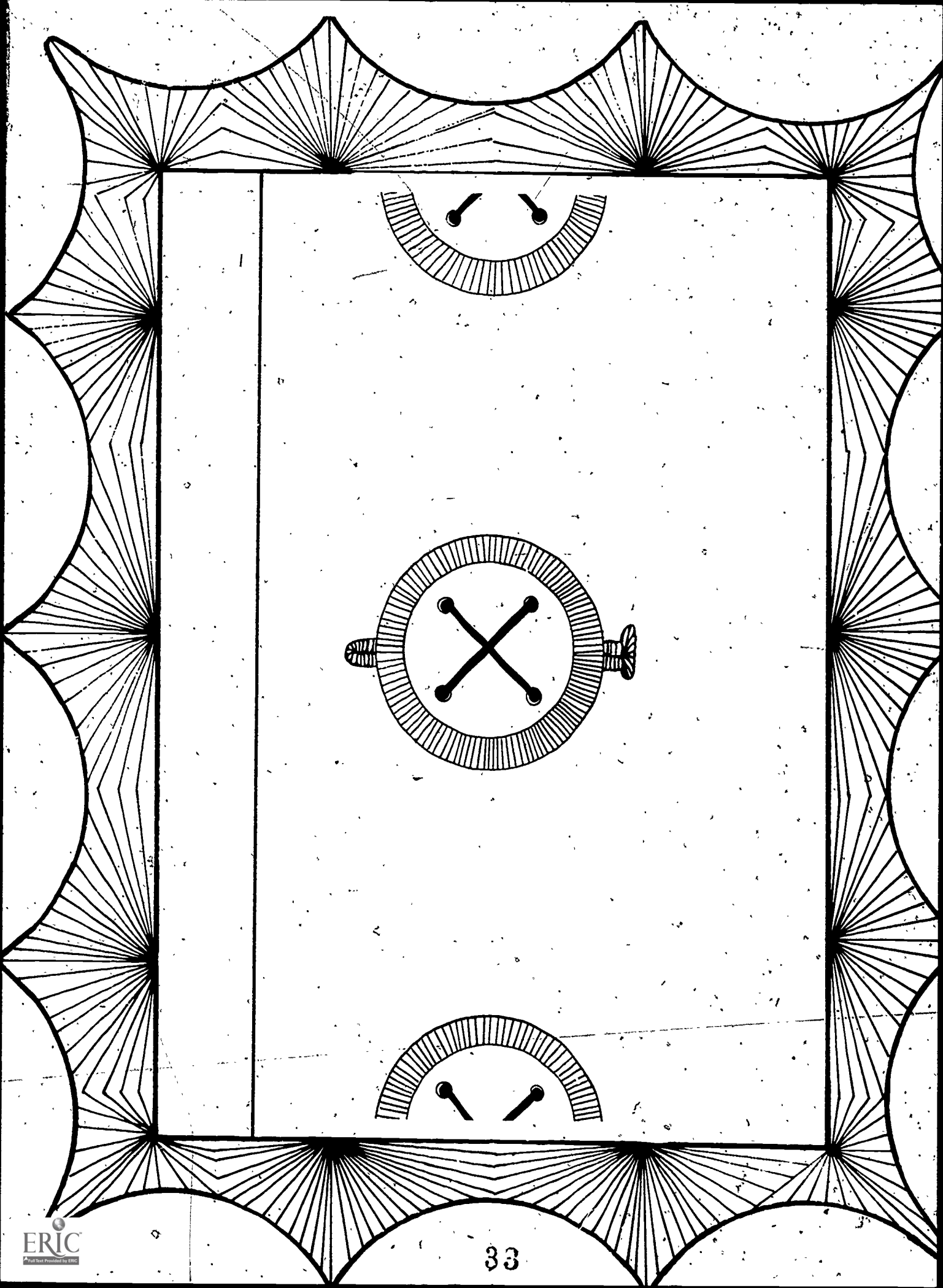


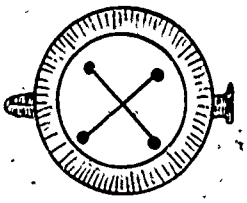


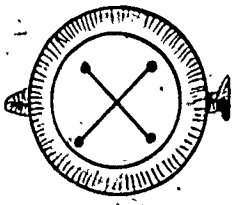


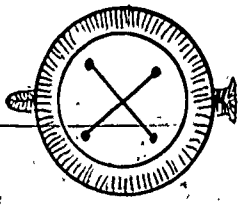


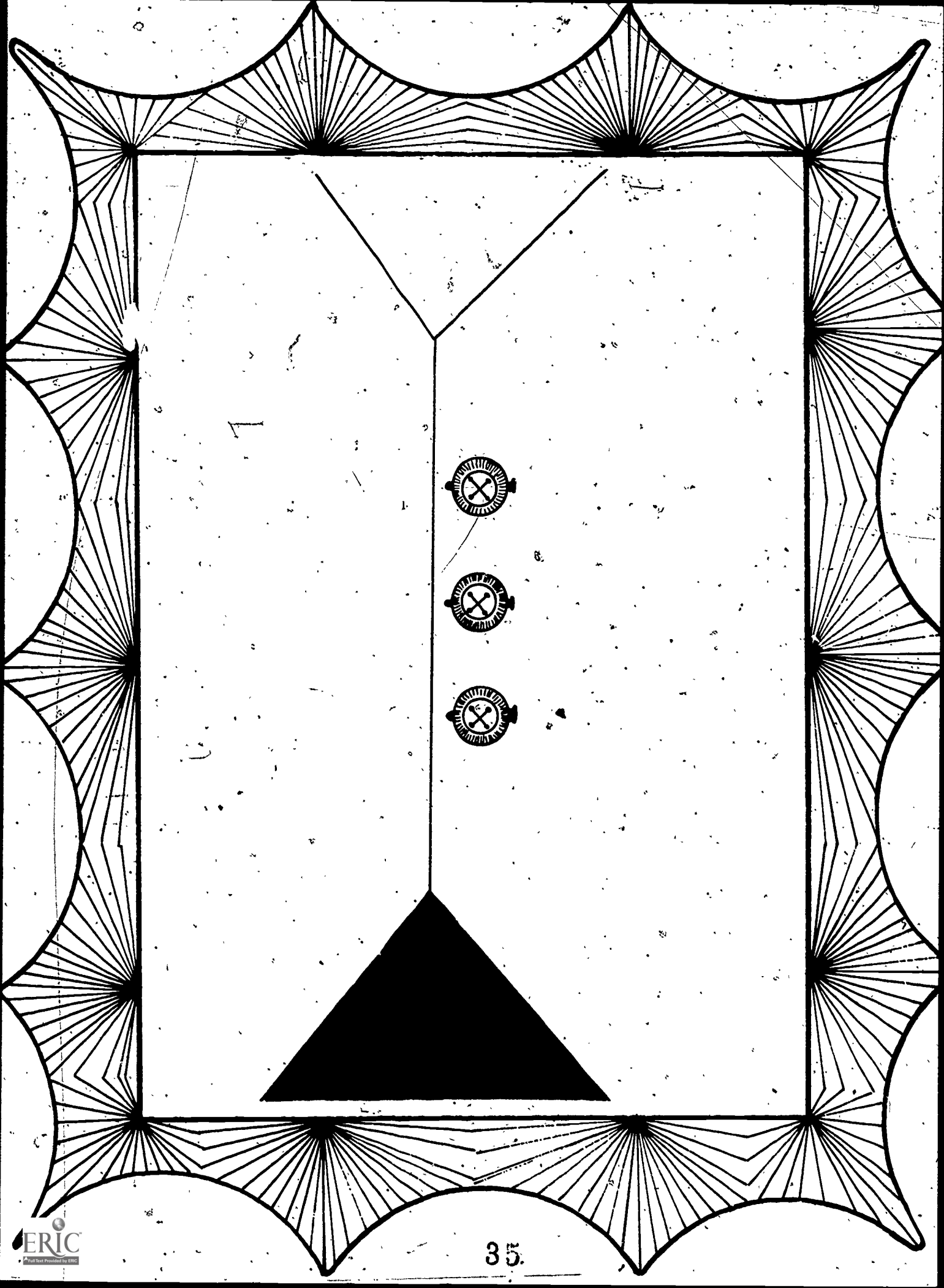












Lesson 3: ENLARGING PATTERNS

An overhead projector is used to project silhouettes onto the chalkboard and onto a grid. When the projector is moved back, the silhouette image becomes larger, but other properties of the image remain the same.

The children apply this concept when they scale up the dimensions of objects by drawing on grids of two different sizes. Then scale factor is introduced: the children enlarge other patterns by making two units of length of the patterns they draw represent one unit of length of the original patterns. (This work is done on grids of equal size.) The multiplication equation for the length of the side is presented.

Introduce the word "dimensions" when measuring the length of the sides. This is important because when two dimensions of a shape are scaled up by a factor of 2, the area of the shape increases by a factor of 4. Thus it is necessary to emphasize that we are scaling up the dimensions, not the area.

If a child notices that the area of the shape he is working with increases by more than a factor of 2 when the length of a side is doubled, tell him that he is right, and that we will work with area later in the unit.

Be sure that the children understand that the images projected in the course of these lessons are representations of the actual objects.

For all the activities involving projected images, be sure that projector lens and center of image are the same distance from the floor to avoid distortion of the image.

MATERIALS

- overhead projector
- transparencies of Worksheets 7 and 9
- silhouettes of man, mouse, rabbit, and 1-inch square
- 1-inch grid paper
- Worksheets 7, 8, 9 and 10.

PROCEDURE

Activity A

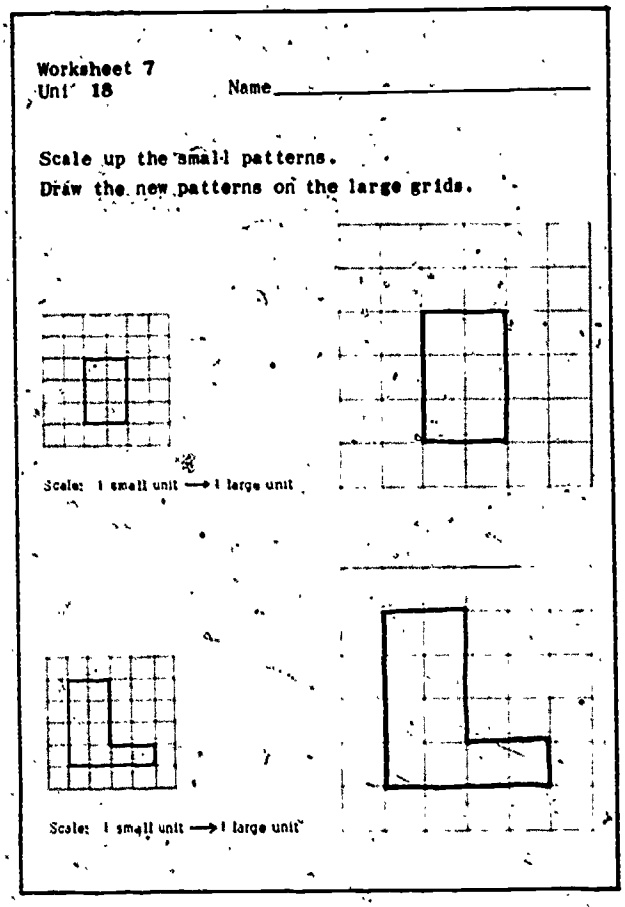
Set the overhead projector about 30 inches from the chalkboard and place the silhouette of the man on its stage. Bring the image into focus on the board. Ask the children what you might do to get a larger image of the man on the board. If no one suggests that you can move the projector back to enlarge the image, demonstrate this. Then ask the children what things are the same about the small image and the large image. (The shape is the same—they both have two arms, two legs, etc.) Next ask what is different about them. (The dimensions are greater on the second one—the image is taller and wider; the second image is bigger. Some children who have had MINNEMAST Unit 12 may say that the area is greater.)

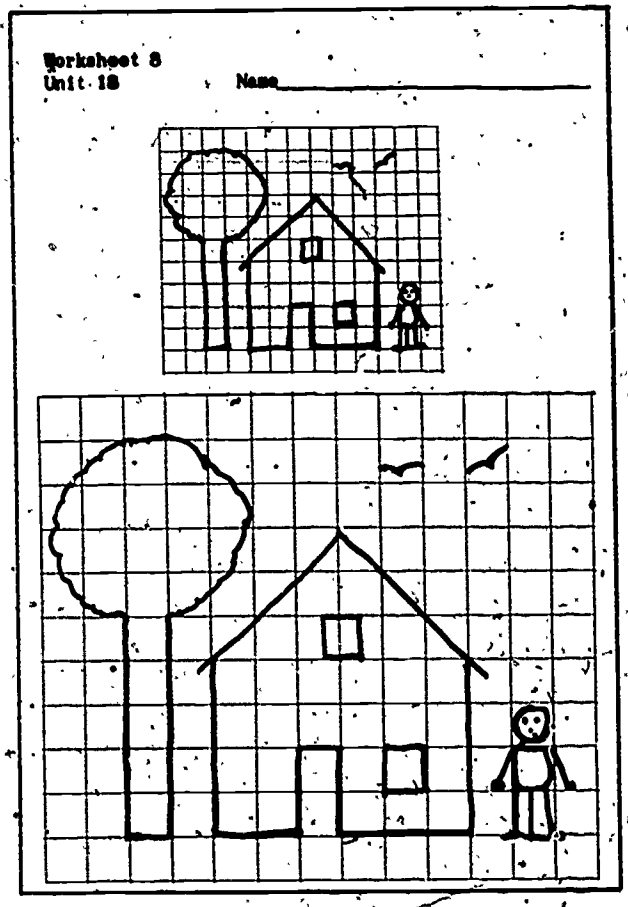
Repeat the above procedure using the silhouettes of the small house and the rabbit.

Return the projector to 30 inches from the board. Place the one-inch square on the stage of the projector. Have the children tell you how to make the image of the square look larger. If possible, let a child move the projector back to show how to enlarge the image of the square.

Ask the children what things are the same about the small image and the large image. (They are both dark, have four corners, are the same shape.) Then ask what is different about them. (The sides are longer; the square is bigger or the area is greater.)

Have the class turn to Worksheet 7. Use the transparency of the worksheet on the overhead. With the help of the class do





the first pattern on the transparency while the children do it on their worksheets. Ask the children in what way the worksheet is similar to what they saw with the projector. (The shape of the new pattern is the same but the dimensions are greater.)

Have the children complete the second pattern independently. Check their work, and help those who need it.

Worksheet 8 shows two grids of different sizes. Have the children make their own drawings on the small grid and then scale up the dimensions (the lengths of the sides) on the larger grid, using the scale of one small unit → one large unit. When they are finished, let the children show their patterns to the class and tell how they enlarged them.

Activity B

Preparation:

Attach a 1-inch grid paper to the chalkboard. Put the 1-inch

square on the projector stage, and line up the projector so that the image of the square covers four units on the grid attached to the board. (The projector should be about 30 inches from the board.) Turn off the projector.

Procedure:

Tell the children that you have put a grid on the board so that they can find out how much longer the sides of the square become when you move the projector. Turn on the projector and ask the class:

HOW LONG ARE THE SIDES OF THE SQUARE? (2 units each.)

Record the answer on the chalkboard. Ask the children how to make the sides of the image twice as long. (They should suggest moving the projector away from the board until the sides of the shadow are each 4 units long.) If possible have a child move it back as far as he thinks he should for the sides to be twice as long.

HOW MANY UNITS LONG ARE THE SIDES OF THE SQUARE NOW? (4 units each.)

Tell the children that they have scaled up the sides of the square by a factor of two. The sides of the square are now twice as long as they were. Explain that the length of the sides has been multiplied by two, and every one unit of length along the edges of the square has become two units. Record on the board the change in the length of the sides of the square when they are scaled up by a factor of two:

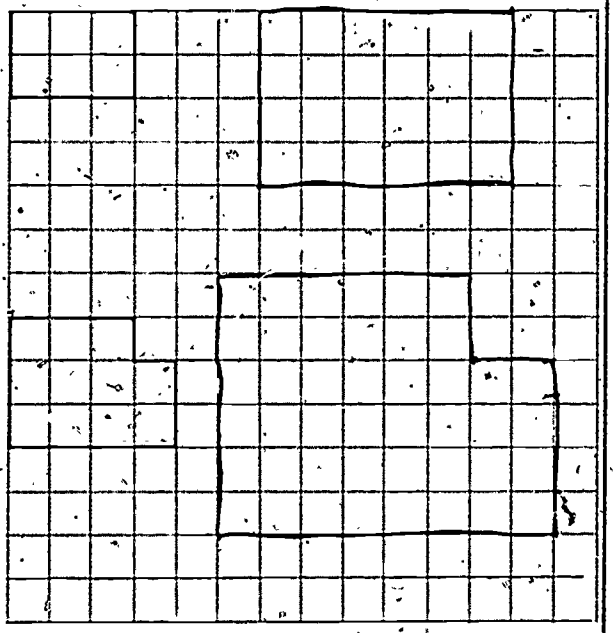
$$2 \times 2 = 4$$

Have the children turn to Worksheet 9. Do the first pattern with the children. (A transparency is provided.) They are to do their drawings on the grid to the right of each shape. Guide the children to notice that each unit along the side of the shape will become two units along the side of the drawing that they make. (One unit will become two units; two units will become four units, etc.)

The children should be able to enlarge the second pattern by themselves. When they have finished, ask a few children to show the patterns they drew and tell what they have done.

Worksheet 9
Unit 18, Name_____

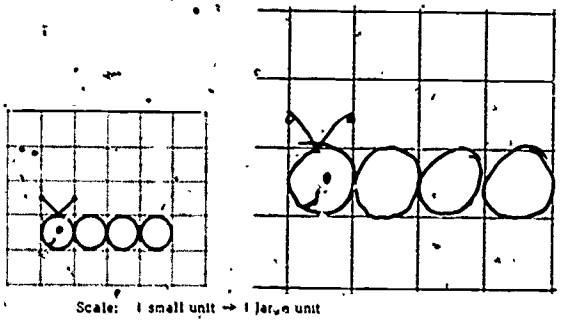
Scale up the dimensions (the length of sides) of the patterns by a factor of 2.
Draw the new patterns on the grid.



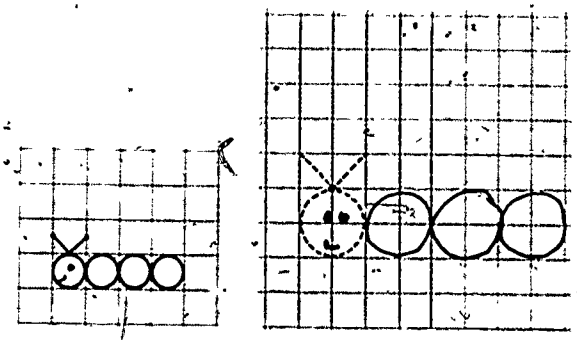
Scale: 1 unit → 2 units

Worksheet 10
Unit 18 Name_____

Enlarge the caterpillars.
Use the scale shown below each one.



Scale: 1 small unit → 1 large unit



Scale: 1 unit → 2 units

Activity C

In Activity A the children scaled by using units of different size, and in Activity B by using a 1 → 2 scale factor. In Worksheet 10 they enlarge the same picture by both techniques.

Call the children's attention to the two scales shown and ask them what the scales tell them to do. After they have drawn the caterpillars, guide them to compare the two enlarged drawings. (They should be the same size and shape.)

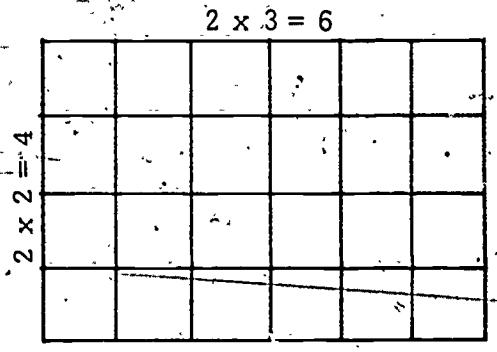
Activity D

Ask how the larger caterpillar compares with the smaller one. (It is twice as long and twice as wide.) Together with the class, develop the multiplication equation for the length and width of the larger caterpillar. Call attention to the second pair of caterpillars to illustrate the increase, as follows:

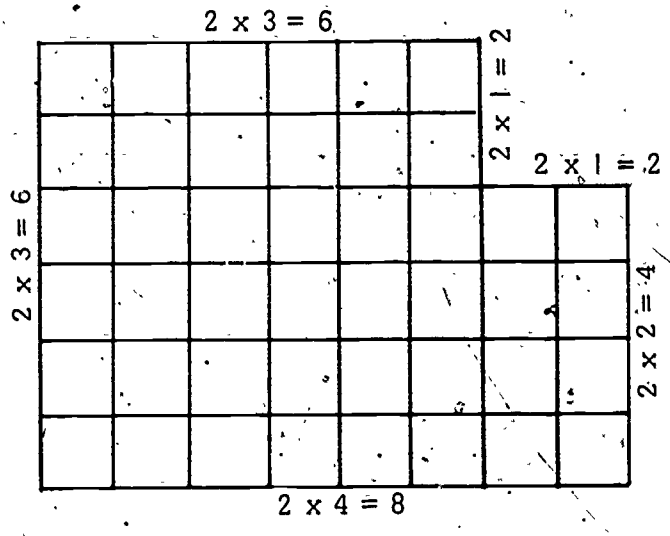
EACH 1 UNIT OF LENGTH BECAME 2 UNITS. THE CATERPILLAR WAS 4 UNITS LONG AND BECAME 8 UNITS LONG. THE MULTIPLICATION EQUATION THAT DESCRIBES THIS IS:

$$2 \times 4 = 8$$

Have the children turn back to Worksheet 9. Together, develop equations for the change in length of the sides of the first pattern.



The second pattern is a more complicated shape—that is, it has more sides. But the change in the length of each side is described in the same way:



Call attention to the fact that the scale factor is the same, 2, in each of the equations.

Lesson 4: REDUCING PATTERNS

The overhead projector is used to project silhouettes onto the chalkboard and onto a grid sheet. When the projector is moved closer to the screen, the silhouettes become smaller. Following this illustration of the fact that the shapes are the same in every detail except size, the children learn how to scale down the dimensions of objects by using paper with grids of two different sizes. Then they go on to reduce patterns using scale factor. They make one unit of length of the patterns they draw represent two units of length of the original pattern.

MATERIALS

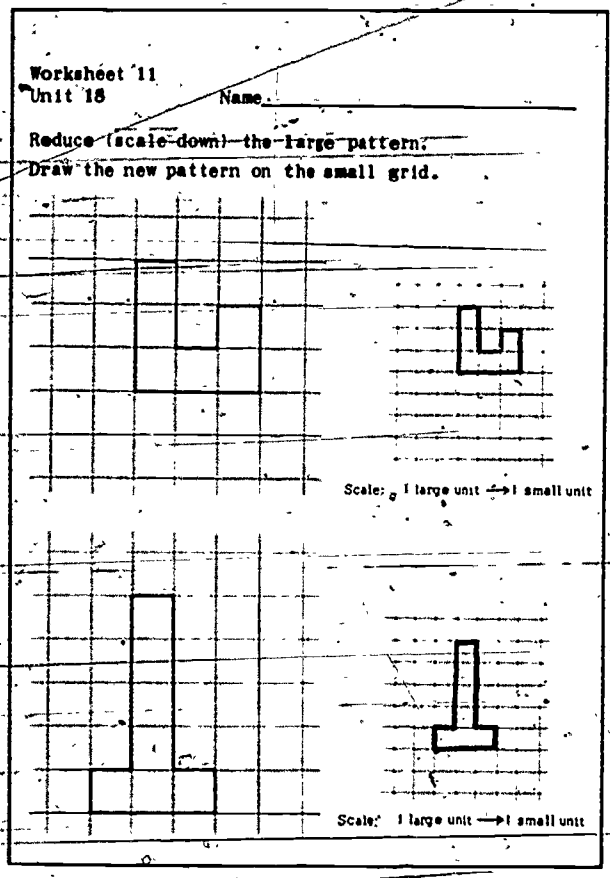
- overhead projector
- silhouette shapes of man, rabbit, house, and 1-inch square
- 1-inch grid paper
- transparencies of Worksheets 11, 13
- Worksheets 11, 12 and 13

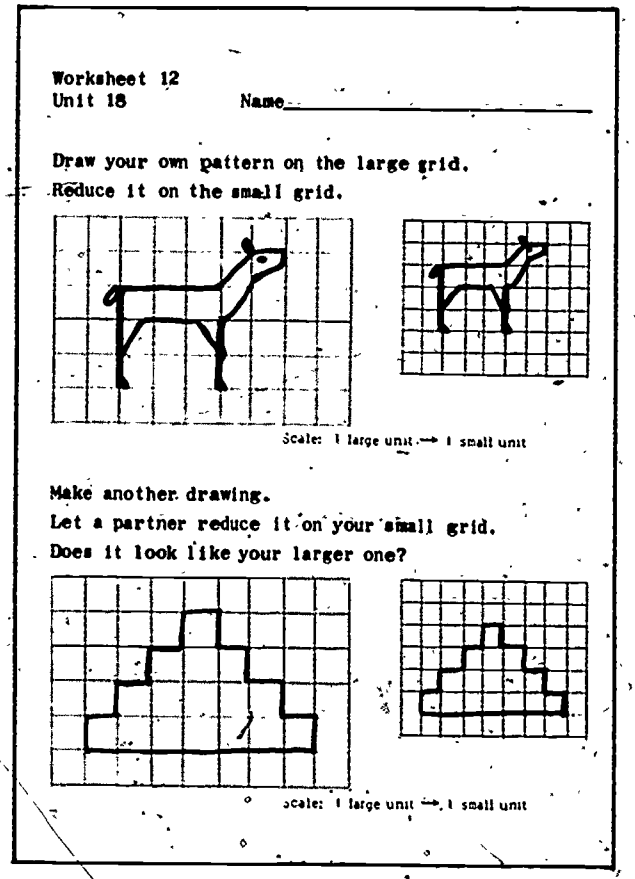
PROCEDURE

Activity A

Follow procedures similar to those in Lesson 3, but this time start with the overhead projector about 60 inches from the board. Place the silhouettes on the projector stage one at a time and ask the children how they can be made to look smaller. If the children do not suggest moving the projector closer to the board, demonstrate this. Then ask the children what things are the same about the large and the small silhouettes and what things are different.

Have the class turn to Worksheet 11. Ask in what way Worksheet 11 is similar to what the children saw with the projector. Use the transparency of Worksheet 11 on the overhead projector, and work with the class to do the first pattern. Ask the children to complete the second pattern by themselves. If they seem to understand what they are doing, let them do





Worksheet 12 independently. Give additional help if needed. The children are to make a drawing on the large grid and then reduce it on the small grid. Then they will make another drawing and exchange papers with a friend. This time they are to reduce each other's drawing. Each may check the other's paper.

Activity B

Preparation:

For this activity the 1-inch grid paper should be attached to the chalkboard. Put the 1-inch square on the projector stage, turn on the projector, and line it up so that the image of the 1-inch square covers an area of the grid paper with 4 units on each edge of the square. (The projector will be about 60 inches from the board.) Turn off the projector.

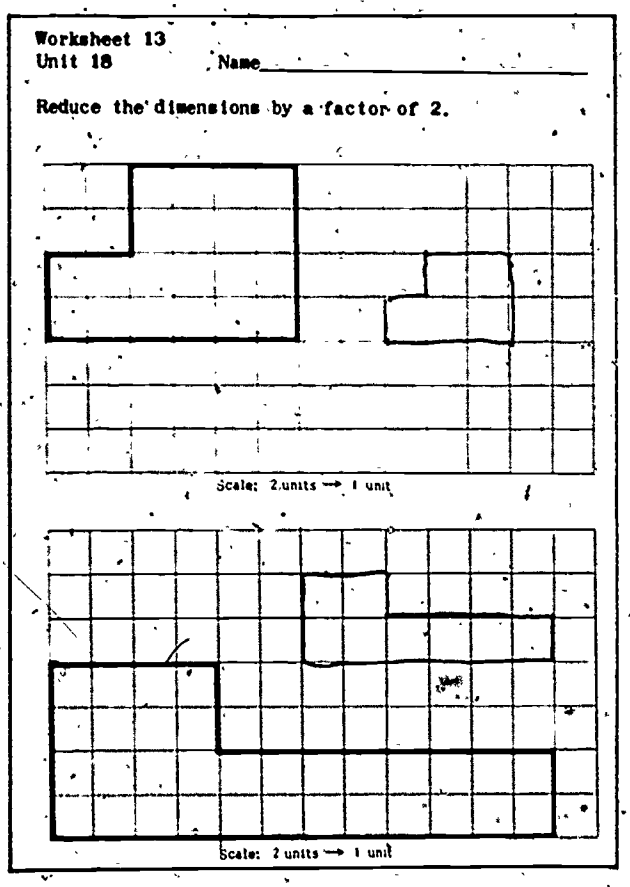
Procedure:

Tell the children that you have put a grid paper on the board so that they can find out how much shorter the sides of the square are when the square becomes smaller. Turn on the projector and ask the class:

HOW LONG ARE THE SIDES OF THE SQUARE? (4 units.)

Record the answer on the chalkboard. Ask the children how to make the sides of the square half as long. They should suggest moving the projector closer to the board until the sides of the square are each two units long. If the projector is on a movable cart ask a child to move it as close as he thinks he should to make the image half as long.

HOW MANY UNITS LONG ARE THE SIDES OF THE SQUARE NOW? (2 units.)


Worksheet 13
Unit 18 Name_____

Reduce the dimensions by a factor of 2.

Scale: 2 units → 1 unit

Scale: 2 units → 1 unit

Tell the children the sides of the square have been scaled down by a factor of two; two units have become one unit. The sides of the square are now half as long as they were. The length of the sides has been divided by two. (Note that we use the term "division" here, but we do not write division equations.)

Have the children turn to Worksheet 13. Do the first pattern with the children. (A transparency is provided.) Guide the children to notice that, as in the projected square, each two units along the sides of the shapes on the worksheet will become one unit. Two units become one unit; four units become two units.

Have the children complete the second pattern independently. Check their work.

Lesson 5: ENLARGING AND REDUCING PATTERNS

The children practice enlarging and reducing patterns using the methods they have studied in the two previous lessons. First they scale up and down using grids with different sized squares, and then they use grids with the same size squares to scale by factors of 2, 3 and 4.

Finally they scale a figure up, and then scale the enlarged figure down again by the same factor, arriving at the original figure. This illustrates the fact that scaling up and down by the same factors undo each other, and so lays the groundwork for the concept that multiplication and division are opposite—that they undo each other. Because division has been barely introduced in Unit 17, the division concepts are mostly intuitive here. We say that the sides of a pattern that are scaled down by 2 are half as long as the sides of the original pattern, but we do not write division equations.

MATERIALS

- overhead projector
- geometric shapes: squares, diamonds, triangles, circles
- Worksheets 14 through 19

PROCEDURE

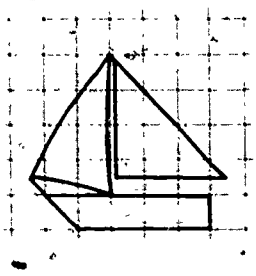
Activity A

Set the overhead projector about 60 inches from the chalkboard, focus it, and place one of the geometric shapes on the stage. Ask a child to trace around the projected outline on the chalkboard. Move the machine further away from the board, focus the image again, and ask another child to trace the outline of the projected representation. Then move the machine quite close to the board, focus it, and again ask a child to trace the outline of the projected image. Ask the children how the dimensions of the image changed when the machine was moved farther from and then closer to the board. Which properties remained the same?

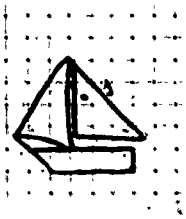
Repeat the activity using the other geometric shapes, and also things from their desks or around the room that the children might suggest. Ask a child to put his hand on the stage of the projector and have another child trace its shadow on the chalkboard. Repeat with the machine at another distance from the board. In each case discuss with the children which properties remain the same and which properties change.

Ask the class to turn to Worksheet 14 and look at the sailboat representation on the middle-sized grid. Have the children tell you what they would have to do to reproduce the sailboat on the two other grids. Guide them to the procedure of counting the number of units for each part of the sailboat before reproducing that part on another grid. Discuss Worksheet 15 in the same way.

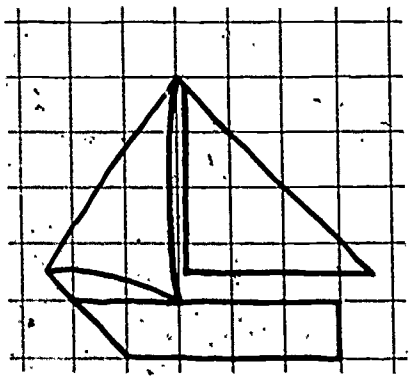
Worksheet 14
Unit 18 Name_____



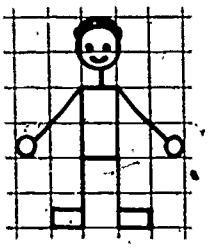
Reduce the sailboat on the small grid.



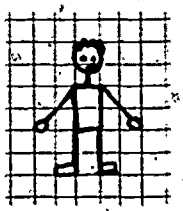
Enlarge the sailboat on the large grid.



Worksheet 15
Unit 18 Name_____

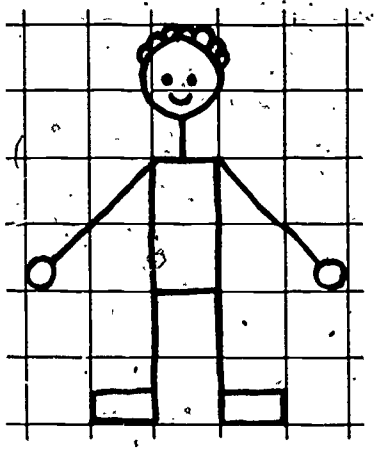


Reduce the representation of the man on the small grid.

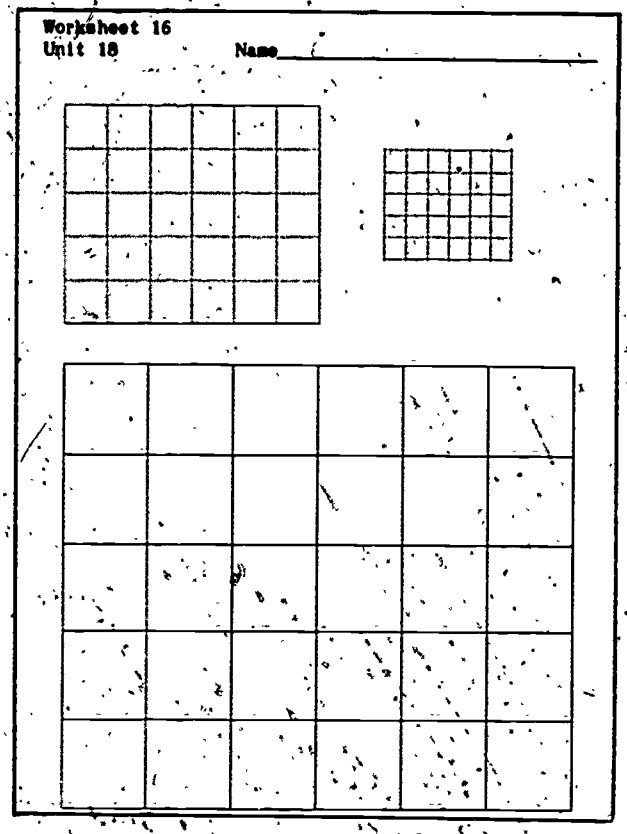


Scale: 1 middle-sized unit → 1 small unit

Enlarge the man on the large grid.



Scale:
1 middle-sized unit →
1 large unit



Check the finished worksheets. Then have the children turn to Worksheet 16. On this sheet they are to make their own patterns on the middle-sized grid, and then scale them up and down. Ask some children to exhibit their work and tell what they did.

Activity B

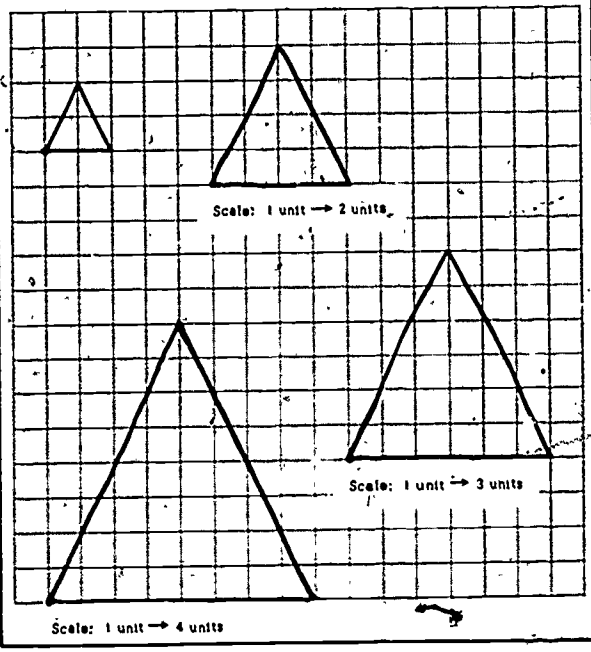
Discuss with the children what it means to them to scale up by a factor of 2, 3, etc. The children should understand that when the dimensions of an object are scaled up by a factor of 2, the lengths of the sides become twice as long as the sides of the original; when the scale factor is 3, the lengths of the sides become three times as long.

Have the children turn to Worksheet 17 and scale up the drawing first by a factor of 2, then by factors of 3 and 4, making the sides twice as long as in the original pattern and then 3 and 4 times as long. Be sure that they understand that they are to start at the dots and work to the right and up. Work with the children as they scale up the triangle by a factor of 2. When they work on the triangle, some children may find it easier to use a ruler to measure and then double the lengths of the sides, while others may continue to scale by counting grid units. If possible the worksheet should be completed independently.

When the children have finished scaling up the triangles, have them write equations for the length of the sides of the new figures.

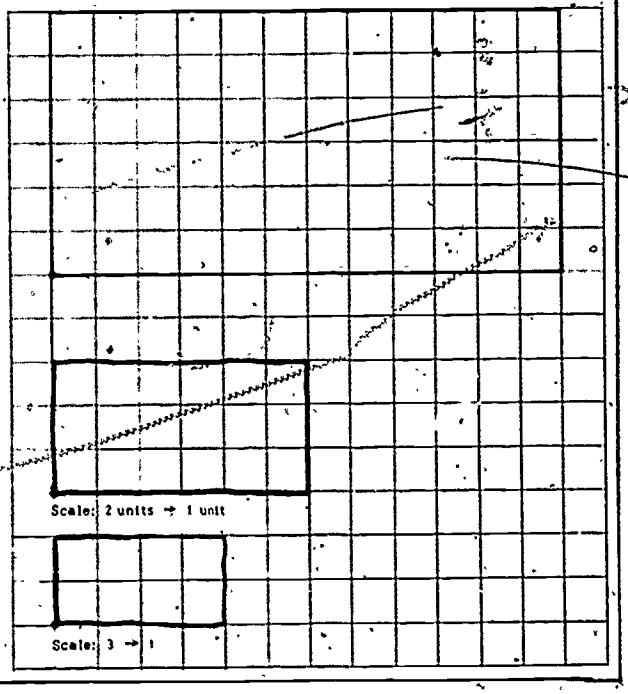
Worksheet 17
Unit 18 Name_____

Make larger triangles to scale.
Start at the dot each time.
Work to the right.
Use the scale shown below each dot.



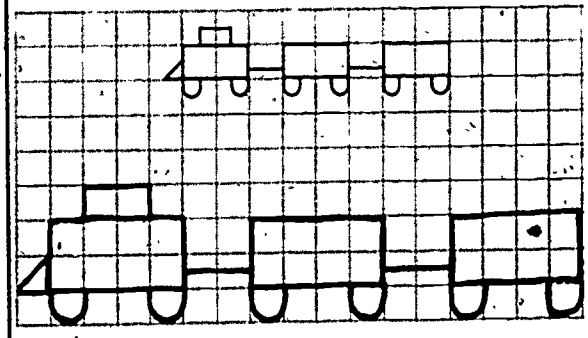
Worksheet 18
Unit 18 Name_____

Scale down the large rectangle.
Start each new rectangle at the dot.

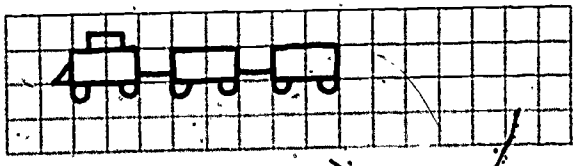


Worksheet 19
Unit 18 Name_____

Scale up this picture by a factor of 2.



Now scale down the new picture by a factor of 2.



Is this picture the same size as the one you started with?

Yes ☑ No ☐

Next have the children turn to Worksheet 18, and ask them to scale down the rectangle first by a factor of 3 and then by a factor of 2.

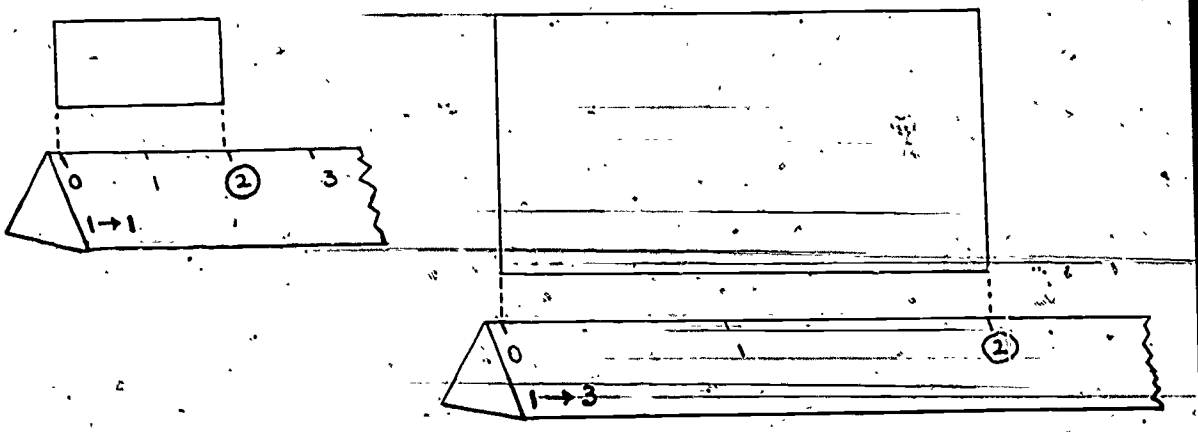
Have the children do Worksheet 19 without any preliminary discussion. Afterwards have them discuss what they have done. They should observe that in scaling the same representation up and then down again, one operation undoes the other.

WHEN WE SCALE UP BY A FACTOR OF TWO, THE SIDES BECOME TWICE AS LONG. WHEN WE SCALE DOWN BY A FACTOR OF TWO THE SIDES BECOME HALF AS LONG. WHEN WE SCALE DOWN WE UNDO SCALING UP.

Scaling up is a multiplication process; scaling down is a division process. We hope to develop an intuitive notion of multiplication and division as inverse processes. But it is not necessary to spell this out to the children. The extension of these division concepts will be made in third grade.

Lesson 6: SCALING WITH AN ARCHITECTS RULER

An architects ruler is made up of six different scales, in various multiples of a 1 → 1 scale. To make a representation of a rectangle in a scale of 1 → 3, you measure the original with the 1 → 1 scale and then draw the representation, using the same number of units on the 1 → 3 scale.



With this handy tool you can increase the dimensions of the original shape by a factor of 3, without using grid paper, counting units, or solving any multiplication equations.

The children are provided a printed sheet from which they make their own rulers. They first learn to use them, and then find out why they work. When they use their own standard units to mark off scales along blank lines, it helps them to see how one scale is a multiple of another. The architects ruler is another form of the parallel number lines the children were introduced to in Unit 17, where one line represented the number of kangaroo jumps and the other represented the number of units in each jump.

Prepare an architects ruler from the printed construction sheet facing page 42 before you teach this lesson.

MATERIALS

- architects ruler (made from Worksheet 20)

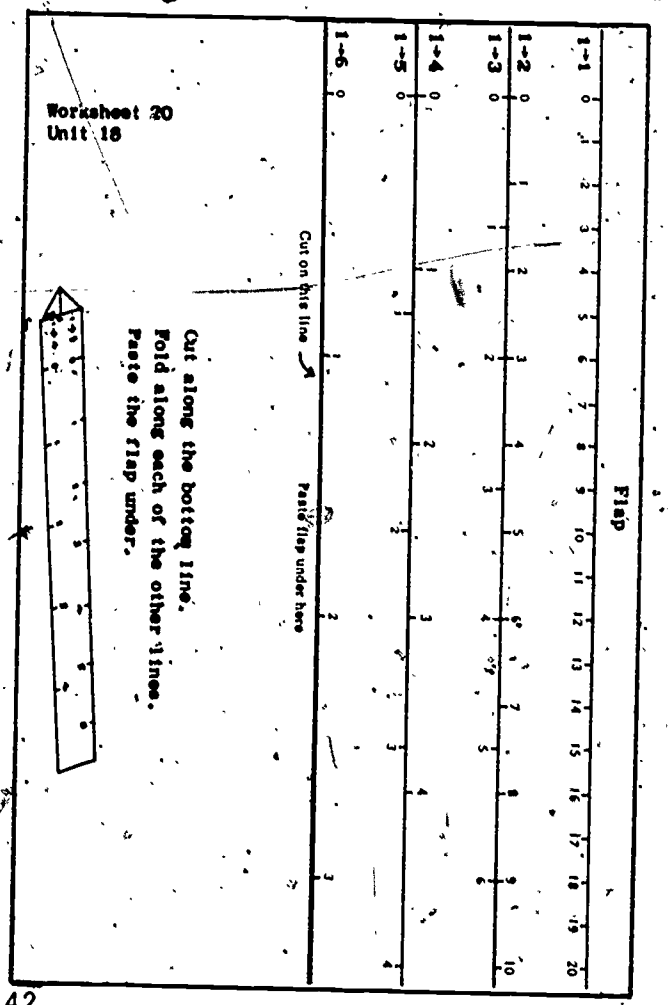
-- for each child --

- scissors
- strip of construction paper, 1 x 9 inches
- Worksheets 20, 21, 22, and 23

PROCEDURE

Activity A

Using the 1 → 1 scale on an architects ruler, draw a rectangle 4 units by 6 units on the chalkboard. Ask the class how they might scale up its sides by a factor of 2. Discuss and try some of the children's ideas.



Then erase all but the original rectangle and have the class turn to Worksheet 20, the scaling ruler construction sheet. Tell them that they are going to make a special tool for scaling.

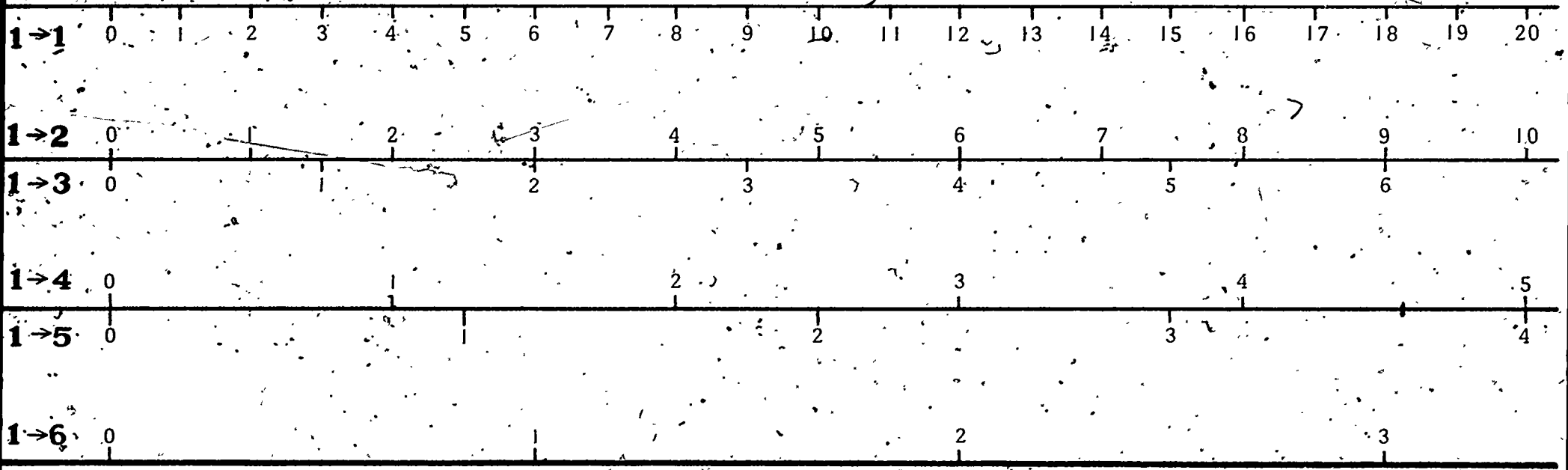
Directions for making architects ruler:

1. Cut carefully along the bottom line of the 1 → 6 scale. The top edge of the paper is the tuck-in flap; no cutting is necessary.

2. Fold carefully along each line so that the numbers face outward. (The children may need help with the folding.)

3. Paste the flap above the 1 → 1 scale to the underside of the 1 → 6 scale, to form a triangular ruler.

Flap

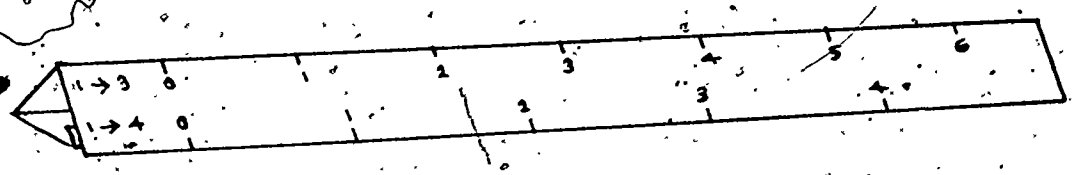


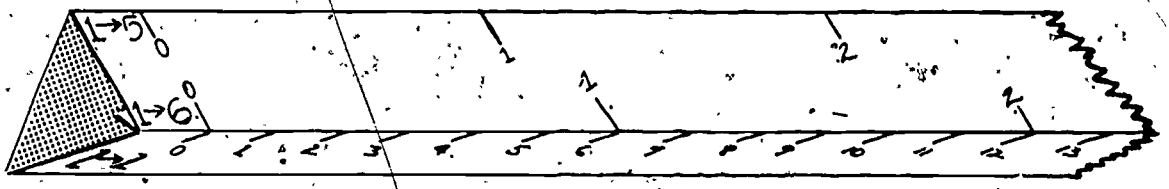
Cut on this line

Paste flap under here

Worksheet 20
Unit 18

Cut along the bottom line.
Fold along each of the other lines.
Paste the flap under.

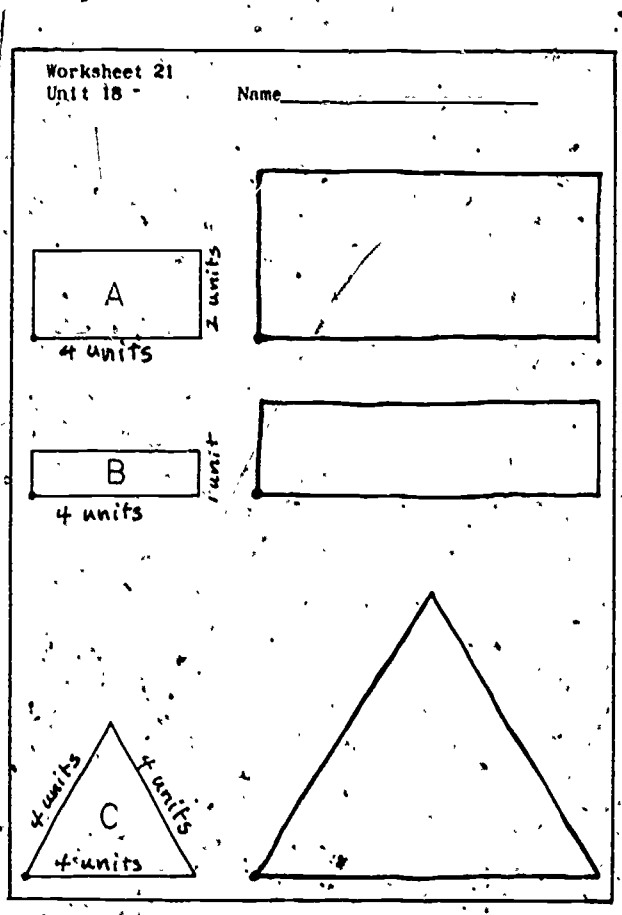




Now use the architects ruler to scale up the sides of the rectangle on the board by a factor of 2. Direct the children to look at the 1 → 1 scale on their rulers, and ask a child to measure one of the longer sides of the rectangle on the board using this scale. Have him write the length next to that side. (6 units.)

Then ask the children to look at the 1 → 2 scale. Select one child to use this new scale to draw a line segment that is 6 units long on the chalkboard. This 6-unit length will be twice as long as the longer side of the rectangle.

Ask a child to use the 1 → 1 scale to measure a shorter side of the rectangle on the chalkboard and write the length next to it. (4 units.) Then have another child draw a side on the new rectangle, using 4 units of the 1 → 2 scale. Finish the other sides.



Have the class turn to Worksheet 21 and measure the sides of patterns A, B, and C, using the 1 → 1 scale on their rulers. They are to record the measurements along the sides of the shapes. Then they are to turn their rulers to the 1 → 2 scale and draw each shape, using the same number of units on the 1 → 2 scale. The dimensions of the new shapes should be twice as long as the originals.

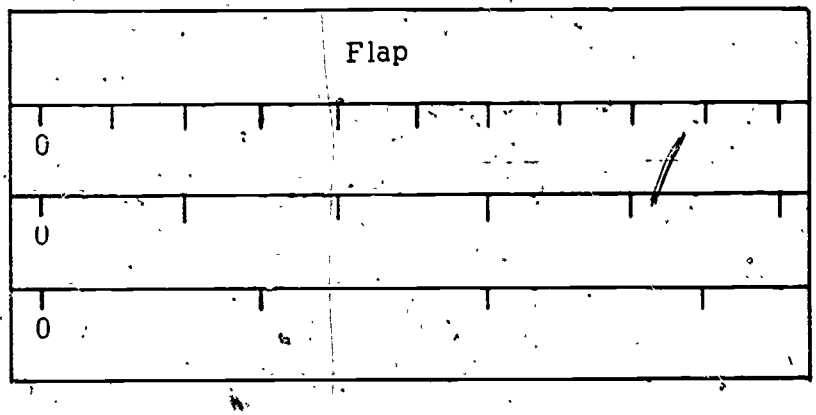
On another sheet of paper the architects ruler can be used to draw these shapes to different scales, such as 1 → 3, 1 → 5. Encourage the children to make drawings of their own and scale them up by a factor of 2, 3, etc. (Use the term "scale by a factor of ..." whenever appropriate, to relate scaling to multiplication.)

After the children have used a few of the scales on their architects rulers, ask them if they have figured out why the rulers can be used to scale up in this way. If no one notices that the units on the different scales are multiples of the 1 → 1 scale, do not try to point it out now. Go right into the next activity, which should help them discover this for themselves. If the children already see the relationship, Activity B will reinforce their understanding.

Activity B

In this activity the children will make architects rulers that have only three number lines, but they will mark off the units themselves. Two of the lines scale up from the first. Worksheet 22 shows a ruler with neither units nor numbers on it.

Give each child a strip of construction paper 1 x 9 inches. Have the children measure a 1-inch piece at the end of the strip, mark it carefully, and cut it off. This is their unit marker. They are to place the 1-inch unit marker at the zero of the ruler's first line and mark off a unit length, then move the marker to that mark and mark off another unit length, and so on, until the complete line is marked off into equal units. The units should be marked below the line.



On the next line the children are to make a 1 → 2 scale. With the one-unit piece, they mark off two units at the end of the construction paper strip. They cut this off and use it to mark off the new larger units on the second line, again starting at zero.

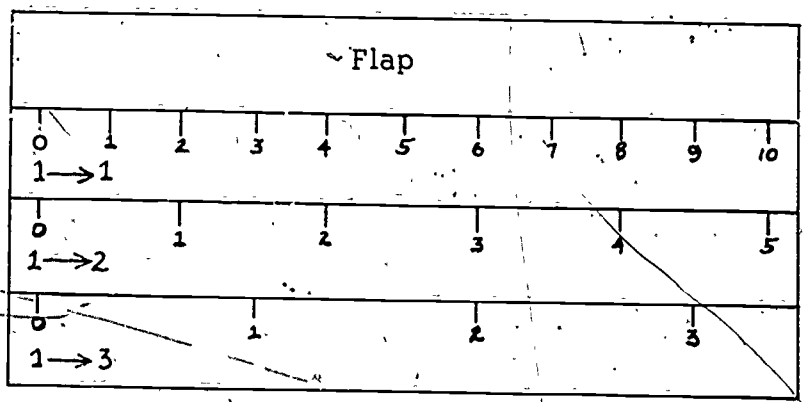
Next they use the one-unit piece to measure off 3 units at the end of the construction strip. They cut this off and use it to mark off still larger units on the third line.

When the children have finished marking off units on the three scales, ask how the length of a unit on the second line compares with the length of a unit on the first line. (It is twice as long.) Explain that this is why we call the second line a 1 → 2 scale. Have them label the scale.

Ask how the length of a unit on the third line compares with the length of a unit on the first line. (It is three times as long.) The scale is therefore 1 → 3. Label it.

It may be more difficult for some children to understand why the first scale of standard units on which the other scales are based is called the 1 → 1 scale. If so, go back to the original rectangle on the board and have a child measure it with the 1 → 1 scale and then draw a new one also using the 1 → 1 scale. They will see that the new rectangle is the same size as the original. This should help them understand why a scale that produces a drawing in which 1 unit represents 1 unit of the original is called a 1 → 1 scale. Have them label the scale.

Now they should number each of their scales.



Activity C

Have the children look at Worksheet 22 and at Worksheet 23, which is a duplicate of the one from which they made their

architects ruler. Point out that these parallel number lines are similar to the ones in Unit 17, Lesson 3. Each larger scale is related to the 1 → 1 scale according to a multiplication (scale) factor. Since one unit on the 1 → 2 scale is twice as long as one unit on the 1 → 1 scale, one unit on the 1 → 2 scale is equivalent to two units on the 1 → 1 scale, 2 units on the 1 → 2 scale is equivalent to 4 units on the 1 → 1 scale, etc.

After the children have had an opportunity to examine and discuss the scales opened flat, if they wish to they can make another architects ruler from Worksheet 22 by cutting along the bottom line, folding the flap in and pasting as before. The children can make 1 → 4, 1 → 5, 1 → 6, and 1 → 7 scales on larger paper if they would like to, and use these to scale up their own drawings.

The architects rulers made from Worksheet 20 should be saved for use in Lessons 9 and 12.

Lesson 7: SCALING WITH MINNEBARS

In this lesson the children will use Minnebar shapes and representations of Minnebar shapes to help reinforce their understanding of scaling up and scaling down. Work with area is introduced here, but this is secondary to the main purpose of learning to scale lengths. Simple arrays are used to illustrate the use of multiplication for finding area. The children first multiply length and width; then they can check their answers by counting the Minnebar units in the shapes they build. Note that the work in this lesson calls for scaling in just two dimensions, length and width.

MATERIALS

-- for each child --

- set of Minnebars
- Worksheets 24 through 29

PROCEDURE

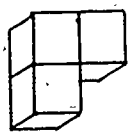
Activity A

Each child should have a box or bag of Minnebars on his desk. Student Manuals should be opened to Worksheet 24.

Have the children place a 1-unit Minnebar in the square on the upper grid of Worksheet 24. Then ask the children to use other Minnebars to make a shape that is similar to the black 1-unit Minnebar but whose sides are twice the length of the sides of the black bar. The larger Minnebar shape should be built in the space provided below the first Minnebar representation. Have the children trace around the larger group of Minnebars.

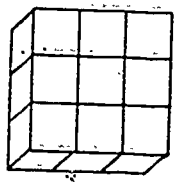
WHAT HAVE YOU DRAWN ON YOUR PAPER? (An outline of a 1-unit Minnebar scaled up by a factor of 2. The sides are twice as long.)

Some children may build a shape that looks like this:



They arrive at this by scaling up two of the sides and stopping at that. In this case, emphasize that the new shape must be the same as the old, and therefore they must scale up all four sides.

On the lower part of Worksheet 24 ask the children to start with the 1-unit black Minnebar again, but this time to scale up the sides by a factor of three. The sides of the new shape they make will be three times as long as the sides of the black Minnebar. Tell them to build the new Minnebar shape on the paper under the black bar, and then to trace around it.



Some children may like to scale the sides of the 1-unit Minnebar up by a factor of 5. This is shown on optional Worksheet 25.

Return to Worksheet 24 and ask how many Minnebar units were needed to build the first shape (1), and how many were needed to scale up the dimensions by a factor of 2 (4).

Draw a representation of the 1-unit shape on the board.

HOW MANY ROWS ARE IN THIS SHAPE? (One.)

HOW MANY COLUMNS ARE IN THIS SHAPE? (One.)

1 x 1 = 1

Write the multiplication equation for the area of the shape on the board.

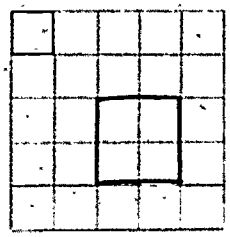
HOW MANY MINNEBAR UNITS WERE NEEDED FOR THE SHAPE? (One.)

Point to the product.

Repeat with the scaled-up shape. Draw a representation on the board.

Worksheet 24
Unit 18 Name_____

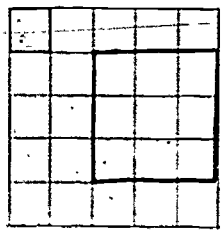
Put a 1-unit Minnebar on the square.
Use more Minnebars to scale up the dimensions (the length of the sides) by a factor of 2.



1 x 1 = 1

2 x 2 = 4

Build another 1-unit Minnebar shape.
Scale up the dimensions by a factor of 3.

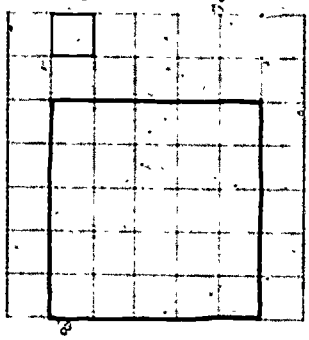


1 x 1 = 1

3 x 3 = 9

Worksheet 25
Unit 18 Name_____

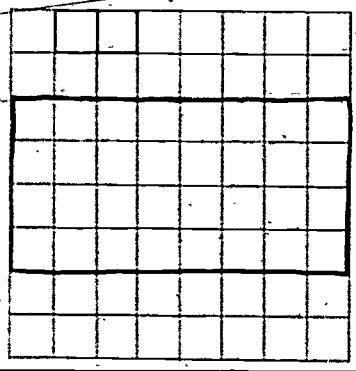
Scale up the sides by a factor of 5.



1 x 1 = 1

5 x 5 = 25

Scale up the dimensions by a factor of 4.

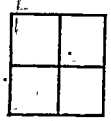


1 x 2 = 2

4 x 8 = 32

HOW MANY ROWS ARE IN THIS SHAPE? (Two.)

HOW MANY COLUMNS ARE IN THIS SHAPE? (Two.)



2 x 2 = 4

Write the multiplication equation for the area of the shape.

HOW MANY MINNEBAR UNITS WERE NEEDED FOR THE WHOLE SHAPE? (Four.)

IF WE MULTIPLY THE NUMBER OF ROWS BY THE NUMBER OF COLUMNS IN THE SHAPE, WE CAN FIND OUT HOW MANY UNITS THERE ARE IN THE WHOLE SHAPE. THIS IS CALLED THE AREA. (Point to each part of the equation as you mention it.)

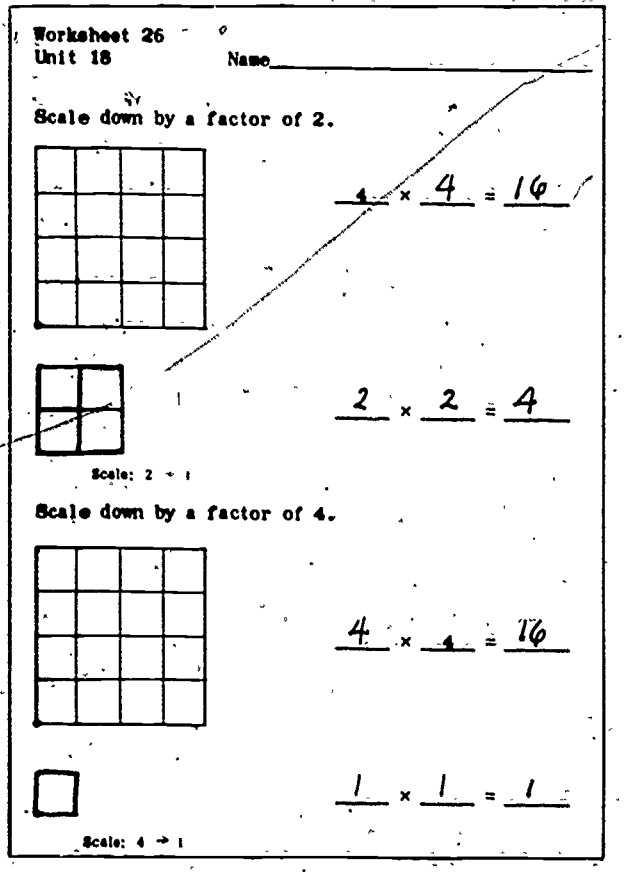
Have the children write the equations for the area next to each shape they have worked with on Worksheets 24 and 25.

You may have to proceed step by step with some students. Have them count the Minnebars along the sides of the shape to find the length of two sides. These are the two factors that they must multiply to find the area. They can count the squares in the entire shape to check their multiplication.

Even if some children have difficulty with area equations, they should move along with the rest of the class, building and scaling the Minnebar shapes, and counting the units to find the area.

Activity B

Have the class turn to Worksheet 26 and build the Minnebar shape shown. Ask the children to count squares on two sides of the shape and tell you what equation they would use to find the area (4 x 4 = 16). Write it on the chalkboard. Then ask the class to build a new shape reducing or scaling down the length of the sides of the shape by a factor of 2. The sides of the original shape are twice as long as the sides of the shape they will make. Each two units will become one unit, and each 4-unit side will become two units long. You can show the class a 2-unit bar and a 1-unit bar and then a 4-unit bar and a 2-unit bar to help them see this. Have them write the equation for the area of the reduced shape. (2 x 2 = 4)


Worksheet 26
Unit 18 Name

Scale down by a factor of 2.

4 × 4 = 16

2 × 2 = 4

Scale: 2 → 1

Scale down by a factor of 4.

4 × 4 = 16

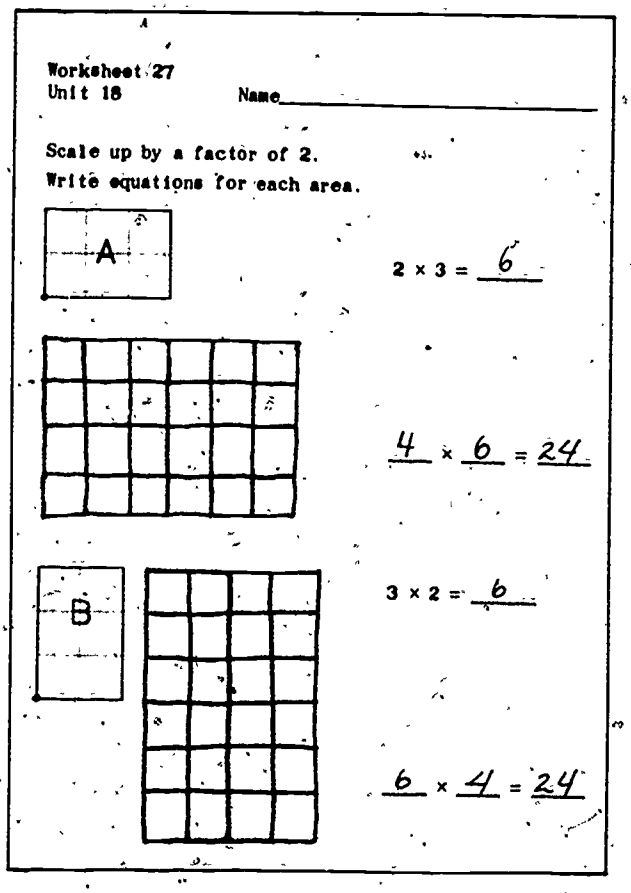
1 × 1 = 1

Scale: 4 → 1

In the second part of the worksheet the children scale down from four to one, or by a factor of four. They begin by using the same shape as before. Remind them that every four units along the edge of the shape should be reduced to one unit. Have them write the multiplication equation for the area of the new shape. (1 x 1 = 1)

Activity C

Discuss Worksheet 27 with the children, and then have them scale up the sides of each representation by a factor of 2. They are to complete the page by writing the equations for the area.



Worksheet 27
Unit 18 Name_____

Scale up by a factor of 2.
Write equations for each area.

A

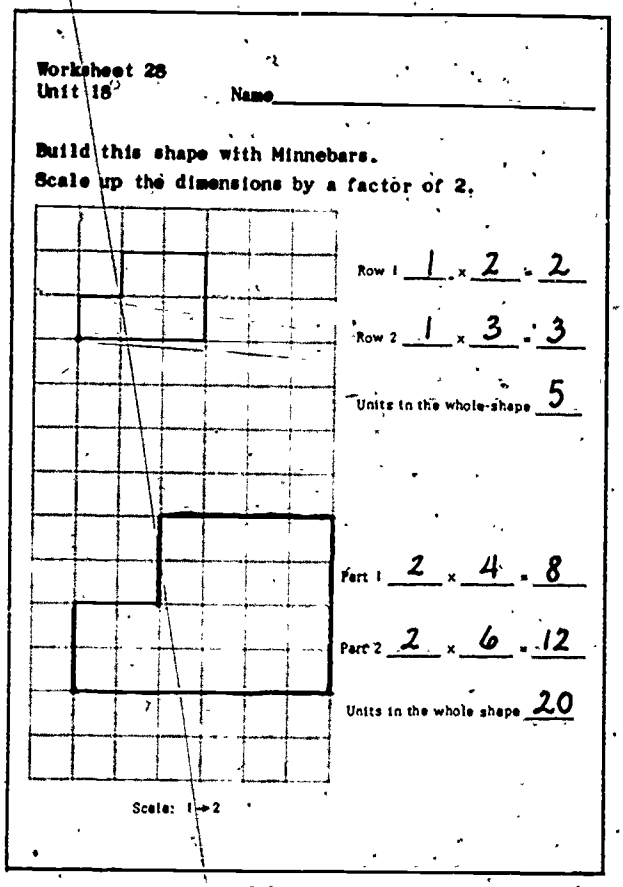
2 × 3 = 6

4 × 6 = 24

B

3 × 2 = 6

6 × 4 = 24



Worksheet 28
Unit 18 Name_____

Build this shape with Minnebars.
Scale up the dimensions by a factor of 2.

Row 1 1 × 2 = 2

Row 2 1 × 3 = 3

Units in the whole shape 5

Part 1 2 × 4 = 8

Part 2 2 × 6 = 12

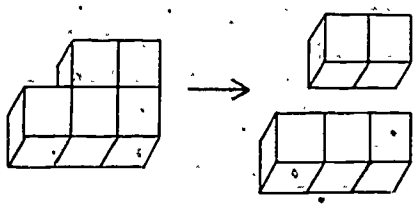
Units in the whole shape 20

Scale: 1→2

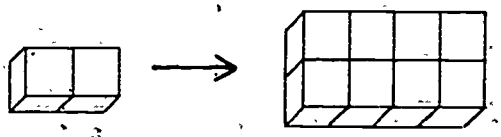
In a class discussion, compare the representations and equations of patterns A and B. Ask what is similar and what is different about them. (Similarities: Same total number of units, same numbers used in the equations, same shape. Differences: Numbers in different position in the equation, rows and columns in reverse positions, different position of shape on the paper.)

Activity D

Have the children turn to Worksheet 28. They have done the same type of activity before on grid paper, but working with Minnebars may present different problems. Ask the children to suggest possible methods for scaling the irregular figures. If no one suggests it, show how each figure can be divided into two rectangles which can be scaled one at a time. It may help the children visualize the problem if they physically separate the two rows of Minnebars.



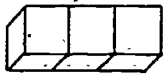
Show a two-part equation, adding the products for each rectangle to find the product for the area of the complete figure.



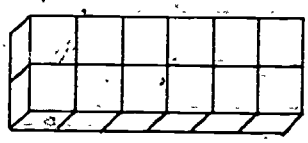
1 x 2 = 2

2 x 4 = 8

$$\begin{array}{r} 8 \\ +\ 12 \\ \hline 20 \end{array}$$



1 x 3 = 3



2 x 6 = 12

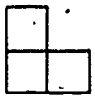
Scale and write equations for the area of the shape step by step with the class. Let them do Worksheet 29 independently.

After the children have scaled up the irregular shapes in sections, have them move the Minnebars together to see what the new shapes look like.

Worksheet 29
Unit 18

Name _____

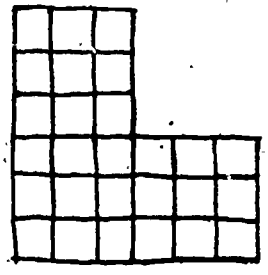
Scale up by 3.



Row 1 1 x 1 = 1

Row 2 1 x 2 = 2

Units in the whole shape 3



Part 1 3 x 3 = 9

Part 2 3 x 6 = 18

Units in the whole shape 27

Lesson 8: THREE-DIMENSIONAL SCALING

In the previous lesson the children scaled two dimensions of Minnebar shapes. They used arrays to help them visualize multiplication for finding area. In this lesson the children will scale all three dimensions of the Minnebars. Three-factor multiplication is introduced as one of the ways to find the number of units in the three-dimensional shapes.

MATERIALS

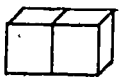
-- for each child --

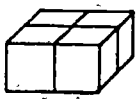
- 1 set of Minnebars
- Worksheets 30, 31, 32 and 33

Activity A

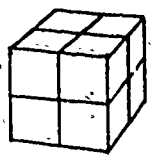
Each child should have a 1-unit Minnebar on his desk. They have scaled two dimensions using Minnebars, and now they are going to scale three dimensions. Ask a child to show how he scaled up two dimensions of the 1-unit Minnebar in the last lesson. Then ask how he might scale up the Minnebar in three dimensions by a factor of 2. Through discussion the children should tell that in the last lesson they went through the following steps:

The one-unit bar 

was made twice as long, 

and twice as wide. 

To scale it in three dimensions they will have to also make it twice as high.



Ask the children to make a shape twice as long, twice as wide and twice as high as the 1-unit bar. After the children have built the shape, ask them which properties of the bar have changed and which have remained the same. (The length of the sides and the total number of units have changed, but the shape has remained the same.)

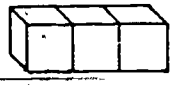
For the first activity the children should build the shapes from separate 1-unit Minnebars, but later they may need to use other bars so that they will have enough units to make particular shapes.

Check the children's work and help those who need it before continuing. Then tell the children to put all the bars aside except for one 1-unit bar. Have them scale it 1 → 3.

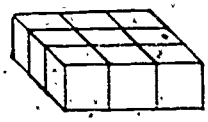
Before they begin to work they should be able to tell you that they will need to make the following changes:

The 1-unit bar will become

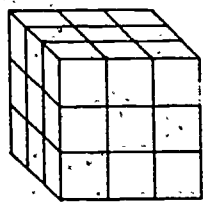
three times as long,



three times as wide,



and three times as high.

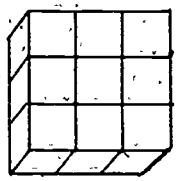


Have the children make a figure that is three times as long, three times as wide and three times as high as the 1-unit bar. They should see that the shape of the scaled-up figure is similar to the shape of the 1-unit bar, but that all the sides are three times as long as the sides of the 1-unit bar.

(Activity B should follow immediately. The children will be using the shape they have just made.)

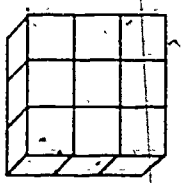
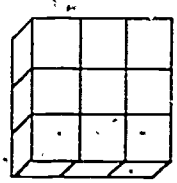
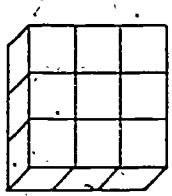
Activity B

Check the 1 → 3 Minnebar shapes made by the children. Ask the children to look at their Minnebar shapes and to think of ways to decide how many Minnebar units are in it. Ask the children to write an equation for the number of units found in the top layer.



3 x 3 = 9

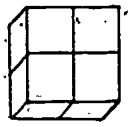
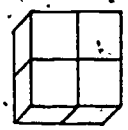
Then ask how many layers there are and how they could show this in equations. The children may suggest laying out the three separate layers, computing the units in each, and then adding them to get the answer.

Layer 1	Layer 2	Layer 3	Total number of units
			9 9 + 9 27
3 x 3 = 9	3 x 3 = 9	3 x 3 = 9	

Someone may come up with the idea that since there are three layers they could use a multiplication equation (3 x 3 = 9 and 3 x 9 = 27, or 3 x 3 x 3 = 27). If no child mentions this method, you may introduce it, but do not stress it. (This is the first introduction of three-factor multiplication in the MINNEMAST program.)

Next have the children put all the bars aside except one 1-unit bar. They are to rebuild the shape they made when one 1-unit bar was scaled up by a factor of 2.

Work with them to develop equations for the layers and find the total number of units in the object.

Layer 1	Layer 2	Total units
		4 + 4 8
2 x 2 = 4	2 x 2 = 4	

or 2 x 2 x 2 = 8

The children can count the units in the shape to see if their calculations are correct.

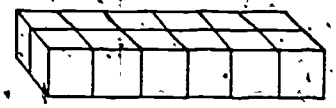
Activity C

Have the class turn to Worksheet 30. Do it with them. First build the shape in a 1 → 1 scale with Minnebars and then scale up the three dimensions by a factor of 2 (1 → 2 scale).

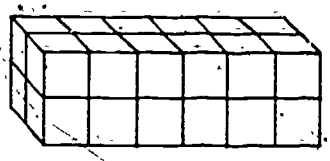
The following changes must be made:

The 3-unit shape becomes 

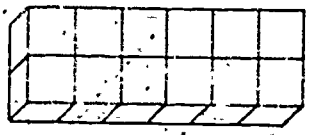
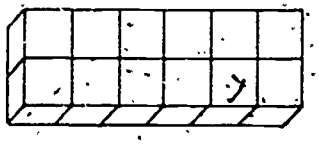
twice as long, 

twice as wide, 

and

twice as high. 

Work with the children to develop equations for the layers in this shape.

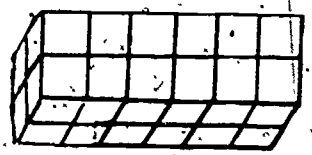
Layer 1	Layer 2	Total number of units
		12 + 12 24
2 x 6 = 12	2 x 6 = 12	

or 2 x 2 x 6 = 24

When Worksheet 30 has been completed, ask the children to select one or more of the plans on Worksheets 31, 32, and 33. They are to build the shapes with Minnebars, and then scale them up by a factor of 2. Although it is more difficult to write multiplication equations for the irregular shapes on Worksheets 32 and 33, some children may enjoy the challenge.

Worksheet 30
Unit 18 Name_____

Build this shape with Minnebars.
Then scale it up by a factor of 2.
Write equations for the number of units in the new one.



Layer 1 2 × 6 = 12

Layer 2 2 × 6 = 12

Total units for shape

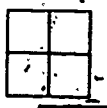
Layer 1 12
Layer 2 +12
24

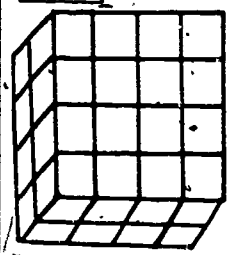
or:

2 × 2 × 6 = 24

Worksheet 31
Unit 18 Name_____

Build this shape with Minnebars.
Then scale it up by a factor of 2.
Write equations for the number of units in the new one.





Layer 1 4 × 4 = 16

Layer 2 4 × 4 = 16

Total units for shape

Layer 1 16
Layer 2 +16
32

or:

2 × 4 × 4 = 32

Activity C

For this activity the children will use their architects rulers. Divide your class into groups of three or four and ask each group to pick an object to scale down so that its representation will fit on Worksheet 36 ($\frac{1}{2}$" grid paper).

Take a few minutes to discuss possible objects to represent. They should be larger than the worksheet. Rectangular objects such as doors, tables, picture frames, or large blocks are easiest to work with. (Children who are skillful enough should not be restricted to rectangular objects.)

The children are to measure their grids with the 1 → 1 scale on the ruler. This tells them what dimensions they will have room for on the paper. (21 by 16) Then they are to measure the objects they have selected with the 1 → 1 scale. Since the dimensions of the object are more than 21 by 16, the worksheet page will be too small for a 1 → 1 representation. They should therefore try measuring the object with the 1 → 2 scale on the architects ruler. If the dimensions of the objects are less than 21 by 16 of these larger units, a 2 → 1 representation will fit on the worksheet. If the object measures more than 21 units long, they will have to try measuring with the 1 → 3 scale, and so on, until they find a scale that fits.

Now they can make their representations, measuring the object with, for example, the 1 → 3 scale, and then making the drawing with each 1 → 3 unit represented by one unit of the 1 → 1 scale. Have the children in each group show their work to the class and explain what they did.

Note that to scale down with an architects ruler, you never start with the 1 → 1 scale as you do for scaling up. Because you are scaling down you always start with larger units (such as 1 → 4) and make your representation in smaller units (such as 1 → 1).

Lesson 9: SCALING IN DIFFERENT KINDS OF UNITS

In previous lessons the children acquired a feeling for enlarging and reducing by scale factors of 2 and 3. This lesson is designed to show that it is possible to make representations of objects using any scale. In the first activity the children begin with paper clips as scale units. Then they select other scale units to use, including unsharpened pencils, thumb widths, or any other units of their choice. Finally they measure objects in the room in feet and draw them on their papers in inches.

MATERIALS

- several unsharpened pencils
- rulers
- yardsticks

-- for each child --

- 10 jumbo paper clips
- 10 #1 paper clips
- architects' ruler (from Lesson 6)
- Worksheets 34, 35, and 36

PROCEDURE

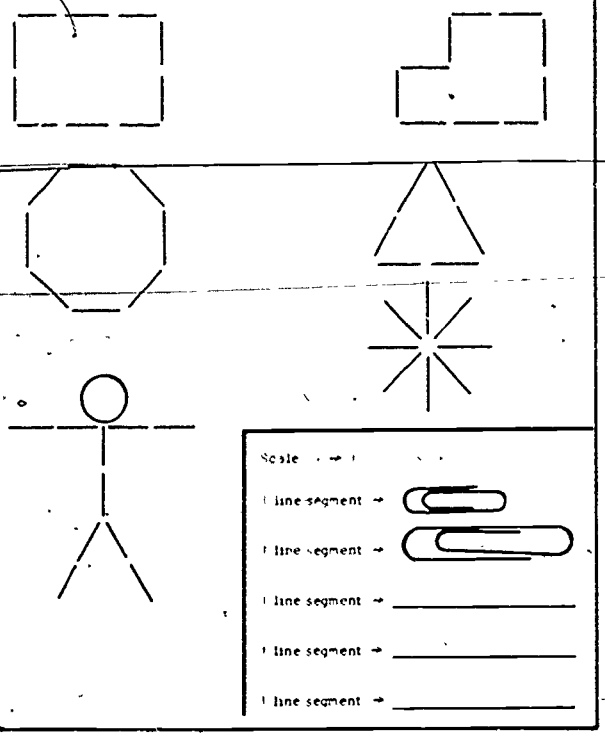
Activity A

Tell the children that they are going to use a new unit of measurement to make scale representations. Worksheet 34 shows a number of patterns they will work with.

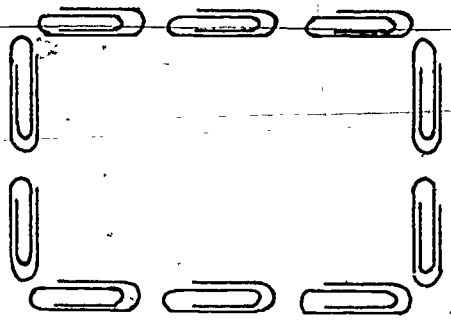
HOW MANY UNITS LONG ARE THE SIDES OF THE RECTANGLE? (2 units by 3 units.)

Ask the children how they could make a representation of the rectangle using paper clips. (Use 1 clip to represent each line segment on the edges of the rectangle.) Distribute 10 #1 paper clips to each child and have them make paper clip representations of each pattern on Worksheet 34.

Worksheet 34
Unit 18 Name_____



The first one should look like this:



And here is another scale unit:

Next give each child 10 jumbo clips, and have them follow the same plans using a jumbo clip scale. Have them make some patterns side by side in regular clip and jumbo clip scale. Leave it to the children's ingenuity to find disks in the appropriate scales for the man's head. When they have finished, collect the paper clips.

Call attention to the scale at the lower right hand corner of the worksheet. The first two scales are shown there.

Ask the children to suggest other possible standard scale units —thumb widths, unsharpened pencils, new crayons, rulers, children, etc. Let different groups of 2 to 4 children use different objects for a standard unit to make representations of the star pattern. They should mark their scale unit on Worksheet 34, below the paper clip scale. The children may need to work on a table or on the floor for the whole pattern to fit. It is important for them to notice that some representations are too large to be made on their own desks.

Activity B

Ask the children to help you solve a problem. Tell them you wish to make a representation of the chalkboard (or another large rectangular object) on your paper. Turn to Worksheet 35 (1-inch grid paper) and then ask them what facts they would need to know before they could decide on a scale. (The length of two dimensions of the object, and the length of two dimensions of the paper.) Make the measurements and record them on the board.

HOW CAN WE DECIDE WHAT SCALE WE SHOULD USE?

After some discussion you might suggest that the children try the scale 1 foot → 1 inch. Let one foot along the edge of the object correspond to one inch on their paper. If the paper is too small, increase the scale so that 2 feet → 1 inch. Continue in this way until you find a suitable scale. Now have the children draw their representations of the chalkboard on Worksheet 35 and record the scale at the bottom.

Activity C

For this activity the children will use their architects rulers. Divide your class into groups of three or four and ask each group to pick an object to scale down so that its representation will fit on Worksheet 36 ($\frac{1}{2}$" grid paper).

Take a few minutes to discuss possible objects to represent. They should be larger than the worksheet. Rectangular objects such as doors, tables, picture frames, or large blocks are easiest to work with. (Children who are skillful enough should not be restricted to rectangular objects.)

The children are to measure their grids with the 1 → 1 scale on the ruler. This tells them what dimensions they will have room for on the paper. (21 by 16) Then they are to measure the objects they have selected with the 1 → 1 scale. Since the dimensions of the object are more than 21 by 16, the worksheet page will be too small for a 1 → 1 representation. They should therefore try measuring the object with the 1 → 2 scale on the architects ruler. If the dimensions of the objects are less than 21 by 16 of these larger units, a 2 → 1 representation will fit on the worksheet. If the object measures more than 21 units long, they will have to try measuring with the 1 → 3 scale, and so on, until they find a scale that fits.

Now they can make their representations, measuring the object with, for example, the 1 → 3 scale, and then making the drawing with each 1 → 3 unit represented by one unit of the 1 → 1 scale. Have the children in each group show their work to the class and explain what they did.

Note that to scale down with an architects ruler, you never start with the 1 → 1 scale as you do for scaling up. Because you are scaling down you always start with larger units (such as 1 → 4) and make your representation in smaller units (such as 1 → 1).

Lesson 10: MAPS AND A WALKING TRIP

This lesson shows the usefulness of maps of different scales to depict the same area in greater or lesser detail. When one inch represents one block of a city on the map, more detail is shown than when one inch on a map of the same size represents one mile of the city. A film Maps—Where Am I? is used to acquaint the children with general mapping ideas. Then they go on to work with maps of different scales.

While the children are working with maps they may enjoy looking at A Map is a Picture by Barbara Rinkoff.

MATERIALS

- film: Maps—Where Am I?
- Barbara Rinkoff., A Map is a Picture
- 24" piece of yarn for each child
- Worksheets 37, 38 and 39

Activity A

The movie Maps—Where Am I? provides background for this lesson in unusually lucid form, and reviews some concepts already introduced in the unit.

At the beginning of the film a comparison is made between a house plan and a map. An aerial photograph of a city fades into a map of the city, to serve as a bridge between the concrete and the abstract. Aerial photographs and maps are shown of a city, a county, a state, the country, and the world. Map symbols for roads, rivers, direction, etc. are introduced. A specific scale is indicated on only one map (1 inch = 48 miles). Near the end of the film a comparison is made between a man and a photograph of him to illustrate scale.

Make every effort to obtain this film, because if you cannot show it, you will need to spend much more time making sure that the children understand the following activities involving maps.

Preview the film with two or three children and ask them what they think are some of the important ideas. Have them select a few things to tell the class to watch for in the movie. (Having something specific to look for can motivate children to watch the film more attentively.) Then show the movie to the class and discuss it briefly with the children. If possible, show the film again after the children have finished Activities B and C.

Activity B

Ask the children to turn to Worksheet 37 in their Student Manuals. Help them study the map by asking them to locate representations of particular highways, rivers, lakes and important places. Tell the children to use the scale to figure out the distances between particular places on the map that they or you select. For measuring the length of a trip from one location to another, give each child a piece of yarn about two feet long to lay along the streets. This makes it easy to measure around corners. Then when they want to figure the distance between the two places they can lift the yarn from the map and stretch it out next to the scale on the map, rather than trying to measure sections of the route with a ruler and adding them.

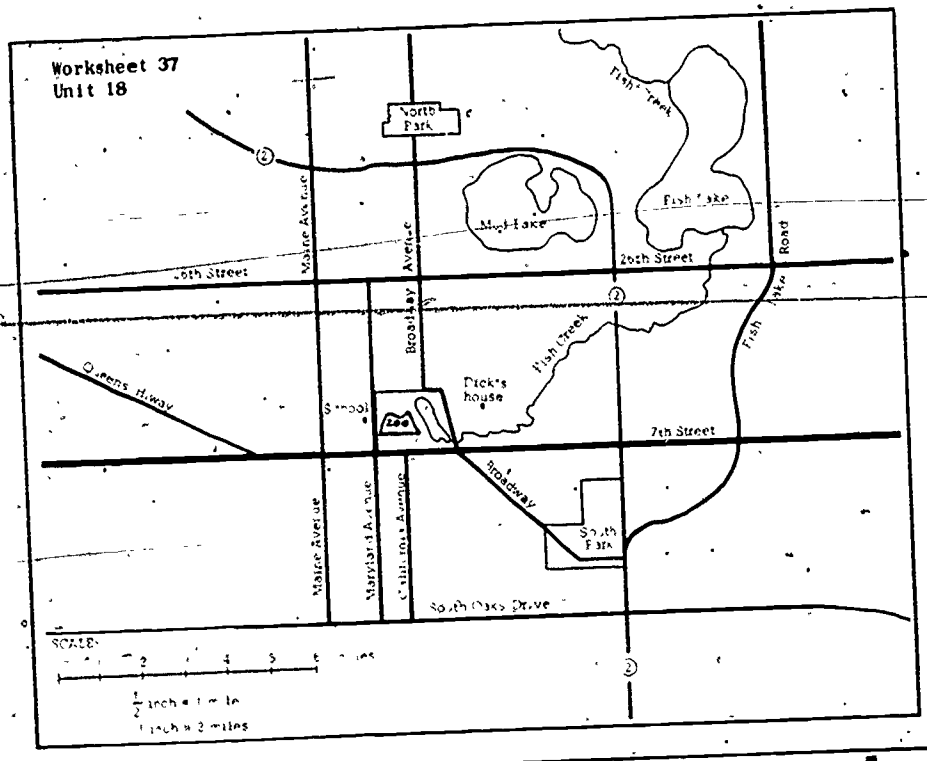
Tell the children that they are going to imagine that this is their city and that they are going to plan a walking trip from their school to the zoo. Ask them the following questions:

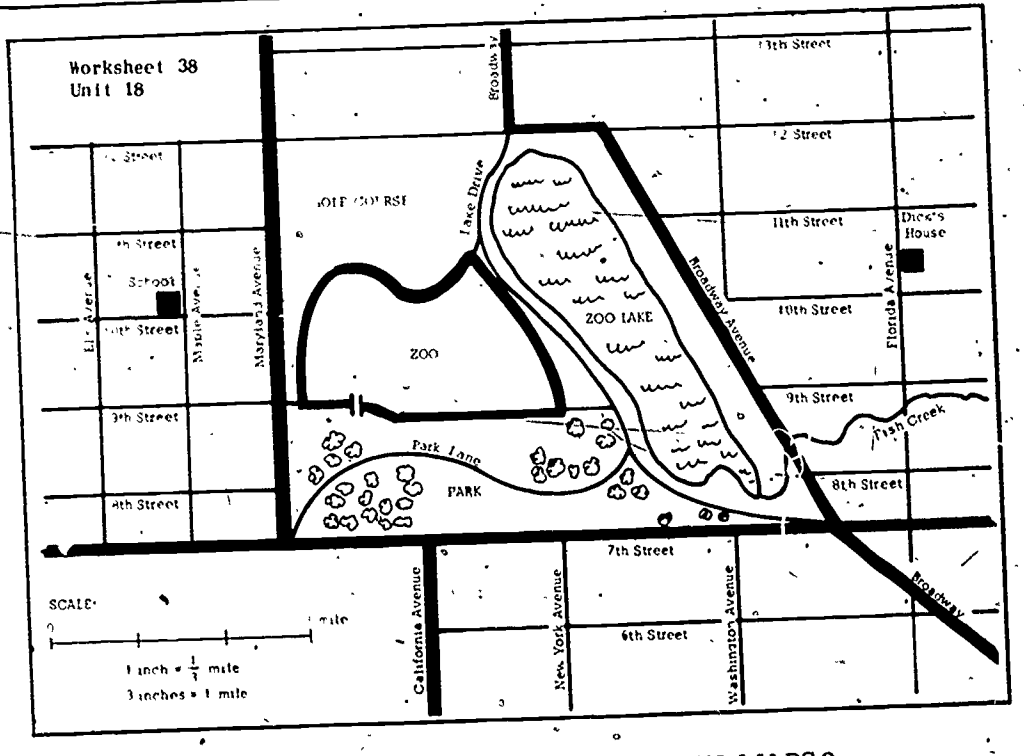
WHAT WILL YOU NEED TO KNOW IN ORDER TO PLAN THIS TRIP? (The location of the school and the zoo, and the distance along each street you must walk on between the school and the zoo.)

HOW CAN YOU USE THE MAP TO HELP YOU?

As the children suggest what to do, let them try some of their ideas. If they do not suggest it, tell them to draw a line showing where they would walk to go from the school to the zoo. Next ask them to turn to Worksheet 38 and study the map showing the area around the zoo and the school.

WHAT DO YOU NOTICE ABOUT THIS MAP THAT IS THE SAME AS THE MAP ON WORKSHEET 37? (Some of the same places are included.)





WHAT ARE SOME STREETS THAT YOU SEE ON BOTH MAPS? (Broadway Ave., California Ave., 7th St., Maryland Ave.)

WHAT ELSE DO YOU SEE ON BOTH MAPS? (Zoo, school.)

WHAT DO YOU NOTICE ABOUT THIS MAP THAT IS DIFFERENT FROM THE MAP ON WORKSHEET 37? (A much smaller area is included, and that area is shown in greater detail.)

WHAT ARE THE NAMES OF SOME STREETS THAT ARE NOT ON WORKSHEET 38 THAT WERE ON 37? (Maine Avenue, Queens Road, 26th Street.)

WHAT CAN YOU FIGURE OUT ABOUT YOUR WALK THAT YOU COULDN'T SEE SO EASILY BEFORE? (Which streets to take.)

Tell the children to plan the trip from the school to the zoo and draw the route on Worksheet 38. Then point out the scale on Worksheet 38 and ask the children to use it to help them figure how far they will have to walk. They may use yarn to measure the distance and lay it out against the scale.

LOOK AT THE ROUTE YOU MARKED ON WORKSHEET 37. ON WHICH MAP DO YOU THINK YOU CAN FIGURE THE DISTANCE MORE ACCURATELY? (On the map showing the smaller area.)

WHY? (Because you can see more detail.)

After the children have planned and figured the length of the trip to the zoo, some of them might like to figure out how far it would be to walk to the zoo from Dick's house or other locations.

(You may wish to stop here for the day.)

Activity C

Tell the children that they will now plan a tour through the zoo.

WHAT WILL YOU NEED TO KNOW TO PLAN A TOUR? (The locations of the animals.)

Have them turn to Worksheet 39.

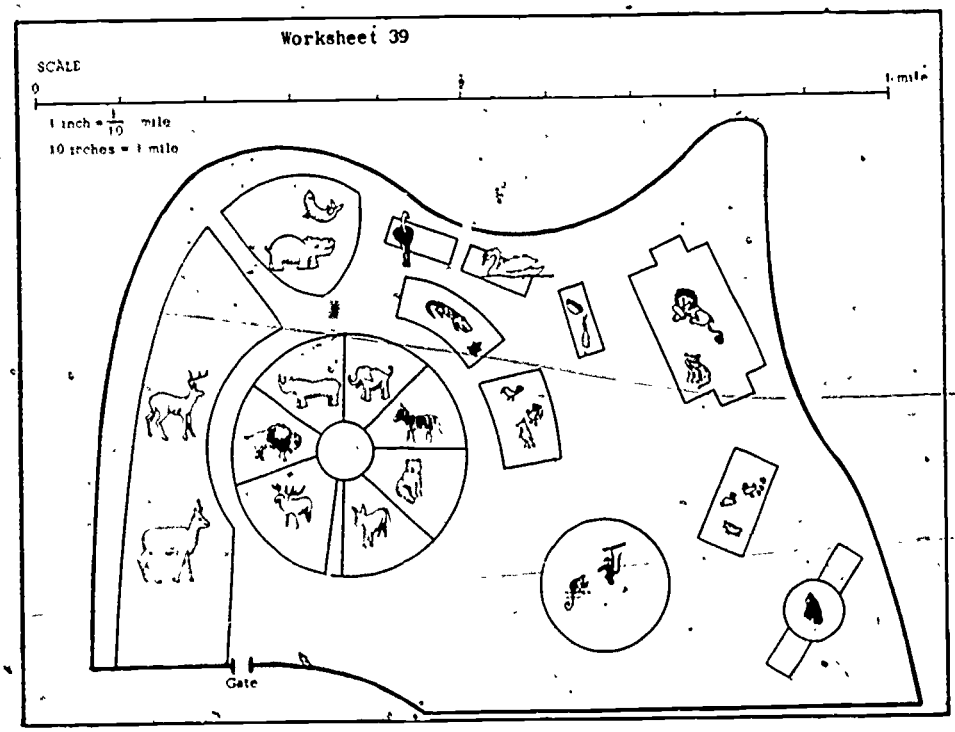
HOW IS THE SCALE ON THIS MAP DIFFERENT FROM THE SCALES ON THE OTHER TWO MAPS? (It shows a much smaller area in greater detail.)

Then introduce the planning of the tour by reading the following paragraph or making up one of your own.

Let's pretend that you have just one hour for your tour, so you can't see all the animals. (For some classes it may be easier to give a specific number of animals to be seen.) All tours will start and end at the gate. Decide where you would like to go in the hour allowed, make an X near each of the animals you want to see, and draw a route on the map. Then use your yarn and the scale to find out how far you would walk if you took the tour you planned.

When they have finished the activity some children may like to tell the class about the tour they planned.

At this point it would be valuable to show the film again. The children will notice many more things in it, now that they have actually worked with maps.



Lesson 11: VALUE OF SCALING

In this lesson the children consider chairs of different sizes, as they are scaled to be comfortable for children of different sizes. The class makes a histogram of children's leg lengths, in order to decide what size chair would be comfortable for the largest number of second graders.

MATERIALS

- a kindergarten chair, a second grade chair and a teacher's chair
- yardsticks or rulers
- story: "Goldilocks and the Three Bears"

PROCEDURE

Activity A

Arrange in advance to have a small-sized kindergarten child available during this lesson.

Line up the three chairs facing sideways in front of the class.

LET'S LOOK AT THESE CHAIRS AND WATCH CHILDREN SIT ON THEM, TO SEE IF WE CAN FIND OUT WHY CHAIRS ARE MADE IN DIFFERENT SIZES.

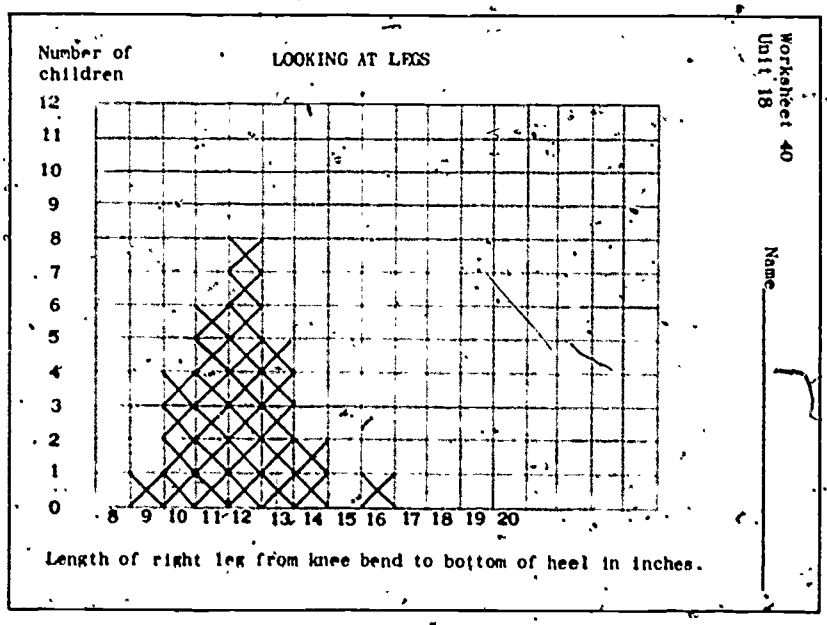
Ask an average-sized student in your class to sit on the second grade chair at the front of the room, so that the side of the chair faces the class. Then ask the rest of the class to tell you what they observe. List the children's observations on the chalkboard. (Typical observations will include the length of the child's leg from his foot to his knee in relation to the length of the chair legs; the length of the child's leg from hip to knee in relation to the width of the seat; the height of the child's shoulder in relation to the height of the chair back.) Next ask the child to sit on the teacher's chair, and list the children's observations. Then have him sit on a kindergarten chair. Repeat the observations with a kindergarten child if possible, and then with an adult sitting on each of the three chairs in turn.

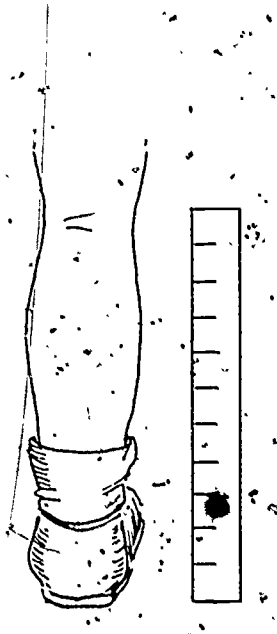
Discuss the observations recorded on the chalkboard and decide what generalizations can be made. (For example, when someone is sitting on a chair the correct size for him, his feet will touch the floor.)

Ask the children to decide what information would be most useful to a chair manufacturer who wanted to make chairs for second graders. (The children's measurements from seat to shoulder, from hips to knee, and from knee to heel.)

Divide the class into pairs of children. Have each child measure and record the length of the back of his partner's right leg from the knee bend to the bottom of the heel.

Record this data on a class histogram on the chalkboard while the children work on Worksheet 40. Then decide what height chair would fit the greatest number of children in the class.





Then ask the class how this information could help a chair manufacturer who wants to make just one size chair that would fit second graders. (The size should suit the greatest number of children.)

Discuss other objects that are scaled to different sizes and why they are more useful than if they were made in only one size. (Clothing, toys, etc.)

Note that pictures in dictionaries frequently have the scale indicated, so that you can tell the size of the real object.

Activity B

Read the story, "Goldilocks and the Three Bears."

Lesson 12: MAKING THREE-DIMENSIONAL SCALE MODELS

We now draw together many of the ideas developed in this unit. Several experiences are designed to reinforce the children's understanding of why some kinds of objects must be scaled to different sizes. Then the children measure and determine the scale factor between the two members of a pair of objects. They go on to construct three-dimensional paper models of doghouses, garages, etc, to scales different from the plans with which they are provided. This involves choosing the appropriate scale factor and selecting techniques for enlarging the plans.

MATERIALS

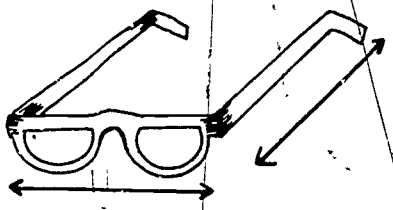
- sunglasses, child's and adult's sizes
- footballs, small and large
- sweaters, child's and adult's sizes
- carriages, small doll's and large doll's sizes
- bicycles, small child's and adult's sizes
- shoes, child's and man's sizes
- ruler for each child
- scissors for each child
- 24" piece of yarn for each child
- infant's shoe, $3\frac{1}{2}$ inches or less
- toy trucks, 6 inches and $2\frac{1}{2}$ inches
- construction paper
- architects ruler (from Lesson 6)
- children's edition, Swift's Gulliver's Travels
- Worksheets 41 through 48

PROCEDURE

Activity A

Introduce this activity by showing the children two pairs of sunglasses, one children's and one adult's. Ask the children which pair of sunglasses would be more comfortable for second

graders. Have several different children try on both pairs. Ask which dimensions of the sunglasses influence whether they fit correctly. (The distance across the front and the length of the earpiece.) Have the children measure these two lengths on each pair and compare them.



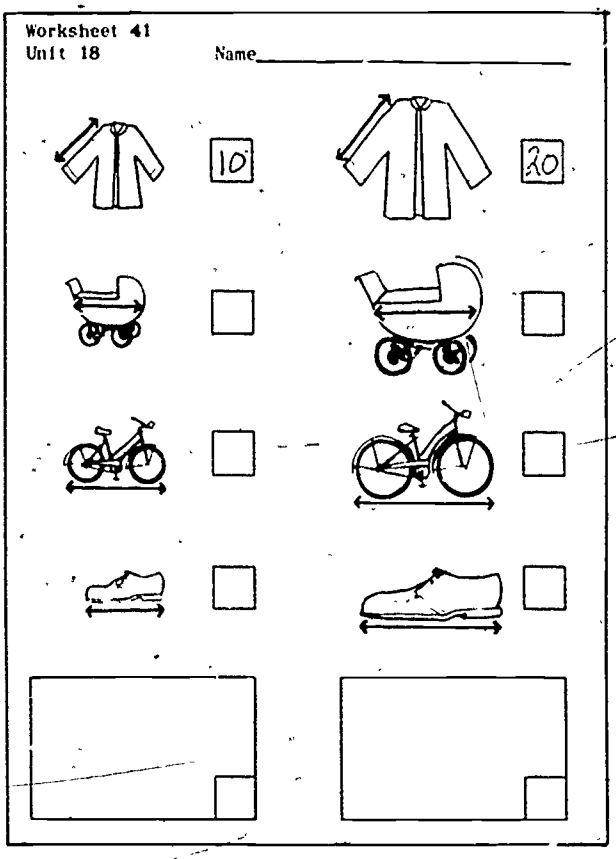
Then show the children footballs of different sizes. Ask several children, one at a time, to hold each of the footballs in throwing position. Ask which is more comfortable for a second grader to hold and throw. Have the children measure the distance between the points on the footballs and the circumference at the widest part and compare the dimensions of the two balls.

Activity B

Gather several of the pairs of objects listed below or other similar pairs. Use those that are easiest for you to obtain, but try to offer the children a choice.

- child's and adult's sweaters
- small and large doll buggies borrowed from the kindergarten or brought from home
- baby's or small child's shoe and man's shoe
- 2 bicycles, one small and one full size

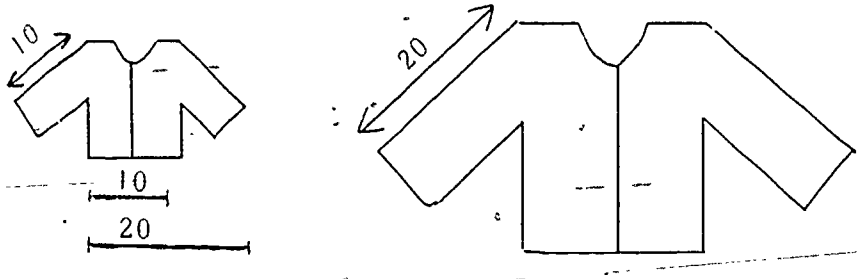
Tell the children to look at Worksheet 41, which has drawings of the four pairs of objects listed above. We also provide a pair of blank boxes in case you substitute other objects. Tell the children that they are to measure the objects along the dimensions shown by the arrows on the worksheet and write the measurements in the spaces provided. Be sure they understand that they are to record the measurements of the actual objects, not of the drawings. Allow the children to select one pair of objects to measure. They may work alone or in small groups. (The children will need something else to work on at their seats while waiting for their turn to measure.)



After each child has measured a pair of objects, have the children tell you what the measurements are and record them on the chalkboard. Work with the children to help them figure what the approximate scale factor is between the two objects in each pair. For example, suppose the length of the sweater sleeves are about 10 inches and 20 inches.

Draw two line segments on the board one below the other, one 10 inches long and the other 20 inches long. The children should be able to decide that the length of the sleeve of the grownup's sweater is about twice as long as the length of the sleeve of the child's sweater. You may want them to use their pieces of yarn to measure the length of the smaller sleeve and count off how many of these match the length of the larger sleeve. This number is the scale factor. Follow the same procedure with the other pairs of objects.

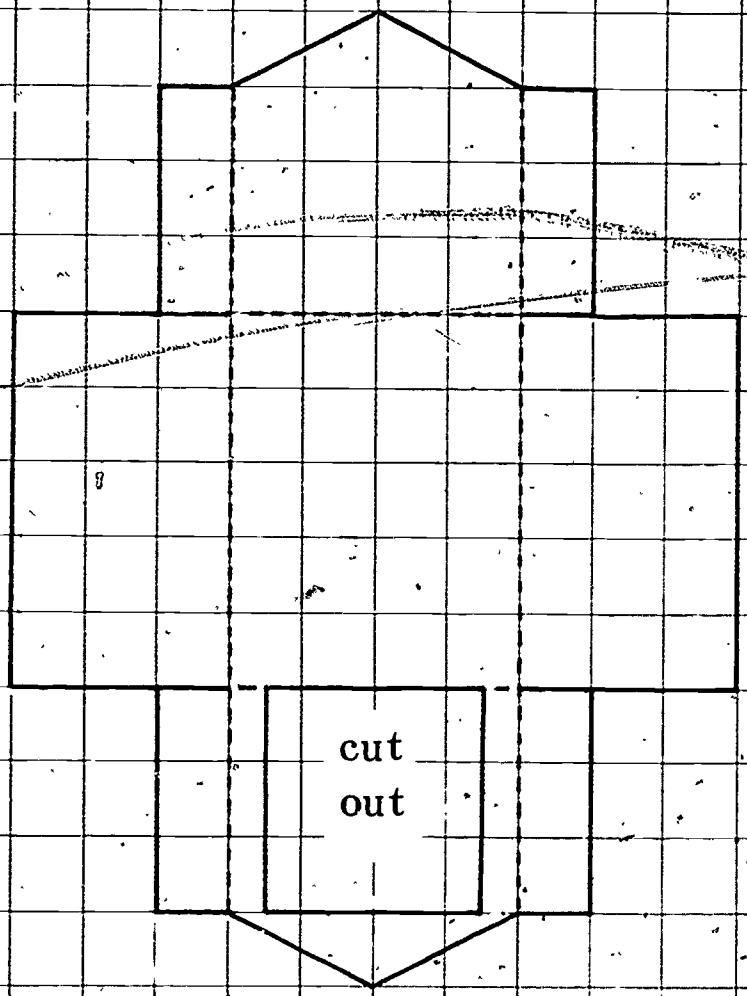
Some objects will have easily identifiable scale factors such as $1 \rightarrow 2$ or $1 \rightarrow 4$. In more difficult situations it will suffice to have children draw line segments on the board, one below the other, indicating the length of each of the pairs of objects.



Activity C

Before beginning this activity, cut out and assemble the small doghouse facing page 74 (Worksheet 43). Tell the children to cut out the dogs on Worksheet 42 and fold them so that they will stand. Then show the children the small doghouse. Ask

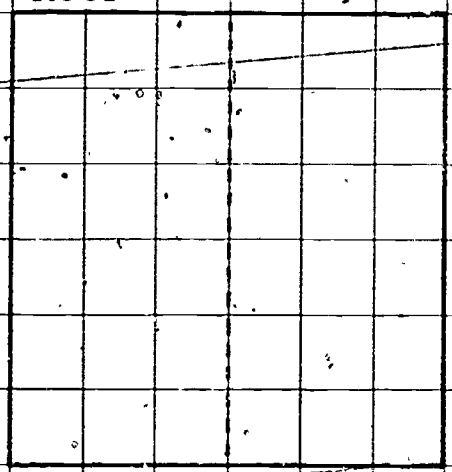
Worksheet 43
Unit 18 Name_____

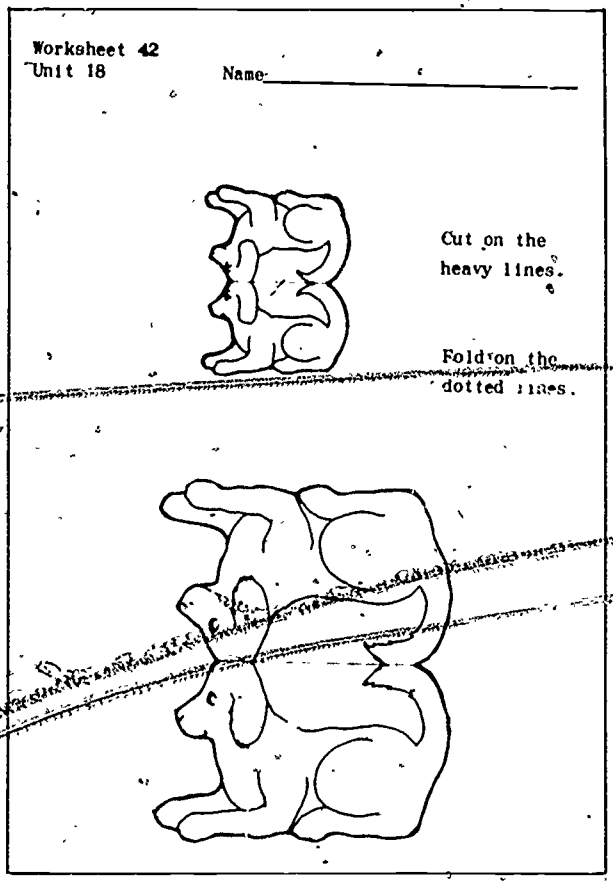


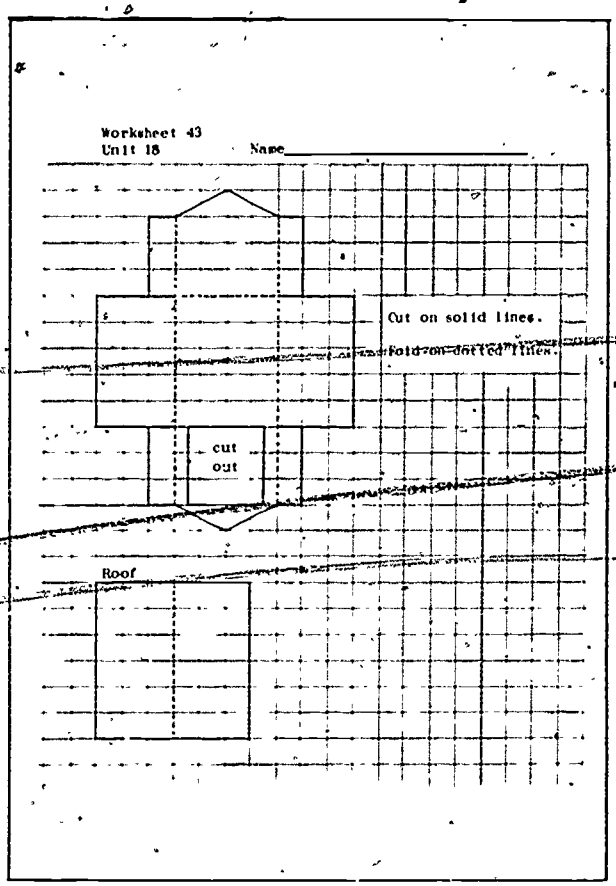
Cut on solid lines.

Fold on dotted lines.

Roof





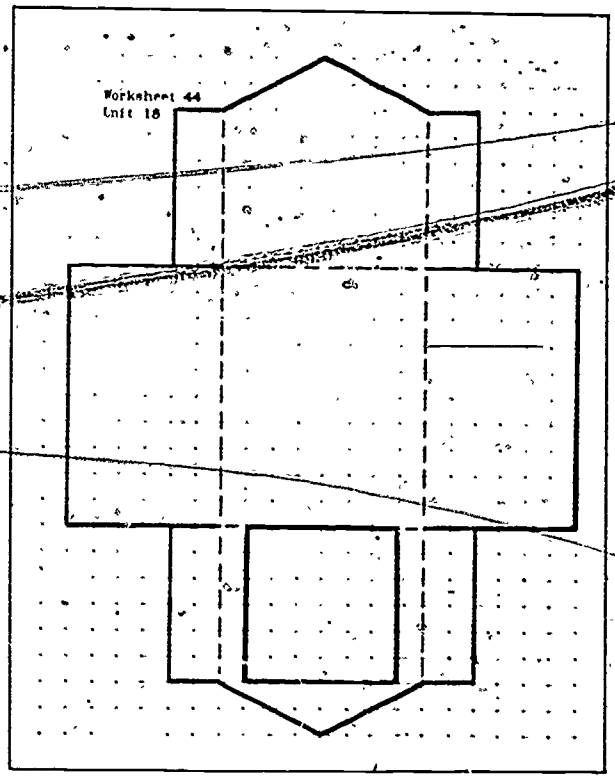


if this house seems to be about the right size for the small dog. Select a child to place his small dog in the house to check the size.

Tell the children that the larger dog needs a house, too. Have them look at Worksheet 43, a flat plan of the small doghouse. Ask what they could do to make a house just like the small one that would fit the larger dog. If no one suggests scaling up the plan, ask:

FROM WHAT YOU KNOW ABOUT SCALING, HOW CAN YOU USE THE SMALL DOGHOUSE TO HELP YOU MAKE THE LARGE ONE?

The procedure should involve measuring the length of the two dogs to find the scale factor between them (2), and then applying that scale factor to the plan of the small doghouse in order to enlarge it. Have the class work together, each child making a large doghouse from the centimeter grid paper on Worksheet 44. They should use the unused space on Worksheet 43 or the very last grid sheet at the back of their manuals for the roof.



Activity D

Preparation:

The plans on the following pages are provided so that you can construct the shoe box and garage beforehand and be better prepared to guide the children.

Cut out the parts of each along the solid lines and fold along the dotted lines. Note the short cut-lines at each end of the horizontal fold-lines of the largest section of each plan and on the cover of the shoe box.

Worksheet 45
Unit 18 Name_____

Would you like to make a  for your 

or a  for a  ?

If you chose the  , stay on this page.

If you chose the  , turn to Worksheet 46

Worksheet 47 has plans for a  for a baby's 

A  for your  must be larger.

Take the  and put it next to your 

About how many lengths of the  does your

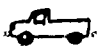
 measure?. _____

That number is your scale factor.

Scale up the plans for the  on Worksheet 47.

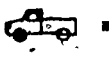
Worksheet 46
Unit 18 Name_____

Worksheet 48 has plans for a  for a 

A  for a  must be larger.

Take the  and put it next to the 

About how many lengths of the  does the

 measure? _____

That number is your scale factor.

Scale up the plans for the  on Worksheet 48.

Plans for a Shoe Box

Worksheet 47
Unit 18

SIDE

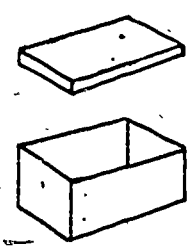
BOTTOM

END

SIDE

END

COVER



Plans for a Garage

Worksheet 48
Unit 18

SIDE

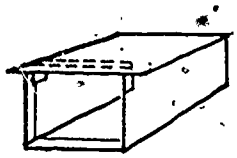
FLOOR

END

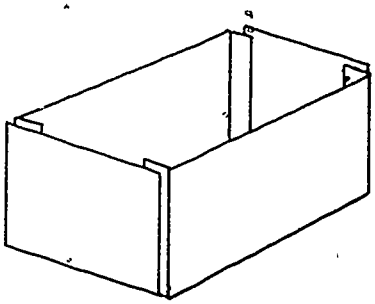
SIDE

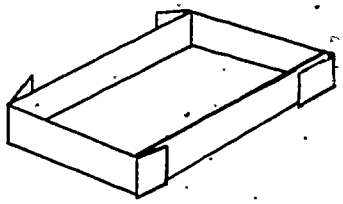
ROOF SUPPORT

ROOF

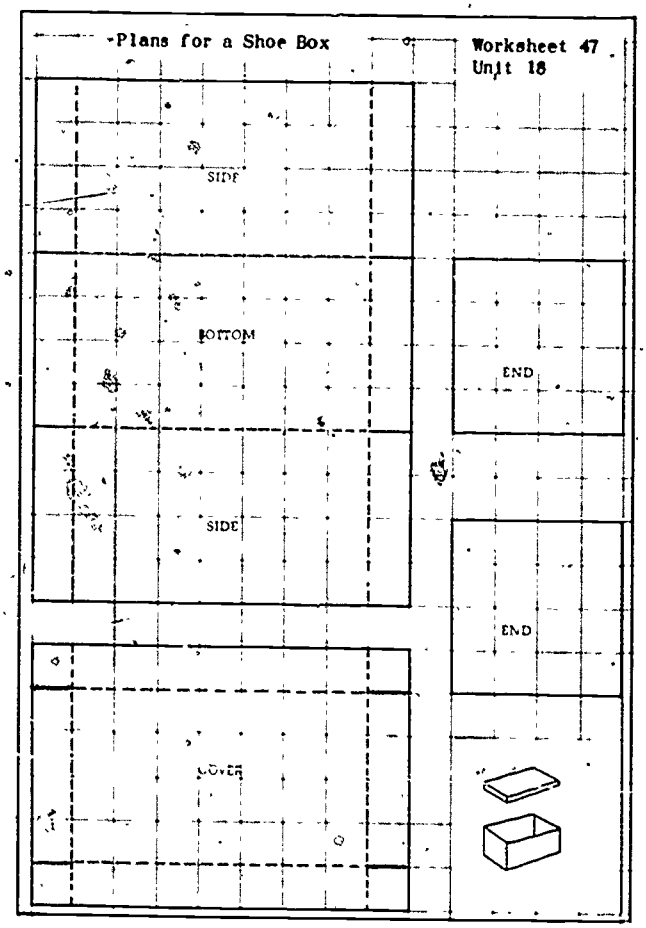


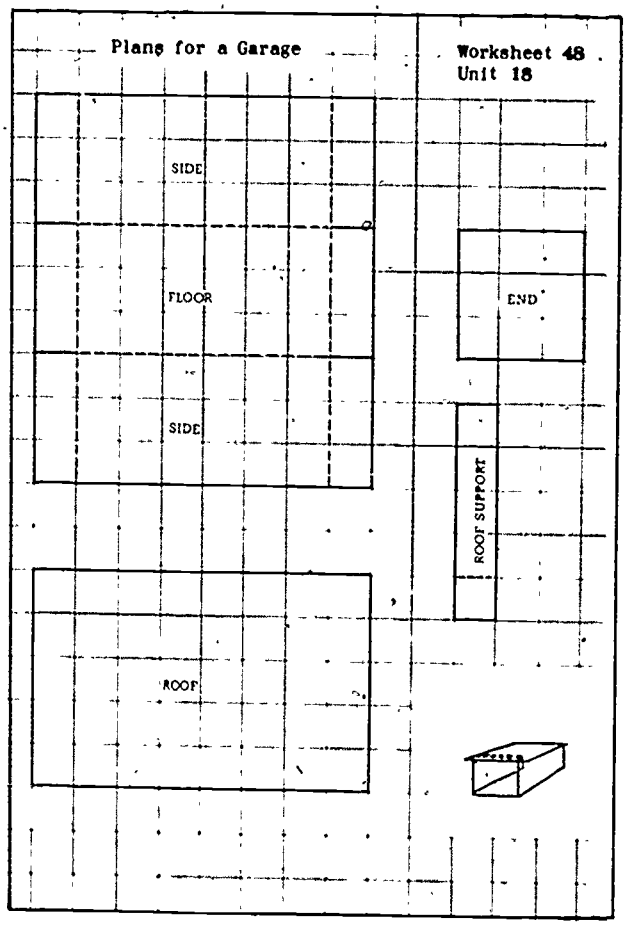
The narrow folded flaps on the largest sections are for pasting. To assemble the shoe box, paste the flaps to the end pieces. For the cover, paste the square tabs as shown in the diagram.



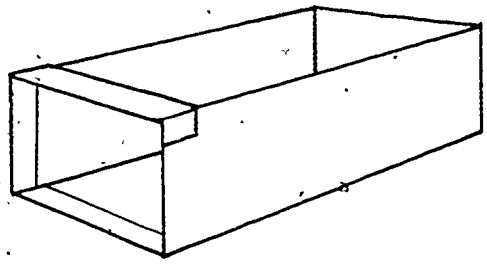


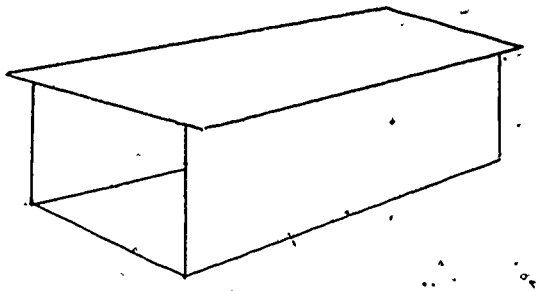
For the garage, paste just one end in place. At the other end, fold the flaps all the way back and paste them against the sides and floor for reinforcement. This open end will be the door of the garage. Place the roof support across the top as shown. Paste the square tabs to the outer surface of the





sides. The roof rests flat on the top of the garage with an overhang projecting on all sides. Put some paste on the top of the roof support to hold the roof in place.





Procedure:

Have each child turn to Worksheet 45 and choose one of the construction projects shown—either a shoe box or a garage for a truck. Ask some child to bring an infant's shoe to class to fit into the small box. Those who choose to make shoe boxes will work with their own shoes. Those who choose to make a garage should bring toy trucks from home or work with a pair you have in the classroom. A "Matchbox" type truck will fit in the small garage. The large truck should be about six inches long.

Worksheet 47 shows a plan for a shoe box and Worksheet 48 has one for a garage. The children will have to scale these plans up in order to make shoe boxes or garages to fit the larger shoes or trucks. They must first determine their scale factors, which will vary. Have them place the small shoes or trucks against the larger ones to find out about how many lengths of the small objects the larger ones measure. These numbers are the factors by which the plans must be scaled up. Help the individual children who have fractional measurements choose the next larger whole number scale factor. (A box can still be used if it is too big, but it is useless if too small.)

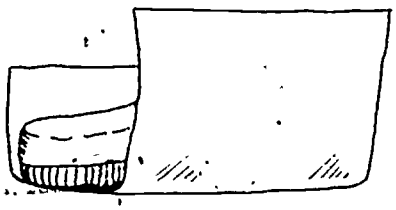
Note that scale factor in this activity is determined by comparing only one dimension, the length of the two objects.

Space is provided at the bottom of the worksheets so that those children whose measurements come out in even multiples can write multiplication equations.

_____	x	_____	=	_____
scale factor		length of small plan		length of large plan

Let each child choose his own method for scaling up the plan. He may use an ordinary ruler, an architects ruler, or the grid paper provided after Worksheet 48. He may make his box or garage out of grid paper or construction paper.

Some children may prefer to work directly from the larger truck or shoe and build the garage or box to fit it. In that case they may trace the outline of truck or shoe to find the dimensions, or simply fold the paper up around the bottom and sides to make it fit.



Guide the children in assembling their boxes or garages. They should check to see that the shoes or trucks will fit.

Activity E

To conclude this unit, read any children's edition of Swift's Gulliver's Travels to the class.

SUMMARY

In this unit the children have been introduced to the following ideas:

1. Objects can be represented by outlines or shadows of their shapes. These outlines or drawings represent only some of the properties of the objects.

2. Representations need not be the same size as the objects. They may be larger or smaller than life-size.

3. The difference in size between an object and its enlarged representation can be described numerically by a mathematical equation, in which the scale factor times the length of the original side equals the new length.

4. Scaling up and scaling down are inverse processes. One undoes the other.

5. Scaling up is related to multiplication and scaling down is related to division. The concept of division is developed only on an intuitive level with no division equations given.

6. Area can be found by multiplying length and width, and volume can be found by multiplying length, width and height. (These concepts are secondary to the main purpose of the unit and complete understanding is not essential.)

7. The process of scaling can be used to make similar objects in different sizes according to need.