

DOCUMENT RESUME

ED 127 182

SE 021 219

AUTHOR Blair, Kay W.; Forseth, Sonia D.
 TITLE Exploring Symmetrical Patterns: MINNEMAST Coordinated Mathematics - Science Series, Unit 14.
 INSTITUTION Minnesota Univ., Minneapolis. Minnesota School Mathematics and Science Center.
 SPONS AGENCY National Science Foundation, Washington, D.C.
 PUB DATE 71
 NOTE 115p.; For related documents, see SE021201-234; Photographs may not reproduce well; Colored transparencies removed from document due to poor reproducibility
 AVAILABLE FROM MINNEMAST, Minnemath Center, 720 Washington Ave., S.E., Minneapolis, MN 55414
 EDRS PRICE MF-\$0.83 HC-\$6.04 Plus Postage.
 DESCRIPTORS *Curriculum Guides; Elementary Education; *Elementary School Mathematics; *Elementary School Science; Experimental Curriculum; *Interdisciplinary Approach; Learning Activities; Mathematics Education; Primary Grades; Process Education; Science Education; *Symmetry; Units of Study (Subject Fields)
 IDENTIFIERS *MINNEMAST; *Minnesota Mathematics and Science Teaching Project

ABSTRACT

This volume is the fourteenth in a series of 29 coordinated MINNEMAST units in mathematics and science for kindergarten and the primary grades. Intended for use by first-grade teachers, this unit guide provides a summary and overview of the unit, a list of materials needed, and descriptions of five groups of lessons and activities. The purposes and procedures for each activity are discussed. Examples of questions and discussion topics are given, and in several cases ditto masters, stories for reading aloud, and other instructional materials are included in the book. This unit continues the study of symmetry begun in Unit 7 of this series. The five sections are devoted to: (1) rotational symmetry, (2) repeating patterns and translational symmetry, (3) bilateral symmetry, (4) symmetry in sound and movement, and (5) other interesting patterns.

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EXPLORING SYMMETRICAL PATTERNS

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UNIT

14



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	8. OBSERVING PROPERTIES
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FIRST GRADE	10. DESCRIBING LOCATIONS
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	12. MEASUREMENT WITH REFERENCE UNITS
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	25. MULTIPLICATION AND MOTION
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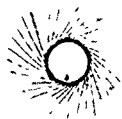
UNIT **14**

MINNESOTA MATHEMATICS AND SCIENCE TEACHING PROJECT

The Minnesota Mathematics and Science Teaching Project

developed these materials under a grant from the

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Fifth Printing

1971

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MINNESOTA MATHEMATICS AND SCIENCE TEACHING PROJECT

Director

DR. JAMES H. WERTZ, JR.
Associate Professor of Physics
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This unit, Exploring Symmetrical Patterns, was developed from previous units and revised in the light of the experience of the many teachers who have tried the activities in the classroom. This trial edition, Unit 14 of the MINNEMAST Coordinated Mathematics-Science Series, was produced under the leadership of:

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Complete List of Materials for this Unit

(Numbers based on class size of 30.)

(Art materials will depend on your selection of Art Activities.)

total number required to teach listed lesson(s)	item	lessons in which item is used
30	** Student Manuals with transparency	
5	*** sets of property blocks	1, 6, 13
1	roll of masking tape	1
1	marker or tape	2
1	overhead projector (optional)	2, 5, 10
5-6	rotationally symmetric objects, e.g. starfish, an S or H shape	3
5-6	objects which are not rotationally symmetric, e.g. most leaves, an L or V shape	3
1	box of colored chalk	4, 7, 13
1	* pegboard (optional)	4, 6, 13
several	construction sets, e.g. Tinkertoys (optional)	4
30	boxes of crayons	4, 7, 8, 14
30	sheets of typing paper	Art, Section 1
30	sheets of black construction paper	Art, Section 1
90	pieces of 6" x 11" onionskin paper	Art, Section 1
1-2	spools of thread	Art, Section 1
	paste or glue	Art, Section 1
120	sheets of colored construction paper	Art, Section 1
30	scissors	Art, Section 1
1	flannel board with several sets of shapes (optional)	6
5-6	objects exhibiting repeating patterns, e.g. a caterpillar, fence, or strip of lace	9
5-6	objects with several parts that do not repeat exactly, e.g. row of unmatched books	9

30	sheets of 12" x 18" construction paper	Art, Section 2
170	sheets of colored construction paper	Art, Section 2
	tempera paint or printer's ink	Art, Section 2
30	pieces of wood and roll of string	Art, Section 2
30	pieces of sponge and/or soap erasers	Art, Section 2
30	pieces of potato	Art, Section 2
	popcorn kernels or seeds	Art, Section 2
	paste or glue	Art, Section 2
30	3" x 5" index cards	Art, Section 2
30	* small mirrors	12, 13
1	large mirror	12
1 set ea.	cards, labeled 0-9 and labeled A-Z	12
30-40	assorted small solid objects	12, 13
	Mirror Cards (optional)	12
30	sheets of graph paper	13
15-25	assorted objects to be tested for symmetries	15
330	sheets of construction paper	Art, Section 3
60	pieces of tagboard	Art, Section 3
	glue, yarn, sequins, pebbles, etc.	Art, Section 3
	paint or ink	Art, Section 3
1-2	spools of thread	Art, Section 3
30	crayons or soft, compressed coal	Art, Section 3
	objects with raised patterns, e.g. leaves	Art, Section 3
30	sheets of newspaper	Art, Section 3
	yarn; yarn needles	Art, Section 3

	percussion instruments	Section 4
	shells; crystals	Section 5
60	sheets of lightweight paper	Section 5
†	honeycomb (optional)	Section 5

* kit items as well as
 ** printed materials available from Minnemath Center,
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 *** available from The Judy Company,
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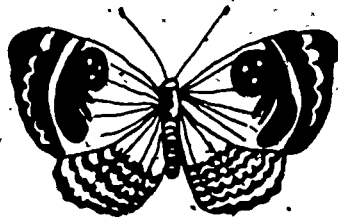
INTRODUCTION

In popular usage the word, symmetry often carries the implication that a thing is pleasantly proportioned or well-balanced. To many, symmetry implies a type of beauty and perfection. In precise usage, a symmetric pattern is one which remains unchanged under a certain rigid motion. One can intuitively notice symmetry in many things such as:

- butterflies
- leaves
- tile floors
- stanzas of poetry
- snowflakes
- the arrangement of seeds in a sunflower
- melodies
- rhythms
- the weaving in straw baskets
- slices of citrus fruits
- mathematical principles

Certain types of symmetry can be given precise geometric descriptions. Three of these types are emphasized in this unit: (1) rotational or turning, (2) translational and repetitive, (3) bilateral or symmetric about a line. A detailed discussion of them will conclude this commentary. It will be found that symmetric patterns are unchanged by certain motions.

You need not feel that a complete mastery of the introductory material is necessary before you start to teach the unit. Each idea is demonstrated and developed as the lessons progress. This introduction should be regarded as a source of information to which to refer for information during the various studies.



Why Have Symmetry Units in the Primary Grades?

Both the intuitive ideas of symmetry and the precisely defined mathematical ideas of symmetry were considered in the following reasons for having small children study symmetry.

1. The activities provide practice in abstracting concepts, e.g., seeing that several different patterns have the same type of symmetry.
2. The concepts discussed will be used in later topics such as geometry, measurement, wave motion, and classification of biological organisms. Symmetry is a tool in understanding the universe.
3. The principles of symmetry are often applied in art, poetry, and music. Awareness of these principles heightens appreciation of the arts.
4. The activities provide perceptual training for reading readiness. For example, the tracing of patterns aids perception through muscular activities. Practice is given in moving from left to right.
5. The children enjoy the activities and the awareness of symmetry that they develop. Many natural and man-made shapes in the children's environment have some type of symmetry that they can recognize.

Objectives

On completion of the unit the child should:

1. Recognize symmetry in nature, music, poetry, and art.
2. Recognize turning symmetry (rotational symmetry), repeating patterns (translational symmetry) and symmetry about a line (bilateral symmetry).
3. Be able to make the necessary tests and comparisons for the three types of symmetry studied.

Rotational Symmetry or Turning Symmetry

Consider the letter S. Think of it as a shape that can be moved and which leaves its image behind. Can the S be moved in a definite way so that after it has been moved it will coincide with its original image?

1. Simply picking the S-shape up and laying it back down is one such operation. This is trivial because it can be done for any movable object.
2. Turning the S-shape a full turn, i.e., through 360° , is another move that will leave the shape in coincidence with its original image. This is also trivial because it can be done for any shape.
3. Turning the S-shape through one half of a full turn, i.e., through 180° , will also cause the S to coincide with its original shape.

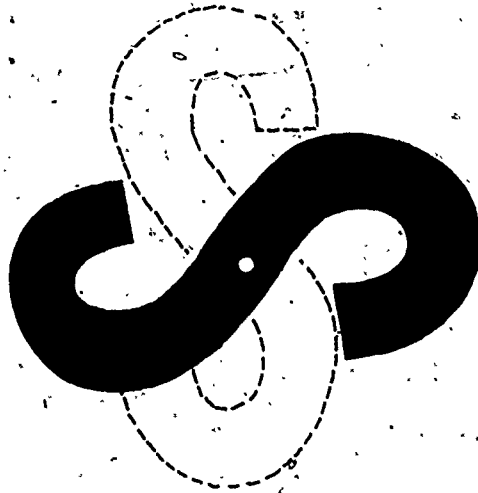



Figure 1

This third operation shows that the S-shape is symmetric with respect to rotation about an axis perpendicular to the shape at the center. The shape is said to have rotational symmetry or turning symmetry. Note that if the S is flipped over to the position  it cannot be put into coincidence with its original shape.

Another example of a shape that possesses rotational symmetry is a triangular block with equal sides. The block outlines in Figure 2 may help in visualizing the rotations that leave the block in coincidence with its original image. The rotations are one-third of a full turn, two-thirds of a full turn, and the trivial ones of no turn and of a full turn; rotations of 0° , 120° , 240° and 360° leave the pattern of the block unchanged.

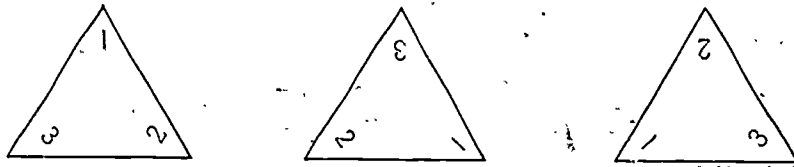


Figure 2

Note that in considering rotational symmetry of a pattern, only rotations about an axis through the center of the pattern are considered.

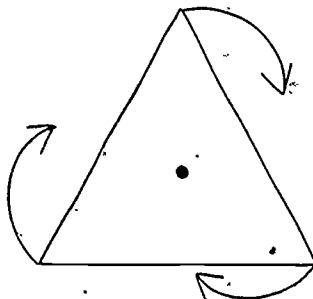


Figure 3

The pattern is not flipped or turned about an axis across the pattern.

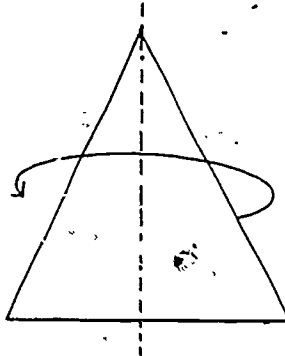
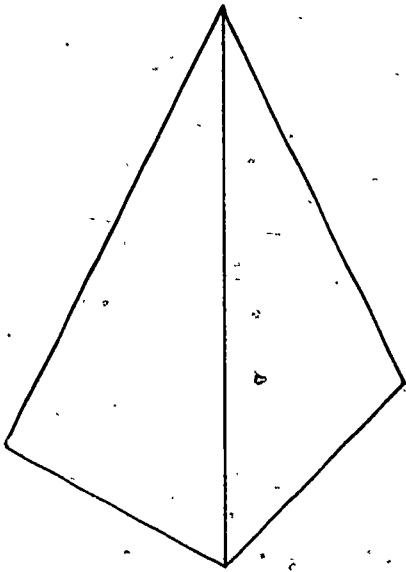
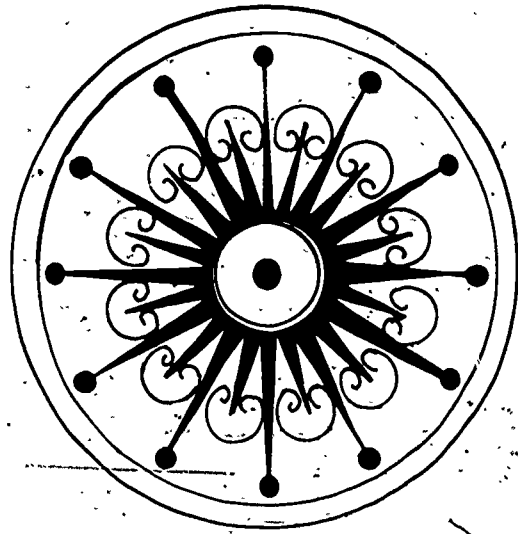


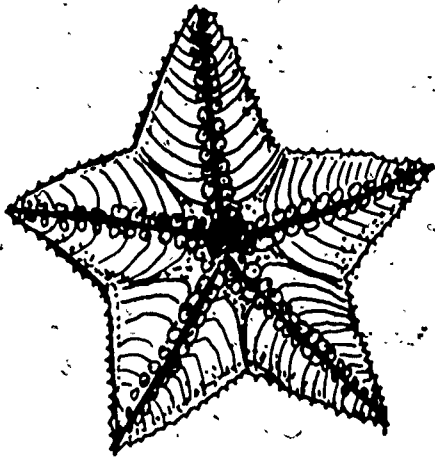
Figure 4



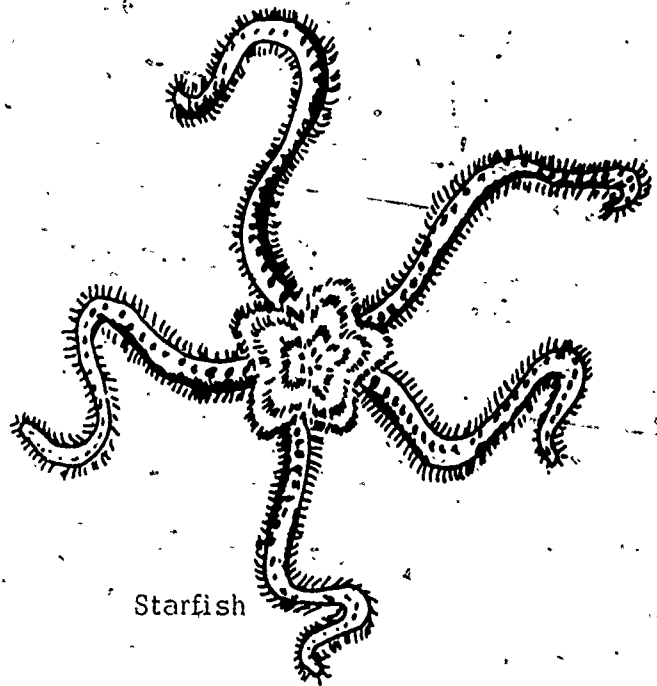
Pyramid



Plate



Starfish





Starfish

Figure 5
EXAMPLES OF ROTATIONAL SYMMETRY

A rotation through any amount less than a full turn that yields coincidence with the original position is a test for the presence in a pattern of rotational symmetry or turning symmetry. Further examples of simple patterns exhibiting rotational symmetry are stars, squares, ovals, and circles.

A three-dimensional object can't be tested for rotational symmetry exactly as done above, because a solid object won't coincide with a surface pattern. However a variation of the test can easily be made. The solid object can be turned and compared to an exact copy of itself. If, after the object has been turned an amount less than 360° , the object and its copy cannot be distinguished, the object has rotational symmetry. Figure 5 shows a pyramid and a plate, each with exact rotational symmetry, and two starfish showing approximate rotational symmetry.

A slightly different point of view is to consider a rotationally symmetric pattern as one that can be generated by rotating an element of the pattern. For example, the shape  can be generated by rotating the shape  into three positions.

Repeating Patterns and Translational Symmetry

Suppose a stencil is used to make the repeating pattern in the strip design shown in Figure 6.

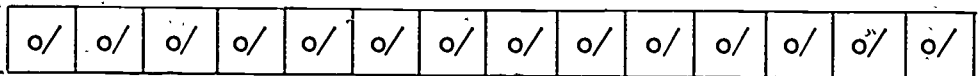


Figure 6

The dotted lines in the figure show that the pattern could be continued indefinitely. If the pattern is imagined as being infinitely long, it can be picked up, moved through a certain distance, and laid down again to cover the entire original pattern. The pattern would then look exactly as it did before it was moved. This is possible because an infinite pattern has no beginning and no end, but extends infinitely in either direction. A mathematician might say that a translation carries the pattern into itself. Because of this, the pattern is said to possess translational symmetry.

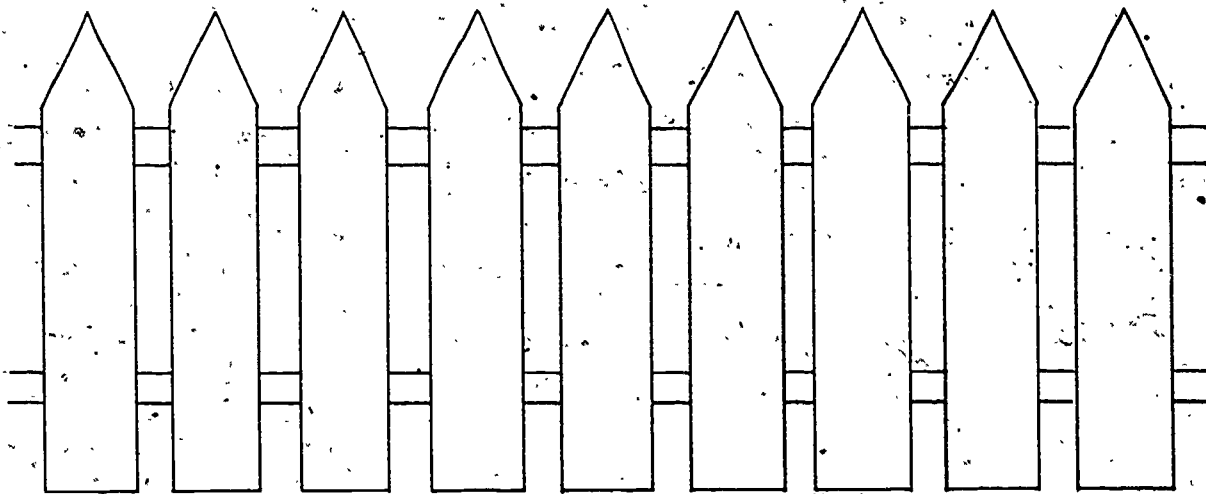
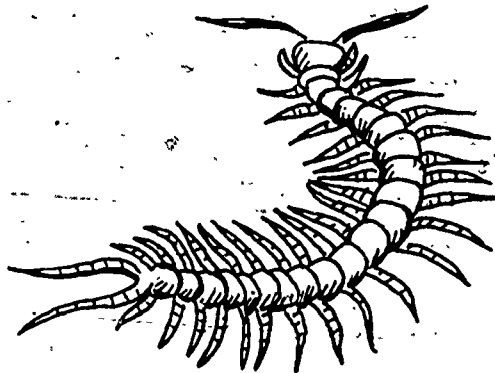
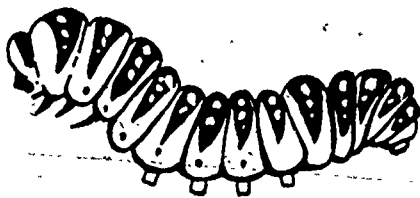
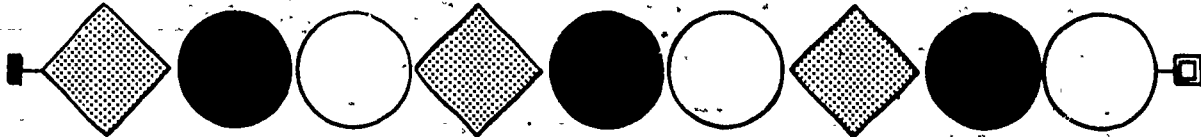




Figure 7
EXAMPLES OF REPEATED PATTERNS

Repeating patterns are easy to find. For example, they are seen in wallpaper, brick walls, and strings of beads. These patterns repeat only a finite number of times, but they may be thought of as "potentially infinite." This means that the pattern can be imagined to extend indefinitely in both directions. Thus the term "translational symmetry" is sometimes used in talking about repeating patterns which are finite but can be imagined to extend infinitely.

A pattern is a repeating pattern if some part of the pattern (1) can be moved a definite distance along a line to cover an identical part of the same pattern and (2) with a succession of such moves will cover the entire pattern. As in rotational symmetry, comparisons are made after some part of the pattern is moved. This time a part of the pattern is moved in equal-sized jumps and compared to other parts of the same pattern to determine coincidence. The part of the pattern that is repeated can be called a "block," a "cell," or an "element."

In testing the string of beads shown below for a repeating pattern, the block to be compared could be .



The block to be compared could also be chosen as . This would again cover the pattern when repeatedly moved in steps along the string. If the beads are formed into a circle, they will show a rotationally symmetric pattern. If actual beads were used, instead of a drawing of beads, the comparison block of beads would of course not be laid on top of another block. It would be placed beside a corresponding block for the comparison.

There are many (imperfect) repeating patterns to be observed in nature. Two examples are bamboo stems and caterpillars. Often only a portion of the object shows the repeating pattern, as in the caterpillar where the middle segments are approximately of equal size and shape but the two ends are not. (See Figure 7.)

The activities in this unit will extend the concept of repeating patterns to poetry and music. Repeating sound patterns are common in both poetry and music.

Bilateral Symmetry or Symmetry about a Line or Plane

Consider the letter A in Figure 8. Any line drawn perpendicular to the dotted line in the figure cuts the pattern in pairs of

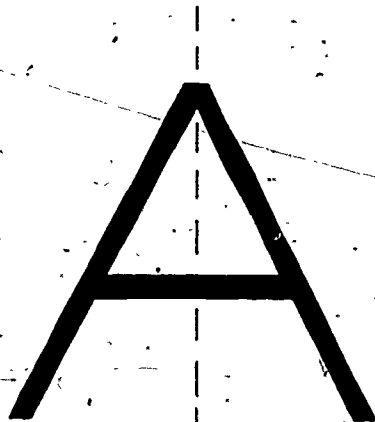


Figure 8

points that are equidistant from the dotted line. For example, the points a and b in Figure 9 are at equal distances from the dotted line.

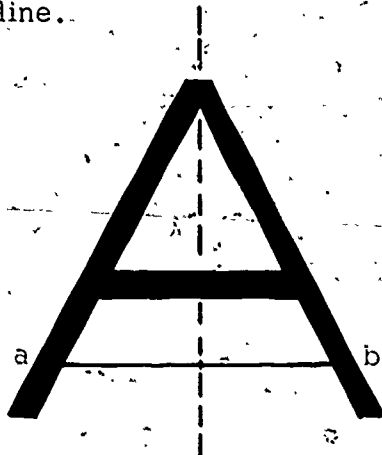


Figure 9

The letter A is said to exhibit symmetry about a line, or bilateral symmetry. The dotted line is called the line of symmetry or the axis of symmetry.

Any pattern through which a line can be drawn such that every line perpendicular to this line cuts the pattern into pairs of points equidistant from the line is said to exhibit symmetry about a line or bilateral symmetry.

A simpler but less exact statement of this idea is that for every point on one side of the line of symmetry there is a corresponding point on the other side. The paired points must be at equal distances from the line of symmetry. Figure 10 shows some almost exactly bilaterally symmetric patterns that occur in nature. There are tests one can use to determine whether or not a pattern possesses bilateral symmetry. Two of these tests are described below.

If a mirror is placed in a vertical position along the dotted line in Figure 8, the pattern viewed (half of A and its image in the mirror) coincides precisely with the original letter A. The mirror test consists of holding a mirror on the axis of symmetry to show that the pattern can be divided into two parts identical in size, one part being the mirror image of the other. In any bilaterally symmetric pattern, one half of the pattern is the mirror image of the other half. This is the reason that bilateral symmetry is also referred to as mirror symmetry. It is interesting to test some non-symmetric patterns with a mirror.

Bilaterally symmetric patterns may also be folded along the axis of symmetry to test for the coincidence of the two halves of the pattern. This gives another test, the folding test, for bilateral symmetry.

There are some differences in examining the symmetry of patterns on plane surfaces and of three-dimensional objects. Corresponding to the line of symmetry of plane patterns, bilaterally symmetric three-dimensional objects have a plane of symmetry. Every line perpendicular to this plane cuts the figure in pairs of corresponding points that are at equal distances from the plane. Obviously, a solid object cannot be folded to test for bilateral symmetry. However, a mirror can be placed along the plane of symmetry to show the existence of bilateral symmetry. The human face is an interesting object that has approximate symmetry with respect to a plane. Other objects that might be tested are a cone, a beetle, and toy figures of people or animals.

A picture of a three-dimensional object is, of course, a two-dimensional pattern. Thus the symmetry of a picture of a three-dimensional object may be tested by either the mirror or folding test. Generally the kindergarten and first grade activities will involve only two-dimensional patterns or pictures.

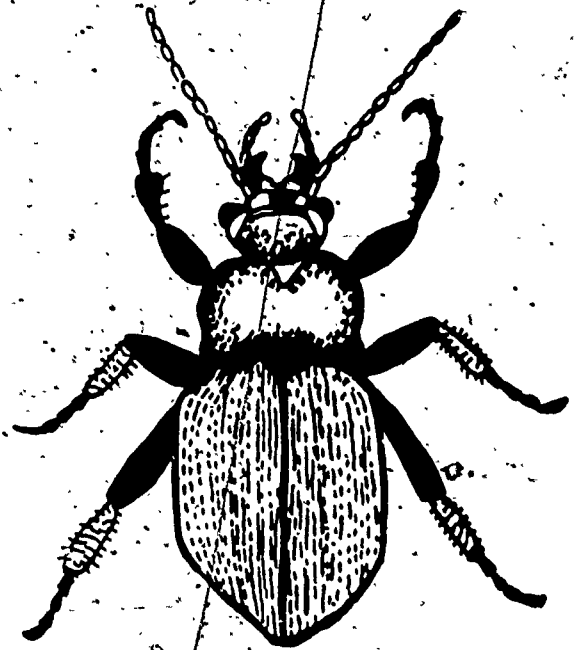
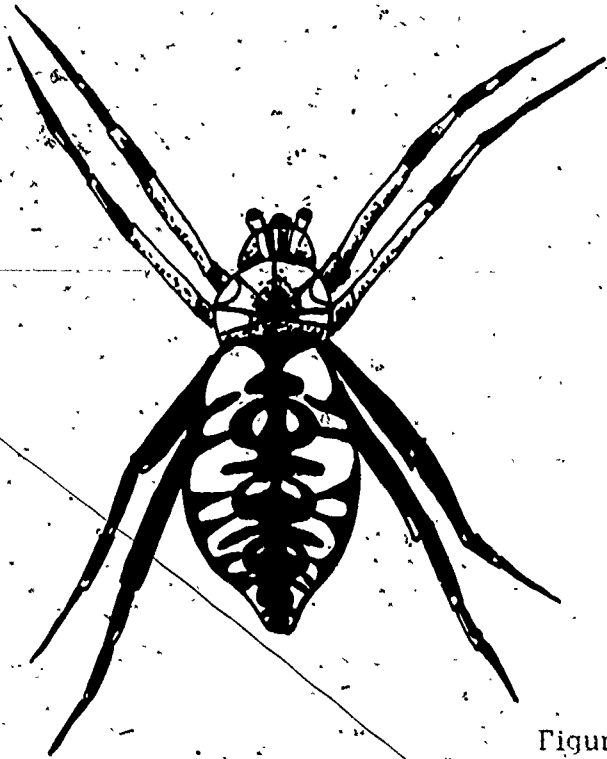
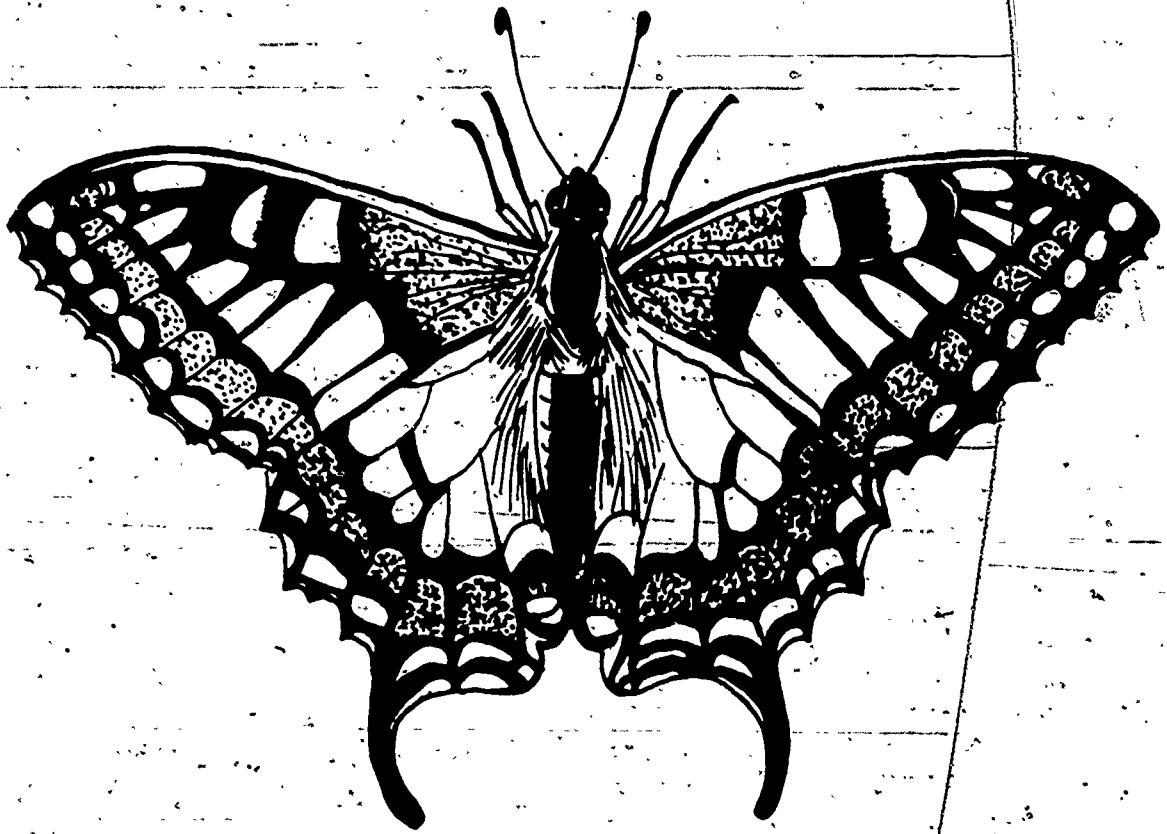


Figure 10
EXAMPLES OF BILATERAL SYMMETRY

Other Symmetries

An object may display more than one of the symmetries that have been discussed. For example, a star such as the one in Figure 11 displays both rotational and bilateral symmetry. Notice that each of the five dotted lines is an axis of bilateral symmetry.

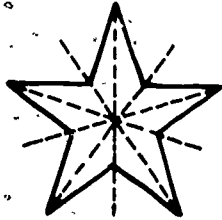


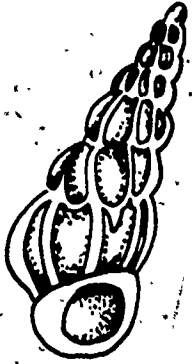
Figure 11

It is not necessary for a child to recognize more than one symmetry in an object, but it is important for you to be aware of the possibility of more than one, so that you may guide the children properly.

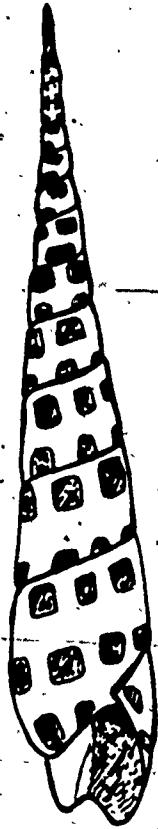
There are other symmetries which may be precisely described mathematically. You may be interested in noting these, but they will not be explicitly presented to the children. One such symmetry is a translation plus dilation. It is illustrated in Figure 12 by the shells of the common wentletrap, an auger, and the giant conch. Another symmetry which is a rotation plus dilation is illustrated in Figure 13 by the shells of the chambered nautilus and the sunburst carrier.

A more complicated kind of symmetry is found in certain two-dimensional patterns, which are called ornamental patterns. A common pattern of this kind is formed by fitting hexagons together as in a honeycomb or in a tiled floor. Ornamental patterns are also often seen in brick walls. (See Figure 14.) There is a precise mathematical theory underlying the ornamental pattern, but it is not necessary to understand this theory to appreciate or create ornamental patterns. Extremely intricate and beautiful tiled patterns have been produced by artists who instinctively used the laws of symmetry. The three-dimensional arrangements of atoms in a crystal are related to ornamental patterns.

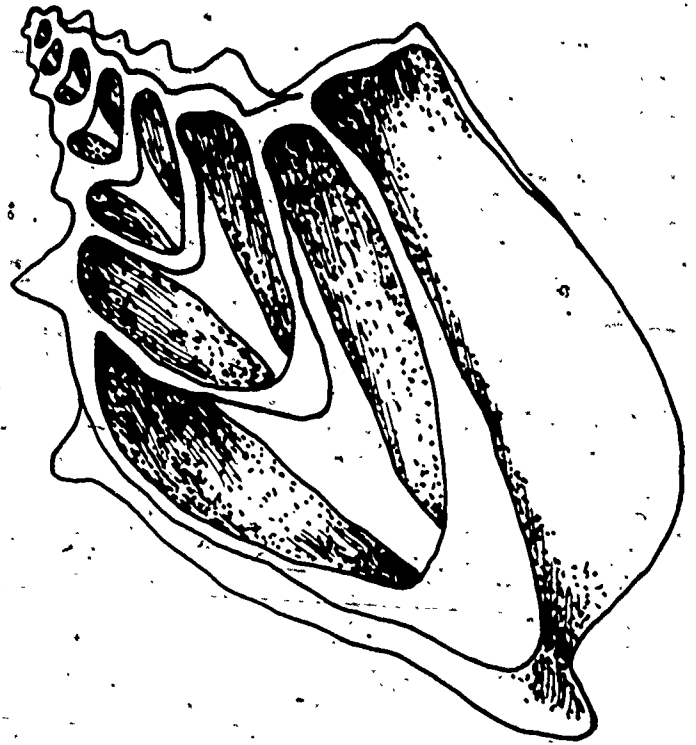
There are other patterns possessing symmetries not discussed here. It is hoped that the child will recognize some of them in his environment. The child should not be expected to describe precisely what he sees, but simply to be aware of and enjoy the symmetric patterns about him.



Common Wentletrap



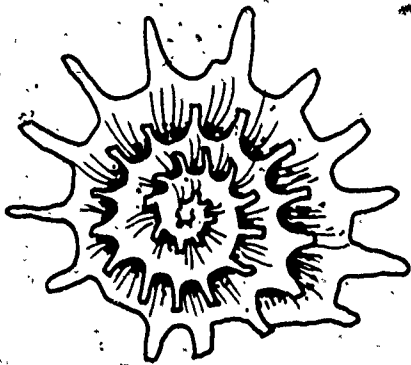
Auger



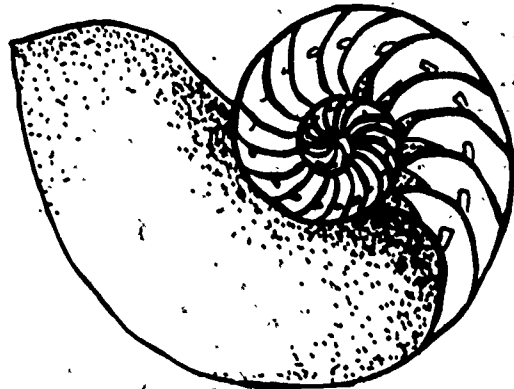
Giant Conch (Cross section)

TRANSLATION PLUS DILATION

Figure 12



Sunburst Carrier



Chambered Nautilus (Cross section)

ROTATION PLUS DILATION

Figure 13

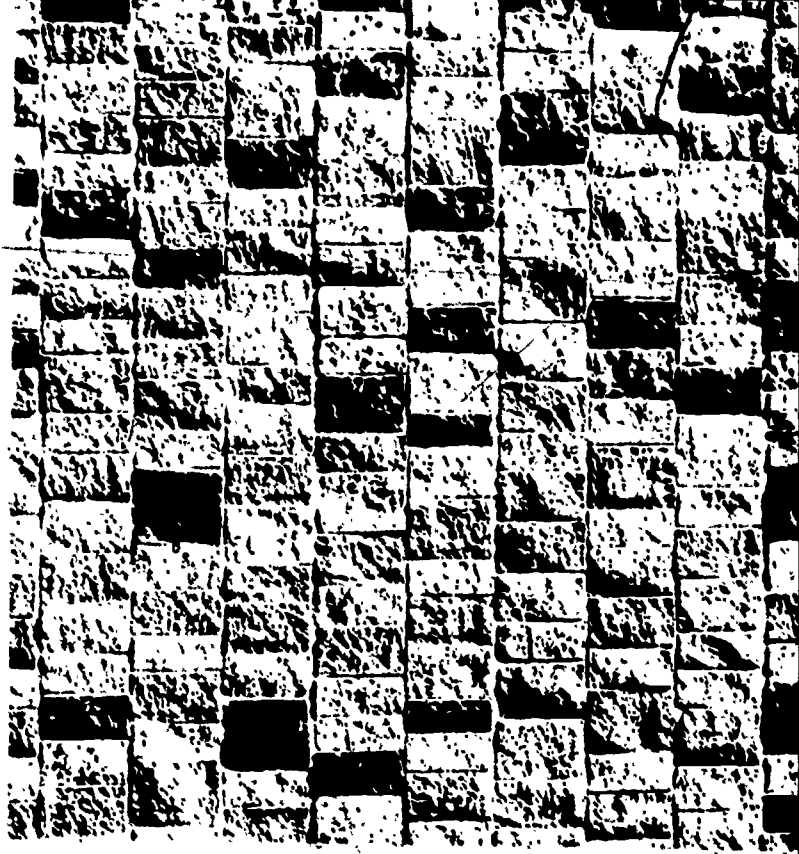
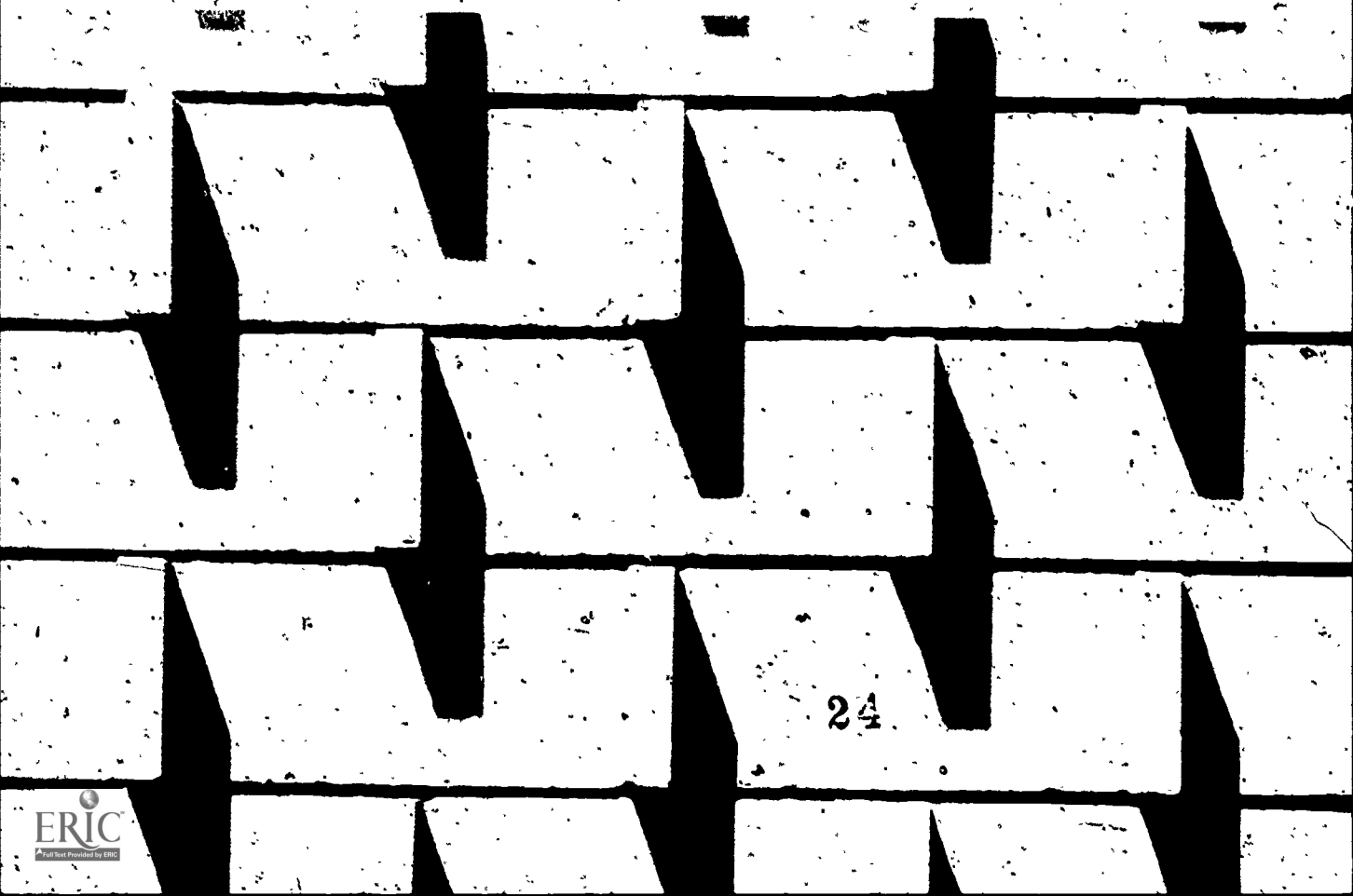


Fig. 14. SYMMETRY IN BRICK AND STONE.

These three photographs show some of the many varied patterns used by architects and builders. Above left -- a wall built with plain bricks; above right -- the stone face of an office building; below -- shadows contribute their symmetry, too.



Summary

In summary the tests for rotational, translational, and bilateral symmetry are given:

1. Rotational symmetry test: an object is rotated about an axis and compared with a copy of the original object.
2. Repeating pattern test: a portion of a pattern is duplicated, moved along the pattern, and compared to parts of the pattern at equally spaced locations to determine coincidence.
3. Bilateral symmetry test:

3-D: an object is compared to the half-object, half-image pattern obtained by using a mirror and is judged for coincidence.

2-D: either the above mirror test is used, or the object is folded to test for coincidence of the two halves!

The use of a copy or replica of the object, or part of the object, in the tests eliminates the need for the tester to imagine the object or part to be moved and compared. You will probably carry out the tests mentally, but five- or six-year-old children may not be successful in making such abstract comparisons. If some children begin to form conclusions quickly concerning the symmetry of a particular object, this should not be discouraged. If they are correct, accept them with joy. If they are incorrect, a direct test for symmetry should be made, involving the concrete operations and comparisons necessary.

DIFFERENCES BETWEEN THE KINDERGARTEN AND FIRST-GRADE SYMMETRY UNITS

If you look at copies of the kindergarten and first-grade symmetry units you will notice that the first four sections of each consider rotational symmetry, repeating patterns, bilateral symmetry, and symmetry in movement, poetry, and music. How then do they differ? Some of the ways are given in the following paragraphs.

An obvious difference is that the first-grade unit contains worksheets and an additional section. On many of the worksheets the children are asked to do more precise and accurate work than that asked of kindergarteners. The additional section for first grade considers crystals and ornamental patterns.

The first-grade unit considers the three basic symmetries in more depth. More complicated patterns are considered. At the end of Section 3 the first grader is asked to identify symmetries when all three types are present in the same object.

Although the major emphasis is on observation and description in both units, the first-grade unit includes some work involving measurement and generalization. The first-grade unit is more quantitative. The children are asked questions such as "How many positions are there where the pattern looks just the same?" and "How far is the star from the line of symmetry?"

You will note that the introductory material in the two units is the same, since both units cover the same subject matter.

COMMENTS ON THE TEACHING OF THIS UNIT

The lessons in this unit are usually planned to take one class period. The questions in capital letters that appear in some lessons are to be considered a guide to the type of questions you will use. Feel free to modify them and any other part of the unit to suit the needs of your own class.

The children should use and understand the concepts of the symmetries considered, but they should not be expected to become proficient in verbalizing them. They will not always be precise in their work. We hope that both you and your class will relax and enjoy this unit.

Art Activities

This unit contains many art activities that complement the more directed classroom experiences. The child's creation of his own designs reinforce his concepts of symmetry and also develop an appreciation of symmetry. These design activities are in contrast to the design activities on the worksheets, which usually should be considered experiences in translation of points on grids rather than creative art.

The choice and timing of the art is generally up to you any time after the appropriate symmetry has been introduced. Provide only necessary direction. Considerable originality results when children are given latitude in the use of colors and materials and free rein to use their own creative powers.

SECTION 1 ROTATIONAL SYMMETRY

This section develops the concept of rotational symmetry. The lessons start with the manipulation of physical objects, and the creation of rotationally symmetric patterns. The children discover patterns that are left unchanged by the rigid motion of rotation. As they do the activities of this section, they should gradually abstract the concepts and the properties of rotationally symmetric patterns.



Snowflake

Lesson 1: TURNING BLOCKS

The manipulation of blocks is used in this lesson to introduce the concept of rotational symmetry to those children who have not previously encountered it. For those children who have had the MINNEMAST kindergarten program, the lesson will be partly review but will also present the counting of positions of coincidence.

MATERIALS

- 1 pair of triangular blocks for each child.
- 1 pair of square blocks for each child (The blocks in each pair should be of the same size, but the different pairs may be of different sizes.)
- masking tape
- assorted blocks for Activity C

PROCEDURE

Activity A

Each child should have a matched pair of triangular blocks and one of square blocks. One corner of one side of each block should be marked with masking tape. The tape helps to keep track of the position of the block and also designates one surface of the block as the front. This is important because, in testing a pattern for rotational symmetry, the pattern must not be turned over to bring the back side up.

Have each child place one of his triangular blocks on top of the other with the taped corners matching.

TURN THE TOP BLOCK UNTIL IT FITS THE BOTTOM BLOCK AGAIN. DO NOT TURN IT OVER.



DID YOU TURN THE TOP BLOCK A WHOLE TURN?
(No, only part of a turn -- or -- No, only one-third
of a turn.)

TURN THE TOP BLOCK MORE UNTIL IT AGAIN FITS
THE BOTTOM BLOCK.

HAVE YOU TURNED IT A WHOLE TURN NOW? (No,
still only part of a turn.)

AGAIN TURN THE TOP BLOCK UNTIL THE BLOCKS FIT.
HAVE YOU TURNED IT A WHOLE TURN NOW? (Yes.)

HOW DO YOU KNOW? (The taped corners match once
again.)

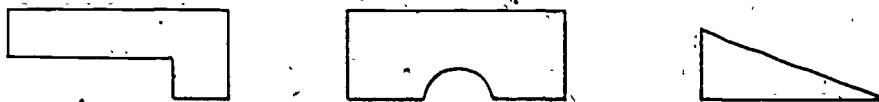
Explain that when we can turn a pattern or an object part way
around and have it look the same way it did before it was
turned, the pattern or object has turning symmetry or rotation-
al symmetry.

Activity B

Repeat Activity A with pairs of square blocks.

Activity C

The concept of rotational symmetry may be strengthened by
consideration of some shapes that are not rotationally sym-
metric. Pairs of blocks of the following rotationally non-
symmetric shapes are often available, or you can prepare
cardboard shapes.



In each case have a child see whether he can rotate the top
block part of a full turn and have it fit the one that has not
been rotated. Since in each case it is not possible to carry
out the test successfully, the shapes do not possess rota-
tional symmetry. (If you do not have pairs of these asymmet-
ric blocks, trace around the block being tested and rotate the
block against its outline.)

Lesson 2: ROTATING TRANSPARENCIES

Here transparent patterns are used to illustrate rotational symmetry further. The lesson also introduces the idea that one part of a rotationally symmetric pattern may be rotated to generate the entire pattern.

MATERIALS

- 1 blue and 1 yellow star transparency
- 1 blue star point transparency
- marker or tape
- overhead projector (very desirable, but not necessary)
- transparent Pattern A, for each child
- Worksheet 1, for each child

PROCEDURE

Activity A

It is assumed that you will use an overhead projector. If you do not, you can hold the transparencies against a window to be viewed.

Project the blue and the yellow stars separately and then superimposed. Check to see that the children realize that an all-green star shows the coincidence of the blue and the yellow star. Rotate the upper star slowly over the lower star. As this is done some yellow, some blue, and some green will be seen. After one fifth of a complete turn, the stars will again coincide and an all-green star will be seen again. The children should understand that this coincidence implies the star possesses rotational symmetry.

**IN HOW MANY POSITIONS WILL THE STARS FIT TOGETHER?
LET US COUNT THEM. TO HELP US KEEP TRACK OF THE
TURNS WE WILL MARK ONE CORNER OF EACH STAR.**

Mark the corners with a marker or tape. Rotate and count the five positions where the star patterns fit together exactly.

PAGES 21-30 WERE REMOVED FROM
THIS DOCUMENT PRIOR TO ITS BEING SUBMITTED TO
THE ERIC DOCUMENT REPRODUCTION SERVICE
DUE TO POOR REPRODUCIBILITY.

Another way of viewing a rotationally symmetric pattern is to notice that the entire pattern can be generated by turning one part of the pattern into equally spaced successive positions. For example, the star pattern can be covered by one point of the star rotated into five successive positions.

Don't expect the children to express this idea fluently, but demonstrate it by projecting the yellow star and the blue star point.

Activity B

Have the children cut out their transparent copies of Pattern A. They need not cut exactly on the dotted circle. Ask them to put their patterns on their desks, look at them carefully, and then turn them one third of a whole turn.

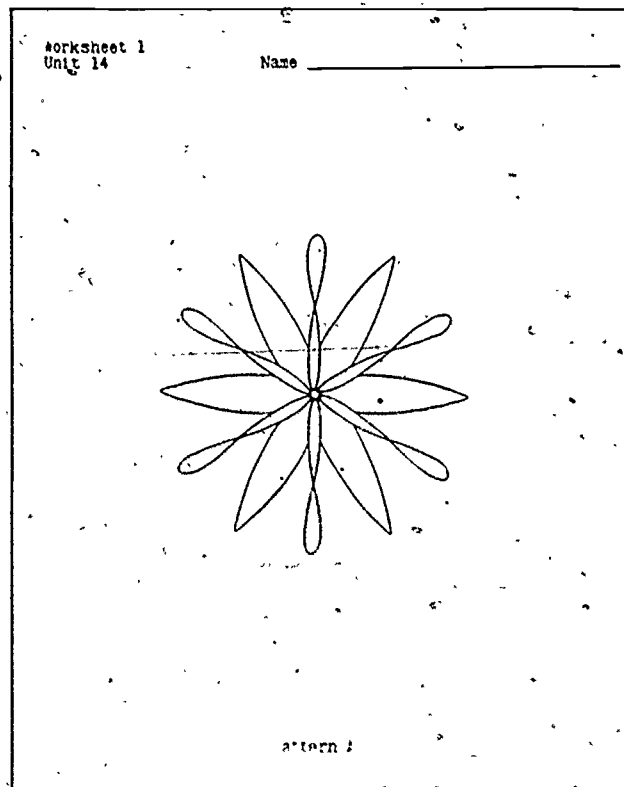
DOES YOUR PATTERN LOOK JUST AS IT DID BEFORE YOU TURNED IT? (Yes..)

HOW CAN YOU TELL FOR SURE? (By making some kind of record of the way it first looked.)

Tell the class that this record has been printed for them on Worksheet 1. Suggest that each child fit his transparent

pattern over his printed pattern. He can then turn the transparent pattern to find the positions in which it looks just as it did before it was turned.

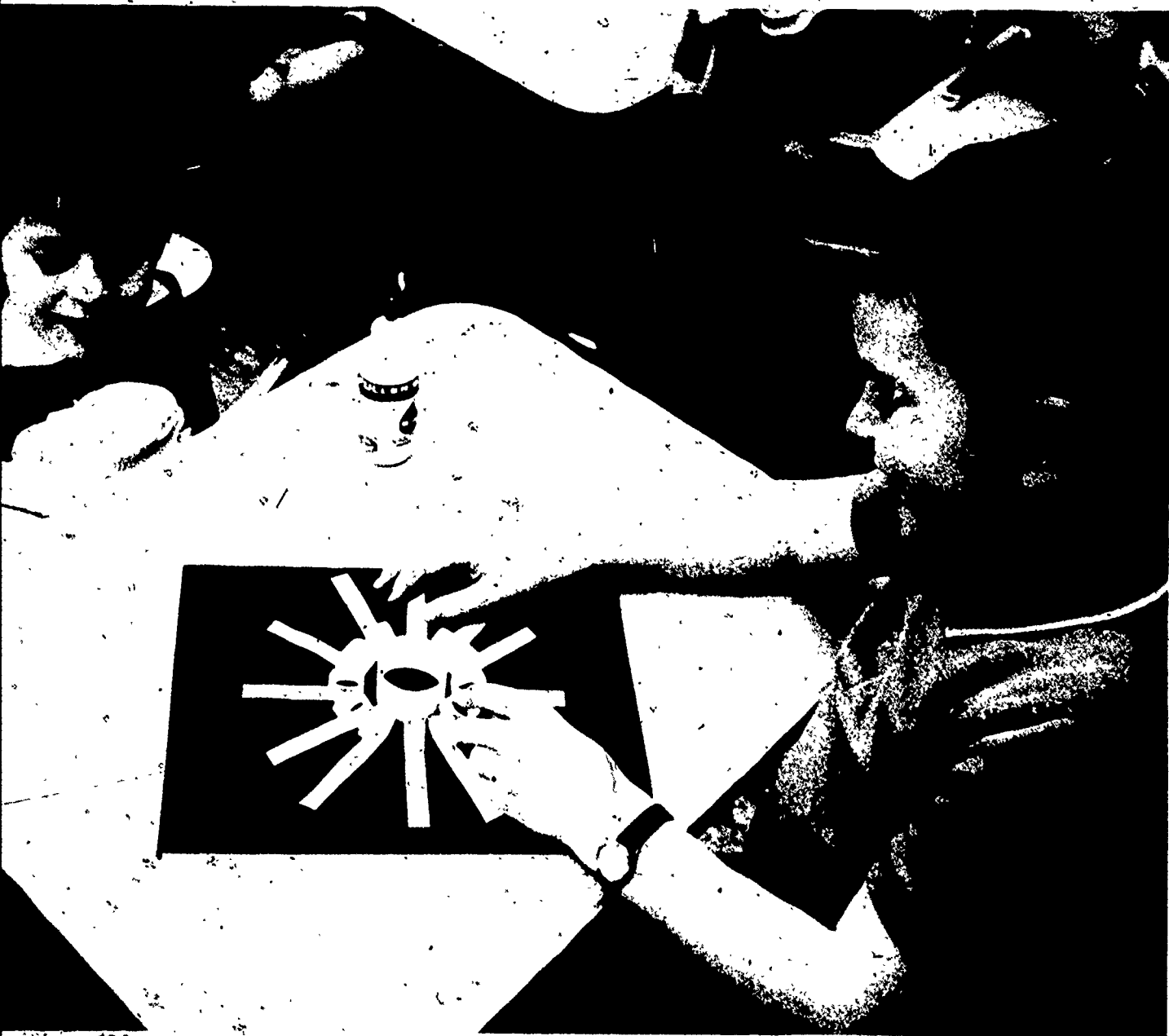
HOW MANY POSITIONS ARE THERE IN WHICH THE PATTERN LOOKS JUST THE SAME? (Six.)



- Activity C (Optional)

Discuss the fact that the whole pattern can be generated (or made) by turning a part of the pattern into six different positions.



At the conclusion of this lesson, you may have the children use any of the art activities for this section (pages 39-50).



Lesson 3: FINDING ROTATIONALLY SYMMETRIC OBJECTS

In this lesson the children's concepts of rotational symmetry will be applied to a wide variety of objects. Sometimes they will test two-dimensional shapes and sometimes three-dimensional shapes. They will also test natural objects that have only approximate rotational symmetry.

MATERIALS

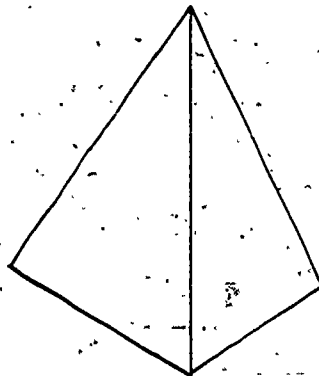
- an assortment of rotationally symmetric objects. Some examples are certain flowers (real or artificial), certain shells, an apple or orange cut across, starfish, gears, wheels, designs made from Tinkertoys or other construction sets, and cardboard shapes (S, H, .
- an assortment of objects that are not rotationally symmetric. Some examples are most leaves, certain flowers, certain shells, certain blocks, pencils, designs made from Tinkertoys or other construction sets, and cardboard shapes (, L, V).

PROCEDURE

Activity A

Prepare a shape table for the class. Place on the table some objects that are rotationally symmetric and some that are not. Some, but not all, of the objects should be in pairs. Other rotationally symmetric and non-symmetric objects should be placed about the room. Suggestions for the objects are given in the materials list.

Have the children determine which objects on the shape table have rotational symmetry. Although many children will immediately recognize rotationally symmetric objects, they should be able to test the objects if requested to do so. Discuss with them the fact that the test for rotational symmetry in a solid object is a little different than the test for a flat (plane) pattern. For example, note the pyramid-shaped block sketched on the next page.



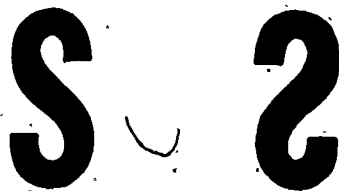
It cannot be fitted over another identical block, but it can be placed next to it and compared. After being rotated one-fourth of a full turn, it will look the same as before it was turned.

When only one object is available, the test for rotational symmetry is again slightly different. Either the original appearance of the pattern must be remembered or it can be recorded by a sketch. Perhaps you or a child could trace around a starfish (or other object) on the board, rotate the starfish, and compare the outlines. The children will notice that the rotational symmetry of natural objects is usually not exact.

After the objects on the shape table have been examined, have the children look about the room for other rotationally symmetric objects. They can also be asked to think of symmetric objects they have seen in other places.

Activity B (Optional)

Some children may notice that although a certain object may look the same when rotated a certain amount, it will look different when flipped over. This may be shown with an S-shape. On the other hand, an H-shape does look the same when flipped over.



This activity should be used here simply to provide additional experience in observing patterns. In Section 3, the effects of turning a pattern over will be considered further.

Lesson 4: KEEP THE SYMMETRY

In this lesson the children are asked to design, complete, or alter patterns as they wish, subject to the restriction that the rotational symmetry of the pattern be maintained. This provides further practice and also gives you an indication of their concepts of this symmetry. Color is used in some of the patterns.

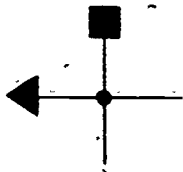
MATERIALS

- colored chalk
- pegboard (optional)
- construction sets such as Tinkertoys (optional)
- Worksheets 2 and 3

PROCEDURE

Activity A

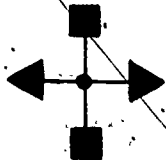
With white chalk draw a pattern on the board such as this:



IS THE PATTERN ROTATIONALLY SYMMETRIC? (No.)

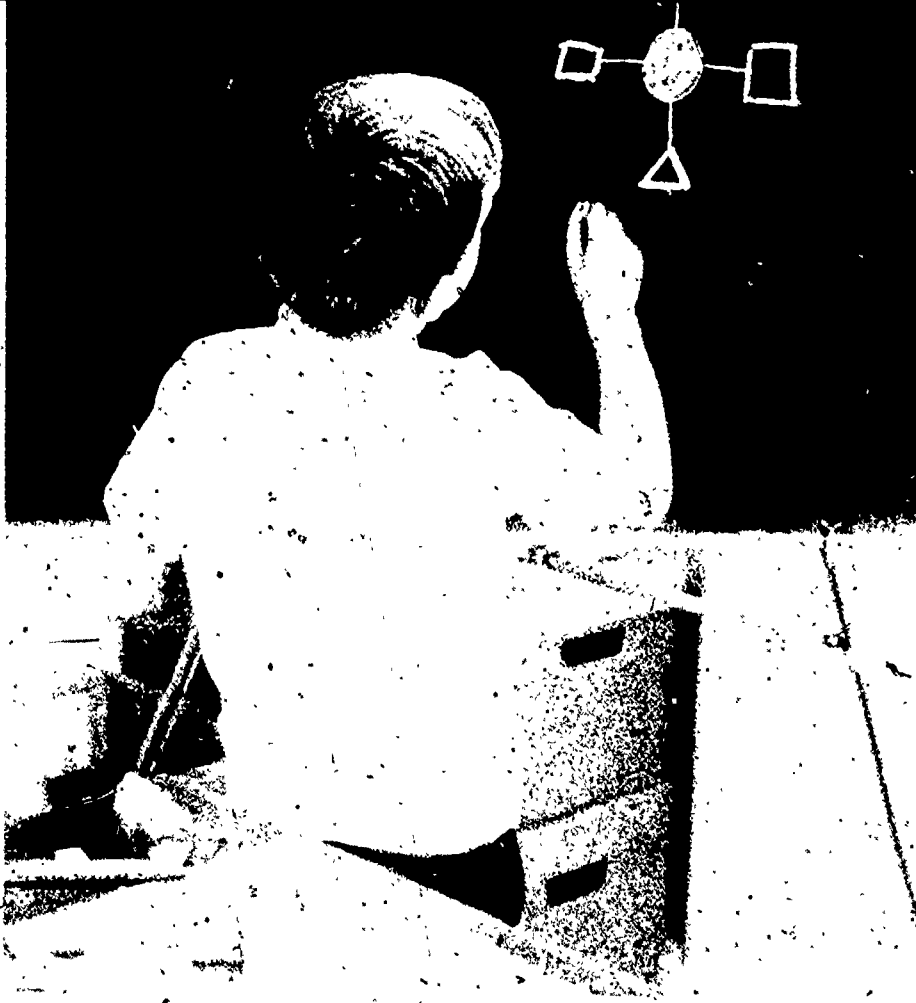
CAN SOMEONE MAKE IT ROTATIONALLY SYMMETRIC?
(Yes.)

Have a volunteer do so.



CAN SOMEONE ADD SOME MORE PARTS TO THE PATTERN? (Yes.)

Have the children do so.

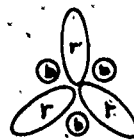


Draw another incomplete pattern with colored chalk. Again have volunteers complete and elaborate the pattern. A possibility would be:



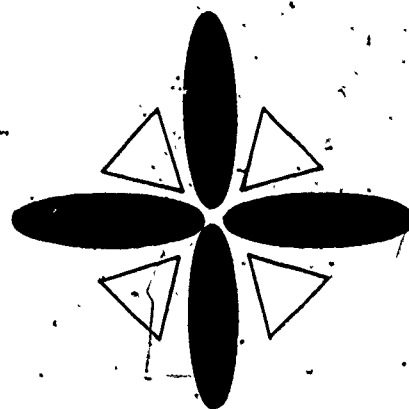
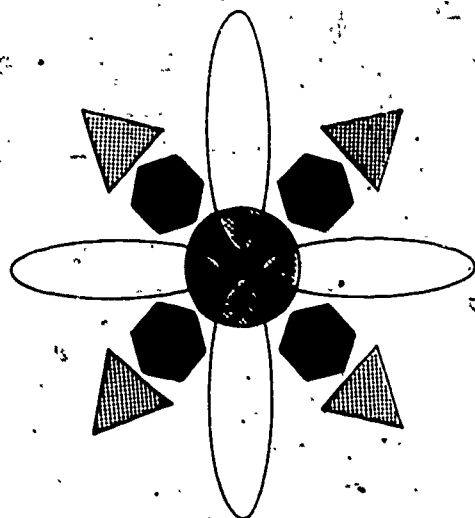
r = red

b = blue



Have the children continue this activity as a game. They can play in pairs or small groups, taking turns in adding elements to the designs. They may play on the chalkboard or on paper, using colors. The "keep the symmetry" rule should be very clear to the players.

Sample patterns with different degrees of complexity are shown below.



The activity may go on as long as the children can find elements they wish to add, or you may set a time limit on it.

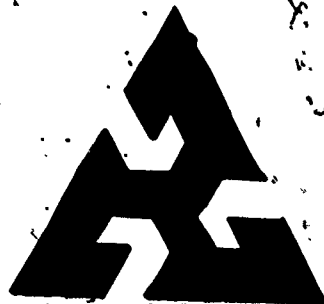
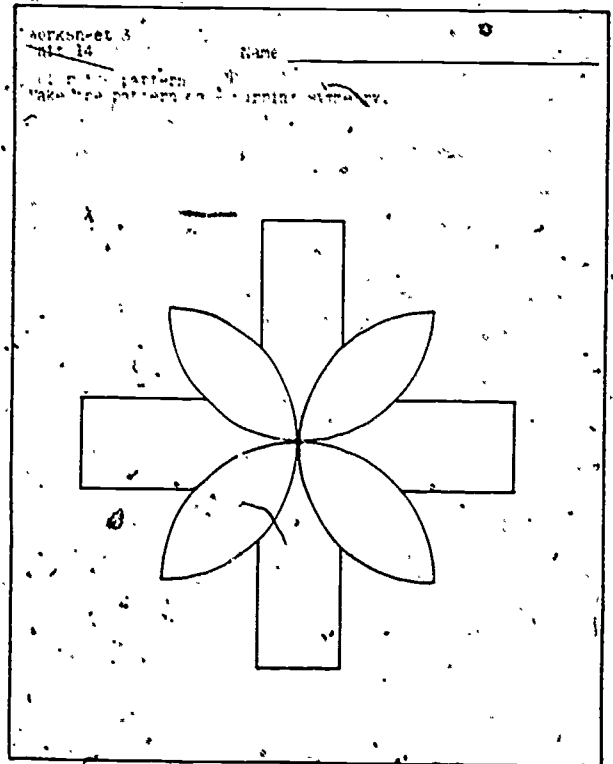
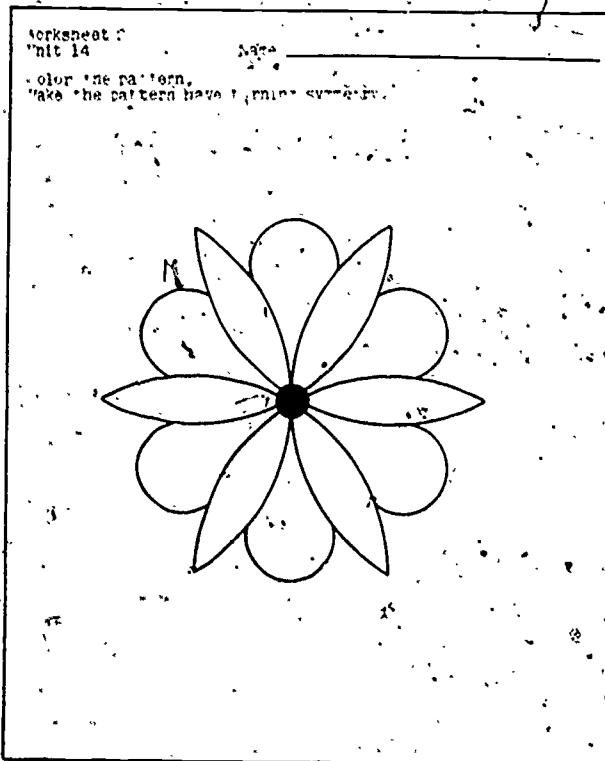
Activity B (Optional)

A pair of children may play a game very similar to that discussed in Activity A by making patterns on a pegboard or with a construction set such as Tinkertoys.

Activity C

The class should now be ready to color the patterns on Worksheets 2 and 3. Again the "keep the symmetry" rule should be explained, but do not insist on complete accuracy. Most children should be able to color the patterns with freedom and imagination. However, if a child seems unable to get started, you could suggest a simple alternation of two colors.

Children enjoy seeing the colored, rotationally-symmetric patterns formed in kaleidoscopes.



ART ACTIVITIES FOR SECTION 1: ROTATIONAL SYMMETRY

A number of art activities are suggested here. Any selection of them may be used to provide review and extension of the ideas of rotational symmetry. These art activities, as well as those in other sections, can usually be modified rather easily and used again for other types of symmetry.

Two toys that mechanize the drawing of geometric patterns, most of which are rotationally symmetric, are listed below. A large variety of patterns may be produced fairly easily. A ball-point pen usually gives better results with these toys than does a pencil.

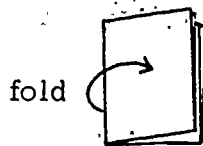
Magic Designer 8245 (Lakeside Toys, Division of Lakeside Industries, Minneapolis, Minn., \$4.95)

Spirograph No. 401 (Kenner Products Co., Cincinnati, Ohio, \$3.98)

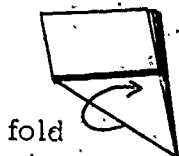


Activity A: Design Silhouette

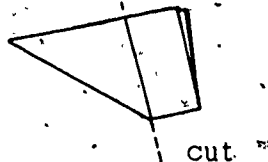
1. Fold a sheet of typing paper into quarters.



2. Fold again along the diagonal.



3. Cut as shown to form a square.



4. Cut out design from folded paper. (If you wish, you may draw a design before cutting.)

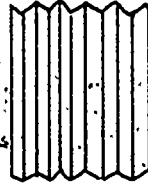


5. Unfold design and paste on black construction paper.



Activity B: Rosettes

1. Cut a sheet of onionskin paper to measure 6" x 11".
2. Fold pleats into the paper across the narrower dimension. The pleats should be about $\frac{1}{2}$ " to 1" apart. Roll a pencil over the pleated paper to insure deeply creased pleats.



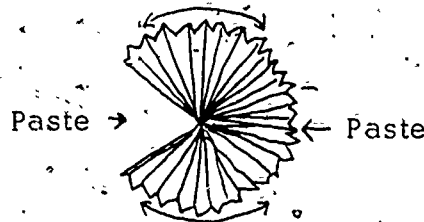
3. Fold pleated paper in half to crease.



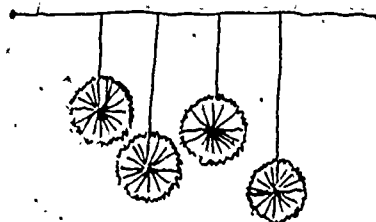
4. Bind at creased center with white thread, and knot.



5. Fan ends out and paste edges together to make a full circle.

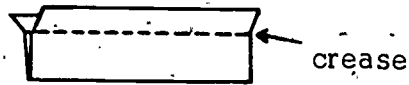


6. These rosettes make nice classroom decorations. Attach a thread to the top of each design and hang it from a string stretched across the classroom.

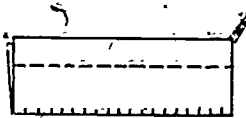


Activity C: Paper Flowers

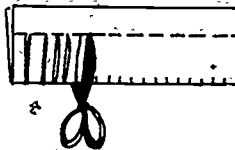
1. Fold colored construction paper in half lengthwise.
2. Crease paper about 1" from top.



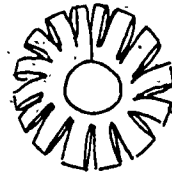
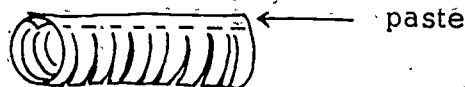
3. Mark off every $\frac{1}{2}$ " along the length of the fold.



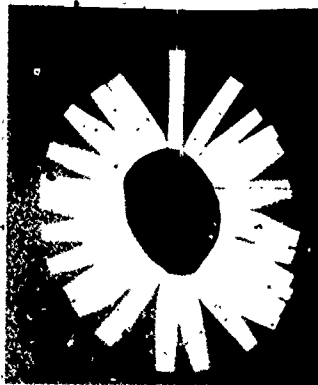
4. Cut at $\frac{1}{2}$ " marks as far as the crease. Do not cut to edge.



5. Overlap uncut edges and paste together.

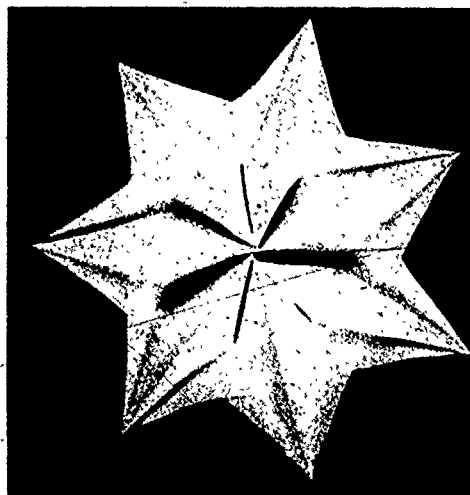
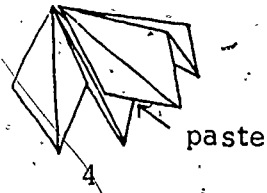
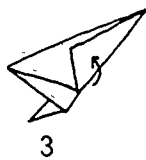
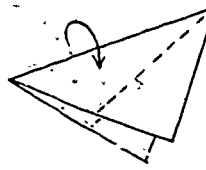
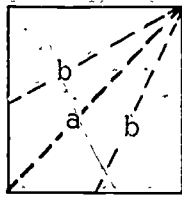


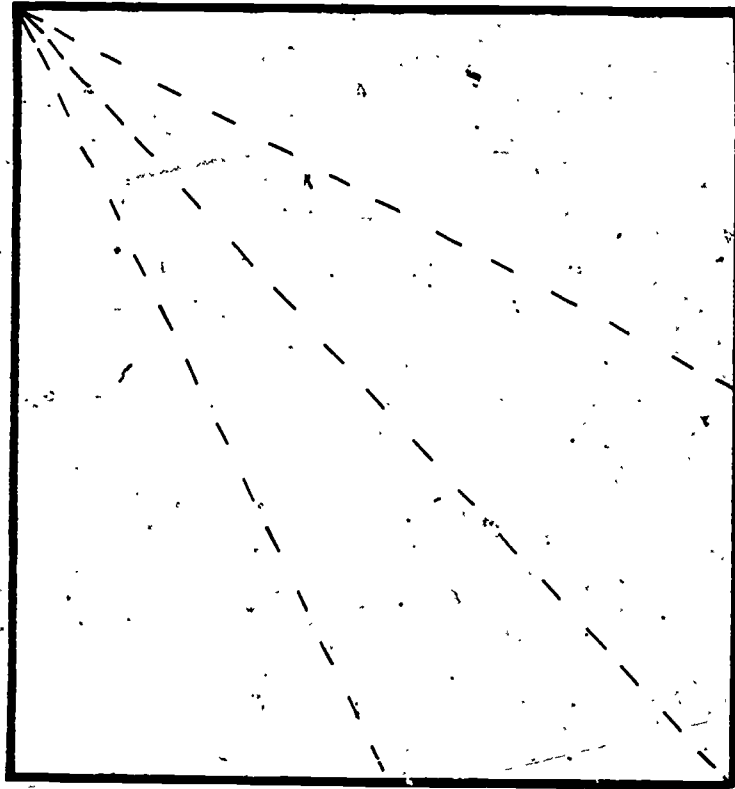
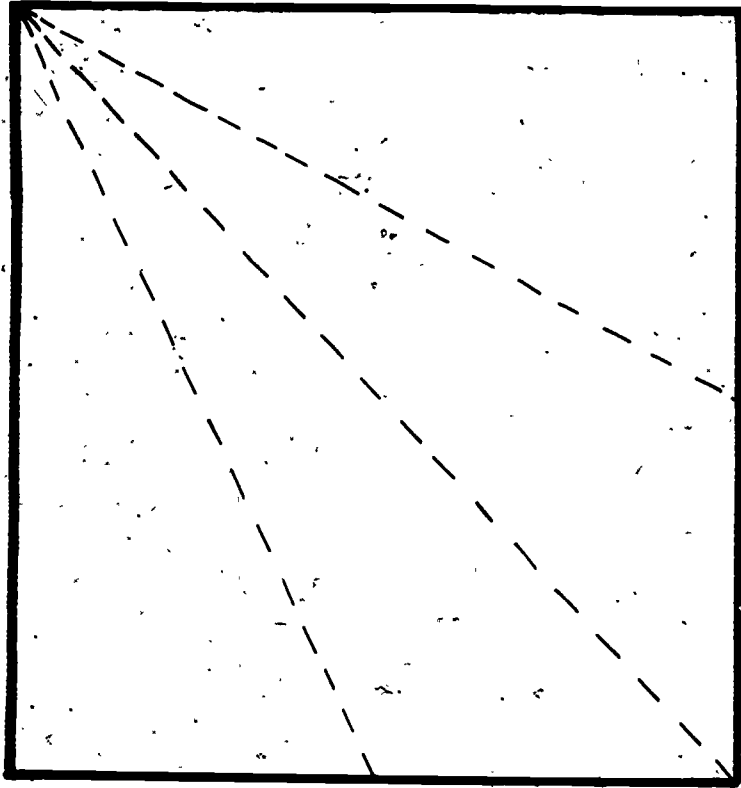
6. Loop around and paste ends together.
7. Attach a thread to top of design and hang from a string running from one end of the classroom to the other.



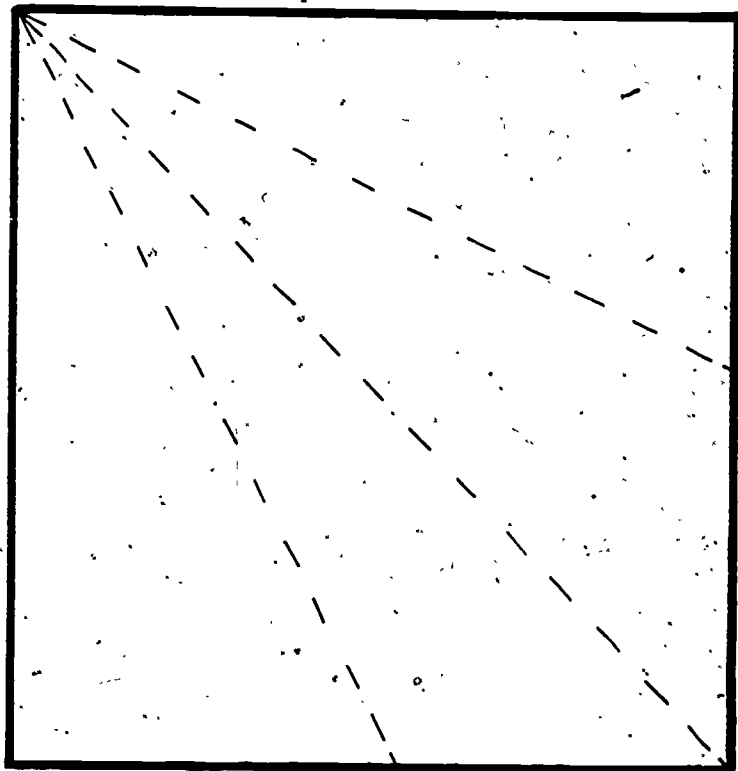
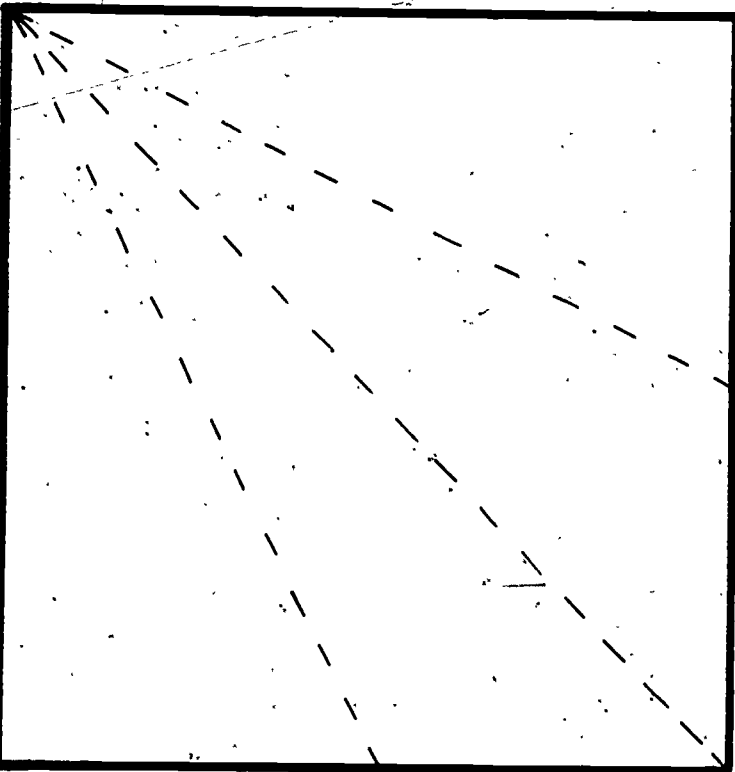
Activity D: Stars

1. Cut out squares on heavy lines.
2. Fold on dotted lines as indicated. Fold forward on a and fold back on b.
3. Paste corner flaps of 2 squares together as shown in illustration.
4. Continue pasting one square to the next; the last square is to be pasted to the first one.



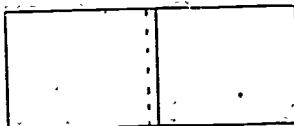


Two pages of these patterns for Activity D are provided for each child in the student manual.

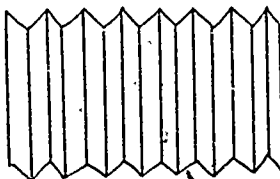


Activity E: Lanterns

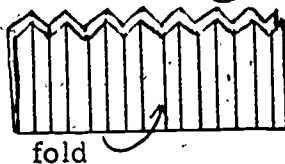
1. Paste 2 sheets of onionskin together at the narrow end to make a strip about $8\frac{1}{2} \times 22$.



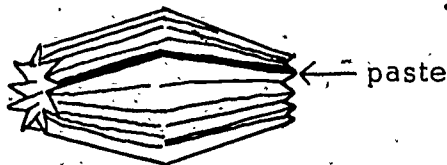
2. Pleat the strip of onion skin across the narrower dimension. Allow $\frac{1}{2}$ " to 1" for each pleat. Roll pencil over pleated paper to insure deeply creased pleats.



3. Unfold the pleated paper. Fold across the length and crease again.



4. Unfold creased paper. Bring the narrower ends together and paste to form a tube. Onion skin paper holds the crease so well that the edges will pleat together and the middle will bulge to make a lantern shape.

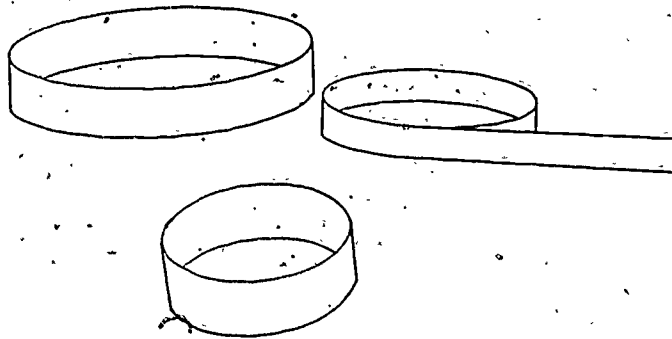


5. Cut a strip of paper and attach to top of lantern for the handle.

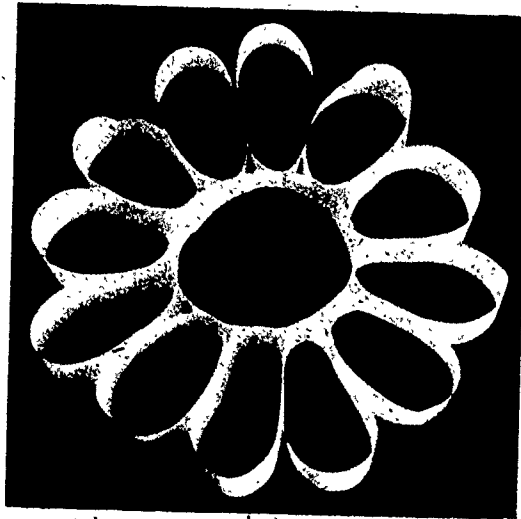


Activity F: Loop-de-doos

1. Cut strips of colored construction paper into $\frac{1}{2}$ " , $\frac{3}{4}$ " and 1" widths.
2. Make loops of different sizes.

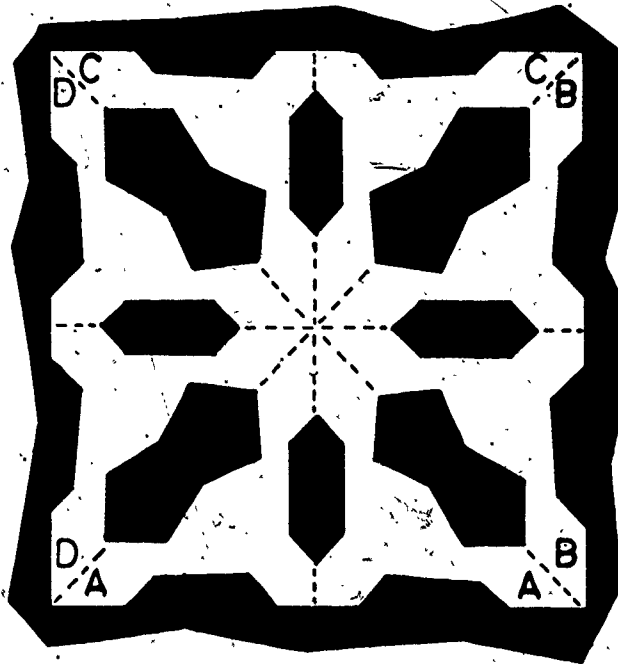


3. Arrange loops in rotationally symmetric patterns on construction paper, using all-purpose glue.

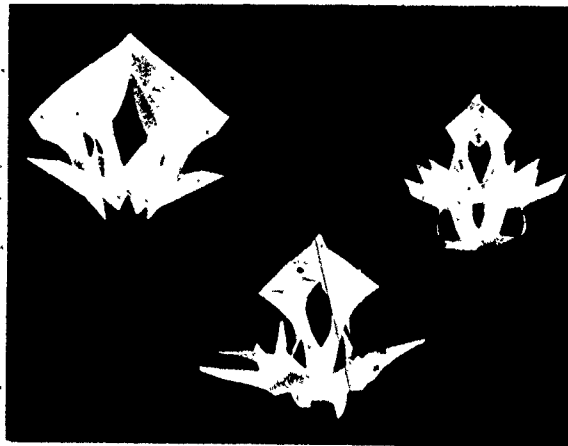


Activity G: Party Decorations

1. Cut out a four-sided paper design following the directions in Activity A.
2. Paste A to A, B to B, etc. This will bring the four corners into a cluster.

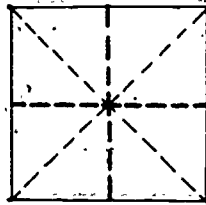


3. Insert thread to suspend.

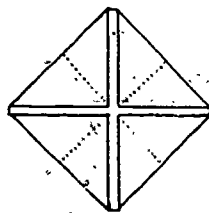


Activity H: 4-Pocket Baskets

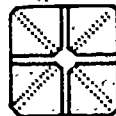
1. Begin with a 9" x 9" square sheet of construction paper.
2. Fold the paper along the diagonals and then open flat.
3. Then fold in half, both ways, and open flat.



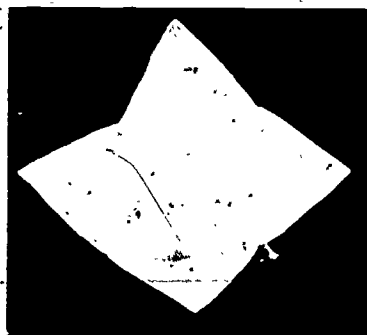
4. Fold corners to the center.



5. Turn folded square over. Working from the back, fold new corners to the center again.

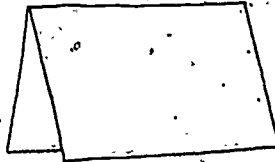


6. This forms pockets into which fingers of left hand can be slipped. With right hand, push from behind to make a peak in the center.
7. Paint a rotationally symmetric design on the flaps.



Activity I: Baskets

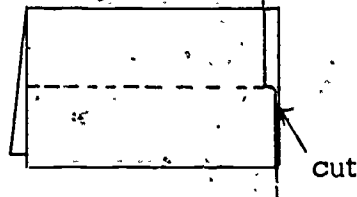
1. Fold a sheet of construction paper in half.



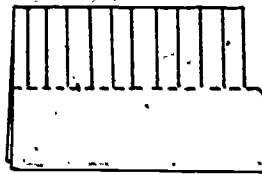
2. Fold the sheet in half again, thus making a crease.



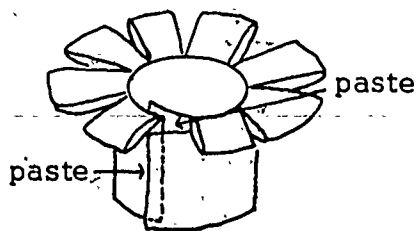
3. Open the second fold and make a tab by cutting from the center fold to the crease as shown in the diagram.



4. Cut from center fold into crease, making strips $1/2-3/4$ " wide.



5. Paste the tab end to the other end, making a cylinder.



6. Bend the petals down and open them.
7. Glue a colored cupcake paper to the inside of the basket.
8. Paste a paper handle to the inside of the basket.



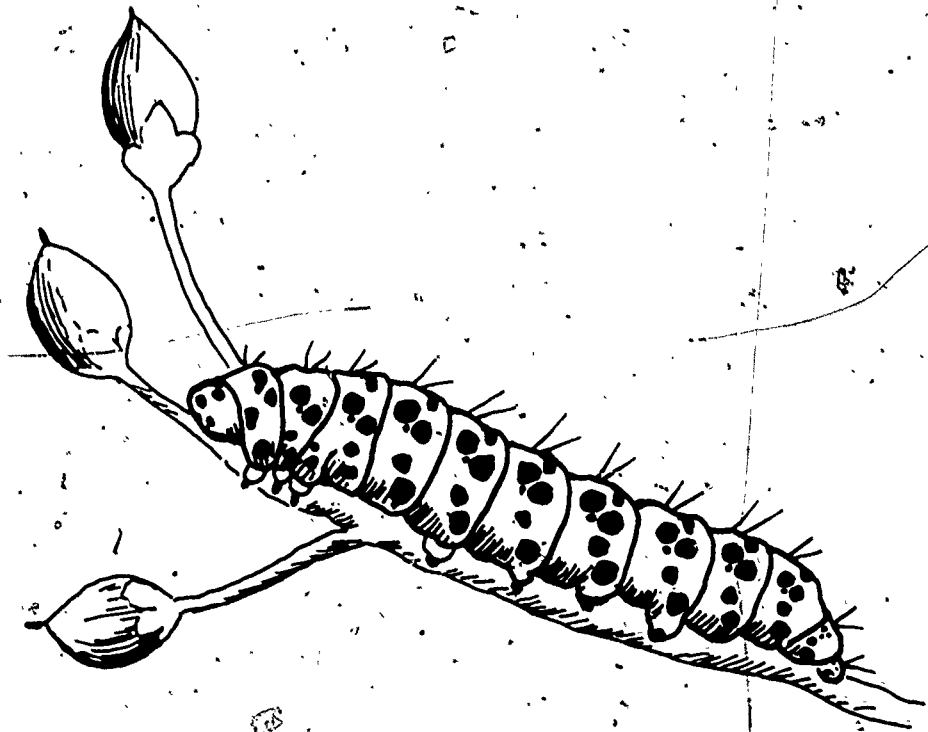
Variation

The children may wish to fringe the ends of the petals.

SECTION 2 REPEATING PATTERNS AND TRANSLATIONAL SYMMETRY

In this section the class studies repeating patterns. If the repeating patterns are imagined as infinitely long, they will be translationally symmetric. The concept of translational symmetry, which is discussed in the introductory part of the unit, is treated in Lesson 8. However, it is designated as optional, for it may be too abstract for some first-grade children.

The activities in the section provide for the creation of many repeating designs by the children. Whenever possible, the lessons are based on real objects and concrete operations. Both man-made exact repeating patterns and approximate repeating patterns from nature are used as illustrations for lesson ideas.



Cucullia verbasci Caterpillar

Lesson 5: TRANSPARENT REPEATING PATTERNS

This lesson introduces the idea that a repeating pattern is a pattern that can be generated by one part of the pattern being repeated at equal distances along a straight line.

MATERIALS

- one strip transparency of dogs (blue)
- one dog transparency (yellow)
- sailboat transparency (Pattern B) for each child
- Worksheet 4 for each child
- overhead projector (desirable, but not required)

PROCEDURE

Activity A

Again it is assumed that you will have an overhead projector available. Although it is more effective to project the transparencies, they may be viewed against a window.

Project the transparent yellow dog pattern and the blue dog strip pattern separately. Next project the transparencies together with the single figure on top. Bring the single dog into coincidence with the first dog in the strip. The first square will now appear green and the dog figures will fit exactly. Continue moving the single figure until each figure in the strip has been exactly covered. This will demonstrate that the dog figures in the strip are all identical.

ALL THE PARTS OF THIS DESIGN ARE EXACTLY THE SAME.
ALL THE PARTS ARE EXACTLY THE SAME DISTANCE APART.
THE DESIGN IS A REPEATING PATTERN.

Activity B

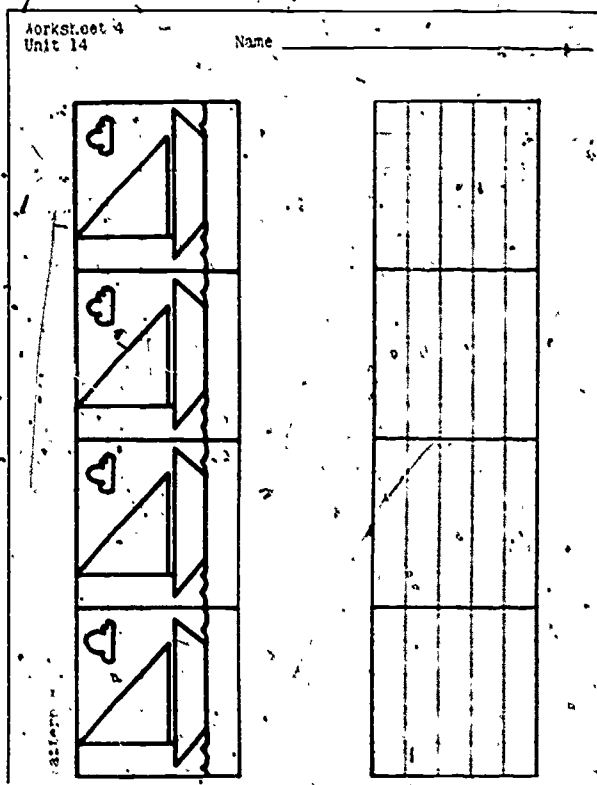
Have the children examine the sailboat pattern on Worksheet 4.

ARE ALL THE SAILBOATS EXACTLY ALIKE? (Yes.)

HOW CAN YOU TELL FOR SURE? (By using a copy of one of the sailboats for comparison.)

When the children understand the meaning of the answer to the last question (although these exact words certainly need not be used), tell the children that you have a copy of the sailboat pattern for them.

Each child should have a transparent copy of the sailboat (Pattern B). Give them time to experiment with the transparent pattern and Worksheet 4. Let them find that all the sailboat designs are identical. Then ask questions to lead to the idea that all the points on the first sailboat design are moved the same distance to match the corresponding points on the second design, all the points on the second design are moved the same distance to match the corresponding points on the third design, etc. Some appropriate questions follow:



HOW MANY SQUARES IS IT FROM THE TOP OF THE FIRST SAIL TO THE TOP OF THE SECOND SAIL? (Five squares.)

HOW MANY SQUARES IS IT FROM THE TOP OF THE SECOND SAIL TO THE TOP OF THE THIRD SAIL? (Five squares.)

HOW MANY SQUARES IS IT FROM THE TOP OF THE THIRD SAIL TO THE TOP OF THE FOURTH SAIL? (Five squares.)

HOW FAR IS IT FROM THE FRONT OF THE FIRST BOAT TO THE FRONT OF THE SECOND BOAT? (Five squares.)

After this lesson, any of the art activities for this section may be used.

Note: Worksheet 4 will be used again in Lesson 7.

Lesson 6: REPEATING PATTERN GAMES

In the games and activities suggested here, the children create designs with various materials. The only restriction is that all the designs should show a repeating pattern. Some designs will be altered according to rules that preserve the repeating property of the design. This changing of one design to another in a definite way is a forerunner of the mathematical idea of transformation.

MATERIALS

Possible materials from which repeating patterns may be made are:

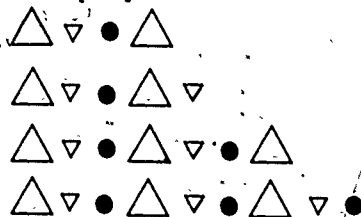
- colored chalk
- flannel board with several sets of shapes
- property blocks
- pegboards with pegs

PROCEDURE

Activity A

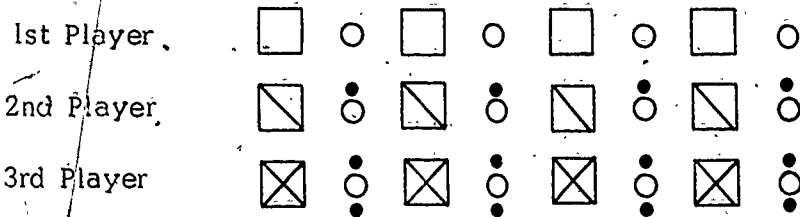
The rules for three games are outlined here. They may be varied in many ways. The designs may be drawn or assembled from objects similar to those in the materials list. The games may be played by the class as a whole, by teams, or by partners. The children may invent scoring rules.

Go on with the Pattern: One player begins a pattern. Successive players add to it, producing a repeated pattern. A rule can limit the number of elements each player adds; e.g., the starting player can use four elements in his part and each successive player can add one or two elements. An example of a sequence of play is:

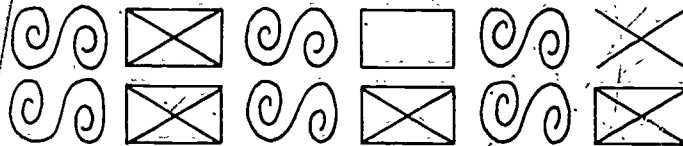


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Add to the Pattern: One player creates a repeating pattern. The next player adds to the pattern being sure to maintain the repeating property. An example of a sequence of play is:



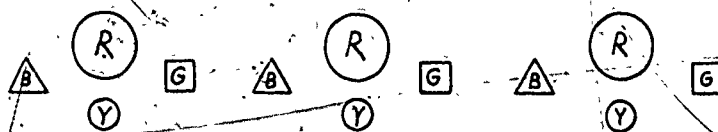
Make It Repeat: One player (either the teacher or a skilled child) makes a pattern that isn't exactly repeating. Another player completes it so that each portion of the design is repeated. An example of a sequence of play is:



Activity B

Here the children transform one repeating pattern into another by a specific procedure. The activity provides background for the mathematical concept of transformation or mapping, which will be developed in later grades,

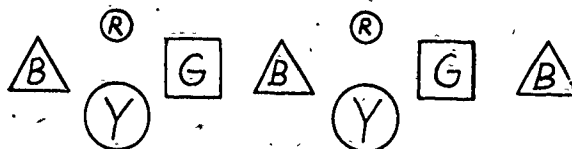
Suppose a group of children has made the following pattern from property blocks.



(The letters on the blocks indicate the color. Coloring the drawings may make the pattern clearer.)

LET'S CHANGE THIS PATTERN BY FOLLOWING A RULE. THE RULE WILL BE TO CHANGE ALL THE BIG BLOCKS TO SMALL BLOCKS AND THE SMALL BLOCKS TO BIG BLOCKS. (Large \leftrightarrow Small.)

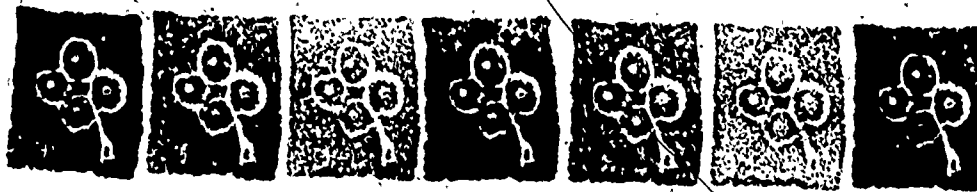
After the change, the pattern will look like this:



IS THE NEW DESIGN A REPEATING PATTERN? (Yes.)

NOW YOU MAKE UP A RULE AND CHANGE THIS PATTERN.

This activity may also be varied. It is good to leave the original design undisturbed and construct the transformed design below it. The rules used may be very simple (such as thick \leftrightarrow thin) or quite complicated ($\bigcirc \leftrightarrow \square$ and red \rightarrow yellow \rightarrow blue \rightarrow green \rightarrow red). Although property blocks are especially suited for this work, other materials may be used for the designs. In all cases the transformed design will retain the repeating property.



Lesson 7: REPEATING PATTERNS ON GRIDS

The worksheets that constitute this lesson give the children practice in drawing repeating patterns. These design activities are considered less as creative art than as exercises in translation of sets of points on grids.

MATERIALS

— Worksheets 4, 5, and 6

PROCEDURE

Ask the class to complete the pictures on Worksheet 5 so that the picture in the first cell is exactly repeated. Questions such as those below may help the children see that corresponding distances remain the same throughout an entire repeating pattern.

HOW MANY SQUARES HIGH IS THE HOUSE TO THE TOP OF THE ROOF? (Six squares.)

THE SECOND HOUSE? (Six squares.)

THE THIRD HOUSE? (Six squares.)

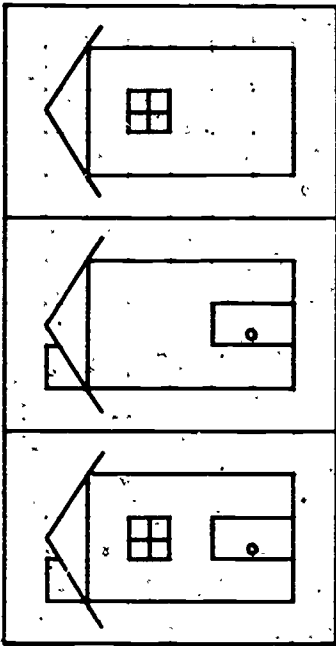
HOW FAR IS IT BETWEEN THE DOORKNOB ON THE FIRST HOUSE AND THE DOORKNOB ON THE SECOND HOUSE? (Five squares.)

BETWEEN THE DOORKNOBS ON THE SECOND AND THIRD HOUSES? (Five squares.)

Have the children draw their own designs on the blank strip on Worksheet 4, which was first used in Lesson 5. By watching them work you will see which children need help in drawing and spacing.

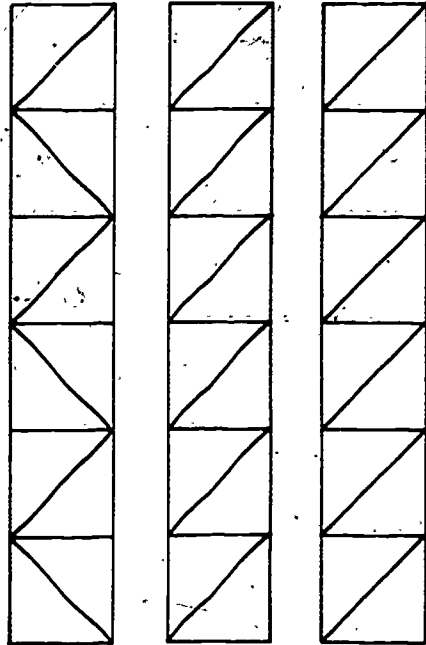
Ask the children to color the strip patterns on Worksheet 6 so that each strip has a repeating pattern.

Name _____



Make each house just like the first one.

Name _____



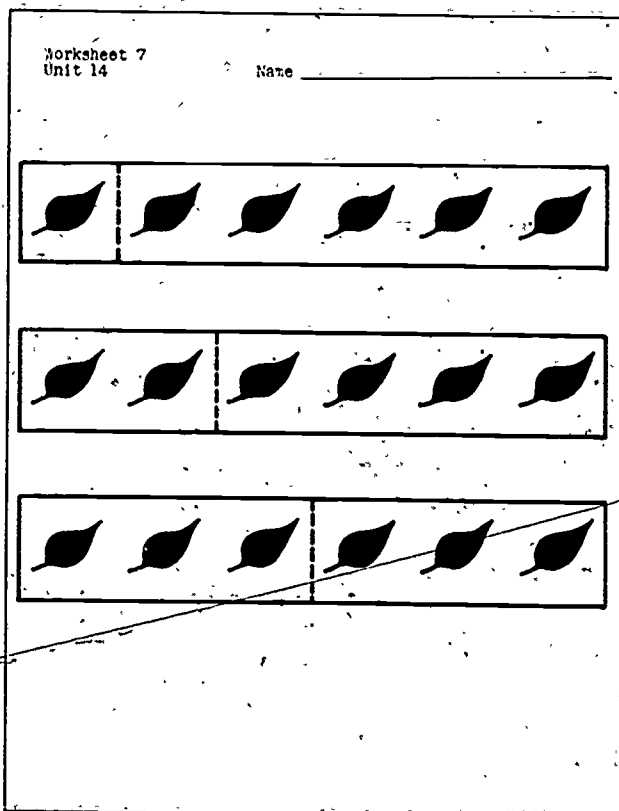
Color each strip. Make each pattern repeat.

Lesson 8: TRANSLATION

The first activity of this lesson shows that a strip pattern can be generated by translating a part of the pattern in uniform steps along a straight line. In the optional second activity the concept of translational symmetry is introduced.

MATERIALS

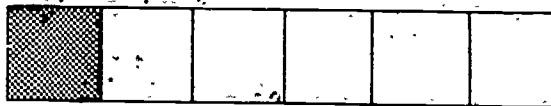
- Worksheet 7



PROCEDURE

Activity A

Worksheet 7 has three identical strip patterns, each having an elemental design repeated six times. Ask the children to color lightly the first square in the first strip.



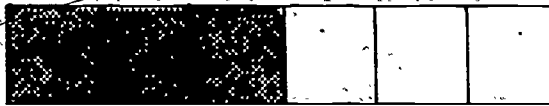
Bring out the idea that the pattern of the whole strip can be made by repeating the design on the colored square six times.

Next have the children color the first two squares in the second strip.



This time they should note that the pattern of the whole strip can be made by repeating the colored design three times.

Finally have the first three squares of the third strip colored.



This should help show that the pattern of the whole strip can be made by repeating the colored design two times.

WE CAN CHOOSE DIFFERENT PARTS OF THE PATTERN TO REPEAT OVER AND OVER TO GET THE WHOLE PATTERN, BUT USUALLY WE CHOOSE THE SMALLEST POSSIBLE PART AND TALK ABOUT REPEATING IT.

Activity B (Optional)

If you feel your class is able to comprehend the following material, use it. It is the introduction to translational symmetry and its connection with repeating patterns.

Place a simple repeating pattern on the chalkboard. Ask for successive volunteers to extend the pattern across the entire board. A possible design would be:

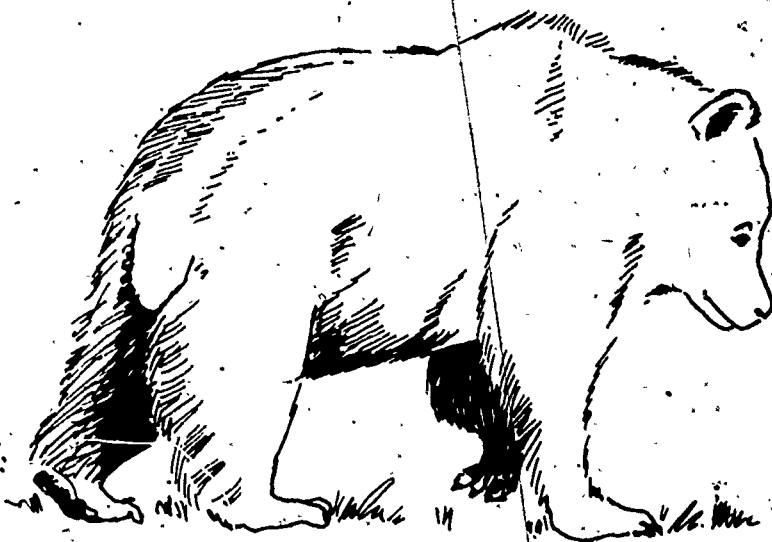


IF THE CHALKBOARD WENT STRAIGHT ON AND ON, COULD WE MAKE THE PATTERN AS LONG AS WE WISHED? (Yes.)

PRETEND THE PATTERN GOES ON FOREVER IN BOTH DIRECTIONS. THEN WE WOULD SAY IT IS INFINITELY LONG. PRETEND YOU PICK UP THE WHOLE PATTERN, MOVE IT ONE DESIGN OVER, AND PUT IT DOWN. WILL THE PATTERN NOW LOOK ANY DIFFERENT? (No.)

BECAUSE THE PATTERN LOOKS THE SAME AFTER IT IS MOVED ALONG A LINE, IT HAS A SPECIAL KIND OF SYMMETRY CALLED TRANSLATIONAL SYMMETRY. "TRANSLATE" IN THIS CASE SIMPLY MEANS TO MOVE FROM ONE PLACE ON THE LINE TO ANOTHER.

A discussion can bring out the idea that if a repeating pattern is imagined to be infinitely long, it can be thought of as having translational symmetry.



Lesson 9: FINDING REPEATING PATTERNS

Here the children look for repeating patterns in a specially prepared classroom. They have opportunities to discuss approximate, finite, and infinite repeating patterns.

MATERIALS

- an assortment of objects, pictures, and designs exhibiting repeating patterns. Some examples are a caterpillar, a compound leaf, animal tracks, a fence, a chain (metal or paper), a piano keyboard, a strip of lace, a dress border, a necklace of colored beads, a wallpaper border, and a set of books.
- an assortment of objects, pictures, and designs that have several parts that do not repeat exactly. Some examples are a paper chain with randomly colored links, a set of non-matching books, and a strip design you have made for this lesson.

PROCEDURE

Place on the shape table, bulletin board, and around the room some objects, pictures, and designs that exhibit repeating patterns and some that do not, as suggested in the materials list. Any completed art projects for this section can also be displayed.

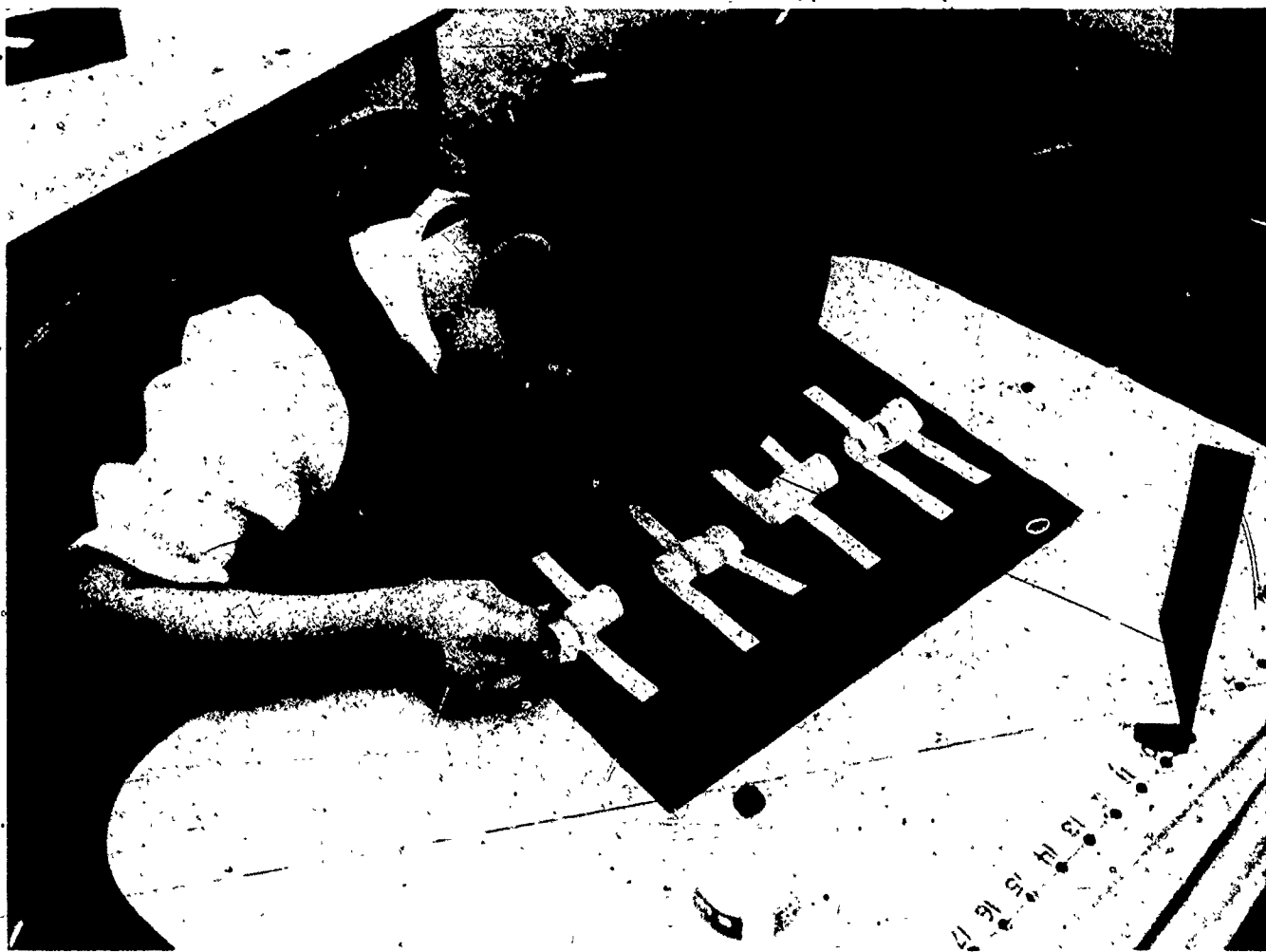
Ask the children to find examples of repeating patterns in the room. A child who finds one should see that one part of the pattern is repeated at equal intervals along a straight line. You might help him express his ideas by asking questions similar to "Are all the flowers in this border alike?" and "How many beads are between each red bead and the next red bead?"

In many natural objects only part of the pattern repeats. For example, the ends of a caterpillar must be excluded if it is to be considered to have a repeating pattern. The children may be interested in patterns that repeat only approximately.

If you discussed infinite patterns and translational symmetry in Lesson 7, you may wish to consider the concepts again here.

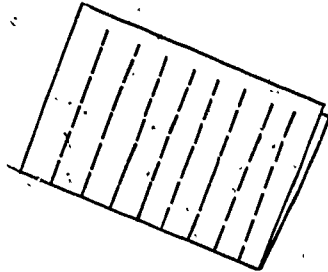
ART ACTIVITIES FOR SECTION 2: REPEATED PATTERNS

The ideas of repeated pattern are used in the art activities described here. Any of the activities may be used any time after Lesson 5 has been presented.

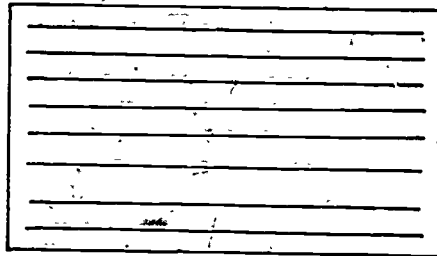


Activity A: Place Mats

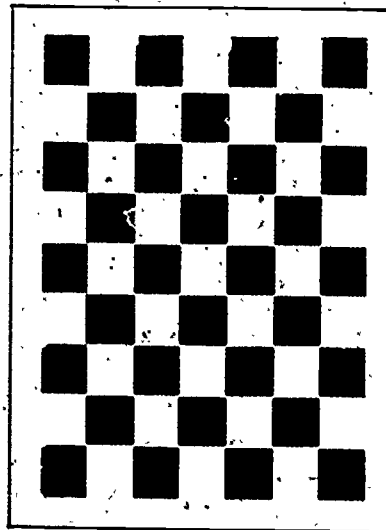
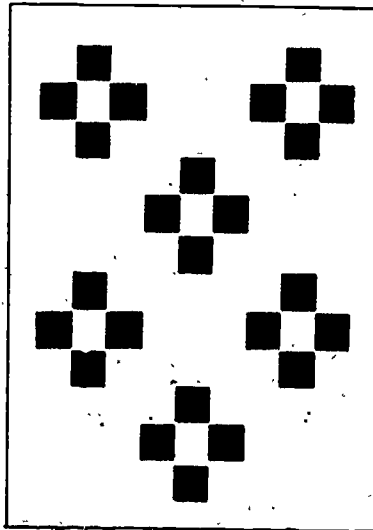
1. Fold a sheet of 12" x 18" construction paper in half. Cut from folded edge to 1" from outer edge.



2. Unfold. You will have a sheet with parallel slits that do not touch the edge of the paper.

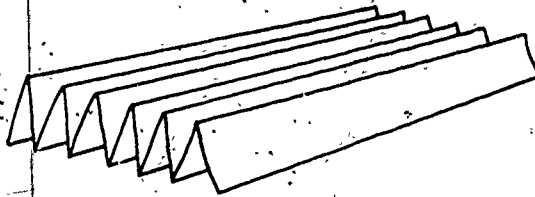


3. Cut construction paper of another color into 1" strips. Weave these through the slit sheet you have prepared.

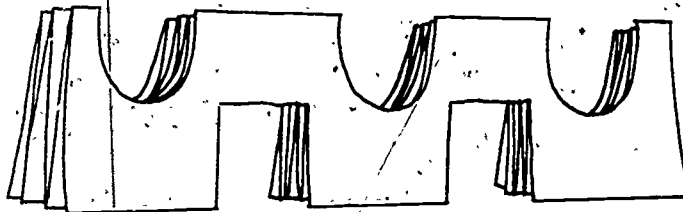


Activity B: Cut-Paper Designs

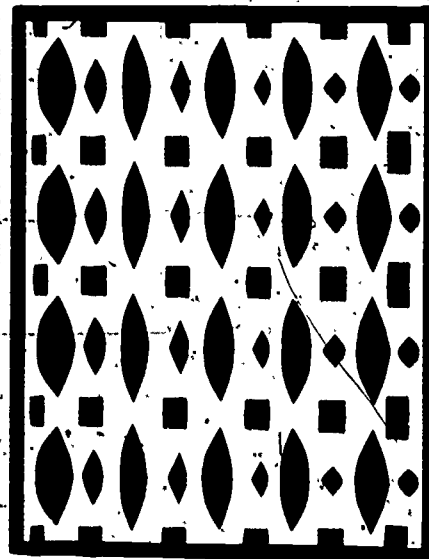
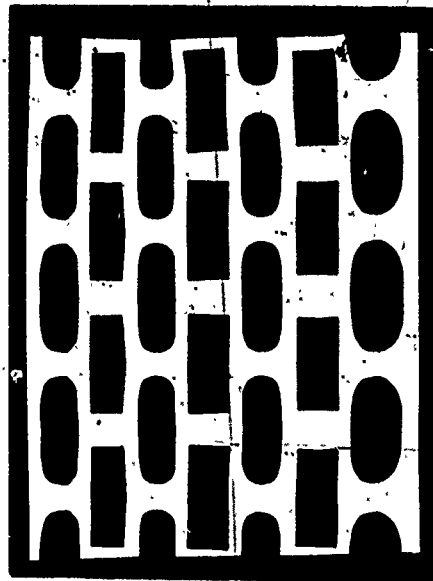
1. Fold paper into 2" pleats.



2. Cut notches of different shapes in the folded edges.



3. Unfold and paste on colored construction paper.



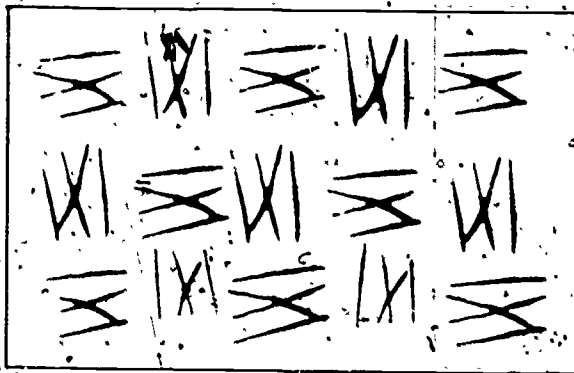
Activity C: Print-Making

Any of the methods described below may be used for printing repeated patterns. Tempera may be brushed on to the printing surface, or water-base printer's ink may be rolled on with a brayer. If you use a brayer, squeeze a 3-inch ribbon of paint on a piece of glass and roll the brayer back and forth over it until it sounds sticky. Then roll the brayer over the printing surface.

Newsprint or construction paper are best for printing, since they are more absorbent than other paper.

String Prints

Wind a string around a piece of wood. Ink one side. Press inked string on newsprint to make a repeated pattern.



Sponge Prints

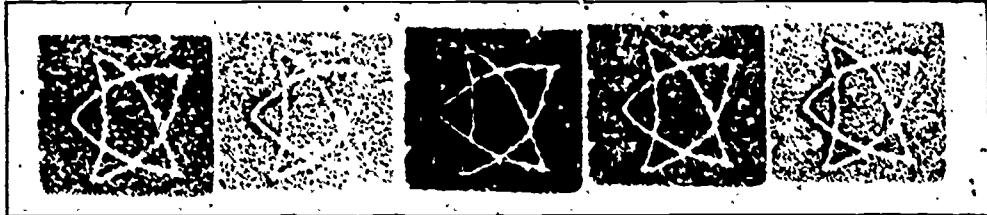
Ink a piece of sponge. Press the sponge on newsprint repeatedly.



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Soap-Eraser Prints

Cut a design into one surface of a soap-eraser, ink it, and print.



Gadget Prints

Ink kitchen gadgets such as a potato masher, fork, bottle cap, to make prints.

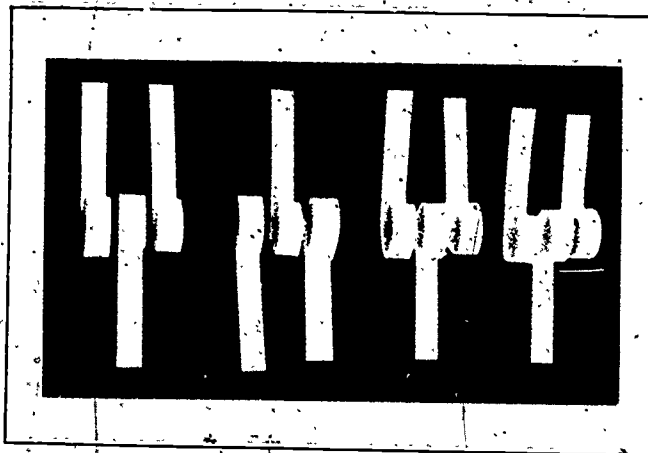
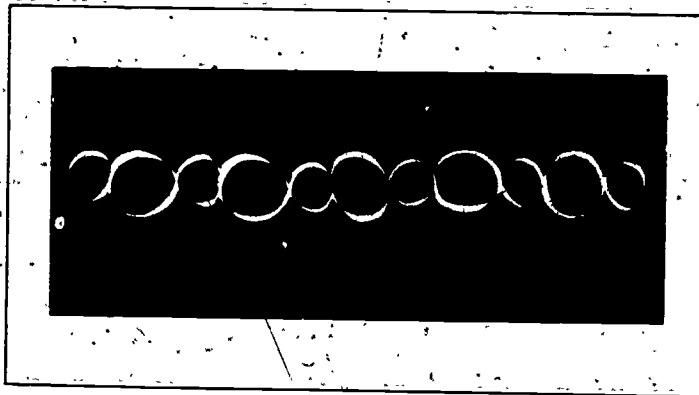


Potato Prints

Cut a potato in half. Have the children scratch a design on the cut surface with a pointed pencil or an orangewood stick. Water color or tempera should be applied to the potato surface with a brush, rather lightly, because the potato provides a good deal of moisture for the printing. Press the potato firmly on the paper.

Activity D: Loop-de-doos

Make loop-de-doos of various sizes (see p.46) and glue them on construction paper in a repeating pattern.

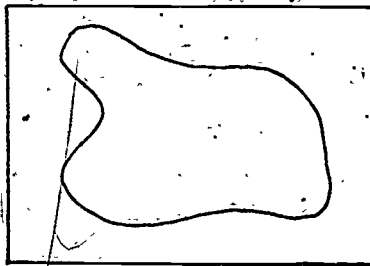


Activity E: Seed Designs

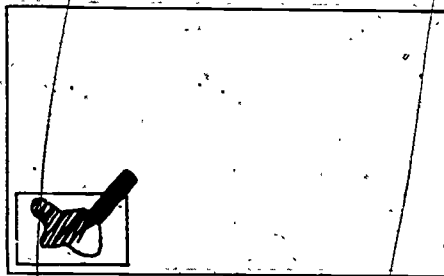
Use a white, all-purpose adhesive such as Elmer's glue to paste popcorn kernels, pumpkin or sunflower seeds, or split peas on tagboard to make repeating patterns. These may be strip patterns or, for the more ambitious child, may cover the whole piece of tagboard.

Activity F: Patterns Using Stencils

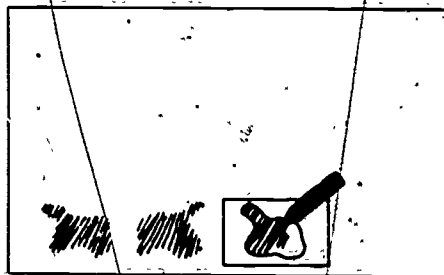
1. Cut out a design from the center of a 3" x 5" white index card. This design should not be symmetric. An unusual hole or odd shape will do. (If a child cuts through the boundaries of the card, place a piece of tape over cut.)



2. Place card on construction paper. Use it as a stencil, rubbing crayon over design to produce image.



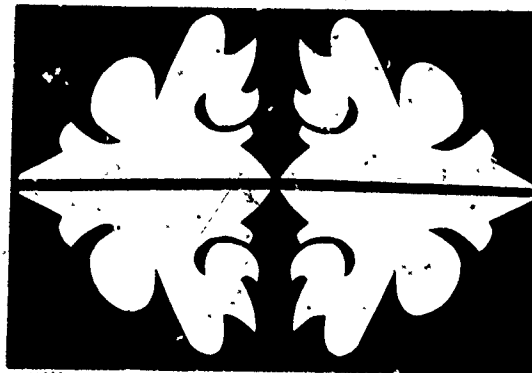
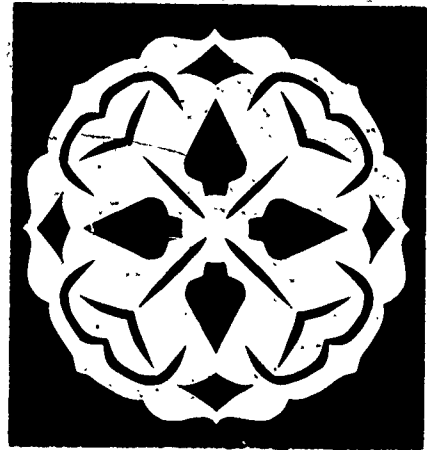
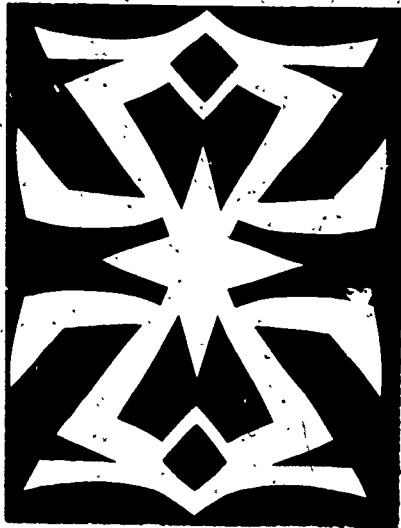
3. Show the children how to flip the stencil and repeat the rubbing. They can make a repeated pattern by flipping the pattern over and over in a straight line.



SECTION 3 BILATERAL SYMMETRY

The lessons in this section of Unit 14 develop the concept that bilaterally symmetric patterns are those that are unchanged by reflection about an axis or line of symmetry. It is shown that a bilaterally symmetric pattern is made of pairs of corresponding points which are equidistant from the line of symmetry. Bilateral symmetry is sometimes called folding or mirror symmetry because two tests for it use folding and mirror images.

As in all parts of the unit, the children's understanding of the concepts will come in large part from observation and manipulation of objects and from art activities. There will be some patterns and objects in this section that possess both bilateral and rotational symmetry. You may or may not choose to discuss this with your class. Always be clear about the type of symmetry you are considering at the time.



Lesson 10: INTRODUCING BILATERAL SYMMETRY

This first lesson of Section 3 introduces the concept of bilateral symmetry. It also demonstrates the fact that half of a bilaterally symmetric pattern may be reflected to generate the entire pattern.

MATERIALS

- transparent yellow butterfly
- transparent blue half-butterfly
- overhead projector (desirable, but not required)
- transparent Pattern C for each child

PROCEDURE

Activity A

As in earlier lessons where transparent figures are used, it is possible to view the transparencies against a window if an overhead projector is not available.

Project the yellow butterfly.

DO THE TWO SIDES OF THE BUTTERFLY LOOK ALIKE
IN SOME WAYS? (Yes.)

Any reasonable response is acceptable, e.g., the two wings are the same size.

ARE THE TWO SIDES OF THE BUTTERFLY EXACTLY ALIKE?

There may be either yes or no answers from the children. Have them wait until after the next part of the demonstration to decide which answer is correct.

Project the blue half-butterfly beside the whole butterfly. Suggest that it can be used to compare the two halves of the yellow butterfly. Move the blue figure until it coincides with one side of the whole butterfly. That side will now look green.

IS THE BLUE HALF-BUTTERFLY AN EXACT COPY OF ONE-HALF OF THE YELLOW BUTTERFLY? (Yes.)

HOW CAN YOU TELL? (The edges and spots match, half of the butterfly is green now, or similar observations.)

Slide the blue half-figure to the other side of the whole figure.

DOES THE BLUE HALF-BUTTERFLY EXACTLY FIT ON THIS HALF OF THE YELLOW BUTTERFLY? (No.)

HOW CAN I MAKE IT FIT? (Turn it over.)

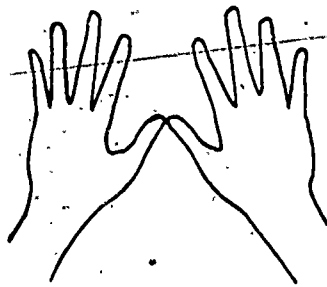
Turn the half-figure over, making the motion clear to the class.

DOES THE BLUE HALF-BUTTERFLY EXACTLY FIT OVER THIS HALF OF THE YELLOW BUTTERFLY NOW? (Yes.)

Explain that this demonstration shows that the two halves of the yellow butterfly would fit exactly if one of them were folded over the other.

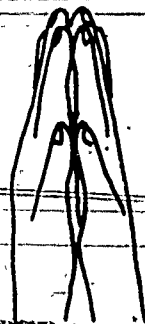
Activity B

Have each child place his hands on his desk with the palms down and the thumbs just touching.



ARE YOUR HANDS EXACTLY ALIKE? (No, the fingers are in a different order on the two hands.)

Ask each child to fold his palms together.



DO YOUR HANDS FIT TOGETHER EXACTLY? (Yes.)

Explain that the way the fingers match when the hands are put together shows that the parts of the hands are alike but are arranged in opposite order. Tell the class that both the butterfly and their hands have a special kind of symmetry called bilateral symmetry.

Activity C

Have each child cut his Pattern C along the center line. He can then experiment with the two parts as he wishes.



Lesson 11: FOLDING PATTERNS

This lesson continues the development of the folding test for bilateral symmetry. The investigations of the properties of bilaterally symmetric patterns are continued.

MATERIALS

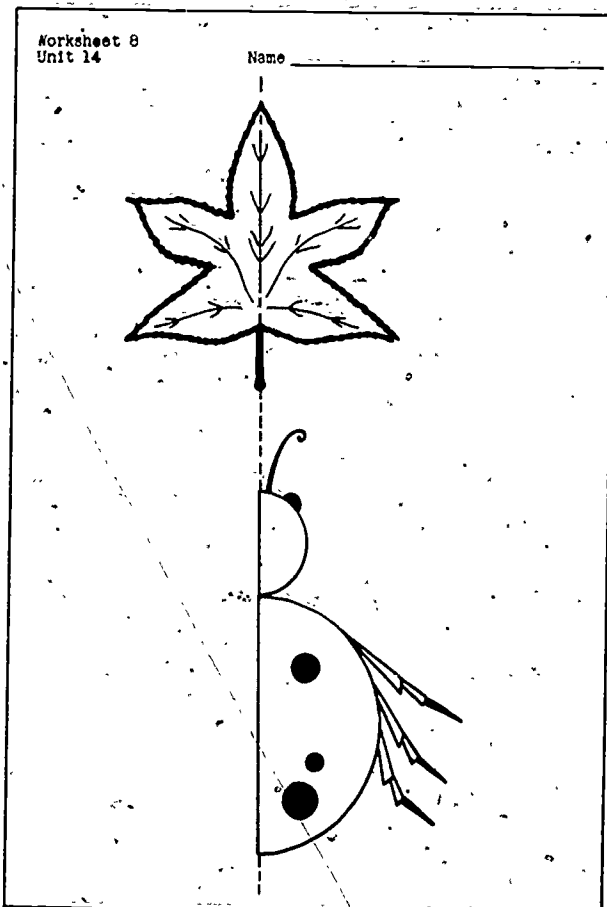
— Worksheets 8, 9, 10 and 11

PROCEDURE

Activity A

Have each child fold his copy of Worksheet 8 along the dotted line. Because this fold must be made rather accurately, you may need to help some children. After the class has had time to see the way the two sides of the leaf pattern fit exactly when folded, you may tell them that this is true of all bilaterally symmetric patterns. Thus a folding test can show whether or not a pattern has bilateral symmetry.

Activity B



Suggest that each child fold Worksheet 8 (which is printed on tracing paper) along the line of symmetry of the bug, trace the half bug, and open the paper.

WHAT DO YOU SEE? (A whole bug.)

IS IT BILATERALLY SYMMETRIC? (Yes.)

HOW DO YOU KNOW? (By the folding test.)

Note: Worksheet 9 is tracing paper without a printed pattern.

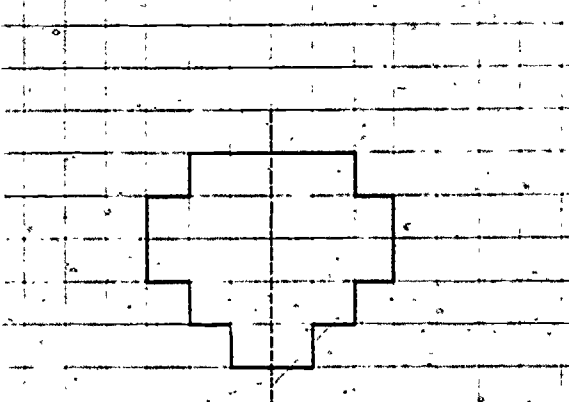
Discuss with the class the following ideas: The dotted line is the line of symmetry of these patterns. For every point on one side of the line of symmetry there is a matching point on the other side. There is a kind of balance of the parts on each side. Each child may make any bilaterally symmetric design he wishes on Worksheet 9 using the fold-and-trace technique. If some child does not spontaneously mention that the patterns look the same from either side of the paper, suggest that they look from both sides and describe what they see.

Activity C

Worksheets 10 and 11 should now be completed. The worksheets are designed to reinforce the concept that all points of the pattern occur in pairs of points that are equidistant from the line of symmetry. Again, the children should not be expected to verbalize this concept but simply to use it.

Worksheet 10
Unit 14

Name _____

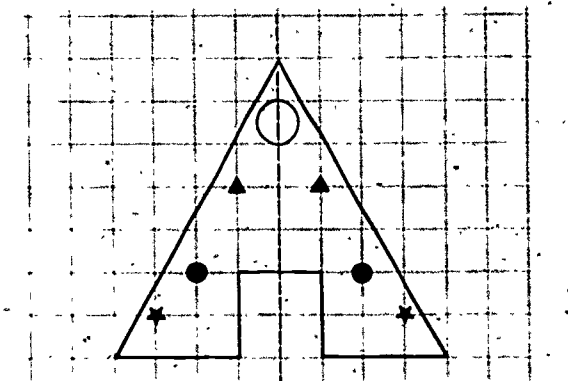


The area of the left side is _____ square units.

The area of the right side is _____ square units.

Worksheet 11
Unit 14

Name _____



The left ★ is _____ units from the line of symmetry.

The right ★ is _____ units from the line of symmetry.

The left ▲ is _____ units from the line of symmetry.

The right ▲ is _____ units from the line of symmetry.

The left ● is _____ units from the line of symmetry.

The right ● is _____ units from the line of symmetry.

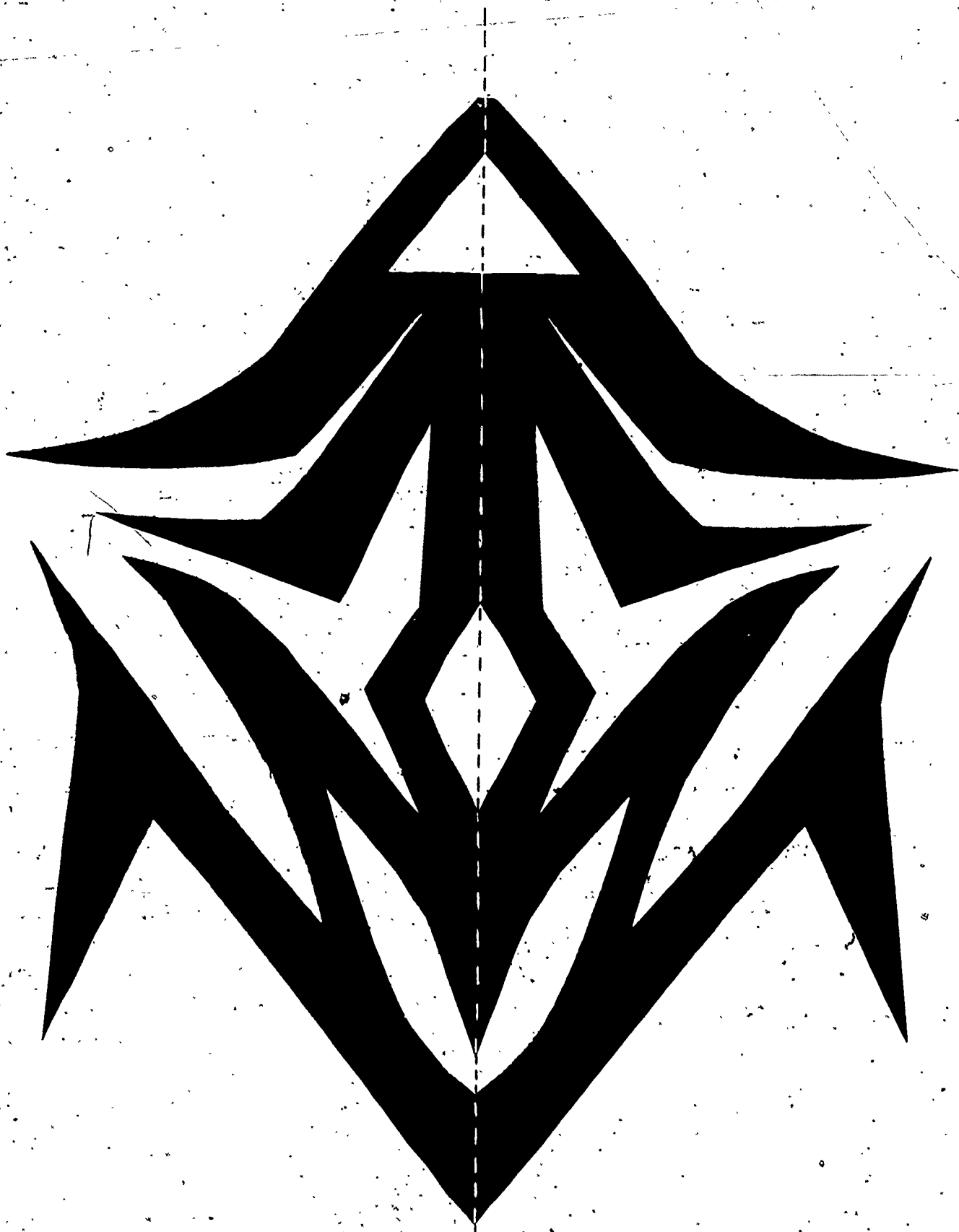


FIGURE A

Lesson 12: MIRROR TEST

In this lesson two-dimensional patterns and three-dimensional objects are tested for bilateral symmetry with mirrors. The lesson probably will take two class periods.

MATERIALS.

- Worksheets 12 and 13
- small mirror for each child
- larger mirror (if necessary, this may be made from two small mirrors)
- cards printed with numerals 0 through 9 and the capital letters (optional)
- assorted small solid objects to be tested with mirrors
- Mirror Cards, Elementary Science Study, 108 Water Street Watertown, Massachusetts 02172 (optional)

PROCEDURE

Activity A

Place Figure A so that the class can see it. Have a child hold the larger mirror on the line of symmetry of the figure and tell what he sees as he looks into the mirror. Either he or a succeeding volunteer should realize that he sees the same pattern with or without the mirror. This is another test to show that a pattern has bilateral symmetry. This demonstration is intended only to show the general procedure. The children will learn much more as they use their own mirrors.

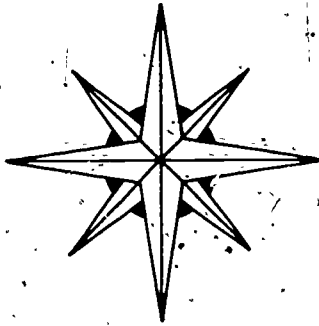
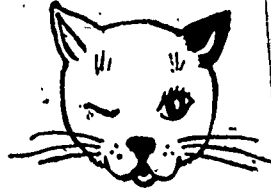
Activity B

Give each child a small mirror and Worksheets 12 and 13. You may also wish to provide other patterns such as sets of numeral and letter cards. It is important that each child be allowed to make his own unhurried investigations of the patterns. However, a brief reminder of the mirror test is in order. If there exists a position for the mirror that will give a view of the pattern just as it was without the mirror, the pattern is bilaterally

Worksheet 12
Unit 14

Name _____

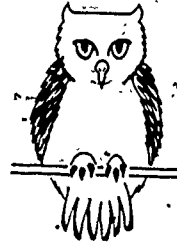
Look at these pictures with a mirror.
Are they bilaterally symmetric?



Worksheet 13
Unit 14

Name _____

Look at these pictures with a mirror.
Are they bilaterally symmetric?



symmetric. The line along which the mirror is placed in a successful test is a line of symmetry of the pattern. Some children will find designs that are symmetric with respect to more than one line.

Many children are fascinated by the Mirror Cards prepared by the Elementary Science Study. The cards make a good addition to this lesson.

Activity C

Show the class a two-dimensional symmetric pattern or picture. (Figure A on page 76 can be used.)

HOW CAN WE TEST THIS PATTERN TO SEE IF IT HAS BILATERAL SYMMETRY? HOW CAN WE TEST IT ANOTHER WAY? (With the mirror test or folding test.)

Demonstrate the two tests. An explanation of the tests is appropriate at this point, but ordinarily the children should not be expected to verbalize the explanation. You can present the substance of the following paragraph, which although not completely precise, should give the idea of bilateral symmetry to most of the children.

Every point in a bilaterally symmetric pattern has a partner on the other side. Each of the points in a pair is the same distance from the line of symmetry. This is the reason that the points of the two sides fit together when the pattern is folded and also the reason that one side of the pattern looks just like the other side reflected by a mirror.

Activity D

Show the class a bilaterally symmetric, three-dimensional object, such as a vase, a doll, or a plaque.

DO YOU THINK THIS IS BILATERALLY SYMMETRIC? CAN WE TEST IT WITH A MIRROR? (Yes.)

Make the test with the larger mirror.

CAN WE TEST IT BY FOLDING? (No, solid objects can not be folded.)

Provide a number of small solid objects for the children to view with the small mirrors. Possible objects could be plastic toy figures, blocks, combs, flowers, and parts of construction sets. They may also look at their fingers, hands, and feet.

Activity E (Optional)

Some unusually perceptive children may be interested in a discussion of the plane of symmetry of an object. Whereas a bilaterally symmetric two-dimensional pattern is made up of pairs of points that are equidistant from a line, a bilaterally symmetric three-dimensional pattern is made up of pairs of points that are equidistant from a plane. The plane of symmetry divides the object into two similar parts.

Lesson 13: KEEP THE SYMMETRY

The games suggested here provide further practice with bilaterally symmetric patterns as well as the fun of creating designs. A review of rotationally symmetric patterns and repeating patterns is included.

MATERIALS

- property blocks
- pegboards
- small objects such as paper clips, sticks, and counters
- colored chalk
- graph paper
- mirrors

PROCEDURE

The games (Activities) outlined below are similar to those in Lessons 4 and 6, and hence the children should need only a brief explanation of the rules before they play. They should be encouraged to experiment with modifications of the games.

The designs called for in the games may be made with property blocks or any small objects such as paper clips, sticks, or counters; they may be made with pegs on pegboards; or they may be drawn with chalk or crayon. The duration of a game may be limited by specifying a number of moves or a time interval for a game. All the games may be played now, or you may wish to use C and D after Lesson 14.

Activity A

What's Wrong? One child or team designs an imperfect bilaterally symmetric pattern. The other child or team alters the pattern to make it bilaterally symmetric.

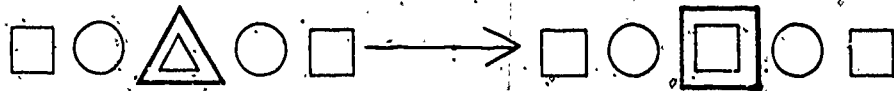
If children have difficulty in making the imperfect patterns, suggest that they secretly make a symmetric pattern and then

change one or two elements. An example of an imperfect pattern and the corresponding symmetric pattern is sketched below.



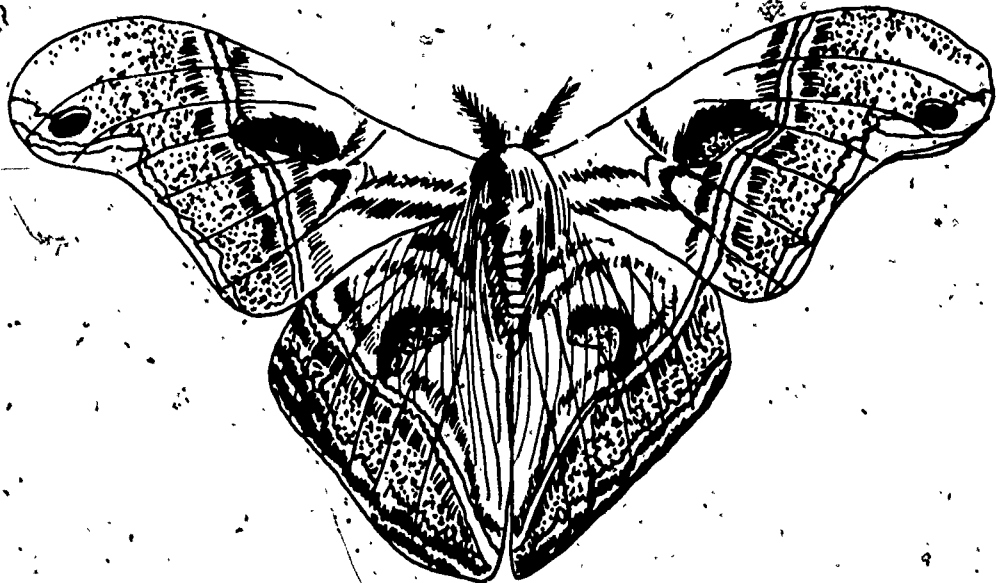
Activity B

Change It. One child or team makes a bilaterally symmetric design. He then specifies a rule by which the other child is to change the design. The purpose and procedure for this game is more fully explained in Lesson 6, Activity B. The sketch below shows a possible transformation. Rule: $\triangle \rightarrow \square$



Activity C

Match the Mirror Image. Two children take turns in drawing parts of a bilaterally symmetric pattern on a grid. After each addition the player checks the pattern with a mirror. If the pattern is correctly drawn, the mirror image of one side is identical with the other side.



Activity D

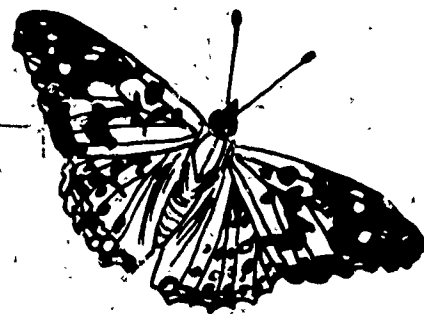
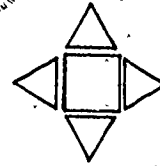
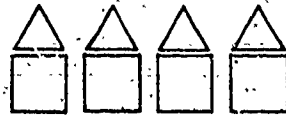
Keep the Symmetry. One child starts with an element of a design, e.g.,



He says, "This is part of a bilaterally symmetric design. Finish it and keep the symmetry." The second child completes the design, e.g.,



The first child may request a repeating pattern or a rotationally symmetric pattern instead of a bilaterally symmetric pattern. Then the completed patterns might be



Lesson 14: DRAWING PATTERNS

In this lesson the children complete, color, and draw bilaterally symmetric patterns on grids. These activities call for somewhat more precision than the art activities suggested for this section.

MATERIALS

→ Worksheets 14, 15, 16 and 17

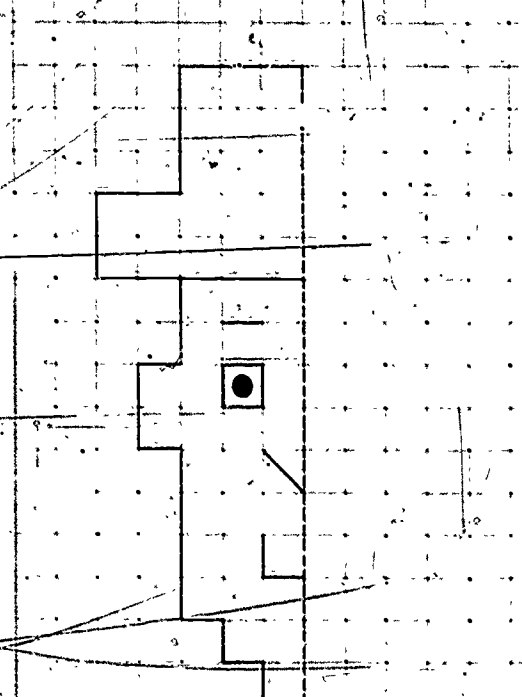
PROCEDURE

Worksheets 14 and 15 are self-explanatory. The child should discover that parts of the pattern can be located by counting squares. He may make any bilaterally symmetric design he wishes on Worksheet 16. Any kind of symmetric design may be made on Worksheet 17.

Worksheet 14
Unit 14

Name _____

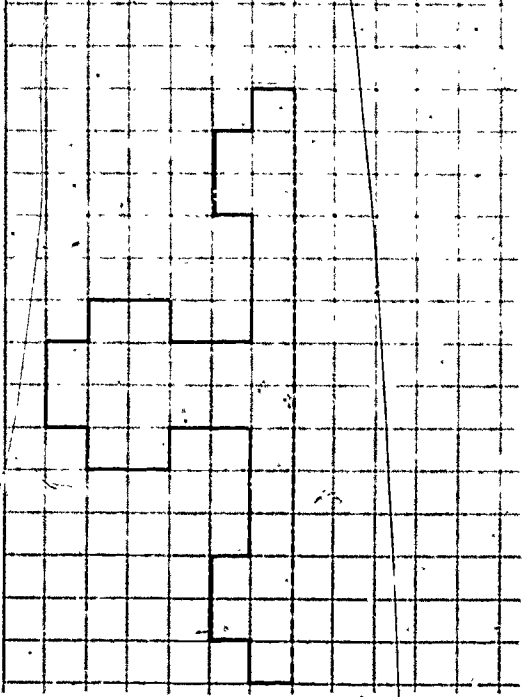
Finish the picture. Make it bilaterally symmetric.



Worksheet 15
Unit 14

Name _____

Finish the picture. Make it bilaterally symmetric.



Worksheets 16 and 17 are on graph paper.

Lesson 15. WHAT KIND OF SYMMETRY?

The children here identify and test the three types of symmetry considered in Sections 1, 2 and 3. This provides for both evaluation and further learning.

MATERIALS

- an assortment of objects, pictures, and designs to be tested for symmetries. Some of these should be similar to those suggested for Lessons 3 and 9. Examples of patterns either bilaterally symmetric or not quite bilaterally symmetric are a key, vase, picture of a face, toy plastic figure, insect, leaf, flower, design from a magazine, ruler, and fruit.
- examples of the children's art work from this unit

PROCEDURE

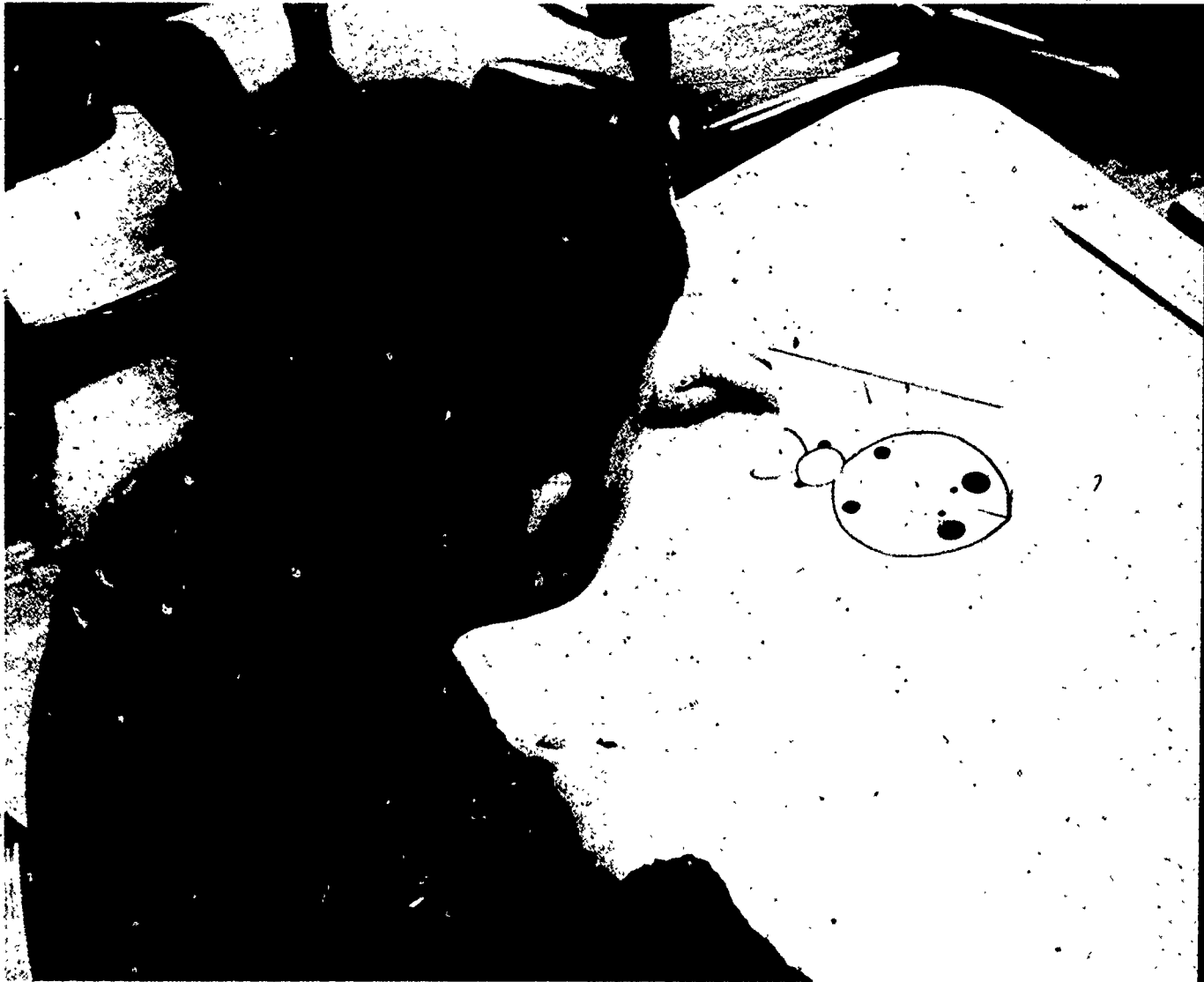
Have a selection of objects, pictures, and designs on the shape table. Examples of the children's art work from this unit should be included. The children may wish to add to the collection later.

Allow the children to investigate and tell about the symmetries of objects they choose from the collection. They should be able to apply tests to determine symmetries about which they are in doubt. Some patterns have more than one kind of symmetry, but a child should not be required to identify the presence or absence of more than one.

Review with the children, if necessary, the various tests for determining symmetries.

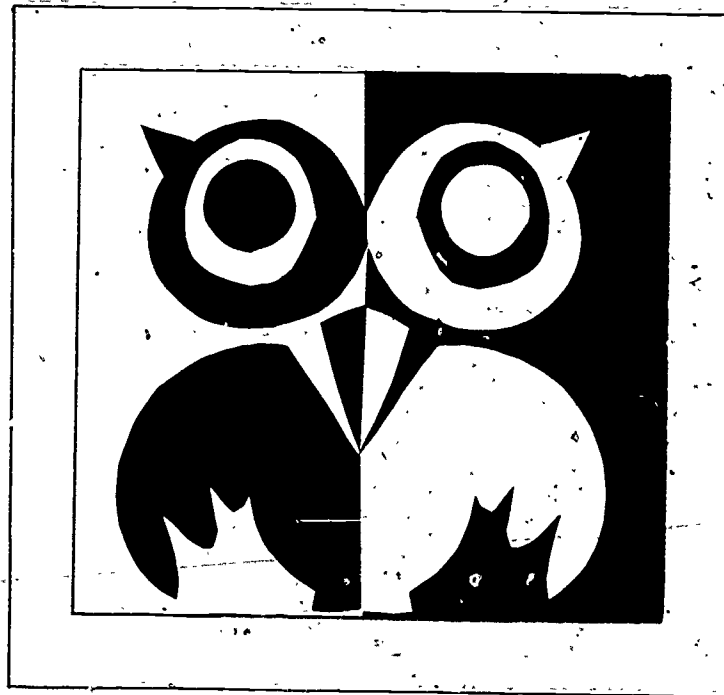
ART ACTIVITIES FOR SECTION 3: BILATERAL SYMMETRY.

A number of art activities are suggested here to reinforce the concept of bilateral symmetry. Use any selection of them any time after Lesson 11 has been completed.



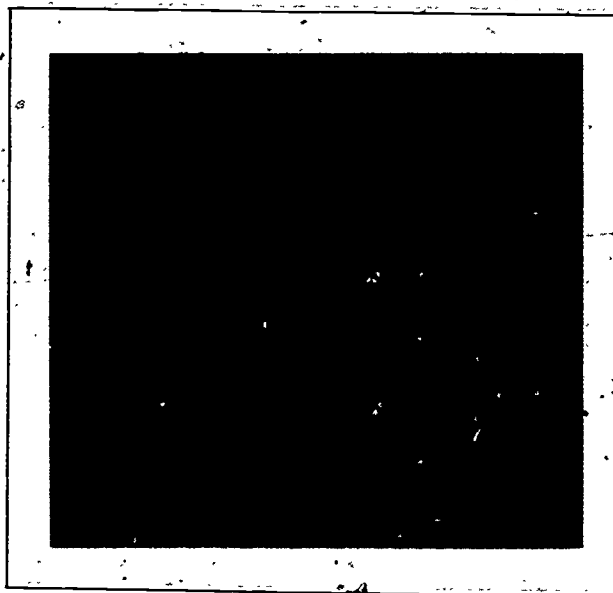
Activity A: Abstract Designs

1. Have the children cut out 2 or 3 shapes from the 12" edge of a sheet of 9" x 12" colored construction paper. Make sure the cuts are on the 12" edge, not the 9" edge.
2. Put the shapes aside and paste the sheet down on a 12" x 18" sheet of contrasting color.
3. Take the cut-out shapes and paste them down on the opposite side of the line of symmetry in a flipped position. Good color combinations are blue paper on yellow, green on orange, or black on white.



Activity B: Making Designs by Punching Holes

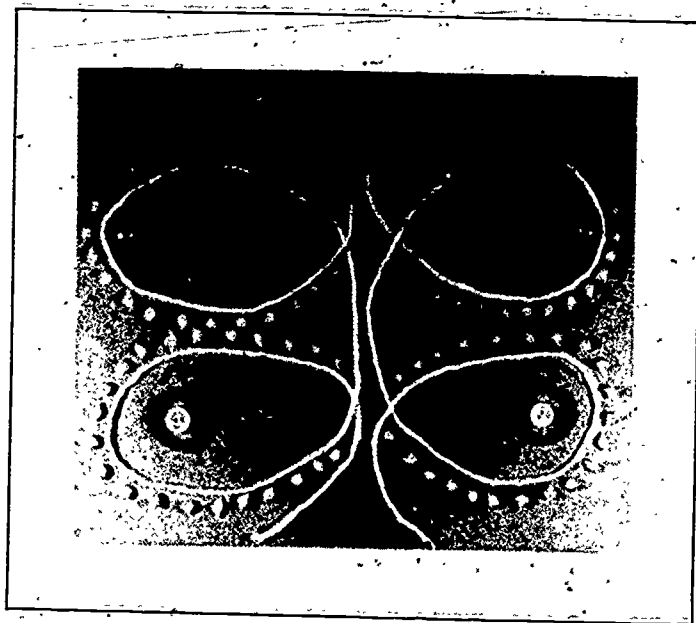
1. Using pencils or pointed sticks, punch holes in a sheet of construction paper to make a pattern exhibiting bilateral symmetry. If children have trouble making the holes, they may place the paper over a pegboard and punch through the paper over a hole.
2. Mount the punched sheet on another sheet of construction paper of contrasting color.



Activity C: 3-Dimensional Designs

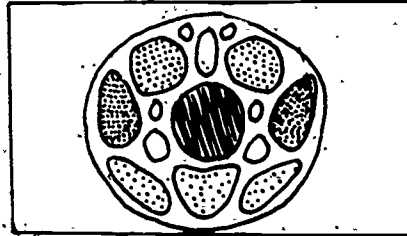
Materials used for this activity include assorted objects such as yarn, sequins, stones, pebbles, and dime store jewelry. You will also need white all-purpose glue and colored tagboard.

1. Draw a line down the center of the tagboard.
2. Make a design on one half of the tagboard by gluing assorted objects on it.
3. Repeat the procedure on the other side, to create a bilaterally symmetric pattern.

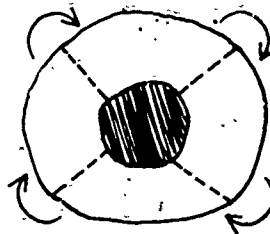


Activity D: Making A Turtle

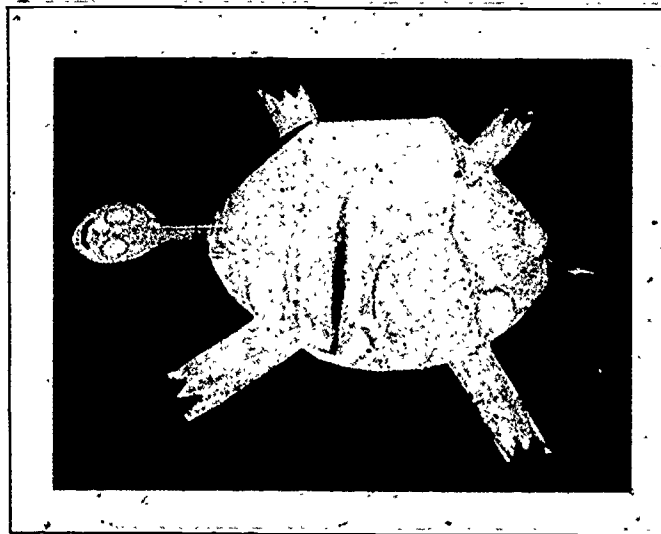
1. Draw a rough circle on a piece of construction paper. Design a turtle shell with bilateral symmetry. Color its spots and the space between the spots.



2. Cut out the circle, then cut four slits as indicated by the dotted lines. Bring the edges of the slits together, as indicated by the arrows, overlapping them a little. Tape, staple or paste the overlaps in place.

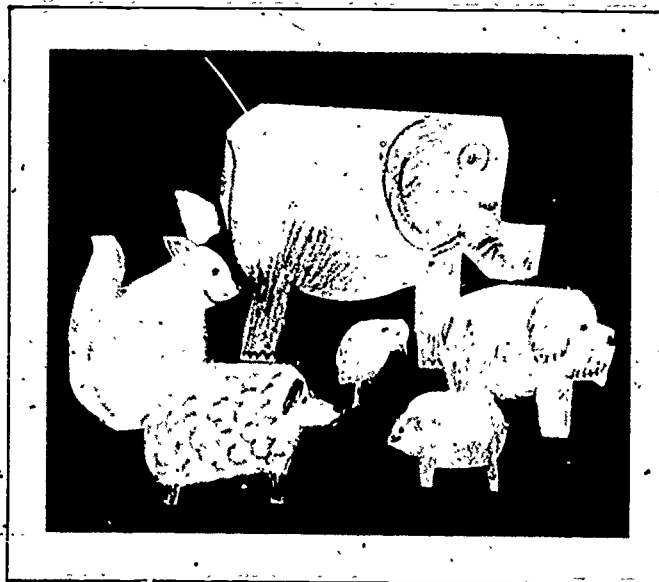


3. Draw four legs, a head, and a tail on piece of construction paper. Cut them out and glue them under the shell of the turtle.



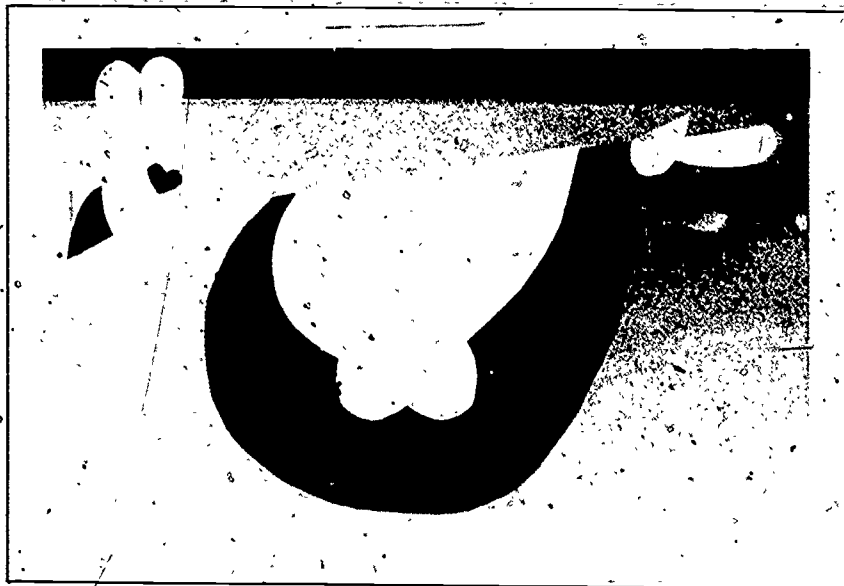
Activity E: Stand-up Animals

1. Fold a sheet of paper in half.
2. Draw an animal in such a way that the top of the drawing coincides with the folded edge. Color the drawing.
3. Cut through both layers of the drawing, turn over and color the other half of the animal.
4. Paste the two layers of the head together. When the paste is dry, the legs can be spread so that the animal will stand. The children can make a farm or a zoo with these animals.



Activity F: Three-dimensional Figure

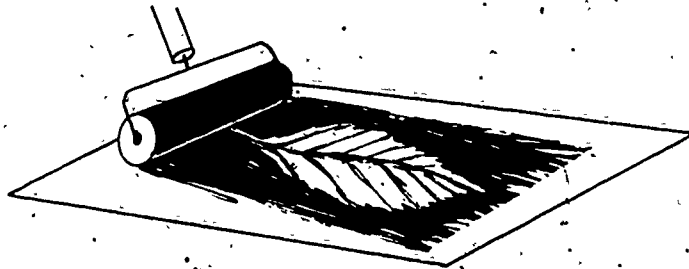
1. Cut out hearts (or other similar shapes) of various sizes and colors.
2. Fold a large heart down the middle for the body. Fold smaller hearts in similar fashion for other body parts (head, beak, etc.)
3. Paste the two halves of the head together, catching a little of the body between the layers so that the bird will stand well. To make the wings, feathers and tail, use smaller hearts pasted on in a bilaterally symmetric pattern.



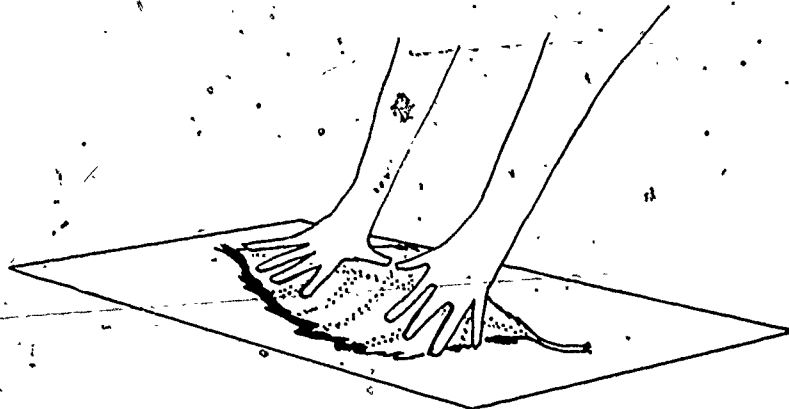
Activity G: Brayer Prints

Materials used here are a brayer, a piece of glass, water-base printer's ink, newspaper and newsprint paper for making prints.

1. Find natural symmetric objects. Leaves, grasses, and weeds usually illustrate bilateral symmetry. Bark often shows repeating patterns.
2. Squeeze a 3-inch ribbon of paint on the glass and roll the brayer back and forth over it until the brayer is smoothly coated with paint and sounds sticky. Lay the object on a piece of newspaper, and with the brayer ink the surface. Do not apply so much ink that the spaces are filled in.



3. Lay the inked object face down on clean newsprint or construction paper and press down with either a clean brayer, your fingers, or the bowl of a spoon. Children often find it easiest to use their fingers.



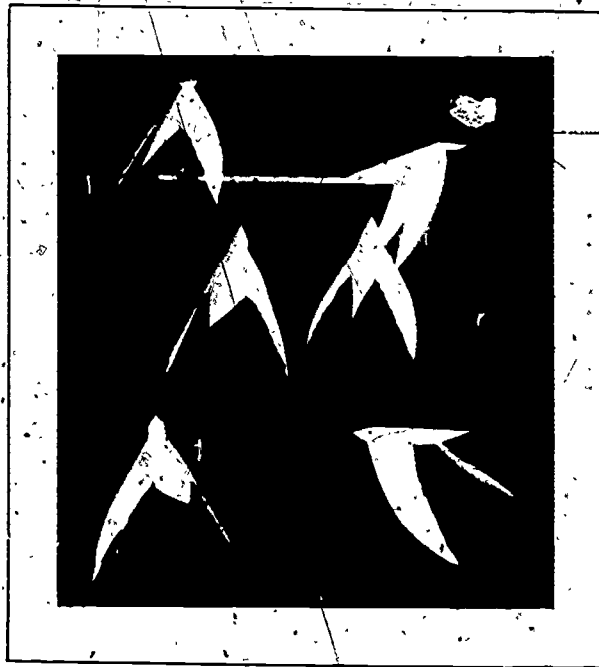
Variation

Place object under a sheet of newsprint and roll firmly over it with inked brayer so the image comes through.

Activity H: Mobiles

Use construction paper, thread, and dowels or wire coat hangers for this activity.

1. Fold construction paper in half and draw half a bird or butterfly, using the center fold as the line that divides the animal. Cut through both layers.
2. Suspend the animal by a thread through its center and tie to a rod of doweling or a wire coat hanger. Mobiles may be made of several dowels and many suspended paper figures, as shown.



Activity I: Rubbings

You will need newsprint, crayons or soft compressed charcoal, and objects with raised patterns.

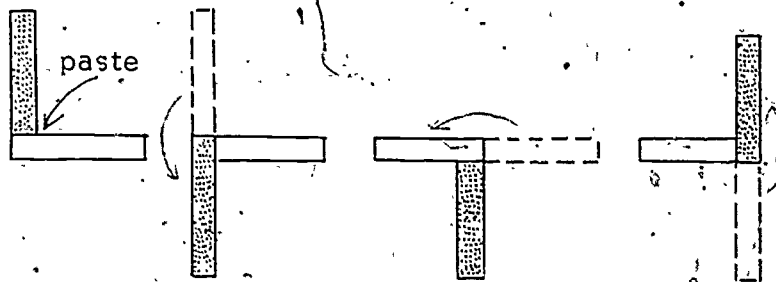
1. Find objects with raised patterns exhibiting bilateral symmetry. Leaves and weeds are excellent for rubbings.
2. Place the newsprint over the object and rub the charcoal or crayon firmly over the paper.
3. Mount the rubbings on a piece of construction paper for display.

The same technique can be used for other types of symmetry. A brick wall or tile floor give good examples of translational symmetry. A bracelet opened flat has a repeated pattern, a brooch may exhibit rotational symmetry.



Activity J: Paper-braid Clown

1. Cut strips of colored construction paper 1" x 18".
2. Paste opposite colors at corners at right angles.
3. Begin folding one color over another to make an accordion pleated braid.



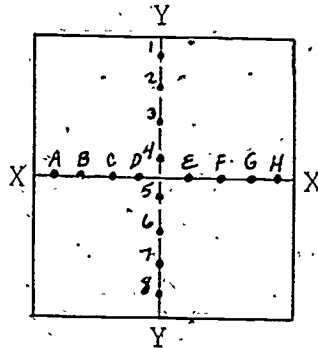
4. Make 6 braids.
5. Glue one braid into a loop for the head. Attach it to the body.
6. Attach arms and legs.
7. Make a small paper cone for a hat.



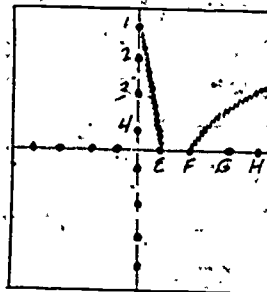
Activity K: Curve Stitching

Materials used are yarn, yarn needles, and light-weight tagboard, 9" x 9".

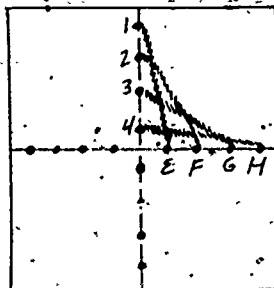
1. Mark tagboard into quarters.
2. Mark along lines at 1-inch intervals. Label X-axis with letters and Y-axis with numbers.



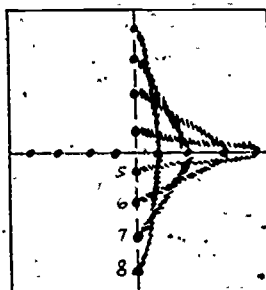
3. Before starting the stitching, the children should poke a hole through the tagboard with the yarn needle at each marked point.
4. Begin stitching from the back of the tagboard. Knot yarn, then pull needle and yarn through 1. Insert in E and come up through F. Demonstrate the stitching to the children, and have them follow you step-by-step.



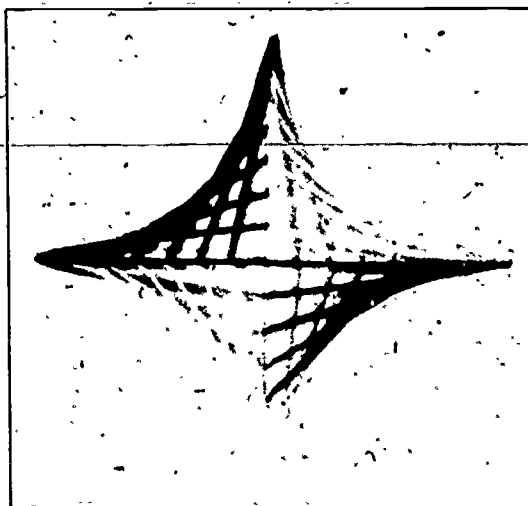
5. Continue sewing curve by stitching from F, over to 2, up through 3, and over to G, up through H, and over to 4.





6. Those children who did not have trouble with this may want to do another section. Number along the lines, as shown. Then stitch next curve as follows: H to 5, 6 to G, F to 7, 8 to E.



7. Sew each quarter until pattern is made.

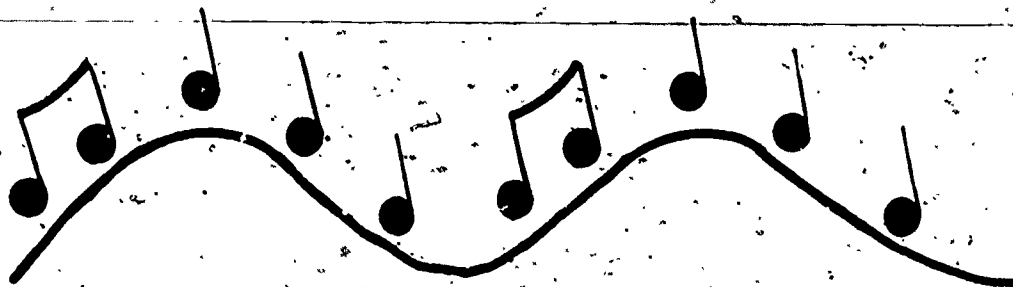


Children who grasp the 1-to-1 correspondence in this activity, may want to try curve-stitching other shapes such as a triangle  or a hexagon . They may use different colored yarns in a symmetric color pattern.

SECTION 4 SYMMETRY IN SOUND AND MOVEMENT

In addition to the visual symmetries they have been studying, first graders will enjoy discovering symmetries in the things they hear around them and in the things they do every day. They might note that they walk symmetrically. There may be repeated patterns in some class recitations. There are symmetries in their addition charts. Some stories, such as "The Little Red Hen," have symmetric structures and repeated word patterns. Symmetries of many sorts can be found in most poetry and in music.

This section has subdivisions according to subject matter rather than by lesson, in order to help you look for symmetries whenever the opportunity arises.



SYMMETRIC MOVEMENT

Many games and physical education activities involve symmetric movements and patterns. As you guide the children in analyzing them, you might find bilateral symmetry, rotational symmetry and repeated patterns. Calisthenics are especially symmetric. These activities are often done to music (e.g., "Chicken Fat"). This serves to reinforce rhythmic perception.

A child who has studied tap-dancing may want to demonstrate for the class.

Rhythm Band

Percussion instruments serve to tie symmetric movement to symmetric sound. The simplest activity is marching around the room while playing the rhythm instruments. Some children who can skip might do so while other children try to accompany the skipping rhythm with their instruments.

It is much more difficult to count a rest than to count a beat, but first graders should be able to accent every third or fourth beat with a triangle or a cymbal. This activity will help them distinguish between stressed and unstressed syllables in poetry.

Clapping

The simplest rhythmic pattern the children can clap is a regular 1-2-3-4. When they have had sufficient practice clapping in unison, have them clap patterns of increasing difficulty, such as those below. The last 2 rhythms call for stress as indicated by the > sign below the note.

The image shows five musical staves, each starting with a 4/4 time signature. The first two staves consist of quarter notes in a steady sequence. The third and fourth staves consist of eighth notes, with an accent (>) placed under the first note of each pair. The fifth staff also consists of eighth notes, with an accent (>) placed under the first note of each pair.

After the class has had enough practice in clapping these rhythms, have some volunteers try to represent the rhythm on the board. This ties together the idea of a rhythmic pattern and a visually repeated pattern. Any notation system a child devises is satisfactory if it visually records the repeated pattern. One method might be to use a long stroke (/) for an accented beat and a dash (-) for an unaccented beat. The length of the dash could indicate the duration of the beat. Thus the last two rhythms given above might be recorded as follows:

The first line of notation is: / - - / - / - - / - / - - / -

The second line of notation is: / - - - / - - - / - - -

POETRY

Have the children clap the rhythm of a poem such as "Pease Porridge Hot." Some pairs of children may be able to clap complicated cross-handed patterns.

Pease porridge hot,
Pease porridge cold,
Pease porridge in the pot,
Nine days old.

Some like it hot,
Some like it cold,
Some like it in the pot,
—Nine days old.

Other symmetries the children may find in this poem are the rhyming of every other line, the repeating pattern of words in the first three lines of each verse, and the repeated last line of each verse.

Another example of poetry rich in symmetry is "Over in the Meadow." There is a parallel structure in all the verses; the number of Mother's little ones increase by one in each verse; within the verses, the third lines have an inner repeated pattern, and the first and fourth lines have an exact repetition of the last parts of the lines; the rhymes follow an a-b-a-b pattern; and so on.

Over in the Meadow

Over in the meadow in the sand in the sun
Lived an old mother turtle and her little turtle one.
Dig said the mother, We dig said the one,
So they dug all day in the sand in the sun.

Over in the meadow where the stream runs blue
Lived an old mother fish and her little fishes two.
Swim said the mother, We swim said the two,
So they swam all day where the stream runs blue.

Over in the meadow in a hole in a tree
Lived an old mother owl and her little owls three.
Tu-who said the mother, Tu-who said the three,
So they tu-whoed all day in a hole in a tree.

Author Unknown

You will find some symmetries in most poems you examine, but not all poems display symmetry, as the following lines of haiku indicate:

My window is broken,
But I see the moon through it,
And the stars, too!

MUSIC

All music is rich in symmetries of many sorts. Any songs the children know can be analyzed for repeated rhythmic patterns by clapping or using rhythm band instruments. The melodies can be examined to discover repeated phrases. The book of simple piano pieces, Copy Cats by John LaMontaine, is excellent for demonstrating musical symmetries.

If your children are able to sing rounds, they can illustrate a very obvious example of a basic type of musical symmetry -- overlapping repeated patterns. (This is developed to great complexity in the fugue.) We have printed "Row, Row, Row your Boat" in such a way as to emphasize the patterned entry of another group of voices every four measures. Even though most first graders cannot read music, it should be easy for them to spot the visual symmetries, especially the groups of three eighth-notes for the word "merrily."

SECTION 5. OTHER INTERESTING PATTERNS

The scientist proceeds with the assumption that nature is basically orderly and symmetric. His task is to find the order and symmetry. The artist, on the other hand, creates his own patterns of symmetry.

In this section, more complicated patterns are examined by the children. Some of these may possess one or more of the symmetries already familiar to the child, but most do not. The section is intended to serve as an open door into the realm of unusual symmetry for the child and, as such, should not be construed as specifying well-defined objectives.

The shape of a crystal exhibits symmetry, but often its symmetry is not obvious. One may infer order within the crystal from the appearance of the exterior faces, but this is a rather sophisticated activity, more suitable for older children. A honeycomb and a brick wall display translational symmetry of a type discussed in the unit commentary. The pattern of bricks in a patio-flocc provides another example. A pyramid of oranges or grapefruit, as seen in a market, has repeating patterns in various dimensions. Observation of such patterns stimulates children's thinking, but the teacher should at all times refrain from trying to drive home points by mere telling.

Creative art is rich in patterns of shape, texture and color. Even in the absence of clear-cut symmetry, the child may still sense harmonies of many kinds and not be at all able to describe his impressions or enjoyment of them.

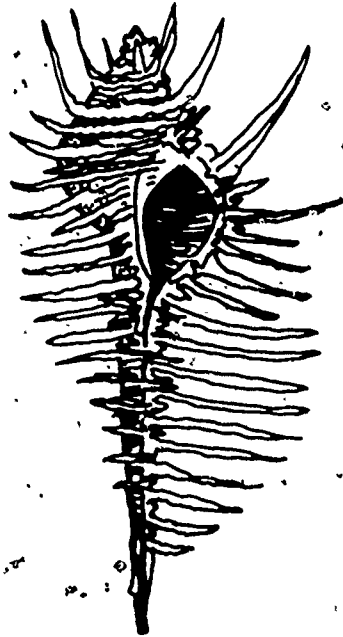
This section is not divided into lessons. Use the parts in any order and with the emphasis that you wish.



Hebrew Cone

Different patterns

Have the class observe some objects that have symmetries of types other than those considered in Sections 1, 2, and 3. This group may include sea shells, leaves, nests, and abstract designs. Allow the children to find basic symmetries, but at the same time encourage them to search for other types of balance, proportion, or any other quality they may consider to be associated with symmetry. Discuss the findings without requiring precise description.



Venus Comb Murex



Ceram Vase

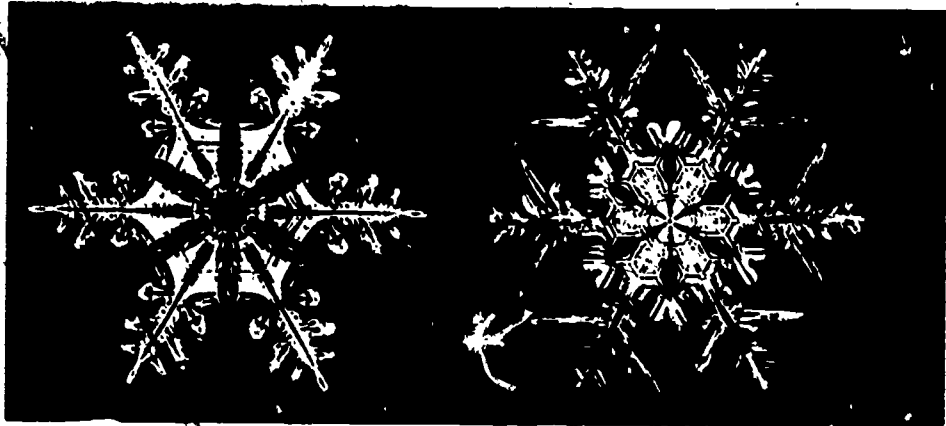
Crystals

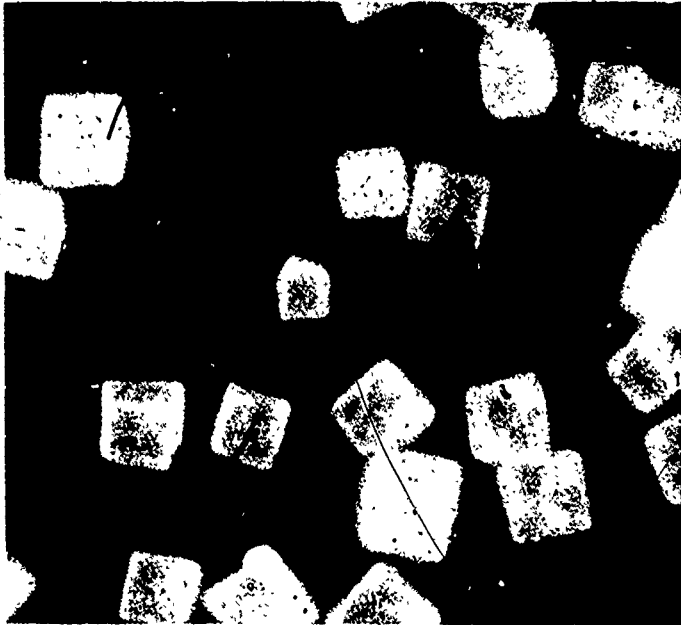
If large crystals of any kind are available, by all means have the children observe these carefully. An alternative is to work with small crystals such as salt, sugar, or epsom salt, using a hand magnifier. Rock-salt crystals can probably be observed without magnification. You can show the photographs on the following pages and later place these on the bulletin board. They are magnified views of table salt, quartz crystals and a snowflake.

Elicit from the children their views of the possible symmetry of the crystal specimens and of those shown in the photos. Ask the children if they know of other crystals to be found at home or elsewhere.

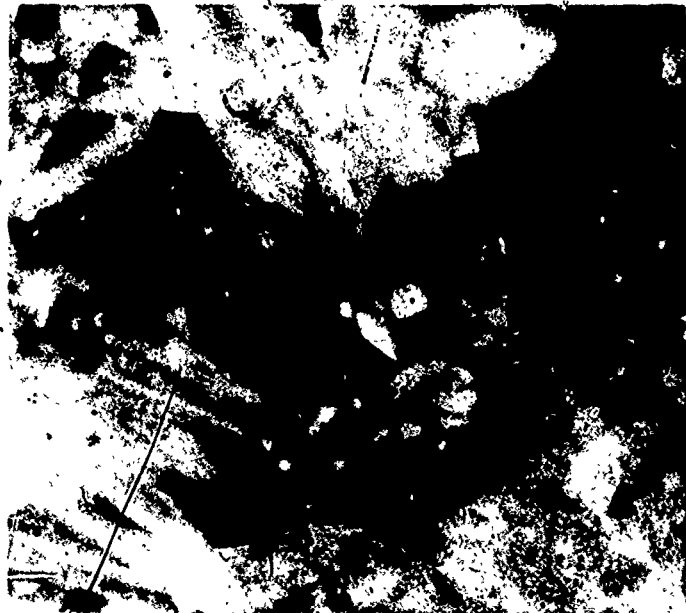
Most children will not realize that snowflakes are groups of tiny ice crystals. An individual snowflake is flat, hexagonal in form, and displays rotational symmetry. A gallery of snowflake photos would show a wide variety of forms, all with a basic hexagonal structure.

Children will enjoy creating their own snowflake designs by cutting a sheet of paper folded in a special way. They will be able to learn to do this following instructions, provided you have the time to repeat the instructions a number of times. Or you may wish to prepare a folded sheet for each child, ready for scissors work.





Magnified Table Salt



Magnified Quartz

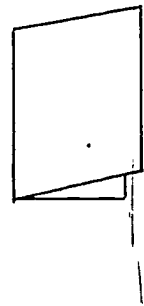


Magnified Snowflake

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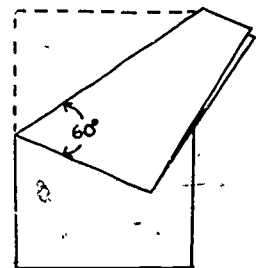
Making a Paper Snowflake

Fold a sheet of $8\frac{1}{2}$ " x 11" onionskin or bond paper in half.

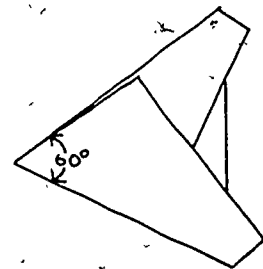


By bending as if to fold the sheet in half again, find center of folded edge and mark with a small crease.

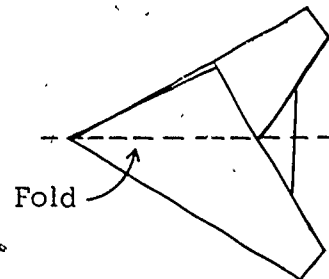
With the paper still folded in half, fold down the upper section, starting at center crease on folded edge, to form a 60° angle.



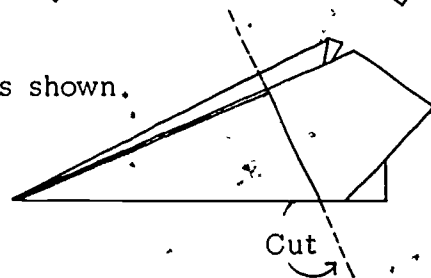
Next fold the bottom portion up in the same way. You will find it very easy to find the 60° angle by folding the upper and lower portions loosely and shifting them against each other to bring the edges in line before you crease them.



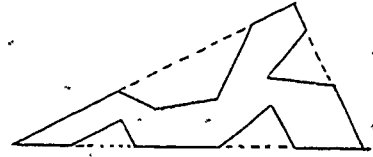
Now fold under along the dotted line.



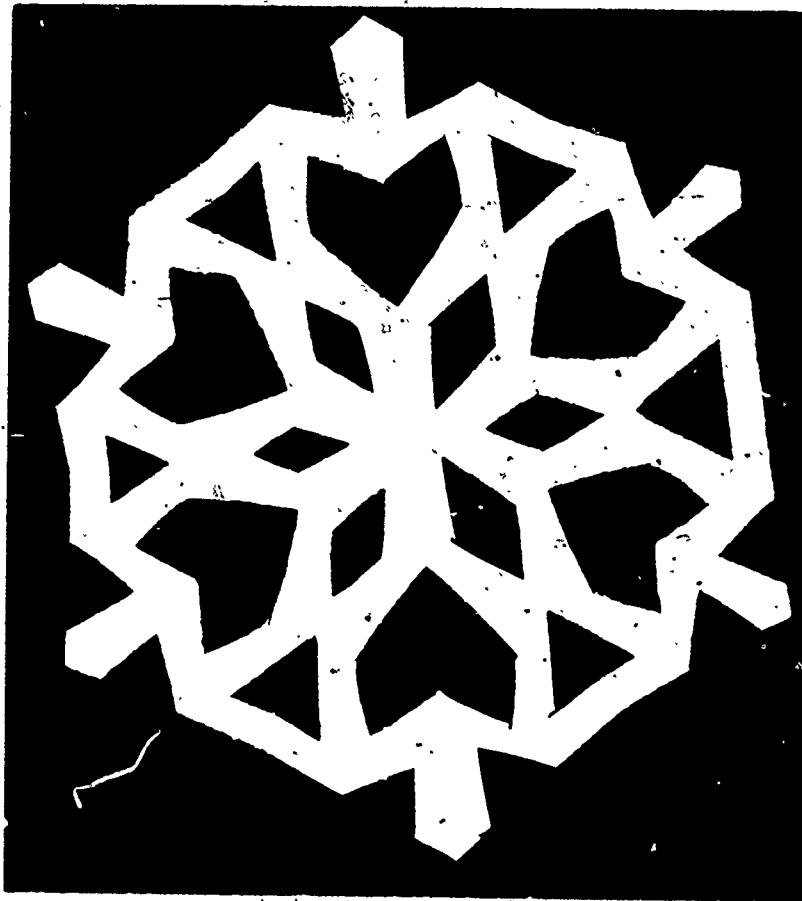
Cut through the folded paper as shown.



The pattern on the snowflake is made by cutting notches of different shapes and sizes along the folded edges.



It is always a surprise for the children to unfold the snowflakes and see what patterns they have made.



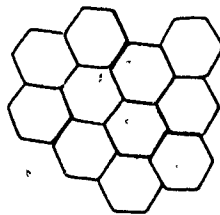
After the completion of the snowflake activity, ask the children about the symmetry of their snowflakes (bilateral and rotational symmetry). Continue with fresh pieces of paper so that each child may make one or more additional patterns.

Honeycomb and Brick Walls

This portion of the section may be taught using, for illustration, either a honeycomb pattern or the pattern shown by an ordinary brick wall. If you discuss the pattern exhibited by a brick wall, work with small blocks or toy bricks. The following procedure refers to the honeycomb.

Try to obtain an actual honeycomb to show the children. If this is not possible, a good picture will be most useful. The children may or may not recognize that the pattern shows a kind of symmetry. They should recognize that the top surface of the comb is the same as the middle or bottom surface of the comb.

Prepare in advance 8 or 10 regular hexagons of construction paper (using the snowflake technique) or prepare 8 or 10 regular hexagons made of poster board by cutting along previously marked lines. Have the children gather around a table both to watch and to participate in the construction and manipulation of the honeycomb pattern. Ask one or more volunteers to construct a honeycomb pattern from the hexagons by fitting the sides together.



Ask the children about the symmetry of the pattern. Tell them that they are free to move one of the figures any way they choose. They may need to be reminded of the test for repeating patterns. Some children will see that one of the hexagons may be moved in one direction along a line in equal-sized steps to make it coincide with other hexagons. If that same hexagon were moved in another direction, it would cover other hexagons. Any cell in the entire pattern can be reached by combining moves in these two directions. Allow ample time for as many children as possible to manipulate the hexagons and to make the new symmetry test for themselves. They may enjoy thinking about how a bee makes such a regular pattern.

Other Ornamental Patterns

The honeycomb and brick wall patterns are examples of ornamental patterns where a surface is covered by shapes arranged in a regular way. Other easily observed ornamental patterns are those in tile floors, wallpaper, and fabrics. Worksheets 18, 19 and 20 are ornamental patterns for the children to color.

Paintings

Prints of paintings that show varying degrees of symmetry can be displayed. Examples from Klee, Miró, Degas, Mondrian, and others may be used. Begin by showing the children a painting with easily observable symmetry and ask, "What does this painting tell you? Why do you like it?" This may lead to a fine discussion of the symmetry of the painting. The children should discover the symmetry for themselves, whether it be symmetry of line, shape, or color. The children may suggest their own titles for each painting. You may continue, using examples with less obvious symmetry.

Give the children the freedom to paint or draw their own designs, using what they know of the various forms of symmetry. Any noteworthy results should be enjoyed, shared and discussed by the entire group.

Sources of art prints include old magazines, calendars, catalogues of various galleries, and unframed prints from galleries.

Art Treasure Hunt

Encourage children to bring in pictures clipped from old magazines, Christmas and other greeting cards, etc., that they find especially pleasing. Ask them to tell why they like the pictures.

Display examples of the best art whenever you can. Change the pictures often and select pictures that express the craft, lore and beauty of the work from many nations and of all people. Prints of masterpieces are now available in every price range, so that it is just as easy to have the finest works on display as inferior ones.

Worksheet 18
Unit 14

Name _____

Color the pattern. Keep it symmetric.

Worksheet 19
Unit 14

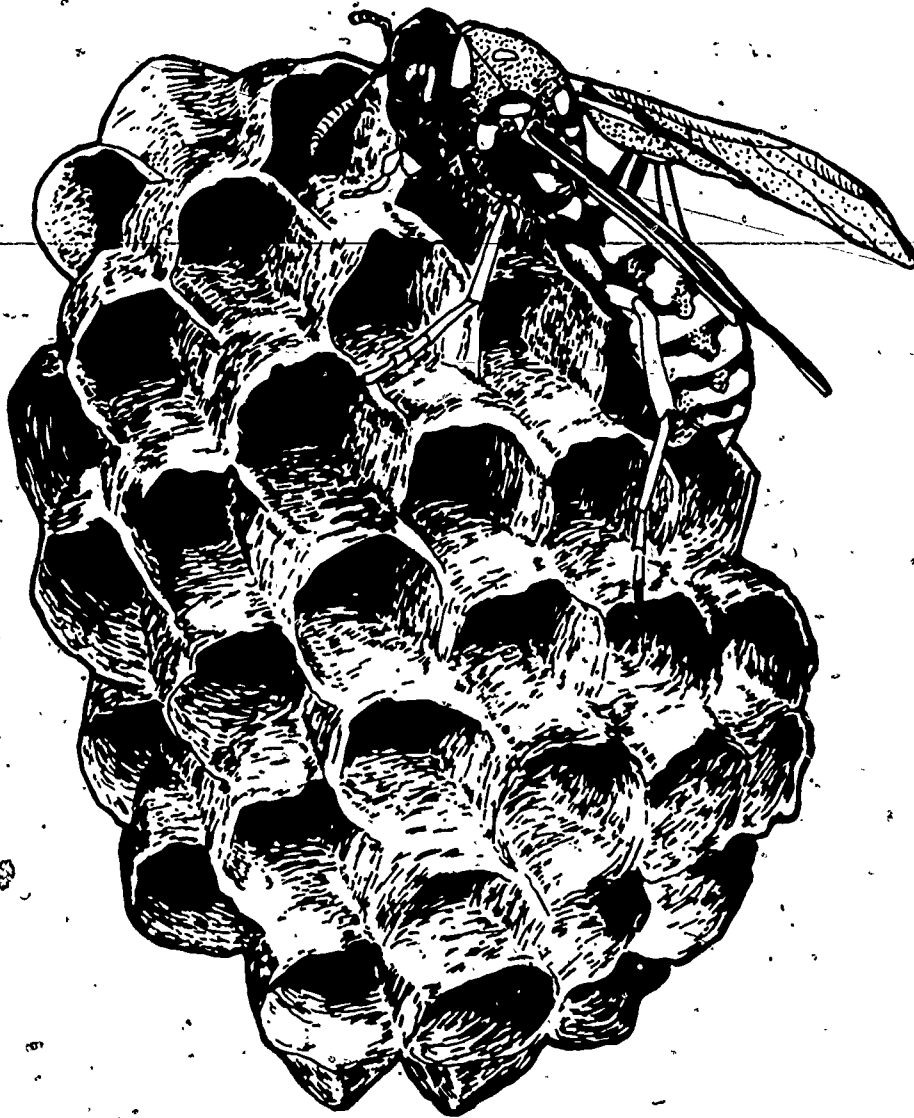
Name _____

Color the pattern. Keep it symmetric.

Worksheet 20
Unit 14

Name _____

Color the pattern. Keep it symmetric.



Wasp and Nest

Lesson 32: FINDING THE RULE (ADDITION)

In this lesson the children are to determine the operation used in going from one number to another. This gives more practice in finding missing addends. It also provides background for mapping and function, concepts that will be developed in later years.

MATERIALS

- demonstration slide rule
- Minnebars
- Worksheet 75

PROCEDURE

A. Write $3 \longrightarrow 5$ on the chalkboard.

HOW DID I GET FROM 3 TO 5 ? (You added 2 to 3.)

WE WILL WRITE THE RULE OVER THE ARROW LIKE THIS:

$$3 \xrightarrow{+2} 5$$

USE THE SAME RULE TO FINISH THESE:

$$4 \xrightarrow{+2} \dots$$

$$1 \xrightarrow{+2} \dots$$

$$5 \xrightarrow{\quad} 7$$

Have the children write the completed series on paper. Then ask for volunteers to complete the series on the board.

Show with the demonstration slide rule that these pairs of numbers can be read from a pair of number lines when one has been moved 2 units to the right of the other. This illustrates that moving one of the number lines 2 units to the right of the other corresponds to adding 2 to each number.

B. Write the following series of number pairs on the board.

$$2 \longrightarrow 5$$

$$3 \longrightarrow 6$$

$$6 \longrightarrow 9$$

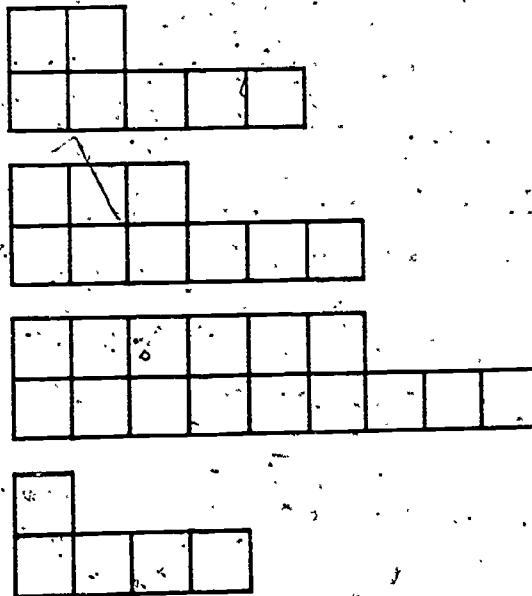
$$1 \longrightarrow 4$$

Tell the children that in every pair you used the same rule to go from the first number to the second number.

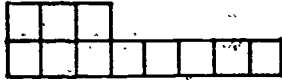
WHAT RULE DID I USE? (+3)

Have the children write +3 over each arrow in the series.

It is helpful for children to see this relation shown by Minnebars. Ask them to arrange their Minnebars to show the pairs in the series on the board.



WHAT PATTERN DO YOU SEE? (The second bar in each pair is 3 units longer than the first bar.)

C. Show the class a pair of Minnebars, e.g.,  Ask them to arrange some of their Minnebars in pairs using the same rule that this pair illustrates. The children can take turns in writing on the board the number pairs shown by their bars.

D. Worksheet 75 has more series of this type. Have the children complete these series. -

Worksheet 75 Unit 13		Name _____	
Each group has a rule. Write the rule over the arrows. Write the missing numbers.			
6 $\xrightarrow{+3}$ 9	1 $\xrightarrow{+5}$ 6	2 $\xrightarrow{+5}$ 7	3 $\xrightarrow{+5}$ 8
0 $\xrightarrow{+3}$ 3	2 $\xrightarrow{+5}$ 7	3 $\xrightarrow{+5}$ 8	4 $\xrightarrow{+5}$ 9
4 $\xrightarrow{+3}$ 7	3 $\xrightarrow{+5}$ 8	4 $\xrightarrow{+5}$ 9	
5 $\xrightarrow{+3}$ 8	4 $\xrightarrow{+5}$ 9		
19 $\xrightarrow{+2}$ 21	10 $\xrightarrow{+1}$ 11	11 $\xrightarrow{+1}$ 12	12 $\xrightarrow{+1}$ 13
20 $\xrightarrow{+2}$ 22	11 $\xrightarrow{+1}$ 12	13 $\xrightarrow{+1}$ 14	14 $\xrightarrow{+1}$ 15
21 $\xrightarrow{+2}$ 23	12 $\xrightarrow{+1}$ 13	14 $\xrightarrow{+1}$ 15	15 $\xrightarrow{+1}$ 16
22 $\xrightarrow{+2}$ 24	13 $\xrightarrow{+1}$ 14	15 $\xrightarrow{+1}$ 16	16 $\xrightarrow{+1}$ 17

Lesson 33: FINDING THE RULE (ADDITION AND SUBTRACTION)

This lesson is an extension of Lesson 32. Again the children will determine the operation used to go from one number in a pair to the other. Both addition and subtraction will be used.

MATERIALS

- demonstration number line
- Worksheets 76 and 77

PROCEDURE

A. Write the following series on the board:

- 4 → 2
- 5 → 3
- 2 → 0
- 7 →

Worksheet 76
Unit 13 Name _____

Find the rule for each group.
Write the missing numbers.

6 $\xrightarrow{-1}$ 5	5 $\xrightarrow{-2}$ 3
4 $\xrightarrow{-1}$ 3	4 $\xrightarrow{-2}$ 2
7 $\xrightarrow{-1}$ 6	3 $\xrightarrow{-2}$ 1
9 $\xrightarrow{-1}$ 8	2 $\xrightarrow{-2}$ 0
8 $\xrightarrow{-3}$ 5	5 $\xrightarrow{-5}$ 0
7 $\xrightarrow{-3}$ 4	7 $\xrightarrow{-5}$ 2
6 $\xrightarrow{-3}$ 3	9 $\xrightarrow{-5}$ 4
5 $\xrightarrow{-3}$ 2	6 $\xrightarrow{-5}$ 1

Explain again that the same rule was used to go from each number on the left to the number on the right. Ask for volunteers to fill in the rule $\xrightarrow{-2}$ and the missing number.

Show the pairs of numbers on the demonstration number line. The second number of each pair in the series is 2 units to the left of the first number.

Use as many other examples with a subtraction rule as you wish before having the class complete Worksheet 76.

- B. Introduce Worksheet 77 by having the class consider several series of number pairs for which the appropriate rule might be either addition or subtraction. Be certain that the children realize that just one rule is used for each series of number pairs. Some suggested pairs are:

$$6 \longrightarrow 3$$

$$7 \longrightarrow 3$$

$$\xrightarrow{+6} 7$$

$$3 \longrightarrow 0$$

$$4 \longrightarrow 0$$

$$\xrightarrow{+6} 6$$

$$4 \longrightarrow 1$$

$$5 \longrightarrow$$

$$\longrightarrow 9$$

$$5 \longrightarrow$$

$$\longrightarrow 6$$

$$\longrightarrow 8$$

Worksheet 77	
Unit 13	Name _____
Find the rule for each group. Write the missing numbers.	
$2 \xrightarrow{+4} 6$ $\xrightarrow{+4} 5$ $3 \xrightarrow{+4} 7$ $5 \xrightarrow{+4} 9$	$4 \xrightarrow{+4} 8$ $3 \xrightarrow{+4} 7$ $3 \xrightarrow{+4} 7$ $5 \xrightarrow{+4} 9$
$18 \xrightarrow{-2} 16$ $11 \xrightarrow{-2} 9$ $7 \xrightarrow{-2} 5$ $13 \xrightarrow{-2} 11$	$11 \xrightarrow{+7} 18$ $7 \xrightarrow{+7} 14$ $5 \xrightarrow{+7} 12$ $0 \xrightarrow{+7} 7$

SECTION 6 ADDING AND SUBTRACTING TWO-DIGIT NUMBERS

(OPTIONAL)

This section reviews the decimal place value notation and introduces the method commonly used to add and subtract multidigit numbers. Only one- and two-digit numbers will be considered, and no renaming of 1's to 10's (carrying) or of 10's to 1's (borrowing) will be required. The material in this section will be presented again in second grade; therefore, feel free to use only those activities which are suited for your class and for which you have time.

COMMENTARY

The justification of finding $50 + 40$ by "adding 0 and 0 and 5 and 4" is that

$$50 + 40 = (5 \times 10) + (4 \times 10) = (4 + 5) \times 10 = 9 \times 10 = 90$$

These steps involve basic principles of multiplication with which the children are not familiar. To avoid the direct use of multiplication we will develop the idea of adding multiples of ten by working with sets of ten. Thus, in these lessons the children will be led to think of $50 + 40$ as 5 tens plus 4 tens.

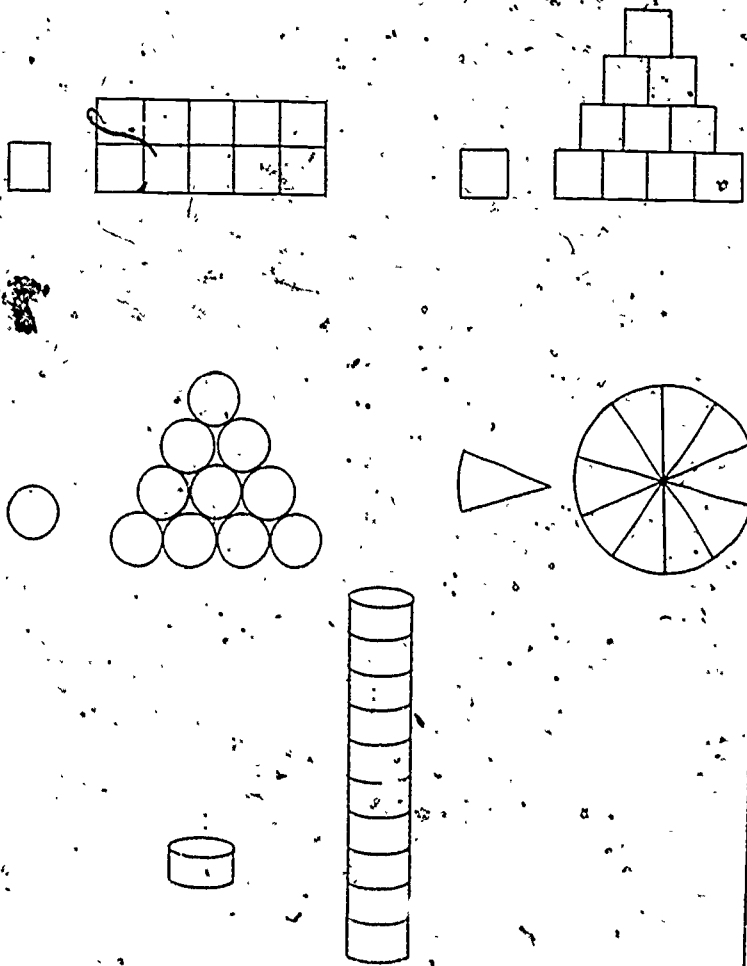
We will also use a set interpretation to introduce more complex problems, such as $54 - 31$. The justification of the common computational method for finding $54 - 31$ would involve the following steps, which of course should not be presented to first graders.

$$\begin{aligned} 54 - 31 &= [(5 \times 10) + 4] - [(3 \times 10) + 1] \\ &= [(5 \times 10) - (3 \times 10)] + [4 - 1] \\ &= [(5 - 3) \times 10] + 3 \\ &= (2 \times 10) + 3 \\ &= 23 \end{aligned}$$

NOTES ON THE TEACHING OF THIS SECTION

Each lesson starts with a demonstration on the board and on an abacus. It is likely that some children will benefit by repeating the demonstrated examples with their own counters.

The demonstrations call for "sets of one" and "sets of ten." The set of one is a single unit shape, and the set of ten is formed by joining ten unit shapes in an easily recognized form. They can be made of heavy paper or tagboard and placed on a blackboard with sticky putty or on a flannelboard. Some shapes that might be used are sketched below.



Nine unit shapes and up to nine of the larger shapes should be prepared in each of the designs selected.

Lesson 34: PLACE VALUE NOTATION

This lesson reviews decimal place value notation. A transition is made from the word "and" to the symbol + in expressing numbers in terms of sets of tens and ones. If a child has much difficulty with this lesson he should review the treatment of place value in Unit 11.

MATERIALS

- paper sets of one and sets of ten
- abacus
- Worksheets 78 and 79

PROCEDURE

Place 2 sets of ten and 4 sets of one on the board. Guide a discussion in which the children note that these are three names for the number of unit shapes displayed:

2 tens and 4 ones $20 + 4$ 24

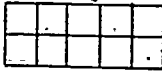
With the 24 shapes still on the board have 24 set up on the abacus. Repeat the process with various numbers until you feel the class is ready for Worksheets 78 and 79.

Worksheet 78

Unit 13

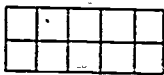
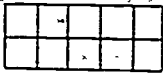
Name _____

Write names for the total number of unit shapes.



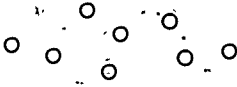
1 tens and 2 ones

$$\underline{10} + \underline{2} = \underline{12}$$



2 tens and 4 ones

$$\underline{20} + \underline{4} = \underline{24}$$



4 tens and 8 ones

$$\underline{40} + \underline{8} = \underline{48}$$



7 tens and 0 ones

$$\underline{70} + \underline{0} = \underline{70}$$

Worksheet 79

Unit 13

Name _____

Write three names for each number.

27 2 tens and 7 ones $\underline{20} + \underline{7}$

56 5 tens and 6 ones $\underline{50} + \underline{6}$

13 1 tens and 3 ones $\underline{10} + \underline{3}$

30 3 tens and 0 ones $\underline{30} + \underline{0}$

42 4 tens and 2 ones $\underline{40} + \underline{2}$

70 7 tens and 0 ones $\underline{70} + \underline{0}$

81 8 tens and 1 ones $\underline{80} + \underline{1}$

99 9 tens and 9 ones $\underline{90} + \underline{9}$

39 3 tens and 9 ones $\underline{30} + \underline{9}$

73 7 tens and 3 ones $\underline{70} + \underline{3}$

60 6 tens and 0 ones $\underline{60} + \underline{0}$

45 4 tens and 5 ones $\underline{40} + \underline{5}$

Lesson 35: SUMS AND DIFFERENCES OF ONE-DIGIT AND TWO-DIGIT NUMBERS

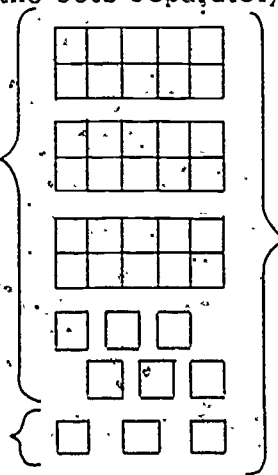
In this lesson a one-digit number is added to or subtracted from a two-digit number by adding or subtracting the one-digit number from the number in the ones place of the two-digit number.

MATERIALS

- paper sets of one and sets of ten
- abacus
- about ten 7" x 5" cards printed with combinations such as
 $36 + 3$, $48 + 1$, $26 + 3$, $11 + 7$, $54 + 5$, $93 - 2$,
 $74 - 3$, $88 - 6$, $17 - 7$, $46 - 0$
- Worksheets 80, 81 and 82

PROCEDURE

- A. Use your paper sets to demonstrate the following problem. First consider the sets separately, then join them.



With the above problem still displayed, have someone set up 36 on the abacus and then add 3 to it.

Repeat the activity with other sums and differences not requiring regrouping (carrying or borrowing).

B. Individual practice in finding sums and differences on the abacus is appropriate here. When you think it will be helpful, provide a child with an abacus and problem cards.

C. Worksheets 80 and 81 contain problems similar to those of A and B in a vertical form. You may wish to do one or two problems as a class activity and use the rest for individual practice. Worksheet 82 allows the children to apply the methods of the lesson to word problems.

NOTE: Worksheet 82 shows a seacow or Manatee eating a water hyacinth. These aquatic mammals live along seacoasts and up river inlets.

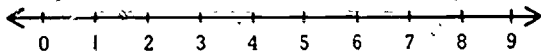
Worksheet 80

Unit 13

Name _____

Find the sums. Use the number line if you need it.

$\begin{array}{r} 30 + 6 \\ + 3 \\ \hline 30 + 9 \end{array}$	$\begin{array}{r} 36 \\ + 3 \\ \hline 39 \end{array}$	$\begin{array}{r} 50 + 1 \\ + 4 \\ \hline 50 + 5 \end{array}$	$\begin{array}{r} 51 \\ + 4 \\ \hline 55 \end{array}$
$\begin{array}{r} 80 + 4 \\ + 3 \\ \hline 80 + 7 \end{array}$	$\begin{array}{r} 84 \\ + 3 \\ \hline 87 \end{array}$	$\begin{array}{r} 90 + 4 \\ + 2 \\ \hline 90 + 6 \end{array}$	$\begin{array}{r} 94 \\ + 2 \\ \hline 96 \end{array}$
$\begin{array}{r} 60 + 5 \\ + 4 \\ \hline 60 + 9 \end{array}$	$\begin{array}{r} 65 \\ + 4 \\ \hline 69 \end{array}$	$\begin{array}{r} 60 + 6 \\ + 1 \\ \hline 60 + 7 \end{array}$	$\begin{array}{r} 66 \\ + 1 \\ \hline 67 \end{array}$
$\begin{array}{r} 50 + 0 \\ + 3 \\ \hline 50 + 3 \end{array}$	$\begin{array}{r} 50 \\ + 3 \\ \hline 53 \end{array}$	$\begin{array}{r} 30 + 2 \\ + 6 \\ \hline 30 + 8 \end{array}$	$\begin{array}{r} 32 \\ + 6 \\ \hline 38 \end{array}$



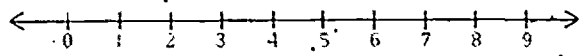
Worksheet 81

Unit 13

Name _____

Find the sums. Use the number line if you need it.

$\begin{array}{r} 11 \\ + 3 \\ \hline 14 \end{array}$	$\begin{array}{r} 25 \\ + 3 \\ \hline 28 \end{array}$	$\begin{array}{r} 91 \\ + 8 \\ \hline 99 \end{array}$	$\begin{array}{r} 48 \\ + 4 \\ \hline 52 \end{array}$
$\begin{array}{r} 62 \\ + 2 \\ \hline 64 \end{array}$	$\begin{array}{r} 18 \\ + 1 \\ \hline 19 \end{array}$	$\begin{array}{r} 21 \\ + 6 \\ \hline 27 \end{array}$	$\begin{array}{r} 72 \\ + 5 \\ \hline 77 \end{array}$
$\begin{array}{r} 53 \\ + 0 \\ \hline 53 \end{array}$	$\begin{array}{r} 37 \\ + 2 \\ \hline 39 \end{array}$	$\begin{array}{r} 12 \\ + 6 \\ \hline 18 \end{array}$	$\begin{array}{r} 82 \\ + 3 \\ \hline 85 \end{array}$
$\begin{array}{r} 95 \\ + 2 \\ \hline 97 \end{array}$	$\begin{array}{r} 24 \\ + 4 \\ \hline 28 \end{array}$	$\begin{array}{r} 53 \\ + 3 \\ \hline 56 \end{array}$	$\begin{array}{r} 12 \\ + 7 \\ \hline 19 \end{array}$



Worksheet 92

Unit 13

Name _____



Tony and Sal looked at a seacow in a river.
They saw it eat 15 plants.
Then they saw it eat 4 more plants.
How many plants did they see it eat?

$$15 + 4 = 19$$

There were 89 plants on the other side of the river.
Another seacow ate 7 of them.
How many plants were left?

$$89 - 7 = 82$$

After this seacow ate the 7 plants it ate 12 more.
How many plants did it eat?

$$7 + 12 = 19$$

Lesson 36: ADDING AND SUBTRACTING MULTIPLES OF TEN

Here the combining and removing of sets of ten illustrate the addition and subtraction of multiples of ten.

MATERIALS

- paper sets of ten
- abacus
- Worksheet 83

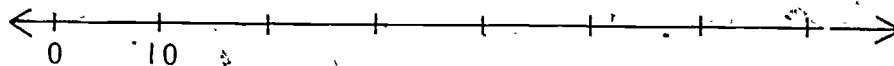
PROCEDURE

- A Have sets of 3 tens and 2 tens placed on the board. Have someone join them and write the following sentences:

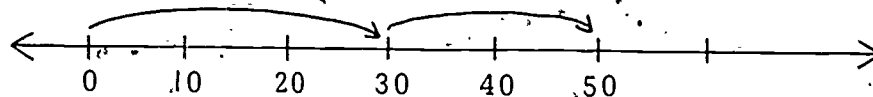
3 tens and 2 tens are 5 tens

$$30 + 20 = 50$$

Sketch a number line similar to this:



Have the children place more numerals on the line. (You may need to review counting by tens.) Have someone mark arrows on the line to show $30 + 20 = 50$.



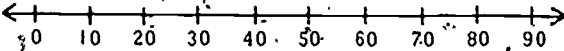
Repeat this procedure with a difference, such as $60 - 30$, and a missing addend problem, such as $50 + \square = 80$.

- B. Have a child show the sum $50 + 40$ on the abacus. Then have $90 - 40$ shown.

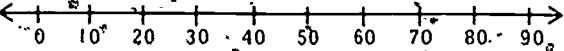
- C. Provide oral practice with combinations of multiples of tens. Each child may use an abacus, a number line marked at the 10's, or sets of ten as aids if he needs them.
- D. Have the children complete Worksheet 83. If a child has difficulty understanding or completing the worksheet, he should have more practice before going on.

Worksheet 83
Unit 13 Name _____

$50 + 20 = 70$ $0 + 50 = 50$
 $60 + 30 = 90$ $20 + 70 = 90$
 $40 + 10 = 50$ $30 + 30 = 60$



50	30	80
$\begin{array}{r} +40 \\ \hline 90 \end{array}$	$\begin{array}{r} +20 \\ \hline 50 \end{array}$	$\begin{array}{r} +10 \\ \hline 90 \end{array}$
60	50	20
$\begin{array}{r} +10 \\ \hline 70 \end{array}$	$\begin{array}{r} +0 \\ \hline 50 \end{array}$	$\begin{array}{r} +60 \\ \hline 80 \end{array}$



Lesson 37: MONEY PROBLEMS

In this lesson problems involving dimes and pennies are considered. These provide practice with the decimal system of numeration, in adding or subtracting a two-digit and a one-digit number, and in adding or subtracting multiples of ten.

MATERIALS

- play pennies and dimes (optional)
- Worksheets 84 and 85

PROCEDURE

Read and find answers to the problems on Worksheets 84 and 85 as a group. Explain that 1 dime is worth 10 cents and that 1 penny is the same as 1 cent.

Play money may be used to help those who have trouble with the problems.

Worksheet 84
Unit 13 Name _____

Jane had 5 dimes.
Mother gave her 2 more dimes.
How many cents did she have? 10¢ 1¢

$50 + 20 = 70$

Dick had 7 dimes.
He lost 1 dime.
How many cents did he have left?

$70 - 10 = 60$

Sam gave 3 dimes to Bill and 5 dimes to Tom.
How many cents did he give away?

$30 + 50 = 80$

John had 5 dimes.
His father gave him 2 dimes and 6 pennies.
How many cents did he have then?

50	or	50
$+ 20 + 6$		$+ 26$
<hr style="width: 100%;"/>		<hr style="width: 100%;"/>
$70 + 6$		76

Worksheet 85
Unit 13 Name _____

Sally had 3 dimes and 2 pennies.
Dick gave her 5 pennies.
She wants a book that costs 35 cents.
Does she have enough money now to buy it? (Yes)

$30 + 2$	or	32
$+ 5$		$+ 5$
<hr style="width: 100%;"/>		<hr style="width: 100%;"/>
$30 + 7$		37

Mary has 4 dimes and 9 pennies.
She buys gum for 5 cents.
How much money does she have left?

$40 + 9$	or	49
$- 5$		$- 5$
<hr style="width: 100%;"/>		<hr style="width: 100%;"/>
$40 + 4$		44

Mike has 50 cents.
He earns 4 dimes.
Now how many cents does he have?

50
$+ 40$
<hr style="width: 100%;"/>
90

Tony spent 32 cents.
How many dimes and pennies did he spend?

3 dimes and 2 pennies

Lesson 38: ADDING AND SUBTRACTING TWO-DIGIT NUMBERS

Two-digit numbers are added and subtracted in this lesson. The numbers are so chosen that no renaming of ones to tens or tens to ones is required.

MATERIALS

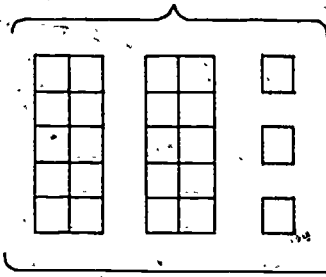
- paper sets of one and sets of ten
- abacus.
- Worksheets 86 and 87

PROCEDURE

- A. Use the paper sets to demonstrate the following sum and as many others as you wish. After an example has been shown on the board, it should also be shown on an abacus.

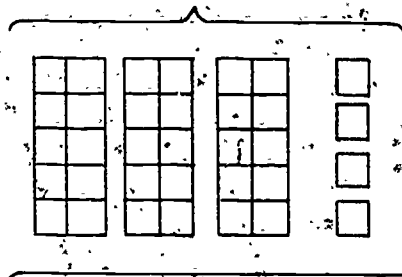
2 tens and 3 ones

$$20 + 3 = 23$$



3 tens and 4 ones

$$30 + 4 = 34$$



(2 tens + 3 tens) and (3 ones + 4 ones)

= (2 + 3) tens and (3 + 4) ones

= 5 tens and 7 ones

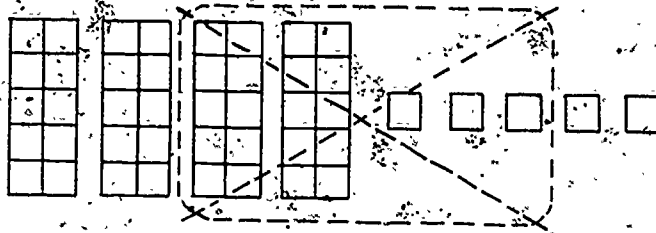
= 50

+ 7

= 57

131

B. The following illustration demonstrates subtraction:



Starting set contains

$$4 \text{ tens and } 5 \text{ ones} \quad \text{or} \quad 40 + 5 = 45$$

Removed set contains

$$2 \text{ tens and } 3 \text{ ones} \quad \text{or} \quad 20 + 3 = 23$$

Remaining set contains

$$(4 - 2) \text{ tens and } (5 - 3) \text{ ones} \quad \text{or} \quad 2 \text{ tens and } 2 \text{ ones}$$

$$(40 - 20) + (5 - 3) = 20 + 2 = 22$$

C. Have the class suggest a way to add 36 and 21 using neither the number line nor sets with 36 and 21 members. Guide them to the idea of renaming 36 and 21 as 30 + 6 and 20 + 1. Then the ones and tens can be added separately and combined. A convenient form to record this process is:

$$\begin{array}{r} 30 + 6 \\ + 20 + 1 \\ \hline 50 + 7 \end{array} \quad \text{or} \quad \begin{array}{r} 36 \\ + 21 \\ \hline 57 \end{array}$$

When the expanded form of a number is subtracted, use parentheses around the number to show that both parts are being subtracted. For example, for $36 - 21$, one can write:

$$\begin{array}{r} 30 + 6 \\ - (20 + 1) \\ \hline \end{array}$$

D. Worksheets 86 and 87 can now be done.

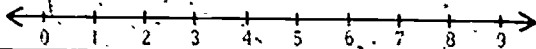
Worksheet 86

Unit 13

Name _____

Find the sums and differences. Use the number line if you need it.

$30 + 5 = 35$ $+ 10 + 2$ $40 + 7 = 47$	$60 + 7 = 67$ $+ 20 + 1$ $80 + 8 = 88$
$80 + 7 = 87$ $- (20 + 2)$ $60 + 5 = 65$	$50 + 5 = 55$ $+ 40 + 2$ $90 + 7 = 97$
$60 + 3 = 63$ $- (40 + 1)$ $20 + 2 = 22$	$50 + 2 = 52$ $+ 30 + 0$ $80 + 2 = 82$
$70 + 1 = 71$ $+ 10 + 3$ $80 + 4 = 84$	$60 + 1 = 61$ $- (20 + 1)$ $40 + 0 = 40$



Worksheet 87

Unit 13

Name _____

Do you remember Jomo and Tanana? On their trip in Africa they counted the storks they saw.

The first week Jomo saw 23 storks.
 The second week Jomo saw 22 storks.
 In the two weeks he saw 45 storks.



The first week Tanana saw 20 storks.
 The second week Tanana saw 14 storks.
 In the two weeks she saw 34 storks.

How many more storks did Jomo see? 11