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ABSTRACT

This monograph on elementary probability for middle school, junior high, or high school consumer mathematics students is divided into two parts. Part one emphasizes lessons which cover the fundamental counting principle, permutations, and combinations. The 5 lessons of part I indicate the objectives, examples, methods, application, and problems for each concept. Part two consists of a combination of 6 lessons and 17 experiments which deal with probability and its applications. The experiments are organized in such a way that they can be converted into task cards. Answers are given for the 47 problems of the entire unit. (JW)

THE MICHIGAN COUNCIL OF TEACHERS OF MATHEMATICS

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Guidelines for Quality Mathematics Teaching

A Monograph Series

ACTIVITIES IN ELEMENTARY PROBABILITY

Daniel J. Fouch

Monograph No. 9
September 1975

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ACTIVITIES IN ELEMENTARY PROBABILITY

Daniel J. Fouch

September, 1975

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ACTIVITIES IN ELEMENTARY PROBABILITY

INTRODUCTION

When I took my first probability course in college, I discovered that probability could be a very challenging subject. There are many notations, formulas, and theorems to be learned. It was not until later, as a student teacher, that I realized most of the probability that an average person will ever need can be learned without much of the formality encountered in the usual probability course.

Not only can the formality be omitted, but probability can even be fun to learn. At the same time, it serves as an exceptional means to reinforce arithmetic skills, including work with fractions and percents. Also, it provides an opportunity to combine theory with practice in the form of lessons and experiments. Often, teachers do not take full advantage of this opportunity.

Thus, what I have attempted to do in this unit is combine the theoretical work essential to understanding basic probability with some experiments which will lend practical insight to that theory. I have attempted to keep the work at a level appropriate for most middle school or junior high school students or even a consumer mathematics class at the high school level.

The unit is divided into two parts. The first part includes the counting techniques which will be needed before the actual work with probability can begin. Covered in part one are the fundamental counting principle and work with combinations and permutations. Some sample problems are included with each lesson. The answers to these problems appear in the back of the unit.

The second part consists of a combination of lessons and experiments. The lessons deal with probability and its applications. Again, sample problems are included. The experiments attempt to reinforce the theoretical work. I have indicated what seems to be a natural order of presentation for both lessons and experiments. Also included are ideas on the techniques I feel work well for written lab reports.

The basis for most of the theoretical work comes from H. R. Jacob's book, Mathematics: A Human Endeavor, (W. H. Freeman and Company, San Francisco, 1970). Mr. Jacob's book is one of the most entertaining and informative mathematics textbooks I have ever read. I would suggest this book for reference and additional problems.

The experiments come from a variety of sources. Some were borrowed from work done by Dr. Albert Shulte of Oakland Schools. Some were adapted from Creative Experiments in Probability, (Midwest Publishers), by Dr. Donald Buckeye of Eastern Michigan University. Some were devised by Mr. Gerald Leckrone and myself to suit a particular situation. Mr. Leckrone, with whom I did my student teaching at Brighton High School, is responsible for many of the ideas in this unit, and I owe him special thanks for his help.

This unit is not an in-depth study of the theory of probability. Its goal is to acquaint the student with a discipline he will encounter, in some form, every day.

INTRODUCTION TO PART I

Following are the outlines for lesson plans which cover the fundamental counting principle, permutations, and combinations. Included are five outlines with the basic development of these ideas and some sample problems.

Remembering that this unit is going to emphasize experiments rather than theory, these lessons explore simple ideas via simple means. In this section, there is only one rule and one definition for students to learn. Complicated presentations and complex developments of the ideas would only confuse a student at this level. They are not necessary and should be avoided.

The best way to begin a lesson is to ask "What if...?", beginning with a simple example, then build up to more complicated examples through class discussion, keeping in mind what it is that needs to be eventually accomplished in the lesson. The problems should reinforce the lesson and perhaps lead to more problems and questions.

These first five lessons should take about one day each.

LESSON 1: THE FUNDAMENTAL COUNTING PRINCIPLE

Objective:

The student can find the total number of decisions to be made by multiplying the number of choices that can be made at each decision.

Example:

Given one true or false question, how many choices are there?

$$\left.\begin{matrix} T \\ F \end{matrix}\right\} = 2 \text{ choices}$$

Given two true or false questions?

$$\left.\begin{matrix} T\ T \\ T\ F \\ F\ T \\ F\ F \end{matrix}\right\} = 4 \text{ choices}$$

Given three true or false questions?

$$\left.\begin{matrix} T\ T\ T & F\ T\ T \\ T\ T\ F & F\ T\ F \\ T\ F\ T & F\ F\ T \\ T\ F\ F & F\ F\ F \end{matrix}\right\} = 8 \text{ choices}$$

Method:

Use the boxes of this make believe test and see how many ways they could be filled in.

one question	2				2 ways = 2^1
two questions	2	2			4 ways = 2^2
three "	2	2	2		8 ways = 2^3
four "	2	2	2	2	? ways = $2^?$

In general, the total number of choices possible is found by multiplying the number of choices which can be made each time.

Application:

Suppose you have 5 multiple choice questions with 4 answers to each question. Apply the above method.

4	4	4	4	4

= 1024 ways.

If a computer dating service had applications from 55 men and 85 women, how many dates can be arranged?

= ? dates.

Problems:

1. How many different Michigan license plates are possible given a three letter-three digit format? (Vowels — a, e, i, o, u — are not used in Michigan license plates.)

   __ __ __ __ __ __

2. A recent letter to Action Line in the Detroit Free Press expressed concern that the Social Security Department may run out of numbers for its cards. Given the standard nine-digit card, how many different cards are possible?

   __ __ __ __ __ __ __ __ __

   Suppose the Social Security Department added a single digit onto the end of the current number. How many cards would then be possible?

   What would be the result if, instead of adding a single digit onto the end, they added a letter?

3. You are working a maze. As you go through it, you find that there are 10 "forks in the road" where you must decide to go left or right. How many different paths are possible in the maze? Is there a good chance that you will get through it on your first try?

LESSON 2. PERMUTATIONS

Objectives:

The student can apply the fundamental counting principle to solve the problem of arranging things in a definite order.
The student can apply the definition of factorial to solve n!, given n.

Example:

Given two people, Alice and Bill, how many ways can they be arranged?

A B
B A } = 2 ways.

How about three people?

A B C A C B
B A C B C A } = 6 ways.
C A B C B A

Given four people, how many ways can they be arranged?

Method:

Use the Fundamental Counting Principle. We have 4 people, so we are concerned with 4 spaces.

The first space can be filled by anyone so there are 4 choices.

4			

But once that first space is filled, there are only 3 choices left for the second seat.

4	3		

Now we are left with only 2 people to place in the next spot.

4	3	2	

And in the last spot is the last person.

4	3	2	1

By the Fundamental Counting Principle, we get the total by multiplying $4 \cdot 3 \cdot 2 \cdot 1 = 24$. This can be verified by writing all of the possibilities as we did above: A B C D — A C D B — A D C B — ....

A shorthand notation for $4 \cdot 3 \cdot 2 \cdot 1$ is $4!$. What would be the shorthand form for $5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$?

Applications:

Suppose that 12 people are coming to Christmas dinner. How many different orders are there for 12 people?

12 · 11 ___ ___ ___ ___ ___ ___ ___ ___ ___ ___

Suppose that 12 people come to a family dinner and that there are only 8 places at the dining room table. The rest have to eat in the kitchen. How many ways can the 8 available seats be filled?

12 · 11 ___ ___ ___ ___ ___ ___

Problems:

4. Calculate: $5!$; $8!$; $3! \cdot 4!$. Is $3! \cdot 4! = 12!$?

5. Suppose a veterinarian has an aardvark, a bear, a cat, a dog, an egret, and a fox and has 6 cages in a row. How many different ways can these animals be arranged?

6. How many ways can 7 people line up at a drinking fountain?

7. At a track meet there are 8 persons running the 100 yard dash. There are awards for first, second, and third place. How many different ways can 1st, 2nd, and 3rd be awarded?

8. How many different ways are there of scrambling the letters of AVERSION?

9. How many different 4-letter arrangements can be made using letters in the word TOLERANCE?

LESSON 3: PERMUTATIONS WITH REPETITION

Objective:

The student can calculate the number of ways to arrange a set of objects when one subset of those objects consists of identical elements.

Example:

Consider the word FEE. How many different 3 letter words could be made from these letters?

Previous work seems to indicate that there could be $3 \cdot 2 \cdot 1 = 6$ different arrangements. Yet, if we listed all 6 "different words", the results might be rather perplexing.

To facilitate the listing, we will number the E's.

$F\ E_1E_2$	$F\ E_2E_1$
$E_1F\ E_2$	$E_2F\ E_1$
E_1E_2F	$E_2E_1\ F$

Now look at the same list without the numbers.

F E E	F E E
E F E	E F E
E E F	E E F

Without the numbers, there is no difference between the two columns. When we worked with people or animals, all the objects were distinct. But when letters are involved, there often is repetition.

Method:

In the above arrangement we, indeed, only have 3 distinct arrangements. Our formula, 3!, gave us 6 "different" arrangements, exactly twice as many as it should have. On closer inspection we see that the 2 E's, when counted as separate letters, can be arranged in 2! = 2 different ways. Therefore, to adjust for repetition, divide out the number of ways that the identical letters can be arranged.

This will give the correct number of different arrangements of the letters in FEE.

$$\frac{3!}{2!} = \frac{3\cdot\cancel{2}\cdot\cancel{1}}{\cancel{2}\cdot\cancel{1}} = 3 \text{ different arrangements.}$$

Application:

How many different 6 letter arrangements are there of the letters in the word SNEEZE?

Since there are 6 letters, there are 6! arrangements. But since there are 3 E's, which can be arranged 3! ways, the number of different arrangements is:

$$\frac{6!}{3!} = \frac{6\cdot5\cdot4\cdot\cancel{3}\cdot\cancel{2}\cdot\cancel{1}}{\cancel{3}\cdot\cancel{2}\cdot\cancel{1}} = 120 \text{ different arrangements.}$$

Problems:

10. How many different arrangements are there for the letters
    in MISSES?
    in UNUSUAL?
    in BRRRRR?

11. How many different numbers can be made by rearranging the digits 6 3 6 1 6 in all possible ways?

12. Suppose 5 identical pennies are placed in a row. How many different arrangements are there in which only 1 coin will be a tail (T)?
    (Hint: How many different ways are there to arrange the letters T H H H H?)

LESSON 4: MORE ON PERMUTATIONS WITH REPETITION

Objective:

The student can calculate the number of ways to arrange a set of objects when there are several different subsets of identical objects within the set.

Example:

Confronted with the word FREEZE, and armed with our knowledge of permutations from Lesson 3, it is obvious that there are

$$\frac{6!}{3!} = \frac{6\cdot5\cdot4\cdot\cancel{3}\cdot\cancel{2}\cdot1}{\cancel{3}\cdot\cancel{2}\cdot1} = 120 \text{ arrangements of these letters.}$$

But in the word FREEZER the addition of the letter "R" to the word FREEZE complicates the matter slightly.

FREEZER not only has E repeated 3 times, but now there are 2 R's.

Method:

It is necessary to expand on the method from Lesson 3. That is, we must divide out not only the 3! ways that 3 E's can be arranged but must also divide out the 2! ways that 2 R's can be arranged.

The number of different ways to arrange the letters in FREEZER would then be:

$$\frac{7!}{3!\cdot2!} = \frac{7\cdot\cancel{6}\cdot5\cdot4\cdot3\cdot\cancel{2}\cdot1}{(\cancel{3}\cdot\cancel{2}\cdot1)\cdot(\cancel{2}\cdot1)} = 420 \text{ different ways.}$$

Application:

How many distinct arrangements can be made from all the letters in the word MISSISSIPPI?

There are 11 letters so there are 11! arrangements. There are

4 I's which can be arranged in 4! ways. There are 4 S's which can be arranged in 4! ways. There are 2 P's which can be arranged in 2! ways. Our final result, dividing out to avoid repetition, will be:

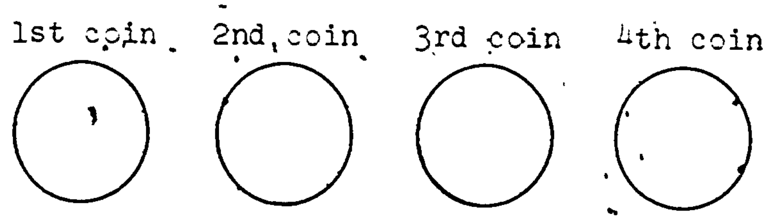
$$\frac{11!}{4!\cdot 4!\cdot 2!} = \frac{11\cdot 10\cdot 9\cdot \cancel{8}\cdot 7\cdot \cancel{6}\cdot 5\cdot \cancel{4}\cdot \cancel{3}\cdot \cancel{2}\cdot 1}{\cancel{4}\cdot \cancel{3}\cdot \cancel{2}\cdot 1\cdot \cancel{4}\cdot \cancel{3}\cdot \cancel{2}\cdot 1\cdot \cancel{2}\cdot 1} = 34,650$$

Problems:

13. How many different arrangements are there for the letters in the word REPETITION? in the word SOCIOLOGICAL?

14. How many different signals can be sent using 6 flags, hung vertically, if 4 of the flags are red and 2 are blue?

15. If 4 pennies are arranged in a row:
   a. How many ways are there for all 4 coins to be Heads (H)? (Hint: How many different arrangements are there for H H H H?)
   b. How many ways can you have 3 Heads (and 1 Tail)?
   c. How many ways can you have 2 Heads?
   d. How many ways can you have 1 Head?
   e. How many ways can you have 0 Heads?
   f. What is the total number of outcomes possible with 4 pennies? (Hint: Use the Fundamental Counting Principle.)



   g. Add up all the answers in parts a through e above. Does your answer agree with part f?

16. Follow a format similar to problem 15 for 5 or more coins.

LESSON 5: COMBINATIONS

Objective:

The student will find the number of combinations by using the Fundamental Counting Principle and then dividing by the number of duplicate arrangements.

Example:

You have joined a record of the month club. As an introductory offer, you may choose any 3 albums from a list of 25. How many ways can you choose?

Method:

Since there are 25 different albums to choose from, it would seem safe to say that there are $25 \cdot 24 \cdot 23 = 13,800$ different ways to choose 3 albums.

But does the order in which these albums are chosen make any difference? The albums come at the same time in the same package. And even though we may enjoy one of the three more than the others, all 3 albums are in a sense, equivalent. There is no advantage to choosing the 3 albums in any particular order. Here, order makes no difference.

To find the number of combinations possible, then, we must use the fundamental counting principle. Then divide by the number of ways in which the things being chosen can be arranged.

Three albums can be arranged in 3! ways. The number of ways of choosing 3 albums from 25 is:

$$\frac{25 \cdot 24 \cdot 23}{3!} = \frac{25 \cdot \overset{4}{\cancel{24}} \cdot 23}{\underset{1}{\cancel{3}} \cdot \underset{1}{\cancel{2}} \cdot 1} = 2300 \text{ ways.}$$

Application:

In a class of 33 students, 4 students are to be chosen for a class presentation. How many ways can these students be chosen?

Choosing 4 students from 33 can be done in 33·32·31·30 ways. Since the order in which they are chosen is irrelevant, we must divide by the number of ways in which 4 students can be arranged, 4!. Our result is:

$$\frac{33\cdot 32\cdot 31\cdot 30}{4!} = \frac{33\cdot \overset{8}{\cancel{32}}\cdot 31\cdot \overset{5}{\cancel{30}}}{\underset{1}{\cancel{4}}\cdot \underset{1}{\cancel{3}}\cdot \underset{1}{\cancel{2}}\cdot 1} = 40{,}920 \text{ ways.}$$

Problems:

17. How many ways can a committee of 6 senators be chosen from 100 senators?

18. There are 12 students eligible to attend a National Leadership Camp, but there are only 5 openings. How many different ways are there to choose the 5 students?

19. A hand of Poker consists of 5 cards dealt from a deck of 52 playing cards. How many different poker hands are there?

INTRODUCTION TO PART II

Now we can begin the experimental part of our unit. First, an introductory lesson on probability, then the first group of experiments.

This part of the unit will take from two to three weeks. After a lesson is introduced, more experiments will be made available to the students.

The student may pick from any of the experiments available. The reason I think they should be passed out a few at a time is that then the student will not be able to do all of the "fun" experiments first. This will also make the decision of which experiment to do easier because he will not have so many to choose from at one time.

I have organized these experiments in such a way that they can be readily converted to task cards. Each page of experiments consists of two task cards. After a lesson is completed, add the experiments included in that lesson to the collection from previous lessons.

One suggestion is to mount and laminate these task cards.[2] This allows for convenience in handling as well as protecting the cards from the rigors of daily classroom use. If unfamiliar with laminating procedures, consult your media specialist. He or she will best be able to help you suit your individual needs.

Each experiment will require a lab report. In keeping with the spirit of criterion referencing, I have devised a "lab check-list" sheet which will help the student make sure his experiment report is complete. I have also provided write-up sheets for each report so that the reports have a more uniform appearance and so that the student will always have a copy of the instructions for the report. It helps to post some of the good reports as examples and as positive reinforcement.

Depending on the depth of each lesson, it should take one to two days to present the lesson with two to three days for experiments. Again, the emphasis is on experimentation, but it seems essential that the student understand some of the basic theory behind what he is doing in lab.

As an added note, I would point out that although I use the word "lab", I, personally, have never been privileged to work in a school where separate math lab facilities were available. When I refer to a lab, I am referring to a classroom in which students, desks, and equipment are conveniently rearranged.

As far as handling the various apparatus, i.e., dice, cards, coins, etc., I ask the students to bring in what they can and warn them that they may not get it back. I then keep the materials in a large shoebox and let students check the shoebox out as they need it.

LESSON 6: PROBABILITY ON A DIE AND DATA COLLECTION

Objectives:

The student will be able to use the definition of probability to find, in fraction form, the probability of a simple event occurring.

The student can plot both a frequency distribution chart and a bar graph, given the appropriate data.

Example:

Consider tossing a coin. What would be the "chances" of getting a head? A tail?

Since there are only two possible ways for it to land, it would seem reasonable that we would get heads half of the time and tails the other half.

If we look at a die, how many ways could we roll 1? How many ways could we roll a 2? a 3? How many numbers are there on a die?

We can say our chances for rolling a 3 would be one chance in six, or as a fraction, $\frac{1}{6}$.

What would the chance be of rolling an even number (since half of the numbers are even)?

Method:

This leads us to the definition of probability. The probability of an event occurring is:

$$P(\text{event occurs}) = \frac{\text{number of favorable ways}}{\text{number of total ways}}$$

Application:

Consider the eight letters in "HOT COCOA". If we write each of these letters on a card, shuffle the cards, then pick one at random:

(a) What are the chances of getting the letter C?

(b) What are the chances of getting a letter in the word CREAM?
Hint: How many of the letters in CREAM appear on the cards? How many cards are there in all?)

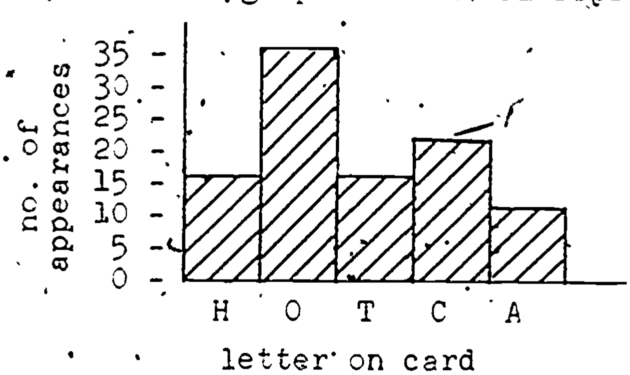
Footnote:

Frequently we will have to keep track of a large amount of data, especially when we are doing experiments. Suppose we want to do an experiment using the above problem. We are going to see if we can get the same results from experimenting as we do in theory. (This may be a good point for discussion).

We are going to pick 100 times from our deck of cards and record the results. One good method is the use of a frequency distribution chart as follows:

no. of H's	卌 卌 卌 ǁ
no. of O's	卌 卌 卌 卌 卌 卌
no. of T's	卌 卌 卌 ǁ
no. of C's	卌 卌 卌 卌 ǀǀǀ
no. of A's	卌 卌 ǀǀǀ

And we can use a bar graph for visual representation of this chart:



Problems:

20. Given a deck of cards, in one draw, what is the probability of getting:
   a. The ace of diamonds?
   b. Any ace?
   c. A spade?
   d. A red card? (heart or diamond)

21. If you roll a die, what is the probability of getting:
   a. A 2?
   b. An odd number?
   c. A multiple of 3?
   d. A seven?

22. If a person rolls a die 6 times, how many times would he/she expect a 6 to appear? (Hint: Since there is a $\frac{1}{6}$ chance for each number on the die to appear, you would expect the number 6 to appear $\frac{1}{6}$ of the time.)
   a. How many times would you expect to get an even number if you roll a die 30 times?
   b. If you roll a die 100 times, how many times would you expect a 3 to appear? What per cent of the time should you expect a 3 to appear in a given number of rolls? (Be accurate to two decimal places.)

23. Suppose a bag contains 16 red marbles, 10 green marbles, 14 blue marbles, and 10 yellow marbles. If a marble is drawn and immediately replaced, what is the probability:
   a. Of getting a red marble?
   b. Of getting a red or a green marble?
   c. Suppose that 5 draws in a row have been red marbles. What is the probability that the sixth draw will yield a red marble? (Remember that the marble is replaced after each draw.)
   d. Out of 100 draws, how many would you expect to be:
   i. Red? ii. Green? iii. Blue? iv. Yellow?

LAB REPORT CHECKLIST

If you can honestly check each one of the following boxes for any given lab report, you will earn an "A" grade. If you use this sheet, please hand it in with your report. It is not mandatory. Use it only if you want to.

- ○ Description of experiment, summarizing your entire procedure.
- ○ A complete list of all data.
- ○ Data neat and well organized.
- ○ Predictions of the general outcome of the experiment.
- ○ Adequate answers to all questions asked.
- ○ If there are any graphs, tables, or charts requested, they are labelled and reflect accurately the data which is submitted.
- ○ Complete sentences, good grammar.
- ○ The report is clearly written, easy to follow from step to step.
- ○ There is a short summary bringing out points of interest and comparing predictions to results.

LAB. REPORT SHEET

EXPERIMENT NO. ______________ NAME ______________

DATE ______________ HOUR ______________

OTHER MEMBERS OF MY GROUP: ______________

______________ ______________

Briefly describe your experiment; what you did, how you did it, and what your findings were. Include answers to any questions that were asked in the experiment. Include predictions of how you expect the experiment to turn out before you actually do it. All answers should be in complete sentences. All graphs should be neat and clearly labelled. (Attach extra sheets or graph paper if necessary.)

EXPERIMENT NO. 1

(Apparatus: Coin)

1. Determine whether a coin is honest by flipping it 100 times and recording your results.
2. What are the results after 50 flips? After 75 flips? After 100 flips?
3. Did you get the same ratio each time?
4. Do you think this coin is honest? Why or why not?
5. Did you ever get 5 heads or 5 tails in a row?

EXPERIMENT NO. 2

(Apparatus: Die)

1. Determine whether a die is honest by rolling it 100 times and recording your results.
2. What are the results after 25 rolls? 50 rolls? 75 rolls? 100 rolls? Is there any one number that consistently seems to appear more often?
3. Did you ever get the same number 3 times in a row?
4. Do you think this die is honest? Why or why not?

EXPERIMENT NO. 3

(Apparatus: Ten pennies, paper cup, box lid)

1. Place the pennies in the cup, put your hand over the top, shake the cup well, and empty the contents into the lid.
2. Count how many coins turned up heads and record this number. (If one should wind up standing against the side of the lid, count the face you see.)
3. Now repeat the above 199 times for a total of 200 tosses.
4. Show the results of the tosses on a frequency distribution chart.
5. With the data collected, draw a bar graph.
6. Do your results seem reasonable?

EXPERIMENT NO. 4

(Apparatus: Thumbtack)

1. Toss or drop a thumbtack onto a hard level surface, where it bounces before coming to rest. When the thumbtack comes to rest, it points up (U) or down (D).
2. Toss the tack 50 times. Record the data and calculate the ratio (fraction) of times the tack falls U.
3. Would you always get the same results?
4. Repeat steps 1 and 2 again and compare your results.
5. Use a slightly slanted surface and do step 2.
6. Does the slanted surface give different results?

EXPERIMENT NO. 5

(Apparatus: Cards)

1. Conceal a red card in one hand and a black card in the other hand.
2. Let your partner tell you which hand holds the red card.
3. Do this 50 times. Keep track of your results.
4. Do you feel that your partner has E.S.P. (extra-sensory perception) or is he just guessing?
5. Now you try the same experiment. Do you have E.S.P.?

EXPERIMENT NO. 6

(Apparatus: New half dollars)

1. Place a new half dollar on edge on the floor, and hold it upright with one hand. Give it a flick with the other hand, and let it spin until it comes to rest.
2. Do this 50 times and record the results. What has happened?
3. Do you think the results would change if you did this 100 times? 1000 times?

EXPERIMENT NO. 7

(Apparatus: Two pennies)

1. Gary the gambler is setting up a friendly game. He suggests to Tony, "Let's toss pennies. I'll toss mine first. If it comes up heads, I win. If it comes up tails, you toss. If your penny comes up heads, I win; if it's tails, you win."
2. Tony says, "That's not fair! You'll win twice as often."
3. Gary: "O.K., then! I get one penny if I win — you get two if you win."
4. Tony: "That's better! Let's play!"
5. Get a partner. One of you be Gary — the other, Tony. Take 20 turns. Who won? How much did he win?
6. Switch places. This time who won?
7. Is the game fair? Why or why not? Record the results.

EXPERIMENT NO. 8

(Apparatus: A coin)

1. The Martingale is an old and commonly used gambling system. It gives the gambler a good chance to win a small amount, balanced against a small chance of taking a large loss.
2. Bet a certain amount x. (For simplicity bet $1.)
3. If you win, bet the same amount ($1).
4. If you lose, double the previous amount bet.
5. Each time you win, go back to the original $1 bet.
6. Toss a coin 20 times to try out the system. Let a head represent a win and a tail a loss. Record your results. (Hint: Keep track of the amount bet on each toss as well as the total amount won.)

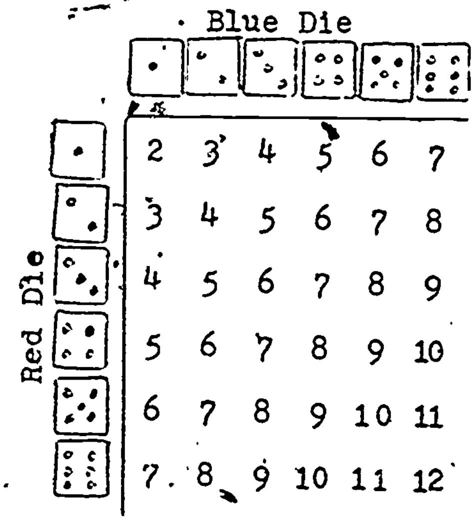
LESSON 7: MORE DICE PROBABILITIES

Objective:

The student will be able to write the probabilities of a particular outcome with more than one die, both as a fraction and as a percent.

Example:

Draw a diagram with two dice and all the possible outcomes. This would be good to draw on a transparency for future use. How many ways can you roll a 7? An 11? A 13? How many outcomes are there?



Method:

The way to compute the chances of getting these numbers is to apply the method from our last lesson:

$$\text{Probability of an event, } P(\text{event}) = \frac{\text{no. of favorable ways}}{\text{total no. of ways}}$$

Application:

What is the most likely to occur when two dice are rolled?
The probability of a 7, $P(7) = \frac{6}{36} = \frac{1}{6}$.
How about a 1? $P(1) = \frac{0}{36} = 0$.

A good way to compare which events are more likely to occur is to convert our fractions to percents.
Example: $P(7) \approx \frac{1}{6} \approx .17 = 17\%$

Footnote:

Often when listening to radio or T.V., we hear someone refer to the ODDS in favor of an event occurring. ODDS are simply a convenient means of expressing probabilities. The way to compute ODDS is:

$$\text{ODDS in favor of an event} = \frac{\text{number of favorable ways}}{\text{number of unfavorable ways}}$$

If a die is rolled, what are the odds in favor of getting a 2?
$\frac{1}{5}$ 1 to 5

What are the odds in favor of getting an even number? $(\frac{3}{3})$

Does the above example help make sense out of the expression "even odds"?

Problems:

24. Given two dice, express the probability of the following events occurring (both as a fraction and a percent):
    a. P(6).
    b. P(2).
    c. P(of getting a multiple of 3).
    d. P(1).

25. A dice game can be played according to the following rules:
    i. You win on the first throw if you throw a 7 or 11.
    ii. You lose on the first throw if you throw a 2 (snake eyes), 3, or 12.
    iii. If you throw any other number on the first throw (4, 5, 6, 8, 9, 10), you must roll the same number again before you roll a 7 in order to win.

    In reference to this game, answer questions 26-31.

26. What is the probability that you will win on the first throw?

27. What is the probability you will lose on the first throw?

28. What is the probability you will neither win nor lose on the first throw?

29. Suppose you roll a 4, 5, 6, 8, 9, or 10 on your first throw. Are you more likely to win or lose the game?

30. What are the odds in favor of winning the game on the first throw? In favor of losing on the first throw?

31. What would be the odds <u>against</u> winning on the first throw? Are they the same as the odds in favor of losing?

EXPERIMENT NO. 9

Apparatus: Two dice)

1. Two dice can come up in any one of 36 ways. Make a table of these ways and include the total number of ways each can come up.
2. Plot these numbers on a bar graph, leaving a blank space between each bar.
3. Roll the dice 3·36 = 108 times. Record the results.
4. Plot your results in a frequency distribution chart.
5. After you find the total of each number that appeared in your throws, divide each one by 3 to get the average over 36 throws.
6. Make a table showing this average.
7. Plot the data on the same graph as the one used in step 2, using a different color.
8. How close are the two sets of data?

EXPERIMENT NO. 10

Apparatus: Styrofoam cup, dice)

1. If you roll two dice 100 times, which total do you think will come up most often? Why? Give reasons to support your guess.
2. Roll both dice from the cup for 100 times.
3. Record the sum of the two numbers showing each time on a frequency distribution chart.
4. After you have completed the chart, make a bar graph of your results.
5. What sum is the mode (the number appearing most)?
6. How does this compare with your prediction?
7. Where does the expression "lucky seven" come from?

EXPERIMENT NO. 11

(Apparatus: A pop bottle)

1. Get at least 3 persons in a group and sit in a circle on the floor. If you were to spin a pop bottle once, what do you think the chances are that it will be pointing at any particular person?
2. Select someone to begin. Spin the bottle. Record who is the closest to where it is pointing. Then that person spins it. Do it 60 times.
3. Draw a frequency distribution chart reflecting the data.
4. Draw a bar graph reflecting the data.
5. Did the bottle favor one person?
6. Do you think it would make a difference if one person spun it each time? Why or why not?
7. If more people played, what would be the result?

EXPERIMENT NO. 12

(Apparatus: Coin)

1. Flip a coin and move one place to the right if it comes up heads and one to the left for tails.
2. Flip the coin 10 times, following the directions it gives you each time. How far from "S" are you?
3. Do this whole procedure ten times. Do you always come out at the same place?
4. Pool your results with the other members of your group and make a bar graph showing how many times you ended up at a given place.
5. Is your graph symmetric about the starting point, S?
6. Calculate the average distance gone from the starting point for your data and your group's.

*	*	*	*	*	S	*	*	*	*	*
5	4	3	2	1		1	2	3	4	5

LESSON 8: BINOMIAL PROBABILITY

Objective:

The student will be able to tell the probability of an event in which several coins are tossed.

Example:

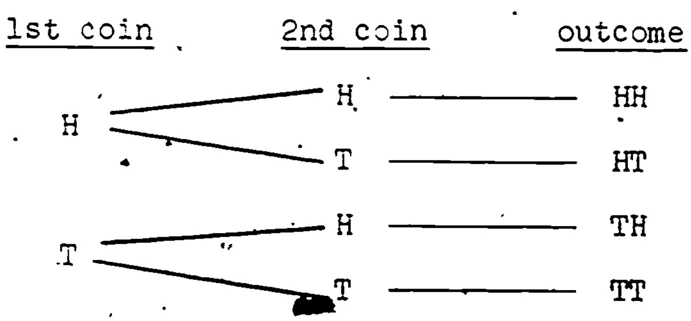
Suppose we toss a coin once. Then $P(H) = P(T) = \frac{1}{2}$. This is called a binomial probability because it has two possible outcomes. Suppose we toss two coins at once. What are the chances that each will be a head? That each will be a tail?

Method:

If we use a tree diagram when discussing binomial probability, it is easier to see our outcomes.

The first coin (of course order does not matter here) could come up H or T. If the first coin is an H, what could the second be? Or if the first coin is a T, what could the second one be?

This is easily kept track of with a tree diagram:

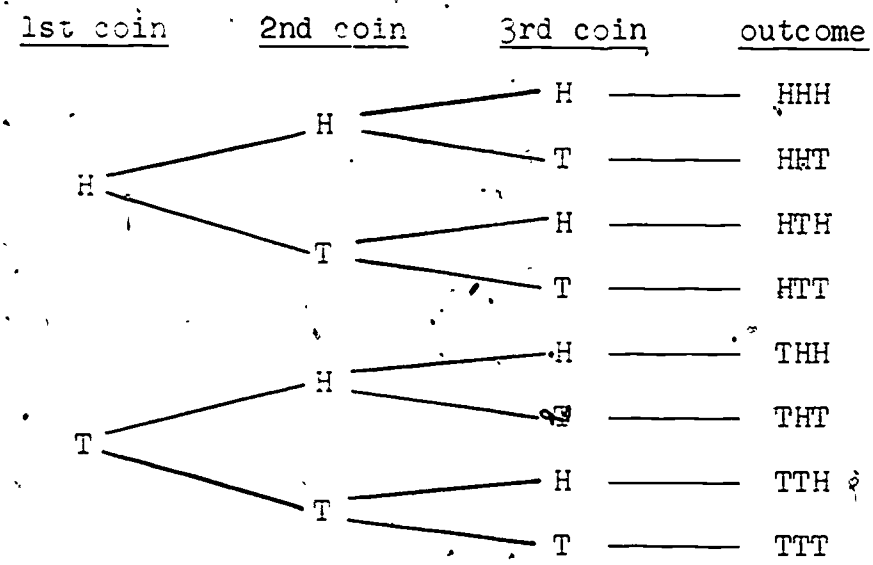


Fill in the following table using the tree diagram above:

no. of HEADS	2	1	0
no. of times	1	2	1
probability	1/4	2/4	1/4

Application:

Suppose we have 3 coins. Again use a tree diagram:



Fill in the table:

no. of HEADS	3	2	1	0
no. of times	1	3	3	1
probability	1/8	3/8	3/8	1/8

Problems:

32. Use the above method to figure out the possibility of there being only one boy in a family of 3 children, assuming that the probability of a boy or a girl is equal.

33. Recalling that one coin has 2 possible outcomes, two coins have 4 possible outcomes, and 3 coins have 8 possible outcomes, how many possible outcomes would you expect 4 coins to have?

34. Construct a tree diagram and a table, similar to those in the lesson, for 4 coins. What is the probability of 4 heads? Of 2 heads?

35. Probability plays an important role in heredity in plants and animals. Let us take a simple example of plant heredity, one of the height of the plant. Each parent plant will contribute a height gene to its offspring. Hence, the offspring will have two genes to determine its height. Let's represent each gene with a letter, T for Tall, t for short. Furthermore, assume that tall will completely dominate short. Hence the combinations TT and Tt will be a tall plant. Only tt will be short. Suppose our parent generation has the following genes: TT and tt. The following diagram shows all possible outcomes of the second generation:

SECOND GENERATION

		2nd parent	
		t	t
1st parent	T	Tt	Tt
	T	Tt	Tt

What is the probability of an offspring in the second generation being short? Tall?

Suppose we cross two plants in the second generation. Fill in the table:

THIRD GENERATION

		2nd parent	
		T	t
1st parent	T	TT	?
	t	?	?

What is the probability of getting a short plant in the third generation? A tall plant? What are the odds in favor of a short plant in the third generation?

LESSON 9: PASCAL'S TRIANGLE

Objective:

The student can use Pascal's Triangle to solve problems in which a tree diagram is not practical.

Example:

Suppose we want to know the probability of a family with ten children having all girls or even three girls and seven boys. A tree diagram is difficult to construct for such large numbers as you can see by examining the tables of the last lesson for one, two, three, and four coins.

If we start each row with the number of ways to get all heads and end with the number of ways to get no heads (all tails), we get:

one coin				1		1			
two coins			1		2		1		
three coins		1		3		3		1	
four coins	1		4		6		4		1

Method:

Are there any patterns in this table? What might be the numbers of a row for 5 coins? 6, 7, or 8?

Look again at the row for three coins. What are the total number of outcomes? How could this row give it to us? Where would the number of outcomes for getting 2 heads and 1 tail be found?

Application:

Fill in this table (Pascal's Triangle) up to ten coins and apply it to the problem of the family with ten children. After filling in the table, add a column on the end for Total Outcomes. In the first row there are 2 outcomes, in the second 4, in the third 8, in the fourth, 16. What pattern is developing here?

Problems:

36. If 11 coins are tossed at once, what is the probability that 5 will be heads and 6 will be tails? What is the probability 5 will be tails and 6 will be heads? What is the probability they will all be heads?

37. Suppose 6 coins are tossed. What is the probability 3 will be tails?

38. If a family has 8 children, what line would we look at in our table to see what the chances of having 3 girls and 5 boys would be?

39. Suppose I have tossed a coin 6 times and it has come up heads every time. What is the probability it will be a head on the next throw if it is a fair coin?

40. Concealed in Pascal's Triangle are the "sequence of squares" and the "Fibonacci Sequence". Can you find them? (Hint: You have to perform some additions to bring them out of hiding.)

EXPERIMENT NO. 13

(Apparatus: cup, 5 pennies)

1. Using the cup, toss the five pennies 100 times. Keep track of the number of combinations of Heads and Tails each toss.
2. Record your results in a table like this:

5(H)	4(H)	3(H)	2(H)	1(H)	0(H)

3. After you have completed the table, take a sheet of paper. Construct a bar graph and answer the following questions:
   a. Which combinations occurred the most? Least?
   b. What percent of the total number of tosses did you get for each of the possible outcomes?
4. Could you predict the results using Pascal's Triangle?
5. How do your results compare to these predictions?
6. Each person must get his own data.

EXPERIMENT NO. 14

(Apparatus: Waste basket, paper balls)

1. Get a partner and roll 10 sheets of paper into balls.
2. Set the waste basket in the middle of the floor. Stand about 5 paces away from it. Throw a ball of paper and try to get it in. Record the results.
3. Repeat the above 50 times. Have your partner retrieve your shots.
4. Now let your partner try it.
5. Move the waste basket into a corner and repeat the above.
6. Calculate the percentage of shots you missed and the percentage of the shots you made in each spot.
7. Is the experiment a "fair" one? Does the position of the basket make a difference? Could you compare it to flipping a coin for "fairness"? List factors which might influence the outcome of this experiment.

LESSON 10: INDEPENDENT & DEPENDENT EVENTS

Objectives:

The student can find the probability of several events occurring by multiplying the probabilities of the individual events together. The students can use this working definition to find the outcome of a series of events, whether they are dependent or independent.

Example:

If we toss a coin 9 times and if it is tails every time, what is the probability that on the next throw it will be tails?

Since the previous tosses have nothing to do with the next toss, we call these events independent.

Suppose we pick a card from a deck of playing cards. (A good aid here is an oversize deck of cards available at most game stores.) What is the P(9 of hearts)? P(9 of any suit)?

Replace the card in the deck. Pick another. What is P(6 of clubs)? P(a heart)? Does it matter what happened on the previous draw?

Suppose a card is drawn, replaced, and then a second card is drawn. What is P(both cards are hearts)?

Method:

Apply the fundamental counting principle to find P(both cards are hearts) $= \frac{13}{52} \cdot \frac{13}{52} = \frac{1}{16}$.

Suppose that the first card had not been put back into the deck. Then the outcome of the second would "depend" on what happened on the first draw. These are called dependent events since what happens previous to an event influences what happens next.

Looking again at our example, if there is no replacement, we need P(Heart, Heart). Of course on the first draw, P(Heart) = $\frac{13}{52}$.

Since we are interested in P(Heart, Heart), assume we got a heart on the first draw. How many hearts are left? How many cards are left? What is P(Heart) on the second draw? $(\frac{12}{51})$

Then the P(Heart, Heart) = $\frac{13}{52} \cdot \frac{12}{51} = \frac{1}{17}$

It is necessary to stress the point that, indeed, we may not have gotten a heart the first time; but the only case we are interested in is the series of events that yields a heart on the first draw and also a heart on the second draw. That is the only case we consider.

Application:

What is the probability of choosing three cards from a deck, without replacement, and getting "three of a kind"? Since it does not matter what the first card is, the probability of getting the first card is $\frac{52}{52}$ (=1). This may seem strange at first, but remember, the first card can be any card. It is the next two that have to match it. Since there are only 4 of each rank of card and since one of them has already been drawn, how many are left? (3) And how many cards total? (51)

So the probability that the second card matches the first is $\frac{3}{51}$.

And the probability that the third card matches the first is $\frac{2}{50}$.

Hence:

$$P(3\text{ of a kind}) = \frac{\overset{1}{\cancel{52}}}{\underset{1}{\cancel{52}}} \cdot \frac{\overset{1}{\cancel{3}}}{\underset{17}{\cancel{51}}} \cdot \frac{\overset{1}{\cancel{2}}}{\underset{25}{\cancel{50}}} = \frac{1}{425}$$

Problems:

41. You are given a jar with 3 red, 2 blue, 5 yellow, and 6 green marbles:
    a. If a marble is drawn, replaced, and another is drawn, what is the probability both marbles were red?

b. If a marble is drawn but is not replaced, and another is drawn, what is the probability both marbles were red?

c. 3 marbles are drawn without replacement. What is the probability that 2 are yellow and 1 is blue?

42. A pair of dice is thrown twice. The result of the first throw is a 7. What is the probability that a 7 will appear on the second throw? Does the second throw depend in any way on the first throw?

43. In each of the following, assume you are given a standard deck of playing cards and that all drawing is done without replacement. What is the probability of drawing:

a. 4 hearts in 4 draws?

b. The jack of diamonds, followed by the king of spades?

c. A pair of aces?

d. Any pair? (Remember that the first card can be any card.)

e. A Royal Flush in 5 draws. (A royal Flush is Ace, King, Queen, Jack, Ten in any one suit. Remember that the first card can be in any suit, but it must be "Royal".)

EXPERIMENT NO. 15

(Apparatus: Box, blue and red marbles)

1. In a box put 10 red and 10 blue marbles.
2. How many marbles do you think you will have to draw, without replacing them, to get two red ones?
3. Try drawing marbles out, one at a time, and count how many you draw to get two red ones.
4. Replace the marbles and repeat the process 49 times.
5. What is the average number of marbles you had to pick to get two red ones? How does this compare to your guess?
6. What would be the largest number of marbles you could pick out before you got two red ones?
7. Would the results change if you simply had to draw two marbles of the same color (either red or blue)? Why or why not?

EXPERIMENT NO. 16

(Apparatus: Paper, toothpicks)

1. Draw parallel lines, equally spaced, on a sheet of cardboard spaced twice the length of a toothpick.
2. Allow a toothpick to fall anywhere on the sheet; do this 100 times.
3. Keep track of the number of times the toothpick touches a line.
4. Determine the ratio between the number of tosses of the toothpick and the number of times it touches any line.

$$\frac{\text{total no. of tosses}}{\text{no. of times it touched a line}}$$

5. Compare this ratio to 3.14. Does this ratio look familiar? What does it have to do with this experiment?

EXPERIMENT NO. 17

(Apparatus: Playing cards)

1. Take 8 cards from a normal deck of playing cards so that you have 4 pairs. (It does not matter which ones they are.)
2. Shuffle them and choose 2 cards, one at a time.
3. What is the probability that the 2 cards you have chosen will be a pair?
4. Put back the 2 cards you have chosen, shuffle them, and choose 2 more until each person in your group has repeated the exercise 50 times.
5. Now combine your data. How many times did you get two cards of the same rank? (A pair)
6. What percent of the total is this number?
7. Make a frequency distribution chart and a bar graph showing the results of your data. (For the purposes of this experiment, assume that the order in which the cards were chosen does not matter.)

LESSON 11: COMPLEMENTARY PROBABILITIES

Objective:

The student can use the definition of complementary probability to solve elementary problems.

Example:

Suppose we take a group of thirty people. What are the chances two of them have a birthday on the same day? Well, it figures out to be about 70%!

Method:

What would be the P(rolling a 5) on a pair of dice? What would be P(not rolling a 5)?

What is P(rolling a 5) + P(not rolling a 5)?

These are called complementary probabililities because any time we add the probability of an event occurring to the probability that it will not happen we get 1. This is intuitively obvious since we know that either an event will or will not happen. We can rearrange this identity to help us solve problems.

P(event happens) + P(event does not happen) = 1.
P(event does not happen) = 1 - P(event does happen).
P(event does happen) = 1 - P(event does not happen).

Application:

P(2 people share birthday) = 1 - P(2 people do not). If we consider two people, how can we choose their birthdays so they do not coincide? The first person can have any day, i.e., $\frac{365}{365}$. The second can have only his choice of $\frac{364}{365}$ days.

P(2 share) = 1 - P(2 do not share)

$$= 1 - \left(\frac{365}{365} \cdot \frac{364}{365}\right)$$

$$= 1 - 0.997 = .003.$$

Suppose we have 3 people:

$$P(\text{2 share}) = 1 - P(\text{2 don't share})$$
$$= 1 - \left(\frac{365}{365} \cdot \frac{364}{365} \cdot \frac{363}{365}\right)$$
$$= 1 - 0.992 = .008.$$

Suppose we have 30 people:

$$P(\text{2 share}) = 1 - \left(\frac{365}{365} \cdot \frac{364}{365} \cdot \frac{363}{365} \cdots \frac{336}{365}\right)$$
$$= 1 - 0.294 = 0.706 \approx 70\%$$

Footnote:

At this point it might be useful to discuss experimental results vs. theoretical results if such a discussion has not already taken place. A good way to lead into such a discussion would be to do a survey in a class with thirty people to see if you can get the desired results for the birthday problem.

Problems:

44. The probabilty of rolling a 7 on a pair of dice is $\frac{6}{36} = \frac{1}{6}$. What is the probability of not rolling a 7? Show two different methods for confirming your answer.

45. If a die is rolled once, what is the probability it will not come up 1?

    If a die is rolled twice, what is the probability it will not come up 1 either time?

46. If you draw a queen from a deck of cards and do not replace it, what is the probability you will not draw a queen on the second draw?

47. If 5 coins are tossed, what is the probability that they will not all be heads?

ANSWERS TO PROBLEMS

1. $21 \cdot 21 \cdot 21 \cdot 10 \cdot 10 \cdot 10 = 9{,}261{,}000$ license plates
2. $10^9 = 1{,}000{,}000{,}000$ numbers

   $10^9 \cdot 10 = 10^{10} = 10{,}000{,}000{,}000$ numbers

   $10^9 \cdot 26 = 26{,}000{,}000{,}000$ numbers
3. $2^{10} = 1024$ choices
4. $5! = 120$

   $8! = 40{,}320$

   $3! \cdot 4! = 144$

   $12! = 479{,}001{,}600$
5. $6! = 720$ arrangements
6. $7! = 7 \cdot 6! = 5040$ ways
7. $8 \cdot 7 \cdot 6 = 366$ ways
8. $8! = 40{,}320$ ways
9. $9 \cdot 8 \cdot 7 \cdot 6 = 3024$ combinations
10. $\frac{6!}{3!} = 120$ arrangements

    $\frac{7!}{3!} = 840$ arrangements.

    $\frac{6!}{5!} = 6$ arrangements
11. $\frac{5!}{3!} = 20$ numbers
12. $\frac{5!}{4!} = 5$ ways
13. $\frac{10!}{(2!2!2!)} = 453{,}600$ different arrangements

    $\frac{12!}{(3!2!2!2!)} = 9{,}797{,}200$ different arrangements

14. $\frac{6!}{4!2!}$ = 15 different signals

15. a. $\frac{4!}{4!}$ = 1 way b. $\frac{4!}{3!}$ = 4 ways c. $\frac{4!}{2!2!}$ = 6 ways

d. $\frac{4!}{3!}$ = 4 ways e. $\frac{4!}{4!}$ = 1 way f. $2^4 = 16$

g. It should agree.

16. For 5 coins the answers would be (going from all heads to all tails): 1, 5, 10, 10, 5, 1 = 2^5 = 32 outcomes.

17. $\frac{100\cdot 99\cdot 98\cdot 97\cdot 96\cdot 95}{6!}$ = 1,192,052,400 ways

18. $\frac{12\cdot 11\cdot 10\cdot 9\cdot 8}{5!}$ = 792 ways

19. $\frac{52\cdot 51\cdot 50\cdot 49\cdot 48}{5!}$ = 2,598,960 ways

20. a. $\frac{1}{52}$ b. $\frac{4}{52} = \frac{1}{13}$ c. $\frac{13}{52} = \frac{1}{4}$ d. $\frac{26}{52} = \frac{1}{2}$

21. a. $\frac{1}{6}$ b. $\frac{3}{6} = \frac{1}{2}$ c. $\frac{2}{6} = \frac{1}{3}$ d. $\frac{0}{6} = 0$

22. $\frac{1}{6} \cdot 6 = 1$ time

a. P(even number) = $\frac{1}{2}$

$\frac{1}{2}\cdot 30 = 15$ times

b. $\frac{1}{6}\cdot 100 \approx 16.67$ times $\approx$ 17 times out of 100 = 17%

23. a. $\frac{16}{50}$ b. $\frac{16}{52} + \frac{10}{50} = \frac{26}{50}$ c. $\frac{16}{50}$ d. 32, 20, 28, 20

24. a. $\frac{5}{36} \approx 14\%$

b. $\frac{1}{36} \approx 3\%$

c. $\frac{12}{36} = \frac{1}{3} \approx 33\%$

d. $\frac{0}{36} = 0\%$

26. $\frac{8}{36} \approx 22\%$

27. $\frac{4}{36} \approx 11\%$

28. $\frac{24}{36} \approx 67\%$

29. You are more likely to lose since a 7 is more likely to come up than any one of those numbers.

30. 8 to 28 = 2 to 7 in favor of winning
    4 to 32 = 1 to 8 in favor of losing

31. odds against an event = $\frac{\text{number of unfavorable ways}}{\text{number of favorable ways}}$

    odds against winning on first throw = 7 to 2
    They are not the same as the odds against losing.

32. $\frac{3}{8}$

33. 16 outcomes

34. $P(4H) = \frac{1}{16}$

    $P(2H) = \frac{6}{16} = \frac{3}{8}$

35. $P(\text{tt in second generation}) = \frac{0}{4} =$

    $P(\text{TT in second generation}) = \frac{4}{4} = 1$

    Third generation:

TT	Tt
Tt	tt

    $P(\text{short in third}) = \frac{1}{4}$

    $P(\text{Tall in third}) = \frac{3}{4}$

    Odds in favor short in third = 1 to 3

36. $P(5 \text{ Heads}) = \frac{462}{2048} \approx 23\%$

    $P(6 \text{ Heads}) = \frac{462}{2048} \approx 23\%$

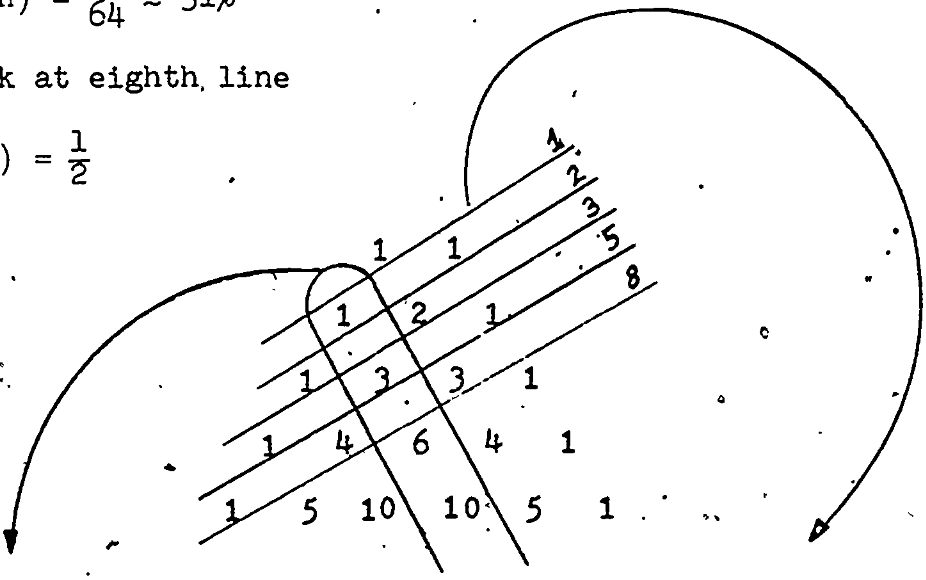
    $P(11 \text{ Heads}) = \frac{1}{2048} \approx .0005$

37. $P(3H) = \frac{20}{64} \approx 31\%$

38. Look at eighth line

39. $P(H) = \frac{1}{2}$

40.



Sequence of Squares:

$1 + 3 = 4$

$3 + 6 = 9$

⋮

Fibonacci Sequence: (1,1,2,3,5,8...)

$1 = 1$

$1 + 1 = 2$

$1 + 2 = 3$

$1 + 3 + 1 = 5$

⋮

41. a. $\frac{3}{16} \cdot \frac{3}{16} = \frac{9}{256} \approx 4\%$

b. $\frac{3}{16} \cdot \frac{2}{15} = \frac{6}{240} \approx 3\%$

c. $\frac{5}{15} \cdot \frac{4}{15} \cdot \frac{2}{14} = \frac{40}{3150} \approx 1\%$

42. $\frac{6}{36} = \frac{1}{6} \approx 17\%$ No, it does not depend in any way on the first throw.

43. a. $\frac{13}{52} \cdot \frac{12}{51} \cdot \frac{11}{50} \cdot \frac{10}{49} = \frac{17,160}{6,497,400} \approx .003$

b. $\frac{1}{52} \cdot \frac{1}{51} = \frac{1}{2652} \approx .0004$

c. $\frac{4}{52} \cdot \frac{3}{51} = \frac{12}{2652} \approx .005$

d. $\frac{52}{52} \cdot \frac{3}{51} = \frac{3}{51} \approx .058$

e. $\frac{20}{52} \cdot \frac{4}{51} \cdot \frac{3}{50} \cdot \frac{2}{49} \cdot \frac{1}{48} = \frac{480}{311,875,200} \approx .000002$

44. P(not rolling a 7) $= \frac{30}{36} = \frac{5}{6}$

or

P(not rolling a 7) = 1 - P(rolling a 7)

$= 1 - \frac{1}{6} = \frac{5}{6}$

45. a. $\frac{5}{6}$

b. $\frac{5}{6} \cdot \frac{5}{6} = \frac{25}{36} \approx 70\%$

46. P(not drawing a queen on second draw) = 1 - P(will draw a queen)

$= 1 - \frac{3}{51}$

$= \frac{48}{51} \approx 94\%$

47. P(not all heads) = 1 - P(all heads)

$= 1 - \frac{1}{32}$

$= \frac{31}{32} \approx 97\%$

The following were members of the Guidelines Committee for Quality Mathematics Teaching during the development of this monograph

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Richard Debelak	Iron Mountain
Theresa Denman	Detroit
Herbert Hannon	Kalamazoo
Dianne Hewitt	Traverse City
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