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ABSTRACT
        This document is one of six which set forth the
mathematics components of the Project SEARCH Articulated Curriculum
developed by the Utica (New York) City School District. Each volume
deals with a broad area of mathematics and lists objectives related
to that area for all grades from K through 12. Each objective listed
is described first in general terms and then in terms of specific
skills which students should exhibit. This volume addresses
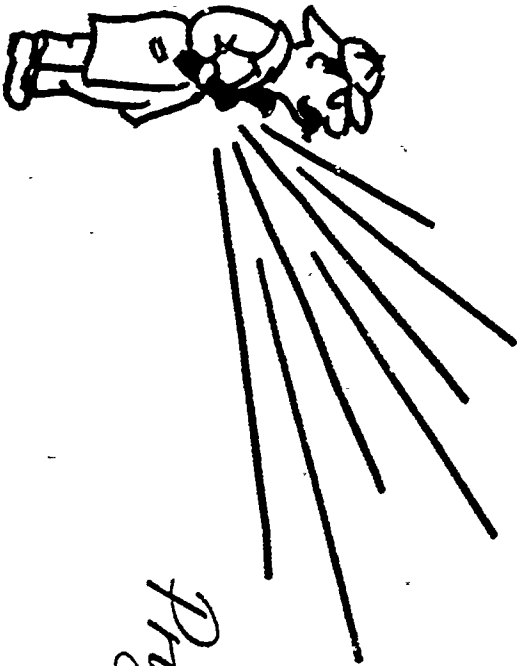techniques of solving problems throughout the curriculum. (SD)

Utica City School District

Articulated Curriculum

Project Search

1975

Draft Copy

This Articulated Curriculum is being printed and bound in this manner to provide for on-going revision. This also serves as evidence of work completed during Phase III of Project SEARCH.

MATHEMATICS

K - 12

Problem Solving

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PROJECT SEARCH

ARTICULATED CURRICULUM

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MRS. ROSE DANELLA and NORMAN I. SIEGEL, both former Board Members deserve
special mention for all their efforts on behalf of Project SEARCH.

UTICA CITY SCHOOL DISTRICT
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MATHEMATICS

The student will know:

Equation Solving

- the relationship of numerals to the number sentence.

- that a problem has a solution.

- the concept of larger and smaller.

- the meaning of problem solving vocabulary.

- that a graph represents comparisons.

- that many Math problems will arise from classroom activities.

- that different problems require different operations.

- the meaning of problem solving vocabulary.

Graphing

- that graphs represent comparisons.

Equation Solving

- that word problems arise from classroom activities.

- that numbers make patterns.

PROBLEM SOLVING

The student will:

- substitute numerals and pictorial representations found in problems.

- orally list solutions.

- compare and contrast concept of larger and smaller.

- arrange materials and orally identify "long, longer, longest," more than, less than, etc.

- construct a bar graph.

Grade 1

- explore problems based on classroom activities.

- combine separate objects as they relate to operations.

- orally identify "on-under" "long, longer, longest" "more-than" "less-than" using various materials.

- identify number patterns in problem solving equations.

- construct simple graphs.

Grade 2

- orally solve problems based on classroom activities.

- demonstrate understanding of number patterns.

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- that word problems can be translated into number sentences.

- that number lines can be used to translate word problems into number sentences.

- that problems can be solved using mental computation.

- the meaning of properties of order and grouping.

- that graphs are used for comparison.

Graphing

Equation Solving

- that open sentences can be used for problem solving.

- that problems can be solved through a series of steps.

- that a graph is used for comparison.

Equation Solving

- that estimation before computation and determination of enough information is necessary before solving problems of comparative relationships and averages.

Grade 2

- translate word problems into number sentences.

- use number lines to solve problems

- solve word problem orally.

- be given a problem regarding number order and will demonstrate understanding by finding the correct solution.

- construct and interpret simple graphs.

Grade 3

- convert word problems into number sentences.

- solve problems using a series of steps using basic operations.

- solve word problems by using a number line, line graph, picture graph, and bar graph.

Grade 4

- estimate before and after computation.

- solve problems of comparative relationships.

- solve problems of finding averages.

- solve verbal problems involving reading--no numbers, unneeded numbers, and not enough information.

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- that word problems can be translated into number patterns.

- that word problems can be translated into graphs.

Graphing

- the proper steps in writing an equation of a given problem.

Equation Solving

- the meaning of the Associative Property and Commutative Property.

- the meaning of an indirect number sentence.

- that graphs show visual representatives of facts and figures.

Graphing

- that there are various types of graphs.

Equation Solving

- that a direct sentence is one where the answer is missing while an indirect sentence has other parts missing. A closed sentence is either true or false while an open sentence is neither true nor false and contains a place holder.

- that a word problem can be translated into a number sentence using a parentheses to indicate the order of operation.

Grade 4

- identify correct number patterns from various problems.

- interpret, determine, construct a given problem into a line graph or bar graph.

Grade 5

- write an equation in two steps or more to solve the problem.

- write more than one equation to solve one problem using the associative and commutative property.

- determine the necessary parts of an indirect number sentence.

- construct a bar graph and line graph using vertical and horizontal scales and label the vertical and horizontal scales correctly.

- select the correct graph to solve the problem.

Grade 6

- solve direct, indirect, closed and open sentences from word problems.

- solve 3-step word problems using parentheses to write number sentences.

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Equation Solving

- that a ratio shows the correspondence between the numbers of two sets (3 to 5 or 3:5 or $\frac{3}{5}$)

- that operations can be performed on integers to solve problems. (e.g. temperature)

- solve word problems using ratios.

- solve word problems involving negative integers.

Graphing

- that graphs can be used to visually represent data in a convenient and organized manner.

- the need to find only the essential information in a problem.

- plot information on line and bar graphs.

Equation Solving

Grade 7 - 8

- the means of arriving at a suitable number sentence (equation) for the solution of a verbal problem.

- discriminate between the necessary and extraneous information in a problem situation and list only the information pertinent to the problems solution.

- the algorithm necessary to the solution of various categories of verbal problems.

- translate the essential information of a problem into an equation using word clues such as:

  "and" +
  "difference" -
  "product" x
  "is" =
  "is not" $\neq$

- that the solution to some problems requires but a single operation, while other problems require more than one operation to attain the solution.

- solve a series of verbal problems given the equation.

- identify from a list of problems those problems whose solution require but one operation and those problems which require more than one operation to attain the solution.

- that some problems by their nature lend themselves to the use of a proportion to attain a solution.

- translate a series of problems in which proportions may be utilized into number sentences (equations).

- how to translate various kinds of problems such as coin problems, age problems, and mixture problems into number sentences in order to attain a solution.

- solve a series of problems such as coin problems, age problems, and mixture problems.

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- certain operations take precedence over others (order of operations).

U.E. Foundations

- the method of solving an arithmetic fractional equation.

- the method of solving an arithmetic equation using whole numbers.

- the method of solving the three basic percentage problems.

- how to solve equations involving: one operation; two operations; combinations of like terms; variables on opposite sides of equal sign; fractional equations; quadratic equations; systems of equations.

- how to solve word problems involving: number; investment; digit; mixture; motion; age; work; consecutive integers.

- how to graph a point on a coordinate system given the coordinates of the point.

- how to find the coordinates given a point on the coordinate system.

- how to graph the solution set of linear equations and inequalities.

- how to find the solution set of a system of equation and inequalities by graphing.

- solve a series of number sentences (equations) following correct operational order.

Grade 9

- solve an arithmetic fractional equation.

- solve an arithmetic equation involving whole numbers.

- solve the three basic percentage problems

- solve equations involving: one operations; two operations; combination of like terms; variables on opposite sides of equal sign; fractional equations; quadratic equations; systems of equations.

- solve word problems of this type: number; investment; digit; mixture; motion; age; work; consecutive integers.

- graph the point given the coordinates of the point.

- give the coordinates of a point on a coordinate system.

- find the solution set of linear equations and inequalities by graphing.

- find the solution set of a system equations and inequalities by graphing.

Equation Solving

- that algorithms are needed to solve problems.

- that various programming techniques (skills) are used by the programmer in order to solve a problem.

- solve a problem using an algorithm.

- develop one program which utilizes a programming technique.

- that computer solutions should always be checked in order to insure that the solution is correct and reasonable.

- that there are a variety of techniques for debugging a program.

- analyze a computer solution in order to determine its "reasonableness."

- debug a given program using a technique learned.

- that a graphic (visual) presentative of any program can be done through flowcharting.

- draw a flowchart for a given problem.

- that a computer can be programmed to make drawings or to draw curves on a coordinate axis system.

- develop a program which instructs the computer to make a drawing.

- angle relationships involving supplementary, complementary, vertical, acute, right, obtuse, etc. angles.

- solve a first degree equation involving relationships between the measure of angles in the given problem.

Advanced Foundations

- the different types of equations.

- a. solve a linear equation in one variable.
- b. solve a proportional equation.

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- a. the quadratic equation $ax^2 + bx + c = 0$
  b. the quadratic formula $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
  c. the discriminant.
  d. the relationship between roots and coefficients.
  e. the quadratic inequalities.

- the following laws of logarithms:
  a. $\log ab = \log a + \log b$.
  b. $\log a/b = \log a - \log b$.
  c. $\log a^n = n \log a$.
  d. $\log \sqrt[n]{a} = 1/n\,(\log a)$

- the method of interpolation of numbers to 4-figure accuracy and values of functions of angles to the nearest minute.

- the following laws of exponents:
  a) $x^a \cdot x^b = x^{a+b}$
  b) $x^a \cdot y^a = (xy)^a$
  c) $x^a \div x^b = x^{a-b}$, $(a > b, x \neq 0)$
  d) $(x^a)^b = x^{ab}$

- the law of sines and law of cosines

- the basic trigonometric identities.

—a. solve the quadratic equation by factoring.
  b. solve a quadratic equation using the quadratic formula

  c. use the discriminant in determining the nature of the roots of a quadratic equation.
  d. derive the quadratic equation from the roots and coefficients.
  e. solve quadratic inequalities.

- evaluate the following expression by using the laws of logarithms.

$$x = \sqrt{\frac{(8.4)\,(7.3)}{(s/n\ 57^\circ\ 10')\,(3)}}$$

- interpolate and compute the values of N to the nearest hundredth using logarithms.

$$N = \frac{(5.124)\ \sqrt[3]{7.298}}{\sqrt{\cos 50^\circ\ 13'}}$$

- solve exponential equations of the type
$4^x = 8$, $3^{x+1} = 27^x$, $N^{1/3} \cdot N^{2/3} = 4^2$

- a. apply the law of sines and law of cosines to verbal problems.
  b. determine the area of triangles using the law of sines.

- a. derive these identities.
  b. use them in the proofs of other identities.

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- the trigonometric identities of the single angle and the double angle.

- that inequalities may be true for all, some, or no elements of the replacement set.

- the definition of absolute value.

- the definition of inverse trigonometric form.

PROBABILITY & STATISTICS

Introduction

- the definition of probability.

- that a sample space may be made to show complementary events.

- that a sample space may be made to show mutually exclusive events.

- that a sample space may be made to show independent events.

- the formula for the probability for the union of two sets.

- the formula for complementary events of a sample space.

-solve trigonometric equations involving two or more functions.
- express any function of an angle in terms of any other function of that angle.

- solve three inequalities for a particular value.

- solve inequalities algebraically, using the definition of absolute value.

- express the roots of a trigonometric equation in inverse trigonometric form.

Grade 11 - 12

- apply the definition of probability to answer a question.
- make a sample space showing complementary events.

- make a sample space showing mutually exclusive events.

- make a sample space showing independent events.

- apply the formula for the union of two sets of a sample space to a specific problem.
$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

- apply the formula for complementary events to a specific problem.
$$P(A) = 1 - P(\overline{A})$$

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Introduction

- the formula for the intersection of two sets of a sample space.

- the definition of independent events.

- apply the formula for the intersection ... of two sets of a sample space.

$$P(A \cap B) = P(A) \cdot P(B / A)$$

- write the formula for independent events.

$$P (A \cap B) = P (A) \cdot P (B)$$

Graphing

- the technique for making random selections

- collect data using random selection techniques.

- the technique for presentation of data.

- organize the data collected.

- that data may be presented by use of a "Frequency Table."

- make frequency table, using the data collected.

- that data may be presented by means of a Dot Frequency Diagram.

- make a Dot Frequency Table using the data collected.

- that data may be presented by use of a cumulative graph.

- construct a cumulative graph using the data collected.

- the technique for finding the quartiles using the cumulative graph.

- find the quartiles using the cumulative graph.

- the technique finding the percentiles using the cumulative graph.

- find certain percentiles using the cumulative graph.

- the technique for making a cumulative polygon.

- make a frequency histogram using graphed data.

- the techniques for misrepresentation of data.

- make a cumulative polygon using the data collected.

- that one may use the Monroe X,Y Plotter to prepare graphs.

- use given graphs to detect deceptive practices.

- that data may be presented by means of a Dot Frequency Diagram.

- construct graphs using the X,Y. Plotter.

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Equations

- that Chebyshev's Theorem may be used to analyze data.

- that a binomial may be expanded.

- that the binomial expansion may be used to solve a probability problem.

- that one may use the binomial distribution tables.

- that one may make histograms using the binomial distribution tables.

- the use of the standard normal curve tables.

- the techniques of changing a normal distribution to a standard normal distribution.

Computer Programming

- the methods and techniques relating to writing computer programs that may be used in problem solving.

- the methods how to properly prepare a computer program flow chart.

- the applications of computers to solving problems relating to conic sections.

- the preparation of a computer program that can apply to conic sections.

- apply Chebyshev's Theorem using collected data.

- a. expand a binomial
  b. find and simplify a specific term of a binomial expansion.

- use the binomial expansion to solve a probability that a baseball player hits fewer than 4 homeruns in his next 10 trips to bat.

- solve a probability problem using the binomial distribution tables.

- make a histogram using the binomial tables.

- use the standard normal curve tables to find the area under the curve between two abscissas.

- change a normal distribution to a standard normal distribution.

Grade 12

- apply the computer to problem solving of conic sections related types.

- prepare input data from a given equation (quadratic) that typifies a specific conic section.

- present this raw data to the computer, either manually (keyboard) or via punched cards.

- operate the computer properly and secure a printout.

- the use of such programs to be put to actual use in problem solving.

- the proper interpretation of output results of the computer output data.

- the method of proving a problem by direct, indirect, and inductive techniques.

- the method of sketching the graph of a given relation

- the definition of permutation and combination.

- the meaning of random variables and probability distribution.

- binomial distribution.

- the method to determine the relative maximum, relative minimum and inflection points of a graph.

- interpret output results into meaningful algebraic format.

- utilize output data to construct graphs representative of the specific related conic section.

<u>Grade 12 X</u>

- solve a given problem using direct, inductive reasoning.

- graph $F(x) = 2 = x + 1$ when $x = 3=x=3$

- use the formula $Pr = \dfrac{n}{(n-r)}$ to find the solution for the given problem with 8 flags of different color, how many possible arrangements of 3 flags can be made.

- find the probability distribution of X given: 2 dice, letting X be the number shown when $(X-2, 3, \ldots, 12)$

- use the formula $C = Pn \, r = \dfrac{n}{r-(n-r)}$ to find the probability of drawing 4 clubs from a well-shuffled deck.

- find the probability distribution of X given: 2 dice, letting X be the number shown when $(X-2, 3, \ldots, 12)$

- find the probability that double heads will appear 4 or more times when 2 coins are tossed simultaneously six times.

- find the maximum value of: $F = (x,y_ ; y = x - 3.125x^2$

10^6

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- the application of derivative formulae to verbal problems: extreme value problems; velocity and acceleration problems; related rates of change problems.

- the application of integration formulae to verbal problems; area and volume problems; growth and decay problems; harmonic motion problems.

- apply the derivative formulae in solving verbal problems: extreme value problems; velocity and acceleration problems; related rate problems.

- apply the appropriate integration formulae in solving verbal problems: area, volume, growth and decay, and harmonic motion problems.

- the derivation of the derivative formulae: polynomial, trigonometric, exponential, logarithmic function; sum, product and quotient of function; composite functions; inverse trigonometric functions.

- apply the specific derivative formulae: polynomial, trigonometric, exponential, logarithmic functions; sum and product and quotient of functions; composite functions; inverse trigonometric functions.

- the mean value theorems for derivative and integrals.

- apply the mean value theorems for derivatives and integrals.

- the Rolle's Theorem.

- apply Rolle's Theorem.

- the solution of differential equations of the first degree.

- solve differential equations of the first degree.

(b) Graphing

Grade 12

- the techniques of graphing polynomial functions through the fourth degree.

- graph polynomial functions through the fourth degree.

- the techniques of graphing circular trigonometric functions.

- graph circular trigonometric functions.

- the techniques of graphing inverse trigonometric functions.

- graph inverse trigonometric functions.

- the techniques of graphing ellipses, circles, hyperbolae, and parabolas.

- graph ellipses, circles, hyperbolae and parabolas.

Grade 12

- the techniques needed to graph an equation and indicate revolution about a line parallel or perpendicular to the x axis.
- graph an equation and illustrate revolution about a line, parallel or perpendicular to the x axis.

- the graph of a problem to illustrate Rolle's Theorem.
- graph a problem to illustrate Rolle's Theorem.

- the techniques of graphing exponential and logarithmic equations.
- graph exponential and logarithmic equations.

- the techniques of analyzing an equation for asymptotes; x and y intercepts; symmetry; and extreme values before graphing the equation.
- analyze an equation and state: asymptotes, symmetry; x and y intercepts; and extreme values before graphing the equation.

- the techniques of graphing, using polar coordinates.
- graph polar equations.