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ABSTRACT

In October 1975 a conference was convened in Euclid, Ohio, by the Basic Skills group of the National Institute of Education (NIE). Position papers presented by the 33 participants and status reports from 11 agencies involved in mathematics education were received and analyzed. On the basis of this analysis, four topics were identified as issues around which working groups would be formed for the remainder of the conference. These were: (1) classroom instruction and teacher education, (2) curriculum development and implementation, (3) goals for basic mathematical skills and learning, and (4) research priorities. Reports of the individual working groups are presented in this volume. These reports make specific recommendations to NIE: recommendations range from statements of general policy and procedural guidelines which the groups felt should be followed to specific projects for which funding should be available. The final paper in this volume is a summary of the conference with comments on 10 broad problems by the co-chairmen. (SD)

volume II: working group reports

conference on basic mathematical skills and learning

euclid, ohio

nie

u.s. department
of health,
education, and welfare

national institute
of education

THE NIE CONFERENCE ON BASIC MATHEMATICAL SKILLS AND LEARNING

VOLUME II: REPORTS FROM THE WORKING GROUPS

OCTOBER 4-6, 1975
EUCLID, OHIO

ACKNOWLEDGMENTS

We are indebted to the firm of Nero & Associates and, in particular, to Nitina Chavan, for making the necessary arrangements before, during, and after the conference.

We also thank the chairpersons and recorders of the working groups and the members of the ~~steering committee~~. Members of the steering committee were helpful both in suggesting names of potential participants before the conference and in assisting in the conduct of the conference itself. In addition, several steering committee members were involved in the final writing of the working group reports. We are especially indebted to Peter Hilton and Gerald Rising, who were involved in the conference from the initial planning stages.

Most of all, we are grateful to the conference participants and observers, whose thoughts and expertise are imbedded in the working group reports of this volume and in the position papers of volume I.

Edward Esty
NIE Associate, Learning Division

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CONTENTS

BACKGROUND AND ORGANIZATION OF THE EUCLID CONFERENCE

Edward Esty

In October 1975, the Basic Skills Group of the National Institute of Education (NIE) sponsored a 3-day conference on basic mathematical skills and learning in Euclid, Ohio. This introductory section describes the genesis of the conference and its organization.

Background

NIE was created in 1972. Its authorizing legislation requires NIE to:

1. Help solve or alleviate the problems of, and achieve the objectives of, American education.

2. Advance the practice of education as an art, science, and profession.

3. Strengthen the scientific and technological foundations of education.

4. Build an effective education research and development system.

In order to aid in meeting these general objectives, the National Council on Education Research (NIE's policymaking body) approved the creation of five priority areas in December 1973. One of the priority areas, originally called the Essential Skills Program, is the Learning Division of the Basic Skills Group. The purpose of that program is

> To investigate, through research and development, ways to aid all children to obtain skills essential for functioning adequately in school and society.

The initial focus of the program was on reading, and reading continues to be a major area of interest. More recently, however, NIE has put increasing emphasis on mathematics.

It is natural that when an entity called the Basic Skills Group turns its attention to mathematics, there immediately arises the question "What are basic mathematical skills?" In recent years, many persons and groups have addressed this matter (See the References for a list of some of the previous work.) Clearly there is disagreement among persons concerned with mathematics education not only on what "basic mathematical skills" should mean but also on appropriate methods for determining the nature of basic mathematical skills.

As an initial step in planning the Basic Skills Group's work in mathematics, it seemed to us appropriate to hold a conference where issues related to basic mathematical skills could be discussed. Peter Hilton and Gerald Rising agreed to act as cochairmen of the conference. In May 1975, they met with Ned Chalker of the Learning Division and me to formulate plans for the conference.

In addition to the disagreement among educators about "basic mathematical skills" one must also contend with the popular notions that mathematics <u>is</u> arithmetic, and that the most basic mathematical skill, perhaps the only one, is fluency with written computational algorithms. This is certainly not my conception of basic mathematics, but Hilton felt that some participants might misunderstand the purpose of the conference unless its title were explicitly broadened. As a result, the title became "Conference on Basic Mathematical Skills and Learning."

Organization

Before the Conference

At our May 1975 meeting we decided that one sensible way to get thoughtful input from participants would be to ask each to write a position paper so that ideas and opinions could be shared even before the conference began. In the invitation letter, therefore, each person was asked to respond (in about 10 pages) to the following questions:

1. What <u>are</u> basic mathematical skills and learning?

2. What are the major problems related to children's acquisition of basic mathematical skills and learning, and what role should NIE play in addressing these problems?

Every participant except the two representing Federal agencies wrote a paper, and these are published in Volume I (<u>Contributed Position Papers</u>).

During the Conference

The first day of the conference was devoted to 11 short status reports from a variety of groups involved in mathematics education. Five of the groups are supported by the Basic Skills Group. The 11 groups are:

1. Conference Board of the Mathematical Sciences (NACOME report)	James Fey
2. Comprehensive School Mathematics Program	Burt Kaufman and Frédérique Papy
3. Education Development Center (Projects ONE and TORQUE)	Jerrold Zacharias and Mitchell Lazarus

4. Learning Research and Development Center — C. M. Lindvall

5. Mathematics Education Development Center — John LeBlanc

6. National Assessment of Educational Progress — Jane Armstrong

7. Research for Better Schools, Inc. — David Helms and Anna Graeber

8. Southwest Regional Laboratory — Aaron Buchanan

9. National Science Foundation — Joseph Payne

10. Project for the Mathematical Development of Children — Eugene Nichols

11. Wisconsin Research and Development Center for Cognitive Learning — Thomas Romberg

The first day's sessions, like the others, were conducted under the capable leadership of James Wilson; I am grateful to him for undertaking this difficult job.

The morning of the second day was devoted to the presentation and discussion of Gerald Rising's summary of all 33 position papers.

On the basis of his analysis of the papers, Rising also suggested four topics around which working groups could be formed:

1. Classroom Instruction and Teacher Education.

2. Curriculum Development and Implementation.

3. Goals for Basic Mathematical Skills and Learning.

4. Research Priorities.

Participants and observers[1] chose which of the four groups they wished to join. The rest of the conference, except for a final plenary session, was spent in the small working groups.

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[1]A number of observers also attended and participated fully in the conference. The difference between "participants" and "observers" was only that NIE could not financially support observers' attendance. Lists of participants and of observers appear at the end of this volume.

Contents of This Volume

The reports of the working groups form the next four chapters of this volume. The groups were in session for only a day and a half, so their reports are not necessarily so polished and thought out as the individual position papers. On the other hand, a considerable amount of work was done by the third and fourth committees, by mail, well after the conference had ended.

The final versions of the working group reports are the products of the four steering committee members who agreed to take on the task of final authorship. They are, respectively, (1) Gerald Rising, (2) Peter Hilton, (3) Ross Taylor, and (4) James Wilson.

The last chapter of this volume is an essay by Hilton and Rising, expressing their views on what they see as some of the chief problems confronting mathematics education today.

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REPORT OF THE WORKING GROUP ON CLASSROOM INSTRUCTION AND TEACHER EDUCATION

Charles Allen
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Anna Graeber
Joseph Harkin
David Helms
Leon Henkin (Chairperson)
Martin Johnson
John LeBlanc
Donald Ostberg
Gerald Rising
Joseph Rubenstein

This report summarizes the recommendations made by various committee members and supplements that summary with the original motions. The latter provide more detail and occasionally a different perspective from the eventual summary.

Summary Recommendations

We identify the critical focus of mathematics education as the classroom where students and teachers engage in the mutually supportive tasks of teaching and learning. We urge NIE to consider as first priority those activities that will strengthen the classroom instructional program. In fact, we believe that NIE should apply, as a rough measure for evaluation of proposed educational research and development enterprises, the distance from the student. In other words, we urge NIE never to lose sight of the person education is designed to serve -- the student in the classroom.

Our recommendations focus on two specific thrusts related to the improvement at all school levels of learning and teaching: (1) efforts to improve the instructional strategies of teachers, and (2) efforts to improve the preparation of teachers.

Specifically, we recommend three areas for NIE support.

1. Exemplary innovative and creative teacher education programs in mathematics for both prospective and experienced teachers that emphasize:

   a. Clinical procedures for gaining better insight into students' mathematical knowledge.

   b. Diagnosis of learning difficulties in mathematics.

   c. Development of problem-solving teaching techniques.

2. Efforts to develop effective diagnostic tools and appropriate remediation materials. Further, this committee believes that clinical centers need to be identified and/or established, as models that other schools might imitate.

3. Development of instructional material and methodology related to problem solving that focus on mathematically important processes.

Efforts to improve teacher education and classroom instruction should be coordinated across the span of educational levels -- children through adults -- and classroom-related roles -- student, teacher, paraprofessional, principal, supervisor, and parent -- to assure total impact. Because of this we urge that all the components we have noted be considered parts of a system with significant internal and external interactions that themselves are worthy of study.

Individual Recommendations

The following specific recommendations are given in essentially unedited form so that they convey the unmodified intent of the author in each case.

1. We see the elementary and secondary school classroom teachers as the most important figures in the total mathematics instructional program. We belive that critical problems relate to the preservice and inservice training and supervision of these teachers. We believe that they are usually hard working and capable but that they have often been misdirected in their mathematics content and pedagogical preparation.

Because of this situation we recommend to NIE that a major focus of their concern with mathematics education be on teacher preparation. (We note in this regard that a substantial majority of the conference papers recommended this, many indicating this as most important.) We recommend that support be given to the development, implementation, and evaluation of new and creative preservice and inservice teacher preparation programs and curricular materials.

We also recommend that support be given to the establishment of exemplary total systems (teacher preparation, classroom instruction, and supervision and curriculum development) in order to establish existence proofs of varied programs as well as to provide bases for visitation, for fair tests of curricular materials and pedagogical techniques, and for research on mathematics teaching and learning.

2. We recommend that NIE examine the present systems for communication to and among mathematics classroom teachers. Where they identify discrepancies in the systems, we recommend that they support development of new communication modes or seek means to modify those in current use.

3. We recommend that NIE establish and support exemplary clinical centers that would both (1) provide diagnosis and remediation to individual students, and (2) serve as centers for the training of mathematics clinical remediation specialists and for research in this area.

4. We recommend that NIE initiate activities that would seek to:

   a. Identify a more realistic list of basic skills.

b. Identify problems -- pulling no punches.

c. Identify responsibilities of university personnel, school administrators, teachers, parents, and students.

d. Identify successful models.

e. Carry out a comparative study of successful and unsuccessful school programs.

f. Establish guidelines for texts and standardized tests.

5. We recommend that action be taken to modify Title I to do the following:

a. Require mandatory inservice courses for teachers.

b. Extend benefits to colleges and universities for remediation centers.

6. The identification of mathematics learning difficulties is essential at all grades so that proper instruction can be designed. In view of the present concern with difficulties children have with learning basic skills, it is increasingly necessary that diagnostic techniques be more refined. It is recommended that NIE provide support for development and fieldtesting of models for diagnosis and remediation in mathematics at all levels. Such endeavors should be joint efforts between public school and university personnel.

7. We recommend that NIE solicit proposals for Cooperative Projects to Improve Mathematics Instruction meeting the following four criteria:

a. Project personnel must include teachers from two of the categories: elementary, secondary, tertiary schools.

b. Project aims must include improvement of instruction at both levels of schools involved.

c. New teaching materials, if developed, must deal simultaneously with subject matter and teaching problems and methods.

d. Proposals involving elementary or secondary schools must include a component designed to teach parents and school administrators as well as students.

8. We recognize that the support and active involvement of parents, school administrators, paraprofessionals, and other interested citizens is essential for children's success in mathematics. Finding realistic methods for enlisting the support of these segments of society and identifying their potential contributors are both activities that may return large dividends for relatively small investments. Such efforts should be mounted by NIE to complement efforts toward improving teacher education and classroom instruction.

9. We recommend that a compilation of proven pedagogical techniques and teaching strategies be assembled and widely disseminated to teachers and other educators. A project of this type should include a literature search, development of means of gathering from the schools unadvertised but valuable techniques and strategies, research aimed at finding new approaches, and a means for communicating effective strategies to mathematics teachers.

10. We recommend that special consideration be given to innovative proposals for inservice teacher-training programs for teachers in inner-city schools.

11. We recommend that NIE support research and development efforts to identify and prepare appropriate instructional materials and procedures for elementary problem-solving activities that focus on mathematically important processes. Further, we recommend that concomitant teacher education support programs be developed.

12. We recommend that NIE explore avenues for identifying and rewarding master teachers of mathematics at all school levels. One possibility to be explored is the current British system for such awards.

13. Classroom instruction is enhanced when teachers have better insights into students' mathematical knowledge. These insights may be fostered through teacher workshops focusing on the assessment of students' mathematical thinking and on teaching strategies derived from such an assessment.

Based on this rationale, we recommend that models for clinical analysis be developed for use in teacher workshops. In these workshops, classroom teachers would (a) learn interview techniques; (b) practice analysis of students through such procedures as viewing video tapes; (c) seek to conceptualize various processes of mathematical thinking; and (d) consider teaching techniques based on this work that would respond to student problems.

14. We recommend that NIE establish the means to identify the instructional program for basic skills in mathematics and the related role responsibilities of classroom teachers, school principals and supervisors, and college mathematicians and mathematics educators.

REPORT OF THE WORKING GROUP ON CURRICULUM DEVELOPMENT AND IMPLEMENTATION

Aaron Buchanan
James Fey (Chairperson)
Peter Hilton
James Jordan
Burt Kaufman
Frédérique Papy
Dorothy Strong
Clifford Swartz
Robert Wirtz

Discussion in the working group on curriculum development and implementation produced suggestions of two types: (1) general policy or procedural guidelines applicable to all potential efforts in research and development; and (2) proposals for research and development related to specific aspects of the curriculum. Though several of the proposals met with nearly unanimous approval within the group, others were clearly minority positions. However, since the discussions were not directed toward achieving consensus, and certainly no vote was taken, the entire list should be viewed as consisting of suggestions, not recommendations of the group.

Basic Issues and Policy Guidelines

The Impact of Psychological Positions in Curriculum Development and Evaluation

- It was suggested that NIE appoint a high level commission to study, in general, the behaviorist—humanist controversy currently prevalent in American education and, in particular, the implications this controversy has on genuine reform at all levels. As a minimum, this commission should include representatives from all major disciplines taught at the precollege level, elementary and secondary teachers, parents, school board members, school administrators, educational philosophers, clinical psychologists and (perhaps) a linguist. This commission should make recommendations to NIE concerning general goals of education, alternative philosophical approaches to reach these goals, the role of evaluation in the educational process and ways to prevent it from interfering with curriculum reform and creative teaching, and appropriate education of the public as to the findings of the commission. (This was an issue that divided the group.)

The Dissemination Process

- Future curriculum development and research must include specific plans for implementation so that findings of success can be replicated in the dissemination process. There is a need to make development results replicable across sites.

- Future curriculum development efforts should lead to production of total packages — text materials, teacher guides, inservice staff development plans (including resource personnel).

The Locus of Development Activity

- Future curriculum development and research should take place in situations resembling as closely as possible the school settings of eventual implementation. Furthermore, the development activity should involve people from all sectors of mathematics education and related disciplines, and accord a prominent role to classroom teachers.

- Future curriculum research should begin to make use of materials and procedures that research has found to be effective.

Proposals for Research and Development

Most of the following areas judged to be in need of development activities have been the subject of recent work. More is needed in each area, but dissemination of existing products is probably even more important. We realize the potential conflict between private publishers' interests and government support of materials production. However, we believe that the leadership role of model government-supported projects can do much to stimulate future improvements in the broad commercial curriculum materials marketplace, without causing serious conflict of interest.

The Impact of Calculators and Computers

Experimentation with and development of instructional materials and teachers' manuals are needed to help educators effectively use calculating aids such as hand and desk calculators and digital and analog computers. This is especially important because, otherwise, it is certain that an enormous investment will be made by institutions in the hardware, without adequate help in the form of software.

- In making plans for needed development in calculator implementation, NIE should refer to the findings of a survey conducted by the mathematics education staff at Ohio State University.

- A variety of studies should be supported in exploring the impact of calculators: (1) alternative sequences for elementary instruction in arithmetic; (2) uses of the calculator as an aid and stimulus for arithmetic instruction; (3) impact of calculator availability on problem-solving instruction; and (4) relative importance of various familiar fraction concepts in an environment of calculators (to include investigation of curriculum topics in later courses such as algebra where the field properties of "fractions" are used).

- As computing equipment becomes cheaper and more miniaturized, it will be used widely at all educational levels and by many segments of the population from the consumer to the professional, making a minimum of computer-literacy a necessary part of the general education of all citizens. This minimum includes exposure to some simple computer language, to the power and limitations of computers, and to the ways in which computers are used in our society. Support should be given to the development of instructional materials fostering this literacy suitable for various educational levels ranging from elementary school to adult education.

- At the secondary level there is a need to reexamine curriculum structures and priorities in light of increasing computer and calculator capabilities to perform traditional computations. This clearly affects the definition of basic skills.

The Interaction of Mathematics With Its Areas of Application

Though there is much current activity in the search for better materials that integrate mathematics and its applications, few high quality products have yet been produced.

- There is a continuing need to collect and/or produce examples of mathematical thinking in social studies, language, etc. in the form of source materials or, possibly, curriculum materials appropriate for various grade levels.

- One possible form of resource would be an extensive, thoughtful analysis of the mathematics required in the skilled trades. This could lead to a resource of ideas for curriculum development; it should not be viewed as a taxonomy of exit skills from school mathematics.

- Curriculums should be explored that begin all instruction with "applied" problems, instead of postponing problem activity until skills have been mastered in purely mathematical settings.

- At the secondary level, development of curricular modules that aim at building student ability to construct bridges between real world situations and mathematical models would be useful. These would not be simply application activities, but exercises followed by reflection on methods to promote student ability with and predisposition to use the ideas of mathematics in decisionmaking and problem-solving situations.

- Development and use of evaluation activities that pose interdisciplinary problems for students could provide baseline data indicating, more deeply than current testing programs, how well students currently can use their mathematical skills as an aid in decisionmaking and problem solving.

The Implementation Process

There was widespread agreement that a major failing of curriculum development efforts during the 1960's was poor implementation strategies. Several models and specific techniques were suggested as alternatives to the laissez-faire style that characterized those earlier efforts.

- It was suggested that NIE increase their present dissemination efforts by several orders of magnitude to bring to the attention of the mathematics education community the fact that there are several NIE-funded projects in mathematics presently available. A mechanism should then be developed to compare these programs on several dimensions -- basic mathematical education (including the acquisition of basic skills), humanistic versus behavioristic philosophy, attention to the affective domain, mastery versus spiral approach -- and this comparison should then be widely diffused to school administrators, teachers, parents, and school board members. School districts should be invited to submit proposals for the try-out and/or adoption of one or several of these programs. Successful proposals should receive not only the necessary financial resources for implementation but should also have available the full cooperation of NIE internal staff and the staff of the project that developed the program. That is, NIE should "advertise" its present storehouse of curriculums and give its complete support (moral, financial, and personnel) in helping local school systems implement that which they wish to implement. Because this storehouse presently contains almost every conceivable type of curriculum that practitioners want, there is little justification for starting new and expensive curriculum projects.

- NIE could train resource persons to be knowledgeable about available alternative curricular materials and skilled in implementing those programs. These resource people could then respond to school system requests for assessment of their current programs and assistance in choosing viable alternatives. It is suggested that these resource people be chosen from relatively highly trained scientific/mathematics persons with a favorable disposition toward service to schools, trained by NIE, and subsequently listed in an NIE or NCTM register with their services available. Also, these resource people should be partially supported by NIE.

- Because commercial publishers play an important role in widespread implementation of new curriculums, they should be included at some point in deliberations about the direction and implementation of new ideas.

- Currently there is much curriculum development in local school districts designed to meet needs of special student populations. NIE might provide funds for broadening these existing programs, making the programs better known, and financially assisting the dissemination process.

- One important consideration in judging which programs are worth disseminating is to estimate their potential marketability and use by large numbers of teachers under a wide range of instructional conditions.

- A genuine information gap exists with respect to what programs (commercial and developmental) are in use across the Nation and the conditions under which they are being used.

- The NSF programs in their Cooperative School and College implementation model deserve further trial, as does the OE identification, validation, and dissemination program in reading.

- One of the most effective parts of the PSSC physics dissemination program was the organized series of monthly regional meetings of teachers and scientists. Other dissemination programs should copy this pattern of meetings. There should be a sequential theme to such meetings. They should be sponsored with money or credit support as available.

Creative Ways of Teaching Basic Arithmetic and Other Mathematical Topics

- Several projects are currently developing ways to teach arithmetic through problem solving instead of mere rote drill of number facts and techniques. What is needed for support of these projects is a supply of good problems.

- There is a similar need for successful methods of teaching arithmetic to students who have passed the grade levels at which those skills are ordinarily acquired -- including early collegiate years.

- Ratios and proportions are another traditionally difficult basic skill topic, and it might be fruitful to approach those topics through a function and graph style as an alternative to the standard methods.

Alternative Curriculum Structures

There was considerable discussion on the need for exploration of more open curriculum structures at the elementary level, breaking down the rigid scope-and-sequence tradition that is a barrier to innovation.

- There should be support for experimentation with alternative developments of the basic arithmetic skills.

- Programs should be supported that force students and teachers to get involved in activities that go beyond the use of paper and pencil and reading normal textbooks to actual manipulation of real world materials.

- Programs that offer alternatives to the linear development of current texts should be developed and tested.

- It might be productive to study the extent to which current "materials selection procedures" perpetuate the tight control of the elementary school curriculum exercised by established textbook publishers.

- There is a need to determine the productivity of management systems approaches to elementary school instruction.

Materials to Assist Individualized, Diagnostic/Prescriptive Teaching

- There is a need to prepare mathematics curriculum materials and procedures that will assist teachers and students in identifying deficiencies in student understanding and performances.

- There is a comparable need for material that will help meet diagnosed needs of individual learners.

Evaluation

There was nearly unanimous agreement in the group that mathematics education must develop better means of evaluating student progress toward acquisition of higher level cognitive skills and attitudes. Failing this, curriculums will probably continue to emphasize those lower level skills that can be more easily measured.

REPORT OF THE WORKING GROUP ON GOALS FOR BASIC MATHEMATICAL SKILLS AND LEARNING

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Introduction

The NIE Conference on Basic Mathematical Skills and Learning at Euclid, Ohio, named a committee to consider goals for basic mathematical skills and learning for the next several years. A reexamination of goals is always in order, and changing educational and technological patterns make this committee's task especially timely.

This report is designed to reflect the thinking of the members of the committee, indicating the goals where there were differences as well as the goals for which consensus was achieved. In light of the limited time and the relatively small number of persons involved, the goals presented here should not be considered as absolute. The committee members anticipate that this report will have some influence on mathematics education, but the committee recognizes that much broader input is necessary in the final selection of goals. While these goals may indicate new directions in the learning of mathematics, they should not be considered as prescriptions to be followed without question.

The goals outlined in this paper mark the mathematical skills and learning that people should acquire on the way to high school graduation.[1] Many of these topics, however, will arise in elementary school. Thus, these goals represent the overall mathematical outcomes appropriate to 12 years of schooling.

There are three categories of goals.

1. General Goals. General goals indicate the advantages that can accrue to a child's life with an understanding of mathematics. In a sense, these are the "why's" of a mathematics education -- in very broad terms, the reasons for studying mathematics.

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[1] A communication from NIE to conference participants reads in part: "...'Skills' is to be interpreted in the widest possible sense, as a kind of shorthand for abilities, understandings, knowledge, and so on."

2. Basic Goals. Basic goals are those which should be attained by adults in our society. This is the mathematics needed by consumers, citizens and voters, and many workers. This kind of mathematics can also broaden possibilities for hobbies and recreation. Provisions should be made to give all students the opportunity to attain these basic goals by the time they graduate from high school. Which of these goals can idealistically be required as minimum essentials for high school graduation would be determined on the basis of program development, implementation, and evaluation. For the small minority of students with severe learning disabilities, attainment of even those basic goals determined to be minimum essential goals would not be realistic.

3. Further Desirable Goals. Further desirable goals will meet the greater needs of students whose interest in mathematics goes beyond the ordinary, either in training for a profession to which mathematics is basic or purely for its own sake. Topics covered by meeting these goals represent a deeper intellectual approach than the basic goals do. But these more advanced goals are not only for gifted or preprofessional students; they are worthwhile for any student interested in pursuing them.

General Goals

The purpose of mathematics education is threefold: (1) to prepare the child for life as a consumer, voter, and citizen; (2) to begin the training for a variety of productive occupations and professions; and (3) to assist the child in developing a rich and rewarding life.

Broadly, then, school mathematics should develop a student's ability to think. In suitable contexts, experience with mathematics can enhance a student's perceptions, help him or her reason constructively, and bring insight to a wide variety of problems and situations. In many situations the contributions of experiences in learning mathematics to one's ability to think are vital to the attainment of a goal or solution of a problem.

Second, education in mathematics should encourage the ability to feel secure in situations calling for reasoning or quantitative thinking. The student should develop the level of self-confidence necessary to operate effectively in a society that makes heavy use of mathematics and mathematical ideas.

Third, mathematics should improve students' ability to do. Mathematics education should include a range of "mathematical tools" useful in practical contexts. These tools help people cope with realistic problems in efficient ways.

Basic Goals

Appropriate Computational Skills

The automation of arithmetic during the past half century has strongly affected educational needs. Hand-held calculators have had the most recent (and potentially drastic) effects. The whole issue of the effect of the calculator on the teaching of arithmetic is a very complex one which deserves considerable investigation and consideration. As a rule, decisions on arithmetic topics should consider both general usefulness to adults in the coming decades and the investment of time that a majority of students need for mastery.

With the increasing availability of calculators, adults will have less need for longhand arithmetic in the future. The time that we currently spend teaching elaborate long division problems and complicated lowest common denomination fraction problems -- often with little success -- could be better spent on more interesting, rewarding, and motivating topics.

However, students ~~should~~ not become completely dependent on calculators. While avoiding endless and mindless drill in computation, we should emphasize the mathematical principles and concepts underlying the computation algorithms. For example, the two-by-one digit multiplication algorithm depends on distributivity. Learning the processes of computation combined with the skills of estimation and approximation is useful in terms of readiness for future learning.

We must find the "right" combination of understandings and skills to enable a student to develop an algorithm when necessary and to use the mechanical and electronic devices when it is efficient to do so. Students must know the basic single-digit number facts, including the multiplication table, and should be fluent at some relatively simple types of computation. Exactly how much, between this "bare bones" minimum and the amount of computation that is currently being taught, is a question that needs further study and far more discussion among a broader base of people.

Links Between Mathematical Ideas and Physical Situations

Students should be able to relate the abstract properties of mathematics to physical situations. This typically involves expressing a real situation in mathematical terms, manipulating the mathematics with an eye to gaining some conclusions about and insight into the real situation, and then translating the result back into realistic terms. When coupled closely with the teaching of mathematical skills and ideas, these relationships can help enhance motivation, provide mental frameworks on which to hang more abstract ideas, and offer ways for students to stay in practice. (The basic goals which follow are closely related to this one.)

Estimation and Approximation

These skills are basic to facility and comfort with quantitative ideas. Students should know some simple techniques for estimating quantity, length, distance, weight, and so on. Also, students should be able to carry out approximate, rapid calculations by first rounding off numbers. Necessary here are a sense of the likely error in various procedures, and of whether a particular result is precise enough for the purpose at hand.

Organization and Interpretation of Numerical Data, Including Using Graphs

Currently, information often takes the form of numbers -- sometimes many numbers at once. Students should know not only how to set up simple tables, charts, and graphs, but also how to read them and draw conclusions. Well-organized charts and graphs are especially helpful in recognizing patterns and trends in a collection of numbers. Moreover, students should be confident enough with numerical data that a mass of numbers per se is not intimidating.

Measurement, Including Selection of Relevant Attributes, Selection of Degree of Precision, Selection of Appropriate Instrument, Techniques of Using Measuring Instruments, and Techniques of Conversion Among Units Within a System

Measurement is central to useful mathematics because measurement is the way people most often express reality in numbers. While it is possible to argue whether measurement is more legitimately a topic of mathematics or of science, no one will dispute its importance. At a minimum, students should know how to measure length, distance, weight, volume, and temperature, and perhaps area and angles as well.

Alertness to Reasonableness of Results

Due to arithmetic errors or other mistakes, results of mathematical work are sometimes wrong. Occasionally they are manifestly unsound. Students should learn to inspect all results, checking for reasonableness in terms of the original problem.

Qualitative Understanding of and Drawing Inferences from Functions and Rates of Change

This refers to a general understanding of how one quantity can "depend" on another, along with a qualitative grasp of rates of change. For example, one's financial condition can be projected on the basis of present condition, rate of expenditure, and rate of income. Graphs and tables can be used to give students a feeling for relationships among quantities.

Notions of Probability

Students should learn enough about probability to be able to meaningfully interpret weather forecasts and other predictions that are presented using notions of probability. Students should be able to rationally apply probability in problem-solving and gambling situations. For example, they should be aware of the notion of independence of events, realizing, for instance, that if a fair coin is flipped the probability of heads is 1/2 regardless of what occurred on previous flips.

Computer Uses: Capabilities and Limitations (Gained through Direct Experience)

It is important for all citizens to understand just what computers do -- and do not do. The "mystique" surrounding computers is disturbing, for it can put people with no understanding of computers at a disadvantage. By far, the best way to become acquainted with computers is to work with them, even if only a little. To gain a sense of what computers do best, and of how much their performance is governed by human planners and programers, there is no substitute for writing, debugging, and running a simple program. A little experience can go a long way toward dispelling the computer mystique.

Problem Solving

Problem solving should be considered as a special goal interrelated with all of the general, basic, and further desirable goals presented here. For example, for computation to be useful, we must be able to determine when to add, subtract, multiply, or divide. Basic goals such as estimation and approximation, organization and interpretation of data including the use of graphs, and alertness to reasonableness of results are important primarily because of the contributions they make to problem solving. Everyone should have a large collection of facts, information, and experiences that can be helpful when confronting a new situation. Changing scale, or changing frame of reference, can make the problem look different, and sometimes easier. Successive approximation can help narrow the problem to a workable solution. There are many other examples of general problem-solving techniques.

Further Desirable Goals

Recognition that Mathematics is a Construct

Mathematics is a product of creative and inquiring minds. It is a live and dynamic discipline with new developments that are stimulated both by practical and theoretical sources. The basic goals previously listed tend to stress relations between the mathematical realm and the real world. However, students should know something about internal considerations of the discipline of mathematics. While mathematicians have great freedom in selecting assumptions upon which mathematics is based, they must develop mathematical structures that are internally consistent. In some cases, such as the development of non-Euclidean geometries, the assumptions selected may appear to be implausible. However, mathematicians do not tend to select their assumptions capriciously. Their work is directed toward contributing to theoretical mathematical knowledge or practical application of mathematics.

Ability to Reason Abstractly

Students should be able to reason in the abstract realm without recourse to the concrete. Students should come to understand the nature of an argument or proof, and should be able to form an opinion about its reasonableness. The ability to construct such arguments, in purely abstract ways, is useful in other fields as well as in mathematics.

Enrichment of the Student's World

Mathematics can be an aid to insight -- a way of looking at events and phenomena that brings increased appreciation, understanding, and creativity. Developing such styles of perception is, or should be, part of what it means to become educated. A student's world can be enriched by gaining knowledge of the contributions that mathematics (and mathematicians) have made to our culture.

Acquaintance with the Natural Notations of Mathematics

Over the centuries, people have worked out certain ways of writing down mathematical ideas. In the process, an international written language has been developed for communicating mathematical ideas. The use of exponents and the development of Hindu-Arabic notation, including the use of the numeral zero, are examples of notations that have facilitated mathematical communication and thought.

Mathematical Modeling

A mathematical model represents, in the abstract realm, certain aspects of some real or hypothetical situation. Its power stems from the relative ease of manipulating the mathematics instead of the real situation. For example, a mathematical model could be created by making mathematical assumptions concerning the size of the whale population in the world and factors affecting birth and death rates of whales. Then the model could be used to predict the growth or decline of the whale population.

The principle of mathematical modeling has been an important element in human progress over the past few hundred years. It is through models that mathematics finds some of its most elegant and useful applications to the changing needs of mankind.

REPORT OF THE WORKING GROUP ON RESEARCH PRIORITIES

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Introduction

Drawing upon the research questions posed by the participants of this conference, the research working group viewed as its task the preparation of a set of recommendations concerning the support of research basic to the learning and teaching of mathematics. The recommendations are directed to NIE to assist them in their future planning; to mathematics educators to assist them in directing their scholarly efforts; and to the educational research community at large. The recommendations have been organized under two headings -- policy recommendations concerning research on the learning and teaching of mathematics, and recommendations as to the priority of questions to be investigated.

Policy Recommendations

Although this working group spent the majority of its time identifying, clarifying, and categorizing the specific questions raised by the conference participants, sentiment on several general policy issues was strongly and repeatedly expressed. Five such policy issues are addressed in the following recommendations.

<u>Recommendation 1</u>. Increased support of research related to the learning and teaching of mathematics is critical.

A common theme, voiced in several ways, appears in almost every paper presented at the conference; namely, there are many important questions about the learning and teaching of mathematics but not very many answers. The sages, soothsayers, and charlatans are and will be proposing answers, but only through scholarly inquiry will real answers be found. We agree with the critics that past research in this area has been inadequate. Past research can be characterized as a plethora of piecemeal studies rather than sets of studies reflecting scholarly chains of inquiry. Too many studies have been based on an inadequate conceptualization of the problem being investigated and have employed poor instruments and inappropriate methodology.

These faults of past and current research are the typical characteristics of emerging fields of inquiry. On the other hand, it is clear that we now know more about the teaching and learning of mathematics than we did some 15 years ago before there was substantial Federal support for educational research. In particular, we have eliminated some options that at one time seemed to be viable but proved to be unproductive; we have developed a much more sophisticated research methodology; and we have identified some potentially promising directions for research. For example, we know not to rely solely on simplistic frameworks such as behavior modification or discovery learning to solve our problems, not to use standardized performance tests as sensitive dependent variables, and not to rely on quasi-experimental designs from agriculture as canons of research methodology.

In summary, we feel that a promising start has been made both in terms of research completed and in terms of improvements in research methods. We believe that continued and increased support of research is the only intelligent way that many of the questions raised by the participants of this conference can be satisfactorily answered.

Recommendation 2. Research efforts supported in mathematics education should be diverse in terms of breadth of problems attacked but should be concentrated and collaborative in terms of resources devoted to those problems selected for study.

The specific research questions raised by the participants (summarized in the next section of this report) clearly indicate the diverse concerns of mathematicians, mathematics educators, psychologists, classroom teachers, and the public. No one constituency's concerns should be supported at the expense of others. Research on diverse questions by persons with diverse approaches and methodologies must be supported.

Once it has been decided to support a line of inquiry, sufficient resources must be provided so that a sequence of related studies can be carried out. Single, isolated studies are rarely of much value. Profitable research proceeds in small steps. By concentrating their resources on a chain of related studies, scholars increase the probability of finding useful answers.

Finally, while through the history of scholarly inquiry one can identify productive scholars who have proceeded independently of others, and support of such scholars must continue, perhaps the greater productive work on complex problems (such as those of education) has occurred in programmatic research environments where persons of differing backgrounds work in collaboration.

Recommendation 3. It is imperative that researchers in mathematics education drastically enrich the procedures by which problems are conceptualized and explore a variety of methods, especially those proved successful in fields other than mathematics education.

It is not our intention to detail a list of such methodologies, but to suggest that a search for them be encouraged by NIE and undertaken by investigators in mathematics education. In an attempt to stimulate such an effort, we present a few examples.

Several of the questions for future research concern the cognitive processes children use in doing mathematics. The main questions refer to—

children's problem-solving procedures, to the rule structures producing both success and failure in mathematical behavior, and to such issues as comprehension of mathematical ideas. Such questions have been approached by various investigators in cognitive psychology who have not been directly concerned with mathematical education.

For example, psycholinguists have used both naturalistic and experimental paradigms to investigate children's comprehension of quantitative terms like "more" and "same." Information-processing psychologists have developed elaborate methodologies for analyzing detailed behavioral and verbal protocols during complex problem-solving episodes involving both children and adults. Piagetian psychologists have explored the use of clinical interview techniques that some researchers in mathematics education have recently found valuable. Experimental psychologists have developed techniques for characterizing aspects of mental representation, including imagery, and investigating their role in problem solving. And cognitive psychologists, particularly those concerned with cross-cultural and social class issues, have clarified the distinction between underlying intellectual competence and performance as influenced by cultural and motivational factors, an analysis resulting in the development of research techniques sensitive to intellectual competencies usually ignored.

We have listed several new approaches to methodology in the area of cognitive processes. Similar points could be made about the other areas of proposed research; it is imperative to consider newly developed research techniques in such fields as political science, organizational decisionmaking, anthropology, and social psychology. For example, in the realm of philosophy, analytic philosophy has been concerned with logical analysis and with meaning clarification (which also are methodological matters). The philosophy of mathematics has been concerned with the foundations of mathematics; ethics and social philosophy have been concerned with matters of value and purpose; and philosophy of education has dealt with purpose and value in education.

At the present state of our knowledge, it seems useful to explore the potential relevance of these procedures for research in mathematics education. In many cases we may find that these procedures are not immediately applicable to our problems; we may need to modify the methods, but should not need to reinvent them. Such improvements may be facilitated by interdisciplinary communication and cooperation in which researchers from other disciplines are challenged by the special problems faced by mathematics educators, and in which the latter attempt to assimilate what is useful in the other disciplines.

Cooperation may be enhanced by the creation of a new institutional framework for the conduct of research in mathematics education. Such a framework might include new publication outlets, interdisciplinary centers, and new professional organizations.

Recommendation 4. Support for the collection and collation of information from prior work in mathematics education is essential.

In several of the papers and in our group discussions, strong sentiment was expressed that merely recommending work on various questions does not take cognizance of the fact that much work has been done directly on, or bearing on, many such questions. The first steps in research are thinking about problems and collating information (which is largely scattered) from prior work. In many cases, those steps in themselves yield valuable answers. There is an urgent need for giving support to certain centers or individuals to address questions critically and inventively rather than to put all resources into further empirical investigations.

Recommendation 5. Particular attention should be paid to problems of instrumentation and evaluation in mathematics education.

Two aspects of this recommendation should be recognized. First, one mark of a mature science is its possession of sophisticated instrumentation and techniques. Basic research on the development of instruments to measure processes as well as achievement in mathematics must be considered. New techniques must be developed and repeatedly used. Research is needed on alternatives to the standardized tests in use today. Our tests are inadequate, but little research is being done to explore acceptable alternatives.

Second, we must have quality evaluation studies in mathematics education. Attention to evaluation studies includes not only instrumentation and the collection of appropriate information, but the proper interpretation and utilization of evaluation or assessment ideas. The use of evaluation techniques for judging the worth of new products and processes is a fertile research domain for the mathematics educator. Also, research is needed on the processes of evaluation and assessment in mathematics education.

Priority Recommendations

As a first step in our deliberations, the research group attempted to identify the research questions posed by the participants of this conference. Next, these questions were organized into meaningful categories. Finally, because there are not adequate human and financial resources to reasonably investigate all of the questions raised by the participants in this conference, we found it necessary to prepare guidelines that NIE, faced with the task of allocating its scarce resources wisely, could use to establish priorities.

During an initial brainstorming session, questions derived from the papers prepared for the conference and questions of personal concern to the committee members were discussed. From this session, 103 specific questions were identified. After examining the questions for redundancy and clarity, this list was reduced to 52 questions.

The questions were classified on two dimensions: (1) focus or area of investigation and (2) levels of mathematical skills.

The guidelines we propose are summarized under the following three recommendations calling for a balanced effort among competing interests.

Recommendation 6. Research in mathematics education should be supported to reflect a balance between investigations directed toward resolving questions of immediate practical urgency and investigations related to understanding learning and teaching.

Many participants at the conference raised critical questions that if answered would directly improve the current practice. Such questions as "What is the current status of the acquisition of skills?" or "What is the effect of different innovative uses of hand calculators?" reflect this stance. On the other hand, the psychologists and the instructional researchers saw a need for basic research dealing with other questions like "How do students process information?" or "To what extent are concepts context bound?" Answers to such questions provide a basis for building a better understanding of learning or instruction.

An analogy to cancer research is useful. There are questions of practical importance (like radiation treatment) that must be examined if current practice is to be improved. There are also basic questions that if explored could lead to a better understanding of the causes of cancer and in the long run could produce more substantial benefits. Both types of research are important and potentially productive. Research in both categories should be given high priority.

The point we want to make by stressing "balance" is that the sorting into immediate practical and long range understanding priorities does not mean a choice must be made as to which is more important. Both should be pursued with equal commitment. In particular, it would be shortsighted to sacrifice understanding for immediate priorities. Indeed, such would forestall any long term successful resolution to most of those priorities, whatever patchwork might be quickly done. This means that a firm understanding of the cognitive processes involved in doing and learning mathematics is a research imperative.

Within the practical and understanding categories, the priorities we have assigned should not be taken as reasons against considering excellent research in low or nonpriority areas. All the questions listed, and no doubt many others, are relevant; and it is the nature of excellent research that its implications reach beyond its superficial boundaries. The priorities are suggestions for the weighting or relative balance of support rather than reasons to exclude any proposed piece of research without consideration of its individual merits.

Recommendation 7. Mathematics education research should be supported on questions that reflect all seven areas of investigation identified, but with preference to the first three areas.

By examining the various questions we were able to identify seven areas of investigations: (A) identification and clarification of specific aspects of research problems; (B) the development of attitudes, concepts, skills, processes; (C) instruction; (D) school context; (E) political and social context of the school; (F) methodologies of research, development, and evaluation; and (G) teacher training.

The fundamental distinction among the areas is the object of the investigation. In Area B the primary object of investigation is the individual child. In Area C questions shift to pedagogy and the emphasis is on the teacher and student-group interaction. In Areas D and E the focus is on the school organization and the political and social context of schooling. One observes in this schema a widening perspective starting with the individual child and expanding to include more comprehensive social structures that encompass the child.

In Areas A and F the focus is on the research process itself. Clearly, our ability to improve educational practice with respect to mathematics is limited by our research methodologies and by our ability to define the problems clearly.

Within the first five areas we see a distinct hierarchy. It is difficult to conceive of any significant progress at any level without a careful definition of the problem. Thus, to some degree, investigations in all areas include Area A activities. Similarly, research in Areas C, D, and E seems to depend upon a firm basis in Area B.

Finally, while only one question was raised with respect to teacher education, it is an important area. Thus, we created Area G to cover teacher training.

<u>Recommendation 8</u>. Mathematics education research should be supported on questions that reflect a balance among different levels of mathematical skills.

The mathematical skills identified by the conference participants could have been subdivided into various categories. However, we felt that three general labels were useful to partition the questions on this dimension: (I) manipulation, (II) quantitative and spatial comprehension, and (III) problem solving.

Manipulation includes a large number of items dealing with recall of facts, application of skills, algorithms, and so on, and their routinization or their relationship to various aspects of mathematics. Quantitative and spatial comprehension refers to questions about how students quantify or model problem situations, such as by measuring. Problem solving refers to questions concerning strategies and heuristics a student uses to solve problems.

The Identification and Classification of Specific Research Questions

The Research Priorities Working Group felt it would be helpful if we could go beyond the eight policy recommendations to the identification of the specific research questions raised by the conference participants, and subsequently to classify each by priority, area, and content as suggested in recommendations 6, 7, and 8. While the classification of most questions met with nearly unanimous approval in the group, others were clearly minority positions. The entire classification should be viewed as merely suggestive.

The final list of research questions derived by the group in its deliberations appears at the end of this section. The questions are grouped by priority classification. Following each question the area of investigation and content of the question is given using the following code:

Area of Investigation

A. Identification and clarification.

B. Development of . . . .

C. Instruction.

D. School context.

E. Political and social context.

F. Methodologies of . . . .

G. Teacher training.

Levels of Skills

I. Manipulation.

II. Quantitative and spatial comprehension.

III. Problem Solving.

Because of their breadth, the questions often span areas, content, and even priority categories. "First priorities" represent those questions where inquiry is some mix of urgent, logically prior, manageable, likely to show some results in a short time, and likely to have a broad payoff. "Second" and "third" priorities are roughly governed by a diminished portion of these categories.

Questions of Practical Urgency

First Priority

1. What do we mean by skills? That is, (1) what do we -- and should we -- have in mind when we talk about basic mathematical skills? Some mapping of the alternative possibilities seems imperative to avoid an overly narrow notion of skills; (2) what does it mean, and require, in psychological terms, to perform in a skilled rather than an unskilled way, i.e., is it a matter of routinization, knowing what to do? A--I, III.

2. Study exceptionally successful teachers, students, and schools. This procedure seems clear and important -- a matter of tapping information resources already on hand. However, it may not be easy to do, especially in light of confusion concerning what constitutes being successful. However, the results of such a study could lead to the identification of teaching strategies or pedagogical methods of great value. C--I, II, III; D--I, II, III.

3. Perception of structures; noticing (relation to taste, sense of aesthetics). This asks whether and when people spontaneously notice in their out-of-math-class experience structures and occasions for tactics learned in class. Such spontaneous noticing is crucial if there is to be good payoff outside the classroom; yet research on curriculum success typically ignores this, and curriculums are rarely designed with this transfer objective continuously in mind. In particular, what are children's tacit and/or spontaneous concepts, processes, skills, beliefs? This is fundamental. We must know where they are as well as where we want to move them before we can decide how to move them. Though many beliefs prevail concerning what attitudes and skills children do or do not have, it became clear at the conference that there is much simple descriptive work that needs to be done along with the more subtle probing of underlying cognitive processes. B--I, II, III.

4. True functional needs (remember Xerox). In the midst of controversy about what skills people ought to have, this question calls for the matter to be studied empirically -- track people's behavior and see where they need which skills. "Remember Xerox" reminds us that "needs" should not merely mean needs exercised by current practices. As with the office copier, there may be unrecognized needs. In particular, the potential of hand calculators must be explored. What are their innovative uses? Learning uses? What would be the long range implications of substituting calculators for pencil-and-paper computation? A--I, III.

5. Why are algorithms so hard to learn? In particular, how do students acquire algorithmic routines from problem situations or instructional algorithms? B--I.

6. What are children able to comprehend? All sorts of claims are made concerning what children of various ages are or are not able to do, and, therefore, about what is or is not possible to teach. The claims are contradictory and need to be resolved. Also, what is the relationship between comprehension and computation? B--I, II, III.

Second Priority

7. Study "passive" vs. "active" learning. For example, TV as it relates to noticing. B--II, III.

8. Do children understand the informative nature of error and when it is informative? How is error information used to solve problems? Can we teach this? B--II, III.

9. Study "real" vs. "contrived" activities. B--II, III; C--III.

10. Study mathematical trauma, "symbol shock," etc. Identify people via clinical studies. B--I.

Third Priority

11. To what extent are manipulatives used in the early grades? C--I, II.

12. What are public attitudes toward mathematics? What are the sources of these attitudes? How do they develop? E--I, III.

13. Study organizational task structure (what teachers are expected to do) and organizational technology (what materials and routines teachers and students are to follow) as related to mathematics performance. D--I, II, III.

Questions Related to Understanding

First Priority

14. What models for learning are generalizable across content areas? That is, what models of learning and learning procedures that one can deliberately practice apply indifferently to many content areas? How good are these models and can we import them to apply to problems of math education? This asks for some heavy thinking about how to apply already developed concepts to the problem at hand. One way to approach this question would be to study elementary information processes -- memory management processes, types of encodings, types of search procedures, basic symbol manipulation repertoires. In particular the study of information processing underlying learning should be contrasted with terminal performance. This is a distinction that has not been explored carefully and might be very fruitful to explore. We dare not assume that the processing involved in a partially mastered skill bears some simple relations to the final process -- e.g., merely slower, less reliable. Rather, the step from halting and inadequate performance to mastery may involve substantial shifts in the organization of the process. B--I, II, III; C--I, II, III.

15. To what extent are concepts context-bound or general, and when? This asks about the nature and problems of transfer -- how readily will concepts mastered in one context carry over to others, and what kind of nudging, explicit pointing out of connections or the like, encourages such transfer. Everyday examples show that the help needed to achieve transfer varies enormously with the sort of material under consideration and that this question needs to be asked repeatedly if education wants to achieve maximum leverage. B--II, III.

16. What techniques of practice are being used and how effective are they? This asks what is known about effective practice -- different patterns, schedules, motivation -- and whether this information is in fact being used in current math programs. One suspects, with regard to learning algorithms,

that there may be valuable parallels with, and information importable from, such areas as musical or athletic training, where routinization is at a premium. B--II.

Second Priority

17. Study search strategies. B--III.

Third Priority

18. What does it mean to comprehend? B--II, III.

19. Study imagery -- visual and other -- in mathematics. Is it needed? How is it used? B--I, III.

20. Study the role of mental arithmetic. A--I, III.

21. Study cognitive styles as guides to alternate methods of instruction. B--I, II, III; C--I, II, III.

Other Questions of Interest

Limited by time constraints and the background of the group participants, several interesting questions raised were not given a priority designation. Simply because these questions were not listed as priorities should not be construed as indicating their lack of importance.

22. Study attitudes -- teachers', children's, parents' -- as a social phenomenon. Particularly, study attitudes concerning "work" versus "pleasure," especially held by teachers.

23. Study the relation between language and mathematics.

24. Study the role of false (partial, approximate) explanations in pedagogy.

25. Probe sociology of the classroom as it relates to (e.g., limits) the introduction of new practices.

26. Study the match between the pedagogical intent of a program and what in fact is acquired and diverges from intent.

27. Study the relations of mathematics to other fields, e.g., art, both abstractly (formal parallels or applications) and in the learning processes involving the child.

28. Study humanizing mathematics -- history of mathematics, biography of mathematicians (as in Project Physics). Relate mathematics to other aspects of culture. Figure out how to create a more humanistic picture for the child. (This relates to public and personal attitude.)

29. Study the factual adequacy and influence of testing. In particular, see if we can move away from testing technology.

30. Study students' preferences and see whether they match our expectations. Study the motivational role of students' choice.

31. Study achievement gaps in mathematics as related to other subjects. Metric problems in science?

32. Study the effect of reward structures and different kinds of rewards.

33. Study teacher education.

THOUGHTS ON THE STATE OF MATHEMATICS EDUCATION TODAY

Peter Hilton and Gerald R. Rising

Introduction: A Personal Disclaimer

When we were asked by NIE to act as cochairmen of a conference on basic skills and learning in mathematics, our enthusiasm for the charge given to us was tempered by our realization that many of those sharing our concern about the state of mathematics education today would believe that our own special interests might lead us to introduce some bias into the proceedings of the conference. This hesitation on our part was fully and unmistakably justified by some of the responses that we received to our initial invitation to our colleagues in mathematics education to participate in the conference. Nevertheless, we hope that the actual conduct of the conference put to rest the fears of those who thought that the proceedings might somehow or other be rigged in order to reach a conclusion favorable to a particular point of view or set of points of view with regard to the problems besetting mathematics education today.

We have also been asked by NIE to provide our own summary of the conference. In view of the fact that within the span of two fairly substantial volumes the proceedings of the conference have been made available to the public, we have thought it neither necessary nor, indeed, helpful to try to provide a summary of these proceedings. Rather, we will set out in this article to express our own thoughts arising from the deliberations of the conference. Certainly these thoughts will have been strongly influenced by our own positions with regard to developments in mathematics education. For the existence of such a bias in our remarks, we make no apology. Our choice of thoughts is our own, but we both gladly testify to the strong effect of the contributions written and oral of the conference participants had on our own thinking. Nevertheless, at the risk of becoming tedious, we feel bound to emphasize that the present article is of a personal nature and commits no one except the authors.

We have grouped our comments under 10 headings. Of course, both the nature of these 10 problems and the means proposed for their attempted solution involve considerable overlap, but we have thought it helpful to separate them in this way and treat them fairly independently.

1. Problems of Communication

The communication problem is perhaps the central one in curriculum development. An enormous amount of work on curriculum development has been done over the past years, but very little of it has become part of the common fund of knowledge of professionals in the field. Many good programs are now gathering dust on the shelves of NSF and NIE. Many poor programs continue to receive support by these agencies when, in our view, a full description of their activities would immediately expose their inherent weaknesses. Projects get repeated and continue to be executed by the same project teams addressing the same audience. The wheel is forever being rediscovered.

As a possible remedy for this situation, which must be very familiar to many people and has become highly topical as a result of recent Congressional hearings on the NSF bill, we suggest a new periodical designed to provide information about developments in mathematics curriculums and in pedagogy. The experience of recent years shows that such reports are not going to be accepted by regular mathematics education journals. Nor, we contend, would this method of publication serve the intended purpose. We envisage a publication that would include extended passages from curricular materials; reports on classroom trials of materials; comparisons among developments; points of view regarding curriculum development; and analyses of and suggestions for classroom teaching practice. Naturally, such a journal would not be restricted to the publication of government-supported projects, but would be open to commercial agencies and authors as well. We believe that the publication of a periodical of the type proposed would not only meet a most serious need in mathematics education today, but would also satisfy many of the Congressional and lay critics of the NSF dissemination program.

2. Science Teaching

At the Euclid conference it was clear that much attention was given to discussions of what science topics should be taught in the mathematics classroom and how they should be taught. Significantly, this attention was by no means confined to those who had come to the conference with an explicit brief to direct the attention of its participants to these problems. Some participants reacted to the emphasis given to science teaching by suggesting that teachers of mathematics could better address themselves to the task of developing basic mathematical skills and learning in their students if their classes were not saddled with a proliferation of science topics such as measurement and the metric system, data collection, approximation, estimation, and applications.

We find ourselves in some sympathy with this rather parochial approach to the subject, but it is a human rather than a professional sympathy. We do not believe that there should exist strong sharp demarcations among disciplines at the elementary level. We do believe that a child should know when he is doing mathematics, but he should surely realize that a principal purpose of doing mathematics is to be able to answer questions which are, in a broad sense, scientific. Thus, we believe that the task of teaching the effective application of mathematics is a task that teachers of mathematics and mathematics educators in general must share with other teachers and other educators. And it is the tendency of these other people to regard the teaching of anything mathematical as the exclusive responsibility of the mathematics class and the mathematics specialist that arouses our sympathy for our already overburdened colleagues. Science teachers, especially at the middle and secondary school level, and science programs at all levels, must concern themselves with the problem of explaining the relationship of mathematics to the acquisition, organization, and utilization of knowledge. We have some evidence that science teachers and science programs do not always willingly share this responsibility. Too often the teachers of science complain that their students arrive lacking the basic mathematical knowledge necessary for the understanding of the science which they want to teach. We claim this point of view is untenable because the learning of mathematics in a useful way presupposes a continuing interaction between the development of the understanding of natural processes and that of mathematics.

We strongly recommend that scientists and mathematicians, science teachers and mathematics teachers, cooperate closely in order to provide the best possible total program in these two closely related areas.

More generally, we wish to emphasize that mathematics, however important, forms only a part of education. The SAT statistics, which have created so much furore recently, show quite clearly that the decline in arithmetical skills is simply a part of the general decline in the effectiveness of basic education. We must insist that it is not fair or reasonable to stigmatize mathematics education as if it were an isolated instance of educational failure. We believe that there will be a really significant improvement in the effectiveness of mathematics education only when the importance of education itself is once again recognized by all the constituencies that education serves; primary among these are the students themselves, their parents and their teachers. And, by the same token, mathematics education will only be effective when it, mathematics education itself, is recognized as important by all teachers regardless of their specialties.

3. Schools in Chaos

We owe to Professor James Wilson the explicit observation, in his position paper, that our schools are in chaos. The comments of many classroom teachers bear out this very somber observation. Teachers today are subject to overwhelming pressure; they are expected to solve, or at least redress, all the problems of modern society and somehow, at the same time, to instruct the children in reading and mathematics. In many schools student attendance is so irregular that it is virtually impossible for teachers to develop a coherent strategy for instruction. Often, in rural and suburban environments as well as in urban schools, vandalism is rife and demeaning attacks on teachers by the students are matched by verbal attacks on the teachers by parents.

It is plain that these conditions are not taken into account when schools are condemned for their failure to communicate basic skills to children. We have two proposals to make. The first is that, in measuring the extent to which schools are accomplishing their mission, it will be more fair to concentrate on the achievements of students who attend school for, let us say, 90 percent of the time or more. Second, and in a more ambitious vein, we recommend that a careful study be made of the forces at work undermining the efforts of the teacher to educate the student. We should try to account for the reasons and consequences of the breakdown in the traditional respect for the value of education; and we believe we would find we are suffering from the effects of the very superficial type of "sales pitch" for education that has been current for the past decades. We should try to estimate the effect of the diet of violence which is served up to the children so liberally by commercial television (Rothenberg, 1975, p. 1043). And we should study ways to restore respect for education and for the teacher's role in our society.

4. Literature Search

Several papers have called for a search of the research literature in order to determine what questions have already been answered. While this sounds reasonable enough, our experience with the research literature of mathematics education has led us to adopt a rather critical view of the quality of that literature and to expect a very modest gain to ensue from a systematic cataloging of its results. We find that there are few firm answers to be drawn from the published research, and even those which we do find would appear to apply to severely limited and not always very practical problems. Moreover, the objective of the research itself has often been of a restricted nature, and has remained in the area of theoretical speculation. We would particularly recommend, however, some of the recent clinical studies which seem to us to provide important information and also to bring investigators into closer contact with schools and students. It is a striking fact that for some researchers this seems to be the first time that they have realized that there is actually a warm blooded student behind those mark-sensed inventories.

5. The Relationship of University Academics to Classroom Teachers

We consider now one of the most serious concerns that was repeatedly raised at the Euclid conference. In its most blunt form it presented itself as a suggestion that, with some money available from the Federal Government, the time has come once again for university academics to move in and "rip off" the classroom teacher. There was even the counter suggestion that there was really no place for the university academic in raising the standard of mathematical education, particularly at the elementary level, and that, indeed, it was better in many respects that the classroom teacher should not himself be a mathematics specialist, for if he were he would have less sympathy and understanding for the difficulties of his students.

It is our strongly held belief that only an insignificant number of members of the mathematics and mathematics education community have ever willfully abused their primary responsibility to students and their secondary responsibility to school classroom teachers. We belive that the members of that community have always understood the nature of the heavy responsibility borne by teachers and have had great respect for their abilities and their courage. This does not mean, however, that we are so foolish as to believe that mistakes of judgment by mathematicians and mathematics educators have not been made, and we recognize that some activities in the past have been poorly conceived, and some have been poorly executed. Naturally we urge that great care should be taken to avoid such errors in the future; and as one response to the problem, we strongly encourage the increased participation by classroom teachers at all levels of grant activity, in particular in reviewing proposals. Need it be added that the participation of classroom teachers in testing curricular proposals should form an organic part of any grant proposal.

As a further response to the attitudes and points of view expressed by some of the Euclid participants, we want to emphasize the importance of institutional cooperation between schools and colleges as much as cooperation between mathematicians and mathematics educators. In this way, we can expect not only that we will be able to reduce the number of errors, but also that the increased input from a variety of sources will lead to the evocation of new and better ideas. Further, we need an understanding and appreciation of the differing roles of our colleagues within the educational community as a whole. At the risk of being trite, we must add explicitly that no one group has all the answers and that each has much to offer.

There is another problem embedded in this discussion. It is clearly more difficult to reach total agreement if people from widely different backgrounds with widely different interests are all brought into the decision-making and testing process. Our response is: So much the better. We should seldom expect total agreement, and should in fact encourage differing points of view. Each of us has had the experience of reacting negatively to a teacher's description of what he planned to teach and then finding our skepticism totally unjustified by our subsequent observation of the teacher's success in the classroom. From this we have learned the lesson that we should not be too quick to reject ideas that are different from our own or that appear to contradict certain a priori views which we hold. To be too conservative is at least as bad, we suggest, as to be too radical in educational reform. Indeed, we should be trying to balance the tendency toward conservatism in this field that is encouraged by the fact that criteria of success often tend to be highly traditional and beyond our control.

6. Standardized Tests

We join our colleagues, Professors Jerrold Zacharias and Banesh Hoffman, in their campaign to suppress and supplant current models of standardized testing. We consider such testing to be not merely suspect, but actually seriously damaging to mathematics instruction.[1] Several conference participants have suggested that these tests are essentially valueless as diagnostic tools, and we are in complete agreement with their point of view.

What, then, are standardized tests good for? As we have seen them used, they serve as superficial support for school programs ("Our students score

[1]We find the NACOME report pusillanimous on this issue -- and only on this issue -- when they write "Many testing programs use instruments that provide such crude measures of achievement that they have limited value for improving instructional programs or assessing an individual student's educational needs" (p. 134). We believe that many instruments have unlimited negative value for these purposes.

well above the State average.") or as threats forcing teachers and students into a tight test-determined lockstep. We find the search by school administrators for the easiest normed test demeaning to the profession, and we are horrified by the frequent examples of cheating (in one direction to satisfy school boards, in the other direction to meet Federal requirements for support programs).

A glance at the tests reveals them for what they are: rock-bottom recall and algorithm-performance measures. Even the problem-solving sections are trivializations of that so often misused term. Somehow it would seem to be claimed that the addition of a few words changes a computational exercise into something else, and something far more valuable and humane. But this is the real problem: The tests subvert any meaningful goals for mathematics learning. They substitute calculation for mathematics and reinforce society's misidentification of the nature of our subject. They misdirect the attention of teachers and students, as well as of parents and legislators.

To forestall any misinterpretation of our point of view, let us immediately repudiate any opposition to the teaching of calculation algorithms. Of course we support the teaching of computation as an important part of the larger mathematics program. What is wrong with the tests is their identification of mathematics solely with that aspect of the program. As has often been said -- what is easy to test gets tested, and one might add that what is difficult to test gets ignored. What teacher will devote a major effort to the teaching of material that does not appear on a standardized test, and what student and what parent will be happy with such a brave teacher if he discovers the teacher engaged in such an enterprise? In fact, it was exactly this concern with the prevalence of a very restrictive view of the nature of what is basic in mathematics education that led us to insist that the title of the Euclid conference be extended to include with basic skills the additional words "and learning." We were fully aware of the test designers' false view of this subject and of its wide prevalence, and we were determined to redress the imbalance.

What can be done in the face of the ever increasing tendency to adopt current models of standardized testing? We support Professor Zacharias' endeavors on project TORQUE to redirect the evaluation efforts of the schools. Further, we urge the mathematics education community to take a strong stand against further use of the current tests. Let us at least have a moratorium on standardized testing in the schools until new, more reliable, and more acceptable measures can be developed.

7. Calculators

If the film "The Graduate" were being remade today, the word "plastics" would surely be replaced in the dialogue by the word "calculators." Inundated by these devices, we seem to have no idea what to do with them in the schools. Some seem to wish to stem the flood by arbitrary restraints on their utilization. Many seem to react by rushing into highly singular dogmatic answers. We are greatly concerned that such irrational reactions have already taken place and will tend to close off interesting avenues for experimentation. For example, we are highly dubious of recommendations that would postpone the use of calculators until after concepts have been taught, because it is plain to us that the calculator can itself play a very positive role in concept development. Again, we oppose those who would restrict the school use of calculators to those instruments that employ so-called algebraic logic, as opposed to Polish logic. We believe that many of these reactions stem from a basic conservatism and fear of change, and we recommend, by contrast, that a wide range of experiments should be encouraged in order to plumb the full potential benefits of this remarkable tool.

We have been encouraged to learn, since the Euclid conference, that at least one major calculator manufacturer is about to produce curriculum materials (software) to be used with their calculators. Clearly, teachers need such support materials in order to make effective use of the instruments. Naturally we are not at this time in a position to evaluate the quality of these particular materials, but we believe that their appearance may well provide a base for successive improvements. We look forward to the entry of others into this field.

However, we feel it necessary to sound a note of caution. We are ourselves sufficiently traditional to maintain that mathematics teachers should be concerned primarily with the teaching of mathematics. As we said in the section on science teaching, the mathematics teacher has already been overloaded with responsibilities which should properly be assumed by the science teacher. Of course, the mathematics teacher will present his students with problems related to the metric system, but it is not our view that it is his primary responsibility to teach the metric system. Similarly, it is not his primary reponsibility to teach computer languages. We hope that the hand-held calculator will be used in the mathematics classroom predominantly to support and extend conceptual understanding of mathematics and to facilitate the application of arithmetical techniques to the solution of real life problems. We certainly believe that it can serve these vital purposes.

Finally, we support the NACOME report recommendation that hand-held calculators be provided to secondary school general mathematics students who have not by that time mastered the so-called basic facts. There can be no conceivable justification for allowing the spectre of those unmemorized tables to continue, at that level, to get in the way of the opportunity to attack reasonable problems.

8. False Dichotomies

We heartily endorse -- and might even claim some priority for (Hilton, 1975) -- the view expressed in the NACOME report that "In the creation, introduction, and support of mathematics programs, neither teachers, educational administrators, parents, nor the general public should allow themselves to be manipulated into false choices between

The old and new in mathematics

Skills and concepts

The concrete and the abstract

Intuition and formalism

Structure and problem solving

Induction and deduction." (Hill, 1975, p. 136)

Indeed, we could add many items to this list. As suggested in section 5, we need many points of view in mathematics education and the best procedure is likely to be a synthesis rather than an extreme position. It is very rarely the case that the best way to improve an existing procedure is to adopt the opposite, though this is often the type of recommendation that attracts maximum publicity.

Moreover, many of the apparent dichotomies are not really opposites at all, but express complementary aspects of a common objective. It is necessary for those concerned with curricular reform and improvement, as with instructional reform and improvement, to seek the opinions of experts in different areas and with different viewpoints.

A particularly dangerous false dichotomy is that between skills and concepts. It has led to the view that we are fulfilling our duty as educators if we present students with the opportunity to acquire efficiency in the execution of the basic arithmetical algorithms. As we have already said, we cannot accept this view. It is useless to be able to execute, for example, a subtraction if we are unable to recognize situations for which subtraction is the appropriate mathematical model. There is no justification for devoting such an inordinate amount of time to arithmetical instruction in our schools if the students are not to understand those contexts of their lives to which arithmetical operations are relevant and appropriate.

Similarly, there is no true antithesis between the concrete and the abstract. In mathematics we use abstract methods to study concrete situations. It would be an appalling weakening of the whole nature of mathematics, even at the elementary level, if we were to present arithmetical operations as being nothing but operations on concrete objects. Plainly, the student must understand the versatility of the mathematics which springs from its nature as an abstract concept.

The implication of all this is that, instead of indulging in a sterile exercise in bogus scientific methodology, we should regard the two aspects set in false dichotomy as complementary in any worthy program of mathematical instruction and should seek, by experiment, the appropriate balance between these two aspects. It is surely very probable that such a balance would depend on a number of factors and that, in an ideal situation, the appropriate balance might even depend on the individual student.

9. Problem Solving

Of the false dichotomies listed in the previous section, we have selected problem solving here in view of its extreme topicality. The charge has been made against the new mathematics that it emphasizes too strongly the importance of mathematical structure, and that it overlooks the necessity to educate the child to be able to solve problems. Certainly the student of mathematics must be able to solve problems. But, the problems are presumably to be those which require mathematics in their solution, and consequently the mathematics must be well understood if it is to be effectively applied. It would be readily acknowledged that it is an important educational experience to be faced with a problem and then to attempt to solve it. But to deny oneself access to the available theory, however elementary the appropriate level, in attempting the solution is to place oneself under a very grave handicap and to diminish enormously the probability of obtaining a good solution in a reasonable amount of time. Problems are most efficiently solved by the application of the appropriate theory; and the place where the theory is most likely to be developed is in response to the desire to solve interesting problems. Thus, the two activities of structure building and problem solving are highly complementary to each other, and, indeed, depend on each other in any well-balanced curriculum.

In particular, we are highly skeptical of research that appears to isolate the solving of problems from the rest of the child's development of mathematical understanding. We hear much today of "problem-solving strategies" and we find certain social scientists endeavoring to convince us that there are such common strategies which can be applied whether or not the problem is susceptible of mathematical analysis. We believe that the solving of problems by mathematics is part of mathematics and that, if problem-solving strategies relevant to mathematics are to be developed and learned, then this process must take place within the context of the mathematics lesson -- but not necessarily exclusively within this context. We would, of course, be happy, as we have already said, if the use of mathematics were also encouraged by other teachers, and then we would expect those other teachers simultaneously to be developing the student's problem-solving abilities.

10. The Nature of Mathematical Usage Today

We believe it necessary to give attention to the question of what mathematics is really needed by today's citizens, and what mathematics is likely to be needed by tomorrow's citizens. It has sometimes been said that, whereas it is clear and noncontroversial that all adults should be able to compute with natural numbers, fractions, and decimals, it is a long time after that in the natural sequence of mathematical instruction that one again meets a mathematical topic which is capable of being applied in everyday life. It seems necessary for us to acquaint ourselves with the current mathematical usages in the skilled trades, so that we should know what minimum exit skills the student should acquire.[2] Beyond that, we should find out more about the mathematics demanded of technicians and those who act as auxiliaries in scientific and technological enterprises. By studying the way in which these needs have evolved over the past years, we may be able to project sufficiently far into the future to be able to anticipate the needs of our

[2] We understand that an article on just this topic, written by Henry Pollak, is to appear in a forthcoming issue of Educational Studies in Mathematics.

present students during their active Lifetime. As a less ambitious but extremely important consequence of such a study we may very well be able to determine that we can no longer justify the time devoted to certain traditional areas of our present curriculum.

Of course such a study must be carried out cautiously. The expectations of mathematical competence will surely themselves be a function of the norm of acquired competence in those who leave school. Thus, it may very well be that employers set their sights low because they know what to expect. It is probable that a study of this kind would be the richer and the more valuable for extending beyond the experience of one country alone.

Finally, we must sound the clear warning against assuming that mathematics which is not going to be directly applied automatically forfeits its place in the curriculum. Mathematics is, itself, a proper study for mankind, and no civilization which underrates it deserves to retain its self-respect or its leadership role. We must be ever mindful of all the needs of our students.

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