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ABSTRACT

Reading difficulties in mathematics may stem more from the abstract and highly symbolic nature of the subject than from inability to recognize or comprehend the words. A review of Piaget's model of cognitive development suggests that many seemingly simple mathematical terms assume cognitive processes which may not be available to many secondary school students. Preliminary research with tenth-through-twelfth grade students has supported this analysis. In order to overcome the resultant reading problems in mathematics, teachers must provide students with a broader set of experiences, centered on the difficult concepts, in order to generate deeper and more specific meanings for the words causing difficulty. (AA)

READING IN MATHEMATICS AND COGNITIVE DEVELOPMENT

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There are many aspects to reading. Amongst them are the following:

- word recognition
- vocalizing the word (oral reading)
- understanding individual words
- understanding groups of words
- understanding sentences and groups of sentences
- remembering

It is with the understanding part that this paper deals. I contend that a student may "know" or may "recognize" or "understand" individual words and be able to "vocalize" words yet not comprehend what a group of words or the sentence(s) is saying. We may apply a word recognition test, a vocabulary test or some other tests to a mathematics passage and the results may indicate the reading level is very low - at least two or three years below the grade level (age level) of the students for which the book is designed. In many instances the students may be reading in other content fields at a much higher reading level. Yet in the mathematics content area these students may just not be able to get the meaning from the passage. The question of course is WHY?

Why might a student who is reading at a level consistent with his age in literature, social studies, etc. not read at a comparable level in mathematics - at least in mathematical material which by many tests may be rated comparable to or lower than that of the student's level? There are a number of reasons which I will present. I'll pass quickly over a number so that I can go into some depth and specifics on one that cuts across many of the others.

Mathematics passages often (1)*

1. are conceptually packed
2. have a high content density factor
3. require other eye movements than left to right (requires vertical movement, regressive eye movement, circular eye movement, from word to chart to word movement)
4. requires a rate adjustment
5. requires multiple readings (to grasp the total idea, to note the sequence or order, to relate two or more significant ideas, to find key question, to determine the operation or process necessary, to conceptualize or generalize)
6. use symbolic devices such as graphs, charts, diagrams, and mathematical symbols
7. are heavily laden with its own technical language which is very precise. Often common words are used with special exact meanings e.g. function.

The reason with which this paper deals is:

Mathematics is an abstract subject. It is usually presented in a very symbolic and generalized form. Its language is highly specific. Reading in mathematics often requires a higher level of cognitive and conceptual development than the students have achieved.

In order to put this all in a logical framework a model of cognitive development is necessary. I shall use a modified Piaget model.

(1)* Numerals in brackets indicate the references at the end of the article.

Proponents of Piaget generally agree that a child, in developing a concept, goes through certain stages. These stages are not discreet but rather blend or meld into one another to form a continuum.

Now about the stages:

1. The order of the stages is fixed.

2. The rate of progression through the stages is not fixed - nor can they be tied to any chronological ages. Where this is done - it is only for referencing.

3. The movement along the continuum can be altered by certain factors - teaching, environment, maturation, etc.

Now for more details:

Different authorities give different names for the stages and often break the continuum into different stages. A general breakdown lists these four stages:

1. <u>SENSORI - MOTOR STAGE</u> (From 0 to 2 years)

   - preverbal - though language does start to develop
   - direct motor action
   - begins to learn to coordinate perceptive and motor functions - see thing with reference to what they do with it

2. <u>PREOPERATIONAL STAGE</u> (2 to 7 years)

   - objects take on qualitative features
     - permanent
     - non permanent
   - tends to ego-centuism
     - tendency to see things from one point of view - his own
     - tendency to see one feature (elation) to the exclusion of others

3. CONCRETE OPERATIONAL STAGE (7 to 12 years)

- Piaget describes operation as a mental act - an internalized action.

  The concrete aspect comes from the fact that the actions (mental or internalized) had their starting point as some real system of objects and relations - that the person could see, feel, hear, etc. (senses) or could imagine.

  Concrete - in the sense it's real - involves no assumptions
- The child can now focus on more than one attribute at a time - two, then maybe multiple ones
- begins to see other person's point of view

This is the stage where the classic operations are developed

- combinativity
- reversibility
- commutativity
- identity
- associativity

For example:

The operation of conservation is not a unitary development.

Conservation of discreet quantities - (number) develops before continuous quantity (substance)

Conservation of weight (force of gravity) and of volume (space occupied) follow in that order.

4. FORMAL OPERATION (12 onward)

- characterize by full, logical thinking
- can now deal with statements that are not known or supposed to be true –

  if.... then

  either.... or

  either or both.... or

  $a > b,\ b > c \rightarrow a > c$

Let me take some examples of how a concept would develop – or rather the manifestations of the development of a concept.

Given a set of beads

– Sensori-motor	– hit, throw	
	– only present when seen	
– Preoperational	– red, round, thick	) one) feature
	– yellow, square, thin)	at a time
– Concrete operational	– red and round and thin	) two or) more at
	– yellow and round and thick)	a time
– Formal operational	– either the red or the square or the thick ones	
	– neither the red nor the round ones	

Another Example:

Equivalent Rate Pairs

$\frac{2}{3} = \frac{\square}{9}$ or $\frac{2}{3} = \frac{x}{9}$ (concrete)

$\frac{23}{49} = \frac{\square}{98}$

$\frac{a}{b} = \frac{x}{d}$ solve for x

$\frac{x-2}{x-4} = \frac{y}{x^2 - 16}$ solve for y (formal)

3rd Example:

An ideal car travels due east for 2 hours at 60 mph

Returns at 30 mph

What is the average speed? 40 mph or 45 mph

You all know two things: $d = rt$ and $r = \frac{d}{t}$ (concrete) (generalization)

$$\text{average} = \frac{\text{sum}}{\text{No of number}}$$ (concrete ??)

In Summary

Piaget maintains a child moves along a continuum - the continuum representing the blending of the stages of development for each concept.

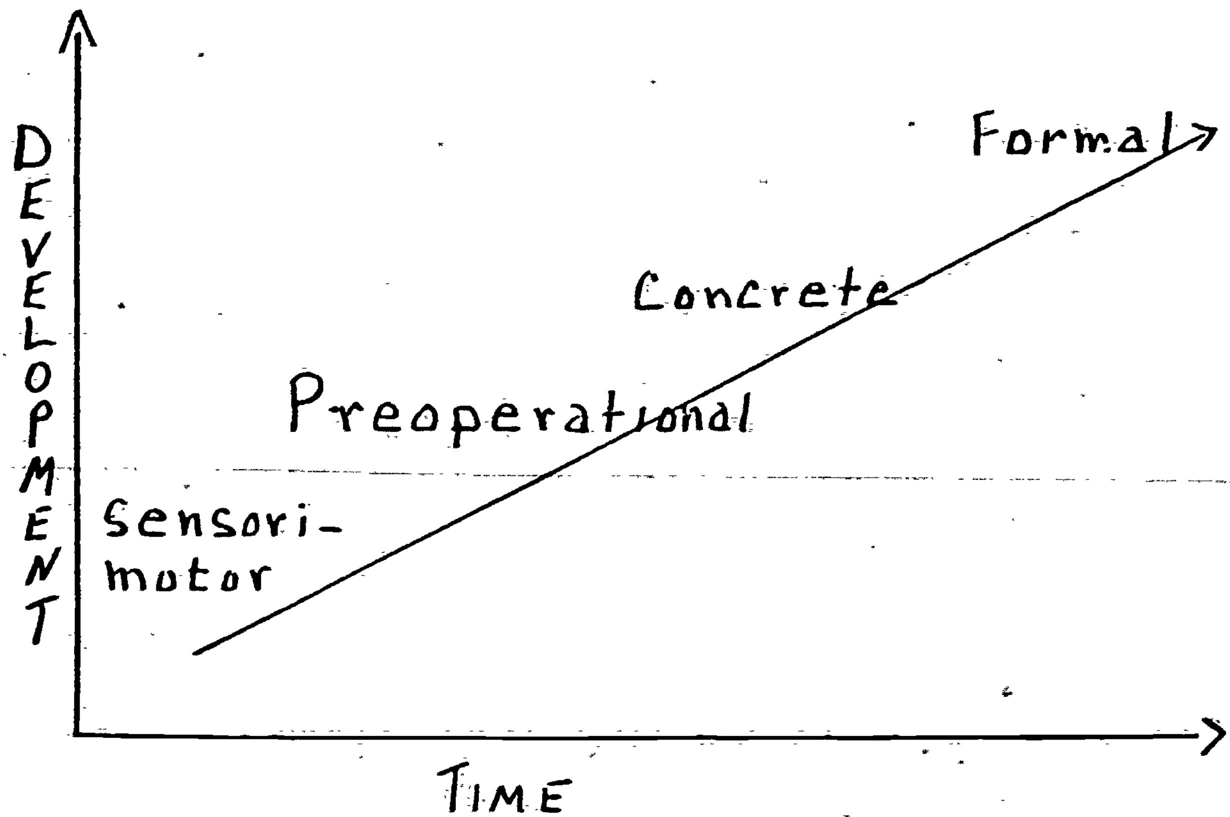
While it is known that some students may enter the formal operational stage at the age of 12, many will not do so until the late teens and some never will. It has been reported from one piece of research that 82% of the 8th and 9th grade students are still in the concrete operational stage while 50% of the University freshman classes are still in the concrete generalization stage (2). Yet many of the mathematical terms are used as though every child is in the formal operation stage. So what does all this mean? If all I have claimed is true, what do we do about it?

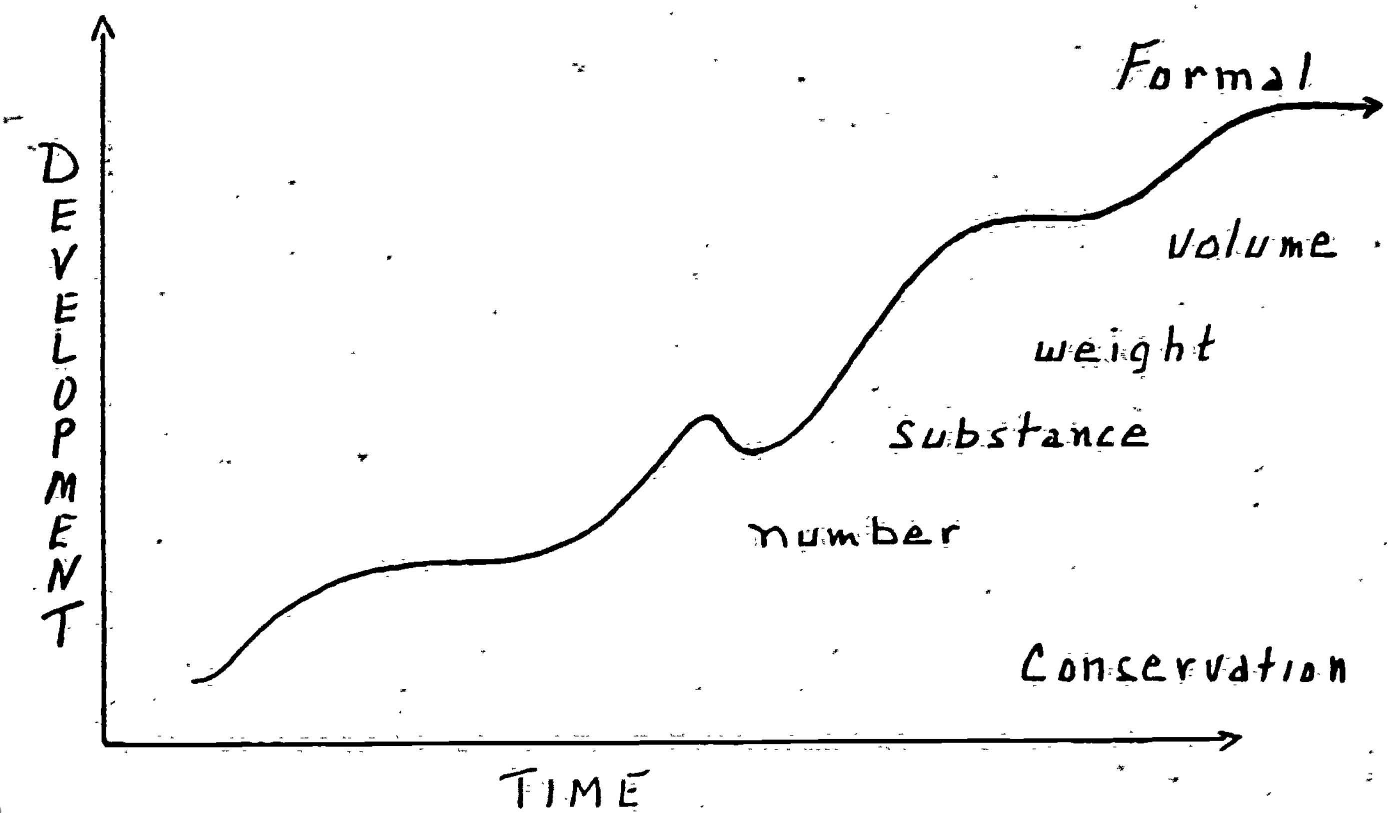
I believe there are two things we must do:

(1) diagnose more specifically the weaknesses than in the past

(2) devise ways and means (a) to bring about greater cognitive development (b) to build more word and word group associations to facilitate greater cognitive development

Let us look at ways of diagnosing weakness. I will deal with diagnosing the lack of interpretation of the printed symbol because of a lack of


Formal
Concrete
Preoperational
Sensori-
motor
DEVELOPMENT
TIME

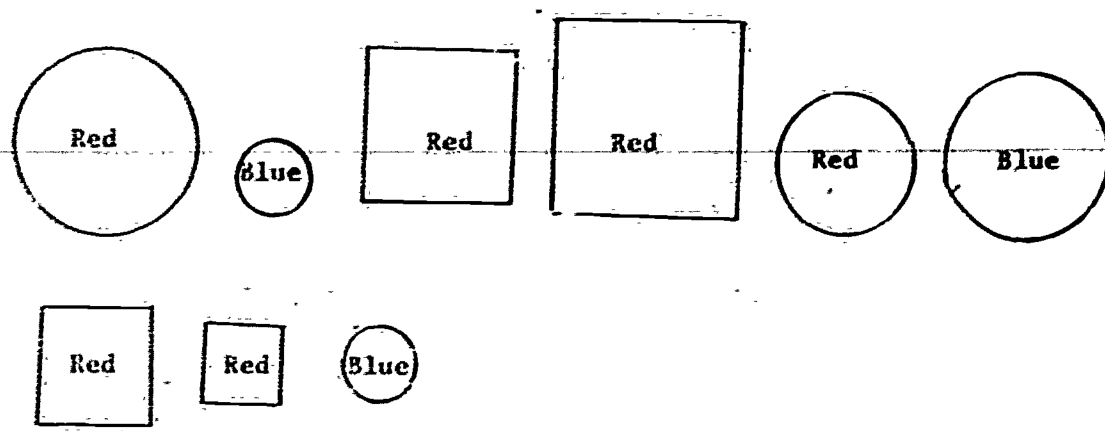

Formal
volume
weight
substance
number
Conservation
DEVELOPMENT
TIME

cognitive development in the conceptual area represented by the symbols.

We will start with two easy words: some, all

Example 1 (3)

Place in front of a student (age 5-9) red squares and blue squares and circles.



Ask the question: "Are all the squares red?"

The reply typical of a 5-6 year old is "No." When pressed the child may reply "No, because there are some red circles too."

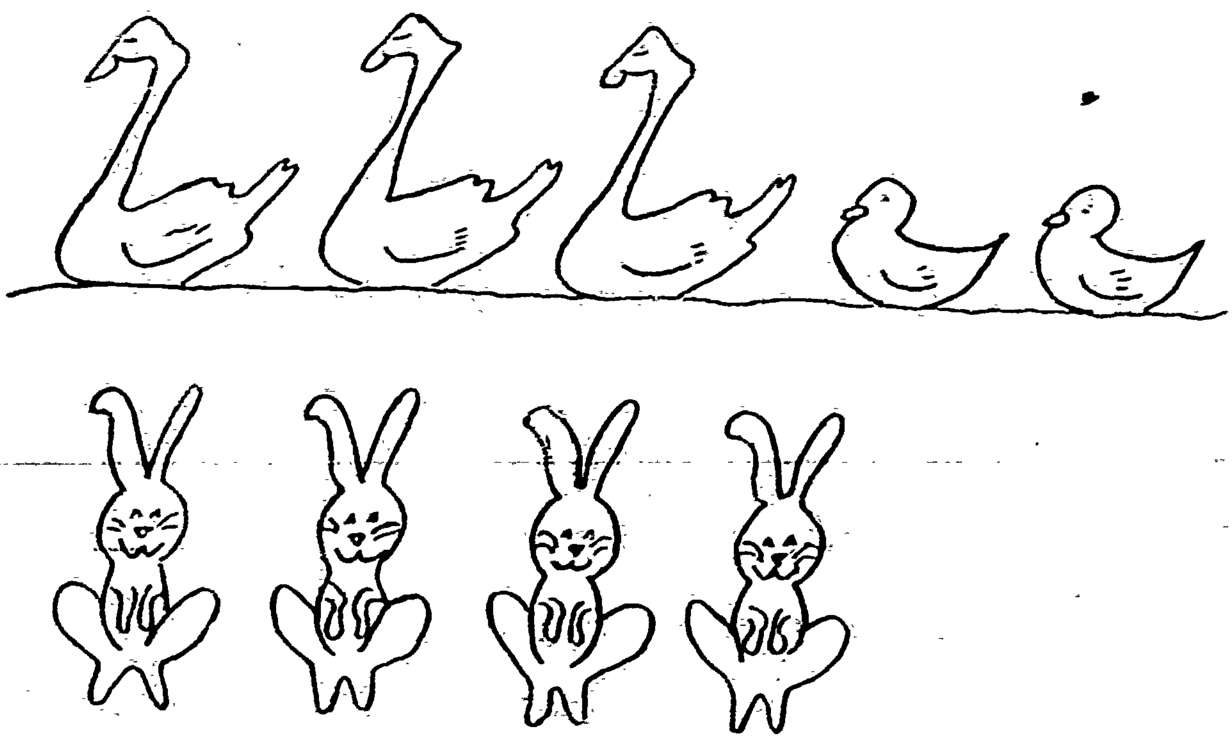
Let us consider extending this. Take the two statements:

Dogs are animals	(Squares are rectangles)
Animals are dogs	(Rectangles are squares)

What do we mean? In one case we mean all dogs are animals, and in the other we mean some animals are dogs. We expect children to make these subtle interpretations.

Example 2 (4)

Place in front of child drawings or photographs of geese, ducks and rabbits (I have changed this example from poodles, dogs and horses, cows, sheep and cats, to geese, ducks and rabbits - since I can draw them better)



Ask the students (age 5-9) these questions:

"Is every goose some kind of bird (animal)?" "Why?"

"Are all geese some kind of bird (animal)?" "Why?"

Then

"Is every bird (animal) some kind of goose?" "Why?"

"Are all birds (animals) some kind of geese?" "Why?"

Typical replies indicate that students realize geese are birds and geese are animals. But they fail to realize that all birds (animals) are not geese. Yet when pressed they will identify a bird (animal) that is not a goose.

With older children similar situations can be structured:

Is every triangle some kind of polygon? Why?

Is every polygon some kind of triangle? Why?

Is every natural number some kind of a rational number? Why?

Is every rational number some kind of natural number? Why?

The concept underlying the two words 'some' and 'all' is that of inclusion. It is not an easy mathematical concept. It n eds to be developed over a long period of time starting during the concrete operational state. Consider the situation of the three geese, two ducks and four rabbits.

Example 3 (3)

Place pictures of three geese, two ducks and four rabbits in front of the child.

Then ask these questions:

"Are there more geese or are there more ducks?" "Why?"

"Are there more geese or are there more birds?" "Why?"

"Are there more geese or are there more animals?" "Why?"

Typical replies indicate that some students will not be able to accept the two classifications - geese and birds. They reply -

"There are more geese because there are only two ducks."

Interviewer: "What did I ask you?"

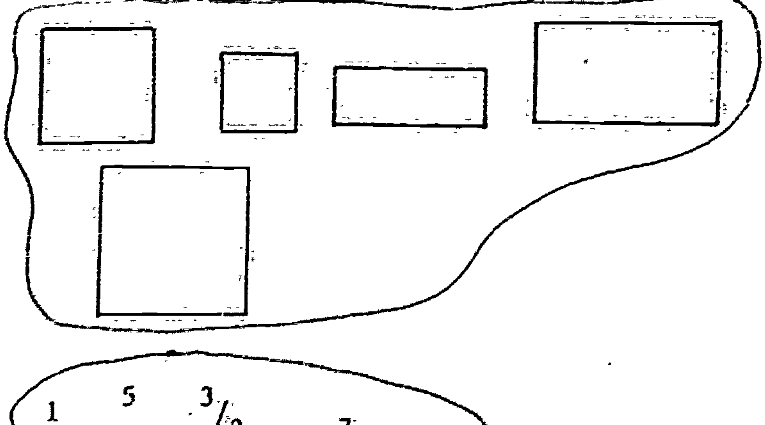
Student: "Are there more geese or more ducks?"

Interviewer: "Are there more geese or are there more birds?"

Student: "There are more geese because there are only two ducks."

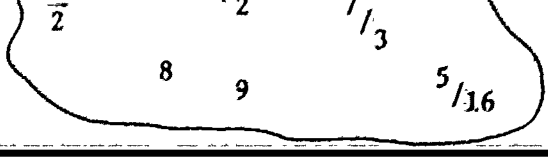
Inclusion is reflected in mathematics through the grades

(a)



Are there more squares or more rectangles in the set shown?

(b)



Are there more natural numbers or more rational numbers in the set shown?

Other words we use casually in mathematics and expect students to automatically have a grasp of are those which indicate relationships.

Example 4 (3)

Each student is given these incomplete sentences and asked to complete them (typical reply in brackets)

Peter went away even though....(he went to the country)

It's not yet night even though....(it's still day)

The man fell from his bicycle because....(he broke his arm)

Fernand lost his pen so....(he found it again)

I did an errand yesterday because....(I went on my bike)

Instances where we use these terms in mathematics are:

$^8/_3$ is not an answer even though $^8/_3 < 10$

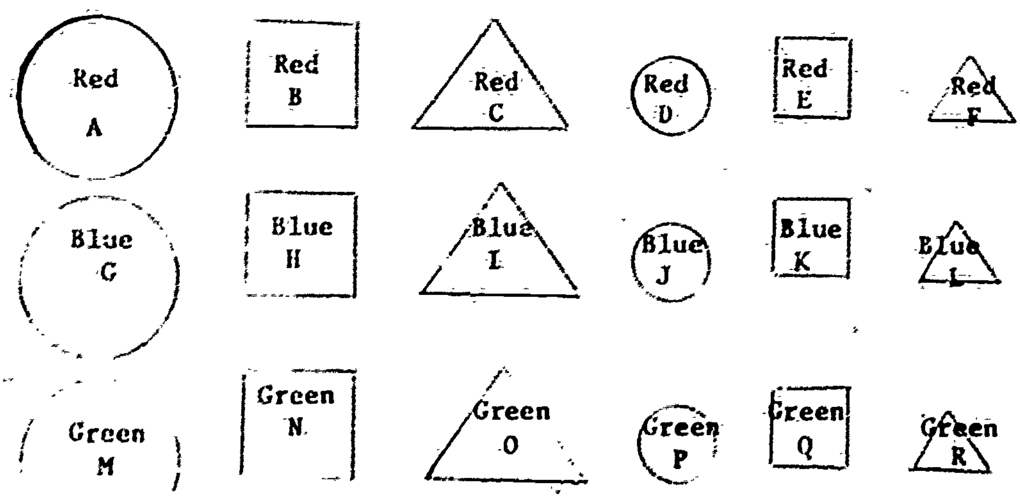
$^8/_3$ is not an answer because it is not a counting number

$^8/_3$ is not an answer so we do not include it in the solution set

We have a tendency in mathematics to put simple words together into very difficult questions.

Example 5 (5)

Each student is shown a set of shapes consisting of large triangles, circles and squares each in red, blue and green. Small ones of each color and shape were also in the set. The shapes are lettered A to R (Simpler versions can be given to younger children).



Questions asked are of this type:

1. (a) Write the letters of all the shapes that are not circles.

   (b) Write the letters of all the shapes that are not blue circles.

   (c) Write the letters of all the shapes that are not small green circles.

2. (a) Write the letters of all the shapes that are not both big and red.

   (b) Write the letters of all the shapes that are not both small and green.

3. (a) Write the letters of all the shapes that are neither big nor red.

   (b) Write the letters of all the shapes that are neither small nor green.

4. (a) Write the letters of all the shapes that are neither small and red nor big and green.

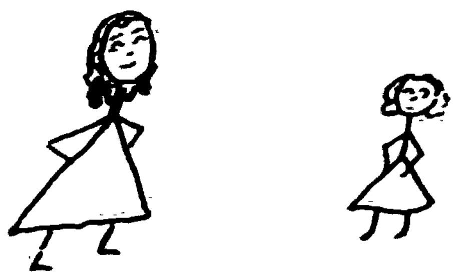
5. (a) Write the letters of all the shapes that are not yellow.

Our preliminary research has indicated that many students in high school had problems with the questions 3(a) and 4. Of one sample of 318 students only 21% of the grade 10's, 34% of the grade 11's and 34% of the grade 12's got all the questions correct.
Nearly 80% of the grade 10's, 65% of the grade 11's and 12's had difficulty with question 4. Apparently students were unable to keep track of four attributes at one time. I believe this is not a reading problem in the phy ical sense but rather a problem of lack of cognitive development to be able to properly respond to the written words.

A final example of simple words that contain a difficult concept is the if....then.... sentences. We all know that much of the logic underlying mathematics hinges on this sentence structure.

Example 6 (5)

A picture of two girls is shown to the students.



Jean Carol

The following statement is presented:

"If Carol goes for a walk, then Jean always goes with her."

The following questions are asked:

(a) Would it be possible for Carol to go for a walk, and Jean to stay at home?

(b) Would it be possible for Carol to go for a walk and for Jean to go for a walk?

(c) Would it be possible for Carol to stay at home and for Jean to stay at home?

(d) Would it be possible for Jean to go for a walk, and for Carol to stay at home?

(e) Would it be possible for Carol to stay at home, and for Jean to go for a walk?

Our test results indicate that many students in the secondary school have difficulty with questions c, d and e. They usually reply "Jean and Carol must always go out for a walk together."

So far in this paper I have only presented a sample of the problems connected with words. There is the problem of interpreting symbols as well. It is a compound one, first the symbol needs to be translated into words (a difficult enough task in itself), second, the words need to be interpreted. (symbol → word → concept). Many students become familiar enough with symbols to omit the word stage (symbol → concept) but often in gaining this familiarity the word stage is used. Many students do not progress to the symbol → concept stage.

Three instances with reference to symbols will be illustrated. The first involves = (equals). Steffe, (6) points out that young children he worked with would write 3 + 4 = 7 but would not write 7 = 3 + 4. The reason given by the children paraphrased here, is that equals always follows an operation - a doing. Is this built into our teaching or is it inherent in the stages of a child's development? We are not certain.

The second example involves basic concepts of union and intersection of sets.

$$\{2,3,4\} \cup \{2,4,5\} = \{2,3,4,5\}$$

$$\{2,3,4\} \cap \{2,4,5\} = \{2,4\}$$

Using a test devised by Bliss (5) we tested a sample of high school students and found many students had a computational facility with union and intersection, yet did not have a usable conceptual understanding of the two.

Finally, Collis (7) has indicated that

$8 \times 3 = 3 \times \triangle$ is of lower cognitive level than $7 - 4 = \triangle - 7$

$8 + 4 - 4 = \triangle$ is of lower level than $4283 + 517 - 517 = \triangle$

$a \div b = 2a \div \triangle$ requires a comparatively high level of cognitive development.

In summary, I have presented a few samples which tend to indicate simple words often convey high level concepts requiring high level cognitive development. These samples are only a few of the many we have. I have tried to point out that there is a definite need to not only teach students words but we must teach students the ideas being represented by the words. We must not assume that because a student does not perform adequately in response to a passage in a mathematics book that it is a reading problem in the physical sense - it may be a cognitive development problem.

I feel we are making progress in identifying the specific nature of some of the weaknesses (objective 1). We must now attack the second objective - that of indicating ways and means to overcome the weaknesses. We have ideas but at this point we have no experimental results to back them up. For what they are worth, let me put forth a couple of ideas related to what I have been saying.

I think we must find ways to build word-meaning associations and especially word groups - meaning associations. Perhaps symbol groups - meaning associations will follow more naturally.

<u>Example 7</u> (8)

Read: $8^2 + b$

Four ways to read this are:

$8^2 + b$ "eight squared plus b"

$8^2 + b$ "the square of eight plus b"

$8^2 + b$ "b is added to the square of eight"

$8^2 + b$ "b is added to eight squared"

Many varied experiences will more likely develop the proper associations than just using one of the four.

A problem that symbols present is that a student may not be able to pronounce the word(s) represented. There are no hints. He can't use the rules of his phonetics class. To establish a phoneme-grapheme relation to one has to recall the following words as spoken by the teacher: the integral of the function of x with respect to x. The student can at least read aloud the word form but may have no clue as to how to read the symbolic form. Utilizing symbols only when necessary or only after students have adequate facility with them will aid the weak reader in mathematics. Drill in symbols is necessary.

Mathematics teachers, along with others, like to quote Lewis Carroll in Through The Looking Glass where Humpty-Dumpty says to Alice "When I use a word it means just what I choose it to mean - neither more nor less." In mathematics words are used with special meanings (we pay them extra - said Humpty-Dumpty). These special meanings often - though not always - are related to the common meaning. Take the word "associative" as in The Associative Property. We can illustrate the relation this way:

mathematics	example
$(8 + 9) + 3 = 8 + (9 + 3)$	Jerry and Harry joined Mary. Jerry joined Harry and Mary.

To sum up, we must provide students with a broader set of experiences centred about the concepts in which weaknesses in reading are reflected. These broader experiences will provide for greater cognitive development in the conceptual area hence the words the students read will have more specific and deeper meanings. This aspect of teaching reading is more of a concern to the mathematics teacher while the mechanics and other aspects of reading not mentioned here maybe of equal concern to both the teacher of mathematics and the teacher of reading.

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