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ABSTRACT

This minicourse was prepared for use with secondary physics students in the Dallas Independent School District and is one option in a physics program which provides for the selection of topics on the basis of student career needs and interests. This minicourse was aimed at two levels in the study of basic machines. The "light" level introduces the basic machines of technical physics. The "heavy" level treats the technical aspects of motion associated with basic machines in a precise and mathematically involved fashion. The minicourse was designed for independent student use with close teacher supervision and was developed as an ESEA Title III project. A rationale, behavioral objectives, student activities, and resource packages are included. Student activities and resource packages involve investigating levers, pulleys, wheel and axle, inclined plane, and screw jack machines, and applying the concept of mechanical advantage. A technical description of forces, moments, equilibrium, and motion emphasizes the vector nature of forces and moments and uses mathematical representations and analyses of these. (GS)

BASIC MACHINES

PRELIMINARY EDITION

THE "NUTS and BOLTS" OF TECHNICAL PHYSICS

CAREER ORIENTED PRE-TECHNICAL PHYSICS

Basic Machines

the

"Nuts and Bolts"

of

Technical Physics

Minicourse

ESEA Title III Project

1974

dallas independent school district

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This Mini Course is a result of hard work, dedication, and a comprehensive program of testing and improvement by members of the staff, college professors, teachers, and others.

The Mini Course contains classroom activities designed for use in the regular teaching program in the Dallas Independent School District. Through Mini Course activities, students work independently with close teacher supervision and aid. This work is a fine example of the excellent efforts for which the Dallas Independent School District is known. May I commend all of those who had a part in designing, testing, and improving this Mini Course.

I commend it to your use.

Sincerely yours,

Nolan Estes

Nolan Estes
General Superintendent

mfs

CAREER ORIENTED PRE-TECHNICAL PHYSICS TITLE III ESEA PROJECT

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CAREER ORIENTED PRE-TECHNICAL PHYSICS

BASIC MACHINES - THE "NUTS AND BOLTS" OF TECHNICAL PHYSICS

MINICOURSE

RATIONALE (What the minicourse is about)

This minicourse comes in two sizes: light and heavy. The first half of the minicourse is "light", and introduces the basic machines of technical physics. Such machines always involve motion. The elemental technical physics of motion is presented in other minicourses, especially in *Physics of Sports* and *Physics of Toys*. But the second half of this minicourse is "heavy", because it treats the technical descriptions of motion in more precise and more advanced mathematical ways than do the other minicourses.

If you are interested in a light treatment of basic machines, study only up to Resource Package 4-2. If you want to try the whole works, lots of luck! It is fun, once you begin to master the "heavier" stuff.

A scholar of jobs in America said recently that the future belongs to those of us still willing to get our hands dirty. What he meant was that an estimated one million skilled persons will be added to the American work force within the next five years to fill new jobs. And certainly there are advantages to working at a "hand skill" career: you can see the immediate results of your work; you don't have to "take your job home with you"; your chances of getting job ulcers are far less; and your prospects for decent salaries and wages are good. A table of hourly pay for 1972, and of estimated job openings per year over the next five years, is presented below (from 1972 U. S. Labor Department Reports):

Career	Approximate New Jobs	Approximate Hourly Pay*
Auto Mechanics	20,000	$5.00
Air Conditioning	20,000	$3.00-$7.00
TV Service	5,000	$3.50-$6.50
Electrician	20,000	$3.50-$4.75
Electronics	10,000	$3.50-$5.50

In this minicourse, some physics of the following basic machines will be introduced: lever, piston, wheel and axle, inclined plane, pulley, screw, and wedge. These machines dominate our everyday lives. Sometimes they are disguised and not readily recognized. For instance, the steps to the school building are modified forms of the inclined plane; and the knob on the door to your classroom is an application of the wheel and axle. Of course, many machines are familiar and recognizable; even the untrained person can identify a bicycle or an automobile. But these are NOT basic machines; they are actually compound machines, since they are combinations of several basic machines. For instance, the sprocket and the handle bars on the bicycle are each examples of the wheel and axle.

As you well know, machines help us to do more work, and to do it easier and faster. They help us in these three essential ways: 1) They allow us to exert a smaller force through a large distance, in order to

* These are average national straight-time hourly pay figures. Certain union shops, commissions, overtime benefits, fringe benefits, etc. result in much higher pay for certain skilled workers.

move a bigger load through a smaller distance. 2) They allow us to exert a bigger force through a small distance, in order to move a smaller load through a larger distance. 3) They allow us to exert a force (effort force, F_E) in a direction opposite to the load (resistance force, F_R).

As you study these basic machines, you will see that to get work from a machine you must put work (energy) into it. Try operating an automobile for a long period of time without suppling chemical energy (gasoline) and observe what happens! We always get less work from a machine than we supply to it, because of friction-caused energy losses.

The ratio of energy output to energy input (or work output to work input) is called efficiency. All machines must operate at less than 100% efficiency, because of the energy losses required to overcome frictional forces between moving parts. For example, the steam locomotive operates at an efficiency of only five (5) to eight (8) per cent. With proper design and lubrication, certain machines can operate at ninety (90) per cent efficiency or even greater.

In addition to RATIONALE, this minicourse contains the following sections:

1) TERMINAL BEHAVIORAL OBJECTIVES (Specific things you are expected to learn)

2) ENABLING BEHAVIORAL OBJECTIVES (Learning "steps" which enable you to reach the terminal behavioral objectives)

3) ACTIVITIES (Specific things to do to help you learn)

4) RESOURCE PACKAGES (Instructions for carrying out the Activities, such as procedures, references, laboratory materials, etc.)

TERMINAL BEHAVIORAL OBJECTIVE

Upon completion of this minicourse you will demonstrate your level of understanding of basic machines by correctly solving at least seven (7) out of ten (10) problems concerning basic machines and their application to the physical world.

ENABLING BEHAVIORAL OBJECTIVE #1:	ACTIVITY 1-1	RESOURCE PACKAGE 1-1
Given three (3) problems concerning the lever, you will correctly solve at least two (2).	Read Resource Package 1-1.	"Lever Machine"
	ACTIVITY 1-2	RESOURCE PACKAGE 1-2
	Complete Resource Package 1-2	"Investigating The First Class Lever Machine"
	ACTIVITY 1-3	RESOURCE PACKAGE 1-3
	Complete Resource Package 1-3	"Investigating The Second Class Lever"
	ACTIVITY 1-4	RESOURCE PACKAGE 1-4
	Complete Resource Package 1-4	"Investigating The Third Class Lever"
	ACTIVITY 1-5	RESOURCE PACKAGE 1-5
	Study Resource Package 1-5	"Mechanical Advantage"
	ACTIVITY 1-6	RESOURCE PACKAGE 1-6
	Do Resource Package 1-6	"Investigating The IMA and The AMA of The Lever"

ENABLING BEHAVIORAL OBJECTIVE #1:	ACTIVITY 1-7	RESOURCE PACKAGE 1-7
See previous page	Complete Resource Package 1-7	"Self-Test On Levers"
	ACTIVITY 1-8	RESOURCE PACKAGE 1-8
	Check answers using Resource Package 1-8	"Self-Test Answers"
	ACTIVITY 1-9	RESOURCE PACKAGE 1-9
	Study Resource Package 1-9	"Applications of The Lever"
ENABLING BEHAVIORAL OBJECTIVE #2:	ACTIVITY 2-1	RESOURCE PACKAGE 2-1
Given three (3) problems involving the pulley, you will be able to solve at least two (2).	Read Resource Package 2-1	"The Pulley Machine"
	ACTIVITY 2-2	RESOURCE PACKAGE 2-2
	Do Resource Package 2-2	"Investigating Pulleys"
	ACTIVITY 2-3	RESOURCE PACKAGE 2-3
	Complete Resource Package 2-3	"Self-Test on Pulleys"
	ACTIVITY 2-4	RESOURCE PACKAGE 2-4
	Compare your answers with the answers in Resource Package 2-4. If you missed more than two of the individual questions, ask your teacher for assistance.	"Self-Test Answers"

ENABLING BEHAVIORAL OBJECTIVE #3:	ACTIVITY 3-1	RESOURCE PACKAGE 3-1
Given five (5) problems involving the wheel and axle, inclined plane, wedge, and screw jack machines, you will work at least four (4) of these problems.	Read Resource Package 3-1	"Wheel-And-Axle Machine"
	ACTIVITY 3-2	RESOURCE PACKAGE 3-2
	Read Resource Package 3-2	"The Inclined Plane Machine"
	ACTIVITY 3-3	RESOURCE PACKAGE 3-3
	Read Resource Package 3-3	"The Wedge Machine"
	ACTIVITY 3-4	RESOURCE PACKAGE 3-4
	Read Resource Package 3-4	"The Screw Jack Machine"
	ACTIVITY 3-5	RESOURCE PACKAGE 3-5
	Complete Resource Package 3-5	"Investigating The Wheel-And-Axle, Inclined Plane, and Screw Jack Machines"
	ACTIVITY 3-6	RESOURCE PACKAGE 3-6
	Study Resource Package 3-6	"Compound Machines"
	ACTIVITY 3-7	RESOURCE PACKAGE 3-7
	Work the problems in Resource Package 3-7	"Wheel-And-Axle, Inclind Plane, and Screw Jack Problems"
	ACTIVITY 3-8	RESOURCE PACKAGE 3-8
	Compare your answers with the answers in Resource Package 3-8. If you have trouble, ask your teacher for assistance	"Answers to Problems"

ENABLING BEHAVIORAL OBJECTIVE #3:	ACTIVITY 3-9	RESOURCE PACKAGE 3-9
See previous page	Complete Resource Package 3-9	"Self-Test"
	ACTIVITY 3-10	RESOURCE PACKAGE 3-10
	Compare your answers with the answers in Resource Package 3-10. If you missed more than one (1), ask your teacher for assistance.	"Self-Test Answers"
	ACTIVITY 4-1	RESOURCE PACKAGE 4-1
	Read Resource Package 4-1	"Energy and Machines"
	ACTIVITY 5-1	RESOURCE PACKAGE 5-1
	Ask your teacher for the final evaluation after you have reviewed the materials within this minicourse, or if you elect the optional sections for further study and understanding of basic machines, complete Resource Package 5-2.	"Final Evaluation"
		RESOURCE PACKAGE 5-2
		"Technical Descriptions of Forces, Moments, Equilibrium and Motion"

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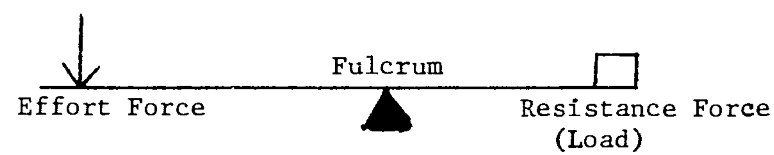
RESOURCE PACKAGE 1-1

LEVER MACHINE

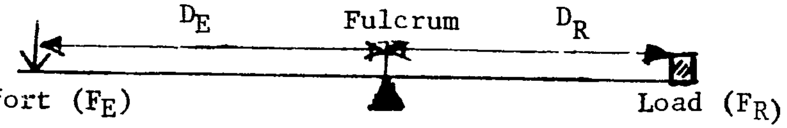
The lever machine is a bar which is pivoted to rotate about a point called its fulcrum. The playground see-saw is a typical lever. In addition to the bar and its fulcrum, it is necessary to identify the effort force (F_E) and the resistance force (F_R). The effort force can be thought of as the force applied to the lever which tends to make it rotate. The resistance force can be thought of as the load or the resistance to the effort force by the object on which work is being done; the resistance force tends to resist rotation of the lever (See Figure 1 below).



LEVER MACHINE
Fig. 1

Sometimes lever machines are divided into three types of classes. These are referred to as first, second, and third class levers, depending upon the relative position of the effort force, the fulcrum, and the resistance force.

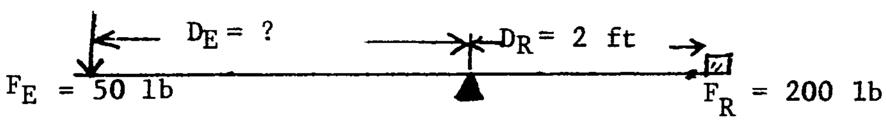
The effort force required to move the load depends upon the relative position of the fulcrum, the point of application of the effort force, and the point of application of the load force. The following ratio, which we call the Lever Formula, will help us to calculate respective forces and/or distances for the lever machine: $D_E F_E = D_R F_R$ (See Fig. 2 below)



LEVER FORMULA VARIABLES
Fig. 2

In Figure 2, the distance from the fulcrum to the effort location is designated D_E, while the distance from the fulcrum to the load is D_R. The effort is designated F_E and the load F_R. If any three of the Lever Formula variables are given, we can always calculate the fourth.

Example #1: Given the diagram as labeled, calculate D_E.



Solution: Start with the Lever Formula, $D_E F_E = D_R F_R$, and solve for D_E by dividing both sides of the equation by F_E:

$$\frac{D_E F_E}{F_E} = \frac{D_R F_R}{F_E}$$

$$DE = \frac{D_R F_R}{F_E}$$

Substitute the given values of the variables F_E, F_R, and D_R to get

$$D_E = \frac{(2\ \text{ft})(200\ \text{lb})}{50\ \text{lb}}$$

$$= \frac{400\ \text{ft}}{50}$$

$$= 8\ \text{ft}$$

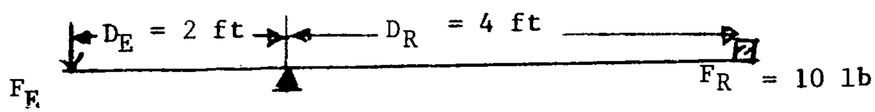
Example #2: Given the following, calculate F_E for the type of lever in Example #1:

$D_R = 4$ ft

$F_R = 10$ lb

$D_E = 2$ ft

Solution: First draw a diagram:



Then use the Lever Formula:

$$D_E F_E = D_R F_R$$

$$F_E = \frac{D_R F_R}{D_E}$$

$$= \frac{(4\ \text{ft})\ (10\ \text{lb})}{2\ \text{ft}}$$

$$= 20\ \text{lb}$$

Note the fulcrum position in both examples. In the first, the fulcrum is nearer to the resistance and this makes the required effort less. Here, force advantage is gained at the expense of distance. The 50-lb effort force moves a load of 200 lb, but the effort force must move down four inches to move the load up one inch! In the second example, the fulcrum is nearer the effort force and this requires that the effort force br proportionally greater. Here distance advantage is gained at the expense of effort force. In this case, the 20-lb effort moves only a 10-lb load but the effort force moves down only one inch to move the load up two inches.

We can design levers to gain mechanical advantage of force, mechanical advantage of distance, or mechanical advantage of direction, depending upon our needs. Incidentally, both levers in the solutions above also gained mechanical advantage of direction; the effort forces moved down while the resistance forces moved up.

The Lever Formula does not consider the weight of the bar itself. For many practical cases the weight of the bar can be ignored and only trivial, unimportant errors will result when using the Formula. These errors are particularly small when the fulcrum is located at or near the center of the bar, and when the bar is uniform throughout so that its center of gravity (balance point) is located at the center of the bar also.

RESOURCE PACKAGE 1-2

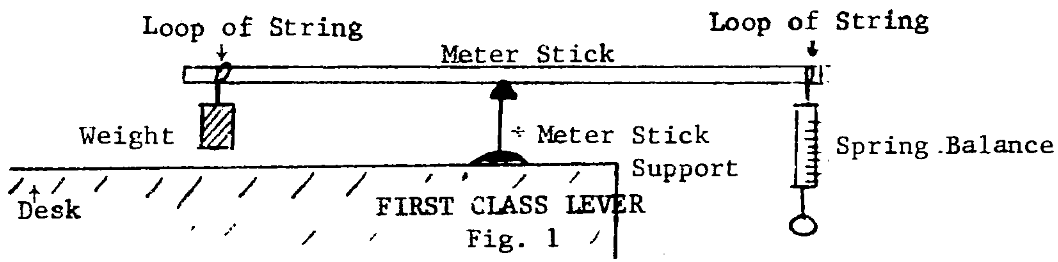
INVESTIGATING THE FIRST CLASS LEVER MACHINE

In the first class lever the fulcrum is located between the effort and the load; and the load, the fulcrum, and the effort are on the same side of the bar (See Fig. 1). Let's investigate this machine.

a) Obtain a meter stick, spring balance, set of standard metric weights, meter stick support, and about 2 ft of string.

b) Examine Figure 1. Make two string loops each about 2 inches in diameter.

c) Draw a diagram of the apparatus as indicated in the figure below. Label each part.



FIRST CLASS LEVER
Fig. 1

d) Place the center of gravity (balance point) of the meter stick in the support. (Can you see that this eliminates the weight of the meter stick from entering into the investigation?) Using a string loop, attach the 1,000-unit weight* at the 10-cm mark. Use the other loop and attach the bottom of the spring balance to the meter stick at the 90-cm mark. Pull down on the spring balance until the meter stick is level. Record the balance reading on a data chart such as the one shown below (This reading represents the effort force):

---

* The weights you use will likely be marked in terms of their inertial mass units, NOT in terms of their weight. (They will likely be marked in gram or kilogram mass units.) In technical physics, weight units are the pound, the newton, and the dyne. See the minicourses Metric System and Slide Rule, Physics of Sports, Physics of Toys, etc. for simple explanations of weight and mass.

Trial	Effort Force (F_E)	Effort Distance From Fulcrum (D_E)	Resistance Force (F_R)	Resistance Distance From Fulcrum (D_R)
c)				
d)				
e)				
f)				
g)				

Next, use the Lever Formula to calculate the effort force. Did your calculated reading and your measured reading come close to 1,000? Your calculation should look like this:

$$D_E F_E = D_R F_R$$

$$F_E = \frac{D_R F_R}{D_E}$$

$$= \frac{40 \text{ cm } (1{,}000 \text{ units})}{40 \text{ cm}}$$

$$= 1{,}000 \text{ units}$$

e) Next, move the spring balance to the 80-cm mark. Read the balance and record your answer on the chart. What is the distance D_R? Use the Lever Formula as you did in part (d), above, and calculate the effort, F_R. How well does your calculation compare with the experimental value?

f) Move the weight to the 35-cm mark and move the spring balance to the 95-cm mark. Read the spring balance and record your answer. Calculate the value for D_R and again compare this with the experimental value read from the balance. Your answers should be close.

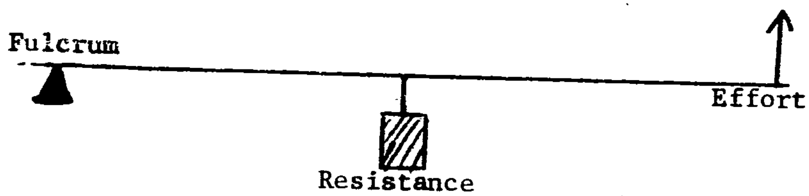
g) Next, move the weight (resistance) to the 5-cm mark, and move the spring balance (effort force) to the 65-cm mark. Proceed with readings and calculations as you did in part (f) above. Do you notice anything different about the effort force this time?

Submit your data, responses, and calculations for evaluation.

RESOURCE PACKAGE 1-3

INVESTIGATING THE SECOND CLASS LEVER

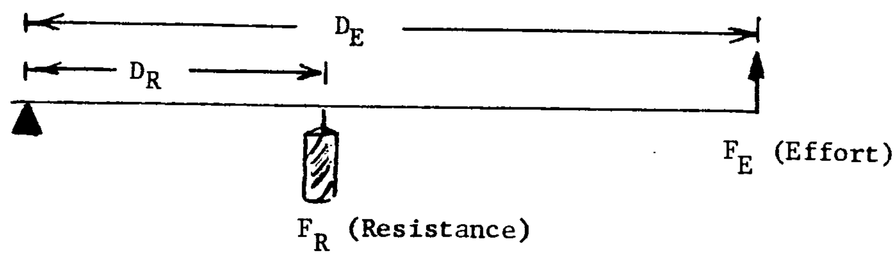
The second class lever is arranged such that the resistance is between the fulcrum and the effort (See Fig. 1 below).



SECOND CLASS LEVER
Fig. 1

This type of lever always gains advantage of force at the expense of distance, because the effort always moves a greater distance than the resistance. Further, with the second class lever no advantage of direction is gained. Notice in Fig. 1 that if the effort moves up, then the resistance will also move up.

The Lever Formula works for all classes of levers. Again we designate the distance from the fulcrum to the effort D_E, the distance from the fulcrum to the resistance D_R, the effort force F_E, and the resistance force F_R (See Fig. 2 below).

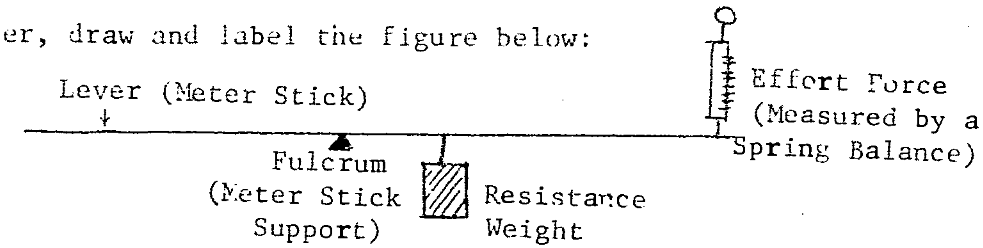


LEVER VARIABLES
Fig. 2

Perform the following:

a) Obtain the same materials used in Resource Package 1-2.

b) On notebook paper, draw and label the figure below:



SECOND CLASS LEVER
Fig. 3

c) Arrange the apparatus so that the meter stick is supported at its center of gravity.

d) Use a loop of the string to hang a 1,000-unit weight at the 70-cm mark. Use the other loop to attach the spring balance at the 95-cm mark. Pull the spring balance by the ring at the top until the lever appears to be level. Read the spring balance and record your answer in a chart such as the one you made up for Resource Package 1-2.

e) Use the Lever Formula and calculate F_R. Compare the calculated force with the experimental force from part (d) above.

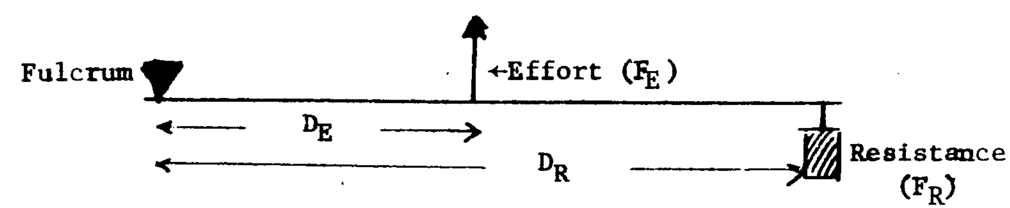
f) Move the weight to the 80-cm mark and move the spring balance to the 90-cm mark. Record the spring balance reading; also, calculate F_E. Does the spring balance support more or less force than it did in part (d) above?

g) Now move the meter stick so that it is supported at the 1-cm mark by the meter stick support. Remove the 1,000-unit weight from the lever, and attach the balance at the 99-cm mark. Pull on the balance ring until the level is level. The balance should read approximately 75 units! How does one account for this? Well, this is due to the weight of the meter stick. This weight acts as if it were concentrated at its center of gravity; therefore, the meter stick weight serves as a resistance force acting at the center of the meter stick (See Fig. 2 below).

RESOURCE PACKAGE 1-4

INVESTIGATING THE THIRD CLASS LEVER

The last type of lever is he third class lever, which has the effort force located between the fulcrum and the resistance (See Fig. 1 below)

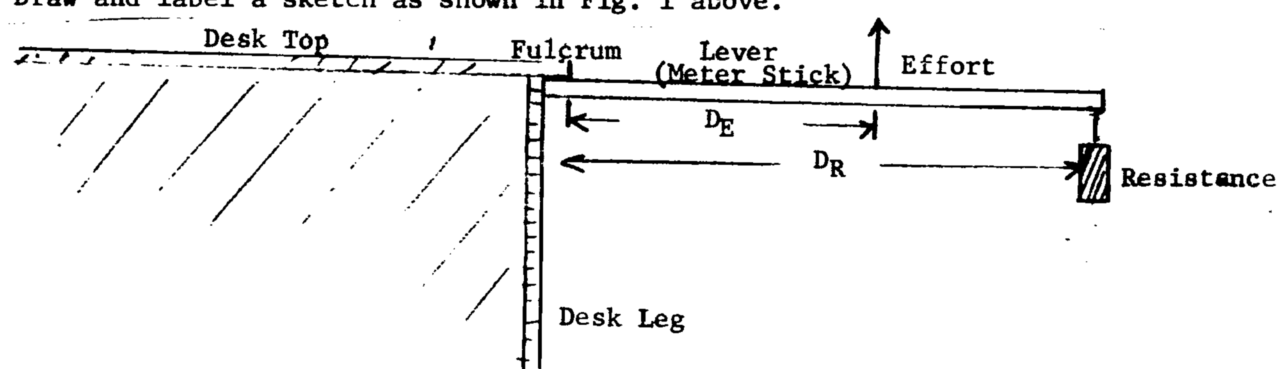


THIRD CLASS LEVER
Fig. 1

The third class lever always gains advantage of distance at the expense of force. Direction advantage is not gained because the resistance always moves in the direction of the effort force.

a) Arrange the apparatus as shown in Fig. 2 below.

b) Draw and label a sketch as shown in Fig. 1 above.



THIRD CLASS LEVER
Fig. 2

c) Place approximately 3 cm of the lever under the desk top. Place the 500-unit weight at the 90-cm mark and support the lever with the spring balance at the 60-cm mark. (Use care to prevent the apparatus from falling!) Record the spring balance reading. Using the very edge of the desk top (See Fig. 2) as the fulcrum, determine D_E and D_R. Calculate F_E and compare it with the measured value.

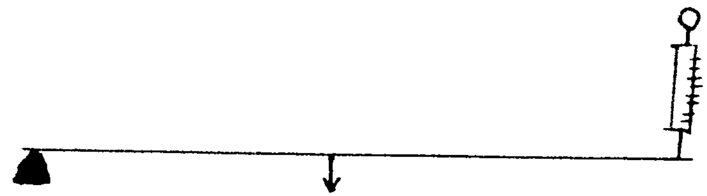
d) Now move the spring balance to the 50-cm mark. Record the spring balance reading. Move the balance to the 40-cm mark and record this reading. What happens to the effort in a third class lever as you move nearer to the fulcrum? Do you gain or lose force advantage?

e) Remove the spring balance and support the lever with your hand at the 40-cm mark. Slowly pull the lever up about 3 or 4 cm. Repeat the motion two or three times, noting the distance moved by the resistance at the 90-cm mark. Is distance advantage gained or lost?

Remember that with any type of machine if the resistance moves farther than the effort, distance advantage is said to be gained. Conversely, if the resistance moves less than the effort, distance advantage is lost. Of course, if the effort is greater than the resistance then force advantage is lost.

As you have observed, distance advantage is always gained and force advantage is always lost in the third class lever.

f) Remove the weight from the lever and attach the spring balance at the 20-cm mark. Record the reading on the balance. This reading is due to the weight of the meter stick acting at its center of gravity (at the 50-cm mark).



Weight of Meter
Stick Acts Here

SECOND CLASS LEVER
Fig. 2

g) Using the apparatus arrangement of part (g) above, attach the weight at 60 cm. Record the spring balance reading. Calculate F_E using the Lever Formula. Compare your calculation with the reading on the spring balance. Can you account for the large discrepancy?

Submit your data, sketches, and notes for evaluation.

RESOURCE PACKAGE 1-5

MECHANICAL ADVANTAGE

The mechanical advantage of a machine is frequently defined in two ways:

1) The ideal mechanical advantage (IMA) is defined as the ratio of the distance over which force acts (S_1) to the distance the resistance is moved (S_2):

$$IMA = \frac{S_1}{S_2}$$

Since S_1 and S_2 are not as easily calculated for the lever as D_E (distance from fulcrum to point of application of effort force) and D_R (distance from fulcrum to point of application of resistance force), the following equivalent definition is frequently preferred: The IMA for a lever is the ratio of the distance from the fulcrum to the effort (D_E) to the distance from the fulcrum to the resistance (D_R). Mathematically,

$$IMA = \frac{D_E}{D_R}$$

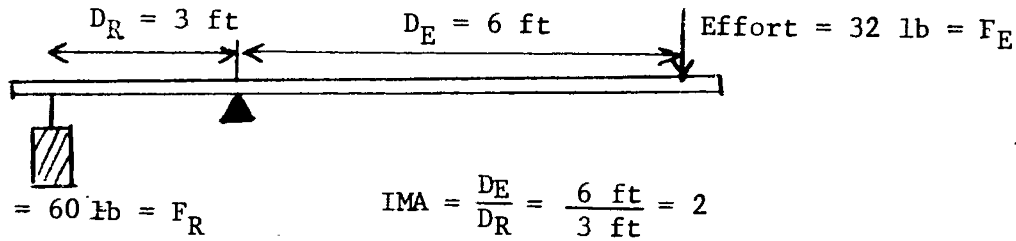
2) The actual mechanical advantage (AMA) is defined mathematically as the ratio of the resistance force (F_R) to the effort force (F_E):

$$AMA = \frac{D_E}{F_E}$$

The IMA assumes that an idealized friction-less condition exists for a machine. The AMA, on the other hand, deals with the actual forces including friction (friction forces exist in all machines to some degree). Both the IMA and AMA are pure numbers; that is, they have no dimensional units such as feet, centimeters, or pounds; and both IMA and AMA are often expressed as either common fractions or decimal fractions.

Example #1: A lever 10 ft long has its fulcrum 3 ft from one end. The resistance is 2 ft from the fulcrum, and the effort is 6 ft from the fulcrum on the opposite side from the resistance. The resistance (load) is 60 lbs, and the effort is 32 lbs. Find the IMA and the AMA.

Solution:



$$IMA = \frac{D_E}{D_R} = \frac{6\ ft}{3\ ft} = 2$$

$$AMA = \frac{F_R}{F_E} = \frac{60\ lb}{32\ lb} = \frac{15}{8} = 1.875$$

The AMA is always less than the IMA, depending upon the amount of friction in the machine. The ratio of the AMA to the IMA represents the efficiency of the machine. Efficiency is frequently expressed as a percent:

$$\text{Efficiency} = \frac{AMA}{IMA}$$

For example, in the solution above, the efficiency is

$$\frac{AMA}{IMA} = \frac{1.875}{2}$$

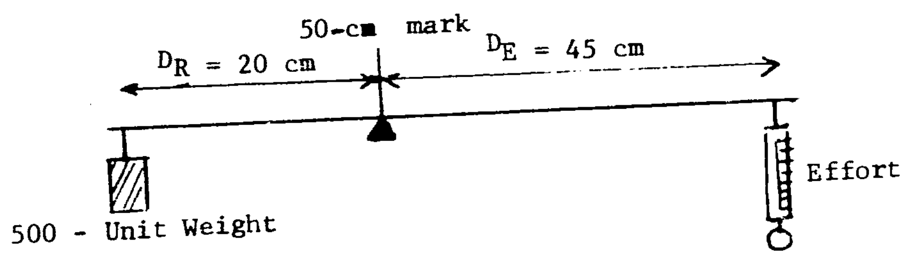
$$\cong .9375$$

or 93.75%

RESOURCE PACKAGE 1-6

INVESTIGATING THE IMA AND AMA OF THE LEVER

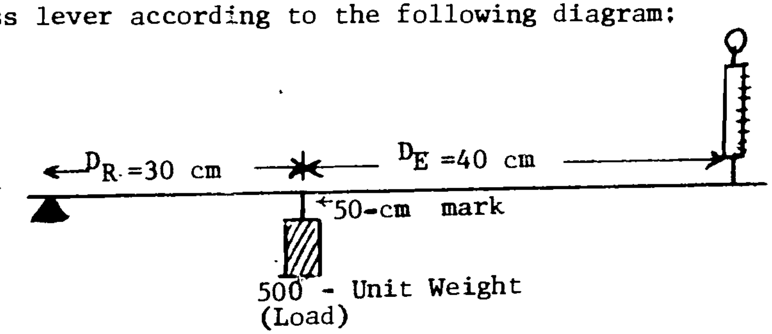
a) Set up a first class lever according to the following diagram;



b) Using D_E and D_R, calculate the IMA. Record your answer.

c) Using the reading from the spring balance, calculate the AMA. Record your answer and compare it with the IMA.

d) Set up a second class lever according to the following diagram:



e) Using D_E and D_R, calculate and record the IMA.

f) From the effort force reading and the load value, calculate the AMA. Compare this with the IMA.

g) Calculate the efficiency of this lever.

Submit your notes, sketches, and calculations for evaluation.

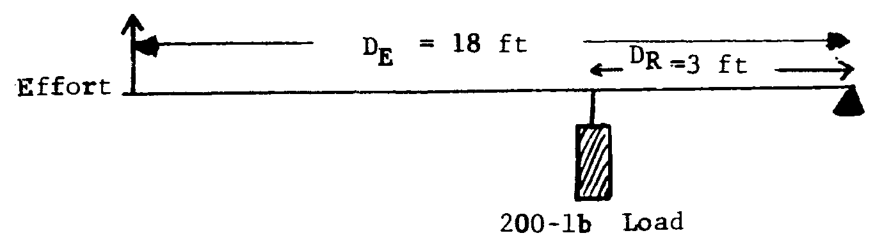
RESOURCE PACKAGE 1-7

SELF-TEST ON LEVER MACHINES

Draw and label a diagram for each example. Show all calculations. Ignore the weight of the levers.

1) A plank 10 ft long is used as a lever with the fulcrum between the effort and resistance. The fulcrum is 2 ft from a 50-lb resistance which is at one end of the plank. What is the IMA of the lever? What force applied at the other end of the plank will counterbalance the 50-lb resistance?

2) A 200-lb weight is supported by a 35-lb effort in the diagram below. Calculate the IMA, AMA, and efficiency.



3) A diver stands at the end of a 12-ft diving board which extends 7 ft from its frame of support. If the diver weighs 180 lbs, calculate the force needed to secure the other end of the board.

RESOURCE PACKAGE 1-8

SELF-TEST ANSWERS

1) IMA = 5

   Effort = 10 lbs

2) IMA = 6

   AMA = 5.7

   Efficiency = .95

   = 95%

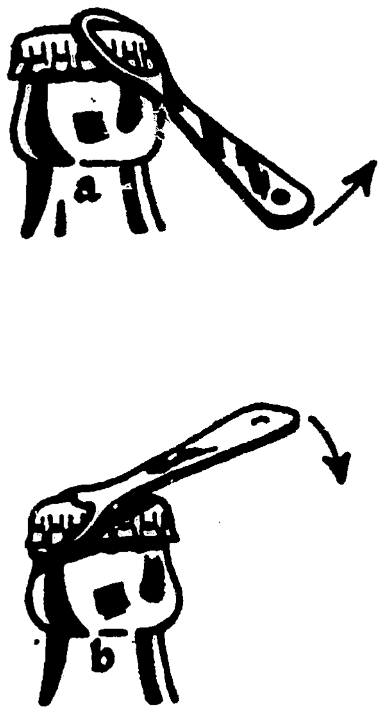
3) Force = 252 lbs

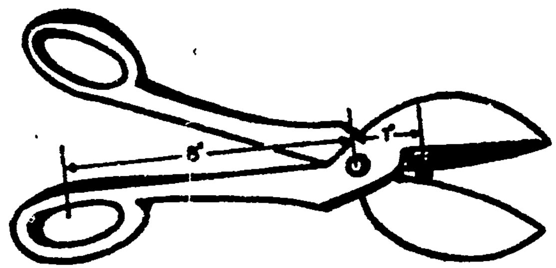
RESOURCE PACKAGE 1-9

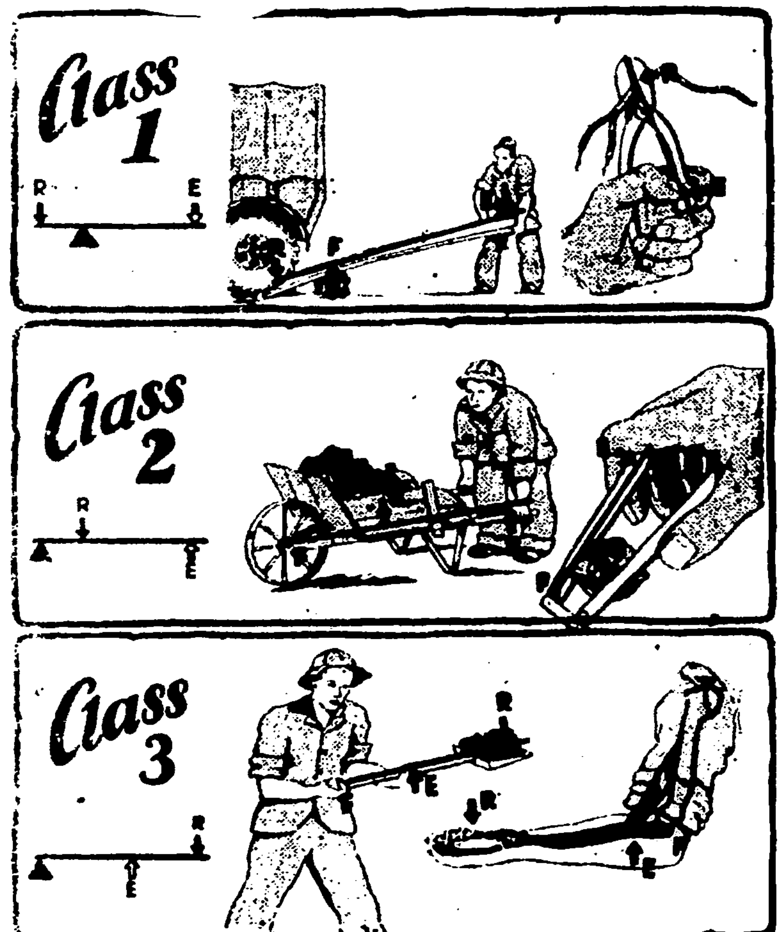
APPLICATIONS OF THE LEVER

Because we live in a highly-industrialized nation, levers play a critical role in our everyday lives. Even such simple tools as the scissors, pliers, or shovel are based upon level principles. Of course, in <u>any</u> kind of society, humans cannot escape levers! The muscle and bone structures of our bodies constitute a number of anatomical (body) levers: the jaw, the arm, the leg, etc.

Examine the following figures and see if you can determine the class of lever for each case:





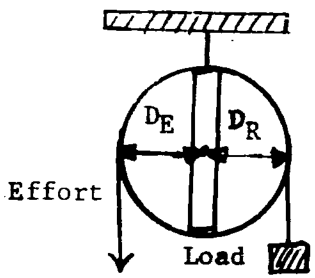


RESOURCE PACKAGE 2-1

THE PULLEY MACHINE

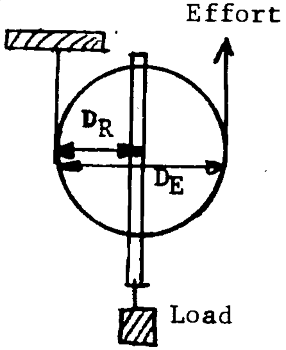
The pulley machine consists of a wheel that turns on an axle, which is mounted on a frame. One or more pulleys enclosed in a frame is usually called a block, and a series of pulleys with a connecting rope or chain threaded through the pulleys is known as a block and tackle.

The simplest pulley is the single fixed pulley illustrated in Fig. 1. This pulley acts as a lever in which D_E equals D_R, and the IMA is therefore one. If friction is neglected and if the effort force of 1 lb moves the pulley rope downward through a distance of 1 ft, then this can raise a resistance force of 1 lb through a distance of 1 ft. In other words, the IMA is one, and a single fixed pulley gains a mechanical advantage of direction only! The single fixed pulley does not gain advantage of speed or of force.



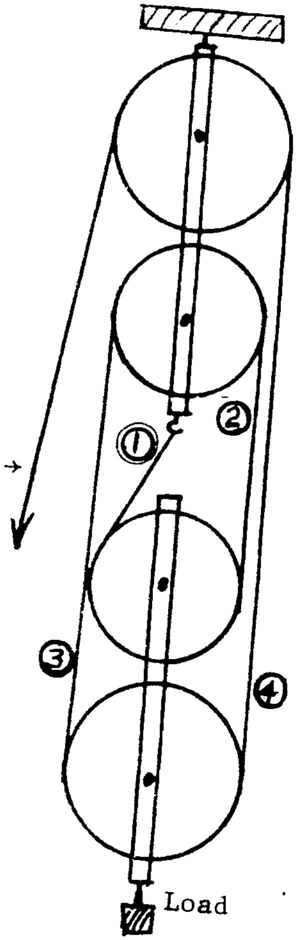
SINGLE FIXED PULLEY
Fig. 1

A single movable pulley is shown in Fig. 2. The effort force acts upon the "lever arm" D_E, which is the diameter of the pulley; and the load acts upon the "lever arm", D, which is the radius of the pulley. Since the diameter is twice the radius, the IMA is 2. When the effort force moves 2 ft. the resistance force is lifted 1 ft. Actually, the effort must also support the pulley, but since the weight of the pulley is generally very small compared to the load, the pulley's weight can usually be ignored. (In the following discussions and exercises, ignore the pulley weight).



SINGLE MOVABLE PULLEY
Fig. 2

There are many combinations of fixed and movable pulleys.

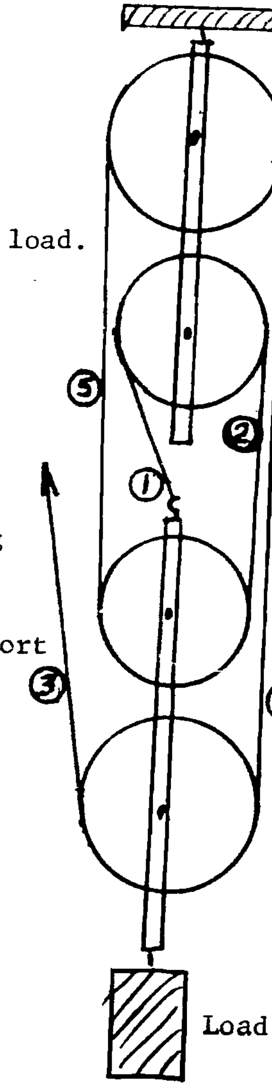


COMBINATION FIXED
AND MOVABLE PULLEY
Fig. 3

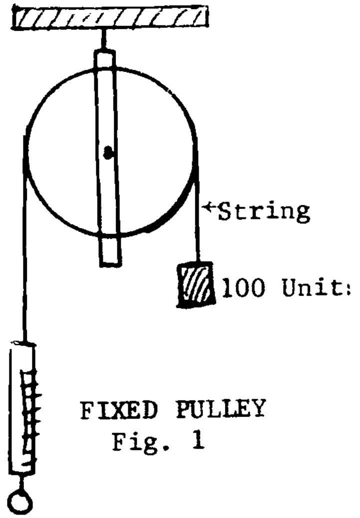
In Fig. 3, the upper block is the fixed block. In this continuous-strand arrangement, the number of "strands supporting the movable block" is 4. The "free" strand on which the effort force acts does not support the movable block, but gains only a mechanical advantage of direction. The IMA for this system is 4; the ratio of D_E to D_R is 4; the number of "strands supporting the movable block" is 4.

In Fig. 4, the "free strand" is attached to the movable block. In this case, the "free strand" on which the effort acts supports the movable block and now a total a total of 5 strands hold up the load. Therefore, the IMA of this system is 5; the number of "strands supporting the movable block" is 5; and the ratio of D_E to D_R is 5.

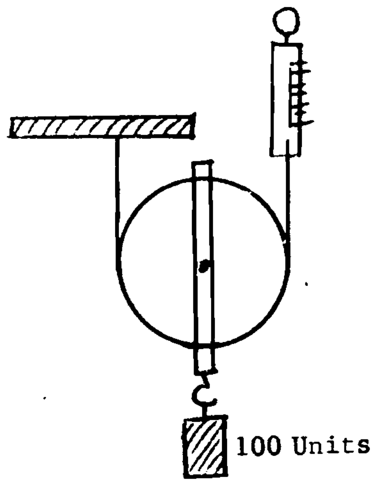
Some texts indicate that "counting the number of strands supporting the load will yield the IMA of a pulley system" Be careful with this suggestion! It is NOT always true that the IMA equals the number of supporting strands. But it is ALWAYS true that the ratio of D_E to D_R yields the IMA.



COMBINATION SYSTEM



FIXED PULLEY
Fig. 1



MOVABLE PULLEY
Fig. 2

RESOURCE PACKAGE 2-2

INVESTIGATING PULLEYS

1) Examine Figs. 1 through 4. Obtain a block with one pulley, two blocks with 2 pulleys each, 4 or 5 feet of string, a spring balance, a set of metric weights, and a means of supporting a pulley system.

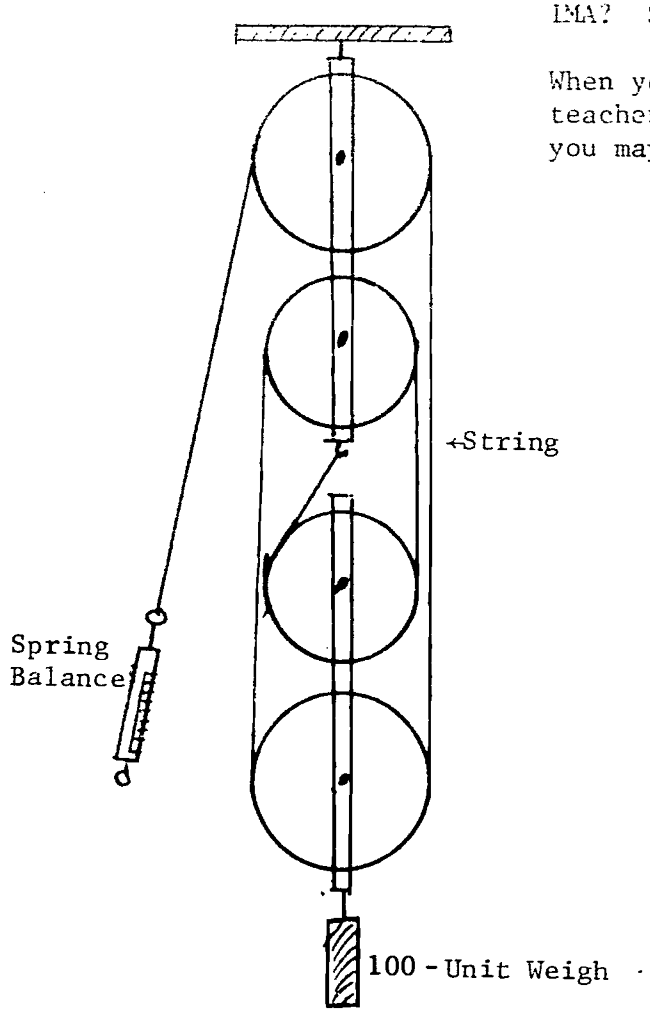
2) Arrange the single fixed pulley as indicated in Fig. 1. Attach a 1,000-unit weight to the string and suspend the system in static equilibrium (at rest) by pulling downward with the spring balance. Record the spring balance reading.

3) Arrange the single movable pulley as indicated in Fig. 2. Attach 1,000-unit weight to the pulley. Read the reading on the spring balance. What is the IMA of this system? How many strings actually support the movable pulley? Use the balance reading and calculate the AMA. (See Resource Package 1-5 for help.)

4) Arrange the two pulley blocks as shown in Fig. 3. Diagram the arrangement on your data sheet. Record the spring balance reading when the system is in equilibrium. What is the IMA of this system? Use the balance reading and calculate the AMA. Calculate the efficiency of the block and tackle.

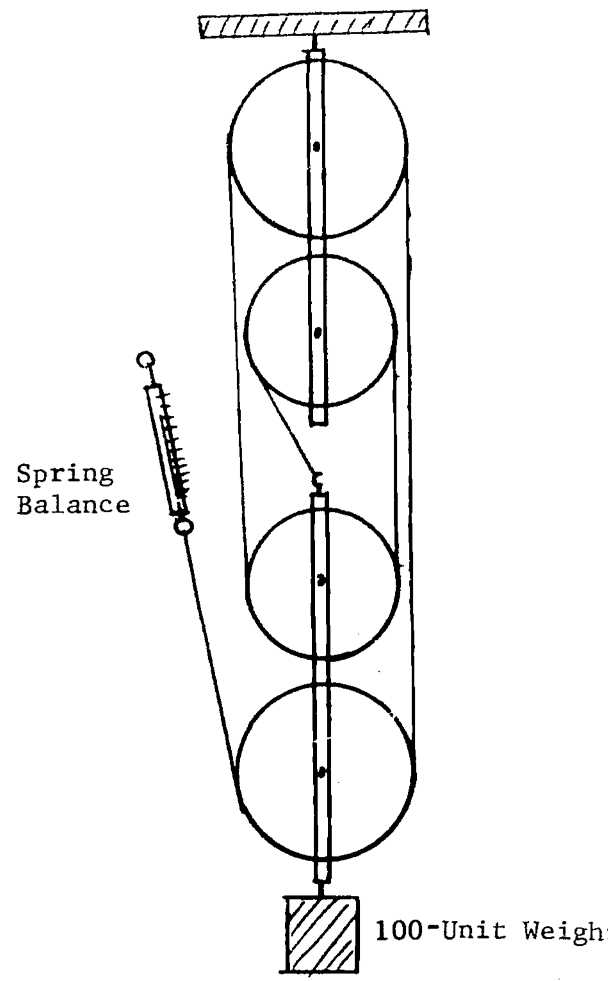
5) Arrange the 2-block system shown in Fig. 4 and sketch the system on your paper. Record the spring balance reading. What is the IMA? Show your work.

When you have finished this lab exercise, submit your work to your teacher for evaluation, and ask for assistance with any problems you may have.



TWO-BLOCK SYSTEM

Fig. 3



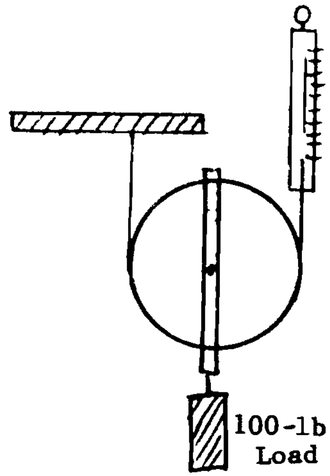
PULLEY SYSTEM

Fig. 4

RESOURCE PACKAGE 2-3

SELF-TEST ON PULLEYS

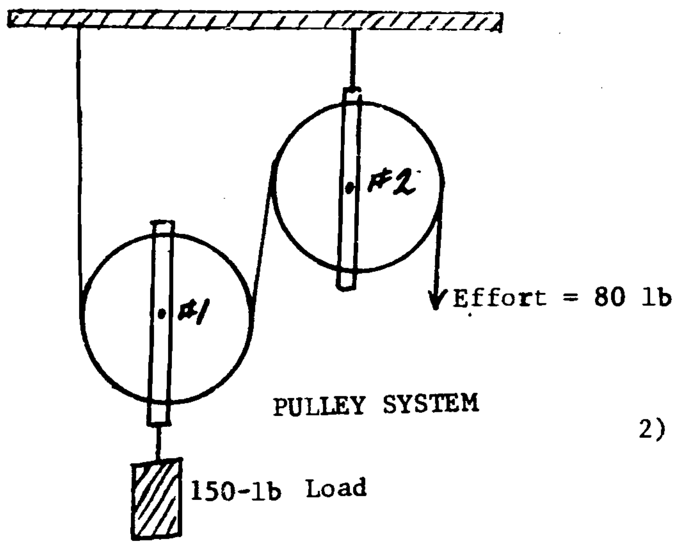
Draw the figures and show all calculations on a separate sheet. Do not write on this page, please!



SINGLE MOVABLE PULLEY

Fig. 1

1) The single movable pulley in Fig. 1 supports a 100-lb load with an effort of 55 lb. What is the AMA, IMA, and efficiency of the system?



PULLEY SYSTEM

Fig. 2

2) Observe Fig. 2. Calculate the IMA and AMA. What purpose does pulley #2 serve?

RESOURCE PACKAGE 2-4

SELF-TEST ANSWERS

1) IMA = 2

   AMA = 1.82

   Efficiency = 91%

2) IMA = 2

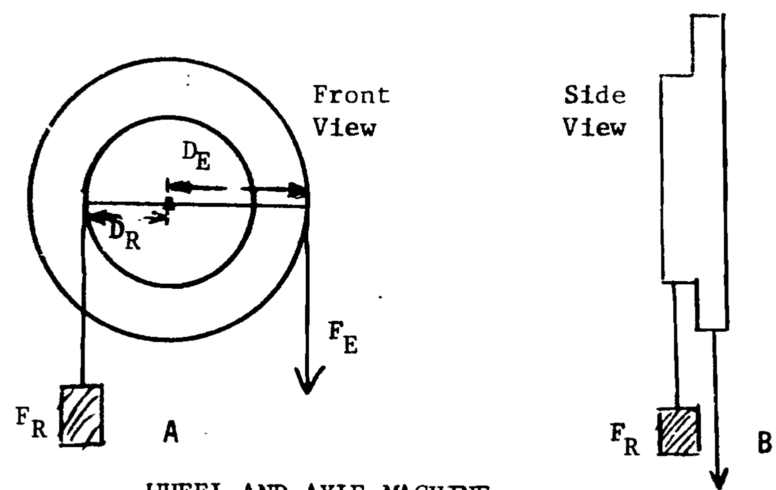
   AMA = 1.875

   Pulley #2 allows for change in direction.

RESOURCE PACKAGE 3-1

WHEEL-AND-AXLE MACHINE

The wheel-and-axle machine consists basically of a wheel or crank rigidly attached to an axle, so that the wheel and axle rotate as a single unit. For example, in Figure 1 the two wheels of unequal diameter are fastened so that they must rotate as one about their common axle.



WHEEL-AND-AXLE MACHINE

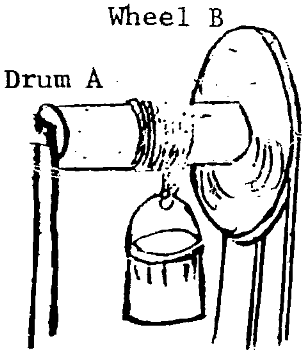
Fig. 1

Can you see that the wheel and axle is simply a modified first class lever, with the axle acting as the fulcrum? In one revolution, F_E moves a distance equal to the circumference of the wheel (C_E). During this same revolution, F_R must act over a distance equal to the circumference of the axle (C_R). The IMA of the wheel and axle is the ratio of the effective "lever arms" C_E and C_R:

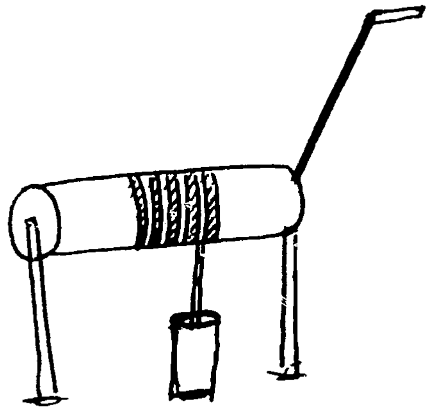
$$IMA = \frac{C_E}{C_R}$$

$$= \frac{2 \quad D_E}{2 \quad D_R}$$ where D_E and D_R are the wheel and the axle radii, reespectively.

The windlass shown in Fig. 2 is an example of a wheel-and-axle machine. The ratio of the radius of wheel B to the radius of drum A, is the IMA. Wheel B may be replaced by a handle, as in Fig. 3. The length of the handle corresponds to the radius of the wheel. In wheel-and-axle machines, a mechanical advantage of force is gained at the expense of distance. The distance over which the forces act are the circles generated by the handle and by the drum. The AMA of a wheel-and-axle machine is F_R/F_E.



WHEEL AND AXLE
Fig. 2



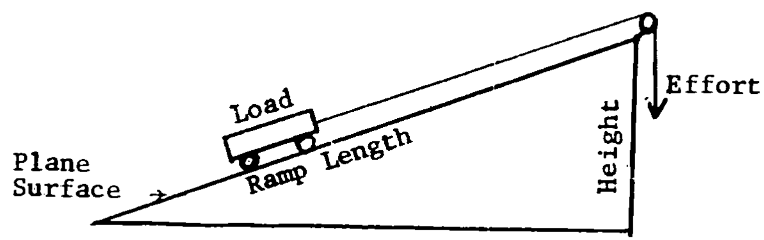
CRANK AND AXLE
Fig.

40

RESOURCE PACKAGE 3-2

THE INCLINED PLANE MACHINE

The inclined plane machine consists simply of a flat surface elevated at one end. (See Fig. 1)

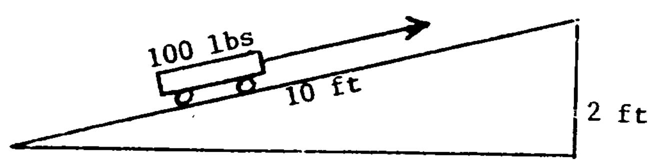


INCLINED PLANE
Fig. 1

We can use an inclined plane to increase the height or elevation of an object without lifting it vertically. The IMA can be calculated from: length/height; and the AMA can be calculated from: load/effort.

Example #1: A board 10 ft long is elevated 2 ft at one end. Neglecting friction, how much force does it take to pull 100 lbs up the inclined plane?

Solution:



Since we are ignoring friction IMA and AMA are equal.

Using the equations:

$$IMA = \frac{\text{length}}{\text{height}}$$

$$= \frac{10 \text{ ft}}{2 \text{ ft}}$$

$$= 5$$

Ignoring friction:

$$IMA = AMA$$

$$5 = AMA$$

$$5 = \frac{F_R}{F_E}$$

$$F_E = \frac{F_R}{5}$$

$$= \frac{100 \text{ lb}}{5}$$

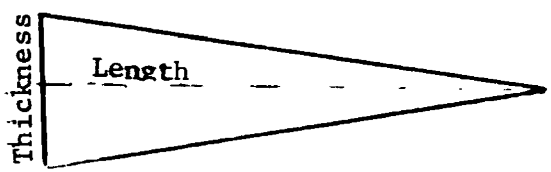
$$= 20 \text{ lb}$$

An effort of 20 lbs will move the 100-lb load up the inclined plane.

RESOURCE PACKAGE 3-3

THE WEDGE MACHINE

The wedge machine is essentially a double-inclined plane formed from two inclined planes set base-to-base (See Fig. 1). The IMA of the wedge is the ratio of its length to its thickness. There is so much friction associated with a wedge that the IMA and AMA differ a great deal. We will not attempt to calculate the AMA of the wedge. You are referred to the Physics of Sports minicourse for a more complete treatment of the wedge machine, its AMA, its IMA, and some of its applications.



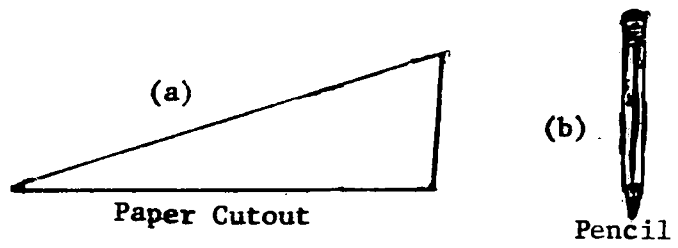
WEDGE MACHINE
Fig. 1

The wedge is used as a splitting tool, as a fastener device, etc.

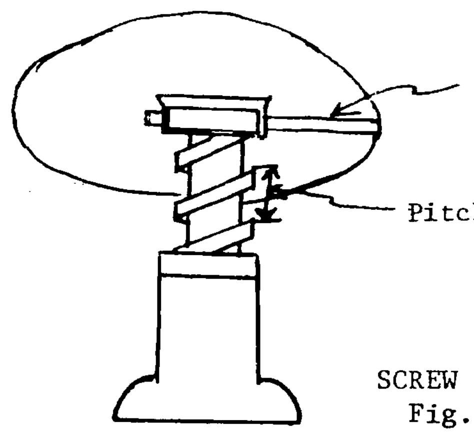
RESOURCE PACKAGE 3-4

THE SCREW JACK MACHINE

A screw-jack machine is an inclined plane wound upon a cylinder. Such a machine combines the advantages of the inclined plane with those of the wheel and axle. Cut a small piece of paper into the form of an inclined plane as shown in (a) below.



Use a pencil, or like instrument, and wrap the paper "inclined plane" around the pencil. Start with the wide end of the plane next to the pencil. Can you see that a screw is basically an inclined plane wrapped around a cylinder? The distance between the threads is called the pitch of the screw. The effort force can be applied to a screw by a lever (screw driver) set in a groove in the head of the screw, or the effort force can be applied by a lever (jack handle) set inside the head of the screw. If the lever is set in the screw head, the device is called a screw jack (See Fig. 1).



SCREW JACK
Fig. 1

As in the wheel-and-axle machine, when the wheel or crank revolves one time the effort force acts through a distance which is the circumference of a circle. But unlike the wheel-and-axle machine, the resistance force acts along a straight line which is along the length of the axle (screw) and is equal to the screw pitch.

The pitch is the distance a screw moves (or a jack raises) each time the handle (lever) is rotated once. The pitch is mathematically equal to the number of threads per unit of length. For example, if there are five threads along each 1-inch length, then we say there are 5 threads per inch and the pitch is 1/5-inch:

$$\text{Pitch} = \frac{\text{Unit of Length}}{\text{Number of Threads}}$$

$$= \frac{1 \text{ inch}}{5 \text{ threads}}$$

$$= \frac{1}{5} \text{ inch}$$

This tells us that every time the screw rotates <u>once</u>, the screw moves linearly 1/5th of an inch; in five (5) turns the screw moves one (1) inch.

If D_E is the length of the lever arm upon which the effort F_E acts, then for one revolution the effort acts over a distance (circumference) equal to 2 D_E. As F_E moves this distance, the load F_R moves the distance D_R, which is the pitch of the screw. For the screw:

$$IMA = \frac{\text{Effort Distance}}{\text{Load Distance}} = \frac{2\pi D_E}{D_R} \qquad AMA = \frac{\text{Load}}{\text{Effort}} = \frac{F_R}{F_E}$$

Example #1: A screw jack has a pitch of ½ inch and is turned by a handle that is 2 ft long. Ignoring friction, what effort is required to lift 2,000 lbs?

Solution:

$$IMA = \frac{2\pi D_E}{D_R} = \frac{(2\pi)(2\ ft)}{(\frac{1}{2}\ in)(1\ ft/12\ in)} = \frac{4\pi}{1/24} = 96\pi \cong 302$$

Therefore, $IMA = \frac{F_R}{F_E}$

$$F_E = \frac{2,000\ lb}{302} = 6.6\ lb$$

Example #2: A jack (screw) with a pitch of 1.8 in and a handle of 1 foot can lift 1,500 lbs with an effort of 20 lbs. What is the IMA and AMA?

Solution:

$$IMA = \frac{2\pi D_E}{D_R} = \frac{2\pi(1)\ ft}{1/96\ ft} = 2\pi(96) = 192(\pi) \cong 603$$

$$AMA = \frac{Load}{Effort} = \frac{1,500\ lb}{20\ lb} = \frac{150}{2} = 75$$

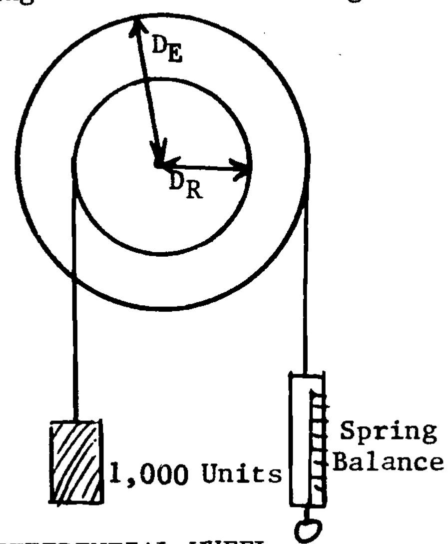
The IMA is approximately 6 times the AMA! This typical loss of theoretical (ideal) mechanical advantage is due to the great friction inherent in jacks and screws of all kinds. Of course, friction can act as an important advantage in many real problems. For example, friction is what keeps a jack from simply unwinding under a load when the effort force is removed. Friction also keeps screws from unthreading, thus making them ideal wood fasteners, etc.

RESOURCE PACKAGE 3-5

INVESTIGATING THE WHEEL-AND-AXLE, INCLINED PLANE, AND SCREW JACK MACHINES

PART A

1) Arrange a differential wheel (wheel and axle), about 5 ft of string, metric weights, and a spring balance, as shown in Figure 1. Draw a diagram of this apparatus on a separate sheet of paper. If your differential wheel consists of a cluster of three different-sized wheels, attach the 1,000-unit weight to the smallest wheel and the spring balance to the largest wheel.



DIFFERENTIAL WHEEL
Fig. 1

2) Record the spring balance reading.

3) Measure the lever arm distances, D_E and D_R, in centimeters. One way to do this is to measure across a wheel (diameter) and then divide the diameter by 2 to get the radius, which is the lever arm distance.

4) Determine the IMA of the wheel and axle.

5) Determine the AMA.

6) On the basis of your results, does the wheel and axle have very much friction?

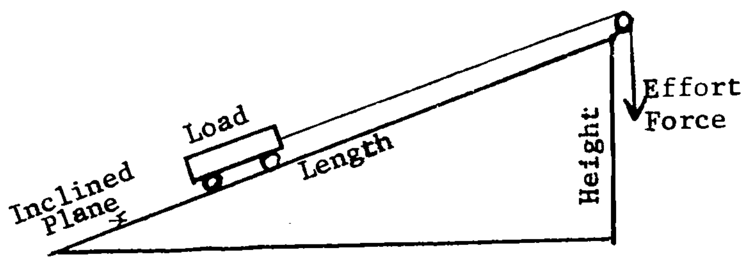
6) Ask your teacher to check your work.

PART B

1) Obtain an inclined plane, a Hall's carriage, about 2 ft of string, a set of metric weights, a spring balance, and meter stick.

2) Determine the weight of the Hall's carriage.

3) Arrange the apparatus as shown in Figure 2. Draw the diagram on your paper.



INCLINED PLANE
Fig. 2

4) Measure and record both the length and height of the inclined plane.

5) Record the balance reading when the carriage is at rest on the plane. Put a load in the carriage if it seems appropriate.

6) Determine the IMA and AMA of the inclined plane.

7) Ask your teacher to check your work.

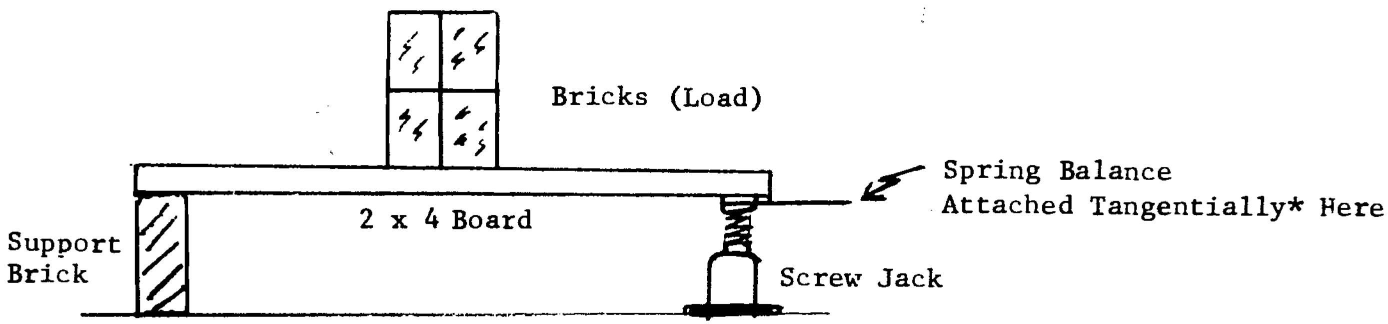
PART C

1) Obtain a small screw jack, a board about 2 ft long (A 2 x 4 would be fine.), a half-dozen or so bricks, a spring balance, a triple beam balance, and a vernier caliper.

2) Use the triple beam balance to determine the weight of the bricks and the board.

3) Using the verier claiper, or other appropriate method, determine the pitch and record it.

4) Arrange the appratus as indicated in Fig. 3 below.



SCREW JACK

Fig. 3

5) Determine and record the length of the handle. This length is measured from the center of the screw jack to the point where the spring balance is attached.

6) Pull on the balance to turn the screw at constant speed through about one half of a turn. As you do, read the spring balance. Record this.

7) The brick under one end of the board supports half the weight load; the jack supports the other half of the weight of the board plus the weight of the bricks. Using the load on the jack, and the spring balance reading, determine the AMA. Also find the IMA. From your calculations, does the screw jack have very much friction?

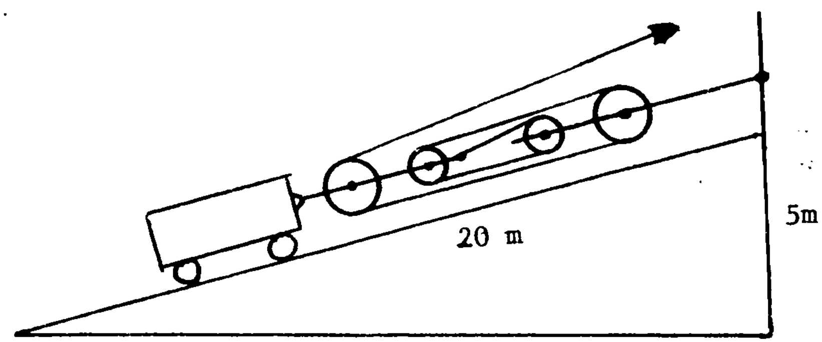
8) Ask your teacher to check your work.

* Your teacher can explain this word further. It means that the spring balance will pull at right angles to the handle end.

RESOURCE PACKAGE 3-6

COMPOUND MACHINES

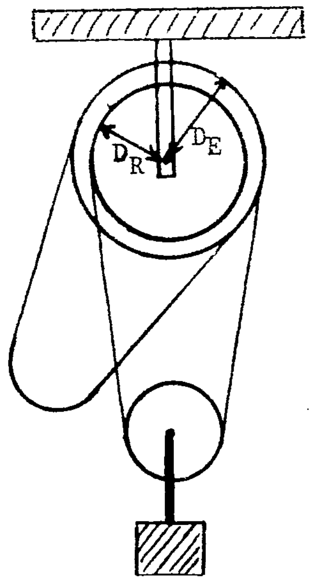
Most of the machines in everyday life are compound machines, composed of two or more simpler machines. The inclined plane and the block and tackle in Fig. 1, form a compound machine. For most cases, the IMA of a compound machine is the product of the IMA's of each simple machine.



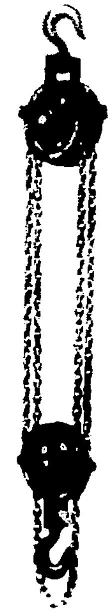
COMPOUND MACHINE
Fig. 1

In Fig. 1, the IMA of the inclined plane is 4, and the IMA of the block and tackle is 5; therefore the IMA of the compound machine is 5 x 4 or 20. Do you understand this reasoning?

Another example of a compound machine is the differential pulley, which combines a single movable pulley and the wheel-and-axle (differential wheel) with an endless chain. (See Fig. 2).



DIFFERENTIAL PULLEY
Fig. 2



CHAIN HOIST
Fig. 3

Again, the IMA of this system is the ratio of distances moved, which is a circumference ratio:

$$IMA = \frac{2\pi D_E}{2\pi D_E - 2\pi D_R}$$

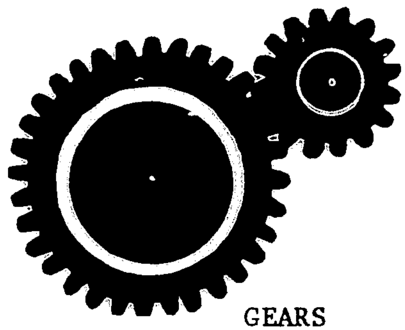
$$= \frac{2\pi D_E}{2\pi(D_E - D_R)}$$

$$= \frac{D_E}{D_E - D_R}$$

$$= \frac{r_E}{r_E - r_R}$$

where r_E and r_R are the radii of the large and small wheels, respectively. The differential pulley, quite often called a chain hoist, has many applications in industry (See Fig. 3 above).

Combinations of gear wheels of different diameters are used to increase or decrease speed. Suppose we mesh a wheel that has 32 gear teeth with one that has 16 teeth, as shown in Figure 4.



GEARS
Fig. 4

If we apply force to wheel #1, wheel #2 will gain a mechanical advantage of speed. If we apply force to wheel #2, wheel #1 will gain a mechanical advantage of force. Write a few short sentences explaining these two preceding statements about mechanical advantages of speed and force.

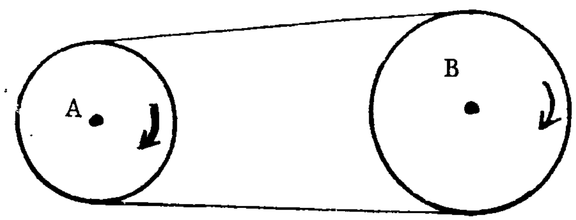
The IMA for a gear is:

$$IMA = \frac{C}{c} = \frac{D}{d} = \frac{R}{r}$$

where C is the circumference of the large wheel and c is the circumference of the small wheel. R and r are the radii of the large and small wheels, respectively. What are D and d? Show mathematically that

$$\frac{C}{c} = \frac{R}{r}$$

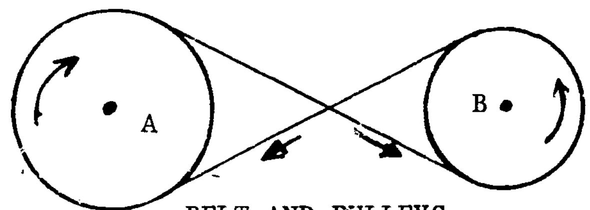
The belt and pulley is used in a manner similar to gears to gain mechanical advantage of speed, force, of direction. In Figure 5 (See next page), if wheel B is the driving wheel force advantage is gained in wheel A at the expense of distance. For instance, if the diameter of B is 2 inches, and if the diameter of A is 6 inches, then wheel A will rotate only one time while B rotates 3 times. But the force on A will be multiplied by a factor of 3. Can you see this?



BELT AND PULLEYS
Fig. 5

If B is the driving wheel, then speed advantage is gained at A at the expense of force at B. This illustrates the basic principle underlying bicycle gears, automobile gears, etc. Notice that both A and B rotate in the same direction.

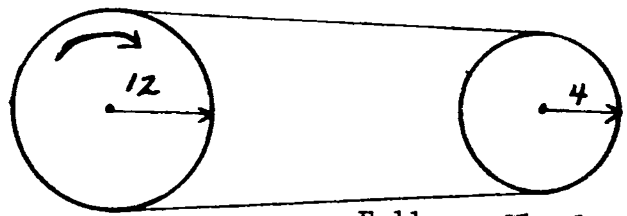
If we cross the belts as shown in Figure 6, then the wheels will rotate in opposite directions.



BELT AND PULLEYS
Fig. 6

The IMA remains the same as before, but a mechanical advantage of direction is gained.

<u>Example</u>: A wheel 12 inches in diameter powers a belt which turns a wheel 4 inches in diameter. If the driving wheel rotates at 5 cycles per second (5 rps), at what speed does the follower wheel rotate?



BELT AND PULLEYS
Fig. 7

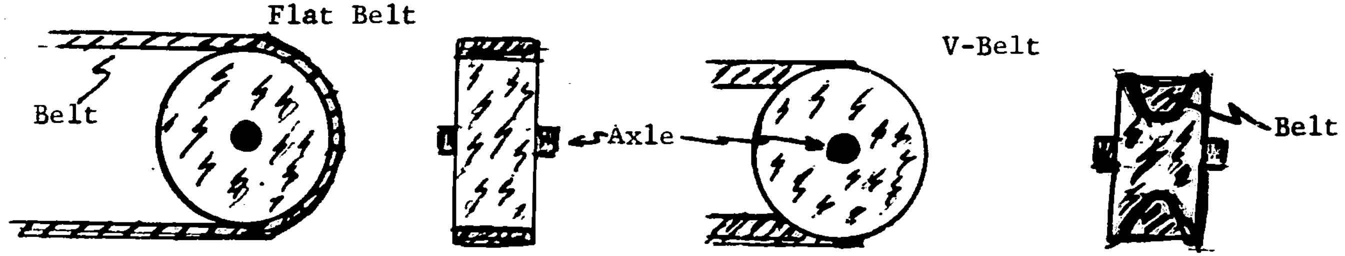
Solution:

$$IMA = \frac{D_E}{D_R} = \frac{2}{2}\frac{(12\ in)}{(4\ in)} = 3$$

55

The speed of the driver wheel = 5 cps. The IMA = 3; therefore, the follower wheel rotates at 15 cycles per second.

Some belt drives use flat belts; others use V-belts. V-belts have advantages, a most prominent one being their ability to not slip off the pulley (See Fig. 8).



BELT TYPES
Fig. 8

RESOURCE PACKAGE 3-7

WHEEL-AND-AXLE, INCLINED PLANE, AND SCREW JACK PROBLEMS

1) The length of the crank handle on a windlass is 10 inches. The diameter of the axle is 1 inch. If a force of 50 lbs on the crank handle can raise a load of 600 lbs, what is the IMA and AMA of the windlass? What is its efficiency?

2) A plank 13 ft long is used as an inclined plane to a platform 5 ft high. What force must be used to push a block of ice weighing 190 lbs up the plank, if friction is ignored?

3) A jack screw has a lever arm 2 ft long. The screw has 36 threads to the foot. If 50 lbs exerted at the end of the lever is needed to raise a load of 10,000 lbs, what is the IMA, AMA, and efficiency of the jack?

RESOURCE PACKAGE 3-8

ANSWERS TO PROBLEMS

1) IMA = 20

   AMA = 12

   Efficiency = 60%

2) Effort $\cong$ 73 lbs

3) IMA $\cong$ 452

   AMA = 200

   Efficiency = 44%

RESOURCE PACKAGE 3-9

SELF-TEST

1) Jane is using a windless to raise a 750-lb weight. The radius of the path of the force applied to the handle is 18 inches and the radius of the axle is 1 inch. What is the IMA? If the efficiency of the machine is 60%, what force does Jane exert?

2) An inclined plane 10 ft long has an IMA of 5. What is the height of the inclined plane? If it takes 50 lbs to pull a 200-lb load up the incline, what is the efficiency of the inclined plane?

3) A housemover's jack screw has a handle of effective length 2 ft and a pitch of ½ inch. If it takes an effort of 50 lbs to lift 6,000 lbs, what is the efficiency of the jack?

4) A gear 1 ft in diameter powers a gear 4 inches in diameter with an efficiency of 80%. If a force of 60 lbs is applied to the large gear, what force will be applied to the small gear?

5) A safe weighing 3,000 lbs is to be pulled up an inclined plane 20 ft long onto a platform 4 ft high. A block and tackle having an IMA of 5 is attached to the safe. If two men, each pulling with a force of 100 lbs move the safe, what is the efficiency of the machine?

RESOURCE PACKAGE 3-10

SELF-TEST ANSWERS

1) IMA $\cong$ 57

   Effort $\cong$ 22 lb

2) Height = 2 ft

   Efficiency = 80%

3) Efficiency $\cong$ 40%

4) Force on small gear is 144 lbs

5) Efficiency = 60%

RESOURCE PACKAGE 4-1

ENERGY AND MACHINES

There is a different kind of treatment of basic machines in The Physics of Sports minicourse, and it is suggested that you read the Resource Packages on machines in the minicourse. But in any study or presentation of machines, the key word is energy.

All machines represent devices in man's struggle to harness energy for advantageous purposes. All machines must obey the laws of the physics of energy. All of the machine equations for IMA, AMA, efficiency, work, power, etc. are derivable in a truly fundamental way from energy considerations. A fundamental tenet (base) in technical physics is The Conservation of Energy Principle. Also basic to machines are the energy-related laws of thermodynamics, that branch of physics which treats heat, temperature, etc.

Force can be simply defined as a push or pull. Work can be simply defined as the energy change occuring when an effort force acts over some distance against a resistance force. And energy can be simply defined as the ability to do work. These definitions are good enough for treating most problems in the mechanical systems of technical physics.

Energy comes in many forms, and can be transformed from one form to another. For convenience sake, energy forms are sometimes classified. Some energy form classifications are: sound, light (electromagnetic), heat, electric, mechanical, atomic, chemical, nuclear, gravitational, etc. Classifications are useful in particular instances, though not always helpful in understanding fundamental physics in general. For example, sound is a special kind of mechanical energy; so-called radiant heat energy is really a kind of electromagnetic energy, etc.

Some devices for harnessing energy to perform useful work include:

a) internal combustion engines
   1) gasoline engines
   2) diesel engines
   3) jet engines
   4) rocket engines
b) external combustion engines
   1) steam engines
   2) steam turbines
c) electric motors
d) nuclear engines
e) ion engines
f) solar cell engines
g) fuel cell engines
h) magnetohydrodynamic engines

In other words, energy is the basis for all work and all power; therefore, any study of machines is basically a study of harnessed energy.

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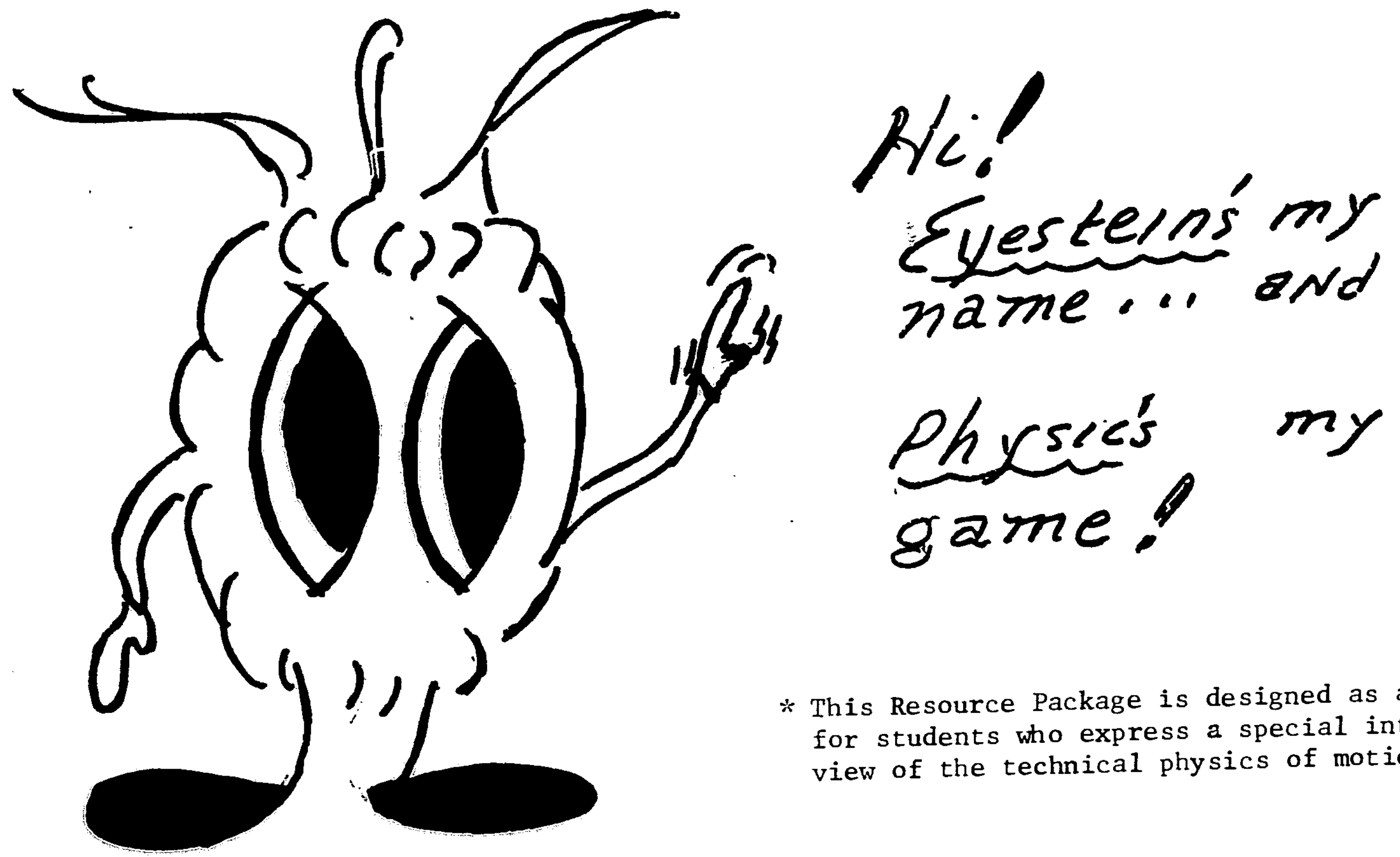
* RESOURCE PACKAGE 5-2

TECHNICAL DESCRIPTIONS OF FORCES, MOMENTS, EQUILIBRIUM, AND MOTION

This Resource Package is devoted to a "heavier" treatment of the technical physics of motion than that found in any of the other minicourses. It is divided into two sections. Section I is a technical treatment of vector force, vector moment, and equilibrium. It should prepare you well for the study of motion, since motion results from forces and moments. Section II is a technical description of motion itself.

Eyestein, to whom you may have been introduced in the minicourse, <u>Metric System and Slide Rule</u>, will be your guide through this "heavier" stuff. If you have not yet met, here's Eyestein:



* This Resource Package is designed as an optional excursion for students who express a special interest in a deeper view of the technical physics of motion.

SECTION I

Section I Goals. Upon completion of this section, you should acquire an understanding of the vector nature of both forces and moments (torques). You should also acquire a knowledge of mathematical representations of forces and of moments. Further, you should be able to relate the concept of force and the concept of moment to the necessary conditions for rigid body equilibrium.

Section I Operational Objectives. Upon completion of this section, you should be able to:

1) define these terms: force, moment (torque), equilibrium, resultant moment, rigid body, equilibrant force, free-body diagram, normal force, scalar physical quantity, vector physical quantity, center of gravity, and equilibrium of a rigid body (neutral, stable, and unstable).

2) resolve a given force into rectangular components.

3) calculate resultant and equilibrant forces, when presented with a set of coplanar forces.

4) construct simple free-body diagrams.

5) construct the free-body diagram for a simple inclined plane problem.

6) determine the values of the normal and parallel-to-the-plane forces in a simple inclined plane problem.

7) calculate moments, if given an axis of rotation and a system of coplanar forces.

Introduction. The reader is reminded that in the real world nature shows neither a preference for rotation (spinning) nor for translation (moving along a line path). Both are equally likely to occur, and an observer of a natural event usually sees a combination of rotational and translational motion.

TIPS, REFRESHER REFERENCES, SELF-QUIZZES, AND OTHER USEFUL STUFF!

TIP FROM EYESTEIN: Section Goals and Objectives help you to set your sights. By keeping these in mind, you can better zero-in on the learning targets.

A few examples of the many bodies in our environment which can exhibit both translational and rotational motion include bowling balls, baby buggies, bicycles, race cars, rickshaws, and roller skates.

To acquire a better technical understanding of motion, it is essential that the concepts of force and moment (torque) be understood. You will learn that force is associated with translational motion and that moment is associated with rotational motion. You will also learn that no change of a body's motional state occurs only when no net force and no net moment act on a solid body. The name you will learn to apply to this condition of "no change of motional state" is equilibrium.

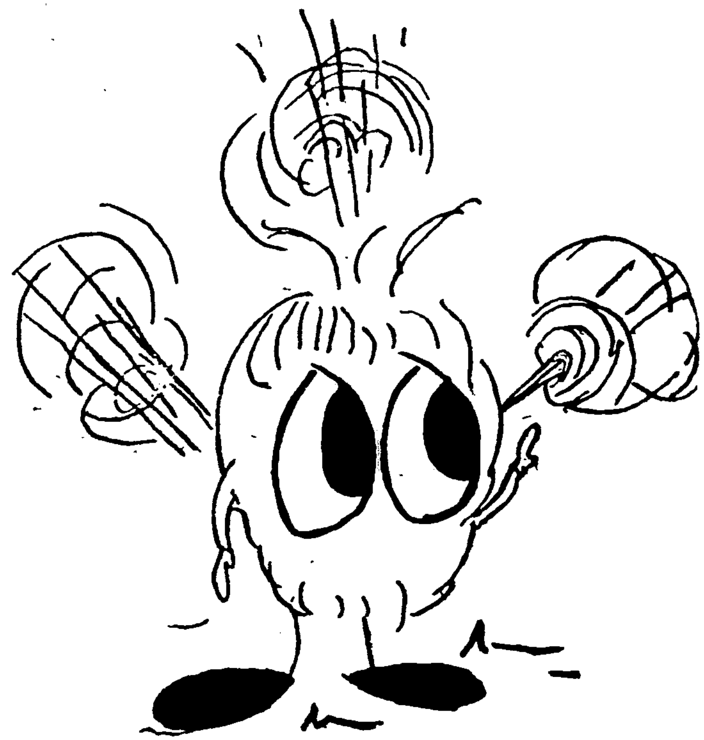
You will also learn some technical physics "tricks" for analyzing real-life kinds of situations involving forces and moments, and some related mathematical "tricks" for describing forces and moments acting on rigid bodies in equilibrium. Later on, these "tricks" will be used to analyze the motions of rigid bodies not in equilibrium (i.e., accelerating rigid bodies).

In addition, you will learn that bodies in equilibrium can be either at rest or in motion! That is to say, bodies are in equilibrium if they are moving at constant speed in a straight line or spinning at a constant rate about some fixed axis, or both (i.e., moving at constant linear speed in a straight path while at the same time spinning at a constant angular speed about an axis whose spatial orientation is unchanged or fixed with respect to the linear motion).

It can also be shown that bodies experiencing forces or moments can not only undergo changes in motional state (i.e., speed up, slow down, change direction, come to rest, etc.), but they can undergo deformations. These deformations are described with such terms as <u>compression</u>, <u>torsion</u>, <u>tension</u>, etc.

This chapter can be both a "mind-blower" and an "eye-opener." Prepare yourself for the unexpected, as well as for learning concepts mostly unfamiliar to the man on the street.

<u>Vector Forces and Vector Moments</u>. Vector forces (pushes or pulls) and vector moments (torques or twists) are to be found everywhere in your environment. The kite pulling on its cord or the man pushing a stalled auto present us with examples of vector forces. The woman twisting a jar lid or turning a door handle presents us with examples of vector moments.



Eyestein says,
"Physics can be a mind blower!
It really opens one's eyes to
Nature's mysterious ways."

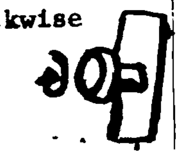
Let us apply our knowledge of vector mathematics to the representation of the vector force due to Eyestein's kite. To represent the kite's pull (vector force) geometrically on a plane, such as on this sheet of paper, one can simply use a <u>directed</u> <u>line</u> <u>segment</u>. The length of this line

Eyestein Demonstrates Vector Forces
(The kite <u>pulls</u> on Eyestein.)

VECTOR MOMENTS

(1) Twist Counter-clockwise

(2) Twist Clockwise

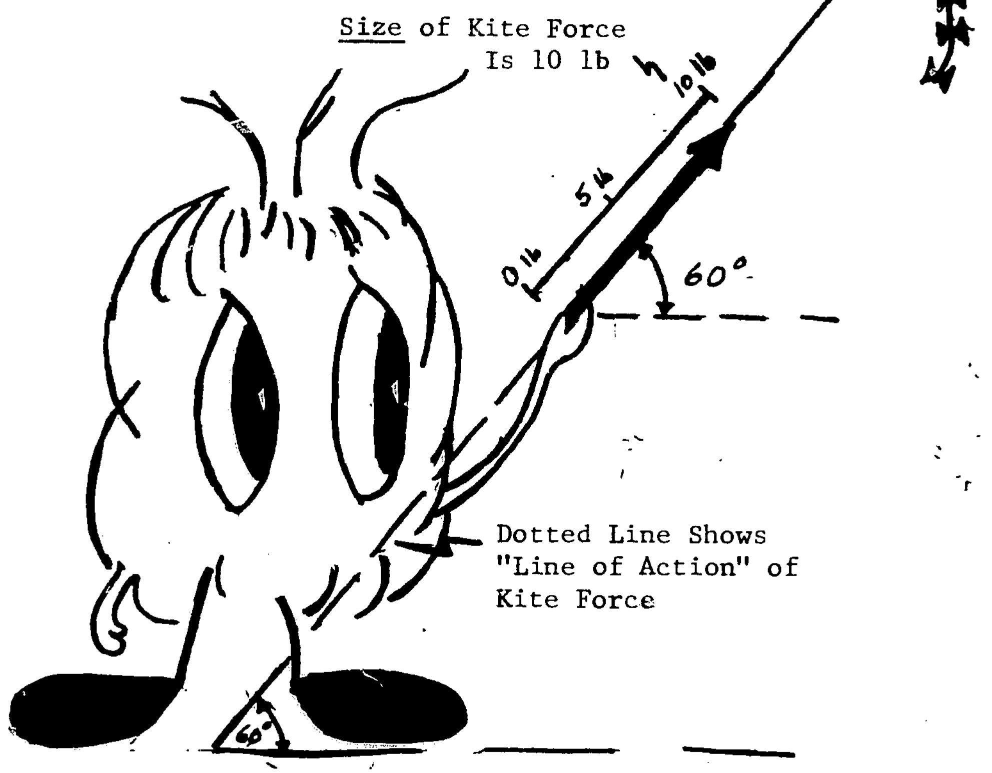


<u>TIP FROM EYESTEIN</u>: Notice that the word, <u>vector</u>, precedes both <u>force</u> and <u>moment</u>. This is to remind us that pushes, pulls, and twists are vector physical quantities. They are classified as vector physical quantities because they are physical quantities whose description requires both a <u>size</u> (magnitude) and a <u>direction</u> and because (mathematically speaking) such physical quantities are elegantly describable by <u>vector</u> <u>numbers</u>.

segment is scaled to show the kite pull magnitude, and the segment's arrow tip aims precisely along the direction of pull. In the picture below the solid arrow in Eyestein's hand represents the kite's pull.

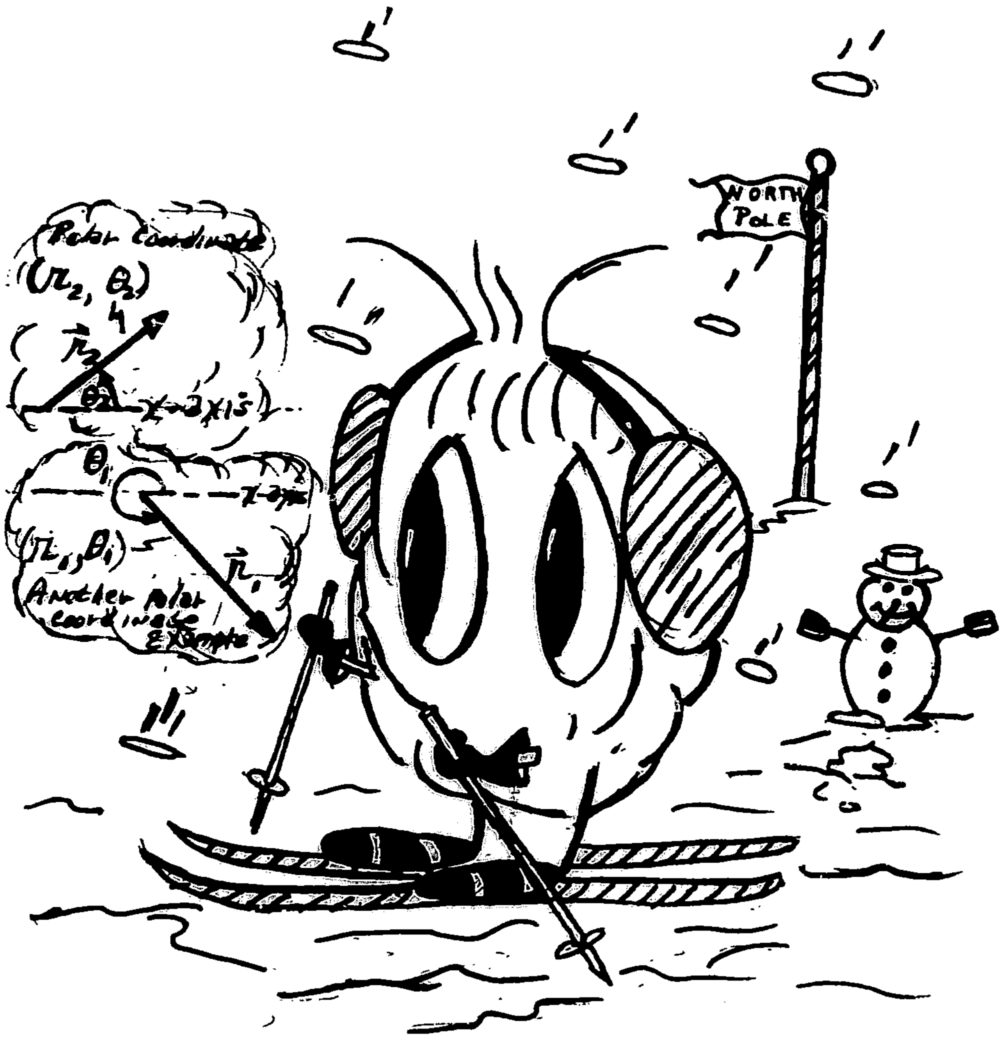
Demonstrating Both The Size and the Direction Properties of Eyestein's Kite Force:

$F = 10$ lb @ 60°
Above Horizontal



Direction of Kite Force Is Determined By the 60° Angle Between the Line of Action of the Kite Force and the Horizontal (ground).

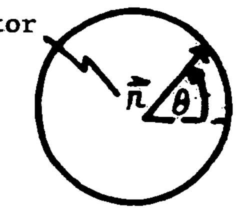
One can also represent the kite's pull force mathematically in polar coordinate notation. This representation consists of a <u>radius vector</u> $\vec{r}$, whose magnitude is given by the radius vector length, $\underline{r}$, and whose direction is given by an angle, (i.e., $F = r \angle \theta$, which for our kite example would be F = 10 lb at 60° above the horizontal). The picture below presents a further explanation of polar coordinates.



Eyestein Views Polar Coordinates

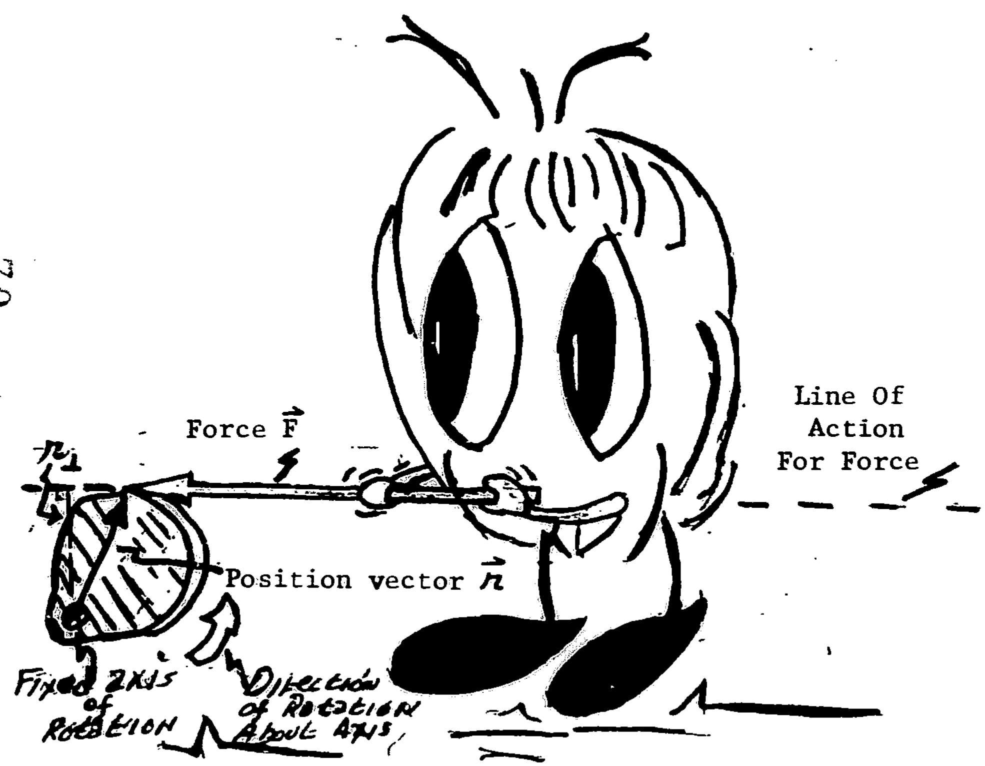
HINT FROM EYESTEIN: You will use polar coordinate notation later on if you study electronics or electrical technology.

Reference circle for polar coordinates (to help refresh your memory):



Next, let us apply our knowledge of mathematics to the representation of a vector moment. The Greek symbol, Gamma, $\vec{\Gamma}$, will be used to represent vector moment. The direction of $\vec{\Gamma}$ is one of rotation about an axis and is frequently designated as clockwise or counterclockwise. The size or magnitude of a vector moment can be determined by multiplying the twisting force size by the perpendicular distance measured from the twist force "line of action" to the axis of rotation (See picture below).

Eyestein Demonstrates Vector Moment
(He pushes with force, $\vec{F}$, to cause a twist of the object counterclockwise about the rotational axis)

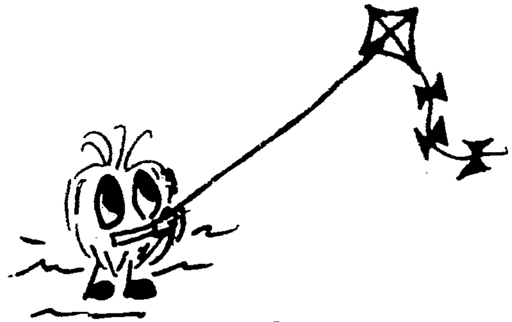


= (size of perpendicular distance from axis of rotation to a line of action of force) x (size of force); and the DIRECTION VECTOR ( ) indicates a counterclockwise twist about the axis of rotation.

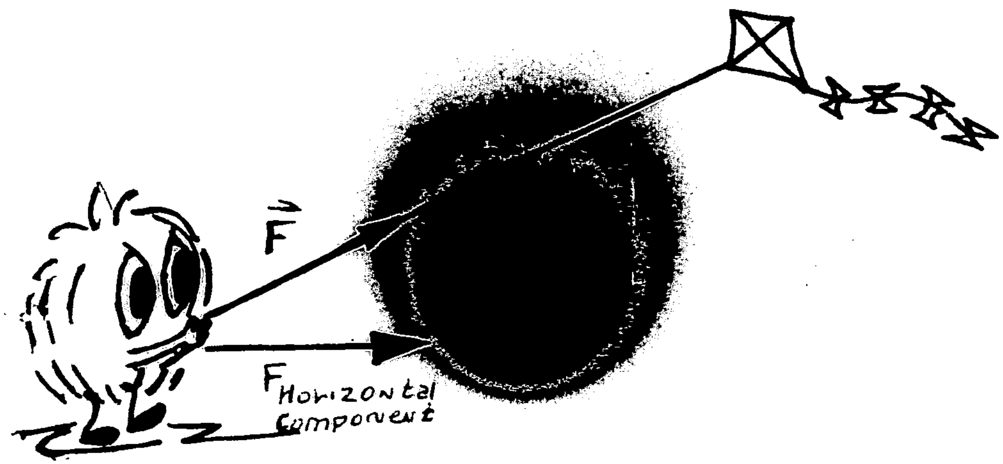
TIP FROM EYESTEIN: The size of the vector moment, $\vec{\Gamma}$, is always the mathematical product of a force value and a distance value.

Examples of Vector Forces and Vector Moments. We will now examine two sample problems.

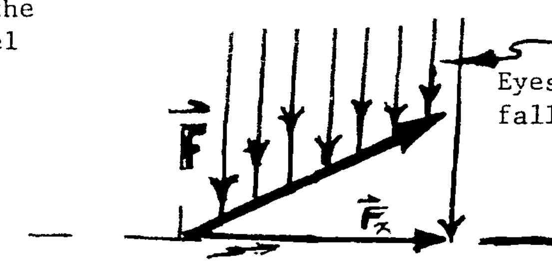
Example 1: In the previous Eyestein kite example, what is the component of the kite force tending to pull Eyestein parallel to the earth?



In other words, we are searching for the horizontal component of the kite force, $\vec{F}_h$.



ANOTHER TIP FROM EYESTEIN: Shown below is another way of looking at the x-component of the kite force F; namely, the force's projection onto the x-axis:



The "shadow" (x-component) of vector $\vec{F}$ projects onto the x-axis ($\vec{F}_h$)

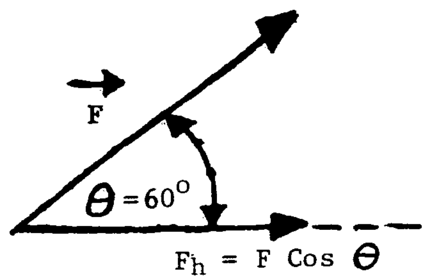
To lay vector $\vec{F}$'s "shadow" (component) on the horizontal axis, we employ the cosine function (the "lay" function). Thus, the <u>size</u> of the horizontal component of vector $\vec{F}$ is given by

$$\begin{aligned} F_h &= F \cos \theta \\ &= (10 \text{ lb}) \cos 60^\circ \\ &= (10 \text{ lb}) \tfrac{1}{2} \\ &= 5 \text{ lb} \end{aligned}$$

The direction of F_h (F-horizontal) is obviously along the earth's surface. We also can say that the horizontal component of this vector force is given by

$$\begin{aligned} F &= 5 \text{ lb @ } 0^\circ \\ &= 5 \text{ lb, parallel to earth} \end{aligned}$$

Geometrically, the solution to the horizontal component problem looks like this:



Because, by definition,

$$\cos \theta = \frac{\text{length of adjacent side}}{\text{length of hypotenuse}}$$

Or,

$$\cos \theta = \frac{F_x}{F}$$

Therefore,

$$\begin{aligned} F_x &= F \cos \theta \\ &= 10 \text{ lb } (\cos 60^\circ) \\ &= 10 \text{ lb } (\tfrac{1}{2}) \\ &= 5 \text{ lb force magnitude} \end{aligned}$$

Let's examine the solution to this same horizontal component problem if one were required to convert from the engineering to the MKS system of measure.

First, one can convert the vector force (given in engineering units) to MKS units:

$$F_{engr} = 10 \text{ lb c } 60^{o}$$

and since 1 lb $\cong$ 4.45 newtons,

$$F_{MKS} \cong 10 \text{ lb } (4.45 \text{ N/lb}) \text{ @ } 60^{o}$$

$$\cong 44.5 \text{ N @ } 60^{o}$$

Second, to find the horizontal component of this MKS force, one simply writes:

$$F_h \cong 44.5 \text{ N } (\cos 60^{o}) \text{ @ } 0^{o}$$

Again, notice that the cosine function ("lay function") is used to "lay" the "shadow" of the vector force onto the adjacent horizontal axis:

$$F_h \cong 44.5 \text{ N } (\tfrac{1}{2}) \text{ @ } 0^{o}$$

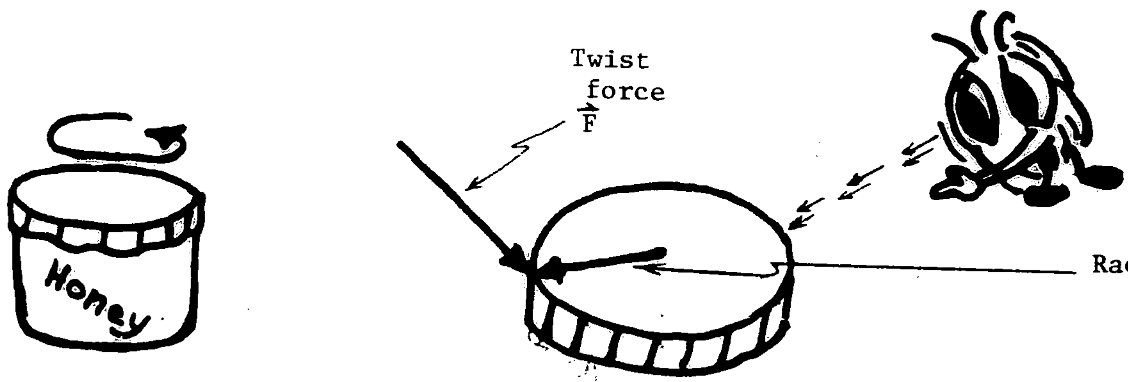
$$\cong 22 \text{ N @ } 0^{o}$$

Example 2: In the previous jar example of a vector moment, suppose a woman applies a 5-lb rim force to unscrew a honey jar lid whose radius is 3 inches. (This would be 3 in of 12 in per foot, or ½ ft.)

TIP FROM EYESTEIN: It is not uncommon to use the dimensional units "force time length" (lb·ft) in the case of vector moment, rather than to use the dimensional units "length x force" (ft·lb). This latter product (ft·lb) is used universally to represent units of work (or energy).

Thus, some people differentiate vector moment and work dimensionally--they re erve ft·lb units for work and lb·ft units for vector moment! Equivalent combinations can be made from m·N (MKS) and cm · dyne (CGS).

Radial distance is equal to the radius size, r, which is equal to the perpendicular distance from the axis of rotation to the line of action of the force.



EYESTEIN STUDIES THE HONEY JAR'S VECTOR MOMENT DIAGRAM

We seek to calculate the vector moment, $\vec{\Gamma}$, in the engineering system of units. The vector moment is obviously directed counterclockwise (as view from above) and the vector moment magnitude equals the twist force size, F, mulitplied by the perpendicular distance, r, measured from the rotational axis to the point of application of the rim force. Thus,

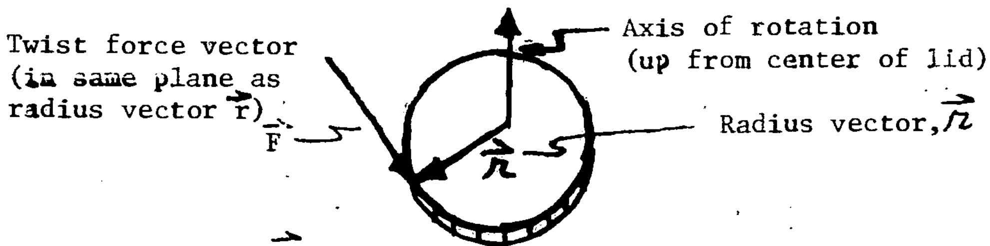
$$\vec{\Gamma} = rF,\ cc$$

$$= \tfrac{1}{4}\ ft\ (5\ lb),\ cc$$

$$= \frac{5}{9}\ lb\cdot ft,\ cc$$

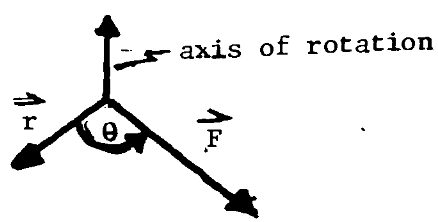
Geometrically, the jar problem and its solution look like this:

TIP FROM EYESTEIN: Let's refresh our memory by re-examining this picture on the next page.

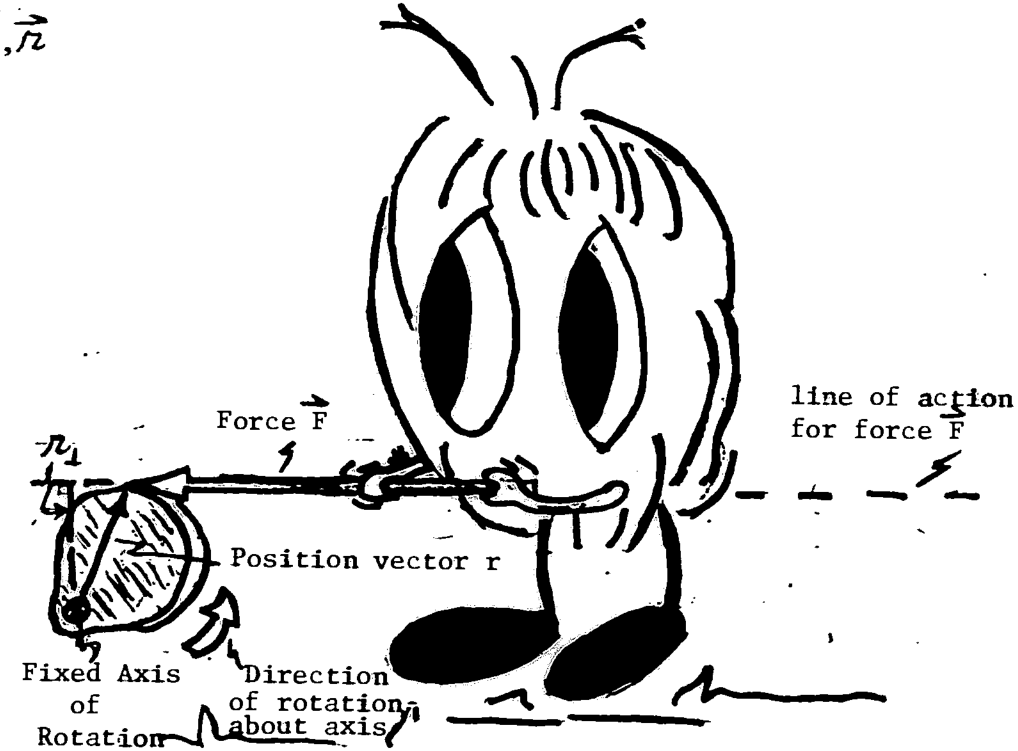


$\vec{\Gamma} = r \sin \theta, \vec{n}$

Where $\vec{n}$ gives the vector moment's direction, and where θ is the smaller angle between vectors $\vec{F}$ and $\vec{r}$ when these vectors are arbitrarily placed "tail-to-tail" on the geometric plane as shown:



Eye-stein Demonstrates Vector Moment, $\vec{\Gamma}$
(He pushes with force, $\vec{F}$, to cause a twist of the object counter-clockwise about its rational axis)



$\vec{\Gamma} = r_\perp F, \vec{n}$ = (size of perpendicular distance, from axis of rotation to line of action of force); and the direction vector n indicates a counter-clockwise twist about the axis of rotation.

The actual calculation would be

$$\vec{\Gamma} = r\ F \sin\theta,\ \vec{n}$$ (where $\vec{n}$ is simply a rotational axis direction indicator)

$$= \tfrac{1}{4}\ \text{ft}\ (5\ \text{lb})\ \sin 90^\circ,\ \vec{n}$$ (where $\sin 90^\circ = 1$)

$$= \frac{5}{4}\ \text{lb}\cdot\text{ft, counterclockwise}$$

The same problem, in MKS system units, would be:

$$\vec{\Gamma} = \tfrac{1}{4}\ \text{ft}\ \left(\frac{\text{m}}{3.28\ \text{ft}}\right)\ \frac{5}{4}\ \text{lb}\ \left(\frac{4.45\ \text{N}}{\text{lb}}\right)\ \sin 90^\circ,\ \vec{n}$$

$$= \frac{22.3}{16}\ \text{m}\cdot\text{N, cc}$$

$$= 1.4\ \text{m}\cdot\text{N, counterclockwise}$$

The reader is asked to pay particular attention to the direction of the vector cross product. Two directional elements are involved. One is the position of the axis of rotation. The other is the direction of rotation about the axis (i.e., clockwise or counterclockwise). The "Karate Chop" method of direction determination is recommended. Imagine the vectors tail-to-tail as shown in (a). With the right hand "karate chop" the first vector (i.e, in $\vec{r} \times \vec{F}$, $\vec{r}$ is the first vector; in $\vec{v} \times \vec{B}$, $\vec{v}$ is the first vector) keeping the thumb fully extended, as shown in (b).

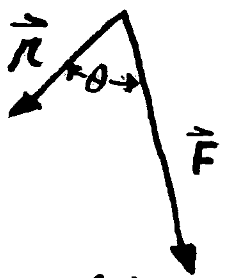
TIP FROM EYESTEIN: For vector moment calculations, imagine the force vector $\vec{F}$ and the position vector $\vec{r}$ placed tail-to-tail, as shown below. Then use the smaller angle between them (θ) for calculating what mathematicians call the "vector cross product."

$$\vec{\Gamma} = \vec{r} \times \vec{F}$$ (read "r cross F")

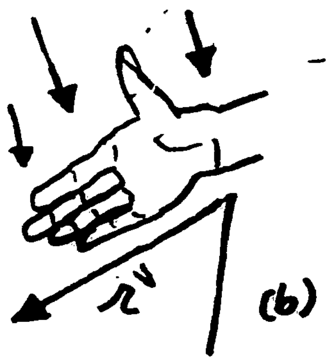
which, by definition, is simply

$$\vec{\Gamma} = rF \sin\theta,\ \vec{n}$$

$$= rF,\ \vec{n}$$

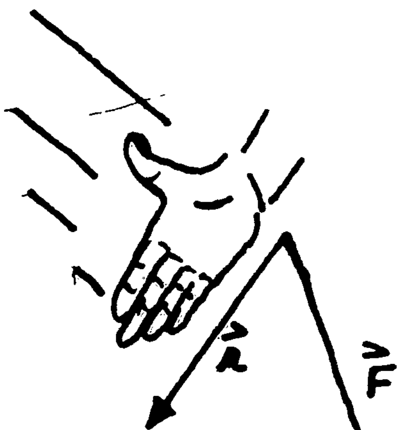


(a)

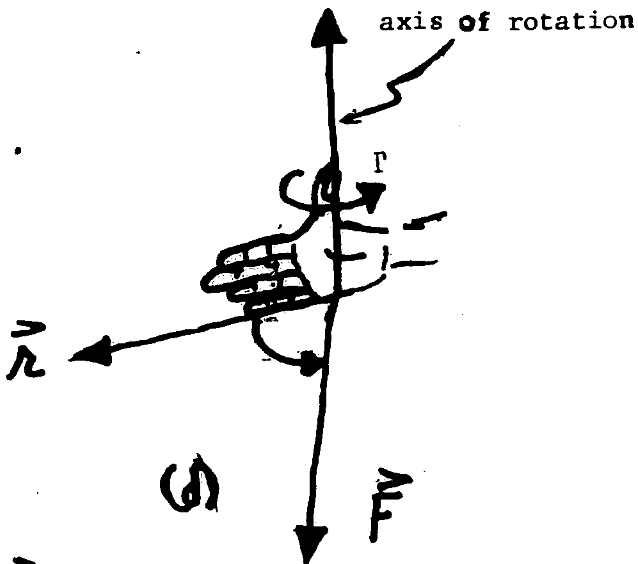


(b)

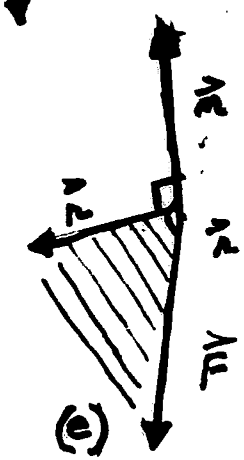
Then, with the smallest finger of the right hand <u>on top of the first</u> vector, rotate the palm and extended fingers of the hand through the smaller tail-to-tail angle (θ), using the outstretched thumb (which is kept at all times perpendicular to the palm and extended fingers) as the hand's rotational axis. See (c) below . The fingers will curl into the direction of the resultant twist (rotation) and the thumb will be aligned along the cross-product rotational axis. See (d) below . Note that the two vectors $\vec{r}$ and $\vec{F}$ will be at right angles to the rotational axis as well as to one another (<u>i.e.</u>, all three are said to be <u>mutually orthogonal</u>, or at right angles to one another). See (d) and (e) below.



(c)



(d)



(e)

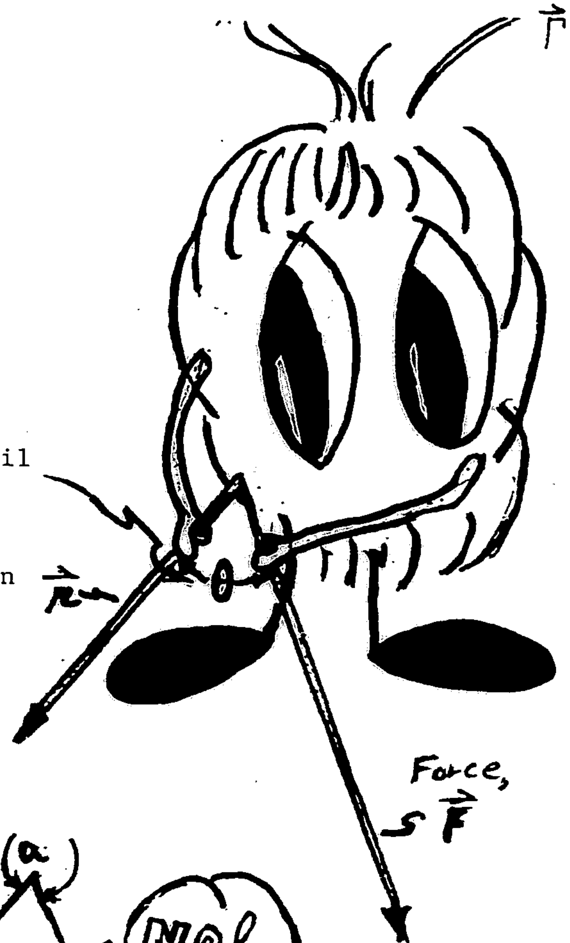
In diagram (e) on the previous page. we see that $\vec{n}$ is perpendicular to $\vec{r}$ and $\vec{F}$ and also to the plane swept out as $\vec{r}$ "crosses" into $\vec{F}$.

Eyestein demonstrates a "first-step" of the vector cross-product: the vectors are "mentally" placed tail-to-tail, and the <u>SINE</u> of the <u>smaller</u> angle between them is used in the equation

$$\vec{\Gamma} = rF \sin \theta, \vec{n}$$

"Tail-to-tail Angle"

Position Vector $\vec{r}$



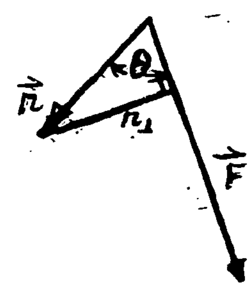
Where:

r = <u>size</u> of the position vector (vector drawn from axis of rotation to point of application of force $\vec{F}$)

F = <u>size</u> of vector force, $\vec{F}$

$\vec{n}$ assigns a rotational orientation in space

sin θ "crosses" the $\vec{r}$ and $\vec{F}$ vectors perpendicularly, as shown below:

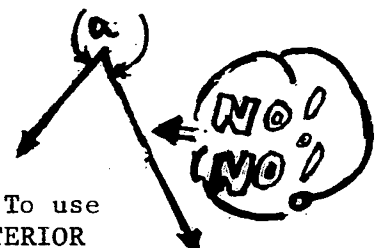


$$\sin \theta = \frac{\text{opp side}}{\text{hyp}}$$

$$\sin \theta = \frac{r_{\perp}}{r}$$

$$r \sin \theta = r_{\perp}$$

= component of $\vec{r}$ "crossing" $\vec{F}$ at right angle



NOTE: To use the EXTERIOR angle, α, would be a real NO-NO!

(Check your progress)

COMPLETION QUESTIONS:

1. __________ numbers are numbers expressing both size (magnitude) and direction.

2. __________ physical quantities are those which can be described by vector numbers.

3. Forces and __________ are examples of vector physical quantities which cause changes in the motional states of objects or which cause deformations of objects.

4. Vector numbers can be represented geometrically by __________ line segments.

5. Geometrically, the __________ of a line segment corresponds to the magnitude of a given physical vector quantity.

6. In polar coordinate notation, vector magnitude is given by the __________ of the position vector $\vec{r}$; and vector direction is given by the __________ measured from the standard position (i.e., from the "x" axis).

MULTIPLE CHOICE AND COMPLETION:

1. Vector moment magnitude is always a mathematical (a) sum (b) difference (c) product (d) quotient (e) none of these.

2. Vector moment direction is most closely related to (a) a vertical axis (b) a horizontal axis (c) the force vector (d) the position vector (e) a rotational axis.

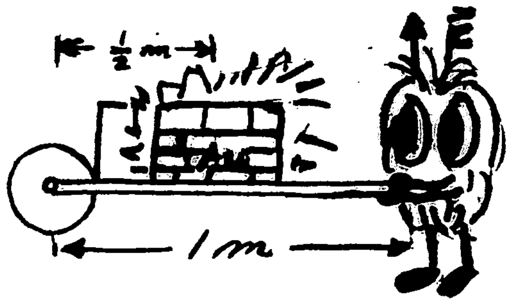
3. Common units for vector moment are (a) lb·sec (b) N·kg (c) lb·ft (d) m·N (e) slug·ft.

4. The mathematical equivalent to the vector moment equation is (a) rF (b) r F (c) r F, $\vec{n}$ (d) rF sin θ (e) rF sin θ, $\vec{n}$. Where r is the __________ of the position vector, __________ is the magnitude of the twisting force, θ is the (smaller/larger) angle between vectors r and F when "placed tail-to-tail," and $\vec{n}$ represents a unit direction vector. The unit direction vector $\vec{n}$, is represented by a directed line segment of units magnitude ($|\vec{n}| = 1$), has its origin at the vector $\vec{r}$-vector $\vec{F}$ tail-to-tail intersection, is at a ____-degree angle to both $\vec{r}$ and $\vec{F}$ and points along the rotational axis of the vector cross product.

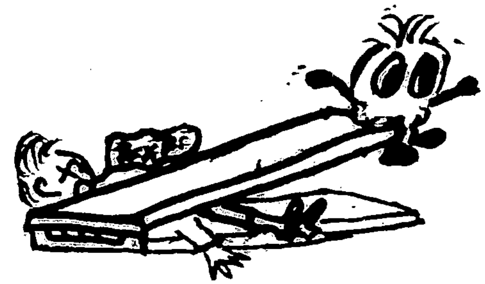
PRACTICE PROBLEMS:

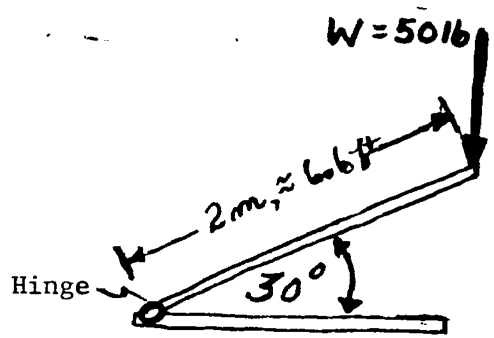
1. The magnitude of the horizontal force component acting on this timid turtle is ________.

2. If the gold bricks in Eyestein's wheelbarrow weight 500 N, and if Eyestein's lifting force has a magnitude of 120 N, find the vector moment about the wheel axis due to Eyestein's effort.



3. Eyestein decides to compress a lovable physics professor as shown. Determine the resulting vector moment about the hinge; report the results in both the MKS and the FSS system (English engineering system).

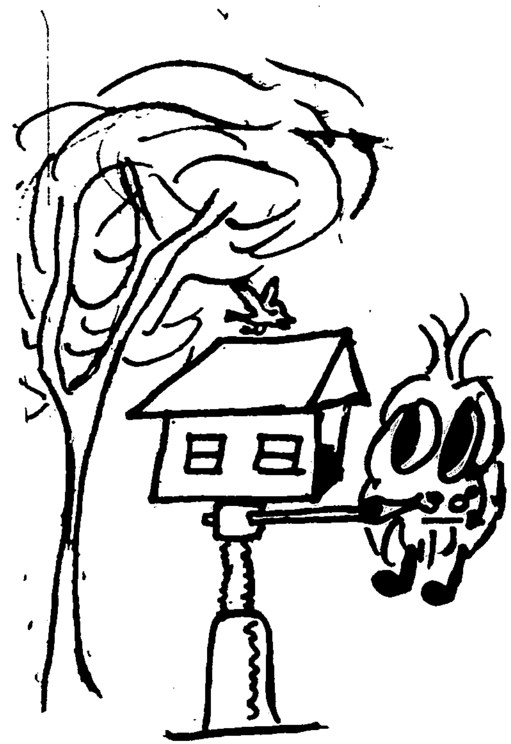




a. MKS system vector moment: ________________

b. Engineering system vector moment: ________________

4. For the house jackscrew shown below:
(a) the vector algebra equation of the Eyestein-produced vector moment is __________, where $\vec{r}$ is called the (b) p__________ v__________ and is measured from the axis of (c) r__________ to the (d) p__________ of contact.



An algebraic (trigonometric) equation for this vector moment would be (e)__________. If Eyestein's applied force is 60 lb, and if the jack handle is 2 feet long, the net vector moment is (f)__________.

COMPLETION ANSWERS:

1. Vector
2. Vector
3. Moments
4. Directed
5. Length
6. Length, angle

MULTIPLE CHOICE AND COMPLETION ANSWERS:

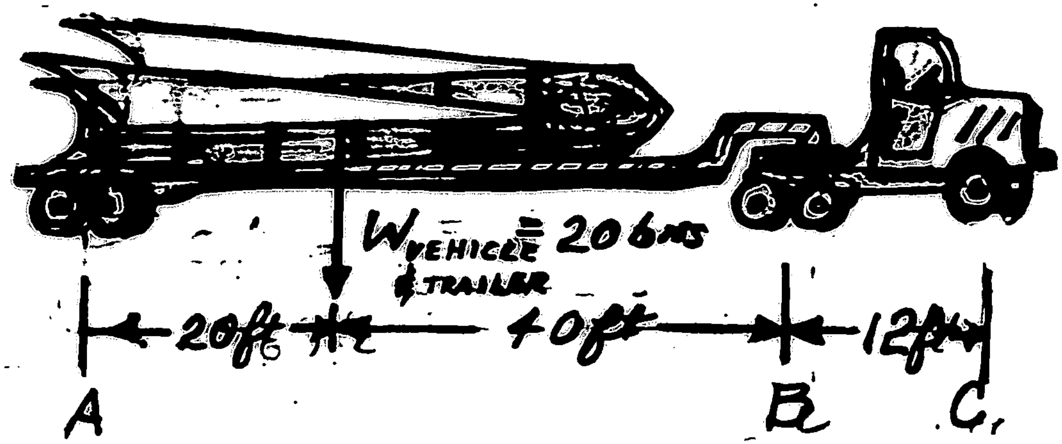
1. (c)
2. (e)
3. (c)
4. (e), magnitude or size, F, smaller, 90^o, axis

ANSWERS TO PRACTICE PROBLEMS:

1. $5\sqrt{3}$ N
2. 500 N·m
3. (a) $50\sqrt{3}(4.45)$ N·m (b) $50\sqrt{3}(3.3)$ lb·ft
4. (a) $\vec{\tau} = \vec{r} \times \vec{F}$
   (b) position vector
   (c) rotation
   (d) point
   (e) $\vec{\tau} = rF \sin \theta \, \vec{n}$
   (f) $\vec{\tau} = rF \sin \theta \, \vec{n}$
   $= 2$ ft (60 lb) $\sin 30^o \, \vec{n}$
   $= 2$ ft (60 lb) $\frac{1}{2}$ cc
   $= 60$ lb, cc

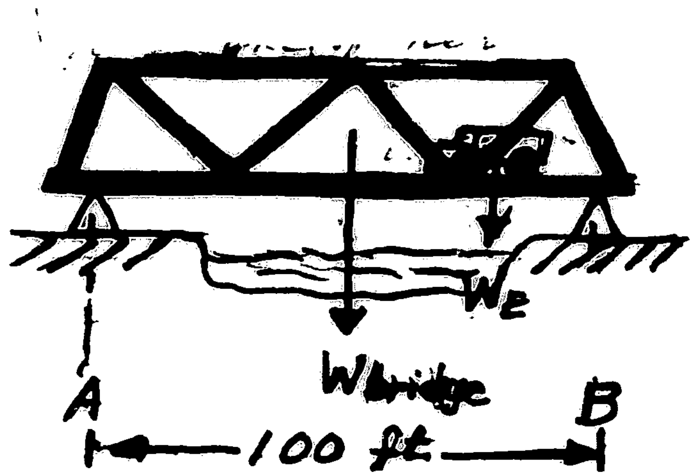
(Note: If you did not show direction, you "blew" this answer. All vector quantities require a direction.)

Suppose that we wish to know the forces the loaded space vehicle sketched below exerts upon the rear axles of the low-boy rig which moves it to the launch area.



Remembering that all forces on a body in equilibrium must sum to zero (Otherwise, the low-boy must collapse or the launch vehicle must mysteriously rise!), we can say that all the forces UP must equal all the forces DOWN. Mathematically, $F_{up} = F_{down}$, and we know immediately that the wheel-and-axle support points, A and B, must push up with a combined force of 20 tons. It is apparent that upward push A plus upward push B equals 20 tons only if we neglect the weight of the low-boy tractor to simplify the problem.

Now, how might we determine just what part of the 20-ton load is supported at point A and what part at point B? The solution is patently simple if we remember that about any point on the launch vehicle or the trailer (such as point A or point B), the vector sum of the moments must be zero.



Weight of the bridge is 100 tons, and a loaded truck weighing 10 tons is stalled 20 ft from support B. This problem will give you an opportunity to apply what you have just learned from analyzing the forces and moments related to the launch vehicle and the low-boy. If you have difficulty, a solution is given on the next page.

A Solution to the Bridge Problem:

Step 1:

$$\sum F_{Up} = \sum F_{Down}$$

$$F_A + F_B = W_{Bridge} + W_{Truck}$$

$$= 100 \text{ tons} + 20 \text{ tons}$$

$$= 120 \text{ tons}$$

Step 2:

Realizing that for a uniformly-constructed bridge, the weight acts at the bridge's center, and further realizing that the moments about any point on the bridge must sum to zero, let's pick point B and sum the moments about it.

Then, $\sum \vec{\Gamma}_{\text{clockwise about B}} = \sum \vec{\Gamma}_{\text{counterclockwise about B}}$

or,

"Weight-of-bridge" moment about B plus "weight-of-truck" moment about B = "support-force-A" moment about B,

Substituting numerical values yields:

$$20 \text{ ft } (20 \text{ tons}) + 50 \text{ ft } (100 \text{ tons}) = 100 \text{ ft } (F_A)$$

$$54 \text{ tons} = F_A$$

Step 3:

Realizing from step 1, above, that $F_A + F_B = 120$ tons, Then,

$$\sum F_{Up} = \sum F_{Down}$$

or

$$F_B + 54 \text{ tons} = 120 \text{ tons}$$

$$F_B = 66 \text{ tons}$$

In other words, pier support A must bear a 54-ton load and pier support B must bear a 66-ton load when the truck is in this location on the bridge.

EYESTEIN ASKS: If the truck moves, will the loads at A and B change? Will the piers A and B ever support equal loads?

Summary of Introduction to Forces and Moments. Forces are "pushes" or "pulls"; moments are "twists." Later, vector force and vector moment will be defined in terms of acceleration. But "push," "pull," and "twist" will remain sound operational definitions throughout our study of physics.

Equilibrium of Rigid Bodies. This section treats the conditions for equilibrium of rigid bodies experiencing vector forces and/or vector moments. We will assume that all forces are coplanar; and a rigid body will be defined as one whose sub-parts (molecules) attempt to maintain a fixed position relative to one another even when the body is subjected to outside forces or moments. We will furhter assume that the two conditions necessary for rigid body equilibrium are: (1) all forces applied to the body must sum to zero vector-wise; and (2) all vector moments experienced by the body must sum to zero vector-wise. Later, after we have treated vector force and vector moment in terms of acceleration, the reader will gain some insight as to why these two conditions must be met before rigid body equilibrium can exist. But for now let us simply accept these two conditions for rigid body equilibrium, plus the following postulates:

1) If a body in translational equilibrium is at rest, it will remain at rest forever; or, if moving linearly, it will continue to move forever at a constant speed in a straight line

2) If a body in rotational equilibrium is at rest, it will remain at rest forever; or, if spinning, it will continue to spin forever at a constant speed about a fixed axis in space.

TIP FROM EYESTEIN: Two conditions must be met before mechanical equilibrium of a rigid body can exist. One is a linear condition; the other is a rotational condition.

The two equilibrium conditions provide powerful tools with which to tackle a multitude of relevant everyday engineering and technical problems.

But before pursuing equilibrium further, the reader is informed that if any part of a rigid body is in equilibrium, then all parts must be in equilibrium. This is equivalent to saying that if such a body is in equilibrium, one could randomly select any point on the body and the following conditions must hold true:

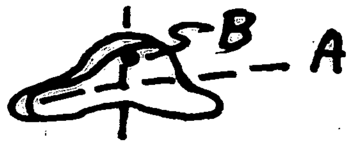
1) The sum of all forces on the point must be zero

2) The sum of all moments about that point must be zero.

Rigid Body's Center of Gravity. The center of gravity of a rigid body can be thought of as that point in space through which the line of action of the gravity force acts when the body experiences a gravity force. Operationally, one might determine the c. g. of a thin uniform slab as follows:

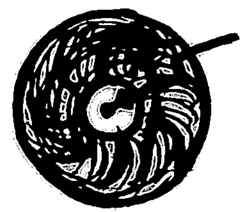
1) The thin slab is suspended at axis point A. A plumb line from axis A is shown.

2) A plumb line is shown when the slab is suspended at axis B. Note that the plumb lines A and B intersect.

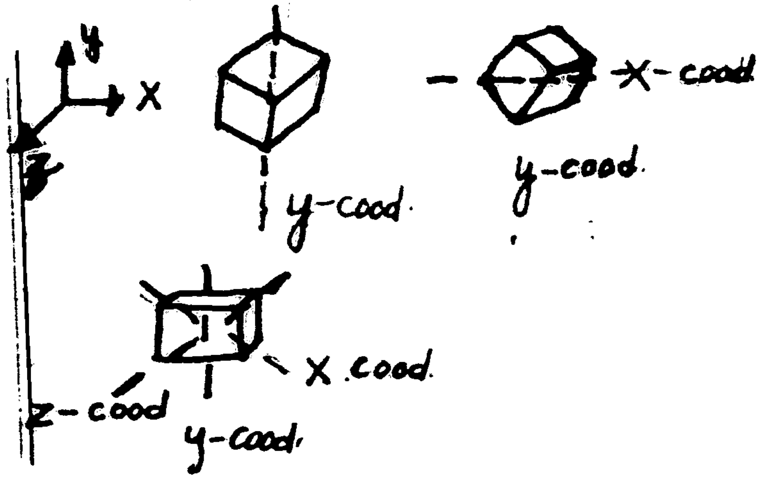


3) All possible suspension points will yield corresponding plumb lines which intersect THE SAME POINT ON THE BODY. This point is the planar center of gravity; which is to say that we have determined a "two-dimensional" c. g. as out center-of-gravity point. The c. g. of any thin slab can be determined by an "x" and a "y" coordinate value. Also, the c. g. of a cube or any three-dimensional object can be determined in like fashion, using "x", "y" and a third "z" coordinate value.

EYESTEIN SAYS: The center of gravity of a rigid body need not be within the body itself. The doughnut (a toroid-shaped rigid body) has its c. g. at its geometric center.



DOUGHNUT'S C. G. IS OUTSIDE ITS DOUGH!

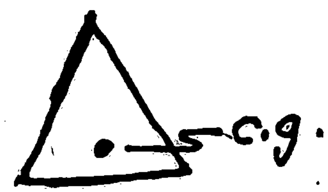


Note that for a three-dimensional object, these plumb lines intersect in what we call "3-space," a point in space whose position is determined by the combination of x, y, and z-coordinates.

VII. Neutral, Stable, and Unstable Equilibrium of Rigid Bodies. One can use the concept of center of gravity to describe the stability states of a rigid body in equilibrium. Classically, the case usually considered is that of the cone. The diagrams below illustrate these three equilibrium states.

1) Stable Equilibrium State:

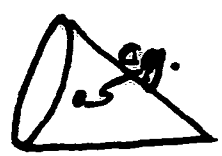
a) Any effort to capsize the cone tends to raise its c. g. above the supporting surface



b) Cone tends to restore itself to original position when disturbing force is removed (c. g. lowers to initial position).

2) Neutral Equilibrium State:

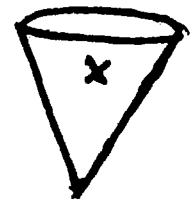
a) Any effort to capsize the cone tends NOT TO ALTER its c. g. relative to the surface (neither raises nor lowers the c. g.)



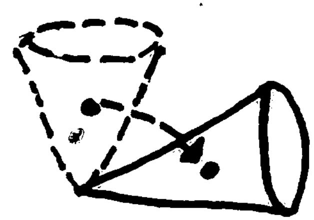
b) Cone tends to merely roll when a disturbing force is applied.

3) Unstable Equilibrium:

a) Even the slightest effort to capsize the cone results in a lowering of c. g.



b) No restoring tendencies exist.

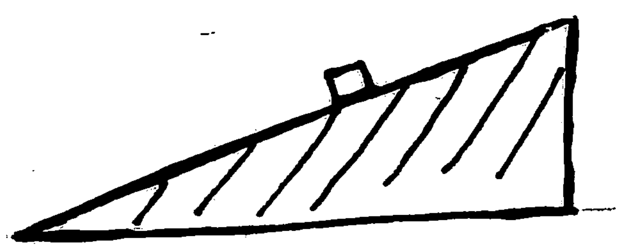


Another common analogy of these equilibrium states is provided in the sketch of the hot wheels track and car below:

Points A and B are positions of neutral equilibrium. Point C is a position of unstable equilibrium, and Point D is a position of stable equilibrium.

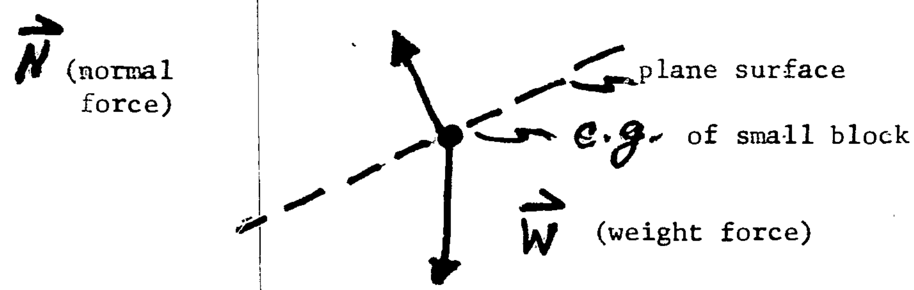
VIII. <u>The Inclined Plane</u>. A classic problem found in nearly all introductory applied physics texts is that of the inclined plane. We will consider this problem because it can involve the resolution of forces, construction of a space diagram, construction of a "free body" diagram, application of equilibrium conditions, and utilization of some plane geometry and trigonometry.

Consider the block on the plane shown below. This is often called a "space diagram" because it exhibits the spatial relationships of the bodies under consideration. (In this case the block and the inclined plane are the bodies considered.) The problem we face is to analyze the forces acting on the block.



Step 1:

The first step in our analysis is to construct a "free-body diagram" of the block. A free-body diagram is one which consists ONLY of representations of FORCES acting ON the body being analyzed. (In this case, only those forces which act on the block can go on our diagram.) The free-body diagram of the block appears below:



We simplify the problem by assuming the plane is frictionless and that we are interested only in an analysis of linear effects, since the block is not likely to roll.

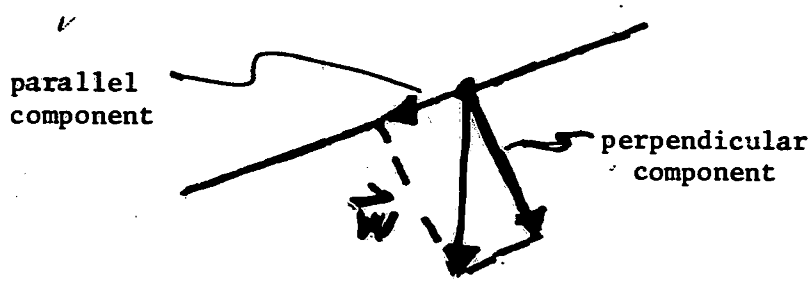
Notice that only two forces act on the block, since we have chosen to simplify our problem by neglecting any frictional and rotational possibilities. The two forces we do consider then are the weight force and the force exerted by the plane's surface on the block. This latter force is called the normal force because it acts at right angles (perpendicular) to the block's contact surface.

Can you see that the normal force results because the gravity force pulls the block against the plane? In other words, the gravity force component which acts against the plane's surface is equal to and is directed opposite from the normal force. In fact, the normal force would not exist without the gravity force; the normal force is merely a reaction force to that part of the gravity force pulling the block against the plane!

Step 2:

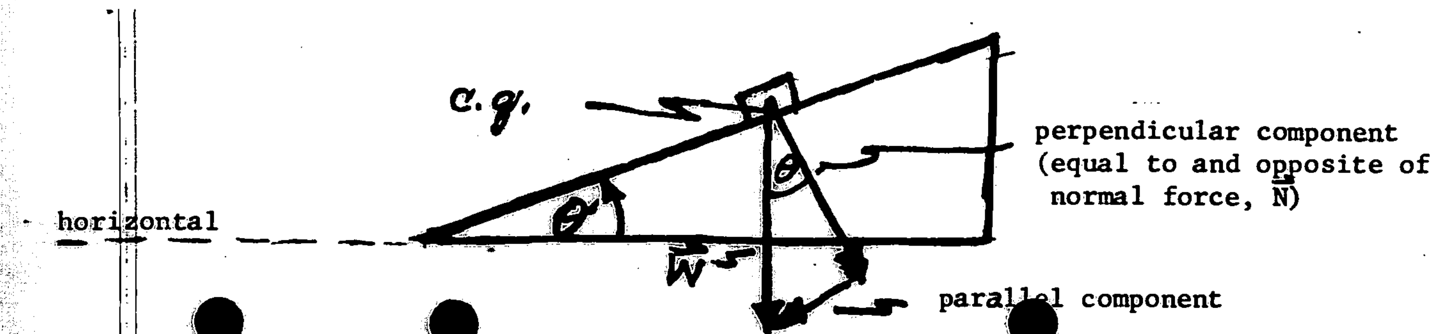
The first step in our analysis was to construct a free-body diagram. The second step is to examine the component of the weight force tending to pull the block down the plane (parallel component) and the component of the weight force acting to pull the block against the plane's surface (perpendicular component).

Graphically, our analysis looks like this:



In other words, vector $\vec{W}$ is to be resolved into a component parallel to the plane and a component normal to the plane.

Before we push on, however, let's look at a diagram which superimposes step 1 onto step 2.

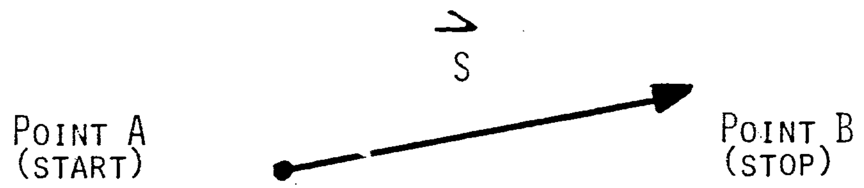


Notice that two angles have been designated as θ and θ'. Because it can be shown from Euclidean geometry that the inclined plane triangle and the weight force component triangle are similar (*i.e.*, two right triangles whose corresponding sides are mutually perpendicular are similar triangles), angles θ and θ' are equal.

SECTION II

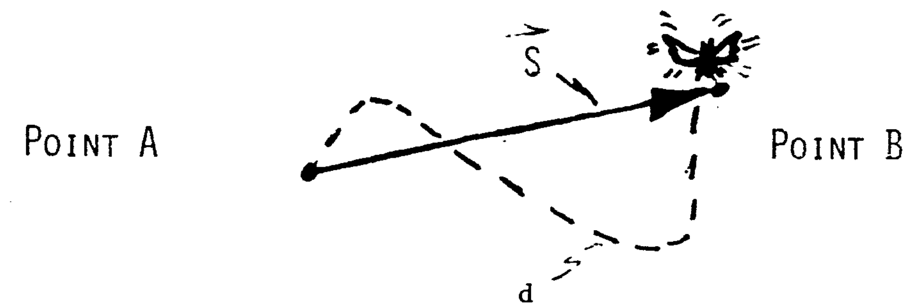
Introduction. Technically, it is possible to describe the translational and rotational motions of an object graphically or algebraically. Motions of concern in this section are the vector motions we call displacement, velocity, and acceleration. These vector motions will be examined for both rotation and translation of rigid bodies. Careful distinctions will be made between distance (a scalar quantity) and displacement (a vector quantity), and between speed (a scalar quantity) and velocity (a vector quantity).

Vector Linear Displacement. A graphical description of an object's linear displacement is a directed line segment whose origin represents the object's starting point (initial position) and whose tip represents the object's stopping point (final position), as shown in the sketch below.



In this case, let it be assumed that the linear displacement vector $\vec{s}$ coincided precisely with the actual straight line path traced out by the object as it displaced from point A to point B.

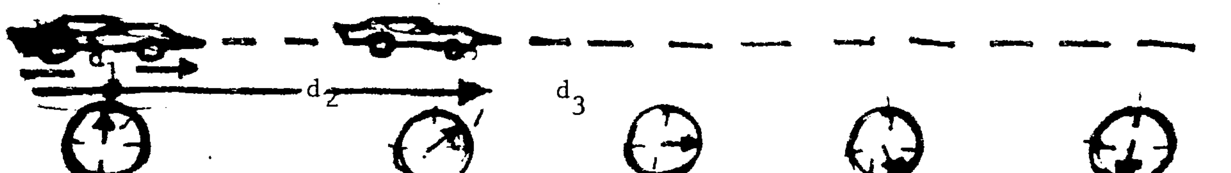
But suppose, in another case, that a butterfly flying from Point A to point B was diverted from a straight line path by certain aromatic flowers. For the sake of discussion, suppose the butterfly actually traced out the broken line path shown below.



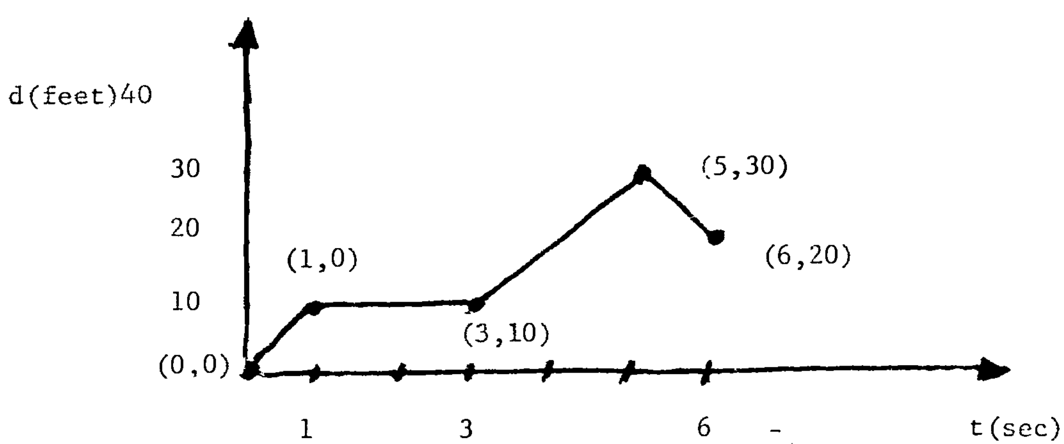
Note that the linear displacement vector $\vec{s}$ (directed line segment representation) and the butterfly's linear path distance "d" (broken line representation) originate and terminate at the same points in space but differ significantly in path length (magnitude). Obviously, linear <u>displacement</u> is a vector physical quantity since it shows both distance and direction, whereas linear <u>distance</u> is a scalar physical quantity. It is further obvious that displacement and distance need not have the same magnitude (size), nor need they be <u>colinear</u> (along the same line). Further, only when the two paths are identical (colinear) is the magnitude of the linear displacement vector equal to the linear distance.

<u>Distance vs Time Graphs</u>. It is often useful to examine the graphical properties of distance and time. Such examinations can reveal mathematical descriptions of an object's motion, can indicate whether or not this motion is uniform, and can show the object's speed.

Suppose we recorded a moving automobile's position at successive times. (See sketch below.)



We begin timing at position d = 0 and at time t = 0. At time t = 1 sec, our automobile has traveled a linear distance d_1. Below is a plot of d vs t, and other respective times and linear distances.



GRAPH (PLOT) OF LINEAR DISTANCE VS TIME

Let's examine several features of our graph. The graph has both positively and negatively sloped intervals. The graph also has an interval whose slope is zero. These slopes constitute a measure of the speed (magnitude of the object's velocity vector).

The special name we give to a change in linear distance with respect to a change in time is linear speed. In a like sense, the special name we assign to change in vector displacement with respect to a change in time is velocity. (Remember, speed is the name assigned to the magnitude of the linear velocity vector.)

If an automobile travels for two hours along a winding road to reach a town 80 actual road miles away, its speed over that distance is found algebraically in this manner:

$d = \bar{v}t$ (where d is distance, $\bar{v}$ is average speed, and t is time)

Solving for average speed, $\bar{v}$

$$\bar{v} = d/t$$

Substituting given values:

$$\bar{v} = 80 \text{ mi}/2 \text{ hr}$$

$\bar{v} = 40$ mi/hr, and no direction of speed is implied

TIP FROM EYESTEIN: Part of our exercise here is to become acquainted with the concept that graphical slope shows how a graph variable (in this case, distance) changes with respect to a change in its related variable (in this case, time).

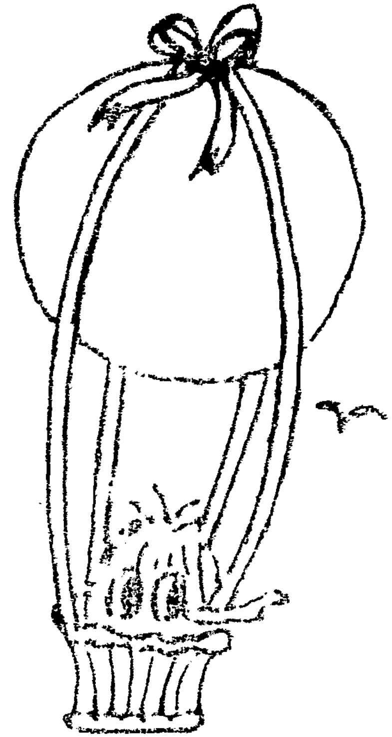
But if this were an aerocar whose flight path was "as the crow flies" (i.e., a straight line path of 60 miles for 3/4 hour), then the average speed would be:

$$\bar{v} = \frac{d}{t}$$

$$= 60 \text{ mi}/ \frac{3}{4} \text{ hr}$$

$$= 80 \text{ mi/hr}$$

and the velocity would be 80 mi/hr (the speed of translation) in a straight line direction between the flight's origin and its terminus at the distant town's airport (the direction of translation).



Eyestein checking on "as the crow flies"

Returning to the automobile's speed-time graph, notice that over the interval between 0 sec to 1 sec, the distance traveled was 10 ft. ... calculate the average speed as follows:

$$d = \bar{v}t$$

$$\bar{v} = \frac{\Delta d}{\Delta t}$$

$$= (d_f - d_i) / (t_f - t_i)$$

$$= \frac{(10-0)\text{ft}}{(1-0)\text{sec}}$$

$= 10$ ft/sec (Since we seek speed, no direction needs to be indicated.)

From the same automobile's speed-time graph, we notice also that the slope over the 0-1 second interval was given by d/t and that this ratio value was a constant value of positive 10 (straight line for this time interval).

TIP FROM EYESTEIN: Remember, slope is defined as y/x, or as rise ÷ run.

Next, consider the average speed over the time interval 1 sec to 3 sec.

$$\bar{v} = \frac{\Delta d}{\Delta t}$$
$$= \frac{d_f - d_i}{t_f - t_i}$$
$$= \frac{(10 - 10)\ ft}{(3 - 1)\ sec}$$
$$= 0\ ft/sec$$

The average speed over this interval is zero, and the slope ($\Delta d/\Delta t$) is zero. Again, the line graphed over this interval is straight, indicating that the ratio, $\Delta d/\Delta t$, was fixed and unchanging over the interval. Thus, we can say that the graph (slope) indicates that speed was constant over the interval.

Over the time interval 3 sec to 5 sec, the average speed is given by:

$$\bar{v} = \frac{\Delta d}{\Delta t}$$
$$= \frac{(30 - 10)\ ft}{(5 - 3)\ sec}$$
$$= 10\ ft/sec$$

Notice that the average speed here is the same as that over the first second interval and that the slope of the graph is exactly the same (a constant positive value of 10).

Last, let's calculate the average speed over the time interval 5 sec to 6 sec.

$$\bar{v} = \frac{\Delta d}{\Delta t}$$

$$= \frac{d_f - d_i}{t_f - t_i}$$

Notice that $t_f = 20$ ft

$$= \frac{(20 - 30)\ ft}{(6 - 5)\ sec}$$

$$= 10\ ft/sec$$

Here the speed (slope) is the same as those over the 1-to-2 sec interval and the 3-to-5 sec interval, except for the algebraic sign and slope direction. This tells us that the average speeds over the three intervals are indentical, but the velocities are different. In the 5-to-6 sec interval, the direction of the speed must be opposite that direction of the 1-to-2 sec and the 3-to-5 sec interval speeds (See Eyestein's tip).

TIP FROM EYESTEIN: The speeds must be opposite because a negative time value is impossible. Time never "runs backward"; i.e., a later time value is always greater than an earlier time value. Therefore, the negative sign and the negative slope must be due to Δ v; so the $-\Delta v$ calls our attention to this "oppositeness" of velocities.

Again, the straight lines over the intervals indicate that all of these speeds remain constant over their respective intervals.

In summary of our foregoing analysis of the distance vs time graph:

1) The slope over any time interval indicates the average speed over that interval

2) A negative slope indicates that direction of motion has been reversed

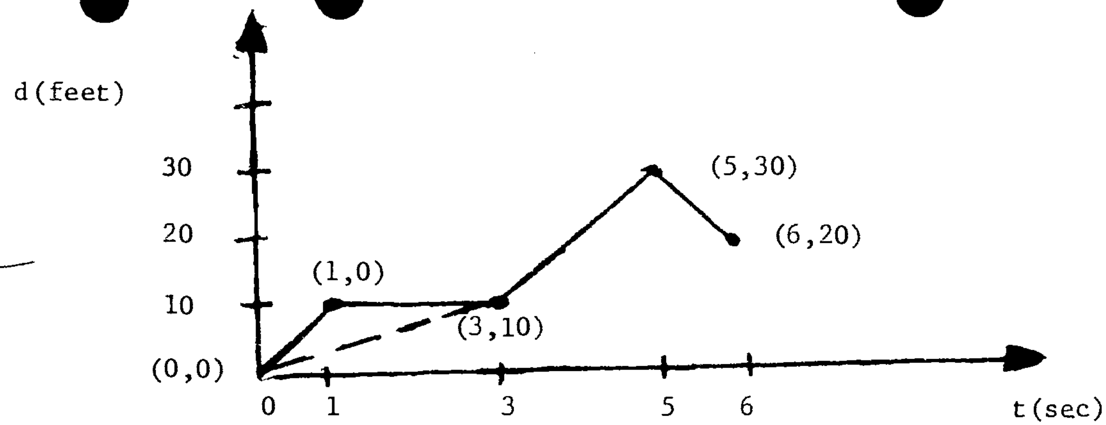
3) A straight line between a time interval indicates a constant speed over that interval.

See Eyestein's tip.

TIP FROM EYESTEIN: Since the slope is zero over the 1-to-3 second interval, the speed is zero and the automobile must have been at rest during this two-second period.

Next, why not examine a situation where speed is not constant but changes over a given time interval? Consider the interval 0 sec to 3 sec. The graph clearly shows a change in slope (from a positive 10 slope over the first second to a zero slope over the second and third seconds). Therefore, the change in slope must indicate a non-constant speed. (Recall that the average speed was calculated as 10 ft per second during the first second of travel and as 0 feet per second during the 2nd and 3rd seconds of travel. Obviously, the speed changed during the interval 1-to-3 seconds!)

We can next ask ourselves, "If the speed changes, what is the average speed over the entire time interval 0 to 3 seconds?" The dashed line in the diagram on the next graph is the slope we now seek.



GRAPH (PLOT) OF LINEAR DISTANCE VS TIME

Again,

$$\bar{v} = \frac{\Delta d}{\Delta t}$$

$$= \frac{(10 - 0)\ \text{ft}}{(3 - 0)\ \text{sec}}$$

$$= \frac{10}{3}\ \text{ft/sec}$$

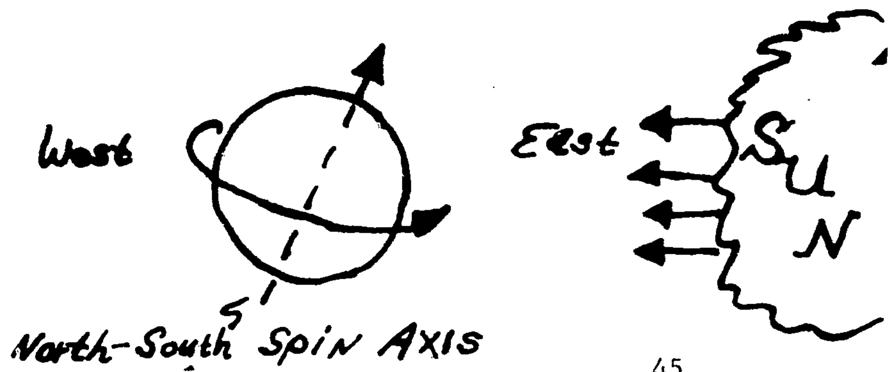
Obviously, this slope is smaller than that for the 0-to-1 sec interval, but is greater than the zero slope for the 1-to-3 sec interval. The dashed line tells us that the average speed for the first three seconds of motion is in a positive direction and has a value of 10/3 ft per sec. Notice that the greater the speed, the greater the slope magnitude displayed for respective intervals on the graph.

Note the following "like forms" (called symmetries or isomorphisms) existing between the speed equations and the vector velocity equations;

Speed	Velocity
$\bar{v}_{Avg} = \frac{\Delta d}{\Delta t}$	$\vec{v}_{Avg} = \frac{\Delta s}{\Delta t}$
$= \frac{d_f - d_i}{t_f - t_i}$	$= \frac{\vec{s}_2 - \vec{s}_1}{t_2 - t_1}$
	where $\vec{s}_2 - \vec{s}_1$ is a <u>vector</u> <u>algebraic</u> subtraction operation.

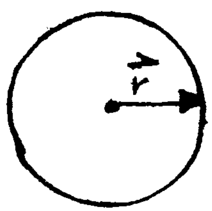
Rotational Analyses of Displacements. In real life, analyses of rotations are as necessary and as useful as analyses of linear translations, so it is important that we examine the algebraic and graphic descriptions of rotation.

First, the vector property of a rotating object is related to the space orientation of the axis about which rotation occurs, and to the direction of the object's rotation about that spatial axis. The earth, for example, rotates direction-wise from west to east. (It turns "into the sun," thus causing the sun to "rise" in the east and to "set" in the west.) The earth also rotates spatially about its north-south spin axis, as shown below.

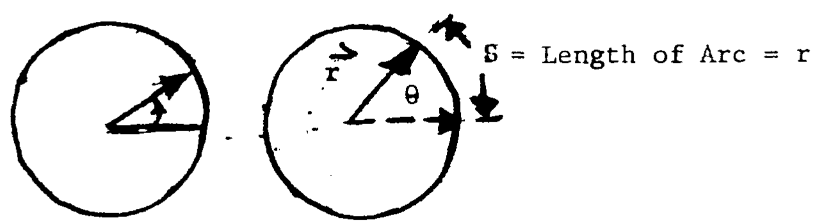


Frequently the Greek symbol theta, $\vec{\theta}$ (pronounced "thay-tuh"), is used with the arrow superscript to represent vector angular (rotational) displacement, and θ (no arrow superscript) to represent rotational distance.

Rotational "distance" can be described or measured in several ways, including number of revolutions, number of circular degrees, or number of radians. Radian measure is widely used in technical work. A radian is defined as that angle formed when the tip (terminus) of a rotor (radius vector) rotates through an arc such that the rotor tip traces out an arc length exactly equal to the radius length.



Rotor (radius vector) in Standard Position



Rotor (radius vector) rotated through 1 radian, θ. (Note that the angle subtended, θ, is such that the length of arc precisely equals the length of the rotor (radius vector) r, thus defining the radian as a unit of circular measure.)

TIP FROM EYESTEIN: There are 2 radians in the circumference of a circle, or

$$2\pi \text{ radians} = 360^\circ$$

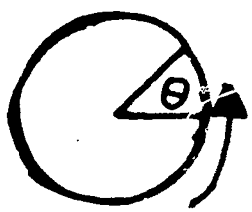
$$\text{radian} = \frac{360^\circ}{2\pi}$$

$$\text{radian} \cong 57.3^\circ$$

When the rotational direction about a specific axis is known, then the angular distance, θ, becomes the vector angular displacement $\vec{\theta}$. Frequently, direction is referred to as clockwise (c) or counterclockwise (cc) about the specified axis.

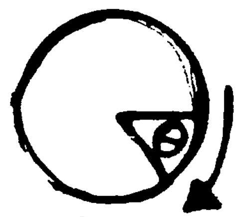
Let's examine some sketches of small angular displacements to see if we can detect their vector nature.

Here the direction is counter clockwise through 60^{o} ($\approx$ 1 radian)



(a)

Here the rotation is clockwise through 60^{o} ($\simeq$ 1 radian)



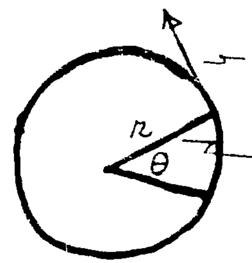
(b)

TIP FROM EYESTEIN: Rotations behave quite like vectors for small angles, but one must use care in applying this kind of analysis to large-angle rotations.

It can be shown that the linear speed of the rotor tip is related to the angular speed by the equation

$$v = r\omega$$

where v is the linear speed, r is the rotor vector magnitude, and ω (omega) is the angular speed in radians' per unit time.

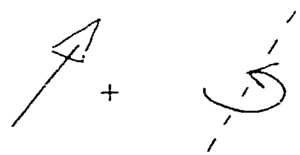


$\omega = \frac{\Delta\theta}{\Delta t}$ (change of angle θ with time)

Where direction is applicable, one can also write

$$\vec{v} = r\vec{\omega}$$

It is evident that the linear velocity $\vec{v}$ is related directly to the product of the scalar length, r, and the angular velocity $\vec{\omega}$. (Note that angular velocity, like linear velocity, has both magnitude and "direction of rotation" about some specific axis.)

$\vec{\omega}$ = 

= direction + angular speed

Vector Linear Velocity. Linear velocity is a vector physical quantity. Physically, it represents the rate of change of the displacement vector. Algebraically, we write

$$\vec{v} = \frac{\Delta \vec{s}}{\Delta t}$$

$$= \frac{\vec{s}_2 - \vec{s}_1}{t_2 - t_1}$$

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PROGRAMMED REVIEW

1) If we observe motions in nature (or in technological applications of physics), we can expect these motions to include both (a) l_________ (sometimes called t________________) motion and (b) r_______________ (sometimes called a______________) motion.

2) F___________ is associated with linear motion and m______________ is associated with rotary motion.

3) Scalar numbers have (only direction/only size/both size and direction).

4) Scalar physical quantities have (only direction/only size/both size and direction).

5) S______________ numbers are used to describe scalar physical quantities, whereas ________________ numbers are used to describe vector physical quantities.

6) Which are scalar physical quantities?
a) hot day temperature of 90° F
b) sound intensity of a "rock" group
c) rpm of a Mazda rotary engine
d) wind velocity in March
e) weight of a lunar landing vehicle

7) The term associated with the motional condition resulting from no net vector forces and no net vector moments acting on a rigid body is ________________________.

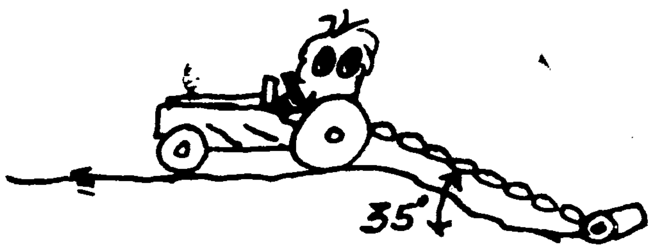
8) Which of these are obviously related to vector moments?
a) electric mixer
b) spinning top
c) spin-stabilized satellite
d) pencil
e) TV set

9) Examine Eyestein's tractor. Then,

a) Sketch the vertical component of the logging chain force.

b) The angle which the vertical component makes with the horizontal is ____________________.

c) If Eyestein's tractor pulls along the chain with a force component of 2,000-lb magnitude, the lifting force on the log is ______________. In polar coordinate notation this lifting force would be (d)__________________.



ANSWERS TO PROGRAMMED REVIEW

1) a. linear, translational
   b. rotational, angular

2) a. Force
   b. moment

3) only size

4) only size

5) scalar, vector

6) (a), (b), (c)

7) (a) equilibrium
(Note that to the scientist or engineer, REST is a special kind of motional state.)

8) (a), (b), (c)

9) (a) $\uparrow$

(b) 90^{o}

(c) $F_V = 2 \times 10^3 \text{ lb} \sin 45^{o}$
$= 2 \times 10^3 \text{ lb} \left(\frac{1}{\sqrt{2}}\right)$
$= 2(.707)\ 10^3 \text{lb}$
$= 1,414 \text{ lb}$

(d) 1,414 @ 0^{o},

or, 1414 $\angle 0^{o}$

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