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AUTHOR Bullock, Bob; And Others
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ABSTRACT

This minicourse was prepared for use with secondary physics students in the Dallas Independent School District and is one option in a physics program which provides for the selection of topics on the basis of student career needs and interests. This minicourse was aimed at providing the student with an understanding of the physics of falling objects, projectiles, and missiles. The minicourse was designed for independent student use with close teacher supervision and was developed as an ESEA Title III project. A rationale, behavioral objectives, student activities, and resource packages are included. Student activities and resource packages involve reading about firearm ballistics, free fall, and acceleration, investigating trajectory, analyzing general projectile motion, investigating conservation of momentum and rocket propulsion, and studying blood ballistics. (GS)

CAREER ORIENTED PRE-TECHNICAL PHYSICS

Ballistics, Bullets and Blood

Minicourse

PRELIMINARY EDITION

1974

dallas independent school district

CAREER ORIENTED PRE-TECHNICAL [illegible]

Ballistics, Bullets & Blood

Minicourse

FSEA Title III Project

1974

dallas independent school district

March 25, 1974

This Mini Course is a result of hard work, dedication, and a comprehensive program of testing and improvement by members of the staff, college professors, teachers, and others.

The Mini Course contains classroom activities designed for use in the regular teaching program in the Dallas Independent School District. Through Mini Course activities, students work independently with close teacher supervision and aid. This work is a fine example of the excellent efforts for which the Dallas Independent School District is known. May I commend all of those who had a part in designing, testing, and improving this Mini Course.

I commend it to your use.

Sincerely yours,

Nolan Estes

Nolan Estes
General Superintendent

mfs

CAREER ORIENTED PRE-TECHNICAL PHYSICS TITLE III ESEA PROJECT

Project Director:	Jesse M. Harris Assistant Director, Math-Science Dallas Independent School District
Project Editor:	Walt Elliott Physics Department California Polytechnic State University
Minicourse Authors:	Bob Bullock North Dallas High School
	Perry Oldham Lincoln High School
	Helen Shafer W. H. Adamson High School
	Hollis Tapp H. Grady Spruce High School
Physics Consultants:	Ben Doughty Physics Department East Texas State University
	Glenn Terrell Physics Department University of Texas at Arlington

CAREER ORIENTED PRE-TECHNICAL PHYSICS

BALLISTICS, BULLETS, AND BLOOD

MINICOURSE

RATIONALE (What this minicourse is about):

This course is an introduction to the physics of falling objects, of projectiles, and of missiles. The mathematics is a little "heavy," and many of the same physical concepts are presented less mathematically in other minicourse (Physics of Sports, Physics of Toys, etc.).

The motions of all objects falling freely can be described by the laws and the mathematics of projectile motion, whether the object be a bullet, a parachutist, an arrow, a football, or components of a Fourth of July fireworks display! So this minicourse will stress some of the ideas and mathematics of the natural laws governing the motion of objects in what physicists call free fall. In addition, of course, some basic ideas and mathematics of missiles will be presented.

You might well ask, "I can 'dig' all this 'heavy' stuff about mathematics and natural laws; but how does this minicourse relate to a technical career?" The answer is that an understanding of these fundamental laws of physics will be a valuable occupational asset for the rest of your life and in any number of trades, occupations, or vocations (Mother Nature can't be fooled, but she doesn't change much, either!) Consider the specific career of police science. Everyone knows that ballistics is the "science of bullets," but how many have heard about "blood ballistics"? Do you know that a criminal investigator's ballistic analysis of a bloodstain can yield just as important and valid a set of clues as his ballistic analysis of a bullet?

Modern policemen and policewomen are frequently specialists working within the broad occupational field of police science. But all police persons have a familiarity with firearms and ballistics. The same familiarity with ballistics applies to many military occupations. Of course, professional athletes work daily with the ballistics of balls, pucks, etc. And for persons considering a career in aerospace or aviation-related technology, a knowledge of the fundamentals of flight characteristics is highly desirable and requires the same knowledge of ballistic physics.

In addition to RATIONALE, this minicourse contains the following sections:

1) TERMINAL BEHAVIORAL OBJECTIVES (Specific things you are expected to learn from the minicourse)

2) ENABLING BEHAVIORAL OBJECTIVES (Learning "steps" which will enable you to eventually reach the terminal behavioral objectives)

3) ACTIVITIES (Specific things to do to help you learn)

4) RESOURCE PACKAGES (Instructions for carrying out the Activities, such as procedures, references, laboratory materials, etc.)

TERMINAL BEHAVIORAL OBJECTIVE:

Upon completion of this minicourse, you will be able to demonstrate some of the basic technical physics ideas and concepts, mathematics, and nomenclature of ballistics by correctly solving six (6) problems from a group of eight (8) simple problems related to projectile motion, free-fall, and momenta.

ENABLING BEHAVIORAL OBJECTIVE #1	ACTIVITY	RESOURCE PACKAGE
Given sufficient data, solve simple problems involving free-falling objects; also, calculate the range of a bullet fired from a gun held in a horizontal position.	<u>ACTIVITY 1-1</u> Study Resource Package 1-1.	<u>RESOURCE PACKAGE 1-1</u> "Firearms Ballistics"
	<u>ACTIVITY 1-2</u> Study Resource Package 1-2.	<u>RESOURCE PACKAGE 1-2</u> "Free Fall and Acceleration"
	<u>ACTIVITY 1-3</u> Complete the activities in Resource Package 1-3.	<u>RESOURCE PACKAGE 1-3</u> "Investigating Trajectory"
	<u>ACTIVITY 1-4</u> Study Resource Package 1-4.	<u>RESOURCE PACKAGE 1-4</u> "Free Fall Equations"
	<u>ACTIVITY 1-5</u> Do Resource Package 1-5.	<u>RESOURCE PACKAGE 1-5</u> "Problems"
	<u>ACTIVITY 1-6</u> Compare your answers with the answers in Resource Package 1-6. If you missed more than four (4), ask your teacher for assistance.	<u>RESOURCE PACKAGE 1-6</u> "Answers to Problems"

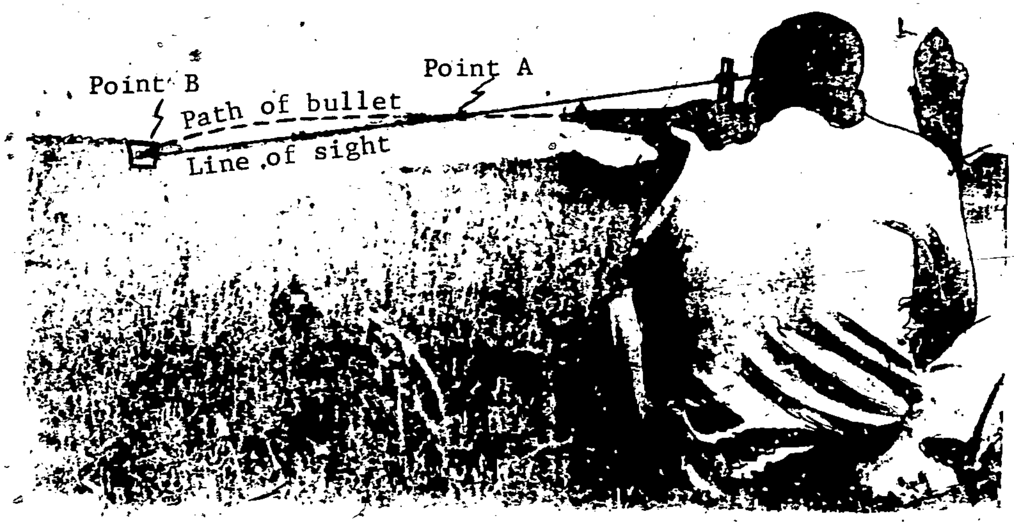
ENABLING BEHAVIORAL OBJECTIVE #2	ACTIVITY 2-1	RESOURCE PACKAGE 2-1
Given sufficient data, solve simple projectile problems involving range, height, and time of flight for a projectile fired at some angle to the horizontal.	Read Resource Package 2-1.	"General Projectile Motion"
	ACTIVITY 2-2	RESOURCE PACKAGE 2-2
	Do Resource Package 2-2.	"Problems"
	ACTIVITY 2-3	RESOURCE PACKAGE 2-3
	Compare your answers with the answers in Resource Package 2-3. If you missed more than four (4) ask your teacher for assistance.	"Answers to Problems"
ENABLING BEHAVIORAL OBJECTIVE #3	ACTIVITY 3-1	RESOURCE PACKAGE 3-1
Given sufficient ballistic data, solve simple problems involving momentum.	Read Resource Package 3-1.	"Rocket Propulsion"
	ACTIVITY 3-2	RESOURCE PACKAGE 3-2
	Do Resource Package 3-2.	"Conservation of momentum Investigation"
	ACTIVITY 3-3	RESOURCE PACKAGE 3-3
	Do Resource Package 3-3.	"Rocket Propulsion Investigation"
	ACTIVITY 3-4	RESOURCE PACKAGE 3-4
	Do Resource Package 3-4. Check your answers using Resource Package 3-5.	"Momentum Problems"
		RESOURCE PACKAGE 3-5
		"Answers to Momentum Problems"

	ACTIVITY 3-6	RESOURCE PACKAGE 3-6
	Do Resource Package 3-6.	"Self-Test"
	ACTIVITY 3-7	RESOURCE PACKAGE 3-7
	Compare your answers with the answers given in Resource Package 3-7. If you missed more than two (2) of the first six (6) ask your teacher for assistance.	"Self-Test Answers"
ENABLING BEHAVIORAL OBJECTIVES #4	ACTIVITY 4-1	RESOURCE PACKAGE 4-1
Be reasonably conversant about blood ballistics and about some relationships between the technical physics of ballistics and possible careers.	Read Resource Package 4-1.	"Blood Ballistics"
	ACTIVITY 4-2	
	Discuss blood ballistics with your instructor. Also discuss relationships between ballistics and occupational careers.	
EVALUATION	ACTIVITY 5-1	RESOURCE PACKAGE 5-1
	If you have no particular questions, ask your teacher for the terminal evaluation. (Final test!).	"Terminal Evaluation"

RESOURCE PACKAGE 1-1

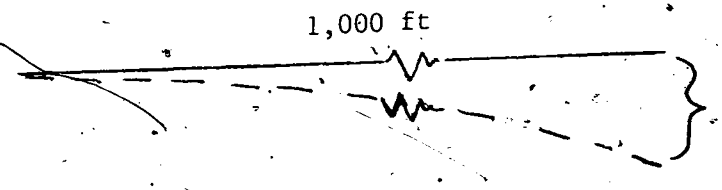
FIREARM BALLISTICS

To aim a rifle at a target, look through the sights and align your eye, the front sight, and the rear sight with the target. The line of sight is a straight line from the eye to the target. But the actual flight path of the (trajectory) bullet is curved all the way from the gun muzzle to the target. This curved flight path intersects the line of sight at two points (See Fig. 1 below). Mathematically speaking, the curved bullet path approximates a parabola. Physically speaking, the actual path would be a true parabola if the firing were done in a vacuum to eliminate frictional effects.



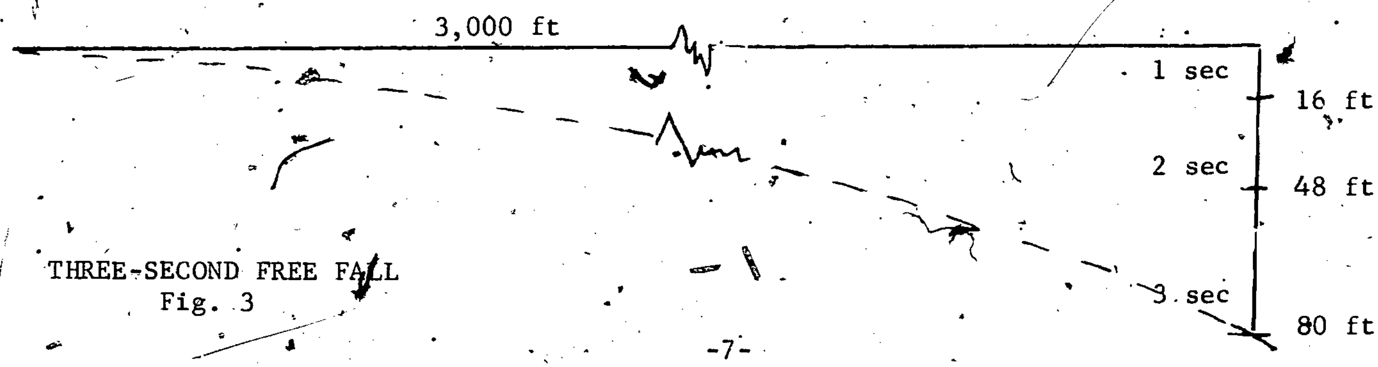
PARABOLIC BULLET PATH
Fig. 1

The bullet crosses the line of sight at point A and falls into the target at point B. It is the gravitational attraction between earth and bullet that pulls the bullet down the instant it is no longer held up by the rifle barrel. If the rifle is fired horizontally, the bullet will fall approximately 16 feet below the barrel during one second of flight. See Fig. 2.



ONE-SECOND FREE FALL
Fig. 2

Assuming the muzzle speed to be 1,000 feet per second (ft/sec) and ignoring air friction, the bullet will travel 1,000 feet horizontally while it drops 16 feet during this second of flight. During the next second, the bullet will drop an additional 48 feet; and during the third second, it will drop 80 feet. See Fig. 3.



THREE-SECOND FREE FALL
Fig. 3

Of course, in most cases, a rifle bullet doesn't remain airborne for 2 or 3 seconds. With a muzzle speed of 1,000 ft/sec, the bullet would travel 2,500 feet (approximately $\frac{1}{2}$ mile) in 2.5 seconds. Most riflemen prefer a target much nearer than $\frac{1}{2}$ mile away!

There are many variables which affect the path of a bullet, causing it to alter from a true parabola. Wind velocity and air friction are two such variables, and these become important when the target is at a distance of 150 feet or more.

If the target is within 50 feet of the muzzle of the gun, the line of sight and the path of the bullet are practically the same. The 1,000 ft/sec bullet will drop only $\frac{1}{2}$ inch during the first 50 feet of flight. If the muzzle speed is increased, the bullet will drop less over a given distance. At a muzzle speed of 3,000 ft/sec the bullet drop would be cut in half (from $\frac{1}{2}$-in to $\frac{1}{4}$-in) and would have a very flat flight path indeed!

In this minicourse we will ignore air friction and wind velocity and will concentrate upon the initial muzzle velocity and upon <u>gravitational force acceleration</u> in calculating various aspects of projectile flight.

In order to better understand projectile motion, you will first investigate and study some related technical physics concepts.

RESOURCE PACKAGE 1-2

FREE FALL AND ACCELERATION

If we drop a rock from a height of 5 feet, it gains speed until it strikes the earth. This is called free fall, because the rock is free of all forces except the earth's gravitational attraction; and the gain in speed is called acceleration because the speed is not constant. If we experimentally measure the change in speed during the time something falls, we would find that the free-fall acceleration value near the surface of the earth is approximately 32 feet per second^2. More commonly this is expressed as 32 ft/sec/sec, or 32 ft/sec^2. In the MKS system of measure, this acceleration value is approximately 9.8 m/sec^2.

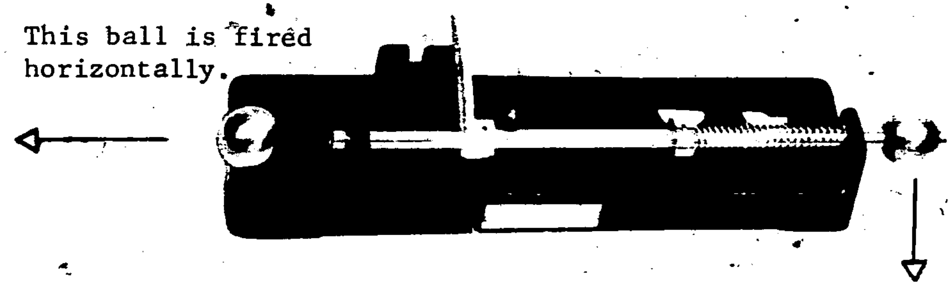
As one leaves the earth's surface, the gravitational force and consequent acceleration decrease. This decrease will be observed if one climbs a high mountain or goes down into a deep mine. But unless the problem requires special treatment, we shall assume that the gravitational acceleration value is 32 ft/sec^2 (9.8 m/sec^2). The symbol g is commonly used for this acceleration.

In the preceding paragraphs, a vacuum condition was assumed. Under actual conditions, objects may be prevented from accelerating ideally. In real life the atmosphere changes the rate of free fall, with lighter objects being generally more affected than heavier objects. In many actual cases, an object may reach a terminal speed (its greatest speed) before it ever strikes the earth. A sheet of paper dropped from a second story window will reach terminal velocity quickly and will then flutter downward toward the earth. A sky diver can jump from a plane, fall faster and faster, and then reach terminal speed, at which speed he can figuratively "float" in the air for several seconds before opening the parachute. While "floating" during free fall, clothing and body offer frictional resistance to the atmospheric molecules; this friction force increases with speed, and at terminal speed is sufficient to balance the earth's gravity force. When this occurs, the terminal speed of a diver is approximately 120 miles per hour. When the parachute is opened, the air resistance is increased tremendously and the terminal speed is further reduced to perhaps 15 mi/hr (a much safer landing speed!).

RESOURCE PACKAGE 1-3

INVESTIGATING TRAJECTORY

1) Ask your teacher for the apparatus shown in Fig. 1, and for an explanation of how the apparatus works.



SECOND LAW OF MOTION APPARATUS

Fig. 1

2) Place the apparatus on the corner of your desk and release the steel balls.

   a) Do the balls strike the floor at the same time? (Listen for two different impact sounds.)

3) Repeat the exercise several times, with the launching spring cocked at the two different tension positions.

   a) Does the ball which follows the curved path strike the floor at the same time as the ball which falls vertically, regardless of the tension on the spring?

4) Draw a diagram which illustrates the motion of the balls.

5) Record your observations on paper, and ask your teacher to examine your diagram and recorded observations.

RESOURCE PACKAGE 1-4

FREE FALL EQUATIONS

Using the results obtained in Resource Package 1-3, and recalling the discussions in Resource Packages 1-1 and 1-2, you can get a little of the "feel" of projectile motion. Now we consider such questions as "How can one find the range or height of a projectile trajectory?" "If a rock is dropped from a known height, how long will it be in free fall?" These questions (and many others) can be answered with the help of the simple mathematical equations listed below*:

Equation 1: $V_f = V_i + gt$

Equation 2: $d = V_i t + \frac{1}{2}gt^2$

Equation 3: $V_f^2 = V_i^2 + 2gd$

where V_f = size of the final velocity (final speed)

V_i = size of the initial velocity (initial speed)

g = size of the gravitational acceleration

t = time lapse (time interval)

d = distance traveled (distance interval)

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* These equations ignore friction and assume that the value of g is constant, safe assumptions for a large variety of technical physics problems. Be careful; V is a symbol for speed in these equations; the velocity symbol ($\vec{v}$) will be discussed later.

Now recall the lab exercise in Resource Package 1-3. Assume a launching platform desk to be 2.5 feet above the floor level and assume we wish to find the speed with which the vertical ball strikes the floor. We can use Equation 3, since we know the fall distance d and the acceleration g:

$$V_f^2 = V_i^2 + 2gd \qquad \text{(line 1)}$$

$$= 0^2 + 2(32\ ft/sec^2)\ 2.5\ ft \qquad \text{(line 2)}$$

$$= 0 + 64\ (2.5)\ ft^2/sec^2 \qquad \text{(line 3)}$$

$$= 160\ ft^2/sec^2 \qquad \text{(line 4)}$$

$$V_i \doteq 12.7\ ft/sec \qquad \text{(line 5)}$$

The symbol, ≐, indicates that the speed with which the ball strikes the floor is approximately equal to 12.7 ft/sec. Note the initial speed, V_i, was zero because the ball started from the rest position.

With the aid of your text and your instructor, study the solution steps of the preceding problem. Concern yourself with such questions as: (1) Where did the equation of line 1 come from? (2) In line 3, where did the dimensional units, ft^2/sec^2 come from? (The answer is really simple, once you see it!) (3) In line 5, how did the units, ft/sec, suddenly appear?

We can now use Equation 1 to determine the time of flight (the time the ball was in free fall). The equation $V_f = V_i + gt$ has four variables (letters representing the physical quantities of the problem). We already calculated V_f (12.7 ft/sec); V_i is zero (because the ball starts from rest); and g is 32 ft/sec^2. Therefore, the time interval, t, is the only unknown. Shown below is a solution of Equation 1 for the variable t:

$$V_f = V_i + gt$$

$$V_i + gt = V_f \quad \text{(By symmetry of equality)}$$

$$gt = V_f - V_i \quad \text{(By subtracting } V_i \text{ from both sides)}$$

Equation 4: $$t = \frac{V_f - V_i}{g} \quad \text{(By dividing both sides by g)}$$

Equation 4 now has the variable t expressed in terms of the known quantities, V_f, V_i, and g. If you need assistance with the algebra, ask your teacher for help. Make sure you can follow the arithmetic steps used to derive Equation 4. Last, we plug the known values into Equation 4:

$$t = \frac{12.7 \text{ ft/sec} - 0 \text{ ft/sec}}{32 \text{ ft/sec}^2} \quad \text{(line 1)}$$

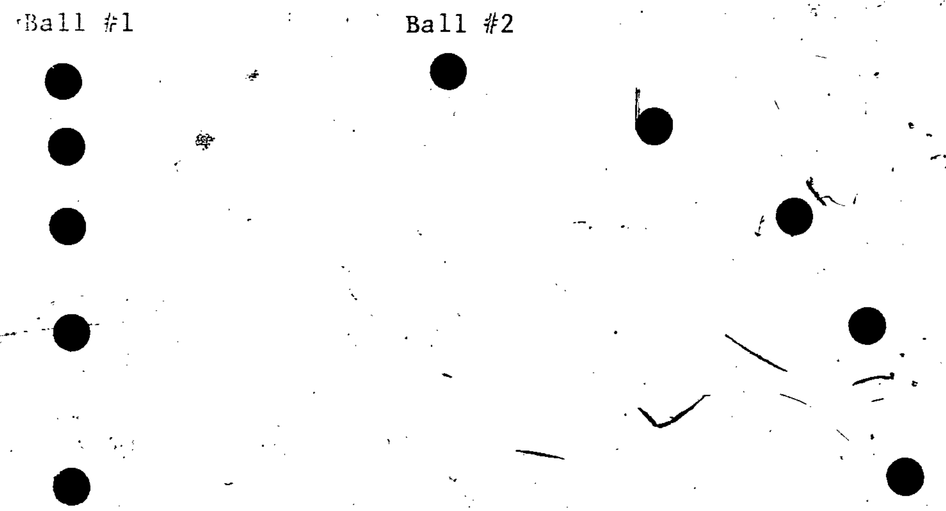
$$= \frac{12.7 \text{ ft/sec}}{32 \text{ ft/sec}} \quad \text{(line 2)}$$

$$\cong .4\left(\frac{\text{ft}}{\text{sec}}\right)\left(\frac{\text{sec}^2}{\text{ft}}\right) \quad \text{(line 3)}$$

$$\cong .4 \text{ sec} \quad \text{(line 4)}$$

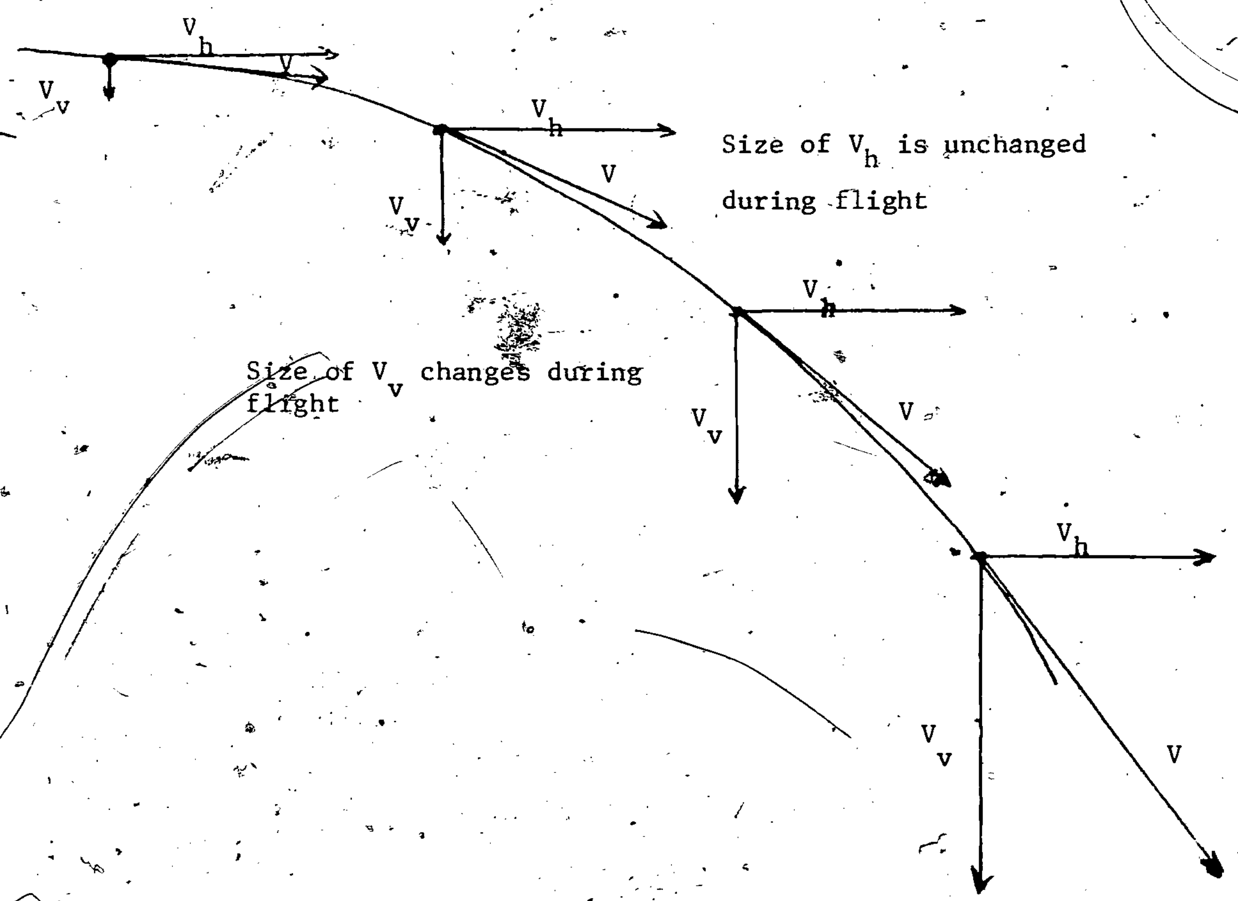
Therefore, it took approximately 4/10 sec for the ball to strike the floor. In the numerical solution for t, above, can you follow the units change between lines 1 and 2? Between lines 2 and 3? Between lines 3 and 4? When you complete any calculation, always check to see that the units on the right side of the equation agree with the physical quantity on the left side. In this calculation we recognize that the dimensional unit, second, is correct for a time lapse, t.

Now, we will turn our attention to the steel ball which follows the curved path. Figure 1, below, is a facsimile of the flight path of the two balls. The positions of each ball are shown for the same time lapse.



POSITIONS OF BALL #1 AND BALL #2
AT SUCCESSIVE TIME INTERVALS
Fig. 1

Can you see that the dropped Ball #1 moves in one direction only (downward), while the launched Ball #2 moves in two directions (downward and sideward)? We say that Ball #2 has both a <u>vertical</u> (downward) and a <u>horizontal</u> (sideward) motion. These motions are obviously <u>simultaneous</u> (take place at the same time) and the real path appears as a <u>parabolic curve</u> motion. In technical physics, such a description of motion involves <u>velocity</u> (both <u>speed</u> and <u>direction</u> are specified). In this case, the vertical velocity motion associated with Ball #2 changes during fall in <u>exactly the same way</u> as does the vertical velocity of Ball #1 (Study Figure 1). The horizontal velocity motion of Ball #2, however, remains <u>fixed</u> throughout the flight. Can you see that the actual path of Ball #2 can be thought of as the combined effects of <u>both</u> its horizontal and vertical velocities? Examine Figure 2.

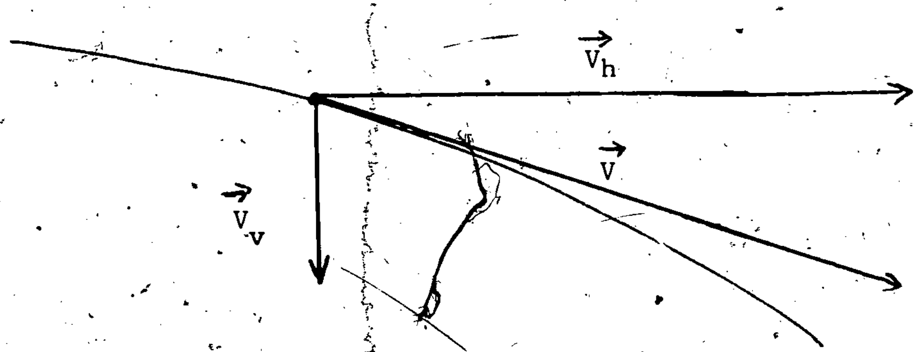


HORIZONTAL AND VERTICAL COMPONENTS OF VELOCITY

Fig. 2

The horizontal speed of Ball #2, V_h, is established when the metal rod strikes Ball #2; and this V_h remains constant during the entire free fall. The vertical speed, V_v, has an initial value of zero the instant it begins free fall; however, it does not remain constant but obviously increases in speed (is accelerated at 32 ft/sec^2) right up to the instant the ball strikes the floor.

The instantaneous velocity V of Ball #2 at any point along its path is determined by its component vectors V_v and V_h. Since V varies, instantaneous V_v will also vary (since V_h is constant). Study Fig. 3 below. This is a velocity component diagram.

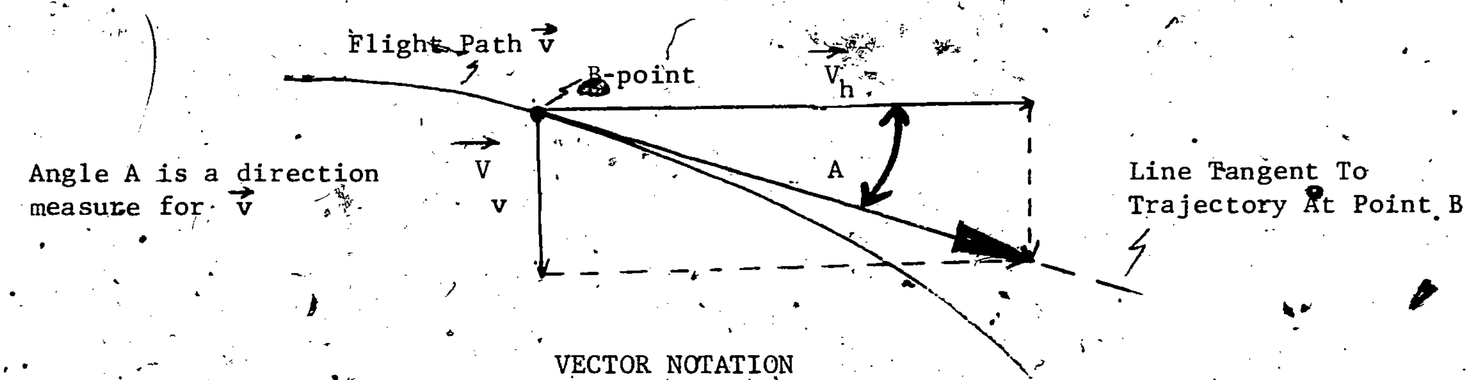


VELOCITY COMPONENTS
Fig. 3

Notice that V_h, V_v, and V all have lengths (sizes) and all point in specific directions. A physical quantity which has both size (magnitude) and direction (such as this velocity quantity) is called a vector quantity. By contrast, any physical quantity that has only magnitude or amount (without direction) is called a scalar quantity. Volume is an example of a scalar quantity. Five gallons of water doesn't have a direction, such as north or east associated with it. Notice that in Figs. 2 and 3 the velocity direction was shown by arrowheads. This is an accepted practice in technical physics, as is the practice of placing a small arrow above any symbolized quantity to indicate it as a vector quantity (Examples: Force $\vec{F}$, velocity $\vec{v}$, acceleration $\vec{a}$; weight $\vec{w}$; etc.). The direction for $\vec{V}_v$ is straight down (toward the center of mass of the earth); while $\vec{V}_h$ has a direction parallel to the floor. Velocity $\vec{V}$ has a direction that constantly varies, but is always tangent to the curved flight path at any instant of flight.

In many textbook discussions, the terms velocity and speed are used interchangeably. Be careful! Such loose usage can confuse you. Velocity is a vector quantity; it has both a magnitude and a direction. Speed is simply the magnitude (size) of the velocity vector. If we are traveling 50 mi/hr in a car, we are speaking of the scalar quantity speed. Suppose we leave home and travel in a straight line for one hour at the rate of 60 mi/hr. The distance (a scaler quantity) traveled will be 60 miles; but without direction we could be located at any point on a circle of radius 60 miles and with its center at our home. However, if we travel 60 mi/hr due north for one hour, we are now able to locate our position precisely. Our velocity was 60 mi/hr, north. The distance was 60 miles and the displacement was 60 mi north. Displacement, too, is a vector quantity; it tells how far and in what direction! Make very sure you understand the difference between distance and displacement, and between speed and velocity.

Now that we have some knowledge of vector quantities and their notation, can you see that a more technical representation of the Ball #2 flight path might look like Fig. 4?



VECTOR NOTATION
Fig. 4

Let's assume that Ball #2 had an initial horizontal speed (V_h) of 20 ft/sec. Let's calculate the vertical speed (V_v) after ¼-second of free fall. We can use Equation 1, where $V_f = V_v$:

$$V_v = V_i + gt$$

$$= 0 \text{ ft/sec} + (32 \text{ ft/sec}^2)(\tfrac{1}{4} \text{ sec})$$

$$= 0 \text{ ft/sec} + 8 \text{ ft/sec}$$

$$= 8 \text{ ft/sec.}$$

The downward speed of Ball #2 is 8 ft/sec and the horizontal speed is 20 ft/sec. The vectors $\vec{V}_h$ and $\vec{V}_v$ can be represented by the two sides of a rectangle whose sides form right angles at the ball. See Fig. 4. We can complete the rectangle by drawing the dotted line segments representing $\vec{V}_v$ and $\vec{V}_h$ as shown in Fig. 4. The velocity vector $\vec{V}$ is now represented by the diagonal of the rectangle. Its length, which represents the magnitude of $\vec{V}$, can be calculated using the Pythagorean Theorem:

<u>Equation</u> 5:

$$
\begin{aligned}
V_f^2 &= V_h^2 + V_v^2 \\
&= (20\ ft/sec)^2 + (8\ ft/sec)^2 \\
&= 400\ ft^2/sec^2 + 64\ ft^2/sec^2 \\
&= 464\ ft^2/sec^2 \\
V_f &= (464\ ft^2/sec^2)^{\frac{1}{2}} \\
&= 22\ ft/sec
\end{aligned}
$$

The speed of the ball along its path is approximately 22 ft/sec after ½ second of flight.

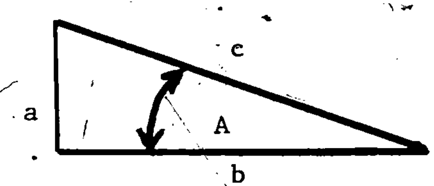
From previous calculations in this Resource Package, Ball #1 was found to be in free fall for 4/10 seconds (assuming the launch desk was 2.5 feet high). The speed of Ball #1 was found to be 12.7 ft/sec, which is also the vertical speed of Ball #2 the instant it makes contact with the floor. Using this known vertical speed of 12.7 ft/sec and the known constant horizontal speed of 20 ft/sec, The Pythagorean

Theorem (Equation 5) will yield the speed (directed along the actual flight path) with which Ball #2 strikes the floor.

$$V_f^2 = V_h^2 + V_v^2$$
$$= (20 \text{ ft/sec})^2 + (12.7 \text{ ft/sec})^2$$
$$= 400 \text{ ft/sec}^2 + 161 \text{ ft}^2/\text{sec}^2$$
$$= 561 \text{ ft}^2/\text{sec}^2$$
$$V_f = 24 \text{ ft/sec.}$$

The quantity found above is a scalar quantity, since no direction is given.

The direction of the ball is shown by the angle A, as seen in Fig. 4. The calculation of angle A involves some simple trigonometry. In Fig. 5 we have a right triangle with sides a, b, c, and angle A.



RIGHT TRIANGLE
Fig. 5

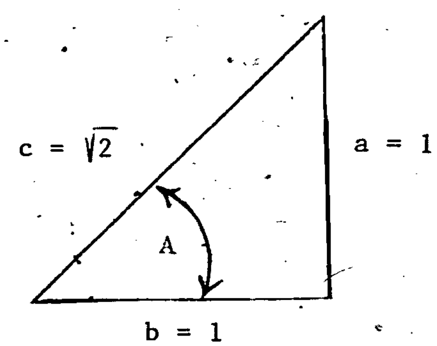
The ratio of the length of side a to the length of side c is called the sine of A, and is written:

$$\sin A = \frac{a}{c}$$

The ratio of the respective lengths of sides b and c is called the cosine of A, and is written:

$$\cos A = \frac{b}{c}$$

Angle A is expressed in degrees. For a given value of A, the sin and cos always have the same values regardless of the size of the triangle. Consider a triangle with sides a and b equal to 1 (a = b = 1). From the Pythagorean Theorem, side c = $\sqrt{2}$ ($c^2 = a^2 + b^2$; $c^2 = 1^2 + 1^2$; $c^2 = 1 + 1$; $c^2 = 2$; $c = \sqrt{2}$; and $\sqrt{2} \cong 1.414$). Study Fig. 6.



45-45-90 TRIANGLE
Fig. 6

$$\sin A = \frac{1}{\sqrt{2}}$$

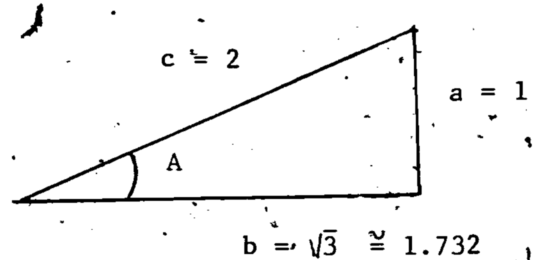
$$= \frac{1}{1.414}$$

$$\cong .707$$

In this triangle, A is 45°; therefore:

$$\sin 45^{\circ} \cong .707$$

and this value is completely independent of the actual length of the sides of the triangle, as long as the ratio of these lengths does not change. Next, consider a triangle with a = 1 and c = 2. Side b is calculated from The Pythagorean Theorem to be $\sqrt{3}$ or approximately 1.732. (See Fig. 7)



30-60-90 TRIANGLE
Fig. 7

In this case, sin A is ½ or 0.5. It can be shown that A in this triangle is 30°. Therefore, $\sin 30^{\circ} = 0.5$; cos A is

$$\cos 30^{\circ} = \frac{\sqrt{3}}{2} = \frac{1.732}{2} \cong .866$$

The values for the sin and cos of an angle from 0 to 90 degrees have been calculated, tabulated, and included in mathematics, engineering, and physical science books. Examine such a table. If we know an angle, we can look up its sin or cos value in such a table. For example, sin 21° is .3584, and cos 39° is .7771. Conversely, by knowing an angle, we are able to look up its sine or cosine value.

Let's see how such tables can enable us to determine the <u>impact</u> <u>angle</u> of Ball #2. In Fig. 4, the vertical component of the velocity $\vec{V}_v$ has a magnitude of 12.7 ft/sec; and the velocity $\vec{V}$, has a magnitude of 24 ft/sec. Therefore:

$$\text{Sin A} = V_v/V_h$$

$$= \frac{12.7 \text{ ft/sec}}{24 \text{ ft/sec}}$$

$$= .54$$

From the trig table we can determine that angle A has a value of approximately 32°. Therefore, we know the impact speed of $\vec{V}$ to be 23.7 ft/sec and the direction to be 32° below the horizontal. $\vec{V}$ is a vector quantity and we have specified both its size and direction.

Example #1:

A man holds a rifle horizontally 5 feet above the earth and fires it. If the muzzle speed is 3,000 ft/sec, and if air friction is neglected, how far will the bullet go before it strikes the earth?

We calculate vertical speed from Equation 3, since the bullet will have the same vertical speed as if it were dropped from a height of 5 feet:

$$V_f = \sqrt{V_i^2 + 2\ gd}$$

$$= \sqrt{0^2 = 2(32\ ft/sec^2)\ \ (5\ ft)}$$

$$= (320\ ft^2/sec^2)^{\frac{1}{2}}$$

$$\cong 17.9\ ft/sec$$

Since 17.9 ft/sec is the <u>vertical speed</u> V_v we can substitute this value into <u>Equation 1</u> for V_f to find the time of flight, t:

$$V_f = V_i + gt = V_v$$

or

$$V_v = V_i + gt$$

Our equation for this portion of the problem then becomes:

$$V_v = 0 + gt$$

Because the <u>initial</u> vertical speed is zero. Then,

$$V_v = gt \quad \text{(Divide both sides by g)}$$

$$t = \frac{V_v}{g}$$

$$= \frac{17.9 \text{ ft/sec}}{32 \text{ ft/sec}^2}$$

$$\cong .56 \text{ sec.}$$

We may now use Equation 2 to calculate the horizontal distance d, which is called range.

$$d = V_i t + \tfrac{1}{2} gt^2$$

Horizontally, g is always zero (since gravity cannot pull anything sideways) so Equation 2 becomes simply:

$$d = V_i t$$

$$= 3000 \text{ ft/sec} \cdot (.56 \text{ sec})$$

$$\cong 1677 \text{ ft.}$$

The range is approximately 1677 ft if the rifle is fired horizontally and if the ground is level.

PROBLEMS

Solve the following problems on a separate sheet of paper. Show all diagrams and calculations.

1) A cannon with a muzzle speed of 1000 ft/sec is fired horizontally from a hill 50 feet above a plain. How far from the foot of the hill does the cannon ball strike the earth?

2) Find the sin and the cos of the following angles in a table: (a) 40°, (b) 22°, (c) 50°, (d) 75°.

3) Let angle $A = \theta$, (This is a Greek letter which is commonly used to represent an angle; it is written <u>theta</u>.) Find angle θ if (a) $\sin\theta = .1219$, (b) $\sin\theta = .5446$, (c) $\sin\theta = .8090$, (d) $\cos\theta = .8910$, (e) $\cos\theta = .6947$, (f) $\cos\theta = .1908$.

4) An airplane is flying at an altitude of 2000 ft. If its speed relative to the ground is 200 mi/hr, how many feet ahead of the target must a bomb be released if it is to strike the target? How long will the bomb be in free fall? What is the speed of the bomb along its path the instant it strikes the target (its impact speed)?

ANSWERS TO PROBLEMS

1) 1768 ft

2) (a) $\sin 40^\circ = 0.6428$; $\cos = 0.7660$.

   (b) $\sin 22^\circ = 0.3746$; $\cos = 0.9272$.

   (c) $\sin 50^\circ = 0.7660$; $\cos = 0.6428$.

   (d) $\sin 75^\circ = 0.9659$; $\cos = 0.2588$.

3) (a) 7°

   (b) 33°

   (c) 54°

   (d) 27°

   (e) 44°

   (f) 79°

4) 3,285 feet ahead of target.

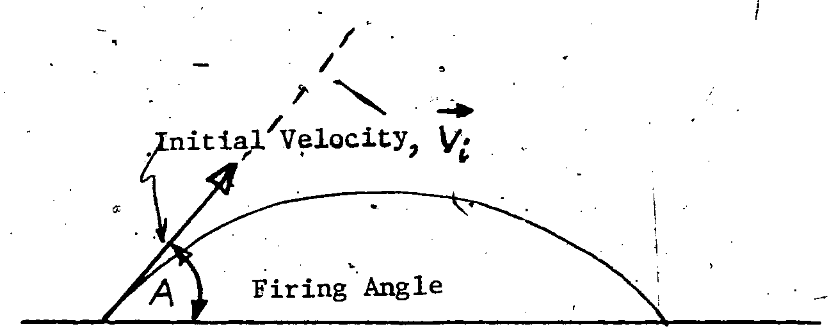
   11.2 sec in free fall.

   463 ft/sec is the impact speed.

RESOURCE PACKAGE 2-1

GENERAL PROJECTILE MOTION

We have calculated the time of flight and the range of a projectile fired horizontally. In other words, we knew in advance its maximum height (Make sure you understand this first sentence!) But what happens if we fire a rifle at some angle other than the horizontal? What will its path look like? How will its speed vary? We'll begin with a simple case where a rifle is fired at an angle of 45° above the horizontal. The projectile has an initial velocity above the horizontal and its general path is indicated in Fig. 1 below.

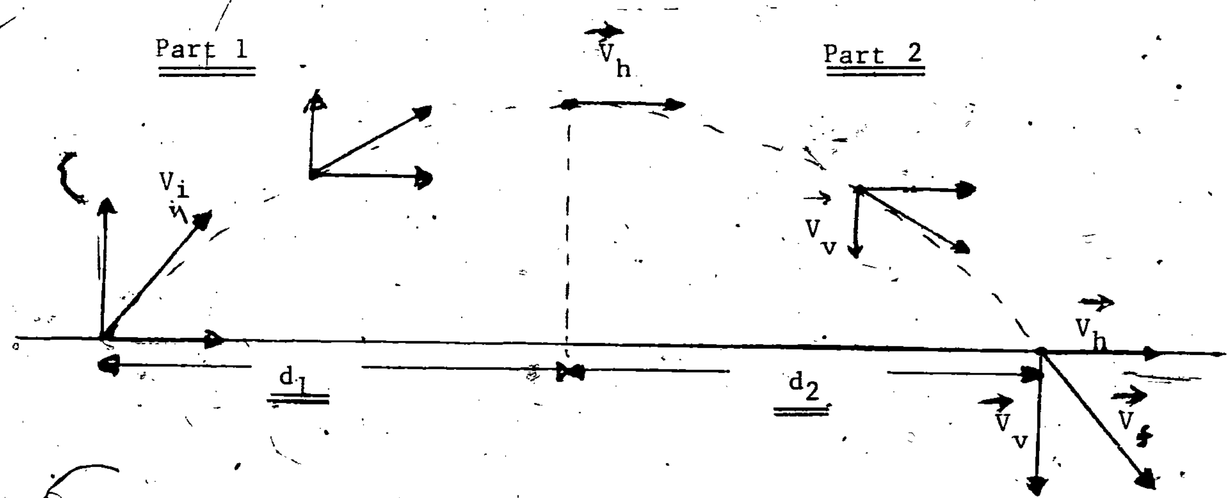


PROJECTILE FIRED AT UPWARD ANGLE

Fig. 1

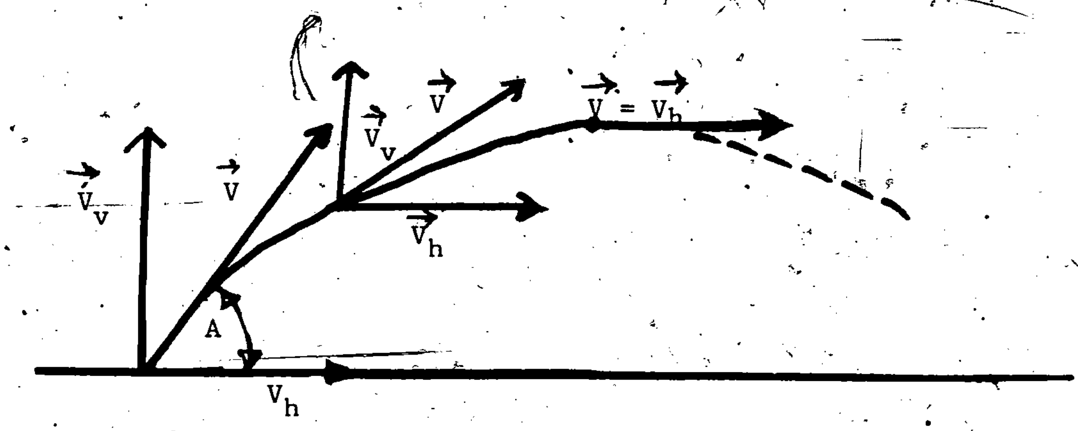
The initial velocity direction will be determined by the gun barrel. The instant the bullet leaves the barrel it is subject to free fall and begins to fall below the initial velocity vector line of

Fig. 1. However, the bullet continues to gain altitude until it reaches a maximum height; then it begins to fall back toward the earth. If air friction is ignored, the first half of the path (to the peak) is a mirror image of the second half (to the ground). In other words, the entire flight path has a parabolic symmetry. (See Fig. 2).



PROJECTILE PATH
Fig. 2

After the bullet reaches its peak, we have the identical problem encountered (Resource Package 1-4) for horizontal firing. The initial (muzzle) velocity discussed in Resource Package 1-5 is identical to $\vec{V}_h$ shown at the peak in Fig. 2. The horizontal velocity, $\vec{V}_h$, remains constant; the vertical velocity, $\vec{V}_v$, increases in size as the bullet falls to the earth. The bullet velocity, $\vec{V}$, is calculated as was done previously. The time of flight for the first part of the path is equal to the time of flight for the second part. The range, d_1, for Part 1 of the path is equal to the range, d_2, for Part 2 of the path. When a gun is fired at an angle, bullet velocity, V, continually changes magnitude and direction. Of course, the vertical velocity component decreases in size from maximum at firing to zero at the peak, then reverses direction and increases in size to its original maximum; the horizontal velocity component remains constant over its entire flight. (See Figure 3)

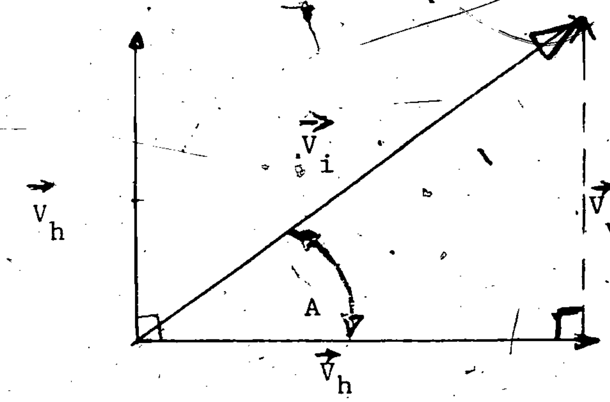


PROJECTILE VELOCITY COMPONENTS

Fig. 3

When a projectile problem involves firing at an angle, the two variables usually given are the initial speed, V, and the angle of elevation, A. (This is because one usually knows the initial velocity $\vec{V}$.) Let's now calculate $\vec{V}_v$ and $\vec{V}_h$.

Consider Fig. 4



RESOLUTION OF MUZZLE VELOCITY

Fig. 4

The initial velocity vector, $\vec{V}_i$, may be broken up into the two parts we have called components, and are designated $\vec{V}_v$ and $\vec{V}_h$ in Fig. 4. Inspection of Fig. 4 will reveal that these vectors are similar to the legs of a right triangle, with $\vec{V}_i$ as the hypotenuse. Knowing ~~the hypotenuse~~ and Angle A, we can use the sin and cos ratios to find the magnitude of V_h and V_v.

$$\sin A = \frac{V_v}{V}$$

$$\cos A = \frac{V_h}{V}$$

In figure 4, assume A = 30° and the magnitude of $\vec{V}$ to be 1000 ft/sec. Substituting these quantities in the sin ratio, we have:

$$\sin 30^\circ = \frac{V_v}{1000 \text{ ft/sec}}$$

Using a trig table, we find sin 30° to be .500. Therefore, we have:

$$.500 = \frac{V_v}{1000 \text{ ft/sec}}$$

Solving for V_v,

$$V_v = 1000 \text{ ft/sec } (.500)$$

$$= 500 \text{ ft/sec (magnitude of } V_v)$$

From the cos ratio we have

$$\cos 30^\circ = \frac{V_h}{1000 \text{ ft/sec}}$$

The trig table shows that cos $30^{o} = .866$.

$$.866 = \frac{V_h}{1000 \text{ ft/sec}}$$

$$V_h = .866\ (1000) \text{ ft/sec}$$

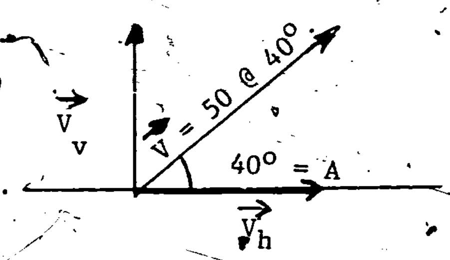
$$V_h = 866 \text{ ft/sec (magnitude of } V_h).$$

We made the calculations above using these rules:

Rule 1) To find the size of the horizontal component, multiply the magnitude of the vector by the cos of the elevation Angle A.

Rule 2) To find the size of the vertical component, multiply the magnitude of the vector by the sin of the elevation angle A.

Example 1: Assume that a vector has a magnitude of 50 and an angle A of 40^{o}, as shown in the diagram. Find the magnitude of $\vec{V_h}$ and $\vec{V_v}$. Solution:



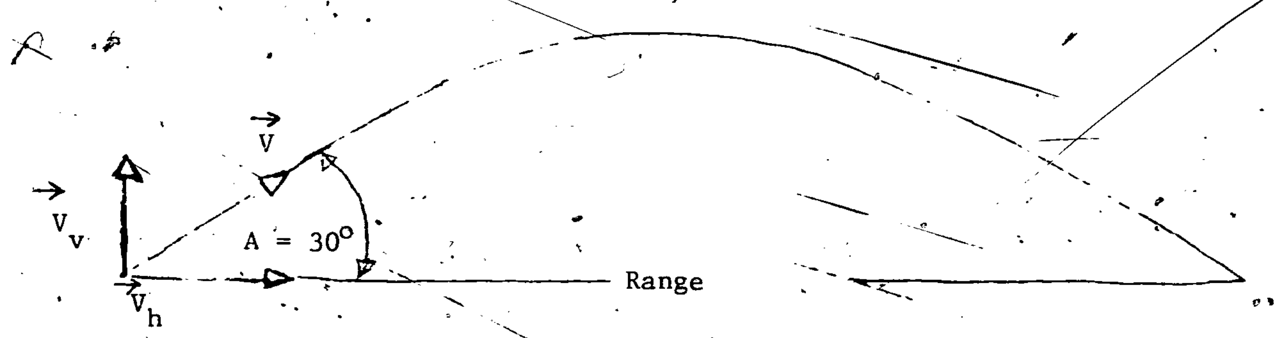
To find V_h, use Rule 1:

$$V_h = \cos 40^\circ(50)$$
$$= .766(50)$$
$$= 38.3, \text{ size of } V_h$$

To find V_v, use Rule 2:

$$V_v = \sin 40(50)$$
$$= .6428(50)$$
$$= 3[illegible].14, \text{ size of } V_v$$

Example 2: Consider a rifle fired at an angle of 30° and with a muzzle velocity of 1500 ft/sec. Assuming no air friction, what is the range, time of flight, and maximum altitude of the bullet?



First we need to calculate the sizes of V_h and V_v:

$$V_h = \cos 30^\circ\ (1500\ \text{ft/sec})$$
$$= .866(1500\ \text{ft/sec})$$
$$= 1299\ \text{ft/sec}$$

$$V_v = \sin 30^\circ (1500\ \text{ft/sec})$$
$$= .500(1500\ \text{ft/sec})$$
$$= 750\ \text{ft/sec.}$$

The horizontal component, $\vec{V}_h$, remains constant at 1299 ft/sec. The initial vertical component, $\vec{V}_v$, of 70 ft/sec decreases to zero as the bullet reaches its peak. At the peak, the velocity along the curve, $\vec{V}$, is <u>equal</u> to the horizontal component, $\vec{V}_h$. As the bullet goes through the peak and starts on its downward path, its vertical speed increases until it strikes the earth; at which time, the final downward (vertical) speed of the bullet is equal to the initial upward (vertical) speed the bullet had when it left the muzzle of the gun.

We found the initial upward <u>speed</u> (V_v) of the bullet in this problem to be 750 ft/sec. The final downward speed (V_v) must also be 750 ft/sec, which is the same speed the bullet would have if it were simply dropped from the peak and fell straight to the earth. The time of flight of this dropped bullet, and that of the bullet along the descending part of the flight path (part 2, Fig. 2) will also be the same. <u>Equation 1</u> can be used to calculate the time of flight for the bullet which is dropped straight down from the peak; this will be one-half of the projectile flight time, where the bullet first "falls upward" to the peak and then falls downward to ground:

$$V_i + gt = V_f$$

$$gt = V_f - V_i$$

$$t = \frac{V_f - V_i}{g}$$

Putting in values,

$$t = \frac{750 \text{ ft/sec} - 0 \text{ ft/sec}}{32 \text{ ft/sec}^2}$$

$$t = 23.44 \ \frac{(\text{ft})}{(\text{sec})} \ \frac{\text{sec}^2}{\text{ft}}$$

$$t = 23 \text{ sec.}$$

The time of 23 sec is only the time required for the bullet to fall from its peak to the earth. Therefore, the total time of flight is twice as much, or 46 sec.

The maximum altitude of the bullet will be equal to the distance a bullet can start from rest and free fall for 23 seconds. Therefore, we use Equation 2 to find the distance:

$$d = V_i t + \tfrac{1}{2} g t^2$$

$$= 0(23 \text{ sec}) + \tfrac{1}{2}(32 \text{ ft/sec}^2)\ (23^2 \text{sec}^2)$$

$$= 0 + (16 \text{ ft/sec}^2)\ (529 \text{ sec}^2)$$

$$= 8464 \text{ ft}$$

Therefore, the maximum altitude of the bullet is 8464 feet, or about 1.6 miles. The <u>range</u> of the bullet can also be determined from <u>Equation 2</u>.

$$d = V_i t + \frac{1}{2} g t^2$$

For this problem V_i = 1,299 ft/sec, t = 46 sec, and g = 0 (since gravity cannot act sideways):

$$d = 1{,}299 \text{ ft/sec } (46 \text{ sec}) + \frac{1}{2}(0)\ (46 \text{ sec})^2$$

$$d \cong 62{,}400 \text{ feet}$$

The range is 62,400 feet, or about 11.8 miles.. This range is <u>not</u> realistic for a rifle, because in a real situation the bullet is subject to air friction and wind drift. But if the rifle were fired in a vacuum, the bullet would indeed have a range of over 60,000 feet!

Even with air friction, a projectile fired from a military howitzer has a long range. An 8-inch artillery weapon is accurate at a range of about 15 miles. A battleship's 16-inch guns could hit a target about 18 miles away.

RESOURCE PACKAGE 2-2

PROBLEMS

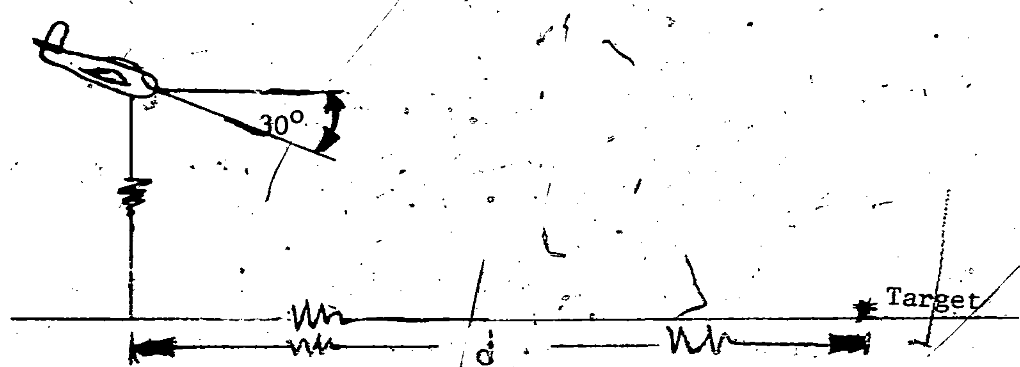
Solve the following problems. Show all work and diagrams. Neglect air friction.

1) Define: vector quantity; scalar quantity.

2) If a baseball is thrown with a speed of 80 ft/sec at an angle of 60^{o}, what is its maximum altitude and range?

3) A cannon fires a cannonball at an angle of 30^{o} with a speed of 640 ft/sec and strikes a target on a hill 1,200 feet above the plane of the cannon. How long does the cannonball stay in flight? With what final velocity magnitude (velocity along the path) does the cannonball strike the target?

4) A plane flying in level flight goes into a dive 30^{o} below the horizontal. At an altitude of 2,000 feet and a speed of 240 mi/hr, the plane drops a bomb. At what horizontal distance away from the target must the bomb be released if it is to strike the target? What is the vertical speed of the bomb on impact?



5) A 22-calibre rifle has a muzzle speed of approximately 1000 ft/sec. Assume the rifle to be fired from an angle of 30^o, then 45^o, then 60^o. Calculate the range, time of flight, and maximum altitude for the projectile in each case. From your calculations can you determine at what angle a gun or rifle should be fired for maximum range?

RESOURCE PACKAGE 2-3

ANSWERS TO PROBLEMS

(1) Vector quantity - physical quantity having both magnitude and direction.

Scalar quantity - physical quantity having magnitude only.

(2) maximum altitude = 75 ft.

range = 173 ft.

(3) time = 15 sec.

V = 576.8 ft/sec.

(4) d = 1,939 ft.

V = 379.6 ft/sec.

(5) (a) range = 27,062 ft.

time = 31.2 sec.

altitude = 3,900

(b) range = 31,240 ft.

time = 44 sec.

altitude = 7,744 ft.

(c) range = 27,000 ft.

time = 54 sec.

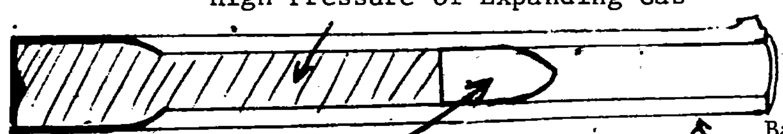
altitude = 11,662 ft.

RESOURCE PACKAGE 3-1

ROCKET PROPULSION

In previous Resource Packages we considered the ballistics of projectiles whose horizontal speeds were constant after firing. Rocket-propelled missiles differ from freely-falling projectiles because rockets have the capacity to change speed (and sometimes direction) after launch. In this Resource Package we will consider some aspects of rocket ballistics.

For a bullet to reach muzzle velocity it starts from rest the instant the cartridge is fired, and it gains speed until it reaches the end of the gun barrel. This increase in speed is called acceleration. But what causes the acceleration? When the cartridge is fired, the gun powder in the casing oxidizes to produce a gas at a very high pressure. The gas pressure energy does work on the bullet and forces the bullet out along the gun barrel. See Figure 1 below.



RIFLE SYSTEM
Fig. 1

This high pressure gas can be said to do all of the following inter-related activities: exert a force on the bullet, accelerate the bullet, do work on the bullet, change the energy of the bullet, and cause a change in the momentum of the bullet.

The high pressure gas force acts upon the bullet until emerges from the muzzle of the gun. Because this pressure is exerted equally in all directions within the enclosed combustion chamber, pressure is exerted on the rear of the firing chamber as well as on the back side of the bullet. The force resulting from the gas pressure against the rear of the firing chamber is transmitted through the stock of the gun to the shoulder. We call this the kick when we fire a gun.

This kick phenomenon (event) is an example of an important physical quantity known as linear momentum. The linear momentum of an object can be calculated as the product of its velocity and its inertial mass. The notation for linear momentum is $\vec{P}$; mathematically, $\vec{P} = m\vec{v}$. Momentum is a vector quantity with the size mv, and with the direction of $\vec{v}$. The term inertial mass (or mass) is simply a term for the universal property of all objects to resist change in motional condition or rest condition.

This leads us to a fundamental conservation law of physics, Conservation of Momentum. This Law tells us that if we isolate things and watch their behavior when they bang together, fly apart, or otherwise interact forcewise, then we can rest assured that the initial momentum of the system of these isolated objects before they interact will be precisely equal to the momentum of the system after interaction.

Mathematically, we write

$$\vec{P}_{before} = \vec{P}_{after}$$

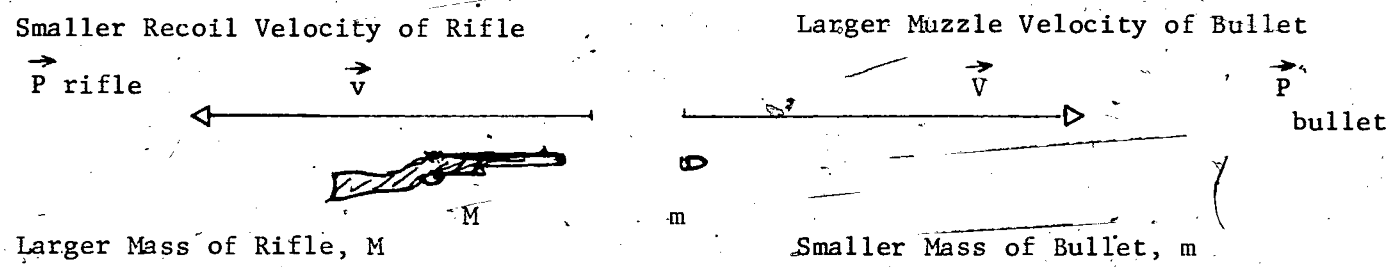
$$\vec{mv}_{before} = \vec{mv}_{after}$$

Where $\vec{mv}$ is the <u>vector sum</u> of all the individual linear momenta of each object in the system.

Realize that this discussion has dealt only with linear momentum. There exists, also, an equivalent <u>angular momentum</u>. It, too, is conserved, and is treated as a vector quantity. Mathematically, angular momentum $\vec{L}$, is the product of angular velocity ($\vec{\omega}$) and <u>inertial moment</u> (I), where the latter is a measure of the resistance all bodies have to a change in their spin condition. We write $\vec{L} = I\vec{\omega}$.

For a more comprehensive treatment of linear and angular momenta, look at the minicourses <u>Physics of Sports</u> and <u>Physics of Toys</u>.

Let us apply the Conservation of Linear Momentum Law to the firing of our rifle. Isolating the system in terms of gun and bullet, the <u>linear momentum before</u> firing is zero (Why? Because both the gun and bullet are at rest relative to each other.) Conservation of Momentum Law implies that the momentum after firing must also be zero. Because linear momentum is a vector quantity, two such vectors can add to zero if and only if they act along the same line, are equal in size, and have opposite directions. In other words the $\vec{mv}$ of the bullet must be equal and opposite to the $\vec{mv}$ of the rifle at all times. See Fig. 2.



$$Mv \text{ (Rifle)} = mV \text{ (Bullet)}$$

Consider this example:

A bullet of 10-gram mass property has a muzzle velocity of 300 meters per second when fired from a rifle which has a mass property of 2 kilograms. What is the <u>magnitude</u> of the linear momentum of the bullet? What is the recoil velocity of the gun?*

---

* If the dimensional units meter, kilogram, etc, bother you, perhaps you should examine the section on the metric system in the minicourse, <u>Metric System and Slide Rule</u>?

Solution: First, convert all measurements to the same system of measure. We will use the MKS system, since it is preferred by scientists and since the United States is "going metric" to keep up with the rest of the world.

$$10 \text{ gm} = .01 \text{ kgm}$$

Second, the size of the linear momentum $\vec{P}$ (of the bullet) $= mV$

$$= (.01\text{kg}) \quad (300 \text{ mi/sec})$$

$$= 3.00 \frac{\text{kg-m}}{\text{sec}}$$ (A scalar since only magnitude asked for.)

Third, the linear momentum $\vec{P}$ (of the gun) has a size $= Mv$

$$= 2 \text{ kg } (v)$$

Momentum conservation assures us that the size of these two momenta is the same and that their directions are opposite; therefore, the size of the momentum of the bullet = the size of the momentum of the gun or,

$$mV = Mv$$

$$2 \text{ kg } (v) = \frac{3\text{kg - m}}{\text{sec}}$$

$$= \frac{3}{2} \frac{\text{kg - m}}{\text{kg - sec}}$$

= 1.5 m/sec, in a direction opposite to the bullet.

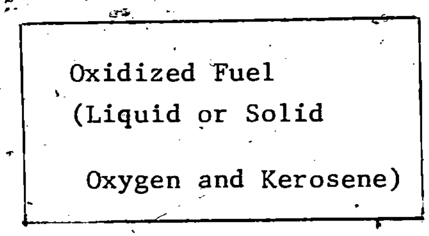
Therefore, the maximum speed of the gun is 1.5 m/sec, or about 5 ft/sec.

If the gun were floating somewhere in outer space when fired, with little friction and little gravitational force, the gun would actually reach this recoil speed! But in real life, we hold a rifle to our shoulder. Under these conditions, the mass property of our body is added to the mass property of the gun and the recoil momentum is thus imparted to both. Because the gun-plus-shoulder constitutes a relatively large mass, the kick <u>speed</u> is reduced even though the recoil momentum <u>remains the same</u>. The kick from a small calibre gun is not relatively great because the mass property of the bullet is very small when compared to the combined mass of the gun and the person firing it. Conversely, the kick from a large-bore gun is relatively great because of the larger mass of the bullet (which usually has a higher muzzle velocity, too!). The firing of large-bore naval guns can cause an entire ship to recoil.

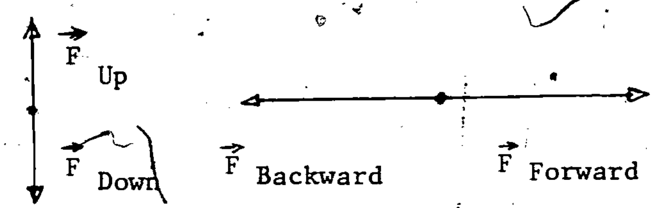
This momentum problem was restricted to linear analysis. But real rifles derive their name from the fact that when the rifle barrel is <u>bored</u>, screw-like ridges called <u>riflings</u> are left as an inner lining of the barrel. These riflings cause a bullet to spin as it moves linearly down the gun barrel toward the muzzle. When a bullet leaves a rifled barrel, it really has <u>two</u> velocities; a linear velocity, $\vec{V}$, and an angular velocity, $\vec{\omega}$. Obviously, it must then have <u>two</u> momenta: the linear momentum $\vec{P} = m\vec{v}$, and the angular momentum $\vec{L} = I\vec{\omega}$. It is the spin momentum which <u>stabilizes</u> the bullet in flight (See the minicourse, <u>Physics of Sports</u> or <u>Physics of Toys</u> for more about spin momentum). Further, to conserve spin momentum the rifle must rotate opposite to the rotation of the bullet!

<u>A Thought to Ponder</u>: Probably you have observed the firing of machine guns and heavy artillery in war films. You likely noticed that the machine gun recoil caused the gun to bounce around as it spewed forth its bullets at a rate of 600 rounds per minute, or more. On the other hand, the artillery firing of large projectiles resulted in a much slower recoil over a much larger distance (the barrel may have moved back several feet!). Can you accound for these two different kinds of recoil activity?

We will now apply the momentum concept to rocket propulsion. The rocket engine contains a fluid consisting of hot gas molecules. These high-speed gas molecules exert an explosive pressure (force) <u>equally</u> on the walls of the engine's <u>combustion chamber</u>. But <u>because there is a hole</u> (exhaust nozzle) <u>in the rear engine wall</u>, the gas force on the forward engine wall is greater than on the rear wall. The rocket must respond to the unbalanced forward force by moving off in the forward direction. The net forward force is called <u>thrust</u>, and the thrust accelerates the rocket quite like exploding gunpowder accelerates a rifle bullet. By symmetry, it is obvious that the gas force on the upper wall is exactly balanced by the force on the lower wall. Study the diagrams below:

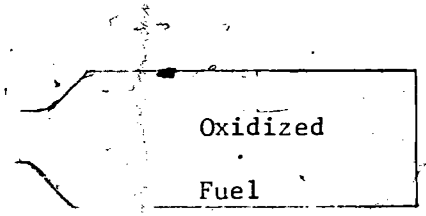


CLOSED ENGINE CHAMBER

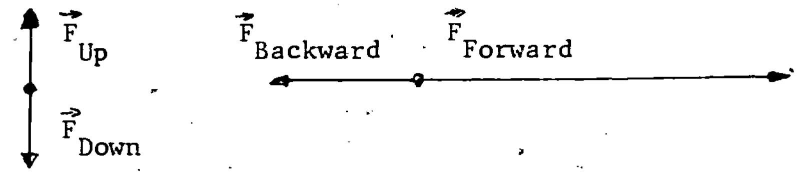


BALANCED FORCES WITHIN CHAMBER

The forces up, down, forward and backward cancel one another. No thrust results.



OPENED ENGINE CHAMBER



UNBALANCED FORCES WITHIN CHAMBER

The forces up cancel the forces down, but the force on the forward wall of the engine is greater than the force on the backward wall of the engine. Thrust results! From a momentum conservation point of view, the hot gases expelled from the exhaust orifice (port) acquire linear momentum in the backward direction; therefore, the rocket must acquire an equal linear momentum in the forward direction.

The mass property of the entire rocket at any given instant is a variable, because the mass of the fuel is decreased as it is oxidized and ejected from the rocket. But the rocket thrust is a constant when its engine oxidizes a fixed amount of fuel in equal intervals of time. Therefore, as the mass of the rocket fuel decreases, the payload and airframe of the rocket gain more and more momentum until burn out occurs. This momentum increase can result in a terminal speed of hundreds or thousands of miles per hour!

Now let us look at some technical physics of a near-earth (inner space) vertical rocket launch, where we must take into account the force of gravity. The thrust upward will be constant if the rate at which the oxidation gases ejected is constant and if we neglect any change in *g* as the rocket moves away from the earth's surface. Assume the rate of fuel ejection to be a constant 665 lb/sec, and to yield an average force of 60,000 lb thrust from ignition to burn out. This fixed upward force (thrust) will be opposed by the downward gravity force (weight). Assume this weight force initially of rocket and fuel to be 30,000 lb, and to be 10,000 lb finally (weight of rocket after fuel is consumed). Hence, the net upward force initially is 30,000 lb (60,000 - 30,000); and the net upward force finally is 50,000 lb (60,000 - 10,000)! The ratio of the net upward force to the force of gravity is

$$\text{initially:} \quad \frac{30{,}000 \text{ lb}}{30{,}000 \text{ lb}} = 1:1$$

$$\text{and, finally:} \quad \frac{50{,}000 \text{ lb}}{10{,}000 \text{ lb}} = 5:1$$

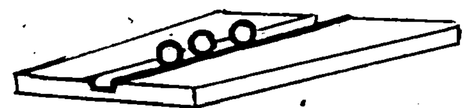
In other words, during the first moment of launch the thrust force upward just equals the rocket weight force downward. The rocket appears to hover on its launch pad. But as the fuel is consumed, the rocket thrust force upward (though <u>constant</u>) <u>becomes increasingly greater than the weight force downward</u> (weight force <u>decreases</u> as burning goes on). Now can you understand why rockets appear to hover, and then to rise slowly from their launch pads during <u>blast off</u>? The Atlas, a U.S. intercontinental missile, has an initial upward thrust which gives it only a net acceleration of 6 or 7 ft/sec^2; whereas, its final acceleration may be 30 times greater than this!

RESOURCE PACKAGE 3-2

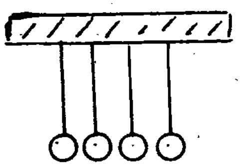
CONSERVATION OF MOMENTUM INVESTIGATION

1) Obtain a collision apparatus from your instructor. This Resource Package assumes you will have either a device consisting of a grooved board and some elastic balls, or a device consisting of elastic balls suspended by strings. You may investigate either the track type (the balls roll along the groove in the board) or the suspension type (the balls are suspended from strings attached to a supported stand), or both types of apparatus.

2) If you have the track type, go to step 8.



TRACK TYPE



SUSPENSION TYPE

3) If you have the suspension type, line the balls up so that their centers lie along the same line.

4) Pull one of the end balls away from the others and release it. How many balls are set in motion, and which one(s)?

5) Repeat step 4, pulling two (2) balls away from the others and releasing them together. Record your observations.

6) Now pull three (3) balls away, and repeat step 5. Record your observations. In each

experiment you should have noticed and recorded that the number of balls set in motion after the collision was equal to the number of balls pulled to one side. Also, you should have noticed and recorded that the balls after the collision moved about the same distance as the balls moved before the collision. How do these observations relate to the Law of Conservation of Linear Momentum? (Write out a short statement.)

7) When two (2) balls were released, the same number were also set into motion. From a mathematical point of view, why couldn't one ball be set in motion with twice the speed of the initial balls? The answer is not immediately obvious. It would seem at first glance that if linear momentum magnitude equals mass times velocity (mv), then if the mass property is cut in half ($\frac{m}{2}$), while the speed of the remaining ball is doubled (2V), the product $mv = (\frac{m}{2})(2v)$ remains the same and momentum is conserved! Not so! Momentum would be conserved, but we have overlooked yet another conservation law: Conservation of Energy.

The analysis of this problem brings up another physical concept, energy. Just as momentum is conserved for an isolated classical system, so is energy. If one ball moves away with twice the speed, energy cannot be conserved. Energy is discussed in greater detail in other minicourses, but this kind of energy is mathematically expressed as $\frac{1}{2}mv^2$. Can you see that doubling the speed would quadruple (increase by four) the energy? Go to step 10.

8) Arrange the balls so that they contact each other and are about midway between the ends of the groove. Separate one of the end balls and set it in motion so that it strikes the others. Record your observations.

9) Go to step 5, above, and complete the investigation procedures 6) through 10), inclusive.

10) Turn in your observations, sketches, etc., for evaluation.

RESOURCE PACKAGE 3-3

ROCKET PROPULSION INVESTIGATION

1) Blow up a toy balloon. The higher pressure on the inside of the balloon corresponds to the hot gas pressure inside a rocket motor. Release the balloon. As the pressurized air inside escapes, the air molecules gain momentum away from the balloon; to conserve momentum the balloon must gain an equal momentum away from the air molecules. Some people like to call this an action-reaction phenomenon ("happening").

Repeat the investigation a time or two to see if you can get the balloon to fly straight. What is missing on the balloon that is built onto a rocket to insure that it follows a desired course? List some of these missing components.

2) Try to design something that will cause the balloon to better follow a prescribed course.

3) Read about rocket propulsion in the minicourse Physics of Toys.

4) Submit your notes, and answers, and design. They will be evaluated by your instructor.

RESOURCE PACKAGE 3-4

MOMENTUM PROBLEMS

Solve the following on separate paper. Show all calculations and diagrams. Please do not write on this sheet.

1) Discuss this statement: "A rocket cannot move under its own power in outer space because there is no air for its exhaust to push against."

2) The speed of the final stage of a multi-stage rocket is much greater than the speed of a single-stage rocket of the same total launch weight and fuel supply. Account for this.

3) A rocket is set for a vertical firing. It is to be spin-stabilized when in flight. List the kinds of momenta conservation involved.

4) What is the magnitude of the momentum of a race car of mass 1,000 kg and of speed 30 m/sec? At what speed would a bus of 5,000 kg mass have the same linear momentum magnitude as this race car?

5) A 200-lb person standing on a surface of negligible friction kicks forward a 0.1-lb stone lying at his feet so that the stone acquires a speed of 10 ft/sec. What

velocity does the person acquire? Remember that lb is NOT a mass unit; to convert from the force unit lb to the inertial mass unit slug, one divides lb by 32 ft/sec^2. In this special case; however, the answer will be correct if you simply leave the units of lb as is, and plug into the appropriate equation.

RESOURCE PACKAGE 3-5

ANSWERS TO MOMENTUM PROBLEMS

1) Conservation of momentum does not require that the ejected gas push against anything external to the rocket. In fact, air friction would serve to slow the rocket down!

2) The single-stage rocket carries all of the rocket weight (and equivalent mass) throughout the flight; therefore, acceleration must be less than that of a multi-stage rocket whose empty fuel stages (sections) are jettisoned as the fuel is consumed.

3) 1) linear momentum
   2) angular momentum

4) a) 30,000 $\frac{kg - m}{sec}$
   b) 6m/sec

5) $\frac{1}{200}$ ft/sec, in a direction opposite to the stone. Your answer MUST have included direction, since velocity, not speed, was asked for!

RESOURCE PACKAGE 3-6

SELF-TEST

1) Define a) muzzle velocity, b) acceleration due to free fall, c) vector quantity, d) scalar quantity, e) angular momentum, and f) linear momentum.

2) A rock is dropped from a bridge 96 feet high. How long will it take it to strike the water?

3) An airplane flying horizontally at 240 mi/hr drops a bomb from an altitude of 2,000 feet. How far ahead of the target must the bomb be released? What will be the vertical speed of the bomb when it hits the target?

4) An auto of 1,000 kg mass is traveling at 30 m/sec. Suddenly, the auto is accelerated to 60 m/sec. What is the magnitude of the change in the auto's linear momentum?

5) A cannon of 4,000 kg mass is mounted on a railroad car of 16,000 kg mass. The cannon fires a 50-kg projectile with a muzzle speed of 500 m/sec at an angle of 10^{o}. Assume no friction. a) What is the horizontal <u>momentum</u> of the bullet? b) How far from the initial position of the car will the bullet strike the earth? c) How far will the car roll before the bullet strikes the earth? d) What will be the initial and the final speed of the railroad car?

SELF-TEST ANSWERS

1) a) Muzzle velocity - velocity at which a projectile leaves a gun barrel (muzzle speed and direction).

   b) Acceleration due to free fall - acceleration toward the earth's center of mass due to the earth's gravitational attraction (32 ft/sec^2; 9.8 m/sec^2; 980 cm/sec^2).

   c) Vector quantity - physical quantity which has both direction property and magnitude property.

   d) Scalar quantity - physical quantity which has magnitude property only.

   e) Angular momentum - inertial moment times angular velocity ($I\vec{\omega}$); a vector.

   f) Linear momentum - inertial mass times linear velocity ($\vec{mv}$); a vector.

2) 2.45 sec.

3) a) 2,957 ft ahead of target.

   b) vertical speed is 358 ft/sec.

4) 300,000 $\frac{kg\text{-}m}{sec}$

5) a) 24,620 $\frac{kg\text{-}m}{sec}$

   b) 8,729.5 meters

   c) 21.8 meters

   d) 1.23 m/sec for both cases (no friction!)

RESOURCE PACKAGE 4-1

BLOOD BALLISTICS

If a person learned about life only from watching television, it would be rather common knowledge that an astute ("sharp") criminologist can come up with a variety of clues to a shooting by means of bullet ballistics. Such clues might include the kind of weapon fired, positive identification of a specific weapon, probable angle of firing, probable flight path and range of the bullet, probable relative location of gunman and victim, etc.

Let us now consider a newer and relatively unknown kind of ballistics technology, blood ballistics. Whenever an investigation is made for evidence pertinent to an act of violence involving bloodshed, a detailed study of bloodstains can yield clues as vital as those of bullet ballistics. Bloodstain evidence often permits reconstruction of important conditions existing at the moment of bloodshed which, in turn, may answer questions vital to understanding the nature of the violent act itself.

Blood ballistics is a scientific endeavor. Given a sufficiently large bloodstain, the expert can draw firm conclusions regarding the conditions necessary for its formation. The phrase "firm conclusions" is used herein to imply proof within a reasonable scientific certainty, and therefore substantial enough for presentation as physical evidence in a courtroom.

For the purposes of blood ballistics, what are some of the pertinent characteristics of blood and bloodstains? Such characteristics would include:

a) size of spot

b) shape of spot

c) target surface and related spatter (See Fig. 1)

d) evidence of horizontal motion (See Fig. 2)

e) properties of blood in flight

f) impact angle (See Fig. 2)

g) impact velocity

Taken in combination, these and other pertinent blood characteristics, can yield answers to such questions as:

a) distance between the impact surface and the victim at the time the blood was shed

b) sites of origin of the blood

c) movements and spatial positions of persons and objects <u>while they were shedding blood</u>

d) position of victim and objects during bloodshed

e) number of blows struck or shots fired

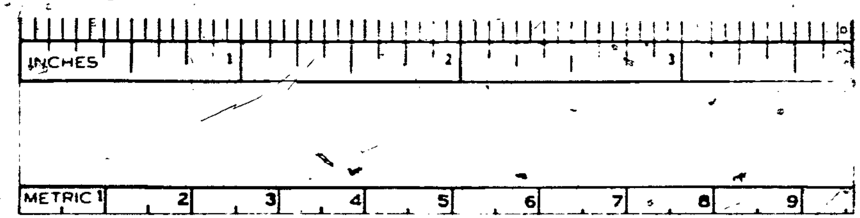


FIGURE 4.—*Bloodstain from a single drop of human blood that struck a hard, smooth, glossy, cardboard surface after falling eighty feet.*

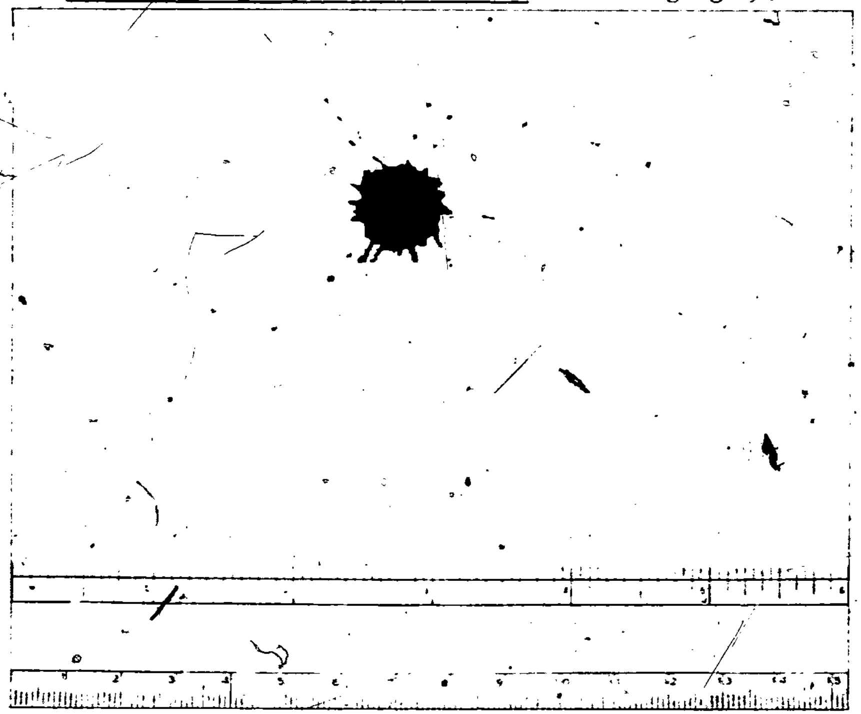


FIGURE 5.—*Bloodstain from a single drop of human blood that struck a blotter after falling eighteen inches*

TARGET SURFACE AND
RELATED SPATTER

Fig. 1

HORIZONTAL MOTION AND
IMPACT ANGLE PATTERNS

Fig. 2

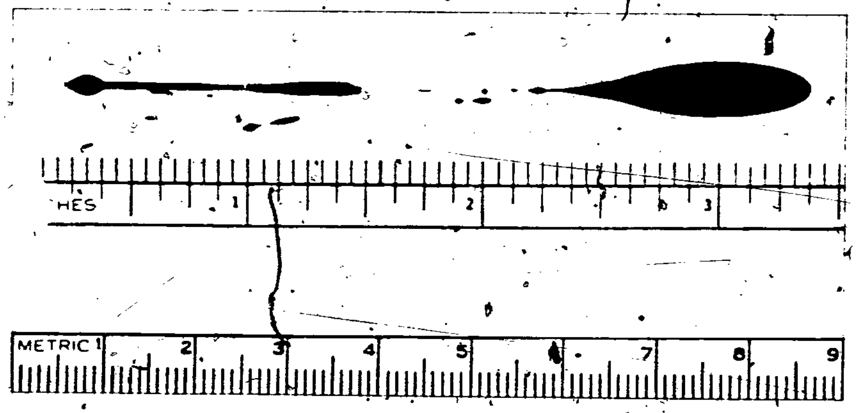


FIGURE 14.—*Bloodstain produced by a drop of human blood striking hard, smooth cardboard with a horizontal motion of about ten feet per second at an angle of approximately sixteen degrees.*

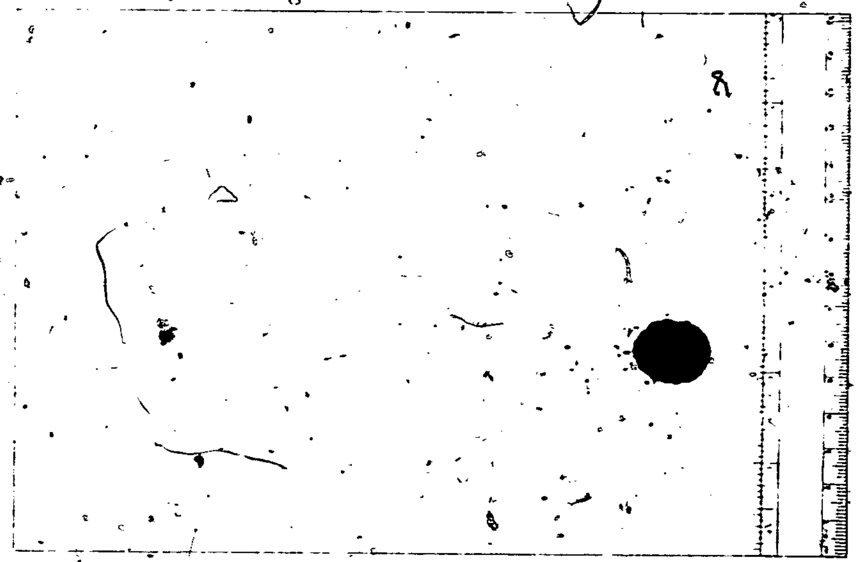
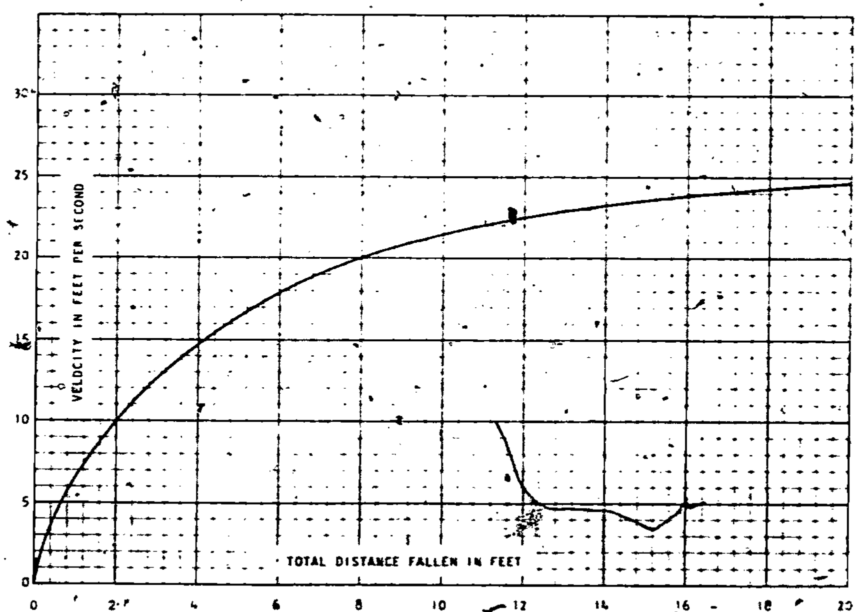


FIGURE 15.—*Bloodstain produced by a drop of human blood striking hard, smooth cardboard with a horizontal motion of about four feet per second at an angle of approximately fifty-six degrees.*

When a spatter or a drop of blood leaves a body, its behavior obeys the laws of projectile motion. Some interesting technical data are used by experts to evaluate blood in flight. Some of these are enumerated below:

a) The volume (size) of a single drop of blood in free fall is always the same (about 0.05 ml.)

b) A single blood drop's terminal speed is about 25 ft/sec (See Fig. 3)



Velocity of a single drop of human blood as a function of distance fallen.

TERMINAL SPEED OF BLOOD DROP
Fig. 3

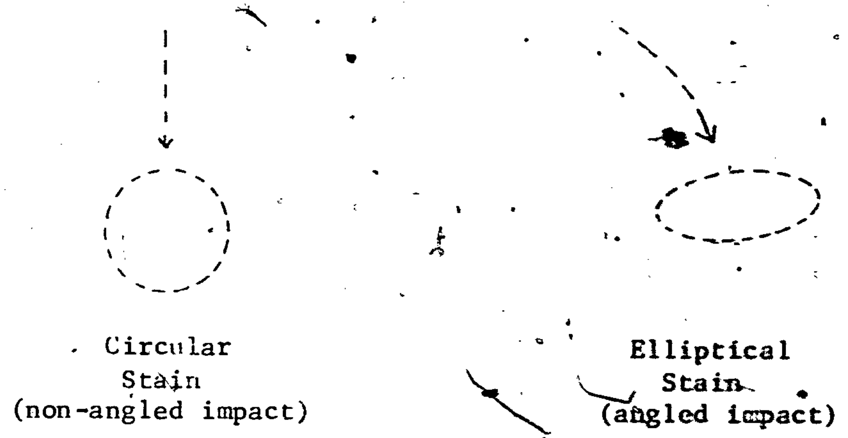
c) A single blood drop will not break up into smaller droplets in free fall. It must first strike an object, or be otherwise acted upon, for break-up to occur.

d) Accurate correlation (connection) can be established between

   1) blood spot diameters and the distance blood falls

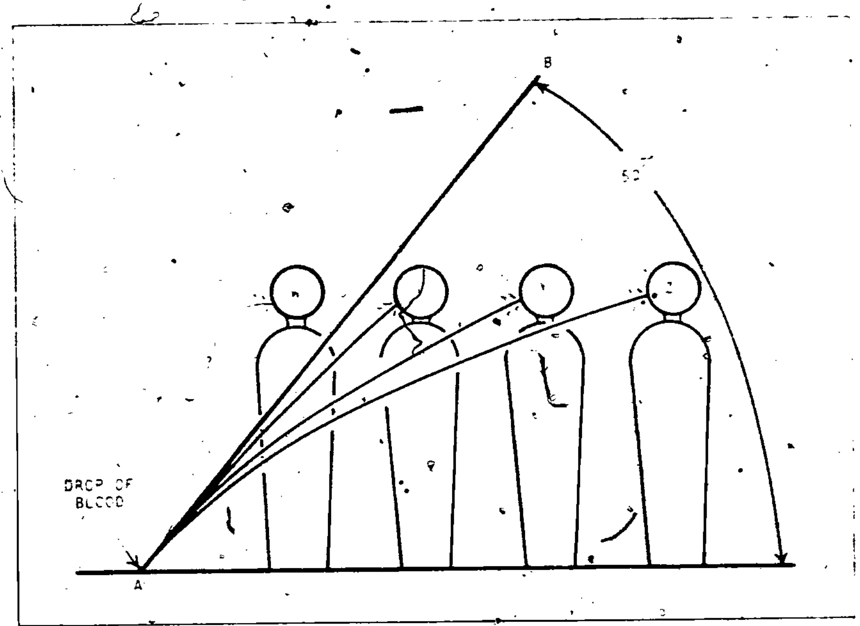
   2) blood spot edge characteristics (spines, spatters, etc.), distance blood falls, and the direction of the blood at impact

e) Blood spatters more on impact with porous surfaces than with harder, smoother surfaces. Blood dropping on a flat surface at an angle produces a more elliptical stain; whereas, a non-angled impact yields a circular stain. Further, the degree of elliptical distortion depends in a measureable way upon the angle of incidence (See Fig. 4)



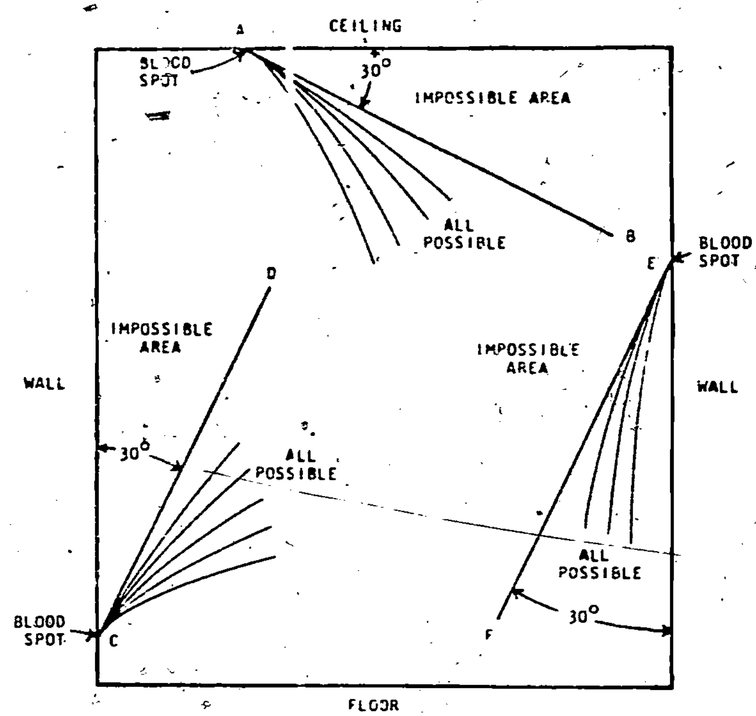
IMPACT ANGLES AND BLOOD DROP PATTERNS
Fig. 4

The list of established technical correlations goes on and on, but this discussion should give you some idea of how a knowledge of projectile motion is basic to blood ballistics. Figures 5 and 6 further exemplify bloodstain ballistics.



Proposed position of a victim from a bloodstain found on the floor. All to the right (x, y, z) of the impact angle are possible, the one to the left (w) is not possible.

CORRELATING (CONNECTING) BLOODSTAINS
Fig. 5



Diagrammatic representation of possible flight paths that could result in thirty degree angles of incidence. Ceiling bloodspot angle of incidence projection (line AB) for an upward spatter; downward or falling wall bloodspot angle of incidence projection (line CD); upward or rising wall bloodspot angle of incidence projection (line FG).

BLOODSTAIN CORRELATIONS
Fig. 6

INVESTIGATION: Characteristics of Simulated Bloodstains. First, prepare the following simulated blood solution:

= 1 liter water

≅ 15 grams salt

≅ 30 grams sugar

Some food coloring

Next, use a pipette or medicine dropper to:

1) Drop a "blood" drop from a known height onto

a) a glass surface

b) a blotter or paper towel surface

2) Repeat step 1), above, while moving your arm (and dropper) horizontally at a constant speed. Again, repeat these trials on the two unlike impact surfaces, but change to a different constant horizontal speed.

Record your observations. Indicate types of impact surfaces used, heights dropped, etc. A sketch should accompany each set of data. Turn all this in for evaluation.

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