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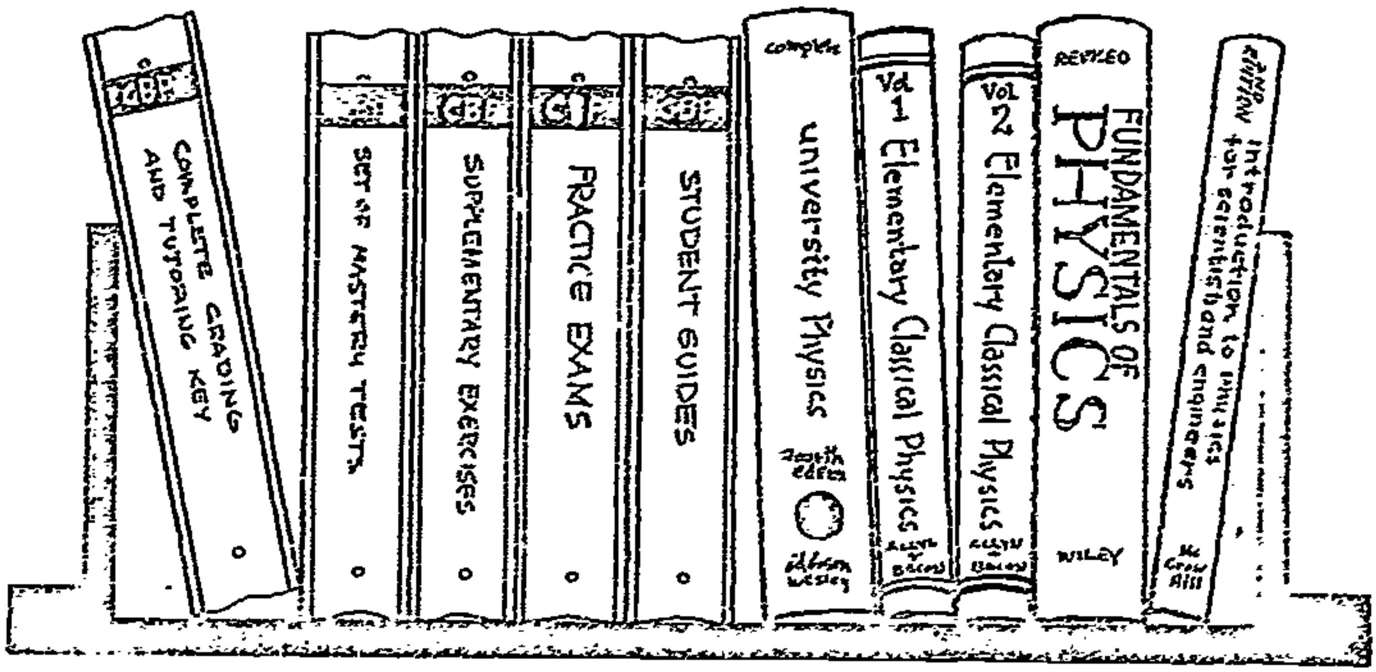
ABSTRACT

This is part of a series of 42 Calculus Based Physics (CBP) modules totaling about 1,000 pages. The modules include study guides, practice tests, and mastery tests for a full-year individualized course in calculus-based physics based on the Personalized System of Instruction (PSI). The units are not intended to be used without outside materials; references to specific sections in four elementary physics textbooks appear in the modules. Specific modules included in this document are: Module 8--Conservation of Energy, Module 9--Impulse and Momentum, and Module 10--Rotational Motion. (CP)

STUDY MODULES FOR CALCULUS-BASED GENERAL PHYSICS*



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Comments

These modules were prepared by fifteen college physics professors for use in self-paced, mastery-oriented, student-tutored, calculus-based general physics courses. This style of teaching offers students a personalized system of instruction (PSI), in which they increase their knowledge of physics and experience a positive learning environment. We hope our efforts in preparing these modules will enable you to try and enjoy teaching physics using PSI.

Robert G. Fuller
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COMMENT TO USERS

In the upper right-hand corner of each Mastery Test you will find the "pass" and "recycle" terms and a row of numbers "1 2 3 ..." to facilitate the grading of the tests. We intend that you indicate the weakness of a student who is asked to recycle on the test by putting a circle around the number of the learning objective that the student did not satisfy. This procedure will enable you easily to identify the learning objectives that are causing your students difficulty.

COMMENT TO USERS

It is conventional practice to provide several review modules per semester or quarter, as confidence builders, learning opportunities, and to consolidate what has been learned. You the instructor should write these modules yourself, in terms of the particular weaknesses and needs of your students. Thus, we have not supplied review modules as such with the CBP Modules. However, fifteen sample review tests were written during the Workshop and are available for your use as guides. Please send $1.00 to CBP Modules, Behlen Lab of Physics, University of Nebraska - Lincoln, Nebraska 68588.

FINIS

This printing has completed the initial CBP project. We hope that you are finding the materials helpful in your teaching. Revision of the modules is being planned for the Summer of 1976. We therefore solicit your comments, suggestions, and/or corrections for the revised edition. Please write or call

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CONSERVATION OF ENERGY

INTRODUCTION

Imagine a bicycle rider coasting without pedaling along a road that is very smooth but has a lot of small hills. As he coasts up a hill, the force of gravity will, of course, slow him down; but it speeds him up again as he goes down the other side. We say that gravity is a conservative force because it gives back as much kinetic energy (KE) to the cyclist when he returns to a lower level, as it took away when he ascended to the top. We therefore assign a gravitational potential energy (PE) U_g to the cyclist, which depends only on his elevation. The lost kinetic energy is converted into this U_g. We then find to our delight that the sum $E = K + U_g$ is (approximately) constant: U_g is larger at the top of each hill, and smaller at the bottom, in just such a way that its change compensates for the change in the kinetic energy K! This is an example of the conservation of mechanical energy.

However, if we watch the cyclist for some time, we are disappointed to find that $K + U_g$ is only approximately conserved: frictional forces gradually slow the cyclist down; and after awhile he starts pedaling again, thereby increasing $K + U_g$. But still, all is not lost. The energy-conservation law can be saved by defining other kinds of energy (for example, chemical, thermal, and nuclear) that are produced by the action of so-called nonconservative forces. If we call these nonmechanical energy forms E_{nc}, then $E = K + U + E_{nc}$ is exactly conserved. In fact, energy conservation is one of the great principles of physics, and one that holds even outside the domain where Newton's laws are valid.

Another example of energy transformation is provided by hydroelectric power production, beginning with the water stored behind a high dam. As the water rushes down the intake pipes it gains kinetic energy, then does work on the turbine blades to set them in motion; and, finally, the energy is transmitted electrically to appear as heat in the oven in your kitchen. Experience with energy transformations of this kind led to the formulation of the law of conservation of energy in the middle of the nineteenth century: ENERGY CAN BE TRANSFORMED, BUT NEITHER CREATED NOR DESTROYED. This law has survived many scientific and technological developments since that time, and our conception of the possible forms of energy has been enlarged. Whenever it seemed that energy was created or destroyed, physicists ultimately have been able to identify a new energy source (for example, thermonuclear energy in the sun) or a new energy receiver (such as neutrinos in beta decay).

In this module we shall be concerned only with mechanical energy and energy exchanged by doing work. We shall therefore be describing examples of mechanical energy conservation - the case of the ideal bicycle rider, and nonconservation of mechanical energy - the case of the real bicycle rider or the hydroelectric power plant. As a matter of fact, all practical, physically realizable phenomena involve friction, air resistance, and similar effects that result in some heating and a corresponding loss of mechanical energy. We shall therefore deal with idealized situations in which frictional forces are absent or are of a simple form. Since these forms of mechanical energy loss are often very small, our descriptions will be adequate approximations for many phenomena, and they will illustrate the law of conservation of energy as applied to mechanical processes.

PREREQUISITES

Before you begin this module, you should be able to:	Location of Prerequisite Content
*Calculate the work done by constant or variable force (needed for Objectives 3 and 4 of this module)	Work and Energy Module
*Apply the work-energy relationship to solve problems involving conservative and/or nonconservative forces (needed for Objectives 3 and 4 of this module)	Work and Energy Module

LEARNING OBJECTIVES

After you have mastered the contents of this module, you will be able to:

1. Forces - Define a conservative or a nonconservative force, or distinguish between them in problems.

2. Potential energy - Calculate the potential energy function $U(x)$, given a conservative force $F(x)$ depending on one coordinate; or conversely, given $U(x)$, find $F(x)$.

3. Conservation of mechanical energy - Use the law of conservation of mechanical energy for conservative forces to solve problems involving particle motion in one dimension.

4. Conservation of total energy - Apply the law of conservation of total energy, specifically including frictional forces, in the solution of problems of particle motion in one dimension.

TEXT: Frederick J. Bueche, Introduction to Physics for Scientists and Engineers (McGraw-Hill, New York, 1975), second edition

SUGGESTED STUDY PROCEDURE

Read Chapter 8, Sections 8.5 through 8.8 and Chapter 9, Sections 9.6 through 9.9. Bueche treats the objectives in this module in different order from most texts. We would suggest that you read Sections 8.5 and 8.6 in which the author shows that the ability of the gravitational field to do work on a man can be defined as gravitational potential energy, Eq. (8.6); then skip to Chapter 9 and read Section 9.6 up to Illustration 9.5, in which the potential energy of a spring is derived in Eq. (9.16). These are the two forms of potential energy we will use most often. Many others exist in nature, however, and the point of Objective 2 is the general relation between force and potential. At the end of Section 8.5 (p. 119), the work of gravity is shown as a sum of increments over a path in y. This would result, in the limit, in a general relation for an arbitrary conservative force:

$$U(x) = -\int_{x_1}^{x_2} F(x)\,dx.$$

BUECHE

Objective Number	Readings	Problems with Solutions: Study Guide	Problems with Solutions: Text	Assigned Problems: Study Guide	Additional Problems
1	Sec. 8.5	A			
2	Sec. 9.9	B, C		M	Chap. 8, Prob. 2, Chap. 9, Probs. 30, 31
3	Secs. 8.5-8.7	D, E	Illus.* 8.6, 8.7, 9.5	N, O, P	Chap. 8, Quest.* 6, Probs. 17, 23, 25; Chap. 9, Probs. 21, 25, 27
4	Sec. 8.8	F	Illus. 8.8, 8.9, 9.6, 9.8	Q, R	Chap. 8, Quest. 8, 14, Probs. 16, 18, 21, 22

*Illus. = Illustration(s). Quest. = Question(s).

Now read Section 9.9, which derives the differential relation between F(x) and U(x):

$$F = - dU/dx.$$

(You need not worry about more than one dimension.) With these equations the potential energy function can be found for an arbitrary force, and conversely F(x) can be found for a given U(x). Problems B and C in this module and Problems 30 and 31 in Chapter 9 of the text are keyed to Objective 2. You have already read about conservative and nonconservative forces in Section 8.5. Conservation of mechanical energy is treated in Section 8.7. Note that the author does not use the term "mechanical energy," preferring to use the symbolic form (U + K) instead. In Objective 3, "(U + K)" is equivalent to "mechanical energy."

The last logical step is to extend conservation ideas to nonconservative forces. This is done in Section 8.8. It should be pointed out that in Illustration 8.8, the "friction work" is the energy output equivalent of the actual work done by friction on the body (which is a negative quantity).

Read the General Comments; study the solutions to Problems A through F; and solve Problems M through R. If you need more practice, you may wish to work some of the optional problems listed above. Take the Practice Test before attempting a Mastery Test.

TEXT: David Halliday and Robert Resnick, Fundamentals of Physics (Wiley, New York, 1970; revised printing, 1974)

SUGGESTED STUDY PROCEDURE

Read all of Chapter 7 except Sections 7-6 and 7-9. You should have no difficulty with Objectives 1 and 2, which are well described in the text. The important principles of energy conservation are developed logically but rather briefly in Sections 7-7 and 7-8. You should note that Eq. (7-12b) is a statement of conservation of mechanical energy and is applicable to problems that do not involve friction. Section 7-8 discusses how to include friction or any other nonconservative force in the conservation principle. The last equation on p. 125 is a statement of conservation of total energy. The formulas given in General Comments may help you solve problems.

Read the General Comments; study the solutions to Problems A through F; and solve Problems M through R. If you need more practice, you may work some of the optional problems listed below. Take the Practice Test before attempting a Mastery Test.

HALLIDAY AND RESNICK

Objective Number	Readings	Problems with Solutions		Assigned Problems	Additional Problems
		Study Guide	Text	Study Guide	(Chap. 7)
1	Secs. 7-2, 7-7	A			Quest.* 2, 3, 4
2	Secs. 7-3, 7-4	B, C	Ex.* 1, 2, 3	M	Probs. 2(a), 3(a), 15(a), 22
3	Secs. 7-3 to 7-5	D, E	Ex. 5	N, O, P	5, 7, 17, 18, 25, 28, 30
4	Secs. 7-7, 7-8	F		Q, R	23, 32, 33, 37, 39 [part (d) is difficult]

*Ex. = Example(s). Quest. = Question(s).

TEXT: Francis Weston Sears and Mark W. Zemansky, University Physics (Addison-Wesley, Reading, Mass., 1970), fourth edition

SUGGESTED STUDY PROCEDURE

Read Chapter 7, Sections 7-4 through 7-8. The authors develop the principle of conservation of mechanical energy but do not state it in a form directly applicable to Objective 3. Equation (7-14) is a statement of total energy conservation (Objective 4) in which W contains the nonconservative forces. As the authors state, when $W = 0$; $\Delta E_k + \Delta E_p = 0$ = conservation of mechanical energy. Some further discussion of this is given in the General Comments. Note that the text notation is different from the more common notation we have used in the General Comments; $E_p \equiv U_p$, $E_k \equiv K$, $W \equiv W_{nc}$. Note that Objective 1 is not covered in the text until Section 7-6. This change in order from other texts is unimportant and should cause you no difficulty.

Guidance for Objective 2: In the previous module Work and Energy, the work-energy theorem was introduced in terms of the kinetic energy;

$$\Delta E_k = \int_{s_1}^{s_2} \vec{F} \cdot d\vec{s}.$$

SEARS AND ZEMANSKY

Objective Number	Readings	Problems with Solutions: Study Guide	Problems with Solutions: Text	Assigned Problems: Study Guide	Additional Problems
1	Sec. 7-6	A			
2	Secs. 7-4, 7-5, 7-7, 7-8	B, C		M	7-11, 7-12, 7-13, 7-34
3	Secs. 7-4, 7-5, 7-7, 7-8	D, E	Sec. 7-4, Ex.* 1, 2, 4, 5 Sec. 7-8, Ex. 3	N, O, P	7-22, 7-23a, 7-26, 7-29
4	Secs. 7-4, 7-5, 7-7, 7-8	F	Sec. 7-4, Ex. 3	Q, R	7-19, 7-24, 7-27

*Ex. = Example(s).

In Section 7-4, the text introduces the potential energy, associated with the ability of a body in a force field to do work, by considering as an example the energy of a body in a gravitational field. The text then generalizes the concept by introducing elastic poential energy. This treatment leads easily to the conservation-of-energy law, but obscures a useful relationship between force and potential energy. As you read Section 7-4, note that if $W' = 0$ in Eq. (7-7), then (rearranging terms)

$$mgy_2 - mgy_1 = -(\tfrac{1}{2}mv_2^2 - \tfrac{1}{2}mv_1^2),$$

or in general $E_p = - E_k$, and from Work and Energy

$$E_p = - \int_{s_1}^{s_2} F(\vec{s}) \cdot d\vec{s}.$$

Alternatively, this relationship may be stated in differential form:

$$F(s) = -dE_p(s)/ds.$$

You will be assisted in mastering this objective by studying Problems B and C.

Read the General Comments; study the solutions to Problems A through F; and solve Problems M through R. If you need more practice you should work some of the optional problems listed in the Table. Try the Practice Test before attempting a Mastery Test.

TEXT: Richard T. Weidner and Robert L. Sells, Elementary Classical Physics (Allyn and Bacon, Boston, 1973), second edition, Vol. 1

SUGGESTED STUDY PROCEDURE

Read Chapter 10, Sections 10-1 through 10-5. Although the order of presentation is different from the order of the objectives, we suggest you read the text in order. You may omit Eq. (10-7), since we are limiting this module to motion in one variable. All of the material in Sections 10-1 through 10-4 is a development of the concept of potential energy, although Section 10-4, Property 5 will help you the most to develop problem-solving skills for Objective 3.

In Section 10-5, the conservation of total mechanical energy is defined, $E = K + U =$ constant. The total energy of an isolated system is always conserved, but mechanical energy is conserved only in the absence of friction and other nonconservative forces. Read the General Comments; study the solutions to Problems A through F; and solve Problems M through R. If you need more practice, you may work some of the optional problems listed below. Try the Practice Test.

WEIDNER AND SELLS

Objective Number	Readings	Problems with Solutions: Study Guide	Problems with Solutions: Text	Assigned Problems: Study Guide	Assigned Problems: Text	Additional Problems
1	Sec. 10-4	A				
2	Secs. 10-1 to 10-4	B, C		M		10-17, 10-18
3	Sec. 10-5	D, E	Ex.* 10-1 to 10-6	N, O, P		10-2, 10-3, 10-4, 10-9, 10-10, 10-11, 10-14
4	Sec. 10-5	F		Q, R		10-16, 10-19, 10-20

*Ex. = Example(s).

GENERAL COMMENTS

The texts vary somewhat in their presentation of formulation for conservation of mechanical energy and total energy convenient for solving problems. We suggest the following for problems involving single springs and single masses (or multiple masses moving at the same velocity).

(1) Mechanical energy conservation: $K_i + U_i = K_f + U_f$ or

$$(1/2)mv_i^2 + (1/2)kx_i^2 + mgh_i + U_i = (1/2)mv_f^2 + (1/2)kx_f^2 + mgh_f + U_f,$$

where the subscripts i and f refer to initial and final states of the motion.

(2) Total energy conservation where friction is the only nonconservative force:

$$K_i + U_i = K_f + U_f + E_{nc},$$

$$(1/2)mv_i^2 + (1/2)kx_i^2 + mgh_i + U_i = (1/2)mv_f^2 + (1/2)kx_f^2 + mgh_f + \int_{x_1}^{x_2} f\ dx.$$

The absolute magnitude sign has been used for the work of the frictional force to avoid sign confusion, which arises because

$$E_{nc} = -W = -\int_{x_1}^{x_2} \vec{f}\cdot d\vec{x}.$$

But since f and dx are always oppositely directed, $\vec{f}\cdot d\vec{x} = -f\ dx(\cos\theta)$; Hence $E_{nc} = f \cos\theta\ dx$. It should be pointed out that because one is always concerned only with differences in potential energy, the choice of the zero of potential energy is arbitruary. If the force associated with the potential energy is constant, as in the case of the gravitational force on an object close to the earth, we may choose any convenient horizontal level (usually the lowest, or ground level) as the zero for gravitational potential energy. If, on the other hand, the force varies with displacement, as in the case of the spring, it is customary to choose the zero of potential energy as that displacement for which the force is zero.

Objective 1 will be satisfied by the following statement: A force is <u>conservative</u> if the work done by it on a particle that moves between two points depends only on these points and not on the path followed. A force is <u>nonconservative</u> if the work done by that force on a particle that moves between two points depends on the path taken between those points. (See Problem A for an alternative definition.)

PROBLEM SET WITH SOLUTIONS*

A(1). Define a nonconservative force and give an example of one.

Solution

A force is nonconservative if the work done by the force on a particle that moves through any round trip is not zero. (See General Comments for alternative definitions.)

Example: friction

A block sliding on a flat surface is projected against a spring mounted on the wall. If the surface is frictionless, the block will be brought to rest momentarily by the spring. The work done by the spring on the body is

$$W_1 = \int_i^f \vec{F} \cdot d\vec{x} = -\frac{1}{2}kx^2.$$

The motion reverses; on the outward path the work done is $W_2 = +(1/2)kx^2$, and the total work $W_1 + W_2 = 0$. If the surface is not frictionless, then the work done by the frictional force is

$$W_f = \int_i^f \vec{f} \cdot d\vec{x},$$

which is negative in both directions, and $W_1 + W_2 \neq 0$ for the round trip. Frictional forces, therefore, are an example of nonconservative forces. Conversely, for a conservative force the round-trip work is zero.

B(2). A body moving along the x axis is subject to a force given by $F(x) = -kx + cx^2$. Find the potential energy function for this force. Let $U(x) = 0$ at $x = 0$.

Solution

The potential energy is defined as $U(x) = -\int_{x_0}^{x} F(x)dx + U(x_0)$.

In this case $x_0 = 0$ and $U(x_0) = 0$.

$$U(x) = \int_0^x kx\,dx - \int_0^x cx^2\,dx, \qquad U(x) = \frac{1}{2}(kx^2) - \frac{1}{3}(cx^3).$$

C(2). What force corresponds to a potential $U(x) = -k_1m_1m_2/x + k_2x$?

*Problems satisfying Objective 4 also satisfy Objective 1 (i.e., Problems F, Q, R).

Solution

For one-dimensional motion $F(x) = -dU(x)/dx$. Therefore,

$$F(x) = \frac{d}{dx}\left(\frac{k_1m_1m_2}{x}\right) - \frac{d}{dx}(k_2x) = -\left(\frac{k_1m_1m_2}{x^2}\right) - (k_2).$$

D(3). A 1.00-kg object coasts along a smooth horizontal surface at 2.00 m/s and strikes a spring with force $k = 25.0$ N/m whose right end is firmly attached to the wall. What is the maximum amount by which this spring is compressed? (See Fig. 1.)

Solution

Use conservation of mechanical energy, taking i to be the point of first contact with the spring and f as the point of maximum compression:

$$K_i + U_i = K_f + U_f, \quad (1/2)mv^2 + 0 = 0 + (1/2)kx^2,$$

$$x = (m/k)^{1/2}(v) = (1.00\ \text{kg}/25.0\ \text{N/m})^{1/2}(2.00\ \text{m/s}) = 0.40\ \text{m}.$$

Of course, when the object is at rest, the spring continues to push to the left, and the object is accelerated to the left. When it returns to its initial point the energy conditions will be the same as in i above, although v will be oppositely directed.

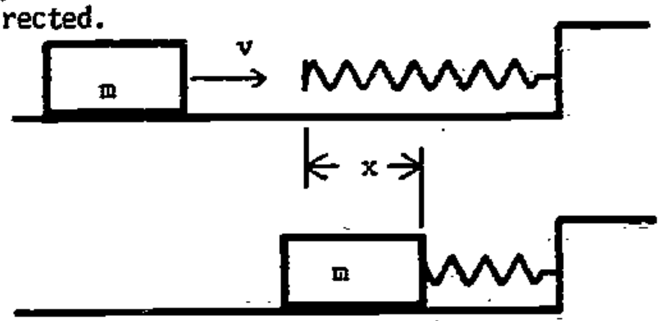


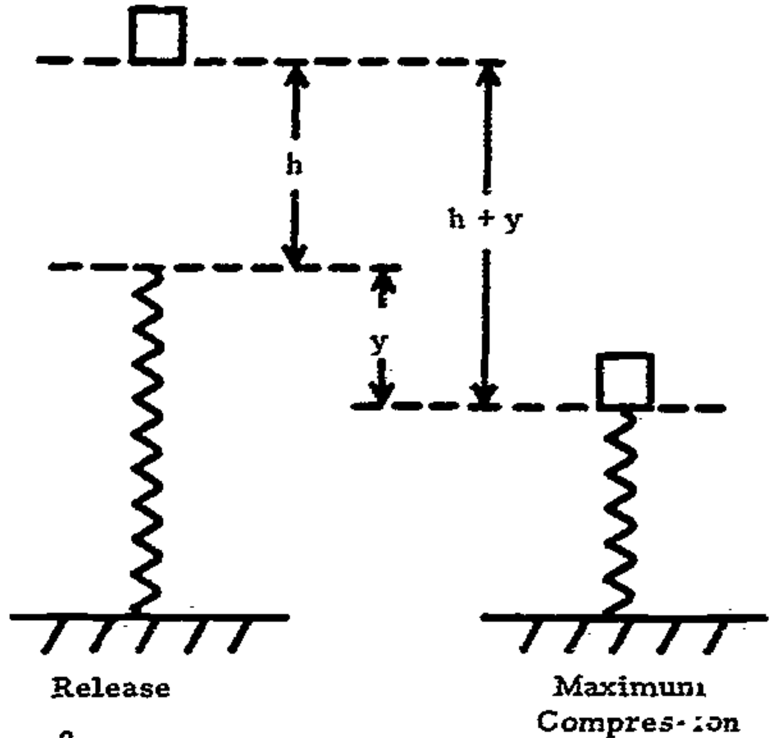
Figure 1

E(3). A block of mass 1.00 kg, initially at rest, is dropped from a height $h = 1.00$ m onto a spring whose force constant is $k = 50$ N/m. Find the maximum distance y that the spring will be compressed. (See Fig. 2.)

Solution

This is a process for which the principle of the conservation of mechanical energy holds. At the moment of release, the kinetic energy is zero. At the moment when maximum compression occurs, there is also no kinetic energy. Hence, the loss of gravitational potential energy of the block equals the gain of elastic potential energy of the spring:

Figure 2



$$mg(h + y) = (1/2)ky^2 \quad \text{or} \quad y^2 - 2mgy/k - 2mgh/k = 0.$$

Therefore,

$$y = \frac{mg}{k} \pm \frac{1}{2}\left[\left(\frac{2mg}{k}\right)^2 + \frac{8mgh}{k}\right]^{1/2}, \qquad \frac{mg}{k} = \frac{(1.00\ \text{kg})(9.8\ \text{m/s}^2)}{50\ \text{N/m}} = 0.196\ \text{m},$$

$$y = 0.196\ \text{m} \pm (1/2)(0.154 + 1.568)^{1/2}\ \text{m} = 0.196\ \text{m} \pm 0.656\ \text{m} = 0.85\ \text{m}.$$

(The negative solution corresponds to stretching the spring and is an unphysical solution since the block is not attached to the spring.)

F(4).

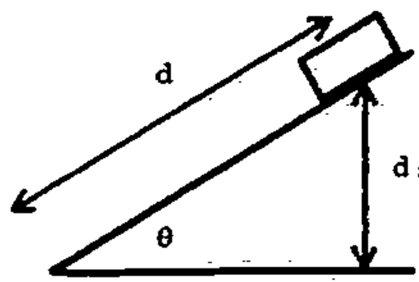


Figure 3

A 5.0-kg body is given an initial speed up an incline plane of 4.0 m/s. It coasts up the plane, comes momentarily to rest, and then coasts down again, its speed at the base of the incline being 3.00 m/s on return. (a) How much energy is dissipated in friction? (b) If the angle of the incline is 37°, what distance does the body travel up along the incline, assuming one-half of the energy found in part (a) is expended as the block goes up the plane?

Solution

(a) We find the loss in mechanical energy by comparing the initial and final mechanical energies. The potential energy is the same in the initial and final states, the body being, both initially and finally, at the base of the incline. Therefore, the decrease in the mechanical energy is simply the change in the kinetic energy:

$$E_{nc} = (k_i + U_i) - (K_f + U_f) = (1/2)m\,(v_i^2 - v_f^2)$$
$$= (1/2)(5.0\text{ kg})(16.0\text{ m}^2/\text{s}^2 - 9.0\text{ m}^2/\text{s}^2) = 18.0\text{ J}.$$

(b) Now we choose as our final state that at which the body is momentarily at rest on the incline. From part (a) we know that 9.0 J of mechanical energy have been converted into nonmechanical energy as the body increases its vertical displacement by $y = d \sin\theta$, as shown in the figure.

$$E_{nc} = (K_i + U_i) - (K_f + U_f),$$
$$9.0\text{ J} = [(1/2)\,mv_i^2 + 0] - (0 + mgd\sin\theta)$$
$$= (1/2)(5.0\text{ kg})(16.0\text{ m}^2/\text{s}^2) - (5.0\text{ kg})(9.8\text{ m/s}^2)d(\sin 37°),$$
$$d = (40 - 9.0)/29.4 = 1.05\text{ m}.$$

Problems

M(2). Find the potential energy function corresponding to this force field often used to describe the interaction of two atoms in a molecule:

$$F(r) = -A/r^7 + B/r^{13}$$

(r is the distance between atoms, A and B are constants). Let the potential energy be zero when the atoms are infinitely far apart. Draw a rough sketch of the force and potential energy function.

N(3). A particle is suspended at one end of a taut string whose upper end is fixed in position (a simple pendulum). The string's length is 12.5 m, and the particle passes through the lowest point at a speed of 7.0 m/s. What is the angle between the string and the vertical when the particle is momentarily at rest?

O(3). A 2.00-kg block and a 1.00-kg block are attached to opposite ends of a massless cord 2.00 m long. The cord is hung over a small frictionless and massless pulley a distance of 1.50 m from the floor, with the 1.00-kg block initially at floor level as shown in Figure 4. Then the blocks are released from rest. What is the speed of either block when the 2.00-kg block strikes the floor?

Figure 4

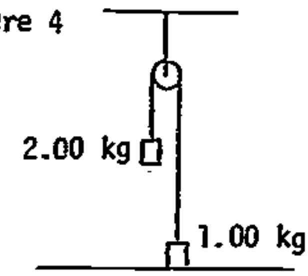
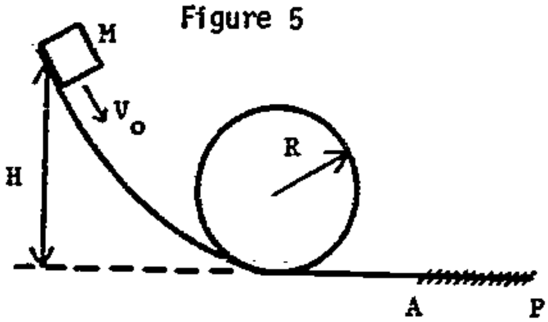


Figure 5



P(3). A 2.00-kg block is dropped from a height of 40 cm onto a spring whose force constant k is 1960 N/m. Find the maximum distance the spring will be compressed.

Q(4). A block of mass M slides on a frictionless track that is bent as in Figure 5. The radius of the loop is R. The block starts its journey at a height H above the floor with an initial speed (at height H) of v_0.

(a) How fast is the block traveling when it is upside down at the top of the loop?
(b) How fast is the block traveling after it has completed the loop?
(c) At point A the block starts sliding on a rough portion of the floor. The force of friction is F. How far beyond A does it travel before it stops at point P?

R(4). A 4.0-kg block starts up a 30.0° incline with 128 J of kinetic energy. How far will it slide up the plane if the coefficient of friction is 0.300?

Solutions

M(2). $U(r) = -A/6r^6 + B/12r^{13}$.

N(3). 37°.

O(3). 2.60 m/s.

P(3). 10.0 cm.

Q(4). (a) $[v_0^2 + 2g(H - 2R)]^{1/2}$.

(b) $(v_0^2 + 2gH)^{1/2}$.

(c) $(mv_0^2 + 2mgH)/2F$.

R(4). 4.3 m.

PRACTICE TEST

1. A particle of mass 16.0 kg constrained to move along the z axis is subject to a conservative force field given by $F(z) = -Az^3 + Bz$, as in Figure 6. (F is in newtons; the numerical values of A and B are A = 8.0, B = 1.00.)

   (a) What are the dimensions of A and B?

   (b) Find the potential energy as a function of z and sketch it. [U (0) = 0.]

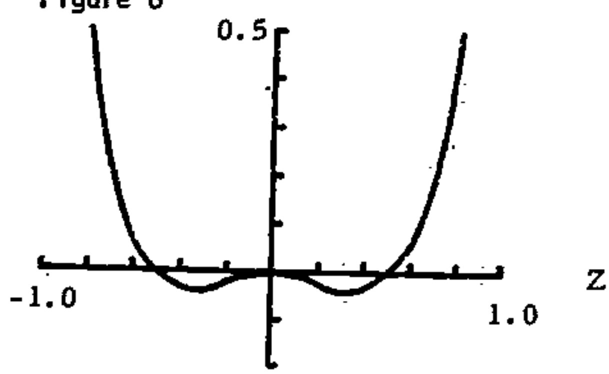
   (c) With what speed will the particle arrive at z = 0 if it starts from rest at z_0 = 4.0 m?

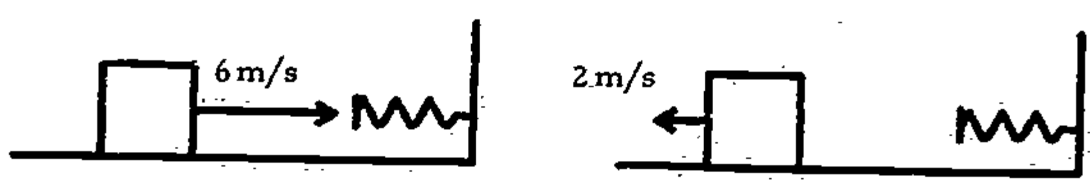
   (d) Do the same for the particle starting at z_0 = 0.100 m.

2. What is meant by a conservative force?

3. A 16.0-kg block traveling at 6.0 m/s in a horizontal direction collides with a horizontal weightless spring of force constant 5.0 N/m. The block compresses the spring a distance s. When the spring is back to the uncompressed position, the block is traveling with a speed of 2.00 m/s. If the coefficient of friction between the block and surface is 0.40, determine the energy expended by nonconservative forces. (See Fig. 7.)

Figure 6





k = 5 N/m

Figure 7

Practice Test Answers

1.(a) A: M/L^2T^2 B: M/T^2, since F: ML/T^2.

(b) $U(z) = -\int_0^z F(z)dz = (\frac{A}{4})z^4 - (\frac{B}{2})z^2$,

$U(0) = 0 = 2z^4 - (1/2)z^2$.

(c) $(1/2)mv^2 + U(z) = E = \text{const}$, $E = U(4) = 512 - 8.0 = 504$ J,
at $z = 0$, $U(0) = 0$, $E = (1/2)mv^2$, $v = 2E/m = (2 \times 504)/16.0 - 63 = 7.9$ m/s.

(d) $(1/2)mv^2 + U(z) = E = \text{const}$, $E = U(0.100) = (2 \times 10^{-4}) - (0.50 \times 10^{-2})$
$= -0.0498 < 0$.

Since $U(0) = 0$, and $(1/2)mv^2 \geq 0$, the particle will not arrive at origin (it is repelled).

2. A force is conservative if the work done by the force on a particle that moves through any round trip is zero.

3. $K_i + U_i = K_f + U_f + E_{nc}$, $E_{nc} = (K_i - K_f) + (U_i - U_f) = (1/2)M(v_i^2 - v_f^2)$
$+ (0 - 0) = (1/2)(16.0\text{ kg})(36 - 4.0)\text{m}^2/\text{s}^2 = 256$ J.

Date ____________

CONSERVATION OF ENERGY

pass recycle

Mastery Test Form A 1 2 3 4

Name ______________________ Tutor ______________________

1. (Note: The following experiment takes place in an orbiting space ship.) A 1.00-kg body is hurled against a special spring that exerts a restoring force given by the function $F(x) = -k_1 - k_2x^2$ when deformed from equilibrium. What was the body's initial speed if it compresses the spring by 0.200 m before being stopped? The constants in the force function are $k_1 = 100$ N and $k_2 = 2000\ \mathrm{N/m^2}$.

2. Two 10.0-kg blocks are connected together by a massless rope strung over a massless, frictionless pulley. The table exerts a 20.0-N frictional force on m. The blocks start from rest at t = 0 and are allowed to accelerate (see Fig. 1).
(a) Using only energy considerations, calculate the speed of M after it has fallen a distance of 2.00 m.
(b) Calculate the kinetic energy of the two blocks after M has fallen a distance of 2.00 m.
(c) Give a definition of a nonconservative force. Identify all forces that do not work in this situation. Which are conservative? Which are nonconservative?

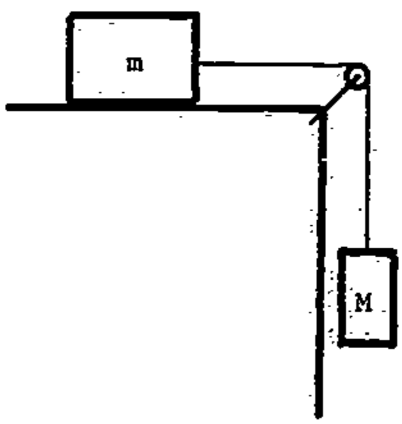


Figure 1

Date ______________

CONSERVATION OF ENERGY

pass recycle

Mastery Test Form B

1 2 3 4

Name ______________________ Tutor ______________________

1. A 2.00-kg object, initially at rest, slides down a frictionless segment of the track from A to B, as illustrated in Figure 1.
   (a) Calculate the speed of the object at B.
   (b) The track between B and C is sufficiently frictional that the object, after continuing past B, comes to rest at point C. Calculate the work done by friction in slowing the object.
   (c) Define a conservative force.
   (d) Calculate the net work done by conservative forces for travel between points A and C.
   (e) Calculate the net work done by nonconservative forces for travel between points A and C.

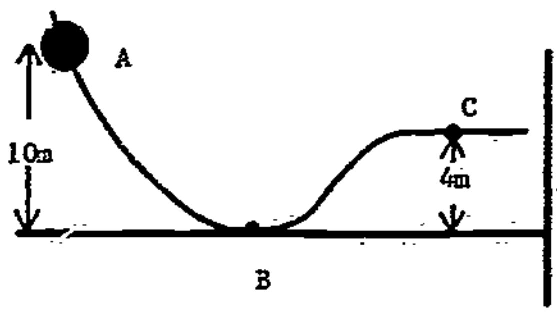


Figure 1

2. The potential energy of a particle of mass m constrained to move along the z axis is $U(z) = Az^2 + B$.
   (a) What is the physical significance of B?
   (b) Find the force field F(z) experienced by the particle, and sketch it as a function of z.
   (c) With what speed will the particle arrive at the point z = 1.00 m if it starts with zero speed at the point z = 6.0 m? Let A = 2.50 kg/s^2; m = 7.0 kg.

Date ____________

CONSERVATION OF ENERGY

pass recycle

Mastery Test Form C

1 2 3 4

Name ____________________ Tutor ____________________

1. Define a nonconservative force, and give an example.

2. A block of 0.200 kg is released from rest from its position of being pressed against a spring whose length is initially 0.100 m shorter than its relaxed length and whose spring constant is 1200 N/m. The block slides without friction along the horizontal and up a ramp that makes an angle of 30.0° with the horizontal and whose top edge is 1.30 m above the level of the horizontal surface. (See Fig. 1.) Determine the velocity of the block as it flies off the ramp.

3. A certain peculiar spring obeys the force law $F_x(x) = -Ax - Bx^2$, where $A = 22.0$ N/m and $B = 18.0$ N/m^2. (See Fig. 2. Note this is not Hooke's law!)

   (a) Compute the potential energy function $U(x)$, taking $U(0) = 0$.

   (b) One end of this spring is fastened to a wall and the other end is fastened to an object of mass $M = 1.20$ kg resting on a rough horizontal surface. The object is moved to the right, stretching the spring by 1.00 m, and then released. If $\mu_k = 0.50$, what is the speed of the object when it reaches the point at which the spring is unstretched?

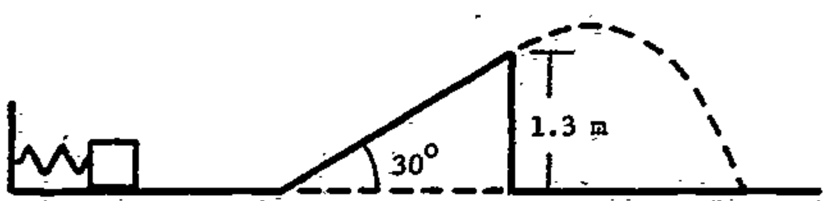


Figure 1

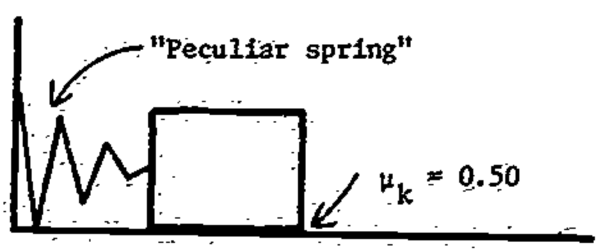


Figure 2

MASTERY TEST GRADING KEY - Form A

What To Look For | **Solutions**

1. (a) First step is to find the potential. Check the minus sign.

1. (May be done in two steps.)
(a) Find the potential energy from the force

$$U = -\int_{x_1}^{x_2} F(x)\,dx = \int_0^x (k_1 + k_2x^2)\,dx,$$

$$U(x) = k_1x + (k_2/3)\,x^3.$$

Gravitational forces do not enter into this "weightless" environment.

(b) Make sure all terms in the conservation equation are present.

(b) Using conservation of mechanical energy, find v:

$$(U_i + K_i) = (U_f + K_f),$$

$$k_1x + (1/3)k_2x^3 + 0 = 0 + (1/2)mv^2,$$

$$v^2 = 2k_1x/m + 2k_2x^3/3m$$

$$= \frac{(2)(100\ N)(0.200\ m)}{(1.00\ kg)} + \frac{(2)(2000\ N/m^2)(0.200\ m)^3}{(3)(1.00\ kg)}$$

$$= 40 + 10.7 = 50.7 = 7.1\ m/s.$$

2. (a) Complete energy balance, i.e., make sure all terms are present. Common error is omission of $(1/2)mv^2$. Check free-body diagram.

2. (a)

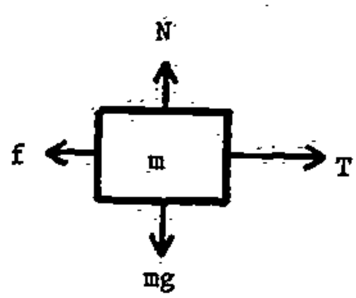


Figure 12

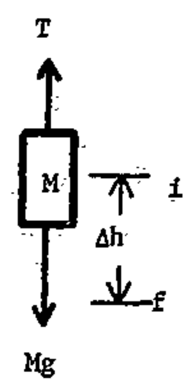


Figure 13

What To Look For	Solutions
	Conservation of total energy: $U_i + K_i = U_f + K_f + E_{nc}$, $mgh' + Mgh_1 + 0$ $= mgh' + Mgh_2 + (1/2)mv^2 + (1/2)Mv^2 + fh$, $(1/2)(m + M)v^2 = (mg - f)h$, $v^2 = 2(Mg - f)h/(m + M) = 16.0\ m^2/s^2$, $v = 4.0$ m/s. (b) $K = (1/2)(m + M)v^2 = (1/2)(10 + 10)16$ $= 160$ J.
(c) Look for alternative definitions of non-conservative force.	(c) For a nonconservative force, the work done in a round trip is not zero: Mg - conservative, f - nonconservative. Alternative definition: nonconservative force -- a force is nonconservative if work done by it integrated around a closed path is not zero.

MASTERY TEST GRADING KEY - Form B

What To Look For | **Solutions**

1. (a) By conservation of total energy:

$$U_i + K_i = U_f + K_f + E_{nc}.$$

There are no nonconservative forces so

$$E_{nc} = 0, \quad mgh_1 + 0 = 0 + (1/2)mv^2,$$

$$V = (2gh_1)^{1/2} = [(2)(9.8\ m/s^2)(10.0\ m)]^{1/2}$$

$$= 14.0\ m/s.$$

1. (b) The energy used up in friction is equivalent to a negative work on the particle by frictional force.

(b) $U_i + K_i = U_f + K_f + E_{nc}$,

$$0 + (1/2)mv^2 = mgh_2 + 0 - W_{nc},$$

$$W_{nc} = mgh_2 - (1/2)mv^2 = (2.00\ kg)(9.8\ m/s^2)(4.0\ m) - (1/2)(2.00\ kg)(196\ m^2)$$

$$= 1.20 \times 10^2\ J.$$

(c) Look for acceptable alternate definitions of conservative force.

(c) If the total work done by a force in a round trip is zero, the force is conservative.

(d) $W_{consv} = -\Delta U = U_i - U_f = mg(h_1 - h_2)$

$$= -1.20 \times 10^2\ J.$$

(e) $W_{nc} = -1.20 \times 10^2$ J; see part (b).

2. (a) Question the student to see if he is aware of the concept but does not relate it to the question in this form.

2. (a) Physical significance - an arbitrary constant can always be added to the potential energy. Note it cancels in part (c).

(b) Look for lack of negative sign in definition.

(b) $F = -dU(z)/dz = -(d/dz)(Az^2 + B) = -2\ Az.$

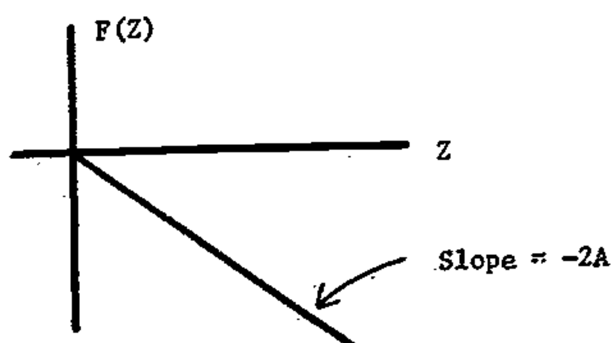


Figure 14

What To Look For	Solutions
	(c) Conservation of mechanical energy:
	$U_i + K_i = U_f + K_f$,
	$Az_1^2 + B + 0 = Az_2^2 + B + (1/2)mv_f^2$,
	$v_f^2 = (2A/m)(z_1^2 - z_2^2) = [2(2.50)/7.0](36 - 1)$
	$= 5.0(35)/7.0 = 25.0$,
	$v_f = 5.0$ m/s.

MASTERY TEST GRADING KEY - Form C

What To Look For	Solutions

1. If the work done by a force in a round trip is not zero, the force is nonconservative. Friction and air resistance are common examples.

2. Releasing the spring transfers the potential energy stored in the spring to kinetic energy of the block. By conservation of mechanical energy:

   $U_i + K_i = U_f + K_f$,

   $(1/2)kx^2 + 0 = 0 + (1/2)mv^2, \quad v^2 = kx^2/m.$

   As the block goes up the ramp, kinetic energy is transformed to gravitational potential energy:

   $U_i + K_i = U_f + K_f$,

   $0 + (1/2)mv^2 = mgh + (1/2)mv_f^2, \quad v_f^2 = v^2 - 2gh.$

   From above, $v^2 = kx^2/m$.

   $v_f^2 = kx^2/m - 2gh = (1200\ N/m)(0.100\ m)^2/(0.200\ kg) - 2(9.8\ m/s^2)(1.30\ m) = 60 - 25.5 = 34.5,$

   $v = 5.9$ m/s.

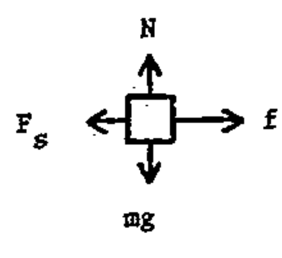
3. (a) Using $U(x) = -\int_{x_1}^{x_2} F(x)\,dx$,

   Find $U(x)$ for $F = -Ax - Bx^2$:

   $U(x) = +\int_0^x (Ax + Bx^2)\,dx = \frac{Ax^2}{2} + \frac{Bx^3}{3} + U_0(0),$

   $U(x) = 11x^2 + 6x^3.$

(b) Free-Body Diagram:



$N = mg, \quad F_s = f = \mu mg$. By conservation of total energy;

$$U_i + K_i = U_f + K_f + E_{nc} = U_f + K_f - W_{nc}, \quad 11x^2 + 6x^3 + 0 = 0 + K_f - \int_x^0 f\,dx,$$

$$11x^2 + 6x^3 = K_f + \mu mgx, \quad \text{or} \quad K_f = 11x^2 + 6x^3 - \mu mgx = 11 + 6 - (1/2)(1.20\ kg)$$

$$\times (9.8\ m/s^2)(1.00\ m) = 17 - 5.88 = 11.1\ J,$$

$v = [2(11.1\ J)/(1.20\ kg)]^{1/2} = 4.3$ m/s.

STUDY GUIDE

IMPULSE AND MOMENTUM

INTRODUCTION

You have already learned that you stub your toe harder trying to kick larger masses. Now imagine another unpleasant activity: catching a bowling ball. This gets harder to do as the ball is dropped from higher places. The difficulty depends both on the ball's mass and its velocity just before you apply the stopping force. This force can be applied in different ways. Any winner of an egg-throwing contest will tell you the way to stop an object with the least force is to spread the stopping process out over a maximum time.

This module will develop the above "folk physics" into a system of concepts and equations; and even a new law that is believed to be more fundamental than the laws from which it will be derived. This wonderful anomaly will not be further explored in this module, but it does indicate some of the philosophical richness and curiosity that continues to be part of this science. The concepts are "center of mass" and "linear momentum"; the law is called "conservation of linear momentum."

PREREQUISITES

Before you begin this module, you should be able to:	Location of Prerequisite Content
*Describe the motion of a body moving in a plane (needed for Objectives 1 to 3 of this module)	Planar Motion Module
*Solve problems requiring the application of Newton's second and third laws (needed for Objectives 3 and 4 of this module)	Newton's Laws Module

LEARNING OBJECTIVES

After you have mastered the content of this module, you will be able to:

1. Center of mass - Write the formulas for the center of mass (c.m.) of a system and explain all the terms. Write the formulas for the linear momentum of a system. Explain all the terms.

2. Linear momentum - Given the masses, positions, and velocities of all particles in a system, find the position and velocity of the center of mass, and the total (vector) linear momentum.

3. <u>Impulse</u> - Given a force versus time graph or function for a system, calculate the change of the system's linear momentum.

4. <u>Linear-momentum conservation</u> - Recognize conditions for which the linear momentum of a system is conserved.

TEXT: Frederick J. Bueche, Introduction to Physics for Scientists and Engineers (McGraw-Hill, New York, 1975), second edition

SUGGESTED STUDY PROCEDURE

Read Chapter 9, Section 9.1 first. This explains the center of mass. Then read Chapter 7, Section 7.1, which introduces linear momentum and impulse and the relation between them.

Now read Section 9.2, which partially shows how to calculate the velocity of a system's center of mass; You solve Eq. (9.3) for $v_{x(c.m.)}$ and similar but unspecified equations for $v_{y(c.m.)}$ and $v_{z(c.m.)}$. The velocity components are then combined into a single velocity: $\vec{v}_{c.m.} = v_{x(c.m.)}\hat{i} + v_{y(c.m.)}\hat{j} + v_{z(c.m.)}\hat{k}$. This section also relates the external forces acting on a system to the motion of the system's center of mass.

Although Section 7.1 describes a single particle, Sections 9.1 and 9.2 have shown how to reduce a system of masses into a somewhat equivalent single particle. This particle has the total mass of the system and is located at the center of mass of the system. It is equivalent only in that it has the same linear momentum as the system. If you do not need to know details of the internal structure of the system, you can apply the ideas of Section 7.1 to systems of masses by treating them as a single particle located at the center of mass. For example, the earth is regarded as a particle by the scientists who calculate very high-altitude satellite orbits, but such an earth would be without meaning to most geologists.

BUECHE

Objective Number	Readings	Problems with Solutions		Assigned Problems		Additional Problems
		Study Guide	Text* (Illus.)	Study Guide	Text* (Illus.)	
1	Secs. 9.1, 7.1	A		A		
2	Secs. 9.1, 9.2	A, B	9.1	A, B	9.1	Chap. 9, Probs. 1, 3, 4
3	Secs. 7.1, 7.2	C	7.1, 7.2	C	7.1, 7.2	Chap. 7, Quest.* 10, 12
4	Sec. 7.4	D	7.3, 7.4, 9.1	D	7.3, 7.4, 9.1	Chap. 7, Quest.* 5, 7

*Illus. = Illustration(s). Quest. = Question(s).

Continue to read Sections 7.2, 7.3, and 7.4. Keep in mind that now when the text specifies a particle it can also be interpreted as referring to the c.m. of a system of masses.

Read the General Comments; and read and understand how to solve the problem set. If you need additional help work some of the Additional Problems.

Try the Practice Test.

TEXT: David Halliday and Robert Resnick, Fundamentals of Physics (Wiley, New York, 1970; revised printing, 1974)

SUGGESTED STUDY PROCEDURE

Read all of Chapter 8. Understand and know Eq. (8-3b). It contains all the previous equations in this chapter. Equation (8-4b) is Eq. (8-3b) in calculus form. The next important equation is (8-8). It also sums up the arguments of Section 8-2. Section 8-3 starts out with a definition you must memorize: the linear momentum of a particle, Eq. (8-9). Equation (8-10) will be used in Sections 8-4 and 8-5. It is a statement of Newton's second law. Section 8-4 shows how to calculate the total linear momentum of a system of particles: Eq. (8-12). Equation (8-13) can be used to find $v_{c.m.}$. Equation (8-10) appears again in a more restricted form as Eq. (8-15): the internal forces have been eliminated. You should know why. Section 8-5 begins with the important equations describing the conservation of linear momentum, but paradoxically does not give them numbers.

Read and understand Examples 1 to 6 in this chapter. Then read Section 9-2 in Chapter 9. Read the General Comments; and read and understand how to solve the Problem Set. If you need additional help, work some of the Additional Problems.

Try the practice test.

HALLIDAY AND RESNICK

Objective Number	Readings	Problems with Solutions: Study Guide	Problems with Solutions: Text	Assigned Problems: Study Guide	Additional Problems
1	Sec. 8-1	A			
2	Secs. 8-1, 8-2, 8-4	A, B	Chap. 8, Ex.* 1, 2, 3	A, B	Chap. 8, Quest. 1 to 4, 10; Probs. 1 to 13, 21, 22
3	Sec. 9-2	C	Chap. 9, Ex. 1 to 4	C	Chap. 9, Quest. 3, 4, 8; Probs. 1 to 13
4	Sec. 8-5	D	Chap. 8, Ex. 4, 5, 6	D	Chap. 8, Quest. 5, 8, 9; Probs. 23 to 39

*Ex. = Example(s). Quest. = Question(s).

TEXT: Francis Weston Sears and Mark W. Zemansky, University Physics (Addison-Wesley, Reading, Mass., 1970), fourth edition

SUGGESTED STUDY PROCEDURE

University Physics does not include center of mass, and therefore other texts must be suggested for this important topic. Read the indicated sections in one of the three texts listed below.

Author and Text	Topic	Section
Frederick J. Bueche, Introduction to Physics for Scientists and Engineers (McGraw-Hill, New York, 1975), second edition	center of mass motion of c.m.	9.1 9.2
David Halliday and Robert Resnick, Fundamentals of Physics (Wiley, New York, 1970; revised printing, 1974)	center of mass motion of c.m.	8-1 8-2, 8-4
Richard T. Weidner and Robert L. Sells, Elementary Classical Physics (Allyn and Bacon, Boston, 1973), second edition, Vol. 1	motion of c.m. center of mass	6-1, 8-4 6-2

SEARS AND ZEMANSKY

Objective Number	Readings	Problems with Solutions: Study Guide	Problems with Solutions: Text	Assigned Problems: Study Guide	Additional Problems†
1	Use one of the readings recommended above. Sec. 8-1	A	Ex.* 1, 2 (Sec. 8-2)	A	
2	Use one of the readings recommended above	A, B		A, B	B: Chap. 9, 1, 3, 4 (non-calculus), 5-8 (calculus) HR: Chap. 8, Quest.* 1-3, 10; Probs. 1-13, Ex. 1-3 WS: 6-1, 6-3 to 6-8 (calculus)
3	Sec. 8-1	C	Ex. 1 (Sec. 8-1)	C	8-3, 8-4 (calculus)
4	Sec. 8-2	D		D	8-5

*Ex. = Example(s). Quest. = Question(s).
†B = Bueche. HR = Halliday and Resnick. WS = Weidner and Sells.

The above readings develop the idea of replacing a system of masses by an imaginary but somewhat equivalent mass particle. This particle has the same total mass as the system and is located at a place called the center of mass (c.m.) of the system. In Chapter 8, Sections 8-1 and 8-2, University Physics develops the ideas of momentum and impulse for particles, and these concepts can be applied to all systems of masses since you now know how to reduce systems to equivalent particles.

Read Chapter 8. Examples 1 and 2 in Section 8-2 show that the momentum of a system of particles is the sum (vector) of the momenta of the particles. Read the General Comments; and read and understand how to solve the Problem Set. If you need additional help, work some of the Additional Problems.

Try the Practice Test.

TEXT: Richard T. Weidner and Robert L. Sells, Elementary Classical Physics (Allyn and Bacon, Boston, 1973), second edition, Vol. 1

SUGGESTED STUDY PROCEDURE

Read Chapter 5, Section 5-5. This section defines the linear momentum of a particle: Eq. (5-2). It furthermore shows by a worked example that in a particular collision the total linear momentum of the two colliding particles does not change. The total linear momentum of a system of particles is defined in Eq. (5-5). Work through Examples 5-1 to 5-3. You should draw figures showing the linear-momentum vectors of the systems before and after the collision.

Now read Chapter 6, Sections 6-1 and 6-2. These sections show you how to calculate the velocity and the location of a system's center of mass (c.m.). They also give you a method of reducing any collection of masses to a single somewhat equivalent particle located at the c.m. Now you have the tools to apply any rules for particles to collections of masses. The remainder of this module will show you how to use these tools.

Read Section 7-4 in Chapter 7. Here you are shown the relationship between the linear momentum of a system and the forces acting on the system: Eq. (7-6). Example 7-3 should be understood.

Read Chapter 8, Section 8-4. This is an important section because it tells you the general requirements for a system in order that its linear momentum be conserved. Example 8-8 is a good review of the ideas presented in this module. There are summaries at the ends of the chapters that gather the ideas into a few lines.

Read the General Comments; and read and understand how to solve the Problem Set. If you need additional help, work some of the Additional Problems.

Try the Practice Test.

WEIDNER AND SELLS

Objective Number	Readings	Problems with Solutions		Assigned Problems		Additional Problems
		Study Guide	Text	Study Guide	Text	
1	Sec. 5-5, Chap. 6, Sec. 8-4	A		A		
2	Sec. 5-5, Chap. 6, Sec. 8-4	A, B		A		5-3, 6-1 to 6-8, 6-5 (calculus)
3	Sec. 7-4	C		C		7-2 to 7-9, 7-11, 7-12
4	Sec. 8-4	D		D		

GENERAL COMMENTS

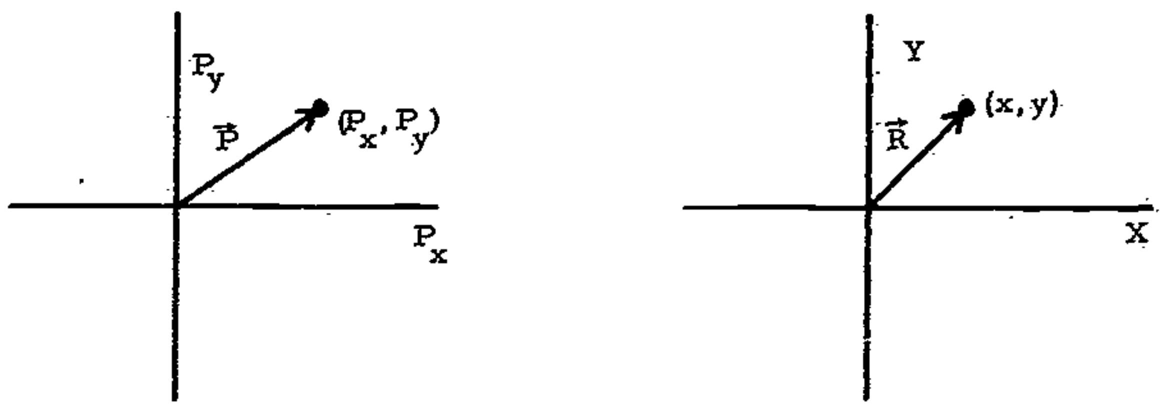
1. Center of Mass

The center of mass (c.m.) of a system is an intrinsic property of that system. Although the formulas for the position of the c.m. give it as the distance from the origin of a coordinate frame, the position of the c.m. does not move with respect to the system if the origin is moved. The distances from the origin to the c.m. will change, of course. But this is only because the origin or the system as a whole has moved.

This property allows you to pick any point as the origin in a problem where you have to find a c.m. A clever choice may simplify your calculation. Look for a symmetry axis in your system and place the origin somewhere on it. Or you can place the origin on one of the system's particles.

2. Linear Momentum

Linear-momentum vectors are added like any other vectors, but they exist in linear-momentum space (see Figure 1). Two-dimensional examples of a linear-momentum space and an ordinary coordinate space are shown below. Points in ordinary space show position, and vectors in this space show displacements. Points and vectors in linear-momentum space represent linear momenta. You have no information about the position of anything in linear-momentum space. Sometimes when solving problems you may draw displacement and linear-momentum vectors in the same figure and not realize that they are superimposing ordinary and linear-momentum spaces. Watch out! This can lead to mistakes such as attempts to add displacement and linear-momentum vectors.



Momentum Space Coordinate Space

Figure 1

3. Coordinate System

You will learn that the total linear momentum of a system is the product of the total mass of the system and the velocity of the system's c.m. If the system has zero resultant external forces acting on it, its c.m. is not accelerated. Thus the c.m. moves at constant velocity, and a coordinate system whose origin is placed at the c.m. and moves with it will be an inertial coordinate system. Furthermore, the total linear momentum of the system relative to that origin is zero because the velocity of the c.m. relative to the origin (at the c.m.) is zero. Here is another case where a good choice of coordinate-system origin may sometimes simplify a problem. By placing the origin at the system's c.m. (providing the c.m. is not accelerating) you can use the fact that the total linear momentum of the system is zero.

This placement of the coordinate system's origin establishes what is called the center-of-mass coordinate system or center-of-mass reference frame.

PROBLEM SET WITH SOLUTIONS

A(1). (a) A system of several particles is shown in Figure 2. You are told the mass of the particles and their position coordinates relative to the coordinate axis. Explain how to find the center of mass of the system.

(b) At the instant shown in Figure 2 the particles are in motion. m_1 is moving upward (+z direction) with speed v_1. m_2 is moving to the left (-y direction) with speed v_2. m_3 is moving toward m_1 with speed v_3. Explain how to find the total linear momentum of the system.

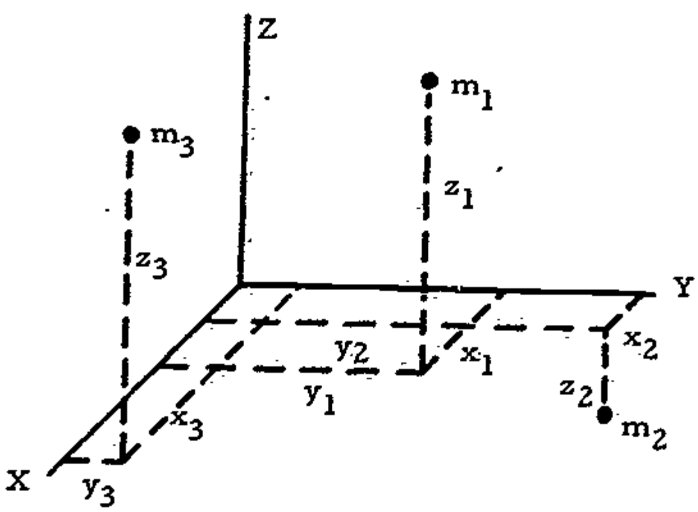


Figure 2

Solution

(a) By inspection of the figure you can see that the c.m. will be at a place that has x, y, and z coordinates. Your texts give formulas for each of these coordinates:

$$x_{c.m.} = \sum_{i=1}^{N} x_i m_i / \sum_{i=1}^{N} m_i = \frac{1}{M} \sum_{i=1}^{N} m_i x_i = X,$$

$$y_{c.m.} = \sum_{i=1}^{N} y_i m_i / \sum_{i=1}^{N} m_i = \frac{1}{M} \sum_{i=1}^{N} m_i y_i = Y,$$

$$z_{c.m.} = \sum_{i=1}^{N} z_i m_i / \sum_{i=1}^{N} m_i = \frac{1}{M} \sum_{i=1}^{N} m_i z_i = Z.$$

Since this problem uses particles and not extended bodies you can use the c.m. formulas for particles. If the masses were not particles you would have to use the calculus formulas for c.m. coordinates.

The c.m. of the system (see Figure 3) is located at the point having coordinates ($x_{c.m.}$, $y_{c.m.}$, $z_{c.m.}$). You can also express the position of the c.m. by specifying the vector $\vec{r}_{c.m.}$. Use unit vectors and write

$$\vec{r}_{c.m.} = x_{c.m.}\hat{i} + y_{c.m.}\hat{j} + z_{c.m.}\hat{k}.$$

The problem could also have been given to you in terms of the masses and their position (or displacement) vectors. You would then have the additional first step of resolving the position vectors into their components, i.e., finding the x_1, y_1, z_1, x_2,....

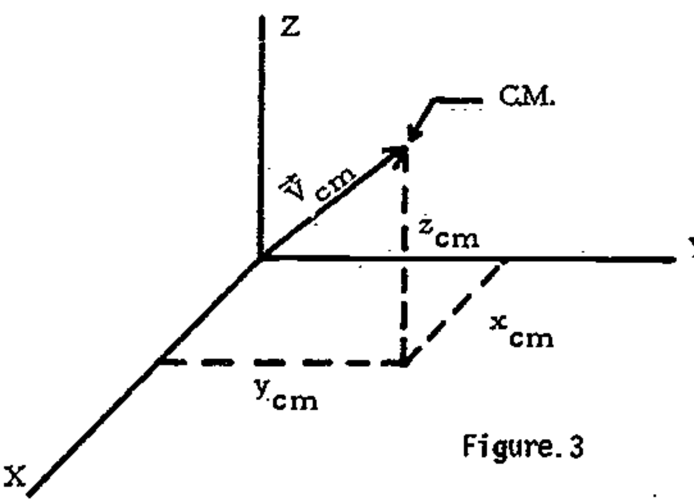


Figure. 3

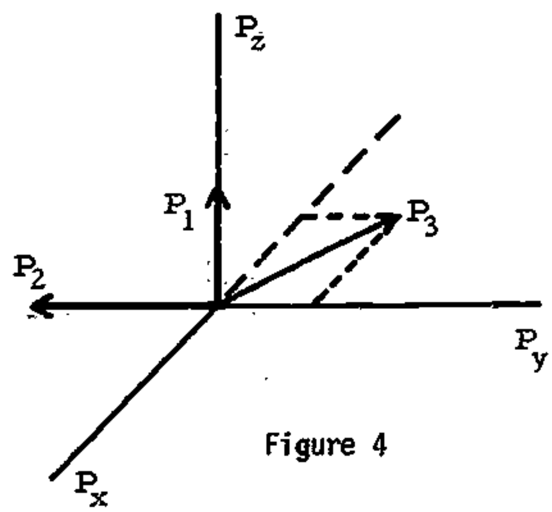


Figure 4

(b) The total linear momentum of the system is the sum (vector) of the momenta of its parts. In this problem you must calculate the linear momentum of each particle and then add them to obtain the total linear momentum. The momentum of each particle is $\vec{p} = m\vec{v}$, where m is the mass and v is its velocity. The momenta vectors would look as shown in Figure 4, and the total linear momentum is

$$\vec{P} = \vec{p}_1 + \vec{p}_2 + \vec{p}_3 = M\vec{V} = m_1\vec{v}_1 + m_2\vec{v}_2 + m_3\vec{v}_3.$$

Since the three momentum vectors of the particles are not collinear nor even coplanar, the best way to add them is by their components. Resolve the velocity vectors into their x, y, and z components and add them:

$$V_x = v_{1x} + v_{2x} + v_{3x},$$

$$V_y = v_{1y} + v_{2y} + v_{3y},$$

$$V_z = v_{1z} + v_{2z} + v_{3z}.$$

Then

$$\vec{V} = V_x\hat{i} + V_y\hat{j} + V_z\hat{k},$$

and finally

$$\vec{P} = M\vec{V}.$$

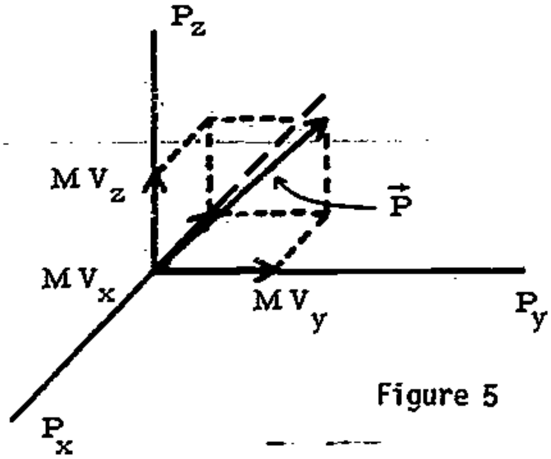


Figure 5

The total linear momentum and its components would look as shown in Figure 5. Although it's not asked for in this problem, you should recognize that <u>V is the velocity of the center of mass of the system</u>.

B(2). Three particles are moving radially outward from the coordinate origin, at angles of 120° to one another in the xy plane. Their masses are $m_1 = 1.00$ kg, $m_2 = 2.00$ kg, and $m_3 = 0.80$ kg, and their speeds are $v_1 = 6.0$ m/s, $v_2 = 2.00$ m/s, and $v_3 = 10.0$ m/s, respectively.

(a) Sketch the system in a coordinate frame.

(b) Which particle has the momentum of greatest magnitude?

(c) What is the total momentum of the three-particle system?

(d) Find the velocity of the c.m.

<u>Solution</u>

(a) Draw an xy coordinate system and show the particles and their motions on it (Figure 6). There are many ways to place the particles on the coordinate system. In Figure 6 one of the particle's trajectories was placed along the y axis. This will save effort later if it becomes necessary to resolve the velocities or momenta into components in the x and y directions.

Also draw the momentum vectors for the three particles (Figure 7). The Magnitudes may be wrong in this figure because the linear momenta have not yet been calculated; but this gives you an idea of the momenta and their directions. Now start answering the questions.

(b) Linear momentum is $\vec{p} = m\vec{v}$, and its magnitude is $p = mv$. For each particle:

$p_1 = m_1v_1 = (1.00\text{ kg})(6.0\text{ m/s}) = 6.0\text{ kg m/s}$,

$p_2 = m_2v_2 = (2.00\text{ kg})(2.00\text{ m/s}) = 4.0\text{ kg m/s}$,

$p_3 = m_3v_3 = (0.80\text{ kg})(10.0\text{ m/s}) = 8.0\text{ kg m/s}$.

Particle 3 has the largest linear-momentum magnitude. We can now redraw Figure 7 to scale as Figure 8.

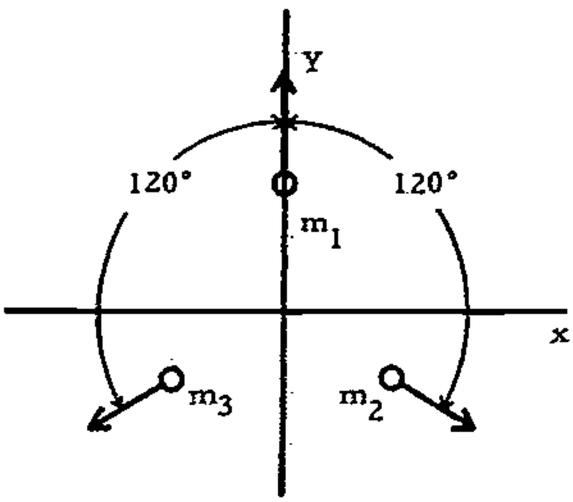


Figure 6

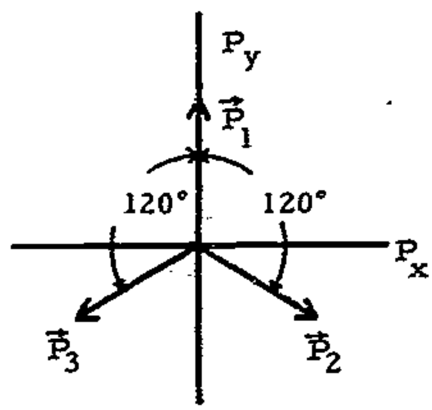


Figure 7

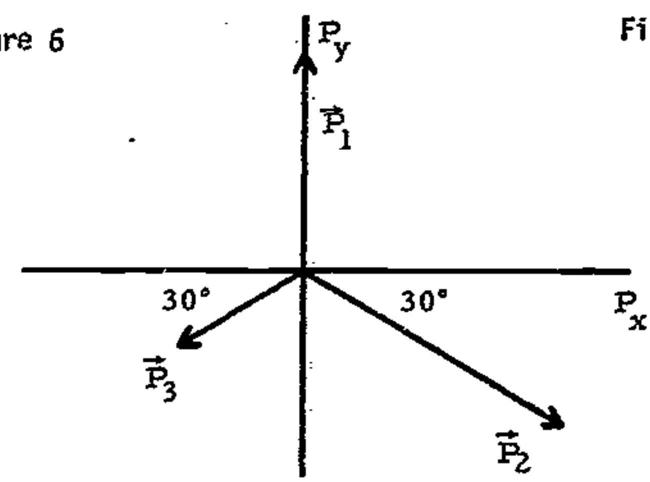


Figure 8

(c) The total linear momentum of the system is

$$\vec{P} = \vec{p}_1 + \vec{p}_2 + \vec{p}_3.$$

In pictorial form this addition is shown in Figure 9, and you see from inspection that $\vec{P} \neq 0$. Pity.

Unless you prefer to work directly with the polygon in Figure 9 the best method to find P is to find its x and y components and add (vector) them. This will require you first to resolve the momenta of the three particles into their x and y components (refer to Figure 7):

$$\vec{p}_1 = p_1\hat{j} = m_1v_1\hat{j} = (6.0\hat{j})\ \text{kg m/s},$$

$$\vec{p}_2 = p_2 \cos(30°)\hat{i} + p_2 \sin(30°)(-\hat{j})$$

$$= [(8.0)(0.866)\hat{i} - (8.0)(0.500)\hat{j}]\ \text{kg m/s},$$

$$\vec{p}_3 = p_3 \cos(30°)(-\hat{i}) + p_3 \sin(30°)(-\hat{j})$$

$$= [(4.0)(0.866)\hat{i} - (4.0)(0.500)\hat{j}]\ \text{kg m/s}.$$

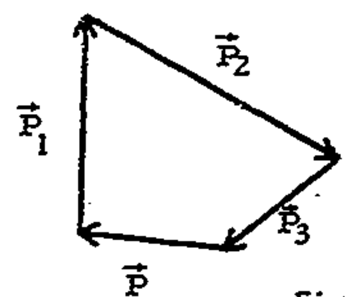


Figure 9

Use

$$\vec{P}_x = \vec{p}_{1x} + \vec{p}_{2x} + \vec{p}_{3x} = 0 + (8.0)(0.866)\hat{i} - (4.0)(0.866)\hat{j}$$

$$= 3.5\hat{i}\ \text{kg m/s}.$$

Similarly

$$\vec{P}_y = \vec{p}_{1y} + \vec{p}_{2y} + \vec{p}_{3y} = [6.0 - (8.0)(0.500) - (4.0)(0.500)\hat{j}]\ \text{kg m/s} = \vec{0}$$

for this case. Thus for the total linear momentum,

$$\vec{P} = [(2.00)(1.732)\hat{i}] = 3.5\hat{i}\ \text{kg m/s}.$$

(d) The velocity of the center of mass is the total linear momentum of the system divided by the system's mass:

$$\vec{V}_{c.m.} = \vec{P}/M = \Sigma m\vec{v}/\Sigma m = \vec{V} \quad \text{or} \quad Mv_{x(c.m.)} = m_1v_{x1} + \cdots + m_nv_{xn}.$$

Thus $\vec{V}_{c.m.}$ or $\vec{V} = [(2.00)(1.732)\hat{i}\ \text{kg m/s}]/3.8\ \text{kg} = (0.53)(1.732)\hat{i}\ \text{m/s} = 0.92\hat{i}\ \text{m/s},$

$$v_{x(c.m.)} = [(2.00)(1.732)\ \text{kg m/s}]/3.8\ \text{kg} = 0.92\ \text{m/s},$$

which is the magnitude of the velocity of the c.m.

C(2). A croquet ball (mass 0.50 kg) initially at rest is struck by a mallet, receiving the impulse shown in Figure 10. What is the ball's velocity just after the force has become zero? Assume the graph is a parabola.

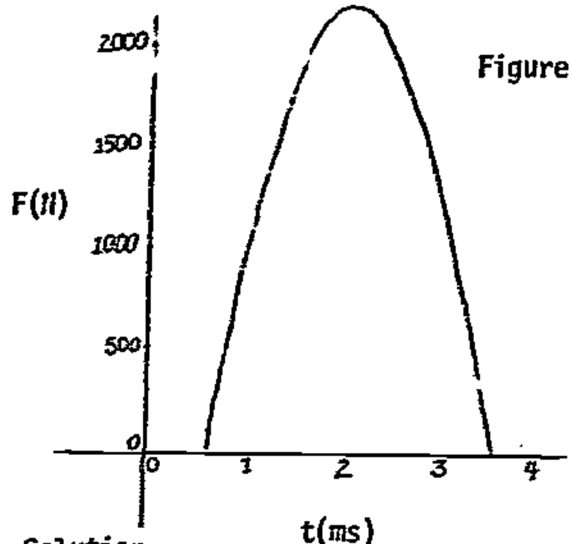
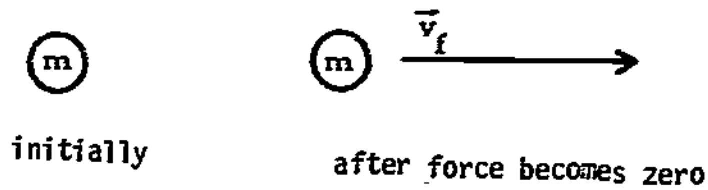


Figure 10

Figure 11



Solution

This is an impulse problem. The croquet ball's initial velocity (and therefore its initial momentum) is zero, and you want to find its final velocity. Since you are given the ball's mass, you must find its final momentum and divide by the mass: $\vec{v}_f = \vec{p}_f/m$.

Note that the problem is one dimensional, and you are not told the direction the ball rolls. However, you are asked for the ball's velocity (vector), and you must just assume a direction such as "horizontally to the right," and proceed to find the speed. Start with

$$\int_{t_i}^{t_f} \vec{F}\,dt = \Delta\vec{p} = m\vec{v}_f - m\vec{v}_i,$$

and apply this equation to the croquet ball. Since $\vec{v}_i = 0$, you can solve for $\vec{v}_f$ algebraically:

$$\vec{v}_f = \frac{1}{m}\int_{t_i}^{t_f} \vec{F}\,dt.$$

You know m and the integral is the area under the force versus time graph. Here are two ways to integrate this area:

(1) <u>Square counting</u>: See Figure 12. Each small square has an area of (100 N) (0.200×10^{-3}) = 0.0200 N s. Now you simply count the number of squares under the graph and multiply this by the area of one square. A count accurate to 1%

gives 210 squares. Thus the total area is

$$\int_{t_i}^{t_f} F\,dt = (210\text{ squares})(0.0200\text{ N s/square}) = 4.2\text{ N s}.$$

Since this is a one-dimensional problem the vector notation has been dropped:

$$v_f = \frac{1}{m}\int_{t_i}^{t_f} F\,dt \quad\text{and}\quad v_f = \frac{4.2\text{ N s}}{0.50\text{ kg}} = 8.4\text{ m/s}.$$

(2) Calculus: An equation for a parabola is $y = kx^2$. However, this parabola passes through the origin, but your graph does not have F at zero when t is zero; thus the equation $F = kt^2$ will not work for you. The equation

$$(F - 2200\text{ N}) = k(t - 2.00 \times 10^{-3}\text{ s})^2$$

will work, if you evaluate k. Pick a point such as $F = 0$ and $t = 0.50 \times 10^{-3}$, and plug these values into the above equation. This gives

$$k = -9.78 \times 10^8\text{ N/s}^2.$$

Now

$$F = (9.78 \times 10^8\text{ N/s}^2)(t - 2.00 \times 10^{-3}\text{ s})^2 + 2200\text{ N}$$

and

$$\int_{t_i}^{t_f} F\,dt = \int_{0.50 \times 10^{-3}\text{ s}}^{3.4 \times 10^{-3}\text{ s}} [(-9.78 \times 10^8\text{ N/s}^2)(t - 2.00 \times 10^{-3}\text{ s})^2 + 2200\text{ N}]dt$$

$$= (-9.78 \times 10^8\text{ N/s}^2)\int_{0.50 \times 10^{-3}\text{ s}}^{3.4 \times 10^{-3}\text{ s}} (t - 2.00 \times 10^{-3}\text{ s})^2\,dt$$

$$+ (2200\text{ N})\int_{0.50 \times 10^{-3}\text{ s}}^{3.4 \times 10^{-3}\text{ s}} dt.$$

A change of variable will simplify the first integral. Let $t - 2.00 \times 10^{-3}\text{ s} \equiv \tau$.

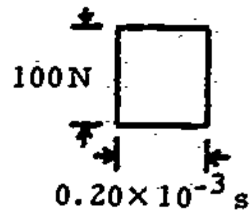


Figure 12

Then $dt = d\tau$, and changing the limits of integration gives us

$$F\,dt = (-9.78 \times 10^{8}\ \mathrm{N\,s^{2}}) \int_{-1.50 \times 10^{-3}}^{1.4 \times 10^{-3}} \tau^{2}\,d\tau + 2200\ \mathrm{N} \int_{0.50 \times 10^{-3}}^{3.4 \times 10^{-3}} dt = 4.38\ \mathrm{N\ s}.$$

Note here that this value is a bit more than the one obtained from square counting. Possibly the assumption that the graph was a parabola symmetric about 2×10^{-3} s was not correct. As before,

$$v_f = 4.38\ \mathrm{N\ s}/0.50\ \mathrm{kg} = 8.76\ \mathrm{m/s} = 8.8\ \mathrm{m/s}.$$

D(2). A child runs and leaps into a stationary wagon. The wagon can roll without friction on the level, rough driveway, but it is not headed in the direction the child was running. The wagon and child move in the direction the wagon was pointed.

(a) Is the linear momentum of the child-wagon system conserved in this process? Explain why.

(b) Are any linear-momentum components of this system conserved? Explain why.

Solution

(a) Momentum is conserved if the total linear momentum of the system does not change. Before the child jumps on the wagon she alone has some linear momentum. The total linear momentum of the system then is in the direction the child is running. When the child is aboard the rolling wagon the total linear-momentum vector of the system points in the direction the wagon is rolling. This is not the direction in which the child was running, and thus the linear momentum of the system cannot be the same before and after the child jumped on the wagon. It is impossible for two vectors to be equal if they are not in the same direction. The linear momentum of the system has not been conserved.

(b) Another way to identify momentum conservation is by the use of the equation

$$\int_{t_i}^{t_f} \vec{F}_{ext}\,dt = \Delta\vec{p}.$$

If $\vec{F}_{ext}$ is zero, then so will be $\Delta\vec{p}$.

Furthermore, if there is any direction in which some component of $\vec{F}_{ext}$ is zero, the component of $\Delta\vec{p}$ in that direction will also be zero. In your problem the wagon rolls with friction in the direction it is pointed. There can be no external forces acting on the system in this direction, and the component of the system's linear momentum in this direction is conserved. You should realize that there is a considerable external force on the system when the child lands in the wagon: the frictional force between the wheels and the rough driveway. This is the force that causes the momentum of the system to change.

An often overlooked, but not always trivial, direction in which momentum might be conserved is the vertical. The external vertical forces acting on the child-wagon system add to zero (assuming the child's c.m. moves horizontally); and the system's change in momentum in the vertical direction is thus also zero. The system's linear-momentum component in the vertical direction remains zero during the process.

PRACTICE TEST

1. Explain how conservation of momentum applies to a handball bouncing off a wall.

2. Three particles floating in space are attached to one another with springs. Their masses are 5.0 kg, 7.3 kg, and 12.2 kg, respectively. One of the particles is hit by a meteorite. The force-time graph of this collision is shown in Figure 13. Calculate the change in momentum of the three-particle system.

3. (a) Write the formula for the center of mass of any system. Explain what you would need to know about the system in order to calculate it.

   (b) Write the formula for the linear momentum of any system whose parts are moving in straight lines and not rotating. Explain what you would need to know about the system in order to calculate it.

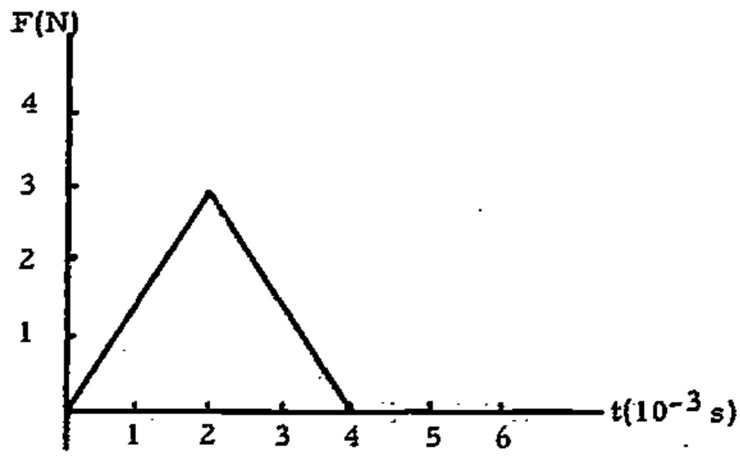


Figure 13

Practice Test Answers

1. The linear momentum of the ball is certainly not conserved. The ball's momentum has flipped direction owing to the force from the wall. If you include the wall and everything it is attached to in your system then momentum must be conserved: no external forces act. It is not easy to visualize the wall's momentum changing during the collision, but it does. Its great mass permits a very small velocity change.

2. Use $\int_{t_i}^{t_f} \vec{F}_{ext}\, dt = \Delta\vec{P}$.

The system is the three masses and the springs. The spring forces are internal. The only external force is provided by the meteorite.

$\int \vec{F}\, dt = 3.00$ N s.

Thus, $\Delta\vec{P} = 3.00$ N s in the direction of the external force.

3. (a) $\vec{r}_{c.m.} = x_{c.m.}\hat{i} + y_{c.m.}\hat{j} + z_{c.m.}\hat{k}$, where

$$x_{c.m.} = \sum_{i=1}^{N} xm/\sum_{i=1}^{N} m + \sum_{i=1}^{n} (\int x\, dm/\int dm),$$

$$y_{c.m.} = \sum_{i=1}^{N} ym/\sum_{i=1}^{N} m + \sum_{i=1}^{n} (\int y\, dm/\int dm),$$

$$z_{c.m.} = \sum_{i=1}^{N} zm/\sum_{i=1}^{N} m + \sum_{i=1}^{n} (\int z\, dm/\int dm)$$

for particles and extended bodies. You must know all the masses and positions of the particles; and the positions, shapes, and density distributions of the extended bodies.

(b) $$\vec{P} = \underbrace{\sum_{i=1}^{N} m\vec{v}}_{\text{particles}} + \underbrace{\sum_{i=1}^{n} m\vec{V}_{c.m.}}_{\text{extended bodies}}$$

You must know the masses and velocities of the particles; and the masses and velocities (of any portion) of the extended bodies.

IMPULSE AND MOMENTUM

Mastery Test Form A

Date ______________

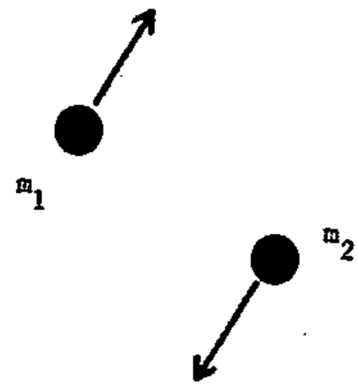
pass			recycle
1	2	3	4

Name ______________________ Tutor ______________________

1. (a) Write the formulas for the center of mass of any system. Explain all the terms.

   (b) Write the formulas for the linear momentum of any system that has no rotation. Explain all the terms.

2.



Two particles are moving apart as shown in the figure above. The mass m_1 is 5.0 kg and m_2 is 3.00 kg; at the instant shown they are 6.0 m apart. Each has a speed of 15.0 m/s.

   (a) Find the center of mass of this system at the instant shown.

   (b) Find the total linear momentum of this system at the instant shown.

   (c) Find the velocity of the center of mass of this system at the instant shown.

3. A 2.0-kg ball falls at a constant speed of 0.100 m/s through a viscous fluid. What is the force of the fluid on the ball?

4. Attack or defend the following statement: If a system is made large enough its linear momentum is always conserved.

IMPULSE AND MOMENTUM

Date ____________

pass recycle

Mastery Test Form B

1 2 3 4

Name ______________________ Tutor ______________________

1. (a) You are shown a system of particles and larger bodies in motion. What would you have to know about the system to calculate its center of mass? How would you calculate the center of mass of this system?

   (b) None of the bodies in the above system is rotating. What would you have to know about the system to calculate its linear momentum? How would you calculate the linear momentum of the system?

2. (a) Starting with a statement for the conservation of linear momentum, show that it takes an external force to accelerate the center of mass of a system with constant mass.

   (b) You are a prisoner in a 4.0-m-long boxcar whose frictionless wheels are 1.50 m from the top of a downhill grade (see Figure 1). If you can get the car to start rolling downhill, you can escape to friendly territory. The end of the car nearest the grade is stacked directly over the wheels with 1000 50-kg gold bars. The car has a mass of 40 000 kg. How many bars must you move to escape? Assume you can move the gold the full 4.0 m and ignore your mass.

3. A vertical rod is connected to a 40-kg particle (see Figure 2). The rod exerts a time-varying force on the particle, which can be calculated from the function

   $\vec{F} = (200 + 300t)\hat{j}$ N.

   (a) What is the force on the particle at 0 s? Do not neglect gravity.

   (b) Calculate the particle's change of linear momentum between the first and second seconds.

   (c) Is there a time when the linear momentum of the particle is not changing? If so, calculate this time.

   (d) Is there any component of the particle's linear momentum that is always conserved? If so, what is it and why is it always conserved?

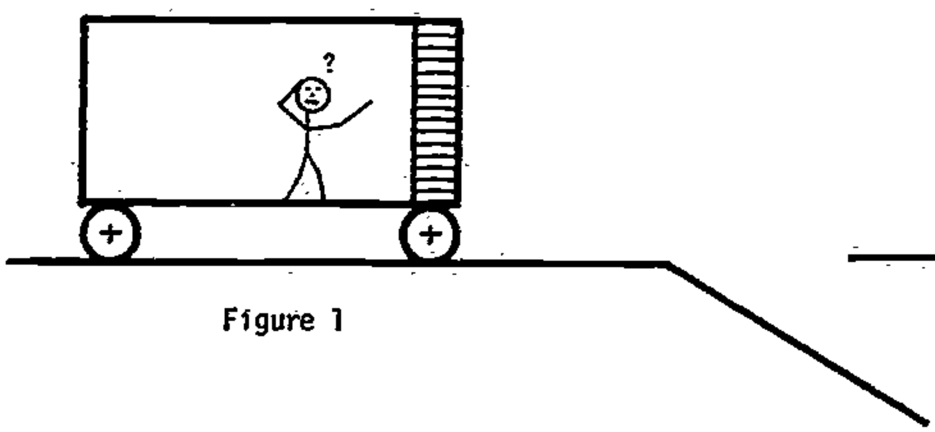


Figure 1

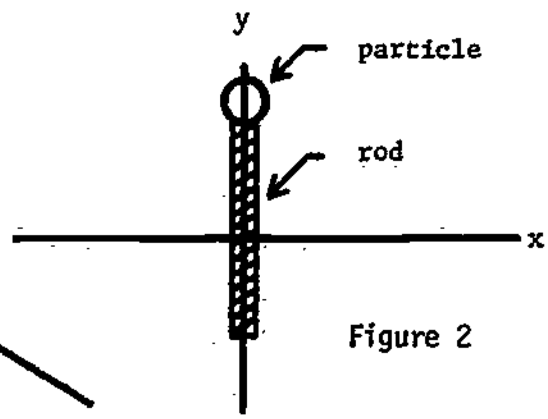


Figure 2

IMPULSE AND MOMENTUM

Date ____________

pass recycle

Mastery Test Form C

1 2 3 4

Name ____________ Tutor ____________

1. (a) Write the formula for the center of mass of any system. Explain what you would need to know about the system in order to calculate it.

   (b) Write the formula for the linear momentum of any system of nonrotating masses moving in straight lines. Explain what you would need to know about the system in order to calculate it.

2. A 30.0-kg particle is suspended by a string. The string is pulled upward and exerts a time-varying force on the particle. The magnitude of this force is given by the function

   $F = (350 + 150t^2)\ N.$

   (a) What is the force on the particle at 0 s?

   (b) Calculate the change in the linear momentum of the particle between the second and third seconds.

   (c) Is the change of the particle's linear momentum per second constant? Why?

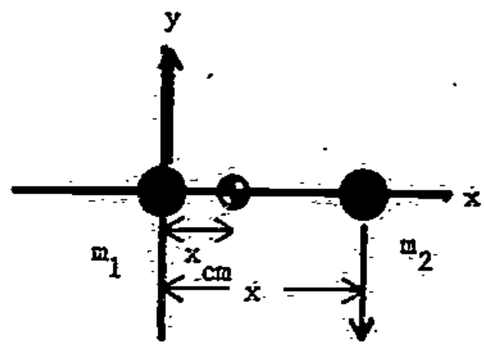
3. For which of the following systems (underlined) is linear momentum conserved? Justify your answers.

   (a) <u>Two colliding billiard balls</u>, rolling on a pool table.

   (b) <u>A canoe with three nonpaddling occupants</u> on a smoothly flowing river.

MASTERY TEST GRADING KEY - Form A

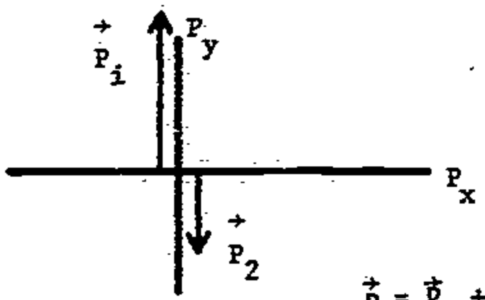
What To Look For	Solutions
1. System includes particles <u>and</u> extended bodies. The student must show some realization that the location of the c.m. is with respect to a set of axes. Two- or three-dimensional answer. If in part (b) the student introduces the troubles with nonrigid bodies, you must make a comment to him that this is too advanced for this course.	1.(a) The answer must combine the c.m. coordinates for all the masses into a single set of coordinates: either $X_{c.m.}$, $Y_{c.m.}$, $Z_{c.m.}$ or $\vec{r}_{c.m.}$ $X_{c.m.} = \sum_{i=1}^{N} X_i m_i / \sum_{i=1}^{N} m_i + \sum_{i=1}^{n} (\int X\,dm/\int dm)$, for $Y_{c.m.}$ and $Z_{c.m.}$, or $\vec{r}_{c.m.} = X_{c.m.}\hat{i} + Y_{c.m.}\hat{j} + Z_{c.m.}\hat{k}$. (b) $\vec{p} = (\sum_{i=1}^{N} m_i \vec{v}_i)_{\text{particles}} + (\sum_{j=1}^{N} M\vec{V}_{c.m.})_{\text{extended bodies}}$. Other forms and an answer in components are OK.
2. The student's choice of coordinates. Comment on an awkward choice, and suggest a better one. However, a poor choice will <u>not</u> make the problem wrong. Directions should be given for all vector quantities.	2.(a)



$y_{c.m.} = 0$; placement of coordinate system.

$$x_{c.m.} = (0 + m_2 x)/(m_1 + m_2)$$
$$= (3.00\text{ kg})(6.0\text{ m})/8.0\text{ kg} = 2.3\text{ m}.$$

2.(b)



$$\vec{p} = \vec{P}_1 + \vec{P}_2 = m_1\vec{v}_1 + m_2\vec{v}_2.$$

$$\vec{P} = m_1v_1\hat{j}_1 - m_2v_2\hat{j} = (m_1v_1 - m_2v_2)\hat{j}$$

$$= [(5.0\text{ kg})(15.0\text{ m/s}) - (3.00\text{ kg})(15.0\text{ m/s})]\hat{j}$$

$$= 30\ \hat{j}\text{ kg m/s}.$$

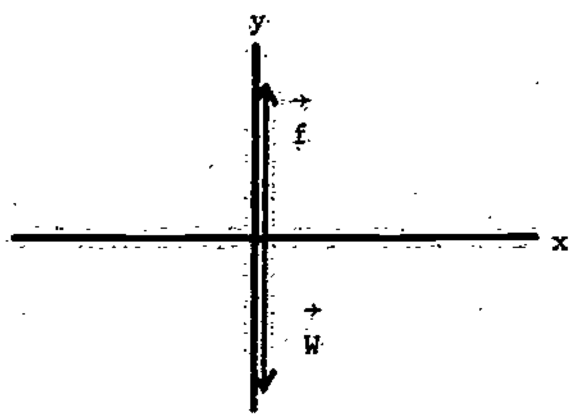
(c) $\vec{P} = M\,\vec{V}_{c.m.}$ and

$$\vec{V}_{c.m.} = \frac{\vec{P}}{M} = \frac{3.00\hat{j}\text{ kg m/s}}{8.0\text{ kg}} = 3.75\hat{j}\text{ m/s}.$$

3.

The velocity of the ball is constant, and thus so is its linear momentum. The total external force on the ball is zero. There are two external forces acting on the ball: its weight and the force from the liquid.

$$\vec{f} = \vec{W} = m\vec{g} = (2.00)(9.8)\hat{j}\text{ kg m/s}^2 = 19.6\hat{j}\text{ N}.$$



The statement is true. By increasing the size of the system, more and more of the forces become internal forces. Some students are clever enough to attack the statement successfully. You must be equally clever in analyzing their arguments. For example, if it is argued that very small external forces will cause unmeasurable momentum changes to large masses, and therefore all the forces do not have to be internal forces, you must accept this. However, you might mention that an improvement in the technology of momentumometers might make their argument wrong. Do not accept arguments based on one-particle universes.

MASTERY TEST GRADING KEY - Form B

What To Look For	Solutions

1\. System includes particles and extended bodies. The student must show some realization that the location of the c.m. is with respect to an axis. Two- or three-dimensional answer. If in part (b) the student introduces the troubles with nonrigid bodies, you must make a comment to him that this is too advanced for this course.

1.(a) The answer must combine the c.m. coordinates for all the masses into a single set of coordinates: either $X_{c.m.}$, $Y_{c.m.}$, $Z_{c.m.}$ or $\vec{r}_{c.m.}$

$$X_{c.m.} = \sum_{i=1}^{N} X_i m_i / \sum_{i=1}^{N} m_i + \sum_{i=1}^{n} (\int X\, dm/\int dm),$$

for $Y_{c.m.}$ and $Z_{c.m.}$, or

$$\vec{r}_{c.m.} = X_{c.m.}\hat{i} + Y_{c.m.}\hat{j} + Z_{c.m.}\hat{k}.$$

You must know mass and position of all particles; and position, shape, and density distribution of the extended bodies.

(b) $$\vec{P} = \left(\sum_{i=1}^{N} m_i \vec{V}_i\right)_{\text{particles}} + \left(\sum_{j=1}^{n} M\vec{V}_{c.m.}\right)_{\text{extended bodies}}.$$

You must know mass and velocity of all particles. For extended bodies you only need to know the total mass and the velocity of any part of the body.

2.(a) Start with

$$\vec{F}_{ext} = M\,(d\vec{V}_{c.m.}/dt) \quad \text{or}$$

$$\int \vec{F}_{ext}\, dt = \Delta\vec{P} = \Delta(M\vec{V}_{c.m.}),$$

and argue that if the left-hand sides of these equations are not zero, neither will the right-hand sides be zero. Recognize that the right-hand sides, which all have a change in velocity, contain the acceleration of the center of mass.

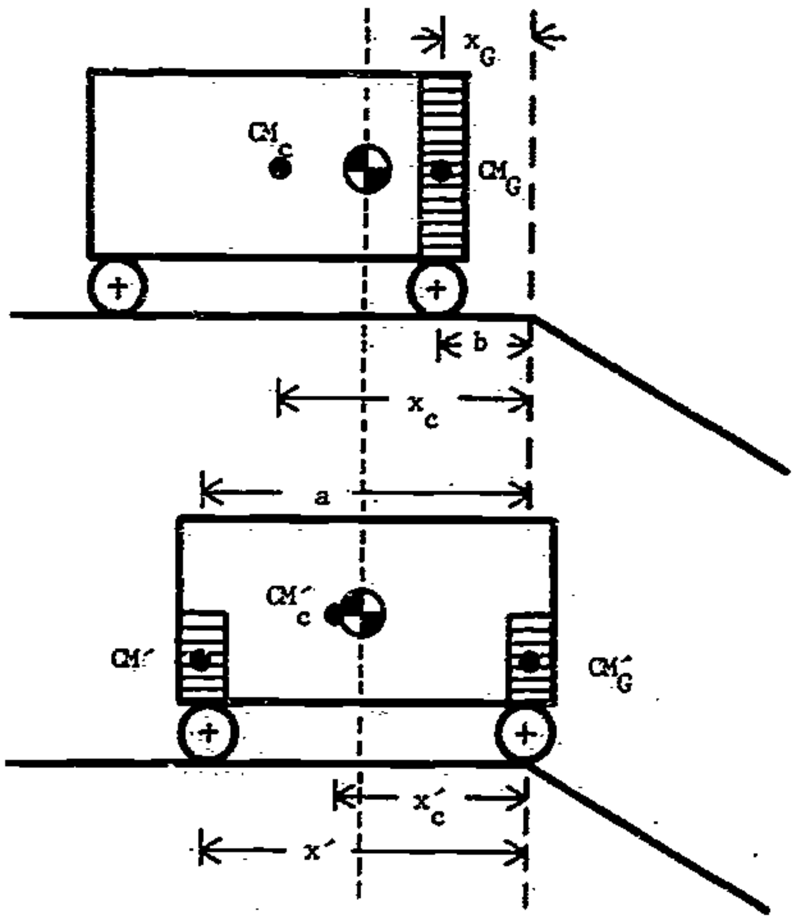
2.(b) The student should realize that here is a case where the center of mass will not accelerate. Since it is initially at rest it will continue to be at rest relative to some place outside the system. Of

(b) The c.m. of the system does not move. As the bars are moved to the left the car must roll to the right.

$$X_{c.m.} = (M_C X_C + M_G X_G)/(M_C + M_G),$$

$$X'_{c.m.} = (M_C X'_C + M_G X'_G + M' X')/(M_C + M_G + M').$$

What To Look For	Solutions
course, once the front wheels move over the edge of the incline a net external force acts, and the center of mass accelerates.	



Neglect prisoner's mass.

$X_{c.m.} = X'_{c.m.}$, $X_c = a/2 + b$,

$X_G = b$, $X'_C = a/2$, $X'_G + 0$,

$X' = a$, $M_C + M_G = M_C + M'_G + M'$,

$M_C\,(a/2 + b) + M_G b = M_C\,(a/2) + 0 + M'a.$

Solving for M':

$M' = (M_C b + M_G b)/a = 33\ 750$ kg.

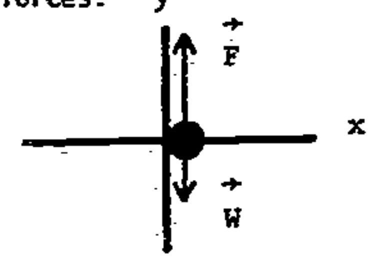
What To Look For | Solutions

Number of blocks moved is

$$n = \frac{31\ 250\ \text{kg}}{50\ \text{kg/block}} = 675 \text{ blocks.}$$

3. Magnitudes and directions of vectors. Recognition that this is mainly an impulse problem. In part (c) there will be an instant when the particle's momentum change is zero. It is poor practice to say that momentum is conserved then.

3.(a) Two forces:



At $t = 0$, $\vec{F} = 200\hat{j}$ N, $\vec{W} = mg(-\hat{j})$.

Total force $= \vec{F} + \vec{W} = -192\hat{j}$ N.

(b) Use $\int_{t_i}^{t_f} \vec{F}\,dt = \Delta\vec{p}$,

$$\int_{1\text{ s}}^{2\text{ s}} [(-192 + 300t)\hat{j}]\,dt = 258\hat{j} \text{ kg m/s.}$$

(c) Yes, when $\vec{F} = 0$. $\vec{F} = [(-192 + 300t)]\hat{j} = \vec{0}$

when $t = 192/300 \approx 0.64$ s.

(d) There are no horizontal forces acting on the particle. Thus the horizontal components of the linear momentum remain constant.

$$\int \vec{F}\,dt = \Delta\vec{P}$$

and if $\vec{F} = \vec{0}$, $\Delta\vec{P} = 0$.

MASTERY TEST GRADING KEY - Form C

What To Look For	Solutions

What To Look For

1. System includes particles and extended bodies. The student must show some realization that the location of the c.m. is with respect to an axis. Two- or three-dimensional answer. If in part (b) the student introduces the troubles with nonrigid bodies, you must make a comment to him that this is too advanced for this course.

Solutions

1.(a) The answer must combine the c.m. coordinates for all the masses into a single set of coordinates: Either $X_{c.m.}$, $Y_{c.m.}$, $Z_{c.m.}$ or $\vec{r}_{c.m.}$.

$$X_{c.m.} = \sum_{i=1}^{N} X_i m_i / \sum_{i=1}^{N} m_i + \sum_{i=1}^{n} (\int X\, dm/\int dm)$$

for $Y_{c.m.}$ and $Z_{c.m.}$, or

$$\vec{r}_{c.m.} = X_{c.m.}\hat{i} + Y_{c.m.}\hat{j} + Z_{c.m.}\hat{k}.$$

You must know mass and position of all particles; and position, shape, and density distribution of the extended bodies.

(b) $$\vec{P} = \left(\sum_{i=1}^{N} m_i\vec{V}_i\right)_{\text{particles}} + \left(\sum_{j=1}^{n} M\vec{V}_{c.m.}\right)_{\text{extended bodies}}$$

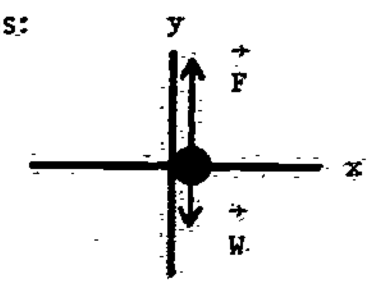
You must know mass and velocity of all particles. For extended bodies you need only know the total mass and the velocity of any part of the body.

What To Look For

2. Magnitudes and directions of vectors. Recognition that this is mainly an impulse problem.

Solutions

2.(a) Two forces:



At $t = 0$, $\vec{F} = 350\hat{j}$ N, $\vec{W} = mg(\hat{j})$.

Force: $\vec{F} + \vec{W} = 56\hat{j}$ N.

(b) Use $\int_{t_i}^{t_f} \vec{F}\, dt = \Delta\vec{P}$.

$$\int_{2\,s}^{3\,s} [(56 + 150t^2)\hat{j}]\, dt = [(56 + 950)]\hat{j}$$
$$= 1006\hat{j} \text{ kg m/s}.$$

What To Look For	Solutions
3. The student must understand that his answer will depend on the existence of external forces acting on the system.	3. Depending on the viewpoint, both yes and no can be justified: (a) Yes - internal forces between the balls are involved and dominate at the moment of collision; external forces are comparatively small. No - over a slightly longer time interval, the interaction of the balls with the table must be considered; since the collision will result in some slippage of the balls on the table, they are subject to a friction force that may change the momentum of the two-ball system. (b) Yes - the net force is zero, vertical forces of water and gravity cancel, no paddling means zero horizontal force; motion of river is smooth, does not give rise to force. No - if river flows around curve.

Module ___

STUDY GUIDE

ROTATIONAL MOTION

INTRODUCTION

There is a motion of a system of masses that is as simple as the motion of a point mass on a straight line. It is the rotation of a rigid body about a fixed axis. For example, we live on a rotating earth, use rotating devices such as a potter's wheel or a phonograph turntable, and test our luck with a spinning roulette wheel. All of these are objects whose motion is described by the time dependence of a single variable, the angle of rotation. We shall study the angular equivalent of uniformly accelerated motion for some rotating objects.

This module also begins the study of rotational dynamics by introducing the dynamical quantities torque and angular momentum for a point mass moving in a plane.

PREREQUISITES

Before you begin this module, you should be able to:	Location of Prerequisite Content
*Calculate the vector product of two given vectors (needed for Objectives 1 and 2 of this module)	Vector Multiplication Module
*Describe the motion of a body in uniform circular motion (needed for Objectives 1 and 2 of this module)	Planar Motion
*Mathematically describe the change of linear momentum of a particle or system of particles as a function of time (needed for Objective 3 of this module)	Impulse and Momentum Module
*Apply Newton's second law to the solution of mechanical problems (needed for Objective 3 of this module)	Newton's Laws Module

LEARNING OBJECTIVES

After you have mastered the content of this module, you will be able to:

1. Rotational kinematics - Define angular displacement, velocity, and acceleration for the case of rotation of a rigid body about a fixed axis; for the case of constant angular acceleration, use the relation among these quantities to solve problems in rotational motion.
2. Angular-linear relation - Using the solution of a problem in angular variables, determine the linear displacement, velocity, and acceleration of a point on the rotating body.
3. Rotational dynamics - Define torque and angular momentum and apply them to a point mass moving in a plane. For some specific examples, calculate torque and angular momentum from force and velocity; show in such examples that the time rate of change of angular momentum is equal to the torque.

TEXT: Frederick J. Bueche, Introduction to Physics for Scientists and Engineers (McGraw-Hill, New York, 1975), second edition

SUGGESTED STUDY PROCEDURE

Read the General Comments. Then read Chapter 10; be sure you understand the definition of angle measured in radians as introduced in Section 10.1 and used in Eq. (10.6) and Figure 10.6. Section 10.2 is interesting but not relevant, since we limit this module to rotations about a single fixed axis. The relations of angular kinematics are summarized in Eqs. (10.2), (10.4), and (10.6). Illustrations 10.1 and 10.2 show how these equations are used, as do Problems A and C of this module. Sections 10.5 and 10.6 show how to determine linear accelerations from the angular quantities. This completes Objectives 1 and 2.

Section 11.1 in Chapter 11 introduces the dynamic relation between torque and angular acceleration. As noted in the General Comments, $mr^2\alpha$ is just the time rate of change of angular momentum if r is constant. Section 11.2 reminds you of the vector product; with this in hand, you can then go to Section 12.5 in Chapter 12 where the basic relation $\vec{\tau} = d\vec{J}/dt$ is developed ($\vec{J} = \vec{L} \equiv$ angular momentum). See Problems B and D for applications of their result. Remember that if $\vec{r}$, $\vec{F}$, $\vec{v}$ are all in the same plane, then $\vec{\tau}$ and $\vec{J}(\vec{L})$ are perpendicular to that plane. Work the problem below before taking the Practice Test.

Problem

A 3.00-kg particle is at x = 3.00 m, y = 8.0 m with a velocity of $\vec{v} = (5.0\hat{i} - 6.0\hat{j})$ m/s. It is acted on by a 7.0-N force in the negative x direction. (a) What is the angular momentum of the particle about the origin? (b) What torque about the origin acts on the particle? (c) At what rate is the angular momentum of the particle changing with time? (See Answer below.)

BUECHE

Objective Number	Readings	Problems with Solutions		Assigned Problems		Additional Problems
		Study Guide	Text*	Study Guide	Text	
1	Secs. 10.1 to 10.4	A	Illus. 10.1, 10.2	C	Chap. 10, Probs. 3-5	Chap. 10, Probs. 1, 2, 6
2	Secs. 10.5, 10.6	A		C	Chap. 10, Probs. 7, 8	
3	Secs. 11.1, 11.2, 12.5	B		D	Chap. 11, Probs. 1, 2; Chap. 3, Prob. 16	

*Illus. = Illustration(s).

Answer: (a) $-174\hat{k}$ kg m^2/s. (b) $+56\hat{k}$ N m. (c) $+56\hat{k}$ kg m^2/s^2.

TEXT: David Halliday and Robert Resnick, Fundamentals of Physics (Wiley, New York, 1970; revised printing, 1974)

SUGGESTED STUDY PROCEDURE

Read the General Comments. Then read Chapter 10; be sure you understand the definition of θ in radians, and remember that our relations between linear and angular quantities are true only for θ, ω, α in radian measure. Table 10-1 summarizes the relations between kinematical variables, but beware of Eqs. (3-14) and (10-4); v and ω are functions of time, as you see in Eqs. (3-12) and (10-3). Example 1 illustrates the use of these equations. Section 10-4 relates the linear velocity and acceleration to the appropriate angular quantities. Figure 10-5 and Example 2 should make these relations clear. Now do Problems A and C, and the assigned problems in Chapter 10. This completes Objectives 1 and 2.

Before starting Chapter 11, you may want to review the definitions of vector (cross) product in Section 2-4 (pp. 19, 20). As you read Sections 11-2 and 11-3, keep in mind that if $\vec{F}$, $\vec{r}$, and $\vec{v}$ all lie in a plane, then the torque $\vec{\tau}$ and angular momentum L are always perpendicular to that plane. See Figures 11-1 and 11-3. Example 1 illustrates these concepts in a nontrivial case (same as Problem 8). Also work through Problem D and the assigned problems of Chapter 11. Take the Practice Test.

HALLIDAY AND RESNICK

Objective Number	Readings	Problems with Solutions		Assigned Problems		Additional Problems
		Study Guide	Text*	Study Guide	Text	
1	Secs. 10-1 to 10-3	A	Chap. 10, Ex. 1	C	Chap. 10, Probs. 9, 10, 13	Chap. 10, Probs. 7, 11, 15, 16
2	Sec. 10-4	A	Chap. 10, Ex. 2	C	Chap. 10, Probs. 1, 4	Chap. 10, Probs. 16, 19, 20
3	Secs. 11-1 to 11-3	B	Chap. 11, Ex. 1	D	Chap. 11, Probs. 1, 3, 7	Chap. 11, Probs. 2, 4-6

*Ex. = Example(s).

TEXT: Francis Weston Sears and Mark W. Zemansky, University Physics (Addison-Wesley, Reading, Mass., 1970), fourth edition

SUGGESTED STUDY PROCEDURE

Read the General Comments. Then read Sections 9-1 through 9-4 in Chapter 9. Be sure you understand the definition of angular measure in radians; the relations found in Section 9-5 are true only for θ measured in radians. The relations between the kinematic variables are derived in Section 9-4. The example in Section 9-4 illustrates their use, as do Problems A and C of this module. The relation between linear accelerations of a point on a rotating body and the angular variables is developed in Section 9-5. This completes Objectives 1 and 2.

Torque is defined in Section 3-1 of Chapter 3, and applied to the rotational dynamics of a point mass m_i in a rotating body in Section 9-6. We shall defer discussion of an extended body and the moment of inertia I until the module Rotational Dynamics. For a point mass, I is mR^2. Angular momentum and torque are discussed for a point mass in Section 9-9. Notice that if $\vec{r}$, $\vec{F}$, and $\vec{v}$ all lie in the same plane, then the torque and angular momentum are parallel to each other and perpendicular to the plane. Problems B and D illustrate these ideas. Work the problems below before taking the Practice Test.

SEARS AND ZEMANSKY

Objective Number	Readings	Problems with Solutions		Assigned Problems		Additional Problems
		Study Guide	Text	Study Guide	Text	
1	Secs. 9-1 to 9-4	A	Sec. 9-4, Example	C	9-2 to 9-4	9-1
2	Sec. 9-5	A		C	9-6, 9-9	9-5, 9-7, 9-8
3	Secs. 3-1, 9-6, 9-9	B		D	3-1, 9-42	

Problems

1. A 2.00-kg particle is initially at the location $x = 4.0$ m and $y = 0$ m. The particle is subject to a constant force of 6.0 N in the negative y direction. Relative to the point (10.0 m, 0 m): (a) What is the particle's angular momentum, and (b) the torque on the particle, both as functions of time? (c) Show for this particular example that torque equals time rate of change of angular momentum. Notice that this is just like Problem B in the Problem Set.

2. A 3.00-kg particle is at $x = 3.00$ m, $y = 8.0$ m with a velocity of $\vec{v} = (5.0\hat{i} - 6.0\hat{j})$ m/s. It is acted on by a 7.0-N force in the negative x direction. (a) What is the angular momentum of the particle? (b) What torque acts on the particle? (c) At what rate is the angular momentum of the particle changing with time?

Answers: 1. (a) $(36t)\hat{k}$ kg m^2/s. (b) $36\hat{k}$ N m.

2. (a) $-174\hat{k}$ kg m^2/s. (b) $+56\hat{k}$ N m. (c) $+56\hat{k}$ kg m^2/s^2.

TEXT: Richard T. Weidner and Robert L. Sells, Elementary Classical Physics (Allyn and Bacon, Boston, 1973), second edition, Vol. 1

SUGGESTED STUDY PROCEDURE

Read the General Comments. Then go to Section 4-5 in Chapter 4 for the definitions of angle and angular speed. Section 12-1 in Chapter 12 discusses linear and angular motion with constant acceleration. Remember that the equations that relate linear and angular quantities (e.g., $s = R\theta$) must have angles in radians. Equations (12-8) and (4-16) relate the components of linear acceleration to the angular variables and radius. This is a good place to go over Problems A and C, and work the problems in Chapters 4 and 12. This completes Objectives 1 and 2.

Sections 11-1 and 11-2 of Chapter 11 give a general discussion of angular velocity and angular momentum. You will probably want to review the definition of the vector (cross) product in Sections 2-6 and 2-7 before trying Sections 11-3 and 11-4. Remember that for the case of a particle moving in a plane, the angular momentum about a point in the same plane is always normal to the plane. The same is true of torque if the force lies in the plane; Figures 11-9 and 11-10 illustrate this point. At this point, study Example 11-1 and Problems B and D. Try the Practice Test.

WEIDNER AND SELLS

Objective Number	Readings	Problems with Solutions		Assigned Problems		Additional Problems
		Study Guide	Text*	Study Guide	Text	
1	Secs. 4-5, 12-1	A		C	12-1, 12-3	12-4
2	Secs. 4-4 to 4-6, 12-1	A		C	4-23	12-2, 12-6
3	Secs. 11-1 to 11-4	B	Ex. 11-1	D	11-1, 11-3	11-2, 11-4, 11-8, 11-9

*Ex. = Example(s).

GENERAL COMMENTS

For rotation about a fixed axis, each point in a body moves in a circle concentric with the axis of rotation (see Fig. 1). This radius R of the circle is the perpendicular distance of the point from the axis. The linear and angular motion have a simple relationship:

$$s = R\theta, \quad ds/dt = v = R\omega = R\, d\theta/dt, \quad dv/dt = a = R\alpha = R\, d\omega/dt,$$

where s, v, and a are tangential linear distance, velocity, and acceleration, respectively; θ, ω, α are the corresponding angular quantities. The above equations imply that angles θ are measured in radians; other common units for angle are degrees (e.g., 45°) or revolutions (e.g., 10 r). They are related by 2π rad = 360° = 1 r. You may wish to review the relations for linear motion given in Rectilinear Motion. Because of the above relationships between linear and angular variables, one can make the table (constant acceleration):

Linear	Angular
$s = v_0 t + (a/2)t^2$	$\theta = \omega_0 t + (\alpha/2)t^2$
$ds/dt = v = v_0 + at$	$d\theta/dt = \omega = \omega_0 + \alpha t$
$v^2 = v_0^2 + 2as$	$\omega^2 = \omega_0^2 + 2\alpha\theta$

Note that a is tangetial acceleration. A particle moving on a circle of radius R also has a radial acceleration $a_r = -v^2/R = -\omega^2 R$. In either the linear or angular case, the first two equations arise by integration of $d^2s/dt^2 = a$ (const) or $d^2/dt^2 = \alpha$(const). The last equation is obtained by eliminating t from the first two; it also follows from conservation of energy.

In order to discuss dynamics, i.e., the relation between forces and acceleration, we start from Newton's second law:

$$F = (d/dt)m\vec{v} = d\vec{p}/dt$$

for a point mass m. In discussing rotations, we note that a force $\vec{F}$ is most effective in producing an angular acceleration if it is applied far from the axis of rotation, and is directed perpendicular to both a line from the axis of rotation and the axis of rotation. For example, a door is most easily opened if the knob is as far from the hinge as possible, and the force is perpendicular to the plane of the door.

The mathematical quantity that has those properties is $\vec{r} \times \vec{F}$, which is called torque. The vector $\vec{r}$ is measured from the axis of rotation or from an origin. The vector product $\vec{r} \times \vec{F}$ is largest when $\vec{r}$ and $\vec{F}$ are perpendicular, and the direction of $\vec{r} \times \vec{F}$ is along the axis of the rotation caused by $\vec{F}$. We can introduce torque into the equation of motion of a point mass as follows (see Fig. 2). Start with Newton's second law:

$$\vec{F} = m\vec{a} = m(d\vec{v}/dt).$$

Form the vector product of $\vec{r}$ with each side:

$$\vec{r} \times \vec{F} = \vec{r} \times m(d\vec{v}/dt).$$

Now introduce the angular momentum $\vec{L}$ defined by $\vec{L} = \vec{r} \times m\vec{v}$, and notice that

$$d\vec{L}/dt = (d/dt)(\vec{r} \times m\vec{v}) = (d\vec{r}/dt) \times m\vec{v} + \vec{r} \times m(d\vec{v}/dt), \; d\vec{L}/dt = \vec{r} \times m(d\vec{v}/dt).$$

The last equation follows from the fact that

$$(d\vec{r}/dt) \times m\vec{v} = \vec{v} \times m\vec{v} = 0.$$

(The vector product of parallel vectors is zero.) We can thus write

$$\vec{r} \times \vec{F} = (d/dt)(\vec{r} \times m\vec{v}) = d\vec{L}/dt, \quad \text{TORQUE} = (d/dt)(\text{ANGULAR MOMENTUM}).$$

For the case of motion and forces in a plane, torque and angular momentum are always perpendicular to the plane; the angular momentum can be expressed simply in terms of angular velocity as follows:

$$\vec{r} = (r\cos\theta)\hat{i} + (r\sin\theta)\hat{j},$$

$$\vec{v} = [(dr/dt)(\cos\theta) - (d\theta/dt)(r\sin\theta)]\hat{i} + [(dr/dt)(\sin\theta) + (d\theta/dt)(r\cos\theta)]\hat{j},$$

$$\vec{L} = \vec{r} \times m\vec{v}$$
$$= [(r\cos\theta)\hat{i} + (r\sin\theta)\hat{j}] \times m\{[(dr/dt)(\cos\theta) - (d\theta/dt)(r\sin\theta)]\hat{i} + [(dr/dt)(\sin\theta) + (d\theta/dt)(r\cos\theta)]\hat{j}\}.$$

All terms with dr/dt add to zero because the radial component of $\vec{v}$ is parallel to $\vec{r}$ (see Fig. 3):

$$\vec{L} = [mr^2\cos^2\theta(d\theta/dt) + mr^2\sin^2\theta(d\theta/dt)]\hat{k} = (mr^2\omega)\hat{k}.$$

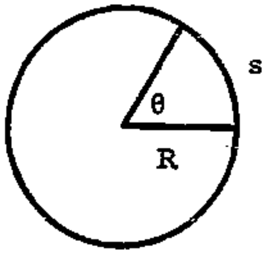


Figure 1

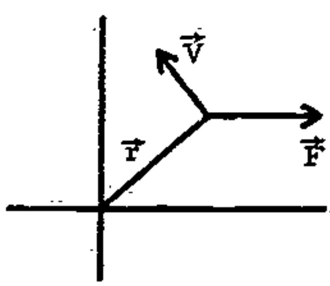


Figure 2

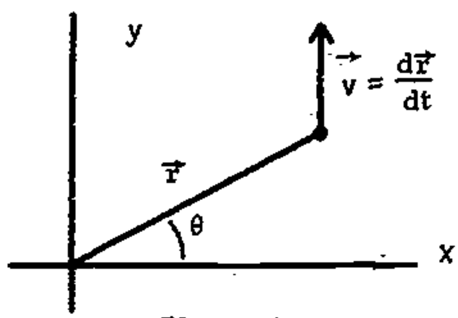


Figure 3

This result is very important and is used in the following module Rotational Dynamics. In the special case that r is constant, we get

$$\tau_z = dL_z/dt = mr^2(d\omega/dt) = mr^2\alpha.$$

This result for a point mass m moving in a circle of radius r is the basis for the treatment of an extended rigid body in Rotational Dynamics. It says that the angular acceleration depends not only on the torque and mass, but also on the distribution of mass (distance from the axis of rotation). The term mr^2 is the rotational inertia, or moment of inertia (I) for a point mass.

PROBLEM SET WITH SOLUTIONS

A(1,2). A phonograph turntable is turning at 3.49 rad/s (33 1/3 r/min) and has a radius of 0.150 m. A friction brake brings it to rest with uniform acceleration in 15.0 s.

(a) What is the angular acceleration of the turntable?

(b) In how many revolutions does it stop?

(c) At ω = 3.00 rad/s while slowing down, what are the radial and tangential accelerations of a point on the turntable rim?

Solution

(a) $\omega = \omega_0 + \alpha t$ has the right variables.

$$\alpha = (\omega - \omega_0)/t = \frac{0 - 3.49\ \text{rad/s}}{15.0\ \text{s}} = -0.233\ \text{rad/s}^2.$$

(b) $\theta = \omega_0 t + (\alpha/2)t^2 = (3.49\ \text{rad/s})(15.0\ \text{s}) + (1/2)(-0.233\ \text{rad/s}^2)(15.0\ \text{s})^2$

$= 26.1\ \text{rad} = 26.1\ \text{rad}\ (1\ \text{r}/2\pi\ \text{rad}) = 4.16\ \text{r}.$

(c) $a_r = -\omega^2 r = -(3.00\ \text{rad/s})^2(0.150\ \text{m}) = -1.35\ \text{m/s}^2.$

$a_t = \alpha r = (-0.233\ \text{rad/s}^2)(0.150\ \text{m}) = -0.035\ \text{m/s}^2.$

B(3). A mass m falls under gravity as shown in Figure 4. The motion is given by $x = s$,

$$y = -(g/2)t^2, \quad V_y = dy/dt = -gt.$$

Calculate the torque and angular momentum about 0 and show that

TORQUE = (d/dt)(ANGULAR MOMENTUM).

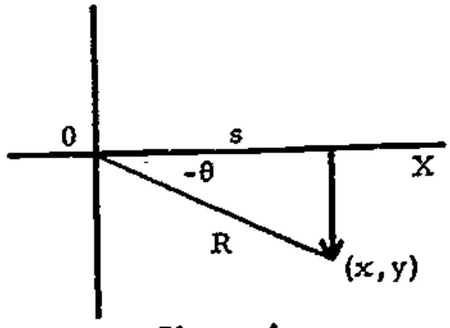


Figure 4

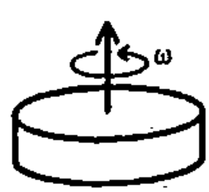


Figure 5

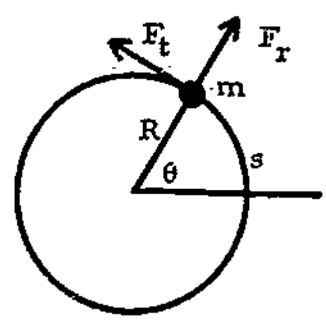


Figure 6

Solution

Since the motion is in a plane all torques and angular momenta are normal to the plane; let $\hat{k}$ be a unit vector out of the paper. Then

$$\vec{\tau} = \vec{r} \times \vec{F} = (S\hat{i} + y\hat{j}) \times (-mg\hat{j}) = -mgS\hat{k},$$

$$\vec{L} = \vec{r} \times \vec{p} = (S\hat{i} + y\hat{j}) \times (-gt\hat{j})m = -mgSt\hat{k}.$$

Notice that $\vec{L}$ is just t multiplied by a constant. Differentiate with respect to time:

$$d\vec{L}/dt = (d/dt)(-mgst\hat{k}) = -mgs\hat{k} = \vec{\tau}.$$

Notice that we can write $\vec{L}$ in terms of angular variables. From Figure 4, we see that

$$y = s \tan\theta, \qquad V_y = dy/dt = (dy/d\theta)(d\theta/dt) = s\omega/\cos^2\theta.$$

Substituting, we find

$$\vec{L} = mV_y s\hat{k} = m(s^2/\cos^2\theta)\hat{k} = mR^2\omega\hat{k}.$$

This is a general result mentioned in the General Comments.

Problems

C(1,2). You live on a disk-shaped asteroid of radius 200 m (see Figure 5).

(a) If the radial acceleration at the edge is 9.8 m/s^2, what is the angular velocity?

(b) The acceleration to the above velocity from rest is done with a tangential linear acceleration of 0.50 m/s^2. How long does it take?

D(3). A point mass m moving on a circle of constant radius R is accelerated from rest ($s = v = 0$ at $t = 0$) by a force whose tangential component F is constant. See Figure 6.

(a) What is the torque about 0.

(b) What is the angular momentum about 0 at time t?

(c) Show that $\vec{\tau} = d\vec{L}/dt$.

Solutions

C(1,2). (a) $a_r = -\omega^2 R$,

$$\omega = (-a_r/R)^{1/2} = (9.8\ m/s^2/200\ m)^{1/2} = 0.221\ rad/s.$$

(b) $a_t = \alpha R$, $\alpha = 0.0025\ rad/s^2$, $\omega = \alpha t$,

$$t = \frac{\omega}{\alpha} = \frac{0.221\ rad/s}{0.0025\ rad/s^2} = 89\ s.$$

D(3). (a) Notice that whereas there is usually a radial component of the force it does not contribute to the torque. Only the component perpendicular to R contributes, so $\vec{\tau} = \hat{k}RF_t$.

(b) Use Newton's second law to get a:

$\vec{F}_t = m\vec{a}$, $a = (F_t/m)t$, then $V = at = (F_t/m)t$.

$\vec{L} = \vec{r} \times m\vec{V} = Rm(F_t/m)t\hat{k} = RF_t t\hat{k}$.

(c) As in the last example, notice that $\vec{L}$ is just a constant multiplied by t:

$d\vec{L}/dt = (d/dt)\hat{k}RF_t t = RF_t\hat{k} = \vec{\tau}$.

PRACTICE TEST

1. While waiting to board a helicopter, you notice that the rotor angular velocity went from 300 r/min to 225 r/min with constant acceleration in one minute. The rotor radius is 5.0 m.

   (a) What is the angular acceleration?

   (b) Assuming constant angular acceleration, how long will it take to stop from 225 r/min?

   (c) How many revolutions will it make in this time?

   (d) At 225 r/min, what are the radial and tangential accelerations at the rotor tip?

2. A planet of mass m moves in a circular orbit about the sun at constant speed v. The orbit has radius R, and a constant magnitude force F is directed toward the sun (see Figure 7).

   (a) What is the torque about the sun?

   (b) What is the angular momentum of the planet about the sun?

   (c) Show that $\vec{\tau} = d\vec{L}/dt$.

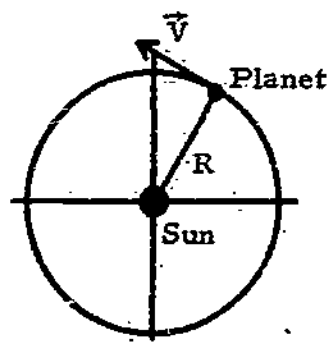


Figure 7

Practice Test Answers

1. (a) $\alpha = -75\ \text{r/min}^2$,
   $\alpha = -0.131\ \text{rad/s}^2$.
   (b) $t = 3.00\ \text{min} = 180\ \text{s}$.
   (c) $\theta = 337.5\ \text{r} = 338\ \text{r}$.
   (d) $a_t = 0.67\ \text{m/s}^2$,
   $a_r = 2776\ \text{m/s}^2$.

2. (a) $\vec{\tau} = 0$, $(\vec{r} \times \vec{F}) = 0$ because $\vec{r}$, $\vec{F}$ are parallel.
   (b) $\vec{L} = mvR\hat{k}$.
   (c) $\vec{\tau} = 0 = d\vec{L}/dt$.

ROTATIONAL MOTION

Mastery Test Form A

Date________________

pass recycle

1 2 3

Name ____________________

Tutor ________________

1. The spin drier of a washing machine initially turning at 20.0 rad/s slows down uniformly to 10.0 rad/s in 50 revolutions. The drier is a cylinder 0.300 m in radius.

   (a) What is the angular acceleration?

   (b) What is the time required for the 50 revolutions?

   (c) What are the radial and tangential accelerations on the side of the drier as it begins to slow down?

2. A point mass m at $r = x\hat{i} + y\hat{j}$ has a velocity $v = v_x\hat{i} + v_y\hat{j}$ and is accelerated by a force $\vec{F} = F\hat{j}$. If $\vec{\tau}$ and $\vec{L}$ are referred to the origin,

   (a) Calculate the torque $\vec{\tau}$ about the origin.

   (b) Calculate the angular momentum $\vec{L}$ about the origin.

   (c) Show that $\vec{\tau} = d\vec{L}/dt$. Remember that $\vec{v} = d\vec{r}/dt$ and $\vec{F} = m\vec{a}$.

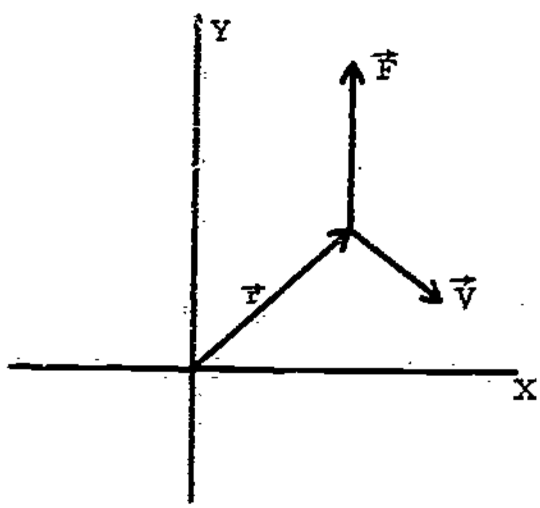


Figure 1

ROTATIONAL MOTION

Mastery Test Form B

Date ____________

pass recycle

1 2 3

Name ____________ Tutor ____________

1. According to measurements made by cesium clocks, the earth is slowing down at a rate $\alpha = -3 \times 10^{-9}$ r/d yr. If this rate is constant,

   (a) How many days (of current length) will it take to stop? 1 d = 86 400 s.

   (b) How many revolutions (sidereal days) will it make in stopping?

   (c) What is the present radial acceleration of a point on the earth's equator?

   ($R = 6.4 \times 10^6$ m, 1 d = 86 400 s.)

2. A pendulum has a mass m on the end of a massless rod of length R and moves under gravity. Using θ as a coordinate, and the result

   $|\vec{A} \times \vec{B}| = |\vec{A}| \cdot |\vec{B}| \sin \alpha,$

   (a) Calculate torque about O.

   (b) Calculate angular momentum about O.

   (c) Show that $d^2\theta/dt^2 = -(g/R) \sin \theta$.

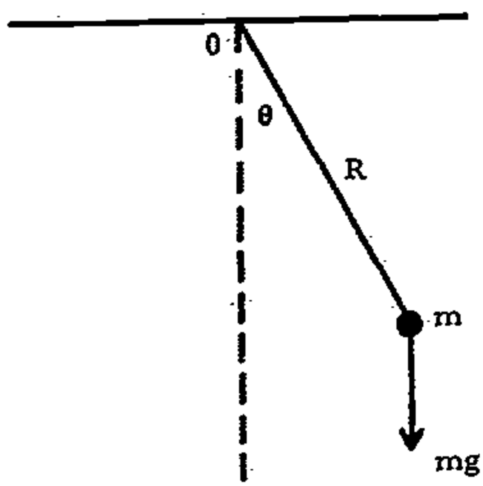


Figure 1

ROTATIONAL MOTION

Mastery Test Form C

Date ____________

pass recycle

1 2 3

Name ____________ Tutor ____________

1. A particle of mass 8.0 kg moves through the point $\vec{r} = (-4.0\hat{i} - 6.0\hat{j})$ m with the velocity $\vec{v} = (6.0\hat{i} + 4.0\hat{j})$ m/s. A force $\vec{F} = (2.00\hat{i} - 3.00\hat{j})$ N acts on the particle.

   (a) What is the torque on the particle with respect to the origin?

   (b) What is the angular momentum of the particle with respect to the origin at this time?

   (c) What is the rate of change of the particle angular momentum?

2. Astronaut training can include work in a centrifuge (rotating cylinder) of 6.0 m radius that spins with an angular velocity $\omega = 2.00$ rad/s.

   (a) What is the radial acceleration at $R = 6.0$ m?

   (b) The angular velocity is increased to 3.00 rad/s in 15 r. What is the angular acceleration, and how long does it take?

Figure 1

MASTERY TEST GRADING KEY - Form A

What To Look For	Solutions
1. Choose correct equation. Convert θ to radians. Correct units and signs.	1. (a) $\alpha = (\omega^2 - \omega_0^2)/2\theta$, $\omega = 10.0$ rad/s, $\omega_0 = 20.0$ rad/s, $\theta = 100\pi$ rad, $\alpha = -0.48$ rad/s^2.
	(b) $t = (\omega - \omega_0)/\alpha = 20.96$ s.
	(c) $a_r = -\omega^2 r = -120$ m/s^2, $a_t = \alpha r = -0.143$ m/s^2.
2. Definitions of $\vec{\tau}$, $\vec{L}$. Evaluation of cross product. Use $V_x = dx/dt$, $F_x = m(dV_x/dt)$.	2. (a) $\vec{\tau} = \vec{r} \times \vec{F} = (x\hat{i} + y\hat{j}) \times F\hat{j}$, $\tau = xF\hat{k}$.
	(b) $\vec{L} = \vec{r} \times m\vec{V} = (x\hat{i} + y\hat{j}) \times m(V_x\hat{i} + V_y\hat{j})$, $\vec{L} = (xV_y - yV_x)\hat{k}\, m$.
	(c) $\frac{d\vec{L}}{dt} = (\frac{dx}{dt}V_y + x\frac{dV_y}{dt} - \frac{dy}{dt}V_x - y\frac{dV_x}{dt})\hat{k}\, m$ $= (x\frac{dV_y}{dt} - y\frac{dV_x}{dt})\hat{k}\, m = \hat{k} \times \vec{F} = \vec{\tau}$.

MASTERY TEST GRADING KEY - Form B

What To Look For	Solutions
1. Choose correct equation. Convert to SI units or carry given units through equation.	1. (a) $t = (\omega - \omega_0)/\alpha$, $t = \dfrac{1.00 \text{ r/d}}{3.00 \times 10^{-9} \text{ r/d yr}}$ $= 3.30 \times 10^8 \text{ yr} = 1.20 \times 10^{11} \text{ d}$.
	(b) $\theta = \omega_0 t + (1/2)\alpha t^2 = 1.00 \text{ r/d} \times 10^8 \text{ yr}$ $= \dfrac{-3.00 \times 10^{-9} \text{ r/d yr}}{2}(3.30 \times 10^8 \text{ yr})^2$ $= 6.1 \times 10^{10} \text{ r}$.
(c) ω must be in radians per second to calculate a_r.	(c) $\omega = \dfrac{2\pi}{86\ 400 \text{ s}} = 7.3 \times 10^{-5} \text{ rad/s}$, $a_r = -\omega^2 r = -0.034 \text{ m/s}^2$.
2. (a) Definition of $\vec{\tau}$.	2. (a) $\vec{\tau} = \vec{r} \times \vec{F} = mgR \sin\theta$ (into paper).
(b) Definition of $\vec{L}$,	(b) $\vec{L} = \vec{r} \times m\vec{v} = mR^2(d\theta/dt)$ (out of paper).
	(c) $d\vec{L}/dt = \vec{\tau}$, $mR^2(d^2\theta/dt^2) = -mgR \sin\theta$, $d^2\theta/dt^2 = -(g/R) \sin\theta$.

MASTERY TEST GRADING KEY - Form C

What To Look For	Solutions
1.(a) Definition of $\vec{\tau}$. Evaluation of vector product. Units.	1.(a) $\vec{\tau} = \vec{r} \times \vec{F} = (-4.0\hat{i} - 6.0\hat{j}) \times (2.00\hat{i} - 3.00\hat{j})$ $= (12.0 + 12.0)\hat{k}$ N m $= 24.0\hat{k}$ N m.
(b) Definition of $\vec{L}$. Vector product. Units.	(b) $\vec{L} = \vec{r} \times m\vec{v} = (-4.0\hat{i} - 6.0\hat{j}) \times 8(6.0\hat{i} + 4.0\hat{j})$ $= 8(-16.0 + 36)\hat{k}$ kg m^2/s $= 160\hat{k}$ kg m^2/s.
	(c) $d\vec{L}/dt = \vec{\tau} = 24.0\hat{k}$ N m.
2.(a) Choose correct equation.	2.(a) $a_r = -\omega^2 r = -(2.00 \text{ rad/s})^2 (6.0 \text{ m})$ $= -24.0 \text{ m/s}^2$.
(b) Choose correct equation. θ is in radians, not revolutions.	(b) $\alpha = (\omega^2 - \omega_0^2)/2\theta$, $\omega_0 = 2.00$ rad/s, $\omega = 3.00$ rad/s, $\theta = 15.0$ r $= 30.0\pi$ rad, $\alpha = 0.0265 \text{ rad/s}^2$, $t = (\omega - \omega_0)/\alpha = 37.7 \text{ s} \simeq 38$ s.