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ABSTRACT

Matrix tasks to assess multiple classifications and multiple seriation skills were administered to 160 children (40 Ss each from preschool, kindergarten, first and second grade levels). Each child received six matrix subtasks (reproduction and transportation of cross classification I, double seriation, and cross classification II) in one of six orders of presentation. Preliminary analyses indicated a general absence of significant presentation order effects and an absence of sex differences. Grade level comparisons were significant for all subtasks except cross classification I transposition. Significantly superior performances on the reporduction when compared to the counterpart transposition subtask were shown for the cross classification I (first grade, second grade, and combined grades), double seriation (kindergarten, first grade, second grade, and combined grades), and cross classification II (first grade and combined grades) cases. The between matrix difficulties were that in both reproduction and transposition subtasks, cross classification II was more difficult than double seriation and cross classification I. Results suggested that development in classificatory abilities may lag behind relational abilities contrary to the structural prediction of orthodox Piagetian theory. (Author/SB)

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an investigation of matrix task classificatory and seriation abilities

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WISCONSIN RESEARCH AND DEVELOPMENT CENTER FOR COGNITIVE LEARNING

Technical Report No. 348

AN INVESTIGATION OF MATRIX TASK CLASSIFICATORY AND SERIATION ABILITIES

by

Frank H. Hooper and Thomas S. Sipple

Report from the Project on
Children's Learning and Development

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ABSTRACT

Matrix tasks to assess multiple classification and multiple seriation skills were administered to 160 children (40 subjects from each of four levels--preschool, kindergarten, first, and second grades). Each child received six matrix subtasks (reproduction and transposition of cross classification I, double seriation, and cross classification II) in one of six orders of presentation. Preliminary analyses indicated a general absence of significant presentation order effects and an absence of sex differences. Grade level comparisons were significant for all subtasks except cross classification I transposition. Significantly superior performance on the reproduction when compared to the counterpart transposition subtask were shown for the cross classification I (first grade, second grade, and combined grades), double seriation (kindergarten, first grade, second grade, and combined grades), and cross classification II (first grade and combined grades) cases. The between matrix difficulties were: (1) reproduction subtasks--cross classification II > double seriation = cross classification I; and (2) transposition subtasks--cross classification II > double seriation = cross classification I.

These results (1) confirm the previous findings of Bruner & Kenney (1966), MacKay, Fraser, & Ross (1970), and Hooper, Sipple, Goldman, & Swinton (1974) concerning the generally lesser difficulty of matrix reproduction compared to transposition; (2) are in contrast to the previous research of MacKay regarding the difficulty of class and series matrices; and (3) suggest that development in classificatory abilities may lag behind relational abilities contrary to the structural predictions of orthodox Piagetian theory.

I

INTRODUCTION

Among the more commonly acknowledged behavioral indices of concrete operational thought, according to Piaget, is the ability to deal with multiplicative classes and relations. Assessments of these abilities have frequently utilized matrix-type formats. Thus, cross classification abilities with discrete categories (e.g., Inhelder & Piaget, 1964, pp. 151-195) and double seriation understanding with continuous dimensions (e.g., Bruner & Kenney, 1966; pp. 154-167; Inhelder & Piaget, 1964, pp. 269-274) have been investigated. Task format and instructional set variations have been found to influence children's performances on cross classification matrices (e.g., Overton & Brodzinsky, 1972; Overton & Jordan, 1971; Smedslund, 1964, 1967a, 1967b). A number of normative studies employing a standardized measure of multiplicative classification, The Raven Progressive Matrices Test, have been conducted (e.g., Shantz, 1967; Sigel, 1963; Storck, 1974; Zelner, 1974).

Studies of multiple seriation skills include Hamel & Van der Veer (1972); Lagatutta (1970); Lovell, Mitchell, & Everett (1962); Shantz (1967); Smedslund (1964); and Steiner (1974). In addition, attempts have been made to devise instructional strategies directed toward furthering children's matrix classification understanding (e.g., Caruso & Resnick, 1971; Jacobs, 1966; Jacobs & Vanderventer, 1968, 1971a, 1971b; Parker, Rieff, & Sperr, 1971; Parker, Sperr, & Rieff, 1972; Shantz & Sigel, 1967) and matrix seriation skills (e.g., Shantz & Sigel, 1967; Steiner, 1974).

Piagetian theory predicts a close developmental relationship between multiplicative class and relations abilities during the concrete operations period of middle-childhood. This follows directly from the fundamental within-stage correspondence assumption; i.e., the conception of stage as representing a *structure d' ensemble* (cf., Brainerd, 1972; Flavell, 1963, 1970; Hooper, 1973; Hooper & Klausmeier, 1973; Pinard & Laurendeau, 1969; Wohlwill, 1963, 1973). In particular, Piaget has consistently postulated developmental synchrony for performances on task settings derived from the classificatory and relational *groupements* (cf., Inhelder & Piaget, 1964, pp. 278-290; Piaget, 1965, pp. 240-243; 1970a, pp. 723-727; 1970b, pp. 24-27 and 65-66).

There have been a relatively small number of previous studies which have examined the developmental interrelationship among multiplicative classification and relationality abilities. The majority of these investigations have utilized cross-sectional assessment designs (see Bingham-Newman, Saunders, & Hooper, 1975; Stephens, 1972; and Wohlwill, Devoe, & Fusaro, 1971 for examples of longitudinal assessments). Lovell et al. (1962) included measures of matrix double seriation (multiplication of

asymmetric transitive relations) and cross classification in a series of task administrations designed to replicate the earlier Inhelder & Piaget (1964) results. They concluded that stage three operational mobility is achieved for these tasks at about the same time in primary school children, although no direct inter-task comparisons or associated inference tests were conducted (i.e., Lovell et al., 1962, Table 2, p. 178).

Smedslund (1964) found that measures of multiple classification and relationality were of approximately equal difficulty for a sample of 160 children ranging in age from 4 to 11 years; i.e., 81 percent of the children either passed or failed both tasks. Shantz (1967) compared the performances of children 7-1/2, 9-1/2, and 11-1/2 years old on multiplication of classes (assessed by the Raven Colored Progressive Matrices Test), multiplication of relations (assessed by using the "diagonals" of 4 x 4 matrices based on various continuous dimensions), and multiplication of infralogical spatial relations (assessed in an adaptation of Piaget's landscape task). Significant rank order correlations between the multiplicative class and relational matrix tasks were found for the two older subsamples.

In another cross-sectional assessment design study, Lagattuta (1970) examined children's abilities to deal with unidimensional classification and seriation (relationality) and matrix format multiplicative classification and seriation tasks. It was found that a child first develops (5-1/2 years of age) the ability to classify a simple arrangement; somewhat later (5-1/2 to 6-1/2 years) the child can successfully deal with a multiple classificatory matrix. Concurrent with this latter acquisition, simple serial skills develop (6-1/2 to 8-1/2 years of age), while the ability to successfully order a serial matrix was shown by the older subjects (8-1/2 years) only. It was tentatively concluded, in apparent contrast to Inhelder & Piaget (1964), that classificatory skills develop independently of and prior to seriation skills. Comparisons of first and second grade children's multiple classification and seriation performances (matrix task formats) are reported by Hamel & Van der Veer (1972). Significant positive intercorrelations were obtained for both the younger (.72) and older (.66) subsamples.

MacKay, Fraser, & Ross (1970), drawing upon the earlier work of Bruner & Kenney (1966) and Inhelder & Piaget (1964), compared the relative difficulties of multiple classification, multiple seriation, and combined class/series matrix tasks for groups of children 5 to 8 years of age. Each child performed on one of the tasks and was required to reproduce and to transpose the presented matrix. The comparisons regarding multiple classification and seriation were derived from an initial experiment involving 90 children, while a second experiment assessed performance of an additional group of 48 children on the class/series matrix. As anticipated, performances on all the tasks improved significantly over the age interval assessed. Matrix reproduction was easier than transposition for the multiple seriation and multiple class/series cases, and this was most notable for the younger (5 to 7 year old) subjects. Combining data from the two samples, it was shown that reproduction of a multiple seriation matrix was more difficult than reproduction of a class matrix which was, in turn, more difficult than the class/series case. Transposition of the seriation matrix was of greater difficulty than either of the other matrices. Transposition of the class and combined class/series task was of approximately equal difficulty. It was concluded that:

1. The ability to construct a matrix composed of discrete categories is developmentally an earlier acquisition than the ability to construct one composed of relational variables.
2. A matrix composed of discrete categories in both directions is of equivalent difficulty to one constructed of discrete categories in one direction and a relational variable in the other.
3. A matrix composed of discrete categories is no more easily reproduced than it is transposed, while matrices where either one or both variables are continuous are more easily reproduced than transposed.
4. The great majority of children under each condition who reproduce the matrix do so as it is presented (MacKay et al., 1970, p. 795).

These results were essentially replicated in a study of 23 severely subnormal adults (average M.A. of 6 years), although the lesser difficulty of the reproduction tasks compared to the transposition tasks was not found.

The immediate precursory investigation to the present study attempted to replicate the MacKay et al. (1970) findings. As part of a large scale investigation of children's classificatory abilities, Hooper, Sipple, Goldman, & Swinton (1974) utilized identical matrix reproduction and transposition tasks to assess the logical multiplicative abilities of 40 subjects at each of seven age levels: preschool, kindergarten, first, second, third, fourth, and sixth grades (overall N=280). There was a significant increase in the number of subjects passing each of the matrix subtasks across this age-grade range. Comparisons across a more restricted age range comparable to that found in the MacKay et al. (1970) study--i.e., preschool (mean age, 5 years) to second grade (mean age, 8 years 2 months)--were also carried out. For each reproduction and transposition subtask there was a significant increase in the proportion of successful subjects; i.e., all X^2 values exceeded 9.40, $df = 3$, $p < .025$. This is essentially similar to the findings of the earlier investigation.

In the MacKay et al. (1970) study, transposition of the matrices which involved a continuous dimension (double seriation and class/series) was a significantly more difficult task than reproduction. Examination of the Hooper et al. (1974) matrix pass/fail frequencies reveals a concordant pattern of relative task difficulties. Chi-square comparisons of the number of successful subjects on the reproduction subtask versus the transposition subtask of the double seriation matrix showed the former to be significantly easier at the preschool (27 vs. 12), kindergarten (28 vs. 13), and first-grade (38 vs. 24) levels. Combining the preschool through second-grade subsamples (N=160), 130 children passed the reproduction case, and 79 passed the transposition subtask ($X^2 = 35.88$, $df = 1$, $p < .001$). Similar comparisons for the class/series matrix also revealed reproduction to be the easier task; i.e., the frequency of passing subjects for reproduction versus transposition for the preschool to second-grade subsamples was 33 versus 21, 37 versus 25, 38 versus 34, and 40 versus 31, and the

total frequencies were 148 versus 111 ($X^2 = 27.73$, $df = 1$, $p < .001$) for the combined younger subject subsamples. In contrast, there was no significant difference between the number of successful reproduction versus transposition cases for the cross classification matrix at any of the separate age-grade levels, although the comparison for the 160 children in the composite younger group--103 passed the reproduction subtask and 85 passed the transposition subtask ($X^2 = 4.18$, $df = 1$, $p < .05$)--was marginally significant. In the preschool through second-grade composite sample, combining all the reproduction and transposition cases for the three matrices indicated that the former task was significantly easier, i.e., $N = 480$ with 381 versus 275 passing cases ($X^2 = 54.09$, $df = 3$, $p < .001$).

A more direct comparison of the relative difficulty of the reproduction versus the transposition matrix subtasks is shown in Table 1. (Note that for Tables 1, 2, and 3, the comparisons within grade levels are binomial tests with one-tailed probabilities for Table 1 and two-tailed probabilities for Tables 2 and 3. The composite subsample comparisons are McNemar Tests for the Significance of Changes with associated X^2 values and one-tailed probabilities for Table 1 and two-tailed probabilities for Tables 2 and 3.) All of the within-grade subsample comparisons on double seriation significantly favor the easier reproduction task, and only the first-grade subsample comparison fails to indicate a similar significant relative difficulty pattern for the class/series matrix. For the cross classification matrix, only the first-grade subsample and the composite preschool to second-grade sample comparison reach significance. It was concluded that matrix reproduction is significantly easier than matrix transposition, and this was particularly true for the double seriation and class/series matrices.

In considering the relative difficulty of the three basic matrix types, the reproduction and transposition cases were examined separately (Hooper et al., 1974, pp. 40-43). The relevant comparisons for the reproduction case are presented in Table 2. Considering initially the three matrices together, Cochran Q values for the number of passing subjects were 17.04 (preschool), 18.00 (kindergarten), 10.89 (first grade), and 9.25 (second grade), indicating significant differences across the matrix reproduction subtasks (all probabilities less than .01). Reproduction of the cross classification matrix was significantly more difficult than the counterpart class/series case at all of the younger age-grade levels and in terms of the composite sample. A similar case of relatively greater task difficulty for cross classification compared to double seriation is also shown; i.e., only the second-grade comparison fails to reach an acceptable significance level. Finally, the double seriation reproduction task appears to be of significantly greater difficulty than the class/series reproduction case, and this is most notable at the preschool and kindergarten age-grade levels where a sufficient degree of inter-task variability (absence of ceiling effects) permits direct comparisons. Thus, the relative task difficulties for the three matrix reproduction cases are as follows: cross classification > double seriation > class/series.

In the matrix transposition task case, the relative difficulties are somewhat less distinct (see Table 3). Considering initially the three matrices together, Cochran Q values for the number of passing subjects were 7.88 (preschool), 9.91 (kindergarten), and 9.94 (first grade), indicating significant differences across the matrix transposition subtasks (all probabilities less than .05). The double seriation transposition task is clearly

TABLE 1

Comparison of the Relative Difficulties of the Reproduction Versus Transposition Matrix Tasks

	CCT x CCR	CST x CSR	DST x DSR
Preschool	CCR + : 10 (+CCT), 7 (−CCT); − : 4 (+CCT), 19 (−CCT)	CSR + : 21 (+CST), 12 (−CST)[a]; − : 0 (+CST), 7 (−CST)	DSR + : 11 (+DST), 16 (−DST); − : 1 (+DST), 12 (−DST)[a]
Kindergarten	CCR + : 14 (+CCT), 8 (−CCT); − : 4 (+CCT), 14 (−CCT)	CSR + : 24 (+CST), 13 (−CST); − : 1 (+CST), 2 (−CST)[a]	DSR + : 12 (+DST), 16 (−DST); − : 1 (+DST), 11 (−DST)[a]
First Grade	CCR + : 23 (+CCT), 8 (−CCT); − : 2 (+CCT), 7 (−CCT)[b]	CSR + : 34 (+CST), 4 (−CST); − : 0 (+CST), 2 (−CST)	DSR + : 24 (+DST), 14 (−DST); − : 0 (+DST), 2 (−DST)[a]
Second Grade	CCR + : 24 (+CCT), 9 (−CCT); − : 4 (+CCT), 3 (−CCT)	CSR + : 31 (+CST), 9 (−CST); − : 0 (+CST), 0 (−CST)[a]	DSR + : 30 (+DST), 7 (−DST); − : 0 (+DST), 3 (−DST)[a]
Combined	CCR + : 71 (+CCT), 32 (−CCT); − : 14 (+CCT), 43 (−CCT)[a]	CSR + : 113 (+CST), 34 (−CST); − : 2 (+CST), 11 (−CST)[a]	DSR + : 77 (+DST), 53 (−DST); − : 2 (+DST), 28 (−DST)[a]

[a] $p < .01$ (one-tailed).
[b] $p < .05$ (one-tailed).

(from Hooper, Sipple, Goldman, & Swinton, 1974, p. 41)

TABLE 2

Comparison of the Relative Difficulties of the Reproduction Matrix Tasks

	CCR x DST	CCR xCSR	CSR x DSR
Preschool	DSR + / − [a]	CSR + / − [a]	DSR + / −
CCR +	13 / 4	15 / 2	CSR + 26 / 7
CCR −	14 / 9	18 / 5	1 / 6
Kindergarten	DSR + / −	CSR + / − [a]	DSR + / − [a]
CCR +	20 / 2	21 / 1	CSR + 27 / 10
CCR −	8 / 10	16 / 2	1 / 2
First Grade	DSR + / − [a]	CSR + / − [b]	DSR + / −
CCR +	30 / 1	31 / 0	CSR + 37 / 1
CCR −	8 / 1	7 / 2	1 / 1
Second Grade	DSR + / −	CSR + / − [b]	DSR + / −
CCR +	32 / 1	33 / 0	CSR + 37 / 3
CCR −	5 / 2	7 / 0	0 / 0
Combined	DSR + / − [a]	CSR + / − [a]	DSR + / − [a]
CCR +	95 / 8	100 / 3	CSR + 127 / 21
CCR −	35 / 22	48 / 9	3 / 9

[a] $p < .05$ (two-tailed).

[b] $p < .01$ (two-tailed).

(from Hooper, Sipple, Goldman, & Swinton, 1974, p. 42)

TABLE 3

Comparison of the Relative Difficulties of the Transposition Matrix Tasks

	CCT x DST	CCT x CST	CST x DST
Preschool	CCT + : 8, 6 / - : 4, 22 (DST + −)	CCT + : 12, 2 / - : 9, 17 (CST + −)	CST + : 11, 10 / - : 1, 18 (DST + −) [a]
Kindergarten	CCT + : 9, 9 / - : 4, 18 (DST + −)	CCT + : 14, 4 / - : 11, 11 (CST + −)	CST + : 11, 14 / - : 2, 13 (DST + −) [a]
First Grade	CCT + : 17, 8 / - : 7, 8 (DST + −)	CCT + : 25, 0 / - : 9, 6 (CST + −)	CST + : 23, 11 / - : 1, 5 (DST + −) [a]
Second Grade	CCT + : 24, 4 / - : 6, 6 (DST + −)	CCT + : 26, 6 / - : 5, 7 (CST + −)	CST + : 25, 6 / - : 5, 4 (DST + −)
Combined	CCT + : 58, 27 / - : 21, 54 (DST + −)	CCT + : 77, 8 / - : 34, 41 (CST + −) [a]	CST + : 70, 41 / - : 9, 40 (DST + −) [a]

[a] $p < .01$ (two-tailed).

(from Hooper, Sipple, Goldman, & Swinton, 1974, p. 43)

more difficult than the class/series matrix case. In addition, the cross classification transposition task is also of greater difficulty than the class/series counterpart, at least insofar as the preschool, first grade, and overall composite sample comparisons are concerned. There is obviously very little difference in the relative transposition task difficulties for the cross classification and the double seriation matrices. Thus, the relative task difficulties for the three matrix transposition cases are as follows: cross classification = double seriation > class/series.

Returning to the reproduction tasks again, Table 4 presents the number and percentage of children who passed the reproduction tasks and who exactly reproduced the various matrices as originally presented to them. The percentages for the various composite sample totals--i.e., 78.6 percent, 80.4 percent, and 98.5 percent for the cross classification, class/series, and double seriation matrix cases, respectively--closely parallel the earlier results of MacKay et al. (1970, pp. 793-794).

TABLE 4

Number and Percentage of Successful Subjects, Across the Younger Grade Levels, Who Exactly Reproduced the Various Matrices

	Cross Classification			Classification Seriation			Double Seriation		
Grade	No. Passed	Exactly Reproduced No.	% of Passing	No. Passed	Exactly Reproduced No.	% of Passing	No. Passed	Exactly Reproduced No.	% of Passing
Pre	17	9	52.94	33	22	66.7	27	27	100
K	22	13	59.09	37	32	87.49	28	28	100
1	31	29	93.55	38	29	76.32	38	36	94.74
2	33	30	90.91	40	36	90.0	37	37	100
Composite	103	81	78.64	148	119	80.41	130	128	98.46

(from Hooper, Sipple, Goldman, & Swinton, 1974, p. 44)

Table 5 presents the types of error responses (color vs. shape, color vs. height, and diameter vs. height) for the matrix performances. The predominant error category for the cross classificatory tasks is color misplacement; i.e., 67.50 percent and 57.14 percent of the reproduction and transposition error cases, respectively, for the overall combined sample. In contrast, for the class/series matrix tasks the present children made more errors on the height dimension (66.67 percent and 65.43 percent) than on the color dimension (33.33 percent and 34.57 percent). Misplacements based on height were also the predominant error

category (contrasted with the width or diameter dimension) for the double seriation cases; i.e., 66.22 percent and 64.62 percent of the total sample error cases for the reproduction and transposition tasks, respectively. This latter tendency contrasts with the previous findings of MacKay (1972, p. 601) which indicated that correct responses on the double seriation transposition task were more likely to focus upon the height dimension than upon the diameter dimension.

TABLE 5

Number of Subjects Failing Each Dimension of Each Matrix Subtask for the Various Age-Grade Levels

Grade	Cross Classification				Classification Seriation				Double Seriation			
	Reproduction		Transposition		Reproduction		Transposition		Reproduction		Transposition	
	color	shape	color	shape	color	height	color	height	diameter	height	diameter	height
Pre	21	11	19	19	3	7	8	19	9	11	11	28
K	13	7	18	13	1	3	9	13	4	12	17	24
1	7	2	10	8	1	2	1	6	1	2	7	8
2	6	3	10	5	0	0	4	8	2	3	5	8
Composit	47	23	57	45	5	12	22	46	16	28	40	74

(from Hooper, Sipple, Goldman, & Swinton, 1974, p. 44)

The fact that the Hooper et al. (1974) results substantiate the pattern that reproduction tasks are significantly less difficult than transposition tasks as found by Bruner & Kenney (1966) and MacKay et al. (1970) presents some assurance that we are indeed dealing with similar behavioral phenomena. In this regard, while the results concerning the comparative difficulty of cross class and the double seriation matrix cases disagree rather sharply with MacKay et al. (1970), the data are in general accord with the original research of Bruner and Piaget. Bruner & Kenney (1966, p. 158) found that 60 percent of the 5 year olds, 70 percent of the 6 year olds, and 80 percent of the 7 year olds could successfully reproduce the double seriation matrix. The comparison percentages for the Hooper et al. (1974) appropriate age-grade groups are 67.5 percent, 70 percent, and 95 percent, respectively. For the double seriation transposition case, the comparison percentage values are in less clear agreement; i.e., 0.0 percent for Bruner and Kenney vs. (30 percent), 28 percent (32 percent), and 80 percent (60.0 percent) for the 5, 6, and 7 year old subjects. In similar fashion, although Inhelder and Piaget did not utilize any direct counterpart to the present reproduction and transposition subtasks, their contention that "children reach an operational level,

in the multiplication of series about the same period (7 to 8 years) as cross classification (1964, p. 278)" agrees with the present case of equivalent difficulty for cross class and double seriation transposition.

The present investigation was designed in accord with two general objectives: (1) to assess the reliability of the Hooper et al. (1974) and MacKay et al. (1970) results concerning cross classification and double seriation matrices, and (2) to assess the relationship of these measures to a new cross classification task which utilized the double seriation case stimulus materials. Subject sampling procedures and reproduction/transposition instructional sets were identical to those reported in Hooper et al. (1974). In accord with the contentions described above, it was predicted that: (1) performance on all of the matrix tasks would be positively related to age-grade level, (2) all reproduction matrix tasks would be of lesser difficulty than their transposition counterparts, (3) the inter-matrix order of difficulty for the reproduction case would be cross classification II > cross classification I > double seriation, and (4) the inter-matrix order of difficulty for the transposition case would be cross classification II > cross classification I = double seriation.

II

METHOD

SUBJECTS

Subjects for the study were school children from the Jefferson and Madison, Wisconsin, school districts. Forty Ss were drawn from each of four grade levels: preschool, kindergarten, first, and second. Distribution of the subject population by age and sex is given in Table 6.

TABLE 6

Distribution of Subjects by Grade, Sex, and Age

Grade	Subjects	Males	Females	Mean Age	Range
Pre	40	20	20	4-5	3-8 to 5-2
K	40	20	20	6-1	5-6 to 6-9
1	40	19	21	7-5	6-10 to 8-3
2	40	20	20	8-3	7-8 to 9-2

MATERIALS

Each matrix task was arranged on a square wooden board sectioned so as to produce nine individual squares each 110 x 110 millimeters.

a. Cross Classification I Matrix

Stimuli were three square wooden blocks, three circle blocks, and three triangle blocks. One block of each shape was red, one was yellow, and one was blue. Each block had a circumference of 204 millimeters and a thickness of 10 millimeters. The blocks were arrayed on the color and shape dimensions.

b. Double Seriation Matrix

Stimuli were nine cylindrical wooden blocks; three were 100 millimeters high, three were 75 millimeters high, and three were 50 millimeters high. One block of each height had a diameter of 100 millimeters, one was 65 millimeters in diameter, and one was 35 millimeters in diameter. All the cylinders were blue. The blocks were arrayed on the width and height dimensions.

c. Cross Classification II Matrix

Stimuli were identical to those for the double seriation matrix. The cylinders were located such that within each row the height remained constant, and within each column the diameter remained constant. However, neither dimension was seriated across the rows or columns of the matrix. Thus, by definition, this task is distinguishable from the double seriation matrix, described previously, which involves the multiplication of asymmetric transitive relations.

PROCEDURE

Three matrix tasks were adapted to assess the development of the abilities of multiple classification and multiple seriation. The tasks were presented in one of the six possible orders of administration.[1] However, the order of administration of a replacement, a reproduction, and a transposition subtask within each matrix was always fixed in that order.

Instructions were identical for each matrix.

a. Replacement

E removed first one, then two, and finally three (diagonally placed) blocks from the matrix, and each time S was asked to put them back where they were before E removed them.

b. Reproduction

E removed all the blocks from the matrix, and S was asked to put them back so the board looked just the same as it did before E removed them.

c. Transposition

E removed all the blocks from the matrix and then placed the block that had originally occupied the lower left-hand position (of the S) in the upper left-hand position. (For the cross classification II case, E placed the block that had originally

[1] Due to an unfortunate procedural error, the number of Ss assigned to two of the six different orders of administration for the three matrix tasks was much smaller than to the other orders.

occupied the middle-right position (of the S) in the center position.) S was then asked to place the blocks on the board so they made a "pattern like they did before."

In order to pass the cross classification I matrix reproduction subtask, a subject was required to classify one dimension in one direction and the other dimension in the other direction. In order to pass the double seriation matrix reproduction subtask, a subject was required to seriate one dimension in one direction and the other dimension in the other direction. In order to pass the cross classification II matrix reproduction subtask, a subject was required to classify the height dimension in one direction and the width dimension in the other direction. In order to pass each of the transposition subtasks, a subject was required to fulfill the same criteria as for the reproduction cases without moving the replaced block.

III

RESULTS

Initial considerations concern the evaluation of order of presentation effects and possible sex differences in the children's matrix task performances. Deleting the two orders containing substantially lower numbers of subjects, significant order of presentation effects were notably absent, with one exception. That is, none of the frequencies of subjects passing each of the six subtasks were effected by the orders in which they were presented, except the cross classification II involution subtask. A notably higher proportion of Ss passed this task within the two orders where the cross classification II task was presented last in the series: 31 percent passing (cross classification I, double seriation, cross classification II) and 36 percent passing (double seriation, cross classification I, cross classification II) in contrast to 7 percent (cross classification I, cross classification II, double seriation) and 14 percent (cross classification II, double seriation, cross classification I) passing. Chi-square comparisons of the frequency of male versus female subjects who passed the various matrix tasks (see Tables 7 and 8) were consistently nonsignificant for both the reproduction and transposition cases. Consequently the male and female subsamples were combined for all later analyses.

The frequency and percentage of subjects passing the reproduction and transposition subtasks across the present age-grade range are presented in Tables 7 and 8. As anticipated there was a notable positive relationship between age-grade level and the number of subjects passing the various matrix tasks.

The chi-square comparisons of passing frequencies across the four grade levels were significant with one exception--i.e., for the reproduction cases, double seriation $X^2 = 20.51$, $df = 3$, $p < .001$; cross classification I, $X^2 = 12.28$, $df = 3$, $p < .01$; cross classification II, $X^2 = 36.43$, $df = 3$, $p < .001$; and for the transposition cases, double seriation $X^2 = 12.38$, $df = 3$, $p < .01$; cross classification I, $X^2 = 5.30$, $df = 3$, N.S.; cross classification II, $X^2 = 20.77$, $df = 3$, $p < .001$. These age performance trends are also shown in Figures 1 through 5. Pair-wise comparisons between the various grade levels resulted in the following significant distinctions.

Reproduction Cases

Double Seriation:

Pre vs. 1st gr., $X^2 = 8.72$, $df = 1$, $p < .01$

Pre vs. 2nd gr., $X^2 = 18.06$, $df = 1$, $p < .01$

Kdg. vs. 1st gr., $X^2 = 3.85$, $df = 1$, $p < .05$

Kdg. vs. 2nd gr., $X^2 = 8.58$, $df = 1$, $p < .01$

TABLE 7

Number and Percentage of Subjects at Various Grade Levels
Who Passed the Reproduction
Subtasks of Each of the Three Matrix Tasks

Grade Level	Double Seriation		Cross Classification I		Cross Classification II	
	No. Passed	%	No. Passed	%	No. Passed	%
Preschool						
Males (n = 20)	7	35.0	8	40.0	2	10.0
Females (n = 20)	3	15.0	8	40.0	0	0
Total (n = 40)	10	25.0	16	40.0	2	5.0
Kindergarten						
Males (n = 20)	8	40.0	8	40.0	3	15.0
Females (n = 20)	8	40.0	7	35.0	5	25.0
Total (n = 40)	16	40.0	15	35.0	8	20.0
First						
Males (n = 19)	13	68.4	11	57.9	11	57.9
Females (n = 21)	10	47.6	10	47.6	10	47.6
Total (n = 40)	23	57.5	21	52.5	21	52.5
Second						
Males (n = 20)	14	70.0	12	60.0	12	60.0
Females (n = 20)	15	75.0	17	85.0	12	60.0
Total (n = 40)	29	72.5	29	72.5	24	60.0
Combined						
Males (n = 79)	42	53.2	39	49.4	28	35.4
Females (n = 81)	36	44.4	42	51.9	27	33.3
Total (n=160)	78	48.8	81	50.6	55	34.4

TABLE 8

Number and Percentage of Subjects at Various Grade Levels
Who Passed the Transposition
Subtasks of Each of the Three Matrix Tasks

Grade Level	Double Seriation		Cross Classification I		Cross Classification II	
	No. Passed	%	No. Passed	%	No. Passed	%
Preschool						
Males (n = 20)	4	20.0	6	30.0	2	10.0
Females (n = 20)	2	10.0	5	25.0	1	5.0
Total (n = 40)	6	15.0	11	27.5	3	7.5
Kindergarten						
Males (n = 20)	4	20.0	4	20.0	1	5.0
Females (n = 20)	4	20.0	8	40.0	2	10.0
Total (n = 40)	8	20.0	12	30.0	3	7.5
First						
Males (n = 19)	10	52.6	8	42.1	4	21.1
Females (n = 21)	6	28.6	7	33.3	7	33.3
Total (n = 40)	16	40.0	15	37.5	11	27.5
Second						
Males (n = 20)	7	35.0	9	45.0	7	35.0
Females (n = 20)	11	55.0	11	55.0	10	50.0
Total (n = 40)	18	45.0	20	50.0	17	42.5
Combined						
Males (n = 79)	25	31.6	27	34.2	14	17.7
Females (n = 81)	23	28.4	31	38.3	20	24.7
Total (N =160)	48	30.0	58	36.3	34	21.3

Cross Classification I: Pre vs. 2nd gr., $X^2 = 8.58$, $df = 1$, $p < .01$
Kdg. vs. 2nd gr., $X^2 = 9.90$, $df = 1$, $p < .01$

Cross Classification II: Pre vs. 1st gr., $X^2 = 22.03$, $df = 1$, $p < .01$
Pre vs. 2nd gr., $X^2 = 27.58$, $df = 1$, $p < .01$
Kdg. vs. 1st gr., $X^2 = 9.14$, $df = 1$, $p < .01$
Kdg. vs. 2nd gr., $X^2 = 13.33$, $df = 1$, $p < .01$

Transposition Cases

Double Seriation: Pre vs. 1st gr., $X^2 = 6.27$, $df = 1$, $p < .05$
Pre vs. 2nd gr., $X^2 = 8.57$, $df = 1$, $p < .01$
Kdg. vs. 2nd gr., $X^2 = 5.70$, $df = 1$, $p < .05$

Cross Classification I: Pre vs. 2nd gr., $X^2 = 4.27$, $df = 1$, $p < .05$

Cross Classification II: Pre vs. 1st gr., $X^2 = 5.54$, $p < .05$
Pre vs. 2nd gr., $X^2 = 13.07$, $p < .01$
Kdg. vs. 1st gr., $X^2 = 5.54$, $p < .05$
Kdg. vs. 2nd gr., $X^2 = 13.07$, $p < .01$

The relative difficulties of the reproduction tasks contrasted with the transposition cases are presented in Table 9 (note that the probability values associated with the McNemar Tests for the Significance of Changes are for one-tailed inference tests in Table 9 and for two-tailed inference tests in Tables 10 and 11). It may be noted that the reproduction tasks are consistently of lesser difficulty.

The inter-matrix reproduction task relative difficulty comparisons are presented in Table 10. As indicated, there is very little difference between the cross classification I and double seriation reproduction tasks. Each of these measures is significantly easier than the cross classification II task at the preschool level and in terms of the combined sample passing frequencies. These comparisons suggest the following order of difficulty for the three matrix reproduction tasks: cross classification II > cross classification I = double seriation.

The corresponding comparison values for the transposition task cases are shown in Table 11. In this instance also, the double seriation and cross classification I tasks are consistently easier than the cross classification II case, although only the preschool, kindergarten, and combined sample comparisons of cross classification versus cross classification II reach acceptable significance levels. This would suggest the following order of difficulty for the three matrix transposition tasks: cross classification II > cross classification I = double seriation.

TABLE 9

Comparison of the Relative Difficulties
of the
Reproduction Versus Transposition Matrix Tasks

Grade Level	DSR X DST	CCIR X CCIT	CCIIR X CCIIT
	DST	CCIT	CCIIT
Preschool	DSR: Pass/Pass 2, Pass/Fail 8, Fail/Pass 4, Fail/Fail 26	CCIR: Pass/Pass 10, Pass/Fail 6, Fail/Pass 1, Fail/Fail 23	CCIIR: Pass/Pass 1, Pass/Fail 1, Fail/Pass 2, Fail/Fail 36
Kindergarten	DSR: Pass/Pass 4, Pass/Fail 12 [b], Fail/Pass 4, Fail/Fail 20	CCIR: Pass/Pass 8, Pass/Fail 7, Fail/Pass 4, Fail/Fail 21	CCIIR: Pass/Pass 0, Pass/Fail 8, Fail/Pass 3, Fail/Fail 29
First	DSR: Pass/Pass 14, Pass/Fail 9 [b], Fail/Pass 2, Fail/Fail 15	CCIR: Pass/Pass 14, Pass/Fail 7 [b], Fail/Pass 1, Fail/Fail 18	CCIIR: Pass/Pass 10, Pass/Fail 11 [a], Fail/Pass 1, Fail/Fail 18
Second	DSR: Pass/Pass 14, Pass/Fail 15 [a], Fail/Pass 4, Fail/Fail 7	CCIR: Pass/Pass 18, Pass/Fail 11 [b], Fail/Pass 2, Fail/Fail 9	CCIIR: Pass/Pass 12, Pass/Fail 12, Fail/Pass 5, Fail/Fail 11
Combined	DSR: Pass/Pass 34, Pass/Fail 44 [a], Fail/Pass 14, Fail/Fail 68	CCIR: Pass/Pass 50, Pass/Fail 31 [a], Fail/Pass 8, Fail/Fail 71	CCIIR: Pass/Pass 23, Pass/Fail 32 [a], Fail/Pass 11, Fail/Fail 94

[a] $p < .01$) one-tailed inference tests
[b] $p < .05$)

TABLE 10

Comparison of the Relative Difficulties of the Reproduction Matrix Tasks

Grade Level	DS X CCI (CCI)	DS X CCII (CCII)	CCI X CCII (CCII)
Preschool	Pass/Pass: 6; Pass/Fail: 4; Fail/Pass: 10; Fail/Fail: 20	Pass/Pass: 2; Pass/Fail: 8; Fail/Pass: 0; Fail/Fail: 30 [b]	Pass/Pass: 1; Pass/Fail: 15; Fail/Pass: 1; Fail/Fail: 23 [b]
Kindergarten	Pass/Pass: 6; Pass/Fail: 10; Fail/Pass: 9; Fail/Fail: 15	Pass/Pass: 2; Pass/Fail: 14; Fail/Pass: 6; Fail/Fail: 18	Pass/Pass: 3; Pass/Fail: 12; Fail/Pass: 5; Fail/Fail: 20
First	Pass/Pass: 12; Pass/Fail: 11; Fail/Pass: 9; Fail/Fail: 8	Pass/Pass: 13; Pass/Fail: 10; Fail/Pass: 8; Fail/Fail: 9	Pass/Pass: 11; Pass/Fail: 10; Fail/Pass: 10; Fail/Fail: 9
Second	Pass/Pass: 21; Pass/Fail: 8; Fail/Pass: 8; Fail/Fail: 3	Pass/Pass: 19; Pass/Fail: 10; Fail/Pass: 5; Fail/Fail: 6	Pass/Pass: 18; Pass/Fail: 11; Fail/Pass: 6; Fail/Fail: 5
Combined	Pass/Pass: 45; Pass/Fail: 33; Fail/Pass: 36; Fail/Fail: 46	Pass/Pass: 36; Pass/Fail: 42; Fail/Pass: 19; Fail/Fail: 63 [b]	Pass/Pass: 33; Pass/Fail: 48; Fail/Pass: 22; Fail/Fail: 57 [b]

[a] $p < .01$
[b] $p < .05$ two-tailed inference tests

TABLE 11

Comparison of the Relative Difficulties of the Transposition Matrix Tasks

Grade Level	DS X CCI				DS X CCII				CCI X CCII			
	CCI				CCII				CCII			
		Pass	Fail			Pass	Fail			Pass	Fail	
Preschool	DS Pass	3	3		DS Pass	1	5		CCI Pass	1	10	b
	DS Fail	8	26		DS Fail	2	32		CCI Fail	2	27	
Kindergarten	DS Pass	5	3		DS Pass	2	6		CCI Pass	2	10	b
	DS Fail	7	25		DS Fail	1	31		CCI Fail	1	27	
First	DS Pass	10	6		DS Pass	7	9		CCI Pass	8	7	
	DS Fail	5	19		DS Fail	4	20		CCI Fail	3	22	
Second	DS Pass	13	5		DS Pass	11	7		CCI Pass	12	8	
	DS Fail	7	15		DS Fail	6	16		CCI Fail	5	15	
Combined	DS Pass	31	17		DS Pass	21	27		CCI Pass	23	35	a
	DS Fail	27	85		DS Fail	13	99		CCI Fail	11	91	

ap < .01) two-tailed inference tests
bp < .05)

For the reproduction tasks, the frequencies and percentages of subjects exactly reproducing the various matrix arrays are reported in Table 12. For the cross classification I and double seriation cases, these result patterns are in essential accord with the earlier studies of MacKay et al. (1970) and Hooper et al. (1974).

Table 13 presents the number of subjects committing placement errors according to the dimensions in question for each matrix type. The predominant error category for cross classification I is the color dimension; i.e., 65.2 percent of the reproduction and 67.2 percent of the transposition cases (combined grade totals), respectively. In both the cross classification II and double seriation cases there was a tendency to err on the height dimension; i.e., 66.0 percent (reproduction) and 61.4 (transposition) for the former and latter tasks. These results concerning the cross classification I and double series tasks agree with the Hooper, et al. (1974) data and are in contrast to the MacKay (1970) results.

A final consideration concerns the possibility that subjects passing the cross classification I task do so by seriating the color brightness dimension; i.e., yellow, red, and blue. As Table 14 indicates, there was no notable tendency for subjects to do so; i.e., only 7.4 percent of the successful reproduction cases and 27.6 percent of the successful transposition cases produced a rank-ordered brightness array.

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TABLE 12

Number and Percentage of Successful Subjects,
Across the Various Grade Levels,
Who Exactly Reproduced the Various Matrices

Grade	Double Seriation			Cross Classification I			Cross Classification II		
	No. Passed	Exactly Reproduced		No. Passed	Exactly Reproduced		No. Passed	Exactly Reproduced	
		No.	% Passing		No.	% Passing		No.	% Passing
Pre	10	10	100	16	10	63	2	1	50
K	16	16	100	15	9	60	8	8	100
1	23	23	100	21	18	86	21	17	81
2	29	29	100	29	28	97	24	22	92
Combined Grade	78	78	100	81	65	80	55	48	87

TABLE 13

Number of Subjects Failing Each Dimension of Each Matrix Subtask at the Various Grade Levels

Grade	Double Seriation				Cross Classification I				Cross Classification II			
	Reproduction		Transposition		Reproduction		Transposition		Reproduction		Involution	
	Diameter	Height	Diameter	Height	Color	Shape	Color	Shape	Diameter	Height	Diameter	Height
Pre	20	28	26	33	17	12	22	15	19	37	32	33
K	17	19	17	32	22	7	25	12	15	32	23	35
1	13	11	14	23	13	9	17	12	14	17	18	21
2	6	6	11	20	6	3	20	2	4	15	15	19
Combined Grade	56	64	68	108	58	31	84	41	52	101	88	111

TABLE 14

Number of Subjects Who Passed the Cross Classification I Reproduction and Transposition Subtasks Without Seriating on Brightness of Color Dimension

Grade	Reproduction		Transposition	
	No. Passed	No. Passed Without Seriating	No. Passed	No. Passed Without Seriating
Pre	16	14	11	6
K	15	12	12	7
1	21	20	15	12
2	29	29	20	17
Combined Grade	81	75	58	42

IV

DISCUSSION

As predicted, performance on all of the matrix tasks improved significantly over the present age-grade range. The growth of these multiplicative logical skills from approximately four to seven years of age is in essential agreement with previous normative research investigations; e.g., Bruner & Kenney (1966), Hooper et al. (1974), Inhelder & Piaget (1964), Lovell et al. (1962), MacKay et al. (1970), Shantz (1967), and Smedslund (1964). In relative contrast to the Hooper et al. (1974) results, the present subject sample appears to demonstrate lower average ability levels. For example, the percentage of subjects for the combined sample succeeding on the cross classification I (reproduction = 50.6 percent; transposition = 36.3 percent, and double seriation tasks (reproduction = 48.8 percent; transposition = 30.0 percent, are notably lower than the Hooper et al. (1974) comparison values; i.e., cross classification I (reproduction = 64.4 percent; transposition = 53.1 percent) and double seriation (reproduction = 81.3 percent; transposition = 49.4 percent). In addition, in accord with the earlier studies cited above, the present results indicate a distinct absence of significant performance differences based on sex.

The notably greater difficulty associated with the transposition contrasted with the reproduction tasks (see Table 9 and Figures 1, 2, and 3) replicates the previous findings of Bruner & Kenney (1966), Hooper et al. (1974), and MacKay et al. (1970). Additionally, the great majority of children who succeed on the matrix reproduction tasks produce an identical array; i.e., for the combined age-grade subsamples 100 percent, 80 percent, and 87 percent of the double seriation, cross classification I, and cross classification II cases (cf. Hooper et al., 1974; MacKay et al., 1970).

The primary inter-matrix difficulty comparisons were shown in Tables 10 and 11 and Figures 4 and 5. These results agree with the primary findings of Hooper et al. (1974) with one singular exception. The present relative difficulty analyses indicate that there is no difference between cross classification I and double seriation for the reproduction task set. This case of equivalent difficulty was not expected. It should be pointed out, however, that none of the present inter-matrix difficulty patterns agrees with the earlier contentions of Lagattuta (1970, 1974).

Insofar as the double seriation and cross classification I comparisons are concerned (i.e., equivalent item difficulties for both the reproduction and transposition cases), the present study supports the original conclusions of Inhelder & Piaget (1964). In discussing the probable age dependent acquisition points for multiple classification and multiple seriation abilities they stated:

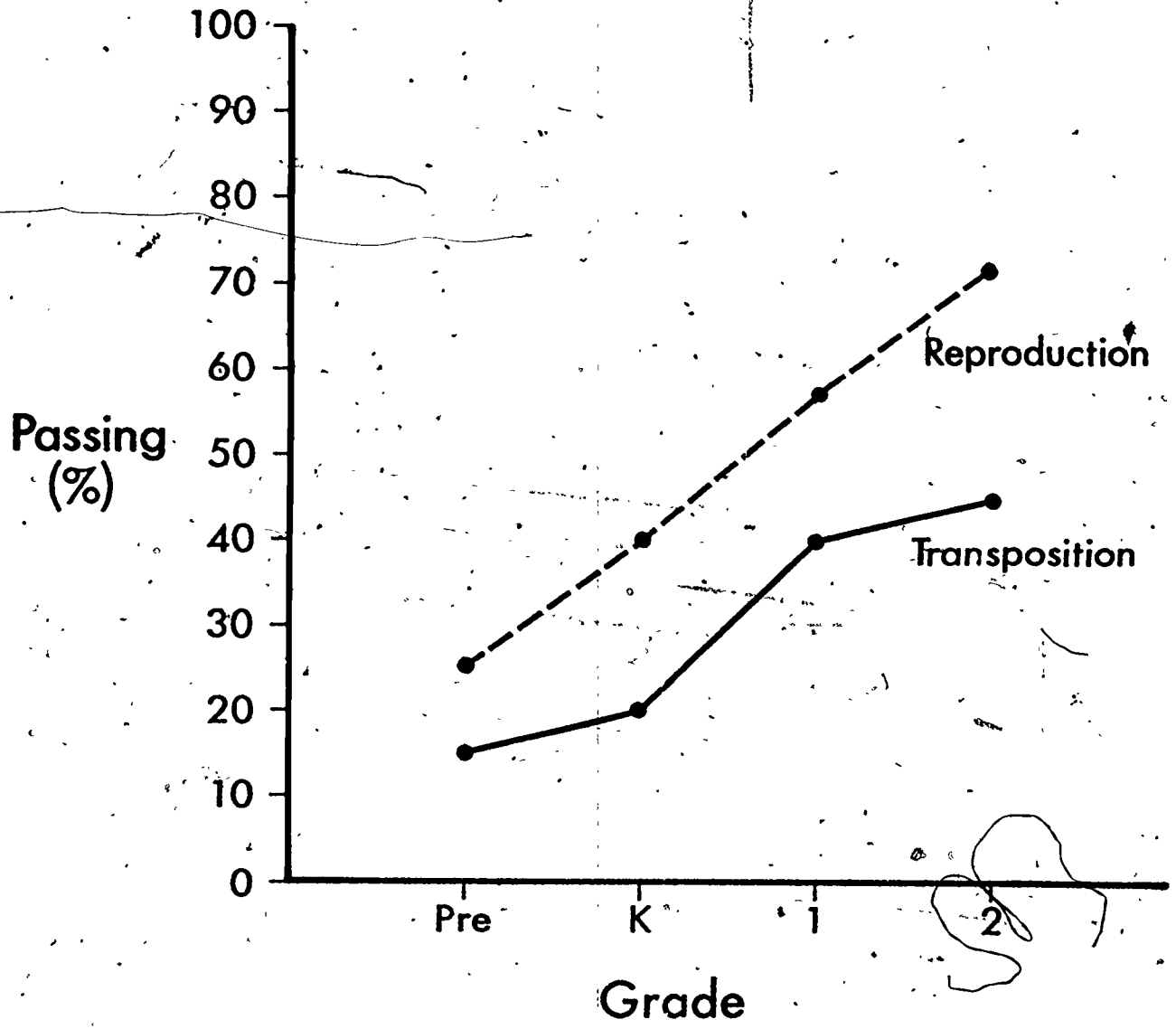

Figure 1. Double seriation subtasks.

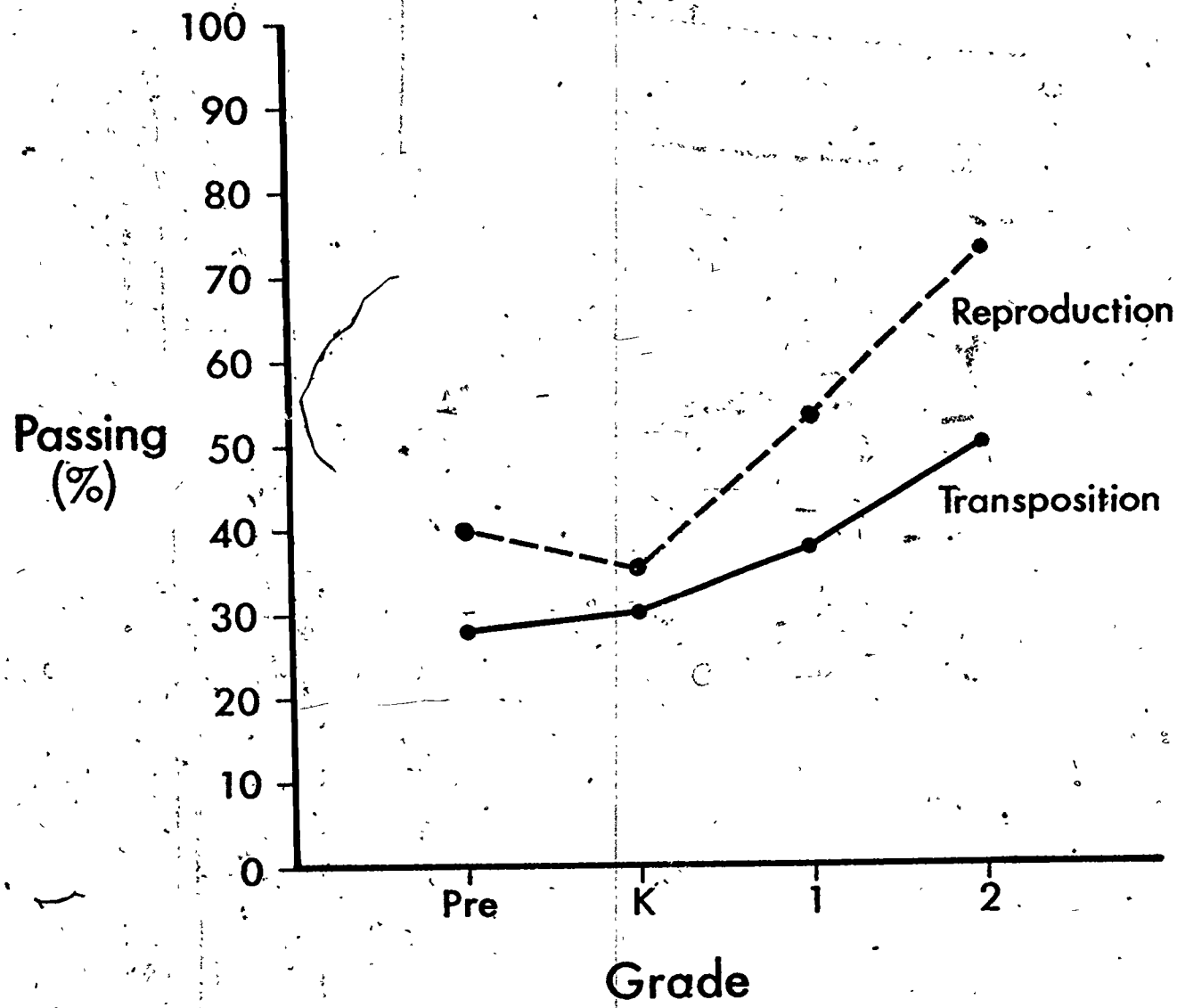

Figure 2. Cross classification I subtasks.

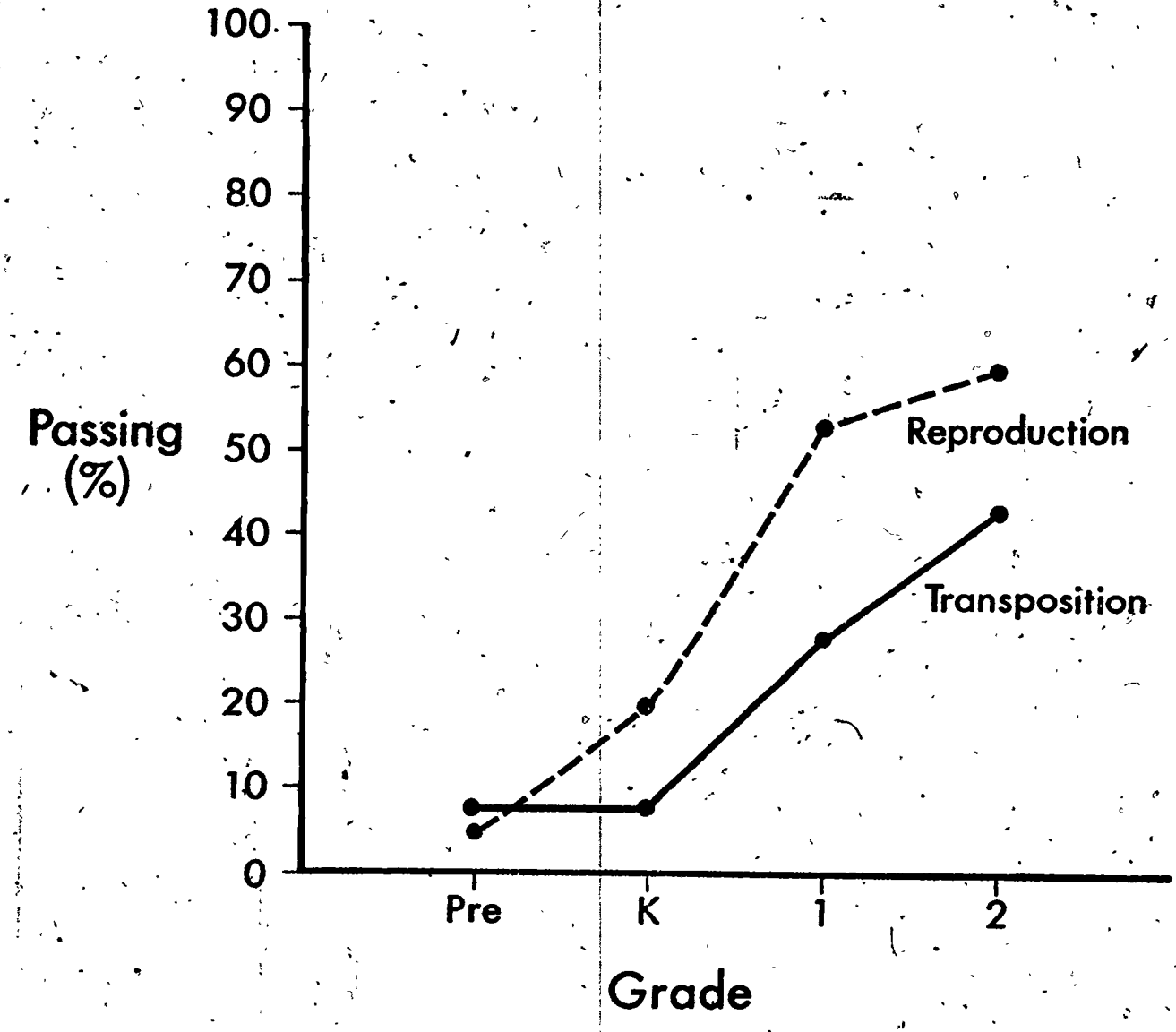

Figure 3. Cross classification II subtasks.

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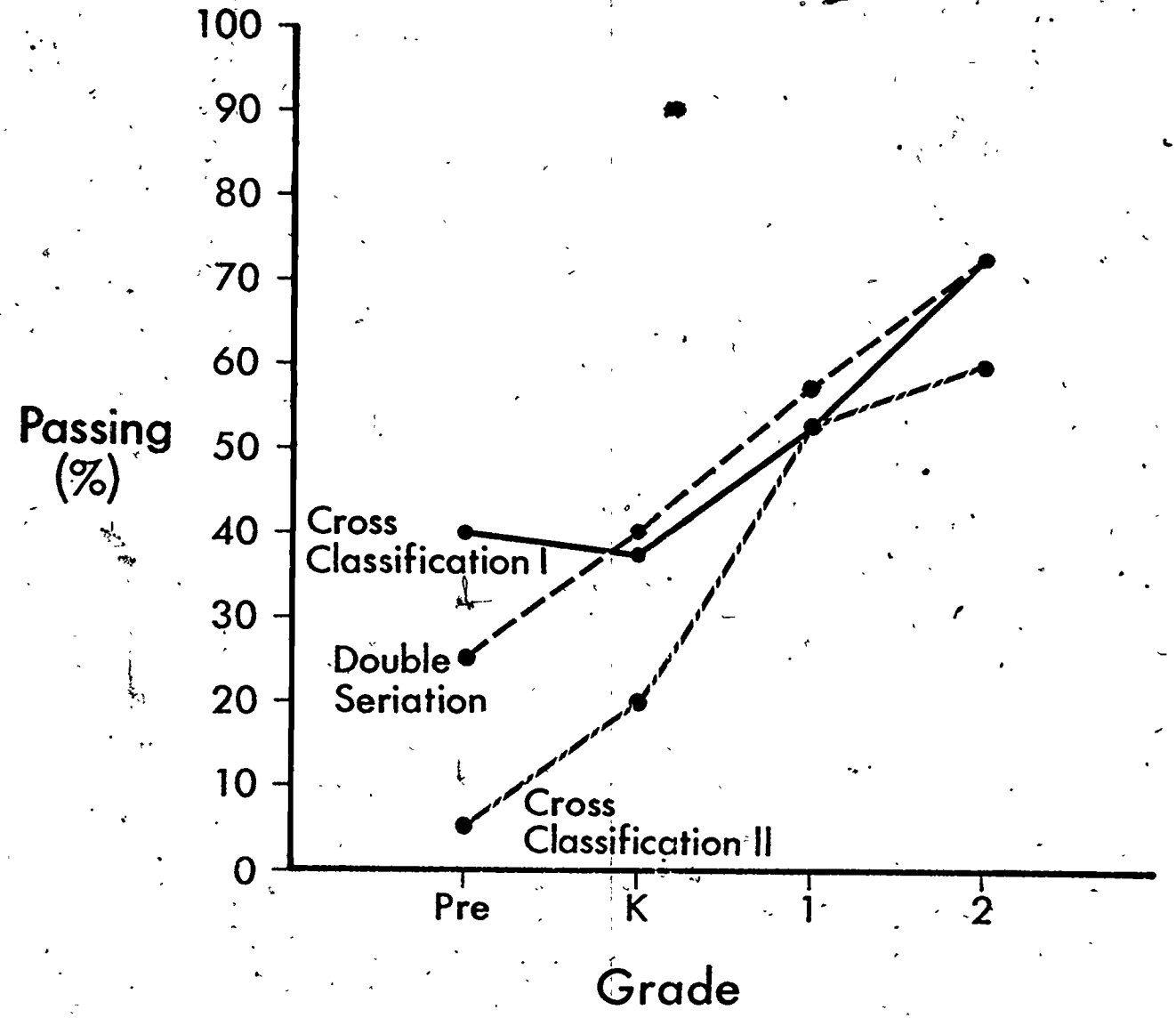

Figure 4. Reproduction subtasks.

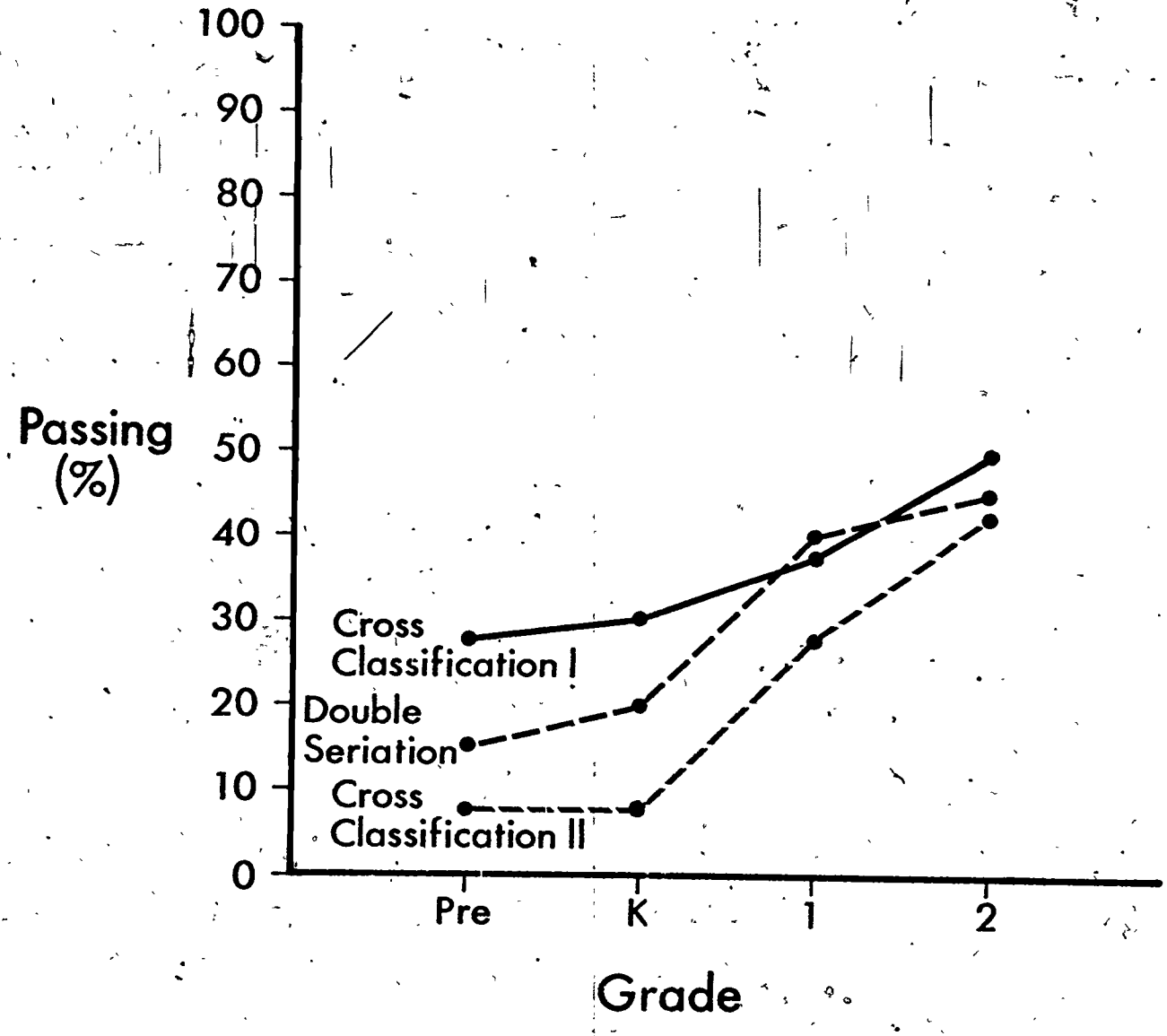

Figure 5. Transposition subtasks.

> Children reach an operational level in the multiplication of series about the same period (7-8) as cross-classification. . . . Finally, there are four principal "groupings" in the logic of classes and relations, corresponding with simple and multiple classification and simple and multiple seriation. It is a most remarkable fact that, in spite of the differences just noted in respect of ease of perceptualization, all four structures become operational at roughly the same period. There are certain minor differences depending on the extent to which the content of a problem lends itself to imaginal representation, but they do not invalidate our main thesis [Inhelder & Piaget, 1964, pp. 278-279].

Thus, it would appear that the present assessments (superior to the original Genevan results since a within-subjects' measurement design was employed) serve to validate the structural synchrony assumptions of orthodox Piagetian theory (cf. Flavell, 1971; Pinard & Laurendeau, 1969).

Yet, an examination of the present overall matrix task array does not support this conclusion. If we accept the assumption that the double seriation and cross classification II tasks represent two multiplicative logical reasoning tasks which employ identical stimulus materials with distinctive instructional sets and associated different initial arrangement conditions, then the markedly different task difficulties are indeed notable. There is little question that the cross classification II tasks (reproduction and transposition) are the more difficult task settings in the present assessment array. Of the 61 subjects for the combined sample reproduction totals (see Table 10) who passed one task while failing the other, 69 percent indicate cross classification II to be the more difficult task. A similar case holds for the more difficult transposition tasks; i.e., of the 40 pass versus fail cases, 68 percent show the cross classification II case to be significantly more difficult. Moreover, we can observe the two cross classification tasks. In the reproduction category 70 cases fell into the pass/fail category (see Table 11). Of these, 69 percent show cross classification II to be the more difficult task. Similarly, 76 percent of the 46 pass/fail transposition cases indicate cross classification II to be significantly more difficult than the original (MacKay et al., 1970) cross classification case.

Wong (1975) has recently completed a study of children's number concepts at the preschool, kindergarten, and first-grade levels. Measures of multiple classification and relationality in matrix formats were included. It was found that the multiple classification tasks were significantly more difficult than the multiple relations (seriation) tasks and these distinctions were most notable at the first-grade level. These results are in essential accord with the generalizations of the present investigation and the earlier findings of Hooper et al. (1974).

The overall implication of these results points to a probable case of lesser general difficulty for multiple seriation (relationality) contrasted with multiple classification concepts. This pattern is substantiated in a number of recent investigations of Piagetian relational concept tasks. Assessment tasks directly based upon the four relational *groupements* (Piaget, 1972; see also Flavell, 1963, pp. 173-195) have been found to be significantly

less difficult than are tasks derived from the classificatory *groupements* (Brainerd, 1972; Dihoff, in preparation, Weinreb & Brainerd, 1975). The relational understanding embodied in the traditional transitive inference task has been found to be a developmentally earlier acquisition than the counterpart concepts of conservation or class inclusion (Brainerd, 1973b; Toniolo & Hooper, 1975). Moreover, these differential item difficulties have been replicated in instructional studies employing transfer of training designs (Brainerd, 1974; Peterson, Hooper, Wanska, & DeFrain, in preparation). It has also been found (Brainerd, 1973a; Gonchar, 1975; Siegel, 1974) that ordinal number understanding (relational concept domain) precedes cardinal number mastery (classificatory concept domain). Finally, these theoretically relevant item difficulty patterns have been substantiated in a number of developmental acquisition sequences in investigations employing longitudinal assessment designs (Dihoff, 1975; Gonchar, in preparation; Toniolo & Hooper, in preparation).

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