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ABSTRACT

This guide is the sixth in a series of publications to assist teachers in using a laboratory approach to mathematics. Twenty activities on probability and statistics for the elementary grades are described in terms of purpose, materials needed, and procedures to be used. Objectives of these activities include basic probability concepts; gathering, tabulating, and interpreting data; making predictions; pattern discovery; formulating hypotheses; and fractions, ratios and basic statistical procedures. The publication includes diagrams, charts, illustrations, and suggested questions to assist the teacher in operating an informal laboratory. (JBW)

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SUGGESTIONS FOR TEACHING MATHEMATICS USING LABORATORY APPROACHES EXPERIMENTAL EDITION

SE



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6. PROBABILITY

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The University of the State of New York
THE STATE EDUCATION DEPARTMENT
Bureau of Elementary Curriculum Development
Albany, New York 12234

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PREFACE

Substantial financial aid to local educational agencies for children of low income families was provided by the Elementary and Secondary Education Act of 1965. Participating school districts have developed a variety of new educational programs to assist children with special educational needs. These programs are based upon local needs assessment with major parental involvement.

The three major priorities for compensatory education programs are bilingual education, reading, and mathematics. One of the highest priority programs under ESEA Title I is the subject matter area of mathematics. Experience has shown that children who have experienced difficulty learning in a traditional program often react with enthusiasm to a mathematics laboratory approach. This publication was developed to provide practical application of this mathematics approach for use by classroom teachers. It should provide practical suggestions for teachers working directly with educationally disadvantaged children.

Irving Ratchick
Assistant Commissioner for Compensatory
Education

FOREWORD

The Bureau of Elementary Curriculum Development and the Bureau of Mathematics Education, in cooperation with the Division of Education for the Disadvantaged, ESEA Title I, have developed a variety of materials on the use of a Mathematics Laboratory approach on the elementary level. This joint effort has resulted in the release of the following publications:

Operating A Math Lab, which gives a short overview of the method.
ESEA Title I, Anatomy of an Elementary Project, which gives a concrete example of the use of a Mathematics Laboratory approach with children with identified weaknesses in the mathematics area of problem solving.

Suggestions for Teaching Mathematics Using Laboratory Approaches
Grades 1-6

1. Number and Numeration
2. Operations
3. Geometry
4. Measurement

These publications provide practical suggestions for teachers who wish to work with their students within the confines of a laboratory approach.

Encouraged by the response of teachers and administrators to these publications, a decision was made to move even further in the direction of providing concrete activities for teachers who wish to move into the humanistic approach inherent in a Mathematics Laboratory program.

Fredric Paul of the Bureau of Mathematics Education and Peter A. Martin of the Bureau of Elementary Curriculum Development began the task of

developing more advanced activities for teacher use. A committee of experienced teachers, consisting of Edward Cunningham, Simone Katz, and Sylvia Orans, under the direction of Elaine Mintz, was engaged as a writing team. Mrs. Mintz is director of elementary mathematics and the other three are classroom teachers. The material completed by this team was field tested in the classroom. As a result of favorable reactions, additional experimental activities have been produced for use by school districts.

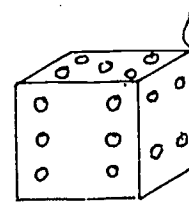
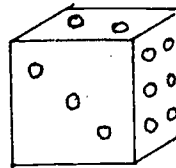
This publication is the sixth which has been developed for teacher use. Each incorporates the latest thinking of the mathematics revision committee and may be utilized with any basic mathematics program. The publication is designed to serve as a stimulant to encourage teachers to employ their imaginations to develop further activities. The classroom teacher in developing a set of "task cards" will adjust vocabulary and choose concrete materials in terms of an intimate knowledge of the ability levels of the children involved and the type and amount of manipulative materials available. Suggestions are welcome and should be sent to Fredric Paul, Bureau of Mathematics Education, State Education Department, Albany, New York 12234.

Peter A. Martin of the Bureau of Elementary Curriculum Development did the final editing, contributed material and illustrations, and prepared the materials for publication.

Robert H. Johnstone, Chief
Bureau of Elementary Curriculum
Development

Gordon E. Van Hooft, Director
Division of Curriculum Development

PROBABILITY



Although the theory of probability owes its early development to the needs of gamblers, today it serves as a powerful tool for such diverse fields as social sciences, physical sciences, industry, and insurance.

Students can approach probability informally through the use of experiments. They examine all possible outcomes, perform repeated trials, and tabulate or graph their results. They then judge which outcomes are most likely (probable). In the process, they become involved with applications for fractional computation. Theoretically, the probability of an event is a number from 0 to 1 that predicts how often an event can be expected to happen.

Theoretically:

$$\text{Probability of an Event} = \frac{\text{Number of FAVORABLE ways the event can happen}}{\text{Total NUMBER of ways the event can happen}}$$

This equation is often written:

$$P(E) = \frac{f}{n} \text{ and is read, "P of E equals f over n."}$$

Experimentally:

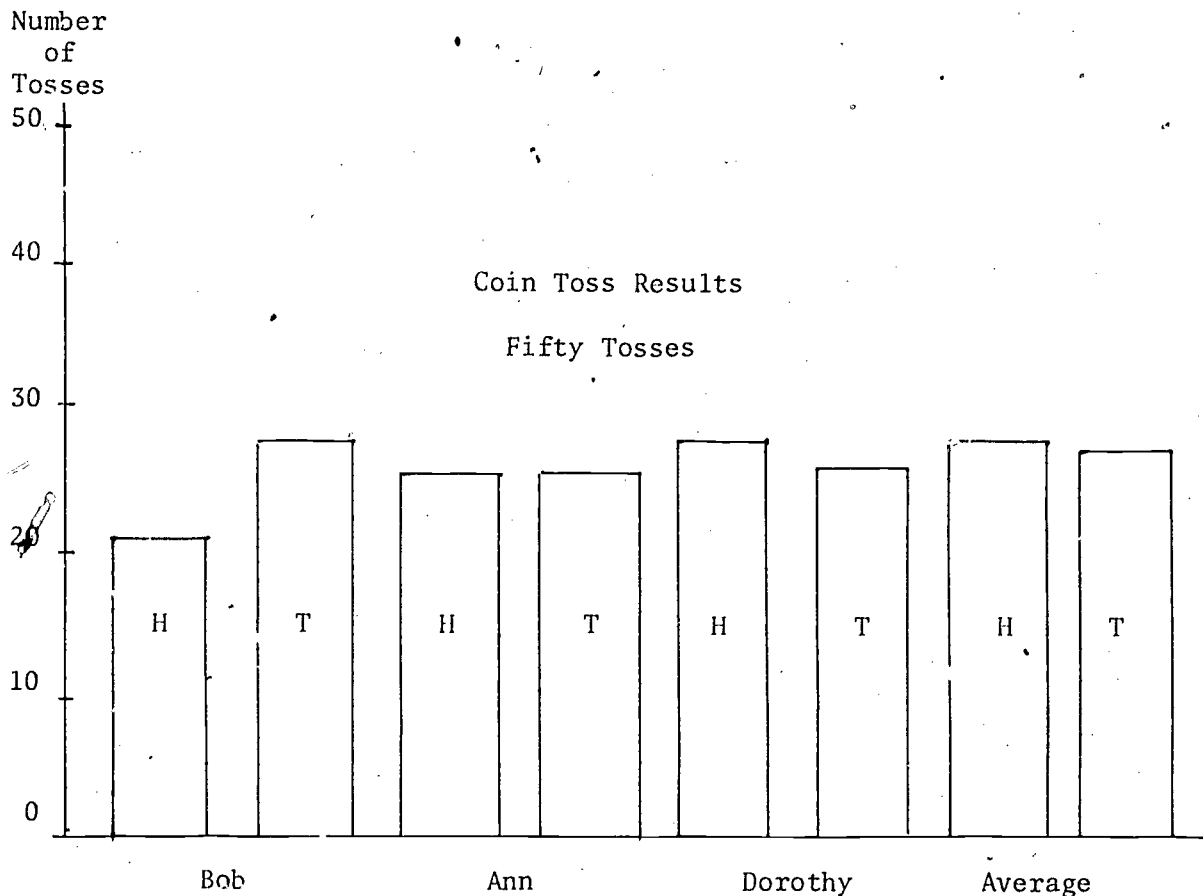
$$\text{Probability of an outcome} = \frac{\text{Number of times the outcome occurred}}{\text{Total number of trials}}$$

When a probability is estimated with experiments, the result is called empirical or statistical probability. It is the statistical approach to probability studies that has practical applications for commerce and science.

Routines for Probability Experiments

Have students use concrete materials when they investigate problems.

- a) The student should guess what to anticipate for an answer (outcome).
- b) A data sheet or graph should be prepared to record all possible outcomes.
- c) The student should perform the task (role dice, pick cards, select marbles, toss coins, etc.) at least fifty times and record the outcome of each trial. (For larger samples, the data obtained by several students can be collated.)
- d) It is possible to determine the probability ratio by comparing the total number of successful outcomes to the total number of trials.
- e) Finally, this ratio should be compared with the original "guesstimate," with the difference recorded.



6-1 Spin, Spin. Purpose: Develop probability concepts, techniques for gathering, tabulating and interpreting experimental data; making predictions; fractions, ratio, percentage

Materials needed: One spinner whose dial is $\frac{1}{2}$ red and $\frac{1}{2}$ green

Procedure: The student should first examine the spinner and try to predict (guess) the different outcomes (possible happenings). Spinner pointer is then pushed and it is recorded whether it stops on red or green. (The spinner should be held flat to insure a fair result. If it stops on the line between the colors it should be spun again.) The activity should be repeated fifty times and the outcomes tabulated (see below).

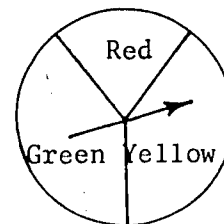
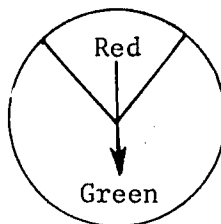
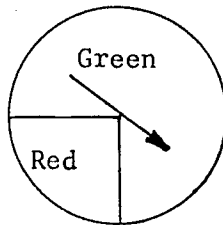
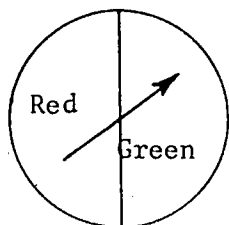
Questions:

- What is the frequency for red? For green?
- What is the difference between the original guess and the actual frequency?
- What fraction of the spins were red?
- What outcomes might be expected after 100 trials? After 1,000 trials?
- What kind of fair game can two children play with this spinner?

Variations: Repeat the above experiment with spinners that have different dial combinations as:

- $\frac{1}{4}$ red and $\frac{3}{4}$ green.
- $\frac{1}{3}$ red, $\frac{1}{3}$ green, $\frac{1}{3}$ yellow.
- $\frac{1}{3}$ red, $\frac{2}{3}$ green, etc.

Outcomes	Tallies	Frequency	Fraction
Red			$\frac{\quad}{50}$
Green			$\frac{\quad}{50}$
Total:		$\frac{\quad}{50}$	$\frac{\quad}{50}$



6-2 Using Numbers to Express Probability. Purpose: Pattern discovery, formulating hypotheses, fractions, percentage

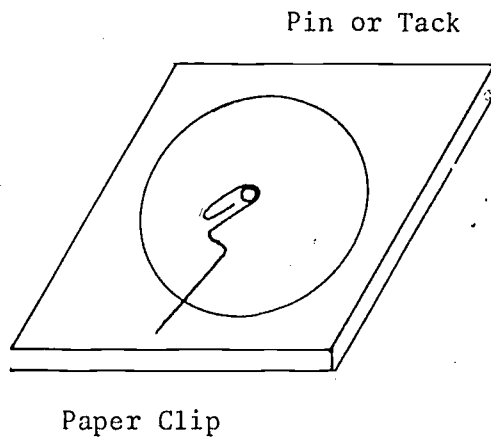
Materials needed: Spinner whose dial is divided into three congruent color sections, data sheet

Procedure: Have the students examine the spinner dial and guess the probable outcome of pointer landings on each color, if the pointer is spun repeatedly. Have them spin the pointer at least thirty times and compare the results with their guesses.

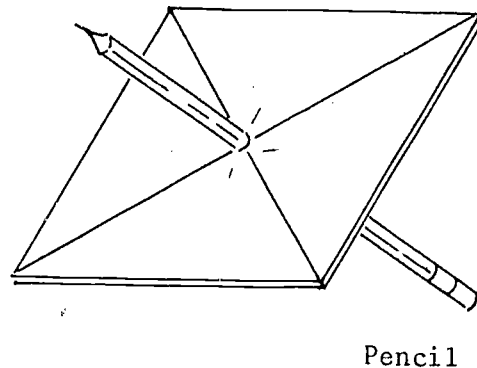
Express the probability of possible outcomes in terms of:

$$P(\text{color \#1}) = \frac{\text{Number of landings on \#1}}{\text{Total number of spins}}$$

Note: Students can make simple spinners or tops.



Plywood or Heavy Cardboard



Cut out a tagboard polygon. Mark it in sections. Pierce the center with a pencil. Use the pencil as a pivot for spinning the top.

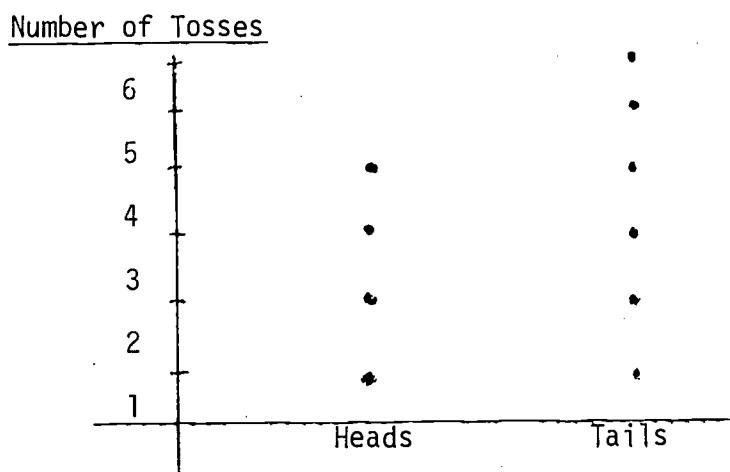
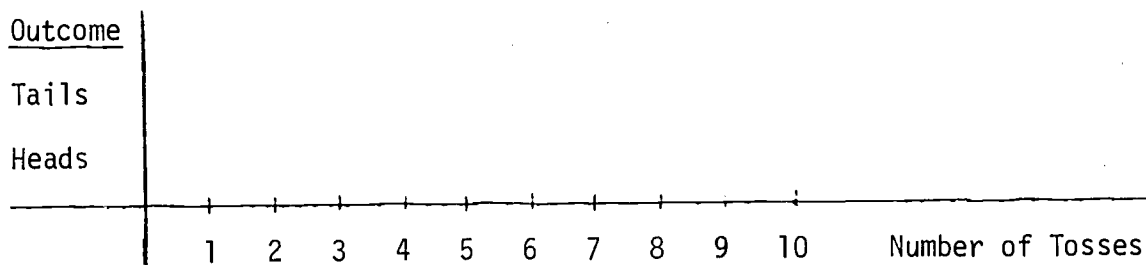
6-3 Coin Toss. Purpose: Gathering, tabulating, graphing and interpreting data from experiments, tallying, probability introduction, independent events

Materials needed: Coins

Procedure: The outcome of a coin toss is either heads or tails. Have student toss a coin and record whether it falls heads or tails. They should tabulate and graph the results. After several trials, have them predict the possible outcomes of numerous coin tosses. Test these predictions where reasonable. Compare the tabulated and graphed data.

Samples:

Outcomes	Tally	Frequency	Fraction
Heads	1111	4	$\frac{4}{10}$
Tails	1111 1	6	$\frac{6}{10}$
	Total	10	$\frac{10}{10}$



Variations: Extend the experiment by increasing:

- a) The number of trials (tosses).

b) *The number of coins used.

c) The number of trials and the number of coins.

*When more than one coin is used, the number of possible outcomes is changed. Have students explore the the outcomes for two coins and three coins (summarized below).

Two Coin Toss Outcomes				
Penny	H	T	H	T
Nickel	T	H	H	T

Note: H represents Heads
T represents Tails

Summary of Outcomes for Coin Tossing

<u>Number of Coins</u>	<u>Possible Outcomes for Coins</u>	<u>Number of Possible Outcomes</u>
1	H, T	2
2	(H H), (H T), (T H), (T T)	4
3	(H H H), (H H T), (H T H), (H T T), (T H H), (T H T), (T T H), (T T T)	8

Students can compare the above table of outcomes for repeated trials with other experiments of independent events. They might also try to find the pattern of outcomes when more coins are tossed.

(Number of possible different outcomes for one throw of n coins = 2^n)

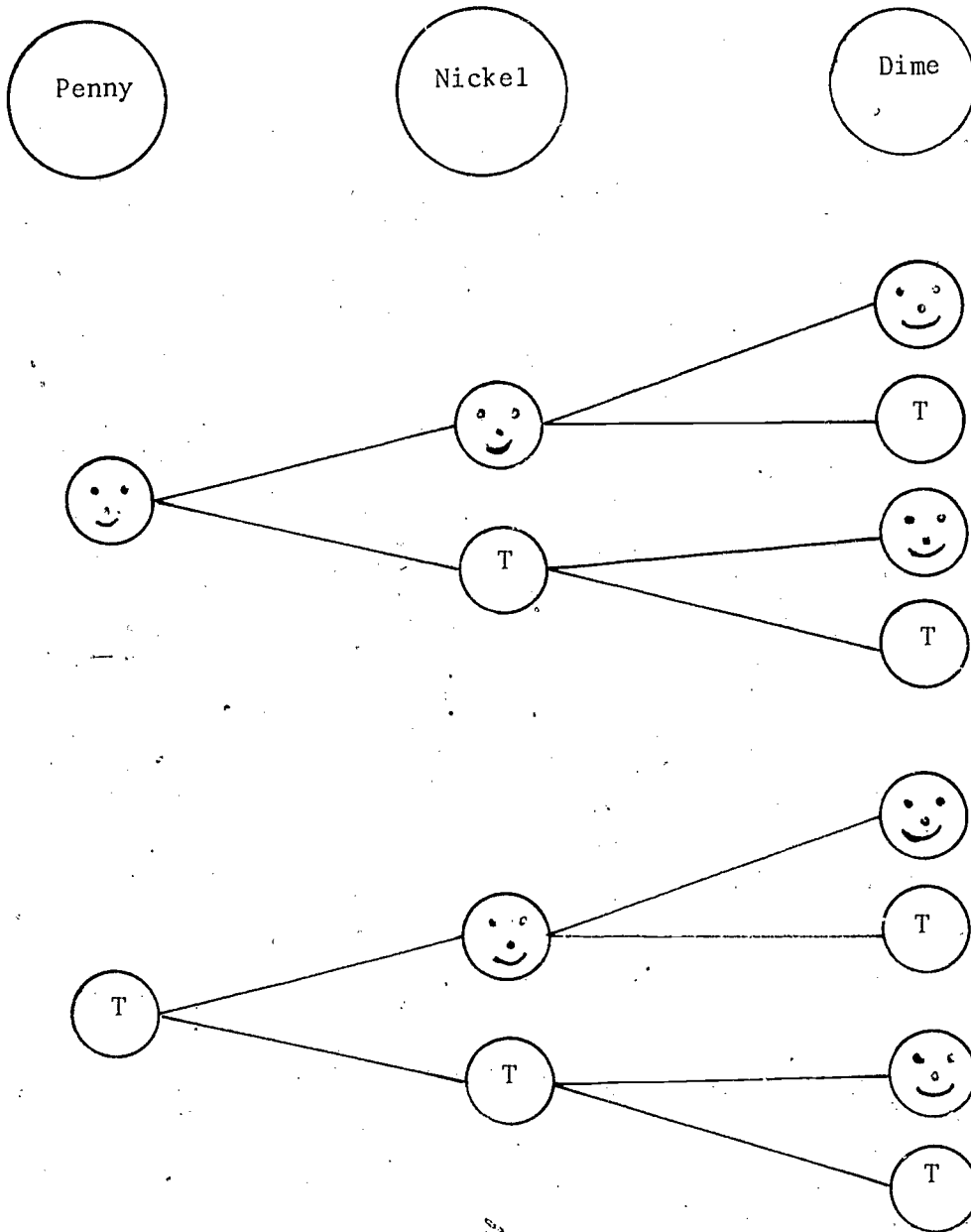
Number of possible different outcomes for n throws of 1 coin = 2^n)

Alternate Tabulation (Outcomes for 1, 2, and 3 coin tosses)

	0 Heads	1 Head	2 Heads	3 Heads
1 coin	T	H		
2 coins	TT	HT, TH	HH	
3 coins	TTT	HTT, THT, TTH	HHT, HTH, THH	HHH

Successive Choices (Independent Events)

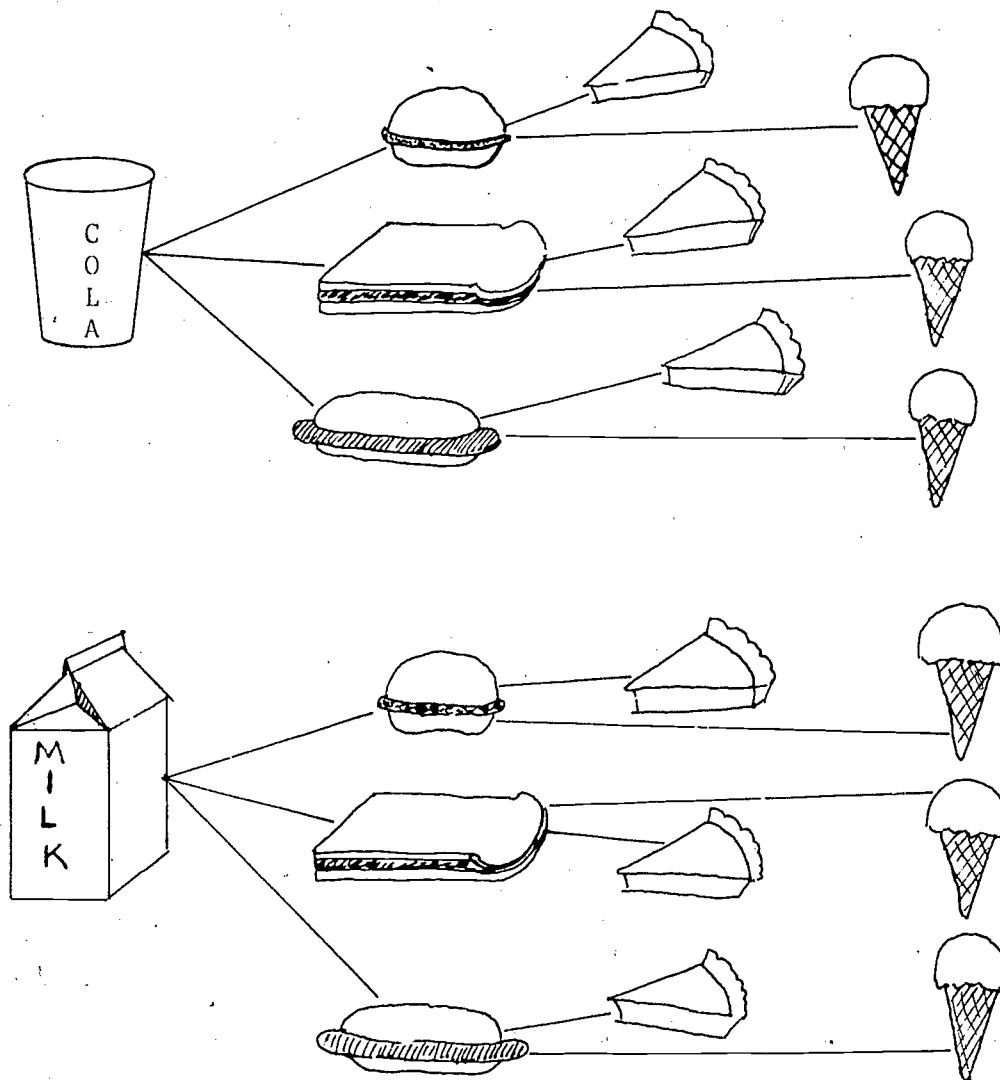
Below is a Tree Diagram for all possible combinations when three coins (as penny, nickel, dime) are tossed.



Tree Diagrams are helpful for indicating distinct arrangements of objects in a special order (permutations).

Have students use the Tree Diagram technique for describing various combinations of outfits (choose from two pairs of slacks, four ties, six shirts), menus (choose from two beverages, three sandwiches, two desserts), auto options, school course options, team players, chromosome distribution, etc.

Encourage students to find the relationship between Tree Diagrams of independent events (choices) and multiplication.



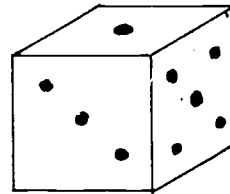
6-4 Rolling a Die. Purpose: Determine the probability of throwing an odd number with a die, determine P (even number), tallying, data collection and interpretation, fractions, ratio

Materials needed: One cubical die (If other polyhedrons are available, comparable experiments can be performed with them.)

Procedure: Before doing the experiment, the student should guess what result is anticipated. Then, proceed to roll a die fifty times. Record the outcome of each throw -- whether the die turns up on an odd or an even number. Then, obtain the total number of odd outcomes and the total number of even outcomes in order to compare actual results with the original "guestimate."

$$P(\text{odd number}) = \frac{\text{Total \# Odd Throws}}{\text{Total \# Throws}}$$

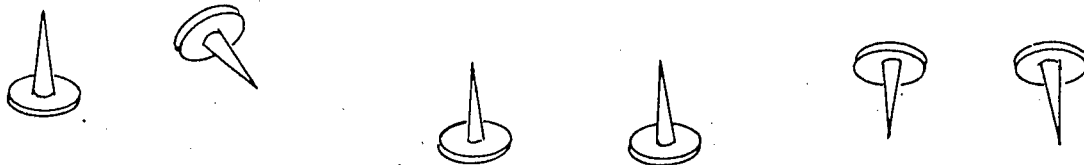
$$P(\text{even number}) = \frac{\text{Total \# Even Throws}}{\text{Total \# Throws}}$$



6-5 Tacky Business. Purpose: Develop probability concepts, techniques for gathering, tabulating, and interpreting experimental data, making predictions, fractions, ratio, percentage

Materials needed: Ten thumb tacks, shaker cup

Procedure: Students should place thumb tacks in the cup, shake the cup gently and roll the tacks out. The two possible outcomes are the tack with its point up or down. Once the students determine the number of trials for their experiments, they should predict the outcome. Then, they perform the trials and record the actual results.



6-6 Compare the Results of Two Chance Events. Purpose: Collect data and compare the results of two chance independent events, ratio

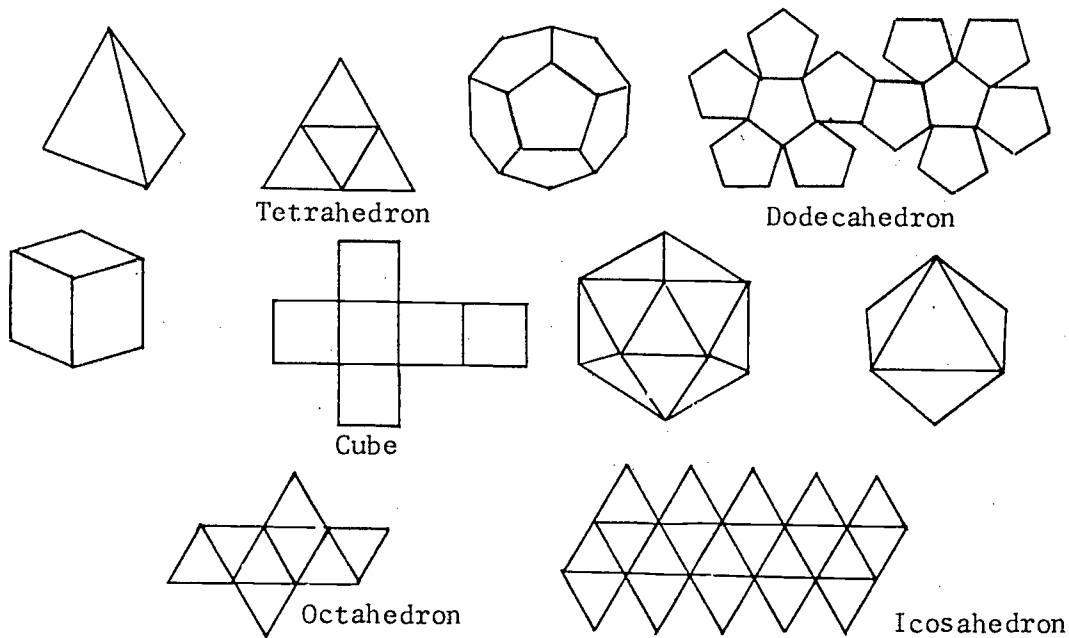
Materials needed: One cubical die, one spinner with dial divided into six equal parts labeled from one to six

Procedure: While one student spins the pointer of the spinner fifty times and records the outcomes, another rolls the die fifty times and records the outcomes. The results of both activities are then compared. The teacher should encourage the formation of generalizations based on actual observations.

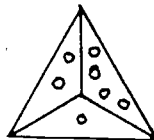
6-7 The Die Is Cast. Purpose: Determination of possible outcomes, sample space, elements of events, Platonic solids

Materials needed: Various polygonal dice

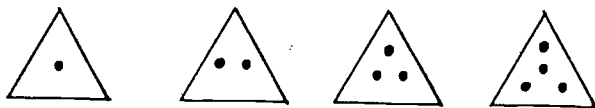
Procedure: Have students construct the five classical Platonic solids and use them as dice. Construction patterns are shown below:



a) A tetrahedron die has four faces, each an equilateral triangle.

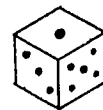


The possible outcomes of one toss are:

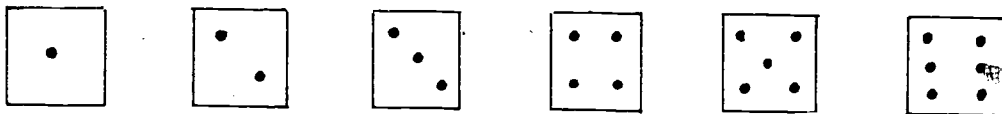


That is, the sample space is 1, 2, 3, 4

b) A cubical die has six faces, each a square.



The possible outcomes of one toss are shown below:



6-8 Sum Dice Production. Purpose: Whole number operations, preparations for sample space, examination, data tabulation, commutative properties

Materials needed: Two dice

Procedure: Have students analyze the combinations obtainable from adding (or multiplying) the amounts on each die face as:

a) Sum = 5

First Die

1
2
3
4

Second Die

4
3
2
1

	Ways of Getting Each Sum										
Sum	2	3	4	5	6	7	8	9	10	11	12
Number of Ways	4										

b) Product = 12

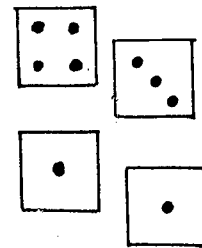
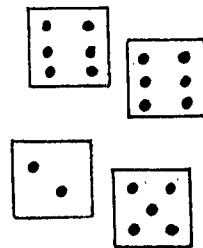
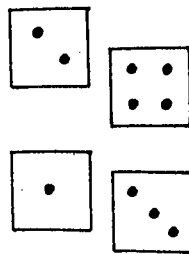
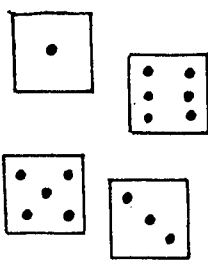
First Die

2
3
4
6

Second Die

6
4
3
2

	Ways of Getting each Product																	
Product	1	2	3	4	5	6	7	8	9	10	12	15	16	18	20	24	25	36
Number of Ways																		



6-9 Die Consequences Purpose: Determine different consequences or outcomes to be gotten from one toss of a pair of dice (or two tosses of a single die), sample space, inequalities, fractions, ordered pairs





















































































Materials needed: Pair of dice (preferably each a different color)

Procedure: Have students roll the dice at least fifty times and record the outcomes. Students should consider problems you suggest, and make up their own. Examples: What is the chance that:

- a) both dice show the same number?
- b) the sum of the faces is greater than 5? less than 5?
- c) the same sum is thrown three times in a row?
- d) the product of the faces is odd? etc.

Of course, "guestimation" before the actual experimentation makes the project more exciting and fun.

Students should be encouraged to make an orderly arrangement of all possible outcomes.

							(1,6) (2,6) (3,6) (4,6) (5,6) (6,6)
							
							(1,5) (2,5) (3,5) (4,5) (5,5) (6,5)
							
							(1,4) (2,4) (3,4) (4,4) (5,4) (6,4)
							
							(1,3) (2,3) (3,3) (4,3) (5,3) (6,3)
							
							(1,2) (2,2) (3,2) (4,2) (5,2) (6,2)
							
							(1,1) (2,1) (3,1) (4,1) (5,1) (6,1)
							
							
							1 2 3 4 5 6

Arrays for tossing of two dice

6-10 Spin More. Purpose: Formulating hypothesis from educated guesses, probability concepts, fractions

Materials needed: Five spinners or polygonal tops with faces such as: 1/2 red, 1/2 green, 1/5 red, 4/5 green, 1/5 green, 4/5 red, 1/3 red, 2/3 green, 5/8 red, 3/8 green

Procedure: Students study and guess the spinners whose pointers are more likely to stop on red than green. They should then devise experiments to confirm or refute their guesstimates.

6-11 Switch. Purpose: Probability sampling with replacement, tallying, ratio

Materials needed: Bag with two cubes which differ only in color

Procedure: Without looking into the bag, a student draws out one of the cubes. Next, record its color and replace it in the bag. This experiment is repeated at least 50 times. The student should decide what chance exists of picking a particular cube. The teacher should encourage students to compare the results obtained in this experiment with those found in the coin, spinner, and dice experiments.

Variations: Change the number of cubes in the bag. The student should investigate how the chances of selecting a particular colored cube are altered when the sample from which the cube is chosen is changed.

6-12 Sticky Pi. Purpose: Experimentally determine the value of π ; division and ratio, parallel line drawing

Materials needed: Ten one-inch long sticks (toothpicks), large sheet of paper, ruler, pencil, data sheet

Procedure: Have the students cover a paper with a series of parallel line segments drawn two inches apart. Next, hold the sticks about one foot above the center of the paper and then, drop them. Count the number of sticks that either touch or cross a line. Record this number. The students repeat the experiment described at least 100 times. This is equivalent to having dropped at least 1,000 sticks.

Discuss why:
$$\frac{\text{Total Number of Sticks Tosses}}{\text{Total Number Sticks Crossing or Touching a Line}}$$

is approximately equal to π . (This is a classic mathematical experiment known as Buffon's needle problem.)

	No. Toothpicks Touching a Line	+ No. Toothpicks Not Touching Any Line	= Total No. of Toothpicks
Trial 1			
Trial 2			

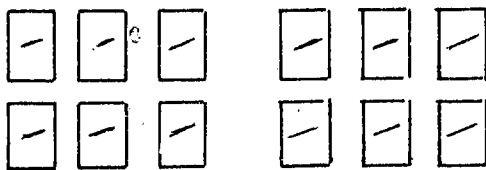
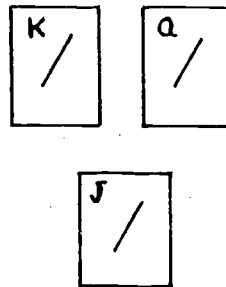
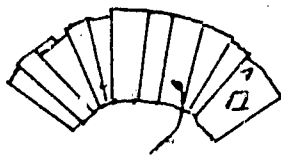
6-13 Royal Welcome. Purpose; Determine the probability for independent events, data collection, fractions, ratio

Materials needed: Deck of playing cards

Procedure: Have a student shuffle the deck of cards and guess the number of cards that must be turned up before getting to a picture card or an ace. The experiment can be repeated as often as desired. Another approach is to have the student pull a card from out of the deck and decide what chance exists of picking a picture card or an ace.

Theoretically:

$$P(\text{Picture card}) = \frac{\text{Total number of picture cards}}{\text{Total number of cards in deck}} = \frac{12}{52} = \frac{3}{13}$$



1	2	3	4	5	6	7	8	9	10	J	Q	K
A	2	3	4	5	6	7	8	9	10	J	Q	K
2	3	4	5	6	7	8	9	10	J	Q	K	A
3	4	5	6	7	8	9	10	J	Q	K	A	2

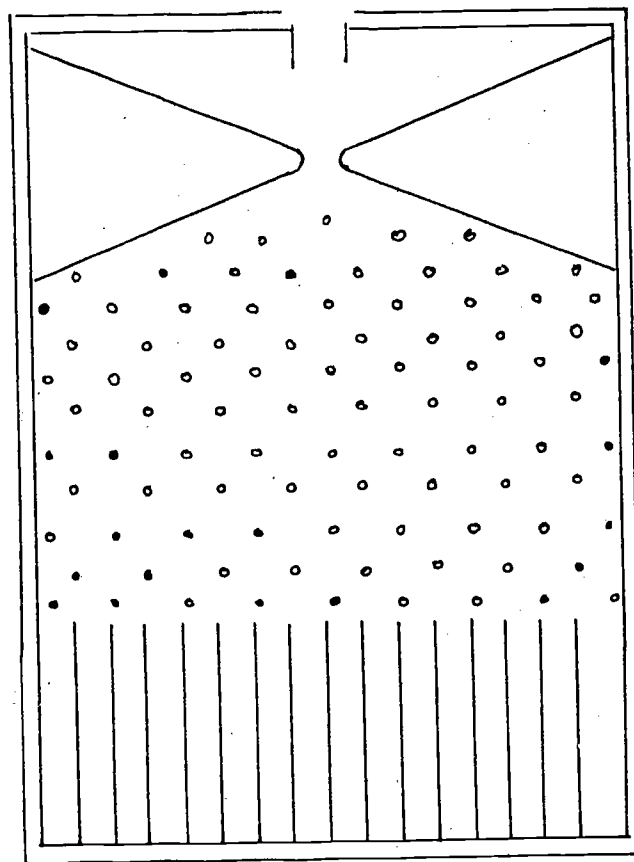
1	2	3	4	5	6	7	8	9	10	J	Q	K
2	3	4	5	6	7	8	9	10	J	Q	K	A
3	4	5	6	7	8	9	10	J	Q	K	A	2
4	5	6	7	8	9	10	J	Q	K	A	2	3
5	6	7	8	9	10	J	Q	K	A	2	3	4
6	7	8	9	10	J	Q	K	A	2	3	4	5
7	8	9	10	J	Q	K	A	2	3	4	5	6
8	9	10	J	Q	K	A	2	3	4	5	6	7
9	10	J	Q	K	A	2	3	4	5	6	7	8
10	J	Q	K	A	2	3	4	5	6	7	8	9
J	Q	K	A	2	3	4	5	6	7	8	9	10
Q	K	A	2	3	4	5	6	7	8	9	10	J
K	A	2	3	4	5	6	7	8	9	10	J	Q
A	2	3	4	5	6	7	8	9	10	J	Q	K

6-14 Probably, It Will Work. Purpose: Encourage formulation of statistical hypotheses, implementation of normal distribution, concepts of frequency, probability, bell curve, research on Pascal's Triangle

Materials needed: Base of one-quarter inch plywood board about 12"x18", one-eighth inch plywood for three inch framing about the base, for dividers (shown at the bottom of diagram), and for the funnel-like barrier (shown at the top of the diagram, BB shot or marbles, nails

Procedure: Nails are driven into the board in rows of increasing numbers below the funnel. The first row will have one nail, right below the funnel. The second row will have two nails, the third three, and so on.

Hold the board vertically. Roll the shot or marbles through the funnel. They will work their way down to the bottom and fill the slots according to a normal curve. Have students perform this experiment and record their results.



1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16

6-15 Coins. Purpose: Experimentally relating probability and statistics in dot distribution chart, histogram, normal distribution, central tendencies

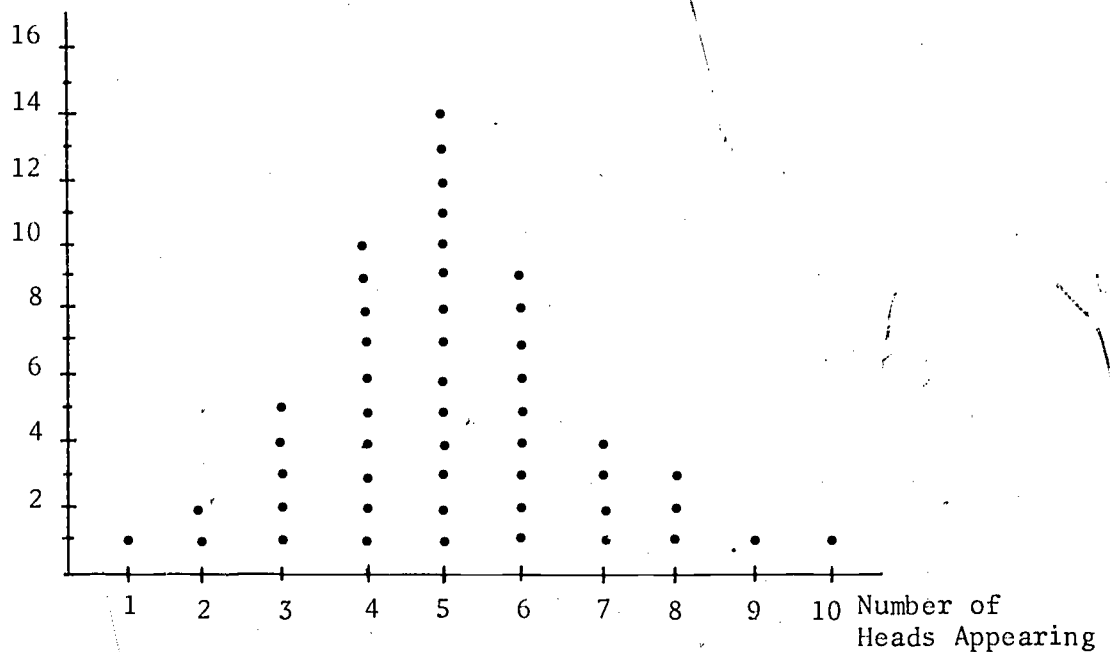
Materials needed: Ten coins of the same type

Procedure: Students should place ten coins in a cup, shake it and empty it. The number of heads (or tails) should be counted and recorded. (See sample below.) The experiment should be repeated for at least 50 trials.

The graphic portrayal of results can serve as a springboard for discussions about central tendencies (mean, median, mode) and the bell-shaped or normal curve.

Dot Distribution of a Sample Experiment with Ten Coins Tossed 50 Times

Frequency of Outcome



STATISTICS

Statistics involves the collection, organization, and analysis of numerical facts. Scientists, sportsmen, business executives, scholars, and citizens rely on statistics inferences to recognize current trends and to predict future affairs. Since, in real life, all possible variations are seldom known in advance, the statistician has a difficult chore in presenting the probability of future events. We work to improve reliability by collecting sample information in an unbiased, random system.

For students, working with statistical data can make mathematics relevant and meaningful. It can also provide problems for applying and practicing computational skills. A typical problem is examined below:

Data Collection

Suppose the following are heights in centimeters of mothers of children in the class: 150, 159, 162, 165, 162, 153, 159, 168, 156, 162, 159, 159, 165, 174, 162, 162, 153, 156, 159, 162, 156, 165, 162, 168, 159, 162, 165, 171, 150, 165.

Data Organization

Since the numbers are not arranged in any discernible order, little information can be gotten from them. Putting the raw data in order from smallest to largest and making a frequency distribution is a convenient way to reveal possible patterns.

<u>Heights</u>	<u>Tally Marks</u>	<u>Frequency</u>	<u>Ratio</u>
150	11	2	2/30 or 1/15
153	11	2	2/30 or 1/15
156	111	3	3/30 or 1/10
159	1111 1	6	6/30 or 1/5
162	1111 111	8	8/30 or 4/15
165	1111	5	5/30 or 1/6
168	11	2	2/30 or 1/15
171	1	1	1/30
174	1	1	1/30
		Total = <u>30</u>	

Data Analysis

The size of the sample is 30.

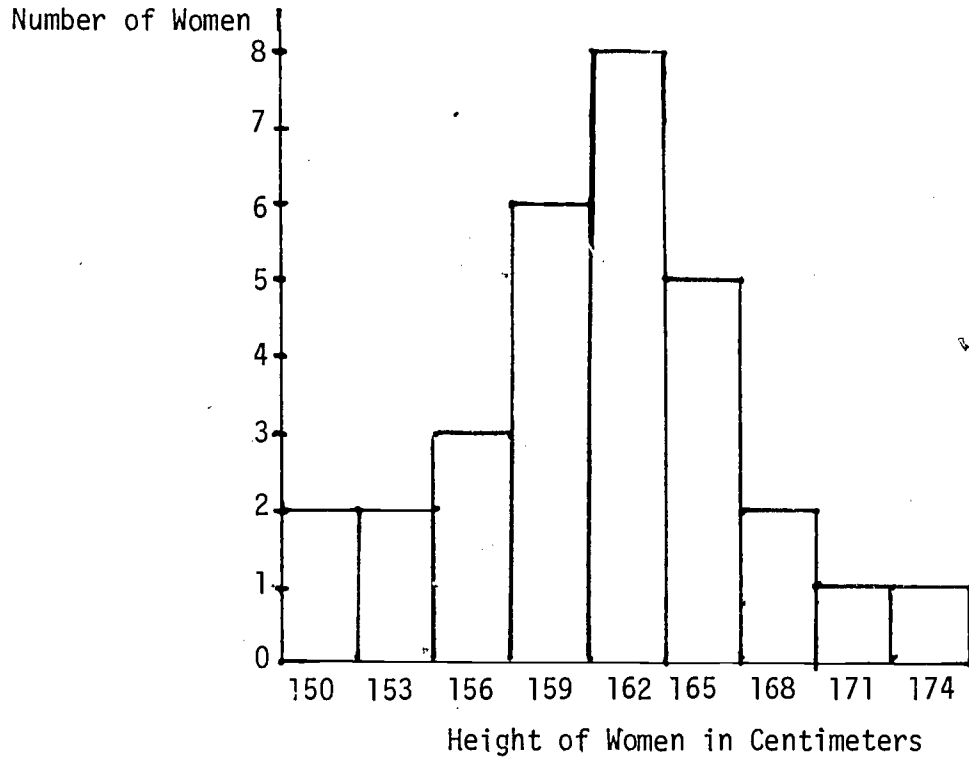
The range of heights goes from 150 cm. to 174 cm. The mean, median and mode are called "Central Tendencies."

The median (middle) height is 162 cm.

The mode (most frequent) height is 162 cm.

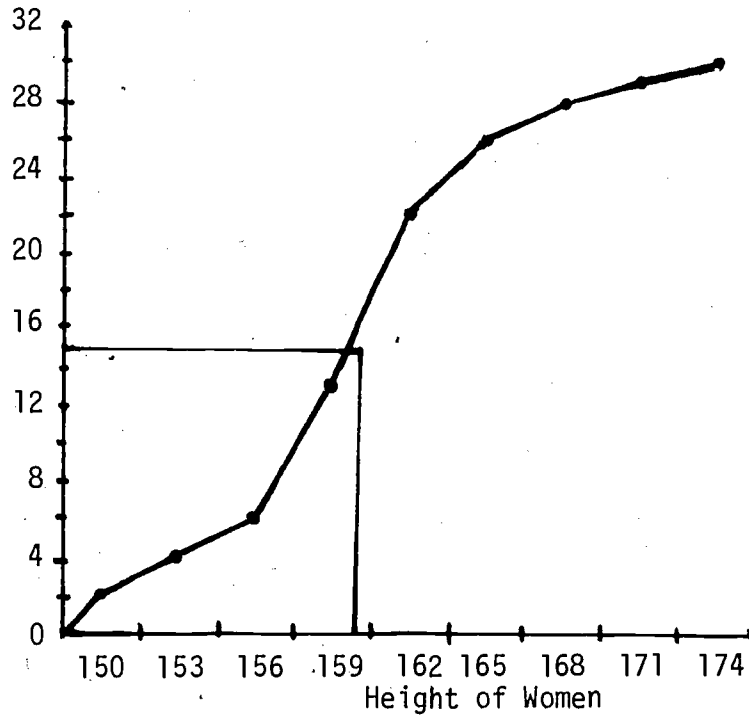
The mean (average) height is 161 cm. (Total of 4830 cm. divided by 30.)

HEIGHT HISTOGRAM



Cumulative Frequency

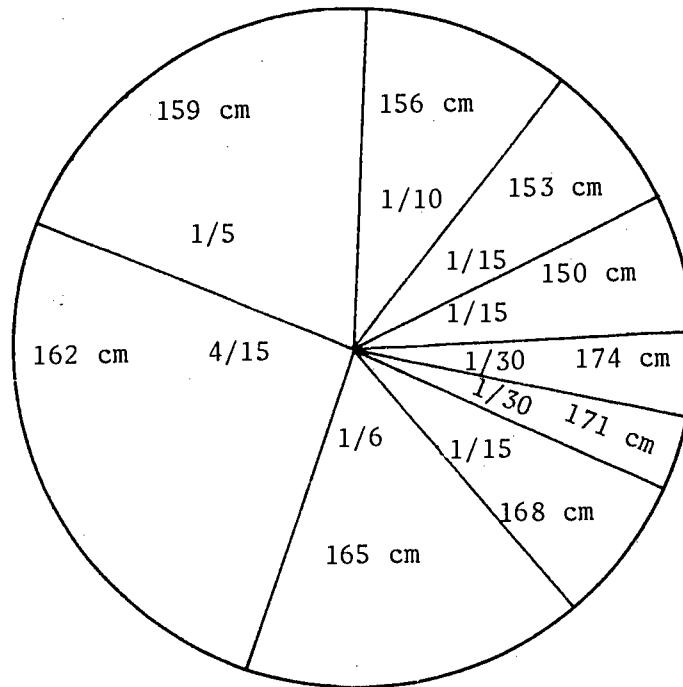
CUMULATIVE FREQUENCY GRAPH



Data Interpretation

The information from the sample population, when graphed, forms an almost bell-shaped or normal curve. This regular pattern is generally found to describe variations in statistical studies. The sample population appears to be typical of adult women regarding height.

Height Distribution Among a Random Sampling of Adult Women



6-16 Problems About Problems. Purpose: Problem solving, computational skills, numeration, data collection, organization and analysis

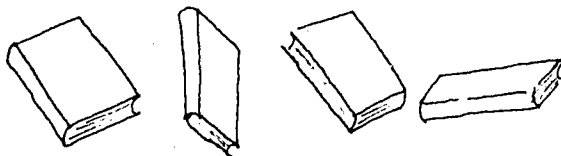
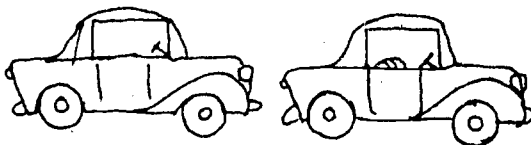
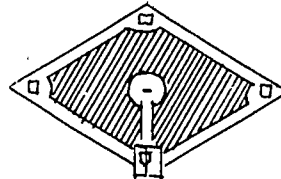
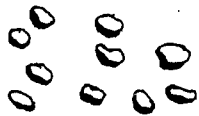
Materials needed: Survey sheets, graph paper

Procedure: Have students discuss problems they may have in getting random samples. Why wouldn't the following sets be random samples?

- a) People entering the Waldorf Astoria Hotel in New York City
- b) Names selected from the Albany telephone book
- c) Readers of "The New York Times"
- d) Motorists
- e) Members of the Environmental Council.

Have students discuss how experiments could be designed to find out:

- a) How many raisins are in 100 boxes of raisins?
- b) How many worms are in the ball field?
- c) How many cars pass through a toll gate in a year?
- d) How many grains of sand are in an acre at the beach?
- e) How many library books are in the school library?



6-17 Surveys. Purpose: Data collection, organization and analysis, measurements, graphing, computation, research

Materials needed: Survey sheets, graph paper

Procedure: Have students devise projects for obtaining data from members of the class, other classes, teachers, parents, neighbors, people at games, movies, concerts, libraries, diners, supermarkets, post offices, etc. A survey form can specify whether questions are asked of a male or female. It may involve data about height, weight, hair and eye color, age, family siblings, vacations, family cars, favorite season, subject, sport, color, television habits, pets, politics, consumption of sugar, fuel, paper, etc.

Have students obtain data from personal observations:

- a) Analyze the traffic on a busy thoroughfare.
- b) Count customers to determine the best time to sell hamburgers.
- c) Determine which class has the most refuse during a two week period (cooperation with the custodial staff is suggested here).
- d) Collect weather facts.
- e) Collect statistics from the sports and finance sections of the newspaper and magazines.

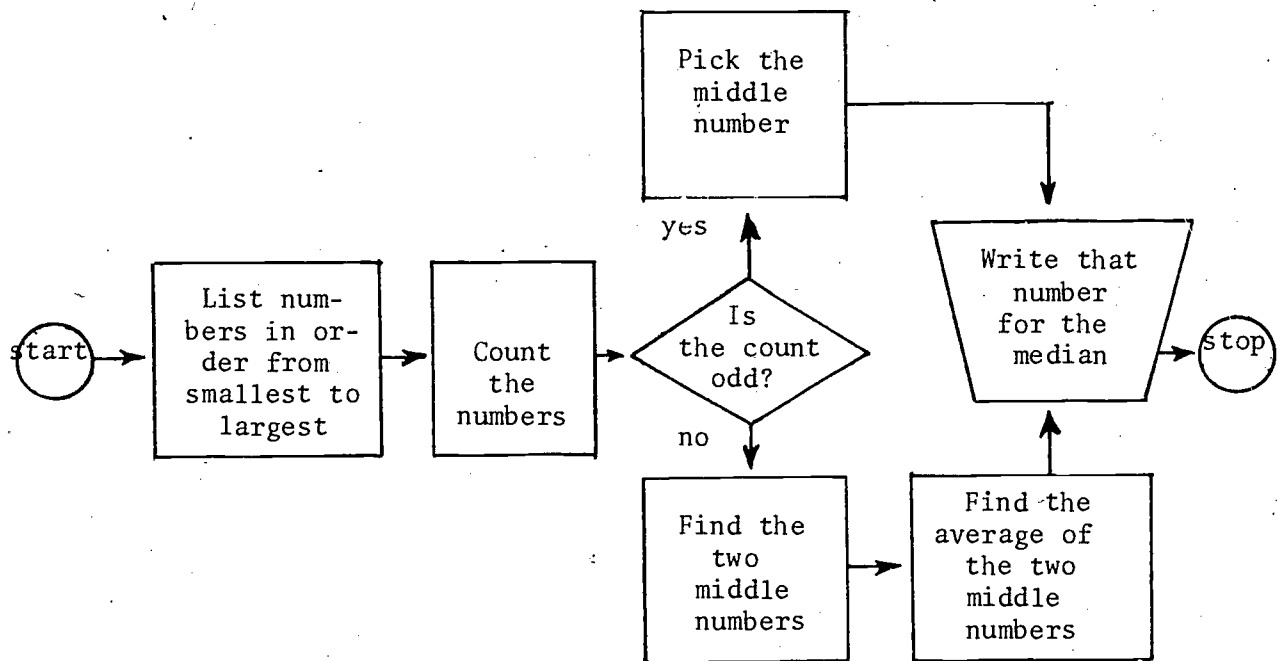
Vehicles on School Street from 1:00 - 2:00 p.m., April 3, _____

Vehicles	Tally	Frequency	Percent of Total
Bicycles			
Buses			
Cars			
Motorcycles			
Trucks			
TOTAL			

6-18 Flow Chart the Median. Purpose: Develop an orderly way to locate the median

Materials needed: Set of data, flow charting template

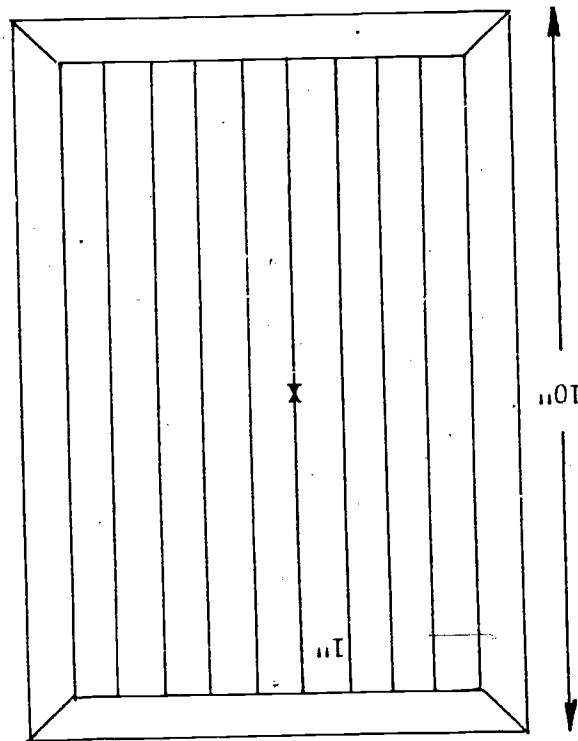
Procedure:



6-19 Wedding Rice. Purpose: Experimentally create a bell-shaped curve, data collection, organization, and analysis, tallying, histogram

Materials needed: Rice, box, ruled paper, data sheet

Procedure: Have students divide a sheet of paper into ten one-inch columns and mark an X in the middle of the paper. Number the columns 1-10. The paper is placed in a box. They then count 100 grains of rice and drop them over the paper. The X is used as a target. They now record the number of grains of rice that land in each column. Rice that falls on a line should be thrown again or counted in the section where more of the rice lies. The experiment should be repeated by several students. Their results should all be recorded on a single graph.



6-20 License Plates. Purpose: Numeration, ordering, permutations, independent events, exponents, multiplication

Materials needed: None

Procedure: In New York State, the ordinary passenger car license identification has three numerals followed by three letters of the alphabet. Have students find the total number of different identifications that could be made with that combination. Assume any digit in the set (0, 1, 2, 3, 4, 5, 6, 7, 8, 9) may be used in any of the first three places and any letter of the alphabet may be used in any of the next three places.

Note: The answer is $10 \times 10 \times 10 \times 26 \times 26 \times 26$ or $10^3 \times 26^3$ or 1,756,000.

Students can also explore different arrangements that might be feasible.

000 - AAA

520 - CEX

099 - ABC

001 - AAB

666 - AAA

222 - DDD

456 - ADW

356 - XXZ

999 - ADF

711 - BCF