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ABSTRACT

Piaget has hypothesized that concrete and formal operations can be described by specific logical models. The present study focused on assessing various aspects of four concrete operational groupings and two variations of two formal operational characteristics. Six hundred twenty-two 9-14 year old students participating in the Human Sciences Program designed by Biological Science Curriculum Study were the subjects. Two 15-item written tests were given on two consecutive days. Twelve items included drawings, 19 were open-ended, and 11 were multiple choice. For analysis seven items were eliminated because of deviant difficulty indices; the remaining 23 items were subjected to an image analysis with the initial factor matrix obliquely transformed using Hofmann's orthotran. Three factors were exceptionally clean: one including formal operational systematic permutations; one measuring concrete operational addition of increasing asymmetrical relations; and one involving the formal operational logic of making correct implications and denying incorrect implications. Four other factors were mixed within developmental period and across developmental period. Results are discussed relative to tentative support for some of Piaget's logical models, the robustness of Piagetian theory, and the feasibility of a written test of cognitive development. (Author/RC)

THE FACTOR STRUCTURE OF CONCRETE AND FORMAL OPERATIONS: A CONFIRMATION OF PIAGET

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Six hundred twenty-two 9-14 year old students participating in the Human Sciences Program designed by Biological Science Curriculum Study were the subjects. Mean age was 12.09 years, with s = .53.

Two 15 - item written tests were given on two consecutive days. Twelve items included drawings, 19 were open-ended, and 11 were multiple choice. For analysis seven items were eliminated because of deviant difficulty indices; the remaining 23 items were subjected to an image analysis with the initial factor matrix obliquely transformed using Hofmann's orthotran.

Three factors were exceptionally clean: one including formal operational systematic permutations; one measuring concrete operational addition of increasing asymmetrical relations; and one involving the formal operational logic of making correct implications and denying incorrect implications. Four other factors were mixed within developmental period and across developmental period.

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Piaget has hypothesized that concrete operations and formal operations have as their defining characteristics specific types of reasoning (Inhelder & Piaget, 1955/1958, 1959/1969; Piaget, 1953/1957). Concrete operational reasoning utilizes class and relational logic within eight models of thought termed groupings. These models possess the mathematical group characteristics of composition, associativity, reversibility, and identiy plus the special characteristics of tautology and special identity (i.e., absorption and resorption) (Flavell, 1963; Boyle, 1969). A concrete operational person can solve concrete problems which demand the use of these models of thought, but cannot solve problems which are abstract and/or demand flexible thinking.

Formal operations are described as a lattice (Piaget, 1953/1957) integrated with a set of four reversible transformations that satisfy the requirements of a group (Inhelder & Piaget, 1955/1958; Piaget, 1953/1957). This integration provides for very flexible thought that can produce and solve problems involving abstractions as well as solve concrete problems. Given that the mathematical models are the heart of the operational periods, one would expect that much research would utilize them by specifically testing different facets of the models and/or interpreting results within their framework. Unfortunately, much Piagetian research has tended to ignore the theoretical operational structures and concerned itself with developmental acceleration, replication, or stimulus conditions affecting operational development. Longeot (1962, 1964), Bart (1971), and Gray (1975) provide some alternative to this trend, as they have attempted to utilize and/or assess various aspects of the logical models. The present research is part of a continuing program directed toward evaluating the validity of the various concrete and formal operational logical models via traditional Piagetian tasks and written tasks. Along with the theoretical evaluation, a second, long-range goal is the development of a written test of operational thinking that could be used by classroom teachers to efficiently determine the general reasoning level of students. For the present study, various aspects of four concrete operational groupings and two variations of two formal operational characteristics were assessed.

Method of Investigation

Subjects

Five hundred seventy-eight 9-14 year old students participating in the HUMAN SCIENCES PROGRAM curriculum designed by Biological Sciences Curriculum Study (BSCS) were the subjects. Average age was 12.09, with a standard deviation of .53.

Instrument

Two 15-item written tests that assessed various concrete and formal operational structures were given on two consecutive days. Twelve of the thirty items included drawings for concrete reference, nineteen were open ended, and the remainder were multiple choice. For analysis, both forms were combined and considered as one test. Four items from Form A and three from Form B were eliminated, as they had difficulty indices widely discrepant from those expected (i.e., some concrete operational items were found to be extremely difficult, while some formal operational items were found to be extremely easy). The remaining twenty-three items were subjected to an incomplete image analysis (Harris, 1962), with the initial factor matrix obliquely transformed using Hofmann's Orthotran (Hofmann, Note 1). Structures assessed by the remaining items included the concrete operations of Bi-Univocal (one-to-one) Multiplication of Classes, Co-Univocal (one-to-many) Multiplication of Classes, Addition of Asymmetrical Relations, and Bi-Univocal (one-to-one) Multiplication of Relations and the formal operations of hypothetical-deductive and combinatorial thinking. Table 1 provides a listing of the structures and their variations that were assessed,

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Insert Table 1 about here.

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as well as the logic for specific questions that exemplify the structures. Table 2 is a distribution of the items according to logical structure. Items assessing the same structure and/or

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Insert Table 2 about here.

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items utilizing the same operation, and/or items measuring classes or measuring relations were expected to produce identifiable factors. For example, any of the following combinations of items could define a factor: B6 and B11, since they were a one-to-one multiplication of classes; B6, B11, A10, B4, and B5, as multiplication of classes; or B6, B11, A14, A6, A7, B3, and B7, as one-to-one multiplication. The first set of items assess a specific structure; the second, a class-oriented operation; and the third, an operation applied to both class and relational data. Any one of the three item sets could define a factor and still provide support for Piagetian theory. It was hypothesized that each concrete operational grouping tested and each formal characteristic tested would produce identifiable factors.

Results[1]

Table 3 is the correlation matrix for the twenty-three remaining items and Table 4 is the obliquely transformed factor pattern matrix with a normalized solution. Interpretation of

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Insert Tables 3 & 4 about here.

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the pattern matrix involved determining the highest loading for each variable and then for each factor, taking the lowest loading that had been determined by the preceding procedure and finding any other loading that was larger. For example, the highest loading for A9 is .258, which is on factor four. Items A12, B11, and B13 also have their highest loadings on factor four. However, A9 has the lowest substantial loading on factor four and B6 has a higher loading than A9; consequently, B6 is considered to load on factor four as well as on factor three. After the appropriate loadings for each factor were identified, factors 8-11 were eliminated; factors 10 and 11 had been transformed so that the loadings were trivial; while 8 and 9 were considered pseudo-specific, since each had only one item that loaded on it (A7 on 8 and B7 on 9), the loadings were low, and each of the loaded items had similar loadings on at least one other factor. Thus, for interpretation, seven factors were retained.

Three factors--five, six, and one--were exceptionally clean. Factor five included items A15 and B15, both measuring the permutation or systematic aspect of combinatorial thinking. Factor six appeared to be a factor of addition of increasing asymmetrical relations, as it had its major loadings on A1 and A4. Items A3, A5, B8, and B14 loaded on factor one, indicating that it involved the beginning formal operational logic of making correct implications (Inhelder & Piaget, 1955/1958) and the secong stage formal operational logic of denying incorrect implications (Inhelder & Piaget, 1955/1958). The remaining factors were not as clear as the previous three.

Factor two was loaded on by items involving addition of decreasing asymmetrical relations (B1, B2) and inverse correspondence of a decreasing series and an increasing series (B3). Table 1, structure 7C4, provides an example of the logic involved in B3. Because of the initial series (i.e., decreasing), subjects apparently were treating B3 more as a decreasing series stead of an inverse correspondence of two series. The remaining items measuring an inverse correspondence of a decreasing series and increasing series (A7, B7) did not provide any interpretable loadings. A7 was split between factors four and eight, while B7 was split among factors two, four, and nine, the largest of any of the loadings being .147. This fragmentation of items, the partial split of A14 between factors three and seven, and the loading of A6 with A10 on factor seven indicates that subjects were not treating the variations of Bi-Univocal Multiplications of Relations the same; and, more likely, the items were not written in a manner that would demand similar types of reasoning.

Factors three and four are more difficult to interpret. Items A9, A12, and B13 seem to indicate that factor four is a formal operational combination factor; however, the loadings of B6 and B11 indicate concrete operational Bi-Univocal Multiplication of Classes was also being measured. Both types of items used pair-wise combination of entities (see Table 2, structures 3 and CC); this; coupled with the age of the subjects, may indicate that many subjects were transitional with respect to combinatorial thinking and looked upon items demanding the pairing of entities as being the same regardless of their hypothesized level. This is quite reasonable, since Bi-Univocal Multiplication of Classes is prerequisite to formal operational combinatorial thinking (Inhelder & Piaget, 1955/1958). Factor three appears to be a combination of Co-Univocal Multiplication of Classes (B4, B5), Bi-Univocal Multiplication of Classes (B6, B11), and Bi-Univocal Multiplication of Relations (A14). These could be described as a generalized concrete operational structure utilizing multiplication; or, if A14 is ignored, as a structure involving multiplication of classes.

Factor seven is uninterpretable --the groupings of both items (A6, A10) have only the operation of multiplication and their content of fishing poles in common.

The factor intercorrelation matrix is provided in Table 5.

Insert Table 5 about here.

As would be expected by looking at the pattern matrix, the substantial correlations exist among the first seven factors. Although factors five, six, one, and possibly two were clear relative to loading on specific types of items, they are substantially related to each other and to the remaining interpretable factors. The correlations among the factors, especially the four purest ones, seem to provide some support for the Piagetian concept of structure d'ensemble--that is, the factors measure structures which involve different types of reasoning, but they are integrated into an organized system of reasoning about the world.

Discussion

Several implications can be derived from the results. First, and theoretically most important, was a confirmation of several logical thought models postulated by Piaget. The fact that the items were designed to duplicate the logic of some of Piaget's logical models and that each of four factors (5, 6, 1, 2) could be clearly described as representing a different logical structure provides this support. Specifically, the formal operations of systematic thinking (permutations) and making correct and denying incorrect implications, and the concrete operational groupings of Addition of Asymmetrical (increasing) Relations and Addition of Asymmetrical (decreasing) Relations were clearly represented in the data.

A second theoretical implication focuses on the robustness of Piagetian theory. Traditional assessment of operational thought has been via concrete physical manipulable tasks. Although a variety of studies have used non-concrete tasks, very few have specifically focused on the logical models themselves, thus restricting the generalizability of the Piagetian logical models. The results of this study clearly support the generalizability of Piagetian theory in explaining the responses to written situations. This generalization can also be extended to written items that were initially designed without concern for the logic involved in them (Gray, Note 2).

Finally, the success with the written items indicates that a written test of cognitive development is feasible, if the items accurately duplicate the logic of the hypothesized developmental cognitive structures. Obviously, such a test would be useful in psychological research as well as in classrooms where it could provide teachers with information on the types of reasoning their students can or cannot engage in.

Reference Notes

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Footnotes

[1] Data provided by the Human Sciences Program of the Biological Sciences Curriculum Study; Boulder, Colorado.

Table 1

Logic of Concrete Operational and Formal Operational Structures

Structure	Example Logic
3	(A, B, C) x (L, M, N) = AL AM AN BL BM BN CL CM CN
4	$G \times (S_1, S_2, S_3, S_4, S_5, S_6, S_7, S_8) =$ $GS_1, GS_2, GS_3, GS_4, GS_5, GS_6, GS_7, GS_8$
5A	$K < J < H \therefore K < H$
5B	$P > K > J > R \therefore P > R$
7B	$B_1 = B_2$, $(G_1 \uparrow G_2 = G_2 \downarrow G_1)$, $(G_1 \leftarrow G_2 = G_2 \rightarrow G_1)$ where $\uparrow$ = taller than, $\downarrow$ = shorter than, $\leftarrow$ = thinner than, $\rightarrow$ = wider than, T = transformed into B_1 T G_1 B_2 T G_2 $\therefore (G_1 \uparrow \leftarrow G_2) = (G_2 \downarrow \rightarrow G_1)$
7C1	$D < J < G < R$ $\updownarrow \quad \updownarrow \quad \updownarrow \quad \updownarrow$ $F_1 < F_2 < F_3 < F_4$ $\therefore J \updownarrow F_2$

Table 1 (Cont'd.)

7C4	$B > T > F > J$ $\updownarrow \quad \updownarrow \quad \updownarrow \quad \updownarrow$ $C_1 < C_2 < C_3 < C_4$ $\therefore \; B \updownarrow C_1$
HC	$p \wedge q \wedge \bar{r} \wedge x$ $\bar{p} \wedge q \wedge r \wedge x$ $p \wedge \bar{q} \wedge \bar{r} \wedge \bar{x}$ $\therefore \; q[p \vee r] \rightleftarrows x$
HD	$p \wedge \bar{q} \wedge r \wedge x$ $\bar{p} \wedge q \wedge \bar{r} \wedge \bar{x}$ $\bar{p} \wedge q \wedge \bar{r} \wedge x$ $p \wedge \bar{q} \wedge r \wedge \bar{x}$ $\therefore \; [p \vee q \vee r] * x$
CC	(A, D, L, M, N, S) x (A, D, L, M, N, S) = AD AL AM AN AS DL DM DN DS LM LN LS MN MS NS
CP	FGIP GFIP IFGP PFGI FGPI GFPI IFPG PFIG FPGI GPFI IPFG PIFG FPIG GPIF IPGF PIGF FIPG GIPF IGPF PGIF FIGP GIFP IGFP PGFI

Note. 3 = Bi-Univocal Multiplication of Classes (complete matrix)
4 = Co-Univocal Multiplication of Classes (one-to-many correspondence)

5A = Addition of Asymmetrical Relations (increasing)
5B = Addition of Asymmetrical Relations (decreasing)
7B = Bi-Univocal Multiplication of Relations (conservation of continuous quantity)
7C1 = Bi-Univocal Multiplication of Relations (correspondence: direct, increasing)
7C4 = Bi-Univocal Multiplication of Relations (correspondence: inverse, decreasing - increasing)
HC = Hypothetical-Deductive Thinking (make correct implication)
HD = Hypothetical-Deductive Thinking (deny incorrect implications)
CC = Combinatorial Thinking (pair-wise combination)
CP = Combinatorial Thinking (permutation of four entities)

The term 'implication' used in describing HC and HD type logic is that of Inhelder & Piaget (1955/1958).

Table 2

Item Distribution

	Piagetian Logical Structure										
	3	4	5A	5B	7B	7C1	7C4	HC	HD	CC	CP
Item	B6	A10	A1	B1	A14	A6	A7	A3	B14	A9	A15
	B11	B4	A4	B2			B3	A5		A12	B15
		B5					B7	B8		B13	

Note. 3 = Bi-Univocal Multiplication of Classes (complete matrix)
4 = Co-Univocal Multiplication of Classes (one-to-many correspondence)
5A = Addition of Asymmetrical Relations (increasing)
5B = Addition of Asymmetrical Relations (decreasing)
7B = Bi-Univocal Multiplication of Relations (conservation of continuous quantity)
7C1 = Bi-Univocal Multiplication of Relations (correspondence: direct, increasing)
7C4 = Bi-Univocal Multiplication of Relations (correspondence: inverse, decreasing increasing)
HC = Hypothetical Deductive Thinking (make correct implication)
HD = Hypothetical-Deductive Thinking (deny incorrect implications)
CC = Combinatorial Thinking (pair-wise combination)
CP = Combinatorial Thinking (permutation of four entities)
The term 'implication' used in describing HC and HD type logic is that of Inhelder & Piaget (1955/1958).

Table 3

Interitem Correlation Matrix

Item	A1	A3	A4	A5	A6	A7	A9	A10	A12	A14	A15
A1	1.000										
A3	.208	1.000									
A4	.393	.159	1.000								
A5	.190	.692	.123	1.000							
A6	.083	.047	.027	.047	1.000						
A7	.066	.167	.055	.127	.087	1.000					
A9	.101	.170	.118	.153	.015	.087	1.000				
A10	.079	.017	.056	.054	.264	.100	-.062	1.000			
A12	.172	.194	.184	.261	.066	.183	.283	.146	1.000		
A14	.094	.059	.093	.064	.113	.049	.025	.163	.158	1.000	
A15	.025	.130	.070	.111	.024	.042	.093	-.033	.117	.003	1.000
B1	.095	.097	.122	.112	.041	.053	.131	.005	.135	.142	.070
B2	.113	.148	.103	.140	.111	.036	.154	.051	.139	.123	.038
B3	.070	.075	.066	.082	.026	.202	.145	-.001	.130	.063	.027
B4	-.005	.066	.053	.099	.011	.083	.060	.101	.125	.213	.042
B5	.014	.123	.109	.112	.119	.082	.004	.058	.105	.217	.051
B6	.055	.122	.133	.138	.118	.062	.107	.118	.280	.177	.042
B7	.035	.148	.041	.151	.091	.038	.126	.090	.126	.053	.075
B8	.156	.464	.145	.523	.040	.098	.113	.015	.170	.080	.104
B11	.119	.159	.144	.242	.098	.123	.140	.035	.313	.190	.097
B13	.127	.233	.199	.299	.067	.171	.252	.043	.597	.133	.106
B14	.116	.247	.067	.325	.064	.110	.153	.085	.197	.080	.102
B15	.099	.164	.095	.209	.031	.077	.182	.066	.173	.068	.436

Table 3 (Contd.)

Interitem Correlation Matrix

Item	B1	B2	B3	B4	B5	B6	B7	B8	B11	B13	B14	B15
A1												
A3												
A4												
A5												
A6												
A7												
A9												
A10												
A12												
A14												
A15												
B1	1.000											
B2	.678	1.000										
B3	.262	.328	1.000									
B4	.089	.171	.196	1.000								
B5	.038	.111	.038	.331	1.000							
B6	.084	.132	.147	.207	.239	1.000						
B7	.129	.201	.225	.129	.025	.215	1.000					
B8	.197	.212	.145	.070	.067	.166	.142	1.000				
B11	.184	.239	.173	.255	.204	.402	.166	.177	1.000			
B13	.159	.213	.200	.190	.178	.385	.220	.211	.429	1.000		
B14	.110	.124	.106	.074	.045	.110	.076	.261	.101	.163	1.000	
B15	.092	.133	.102	.058	.070	.099	.012	.167	.135	.159	.156	1.000

Table 4

Primary Pattern Matrix

Item \ Factor	1	2	3	4	5	6	7	8	9	10	11
A1	.080	.004	-.052	-.025	-.003	.362	.052	.012	.004	-.003	.000
A3	.694	-.045	.005	-.049	.011	.043	-.024	.009	.004	.020	.008
A4	-.021	-.003	.051	.017	.019	.369	-.003	-.002	.007	.002	.000
A5	.719	-.057	.012	.019	.015	.003	-.011	-.009	-.008	.008	.001
A6	-.002	.021	.009	-.022	.002	-.003	.277	-.005	.016	.019	-.000
A7	.078	-.008	.008	.106	.012	-.004	.075	.147	-.004	.010	.009
A9	.033	.086	-.076	.258	.097	.046	-.057	.039	.008	-.011	.029
A10	-.014	-.042	.011	-.005	-.012	.004	.326	.015	-.009	-.010	.004
A12	.006	-.045	.003	.543	.043	.059	.062	.010	-.061	-.005	.007
A14	-.041	.040	.251	-.029	-.008	.054	.126	-.014	-.046	-.020	.003
A15	-.000	-.020	.001	-.009	.411	-.009	-.021	-.008	.012	.008	-.002
B1	-.007	.666	-.027	-.047	.010	.034	-.003	-.038	-.053	.005	-.002
B2	.015	.668	.029	-.047	.002	.012	.022	-.028	-.021	.009	.003
B3	-.027	.319	.047	.075	.004	-.020	-.018	.118	.087	-.014	-.003
B4	-.030	.052	.408	-.023	-.000	-.042	-.001	.050	.019	-.013	.014

Table 4 (Contd.)

Item \ Factor	1	2	3	4	5	6	7	8	9	10	11
B5	.009	-.054	.429	-.076	.014	.015	.026	.001	-.026	.015	.009
B6	-.030	-.031	.276	.260	-.012	-.002	.058	-.048	.069	-.002	-.020
B7	.081	.143	.024	.134	-.027	-.065	.056	.009	.137	.000	.012
B8	.524	.092	.003	-.038	.019	.025	-.015	-.018	.019	-.011	-.015
B11	.008	.063	.275	.257	.015	.023	-.008	-.037	.027	.012	-.025
B13	.030	.000	.115	.549	.010	.028	-.008	-.006	-.010	.010	-.002
B14	.282	.032	-.044	.054	.070	-.013	.072	.028	-.019	-.042	.003
B15	.043	.013	.000	.002	.409	.019	.020	.007	-.011	-.005	-.002

Table 5

Factor Intercorrelation Matrix

Factor	1	2	3	4	5	6	7	8	9	10	11
1	1.000										
2	.352	1.000									
3	.408	.435	1.000								
4	.507	.426	.608	1.000							
5	.442	.292	.314	.424	1.000						
6	.492	.353	.342	.506	.339	1.000					
7	.283	.286	.519	.375	.204	.364	1.000				
8	.176	.098	.146	.217	.149	.114	.123	1.000			
9	.129	.108	.183	.172	.051	.024	.087	.010	1.000		
10	.057	.000	.033	.006	-.019	.013	-.029	-.163	.080	1.000	
11	-.023	-.025	-.097	-.065	.001	-.021	-.049	.212	-.210	-.187	1.000