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ABSTRACT

This study empirically determined the optimizing weight to be applied to the Wrongs Total Score in scoring rubrics of the general form $S = R - kW$, where S is the Score, R the Rights Total, k the weight and W the Wrongs Total, if reliability is to be maximized. As is well known, the traditional formula score rests on a theoretical framework which is of dubious validity. Two instruments, variant approaches to the assessment of mathematical knowledge, were administered to approximately 1,700 entering college freshmen during an orientation period. The method consists of an iterative computer procedure for calculating split-half reliability of the tests as the weights are systematically varied throughout the region of maximization as determined by essentially canonical approaches. The results indicate that in contrast to the negative weight for the a priori formula score, a sizable positive weight maximizes reliability. The implications for rate of work as the single most reliable aspect of test performance seem clear. The validity of much educational testing rests on assumptions of fairness to those tested, achieved through optimization of standardized conditions. The study suggests that factors which alter rate-of-work characteristics of performance may be most detrimental to candidate success.
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An Optimizing Weight for "Wrong" Scores

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An Optimizing Weight for "Wrongs" Scores*

In scoring a multiple-choice test, the "formula score" or "correction for guessing" is the most widely used alternative to the simple count of the total number of right answers. The formula is

$$\text{F.S.} = R - \frac{W}{k-1},$$

where

F.S. = Formula Score

R = Total number right

W = Total number wrong

k = Number of choices per test item

The basic assumption which underlies this formula is that responses fall into two categories: those based on knowledge sufficient to determine a correct answer, and those based on knowledge insufficient to provide any basis for response better than chance responding. The value of R, the total number right, is a combination of the two categories, but the value of W, the total number wrong, reflects only responses based on insufficient information. The size of W is used to "correct" the observed value of R, to estimate the true value of the number of responses based on knowledge, for the chance behaviors are assumed to be randomly spread equally across the k choices per item, so that $\frac{k-1}{k}$ of them will be wrong answers, summing to the observed W score, and $\frac{1}{k}$ of them will be right answers, "buried" in the R score. The ratio of "buried" wrong answers to "observed" wrong answers is thus $\frac{1}{k-1}$.

Thorndike (1971) has discussed this correction, emphasizing its logical flaws and some of its merits. Ebel (1972) has presented research

* The author is indebted to David R. Saxe for the computer operations and for his interest in the problem.

evidence on the superior reliability of tests when they are scored with a formula correction. More recently, Lord (1975) has focussed on examinee behavior under different sets of instructions: formula scoring directions and number-right directions. He states an assumption that under number-right scoring candidates replace "Omit" responses by random marks on the answer sheet. The impact of this random responding is to reduce the sampling error of the formula score when contrasted with the number right score. This point is established by considering not $F.S. = R - \left(\frac{1}{k-1}\right) W$, but $F.S.' = R + \frac{O}{k}$, where O = the number of items unanswered, and R and k are as before. It has long been known that since R , W , and O sum to a constant, (T , the total number of items) the two values of the formula scores, $F.S.$ and $F.S.'$, are perfectly correlated.

But the assumption of random responses is not an attractive one. Lord is clearly concerned that the assumption be recognized for its crucial role and that instructions be developed to insure that any omissions under formula scoring are truly items for which candidates have only a chance, random, potential for success. But the theory is not strongly substantiated by our evidence on candidate behavior. Guessing on tests is in the main not random activity.

If the theoretical underpinnings of the formula score are so unattractive, why are we constrained to the weight, $\frac{1}{k-1}$, which it leads to for W ? What other weight might we use, and to what purpose? One purpose might clearly be the development of a maximum reliability for the score from a test. In an unpublished study by Fischer and Jackson (1971),

the maximization of reliability was taken as the rationale for determining the best weight, x , for the wrongs. Taking Dressel's (1940) formula for the Kuder-Richardson reliability of a formula-scored test, Fischer and Jackson differentiated the equation with respect to the weights for the wrong answers when the right answers are weighted unity. That is, defining a weighted score as

$$W.S. = R + xw$$

where x may take any value, positive or negative, for what value is the reliability of the W.S., the weighted score, a maximum?

Somewhat to their surprise the authors found that the value of x was positive; the sum of the rights and a fraction of the wrongs was the most reliable score. Further, the Rights score alone was more reliable than the conventional formula score in each of four separately--timed subtests, comprising a form of the College Board Scholastic Aptitude Test (SAT), were two verbal and two mathematical sections with x -values of + .295 and + .585 for the mathematical material and + .639 and + .720 for the verbal.

Lord, in discussing this result observed that "This does not mean that we should give bonuses for wrong answers. It merely means that that trait of omitting items is a trait that can be quite reliably measured." This trait of omitting items, however, may be the trait of working on test material with a consistent speed. Lord, states in his discussion that his theoretical development will work best for unspeeded tests. But the test studied by Fischer and Jackson was a standard SAT form, moderately speeded. There is a possible difference between omitting an item and

not reaching it. In the standard ETS item analysis, an item is considered omitted if there is a response to a later item; it is considered Not Reached if there are no responses to later items. If the preponderance of omitting in Fischer and Jackson's paper was due to a failure to complete the test, to Not Reaching, this would be evidence that the trait which is reliably measured is rate of work, not tendency to omit due to conservatism or caution.

Fischer and Jackson used a generalized internal consistency approach, via Dressel's formula, and determined the maximum reliability by differentiation with respect to the weight for wrongs. The present study extends this work by an empirical determination of the correlation between two half tests on two 50-item mathematics tests. Each half test was scored $R + kW$, (k here is simply the weight in wrongs, exactly equivalent to Fischer and Jackson's x) and the correlation between them computed. This was systematically followed throughout the region $-5.0 < k < 5.0$. The result was the two empirical curves presented in Figure 1 and Figure 2. Each of these curves shows a maximum for a positive weight somewhat less than unity. Tables 1 and 2 provide the data upon which the graphs were based.

This result supports the finding of Fischer and Jackson. The two curves reflect slightly different treatments, however. The curve in Figure 1 was based on a 50-item mathematical test which consisted of data sufficiency items. The curve in Figure 2 is based on a 50-item mathematical test which consisted of "regular math" problems. The data sufficiency items have the form of two statements and a question. The respondent is

Table 1

Interform Reliability (R) of the Score R + kW
for Selected Values of k : Data Sufficiency Tests

(k)	(R)	(k)	(R)	(k)	(R)	(k)	(R)
1	0.611740 00	-0.160000 01	0.615620 00	35	0.190000 01	0.702700 00	69
2	0.611720 00	-0.150000 01	0.616260 00	36	0.190000 01	0.693670 00	70
3	0.611710 00	-0.140000 01	0.617020 00	37	0.200000 01	0.685920 00	71
4	0.611700 00	-0.130000 01	0.617900 00	38	0.210000 01	0.679250 00	72
5	0.611690 00	-0.120000 01	0.618940 00	39	0.220000 01	0.673490 00	73
6	0.611680 00	-0.110000 01	0.620160 00	40	0.230000 01	0.668490 00	74
7	0.611680 00	-0.100000 01	0.621500 00	41	0.240000 01	0.664140 00	75
8	0.611680 00	-0.900000 00	0.623320 00	42	0.250000 01	0.660330 00	76
9	0.611680 00	-0.800000 00	0.625360 00	43	0.260000 01	0.656980 00	77
10	0.611690 00	-0.700000 00	0.627620 00	44	0.270000 01	0.654020 00	78
11	0.611690 00	-0.600000 00	0.630780 00	45	0.280000 01	0.651400 00	79
12	0.611700 00	-0.500000 00	0.634370 00	46	0.290000 01	0.649050 00	80
13	0.611720 00	-0.400000 00	0.638750 00	47	0.300000 01	0.646960 00	81
14	0.611740 00	-0.300000 00	0.644110 00	48	0.310000 01	0.645070 00	82
15	0.611760 00	-0.200000 00	0.650700 00	49	0.320000 01	0.643370 00	83
16	0.611790 00	-0.100000 00	0.658830 00	50	0.330000 01	0.641820 00	84
17	0.611820 00	0.0	0.668860 00	51	0.340000 01	0.640420 00	85
18	0.611870 00	0.100000 00	0.681210 00	52	0.350000 01	0.639140 00	86
19	0.611920 00	0.200000 00	0.696280 00	53	0.360000 01	0.637970 00	87
20	0.611980 00	0.300000 00	0.714310 00	54	0.370000 01	0.636900 00	88
21	0.612050 00	0.400000 00	0.735210 00	55	0.380000 01	0.635910 00	89
22	0.612140 00	0.500000 00	0.758140 00	56	0.390000 01	0.635000 00	90
23	0.612230 00	0.600000 00	0.781190 00	57	0.400000 01	0.634160 00	91
24	0.612350 00	0.700000 00	0.801350 00	58	0.410000 01	0.633380 00	92
25	0.612480 00	0.800000 00	0.815150 00	59	0.420000 01	0.632660 00	93
26	0.612630 00	0.900000 00	0.8270160 00	60	0.430000 01	0.631980 00	94
27	0.612800 00	0.100000 01	0.816080 00	61	0.440000 01	0.631360 00	95
28	0.613010 00	0.110000 01	0.804600 00	62	0.450000 01	0.630770 00	96
29	0.613240 00	0.120000 01	0.789210 00	63	0.460000 01	0.630220 00	97
30	0.613510 00	0.130000 01	0.771990 00	64	0.470000 01	0.629700 00	98
31	0.613810 00	0.140000 01	0.755030 00	65	0.480000 01	0.629220 00	99
32	0.614170 00	0.150000 01	0.739350 00	66	0.490000 01	0.628760 00	100
33	0.614580 00	0.160000 01	0.725390 00	67	0.500000 01	0.628330 00	101
34	0.615060 00	0.170000 01	0.713210 00	68			

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Table 2

Interform Reliability (R) of the Score R + kW
for Selected Values of k : Math Achievement Test

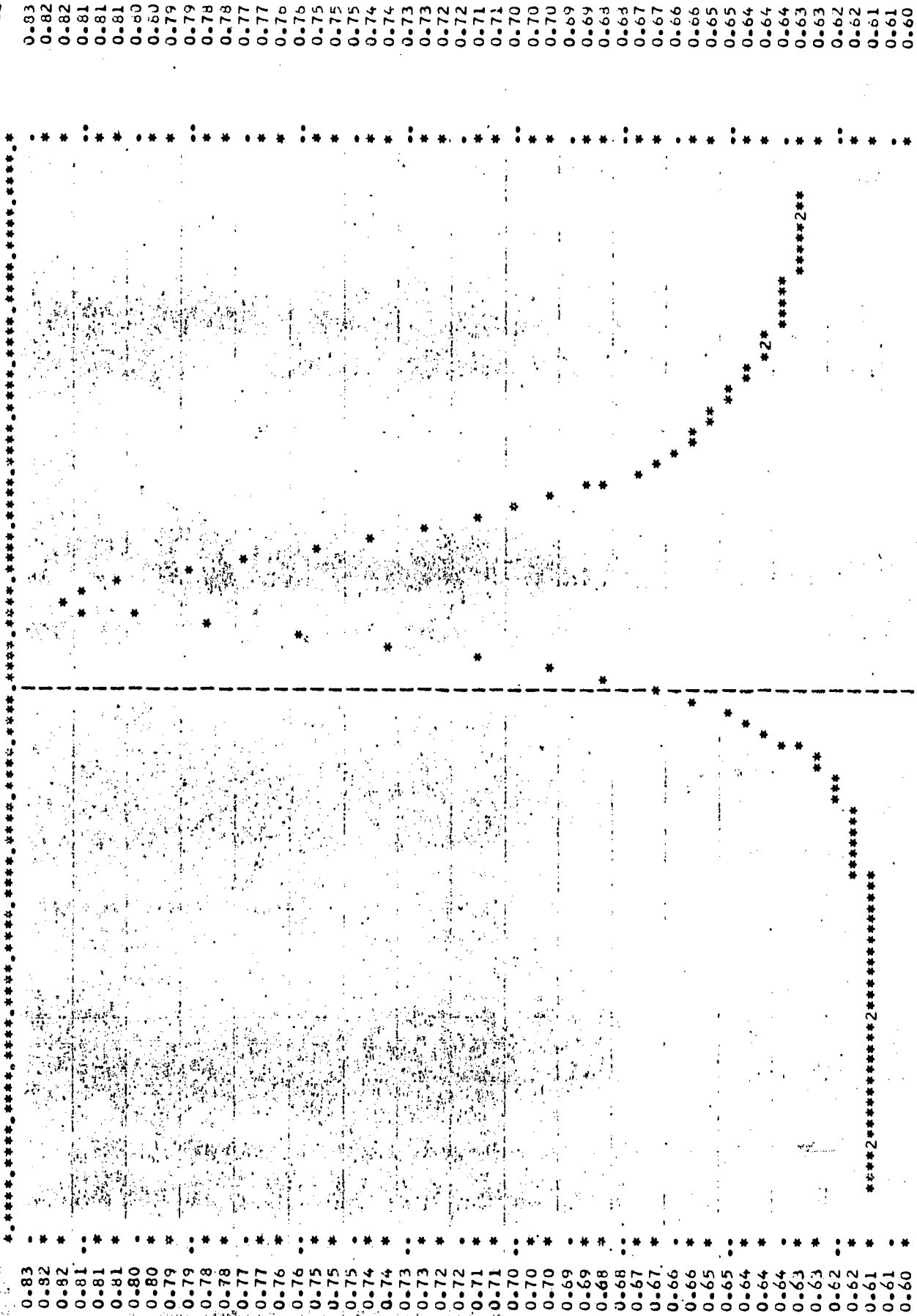
(k)	(R)	(k)	(R)	(k)	(R)	(k)	(R)
1	0.724940 00	1	-0.160000 01	35	0.738140 00	69	0.804600 00
2	0.724990 00	2	-0.150000 01	36	0.739720 00	70	0.799100 00
3	0.725050 00	3	-0.140000 01	37	0.741500 00	71	0.794050 00
4	0.725110 00	4	-0.130000 01	38	0.743520 00	72	0.789440 00
5	0.725180 00	5	-0.120000 01	39	0.745820 00	73	0.785250 00
6	0.725260 00	6	-0.110000 01	40	0.748440 00	74	0.781430 00
7	0.725340 00	7	-0.100000 01	41	0.751420 00	75	0.777960 00
8	0.725430 00	8	-0.090000 00	42	0.754820 00	76	0.774800 00
9	0.725530 00	9	-0.080000 00	43	0.758710 00	77	0.771920 00
10	0.725640 00	10	-0.070000 00	44	0.763150 00	78	0.769300 00
11	0.725760 00	11	-0.060000 00	45	0.768200 00	79	0.766900 00
12	0.725890 00	12	-0.050000 00	46	0.773930 00	80	0.764710 00
13	0.726030 00	13	-0.040000 00	47	0.780420 00	81	0.762700 00
14	0.726190 00	14	-0.030000 00	48	0.787690 00	82	0.760850 00
15	0.726360 00	15	-0.020000 00	49	0.795750 00	83	0.759160 00
16	0.726550 00	16	-0.010000 00	50	0.804550 00	84	0.757590 00
17	0.726760 00	17	0.00	51	0.813970 00	85	0.756150 00
18	0.726990 00	18	0.100000 00	52	0.823760 00	86	0.754820 00
19	0.727240 00	19	0.200000 00	53	0.833560 00	87	0.753580 00
20	0.727520 00	20	0.300000 00	54	0.842680 00	88	0.752430 00
21	0.727820 00	21	0.400000 00	55	0.851170 00	89	0.751360 00
22	0.728160 00	22	0.500000 00	56	0.857850 00	90	0.750370 00
23	0.728530 00	23	0.600000 00	57	0.862440 00	91	0.749440 00
24	0.728940 00	24	0.700000 00	58	0.864650 00	92	0.748570 00
25	0.729400 00	25	0.800000 00	59	0.8664420 00	93	0.747760 00
26	0.729900 00	26	0.900000 00	60	0.861950 00	94	0.747000 00
27	0.730470 00	27	0.100000 01	61	0.857620 00	95	0.746290 00
28	0.731090 00	28	0.110000 01	62	0.851900 00	96	0.745610 00
29	0.731790 00	29	0.120000 01	63	0.845270 00	97	0.744980 00
30	0.732570 00	30	0.130000 01	64	0.838170 00	98	0.744390 00
31	0.733440 00	31	0.140000 01	65	0.830930 00	99	0.743820 00
32	0.734420 00	32	0.150000 01	66	0.823820 00	100	0.743290 00
33	0.735510 00	33	0.160000 01	67	0.817000 00	101	0.742790 00
34	0.736750 00	34	0.170000 01	68	0.810570 00		

BEST WEIGHTED WRONGS DETERMINATION

Figure 1

DATA SUFFICIENCY PARALLEL FORMS RELIABILITY: WRONGS WT. (X AXIS) VS. RELIABILITY (Y AXIS)

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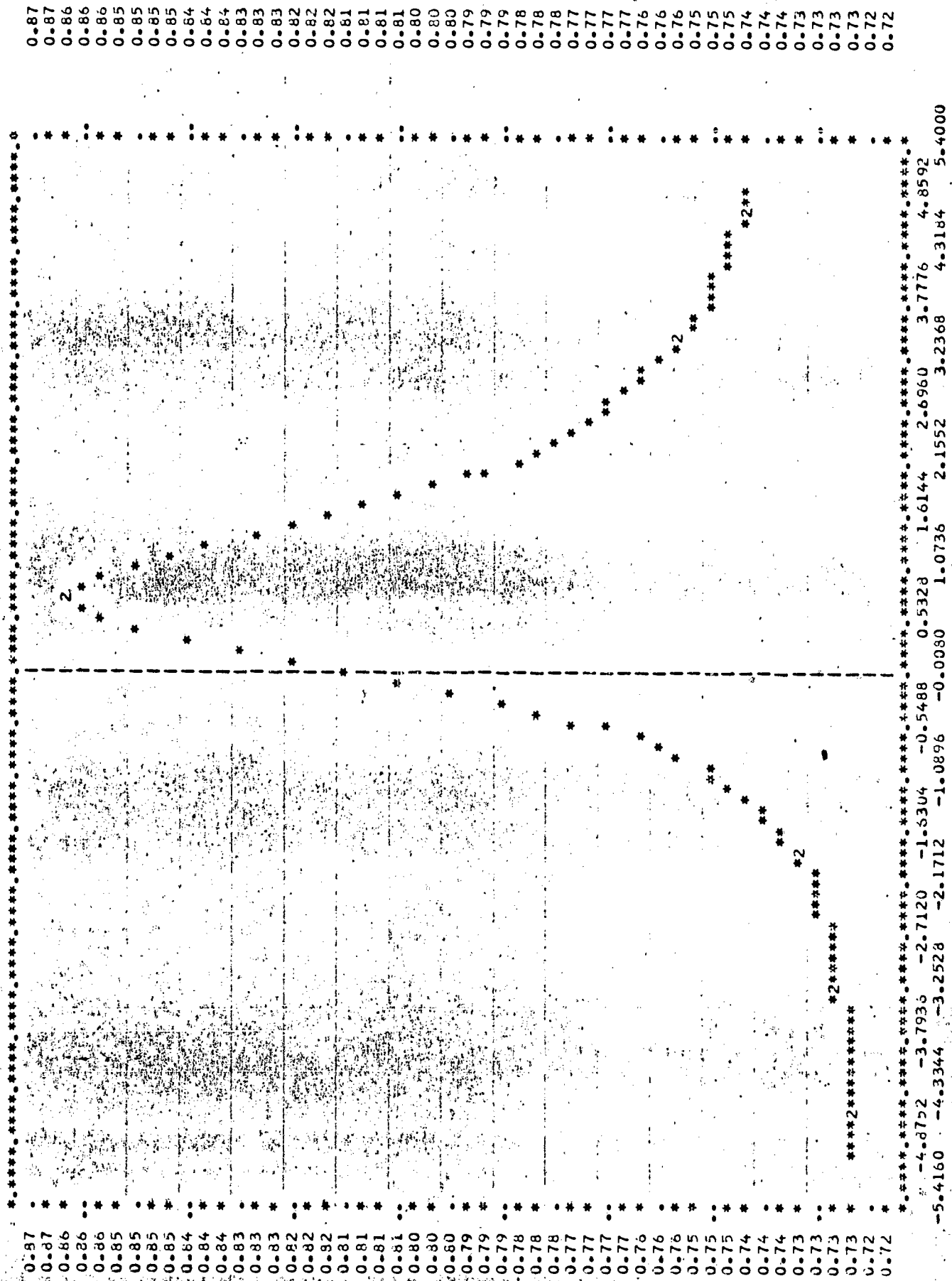
*****2*****2*****
-4.8752 -3.7936 -2.7120 -1.6304 -0.5488 0.5328 1.6144 2.6960 3.7776 4.8592
-5.4160 -4.3344 -3.2528 -2.1712 -1.0896 -0.0080 1.0736 2.1552 3.2368 4.3184 5.4000

BEST WEIGHTED WRONGS DETERMINATION

Figure 2

MATH PARALLEL FORMS RELIABILITY: WRONGS WT. (X AXIS) VS. RELIABILITY (Y AXIS)

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to indicate whether or not there is sufficient information in the statements to answer the question. An example would be as follows:

If x is a whole number, is it a two-digit number?

(1) x^2 is a three-digit number.

(2) $10x$ is a three-digit number.

- (A) if statement (1) ALONE is sufficient but statement (2) alone is not sufficient to answer the question asked,
- (B) if statement (2) ALONE is sufficient but statement (1) alone is not sufficient to answer the question asked,
- (C) if both statements (1) and (2) TOGETHER are sufficient to answer the question asked, but NEITHER statement ALONE is sufficient,
- (D) If EACH statement is sufficient by itself to answer the question asked,
- (E) if statements (1) and (2) TOGETHER are NOT sufficient to answer the question asked and additional data specific to the problem are needed.

This difference in the item format was accompanied by differences in test content. The data sufficiency material was parallel in content to the College Board SAT, which used about 30% items of this type at that time. The regular math test was parallel to the College Board basic-level achievement test in mathematics. This test has a more advanced content than the Scholastic Aptitude Test.

A third difference between the two tests (in addition to format and content) concerns the development of the half-tests. The data sufficiency test was developed as two separately timed subtests of 25 items each. These were the two half-tests correlated in the current study. The mathematics achievement test was administered with a single time limit and

divided into two half tests consisting of all the odd items and all the even items.

The role of these different factors on the somewhat different outcomes for the two tests is difficult to determine. The maximum value for the data sufficiency test was approximately + 0.90 as a weight for the wrongs. The maximum value for the regular math test was + 0.70. These empirical values contrast with the values of + 0.295 and + 0.585 observed for SAT mathematics subtests in the Fischer and Jackson study.

Table 3 presents the means, standard deviations and intercorrelations for the four half-tests considered in the study. The pattern of intercorrelation is consistent across the two tests. The interhalf reliability of the data sufficiency Rights score was .67, versus the value of .81 for the math achievement. Similarly the wrongs score for the math achievement test was more reliable, .73 versus .62. The cross-score correlations, $R_1 - W_2$ and $R_2 - W_1$, were - .46 and - .45 for the data sufficiency test and - .34 and - .35 for the math achievement; with similarly lower cross-score correlations for the intratest comparisons ($R_1 - W_1$, $R_2 - W_2$) for the math achievement test.

While omits were not distinguished from Not Reached in the present study, the general trait of omissiveness can be gauged somewhat by considering the numbers of items not responded to in each of the four half-tests studied. The values can be derived from Table 3 as follows:

Table 3

Means, Standard Deviations and Intercorrelations
for the Half-Test Scores

Data Sufficiency

	R_1	W_1	R_2	W_2
R_1	1.00	-0.75	0.67	-0.46
W_1	-0.75	1.00	-0.45	0.62
R_2	0.67	-0.45	1.00	-0.79
W_2	-0.46	0.62	-0.79	1.00
Mean	12.15	11.02	12.26	11.24
S.D.	3.82	3.57	3.50	3.36

Math Achievement

	R_1	W_1	R_2	W_2
R_1	1.00	-0.48	0.81	-0.34
W_1	-0.48	1.00	-0.35	0.73
R_2	0.81	-0.35	1.00	-0.51
W_2	-0.34	0.73	-0.51	1.00
Mean	12.51	6.70	12.97	6.60
S.D.	4.43	3.75	4.57	3.67

Average Number of Items Not Responded To

Data Sufficiency Half-Test 1	1.82
Data Sufficiency Half-Test 2	1.50
Math Achievement Half-Test 1	5.79
Math Achievement Half-Test 2	5.42

Clearly, the mathematics achievement test was characterized by a greater tendency to omit. Whether this was due to its greater speededness or to a true lack of knowledge of the material on the part of the subjects cannot be determined from this data. Either is plausible, since it is a characteristic of data sufficiency items that they are processed more rapidly by subjects. Referring to the Fischer and Jackson weights for mathematics tests, which were .295 and .585, the higher weight was achieved by the section which had a sizable set of data sufficiency items (18 of its 35-item total) and a slightly more generous time allotment, .77 seconds per item versus .72 seconds per item. This suggests that the weight approaches unity as the test is unspeded. However, the general parity of the number correct on the various half-tests, versus the differences in number wrong, suggests that there may be a greater tendency to give a response to the data sufficiency items, to guess at an answer, than to respond to the mathematics achievement items. This implies a more complex cause for the differences in weight than simply rate of work.

It is interesting to contrast the curves in Figures 1 and 2 with one provided by Fischer and Jackson, presented as Figure 3. In the present study, using empirically determinal curve, there is no suggestion of the minimum point for reliability which is clear in the Fischer and Jackson development. Whether this point would occur outside of the range observed

Figure 3

Reliability For R_{TxW}
(QSA43 - Verbal - 40)

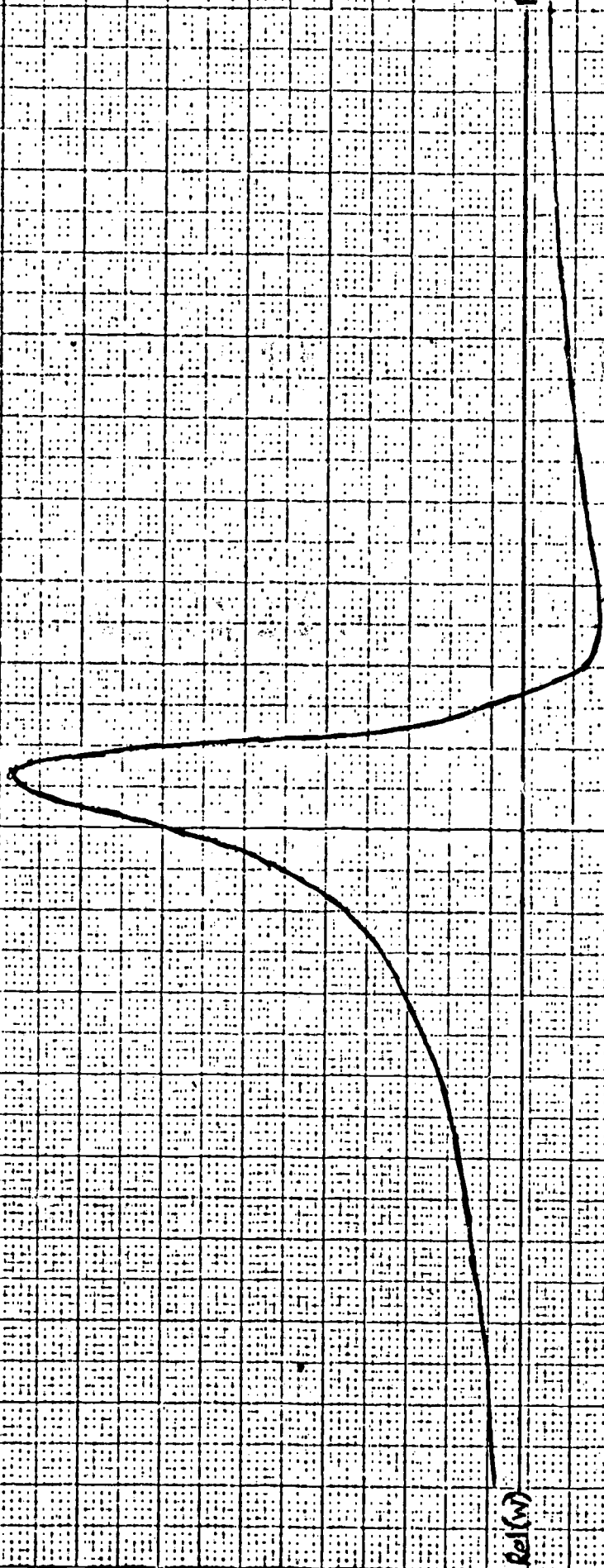
Reliability

.900
.890
.880
.870
.860
.850
.840
.830
.820
.810
.800

$ReI(w)$

$ReI(w)$

-8 -7 -6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6 7 8 9 10 γ



is not clear since no theoretical analysis of the intercorrelations of the tests in this study was undertaken.

Findings of a maximized reliability through a positive weight would seem to indicate that the most reliable aspect of a test performance is the total number of marks which are made. This hardly seems a worthwhile characteristic to focus on, since it would have little implication for validity. However, it is possible that further study of omissiveness would lead to an understanding of the reliability of the two forms of omissiveness: Omits and Not Reached. The best current data on this reliability is available from a study by Flaughner, Melton and Myers (1966), which shows the correlation between a mathematical section of the Scholastic Aptitude Test and each of four other, parallel sections introduced experimentally. The results are summarized in Table 4.

Table 4

Parallel Form Reliability for Four Scores:
Rights, Wrongs, Omits and Not Reached*

Parallel Forms	Correlations with Master Form			
	Rights- Rights	Wrongs- Wrongs	Omits- Omits	Not Reached- Not Reached
1	.790	.700	.628	.452
2	.785	.713	.536	.485
3	.776	.720	.648	.464
4	.770	.710	.576	.446

*From Flaughner, Melton and Myers (1966)

This data suggests that the Not Reached score is not as reliable as the Omits score. While this cannot be generalized too broadly, it

bears on the meaning of the positive weight for the maximally reliable composite score. To the extent that the number of omits on parallel forms reflects a reliable tendency not to know a certain proportion of the answers, it is surprising that this would be a more reliable characteristic of an individual than rate-of-work would be. Even with major efforts at content and difficulty parallelism, most parallel forms vary a good deal, so that one would not readily predict that individuals would find comparable numbers of items they would decide not to attempt. Further research seems indicated to clarify the degree to which the Omit response is determined by rate of work.

This paper has confirmed the determination by Fischer and Jackson of a positive weight for the wrongs as a reliability maximizing score. The parallel-forms technique in the present study varied somewhat from the internal-consistency approach which they used. The implications of this weight, as Lord suggests, are that the trait of omissiveness is a reliable one. The source of this reliability and the implications for work on test speededness could be meaningful future areas for research.

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