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ABSTRACT

Two experiments which investigated the young child's ability to deal with multiplicative classes and relations (considered behavioral indices of concrete operations thought) in double series and cross class matrices are described and discussed. In the initial study, 160 children from preschool through grade 2 received six matrix subtasks (reproduction and transposition subtasks of cross classification, classification seriation, and double seriation) in one of six orders of presentation (the reproduction subtask always preceded the transposition case for a given matrix). The second experiment followed much the same design as the first, except that the class/series matrix was not used and stimuli for one cross class task were rearranged so that neither dimension was seriated across rows or columns of the matrix. Results, conclusions, and methodological problems of each experiment are discussed in detail. It was concluded that the pattern of lesser general difficulty for multiple relations contrasted with multiple classification concepts was subtantiated in these experiments (and other experiments (and other experiments also discussed) and that the original structural synchrony conclusions of Inhelder and Piaget need further examination. (ED)

A Critical Analysis of Matrix Task Classificatory and Seriation Abilities

by

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A Critical Analysis of Matrix Task Classificatory and Seriation Abilities

Among the more commonly recognized behavioral indices of concrete operations thought, according to Piaget, is the ability to deal with multiplicative classes and relations. Matrix-type formats have been used to assess these abilities by researchers such as Inhelder and Piaget (1964) and Bruner and Kenny (1966). Piagetian theory predicts synchronous acquisition of these class and relations abilities during the concrete operations period of middle-childhood. Recently, however, MacKay, Fraser, and Ross (1970) have reported evidence of greater difficulty for double series compared to cross class matrices for both a reproduction case and a transposition case.

An initial study was designed to replicate the MacKay et al. (1970) procedures in a within-subjects assessment design. Distribution of the subjects by grade, sex and age is given in Table 1. Subjects were 160 children (40 from each of four grade levels: preschool, kindergarten, first, and second) who received six matrix subtasks (reproduction and transposition

Table 1

DISTRIBUTION OF SUBJECTS BY GRADE, SEX, AND AGE

Grade	Subjects	Males	Females	Mean Age	Range
Pre	40	23	17	5-0	4-1 to 5-9
K	40	23	17	6-3	5-6 to 6-9
1	40	21	19	7-3	6-8 to 8-1
2	40	23	17	8-2	7-9 to 8-9

subtasks of cross classification, classification seriation, and double seriation) in one of six orders of presentation (the reproduction subtask always preceded the transposition case for a given matrix). Stimulus materials consisted of a three x three matrix board, wooden blocks of three shapes and three colors (cross class task), solid wooden cylinders varying in three values of height and in three values of diameter (double series task), and solid wooden cylinders of three colors varying in three values of height (class/series task). In the reproduction subtasks the completed matrix was shown to the subject, the blocks were removed by the experimenter, and the subject was asked to put the blocks back so that the board looked the same as it did originally. In the transposition subtasks the completed matrix was shown to the subject, the blocks were removed, the experimenter relocated one of the corner blocks, and the subject was asked to put the blocks back so they made a pattern like they did originally. In order to pass the cross class matrix reproduction subtask, a subject was required to classify one dimension in one direction and the other dimension in the perpendicular direction. In order to pass the double series matrix reproduction subtask, a subject was required to seriate one dimension in one direction and the other dimension in the perpendicular direction. In order to pass the class series matrix reproduction subtask, a subject was required to classify color in one direction and to seriate height in the perpendicular direction. In order to pass each of the transposition subtasks, a subject was required to fulfill the same criteria as for the reproduction cases without moving the replaced block.

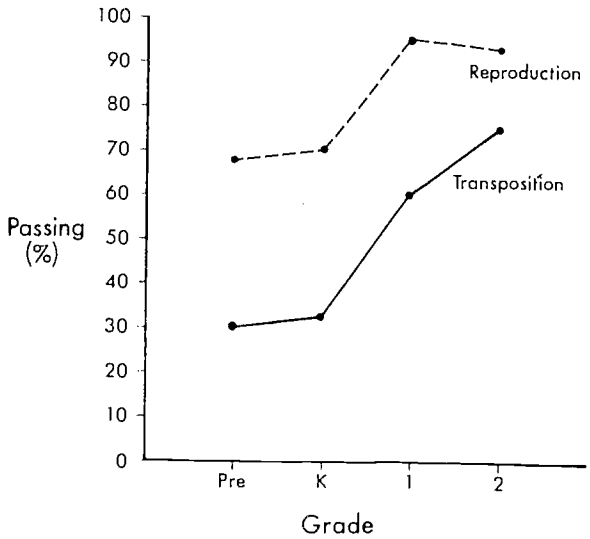
Preliminary analyses indicated a lack of significant presentation order effects and a general absence of sex differences (significant female subject superiority was shown for the cross class reproduction case only). As

anticipated across this age range, there was a significant increase in the proportion of subjects who passed each of the reproduction and transposition subtasks, i.e., all χ^2 values exceeded 9.40, df=3, $p < .025$.[1] This is essentially similar to the findings of the MacKay et al. (1970) investigation.

MacKay et al. (1970) concluded from their data that transposing a matrix which involved a continuous dimension (i.e., double series and class/series) was significantly more difficult than reproducing it. In agreement with this finding, our initial study's pass/fail frequency data showed the transposition subtask of the double series matrix to be significantly more difficult than its reproduction counterpart at each grade and for combined grades (all probabilities less than .01). Comparisons for the class/series matrix also revealed transposition to be a significantly more difficult subtask than reproduction for combined grades and at each grade level except first ($p < .01$). For the cross class matrix, comparisons of reproduction with transposition reached significance at first grade and for combined grade subsamples ($p < .05$) [See Figures 1, 2, and 3].

Figure 1

Double Seriation Subtasks



[1] All comparisons of task difficulty employed the χ^2 test for between subsamples and the McNemar Test for the Significance of Change for within subsamples.

Figure 2

Classification Seriation Subtasks

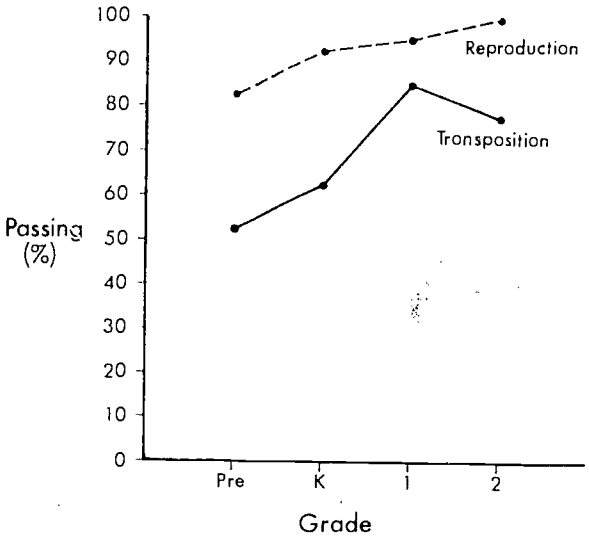
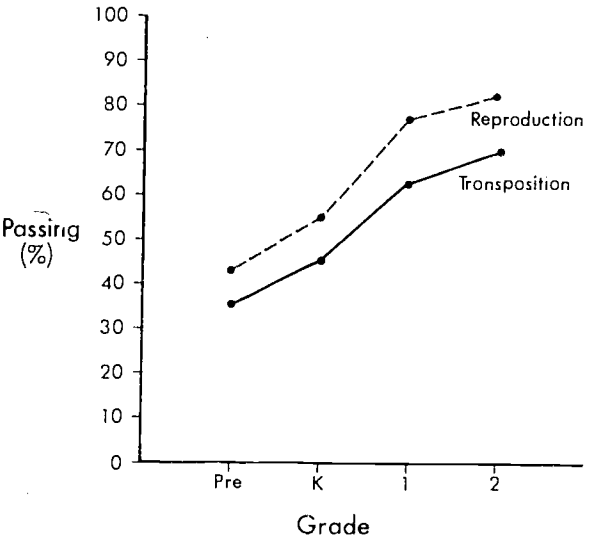


Figure 3

Cross Classification I Subtasks



A major conclusion of the MacKay et al. (1970) study was that the ability to construct a matrix composed of discrete categories (cross class) is developmentally an earlier acquisition than the ability to construct

one composed of relational variables (double series). In contrast to this finding, our initial investigation showed that reproducing a cross class matrix was significantly more difficult than reproducing a double series matrix at each of the lower three grade levels, and when combining all subsamples within the age range (all probabilities less then .05) [See Figure 4]. Very little difference in difficulty in transposition occurred between the cross class and double series matrices [See Figure 5].

Another conclusion of MacKay et al. (1970) was that a matrix constructed of discrete categories in both directions (cross class) is of equivalent difficulty to one constructed of discrete categories in one direction and a relational variable in the other (class/series). Results of our initial study, however, showed (1) reproducing the cross class matrix to be more difficult than reproducing the class/series matrix for combined grades as well as at each grade level ($p < .05$), and (2) transposing the cross class matrix to be more difficult than transposing the class/series matrix at first grade and for combined grades ($p < .05$) [See Figures 4 and 5].

Figure 4

Reproduction Subtasks

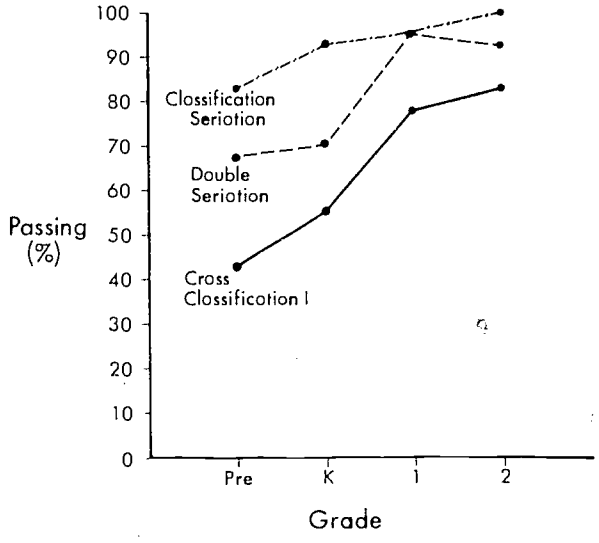
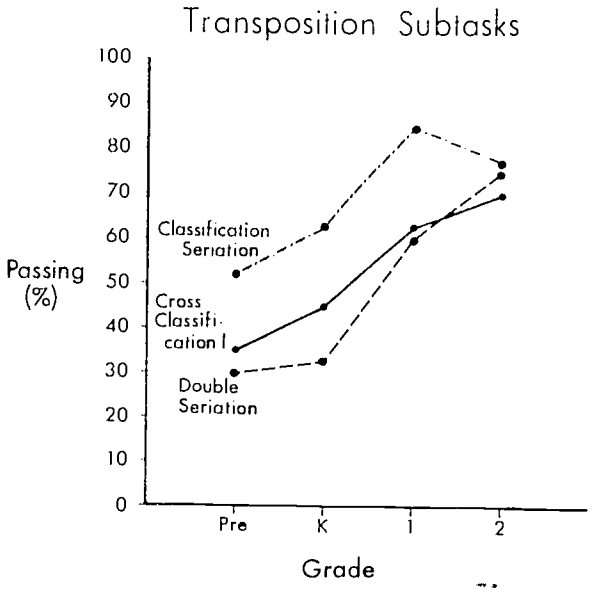


Figure 5



Reproduction subtask data from the initial study were examined to determine what percentages of passing subjects exactly reproduced the various matrices as originally presented to them. For the combined subsamples, the percentages of 79 for cross classification, 98 for double seriation, and 80 for class/series closely parallel the results of MacKay et al. (1970).

The fact that the results of our initial study substantiate the pattern that matrix reproduction subtasks are significantly less difficult than counterpart transposition subtasks as found by Bruner and Kenny (1966) and MacKay et al. (1970) presents some assurance that we are dealing with similar behavioral phenomena. In this regard, while the results of the relative difficulty of the cross class and double series cases disagree rather sharply with MacKay et al. (1970), the data are in general accord with the original research of Bruner and Piaget. Although Inhelder and Piaget did not use any direct counterpart to the matrix reproduction and transposition cases, their contention that "children reach an operational level in the multiplication of series about the same period (7-8 years) as cross classification

(p. 278)" agrees with our initial study's result of equivalent difficulty for transposing the cross class and double series matrices.

A second study was designed with two general objectives: (1) to assess the reliability of the initial study's results and the MacKay et al. (1970) results concerning cross class and double series matrices, and (2) to assess the relationship of these measures to a second cross class task which utilized the double series stimulus materials. In accord with the earlier investigation, it was predicted that: (1) performance on all of the matrix subtasks would be positively related to age level, (2) each reproduction subtask would be of lesser difficulty than its transposition counterpart, (3) the between matrix order of difficulty for the reproduction cases would be cross class II > cross class I > double series, and (4) the between matrix order of difficulty for the transposition cases would be cross class II > cross class I = double series.

The same number of subjects at each of the same grade levels were sampled for the second study. Distribution of subjects by grade, sex and age is given in Table 2. Stimuli, task instructions, and passing criteria were also identical to the initial study for the cross class I and double

Table 2

DISTRIBUTION OF SUBJECTS BY GRADE, SEX, AND AGE

Grade	Subjects	Males	Females	Mean Age	Range
Pre	40	20	20	4-5	3-8 to 5-2
K	40	20	20	6-1	5-6 to 6-9
1	40	19	21	7-5	6-10 to 8-3
2	40	20	20	8-3	7-8 to 9-2

series subtasks. The class/series matrix was not used in the second study. Stimuli for cross class II were the identical cylinders used for double series, but were located on the board such that within each row the height remained constant, and within each column the diameter remained constant. However, neither dimension was seriated across the rows or columns of the matrix. Thus, by definition, this task is distinguishable from the double series task, the latter which involves the multiplication of asymmetric transitive relations. As in the initial study, the tasks were presented in one of six possible orders of administration, the reproduction subtask always preceded its transposition counterpart for each matrix task, and the instructions to the subject were identical for each matrix. For the cross class II transposition case, after removing all the blocks from the matrix board, the experimenter replaced the block that had originally occupied the subject's middle-right position in the center position. In order to pass the cross class II reproduction subtask, a subject was required to hold constant the height dimension in one direction and the width dimension in the perpendicular direction. In order to pass the transposition subtask of this matrix, a subject was required to fulfill the same criteria without moving the replaced block.

Initial considerations concern the evaluation of order of presentation effects and possible sex differences in the children's matrix task performances. Significant order of presentation effects were notably absent, with one exception. That is, none of the frequencies of subjects passing each of the six subtasks were effected by the orders in which they were presented, except the cross classification II transposition subtask. A notably higher proportion of subjects passed the cross classification II

transposition task within the two orders where it was presented last. Chi-square comparisons of the proportions of male versus female subjects passing the various matrix subtasks were consistently nonsignificant for each reproduction and transposition case.

As grade level increased, there was a significant increase in the proportion of subjects passing each reproduction subtask (i.e., all χ^2 values exceeded 12.28, $df=3$, $p < .01$) and in the proportion of subjects passing the double series transposition ($\chi^2=12.38$, $df=3$, $p < .01$) and cross class II transposition ($\chi^2=20.77$, $df=3$, $p < .001$) subtasks. Also, in close agreement with the initial study, the transposition subtask of the double series matrix was significantly more difficult than its reproduction counterpart when combining grades and at all grade levels except preschool (all probabilities less than .05) [See Figure 6]. Transposing the cross class I matrix was significantly more difficult than reproducing it when combining grades and at the first and second grades ($p < .05$); and transposing the cross class II matrix was significantly more difficult than reproducing it when combining grades and at the first grade level ($p < .05$) [See Figures 7 and 8].

Figure 6

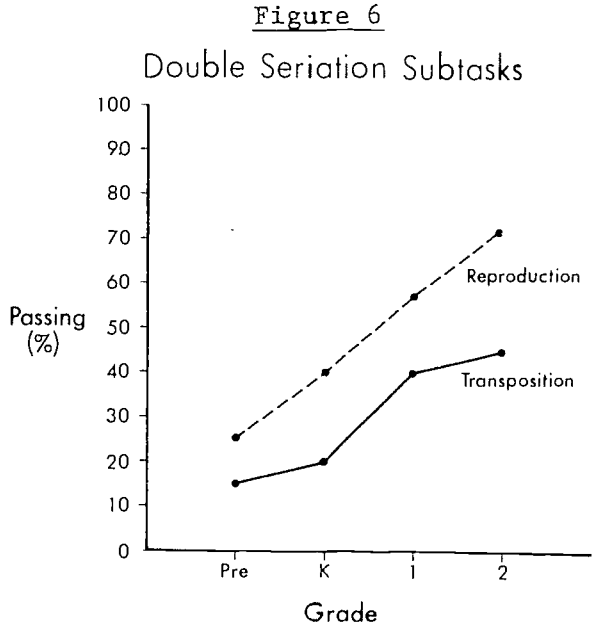


Figure 7

Cross Classification I Subtasks

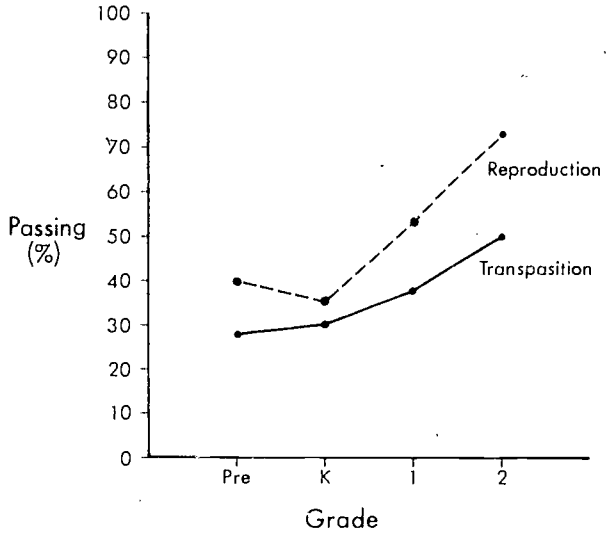
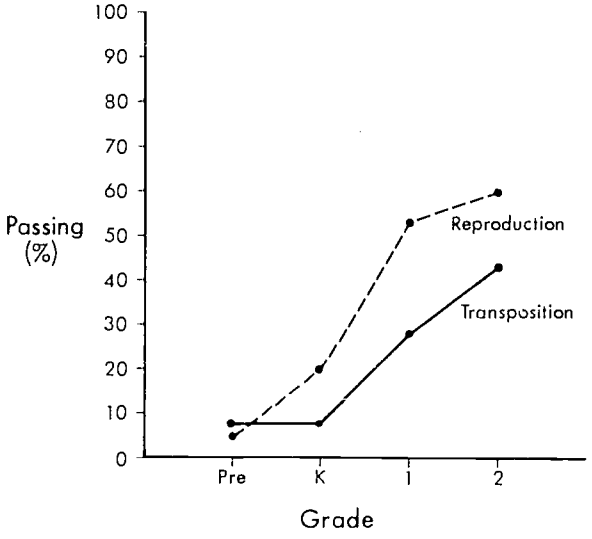


Figure 8

Cross Classification II Subtasks



In disagreement with the initial study, there was very little difference between proportions of subjects passing the cross class I and double series reproduction subtasks. Each of these measures is, however, significantly

less difficult than the cross class II reproduction case at the preschool level and for the combined subsamples ($p < .05$) [See Figure 9]. The corresponding comparison values for the transposition cases also showed no significant differences between the cross class I and double series subtasks. The cross class I transposition case was significantly less difficult than the cross class II transposition subtask when combining grades and at the preschool and kindergarten levels ($p < .05$) [See Figure 10].

Figure 9

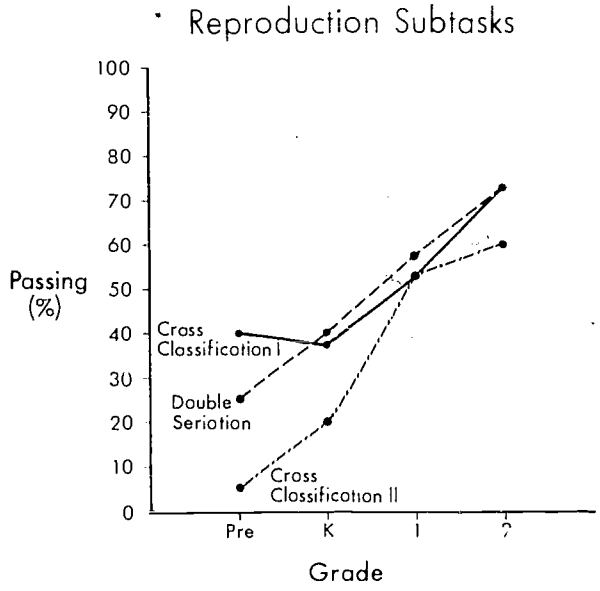
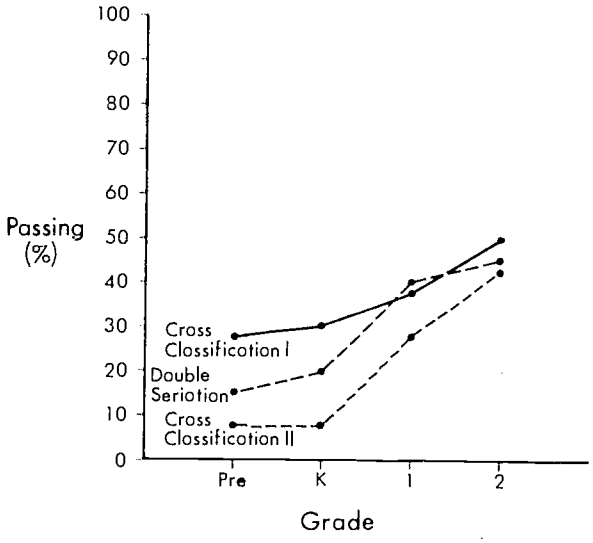


Figure 10



In addition, the percentages of passing subjects who exactly reproduced the cross class I and double series matrices were in very closs accord with the earlier study of MacKay et al. (1970) and with our initial study, i.e., 80% for cross class I and 100% for double series.

While it may be contended insofar as the double series and cross class I comparisons are concerned that the original structural synchrony conclusions of Inhelder and Piaget (1964) are validated; an examination of all data from the second study does not support this conclusion. If we accept the assumption that the double series and cross class II tasks represent two multiplicative logical reasoning tasks which employ identical stimulus materials, then the markedly different task difficulties are indeed notable.

Post-hoc inspection of all these employed matrix task formats, however, yields serious methodological deficiencies. These deficiencies concern the supposedly equivalent task requirements for the cross class and double series matrix cases. One would presume that all task solution requirements and instructional sets would be comparable between the reproduction problems of the various matrices, as well as between the transposition cases. Therefore, any performance differences between the cross class and double series matrices would be attributable to stimulus array differences stemming from the logical categories at issue, i.e., discrete class exemplars representing class intersections (e.g., red circles) versus ordered items of continuous dimensions. However, this is clearly not the case with the present formats. For example, the cross class I matrix subtasks may be "solved" in a number of ways so long as the color and shape placements are consistent across the three columns and the three rows. The

essential arbitrariness of the order of the placements of the class dimensions permits a much greater number of correct patterns than is true for the double series counterparts. (There are 72 correct reproduction patterns for each cross class matrix while only eight for double series; and there are eight correct transposition patterns for each cross class matrix while only two for double series.) What is required is a task format which would operationally equate the cross class and double series problems.

Other deficiencies concern the confounding variable of visual memory in the reproduction cases, and the component of spatial ability, i.e., the ability to rotate a visual image in short-term memory in the transposition cases. With these present matrix formats we find a set of correct responses that are identical to the original presentation and others that are reflections or rotations of it. These solutions do not provide unequivocal evidence that the subject has the multiplicative abilities in question. In collaboration with our colleagues at the Wisconsin R & D Center and with Spencer Swinton of Educational Testing Service, we have developed, but not empirically employed at this point in time, a new task format. The blue cylinders are again used and presented to the child in the cross classified manner identical to the original display in cross class II, i.e., located such that within each row the height remained constant and within each column the diameter remained constant, yet neither dimension seriated across the rows or columns (see Table 3). The blocks are removed, but the experimenter replaces to the center of the matrix a block that had occupied a corner position. There are eight possible correct solutions to this new cross class task, none of

which can be the original pattern nor reflections or rotations of it (see Tables 3 and 4) [Note also that none can be a seriated array]. The subject's placements are then removed from the board and the experimenter places the cylinder of intermediate height and intermediate width in the center position, thus forcing all eight possible correct cross classification responses to be double seriations. Thus we have our new double series task and as in the new cross class task, none of the correct responses have been shown to the subject and none are reflections or rotations of the original array (see Tables 3 and 5). If this new double series task is easier than the new cross class task, it can be so only because of the perceptual cue of serial order. Logically, it is clear that the new cross class task cannot be easier than the new double series task. In accord with the orthodox Piagetian prediction, subjects who determine the relationship between the stimuli when first presented, can classify the matrix in the new cross class format, logically will seriate the matrix in the new double series format. That is, when stimulus materials, number of correct solutions, components of visual memory and spatial ability are held constant across the matrix class and relations formats, the question of relative difficulty admits of only two possible answers: (1) either double seriation is no different from any other cross classification, or (2) the additional perceptual cue of seriated values will enhance performance on the double seriation task. Empirical examples of double series being more difficult than cross class in these new appropriately equated formats could occur only through errors of measurement.

Table 3

Original presentation for new cross class and double series tasks

S

F-3	S-3	M-3
F-1	S-1	M-1
F-2	S-2	M-2

E

Table 4

Display of all (8) possible correct solutions to new cross class task

S

S-1	F-1	M-1
S-3	F-3	M-3
S-2	F-2	M-2

E

S

S-2	F-2	M-2
S-3	F-3	M-3
S-1	F-1	M-1

E

S

M-1	F-1	S-1
M-3	F-3	S-3
M-2	F-2	S-2

E

S

M-2	F-2	S-2
M-3	F-3	S-3
M-1	F-1	S-1

E

S

S-2	S-3	S-1
F-2	F-3	F-1
M-2	M-3	M-1

E

S

S-1	S-3	S-2
F-1	F-3	F-2
M-1	M-3	M-2

E

S

M-2	M-3	M-1
F-2	F-3	F-1
S-2	S-3	S-1

E

S

M-1	M-3	M-2
F-1	F-3	F-2
S-1	S-3	S-2

E

S=skinny, M=medium width, F=fat; 1=short, 2=medium height, 3=tall

Table 5

Display of all (8) possible correct solutions to new double series task

S

S-1	M-1	F-1
S-2	M-2	F-2
S-3	M-3	F-3

E

S

S-3	M-3	F-3
S-2	M-2	F-2
S-1	M-1	F-1

E

S

F-1	M-1	S-1
F-2	M-2	S-2
F-3	M-3	S-3

E

S

F-3	M-3	S-3
F-2	M-2	S-2
F-1	M-1	S-1

E

S

S-3	S-2	S-1
M-3	M-2	M-1
F-3	F-2	F-1

E

S

S-1	S-2	S-3
M-1	M-2	M-3
F-1	F-2	F-3

E

S

F-3	F-2	F-1
M-3	M-2	M-1
S-3	S-2	S-1

E

S

F-1	F-2	F-3
M-1	M-2	M-3
S-1	S-2	S-3

E

S=skinny, M=medium width, F=fat; 1=short, 2=medium height, 3=tall

In conclusion, the pattern of lesser general difficulty for multiple relations concrasted with multiple classification concepts is substantiated in a number of recent investigations. Assessment tasks directly based upon the four relational *groupements* have been found by Brainerd (1972) and Dihoff (1975b) to be significantly less difficult than tasks derived from the class *groupements*. The relational understanding embodied in the traditional transitive inference task has been found to be a developmentally earlier acquisition than the counterpart concepts of conservation or class inclusion (Brainerd, 1973b; Toniolo & Hooper, 1975b). Moreover, these differential item difficulties have been replicated in instructional studies employing tansfer of training designs (Brainerd, 1974; Peterson, Hooper, & DeFrain, 1975). It has also been found (Brainerd, 1973a; Gonchar, 1975a; Siegel, 1974) that ordinal number understanding (relational concept domain) precedes cardinal number mastery (classificatory concept domain). Finally, these theoretically relevant item difficulty patterns have been substantiated in a number of developmental acquisition sequences in investigations employing longitudinal assessment designs (Dihoff, 1975a; Gonchar, 1975b; Toniolo & Hooper, 1975a).

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