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ABSTRACT
Fifteen research reports related to mathematics education are abstracted and analyzed. The reports abstracted were selected from nine educational journals including several published outside the United States, and deal with a wide variety of topics. Three articles deal with concept formation, three with task analysis or instructional sequencing, and two with other aspects of instruction as it relates to the learning process. Two articles deal with the interaction of personality variables with decision making and performance, respectively, one with testing, and one with student evaluation of teaching. Other reports abstracted concern ability, dyscalculia, and evaluation of a kindergarten program. Research related to mathematics education which was reported in RIE and CIJE between January and March 1974 is listed. (SD)

Vol. 7, No. 2 Spring 1974

INVESTIGATIONS IN MATHEMATICS EDUCATION

INVES
IN
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Expanded Abstracts
and
Critical Analyses
of
Recent Research

Center for Science and Mathematics Education
The Ohio State University
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MATHEMATICS EDUCATION RESEARCH STUDIES REPORTED IN RESEARCH IN EDUCATION
January - March 1974

ED 081 174 Smith, M. Daniel, Learning Difficulty, Transfer, and Retention as Functions of Two Types and Two Levels of Redundancy in a Sequence of Concept Formation Tasks in Mathematics Involving Computer Assistance. Final Report. 135p. MF and HC available from EDRS.

ED 081 470 Riley, Christine A.; Trabasso, Tom, Logical Structure Versus Information Processing in Making Inferences. 12p. MF and HC available from EDRS.

ED 081 511 Otaala, Barnabas, The Development of Operational Thinking in Primary School Children: An Examination of Some Aspects of Piaget's Theory Among the Iteso Children of Uganda. 117p. Not available from EDRS. Available from Teachers College Press, 1234 Amsterdam Avenue, New York, New York 10027 ($3.95).

ED 081 620 Appel, Ida J., The Relationship Between the Sequence of Graph Interpretation Skills and the Sequence of Mathematics Skills for Kindergarten Through Ninth Grade. 690p. Not available from EDRS. Available from University Microfilms (73-16,044)

ED 081 621 Tupesis, Janis Arvaldis, Mathematics Learning as a Consequence of the Learner's Involvement in Interactive Problem-Solving Tasks. 182p. Not available from EDRS. Available from University Microfilms (73-9295).

ED 081 622 Diamond, Hannah, An Investigation of the Efficacy of Piaget Curricular Elements Integrated into a Traditional Head Start Program. 118p. Not available from EDRS. Available from University Microfilms (73-15,733).

ED 081 623 Ball, Linda Virginia, Student Contracting for Achievement Grades in Ninth Grade General Mathematics. 146p. Not available from EDRS. Available from University Microfilms (73-16,709).

ED 081 624 Walek, Bruce Peter, A Study of the Relationship Between Conceptual Tempo and Problem-Solving Abilities of Fourth-Grade Children. 83p. Not available from EDRS. Available from University Microfilms (73-15,550).

ED 081 625 Ryoti, Don Eino, Student Responses to Equivalent Inference Schemes in Class and Conditional Logic. 98p. Not available from EDRS. Available from University Microfilms (73-17,395).

ED 081 626 Albig, David L., A Study of the Effects of Verbalization on Concept Formation in Mathematics. 200p. Not available from EDRS Available from University Microfilms (73-18,303).

ED 081 627 Richer, Howard Marshall, Peer Teaching as a Facilitator of Learning: Using Conservation of Substance as a Measure. 86p. Not available from EDRS. Available from University Microfilms (73-18,646).

ED 081 628 McLaughlin, Jane Ann, The Relationship of Open-Ended vs. Closed-Ended Mathematics Laboratory Activities to the Divergent Thinking Ability of Pre-Service Elementary Teachers. 198p. Not available from EDRS. Available from University Microfilms (73-19,351).

ED 081 629 Todd, Howell Wayne, Moves and Strategies in a Skill Venture in Secondary School Mathematics. 93p. Not available from EDRS. Available from University Microfilms (73-17,449).

ED 081 630 Braxton, Loretta Murray, The Effects of Instruction in Sentential Logic on the Growth of the Logical Thinking Abilities of Junior High School Students. 137p. Not available from EDRS. Available from University Microfilms (73-18,994).

ED 081 631 Carney, Harold Francis, The Relative Effectiveness of Two Methods of Teaching the Addition and Subtraction of Rationale Numbers. 146p. Not available from EDRS. Available from University Microfilms (73-19,411).

ED 081 632 Taloumis, Thalia, The Relationship of Area Conservation to Area Measurement as Affected by Sequence of Presentation of Piagetian Area Tasks to Boys and Girls in Grades One Through Three. 220p. Not available from EDRS. Available from University Microfilms (73-19,450).

ED 081 637 Shumway, Richard J., Human Concept Formation: Negative Instances. Final Report. 91p. MF and HC available from EDRS.

ED 081 640 Scandura, Joseph M., and others, Higher Order Characterization of Heuristics for Compass and Straight Edge Constructions in Geometry. Report No. 70. 85p. MF and HC available from EDRS.

ED 081 821 Lindstrom, David R.; Shipman, Virginia C., Form Reproduction; Technical Paper 8. Disadvantaged Children and Their First School Experiences. ETS-Head Start Longitudinal Study. Technical Report Series. 13p. MF and HC available from EDRS.

ED 081 835 Meissner, Judith A., and others, Spontaneous Numerical Correspondence Test; Technical Report 22, Disadvantaged Children and Their First School Experience. ETS-Head Start Longitudinal Study. Technical Report Series. 18p. MF and HC available from EDRS.

ED 082 327 Meyer, Jo Ann; Wurster, Stanley R., The Effect of Three Noise Levels on Task Attention and Performance in Reading and Math With Fifth and Sixth Grade Children. 30p. MF and HC available from EDRS.

ED 082 510 Bjerstedt, Ake, Ed., Subject-Matter Oriented Research: Some Current Projects at the Malmo School of Education. 42p. MF and HC available from EDRS.

ED 082 742 Greenberg, Barry; Tuckfield, Gloria, An Investigation of the Differences Between "Successful" and "Unsuccessful" Students in Trigonometry at Miami-Dade Community College, South Campus. 4p. MF and HC available from EDRS.

ED 082 947 Broomes, Desmond Rodwell, Psychological and Sociological Correlates of Mathematical Achievement and Ability Among Grade 9 Students. 415p. Not available from EDRS. Available from National Library of Canada.

ED 083 005 Leiderman, Gloria F., A Longitudinal Study of Mathematical Achievement in the Primary School Years: Description of Design, Sample, and Factor Analyses of Tests. 157p. MF and HC available from EDRS.

ED 083 006 Rosenthal-Hill, Irene, A Longitudinal Study of Mathematical Achievement in the Primary School Years: Curriculum and Socio-Economic Comparisons and Predictions from Previous Achievement. 442p. MF and HC available from EDRS.

ED 084 112 Romberg, Thomas A.; Wilson, James W., NLSMA Reports, No. 7, The Development of Tests. 391p. Not available from EDRS. Available from A. C. Vroman, Inc.

ED 084 113 Wilson, James W., Ed., and others, NLSMA Reports, No. 8, Statistical Procedures and Computer Programs. 221p. Not available from EDRS. Available from A. C. Vroman, Inc.

ED 084 114 McLeod, Gordon K.; Kilpatrick, Jeremy, NLSMA Reports, No. 12, Patterns of Mathematics Achievement in Grades 7 and 8: Y-Population. 148p. Not available from EDRS. Available from A. C. Vroman, Inc.

ED 084 115 Kilpatrick, Jeremy; McLeod, Gordon K., NLSMA Reports, No. 13, Patterns of Mathematics Achievement in Grade 9: Y-Population. 101p. Not available from EDRS, Available from A. C. Vroman, Inc.

ED 084 116 McLeod, Gordon K.; Kilpatrick, Jeremy, NLSMA Reports, No. 14, Patterns of Mathematics Achievement in Grade 10: Y-Population. 109p. Not available from EDRS. Available from A. C. Vroman, Inc.

ED 084 117 Kilpatrick, Jeremy; McLeod, Gordon K., NLSMA Reports, No. 15, Patterns of Mathematics Achievement in Grade 11: Y-Population. 91p. Not available from EDRS. Available from A. C. Vroman, Inc.

ED 084 118 Wilson, James W., NLSMA Reports, No. 16, Patterns of Mathematics Achievement in Grade 10: Z-Population. 115p. Not available from EDRS. Available from A. C. Vroman, Inc.

ED 084 119 Wilson, James W., NLSMA Reports, No. 17, Patterns of Mathematics Achievement in Grade 11: Z-Population. 140p. Not available from EDRS. Available from A. C. Vroman, Inc.

ED 084 120 Romberg, Thomas A.; Wilson, James W., NLSMA Reports, No. 18, Patterns of Mathematics Achievement in Grade 12: Z-Population. 78p. Not available from EDRS. Available from A. C. Vroman, Inc.

ED 084 121 Travers, Kenneth J., NLSMA Reports, No. 19, Non-Intellective Correlates of Under- and Overachievement in Grades 4 and 6. 330p. Not available from EDRS. Available from A.C.Vroman,Inc.

ED 084 122 Crosswhite, F. Joe, NLSMA Reports, No. 20, Correlates of Attitudes Toward Mathematics. 111p. Not available from EDRS. Available from A. C. Vroman, Inc.

ED 084 123 Wilson, James W., Ed.; Begle, Edward G., Ed., NLSMA Reports, No. 21, Parts A, B, and C, Correlates of Mathematics Achievement: Attitude and Role Variables. 1,453p. Not available from EDRS. Available from A. C. Vroman, Inc.

ED 084 124 Wilson, James W., Ed.; Begle, Edward G., Ed., NLSMA Reports, No. 22, Parts A, B, and C, Correlates of Mathematics Achievement: Cognitive Variables. 1,485p. Not available from EDRS. Available from A. C. Vroman, Inc.

ED 084 125 Wilson, James W., Ed.; Begle, Edward G., Ed., NLSMA Reports, No. 23, Parts A, B, and C, Correlates of Mathematics Achievement: Teacher Background and Opinion Variables. 1,399p. Not available from EDRS. Available from A. C. Vroman, Inc.

ED 084 126 Wilson, James W., Ed.; Begle, Edward G., Ed., NLSMA Reports, No. 24, Parts A, B, C, and D, Correlates of Mathematics Achievement: School-Community and Demographic Variables. 1,883p. Not available from EDRS. Available from A. C. Vroman, Inc.

ED 084 127 Wilson, James W., Ed.; Begle, Edward G., Ed., NLSMA Reports, No. 25, Parts A, B, and C, Correlates of Mathematics Achievement: Teacher Assigned Grades. 1,367p. Not available from EDRS. Available from A. C. Vroman, Inc.

ED 084 128 Wilson, James W., Ed.; Begle, Edward G., Ed., NLSMA Reports, No. 26, Correlates of Mathematics Achievement: Summary. 213p. Not available from EDRS. Available from A. C. Vroman, Inc.

ED 084 129 Begle, Edward G., NLSMA Reports, No. 27, The Prediction of Mathematics Achievement. 144p. Not available from EDRS. Available from A. C. Vroman, Inc.

ED 084 130 Begle, Edward G.; Geeslin, William Edward, NLSMA Reports, No. 28, Teacher Effectiveness in Mathematics Instruction. 146p. Not available from EDRS. Available from A. C. Vroman, Inc.

ED 084 131 Branca, Nicholas A., NLSMA Reports, No. 30, Follow-up Study of NLSMA Z-Population. 143p. Not available from EDRS. Available from A. C. Vroman, Inc.

ED 084 132 Dodson, Joseph W., NLSMA Reports, No. 31, Characteristics of Successful Insightful Problem Solvers. 139p. Not available from EDRS. Available from A. C. Vroman, Inc.

ED 084 133 Bridgham, Robert G., NLSMA Reports, No. 32, The Effects of Different Mathematics Curricula on Science Achievement in the Secondary School. 13p. Not available from EDRS. Available from A. C. Vroman, Inc.

ED 084 134 Wilson, James W., Ed.; Begle, Edward G., Ed., NLSMA Reports, No. 33, Intercorrelations of Mathematical and Psychological Variables. 106p. Not available from EDRS. Available from A. C. Vroman, Inc.

ED 084 139 Phillips, John Durwood, The Relationship Between Selected Piagetian Tasks and Knowledge of the Content Areas in Fifth-Grade Children. 96p. Not available from EDRS. Available from University Microfilms (73-14,288).

ED 084 154 Geeslin, William Edward, An Exploratory Analysis of Content Structure and Cognitive Structure in the Context of a Mathematics Instructional Unit. 213p. MF and HC available from EDRS.

ED 084 156 Townsend, Neal R.; Wheatley, Grayson H., Frequency of Tests and Feedback of Test Results in Calculus Classes. 10p. MF and HC available from EDRS.

ED 084 158 Searle, Barbara W., and others, Structural Variables Affecting CAI Performance on Arithmetic Word Problems of Disadvantaged and Deaf Students. 32p. MF and HC available from EDRS.

MATHEMATICS EDUCATION RESEARCH STUDIES REPORTED IN JOURNALS AS INDEXED BY CURRENT INDEX TO JOURNALS IN EDUCATION
January - March 1974

EJ 083 215 Yeshurun, Shraga. "A Reinforcement Study on Service Courses in Mathematics and Statistics." International Journal of Mathematical Education in Science and Technology, v4 n2, pp127-132, Apr-Jun 73.

EJ 083 854 Michaels, Shirleyanne; Kieren, T. R. "An Investigation of Open-Book and Closed-Book Examinations in Mathematics." Alberta Journal of Educational Research, v19 n3, pp202-207, Sep 73.

EJ 083 856 Cathcart, W. George; Liedtke, Werner W. "Role of Conceptual Tempo in Concept Formation." Alberta Journal of Educational Research, v19 n3, pp216-223, Sep 73.

EJ 085 112 Scandura, Joseph M. "Deterministic Theorizing in Structural Learning: Three Levels of Empiricism." Journal of Structural Learning, v3 n1, pp21-53, 71.

EJ 085 117 Williams, John D. "Modern Mathematics, Intelligence and Basic Conceptual Skills." Journal of Structural Learning, v3 n2, pp79-88, 72.

EJ 085 120 Williams, John D.; Fogelman, Kenneth R. "Sets, Logic and Intelligence Tests." Journal of Structural Learning, v3 n3, pp25-36, 72.

EJ 085 121 Connes, Bernard. "The Use of Electronic Desk Computers in Psychological Experiments." Journal of Structural Learning, v3 n3, pp51-72, 72.

EJ 085 123 Scandura, Joseph M.; McGee, Robert. "An Exploratory Investigation of Basic Mathematical Abilities of Kindergarten Children." Journal of Structural Learning, v3 n3, pp79-98, 72.

EJ 085 124 Bart, William M.; Wiener, Wilma K. "Transformational Closure for the Concept Clock." Journal of Structural Learning, v3 n3, pp99-107, 72.

EJ 085 125 Mueller, Edward; And Others. "Inherent Perceptual Motivation and the Discovery of Structure." Journal of Structural Learning, v3 n4, pp1-6, 72.

EJ 085 129 Williams, John D. "The Evaluation of Three Maths Courses." Journal of Structural Learning, v3 n4, pp41-79, 72.

EJ 085 137 Connolly, Austin J. "Research in Mathematics Education and the Mentally Retarded." Arithmetic Teacher, v20 n6, pp491-497, Oct 73.

EJ 085 146 Morman, Shelba Jean. "An Audio-Tutorial Method of Instruction vs the Traditional Lecture-Discussion Method." Two-Year College Mathematics Journal, v4 n3, pp56-61, F 73.

EJ 085 208 O'Brien, Thomas C. "Logical Thinking in College Students." Educational Studies in Mathematics, v5 n1, pp71-79, Apr 73.

EJ 085 209 Wallrabenstein, Hartmut. "Development and Signification of a Geometry Test." Educational Studies in Mathematics, v5 n1, pp81-89, Apr 73.

EJ 085 210 Wallrabenstein, Hartmut. "Experiments in Teaching Intuitive Topology in the 5th and 6th Grades." Educational Studies in Mathematics, v5 n1, pp91-108, Apr 73.

EJ 086 751 Callahan, Leroy G.; Robinson, Mary L. "Task-Analysis Procedures in Mathematics Instruction of Achievers & Underachievers." School Science and Mathematics, v73 n7, pp578-584, Oct 73.

EJ 093 515 SE 510 714
THE INFLUENCE OF PROFESSIONAL REFERENCE GROUPS ON DECISIONS OF PRESERVICE SECONDARY SCHOOL MATHEMATICS TEACHERS. Bright, George W.; Carry, L. Ray, Journal for Research in Mathematics Education, v5 n2, pp87-97, Mar 74.
Descriptors--*Decision Making, *Preservice Education, *Research, *Secondary School Mathematics, *Teacher Education, Educational Theories, Mathematics Education, Models

Expanded Abstract and Analysis Prepared Especially for I.M.E. by Thomas J. Cooney, University of Georgia.

1. Purpose

The experimenters investigated the following two hypotheses:

a. The professional reference group of mathematicians and the corresponding group of educators influence the decisions of preservice secondary school mathematics teachers in projected classroom situations.

b. The influence of mathematicians as a reference group is stronger than the influence of educators on the decisions of preservice secondary school mathematics teachers in projected classroom situations.

2. Rationale

The theoretical basis for the study consisted of Heider's and Kretch's work involving what is called balance theory. Heider described the cognitive structure of an individual P in terms of two kinds of relations. The first reflects an individual's feelings about an object or person. The second involves an individual's perceptions of the relation between two objects or other persons.

Kretch, et al. interpret Heider's theory in terms of a triadic relationship which could be represented by a triangular configuration. One vertex could be characterized as a subject P, another as an object O toward which P has a certain attitude, and the third vertex as a person (s) X toward which P also has a certain attitude. The segment between O and X represents an attitude, perceived by P of X toward O.

In the present study the triadic relationship involved the subjects, preservice secondary mathematics teachers, and two professional groups - mathematicians and educators - which might be represented by P, O, and X respectively. The experimenters made two assumptions. (1) subjects had a positive attitude toward mathematicians (P toward O) and (2) the subjects perceived a negative relationship between mathematicians and educators (O and X). A triad is balanced if the product of the signs of the three relationships is positive. Given their two assumptions this led the experimenters to conjecture a negative relationship between subjects and educators. Hypothesis (b) then follows.

3. Research Design and Procedure

Preservice secondary school mathematics teachers were randomly assigned to an experimental group (N = 28) and a control group (N = 33) from two methods classes (no practice - teaching experience) and one student - teaching seminar (engaged in practice - teaching). Thirty-six hypothetical classroom problem situations were depicted along with three equally acceptable resolutions (as best determined by the experimenters through a pilot study) to the problems. The problem situations were equally divided into four categories involving preparing and teaching classes, organizing classes, evaluating courses, texts and teachers, and educational philosophy and policy.

Booklets containing each problem situation followed by the three resolutions randomly ordered were distributed to subjects. For the experimental group the terms "mathematicians" and "educators" were randomly assigned to resolutions in 24 of the 36 problem situations. The remaining 12 were used to help the instrument appear more realistic by placing both the labels "mathematicians" and "educators" adjacent to the same resolution. The subjects were told that the labels indicated which of the three resolutions were judged most acceptable by the two professional groups. The control group reacted to the same problem situations with the same resolutions. However, in the control group the subjects were not given labels attached to the resolutions. After subjects in the control group made their selections, labels were then randomly assigned to the resolutions. Hence it was expected that a subject in the control group would select 8 resolutions labeled "mathematicians," 8 labeled "educators" and 8 without labels.

4. Findings

The mean number of responses in the experimental group was 10.1 for the mathematician-labeled resolutions, 10.5 for the educator-labeled resolutions and 3.4 for the remaining non-labeled items. The corresponding statistics for the control group were 8.5 (mathematicians), 7.9 (educators), and 7.6 (no labels).

To test Hypothesis 1, the Kolmolgorov-Smirnov test was applied using a cumulative frequency distribution. The test yielded significant statistics which supported Hypothesis 1 (p .03 for mathematicians, p .01 for educators). To test Hypothesis 2, the Wilcoxon matched pairs, signed-rank test was applied to the responses of the experimental subjects. The computed Z value (-.28) was not significant at the .05 level.

A post hoc analysis revealed that the two types of subjects in the experimental group (methods course only vs. methods course and student-teaching seminar) seemed to have reversed their agreement pattern. That is, the methods only group favored responses identified with mathematicians (10.5) over those identified with the educators (9.6). The methods and seminar subjects favored the responses aligned with educators (11.5) over the responses aligned with the mathematicians (9.7). An application of the Kolmolgorov-Smirnov test yielded no significant differences.

5. Interpretations

The attaching of labels did influence the responses of the subjects. However, there were no differential effects of the labels as hypothesized. Although the post hoc analysis did not yield a significant statistic, the experimenters felt it would be of interest to investigate further the differences between the two types of subjects.

The experimenters noted several limitations of the study.

a. The sample may have been "atypical of the entire population of all prospective secondary school mathematics teachers."

b. The observed behavior of the subjects were not classroom behaviors. Hence there may be a difference in the alternative a subject might select in written form and how the subject might actually behave in the classroom.

c. Failure to detect a difference between "the effects of the two reference groups may be due to inconsistent interpretation of the words 'educators' and 'mathematician' - these were not defined."

The experimenters conjectured that when teachers attend NCTM meetings that "the group that a speaker represents may be as important in determining the impact of the speech as the content of the speech." The authors also state that it seems evident that the teaching of content and methods should be closely coordinated for preservice secondary mathematics teachers. The experimenters feel it is desirable that prospective teachers not perceive a "polarization of opinion between content-oriented professionals and education-oriented professionals."

The experimenters suggested a similar investigation be conducted using inservice secondary school mathematics teachers as subjects.

Abstractor's Notes

This was an interesting study. The experimenters do an excellent job in identifying potentially serious limiting factors of the investigation. Their identification of the second factor identified above is particularly appropriate. Statements of intended classroom behavior by preservice teachers is not always a reliable indicator of how they will actually behave in live classroom situations. This is not a criticism of the study but rather to accentuate what the experimenters have stated - that we haven't found out anything about how teachers would behave in the classroom. Although the problem would be considerably more complex, it would be interesting to investigate how the reference groups referred to in this study might influence teaching behavior. The third limitation cited above is a prime candidate for a confounding factor. How would subjects likely classify a mathematics educator?

Another concern is the extent to which one can be assured that the reference groups "mathematicians" and "educators" were the influencing factors as opposed to other possible reference groups. That is, would the results have been basically the same had different reference groups been selected - perhaps only remotely related to mathematics education? If further research indicates that reference groups in general influence decisions then conjectures different from those given by the experimenters should be formulated. Given the results that were obtained, the conjectures that the import of a speech may be determined as much by the reference group of the speaker than by the actual content of the speech and that the teaching of content and methods should be closely coordinated seem unjustified in terms of the specific findings of the study.

In generating their hypotheses, the experimenters made two assumptions identified earlier in this abstract. The assumptions themselves might be examined. They certainly raise researchable questions. Finally, can the findings be related in any way to the theory set forth in the beginning of the article? Implications stemming from the study in terms of Heider's and Kretch's work were not discussed.

Thomas J. Cooney
University of Georgia

EJ 086 751 SE 509 553
TASK-ANALYSIS PROCEDURES IN MATHEMATICS INSTRUCTION OF ACHIEVERS AND UNDERACHIEVERS. Callahan, Leroy G.; Robinson, Mary L., School Science and Mathematics, v73 n7, pp578-584, Oct 73.
Descriptors--*Educational Research, *Elementary School Mathematics, *Instruction, *Teaching Methods, *Teaching Procedures, Educational Diagnosis, High Achievers, Low Achievers, Mathematics Education, [Research Reports]

Expanded Abstract and Analysis Prepared Especially for I.M.E. by Mary Ann Byrne, University of Georgia.

1. Purpose

To examine the effectiveness of task-analysis procedures for sixth grade underachievers. The mathematics task was "to decide the value of the two places to the right of the ones place in base ten numerals and to write the value of these places in expanded notation."

2. Rationale

Poor instruction ("failure to provide for individual differences, stress on speed, . . .and faulty assumptions about the transfer of training . . .) and a child's inadequate basic skills are pedagogical factors that contribute to underachievement. A method of instruction based on Gagne's theory of learning hierarchies should be especially appropriate for underachieving students. This method would consist in analyzing a learning task back to basic skills, diagnosing the subordinate tasks that a student is weak in, and providing instruction until a student has mastered each subordinate task.

3. Research Design and Procedure

The design for the study (not explicitly stated) seemed to be a posttest control group design. All sixth grade students at a middle class school were given a question testing the attainment of the task: Expanded notation in tenths and hundredths place. Students who answered correctly were excluded as potential subjects. The posttest also consisted of a question testing the task.

A 2 x 2 classification of four groups was established using the factors of control-experimental and achievers-underachievers. There was a random assignment of students into either a control or experimental group. The treatment provided to the subjects in the experimental group was individual instruction given by the researcher. This instruction included concrete and pictorial material, and was directed toward the subordinate learning tasks which the student had answered incorrectly on an initial test over the subordinate tasks. The control group received no instruction.

Students were classified as achievers or underachievers using the difference between an IQ standard score and a mathematics achievement standard score. The standard scores were based on the sixth grade population of the school. An underachiever was defined as a student with a mathematics achievement score 0.50 lower than the IQ standard score. Other subjects were defined as achievers.

From each of the four groups described, 6 subjects were randomly selected for the study. These 24 subjects were tested over the subordinate tasks, not the final task. The experimental subjects received appropriate instruction until mastery of each subordinate task was attained.

Data was compiled concerning. The number of subjects in each group who answered the posttest correctly, the subordinate tasks missed on the initial test and the instructional time taken by the experimental subjects; and IQ scores for each group.

4. Findings

None of the control subjects answered the posttest correctly. In the experimental group, four of the six underachievers and five of the six achievers completed the task correctly.

On the test over subordinate tasks, the achievers and underachievers demonstrated similar patterns of correct responses. Considering the final task as Level I, there were six levels in the task analysis.

The median instruction time for the underachievers was 57.5 minutes; for the achievers it was 37.5 minutes.

The mean IQ score for the achievers was 101.5; for the underachievers it was 116.4. The highest IQ score for the achievers' group was 109, the lowest for the underachieving group was 107.

5. Interpretations

The method of "diagnosis and instruction based on a hierarchical analysis of subordinate tasks is an effective procedure for students' learning of a mathematical task."

Underachievers require more time to master subordinate tasks than achievers.

Abstractor's Notes

The study as reported identified an educational problem, the instruction of underachievers, and a method of instruction, task analysis. It was proposed that the particular method would be effective in alleviating the problem. But a specific question was not clearly proposed for the study itself. So it is hard to interpret the results and to accept the conclusions reported.

The instruction on subordinate tasks did make a difference to achievement on the mathematics task, the experimental group did better than the control.

It is hard to justify any other conclusion using the posttest data. There was no hypothesis describing what kind of behavior on the posttest was expected as evidence of "decreasing underachievement." The underachievers were probably achieving, or learning from instruction in their regular class, but they were not achieving at the level of their capability. From the data reported it is not clear how the researchers could conclude that the "underachievement was decreased" when underachievers had a mean IQ score 15 points above the achiever's mean IQ score and yet had 4 out of 6 subjects respond correctly to the posttest while the achievers had 5 of 6 subjects to respond correctly. The researchers may have some data supporting their conclusion but it was not reported adequately.

The researchers referred to several studies reporting the use of task analysis for mathematics tasks. The present study as reported seems to be only a replication without specific implication for underachievers.

A final comment - A test of one item is seldom adequate to measure reliably a subject's ability to perform a task. It is possible that the particular numbers used or the way of asking a question may influence a student's response. Several questions on the given task would be appropriate.

Mary Ann Byrne
University of Georgia

EJ 083 856 AA 516 562

ROLE OF CONCEPTUAL TEMPO IN CONCEPT FORMATION. Cathcart, W. George; Liedtke, Werner W., Alberta Journal of Educational Research, v19 n3, pp216-223, Sep 73.

Descriptors--*Concept Formation, *Student Reaction, *Research Design, *Mathematics, Academic Performance, Elementary School Students, Achievement Tests, Tables (Data), Analysis Of Covariance, [*Matching Familiar Figures]

Expanded Abstract and Analysis Prepared Especially for I.M.E. by Edward J. Davis, University of Georgia.

1. Purpose

This study was designed to investigate relationships between the conceptual tempo of 7 1/2-8 year old children and the performance of these children on conservation of length tasks, an intelligence test, and on test of mathematics achievement. Conceptual tempo refers to the rate and accuracy of responses children make when confronted with tasks for which they must choose among a number of responses. Some children tend to respond quickly and incorrectly, others tend to deliberate and be more accurate. Hence the labels impulsive and reflexive are used to describe the conceptual tempo of children.

2. Rationale

Aspects of the relationship of conceptual tempo to intelligence, reading skills, mathematics achievement, and conservation of length have been investigated by other authors. Kagan (1963) found differences in conceptual tempo among children of equal intelligence and that reflective children appear to have an advantage in reading skills of word recognition and word recall. Cathcart and Liedtke (1969) reported results indicating reflective children achieve best in mathematics. Callahan and Passi (1971) report a non-significant tendency for reflective children in grades K-1 to be better able to conserve length.

This study was designed to examine relationships between conceptual tempo, intelligence, mathematics achievement, and conservation of length in a more careful and rigorous way by considering sex, mathematics achievement in terms of understanding concepts, recall of basic facts, and problem solving, and by using intelligence as a covariate.

3. Research Design and Procedure

After selecting 59 second grade students the authors administered 2 practice and 6 test items from Kagan's "Matching Familiar Figures" (MFF) test designed to identify subjects as impulsive or reflective. Subjects scoring below the median time to complete the test (14 seconds) and above the median number of errors (1.4) were classified as impulsive.

Subjects taking longer than 14 seconds to complete the test and making fewer than 1.4 errors were classified reflective. This scheme classified 20 subjects as reflective and 22 as impulsive.

The authors designed a conservation of length test of ten items. Each faced the child with arrangements of either sticks and string, toothpicks, or strips of paper. A procedure of presenting two objects or arrangements with coterminous endpoints, establishing a length relation, and transforming the physical arrangement was employed.

A 31-item mathematics achievement test yielded subscores on knowledge of concepts (from 15 multiple choice items), recall of basic facts (10 items), and problem solving (from 6 verbal problems). These items were taken from the mathematics text used in the children's mathematics program.

Intelligence scores were obtained from the Otis Quick-Scoring Mental Abilities Test (alpha). This test yields a verbal and non-verbal subscores and a total score.

4. Findings

Girls tended to be more reflective and boys more impulsive. Of the 20 reflective subjects, 8 were boys and 12 were girls. Of the 22 impulsive children, 16 were male and 6 were female. The resulting Chi Square for this distribution was significant at the .032 level. This was the only sex related finding (main effect or interaction) in the study that approached or exceeded significance at the .05 level. In relation to intelligence scores, reflexive children were significantly higher (.014 level) on the non-verbal subscore and on the total score (.036 level). However, no significant interaction was found between conceptual tempo, sex, and any of the three IQ measures.

To examine the relationship between sex, IQ, conceptual tempo, and the three components of mathematics achievement a two-way analysis of covariance was used with IQ as the covariate and sex as a blocking variable.

The following table presents the results of the two-way analysis of covariance.

TABLE 1

ANALYSIS OF COVARIANCE BETWEEN GROUPS ON MATHEMATICS ACHIEVEMENT (COVARIATE = IQ)

	Mathematics Achievement Test			
Group	Concept	Problem Solving	Basic Facts	Total
Reflective	7.32*	1.48*	7.96*	16.75*
Impulsive	7.75*	1.03*	6.06*	14.83*
$F_{(1,23)}$	.28	.65	6.20	1.70
Probability	.39	.43	.02	.20

*Adjusted means

The interaction between sex and conceptual tempo was not significant for the criterion measures in this analysis of covariance. The table shows higher adjusted means for reflective subjects on problem solving and basic facts. Only the latter was considered significant. On concepts the adjusted mean for impulsive children was slightly higher. This difference was not significant.

An analysis of covariance was also used to investigate the relationship of conservation of length to conceptual tempo and sex. It was hypothesized a relationship exists between conceptual tempo and conservation. The table below presents the results of this analysis. Intelligence was again the covariate. The table does not show the adjusted means on the 10-item conservation test for reflective and impulsive children. These were 7.61 and 5.51 respectively. This difference was significant at the .04 level.

TABLE 2

ANALYSIS OF COVARIANCE BETWEEN GROUPS ON CONSERVATION OF LENGTH (COVARIATE = IQ)

Source	SS	DF	MS	F	P
Conceptual Tempo	29.99	1	29.99	4.46	.04
Sex	9.95	1	9.95	1.48	.23
Tempo X Sex	0.02	1	0.02	0.003	.09
Within	154.59	23	6.72		
Total	194.55	26			

5. Interpretations

In relation to the findings concerning the three components of mathematics achievement the authors conjecture the superior performance of reflective children in recall of basic facts could be attributed to their taking the time to "figure out" answers using aids such as fingers or an available number line. Some children employed such procedures, others responded immediately or not at all. However, no record of which children used these differing strategies was obtained. The suggestion is made that this issue be studied further. No conjectures are offered for the results indicating non-differential performance of reflective and impulsive children or concepts and problem solving.

The significant difference in the adjusted means of reflective and impulsive children on the conservation test is interpreted as evidence that reflective children conserve length before impulsive children. The suggestion is made that conservation of other properties also be studied, with respect to conceptual tempo. Similar findings would result in important implications for many teachers.

The authors make an observation with respect to intelligence measures:

> If reflective children score higher on intelligence tests than impulsive pupils because they take more time to weigh alternatives before they respond then there exists another reason for questioning the validity of an intelligence test. The question "what does it measure?" seems to have added meaning when considered in the light of the results reported here.

The failure of sex to be significantly related to any of the other variables was considered evidence that sex related differences in conceptual tempo were maturational. The observation was also made that children slow to answer a teacher's questions may differ from their more quickly responding classmates in conceptual tempo and not in intelligence. Many teachers may tend to overlook this possibility.

Abstractor's Notes

While the title of the article is somewhat misleading, the authors should be commended for designing this interesting study, for using intelligence as a covariate, and for considering mathematics achievement to be multi-dimensional. However, the apparent lack of a relationship between conceptual tempo, problem solving, and knowledge of concepts was undoubtedly a surprise. Perhaps the problems were too similar to those the children had practiced in class or the concepts tested were too familiar to result in differential performance.

The failure to be able to classify the number of children taking time to figure out basic facts and those responding immediately as either reflective or impulsive is disappointing. This should be resolved; however it should be noted that the recall of basic facts used here is not a measure of memorized responses. No evidence on the relationship of conceptual tempo to memory has been gathered if subjects are allowed time to work out responses.

It should be emphasized the initial classification of children as reflective or impulsive was based on both time and accuracy of responses to items on Kagans (MFF) test. Hence, those children classified as reflective had demonstrated some intellectual superiority over those classified as impulsive. This appears to be standard practice in defining conceptual tempo and appears to this reviewer to make conceptual tempo of limited value in investigating developmental phenomena. There is another concern about the authors questioning the validity of an intelligence test. Their comment, quoted above, somewhat masks the fact that children they classified as reflective were not accurate in responding to the MFF test as well as more deliberate. This also makes the failure of reflective children to significantly out perform impulsive children in problem solving and concept recognition even more surprising.

The evidence indicating reflective children conserve length before impulsive children, coupled with the tendency by Callahan and Passi's subjects to do the same, is worthy of note. Somewhat puzzling, however, is the author's claim that similar findings for conservation of other properties would have important instructional implications. Just what are the implications? Should children be trained to be more deliberate, or should different instructional strategies be employed for reflective and impulsive children?

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EJ 085 846 CS 706 028

A STUDY OF CHILDREN'S ABILITY TO WORK WITH ELEMENTARY MATHEMATICAL SYSTEMS. Collis, Keven F. Australian Journal of Psychology, v25 n2, pp121-130, Aug. 73.

Descriptors--*Mathematical Concepts, *Learning Processes; *Cognitive Processes, *Student Research, *Cognitive Ability, Instructional Materials, Elementary Education

Expanded Abstract and Analysis Prepared Especially for I.M.E. by James M. Moser, University of Wisconsin.

1. Purpose

The purpose of the study was to gather data relative to the general question of how well children of differing age levels can perform on a task requiring formal-operational reasoning.

2. Rationale

Previous studies using mathematical material and the Piagetian model of cognitive development seemed to imply that subjects are not able to work within a closed abstract system before they have attained the capacity for formal-operational thinking. In recent years, the mathematics teaching in Australian (and American) schools seems to have shifted from an emphasis on the development of the computational skills to the understanding of the structure of mathematics. In many cases, this has resulted in presenting abstract systems formally and then asking children to make within that system. It was considered important by the author to determine whether the general run of children were capable of handling such ideas.

3. Research Design and Procedure

Three tests, modeled on an earlier one used by the author, were devised. Each test set up the same mathematical system by defining an operation a*b = a + 2b, on the set of zero and the positive integers. Children were then required to make deductions about the truth or falsity of the statements in the test items. The items are shown in Table 1.

Table 1

Test Items Elementary Mathematical Systems

Item	Test 1	Test 2	Test 3
1	a*b = b*a	4*6 = 6*4	4728*8976 = 8976*4728
2	a*(b*c) = (a*b)*c	5*(4*6) = (5*4)*6	982*(475*638) = (982*475)*638
3	a*x = a	4*5 = 4	4932*8742 = 4932
4	a*(b+c) = (a*b)*c	3*(4+6) = (3*4)*6	6836*(935+2397) = (6836*935)*2397
5	a+(b*c) = (b*c)+a	3+(4*6) = (4*6)+3	572+(865*749) = (865*749)+572

In Test 1, the subjects were asked to tell when the statements would be true, and in Tests 2 and 3 whether the statements were true or false. In all three tests, the response "can't tell" was also allowed. Tests were given in the order listed.

). In a pilot study 6Ss, two each at ages 10, 13, and 16 were chosen, all of IQ 120+. In the main study 170 boys and 160 girls from schools in lower middle class areas were used. Thirty were taken at each age level from 7, 8, 9, . . ., 17 years of age. The six Ss in the pilot study were examined individually in the presence of the experimenter who encouraged the S to express his thoughts verbally. Upon completion the experimenter also went back over incorrect test items, as well as those the S could not do, to see if some small amount of help would enable the S to respond correctly. In the main test, Ss wrote or marked answers on a sheet provided. Each group of 30 Ss at each age level was given forty minutes to respond to all test items.

The hypotheses tested in the study were:

I. (a) responses to items in all tests for Ss below age 16 would show a tendency to ignore the given mathematical system and to reason by analogy with a familiar system.

(b) Ss under age 16 would have little success on any of the tests.

II. success on Items 1 and 2 in Test 1 would be preceded by success in the parallel items in Tests 2 and 3.

These hypotheses were formulated upon the basis of results in prior studies and the results of the pilot test.

4. Findings

Responses were tabulated according to age and correctness. Since the Ss work was also included on the response sheet, it was possible to analyze their pattern of thinking to some degree. A common approach to items in Test 1 was to ignore the defined operation and to substitute the familiar binary operations of + or X. Three other categories of response patterns were identified. These are indicated by the headings of Table 2.

A correct response to an item by 60% of Ss in an age group was selected as the cut-off point to indicate success on an item. In an examination of the data, success on Tests 2 and 3 was generally achieved by age 16. There was also a marked trend for a large gain to be made between years 15 and 16. Items 3 and 5 were the ones most readily achieved in Tests 2 and 3, but for the Ss below 16, it was suspected that they achieved success by using the substitution of an arithmetic operation and by ignoring the defined system.

Table 2

Proportion of Ss in Response Categories by Age Group

Age	Can't Tell (1)	+/X substitution (2)	Attempts to Work With System Defined (3)	Work Correctly With System Defined (4)
17	.00	.14	.23	.63
16	.06	.52	.16	.26
15	.03	.68	.29	.00
14	.09	.80	.11	.00
13	.10	.87	.03	.00
12	.03	.94	.00	.03
11	.12	.88	.00	.00
10	.17	.83	.00	.00
9	.00	1.00	.00	.00
8	.27	.73	.00	.00
7	1.00	.00	.00	.00

The results of the 16 and 17 year olds indicated that Items 1, 2, and 5 were achieved before Items 2 and 4 in all three tests. Item 4, in general, tended to be the most difficult. It was also found that in these age groups, the percentage of successful Ss on Items 1 and 2 of Tests 2 and 3 was higher than on the same numbered items of Test 1. This was particularly true for age 16.

5. Interpretations

The findings seemed to support the hypotheses. The findings also showed the extreme difficulty that even the oldest Ss had in working within the abstract system in Test 1; it was not likely that they could relate the items on that test to other items in a different test until they had command of the items in the original test. Only some of the 17 year olds referred back to Test 1 when working on Tests 2 and 3. The author wondered about the sequencing of the tests, hypothesizing that the order Test 2, Test 3, then Test 1 might provide the Ss some training that they could use to better achieve on Test 1.

The most important aspect from the educational psychologist's point of view is the fact that over 80% of each age group from 9 to 14 completely ignored the given system with its defined operation and turned to a familiar operation. Perhaps the S is incapable of manipulating the propositions that contain the data, but rather looks for and works with what appears to him to be the reality of the situation.

Abstractor's Notes

In general, I feel the study was carefully done and the results well analyzed. In a study of this type I might offer the suggestion to future researchers to consider some type of sequential testing. For example, rather than test Ss at each age level, consider testing only 7, 12, and 17 year olds. If no great differences occur, as might be expected between 7 and 12, do no further testing of Ss between those bounds. When differences occur as between 12 and 17, try an intermediate step, such as age 15 and then repeat the analysis. This would have been warranted in this study in view of prior research and the results of the pilot study.

In view of the available research and the findings of Piaget and his associates, the results of this study were not at all surprising. Nevertheless, I would have been more comfortable with the authors findings and conclusions if he had divided his subjects and used an alternate abstract system and/or alternate set of test items. Thus, one could be reassured that the results are not due to the make-up of the test and its particular items.

Finally, I wonder whether the dramatic increase in success from age 15 to 16 on the test items is a reflection of the Ss' maturing into the formal-operational stage, or just simply due to the historical fact that they have recently completed a high school course in Algebra. But perhaps the latter is a cause of the former.

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EJ 098 533 SE 511 194
STRUCTURAL MODELS FOR PREDICTING THE DIFFICULTY OF MULTIPLICATION PROBLEMS. Cromer, Fred Eugene, Journal for Research in Mathematics Education, v5 n3, pp155-166, May 74.

Descriptors--*Algorithms, *Elementary School Mathematics, *Multiplication, *Research, *Task Analysis, Complexity Level, Learning Difficulties.

Expanded Abstract and Analysis Prepared Especially for I.M.E. by Harold L. Schoen, The University of Iowa.

1. Purpose

To test the feasibility of using structural models constructed by a least-square method to predict the difficulty of two-factor multiplication problems. In particular, "the study was directed toward (a) exploring the correlations between each of [several] structural variables and the criterion variable, (b) testing the significance of various regression models, (c) searching for a more parsimonious model that would still predict problem difficulty, and (d) testing the significance of the independent contribution of each of the variables in a model."

2. Rationale

This study is based on the research of Suppes and others in developing a "process model for arithmetic." In this model a small number of structural variables and multiple linear-regression techniques were used to assess the difficulty level of simple addition problems. In addition, a review of the literature concerned with difficulty level of arithmetic problems was conducted in order to identify salient structural variables. "In general, these studies found that larger addends or factors increase difficulty, combinations with zero are usually at a high level of difficulty, combinations and their reverses are not equally difficult, combinations with the smaller addend or factor second tend to be less difficult."

3. Research Design and Procedure

Only multiplication problems of the form

$$\begin{array}{r} ab \\ \underline{\times cd} \end{array}$$

were used in this study where ab and cd are two digit whole numbers. Thirteen structural variables were chosen as predictors for the criterion variable, item difficulty (DIFF), i.e., the proportion of students failing to arrive at the correct solution to the item. The structural variables were (1) the tens digit in the first factor (TDF), (2) the units digit in the first factor (UDF), (3) the tens digit in the second factor (TDS), (4) the units digit in the second factor (UDS), (5) the number of operation steps in addition (OA), (6) the number of operation steps in multiplication (OM), (7) the number of digits carried in addition (DCA), (8) the number of digits carried in multiplication (DCM), (9) the largest digit in the factors (LDF), (10) the smallest digit in the factors (SDF), (11) the number of digits in the product (NDP), (12) whether either of the factors had the same tens and units digit (SMD) and (13) whether the second factor was larger than the first factor (LFS). The measurement of OA was defined in the same manner as the number of operations (0) used in the studies conducted by Suppes, mentioned earlier. The number of steps in multiplitation (OM) was defined in a similar way. Briefly, OM reflected the total number of binary operations the subject must complete to solve (ab)x(cd) by the usual algorithm.

The sample consisted of 238 fifth graders, all of the fifth-grade children from two elementary schools in a mid-south city of approximately 25,000. Two forms of an instrument, each containing 84 multiplication problems formed from digits generated randomly by a computer, were constructed. One-half of the students completed Form 1 of the test and the other half completed Form 2. The test was administered by the classroom teachers, at their convenience, and with no time limit specified.

Linear models using (1) all 13 variables, (2) the 4 digit-type variables (TDF, UDF, TDS, UDS), (3) the 9 process-type variables and (4) 3 variables determined by a factor analysis were considered.

4. Findings

The summary tables for the four linear models are reproduced below. In model 4, OM, OA and NDP were used as predictors since they were the variables most highly correlated with three factors identified by a principal axis factor analysis with promax rotations.

MODEL (1)—FULL MODEL WITH ALL 13 PREDICTORS

Predictor	Beta Weight	B Weight	Percent of Independent Variance Contributed	F Ratio	Probability Level
TDF	.075	.003	0.2	1.26	.262
UDF	.273	.010	2.2	15.33	.000
TDS	.226	.008	1.0	7.09	.008
UDS	.187	.007	1.4	9.57	.003
OA	.376	.028	1.9	12.86	.001
OM	-.288	-.014	0.5	3.19	.072
DCA	-.163	-.023	0.5	3.51	.059
DCM	.275	.035	0.5	3.69	.053
LDF	.097	.007	0.4	2.74	.096
SDF	.122	.008	0.3	2.25	.132
NDP	.216	.035	1.3	8.59	.004
SMD	-.156	-.042	2.1	14.62	.000
LFS	-.005	-.001	0.0	.00	.949

Regression Constant = -.131.
Multiple R = .88. R^2 = .78. p = .000.
Note.-For all F ratios. df = 1/153.

MODEL (2)--DIGIT TYPE VARIABLES

Predictor	Beta Weight	Weight	Percent of Independent Variance Contributed	F Ratio	Probability Level
TDF	.210	.008	4.3	22.58	.000
UDF	.477	.017	22.7	118.49	.000
TDS	.503	.018	25.0	130.45	.000
UDS	.391	.015	15.2	79.40	.000

Regression Constant = -.063.
Multiple R = .83, R^2 = .69, p = .000.
Note. -For all F ratios df = 1/163.

MODEL (3)--PROCESS TYPE VARIABLES

Predictor	Beta Weight	Weight	Percent of Independent Variance Contributed	F Ratio	Probability Level
OA	.423	.031	2.5	15.29	.000
OM	-.408	-.020	1.1	6.67	.010
DCA	-.181	-.026	0.6	4.02	.044
DCM	.509	.064	2.6	15.95	.000
LDF	.254	.018	5.0	30.71	.000
SDF	.249	.016	1.8	11.03	.001
NDP	.272	.044	3.2	19.88	.000
SMD	-.157	-.042	2.3	13.97	.001
LFS	.021	.004	0.0	.25	.627

Regression Constant = -.149.
Multiple R = .86, R^2 = .74 p = .000.
Note. -For all F ratios df = 1/58.

MODEL (4)--FACTOR ANALYTIC MODEL

Predictor	Weight	Weight	Percent of Independent Variance Contributed	F Ratio	Probability Level
OA	.318	.023	5.4	22.28	.000
OM	.338	.017	5.5	22.61	.000
NDP	.245	.039	3.5	14.68	.000

Regression Constant = -.048.
Multiple.-R = .78, R^2 = .60, p = .000.
Note.-For all F ratios df = 1/164.

5. Interpretations

Of the 13 structural variables, only SMD did not correlate significantly with DIFF. Each of the four linear models accounted for a significant proportion of the variance in DIFF. However, model 1 accounted for significantly more of the variance in DIFF than did any of the three restricted models. "Thus, in arriving at a more parsimonious and possibly more useful model, some predictive power may be sacrificed."

This study supported most conclusions drawn from early work on problem difficulty with two notable exceptions. First, this study does not support the conclusion that multiplication problems with the smaller factor second are less difficult. Second, combinations involving 0, or the larger digits, did not prove to be more difficult nor were combinations involving "1" or "2" less difficult. In fact, combinations involving "0" were among the easiest problems to solve.

The results of this study clearly indicate that it is possible to predict the level of difficulty of multiplication problems in terms of easily measured structural variables. Furthermore, the techniques used to measure problem difficulty in this study may be easily extended to different types of mathematical problems. Since problem difficulty is one obvious external variable to be considered in building a theory of mathematics learning, the measurement techniques used in this study could have important theoretical significance.

Abstractor's Notes

This research report was written in an exceptionally concise manner. In particular, its relationship to learning theory and earlier research is evident, though it appears to be unduly limited in scope. For example, would it not have been possible to analyze the performance of individual subjects or types of subjects grouped by some learner characteristics? Basic research which goes beyond analysis of large group data is needed before any theory of mathematics learning will approach completeness. Perhaps the greatest contribution made by this study is the technique for measuring the various structural variables.

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EJ 080 048 310 SE 508 789
LEARNING ACHIEVEMENT AND THE EFFICIENCY OF LEARNING THE CONCEPT OF VECTOR ADDITION AT THREE DIFFERENT GRADE LEVELS. Gubrud, Allan R.; Novak, Joseph D., Science Education, v57 n2, pp179-191, Apr/Jun 73.

Descriptors--*Concept Formation, *Instruction, *Learning, *Learning Readiness, *Mathematics Education, Educational Research, Learning Theories, Secondary School Science, Sequential Learning, [Research Reports]

Expanded Abstract and Analysis Prepared Especially for I.M.E. by H. Laverne Thomas, State University of New York, College at Oneonta.

1. Purpose

The purpose of the study was to test Ausubel's subsumption learning theory. The specific problem investigated was that of finding the appropriate grade level for teaching the concept of non-algebraic vector addition. Primary constraints were that learning time to be held constant for each grade level and that "....the initial information learned should be incorporated into cognitive structure with enough clarity and stability that it can be usefully employed by the learner to learn subsequent but related material....".

Two research hypotheses were specifically tested:

1) Tenth grade students will do better than eighth grade students on a posttest following instruction on non-algebraic vector addition.

2) Tenth grade students will do better than eighth grade students on a related materials test following further instruction on material related to non-algebraic vector addition.

2. Rationale

The investigators were motivated by consideration of Bruner's well known hypothesis on concept learning and its limitations in the classroom setting. The learning theory of Ausubel provides additional restrictions on Bruner's hypothesis in that it posits a strong relationship between learning and the availability in the learner's cognitive structure of appropriate subsuming concepts. From this a test for adequate learning of a particular concept is derived as the degree to which it facilitates the learning of subsequent related material.

This research is indirectly related to research on advance organizers. It is also somewhat related to a host of studies indicating the feasibility of teaching a given concept at a given (lower than usual) grade level.

3. Research Design and Procedure

The experimental groups consisted of 96 students from each of grades 8, 9, and 10. All subjects were selected from the school population in Ithaca, New York, a college town of 25,000. Eighth and tenth graders were volunteers who took part in the study during study hall periods whereas ninth graders participated as students in a general science class. The IQ range for the 288 subjects was 100-150.

All subjects received an instructional sequence of four 40-minute lessons on four consecutive days, leading through the addition of vectors in lessons 3 and 4. This sequence was standardized by an independently developed self instructional system making use of multiple learning materials.

Following this instructional sequence an 18-item Posttest (PT) was administered to one-fourth of the subjects at each grade level at intervals of 3, 21, and 42 days. The PT contained nine items on recall of rules for vector addition (drill) and nine items requiring the abstraction of giving a vector representation of a physical situation as well as applying the rules for vector addition (application).

The remaining one-fourth of the subjects at each grade level received an additional lesson on the resolution of vectors into components and a Related Materials Test (RMT) on this lesson. This occurred 42 days after the initial four lessons were completed. The instruction was a programmed booklet for which 25 minutes were allowed with 15 minutes for the 12-item multiple choice Related Materials Test.

Dependent variables were the Posttest and Related Materials Test scores. Apparent independent variables were grade level, retention interval, and ability level.

Graphic comparisons on mean PT scores were made for each component; drill, application, and for the total PT scores for each grade level. Similar comparisons were made between low and high ability students at each grade level. Factorial analysis of variance was also carried out for each of these sets of scores using Grade (3 levels), Time (3 levels), and Ability (2 levels, as measured by IQ) as the independent variables.

For the RMT, graphic comparisons were made on mean scores by grade level and by three levels of "abstract thinking ability." Contrasts were also given between PT and RMT mean scores by ability and grade level. A factorial analysis of variance was done for the RMT scores with grade level and ability level as the independent variables.

4. Findings

The ANOVA for the PT and each subtest (drill, applications) yielded significant main effects for grade level and ability level ($p < .01$). A significant main effect for retention time was found on the PT and the drill subtest ($p < .05$). There were no significant interactions. For the RMT the ANOVA showed a significant effect only for grade level ($p < .01$). These statistical results, together with the extreme differences in mean scores between grades eight and ten were reported as supporting both research hypotheses.

5. Interpretations

The investigators argue in several ways that the results of the study are supportive of Ausubel's subsumption theory. First, the superior performance of the tenth graders on the Posttest is explained by the greater availability in cognitive structure of relevant subsuming concepts. Informal investigation tended to rule out an alternative explanation that students had learned specific concepts of vectors and vector addition in the interval from grade eight to grade ten.

Second, the further superior performance of tenth graders on the RMT is explained in a cumulative fashion based on subsumption theory. That is, tenth graders possessed through cognitive growth relevant subsumers that enabled them to learn the initial material with greater clarity and stability hence facilitating the learning of the related materials.

Third, it is noted that differential effects on the PT scores appear that may be explained or predicted by subsumption theory. The difference between mean scores on the Posttest for high and low ability ninth graders is 4.9 as compared with 2.2 and 3.5 for eighth and tenth graders, respectively. The interpretation is that most high ability ninth graders have a greater availability of relevant subsuming concepts through their study of algebra whereas most low ability ninth graders do not study algebra. This is further supported by an increase in mean score from eighth to ninth grade of only 0.1 for low ability students as compared with an increase of 2.8 for high ability students.

The fact that a significant difference occurs between ability levels on the PT but not on the RMT is also seen as explainable by subsumption theory. The high ability student begins at grade eight with a greater number of subsumers and accumulates them over time at a greater rate than does the low ability student, thus performing better on the PT throughout. This difference disappears on the RMT because the time interval is too short (six weeks) for any factors to operate other than the initial state of the learner and the effect of the learning material in lessons 1-4.

Finally, the subjects were further categorized for analysis into three levels of abstract thinking ability. These levels were defined as (a) all eighth graders and low ability ninth graders, (b) high ability ninth graders and low ability tenth graders, and (c) high ability tenth graders. Group (a) had a mean score on the RMT corresponding to an estimated chance score (3.61) whereas groups (b) and (c) had mean scores 1.80 and 2.81 points above that score. Thus, the criterion for meaningful learning as initially stated is not met for group (a) but is better met by groups (b) and (c).

In sum, the investigators conclude that "Ausubel's contention, that the primary facilitation for new learning is the adequacy of <u>relevant</u> concepts, is supported by our data."

Abstractor's Notes

There are a few apparent errors in the original report that may be puzzling to the reader. In Figures 2 and 3, N should be 216 rather than 192 to agree with the text of the report. The titles for Figures 6 and 7 are reversed. In Table V, the number of items for the Related Materials Test should be, presumably, twelve rather than nine.

This research is of definite interest for mathematics educators although carried out in the context of science education. It would appear that the only parts of the learning materials specifically science related were those that dealt with applications of vectors.

Much prior research, with little productivity, has been directed towards the effectiveness of advance organizers in instruction in mathematics. The research reported takes a different tack in testing subsumption theory on a more basic level and is thereby of great interest to those exploring the application of Ausubelian theory in mathematics instruction. Despite the contention that the data are supportive of the theory, the results are so broad and diffuse as to make the interpretations obvious on the one hand (the gains from the eighth to the tenth grade) and somewhat strained on the other (explaining the superiority of high ability ninth graders over those of low ability). What explains the narrowing of the gap for high ability and low ability tenth graders (a gain of 3.0, the largest single gain for any group)? In what way would they have attained relevant subsuming concepts from ninth to tenth grade of such magnitude when they have not taken algebra?

These and other interpretations made would have been greatly strengthened had significant interactions occurred on the ANOVA. In contrast, why were the significant effects for retention time not discussed? Do they have a specific relationship to subsumption theory? If not, why was this factor built into the study?

An interesting contribution of the study is its approach to testing feasibility of teaching specific concepts at different grade levels. As such, critics might argue that the results might have been more favorable at grade eight if only a better teaching approach had been used. Further studies could incorporate this factor as an experimental variable.

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EJ 099 486 EC 061 607
DEVELOPMENTAL DYSCALCULIA. Kosc, Ladislav, Journal of Learning Disabilities, v7 n3, pp164-177, Mar 74.
Descriptors--*Exceptional Child Research, *Learning Disabilities, *Mathematics, *Testing, Exceptional Child Education, Childhood, Adolescents, Number Concepts, Definitions, [*Dyscalculia]

Expanded Abstract and Analysis Prepared Especially for I.M.E. by Nicholas A. Branca, The Pennsylvania State University.

1. Purpose

The purposes of this paper are to put forth a definition of developmental dyscalculia to distinguish it from other forms of disturbed mathematical abilities and to present the findings of an investigation of mathematical abilities and disabilities in eleven-year old pupils from normal schools in Bratislava, Czechoslovakia.

2. Rationale

The concept of developmental dyscalculia has not received the attention that developmental dyslexia and dysgraphia have. Evidence is given to support the existence of genetic dispositions for mathematics and various definitions of developmental dyscalculia are given. The author considers daycalcula to be a much more complicated disorder than past interpretations have suggested and defines it as "a structural disorder of mathematical abilities which has its origin in a genetic or congenital disorder of those parts of the brain that are the direct anatomico-physiological substrate of the maturation of the mathematical abilities adequate to age, without a simultaneous disorder of general mental functions." Factor analytic studies of mathematical abilities are also cited to support the author's contention that these abilities are not simple and compact and that it is necessary to differentiate between several relatively isolated abilities or factors. Developmental dyscalculia is differentiated from postlesional dyscalculia, acalculia, oligocalculia, and paracalculia and its basic forms of verbal dyscalculia, practognostic dyscalculia, lexical dyscalculia, graphical dyscalculia, ideognostical dyscalculia, and operational dyscalculia are explained.

3. Research Design and Procedure

A sample of 375 pupils (199 boys and 176 girls) selected at random from 14 fifth grade classes in 14 Basic schools in Bratislava were administered two sets of group tests measuring mathematical abilities. The first set consisted of Kalkulia I, developed by the author for research purposes, and a modified version of the Minnesota Paper Form Board (PFB). The second set consisted of tasks testing basic arithmetical operations, sequences, and symbols. Children scoring at or below the 10th percentile of the score distribution on the group tests were considered failures.

After eliminating those students with IQ's below 90, three groups of failures were delineated, those who failed the first set, those who failed the second set, and those who failed both sets. These 66 children were then submitted to a detailed individual psychological and neurological examination. The following tests--numerical triangle, Rey-Osterrieth Complex Figure, arithmetical reasoning, digit memory test, successive subtraction of 7 from 100, numerical square test, and G-test were included among others in the investigation.

4. Findings

In the numerical triangle test the groups with global failures differed significantly (at the .01 level) from both experimental groups in the number of incorrect steps and the testing time.

The Rey-Osterrieth Complex Figure did not distinguish between the experimental groups.

On the arithmetical reasoning problems the lowest average scores and the longest mean time were obtained by the experimental groups who failed in the performance tests.

In the digit memory test all experimental groups were found to have a higher average number of incorrect reproductions on both sets in which the numbers were presented verbally. The greatest number of failures were again recorded in the group of globally failing children.

The test of successively subtracting 7 from 100 was given in three forms (two verbal and one written performance). As far as number and range of faulty steps are concerned, the poorest results were achieved by the two experimental groups who failed all the numerical tests applied in the screening.

The numerical square holds a special position in the test battery used insofar as the poorest results were not attained by the group of global failures but by the group that failed only the tests of numerical character used in the screening stage of this investigation.

The G-test was included in the test battery chiefly to determine to what extent dyscalculia (numerical dyslexia and dysgraphia) occurs simultaneously with literal dyslexia and dysgraphia. The lowest scores were obtained by both experimental groups who failed in all numerical tests employed in the screening stage of the investigation.

Out of 15 children of the group of global failures, 11 children showed at least grave suspicion of an objective pathological neurological finding, whereas in the group of 21 children that failed in the numerical tests only, neurological deficits were found in just two cases. In the last group (failure in the performance tests) there was not a single case with even a suspicion of an objective pathological neurological finding.

Out of the whole sample of 375 children, dyscalculia was found in a total of 24 (6.4%).

5. Interpretations

The author's definition of developmental dyscalculia is more inclusive than earlier definitions and takes into account the relationship between general mental abilities and special mathematical abilities.

The research findings illustrate various differences in children's mathematical abilities. They suggest that approximately 6% of children of a normal population can be expected to have symptoms of developmental dyscalculia as defined in the study.

Abstractor's Notes

The author has undertaken an ambitious study to collaborate his definition of developmental dyscalculia. The paper is extremely informative on a superficial level due not to the study itself but to space limitations in reporting it. Not all tests are described in detail and some used were experimental versions. The author stresses that detailed analyses of the subjects were performed and individual case studies were undertaken before the presence of developmental dyscalculia could be diagnosed. He reports that school marks, indicators from parents' and teachers' questionnaires and neurological examinations were statistically analyzed but that it is not possible to quote them all within the framework of the paper. The data, however, are available from the author on request.

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EJ 092 098 AA 517 522

LEARNING AIDS IN THE CLASSROOM: EXPERIMENTAL EVIDENCE OF THEIR EFFECTIVENESS. Kuhfittig, Peter K. F., Education, v94 n2, pp135-136, Nov/Dec 73.

Descriptors--*Instructional Materials, *Learning Processes, *Achievement Tests, *Retention, *Educational Experiments, Student Ability, Transfer of Training

Expanded Abstract and Analysis Prepared Especially for I.M.E. by Larry K. Sowder, Northern Illinois University.

1. Purpose

To investigate the effects of concrete aids, of a discovery method, and of the interaction between these two variables.

2. Rationale

No research which simultaneously investigates concrete aids and discovery learning seems to have been done.

3. Research Design and Procedure

Four factors, each at 2 levels were used:

1. learning aids (LA)--presence/absence of concrete aids;
2. discovery (D)--intermediate guidance ("a carefully structured sequence of questions")/maximal guidance ("careful explanation of the individual steps");
3. ability (A)--high/low, based on departure of at least one standard deviation from the mean on Metropolitan Achievement subscores;
4. tests--posttest/retention test.

These factors were the basis for a completely crossed, balanced 2x2x2x2 factorial design with repeated measures on the tests factor.

Subjects were 40 seventh-graders, 20 at each ability level; random assignment of Ss at each level gave 5 Ss per LAxDxA cell.

The learning task was conversion from American to old English currency and vice versa. The concrete aids were experimenter-produced models of coins. Instruction to the 4 LAxD groups was given by E; overhead transparencies were used "to insure uniformity of content and to help prevent an unintentional bias toward confirming the hypotheses." That the treatments were different was established by a review of audiotapes of the sessions.

Two days of instruction were followed by 2 days of posttests. After 4 weeks, the same tests were given for retention measures. The tests consisted of 3 parts: achievement, horizontal transfer, and vertical transfer. Reliabilities of .82, .84, and .55, respectively, were reported (the type of reliability was not given). Use of concrete aids was not allowed during the tests.

4. Findings

1. For low ability Ss on posttest achievement score: concrete > abstract. (No such difference was noted for high ability Ss.)

2. For intermediate guidance Ss on posttest transfer scores: concrete > abstract. (No such difference was noted for maximal guidance Ss.)

3. Trends for posttest vertical transfer score: intermediate guidance > maximal guidance for low ability (not so for high ability).

4. The only significant differences on retention test scores: high ability > low ability on achievement and vertical transfer.

5. For achievement and vertical transfer scores: posttest > retention test.

5. Interpretations

"The main conclusion, then, is that low ability subjects benefit from aids more than high ability subjects in mastering abstract skills. Furthermore, students learning by discovery benefit from concrete aids, but whenever the expository method is employed the benefits derived from such aids are not apparent when transferring the acquired skills to novel situations. Finally, for low ability subjects intermediate guidance appears to be preferable on transfer."

Abstractor's Notes

1. The investigation of the joint influences of "discovery" learning and concrete aids makes this a most interesting study. In particular, there has been a lack of work on whether concrete aids with secondary school Ss make any difference. The experimenter was wise to use the terms "intermediate guidance" and "maximal guidance" and might well have avoided the ill-defined term "discovery" altogether.

2. The necessary brevity of journal articles is likely responsible for the lack of definitions of horizontal and vertical transfer (or sample test items to illustrate these), information on the scores reported (means of raw scores? percents?), and a more complete report of the statistical analyses.

3. The small n (5 per cell) should cause considerable concern. Besides the statistical drawbacks, an "off-day" or misunderstanding by only 1 or 2 Ss could cause distortions in results.

4. Coupled with the small n, the 2-day testing might have lead to this remarkable result (remarkable at least to the abstractor--the experimenter did not comment on it): For the low ability, intermediate guidance, abstract group, posttest mean on horizontal transfer = 37.0, but retention test mean = 63.6.

5. The abstractor is biased against a repeated measures design for settings like that of this study. He would prefer ANOCOVA (covariates: the Metropolitan scores and/or the posttest scores), or a multivariate analysis, or even separate ANOVAs for posttest and retention scores. The author did not comment on the apparent dubiousness of homogeneity assumptions.

6. The author's statements given above under Interpretations seem much stronger than the number of significant differences warrants.

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EJ 075 270 ; AA 515 447

PERSONALITY AND PERFORMANCE IN ELEMENTARY MATHEMATICS WITH SPECIAL REFERENCE TO ITEM TYPE. Lewis, D. G.; Ko, Peng-Sim, British Journal of Educational Psychology, v43 pt1, pp24-34, Feb. 73.

Descriptors--*Personality Assessment, *Secondary School Mathematics, *Performance Factors, *Objective Tests, *Item Analysis, Individual Characteristics, Sex Differences, Analysis of Variance, Tables (Data)

Expanded Abstract and Analysis Prepared Especially for I.M.E. by Marilyn N. Suydam, The Ohio State University.

1. Purpose

To ascertain the relationship between personality factors and mathematics performance as evidenced by secondary-school students on varying types of test items.

2. Rationale

The use of objective tests to supplement more traditional kinds of questions has increased. The relationship between such non-cognitive factors as personality and academic success has been studies, based on the assumption that students with different personality characteristics will respond differently to tests; the possible interactive effects of personality variables with item type has been noted. Little attention has been paid to these effects in a mathematical context; therefore this study is an attempt to ascertain the effect of personality factors on attainment on various types of mathematics test items at the Certificate of Secondary Education Level.

3. Research Design and Procedure

Students aged 14-16 years were selected from two modern secondary schools (one boys', one girls' school) and one coeducational comprehensive school. They were initially tested on (1) the Junior Eysenck Personality Inventory and (2) Raven's Standard Progressive Matrices, and were defined by (1) as being introverted or extraverted and neurotic or stable, and by (2) as having high or low ability. Each student was accordingly placed in one of eight categories, and a sample of 80 boys and 80 girls was randomly selected from these categories.

The mathematics tests were constructed in four sections, each using a distinctive item type: multiple-choice, data-sufficiency, multi-facet (true-false), and traditional (constructed response); each section was divided into two content areas, number and space.

Data were analyzed by a six-way analysis of variance, taking account of a double-crossing of a nested variable, that of individual pupils crossed with item type and content areas, within each of the cross-classifications of extraversion, neuroticism, sex, and ability.

4. Findings

The AOV separates 63 distinct sources of variation, 60 of which are of possible psychological interest; of the 60, nine were found to be statistically significant (two main effects, five first-order interactions, and two second-order interactions).

(1) Extraverted boys performed better than introverted boys in all four item types; this was statistically significant only for multi-facet items. Introverted girls performed better than extraverted girls; this was significant for multi-facet and traditional items. Extraverted boys performed significantly better than extraverted girls in all item types except data-sufficiency; no significant differences were found among the introverted.

(2) Neurotic children performed better than stable children for number content on multiple-choice, data-sufficiency, and traditional items, while stable children performed slightly better on multi-facet items; no differences were significant, however. For space content, stable children performed better than neurotic children; only the difference for traditional items was significant.

(3) Among the extraverted, those who were neurotic performed better than those who were stable; among the introverted, stable children performed better than neurotic children. Groups with the best performances were the neurotic extraverts and the stable introverts. No differences were significant, although the difference of the differences was significant.

(4) Among those with high ability, the introverted did better, whereas among the less able, the extraverted did better. There were no significant differences, but the difference of the differences was significant.

(5) Differences in ability levels within item type were markedly unequal, with that for data-sufficiency items noticeably small. The overall difference among these differences was significant.

5. Interpretations

Previous studies suggested that the advantage of the educational effects of extraversion might be over for this age group, and a significant main effect favoring introversion was hypothesized. This was not found. Extraversion-introversion was found to interact with sex and item type, thus necessitating separate conclusions for boys and girls, these in turn being qualified by item type. Any hypothesis of the superiority of introverts survives only if restricted to girls and to their performance on multi-facet and traditional items. For boys extraversion was still an asset. Thus this suggests that a "change-over" occurs earlier with girls.

The findings relative to ability level provide some support for MA rather than CA being the real determinant of when introversion begins to be an advantage.

Somewhat in contrast to the linkage of extraversion with sex and items type is the extraversion-by-neuroticism interaction.

Stability would appear to be an asset to mathematical attainment only in respect to traditional questions in a space content.

Further research is suggested to confirm the need to take account of the form of assessment of mathematical attainment.

Abstractor's Notes

In general, this is a well-presented research report. The review of literature is concise but thorough. By and large, the sample selection appeared to reflect the characteristics of the population. The mathematics tests seem to have been carefully and competently constructed. (Would the inclusion of sample items have helped to confirm this reaction?)

Discussion of the meaning of the findings for schools is not included (probably because the findings from only one investigation do not necessarily warrant this). But this would be interesting to pursue: to reflect the highest attainment, which item types might be selected for which types of students? (Is this an "acceptable" practice?)

If supported by subsequent research, the results of this study could provide an interesting facet to add to planned research. It might be noted in passing that most of the research on this type of topic has been conducted in Britain; relatively little has been done in this country. Would it be plausible to replicate such studies with an American population?

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EJ 083 854 AA 516 560

AN INVESTIGATION OF OPEN-BOOK AND CLOSED-BOOK EXAMINATIONS IN MATHEMATICS.
Michaels, Shirleyanne; Kieren, T. R., Alberta Journal of Educational Research, v19 n3, pp202-207, Sep 73.

Descriptors--*Mathematics, *Research Design, *Scores, *Tests, *Evaluation, Educational Research, Tables (Data), Test Results, Data Collection, Predictor Variables

Expanded Abstract and Analysis Prepared Especially for I.M.E. by Lewis R. Aiken, Sacred Heart College.

1. Purpose

The following questions were investigated:

1. Do examinations written in an open-book setting, as contrasted with those written in a closed-book setting, provide a different assessment of the high school student's achievement in mathematics? That is, does his achievement differ between the two settings?

2. Does a student's level of anxiety differ in the two settings?

3. Does a student's attitude in the two settings differ?

4. Is there a relationship among attitude, anxiety and achievement in a particular setting?

5. Does the reliability, validity and variance of examinations vary between the two settings?

2. Rationale

The collective findings from six studies of the use of open-book examinations with university students are summarized. These findings are: (1) Little or no increase in achievement with open-book examinations; (2) Students liked open-book examinations better than closed-book examinations, presumably because they were less anxious during the former; (3) sounder study preparation resulted from open-book examinations; (4) students taking open-book examinations were tested not only on memory but also on reasoning; (5) compared to closed-book examinations, scores on open-book examinations had slightly higher variances, reliabilities and validities.

3. Research Design and Procedure

The research sample consisted of 600+ students in twelfth-grade mathematics classes in Alberta, Canada. Students were randomly assigned to four groups by classroom lot. Setting and test forms varied from group to group. The following eight scores were obtained on each student: (a) open-book test; (b) closed-book test; (c) anxiety (in neutral, open-book, and closed-book settings); (d) attitude toward testing (in open-book and closed-book settings); (e) attitude toward mathematics (in neutral setting). The neutral setting involved no achievement testing and occurred between the two testing sessions. Each of the two parallel tests, one administered as open-book and the other as closed-book, contained 36 multiple-choice items. It should be noted that the tests used were originally designed for a closed-book setting.

4. Findings

The variance, reliability and validity of the achievement test scores did not vary significantly with test setting (open-book vs. closed-book). A three-way analysis of variance (time of administration X setting X test form) of achievement scores revealed no significant interaction between time of administration and setting or between test form and setting. Significant differences between the overall means for open-book and closed-book scores were found for knowledge items, comprehension items, and total test scores, but not for application items. Analysis of variance of anxiety scale scores across situations (neutral vs. open-book vs. closed-book) revealed a significantly higher mean in the closed-book than in the open-book and neutral situations, and a significantly higher mean in the open-book than in the neutral situation. The differences between means of attitudes toward open- and closed-book examinations were not statistically significant.

Analysis of regression of attitude and anxiety on achievement revealed that the three attitude variables--attitude toward open-book testing, attitude toward closed-book testing, and attitude toward mathematics--were all significantly related to mathematics achievement. The anxiety variables, however, did not significantly add to the prediction of achievement from the attitude scores alone.

5. Interpretations

The writers conclude from these results that the open-book setting increases scores on knowledge and comprehension items but not application items on a mathematics test. The lack of a significant difference for application items is interpreted as being due to their complexity, lack of relation to specific details, or other factors that make it less likely that students would find ready-made answers to the items in their notes.

The findings also indicate that anxiety, although still higher than in a non-test (neutral) situation, is lower in an open-book than in a closed-book situation. Finally, the significant correlations between attitude and achievement are interpreted as suggesting that attitude toward the type of examination is predictive of one's achievement on the examination. In other words, the differential effect on achievement of open-book and closed-book examination settings may vary with the examinee's attitude and other individual difference variables.

Abstractor's Notes

This was a very interesting study, with the finding that open-book examinations produce higher scores with some types of items but not with others being especially provocative. Unfortunately, it suffers from several flaws, both methodological and interpretive. Hindsight is usually better than foresight, but if I were to re-conduct this investigation I should use a two by two factorial design. The open-book vs. closed-book dichotomy would constitute one factor, and the essay vs. objective test dichotomy the other factor. I should also be more specific about my sampling procedure, since it is unclear in reading this paper whether the sampling unit was the individual student or the classroom group. If it was actually the latter, then the subsequent analyses of variance are incorrect. Also with regard to sampling, the writers are not clear about what they mean by the terms "heterogeneous" and "classroom lot" as applied in this situation.

Educational researchers do not always provide sufficient details in describing their research designs, but they usually do a bit better with their instruments. Our writers, however, tell us nothing about the nature of the "attitude scales" employed in the present investigation. Another possible problem is the fact that the achievement tests were designed for an entirely different purpose in another context. These multiple-choice tests were designed earlier for a closed-book testing situation only.

Regarding the analysis of results, I wonder how the writers determined that the Spearman-Brown coefficients for the two settings, and consequently their reliabilities, were not significantly different? Conventional statistical tests based on independent random samples were inappropriate in this situation because the two correlations were calculated on the same sample of people. Regarding the analyses of variance of the three subscores and total score on the achievement tests, I would have opted for multivariate analysis of variance. Scores on the knowledge, comprehension, and application items are undoubtedly positively correlated. Also, tables of means (Table 1) and ANOVA tables (Table 3) should be labeled as such.

Finally, the authors get into difficulty again in the interpretation of their results. After noting that attitude toward the type of examination is correlated with achievement scores, they refer to the attitude scores as predictors (of achievement). They conclude from this finding that there are individual reactions to open- and closed-book examinations. The last statement is undoubtedly a truism (there are always individual differences!), but attitude is not strictly a predictor of achievement in this situation. Nor has it been demonstrated that differences in attitude toward open- and closed-book examinations affect achievement in these two situations differentially. Attitude was measured after achievement, and can be viewed more as an effect than a cause of the latter in this instance.

In addition to correcting the shortcomings noted above, the next time around I'd like to see the authors or their intellectual descendants include pretest measures of mathematics achievement and attitude. This way we could see how the differential effects of open- and closed-book examinations are related to initial standing on the variables in question.

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EJ 098 540 SE 511 248

A KINDERGARTEN MATHEMATICS PROGRAM. Runnels, Patricia; Runnels, L. K. School Science and Mathematics, v74 n5, pp361-365, May-Jun. 74.

Descriptors--*Curriculum, *Elementary School Mathematics, *Kindergarten, *Program Descriptions, *Research, Achievement, Ability, Accelerated Courses, Gifted, Instruction, Learning

Expanded Abstract and Analysis Prepared Especially for I.M.E. by Robert E. Reys, University of Missouri-Columbia.

1. Purpose

To report results of a formal preschool mathematics training program on mathematics achievement of kindergarteners and a follow-up check during the middle of the first grade.

2. Rationale

Considerable growth in basic mathematical concepts occurs before formal schooling begins. Such development usually reflects no planned sequence of learning activities, whereas this study was based on the assumption that a carefully structured kindergarten mathematics program should produce significant growth in mathematics achievement during kindergarten and first grade.

3. Research Design and Procedure

The sample for the investigation was 17 children from an intact kindergarten class. The subjects participated in a six hour kindergarten program for the entire 1971-72 school year. Each day contained three distinct parts of the instructional program devoted to mathematics. Two 20 minute segments were devoted to group instruction (one dealt with number skills and the other reflected topics selected from a textbook series) and one individual session (between teacher and pupil) of five minutes duration which was also devoted to number skills.

Levels of achievement with respect to addition and subtraction (reflecting the highest basic family fact mastered) were reported. Scores on the arithmetic computation and reasoning subtests of the Gray-Votaw-Rogers General Achievement Test given at the end of the semester of the first grade were also reported. These data were correlated with I.Q. scores, age and sex of the subjects.

4. Findings

At the end of kindergarten, all subjects were able to count by ones, twos, fives and tens to 100 and could recognize numerals to 100. All subjects could also show and count correctly any designated number of fingers from 0 to 10. Significant positive correlations ($p < .01$) were reported between I.Q. and arithmetic computation; and between kindergarten addition and arithmetic computation in first grade. It was also found that I.Q. and first grade computation scores were more highly correlated for boys than girls.

5. Interpretations

The strong correlation between the kindergarten achievement levels and the mid year first grade arithmetic scores is of particular interest. This suggests the predictative potential of selected kindergarten mathematics achievement scores. Such information could be used for the early identification of children gifted in mathematics and those with special learning problems.

Abstractor's Notes

The predictative potential of this study holds considerable promise, however only limited generalizations and/or interpretations can be made from this investigation. Any conclusions drawn and/or implications made must be tempered by the following facts reported by the researchers as well as several questions which were not addressed in the article.

The sample was small and not random. It was composed of a total of 17 subjects, only 13 of which had I.Q. and first grade achievement scores available. These data revealed only one I.Q. score below 100, whereas ten subjects had an I.Q. of 120 or above. This was a group of very intelligent children and would not typify the range of abilities found in most classrooms.

No control group of kindergarteners was identified, consequently it was not possible to compare the effectiveness of the Kindergarten Mathematics Program (briefly described in this article) with a regular kindergarten program. In fact the design of this research did not include a pre and post test component, even though this would have provided valuable baseline as well as change data.

The authors acknowledge the scarcity of instruments for early assessment of mathematical competencies of children entering school. It is agreed that more instruments for early assessment in mathematics are needed. There are however several such instruments available (such as Key Math and the Comprehensive Mathematics Inventory), yet no rationale for not using this type of instrument was given.

The entire assessment during the kindergarten year was weighted toward number skills; yet many other mathematical concepts (such as classification) are not only essential but in fact prerequisites for many basic number ideas. No effort was made to assess the cognitive levels at which these kindergarteners were operating. For example, information regarding the subject's ability to conserve would be invaluable. In particular it would be helpful to know if, when and to what degree these kindergarteners conserved number. The relationship between this ability to conserve number and their levels of computations achievement would also be of interest.

In closing it should be noted that this research investigation would be justified only as a pilot study and seems of questionable value for publication.

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ED 072 979 SE 015 798
NEGATIVE INSTANCES AND THE ACQUISITION OF THE MATHEMATICAL CONCEPTS OF COMMUTATIVITY AND ASSOCIATIVITY. FINAL REPORT. Shumway, Richard J. Ohio State University, Columbus. Spons Agency--National Center for Educational Research and Development (DHEW/OE), Washington, D.C., Regional Research Program. Pub Date June 72 Grant-OEG-5-71-0025(509) Note-98p. EDRS Price MF-$0.65 HC-$3.29
Descriptors--*Concept Teaching, *Instruction, Learning, *Learning Theories, Logic, *Mathematics Education, Number Concepts, *Research, Secondary School Mathematics

Also reported in:

NEGATIVE INSTANCES IN MATHEMATICAL CONCEPT ACQUISITION: TRANSFER EFFECTS BETWEEN THE CONCEPTS OF COMMUTATIVITY AND ASSOCIATIVITY. Shumway, Richard J., Journal for Research in Mathematics Education, v5 n4, pp197-212, Nov. 74.

Expanded Abstract and Analysis Prepared Especially for I.M.E. by William E. Geeslin, University of New Hampshire.

1. Purpose

To determine if an instructional sequence consisting of both positive and negative instances as compared to a sequence of only positive instances produces a difference in the acquisition of the concepts of commutativity and associativity. And, if differences exist, to determine if the effects of including negative instances transfer from the acquisition of one concept to the acquisition of another.

2. Rationale

Mathematicians have stated that negative instances are essential to the understanding of advanced mathematical concepts. Some psychologists have recommended the inclusion of negative instances in the teaching of mathematical concepts. However, reviews of research indicate negative instances may have a debilitating effect on learning, particularly on learning of simple concepts. This study investigated the learning of concepts where the concepts were of the more complicated type encountered in mathematics.

3. Research Design and Procedure

The concepts of commutativity (C) and associativity (A) were selected for this study. The four treatments each consisting of 20 instances of each concepts in a fixed but random order, were 1) C + A +, 2) C $\pm$ A +, 3) C + A $\pm$, and 4) C $\pm$ A $\pm$ (a "+" indicates all 20 instances were positive; a "$\pm$" indicates 10 positive instances and ten negative instances). Treatments were administered by computer terminal. Each treatment consisted of three sessions (approximately 25 minutes each): pretest, treatment, and posttest, respectively. Randomly selected ninth grade students

(N = 64) from one junior high school served as subjects. Mean verbal IQ at the school was 105. Subjects were assigned randomly to treatments (16/treatment) and all subjects completed the session within seven days (most within three days). Two pretests, calculations with parentheses and calculations without parentheses, were administered (reliabilities > .80). In addition, total stimulus interval (sum of lengths of time between typing of stimulus and entering of response for each instance) for each subject on each pretest was recorded. During the treatment session, total stimulus interval and total postfeedback interval (sum of lengths of time between the typing of feedback and the subject's hitting return key to receive next stimulus) for each subject on each concept were obtained. Two posttests, commutativity and associativity, were used (reliabilities approximately .5). Total stimulus intervals and total postfeedback intervals on the posttests were recorded for each subject. Multivariate and univariate analyses of variance and covariance were used to analyze the data. Analyses were done separately for each concept. Achievement variables were separated from time variables.

4. Findings

A univariate analysis of variance indicated treatment groups differed on the calculations with parentheses pretest ($p < .05$). No significant differences between treatments on posttest achievement on commutativity were found. Analyses of covariance indicated that inclusion of negative instances in the treatment for commutativity caused subjects to spend more time responding to items on the posttest for commutativity ($p < .05$).

The presence of negative instances for commutativity and associativity both increased performance on associativity at posttest ($p < .025$). This indicated a transfer effect for the treatment of negative instances from commutativity to associativity. The treatments including negative instances appeared to have increased the stimulus interval times.

5. Interpretations

For the acquisition of the concept of associativity, a sequence of positive and negative instances was favored over a sequence of all positive instances.

The acquisition of the concept of associativity was improved by inclusion of negative instances for commutativity. Transfer occurred.

Three possible explanations of the results are: 1) negative instances are a necessary and integral part of concept learning; 2) negative instances teach subjects to be skeptical; or 3) negative instances teach subjects the proportion of criterion instances that should be classified as negative using guessing. Nonetheless, classroom teachers should begin to experiment with the use of negative instances in instruction.

Abstractor's Notes

Efficient and complete learning of mathematical concepts is an extremely important problem. Shumway's study had several nice features: a good review of the literature; a complete description of the study, the experiment was controlled well; variables were defined carefully; questionable statistical procedures were noted and documentation for each step in the analyses was provided; alternative explanations for the results were given; and recommendations to classroom teachers were included. The interested reader should examine both the Final Report to HEW as well as the JRME article.

The low reliabilities of the posttest suggest the study should be replicated before the results are accepted. Negative instances increased achievement on only one of the concepts and tended to increase response time on both concepts. The educator is thus faced with the dilemma of whether the increase in achievement is worth the apparent instructional time loss. No clear guide exists as to which concepts might be better taught with negative instances or what the longer term effects might be. Transfer seemed to occur more strongly in the achievement variables than in the time variables. In the case of achievement, transfer seemed to occur in a rather special case, i.e., computations on associativity were dependent on computations on commutativity. Does transfer occur between more independent concepts? If negative instances help prevent students from over-generalizing, then the concepts need not be so closely related. Finally, the treatment period was quite short and administered by CAI. It is not known whether the results of the study can be replicated in the more usual classroom setting.

One last concern is of a more general nature, but may reflect only this reader's difficulty with the notion of concept learning. It appears that subjects could have responded correctly to the test items by computational means alone. To determine if an operation is commutative, one should rely on a more abstract form of reasoning than computation. For example, if $a*b = 2a + 2b$ the argument would be that $a*b = 2a + 2b = 2b + 2a = b*a$ and then $*$ is commutative. Has the subject who calculates $a*b$ and $b*a$ and then compares results learned the "concept" of commutativity? Future studies could include items on non-commutative operations which have examples making it appear the operation is commutative: e.g., $a*b$ = remainder when $a \div 2$, $7*13 = 1 = 13*7$, $4*28 = 0 = 28*4$ (but $a*b \neq b*a$. Secondly, although psychologists use a series of examples to examine concept formation experimentally, this is not a paradigm for learning. A study could approximate more closely the teaching/learning situation by preceding (or following) the series of examples with a brief discussion of the concept. Results may or may not differ from the present study, but would be much more applicable to the classroom.

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EJ 098 547 SE 511 255
ELEMENTARY TEACHERS LEARN TO SEQUENCE MATHEMATICS INSTRUCTION. Sowell, Evelyn, School Science and Mathematics, v74 n5, pp403-406, May-Jun. 74.
Descriptors--*Instruction, *Mathematics Education, *Research, *Teacher Education, *Teaching Techniques, Inservice Education, Preservice Education

Expanded Abstract and Analysis Prepared Especially for I.M.E. by James M. Sherrill, The University of British Columbia.

1. Purpose

The study was implemented to "design and test a self-instructional program for elementary teachers. Specifically, the program consists of materials to help teachers select and arrange sequences of behaviorally stated objectives."

2. Rationale

The question, "How did you decide what mathematics you would use with your pupils today?", was asked of elementary teachers and graduate mathematics education students and the most frequent answer was, "We did the next page in our math book." Sowell felt that the reply "seems almost incredible" due to the increased emphases in recent years on understanding mathematics and individualizing instruction. Although she feels textbooks are valuable resource materials, relying on them exclusively deprives children of manipulative learning experiences. She also states that a single source of mathematics information is inappropriate for most classrooms since the range of pupil abilities usually widens as the grade level advances.

3. Research Design and Procedure

Self-instructional materials were written to assist teachers in selecting and arranging objectives in sequences. The seven sections were bases for instructional sequences, prenumber operations, numeration, addition, subtraction, multiplication, and division. Each section featured an overview, general goal statements, flow charts containing objectives, explanations (definitions and illustrations of objectives), and self-checking assessments.

A forty item multiple-choice test was developed to assess teachers' abilities to select objectives which are prerequisite to a given terminal objective and to place them in correct sequence. The test, like the principles upon which objectives and sequences were based, was validated by a group of mathematics education persons. Equivalent forms of the test were prepared and the instrument was administered to forty-five pre-service teachers resulting in a Kuder Richardson formula 21 coefficient of internal consistency of 0.887.

Materials were used both with pre- and in-service teachers. The sample of pre-service teachers included two sections of elementary majors who were enrolled in a mathematics methods class. By toss of a coin one group of 31 students became the experimental group (E_p) while the second group of 30 students was the control group (C_p).

No claim for a random sample is made for the in-service teachers. Twenty-six teachers volunteered as the experimental group (E_i) and twenty teachers from the same school district volunteered as controls (C_i). Since all of these teachers were certified, they have credit for at least one course in mathematics education.

The pre-tests were administered to experimental and control groups on the same day. Then control groups continued their regular activities while experimental groups studied the instructional materials. During nine clock hours scheduled for this study, participants worked independently or in groups of their choosing. The investigator was present as an observer at all sessions. Following the study, all subjects took a post-test.

The means of the post-test scores for E_p and C_p were compared using analysis of covariance with pre-test scores serving as the covariate. The same analysis was implemented for the two in-service groups.

4. Findings

The experimental group scored significantly higher than the control group for both the pre-service and the in-service subjects.

5. Interpretations

The materials seemed to achieve the intended purpose, namely, the subjects that used the self-instructional program were assessed as having better "abilities to select objectives which are prerequisite to a given terminal objective and to place them in correct sequence."

Abstractor's Notes

Sowell justifiably states that, "No claim for a random sample is made for the in-service teachers." The in-service groups were all volunteers, which is not unusual for this type of study using in class teachers. In Sowell's study (if the article statement is accurate) the teachers didn't just volunteer to be in the study, but volunteered for the particular group they would be in with 26 in the experimental group and 20 in the control group. The experimental subjects were allowed to work "independently or in groups of their choosing." Questions of randomness of the sample and independence of the data arise concerning the statistics used to compare the post-test means.

As is true of many research articles, no examples from the self-instructional program or the tests are given. Examples could have shed some light on the test, especially when the teachers were asked to arrange the objectives to match some pre-determined correct sequence.

Also the post-test seems to have been a test on how well the subjects learned the material in the self-instructional program, a program the control group never saw and the experimental group studied for nine hours.

The study did, however, successfully design a self-instructional program which assists teachers to learn to sequence stated objectives. A very important future study is suggested by Sowell, ". . . there should be a follow-up study to find out whether teachers who learned to sequence instructional activities for pupils will indeed do so with their pupils."

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EJ 077 051 SE 508 630
STUDENT EVALUATION OF MATHEMATICS INSTRUCTION. Waits, Bert K.; Elbrink, Larry C., Two Year College Mathematics Journal, v4 n2, pp59-66, Spr 73.
Descriptors--*College Mathematics, *Course Evaluation, *Evaluation, *Research, *Teacher Evaluation, Instruction, Mathematics Education

Expanded Abstract and Analysis Prepared Especially for I.M.E. by Phillip S. Jones, University of Michigan.

1. Purpose

To investigate whether students' evaluations of instructors in a college calculus course are biased by the students' achievement in the course.

2. Rationale

There appears to be a growing concern for the evaluation of teaching. Student ratings of teachers are being used to provide data supporting hiring and promotion, to provide feedback to teachers on the basis of which they may improve their work, and to help students in course selection. With such fundamental uses being made of this data, it is important to examine its meaning, validity and implications. The authors cite a survey of the literature of teacher evaluation by Frances Dwyer as characterizing evaluations as subjective and subject to many inherent limitations. They chose to study the extent to which students' achievement in mathematics may bias their evaluation of their mathematics teachers. The authors felt that this biasing effect may be greater in mathematics than in other fields because many students view required mathematics courses as unpleasant, uninteresting, and difficult.

3. Research Design and Procedure

A SET (Student Evaluation of Teaching) questionnaire developed by Dr. Robert W. Ullman of Ohio State University was administered to all students in the sixteen sections of the second course in calculus with economic applications for non-mathematics, non-physical science majors. The questionnaire contained 48 questions divided into three categories--course, instructor, examination. Approximately half the questions were phrased positively and about half were phrased negatively. The students selected one of strongly agree, agree, disagree, or strongly disagree for each question. These replies were scaled as 3, 2, 1, 0, in that order, for positive questions and in reverse order for negative questions. In addition to using these factors to compute a rating for each student in each category, the average midterm score on the departmental examinations was determined for each student.

Three null hypotheses were then tested for each section of the course. These were the correlation between achievement, defined as average midterm examination score, and the course rating, the teacher rating, and the examination rating would each, in turn, be zero. A second set of three null hypotheses were formulated and tested by classifying the achievement scores into five groups, using the classification as the independent variable and the three rating scores in turn as dependent variables, and applying one-way analysis of variance. The hypotheses were that there was no effect of achievement rank on the student ratings. The second procedure was applied to test the possibility that there might exist non-linear relationships which would not be revealed by correlations computations.

4. Findings

The hypothesis that the correlation of achievement with course rating would be zero was rejected for seven of the sixteen sections, but the null hypothesis was rejected for only one section in each case when teacher and examination ratings were compared with achievement.

When the second set of hypotheses were tested using grouped achievement scores rather than raw scores the null hypothesis with respect to course rating, was rejected for only two sections, and the null hypothesis with respect to examination and with respect to teacher ratings was rejected in one section for each. No significant relations not implied by the correlation analysis were implied by the analysis of variance.

5. Interpretations

The writers concluded that some teachers can expect to receive student evaluations unbiased by student achievement, but that other teachers will receive biased ratings on the basis of the fact that although the correlation between achievement and teacher rating was significant at the .05 level in only one section, it was positive and near significance in eight others.

They noted that evaluations of courses and examinations correlated significantly with achievement (good students giving favorable ratings) in about one half the sections. The writers also note that their subjective evaluation of their colleagues would lead to the same ranking as the SET with a few notable exceptions. They concluded with words of warning: they could not recommend any evaluation instrument, administrators should encourage student evaluations only after careful consideration of their psychological and local effects including the possibility that teachers might teach for good ratings.

Abstractor's Notes

The writers' concluding concern for the possible deleterious effect of SET's on teaching is supported by Leon W. Zelby who reported an experiment in teaching paired classes by different methods as reported in Science, volume 183 (29 March 1974) pp. 1267-1270. He felt that he could manipulate his SFE (SET) score, improving it by using procedures which had a greater appeal to students (especially to the less able students), but which were intrinsically less desirable educationally.

The writers' statistical result that there is little correlation between grades and student ratings is supported by other studies as is also their feeling revealed by their phrases, "some teachers might expect to receive unbiased evaluation ratings -- the ratings of some teachers will be biased--". In the article by J. A. Kulik and W. J. Keachie, "The Evaluation of Teachers in Higher Education" prepared for Review of Research in Education, Vol. 3 (1975) edited by F. N. Kerlinger, a 1928 study by Remmers, a 1950 study by Elliott and a 1971 publication by Castin, Greenough, and Menge are cited, all of which found overall correlation between grades and ratings to be very low. However, somewhat deeper analyses tend to show that instructors tend to differ, by choice or nature, in the level of student with whom they are most effective. Hence some instructors may tend to receive better ratings from good students and other instructors may be highly rated by poorer students. This may explain the marked difference in the behavior of one or two of the sections in the study by Waits and Elbrink.

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