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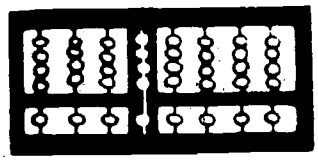
ABSTRACT

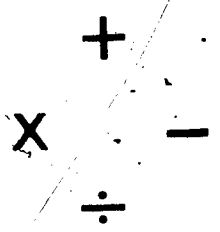
This guide describes activities and materials which can be used in a mathematics laboratory approach for a basic mathematics program for grades 1-6. Twenty-nine activities relate to operations with whole numbers and twenty-five activities pertain to operations with fractions. These activities are described in terms of purpose, suggested grade levels, materials needed, and procedures. Some specific concepts presented are: place value, order of operations, equalities and inequalities, whole number operations, patterns, time measure, number facts, number sentences, ratio, applications, fractional parts and operations, geometric shapes, area, division problem solving, weighing, scale, linear measure, and equivalent fractions. The guide contains a categorical listing of materials such as improvised materials and games, commercial materials and games, general supplies, and other manipulative materials. (JBW)

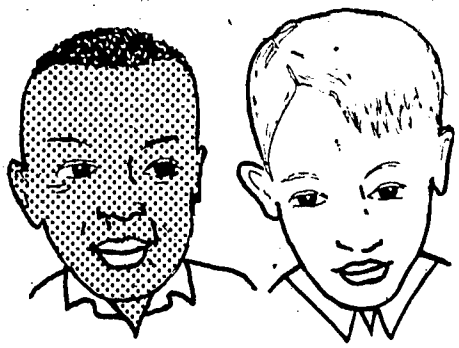
SUGGESTIONS FOR TEACHING MATHEMATICS USING LABORATORY APPROACHES

GRADES 1-6

EXPERIMENTAL EDITION







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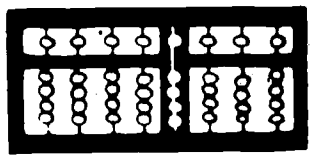

2. OPERATIONS

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THE STATE EDUCATION DEPARTMENT
Bureau of Elementary Curriculum Development
Albany, New York 12224

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GRADES 1-6

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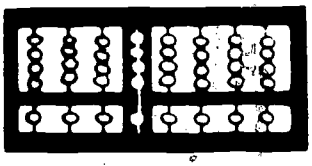

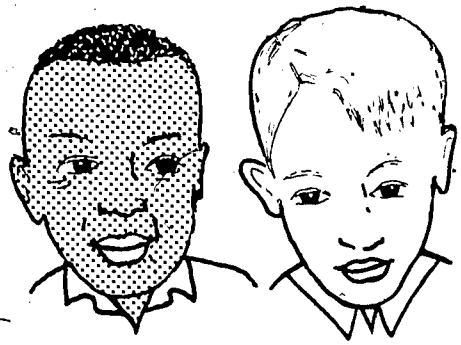

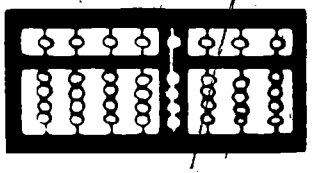

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3 > 1

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PREFACE

Substantial financial aid to local educational agencies for children of low income families was provided by the Elementary and Secondary Education Art of 1965. Participating school districts have developed a variety of new educational programs to assist children with special educational needs. These programs are based upon local needs assessment with major parental involvement.

The three major priorities for compensatory education programs are bilingual education, reading, and mathematics. One of the highest priority programs under ESEA Title I is the subject matter area of mathematics. Experience has shown that children who have experienced difficulty learning in a traditional program often react with enthusiasm to mathematics laboratory approach. This publication was developed to provide practical applications of this mathematics approach for use by classroom teachers. It should provide practical suggestions for teachers working directly with educationally disadvantaged children.

Irving Ratchick
Assistant Commissioner for
Compensatory Education

FOREWORD

The Bureau of Elementary Curriculum Development and Bureau of Mathematics Education in cooperation with the Division of Education for the Disadvantaged, ESEA Title I, have developed a variety of materials on the use of a Mathematics Laboratory approach on the elementary level. This joint effort has resulted in the release of two publications:

Teaching Elementary Mathematics Using Laboratory Approaches, which serves as a short introduction to the method

and

ESEA Title I, Anatomy of An Elementary Project, which gives a concrete example of the use of a Mathematics Laboratory approach with disadvantaged children.

Encouraged by the response of teachers and administrators to the original publications, a decision was made to move further in the direction of providing concrete activities for teachers who wished to move into the humanistic approach inherent in a Mathematics Laboratory program.

Fredric Paul of the Bureau of Mathematics Education and Peter A. Martin of the Bureau of Elementary Curriculum Development began the task of developing activities for teacher use. A committee of experienced teachers consisting of Claire Cohn, Helen Feder, and Pasquale Toscano, under the direction of Elaine Mintz, was engaged as a writing team. Mrs. Mintz is director of elementary mathematics and the other three are teachers in the plainedge School District. The material completed by this team was then sent out for field testing to 11 schools throughout the State for use with children. As a result of a favorable reaction on the part of the teachers who used this material, we have produced

experimental materials for use by school districts.

This publication is the second of four which are being developed for teacher use. Each will incorporate the latest thinking of the mathematics revision committee and may be utilized with any basic mathematics program. This publication is designed to serve as a stimulant to encourage teachers to open their minds and employ their imaginations in developing further activities. The classroom teacher in developing her own set of "task cards" will adjust vocabulary and choose concrete materials in terms of a close knowledge of the ability levels of her own children and the type and amount of manipulative materials available. Suggestions and reactions are welcome, and should be sent to Fredric Paul, Bureau of Mathematics Education, State Education Department, Albany, New York 12224.

Peter A. Martin of the Bureau of Elementary Curriculum, did the final editing and prepared the material for publication.

Robert H. Johnstone
Chief, Bureau of Elementary
Curriculum Development

Gordon E. Van Hooft
Director, Division of Curriculum Development

OPERATIONS

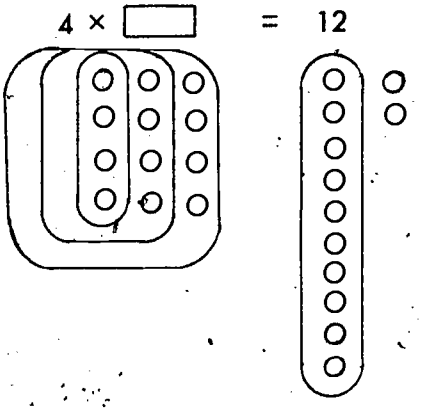
WHOLE NUMBERS

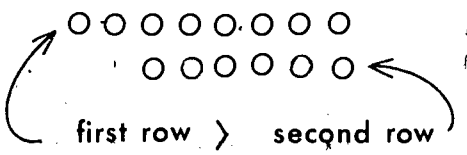
2-1 Open Sesame Sentences. Purpose: Whole number operations, place value, sets

Suggested Grade Level: 3-6

Materials needed: Counters

Procedure: Pupils devise open number sentences. They arrange counters to correspond to these sentences and assist them in finding solutions.





$8 - 2 = 6$

$4 \times 2 = 3 \times 2 + 2$

$8 \div 4 = 6 \div 3$

Counters may be arranged in two rows. Comparisons of the rows are made using number sentences involving all four basic operations.

2-2 The Power of Balance. Purpose: Whole number operations, order of operations

Suggested Grade Level: 3-5

Materials needed: Mathematical balance

Procedure: This activity can be played as a game using teams. First team to find the answer scores the point. The game can be timed giving extra points for speed (to be determined beforehand). Prepare (or have children prepare) many puzzle sentences as follows:

$(4 \times \square) + 7 = 19$ $\quad 7 + (3 \times \square) = 10$

$(2 \times \square) + 4 + 3 = 4 + (3 \times 3) + (2 \times 2)$, etc

Answer must be "proved" on the balance.

2-3 Commute. Purpose: Multiplication, commutative (order) principle
Suggested Grade Level 3-5

Materials needed: Mathematical balance

Procedure: Using the balance, children show that sentences such as the following are true: $5 \times 4 = 4 \times 5$
$7 \times 3 = 3 \times 7$.

2-4 Distribution. Purpose: Multiplication facts, distributive principle
Suggested Grade Level 4,5

Materials needed: Mathematical balance

Procedure: By manipulating weights on the pegs children can verify the distributive principle in several ways. Example: Four weights on "9" will balance: 4 weights on 1 and 4 weights on 8; 4 weights on 2 and 4 weights on 7; 4 weights on 6; 4 wieghts on 4 and 4 weights on 5, etc.

2-5 Statements. Purpose: Equalities and inequalities
Suggested Grade Level 3,4

Materials needed: Mathematical balance

Procedure: Have students work in pairs. Use the balance to help form statements of equality. For example: $8 + 4 = 9 + 3$. Now do the same for inequalities. Write at least ten inequalities. Have your partner check your example, such as: $6 + 5 \neq 4 + 8$.

Now, try with three numbers on each side. $9 + 2 + 3 = 5 + 8 + 1$. Then, try some like this: $2 + 4 + 5 = 9 + 2$.

Complete the following number sentences by finding the missing addend. Use the balance to check your answers.

$6 + \square = 14$

$\square + 11 = 27$

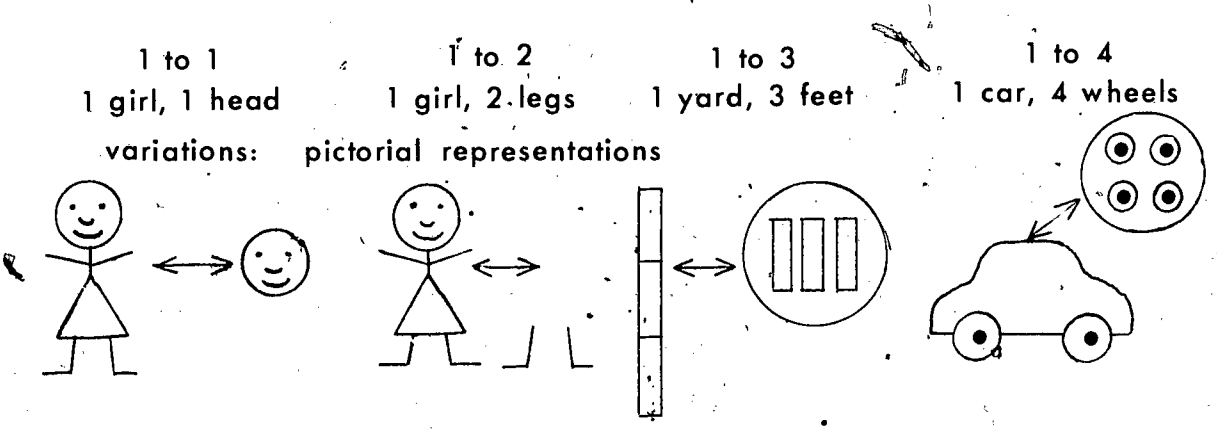
Children can play a game using the balance to check addition and subtraction sentences that they create and challenge each other to complete.

2-6 Set Pairing. Purpose: Set pairing, one to one correspondence, one to many correspondence, ratio readiness

Suggested Grade Level 3, 4

Materials needed: Large tag board chart

Procedure: Prepare a bulletin board with as many examples as the children provide to illustrate correspondence relations.



2-7 See It Happening. Purpose: Whole number operations, place value.

Suggested Grade Level 3-6

Materials needed: SEE calculator

Procedure: Children manipulate the four discs on the calculator. Each disc corresponds to a place value column. Multiplication and division are performed by repeated addition and subtraction respectively. Proper place value exchange is automatic and an intriguing operation to watch. Start anywhere. Add 1, 9, 10, etc. What happens? Show it with an equation. Most youngsters find this "gimmick" fascinating.

2-8 Math Bingo Purpose: Whole number addition, subtraction, multiplication, division

Suggested Grade Level 3, 4

Materials needed: Prepared "Bingo" cards, Paper markers, fact cards

Procedure: Bingo cards can be run off on ditto paper. Depending on objective of the lesson children can fill in numbers with teacher's direction, i.e. Under the B fill in any five numbers from 1-20, etc. Caller calls off number fact such as:

6 x 3
4 x 2
5 x 4

The winner becomes the next caller.

2-9 Regrouping. Purpose: Whole number addition and subtraction with regrouping

Suggested Grade Level 2,3

Materials needed: Squared materials, Dienes M.A.B.

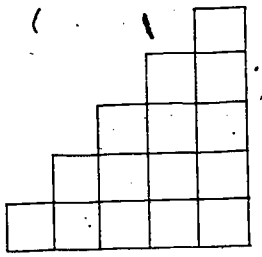
Procedure: Provide many addition and subtraction examples to be solved using squared materials as: select 3 tens and 4 ones. Take away 17. Children should discover necessity of exchanging a ten strip for ten ones and a hundred square for ten tens.

2-10 Staircase. Purpose: Whole number addition, patterns, geometric shapes, counting

Suggested Grade Level 3,4

Materials needed: Cubes or graph paper

Procedure: Build a staircase with cubes that go up to five steps. How many cubes did you use? How many would you need to make a staircase with 6 steps? 10 steps? Experiment with stairs that go up 2, 3, or more blocks at a time.



2-11 True Sentence. Purpose: Whole number operations, time measure

Suggested Grade Level 3, 4

Materials needed: Tuf Game or die with numerals and operation symbols

Procedure: A timed game. Player spills a set of cubes whose faces bear numerals or operational symbols. The player's object is to form a true mathematical sentence using a maximum number of cubes. The number of dice used can vary with student ability.

2-13 Rod Trains Purpose: Whole number addition concept.

Suggested Grade Level 2-4

Materials needed: Cuisenaire Rods or similar materials

Procedure: Introductory activity - Make "Trains" by placing rods in a row, end to end. Write equations by color w + w = r, etc. During the initial lessons with the rods, do not suggest assigning number values to a specific rods. Children may, however, do this on their own.

2-13 Rod Patterning. Purpose: Whole number addition, subtraction, factoring

Suggested Grade Level: 3-5

Materials needed: Cuisenaire Rods or similar materials

Procedure: A rod is selected. Children are challenged to find all combinations of smaller rods which, when placed in a row (this operation is called "making a train"), match the length of the chosen rod. When all combinations have been found, have children take turns hiding one arrangement or a partial one. Another player tries to name the missing rods.

2-14 Rod Check. Purpose: Practice on number facts, solving equations

Suggested Grade Level: 4, 5

Materials needed: Cuisenaire Rods or similar materials

Procedure: Prepare dittos with an assortment of equations. Students use rods to verify their answers. For this activity assign the number one to the white rod.

$9 + \square = 14$

$(2 + 4) + \square = 12$

$15 - (\square \times 8) = 7$

$14 = (3/4 \times \square) + 5$

$\square - (3 \times 4) = 3$

$12 - (1/2 \times 6) = \square$

2-15 What's Missing? Purpose: Whole number facts, number sentences, equalities and inequalities

Suggested Grade Level: 3, 4

Materials needed: Prepare many different number puzzles, as:

$\_\_\_ + \_\_\_ + 6 = 23$

$5 - \_\_\_ + 9 = 10$

$4 \_\_\_ 8 - 3 = 9$

$3 \_\_\_ 4 + 5 = 17$

Procedure: Fill in the missing numbers or the operation symbol to make the sentences true. Can you find more than one solution?

2-16 Face Up. Purpose: Whole number facts, number sentences
Suggested Grade Level: 3, 4

Materials needed: Set of 12 cardboard square inches

Procedure: Prepare the tiles as follows: numbers 1-5, 2 tiles each, the number written only on one side of the tile. Two tiles marked "free" to be used for any number 1-5 as the player wishes. Several children can take part. First player spills the tiles. Any tile that lands face up may be used. First player must compose number sentence equal to one, second player, sentence equal to two, and so on. Any of the four basic operations can be used. Scoring: one point for each tile used. Example: Fourth player must express four. Tiles showing: 4, 4, 5, 1, free; 4 + 4 - 5 + 1, four points; 5 - 1 two points; 5 + 1 - 4 + 4 - (free), 5 points.

2-17 Score with Five. Purpose: Whole number operation, replacement sets, betweenness

Suggested Grade Level 3, 4

Materials needed: Dice, numerals 1-6

Procedure: Two children play using one die each. First child rolls die, makes equation using the five visible numerals. They must use all five numerals once only and any operation (s) he wants to combine them. Let the solution lie between 30 and 50. Scoring corresponds to the solution. Time limits may be set for calculations. After five rounds, high score wins. Examples: 2, 3, 4, 5, 6, show -

(a) $(2 \times 3 \times 4) + 6 + 5 = 35$ - score 35;
(b) $(5 \times 6) + (4 \times 3) + 2 = 44$ - score 44;
(c) $2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 = 720$- score 0.

2-18 Find The Ways. Purpose: Division through repeated subtraction, commutative property, sets, factors
Suggested Grade Level: 3, 4

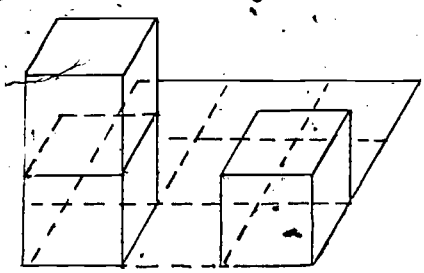
Materials needed: Counters

Procedure: Have a pair of children work with 100 counters. Have them discover the seven possible ways to separate the counters into equal sets (two 50's, ten 10's...). If the children use uniform stacking counters, (cubes, Unifix materials, abacus discs, etc.) then the sets can take the shape of three dimensional arrays.

2-19 Three-D Tic Tac Toe. Purpose: Multiplication by 3, 4, 5.
Suggested Grade Level 3 - 6

Materials needed: 1 inch cubes or Unifix cubes (2 colors - 1 color for each player), grids

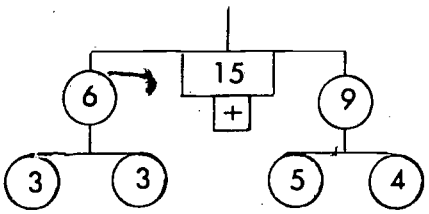
Procedure: Two pupils take alternate turns stacking one cube at a turn on a 3" x 3" grid base. To win, a player has to place three of his cubes in a straight row - horizontally, vertically or diagonally. Variation: On a 4" x 4" grid base, the game ends when 4 cubes are lined up. Similarly, a 5 x 5 grid base would require 5 cubes in a row for scoring. Problem: How many different ways can the cubes be placed for a winning score on the 3 x 3 grid? on the 4 x 4 grid? on the 5 x 5 grid?

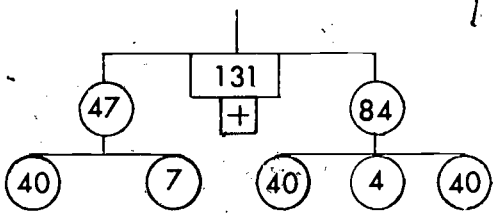


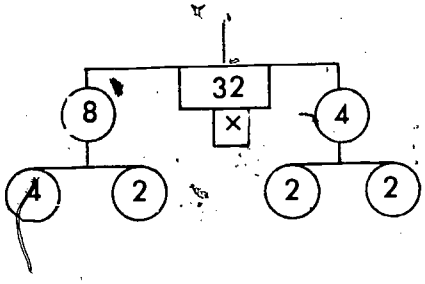
2-20 Mobiles. Purpose: Whole number operations
Suggested Grade Level 3, 4

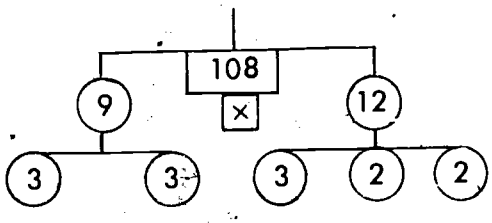
Materials needed: Paper shapes, wire coat hanger, string

Procedure: Make math mobiles for display. These can be used for addition, subtraction, multiplication, division. Examples:









2-21 Switcheroo. Purpose: Whole number operations, commutative associative, distributive principles
Suggested Grade Level: 3-6

Materials needed: Counters

Procedure: Make a rectangular array with a convenient set of counters. How many different true number sentences can you write about the array? Use the symbols =, +, -, x, ÷ Example:

0 0 0 0 0 0

3 x 6 = 18	6 + 6 + 6 = 18
6 x 3 = 18	18 - 6 = 12
(6 x 2) + (6 x 1) = 18	6 + (9 + 3) = (6 + 9) + 3
(3 x 3) + (3 x 3) = 18	18 ÷ 3 = 6 etc.
(4 x 3) + (2 x 3) = 18	

For grades 5 and 6, rearrange counters to reflect place value with number bases other than 10.

2-22 Twenty Questions. Purpose: Whole number operations.
Suggested Grade Level: 3, 4

Materials needed: SEE Calculator (optional)

Procedure: Have children write 20 number sentences for a given number. Can be played as a game with teams. Example: number 24

20 + 4, 21 + 3	addition
10 + 10 + 4	grouping by tens
24 + 0 or 24 - 0	0 property
25 - 1	subtraction
8 + 8 + 8	repeated addition
8 x 3	multiplication

2-23 Catalog Game. Purpose: Numeration, basic arithmetic operations, practical application
Suggested Grade Level: 5, 6

Materials needed: catalog, calculator, timer

Procedure: Each student is "given" $100 to spend. He must list the items of his choice and their exact prices. If his total is above $100 he loses all. Winner is the first student who reaches exactly $100 or is closest to it. Minimum 5 items, no doubles. A reasonable time limit should be set for this kind of investigation.
Variation: Collect menus from local restaurants. Follow comparable procedures as above. Advanced students may be required to include tax and shipping charges in their $100.

2-24 Lattice Multiplication. Purpose: Operations - whole numbers, place value, alternate algorithm

Suggested Grade Level: 4-6

Materials needed: Napier's Rods, reference books

Procedure: Have students study the following multiplication problem to see whether they can find out how it was done.

```
  673
x 824
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554552
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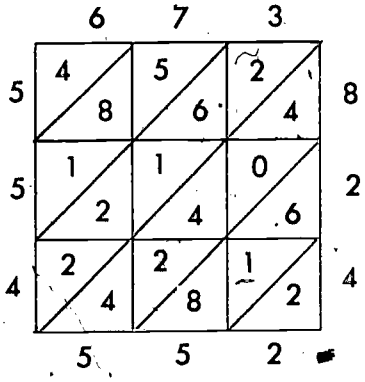


Have pupils research Napier's Rods (Bones). Have them make their own sets of Napier Bones.

2-25 Weight Relativity. Purpose: Whole number and fractional operations, ratio, large numbers, weighing, graphing

Suggested Grade Level: 5, 6

Materials needed: Auto booklets, scale, postal rate schedule

Procedure: Weigh yourself and determine how many persons of equal weight it would take to balance a Volkswagen, Cadillac, other kinds of cars. Graph your results. How much would it cost to mail yourself to a friend in California?

2-26 Chow Time. Purpose: Multiplication, ratio, cost calculation

Suggested Grade Level: 5, 6

Materials needed: Cook book

Procedure: Take a standard recipe for pancakes. How many servings does it make? Make a list of ingredients you would need for serving pancakes if you were the cook at a summer camp with 150 people. Prepare a dinner menu for the same people. What foods would you need? How much would the meal cost?

2-27 Four 4's. Purpose: Whole number operations, order of operations
Suggested Grade Level: 5, 6

Materials needed: None

Procedure: Express the numbers one through ten by combining four 4's with any mathematical operation. Some suggestions are shown below. There are many other solutions.

$1 = \frac{44}{44} = 4^4 \div 4^4$

$2 = \frac{4}{4} + \frac{4}{4} = \frac{4 \times 4}{4 + 4}$

$3 = \frac{4 \times 4 - 4}{4}$

$4 = \frac{4 - 4 + 4}{4}$

$5 = \frac{4 \times 4 + 4}{4}$

$6 = \frac{4 + 4 + 4}{4}$

$7 = \frac{44}{4} - 4 = 4 + 4 - \frac{4}{4}$

$8 = 4 \times 4 - 4 - 4 = 4 + 4 + 4 - 4$

$9 = 4 + 4 + \frac{4}{4}$

$10 = \frac{44 - 4}{4}$

Encourage pupils to devise variations of this technique.

2-28 What Does Our Money Buy? Purpose: Multiplication, division, whole number fractions, recording, standard units, practical applications, weight, monetary values
Suggested Grade Level: 4-6

Materials needed: None

Procedure: Look at the food on the pantry shelf at home. Make a list of things that you find whose labels tell weight in ounces. Find the total price of these food items. How much do they cost per ounce? Variation: Plan an excursion to a local supermarket to obtain data for this project.

2-29 Mechanical Math. Purpose: Whole number operations.
Suggested Grade Level: 3-6

Materials needed: Desk calculator

Procedure: Allow plenty of time for free play with calculator before beginning more structured activities. Practice with different number lists, grocer, arithmetic examples, etc. Teach flow-charting.

FRACTIONS

2-30 Folding. Purpose: Fractional parts of an object
Suggested Grade Level: 3, 4

Materials needed: Paper squares

Procedure: Fold a square in half. Can you fold it in half another way? Record how many halves makes a whole. Repeat activity for fourths, eighths.

2-31. Cut it Out. Purpose: Fractional parts

Suggested Grade Level: 3, 4

Materials needed: Paper, compass, scissors

Procedure: Have children draw a large circle with the compass. The children can then figure out how to cut the circle in half. Fourths, eighths. Compare pieces, discuss how many times will a smaller piece fit on a larger piece. Mount pieces in a suitable arrangement. Have children record observations.

2 32 Piecemeal. Purpose: Fractional operations and concepts, geometry
Suggested Grade Level: 4, 5

Materials needed: Paper, compass, scissors

Procedure: Have pupils draw circles with their compasses and cut them out. Fold and cut one circle into halves; another into quarters, etc., up to sixteenths. Have pupils ask their friends to guess what part of the circle each piece represents. The pupil who gives a correct answer asks the next question.

2-33 Parts. Purpose: Fractional part of a set
Suggested Grade Level: 3

Materials needed: Attribute Materials, Blocks, People Pieces, Creature Cards

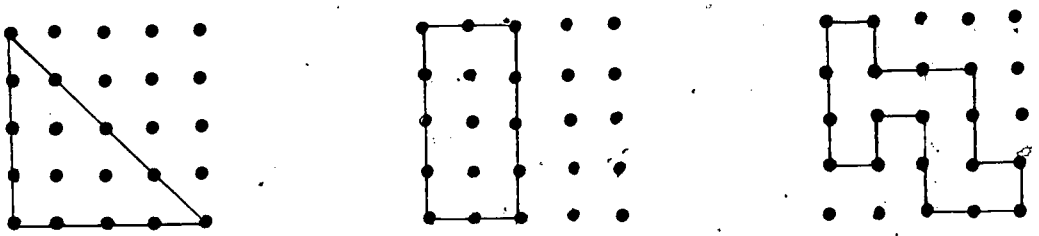
Procedure: Make a set of 4 rounds and 4 squares. What part of the set is round? Vary activity by using color and size discrimination. Creature cards and People Pieces may be used in a similar manner. As enrichment, children may design their own Creature Cards for comparable activities.

2-34 Partways. Purpose: Fractional parts, geometric shapes, area, units of measure

Suggested Grade Level: 3-6

Materials needed: Geoboard, rubberbands

Procedure: Determine a shape on the geoboard that is to be used as a unit of measure. The simplest initial explorations will probably be made by setting the largest possible square as the unit measure. Ask pupils to make a design that shows half. Also, find ways to show 1/4, 1/8, and other fractions.



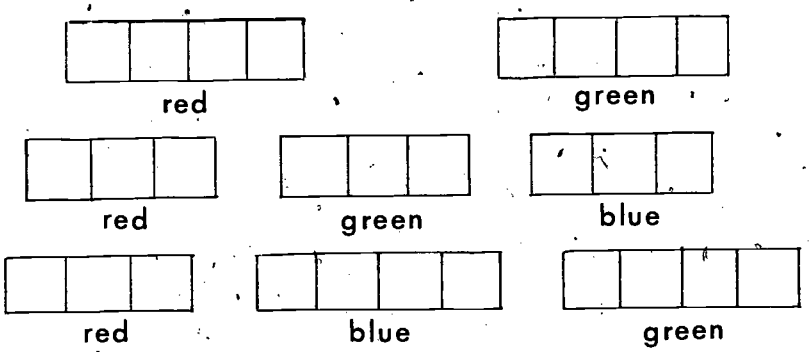
GEOBOARDS

2-35 Color Parts. Purpose: Fractional part of a set

Suggested Grade Level: 3

Materials needed: Color cubes

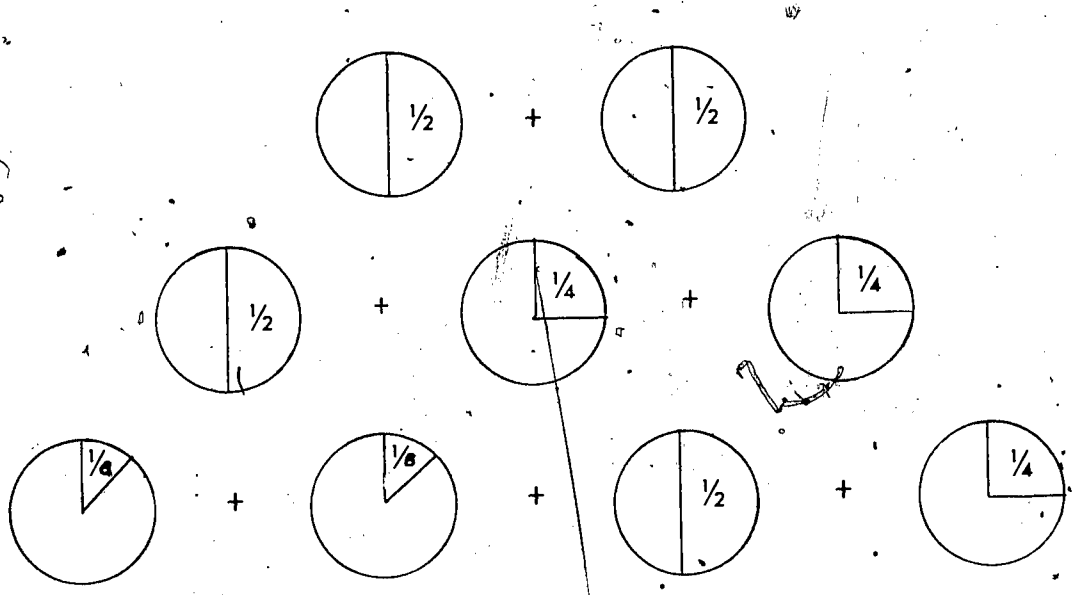
Procedure: Set out 4 red cubes and 4 green cubes. What part of the set is red? etc. Set out 3 red, 3 green, 3 blue cubes. What part of the set is green? blue? etc. Vary the number of set elements to suit the needs of the lesson.



2-36 Make One Game. Purpose: Equivalent fractions, fractional addition
Suggested Grade Level: 3-6

Materials needed: Index cards with fractional values or "Make One Game"

Procedure: A game for 2, 3, or 4 children. Object is to make one, i.e. 1/2 + 1/2, 1/2 + 1/4 + 1/4, 1/8 + 1/8 + 1/2 + 1/4. When all cards are used, child with most sets whose total is one is the winner. Decimal and per cent equivalencies can be used by the older children.



2-37 Parting News. Purpose: Fraction recognition
Suggested Grade Level: 3 - 5

Materials needed: Newspapers or advertising circulars, marking pens

Procedure: At a signal, all pupils look through the newspaper or circular and circle all fractions. Decide on appropriate time period. At the end of the set time, person who has found most fractions wins. May be played as team game.

2-38 Iffy. Purpose: Fractional parts of a whole
Suggested Grade Level: 4, 5

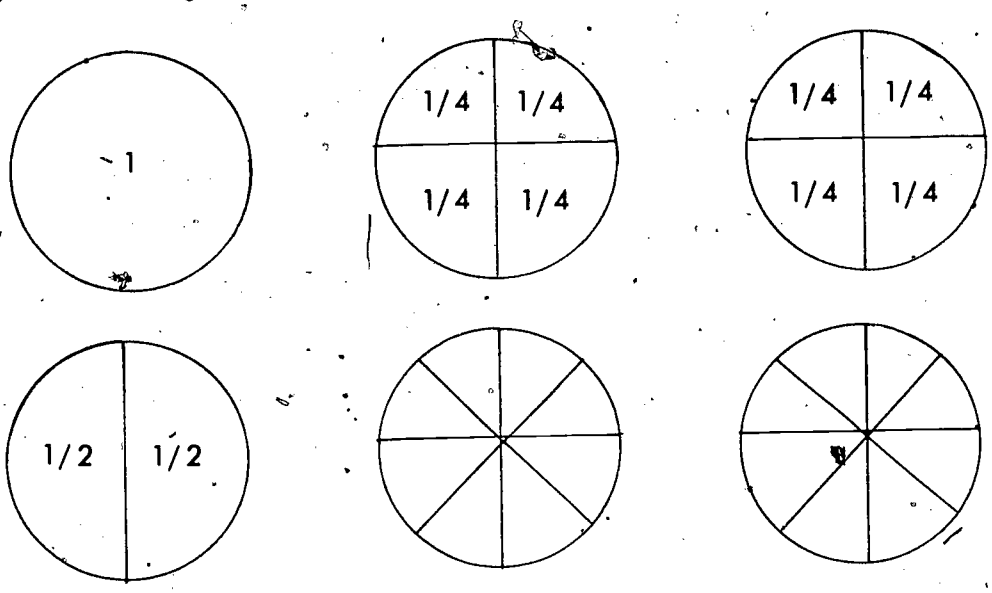
Materials needed: Cuisenaire Rods or similar materials

Procedure: Prepare ditto sheets as follows: If white is 1: Red is, purple is, etc. If red is 1: White is, purple is, etc. Continue for all colors. Many children will quickly discover a pattern. Discuss results and verify them with the rods.

2-39 Make-A-Pie. Purpose: Fractional concepts and operations
Suggested Grade Level: 4 - 6

Materials needed: Uniform sized circles, scissors

Procedure: Teacher should prepare a spiritmaster with 6 circles, each 3 inches in diameter. Directions: Pupils cut out circles; fold them, and label as shown in the diagrams. The circles should then be cut along the folding lines.



Pupils may use the pieces to answer questions such as: a) How many halves (quarters, eighths) make a whole? b) How many quarters make a half, etc. Pupils play in pairs. The challenger sets out several pie pieces and asks his opponent a) How many complete pies can be assembled? b) Write a number sentence to prove his answer in Part a is correct. Example: 7/4 + 1/8 = 15/8 = 1-7/8 Pupils take turns as challenger and responder. Extend the game to other fractional parts. Variation: One pupil picks several pieces and challenges his opponent to find other pieces to complete the pie.

2-40 Boned Down. Purpose: Division problem solving, fractions, checking decimal notation
Suggested Grade Level: 5, 6

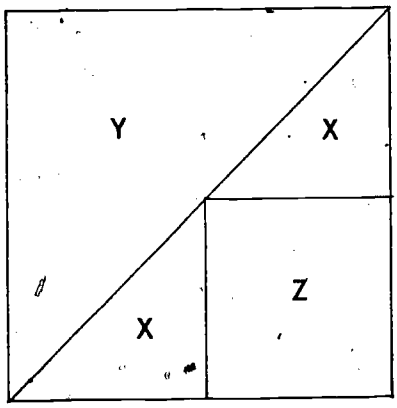
Materials needed: Balance scale, weights, containers, ruler, paper, spools, assorted string, thread, yarn, rope, ribbon, micrometer

Procedure: Find the weight of a single pin, thumb tack, or paper clip. Use at least two different methods to get and to check your answer. Find the thickness of a sheet of paper and a piece of string. Use at least two different methods to get and to check your answers.

2-41 Tiles. Purpose: Congruence, Fractional parts
Suggested Grade Level: 5, 6

Materials needed: Paper, scissors or Pattern Blocks

Procedure: Cut 2 congruent squares. Cut 1 square into pieces as shown on the diagram. If X, Y and square Z were tiles, how many X's would you need to cover the original square? How many Y's? How many square 's? "X" equals what fractional part of whole? "Y" equals what fractional part of whole? "Z" equals what fractional part of whole?



Students can make variations using squares, rectangles, circles, etc., as the whole.

2-42 Clay Ball. Purpose: Comparison, weighing, fractional operations, and concepts

Suggested Grade Level: 4 - 6

Materials needed: Balance, clay, set of weights

Procedure: Have student make 4 clay balls each about the size of a golf ball. Check that they each weigh the same amount. Leave one ball whole. Cut each of the others into halves, quarters and eights respectively. Check each subdivision with a balance. In terms of these clay weights, find weight of objects in room such as a book, eraser, etc. Record answers. Example: 1 book = 1 ball + 1/2 ball + 3/8 balls;
1 book = 1 7/8 balls

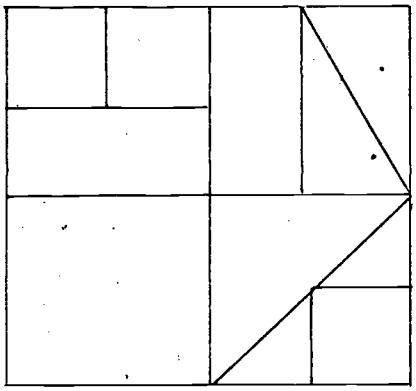
Note: If available, use standard set of weights in addition to clay.

2-43 Segmented Squares. Purpose: Fractional Parts, area
Suggested Grade Level: 3-5

Materials needed: Paper segmented squares, scissors

Procedure: Give pupils a paper square with different regional shapes. (Example: Diagram a) Ask them to express each region as a fractional part of the whole square. After they mark their fractional parts, they can cut out the regions and check their work. They can also use the pieces for tesselation activities.

A)



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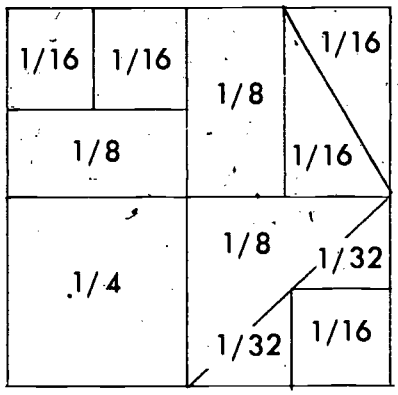


2-44 Average Weight. Purpose: Subtraction, division, weight, fractions, estimation, averaging
Suggested Grade Level: 5, 6

Materials needed: Scale, ruler, congruent cards, micrometer

Procedure: Try to find the weight of a card. Report on what you did and what materials you used. Compare the answers your friends got with yours. Find the average of all the answers. Note: Teacher can have "score board" of results of this problem on bulletin board along with other problems that may have varying answers.

2-45 Illusions. Purpose: Circular and linear measure, estimation, ratio, whole number operations, fractional parts
Suggested Grade Level: 3-6

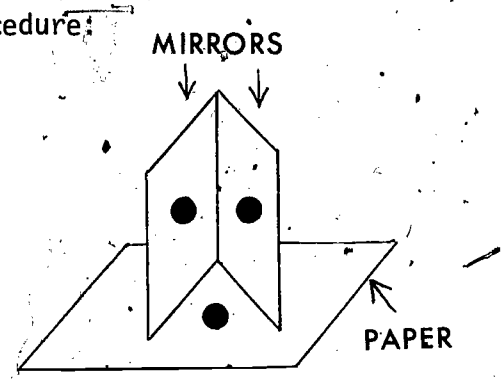
Materials needed: Assorted drinking glasses, ruler, string

Procedure: Bring 6 different sized drinking glasses or cups to your work table. Guess how the circumference at the top of each glass corresponds with with its height. Record your guesses. Measure the glasses and compare your results with your guesses. How good were your guesses?

2-46 Kaleidoscope. Purpose: Fractional parts, equivalent fractions
Suggested Grade Level: 4-6

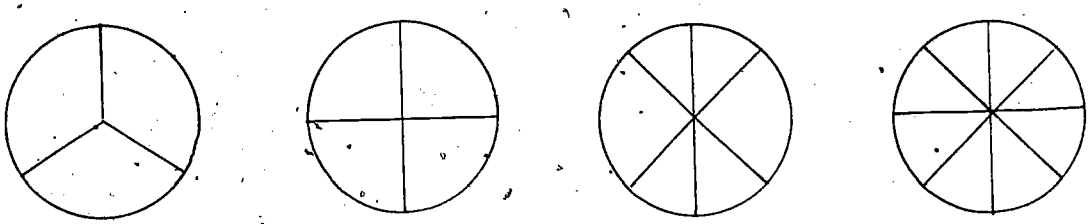
Materials needed: 2 unbreakable mirrors, paper circles (divided as illustrated into 1/3's, 1/4's, 1/6's, 1/8's), counters

Procedure:



Have pupils stand mirrors on a plain paper or desk as shown. Place a counter between the mirrors. What do you notice happen to the number of images as you increase the angle between the mirrors? What happens when more than one counter is used between the mirror?

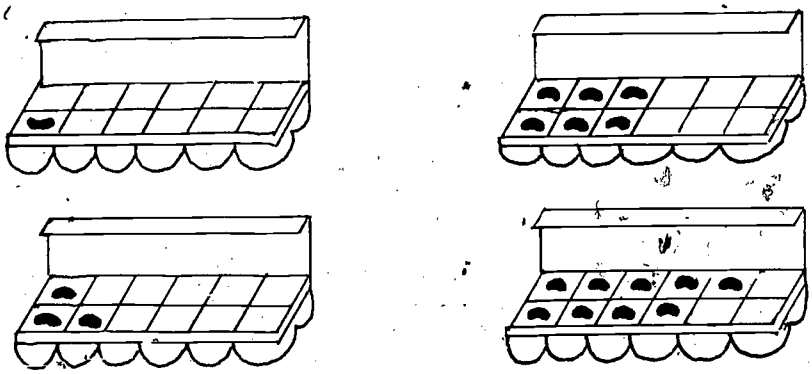
Have pupils follow the same procedure as above but place mirrors so they meet at the centers of the prepared circles.



2-47 Mathematical Egg Cartonry. Purpose: Fractional operations and concepts, recording
Suggested Grade Level: 5, 6

Materials needed: Egg cartons, beans, paper

Procedure: Have a student take an egg carton and some beans or beads. Have him place one bean into one section of egg carton. What fraction of the carton has a bean? Record on chart. Have him place other beans in other sections and record appropriate fractional parts.

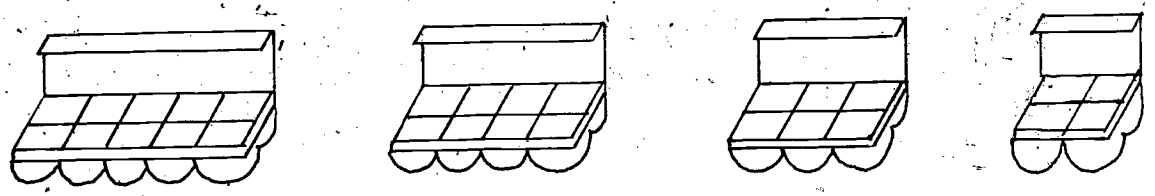


2-48 Truncated Carton. Purpose: Fractional operations and concepts, recording

Suggested Grade Level: 5, 6

Materials needed: Egg cartons, beans, scissors

Procedure: Take egg carton (12 sections) and cut 2 sections off. Now you have fraction box for tenths. Also, cut other cartons for eights, sixth's, quarters. Have students devise these fraction boxes and label each.



2-49 Scrambled Fractions. Purpose: Fractional operations, addition and subtraction, fractions

Suggested Grade Level: 5, 6

Materials needed: Egg cartons, beans, paper

Procedure: Take two egg cartons. Have students add 1/2 + 1/4 = 3/4 by putting one bean in each of the cups for half of the first carton, and into one quarter of the cups of the second carton. Ask how many beans were used? Then, transfer the beans from the second carton to empty cups of the first carton. What fractioned part of the cups in the first carton contain a bean? 9/12 or 3/4. Continue making comparable problems in addition and subtraction involving other fractional parts. Make your own questions.

2-50 Model Cars. Purpose: Inequalities, similarity, ratio, proportion

Suggested Grade Level: 5, 6

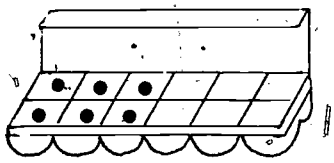
Materials needed: Auto sales booklet or manual, model car, scale

Procedure: Have students measure a model truck or car using an auto sales book or manual. Find whether the model is made to scale. If so, what is the scale? Next, determine if the weight of the model is made to the correct scale.

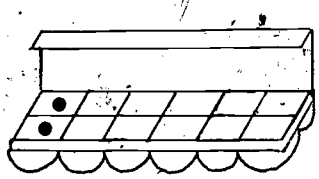
2-51 Egg Carton Computer. Purpose: Fractional operations
Suggested Grade Level: 5, 6

Materials needed: Egg cartons, beans, paper

Procedure: Have student multiply 1/3 x 1/2 with egg cartons. (1/3 x 1/2 = 1/3 of 1/2). Fill half of egg carton, one bean per section. Next, take one third of the beans. What fractional part of whole is this?



1/2



1/3 of 1/2 = 2/12 OR 1/6

Now do these: 1/2 x 1/2 = ? 1/3 x 1/4 = ? 3/4 x 1/3 = ? Can you find a pattern?

2-52 Modeling. Purpose: Ratio
Suggested Grade Level: 5, 6

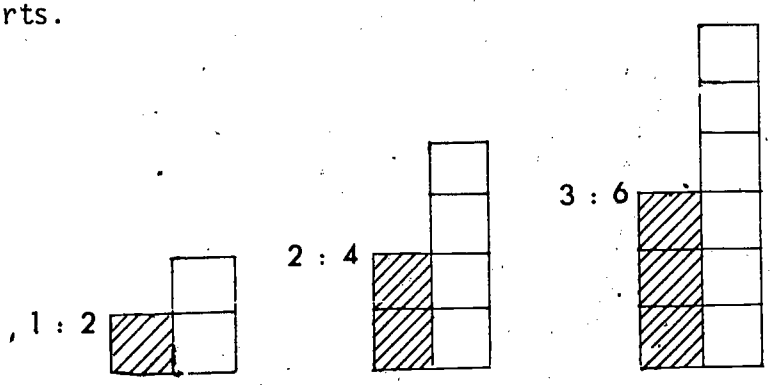
Materials needed: Model car at least 6" long, plasticene

Procedure: Have students measure the wheelbase, length, width and height of model car, boat, plane, etc. If the scale is 1 inch = 2 feet, find find actual measurements of real car. Using plasticene, form a scale model twice (three times, half, ...) as big as your original model. You may call upon the fields of biology, paleontology, architecture, sports, and many others to provide model sources.

2-53 Comparison Shows. Purpose: Ratio
Suggested Grade Level 5, 6

Materials needed: Unifix Cubes (2 colors)

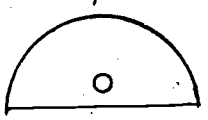
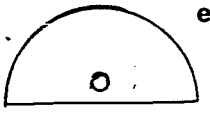
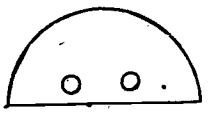
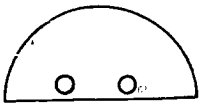
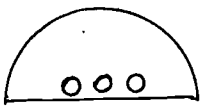
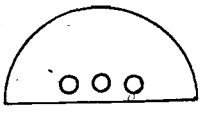
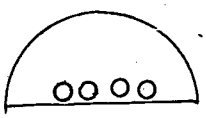
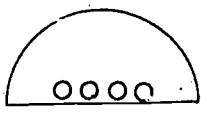
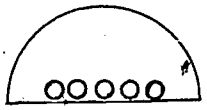
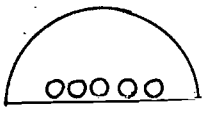
Procedure: Select various combinations of cubes. Arrange in columns. Discuss ways the columns may be compared - Discuss the changing and fixed parts.



2-54 Fractions Have Many Names. Purpose: Equivalent Fractions, concepts
Suggested Grade Level 3, 4

Materials needed: Unifix cubes or counters, circles, cubes, etc.

Procedure: Pretend the circles represent birthday cakes. Plan to put the same number of candles on each piece of cake.

1/2	1/2	Candles on each piece	Total no. of candles	Part of the candles on half a cake
		1	2	1 out of 2
		2	4	2 out of 4
		3	6	3 out of 6
		4	8	4 out of 8
		5	10	5 out of 10

Similarly build other equivalent fractions.

CATEGORICAL LISTING

Operations: Whole numbers and fractions

Improvised materials: Bank books, coins and bills, graphs from newspapers and social studies textbooks, invoices from local businesses, lattices, mailorder catalogs, Napiers rods, nomographs, restaurant menus, score cards, slide rule made from ordinary rulers and yardsticks, spinners and dice, supermarket price lists, baseball batting averages and other sports data, ecological problem studies, egg cartons, industrial and fine arts and home economics project involving measurement, time tables, also, acorns, beans, bottle caps, buttons, classroom equipment (books, erasers, pencils, window panes, desks, chairs, floor tiles, etc.), corks, discs, fingers, foot and handprints, graph paper, horse chestnuts (conkers), ice cream sticks, leaves, money, number lines, pebbles, pine cones, sample swatches of various materials, seasonal paper cut-outs (pumpkins, snowmen, hearts) seeds, shells, straws, telephone directories, tiles, toothpicks, twigs.

Improvised Games: Bingo, Pebbles in the Bag, Postman Stories, What's My Rule?

Commercial Materials: Almanac data, assorted measurement tools, equalizer, flash cards, balance, lattice number boards, Napier bones, number lines and tracks, spring and pan balance scales, also abaci and counting frames, attribute blocks, beads, binary counter blocks, clay or plasticine, calendars, clocks and other timers, clothespins, cubes, dials, directed number slide rule, discs, dice, dominoes, felt shapes and flannel board, funda-math, geoblocks, geoboards, magnetic shapes, mechanical and/or electronic desk calculators, metal washers, meters, number lines, playing cards, pop-it beads, small toys (animals, vehicles, dolls, marbles, checkers, etc.) slide rules, squared materials, structural materials, centimeter rods, Stern material, unifix rods, multibase arithmetic blocks, stencil graph, tally registers, tongue depressors, trundle wheels

Commercial Games: Quizmo, Tiddly-Winks, Monopoly, Yahtzee, Countdown, The Real Numbers Game, Back-Up Three, Numble, Assorted Score Keeping Games, Bingo, Make Ten, Quinto, Three Dimensional Tic-Tac-Toe, The Winning Touch, Ring Toss, Make 1, Fractions, Dominoes, Hexstat, and many others.

General Supplies:

Paper - ruled, unruled
graph - squares (1/10", 1/4", 1/2", 1"), isometric
gummed shapes
construction, art, newsprint, tracing
brown wrapping, wallpaper, carbon
library card - 3" x 5", 5" x 8"
corrugated cardboard

Thumb tacks, paper fasteners, clips pins, cellophane and masking tape, glue, paste, scissors, strings, rubber bands, sponges, laces, yarn, straw, pipe cleaners, plasticine or clay, pencils, paints, brushes, crayons, stapler, stamp pads, filing folders, tool chest, balsa wood, screws, nails, styrofoam forms.

Storage Containers: 1) Cardboard boxes, rectangular and cylindrical (covered with vinyl wallpaper for strength and color): 2) baskets, crates, and bushels from the produce markets; 3) wire hangers and clothespins or clamps for display and paper storage; 4) emptied aluminum, tin, cardboard, plastic food containers - checked for sharp edges and then painted; 5) commercial containers.

Abaci and Counting Frames: These instruments are the ancient forerunners of modern computers. Sliding counting beads may be strung on wire or laces, or placed loosely on a patterned, place value board. A large assortment of excellent commercial models are available. However, they are easy and fun to make. Number bases other than ten can be represented on abaci.

Binary Counter: Mechanical or electrical devices that can display binary numeration. The mechanical counters show combinations with the symbols 0 and 1, while the electrical ones are coded with on-off switches for display lights. Intermediate grade pupils may be challenged to build binary counters.

Dominoes: Many variations on traditional dominoes are now available for teaching concepts of matching, counting, arithmetic operations, and geometric discrimination.

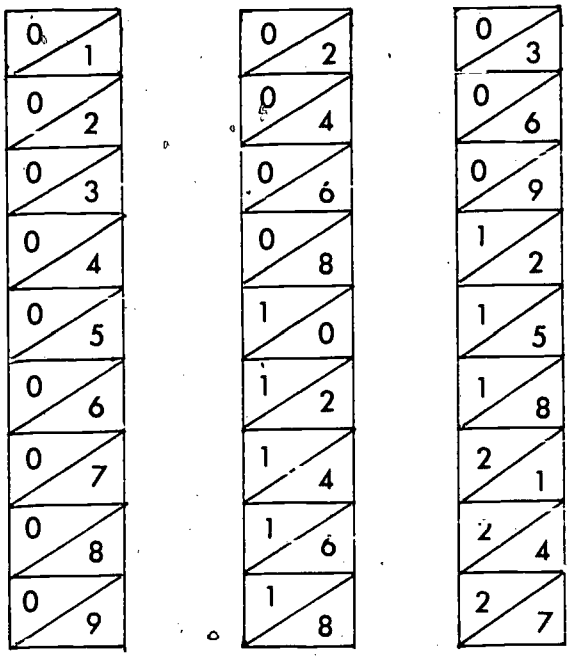
Hundred Chart: Felt on paper chart, can be used on flannel board or tacked up on cork board. This device can be used for skip counting, number patterns, and whole number operations.

Number Lines: A great variety of number lines have been commercially prepared. You can choose ordinary measuring rulers, plastic walk-on number line "carpets", graduated measuring cups, baby bottles, etc... There are perforated rolls of number lines which adhere to desks or shelves. Felt, wood, and plastic number lines are also available. Fractional numbers and directed numbers have been prepared in the form of number lines.

Squared Materials: Cardboard sets consist of single units, strips of ten, and blocks of 100. They may be used to reinforce place value concepts, as concrete representations of numbers, and for addition and subtraction of numbers with and without exchange. Comparable procedures may be developed with graph paper.

Stencil Graphs (Lattices): Perforated plastic sheets come on a window shade roller. The device is hung over a chalk board. The shade is pulled down and rubbed with an ordinary chalking eraser. The shade is then lifted and a chalked lattice is available for further marking.

Napiers Rods or Bones: These devices help students develop a firmer grasp of the role of a partial product in long multiplication. Individual sets can be made easily by each child. Instructional booklets are available.



ADDITIONAL MATERIALS

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