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AUTHOR Shah, Sair Ali
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ABSTRACT

The notions of topological equivalence for one-, two-, and three-dimensional figures, as well as for graphs and networks, are developed for classroom use with children between the ages of three and ten. Properties of open and closed curves are also examined. This manual, addressed to the teacher, describes several activities related to each concept to be introduced. In order to implement this material, the teacher would need plastic clay, colored beads, and wire or pipe cleaners for each student. (SD)

Working Paper No. 18a

TOPOLOGICAL EQUIVALENCE OF OBJECTS

TEACHER'S GUIDE FOR USE WITH STRETCHING AND BENDING

by

Sair Ali Shah
Visiting Professor of Mathematics Education
University of Georgia

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Research and Development Center in Educational Stimulation
University of Georgia Athens, Georgia

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NOTES FOR TEACHERS

OBJECTIVES AND CONTENTS: I. In this unit we want to introduce children to some elementary intuitive ideas in topology.

In topology, we can bend and stretch bodies without tearing, cutting or joining and observe the features which do not change. These we consider as the invariant properties. As we bend and stretch bodies, we say that the bodies are undergoing "elastic motion".

II. This unit is designed for children as young as three years old and may be used up to the intermediate grades.

It is desirable to start from stage I with each group, unless work from this unit was done with previous age groups.

A. Topological Equivalence of Solids

(a) Using two balls of silly putty, we take one ball and bend it and stretch it, without boring holes in it, and compare it with the other ball and observe the features of the two balls, which remain the same. This exercise will give us the equivalence of two solids under "elastic motion" or we may say the topological equivalence of two solids, without holes.

(b) Using two pieces of silly putty - each with one hole, we take one piece and bend and stretch it without filling in the hole, and compare it with the other piece and observe the features, which remain unchanged. This exercise will give us the equivalence of two solids with one hole under elastic motion.

The same is then done with objects with 2,3,4,etc., holes.

B. <u>Topological Equivalence of Sheets</u>

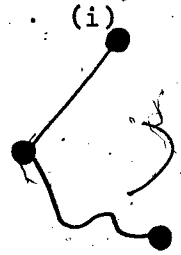
(a) Using two sheets of soft paper or silly putty, we take one and bend it and stretch it and make an object; e.g. a plate, a cup, a bowl, etc., and compare the results with the other sheet and observe the features, which remain unchanged under the "elastic motion".

C. <u>Topological Equivalence of "Lines" and Nets</u>

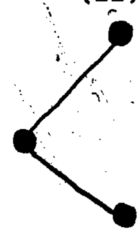
(a) Using two single strips of soft wire we take one and bend it without joining the ends and observe the features which remain the same as in the other strip.

(b) Using small balls made from silly putty and pieces of wire, we make two open nets, each with three balls, and two pieces of wire

(i)

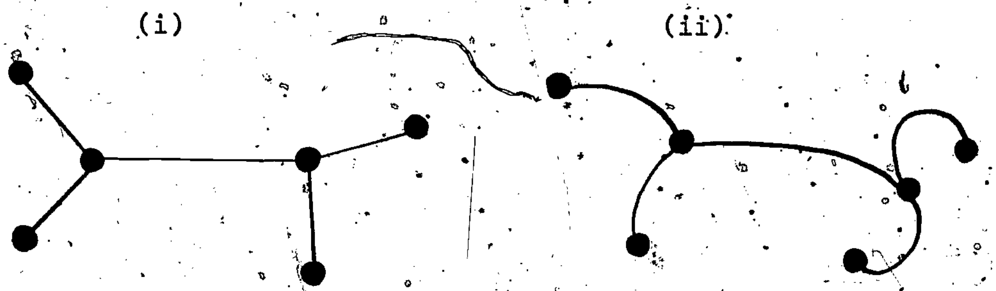


(ii)

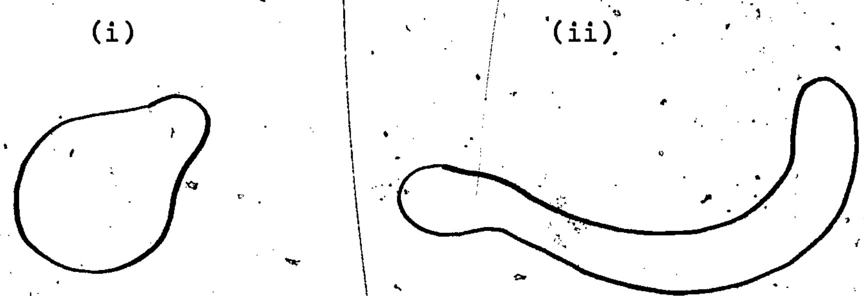


We then take one of the nets, say (i) and bend and stretch the two wires and then compare it with the other net (ii) and observe the features which remain unchanged.

We then make nets with six balls and compare for unchanged features.



(c) We now take two pieces of wire and with each we make a closed figure



We compare (i) and (ii) for invariant features.

(d) We now consider "inside" and "outside" closed figures. We do this by placing an object "inside" a figure and asking where is it, "inside" or "outside"? Similarly we place an object outside a figure and ask where is it, "inside or "outside"?

(e) We now give a set of closed and open figures and ask children to group them.

D. Order of Things on a String After Bending

Here we want to show the idea that points on a line are in the same order after bending, that is, one of the invariant properties of a body undergoing a topological transformation is that the relative points are in the same order.

APPARATUS: Pieces of soft wire, clay, sheets of soft paper, beads, "charm bracelets", pieces of string, rubber bands, objects to place on string, e.g. shapes made from cardboard: triangle, circle leaf, fruit, etc.

PROCEDURE: In this unit we are working with children at a very early age and with children who have no previous experience with these ideas. We therefore recommend the following stages of development be included in your lessons. We illustrate the stages with the content area discussed above.

A. Topological Equivalence of Solids

Stage I. Let children use concrete material to make things as given in the content and get verbal responses from them.

For (A) Topological Equivalence of Solids, give children two balls of silly putty and ask them to make things with them,

(a) without boring holes,

(b) by boring one hole in each ball of silly putty

(c) by boring two holes in each ball of silly putty

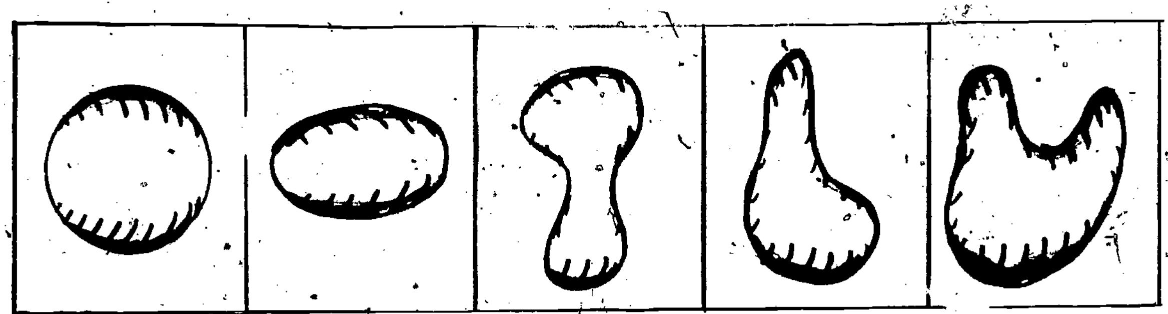
(d) by boring three holes in each ball of silly putty

(e) by boring four holes in each ball of silly putty

Stage II. If children do not understand, then demonstrate as follows: Here we have two balls of clay. I can take one and make a roll, a biscuit, etc. Now notice I did not bore any holes in the clay. Now you can make something.

Stage III. If children still have difficulties then give them pictures of things they can make.

Example:



Stage IV. Ask questions to discover invariants. e.g.

(i) what changed in the ball of silly putty?

(ii) what did not change?

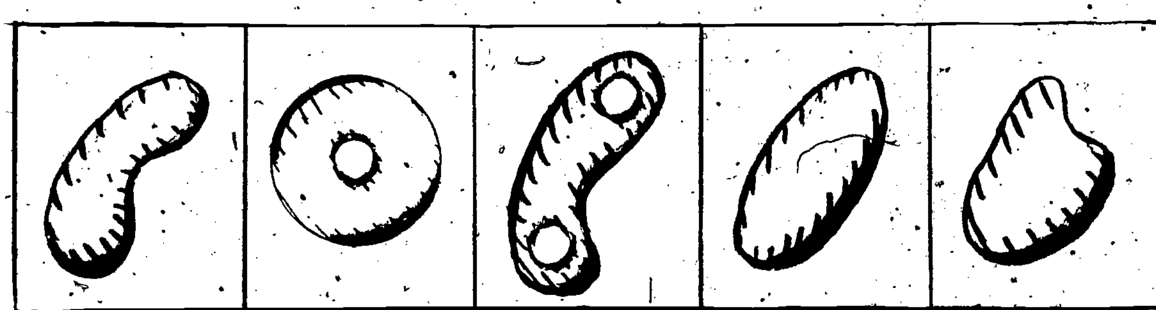
Suggested Activity

For each group of objects; objects without holes, objects with one hole, etc., use an object to help the children see the contrast. For examples (1) for objects with no holes, take a roll from the bakery and ask how it is different from a doughnut. "What must we do to this roll to make it look like a doughnut?" (2) Contrast a cup and another cup with a hole in it. "How are they different?" (3) To contrast an object with two holes with an object with one hole, you might use a button with two holes and another button with one of the holes blocked. (4) In the next group of objects, you might contrast a face mask with three holes to a mask which has one of the three holes blocked. Asking always, "how are they different?"

Stage V. Design test using pictures to discover topological equivalence of objects.

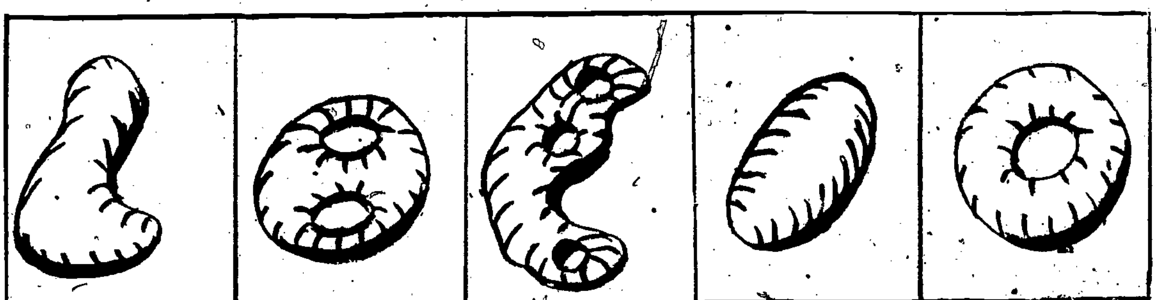
Example 1.

Cross out the things you can make with a ball of clay without boring holes.



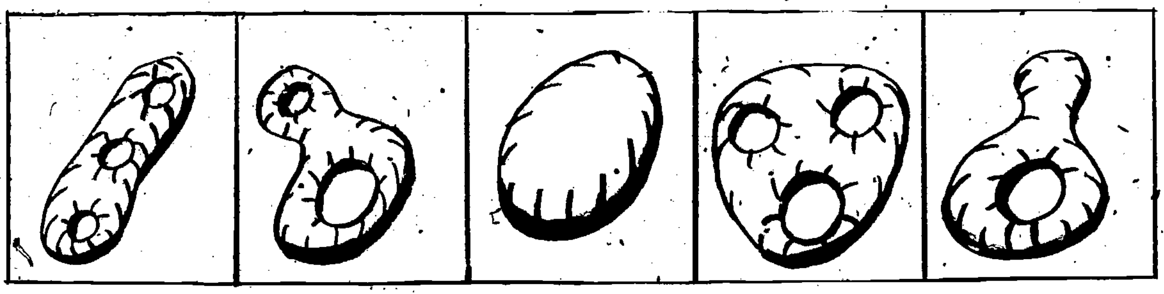
Example 2.

Cross out the objects with two holes.



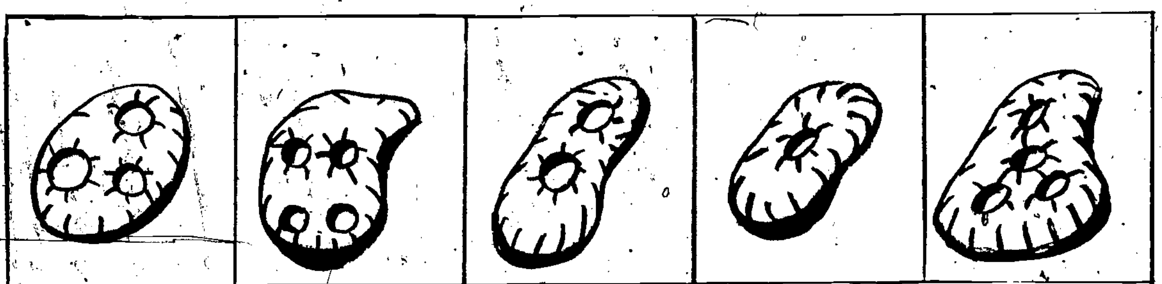
Example 3.

out the objects with three holes.



Example 4.

Cross out the objects with four holes.



Conclusion

For this section we want to conclude that:

(1) All objects without holes are topologically equivalent.

(2) All objects with one hole are topologically equivalent.

(3) All objects with two holes are topologically equivalent.

(4) All objects with three holes are topologically equivalent.

(5) All objects with four holes are topologically equivalent.

B. Topological Equivalence of Sheets

Stage I. Let children use soft paper or a flat piece of silly putty and make things with it without boring holes or without joining the ends of the silly putty.

Stage II. If children do not understand, then demonstrate as follows:

Here we have a flat piece of silly putty. I can take it and make a plate, a bowl, a saucer, a hat, etc. Now you make some things with your silly putty.

Stage III. If children still have difficulties, then give them pictures of things they can make.

Example:

Stage IV. Ask questions to discover invariants

(i) What changed in the ball of silly putty?

(ii) What did not change in the ball of silly putty?

Suggested Activities:

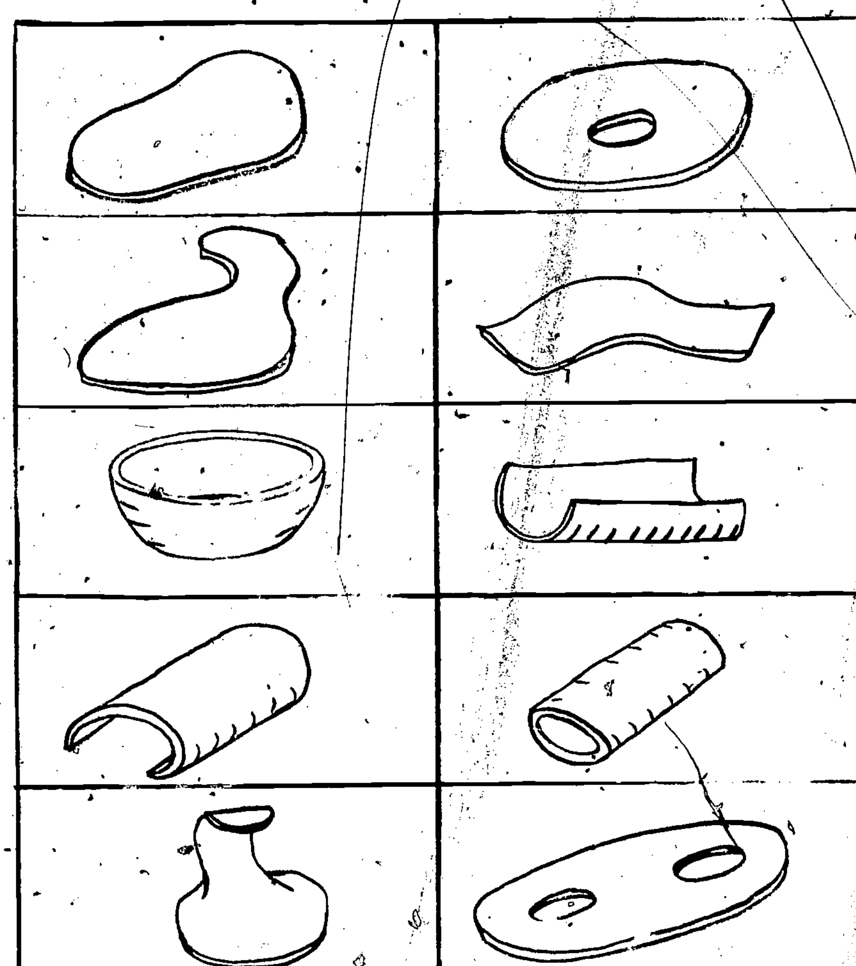
EXAMPLES: (1) To illustrate the difference between a sheet and a sheet with a hole in it, use the example of a sheet on a bed and a sheet with a hole also on the bed. Ask, "What is the difference? "Do both sheets cover the bed completely?"

(2) After a sheet of clay has been molded into a bowl, contrast a bowl and a bowl with a hole in it. How are they different? (Perhaps a student will say, one will hold water, the other will not.)

Stage V. Design test using pictures to discover topological equivalence of sheets.

EXAMPLE:

Pick out the things you can make with a flat piece of clay or a soft sheet of paper without boring holes, or without joining the ends of the clay.



Conclusion:

For this section we want to conclude that all sheets without holes are topologically equivalent.

C. Topological Equivalence of "Lines" and Nets.

(i) Single strip of wire

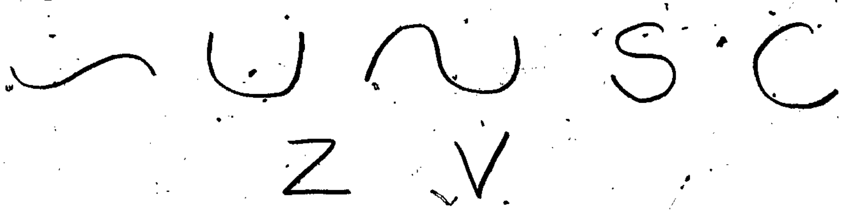
Stage I. Let children use pieces of soft wire to make things without joining or cutting the wires.

Using a single piece of wire, make several things by bending it.

Stage II. If children do not understand, then demonstrate as follows:

Here with this piece of wire, I can make some things by bending it. For example: from  I can make

Stage III. If children still have difficulty then give them pictures of things they can make, e.g.



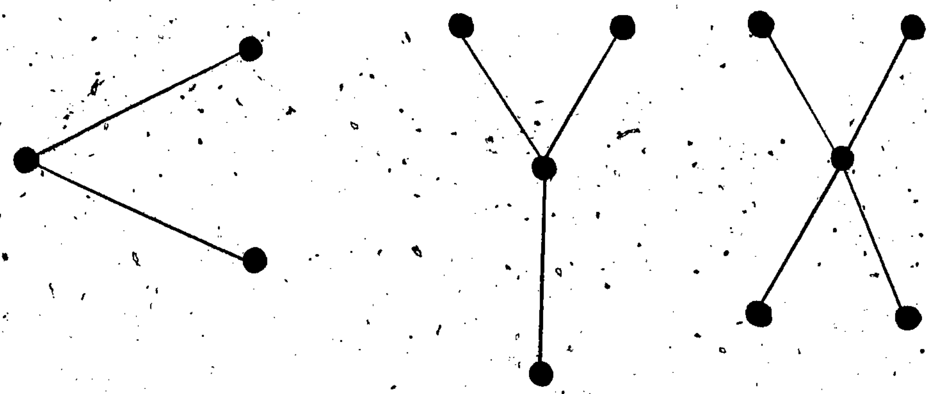
Stage IV. Ask questions to discover invariants.

(i) What changed in the wire?

(ii) What remained unchanged in the wire?

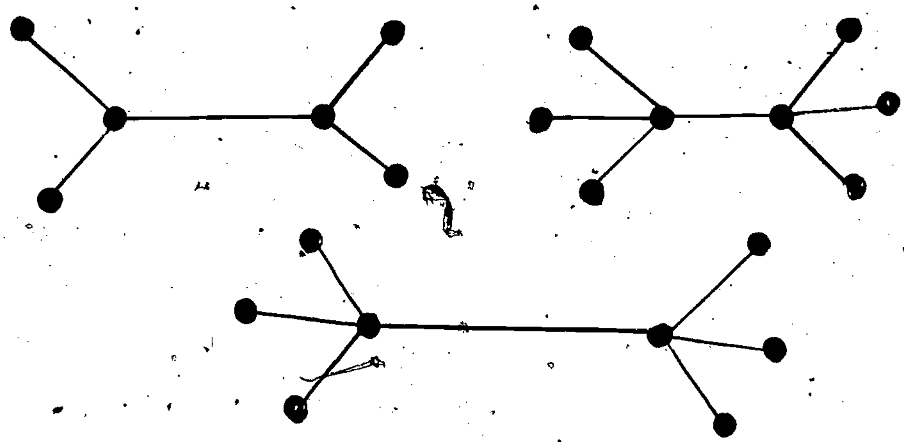
(ii) Bending Wires with one junction

Using the figures below go through stages I to V as above.
(then see Suggested Activities, p. 3)



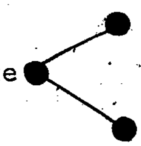
(iii) Bending wires with two junctions

Using the figures below go through stages I to V as above



Suggested Exercises:

I.

1. Take  and bend 

   Ask: Are they the same?

   How are they the same?

2. Lets' give each ball a name:

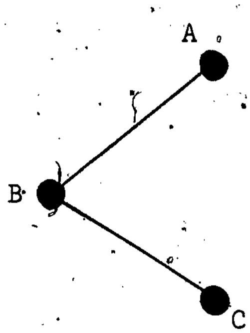


Fig. 1

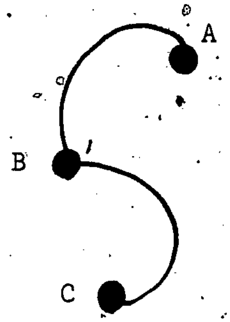


Fig. 2

   Ask: How many ends of string are there in Fig. 1 at A?
   B?
   C?

   How many ends of string are there in Fig. 2 at A?
   B?
   C?

3. Make a chart to show:

BALL	NUMBER OF STRING ENDS	
	FIG. 1	FIG. 2
A	1	1
B	2	2
C	1	1

How many balls have the number 1 or 3?

4. Other examples, same question as 1, 2.

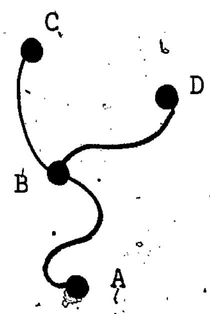


Fig. 3

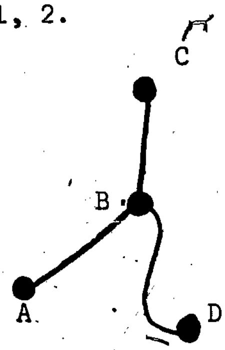


Fig. 4

5. Make a chart to show:

BALL	NUMBER OF STRING ENDS FIG. 3	FIG. 4
A	1	1
B	3	3
C	1	1
D	1	1

How many balls have the number 1 or 3?

6. Other examples, same as question 1, 2.

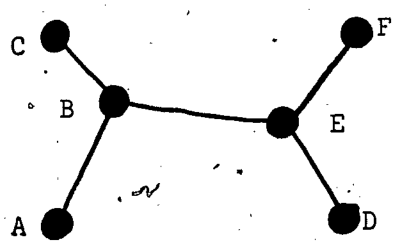


Fig. 5

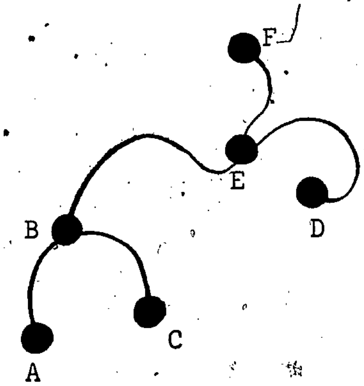


Fig. 6

7. Make a chart to show

BALL	NUMBER OF STRING ENDS Fig. 5	Fig. 6
A	1	1
B	3	3
C	1	1
D	1	1
E	3	3
F	1	1

How many balls have the number 1 or 3?

8. Take your pencil and try to trace all the lines without lifting your pencil or going over any part twice.

In which figures can you do this?

What do you notice from the charts about the number of (odd) vertices with 1 or 3 and being able to trace?

II. To contrast wires with one junction with wires that have two junctions, and variations of these two situations, you might set up the example of children pulling on ropes . . . (each child to one end of the rope)

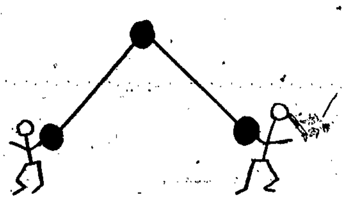


Fig. 1

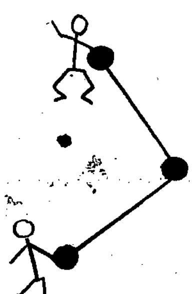


Fig. 2

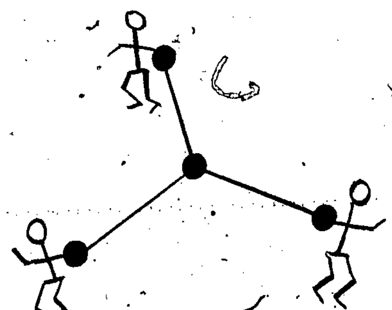


Fig. 3

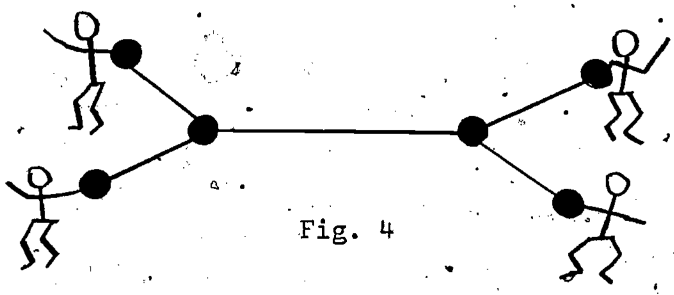


Fig. 4

Ask in each case, "How many children can pull on the ropes?" To answer, how are they the same?

Ask the same question, contrasting Fig. 2, Fig. 3 and Fig. 4. "How are they different?"

Stage V. Design test to discover topological equivalence of lines.

Conclusion:

Here we want to conclude that:

(a) All things with single wires without joining are topologically equivalent.

(b) All things with one junction and the same number of wires are topologically equivalent.

(c) All things with two junctions and the same number of wires at each junction are topologically equivalent.

(d) All nets with more than two odd vertices can not be traced without lifting your pencil or going over some part twice.

D. Open and Closed Figures

Stage I. Give children closed rubber bands and ask them to bend them, twist them, and make different shapes.

Stage II. If children do not understand, then demonstrate.

Stage III. If children still have difficulties, then give them pictures of things they can make.

Stage IV. Ask questions to discover invariants.

(i) What changed in the rubber ring?

(ii) What did not change in the rubber ring?

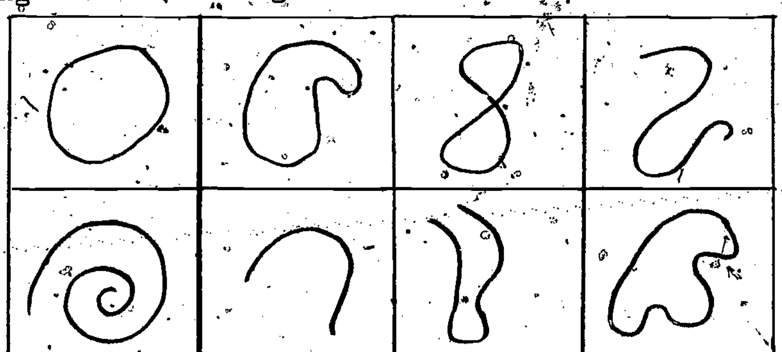
Suggested Activity:

1. To help contrast open and closed figures, use the example of driving around (or tracing around) the figure. Help the children to see that we can "go all the way around to where we started from" in a closed figure, but that is impossible in an open figure.

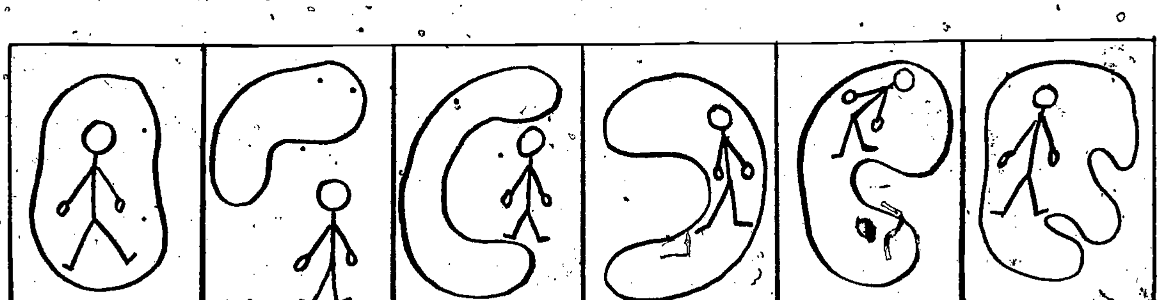
2. To contrast further the concept of open and closed figures, use students and rope. Using a closed rope figure about a student, ask the class, "How can he get out?" or "How can another student get in?" and variations of this.

Stage V. Design test using pictures to discover topological equivalence of objects. Give examples with open and closed figures.

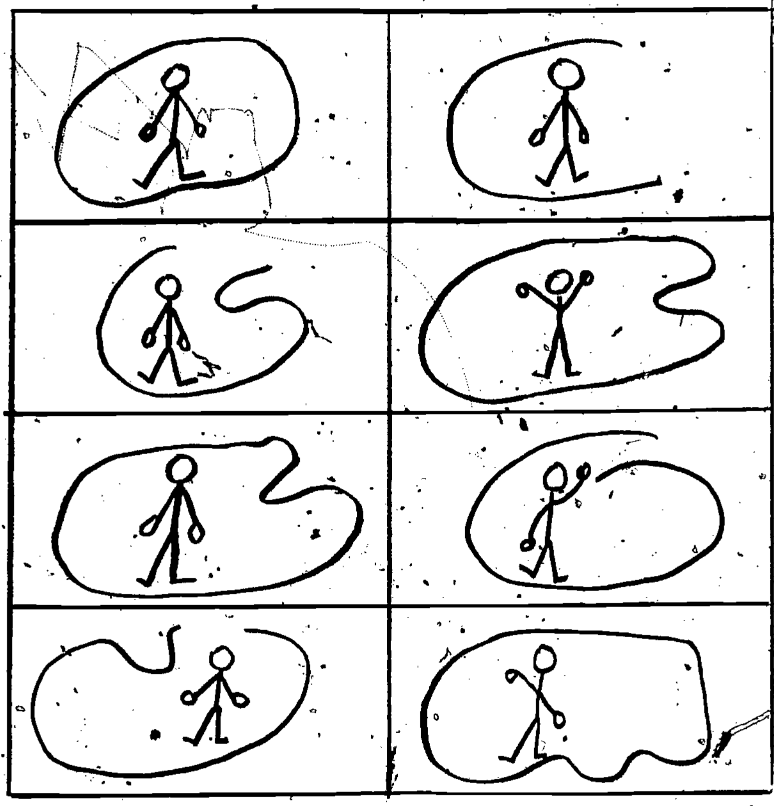
Example I. Cross out the things you can make with a rubber ring without cutting it.



Example II. Say if the doll is "inside" or "outside" the ring.



EXAMPLE III. If the doll is inside the ring, how could it get out? It will have to jump over the fence. Now say in which of the following will the doll have to jump over the fence to get out.



Conclusion

We want to conclude here that:

(a) All open figures are topologically equivalent.

(b) To get from "outside" to "inside" a figure and vice versa, a boundary must be crossed.

E. Order of Things on a Stirng After Bending

Stage I. Give children open strings with two or four objects tied along the string.

Provide children with strings and objects, and ask them to put objects on string in the same order.

Stage II. If they cannot perform the task then demonstrate an example to them and then let them follow and do the given tasks.

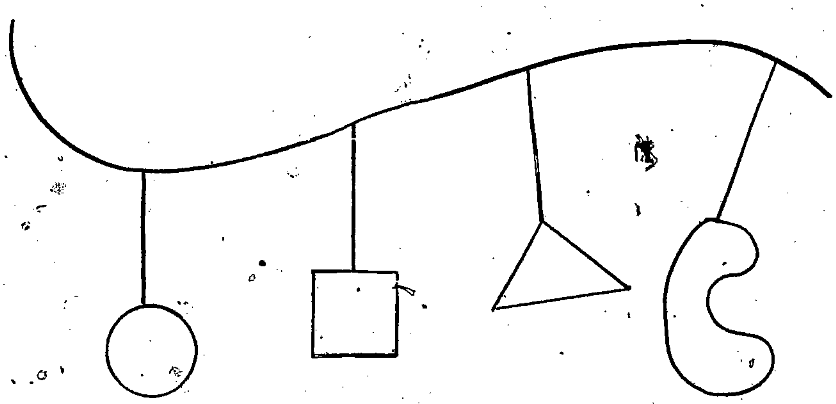
Stage III. If they cannot perform stage II, then give them objects and pictures and demonstrate. Then ask them to perform tasks.

Stage IV. If stage II is successful, then give them pictures and let them perform tasks by using pictures. Here no demonstration is given.

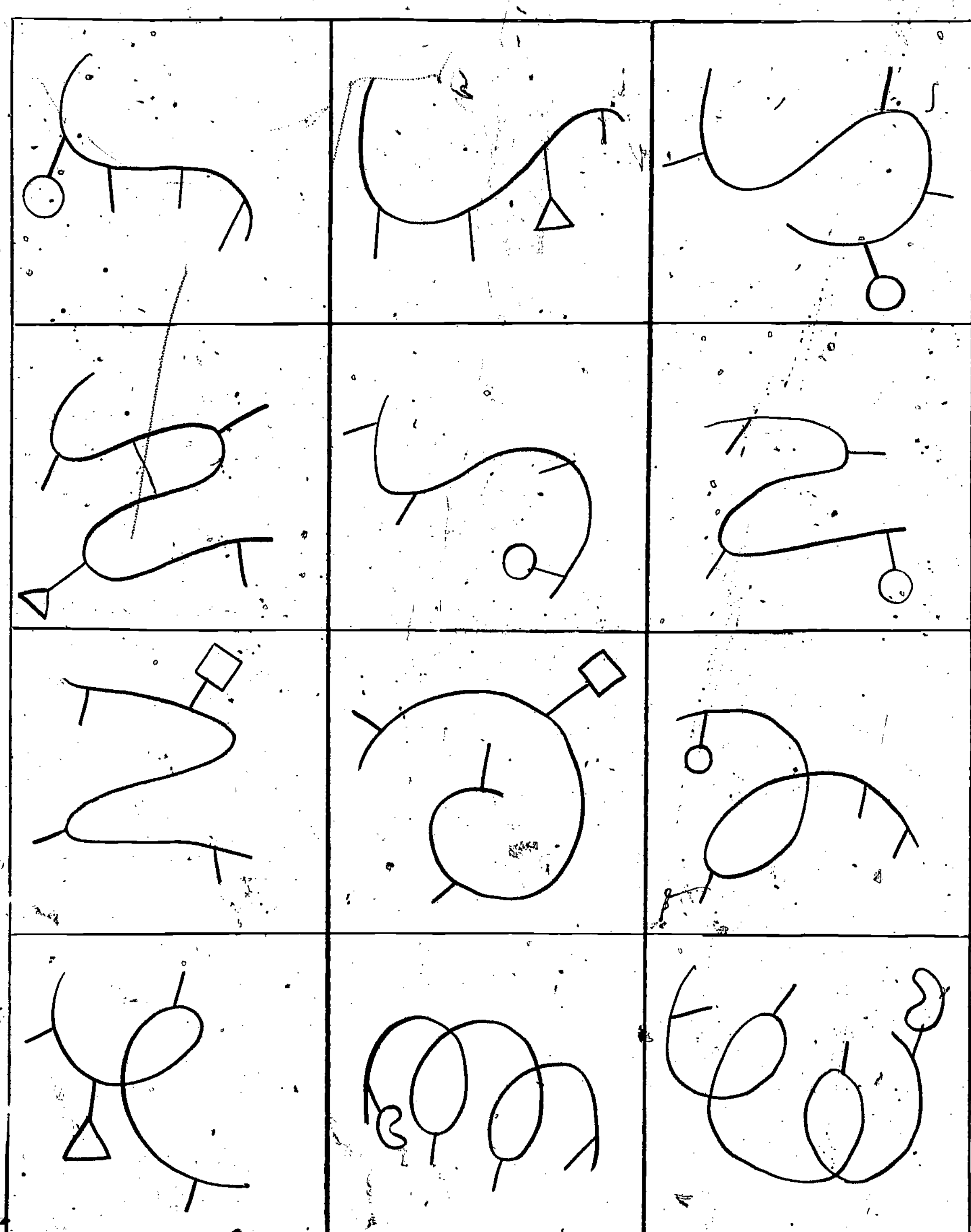
Stage V. Design exercises to discover order.

EXAMPLES:

Here we have some things on a wire



Now with the wires given below put on the things which are missing.



Conclusion

Here we want to conclude that points on a line are in the same order after undergoing a topological transformation, e.g. bending.

Suggested Objects: Tomato, biscuit, potato, doughnut, ring, earring, eye glass frames, binoculars, electric socket plate, Swiss cheese (with holes), notebook paper, mask, hinge, button (with holes), bowl, derby hat, big string for felt board with objects attached.

References

1. Walter Lietzmann, Visual Topology, American Elseview Publishing Company Inc., N.Y. 1965.

2. B.H. Arnold, Intuitive Concepts in Elementary Topology, Prentice-Hall, 1963.

3. Some Lessons in Mathematics, Cambridge University Press.

4. Wolfgang Franz, General Topology, Fred. Ungar Publishing Co., N.Y.

5. Steven A. Gaal, Point Set Topology, Academic Press, N.Y. and London.

Children's Kit

Four ounces of plastic clay (or plasticine)

Ten colored beads

Six pieces of soft wire (or toy pipe cleaners) - each about 4-5 inches long.