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ABSTRACT

This paper directs attention to the use of the EQUATIONS game as a research tool to study certain kinds of mathematical behavior, for example, the kinds of mathematical problems which players prefer to consider and to force their opponents to consider. Mathematical equations that meet certain game rule constraints constitute a problem space. A resource is a component of these equations, i.e., digits and operations symbols. Allocating resources is performed by a player's move within the well defined limits of game rules which may have the effect of altering the problem space. In this paper, forms of the mathematics game EQUATIONS are described and analyzed. Player strategy is analyzed, and the relevance of that analysis to the study of mathematical behavior is discussed. (Author/JBW)

Abstract

RESOURCE ALLOCATION GAMES

AS THE ENVIRONMENT FOR EVALUATION

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Experimental forms of the mathematics game EQUATIONS have been developed as a research tool to study certain kinds of mathematical behavior--for example, the kinds of mathematical problems which players prefer to consider and force their opponents to consider. Such forms of EQUATIONS are described, player strategy analyzed, and the relevance of that analysis to the study of behavior discussed.

Mathematics, a well-defined body of knowledge which can be characterized by a formal set of rules, is fundamental to EQUATIONS. The resources to be allocated are digits and operations. When these and a Goal (the right side of an equation) are preset by the experimenter, a finite number of Solutions (the left side of the equation, and equal to the Goal) can be built from the remaining resources. The finite set of solutions constitutes the problem space and is partitioned into equivalence classes corresponding to the set of resource cubes that can be ordered and grouped into one or more solutions. The experimental games are so designed that the solutions in one subset of equivalence classes contain one kind of mathematical idea, the solutions in the other subset do not contain that idea.

Players take turns moving one resource per turn into forbidden, permitted, and required categories on the playing mat. Cubes placed in forbidden cannot be used in building a solution; cubes placed in permitted may be used but are not required to be used in building a solution; cubes placed in required must be used in building a solution. We cannot observe which in the set of solutions a player is thinking about when he is making cube moves. We can observe the effect produced on equivalence classes by his cube moves. The game itself delineates the problem space, and we can measure the changes in that problem space by the way the game is played.

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Introduction

This paper directs attention to the use of resource allocation games as a research tool. Fundamental to the resource allocation games that we will describe is a body of knowledge which, when well-defined, can be characterized by a formal set of rules, such as mathematics or logic. Statements relevant to the body of knowledge such as mathematical equations that meet certain game rule constraints constitute a problem space. A resource is a component of these statements. In a resource allocation game, a player's move consists of allocating resources within well-defined limits imposed by game rules which may have the effect of altering the problem space.

Several games in Layman Allen's WFF 'N PROOF series have these properties. The body of knowledge fundamental to WFF 'N PROOF[1] is mathematical logic; to the game of EQUATIONS,[2] mathematics, to the ON-SETS[3] game, set theory, and to the ON-WORDS[4] game, word structure. All of these games have the same game rule structure.

We shall direct attention to Layman Allen's EQUATIONS game, describe experimental forms of this game, discuss the analysis of player strategy and the relevance of that analysis to the study of behavior.

Description of EQUATIONS game

In the EQUATIONS game, a set of cubes containing single digits zero to nine and operations (plus, minus, multiplication, division, root, and exponentiation) are thrown and the symbols on the upward faces constitute the Resources for that play of the game.

The first player sets a goal by placing cubes from the Resources onto the playing mat marked GOAL as shown in Figure 1. This number is then the right side of the equation. At this point there are a finite number of solutions that can be built from the remaining resource cubes that can equal that goal. This set of solutions constitutes the problem space. Each player in turn takes one of the resource cubes and places it on one of three areas of the playing mat. If he places the cube in the section marked Required, it means that the cube must be used in building a solution. If he places it in the Permitted section, it means that the cube may be used but does not have to be used in building a solution. If he places it in the section marked Forbidden it means that cube cannot be used in building a solution.

Cube moves are either flubs or non-flubs. Layman Allen has restated the rules to define a flub move as follows:

<u>Flubs</u>: A move can be a P-flub, an A-flub, or a C-flub.

<u>P-flub</u>: A move is a P-flub if it [P]revents every solution from being built no matter how the remaining plays are made.

<u>A-flub</u>: A move is an A-flub if it [A]llows a solution with one more move when the mover could have avoided such a solution without flubbing.

<u>C-flub</u>: A move is a C-flub if the mover fails to [C]hallenge when he could have done so correctly because the previous move was a flub.

The cube moves stop when one of the players challenges that the mover has made a flub move. The player who claims a solution can be built has the burden of proof and must submit a solution within the constraints that

RESOURCES		
Forbidden	**Permitted**	**Required**
(Any cube moved here must not be used in building a solution.)	(Any cube moved here may be used, but does not have to be used, in building a solution.)	(Any cube moved here must be used in building a solution.)

______________________ = ________________

SOLUTION GOAL

Figure 1

EQUATIONS Playing Mat

all cubes in the Required section are used, none in the Forbidden section and any he chooses from the Permitted section. This very briefly describes the Basic game of EQUATIONS.

Game Analysis

The experimental version of the game retains all of these rules. In order to make it more manageable, however, the cubes are not randomly generated, but are preset ahead of time by the experimenter. The goal is also preset. Players then proceed to play as in the Basic game, and a record of each cube move is kept. From the player's point of view, the experimental game is like the Basic game except that he does not throw the resource cubes and set the goal.

The specification of the symbols on the resource cubes and on the goal generates a finite set of solutions and constitutes the problem space. A solution is an expression that is a particular ordering and grouping of some resource cubes and that equals the goal. The set of solutions can be partitioned into equivalence classes. An equivalence class corresponds to the set of resource cubes that can be ordered and grouped into one or more solutions.

In the sample experimental game, the first column in Table 2 lists all possible solutions that can be built with the resource cubes. It will be noted that solutions #1 and #2 use cubes +, 1, 2 and that solutions #3 to #8 use cubes +, -, 0, 1, 2. Thus the cubes used by the first two solutions constitute one cube set as shown in the second column of the Table. The cube sets used by the possible solutions are the criteria for partitioning the set of possible solutions into equivalence classes

Goal = 3

Resources +, -, 0, 1, 2.

Possible Solutions	Cube Sets	Representative Solution From Equivalence Class
1. 1 + 2	{+,1,2}	1 + 2
2. 2 + 1		
3. (1 + 2) - 0	{+,-,0,1,2}	(1 + 2) - 0
4. (2 + 1) - 0		
5. (1 - 0) + 2		
6. (2 - 0) + 1		
7. 1 + (2 - 0)		
8. 2 + (1 - 0)		

Table 2

Sample Experimental Game

of solutions. One equivalence class consists of solutions #1 and #2 and corresponds to the set of cubes (+, 1, 2). The other equivalence class consists of solutions #3 - #8 and corresponds to the set of cubes (+, -, 0, 1, 2). All solutions in a given equivalence class are defined to be variations of each other. Thus the solutions 1 + 2 and 2 + 1 are variations of each other because they are both in the same equivalence class.

The distinction between a solution and an equivalence class of solutions is important in the construction and analysis of an experimental game. A player must submit a solution to sustain the burden of proof. Cube moves, as we shall see, affect equivalence classes of solutions. We cannot observe which in the set of solutions a player is thinking about when he is making cube moves. We can observe the effect produced on equivalence classes by his cube moves.

Consider the effect on the equivalence classes of a move which places a cube in the required section. Assume for the moment that there are no identical cubes in resources. First, all sets of cubes that do not contain that cube (but that correspond to equivalence classes of solutions) are extinguished. This means that all expressions that had been solutions in these equivalence classes can no longer be used by a player to sustain the burden of proof. Secondly, such a move has the effect of reducing by one the number of resource cubes needed to build a solution from any unextinguished equivalence class of solutions.

When a cube is placed in the permitted section, no cube set or corresponding equivalence class of solutions is extinguished, but all cube sets that contained that cube need one less cube from resources to be used to sustain the burden of proof.

When a cube is placed in the forbidden section, all cube sets (and corresponding equivalence classes of solutions) that contained that cube are extinguished, and the solutions in these equivalence classes cannot be used in sustaining the burden of proof. All remaining cube sets need the same number of cubes from resources to sustain the burden of proof as they did before the move.

Thus a player on his turn has the following kinds of control on the sets of equivalence classes of solutions: He can remove equivalence classes of solutions from the problem space by extinguishing the corresponding cube sets with cube moves to required and forbidden. By cube moves to required or permitted, he can reduce the number of cubes in a cube set which are needed from resources to build a solution. If a cube move to required or forbidden extinguishes all equivalence classes of solutions, then no solution can be used to sustain the burden of proof. Such a move is a P-flub and can be correctly challenged. If a cube move to required or permitted has the effect of reducing to one cube the number of cubes needed from resources for at least one cube set, then such a move may be an A-flub that can be correctly challenged. If a cube move follows an unchallenged flub move, then a C-flub has been made and the move can be correctly challenged.

Although we are interested in the kinds of flub moves and what follows them, also of considerable interest is the effect of cube moves on equivalence classes before a flub has been made. It is here that we learn how the game is played.

In the experimental games, the set of equivalence classes has been partitioned into at least two subsets; the solutions in one subset of equivalence classes contain one kind of mathematical idea, and those in the other subset do not contain that idea. By observing which subsets of equivalence classes are removed by cube moves that extinguish cube sets, we hope to discover the kinds of solutions players prefer to consider and force their opponents to consider.

Illustrations of Analyses of Experimental Games

We will illustrate different ways the game may be played in one of thirty odd experimental games my colleague, Mark Plant, has developed. I should like to point out that Mr. Plant is a high school senior who for the past two years has been working on constructing these games and developing the computer program for analysis. He has been playing EQUATIONS and other WFF 'N PROOF games since he was in junior high school and is one of the past champions of the Academic Games Olympics. Such authorities are invaluable in developing measures on how the game is played.

As shown in Table 3, the goal is 9. The resources are specified on the next line. The column marked "Cube Sets" lists all sets of cubes that can be arranged to equal the goal. The next column lists a representative solution of each equivalence class that corresponds to each cube set. The set of equivalence classes has been partitioned into two subsets: Subset M contains equivalence classes of solutions that employ multiplication. Subset L contains equivalence classes of solutions that do not employ multiplication.

The first column under "Moves" lists the number of cubes in each cube set that are in resources before the first move. In the first move of this hypothetical game, Rx, the multiplication cube was placed in the

Goal = 9

Resources +, +, x, 1, 2, 2, 3, 4, 5, 6, 7, 8

Cube Sets	Representative Solution of Equivalence Class		Number of Cubes Needed from Resources to Complete the Solution	Moves: Rx	P3	F+	F+	Cp
{+,x,1,2,7}	(2 + 7) x 1		5	4	4	4	x	
{+,x,1,3,6}	(3 + 6) x 1		5	4	3	3	x	
{+,x,1,4,5}	(4 + 5) x 1		5	4	4	4	x	
{+,+,x,1,2,2,5}	(2 + 2 + 5) x 1		7	6	6	x		
{+,+,x,1,2,3,4}	(3 + 2 + 4) x 1	Subset	7	6	5	x		
{+,x,1,2,4}	1 + (4 x 2)	M	5	4	4	4	x	
{+,x,1,2,3}	3 x (2 + 1)		5	4	3	3	x	
{+,x,2,2,5}	5 + (2 x 2)		5	4	4	4	x	
{+,+,x,2,2,3}	[(2+1) x 2] + 3		7	6	5	x		
{+,1,8}	1 + 8		3	x				
{+,2,7}	2 + 7		3	x				
{+,3,6}	3 + 6		3	x				
{+,4,5}	4 + 5		3	x				
{+,+,1,2,6}	1 + 6 + 2	Subset	5	x				
{+,+,1,3,5}	1 + 3 + 5	L	5	x				
{+,+,2,2,5}	2 + 2 + 5		5	x				
{+,+,2,3,4}	3 + 4 + 2		5	x				

Table 3

Description of a Play of an Experimental Game required section. Because this cube appears in all cube sets corresponding to subset M, the number of cubes needed from resources is reduced by one. All cube sets corresponding to subset L have been extinguished. The problem space has been drastically modified. In this game, the first player chose to deal only with solutions containing the multiplication operation. The next moves illustrate the effect of cube moves to permitted and forbidden. P3 means that the 3 was placed in permitted. No cube sets are extinguished, but those containing a 3 need one less cube from resources.

The next move, F+, places one of the two addition cubes in the forbidden section. It has the effect of extinguishing all solutions that contain both addition cubes. The addition cube remaining in the resources may still possibly be used as a member of the other cube sets. Note that all remaining cube sets need the same number of cubes from resources as they needed before the F+ move. The fourth move is a P-flub. It forbids the remaining addition cube and thus extinguishes all the remaining cube sets and corresponding equivalence classes of solutions. There is no way to play the remaining resource cubes so that a solution can be built. Cp refers to a P-flub challenge made by the next player. Table 4 illustrates how the same game can be played in another way. Here the first move is Fx. The multiplication cube is played into the forbidden section. This has the effect of extinguishing all cube sets corresponding to subset L. All remaining cube sets need the same number of cubes from resources that they needed before the move was made. The P2 move places one of the two 2 cubes in permitted. Again, a permitted move extinguishes no cube sets, but one less cube is needed from resources for each cube set that contains a 2 cube. The third move places a 3 in the required section. It has the effect of extinguishing cube sets that do not contain that cube and reducing the number of cubes needed from resources of each remaining cube set. The fourth move is an A-flub. It places one of the addition cubes in permitted. This extinguishes no cube sets, but reduces one of the cube sets to needing only one cube from resources. Because a 3 is in required and a + is in permitted, the mover has allowed a solution to be built with one more cube from resources -- namely, the 6. Thus, solutions 3 + 6 or 6 + 3 could be used to sustain the burden of proof.

These two sample games illustrate the effect of moves to required, permitted, and forbidden on the cube sets and the corresponding equivalence classes of solutions. They illustrate how P- and A-flubs can be detected by the experimenter. (If other cube moves are made after the flub move, then a C-flub will have been made and can be detected.) Most important, they illustrate ways to play the game such that players are confronted with different mathematical ideas.

This is one suggested approach for developing measures within a dynamic game situation. The game itself delineates the problem space. We can then measure the changes in that problem space by the way the game is played.

In the previous examples, the first move drastically altered the problem space in that an entire subset was extinguished. In using experimental games we can record whether subset M or subset L or either were extinguished before a challenge is made which ends the game. We can also record how early in the game a player may choose to alter the problem space in this way.

Goal = 9

Resources +, +, x, 1, 2, 2, 3, 4, 5, 6, 7, 8

Cube Sets	Representative Solution of Equivalence Class		Number of Cubes Needed from Resources to Complete the Solution	Moves: Fx	P2	R3	P+	Ca
{+,x,1,2,7}	(2 + 7) x 1	Subset M	5	x				
{+,x,1,3,6}	(3 + 6) x 1	Subset M	5	x				
{+,x,1,4,5}	(4 + 5) x 1	Subset M	5	x				
{+,+,x,1,2,2,5}	(2 + 2 + 5) x 1	Subset M	7	x				
{+,+,x,1,2,3,4}	(3 + 2 + 4) x 1	Subset M	7	x				
{+,x,1,2,4}	1 + (4 x 2)	Subset M	5	x				
{+,x,1,2,3}	3 x (2 + 1)	Subset M	5	x				
{+,x,2,2,5}	5 + (2 x 2)	Subset M	5	x				
{+,+,x,1,2,2,3}	[(2 + 1) x 2] + 3	Subset M	7	x				
{+,1,8}	1 + 8	Subset L	3	3	3	x		
{+,2,7}	2 + 7	Subset L	3	3	2	x		
{+,3,6}	3 + 6	Subset L	3	3	3	2	1	
{+,4,5}	4 + 5	Subset L	3	3	3	x		
{+,+,1,2,6}	1 + 6 + 2	Subset L	5	5	4	x		
{+,+,1,3,5}	1 + 3 + 5	Subset L	5	5	5	4	3	
{+,+,2,2,5}	2 + 2 + 5	Subset L	5	5	4	x		
{+,+,2,3,4}	3 + 4 + 2	Subset L	5	5	4	3	2	

Table 4

Description of a Play of an Experimental Game

In the event the first move did not extinguish either subset and was not an A-flub, we can go further. Before the first move, there were nine equivalence classes in subset M and eight in subset L. We can observe the number of equivalence classes in the two subsets after the move. The ratio of M to L equivalence classes after the move divided by the ratio of M to L equivalence classes before the move is a number which reflects the strength of the move in extinguishing M equivalence classes relative to L equivalence classes. Indeed we can classify each potential move as being (1) a flub move, (2) a move that extinguishes only subset M or only subset L, (3) a ratio reflecting the strength of the move in extinguishing M relative to L equivalence classes. Thus, we not only can classify the moves that were made, but also the set of possible moves from which that move was selected.

Discussion

In the resource allocation decision a player must, at least, evaluate his own understanding of the effects of that decision on the problem space and, at most, diagnose his opponent's understanding. In other words, he must (1) recognize whether or not a resource allocation move is a flub move, (2) evaluate his understanding of the possibly altered problem space given his move, and (3) estimate whether or not his opponent(s) will be able to evaluate that altered problem space. Since the problem space is finite and well defined, the uncertainty is in terms of a player's grasp of the problem space and his diagnosis of the other player's understanding of it.

It is to the player's advantage to alter the problem space in such a way that he can still handle it, but that his opponent will incorrectly perceive it and either will make an incorrect challenge or a flub move that the player is prepared to challenge.

In the examples given earlier, if the multiplication operation is new to most players and addition is relatively familiar to all, an adventurous player who sees at least one solution in subset M may choose to make moves that extinguish subset L on the presumption that his opponents do not have the same understanding. A less confident player may choose to alter the problem space by extinguishing subset M. Further, by allocating appropriate resources to required or permitted he may bring his solutions within two cubes of completion and hope an opponent with less understanding will erroneously play one of those cubes in required or permitted commiting an A-flub.

All experimental games are designed under the assumption that solutions corresponding to equivalence classes of one subset are more difficult than those in the other subset. By observing how players alter the problem space we can estimate their willingness to use mathematical concepts in a problem context posed by a game.

In the usual test situation, a student is forced by the test designer to consider a particular problem. The purpose is to measure performance on certain specified tasks where a high score is the reward. In the experimental game situation, more control of the problem to be considered is given to the students in the game. The purpose is to measure the willingness to consider certain problems where the reward is short or long term winning of games. Here the player's selection and solving of mathematics problems is used to solve the larger problem posed by the game.

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