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A GENERAL PROCEDURE FOR APPROXIMATING STANDARD ERRORS OF ESTIMATE IN MULTIPLE MATRIX SAMPLING

David M. Shoemaker

Multiple matrix sampling or, more popularly, item-examinee sampling, is a procedure in which a set of $\underline{K}$ test items is subdivided randomly into $\underline{t}$ subtests containing $\underline{k}$ items each with each subtest administered to $\underline{n}$ examinees selected randomly from the population of $\underline{N}$ examinees. Although each examinee receives only a proportion of the $\underline{K}$ test items, the statistical procedures given by Hooke (1956) and Lord (1960) permit the researcher to estimate parameters of the test score distribtion which would have been obtained by testing all $\underline{N}$ examinees over all $\underline{K}$ test items.

One problem of no minor concern in item-examinee sampling is estimating the standard error of estimate associated with each estimated parameter. Computing the standard error of estimate is essential because knowing the precision with which a parameter has been estimated is a primary importance in any decision-making which occurs after the data have been analyzed. Although all researchers would agree that standard errors should be computed in any multiple matrix sampling investigation, the problem is that few equations are available currently for doing just this. Hooke (1956a, 1956b) has outlined a general procedure for computing the standard error of estimate for lower order moments but expanding this procedure to higher order moments and to other parameters is not a casual undertaking.

What is needed in multiple matrix sampling is a simple procedure for computing standard errors of estimate. The jackknife--a relatively obscure statistical procedure--seems to satisfy this need and the research supporting this statement is described herein.

THE JACKKNIFE

The jackknife procedure was described originally by Quenouille (1956) and put forth as a method for bias reduction in estimating parameters. The name "jackknife" was given subsequently to this procedure by Tukey (Mosteller and Tukey, 1968) to "suggest the broad usefulness of the technique as a substitute for specialized tools that may not be available, just as the Boy Scout's trusty tool serves so variedly." A good description of the jackknife applied to a variety of estimation problems is given by Mosteller and Tukey (1968). Additional descriptions of the procedure are given by Miller (1964), Jones (1965) and Mosteller (1971).

The jackknife operates on a data set which has been divided into subgroups of data and gives a mean estimate of the parameter computed over subgroups and an estimate of the standard error of estimate associated with this estimator. A basic component of the jackknife is the pseudovalue associated with each subgroup which, for each subgroup, is the weighted difference between the statistic computed on all the data and the statistic computed on the body of data that remains after omitting that subgroup. Because the pseudovalues are relatively independent of each other, the standard error of the statistic is computed according to the well-known formula for the standard error of a sample mean. The computations are relatively simple. Let

t = the number of subgroups,

y_{all} = the statistic computed on all the data, and

$y_{(j)}$ = the statistic computed on all the data left after removing subgroup j.

The pseudovalues, y_{*j}, are then equal to

$$y_{*j} = ty_{all} - (t - 1)y_{(j)} \quad \text{for } j = 1, 2, \ldots, t. \tag{1}$$

The jackknifed estimate of the parameter is equal to

$$y_* = (y_{*1} + y_{*2} + \ldots + y_{*t})/t \tag{2}$$

with an estimate of its variance given by

$$s_*^2 = \frac{\sum^{t}(y_{*j} - \overline{y_{*j}})^2}{t(t - 1)} . \tag{3}$$

If the statistics computed on each subgroup are weighted equally, the pseudovalues reduce algebraically to the averages for the subgroups. In this case, y_* is equal to y_{all} and s_*^2 is equal to the variance of the subgroup statistics. When the jackknife is applied to multiple matrix sampling there are t subgroups of data but only one score for each subgroup with that score weighted according to the number of observations nk acquired through that subtest.

An example may be helpful at this point. Shoemaker and Okada (1970) estimated the spelling proficiency of primary grade students through multiple matrix sampling and reported the following subtest results:

Subtest	No. of Observations (0)	$\hat{\mu}$	$\hat{\sigma}^2$
1	180	17.7780	158.1778
2	140	26.4285	53.8430
3	130	23.4615	230.0509
4	130	24.6155	265.5789
5	120	9.5835	169.3378

For this data,

$$\hat{\mu}_{all} = \frac{\overset{t}{\Sigma} O_i \hat{\mu}_i}{\overset{t}{\Sigma} O_i} = \frac{14300.0900}{700} = 20.4287$$

$$\hat{\mu}_{(1)} = \frac{\overset{t}{\Sigma} O_i \hat{\mu}_i - O_1 \hat{\mu}_1}{\overset{t}{\Sigma} O_i - O_1} = \frac{14300.0900 - (180)(17.7780)}{700 - 180} = 21.3462$$

$\hat{\mu}_{(2)} = 18.9287$ $\hat{\mu}_{(3)} = 19.7370$

$\hat{\mu}_{(4)} = 19.4738$, and $\hat{\mu}_{(5)} = 22.6725$.

The computed psuedovalues are equal to

$$\hat{\mu}_{*1} = t\hat{\mu}_{all} - (t - 1)\hat{\mu}_1 = (5)(20.4287) - (4)(21.3462) = 16.7587$$

$$\hat{\mu}_{*2} = 26.4287 \qquad \hat{\mu}_{*3} = 23.1955$$

$$\hat{\mu}_{*4} = 24.2483, \text{ and} \qquad \hat{\mu}_{*5} = 11.4535.$$

The pooled estimate of the parameter, $\hat{\mu}_*$, is

$$\hat{\mu}_* = (16.7587 + 26.4287 + \ldots + 11.4535)/5 = 20.4169.$$

Its associated standard error is the square root of

$$s_*^2 = \frac{\sum^{t} \hat{\mu}_{*j}^2 - \dfrac{(\sum^{t} \hat{\mu}_{*j})^2}{t}}{t\,(t - 1)} = 7.6133$$

and is equal to

$$s_* = 2.7592.$$

Jackknifing $\hat{\sigma}^2$ produces

$$\hat{\sigma}_*^2 = 171.7624 \text{ and } s_* = 34.9603.$$

Confidence intervals for both $\hat{\mu}$ and $\hat{\sigma}$ can be computed using the appropriate standard error of estimate in conjunction with the _t_-distribution having t - 1 degrees of freedom. Other estimators obtained from subtest results are jackknifed similarly to give a pooled estimate of the parameter and its associated standard error of estimate.

METHOD

The computations involved in the jackknife are simple enough, but does the procedure work in multiple matrix sampling? A question such as this is answered easily through post mortem item-examinee sampling with the required data bases generated through a computer simulation model. In post mortem sampling, an entire data base (examinee by item matrix) is generated and the researcher samples both items and examinees from this matrix acting as if only certain examinees had been tested over certain items. To show how post mortem sampling may be used to test the jackknife, consider the following example. Assume that a data base exists and that the parameters of this data base are to be estimated through a ($\underline{t}$=5/$\underline{k}$=10/$\underline{n}$=30) sampling plan. One application of this sampling plan to the data base produces five independent estimates of each parameter which are pooled subsequently to give the single best estimate of each parameter. These five estimates may also be jackknifed (using equations 1 and 3) to give an estimated standard error for each parameter as was done with the Shoemaker and Okada results. So, with one application of the (5/10/30) sampling plan, a pooled estimate of each parameter is produced as well as the jackknifed estimate of its standard error. If this sampling were replicated $\underline{r}$ times, $\underline{r}$ estimates of each parameter would be produced as well as $\underline{r}$ estimates of the jackknifed standard error for each parameter. At the end of $\underline{r}$ replications, <u>two</u> estimates of the standard error of estimate for each parameter may be computed. The first estimate is obtained by computing the standard deviation of the $\underline{r}$ estimates of each parameter; the second, by computing the mean over replications of the jackknifed standard errors for each parameter. If the jackknife works, the standard errors computed in these two ways should be very similar. An additional check is possible for the standard error of the mean test score. When tk is less than or equal to K, the standard error of the mean test score may be estimated by the equation given by Lord and Novick (1968, equation 11.12.3) modified to give the standard error of the mean test score instead of the mean proportion correct score.

Such was the rationale employed in this investigation. Parameters of the data base manipulated systematically were: (a) the number of test items (K = 40, 60), (b) the variance of the item difficulty indices (σ_p^2 = .00, .05), and (c) the degree of skewness in the normative distribution of test scores (distributed normally, markedly negatively-skewed). Additionally, for all negatively-skewed normative distributions, only σ_p^2 = .00 was used. The test reliability of the six normative distributions generated was set at .80. All items were scored dichotomously. For the normal normative distributions, the mean test score was 50 per cent of items answered correctly; for negatively-skewed distributions, 80 per cent correct. Fourteen sampling plans (listed in the left column of each Table) were used with each data base. Each sampling plan was replicated 25 times. It should be noted that, when tk was greater than K, items were sampled randomly but subject to the restriction that each item appears with equal frequency across subtests.

The entire procedure was accomplished by a computer simulation model.[1] The parameters estimated were μ_1' (the mean test score), μ_2, μ_3, μ_4 (the second through fourth central moments) and σ_p^2. The equations used to estimate the moments of the test scores were those given by Lord (1960); σ_p^2 was estimated through a components of variance analysis.

RESULTS

All results are given in Tables 1 through 8. Because the results in each Table are interpreted similarly, only the results for one sampling plan in one Table will be described in detail. In Table 1, for example, are given the standard errors of estimate for the mean test score (μ_1') as a function of K and σ_p^2 using a normal normative distribution of test scores. For the first sampling plan, (02/10/100), the standard deviation of the 25 estimates of the population mean test score was .5149. The mean of the jackknifed standard errors over the 25 replications was .3967 and the standard deviation of these 25 jackknifed estimates was .3085. With this sampling plan, tk was less than K so that the standard error equation for the mean given by Lord and Novick is applicable. Before this equation was used, however, it was modified to give the standard error of the mean test score for an infinitely large examinee population. Because all parameters of the normative distribution were known in advance (they had to be specified to use the computer simulation model), the standard error of the mean test score was computed exactly and found to be .6072 for this sampling plan. (This standard error will be somewhat larger than that computed for a finite number of examinees.) Changing the variance of the item difficulty indices in the data base from .00 to .05 and recomputing these same statistics gives, respectively, 1.6634, 1.9406, 1.4909 and 1.5548.

Considering all results, the jackknife did approximate well the standard error of estimate for each parameter for a given sampling plan. Additionally, the variability of the jackknife was found to decrease with increases in the number of observations tkn acquired by the sampling plan. There was one place, however, where the jackknife did not appear to work well, that is, in estimating the standard error of the mean test score when the variance of the item difficulty indices was equal to .05. Because only a limited number of sampling plans were used in this exploratory investigation, the results do not lend themselves to any statements about the effect of variations in t, k and n on the standard error of the jackknife.

Please insert Tables 1 through 8 about here.

DISCUSSION

It goes almost without saying that the results of this investigation need to be expanded to other sampling plans and to other normative distributions with particular emphasis on the effect of variations in t, k and n on the standard error of the jackknife. The point is, however, that the jackknife did perform surprisingly well in estimating the standard errors for 5 parameters of 6 normative distributions over 14 sampling plans. The jackknife did not work well, however, in approximating the standard error of the mean test score when the variance of the item difficulty indices was equal to .05. The reason for this is not apparent at this time. It should be noted that the similarity between the standard errors of estimate for the mean test score computed over replications and computed exactly by a form of the Lord and Novick equation lends creditability to the simulation model employed in this investigation.

One wonders naturally if, instead of pursuing the jackknife, it would have been more profitable to derive algebraically the standard error equations for other estimators in multiple matrix sampling. The answer to this question is not readily apparent -- and for this reason. In this investigation, all parameters were known in advance and the standard error of estimate for the mean test score could be computed exactly (given tk less than or equal to K). When multiple matrix sampling is used in practice, these parameters are not known and must be estimated. In this case, estimates of parameters are inserted in the standard error equation. This means, simply, that over replications there is a sampling distribution associated with the standard error equation; stated differently, there is a standard error of estimate associated with a standard error of estimate just as there was for the jackknife in this investigation. Such would be the case for any standard error equation. It will probably be the case that, were these standard error equations derived and tested as the Lord and Novick equation was tested here, they would behave in a manner similar to the jackknife. Of course, time will tell. In the meanwhile, the jackknife is ready, willing and seemingly able. Although in one situation it gives a very conservative estimate.

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TABLE 1

Standard Error Of Estimate For $\hat{\mu}'_1$ Approximated By The Jackknife And Computed Over Replications For Selected Sampling Plans As A Function Of K And σ_p^2 For A Normal Normative Distribution

	Sampling Plan (t/k/n)	$\sigma_p^2 = .00$ SE(R)	MN(J)	SD(J)	SE(LN)	$\sigma_p^2 = .05$ SE(R)	MN(J)	SD(J)	SE(LN)
	02/10/100	.5149	.3967	.3085	.6072	1.6634	1.9406	1.4909	1.5548
	04/10/050	.5721	.5999	.2638	.6072	.4016	1.2267	.4775	.5810
	04/10/100	.3765	.3756	.1513	.4125	.2897	1.4118	.6236	.3947
K = 40	10/04/100	.2362	.2786	.1020	.3451	.2828	1.3678	.2811	.3191
	08/10/050	.4697	.3804	.1361		.3423	.9828	.2540	
	08/10/100	.2328	.2631	.0828		.2516	.8989	.2875	
	10/08/100	.2759	.2571	.0865		.1737	.9741	.2625	
	03/10/100	.4117	.4305	.2966	.7056	1.5611	2.3163	.9218	1.9193
	06/10/050	.6092	.5591	.2466	.7056	.5039	1.7860	.4761	.6662
	06/10/100	.4254	.3722	.1213	.4989	.3504	1.6949	.5218	.4711
K = 60	10/06/100	.3177	.3484	.1110	.4573	.3518	1.6956	.3085	.4242
	12/10/050	.3931	.3797	.0901		.4545	1.1611	.2772	
	12/10/100	.3352	.3350	.0850		.2443	1.1725	.3153	
	10/12/100	.3219	.3190	.1062		.2924	1.2865	.3567	

TABLE 2

Standard Error Of Estimate For $\hat{\mu}_2$ Approximated By The Jackknife And Computed Over Replications For Selected Sampling Plans As A Function Of K And σ_p^2 For A Normal Normative Distribution

	Sampling Plan (t/k/n)	$\sigma_p^2 = .00$			$\sigma_p^2 = .05$		
		SE(R)	MN(J)	SD(J)	SE(R)	MN(J)	SD(J)
	02/10/100	15.8331	10.5088	10.1114	7.0780	6.8182	5.9441
	04/10/050	15.9240	10.4511	7.0490	7.3381	8.0812	3.1723
	04/10/100	14.4541	11.4102	4.9912	4.4280	7.7925	2.4018
K = 40	10/04/100	27.5240	19.2121	6.9794	8.2222	9.6352	2.6712
	08/10/050	9.8313	8.0357	2.8034	5.1637	6.2785	1.8165
	08/10/100	9.8782	8.3637	3.2956	3.5200	4.7195	1.1799
	10/08/100	8.2635	8.8302	2.7173	3.8916	5.7127	1.5841
	03/10/100	26.7023	25.7504	11.9774	12.9464	13.3455	6.2556
	06/10/050	32.3783	24.9747	13.1385	12.7801	12.5252	4.9749
	06/10/100	18.5320	18.8286	7.0554	8.6765	11.9152	5.4741
K = 60	10/06/100	39.3374	31.7889	11.0710	8.4013	13.0341	3.8567
	12/10/050	17.7001	16.9813	5.2243	10.6897	10.1299	2.5446
	12/10/100	14.6975	14.9659	4.5392	8.4045	8.7757	2.2705
	10/12/100	19.7716	13.8469	5.6610	5.8985	7.9363	2.3148

TABLE 3

Standard Error Of Estimate For $\hat{\mu}_3$ Approximated By The Jackknife And Computed Over Replications For Selected Sampling Plans As A Function Of K And σ_p^2 For A Normal Normative Distribution

	Sampling Plan (t/k/n)	$\sigma_p^2 = .00$			$\sigma_p^2 = .05$		
		SE(R)	MN(J)	SD(J)	SE(R)	MN(J)	SD(J)
	02/10/100	63.0861	68.7554	46.5141	86.0590	57.6633	45.5443
	04/10/050	77.8591	72.7541	45.6154	61.5731	82.5499	48.7511
	04/10/100	72.7280	50.5585	20.1283	52.8407	63.3402	26.8867
K = 40	10/04/100	149.5387	100.8578	45.9527	107.5407	100.5404	34.7892
	08/10/050	51.5375	53.5500	15.4231	68.7070	57.5452	20.8549
	08/10/100	35.7129	40.2517	14.2071	41.1689	41.7987	12.3384
	10/08/100	40.2367	49.3861	17.2799	42.8286	53.6205	18.3809
	03/10/100	187.0407	231.2016	126.7943	197.9350	164.1716	99.7486
	06/10/050	211.6935	199.3803	94.0182	212.1996	169.5908	54.4030
	06/10/100	111.8895	106.6647	33.8299	96.8829	113.1980	56.9351
K = 60	10/06/100	168.1126	188.3676	53.3423	166.2711	190.8236	62.3389
	12/10/050	137.2293	128.2513	26.1686	133.2780	125.9036	39.1290
	12/10/100	93.9193	106.6978	22.8533	77.1626	113.4519	39.2569
	10/12/100	86.9515	90.1385	38.7612	77.7561	101.1884	32.8962

TABLE 4

Standard Error Of Estimate For $\hat{\mu}_4$ Approximated By The Jackknife And Computed Over Replications For Selected Sampling Plans As A Function Of K And σ_p^2 For A Normal Normative Distribution

	Sampling Plan (t/k/n)	$\sigma_p^2 = .00$ SE(R)	MN(J)	SD(J)	$\sigma_p^2 = .05$ SE(R)	MN(J)	SD(J)
	02/10/100	4296.5078	3271.5923	3349.0579	1556.5723	1502.5593	1328.4954
	04/10/050	3671.2913	2615.9692	2187.2939	2217.8169	1494.3716	628.3235
	04/10/100	2963.8218	2554.9854	1273.9238	959.0288	1270.8542	493.9285
K = 40	10/04/100	13134.2422	12545.5078	3930.3921	3283.7458	3287.2573	954.7429
	08/10/050	1859.1667	1818.9404	613.4961	1190.5420	1305.9075	414.9688
	08/10/100	2022.6479	1943.7612	1212.6816	781.7568	872.4282	283.4258
	10/08/100	1935.1479	2156.4019	870.6963	1145.0813	1249.8762	437.6804
	03/10/100	16098.5625	10821.6055	7669.4609	4468.5508	4531.2539	2805.1099
	06/10/050	11608.2148	9955.4609	7730.3828	5671.8242	4840.6250	2637.9802
K = 60	10/06/100	20033.7500	21470.7852	10955.4258	6534.5430	6678.6523	2013.4229
	12/10/050	8008.4844	7452.0313	4211.0430	4079.5559	3498.9048	1222.9526
	12/10/100	8010.3945	5115.9844	2764.0667	3473.2029	3114.7148	1463.0310
	10/12/100	7379.2734	5468.8867	2975.4890	2762.1653	2586.9832	1078.2485

TABLE 5

Standard Error Of Estimate For $\hat{\mu}'_1$ And $\hat{\mu}_2$ Approximated By The Jackknife And Computed Over Replications For Selected Sampling Plans As A Function Of K For A Negatively-Skewed Normative Distribution

	Sampling Plan	$\hat{\mu}'_1$				$\hat{\mu}_2$		
	(t/k/n)	SE(R)	MN(J)	SD(J)	SE(LN)	SE(R)	MN(J)	SD(J)
	02/10/100	.4256	.2902	.2275	.4858	11.4498	6.4656	6.1730
	04/10/050	.5992	.4595	.2313	.4858	8.3212	7.4218	3.5985
	04/10/100	.2646	.3416	.1293	.3300	5.6491	5.4651	2.8800
K = 40	10/04/100	.2522	.2496	.0960	.2761	9.7724	9.5674	2.8489
	08/10/050	.2939	.3377	.0962		5.1152	5.6191	1.7866
	08/10/100	.1992	.2099	.0691		4.6102	4.5464	1.9151
	10/08/100	.1867	.2040	.0594		5.7240	4.5706	1.5003
	03/10/100	.4586	.4470	.2637	.5644	16.0615	14.1774	7.9461
	06/10/050	.5362	.4804	.1980	.5644	21.3336	16.4572	7.5675
	06/10/100	.2889	.3302	.1331	.3991	14.3128	12.2970	5.5795
K = 60	10/06/100	.3102	.3086	.0829	.3658	17.3286	14.1394	5.1151
	12/10/050	.3389	.3179	.0679		9.4212	9.5727	2.8003
	12/10/100	.2092	.2352	.0763		7.6306	8.2185	2.2768
	10/12/100	.2553	.2404	.0645		8.8187	7.2651	2.6363

TABLE 6

Standard Error Of Estimate For $\hat{\mu}_3$ And $\hat{\mu}_4$ Approximated By The Jackknife And Computed Over Replications For Selected Sampling Plans As A Function Of K For A Negatively-Skewed Normative Distribution

	Sampling Plan (t/k/n)	$\hat{\mu}_3$ SE(R)	$\hat{\mu}_3$ MN(J)	$\hat{\mu}_3$ SD(J)	$\hat{\mu}_4$ SE(R)	$\hat{\mu}_4$ MN(J)	$\hat{\mu}_4$ SD(J)
	02/10/100	208.3827	113.9347	101.6108	6526.7422	3301.1499	3202.2119
	04/10/050	127.4083	124.1658	52.4704	3572.5359	3443.9348	1897.1006
	04/10/100	100.7917	94.0375	48.0740	2729.9880	2690.4424	1381.5217
K = 40	10/04/100	123.8115	142.8001	55.3120	5065.5117	4248.3047	1923.9690
	08/10/050	84.2587	95.8375	32.3467	2200.5508	2726.6399	883.7183
	08/10/100	74.0035	76.0545	29.8502	2296.9358	2166.3672	876.1030
	10/08/100	80.7765	72.[illegible]685	18.9204	2197.3167	2049.6492	640.7891
	03/10/100	232.2791	236.2937	138.5313	10449.1211	7967.1563	5159.2422
	06/10/050	377.0012	377.7117	162.5162	13338.5391	15161.4375	7406.1133
	06/10/100	262.1711	248.2847	101.1652	9623.5820	9707.2305	4254.3164
K = 60	10/06/100	268.8792	254.0711	132.1098	9988.9453	9643.3398	6577.8867
	12/10/050	190.6631	192.2248	72.6866	7796.2500	6956.9023	3216.0015
	12/10/100	147.4538	165.4620	48.2966	6231.8242	6389.8359	1993.4380
	10/12/100	151.0264	163.3851	50.5197	6301.7344	6505.2695	2535.6865

TABLE 7

Standard Error Of Estimate For $\hat{\sigma}_p^2$ Approximated By The Jackknife And Computed Over Replications For Selected Sampling Plans As A Function Of K And σ_p^2 For A Normal Normative Distribution

	Sampling Plan (t/k/n)	$\sigma_p^2 = .00$ SE(R)	$\sigma_p^2 = .00$ MN(J)	$\sigma_p^2 = .00$ SD(J)	$\sigma_p^2 = .05$ SE(R)	$\sigma_p^2 = .05$ MN(J)	$\sigma_p^2 = .05$ SD(J)
	02/10/100	.0011	.0009	.0011	.0103	.0127	.0107
	04/10/050	.0010	.0012	.0006	.0059	.0112	.0060
	04/10/100	.0007	.0006	.0004	.0058	.0115	.0038
K = 40	10/04/100	.0009	.0007	.0004	.0077	.0123	.0029
	08/10/050	.0008	.0009	.0005	.0043	.0088	.0019
	08/10/100	.0007	.0006	.0003	.0034	.0077	.0021
	10/08/100	.0005	.0006	.0003	.0042	.0076	.0018
	03/10/100	.0005	.0004	.0003	.0112	.0100	.0041
	06/10/050	.0010	.0008	.0005	.0060	.0098	.0029
	06/10/100	.0006	.0005	.0003	.0043	.0076	.0025
K = 60	10/06/100	.0006	.0006	.0004	.0040	.0086	.0016
	12/10/050	.0007	.0006	.0003	.0041	.0062	.0015
	12/10/100	.0005	.0006	.0002	.0034	.0063	.0012
	10/12/100	.0006	.0005	.0003	.0039	.0062	.0014

TABLE 8

Standard Error Of Estimate For $\hat{\sigma}_p^2$ Approximated By The Jackknife And Computed Over Replications For Selected Sampling Plans As A Function Of K For A Negatively-Skewed Normative Distribution

	Sampling Plan (t/k/n)	SE(R)	MN(J)	SD(J)
	02/10/100	.0007	.0006	.0006
	04/10/050	.0013	.0011	.0009
	04/10/100	.0006	.0006	.0004
K = 40	10/04/100	.0007	.0005	.0003
	08/10/050	.0007	.0009	.0004
	08/10/100	.0006	.0005	.0003
	10/08/100	.0004	.0004	.0002
	03/10/100	.0008	.0006	.0005
	06/10/050	.0009	.0008	.0006
	06/10/100	.0005	.0004	.0004
K = 60	10/06/100	.0006	.0005	.0003
	12/10/050	.0003	.0005	.0002
	12/10/100	.0004	.0004	.0002
	10/12/100	.0004	.0003	.0002

Footnotes

1. A listing and expanded writeup of the computer program implementing the simulation model is available upon request from the author.