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ABSTRACT

This study examines the effects of using different classes of ratios on adolescent performance on proportionality problems. Problems of the form a/b = c/x varied on four dimensions: size of a/b, equality or inequality of b and c, complex or simple fractions, and form of the test. Tests consisted of six proportionality problems involving sizes of stick figures. Thirty-two seventh-grade students served as subjects. Responses were scored on the 10-point scale developed by Karplus, and the resulting data were submitted to several analyses of variance. Results indicated that only one-fourth of the problems were solved by strategies based on understanding of proportionality. Subjects were able to solve some of the easier problems, suggesting that they had (or were developing) some intuitive understanding of proportionality. The author suggests that content of a proportionality task may strongly affect students' performance on it. (SD)

Adolescent Understanding of Proportionality: The Effects of Task Characteristics

by

Susan Abramowitz

Abstract

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Six proportionality problems involving prediction of the height of stick figures using ratios were administered to 32 seventh grade students. Task characteristics of equal or unequal differences between numbers, size of the unknown number, and type of ratio were investigated. Ss performance was rated on a developmental scale and the effects of subject ability and task characteristics were described Results confirmed the ordering of the scale and showed significant effects for type of ratio and size of the unknown. Student ability was a significant factor in performance. Implications of the findings for developmental theory and further research were discussed.

Adolescent Understanding of Proportionality: The Effects of Task Characteristics

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An understanding of proportionality has been identified with Piaget's stage of formal operational reasoning, which theoretically emerges at the ages of 12 to 13 years (Inhelder & Piaget, 1958). Piaget investigated the child's acquisition of proportionality by examining children's reactions to situations such as equilibrium on a balance and shadow size. He found that younger children (seven to twelve years) dealt with these problems by using arithmetic solutions, whereas adolescents (13 to 17 years old) demonstrated understanding of porportional increase and decrease and reciprocity between various relations. Piaget also found that children demonstrated an intuitive understanding of proportionality before they could deal with it quantitatively.

Studies investigating the developmental acquisition of proportionality have compared children's performance across several tasks to determine under what circumstances and at what age level an understanding of proportionality becomes operational. Several investigators have found that children under the age of fourteen do not have a well-

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developed understanding of proportionality. There is general agreement however, that formal operational thought is prerequisite to the solution of problems involving proportional relationships (Lunzer & Pumphrey, 1966; Lovell & Butterfield, 1966). There is also considerable experimental evidence that children employ an additive strategy prior to the onset of proportional thought regardless of the materials used. (Lunzer & Pumphrey, 1966; Karplus & Peterson, 1970; Karplus & Karplus, 19721 Wollmen, et al., 1973)

A prolific investigator in this area is Robert Karplus, who devised a test to determine the level of abstract reasoning children use in a ratio and proportion task (Karplus & Peterson, 1970). In the Karplus task children were presented with a drawing of a large stick figure (Mr. Tall), the height of which was measured with large paper clips (biggies). A drawing of a small figure (Mr. Short), was then presented and measured with biggies. Subjects were asked to measure Mr. Short with small paper clips (smallies), to predict the height of Mr. Tall in smallies, and to write an explanation of how they arrived at their prediction.

Subsequently Karplus modified his task to prevent children from relying on perceptual cues. In the altered task, the <u>Ss</u> were asked to predict the height of Mr. Tall without seeing the figure. The <u>Ss</u> were supplied with the same information as in the initial task (i.e., Mr. Tall's height in biggies). The important difference was that they were unable to rely on any perceptual comparisons between the two figures to help them solve the problem because they saw only one figure (Mr. Short).

Karplus analyzed Ss protocols and isolated the following response strategies.

1. N - No explanation

2. I - An explanation referring to estimates without reference to the data.

3. IC - An explanation using the data haphazardly.

4. S (scaling) - An explanation based on a change of scale that the subject does not justify in terms of the data.

5. A (addition) - An explanation focussing on a single difference, and solving the problem by addition.

6. AS (addition and scaling) - An explanation in which the difference between measurements is first isolated and then related by multiplication to one of the measurements.

7. IP (incomplete proportion) - An explanation making use of one ratio involved in the proportion, but not applying the ratio correctly.

8. PC (proportion concrete) - An explanation using the correct ratio of measurements but applying it by actually measuring off the ratio on the figure given.

9. AP (addition and proportion) - an explanation using the correct ratio but applying it by addition.

10. R (ratio) - An explanation using a proportion or deriving the scale ratio from the data, and applying the ratio in a proportion.

Karplus classified categories N and I (1-2) as being preoperative and categories PC, AP and R (8-10) as being examples of formal operational thought. The other categories were considered evidence of Ss preference for handling the material in a certain way rather than of developmental stages. Because the distribution of Ss within the categories changed with a change in materials, Karplus suggested that these are transitional categories which cannot be ordered in a developmental sequence in a specific way.

The use of a developmental scale to describe the acquisition of proportionality may be questioned. The scale may be relevant only to comparisons involving simple whole numbers. Alternatively, the use of proportions or the idea of comparing numbers by division may not be spontaneous. It may have to be learned. It is possible that students faced with the limitations of their relational reasoning may be more susceptible to learning how to use proportions, as opposed to naturally or innately conceiving of the idea. Lastly, the problem may be one of discrimination. It is possible that children may know how to solve problems which involve proportionality. They may be simply unable to discriminate those situations in which a particular strategy is appropriate.

The extent to which the stimulus materials controlled responses in the Karplus work also remains to be determined. The problem used by Karplus, $^4/6 = {}^6/x$, may have biased the children who had an incomplete understanding of proportionality towards the use of an additive mode. The difference between the numerators is two, as is the difference between the numerator and the denominator in the first ratio. The repeated difference of two in this problem may have suggested a strategy that children do not generally employ.

The present study was designed to illuminate the effects of several task characteristics on adolescent performance with proportionality problems. The Karplus scale was also studied by application to new data and analysis of results.

Method

Subjects

The subjects were 32 seventh-grade students from a San Francisco Bay area school. All were twelve to thirteen year olds from white, middle class backgrounds.

Teacher assessments were used to classify subjects as high or low ability students.

Materials

Proportionality problems were organized into test booklets. Each subject received a test booklet containing six tasks. Each task was presented on two pages. The first page showed a stick figure (Mr. X) measured by two sets of different colored loops. The subject was asked how many loops of each color it took to measure Mr. X. The second page showed a different sized stick figure (Mr. Y), measured by only one set of colored loops. The subject was asked how many loops of this color it took to measure Mr. Y, to guess how many loops of the other color it would take to measure Mr. Y, and to explain how he/she arrived at that answer.

Stimulus characteristics of the tasks included Difference (Equal/Unequal), Size (Larger/Smaller), and Type (Simple/Complex/Complex Multiple).

The presence or absence of a repeated difference between the measurements was designated as Difference (Equal/Unequal). Values for the problems were chosen so that there was sometimes a repeated difference ($^4/6 = {}^6/x$), and sometimes not ($^4/6 = {}^{10}/x$). The extent to which the numbers used influenced subjects to use a differencing strategy (subtracting the numerator from the denominator or one number from the other) could then be assessed.

The second stimulus characteristic investigated was Size (Larger/Smaller). In some of the problems the unknown number was larger than the numbers already known, in others it was smaller.

The third stimulus characteristic was Type (Simple/Complex/Multiple Complex). Three possible relationships were used: a) small whole numbers involving factors of 2, 3, etc. with the unknown always an integer; b) complex multiples involving factors of 1 1/2, 1 1/4, 2/3, etc. with the unknown always an integer; and c) complex ratios involving more complex factors with the known always a mixed number. It was expected that most Ss would be able to solve the small whole numbers successfully. The complex multiples were expected to present greater difficulty, and the complex ratios the most.

A fourth stimulus characteristic was labeled Materials. Two sets of proportion problems were constructed. Each set included the twelve possibilities in crossing three levels of Type with two levels of Size and two levels of Difference. The sets were designed to provide information about generalizability across particular numbers and were designated as Form (A/B). Task variations and examples are summarized in Table 1.

Insert Table 1 about here

Fractional factorial designs were used to designate what stimulus characteristics and their order went to each subject.

TABLE 1

EXAMPLES OF TASK VARIATIONS

	Form A				Form B			
	Small Number		Large Number		Small Number		Large Number	
	Equal Difference	Unequal Difference	Equal Difference	Unequal Difference	Equal Difference	Unequal Difference	Equal Difference	Unequal Difference
Simple Whole Multiple	$1/2 = 2/x$	$1/2 = 3/x$	$4/2 = 2/x$	$6/3 = 2/x$	$1/3 = 3/x$	$1/3 = 4/x$	$9/3 = 3/x$	$6/2 = 3/x$
Complex Multiple	$4/6 = 6/x$	$4/6 = 14/x$	$9/6 = 6/x$	$12/9 = 8/x$	$4/10 = 10/x$	$4/10 = 14/x$	$25/10 = 10/x$	$25/10 = 15/x$
Complex Ratio	$5/7 = 7/x$	$5/7 = 8/x$	$7/5 = 5/x$	$7/5 = 4/x$	$3/5 = 5/x$	$3/5 = 7/x$	$9/5 = 5/x$	$9/5 = 4/x$

The first two proportionality problems in each test booklet involved proportions containing the ratios of small whole numbers. These two tasks involved variation in only three stimulus characteristics, since just one type of ratio was involved. Sincere there were three factors of two levels each, it was possible to generate eight basic tasks.

Each of the reamining four proportionality tasks in the booklet was described by four factor values, one from each of the following pairs of task characteristics: Size (Small/Large), Difference (Equal/Unequal), Type (Complex/Multiple Complex), Form (A/B). Since there were four factors of two levels each, 16 basic tasks were generated. The porportions of the small whole multiple type were omitted from these problems because they were considered relatively easy for the subjects to solve and therefore relatively insensitive to experimental manipulation.

Procedure

All subjects were presented with the test booklets in one group session. Each subject was asked to read the directions describing the proportionality task in the test booklet while the test administrator read them aloud. Questions were solicited. Before they began, Ss were reminded that the object of the problem set was to determine how they went about solving problems of this nature rather than whether they got a right or wrong answer. The Ss were also reminded not to look back at any figure once they answered questions about it, unless they were directed to do so. They were also told to answer all questions to the best of their ability.

Results

Scaling of Responses

Subject responses to each of the six proportionality problems were scored on the ten-point Karplus scale. Each point on the scale was designed to reflect a different strategy that could be used to solve the proportion problems.

Response categories were derived from the ten-point scale. Scores of 0-2 were considered indicative that the subjects had no ideas how to solve the proportion problems. Scores of 3 to 5 were taken as indicative that the subjects focused on a pattern independent of the ratio of the numbers. Scores of 6 through 9 were thought reflective of subjects who used a ratio to solve the proportion problems. These three categories were designated as non-patterned, patterned (inappropriate) and proportional, respectively.

Two scorers used the nine-point scale to score responses independently. All responses were coded as correct (3), almost correct (2), or incorrect (1). The correlation of the two sets of strategy scale scores was .80, indicating good inter-rater reliability.

An examination of the distribution of total subject response within the patterned category indicated that the most common pattern which subjects used in solving the proportion problems involved subtraction rather than multiplication (Table 2). Thus the differencing strategy serves as a modal point between the scaling and differencing strategies. Degree of correctness was calculated by averaging response codes for correctness (range 0-3).

Insert Table 2 about here

Two pieces of evidence support the ordering into three major and ten -subcategories. The fact that low ability subjects used the scaling strategy much more frequently than the high ability subjects indicates that the scaling strategy is a more primitive response than the differencing strategy. Such a placement was supported. The differential distribution of subjects who used a category three response and those who used a category five response also supports the modification of the Karplus ordering (Table 3).

Insert Table 3 about here

It is evident from the differential distribution of subject responses that those subjects whose responses were coded as three responded more frequently with responses from the lower end of the scale. Alternatively, subjects whose responses were coded as five responded more frequently with responses on the upper end of the scale.

The ordering of the scale was also verified by an examination of the degree of correctness of subject response with respect to each strategy on the scale (Table 2). The distribution of strategies by correctness indicated that subjects who scored at the mid-range of the scale were more likely to arrive at an almost correct solution. This relationship was more apparent when the ten point category scale was collapsed. The degree of correctness corresponding to each of the three categories was 1.36, 1.87 and 2.46. The extremes of both the uncollapsed and collapsed scale were also examined. Subjects

TABLE 2

DISTRIBUTION OF TOTAL SUBJECT RESPONSES IN THE THREE AND NINE CATEGORY SCALES

ABILITY AND DEGREE OF CORRECTNESS

Category Class	Non-Patterned			Patterned			Proportional			
Addition Strategy	No Explanation	Guess	Haphazard	Scale	Difference	Difference Plus Scale	Incorrect Ratio	Ratio	Ratio Plus Addition	Proportion
High ability	0	10	2	2	21	9	6	2	0	12
Low ability	1	17	6	10	12	6	7	4	1	0
Totals	1	27	8	12	33	15	13	6	1	12
Degree of Correctness	1.0	1.3	1.8	2.0	1.8	1.9	1.7	3.0	3.0	3.0

TABLE 3

DISTRIBUTION OF CODE THREE AND CODE FIVE RESPONSES

Number of Items	Strategies				
	Nonpatterned 0-2	Scale 3	Difference 4	Scale Plus Difference 5	Proportional 6-9
Code 3	11	12	8	3	6
Code 5	17	3	8	17	17

classified as using non-patterned responses showed a greater tendency to be incorrect in their answers, indicating not only a lack of facility with the mechanics of proportionality, but also a conceptual unawareness of the proper answer range. Alternatively, Ss classified as capable of solving the problems using a proportional strategy did so correctly.

Task Characteristics

The first two booklet problems contained task characteristics of Size (Large/Small), Difference (Unequal/Equal), and Form (A/B). Only ratios involving simple whole numbers were used in these proportionality problem. The one between subjects' variable investigated was Ability (High/Low). Contrast scores were derived by subtracting performance on one level of a factor from performance on the second level of the factor. A univariate analyses of variance (ANOVA) on contrast scores was used to analyze the data. None of the characteristics varied in these problems appeared to affect subject performance significantly (Table 4). The means of subject scores on the two tasks (Table 5) indicate, however, that subjects did solve problems involving simple fractions with nearly proportional strategies.

Insert Tables 4 and 5 about here

The other four booklet problems varied task characteristics of Type (Complex/Multiple Complex), Difference (Unequal/Equal), Size (Large/Small), and Form (A/B). A univariate ANOVA on contrast scores was again used to analyze the data.

TABLE 4

SUMMARY OF AN ANALYSIS OF VARIANCE FOR THE TASK FACTORS GENERATED FROM SUBJECTS SCORES ON TWO SESSION IA PROBLEMS

Source	d.f.	Mean Square	F
Size	1	0.0	0.0
Difference	1	12.50	3.39
Form	1	1.13	.30
Size x Difference	1	15.12	4.10
Error Term	16	3.69	
Ability	1	128.00	3.61
Error Term	16	35.40	

TABLE 5

MEANS AND STANDARD DEVIATIONS FOR PROBLEMS IN SESSION IA

Factor	Level	Mean Correct	S.D.
Ability	High	7.69	2.78
	Low	5.44	3.64
Size	Large	6.56	3.38
	Small	6.56	3.48
Difference	Unequal	6.69	3.28
	Equal	6.44	3.57
Form	B	6.09	3.65
	A	7.03	3.13

The mean values in Table 6 indicate that for this group of seventh grade subjects responses on the four tasks were primarily patterned (inappropriate). The average range of responses was at the upper end of the patterned (inappropriate) category for the easy level of a factor and at the lower end of the patterned (inappropriate) category for the harder level of a factor.

Insert Table 6 about here

This use of patterned (inappropriate) responses was quite logical from the subject's point of view. Subjects perceived a pattern which exists among the number involved and applied it. Examples of such patterned responses are:

1. Given: Mr. Al is 1 red and 3 blues; Mr. Bob is 4 reds. How many blues does it take to measure Mr. Bob?

   Answer: Seven

   <u>Subject Response</u>: On the first page it took 1 red to measure Mr. Al and 3 blues. Then the second time it took 4 reds, so I figured they just added the reds to tiny chains, so they added 1 + 3 which gives 4 red chains. Then I thought since it takes 3 blues and 4 reds, why not add them and then measure for blues.

2. Given: Mr. Ron is 9 reds and 5 greens; Mr. Sam is 4 reds. How many greens does it take to measure Mr. Sam?

   Answer: 0

   <u>Subject Response</u>: It took 9 reds to measure Mr. Ron. It also took 5 greens which is 4 less. And it took 4 reds to measure Mr. Sam, so I thought you'd subtract 4 from 4 which would give 0.

3. Given: Mr. Lou is 3 blacks and 5 blues; Mr. Moe is 5 blacks. How many blues does it take to measure Mr. Moe?

   Answer: 3

   <u>Subject Response</u>: Just the opposite.

TABLE 6

MEANS AND STANDARD DEVIATIONS FOR PROBLEMS IN SESSION IB

Factor	Level	Mean Correct	S.D.
Ability	High	4.79	2.54
	Low	3.35	2.04
Ratio	Complex ratio	3.72	1.99
	Complex multiple	4.72	2.63
Size	Small	3.72	2.54
	Large	4.44	2.24
Difference	Unequal	3.93	2.18
	Equal	4.21	2.63
Form	B	3.83	2.36
	A	4.33	2.46

Contrary to expectation, Difference was not a significant effect. The Form factor also failed to affect subject performance differentially. The only significant task characteristic effects were Type and Size. (Table 7)

Insert Table 7 about here

The Ss were tested on their ability to solve three types of porpotion problems: simple multiple ($^{1}/2 = {}^{2}/x$), complex multiple ($^{4}/6 = {}^{6}/x$), and complex ratio ($^{5}/7 = {}^{7}/x$). Ss used more sophisticated strategies to solve the simple multiples than they used to solve problems of the other two ratio types. Also a significant difference in performance between the other two types appeared favoring the complex multiples. Table 8 shows the distribution of responses on the three category scale for the ratio factor. Although subject responses were almost equally distributed between the patterned and proportional categories for problems of the complex multiple type, this distribution was skewed to the left for problems of the complex ratio type. Only four subjects used a proportional strategy on both levels of the ratio factor. The other eight subjects who used a proportional strategy to solve complex multiples used either a patterned (inappropriate) or a non-patterned strategy to solve complex ratios.

Insert Table 8 about here

Subjects performed better with proportions in which the unknown was larger than the known numbers than they did with proportions in which the unknown was smaller than the known numbers.

TABLE 7

SUMMARY OF AN ANALYSIS OF VARIANCE FOR THE TASK FACTORS GENERATED FROM SUBJECT SCORES ON FOUR PROPORTIONALITY PROBLEMS

Source	df	MS	F
Size	1	72.00	8.00*
Ratio	1	72.00	22.22**
Order	1	8.00	.88
Error	16	9.00	
Difference	1	8.00	.55
Error	16	14.63	
Size by Difference	1	4.50	.40
Ratio by Difference	1	4.50	.40
Error	16	11.38	
Ability	1	210.00	4.80*
Ability x Ratio	1	28.12	3.12
Ability x Size	1	45.12	5.01*
Error	16	9.00	
Ability x Difference	1	.12	.06
Error	16	14.63	

*$p < .05$

**$p < .01$

TABLE 8

DISTRIBUTION OF RESPONSES ON THE THREE CATEGORY SCALE FOR THE RATIO FACTOR

Item Type	Categories		
	Non-patterned	Patterned	Proportional
complex multiple	13	27	24
complex ratio	23	33	8

Table 9 indicates that half the responses were of a patterned type on proportions involving larger answers, with approximately a quarter of the responses of the non-patterned type and the remainder proportional. This distribution changed when the unknown was smaller than the known numbers. Although the decrease in the proportional category was not great, a considerable number of the subjects who used a patterned strategy reverted to a non-patterned one and were totally unable to solve the problem.

Insert Table 9 about here

The reversion was most noticeable for the low ability subjects as evidenced by a significant ability by size interaction (Figure 1). The performance of the high ability subjects reverted an average of only one category response when they were faced with proportions whose unknown was smaller. Performance of the low ability subjects dropped substantially in this condition; they gave primarily unpatterned responses.

Figure 1

The only between subjects variable which was significant was ability. Subjects designated as high ability students by their teacher performed at the upper end of the patterned (inappropriate) category, whereas lower ability students performed at the lower end of the same category (Table 10). Although there were an almost equal number of responses in the patterned (inappropriate) category for high and low ability subjects, more proportional strategies

TABLE 9

DISTRIBUTION OF RESPONSES ON THE THREE CATEGORY SCALE FOR THE SIZE FACTOR

Item Type	Categories		
	Non-patterned	Patterned	Proportional
Answer Larger	12	34	18
Answer Smaller	24	26	14

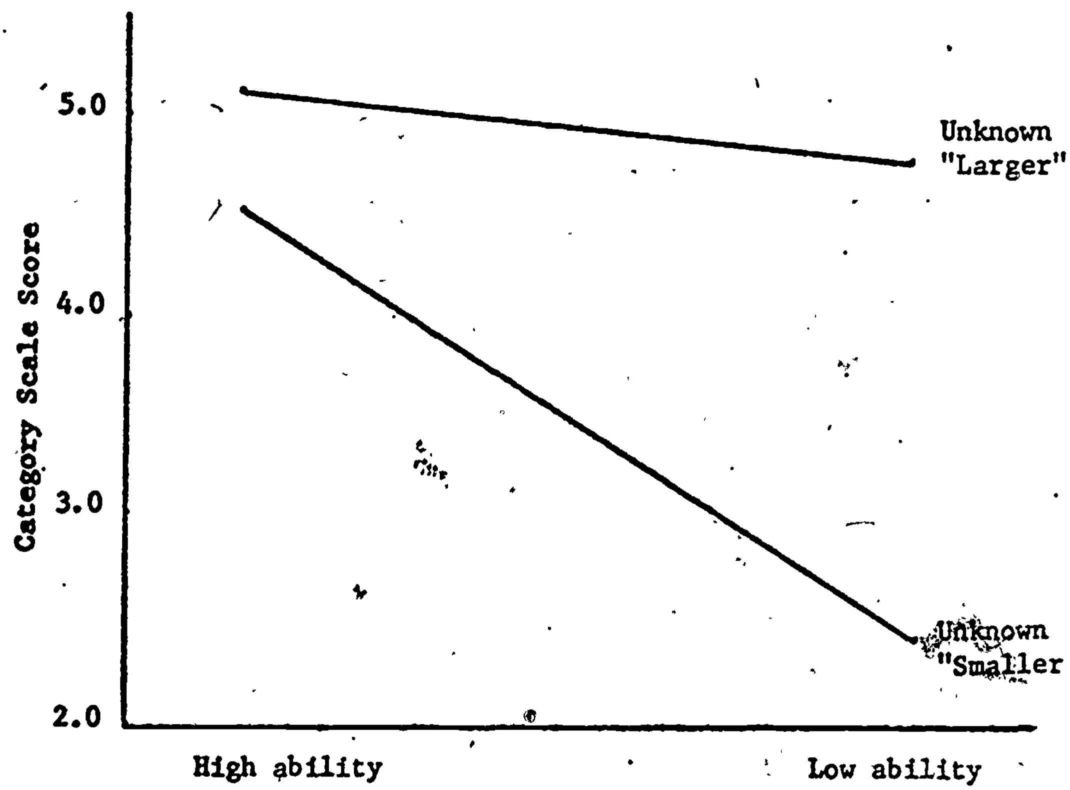

Figure 1. Category score means for high and low ability subjects at each level of the size factor.

were used by high ability subjects and more non-patterned strategies were used by low ability subjects.

Insert Table 10 about here

Discussion

According to Piaget, the ability to understand the concept of proportionality develops between the ages of twelve and thirteen. The results of subsequent research suggest that such concepts develop even later. In the sample of seventh-graders investigated here, only one-fourth of the problems were solved using a strategy that illustrates a well-developed understanding of the concept. And of that 25 percent, only two-third of the problems were solved correctly. These results are consistent with other work describing the concept of proportionality beginning to develop around the ages of thirteen and fourteen.

The seventh-grade subjects demonstrated little flexibility in their understanding of proportionality, as is illustrated by the large effect the task characteristics of Size and Type had on performance. Eighteen percent of the items involving complex multiples were solved using proportional strategies, but only 6 percent of the complex ratios were solved using proportional strategies. Likewise, 14 percent of the proportions involving large answers were solved using proportional strategies, while only 9 percent of the proportions involving small answers were similarly solved.

One question raised by these results is whether the concept of "proportional" reasoning is indicative of abstract thought or merely a component of general ability. Although ability significantly

TABLE 10

MEANS AND DISTRIBUTION OF HIGH AND LOW ABILITY SUBJECTS' RESPONSES ON THE THREE CATEGORY SCALE

Ability	Mean	Categories		
		Non-patterned	Patterned-Inappropriate	Proportional
High	4.80	12	32	20
Low	3.36	24	28	12

affects subject performance, the effect of task characteristics occurred independent of ability for the Type characteristic. Thus even children judged as superior-performing students by their teacher had difficulty solving the more complex proportion. This was not the case, however, for the factor Size. Low-ability subjects who tended to be concrete operational thinking responded to this factor by using preoperational strategies. Future research is needed to separate the effect of general ability from the acquisition of developmental concepts.

It seems possible for Ss to have an intuitive understanding of proportionality without concurrently having the mathematical facility to solve proportion problems. The fact that Ss could solve the easier of the two levels of Size and Type problems indicates that they had some intuitive understanding of proportionality and some mathematical facility with problems of this sort. But there is also a limit either to this intuitive understanding or to their mathematical tools. Which limits which is not clear.

These results suggest that those investigating the developmental acquisition of proportionality must be careful not to generalize too quickly from performance on any one proportion task to the concept of proportionality in general. Subjects, especially those transitional between concrete and formal operational thinking (patterned inappropriate versus proportional strategies), may be quite capable of reasoning through proportions of moderate difficulty. However, when faced with a more demanding task, these same subjects might revert to the use of patterned concrete strategies.

A more valid use of these tasks for assessing the level of competence with proportionality would be to administer at least two proportions -- for example, a complex multiple and complex ratio. Subjects who solved both could be designated as having the concept in hand. Those who solved only the easier of the two could be considered as transitional with respect to an understanding of the concept. The use of such tasks as a diagnostic tool might be especially helpful to teachers of subject matter which requires an understanding of proportionality.

These results also indicate that a child's understanding of proportionality is very dependent on the content of a proportionality task, especially for children transitional between concrete and abstract thought. Before the concept of proportionality can be used as a unitary indicator of formal operational thought, more research must be undertaken with children judged as formal operational reasoners to determine how variable their performance is both within and across different tasks. It may be that adolescents' understanding of proportionality depends to a great extent on the complexity of the physical relations inherent in a science or math task, much as performance on conservation tasks depends on whether the task involves the substance, weight or volume of materials (Uzgiris, 1964). The results of such research may indicate that the use of proportionality occurs in a developmental sequence across a certain set of tasks. Such research has implications for when it would be best to teach various concepts requiring an understanding of proportionality.

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