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ABSTRACT

This paper reports an attempt to teach high school algebra quickly to students in grades 4-7. Twelve boys and 12 girls were taught by professors of the same sex for a total of 37 hours. Instruction took place in the students' regular schools. Twenty-one of the students finished the course, and 18 of these placed between the 49 and 99.4 percentile on a standardized algebra test. Five of the experimental students scored higher on this test than any eight-grade students in their school. Plans for a second-year algebra course are discussed. (Author/SD)

SPECIAL FAST MATHEMATICS CLASSES DURING SCHOOL: ALGEBRA TAUGHT QUICKLY BY COLLEGE PROFESSORS TO FOURTH THROUGH SEVENTH GRADERS

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Special Fast Mathematics Classes During School: Algebra Taught Quickly by College Professors to Fourth Through Seventh Graders

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Abstract

We tried having a year of high-school algebra taught quickly to mathematically apt fourth through seventh graders. Principles and techniques were derived from three classes previously conducted at The Johns Hopkins University with somewhat abler youths. Twelve boys and 12 girls were taught within their school by mathematics professors of their own sex one two-hour session each week, for a total of 37 hours. Twenty-one students finished. On a standardized Algebra I test 18 of them scored between the 49th and 99.4th %ile of national eighth-grade norms. Five beat every eighth-grade algebra student in the school. Quality of instruction was crucially important. Homogeneous grouping can be highly effective. Experimentation with a second-year algebra class is being carried out during the 1974-75 school year. Progress of a college-level fast-calculus class is discussed. Material concerning the Wolfson I class (Fox 1974[I:62), especially their scores on the Differential Aptitude Test, is presented in an appendix.

Chapter VII of Daniel P. Keating (ed.), Intellectual talent: Research and development, an 18-chapter volume to be published in the fall of 1975 by The Johns Hopkins University Press, Baltimore, Maryland 21218, in both paper-bound and hard-cover editions. This book will be Vol. 2 of Studies of intellectual precocity, Vol. 1 of which is Mathematical talent: Discovery, description, and development (JHU Press, 232 pages, $2.95 paper and $10.00 cloth).

This is the report of an initially successful attempt to develop within a single school a program for teaching algebra to mathematically-apt students earlier and much faster than is usually done. In the Study of Mathematically Precocious Youth (SMPY) we had tried such programs outside of school hours—typically, two hours each Saturday morning—with three different rather highly selected groups of junior high school boys and girls. For detailed reports about these, see Fox (1974 [I:6])[2] and Appendix 2 of this chapter, Fox (chapter 9 of this volume), and George and Denham (chapter 6 of this volume).

All three of these earlier efforts drew from the most mathematically talented in a large population of students, most of whom were seventh, eighth, or ninth graders. The primary criterion was upper-1% mathematical reasoning ability in two of the programs, and approximately the upper 2-3% in the third. Other criteria, especially verbal and nonverbal reasoning ability, were also used.

All three programs were completely under the control and supervision of SMPY. None was conducted on school time. Arrangements for credit and accelerated placement in mathematics were worked out with the public schools in which the students were enrolled, but those schools did not prescribe the curriculum, furnish the teacher, supervise the instruction, or prompt the students to work harder. Two programs enrolled both boys and girls in the same classes, whereas the other (Fox, chapter 9 of this volume) was an exploratory study confined to girls.

Need to Try the Program within a Single Large School

SMPY's role is to try out programs in semi-laboratory settings, improve them, and then see whether the principles and practices developed can be used under more typical school conditions. The most direct transfer would be from programs on The Johns Hopkins University campus to a city or county school system that would operate special fast-math classes on Saturdays, in the late afternoon, or evenings; these would draw from the entire county or a sizable portion of it, rather than from just one school. Our programs have involved more than one county or city school system. Other things being equal, the larger the educational unit the more high-level talent is likely to be found and the greater is the need for special classes.

In the fall of 1974 the Montgomery County (Maryland) public school system, situated north of the District of Columbia, set up two such classes on a county-wide basis; these were taught by Joseph Wolfson. He pioneered with us the first two coeducational classes, which we call Wolfson I and Wolfson II. Wolfson I was completed in August of 1973 and Wolfson II in August of 1974. See Fox (1974 [I:6]) and George and Denham (chapter 6 of this volume).

It also seemed desirable to try out modified Wolfson and Fox techniques in a single school. Fortunately, late in 1973 an opportunity to do this arose. We were approached by Leon L. Lerner, the seventh grade guidance counselor of a kindergarten through ninth grade public school and Executive Director of the B'nai B'rith Career and Counseling Services in Baltimore. Having known of SMPY for some time, he suggested that we collaborate to set up special first-year algebra classes in that school, two hours per week for the last half of the school year. We offered to help select the most mathematically talented students in the fourth through seventh grades, organize the classes, and find teachers for them.

Ideally, it seemed to us from Fox's work that there should be two separate classes, one for boys and the other for girls, each with approximately 20 students initially. The least able enrollees should be mathematically talented enough to learn algebra fast and not get far behind the rest of the students in his or her class. The classes would be continued for at least one and one-half years (i.e., through June of 1975) under the school's sponsorship and partial control.

Approval for the program was given by the school principal and faculty. The Parent-Teacher Association (PTA) was involved in the planning for the classes. This was desirable from the standpoint of general parental approval and also, as will be noted below, when deciding how to finance the classes. Parents of children considered for the program were met with several times.

The Selection Process

Characteristics of the School

School R, as we shall designate it, is located in a rather affluent residential community within a large city and near several elite private schools. It draws from the vicinity students for its kindergarten through sixth grades. Many of the youths in that area who come from upper-socioeconomic-level homes attend private schools, however.

Entering its seventh grade are students from approximately 63 different elementary schools in the city. (This number varies radically from year to year, depending on pressures for racial integration.) The abler of these students stay only two years, however. At the end of the eighth grade the more capable students transfer to the ninth-grade "A(dvanced)" college-preparatory curriculum in one of several public or private schools.

Enrollments in the fourth through eighth grades of School R during the academic year 1973-74 were as follows: fourth, 67 ; fifth, 63 ; sixth, 68 ; seventh, 370 ; and eighth, 360.

According to Mr. Lerner, approximately 70% of its students are black. About 5-10% of the students are of Oriental, Mexican-American, or other foreign-language background. Only a few are Jewish.

The students in this school seem, on the average, somewhat abler academically than are students in the typical school of Baltimore City, but probably there is appreciably less high-level intellectual talent in Grades 4-7 of this school than in several schools in the nearby county.

A more direct comparison can be made via results of SMPY's January 1974 Maryland mathematics talent search. Of the 14 students from School R who entered that contest, 10 scored on the College Entrance Examination Board's Scholastic Aptitude Test, Mathematical part, as high as the average male high-school senior does. Of the 14 public middle and junior high schools in the city which participated in that contest, only two had more high scorers than that. Of the 25 junior high schools in the adjacent county that participated, nine had more. Because participation in the contest was voluntary, however, these comparisons can be only suggestive. The number of students from a given school who took the test depended heavily on recruiting within the school by guidance counselors and mathematics teachers. We do know that School R's counselors tried to enroll all eligible students.

Through the seventh grade, students in School R usually study general mathematics or a variant thereof. The better students are permitted to take introductory algebra as eighth graders. Other students who want to take algebra must wait until the ninth grade. As noted above, by ninth grade many of the academically ablest students in the school have left to enter senior high school.

Identifying the Population.

It was decided to locate all boys and girls in the fourth through seventh grades who had scored quite high on the Iowa Tests of Basic Skills (ITBS) achievement battery's arithmetic reasoning section and who also had high total scores. A sliding scale was used. Seventh graders had to score at least the 98th %ile on mathematics and the 95th %ile overall. Sixth graders needed 99 and 97. For fifth graders the required percentiles were 99 and 98. For fourth graders they were 99 and 99.

This pre-screening by Mr. Lerner from the students' records yielded 23 girls and 17 boys to be screened further by SMPY with more difficult tests. They were in the following grades: girls--seven 4th, three 5th, five 6th, and eight 7th; boys--five 4th, two 5th, three 6th, and seven 7th.

Selecting the Students

On 19 December 1973 all but four of the above students took the 1962 version of the Psychological Corporation's Academic Promise Test (APT), which is designed for Grades 6-9. It was administered by the author and William C. George. The next day Mr. George administered the 60-item Raven's Standard Progressive Matrices (SPM). Testing of the absentees was done by Mr. Lerner.

APT consists of four subtests, each of which has 60 items. They are Numerical (N), Verbal (V), Abstract Reasoning (AR), and Language Usage (LU). From previous experience, we knew that those subtests were predictive of success in such a class in the order listed: N most, V next, and AR next. Therefore, in choosing members for the two classes (one for boys, the other for girls) most weight was given to N and to the V + AR + LU sum. An examinee's AR score could be compared for consistency with his or her SPM score.

The scores and other information are listed in the first appendix to this paper, which is Table 7.A1.

A number of facts can be gleaned from that large table, where the rows are in descending order of the 40 N scores ranging from 54 to 5. The highest N score was earned by a seventh grade girl, but eight of the top 10 N scores were obtained by boys. One of these boys, who ranked fourth on N and seventh on APT-Total, was a fifth grader. Another was a sixth grader. Most of the low scorers on N were fourth or fifth grade girls. The seventh grader scoring lowest on N (27) was a boy who ranked 27th out of 40. Only one of the students scored far lower (APT-Total 64) than would be predicted from the pre-screening scores.

Grade and sex differences for APT-N are set forth graphically in Figure 7.1. There the scale of the abscissa is the same as the scale of

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the ordinate, i.e., the 5-54 score range. The two rectangles for each grade are centered over the mean score for that grade. (Near the middle of each rectangle is indicated the mean for that grade-sex group.) The means of the grades were as follows: 18.2 for the 4th, 27.4 for the fifth, 37.6 for the sixth, and 40.8 for the seventh. The figure shows quickly that the sixth and seventh graders differed far less from each other than from the other two grades, which were about as different from each other as the fifth grade was from the sixth grade. The small sixth vs. seventh grade difference might represent some aspects of instruction in arithmetic, but more likely it is due to higher preselection criteria for the former grade than for the latter and some loss of high talent to private schools after the sixth grade.

The figure also indicates that the girls lagged the boys by about one grade. Fifth grade girls, whose mean score was 21.0, were a little below fourth grade boys (23.2). Sixth grade girls (36.2) were about the same as fifth grade boys (37.0). Seventh grade girls (39.2) were about the same as sixth grade boys (40.0). Sex differences in means within the two lower grades were large: 8.6 points for the fourth and 16 for the fifth. They were much less in the sixth and seventh grades, 3.8 and 3.4 respectively. Thus it seems that for mathematically apt youngsters matched on ITBS scores math ability as measured by a more difficult test shows less sex discrepancies in the higher grades. The girls seem to "catch up" with the boys somewhat.

Inspection of the birthdates in the table reveals that not a single one of these 40 highly able students is accelerated in grade placement even one day by local standards (i.e., must become five years old during the calendar year in order to enter kindergarten in September). Three girls--two seventh graders and one fourth grader--are a year behind schedule, however. One of the seventh graders, of Chinese background, was born in July of 1960, so by the above criterion she was more than five months older than the entering minimum. The other girls have October and November birthdays, so they may have been kept out of school an extra year because of their presumed "immaturity." Perhaps, instead, they began their schooling in another school district that had an earlier entrance criterion, such as August 31 or September 30. But in any event it seems to the writer unfortunate that there was no acceleration in school grade but some retardation among the ablest students in a large public school.

Besides the girl of Oriental ancestry, the 40 include four with Spanish surnames, one of them of Philippine background, and two blacks. Also, quite a few of the names seem Germanic.

Of the 23 girls tested, 12 were chosen for the girls' class. (See the Appendix.) Seven of these were seventh graders, and five were sixth graders.[3] No fourth or fifth grade girl scored high enough to be considered ready for the class. All but one of the girls in the class had APT Total scores ranging from the 99th %ile to the 95th %ile of seventh graders; hers (rank 16.5 in the Appendix) was the 85th %ile.

Of the 17 boys tested, 12 were chosen. Half of these were seventh graders. There were three sixth graders, two fifth graders, and one fourth grader. One of the male fifth graders scored two points lower on N and 24 points lower on T than the lowest-N girl accepted, but the other boys chosen were comparable to the girls.[4]

Because considerable attrition must be expected from one year to the next, each class should have had at least 20 students so that the number the second year will be sufficient with which to continue. As noted above, there did not prove to be enough talented youths in the fourth through seventh grades of School R to do this separately by sex, as the work by Fox (chapter 9 of this volume) indicates is probably desirable. This is a severe limitation to conducting fast-math classes within a single school during school hours. Only large schools in high-ability areas (usually upper-middle-class suburbs) are likely to have sufficient students with highly enough developed mathematical reasoning ability to create effective fast-math classes meeting one two-hour period per week and enrolling at least 20 boys and 20 girls.

An alternative possibility is to put the boys and girls together in one class and have it taught by a woman skilled in capitalizing on the socialization needs of the girls. Another is to offer one regular section of Algebra I to especially well qualified boys and girls a year earlier than usual--e.g., in the seventh grade if algebra is ordinarily begun in the eighth. This means that the typical junior high school, with Grades 7-9 and algebra not usually available until the eighth grade, (if then), would need to add plane geometry to its curriculum for this accelerated group--i.e., Algebra I in the seventh grade, Algebra II in the eighth, and geometry in the ninth. It would not seem wise to leave these able accelerants with no mathematics in their last year of junior high school. (Some might prefer to skip the ninth grade, though, and thereby move into senior high school a year early.)

The 24 students chosen for these two classes represented a wide range of family backgrounds and education, as diverse as police sergeant, carry-out shop operator, and university professor. Education of parents ranged from second grade to Ph.D.'s. On the average, this group of students is from a somewhat lower socioeconomic level than the students in the Wolfson I and II classes were. That accords with known differences between this city and its adjacent counties, and with the fact that students in the School R classes are less able, on the average, than were Wolfson's students. Yet even the lowest of them in the class are within the upper few percent of the age group in mathematical ability.

Setting Up the Classes

A female teacher was needed for the girls and a male teacher for the boys. We knew from experience with the Wolfson and Fox classes that these teachers should meet several criteria:

1. They must know mathematics well at a level far above that at which they would teach.

2. They must be bright and alert.

3. They must want to teach elementary algebra fast and well to mathematically apt youths. They must not be easily slowed down or distracted from this central concern. Above all, they must not adjust their pace to the slower members of the class. Instead, they must require these students to fill in gaps in their comprehension of the material between classes by doing a great deal of carefully designed homework.

4. They must hold the students to high standards of homework and class performance.

Those four specifications pointed toward college teachers of algebra and higher mathematics courses, or persons who had extensive graduate work in a related area (such as Mr. Wolfson, who studied physics). Previous experience teaching students of junior high school age was not essential, nor perhaps even desirable, we had found.

For the boys' class we were exceedingly fortunate to get Professor Richard F. McCoart, Chairman of the Department of Mathematics at Loyola College in Baltimore, a well-trained teacher of calculus and other mathematics courses. He knew of our Study and had already volunteered to teach a course such as this.

For the girls we were also exceedingly fortunate to get Miss Ann L. Wagner, an assistant professor of mathematics at Towson State College, near Baltimore. She is an experienced teacher. The first prodigy in our Study had audited her precalculus and calculus courses during the school year 1968-69. Miss Wagner proved to have the warm, friendly manner that seems important for teaching mathematics to sixth and seventh grade girls.

Next we decided on the textbook. The simplest alternative was to choose the book used in the regular eighth grade algebra classes at School R, because it would make transition to that class the following fall easier for those who had not done well enough in the fast class to continue with it. Also, the book could be supplied free to each student. Mr. Wolfson had decided that getting through Algebra I quickly and into a comprehensive Algebra II textbook was important, so for a knowledgeable teacher the vintage of the book was not seen to be crucial. The one used was Smith, Lankford, and Payne (1962).

The special classes met for one two-hour period per week, without a formal intermission. This amount of time was chosen deliberately for the convenience of the teacher and because mathematically apt, interested youths appear to benefit from massing of instruction. They have longer attention spans than average children do. Also, in the school context this one period per week may facilitate scheduling of time and room.

We have often considered whether two separate 50-minute periods per week would distribute learning and homework assignments better. Perhaps so, but because the special class is meant only for students who can rather readily learn to work well on their own between classes the two-hour period seems more efficient.

It might be well to stress here that for the students chosen these special classes are a privilege, not a right. We know from logic and experience that not all of the starters (or their parents) will appreciate the opportunities they afford, so built into the plan are provisions for moving low-achieving students into more suitable classes as early in the course as they become definitely known. All students remained in their regular arithmetic or general mathematics class five periods per week.

Dr. McCoart met his group for the first time on Friday, 18 January 1974, and each Friday thereafter (with two exceptions) through June 7, when during the second hour the standardized test to be described later was administered. Miss Wagner's class began on January 25, and the test for her group came on June 5. The next week in June each teacher reviewed the test results for one hour and then met with the students' parents.

Thus Dr. McCoart taught his boys for 37 hours before the test. Miss Wagner also taught her girls for 37 hours. The classes were conducted independently of each other.

It was not crucial that the boys or girls learn the first year of algebra well in this short period of time, because from the beginning it was planned that they would resume studying Algebra I in the special class(es) during the fall of 1974 before progressing to Algebra II.

Each teacher was paid a set fee per week. The school asked the parents of each child in the special classes to pay $2.00 per week, if able. The Parent-Teacher Association agreed to furnish the rest. In our own programs we have paid instructors from $25 to $75 per two-hour session, depending on the size of class, level of subject matter, and experience of the teacher with that kind of group. Fees students pay have been set at enough per week to meet all or most such costs. The remainder, if any, has come out of our research funds.

Conducting the Classes

Drs. Fox, Keating, and ~~Stanley visited~~ some of the classes and helped the two instructors get acquainted with their bright young students. Dr. McCoart had no teaching experience below the college level, but he quickly proved to be an enthusiastic, ingenious teacher. Miss Wagner molded her girls into a smoothly interacting, well socialized group.

Attendance was splendid. During the semester the teachers at School R went on strike, along with other city teachers. Because Dr. McCoart and Miss Wagner were ad hoc teachers in this school, they continued to come each week. Their students crossed picket lines in order to continue learning algebra.

A boy and a girl dropped out of the classes quickly, and one more boy did so after about 15 weeks. Test scores of the former two, both sixth graders, were rather low in the classes' distribution. (See Footnote 16 of the Appendix.) The other student, a seventh grader, finally quit after persistently not doing any homework. (See Footnote 3 in the text.) These dropouts left 10 boys and 11 girls who continued until the final meetings.

Dr. McCoart and Miss Wagner moved through the algebra textbook fast, operated at a more abstract level than could be done in a usual class, and assigned considerable homework. Dr. McCoart's manner was more intensively forceful and aggressive, whereas Miss Wagner's emphasized group cohesiveness and working together. The latter was intentional, because Dr. Fox had found that girls exposed to a highly theoretical, individualistic, competitive teaching approach tended to do poorly and quit.

It was obvious to persons such as the writer who audited some of the two-hour sessions that these were splendid teachers. They kept the attention and ~~good will of the~~ youths. The next section shows that they were indeed successful.

Evaluating Progress

Results of the standardized test administered to the 10 boys and 11 girls by the writer are shown in the last column of the Appendix table. The boys ranged in percentile rank on national eighth grade norms from 99.4 (a brilliant fifth grader) to 18 (a fourth grader). No sixth or seventh grade boy was below the 68th %ile. No one of the five students who had scored 33 or lower on APT-N exceeded the 49th %ile, whereas all but two of those 16 whose N score was at least 36 achieved or exceeded the 68th %ile.

In summary, only three of the 21 scored below average for eighth graders who have studied algebra for a school year, approximately 175 50-minute periods that total some 145 hours. Two of those (Ranks 20.5 and 26 in the Appendix) were very young, being in only the fourth and fifth grades. An equally young fifth grader (Rank 4) was, however, the best algebra student of the entire group. High score on APT-N seems especially important for students younger than most in the class. Otherwise, they will probably need far too much tutoring and other special attention.

The teachers agreed substantially with the results of the standardized test, which was independent of their own evaluations, except that the lowest-scoring girl was judged to be a better student than her score indicated, and a sixth grade boy (Rank 9.5 in the Appendix) was judged to be less able than his 68th %ile score suggested. The girl had scored relatively low (30) on APT-N. Both of her parents are college graduates, and her father is an engineer. Perhaps she got more help at home than most of the girls did, and this made her homework and class responses seem to show more achievement than she could demonstrate on a test containing 40 multiple-choice items to be answered without assistance in 40 minutes. When retested in the fall with another form of the test she improved greatly, scoring at the 95th percentile.

The boy's "surprisingly high" algebra score tied him for sixth place with two of the other nine boys, whereas he ranked 7.5 among them on APT-N. He had the lowest APT-Verbal score of any boy in the class except one of the fifth graders, whom he tied. This verbal deficit may have caused him to appear less quick-minded in the class than on the test. His mathematics aptitude is considerably higher than his rate of learning. We have encountered several boys like this, who learn mathematics well if given enough time and exposure. They have good mathematical reasoning ability but less high IQ's.

Fortunately, it was possible to compare the algebra test scores of these 21 fourth through seventh graders with those of the eighth graders who took algebra every day in regular class, both sections of which were taught by the same new, inexperienced teacher. (Remember, though, that both Dr. McCoart and Miss Wagner had not previously taught students this young, either.) These were the ablest 18% of the eighth grade students.

At the invitation of the eighth grade counselor, Mr. George and the writer tested the 66 eighth graders on 11 June 1974, nearly a week after testing the special-class students. Most of the special-class students achieved better after 37 hours of instruction than the regular-class, older ones did near the end of a school year. Five of the 21 (24%) scored higher than any of the 66. Twenty-three of the 66 (35%) scored lower than any of the 21. These are startling figures, because the eighth graders themsevles were a selected group that included virtually all of the ablest students in that grade. Less able students wait until the ninth grade, if at all, to begin algebra. These great contrasts in favor of students in the special classes, who were younger and taught only 37 hours, are probably the most salient findings of the within-school study.

The most important factors that produce results such as the above, which were also found at least that strong in our previous fast-math classes, seem to be as follows: a teacher who knows mathematics well, is enthusiastic, has high standards, and moves the group fast; students who have considerable mathematical and verbal aptitude, as determined by standardized tests, and are fairly homogeneous in these respects but not necessarily alike in grade placement or chronological age; interest in learning mathematics quickly and well, which (especially among girls) does not always accompany aptitude; facilitative parents who value the unusual educational opportunity the special class represents and therefore encourage their children to do well; and helpful school personnel who do not try to obstruct the program because they feel threatened by it.

Background Characteristics

Interest can be measured. We did so approximately for it and other aspects by means of a questionnaire filled out early in the course by the girls, because our previous experience had indicated that some of them would probably not be much interested in mathematics. One left the class even before it began, so the questionnaire was not offered her. The other 11 provided information that can be summarized as follows:

Both of the parents of five of the girls were at least college graduates. Only one parent (a mother) did not complete high school. Five of the mothers work outside the home. Three of the fathers (and one of the mothers) are teachers, two are engineers, and one is a lawyer. Other fathers hold positions such as department head in a large steel plant, deputy chief of maintenance at an airport, owner of a carry-out shop, and police sergeant.

None of the girls was an only child. Their number of siblings ranged from one to four. Six of

the girls have no older siblings, but only two of them had no younger siblings. Two had no brothers, and two no sisters. Three of the 11 families matched the stereotype "if you have daughters first, keep on having children until a son is born and then stop." All in all, these sibling relationships seem fairly typical of the types of communities from which the girls came, with perhaps somewhat more tendency for them to be the oldest child. Only one was the youngest child in the family, being four grades lower in school than the closest one of her two brothers and two sisters. In fact, she was the only one of these 11 girls who had any older sisters; she and two others had older brothers.

This analysis of siblings is based on only 11 cases, so it must be considered highly tentative. Astin (1974 [I:4], p. 81) made similar comparisons for six girls in the first Wolfson fast-math class and provided the following statistics: "None of the 17 children [including 11 boys] were only children. Six of the boys were first-borns, but none of the girls. Boys tended to be among the oldest in relatively small families, whereas girls tended to be the youngest in relatively large families. No girls came from two-child families, but four of the boys belonged to such families."

Like Astin's, this sample contained no only children. More than half of the girls were first-borns, however. Two of these 11 girls came from two-child families. Much or all of these discrepancies may be due to sampling fluctuations between small groups drawn from essentially the same population. Some of it might reflect the suburban, extremely high-ability nature of Astin's girls vs. the urban, less high-ability nature of the School R ones. Parents who persist in a somewhat deteriorating city environment may differ in their child bearing and rearing practices from those who move into the surrounding county. Also, the "creaming off" of able children into private schools within Baltimore is probably much more prevalent than in the surrounding counties. The city parents with small families are more likely to send their children (perhaps especially their older sons) to private schools than are those with larger families.

On the questionnaire the girls were asked a number of questions concerning their interest in the course and in mathematics. Eight of these were quantified and a score produced for each girl. The coefficient of correlation between these scores and the algebra scores, with APT-N score partialled out, was .30 . Inspection of the interest scores reveals that the highest scorer performed disappointingly on the algebra test, but one of the lowest scorers also ranked low in the group. The interest items, being in self-report form, may have been quite susceptible to social desirability bias and other atmosphere effects at the start of the course.

It would of course be interesting to have similar questionnaire information for the boys, but that was not collected at the start of the class. The self-report items would not have the same meaning if completed later.

Changes in Fall of 1974

All of the girls were invited to continue in the fall with more Algebra I and then go on with Algebra II. All of the boys except the fourth grader (Rank 20.5 in the Appendix) and one of the fifth graders (Rank 26) were also invited. Continuation in the fall of 1974 is discussed in the next section of this chapter.

The new class in beginning algebra was recruited from incoming seventh graders--those who entered School R from elementary schools--and those persons on the Appendix list from Rank 24-39 who when retested were found to have improved their N and V scores sufficiently. The criteria for this were scores of at least 36 on APT-N and 36 on APT-V. Those whose V scores were already high in December of 1973 had a fair chance to meet these criteria, because the non-class group got special instruction in arithmetic during the spring of 1974.

The Continuing Group

At noted above, 10 boys and 11 girls remained in the class from its inception in January of 1974 until school ended in June. Of these, five boys and nine girls continued in it on September 12. This 33% attrition over the summer seems high, but is probably typical of public schools in a city but outside its center. Because there were not enough students to have separate-sex classes, Miss Ann Wagner took over the whole group.

The five boys who dropped out are accounted for as follows: the brilliant fifth grader who ranked fourth in the Appendix table moved away, the rank 7.5 sixth grader went on a one-semester trip to Europe with his parents, the rank 9.5 sixth grader transferred to a nearby private school, the rank 20.5 fourth grader went on a long trip with his parents (but would have been dropped from the class, anyway, because he did not seem ready to keep up with its pace), and the rank 26 fifth grader was asked to drop out because although conscientious and apparently attentive he was lagging behind the group.

Of the two girls who dropped out, one (the rank 11 sixth grader) attended another school and did not want to make the continued effort to come for the class, and the other (the rank 12 sixth grader) transferred to a nearby private school.

These departures left the class composed of five eighth-grade boys, seven eighth-grade girls, and two seventh-grade girls. A glance at the

Appendix table reveals that one of the seventh graders (rank 24) did quite well on the algebra retest, whereas the other (rank 20.5) scored at the very bottom of the class (33rd %ile). The former's father is an engineer and helps her with homework. That probably partly explains her rise from 39th %ile, lowest of all the 11 girls, on the first test to 95th on the second. Careful doing of homework, with encouragement and preferably some assistance at home, seems highly important, especially for girls.

On the retest the boys had percentile ranks in nearly the same order as on the first test, but averaging 0.23 standard deviations higher. One gained 0.55 s.d., two 0.48, one 0.00, and one -0.36. These do not seem substantial enough gains for the amount of time involved since the previous test, about 24 hours. Too-low ceiling was not a problem except for the top scorer, who missed only one of the 40 items. The other boys scored 31, 28, 28, and 27. It seems that direct review of Algebra I is not as productive as going on into a good Algebra II textbook might be.

The girls gained more than the boys (average of 0.42 s.d.), but their gains were far more variable: 1.92, 1.91, 0.69, 0.67, three 0.00's, -0.41, and -1.03 standard deviations. On the retest the boys averaged 1.33 s.d. above the mean of the national eighth-grade norms, whereas the girls averaged 0.91. Three girls scored considerably lower than any boy, but four girls scored higher than any boy except the top one. At least one of the girls seems unlikely to be able to learn Algebra II fast enough to keep up with the rest of the class.

It is difficult to ascertain what varied factors operated to make the boys achieve better than several of the girls, even though they had a teacher of the opposite sex from their former one, whereas the girls kept the same female teacher from one year to the next. Ranks on APT-N in the Appendix table may give clues to the difficulties that some of the girls are having. The five boys' ranks are 2, 3, 5.5, 7.5, and 9.5. The nine girls' ranks are 1, 5.5, 13.5, 13.5, 15, 16.5, 20.5, 20.5, and 24.

Seven of the nine girls scored 4-13 points lower on APT than any of the five boys did. This difference in numerical aptitude may be more important than even the sex of the teacher and coeducational nature of the class are. But, clearly, most of the girls are doing well, and four of them improved spectacularly from test to retest.

It will be interesting to see how those students who continue in the class until Algebra II is completed, probably by the end of the school year, do on a standardized test. The girls have the advantage of numbers and a familiar female teacher who seems especially good in creating the social atmosphere that Fox (chapter 9 of this volume) believes is needed by most girls in their mathematics classes. The boys have an edge in age (all eighth graders) and numerical aptitude, but they may not be as well motivated by the class atmosphere as most of the girls.

Prepping for the APP Calculus Exam

On 7 September 1974 Dr. McCoart began a new class, rather different from any we had offered before. Meeting two hours each Saturday morning at nearby Loyola College, where he is head of the mathematics department, it was meant to supplement high-school calculus courses so that students would score 4 or 5 on the Level BC (i.e., the higher level) calculus examination of the Advanced Placement Program (APP) in mid-May of 1975. This college-level calculus course would carry no credit. The student's sole reward for taking it would be, we hoped, a better score on the APP exam and therefore a full year of college credit in calculus. The 4 or 5 on a 5-point scale was set as the goal because Johns Hopkins requires at least a 4 to provide 8 credits of Calculus I and II and permission to begin with advanced calculus. Many other universities will accept a 3, and indeed even at Johns Hopkins a 3 on Level BC gives 4 credits and exemption from Calculus I.

Without supplementation, the typical high-school calculus course does not prepare most able students for doing well on the BC level. At best, they are likely to be ready only for the easier level, AB, which usually provides less credit than BC does. For example, one of our most brilliant boys took AB and made a 5, but at Johns Hopkins this automatically earned him only 4 credits and the waiver of Calculus I. (He went into advanced calculus, anyway, and earned a grade of A.)

Fifteen boys--and, regretfully, no girls--signed up for the course. Three of them were regular-age twelfth graders, being three of the four ablest calculus students at a large suburban high school; the fourth (ablest of the group) decided that probably he would not need the supplementation, thereby giving us a strong "control group" of size 1.

One of the other boys, a tenth grader who had skipped the eighth grade, had been an outstanding student in our first fast-math class, taught by Joseph Wolfson (we refer to that class as Wolfson I). In the fall of 1973 as a 13 year old ninth grader he took calculus, a twelfth-grade subject, at a large suburban high school and ranked in the upper two-fifths of an excellent class.

Another tenth grader who had also skipped the eighth grade had been a less successful student in Wolfson I who went into the middle of

Wolfson II and did well.

A third student, an 11-year-old ninth grader taking Level AB calculus in the eleventh grade of a private school and chemistry in the tenth, had done well in Wolfson I at ages 9-10.

Another student, a tenth grader who had skipped the ninth grade, had scored high (SAT-M 700, SAT-V 590) in our January 1974 math-reasoning contest. He had not been in any of our special classes and therefore had less mathematics background than anyone else in the class.

The other eight students who began were graduates of the Wolfson II Algebra I-III, plane geometry, trigonometry, and analytic geometry speeded-up program (see chapter 6 of this volume). Ages of the 15 students ranged from 11 years (two) to 18. Grade placement ranged from ninth (three) to twelfth (three).

Thirteen of these students continued in the class after Christmas of 1974. The 11-year-old mentioned above fell behind and dropped out because he would be taking Level BC calculus in the twelfth grade of his high school. He and his father felt that, despite this boy's extremely high Stanford-Binet IQ (212) and SAT-M ability (730 at age 10), he had enough work in school to keep him busy--being accelerated three years in basic grade placement and more in two subjects.

The other dropout just before Christmas was the boy mentioned above who had done poorly in Wolfson I but better in Wolfson II after taking two mathematics courses in high school as an accelerated ninth grader. He seemed to find getting around to doing his homework difficult, presumably because of lack of motivation and organization. His mathematical and verbal abilities are unusually high even for the SMPY group (SAT-V 720 and SAT-M 680 at age 13), but some of his other cognitive scores such as for nonverbal reasoning, mechanical comprehension, and spatial relationships are less outstanding. His chief academic interest seems to be military history, so perhaps he is simply not "cut out" to choose a field in which high-level mathematical achievement is essential. One wonders, however, how much better he might do in the fast-math classes--and like them more--if his homework time were much more carefully organized.

The Standardized Test

As Table 7.1 indicates, the 13 boys who continued in the class from

To the printer: Please put Table 7.1 about here

its inception until 1 February 1975 (a total of 34 class hours) learned differential and integral calculus extremely well. Only one of them, a regular twelfth grader, scored on a difficult speeded standardized test below the 88th percentile of the exceptionally able group of high-school students--mostly seniors--across the country who elect the calculus course and pursue it five days per week for approximately 180 45-50 minute periods.

Six of the students scored higher than 99% of that norm group, and only two scored less well than 94% of them; they exceeded 88% and 76%, respectively. Even the 11-year-old in the special class outscored 94% of the elite norm groups. Two years earlier he had been a fourth grader!

By comparison with college students who have completed two semesters of introductory calculus the scores of this group are even more impressive. Only two boys scored below the 99.1th percentile of the national college norms; they were at the 98th and 94th percentiles.

Only 10 of the 13 boys are actually accelerated in their mathematical placement. The other three are regular-age high-school seniors who have not skipped a grade. They are in the class as "pacers." One of these earned the highest score on the test, 57 out of the possible 60 points. Another scored 56, being tied by two of the accelerated boys (no one of whom is older than the typical tenth grader). The third twelfth grader ranked 13th in the group, with a score of 42. It seems likely that he had not worked much in the course for two months before the test, because he started off splendidly and then fell behind.[5]

From the results of this standardized testing, it seems quite likely that all of the present group who continue in the course will be splendidly prepared to make 5's or at least 4's on the level BC APP calculus test which they will take in mid-May, 1975. Meanwhile, they will be getting a high-level version of Calculus III, including some coverage of differential equations. By the summer of 1975 nearly all of them should be ready ~~for a strong~~ course in advanced calculus or linear algebra.

The present eleventh grader (who skipped the ninth grade) plans to become a full-time student at Johns Hopkins in the fall of 1975. Some of the tenth graders, and perhaps one or both of the ninth graders, will probably enter in the fall of 1976. As noted several times in this volume, success in SMPY's special fast-math classes leads to much general acceleration.

This book will go to press long before results of the May 1975 APP testing are known, so the outcome of that interesting experiment must await publication elsewhere. Dr. McCoart's "coaching class" for the higher-level APP calculus exam, supplementing as it does regular high-school courses, is an idea that might be applied to a number of other APP exam subjects such as physics, chemistry, biology, and history. Meeting for just one two-hour period per week outside of school hours and serving a large geographical area, it can be both effective and in the long run economical. Students in his class paid $5 per week each, but if there had been 30 students the cost per student could have been cut. Even $150 for the year, plus some $30 for the APP exam, is a bargain, however, if it provides really sound knowledge of the calculus and eight college credits. We expect the students who complete this course to earn 4's or 5's on the APP Level BC calculus exam and go into college advanced calculus courses in the summer or fall of 1975 while most of them are still in high school.

Feasibility of Within-School Programs

<u>Pro</u>

1. They occur during the regular school day and therefore avoid

the transportation problems and absences that classes late afternoons, evenings, or on Saturdays cause.

2. They are part of the school program and therefore should make articulation with other levels of the subject easier. Also, most of the eligible students will probably enroll in the special classes. Few who do well will drop out.

3. Classes are readily available for scrutiny by school personnel.

4. Students and teachers are accessible to guidance counselors. For example, Mr. Lerner developed continuing close relationships with the students and their parents so that counseling and arrangements for tutoring could increase the effectiveness of the teaching.

5. They permit excellent part-time outside teachers to be used inexpensively, or perhaps without cost. Often one can get free teaching by properly assisted community persons such as engineers or housewives who majored in mathematics, either directly or by approaching, say, an engineering firm and asking its president to release a suitable employee for that purpose.

6. They set a model within school for work with gifted in other subjects.

Con

1. It may be difficult to schedule a two-hour period per week, especially across grades, and not interfere much with other classes.

2. Special programs for the intellectually talented often encounter strong overt or covert resistance from teachers, guidance counselors, principals, or parents of children not included in them. Teachers of other subjects such as English may resent absences from

their classes, even though the students are probably superior in those subjects, also. Mathematics teachers may feel threatened by "expert" outsiders who are not certified high-school teachers.[6] Problems of classroom utilization may occur, because most classes meet for 45 or 50 minutes, not two hours. Thus, the school setting is far more complex than the university class.

3. Someone (e.g., school, parents, and/or PTA) must pay the outside instructor, if he or she will not donate the time. Of course, the school might use one of its own teachers, if a suitable mathematics teacher can have two hours per week of time freed. In some junior high schools, however, there will not be any math teacher well enough prepared to continue the program successfully into Algebra II and III, geometry, trigonometry, and analytic geometry.

4. The talent base in Grades 6-7 in the typical public school is too slight to make it possible to start with a large enough class of each sex. Also, most junior high schools do not even have a sixth grade. To find 20 upper-5% boys in the seventh grade of an average school, for example, one would need 400 boys in that grade! If the school has a considerably greater amount of talent than average, 200-300 might suffice, but that would be only for the boys. For a class of 20 girls, too, the enrollment in the seventh grade alone would have to be 400-800.

Especially if one starts with more than a single grade, as School R did, attrition from one year to the next will probably cut the class size down considerably--33% in the present study. Also, the number of years each student can remain in the program will vary; at School R a fourth grader would have five or six years, whereas a seventh grader would have only two or three. Very few fourth or fifth graders will be ready for such a program, so it might be wise to confine the recruiting and selection to not more than two grades, such as sixth and seventh, and to begin the classes at the start of the year rather than in its middle. (One encounters a dilemma here, because although few fourth or fifth graders will qualify, those who do so will tend to be the real stars eventually because of the splendid earlier preparation they can get.)

Attrition occurs because students do not succeed in the special class, lose interest, transfer to other schools within the vicinity, move away, or encounter parental (and often teachers' or counselors') objections to their being accelerated in the school's mathematics program.[7]

5. As noted earlier, the two-hour period may be too long for some students' attention span. The younger or less able the student, the more likely this is to occur. But in our special county-wide classes we have seen a 9-10 year old boy with IQ near 200 proceed happily and well through Algebra I-III, geometry, trigonometry, and analytic geometry in 60 two-hour periods. The next year his 10 year old friend did the same thing; he also earned the highest grade in a college course in computer science, competing with seven of our older math prodigies and 12 adults, and made "A" in a second-level computer course. The more brilliant they are, the earlier they should be identified and facilitated. But there are few nine and ten year olds as able as these. One fifth grader (Rank 4 in the Appendix) at School R was nearly that able, however. It was known in advance that the other two boys in the lower grades (Ranks 20.5 and 26) were not likely to keep up with the rest of the McCoart class. They were admitted on trial and did quite well, con-

sidering their age and grade placement, but scored at the bottom of the 21.

Some of the above five arguments against within-school homogeneously grouped fast-math classes also apply to school-system-wide classes outside school hours. On balance, we prefer that classes be held in the late afternoon, evening, or on Saturday so that they can enroll a more mathematically apt, relatively homogeneous group of 20-30 students of the same sex. Where a school has quite a few talented youths, however, and facilities for working with them in available-size groups,

it would of course be far better to do this than to wait vainly for a suitable system-wide plan.

Conclusion: Quality of Schooling Can Make a Great Difference.

Many interpreters of the "Coleman Report" (Coleman et al. 1966), especially Jencks et al. (1972), seem to say that quality of schooling is not very important. For high-school mathematics, however, it is clear from the special classes we have conducted thus far that type of class and quality of instruction are vital for learning. In far fewer hours the students in these classes have learned far more mathematics well than they would have done in a regular classroom several years later.

A well prepared, fast-pacing instructor is a key element in this instructional package. Homogeneous grouping according to mathematical and verbal reasoning ability is another. High expectations are a third. Concurrent and future opportunities are a fourth; sucessful students are encouraged to skip school grades, take college courses for credit while still in high school, work for advanced placement credit by examination, enter college full-time early, try mathematics competitions, and the like. Our interest in them is meant to be continuous at least over the years from the time they are first identified until they complete graduate school and are employed. We are available for consultation on any aspect of their education.

Small class size may be another important feature, but in other programs we have had similar success when there were 31 students in a class. (See George and Denham, chapter 6 of this volume.)

Well-meaning teachers sometimes try one of three types of "enrichment": so-called busy work, irrelevant material (such as a drama class for boys whose major interests are mathematics and science), or really effective procedures that leave the student even more bored in later grades (such as a splendid modern-math program in Grades K-7 that leads only to conventional algebra in Grade 8). Clearly, we believe that a considerable amount of acceleration in subject-matter and/or grade placement must accompany enrichment, or be employed in lieu of it.

These fast-math classes and other aspects of our Study cater to individual differences in a persistent attempt to find, study, and develop talent. The principles and procedures we have worked out can be used in other schools and for other subjects. Until they are, intellectually gifted students--particularly those with superb mathematical reasoning ability--will for the most part continue to get little that effectively meets their real intellectual needs.

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Footnotes

[1]I thank William C. George and Leon L. Lerner for providing some of the data used in this paper and Daniel P. Keating, Harris J. Silverstone, and Michael Beer for helpful comments.

[2]The citation in brackets indicates that the reference is to a chapter in the previous volume on SMPY. I signifies Volume I of the Studies of Intellectual Precocity; 6 signifies chapter 6 of that book. This convention is used throughout.

[3]Before the course began, one of these, a sixth-grade girl from another school, had joined the group. See Rank 11 in the Appendix.

[4]One of the boys (Rank 16.5 in the Appendix) earned the extremely low score of 17 on AR, even though his scores on the other three subtests were good. No other one of the 40 examinees scored lower than 25 on AR, and that was the lowest-scoring fourth grader (Rank 40). He did a great deal better on SPM, but his pattern of errors was peculiar; he missed a number of easy items and few difficult ones. This boy, alone among the students, proved totally unwilling to do any homework and therefore finally dropped out of the class.

[6]Both Dr. Fox and Dr. Stanley began their teacher careers as mathematics teachers in public high schools.

[7]Year-by year integration actions of school systems may also cause severe attrition because of transfers from the school.

[5]The standardized calculus test was administered by the writer. Dr. McCoart had not seen it; however, two weeks before the test he was given a list of the topics it would cover. This list merely set forth the major topics studied in the usual thorough high-school or college course.

Table 7.1: Test information concerning the McCoart Saturday morning calculus class, September 1974 through 1 February 1975

Coop. Math. Test, Calculus, Form B, Taken 1 Feb. 1975[a]									
Number Right		Percentile Rank of Total Score[b]		Present School Grade	Grade(s) Skipped	Percentage Scores on Prior Tests			
Part I	Part II	High School	National College			1	2	3	4
29	28	99.8	99.97	12		84	52	56	54
28	28	99.5	99.95	12		82	72	88	80
29	27	99.5	99.95	11	9	71	---	---	78
29	27	99.5	99.95	10		88	69	77	71
28	26	99	99.9	10		43	24	76	54
26	28	99	99.9	10		60	52	77	70
28	24	97	99.5	10		55	42	67	56
26	25	97	99.5	10	9	45	49	88	47
27	23	94	99.1	10	8	67	73	78	66
27	23	94	99.1	9	8	54	56	80	77
26	24	94	99.1	9	5, 7, 8	48	45	42	48
24	23	88	98	10		59	17	78	53
25	17	76	94[c]	12		88	59	92	---

[a] This 60-item five-option multiple-choice standardized test consists of two 30-item 40-minute subtests. It was published by the Educational Testing Service, Princeton, New Jersey 08540; copyright 1963.

[b] The total score is the number of the 60 items marked correctly--i.e., the sum of the Part I and Part II scores. You will note that the high-school norms are more stringent than the national college norms; students who take calculus in high school tend to be mathematically abler and better motivated than those who defer it until college. Norms are from pages 51 and 53 of the Cooperative Mathematics Tests Handbook, ETS, 1964.

Of course, norms may have shifted somewhat--though probably not radically--during the dozen years or so since these were developed.

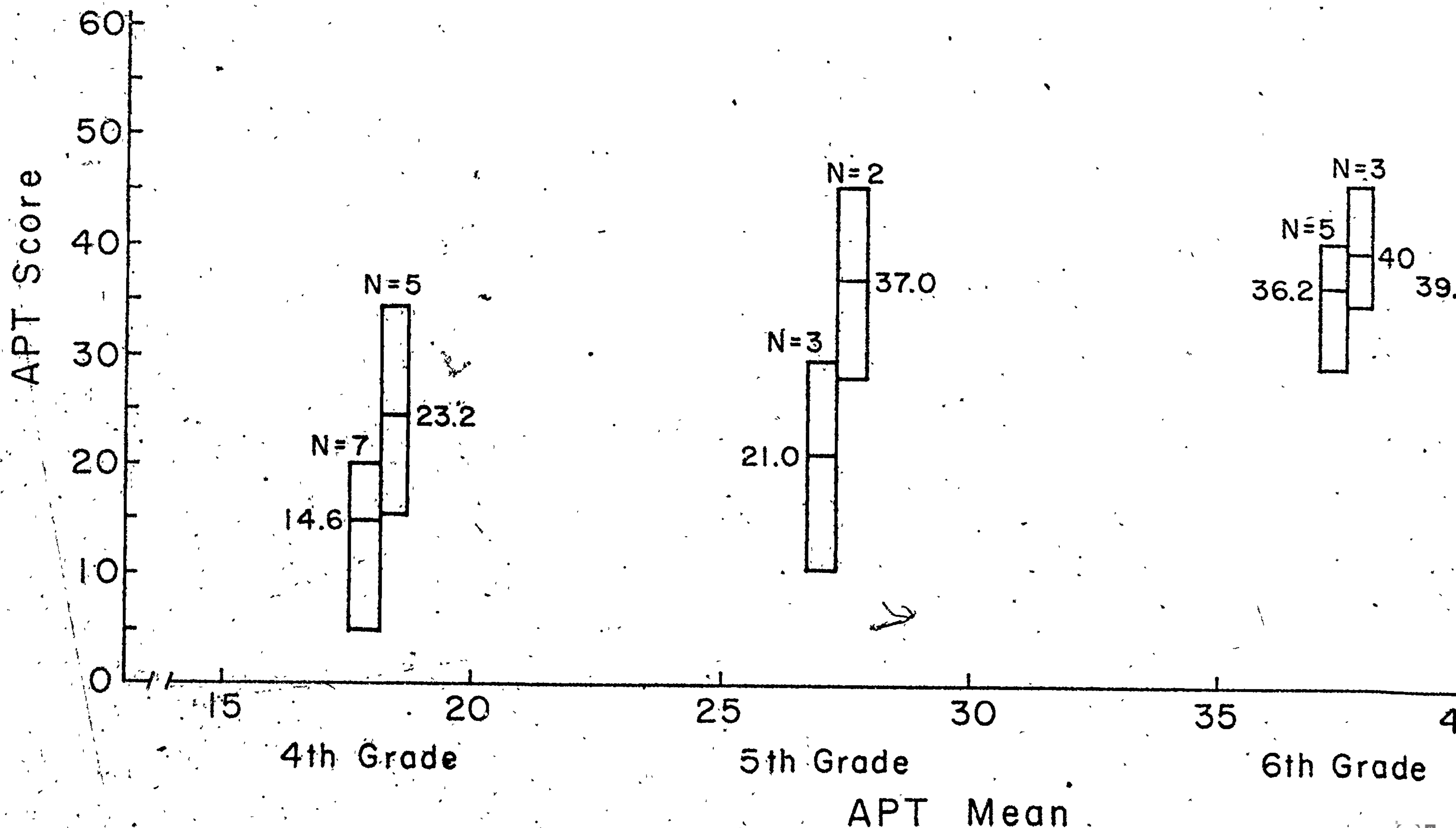
c 95th percentile of the college liberal-arts group. The score of the middle person in this class (52) exceeded that of all but 3 in 1000 students in liberal-arts curricula who have completed two semesters of college calculus.

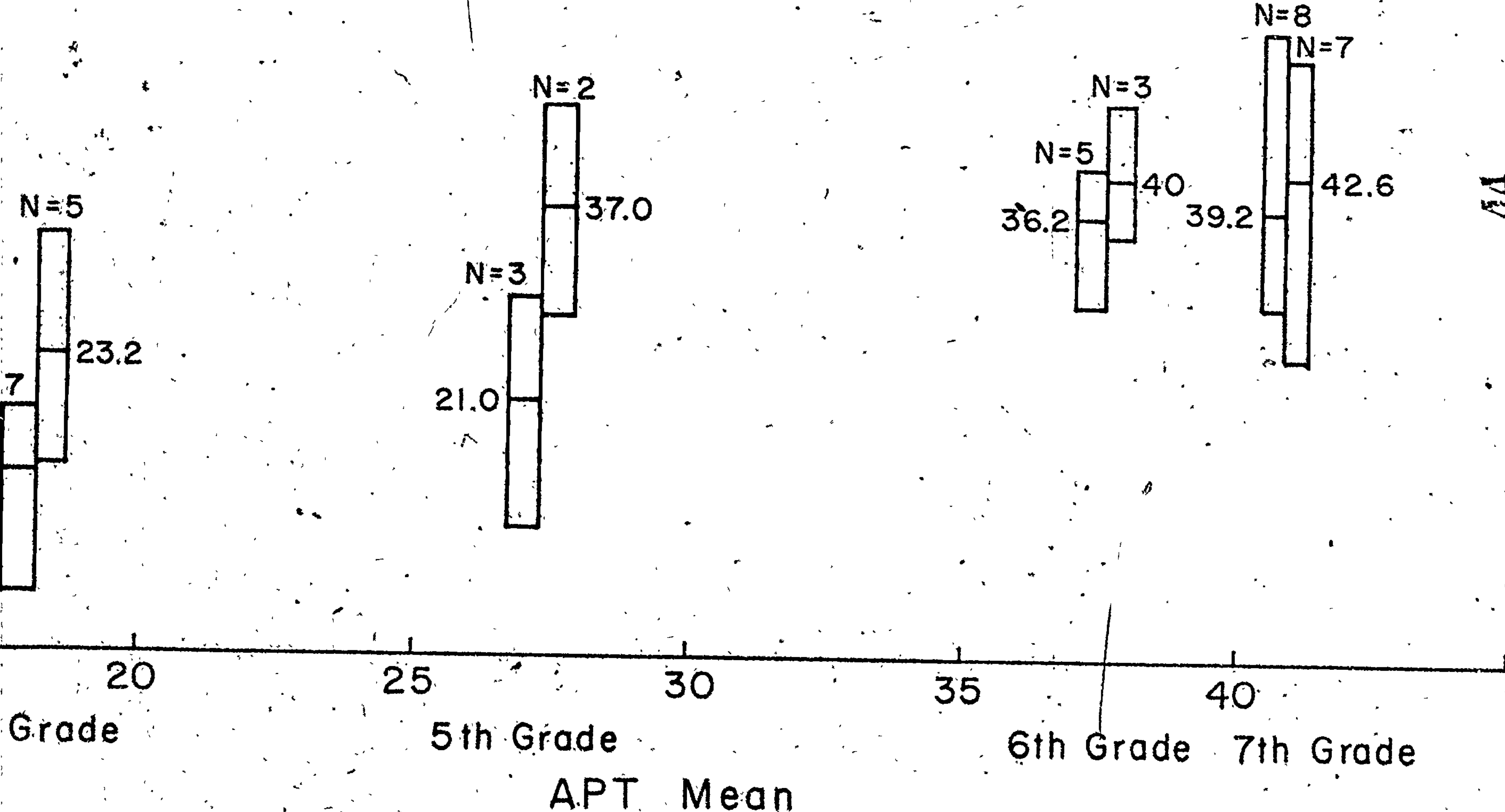
Figure 7.1: <u>The Academic Promise Test numerical scores of the 40 students, by grade and sex.</u> (<u>Left</u> rectangle for each grade represents range of scores for females, with mean also shown. <u>Right</u> rectangles represent males. Each APT subtest has 60 items.)

Appendix 7.1

Table 7.A1: Academic Promise Test (APT) and Raven's Standard Progressive Matrices (SPM) Scores of 17 Pre-selected Boys and 23 Pre-selected Girls in the Fourth through Seventh Grades of School R, 19-20 December 1973

Rank on N	Sex	School Grade Jan. 1974	Birthdate	APT Scores (# Right) N	V	AR	LU	T	SPM Score	Chosen for Class?	June 1974 Alg. I %iles[1]	Nov. 1974 Alg. I %iles[1]
1	F	7	Mar. 61	54[2]	55[3]	37[4]	57	203	56	Yes	95	95
2	M	7	Feb. 61	52	47	47	53	199	50	Yes	68	83
3	M	7	Mar. 61	51[5]	54[6]	48	52	205	60	Yes	99	99.8
4	M	5	Aug. 63	46	51	51	38	186	49	Yes	99.4	---
5.5	M	7	Aug. 61	45[7]	44	55	51	195	57	Yes	68	83
5.5	F	7	Apr. 61	45[8]	46	39	45	175	52	Yes	49	49
7.5	M	7	July 61	44[9]	51[10]	51	47	193	48	Yes	95	90
7.5	M	6	July 62	44	48	51	36	179	51	Yes	73	---
9.5	M	7	July 61	43	46	45	40	174	49	Yes	73	73
9.5	M	6	Oct. 62	43	36	42	41	162	49	Yes	68	---
11	F	6	Sept. 62	42	48	52	49	191	56	Yes	73	---
12	F	6	Dec. 62	41	46	50	47	184	55	Yes	90	---
13.5	F	7	July 61	39	44	50	50	183	51	Yes	95	73
13.5	F	7	Oct. 61	39[11]	47	46	49	181	53	Yes	49	97
15	F	7	July 61	38[12]	40[13]	44	47	169	50	Yes	83	95
16.5	F	7	July 60	36[14]	36	48	31	151	55	Yes	73	90
16.5	M	7	May 61	36	42	17	34	129	51	Yes[15]	---	---
18	F	6	Oct. 62	35	49	45	49	178	45	Yes[16]	---	---
20.5	F	6	Feb. 62	33	47	46	48	174	55	Yes	49	33
20.5	F	7	Jan. 61	33	41	49	48	171	48	Yes	49	49
20.5	M	6	July 62	33	37	52	40	162	50	Yes[16]	---	---
20.5	M	4	Jan. 64	33	43	49	30	155	48	Yes	18	---


APT Score
60
50
40
30
20
10
0
15
20
25
30
35
4
N=7
14.6
N=5
23.2
N=3
21.0
N=2
37.0
N=5
36.2
N=3
40
39.
4th Grade
5th Grade
6th Grade
APT Mean



7
N=5
23.2
N=3
21.0
N=2
37.0
N=5
36.2
N=3
40
N=8
39.2
N=7
42.6
20
25
30
35
40
Grade
5th Grade
6th Grade
7th Grade
APT Mean

Table 7.A1: (Continued) Academic Promise Test (APT) and Raven's Standard Progressive Matrices (SPM) Scores of 17 Pre-selected Boys and 23 Pre-selected Girls in the Fourth through Seventh Grades of School R, 19-20 December 1973

Rank on N	Sex	School Grade Jan. 1974	Birthdate	APT Scores (# Right) N	V	AR	LU	T	SPM Score	Chosen for Class?	June 1974 Alg. I %iles[1]	Nov. 1974 Alg. I %iles[1]
24	F	6	May 62	30	43	48	52	173	47	Yes	39	95
24	F	5	May 63	30	35	43	32	140	43	No	—	—
24	F	7	Oct. 60	30	33	37	33	133	53	No	—	—
26	M	5	Sept. 63	28	36	44	41	149	44	Yes	29	—
27	M	7	July 61	27	41	48	39	155	49	No	—	—
28	M	4	Feb. 64	25	22	36	17	100	43	No	—	—
29	M	4	Aug. 64	23	25	35	17	100	43	No	—	—
30	F	5	Jan. 63	21	26	46	29	122	49	No	—	—
31.5	F	4	Aug. 64	20	32	44	37	133	51	No	—	—
31.5	M	4	July 64	20	30	37	18	105	43	No	—	—
33	F	4	Apr. 64	17	33	45	23	118	47	No	—	—
34.5	F	4	Nov. 63	16	29	30	26	101	49	No	—	—
34.5	F	4	Mar. 64	16	19	45	19	99	46	No	—	—
36.5	M	4	July 64	15	35	42	16	108	46	No	—	—
36.5	F	4	Oct. 64	15	26	31	27	99	35	No	—	—
38	F	4	June 64	13	24	34	20	91	38	No	—	—
39	F	5	Jan. 63	12	24	32	38	106	47	No	—	—
40	F	4	July 64	5	17	25	17	64	37	No	—	—

[1]Ten of the 11 girls were tested on 5 June 1974 with Form B of Educational Testing Service's Cooperative Mathematics Test, Algebra I. The other girl was tested with it on June 11. The 10 boys were tested with Form A on June 7. All testing, except of the absentee girl, was done by Dr. Stanley; Mr. Lerner

Table 7.A1: (Continued) Academic Promise Test (APT) and Raven's Standard Progressive Matrices (SPM) Scores of 17 Pre-selected Boys and 23 Pre-selected Girls in the Fourth through Seventh Grades of School R, 19-20 December 1973

tested her. The percentile ranks shown here are for the national eighth-grade norm group, as provided in the Manual for the test.

The 14 students (9 girls and 5 boys) who continued in the class during the fall were retested on 21 November 1974 by the writer, girls with Form A and boys with Form B. The percentile ranks of their scores are shown in the last column.

[2]By comparison, she made 410 (53rd %ile of a random sample of male high-school juniors and seniors) on the Scholastic Aptitude Test Mathematical (SAT-M), taken in our January 1974 mathematics talent search. This norm and the norms below are from page 5 of College Entrance Examination Board (1973).

[3]She made 580 (94th %ile of a random sample of high-school juniors and seniors) on the Scholastic Aptitude Test Verbal (SAT-V) in the verbal talent search held at The Johns Hopkins University in February of 1974.

[4]This score is curiously low, compared with the girl's other four scores. Note that she scored highest of everyone on N, V, and LU. AR was given first. It is a 60-item test with only a 20-minute time limit, so speed plays an important part. SPM, which also measures nonverbal reasoning, is untimed.

[5]SAT-M 570, 86th %ile.

[6]SAT-V 520, 85th %ile.

[7]SAT-M 450, 63rd %ile.

[8]SAT-M 390, 60th %ile.

[9]SAT-M 470, 67th %ile.

[10]SAT-V 540, 88th %ile.

Table 7.A1: (Continued) Academic Promise Test (APT) and Raven's Standard Progressive Matrices (SPM) Scores of 17 Pre-selected Boys and 23 Pre-selected Girls in the Fourth through Seventh Grades of School R, 19-20 December 1973

[11]SAT-M 330, 41st %ile.

[12]SAT-M 380, 57th %ile.

[13]SAT-V 420, 63rd %ile.

[14]SAT-M 350, 49th %ile.

[15]See Footnote No. 4 in the text.

[16]These two students dropped out soon after the class began. They were two of the three "Yes" students with Spanish surnames.

Appendix 7.2: DAT Scores of Wolfson I Class

In Fox (1974[I:6]) the progress of nine boys and seven girls through SMPY's first special fast-math class was detailed. Not included there were results of the 12 May 1973 testing of those 16 highly able youths with the new version of the Differential Aptitude Tests, published by The Psychological Corporation. At that time one was a fourth grader, 10 were seventh graders, four were eighth graders, and one was a tenth grader. Raw scores, percentile ranks by sex on eighth-grade norms, and other information are shown in Table 7.A2. Rows are

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arranged in decending order of total score on DAT.

It is easy to see from the table that numerical ability ranged from perfect scores (40) for four boys to 33 for a boy. The corresponding percentile ranks were 12 99's and four 97's. This is especially remarkable when one considers that end-of-eighth-grade norms are being used, whereas only five of the 16 students were that high in school. The extremely high scores show how well selected for quantitative aptitude the group was a year earlier and also how stimulated it had been mathematically by Mr. Wolfson, Miss Michaels, and Mr. Bates.

The verbal reasoning scores were nearly as high, ranging from two perfect 50's to a 34 and from 13 99th percentiles to a 90th percentile. Even the fourth grader scored at the 95th percentile on VR.

Only one of the eight subtests, Clerical Speed and Accuracy, proved even mildly difficult for the Wolfson I class. Even there only four students scored below the 50th percentile of end-of-eighth-grade norms.

Physical maturation probably plays a large part in CSA scores.

Only the tenth grader (who had skipped the ninth grade) attained the 99th percentile of eighthgraders on Mechanical Reasoning, but no one scored below the 75th percentile.

The boy who ranked highest on DAT total score entered The Johns Hopkins University as a full-time student with sophomore standing in the fall of 1974 at age 14 years 9 months. During the first semester he took sophomore physics, advanced calculus, introduction to number theory, and American government, making excellent grades.

The boy who ranked only one point lower on DAT-Total also began college advanced calculus that fall, as a part-time student. He did well the first semester and continued with it during the second semester.

The person who ranked generally lowest on the DAT (a seventh grade boy) was also the poorest achiever in the class. The person who ranked seventh on the DAT (a seventh grade girl), was not an excellent achiever; she was next to the bottom of the group by the end of the course period. This occurred even though on APT Verbal Reasoning she earned one of the only two perfect scores. Both of these students who "underachieved" in the Wolfson I class have subsequently moved ahead well in their regular school mathematics classes. The latter skipped the ninth grade.

As of the middle of the 1974-75 school year all 16 of these students seem to be doing well in school. Their grade placement ranges from ninth grade, with tenth grade chemistry and eleventh grade calculus (the former fourth grader), to middle-of-sophomore-year status at Johns Hopkins (the former eighth grader whose DAT scores are shown in the first row of the table). The boys have progressed much faster and better than the girls, only one of whom has even skipped a school grade.

Just two of the nine boys have not skipped at least one grade, and one of those has been taking college courses for credit regularly part-time. It seems clear that this first of our special fast-math classes had enormous facilitating effects on the boys, and moderate ones on some of the girls. Such success is due, we believe, to the superb teaching and stimulating ability of Mr. Wolfson on an extremely able group that with a single exception--the tenth grader--had not yet taken even one algebra course in school.

The success of the Wolfson II, Fox, McCoart, and Wagner classes (see chapters 6, 9, and 7, respectively, of this volume) indicates that the effects were not unique to the first class or to Mr. Wolfson. This out-of-school type class is a powerful way to look after the mathematical needs of quantitative highly apt youths. The concept and techniques should be readily adaptable to other subjects. From many standpoints it would seem desirable (though not necessarily as effective) to have the classes conducted by the school system itself, rather than by an outside agency such as a university. Alternatively, the school system might contract with the outside agency to set up classes and supervise them. Such classes should enroll only unusually able students, not less than the upper one or two percent of the age group in that system. Even most of those probably could not progress as fast as the Wolfson I and II classes, which consisted of students extremely highly selected for mathematical aptitude from several counties.

In addition to the system-wide classes, special classes or groups within individual junior high schools located in talent-rich areas will be needed. The McCoart and Wagner classes at School R and other within-school classes in progress with SMPY's help explore how this can be done best. Of course, the less able the group the less swiftly and well it can move through the mathematics curriculum.

Even the mathematically most apt five or 10 percent within a school system need special opportunities, however, such as having Algebra I available in the eighth or even the seventh grade. Every effort should be made to see that such students are encouraged and helped to complete courses in calculus, computer science, and finite mathematics before being graduated from high school. The most successful of them should take the Advanced Placement Program examination in calculus, offered each year (usually in May), and thereby earn college credit. As Fox points out in chapter 9 of this volume, most quantitatively able girls will probably need more special stimulation and encouragement than the boys.

Table 7.A2: Differential Aptitude Test raw scores (RS) and percentile ranks (PR) of the 16 Wolfson I students on the eight subtests of Form S, administered 12 May 1973

Student[a]	School Grade 1973-74	Sex	Numerical Ability		Verbal Reasoning		Abstract Reasoning		Language Usage		Spelling		Space Relations		Mechanical Reasoning		Clerical Speed and Accuracy	
			RS	PR[b]	RS	PR	RS	PR	RS	PR	RS	PR	RS	PR	RS	PR	RS	PR
6	8[c]	M	40	99	50	99	47	99	49	99	99	99	59	99	58	95	52	90
1	8	M	37	99	48	99	49	99	57	99	97	99	60	99	57	90	48	85
2	10[d]	M	37	99	48	99	48	99	47	97	91	97	60	99	67	99	40	55
3	7	M	40	99	49	99	48	99	53	99	94	99	50	97	57	90	45	75
4	7	F	37	99	46	99	44	97	56	99	94	97	47	97	50	95	57	90
10	7	F	39	99	49	99	40	99	52	99	84	90	53	99	50	95	57	90
11	7	F	37	99	50	99	46	99	55	99	97	99	47	97	53	97	41	40
15	7	F	35	97	47	99	47	99	51	99	94	97	51	99	45	85	53	65
7	8[c]	M	40	99	45	99	49	99	37	90	89	97	55	99	52	80	48	85
9	8	F	40	99	42	97	50	99	43	80	89	95	55	99	55	97	39	35
12	7	M	40	99	46	99	41	90	48	97	87	95	45	95	58	95	38	50
13	7	F	36	99	47	99	45	97	48	97	95	99	41	95	43	75	46	60
5	7	M	35	97	48	99	41	90	46	97	96	99	33	75	53	80	42	65
21	7	F	38	99	45	99	47	99	47	95	94	97	31	75	43	75	36	25
8	4	M	34	97	38	95	46	97	37	90	78	90	43	95	53	80	42	65
16	7	M	33	97	34	90	44	95	39	90	54	50	45	95	55	85	35	35

[a]The code numbers used are the same as in Fox (1974[I:6]). They are the ranks on the Algebra I test administered to the group in August of 1972, after only 18 hours of instruction (except for Nos. 1, 9, and

Table 7.A2: Differential Aptitude Test raw scores (RS) and percentile ranks (PR) of the 16 Wolfson I students on the eight subtests of Form S, administered 12 May 1973 (Continued)

21, who neither took Algebra I in school nor joined the class until September of 1972).

[b]Percentile ranks shown are national spring-of-eighth-grade sex norms, which differ somewhat for boys vs. girls. For example, Student No. 5 scored higher on Numerical Ability than 97 out of 100 male eighth graders do in the spring.

[c]Skipped the seventh grade.

[d]Skipped the ninth grade.