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ABSTRACT

This text is one of the sequence of textbooks produced for low achievers in the seventh and eighth grades by the School Mathematics Study Group (SMSG). There are eight texts in the sequence, of which this is the seventh. This set of volumes differs from the regular editions of SMSG junior high school texts in that very little reading is required. Concepts and processes are illustrated pictorially, and many exercises are included. Chapter 15, the first of two chapters in this volume, concerns measurement. The need for standard units is discussed and, after some work on computation with mixed numbers, both English and metric units are introduced. Measurement of angles using the protractor and computation of area are also discussed. In chapter 16 perfect squares are presented, and the idea of finding the sides of squares with given area is used to motivate an introduction to the real numbers. Computations with radicals, the Pythagorean theorem, and circumference and area of circles are also developed. (SD)

SECONDARY SCHOOL MATHEMATICS

SPECIAL EDITION

Chapter 15. Measurement

Chapter 16. Real Numbers

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MEASUREMENT

Measurement

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Your friend tells you that he has a piece of rope that is 10 args long. Do you have any idea how long his rope is? Probably the first thing you will want to know is, "How long is an arg?" Suppose your friend says, "An arg is 2 yuks long." Even though you still don't have any idea how long an arg or a yuk is, how many yuks long is his rope? ____________ If a yuk is 3 snuks long, how many snuks long is his rope? ____________

Make a list of these answers.

The rope is 10 args long.
The rope is 20 yuks long.
The rope is 60 snuks long.

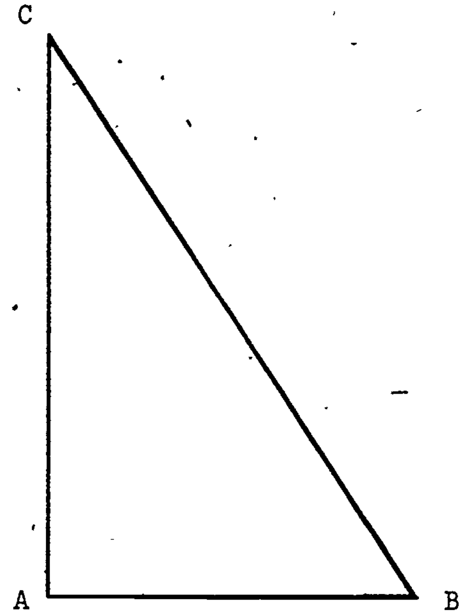
You have been able to associate three different numbers with the length of the rope even though you still don't have any idea of how long it is!

If your friend said, "One yuk is the distance from my nose to my toes," you can now get some idea of the length of the rope.

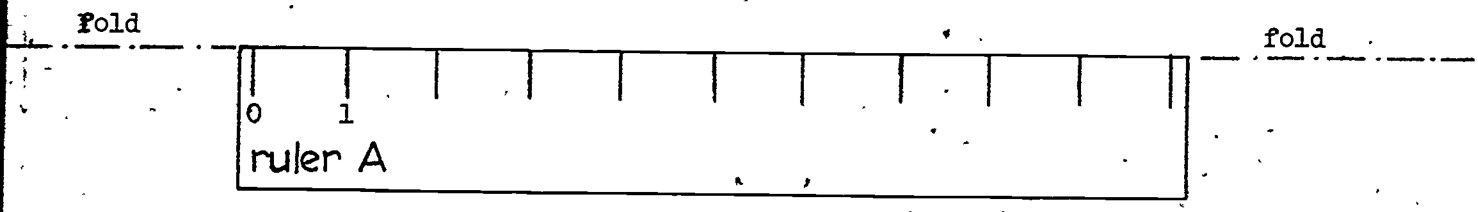
Measurement involves two ideas: First, the idea of unit, and second, the idea of number. We are free to choose any unit we want. The number assigned to the length depends upon the unit we choose.

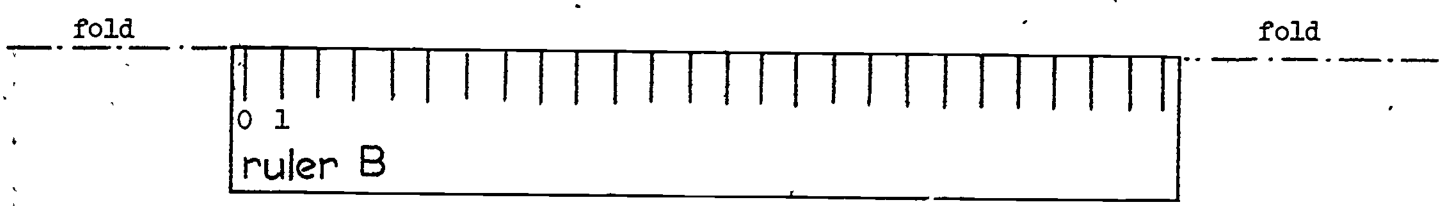
Exercises

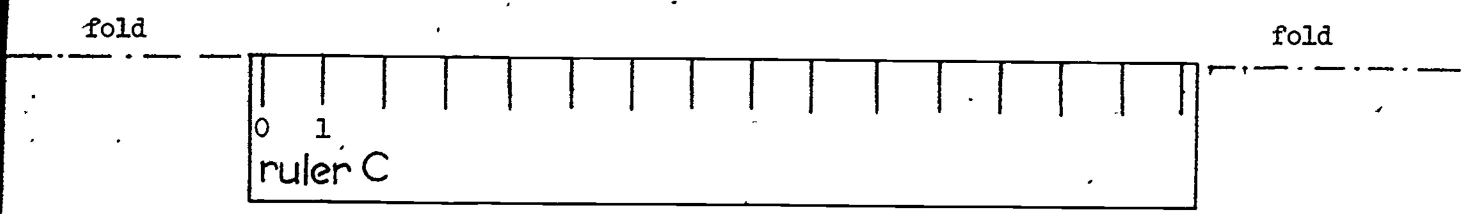
Use each of the rulers on Page 15-1b to measure the sides of this triangle. Read the rulers to the nearest mark. Record your measurements in the table below.

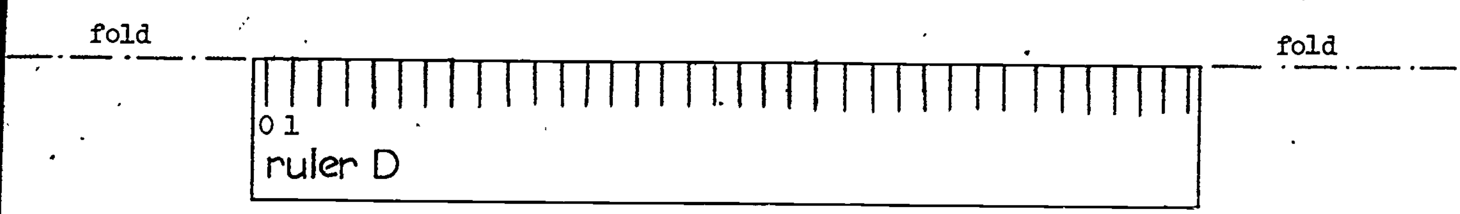


Segment	Ruler A	Ruler B	Ruler C	Ruler D
m $\overline{AB}$				
m $\overline{AC}$				
m $\overline{BC}$				


fold
fold
0
1
ruler A


fold
fold
0
1
ruler B


fold
fold
0
1
ruler C

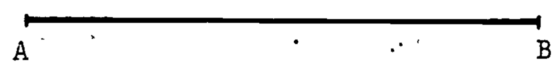

fold
fold
0
1
ruler D

Standard Units

In the last lesson you learned that the number associated with length depended upon the unit you used. In order to have an idea of how long something is, you must know the unit. Think how confusing it would be if everyone used different units. In order to avoid this confusion some agreements on units are needed. In this country some of the units agreed upon are the inch, the foot, the yard, and the mile. Units agreed upon are called standard units.

Class Discussion

On Page 15-2b are several pictures of 6-inch rulers. They are really just parts of portable number lines. The unit on these number lines is one inch. Take Page 15-2b out of your notebook and use "Ruler A" to measure this segment.



Segment $\overline{AB}$ is more than ________ inches long but less than ________ inches long.

Is the length of segment $\overline{AB}$ closer to 2 inches or 3 inches?

____

Is it exactly 3 inches long? ____________

We can say that segment $\overline{AB}$ is "about" 3 inches long. Measurement is always approximate because in using rulers we always read to the nearest mark. If we want a closer approximation to the length of segment $\overline{AB}$, all we need to do is to divide the unit on the ruler into more parts so that the marks are closer together.

Exercises

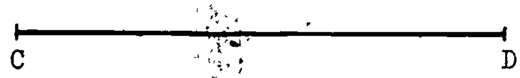
1. (a) Each unit segment of ruler B is divided into _______ parts and each of these parts is _______ inch long.

   (b) Each unit segment of ruler C is divided into _______ parts and each of these parts is _______ inch long.

   (c) Each unit segment of ruler D is divided into _______ parts and each of these parts is _______ inch long.

   (d) Each unit segment of ruler E is divided into _______ parts and each of these parts is _______ inch long.

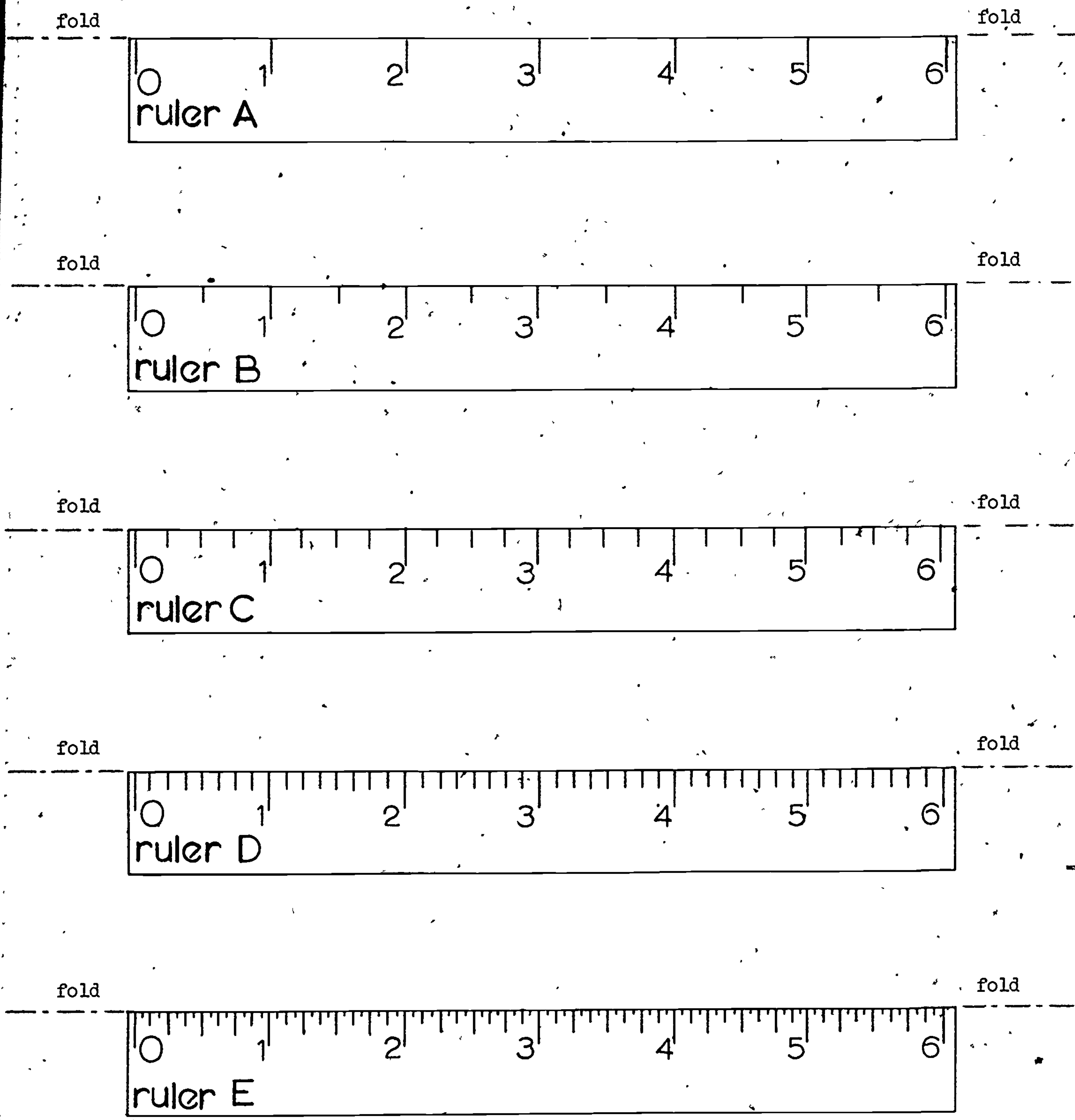
2. (a) Measure segment $\overline{CD}$ using each of the rulers on Page 15-2b and complete the table below. Remember to read the ruler to the closest mark.



Ruler	Length of $\overline{CD}$
A	
B	
C	
D	
E	

   (b) Which ruler is least precise? _________

   (c) Which ruler is most precise? _________


fold
fold
0 1 2 3 4 5 6
ruler A
fold
fold
0 1 2 3 4 5 6
ruler B
fold
fold
0 1 2 3 4 5 6
ruler C
fold
fold
0 1 2 3 4 5 6
ruler D
fold
fold
0 1 2 3 4 5 6
ruler E

Mixed Numbers

Class Discussion

Mixed numbers are commonly used in shops and stores. They are really rational numbers written in a different way. You know that $\frac{7}{2}$ means 7 divided by 2 . If you do the division your answer is 3 with $\frac{1}{2}$ left.

Example.

$$\frac{7}{2} = 3\frac{1}{2}$$

To rewrite a rational number as a mixed number you simply divide the numerator by the denominator and write the remainder as a fraction. Sometimes you can simplify this fraction.

Rewrite these rational numbers as mixed numbers.

1. $\frac{9}{4}$ = __________
2. $\frac{8}{5}$ = __________
3. $\frac{7}{6}$ = __________
4. $\frac{9}{2}$ = __________

To rewrite a mixed number as a fraction you need to find a fraction name for the integer part.

Example.

$$2\frac{1}{3} = 2 + \frac{1}{3}$$

$$2\frac{1}{3} = \frac{6}{3} + \frac{1}{3}$$

$$2\frac{1}{3} = \frac{7}{3}$$

Notice that the fraction name we chose for the integer 2 has a 3 in the denominator. We chose $\frac{6}{3}$ as a name for 2 so that it would be easy to add to $\frac{1}{3}$. We always choose a name for the integer part that has same denominator as the fractional part of the mixed number.

Here is another example.

$$3\frac{3}{7} = 3 + \frac{3}{7}$$

$$3\frac{3}{7} = \frac{21}{7} + \frac{3}{7}$$

$$3\frac{3}{7} = \frac{24}{7}$$

Rewrite these mixed numbers as fractions. Follow the example above.

1. $3\frac{3}{4}$ = ____ + ____

   $3\frac{3}{4}$ = ____ + ____

   $3\frac{3}{4}$ = ________

2. $4\frac{2}{3}$ = ____ + ____

   $4\frac{2}{3}$ = ____ + ____

   $4\frac{2}{3}$ = ________

3. $2\frac{5}{8}$ = ____ + ____

   $2\frac{5}{8}$ = ____ + ____

   $2\frac{5}{8}$ = ________

4. $5\frac{5}{9}$ = ____ + ____

   $5\frac{5}{9}$ = ____ + ____

   $5\frac{5}{9}$ = ________

Addition of mixed numbers is very easy. Simply add the integer parts together and add the fractional parts together.

Example 1.

$$2\frac{1}{3} + 5\frac{1}{3} = 2 + 5 + \frac{1}{3} + \frac{1}{3}$$

$$2\frac{1}{3} + 5\frac{1}{3} = 7 + \frac{2}{3}$$

$$2\frac{1}{3} + 5\frac{1}{3} = 7\frac{2}{3}$$

Example 2.

$$3\frac{2}{7} + 11\frac{3}{7} = 3 + 11 + \frac{2}{7} + \frac{3}{7}$$

$$3\frac{2}{7} + 11\frac{3}{7} = 14 + \frac{5}{7}$$

$$3\frac{2}{7} + 11\frac{3}{7} = 14\frac{5}{7}$$

As long as the fractional parts of the numbers have the same denominators you can probably do this in your head.

Add these mixed numbers. Simplify the fractional part of your answer if possible.

1. $1\frac{5}{8} + 5\frac{1}{8} =$ ________

2. $7\frac{2}{9} + 8\frac{5}{9} =$ ________

3. $12\frac{2}{5} + 17\frac{2}{5} =$ ________

4. $15\frac{3}{10} + 9\frac{5}{10} =$ ________

Sometimes when you add mixed numbers the fractional part of your answer has a numerator that is larger than the denominator. When this happens you rewrite the fractional part of the answer as another mixed number and add again.

Example 1.

$$5\frac{5}{7} + 4\frac{3}{7} = 9\frac{8}{7} \quad \text{but} \quad \frac{8}{7} = 1\frac{1}{7}$$

$$5\frac{5}{7} + 4\frac{3}{7} = 9 + 1\frac{1}{7}$$

So: $$5\frac{5}{7} + 4\frac{3}{7} = 10\frac{1}{7}$$

Example 2.

$$6\frac{7}{8} + 9\frac{5}{8} = 15\frac{12}{8} \quad \text{but} \quad \frac{12}{8} = 1\frac{4}{8}$$

$$6\frac{7}{8} + 9\frac{5}{8} = 15 + 1\frac{4}{8}$$

$$6\frac{7}{8} + 9\frac{5}{8} = 16\frac{4}{8}$$

$16\frac{4}{8}$ simplified is $16\frac{1}{2}$.

To add mixed numbers that have fractional parts with unlike denominators we add the integers and add the fractions separately. To add the fractions you must find a common denominator.

Example.

$$3\frac{3}{4} + 2\frac{4}{5} = 5 + \left(\frac{3}{4} + \frac{4}{5}\right)$$

$$3\frac{3}{4} + 2\frac{4}{5} = 5 + \left(\frac{15 + 16}{20}\right)$$

$$3\frac{3}{4} + 2\frac{4}{5} = 5\frac{31}{20} \qquad \text{but} \quad \frac{31}{20} = 1\frac{11}{20}$$

So: $$3\frac{3}{4} + 2\frac{4}{5} = 6\frac{11}{20}$$

Add these mixed numbers and simplify your answers.

1. $7\frac{5}{8} + 4\frac{3}{4} =$ ________
3. $15\frac{7}{8} + 20\frac{5}{6} =$ ________
2. $5\frac{2}{3} + 12\frac{1}{2} =$ ________
4. $8\frac{2}{7} + 1\frac{1}{2} =$ ________

To multiply an integer times a mixed number it is easiest to write the mixed number as a sum.

Example. To multiply $5\frac{7}{8} \times 2$ we write:

$$\begin{array}{r} 5 + \frac{7}{8} \\ \times\ 2 \\ \hline 10 + \frac{14}{8} \end{array} = 10\frac{14}{8} \quad \text{but} \quad \frac{14}{8} = 1\frac{6}{8}$$

so $$10\frac{14}{8} = 10 + 1\frac{6}{8}$$

$$= 11\frac{6}{8} \text{ or } 11\frac{3}{4}$$

Multiply these numbers.

1. $4\frac{2}{3} \times 7$

   rewritten: $\begin{array}{r} 4 + \frac{2}{3} \\ \times\ 7 \\ \hline \end{array}$

   = ______

2. $7\frac{1}{9} \times 8$

   rewritten: $\begin{array}{r} 7 + \frac{1}{9} \\ \times\ 8 \\ \hline \end{array}$

   = ______

3. $5\frac{2}{5} \times 6$

   rewritten: $\begin{array}{r} 5 + \frac{2}{5} \\ \times\ 6 \\ \hline \end{array}$

   = ______

4. $8\frac{3}{4} \times 5$

   rewritten: $\begin{array}{r} 8 + \frac{3}{4} \\ \times\ 5 \\ \hline \end{array}$

   = ______

Add these numbers.

5. $2\frac{4}{7} + 3\frac{2}{7} =$ ______

6. $8\frac{5}{8} + 6\frac{2}{8} =$ ______

7. $4\frac{5}{6} + 2\frac{1}{6} =$ ______

   = ______

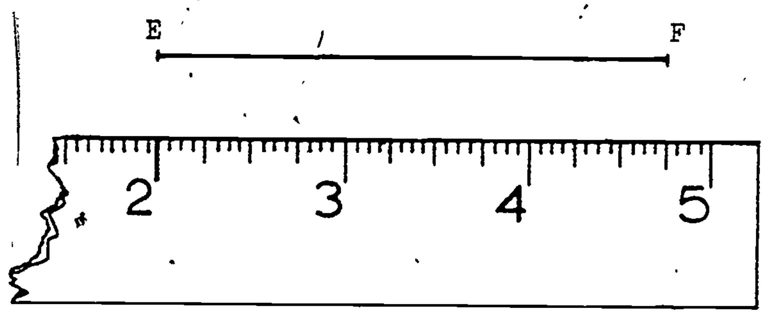
8. $6\frac{7}{9} + 5\frac{4}{9} =$ ______

   = ______

Distance Between Points on the Ruler

Class Discussion

The diagram below shows a broken ruler placed below segment $\overline{EF}$.



1. Is segment $\overline{EF}$ $4\frac{3}{4}''$ long? __________

2. What number should be subtracted from $4\frac{3}{4}$ to get the correct length of $\overline{EF}$? __________

3. The symbol ≈ means "approximately equal to."

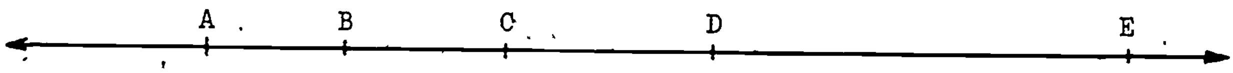
   Is the length of $\overline{EF} \approx 4\frac{3}{4} - 2$? __________

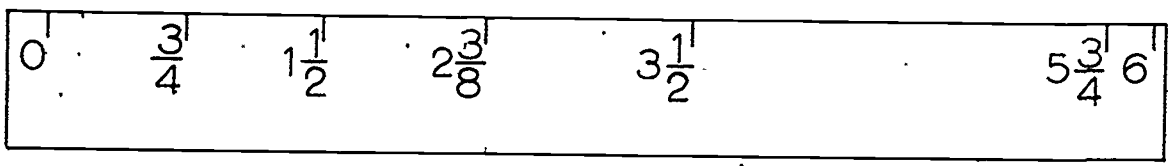
4. $4\frac{3}{4} - 2 =$ __________.

When you place a ruler so that the end of a segment lies on some mark other than zero, you must be careful to subtract the number on the ruler at the left end of the segment from the number on the ruler at the right end. Ends of rulers are often damaged. This method of using a ruler usually gives more accurate measurements.

Exercises

Use this picture to answer the following questions.





Find the lengths of the following segments by writing a subtraction problem and then solving the subtraction problem.

1. $m\ \overline{AB} \approx$ ______ - ______

   $m\ \overline{AB} \approx$ __________

2. $m\ \overline{AD} \approx$ ______ - ______

   $m\ \overline{AD} \approx$ __________

3. $m\ \overline{BE} \approx$ ______ - ______

   $m\ \overline{BE} \approx$ __________

4. $m\ \overline{CE} \approx$ ______ - ______

   $m\ \overline{CE} \approx$ __________

5. $m\ \overline{BD} \approx$ ______ - ______

   $m\ \overline{BD} \approx$ __________

6. $m\ \overline{CD} \approx$ ______ - ______

   $m\ \overline{CD} \approx$ __________

7. $m\ \overline{AC} \approx$ ______ - ______

   $m\ \overline{AC} \approx$ __________

8. $m\ \overline{AE} \approx$ ______ - ______

   $m\ \overline{AE} \approx$ __________

9. $m\ \overline{DE} \approx$ ______ - ______

   $m\ \overline{DE} \approx$ __________

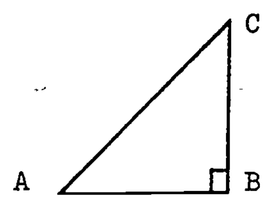
Subdivision of the Inch

The ordinary rulers you have been using were subdivided by repeatedly bisecting the inch. That is, we started with the inch unit and bisected it. We then had divisions of $\frac{1}{2}$-inch. We then bisected each $\frac{1}{2}$-inch so that we had divisions of $\frac{1}{4}$-inch. We again bisected to get divisions of $\frac{1}{8}$-inch and again bisected into $\frac{1}{16}$-inch divisions. Some rulers continue this process until they get $\frac{1}{32}$-inch and then even $\frac{1}{64}$-inch divisions. These rulers are usually made of steel and are called "machinist scales". These "scales" are very hard to read but they are more accurate than the common wood ruler. Even so, they are not nearly precise enough for modern day engineering work. Other ways of subdividing the inch and other measuring instruments are used by engineers and machinists. For this work the inch is divided into tenths, hundredths, thousandths, and even ten-thousandths.

Class Discussion

1. On Page 15-5c is a picture of a ruler with the inch unit divided into tenths. Use it to measure the sides of these triangles. Write your measurements in decimal form.

(a)



$m\ \overline{AB} \approx$ ________

$m\ \overline{BC} \approx$ ________

$m\ \overline{AC} \approx$ ________

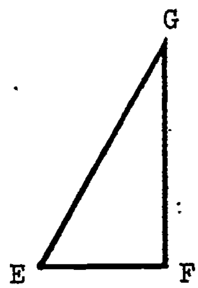
The length of a triangle (distance around) is called the perimeter of the triangle.

The perimeter of $\triangle ABC = m\ \overline{AB} + m\ \overline{BC} + m\ \overline{AC}$

The perimeter of $\triangle ABC \approx$ ____ + ____ + ____

The perimeter of $\triangle ABC \approx$ ________

(b)



m $\overline{EF} \approx$ ________________

m $\overline{FG} \approx$ ________________

m $\overline{EG} \approx$ ________________

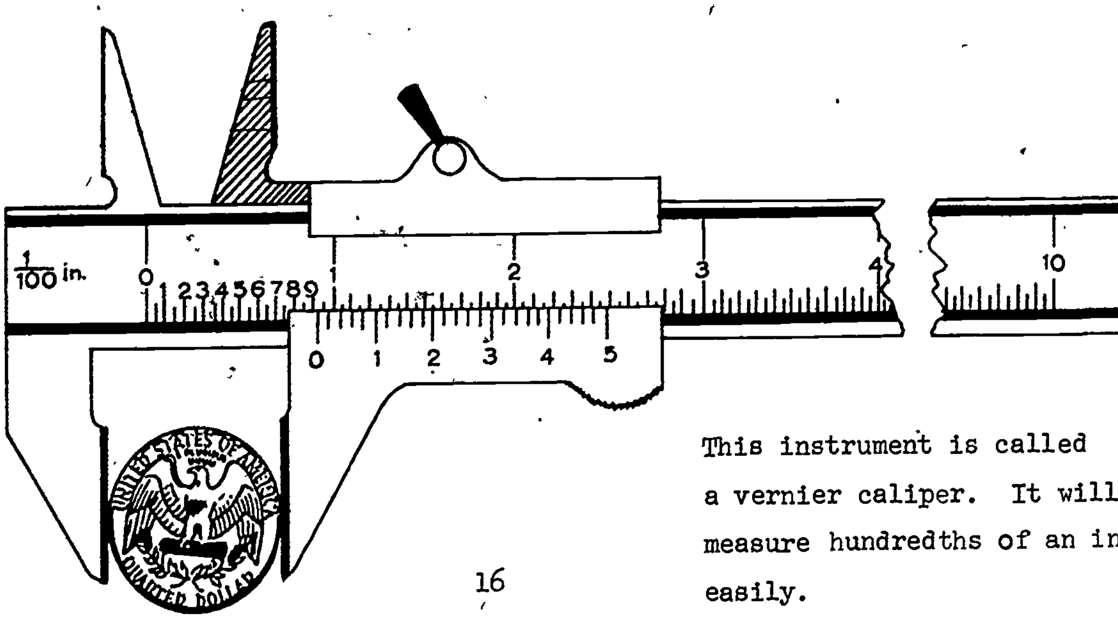
Perimeter of $\triangle$ EFG $\approx$ _____ + _____ + _____

Perimeter of $\triangle$ EFG $\approx$ ____________

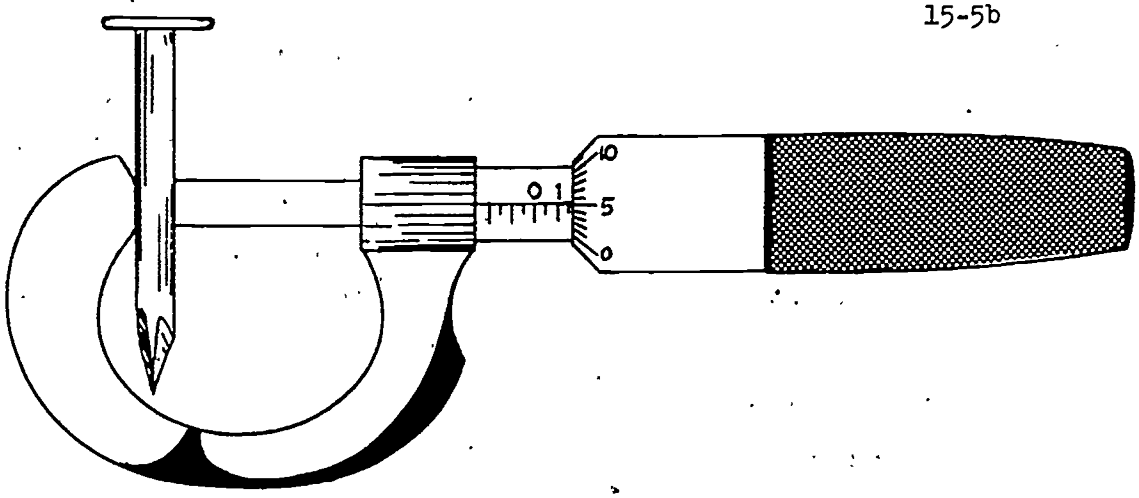
(c) Imagine each tenth of an inch divided into ten parts. Each of these parts would be one ________________ of an inch. They would be very small segments. Such a ruler is very hard to read.

(d) If each of these hundredths were divided into ten parts, each part would be one ______________ of an inch. These segments are so small that they are about the thickness of a human hair. Do you think it would be possible to read such a ruler?

____



This instrument is called a vernier caliper. It will measure hundredths of an inch easily.

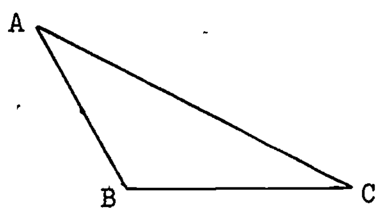


This instrument is called a "micrometer". It is commonly used by machinists. It will measure to thousandths of an inch. You may have a chance to use these kinds of instruments in your science classes. If so, you will be taught how to use them at that time.

Exercises

Use the ruler on Page 15-5c to measure the sides of these triangles. Write your answers in decimal form. Add the lengths of the sides of each triangle to find its perimeter (P).

1.



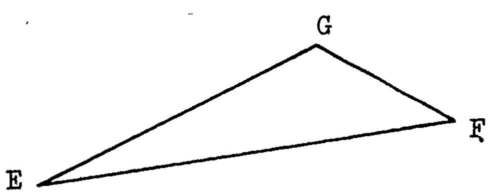
m $\overline{AB}$ ≈ ________

m $\overline{BC}$ ≈ ________

m $\overline{AC}$ ≈ ________

P ≈ ________

2.



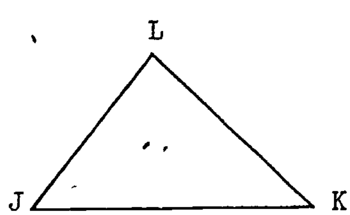
m $\overline{EF}$ ≈ ________

m $\overline{FG}$ ≈ ________

m $\overline{EG}$ ≈ ________

P ≈ ________

3.

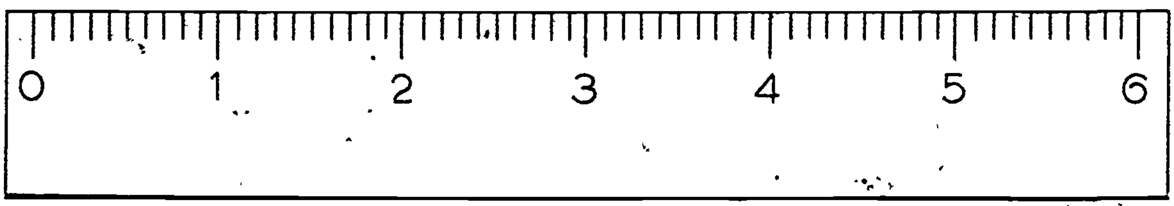


m $\overline{JK}$ ≈ ________

m $\overline{KL}$ ≈ ________

m $\overline{JL}$ ≈ ________

P ≈ ________



Each unit on this ruler is divided into tenths.

The Centimeter Ruler

Most countries use the centimeter as their small unit of measure. The abbreviation of centimeter is "cm". Physicists and other scientists find this unit so handy that they do much of their work using centimeters. Many people in our country want us to adopt centimeters as our small unit of measure.

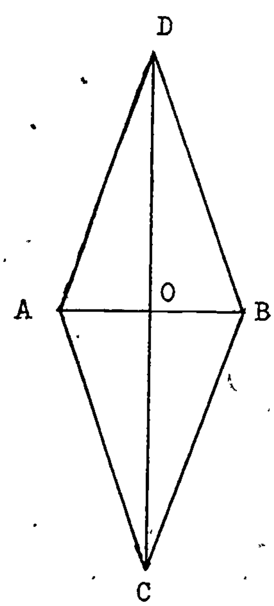
Class Discussion

Take the picture of the centimeter ruler on Page 15-6b out of your notebook.

Into how many parts has each unit centimeter been divided? ________
Can we write our measurements in decimal form? ________

Use your "centimeter" ruler and your "inch" ruler E (Page 15-2b) to measure parts of this rhombus. The parts to be measured are shown in the table below. Write your "cm" measurements in decimal form. Remember to read your ruler to the closest mark.

Segment	Length in inches	Length in cm
$\overline{AB}$	________	________
$\overline{CD}$	________	________
$\overline{AC}$	________	________



Without measuring, how long are $\overline{BC}$, $\overline{AD}$, and $\overline{BD}$? ________ cm

Without measuring, how long is $\overline{AO}$?
________ cm

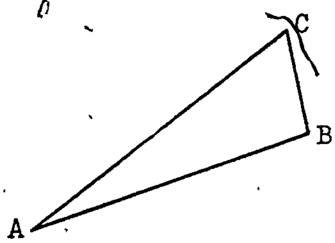
Without measuring, how long is $\overline{DO}$?
________ cm

$\overline{AB}$ is ________ inches long and $\overline{AB}$ is ________ centimeters long.
About how many centimeters are there in one inch?

________ cm $\approx$ 1 inch

Exercises

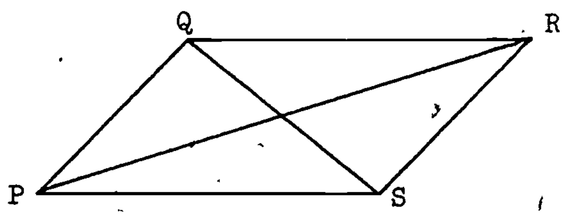
Use your cm ruler to make the following measurements. Write the measurements in decimal form.

1. 

m $\overline{AB} \approx$ ________ cm

m $\overline{BC} \approx$ ________ cm

m $\overline{AC} \approx$ ________ cm

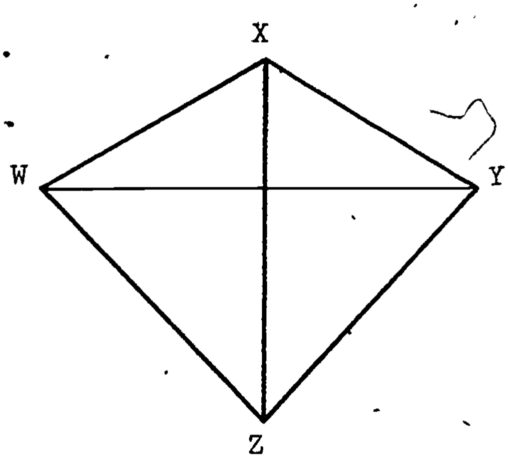
2. 

m $\overline{PQ} \approx$ ________ cm

m $\overline{QR} \approx$ ________ cm

m $\overline{QS} \approx$ ________ cm

m $\overline{PR} \approx$ ________ cm

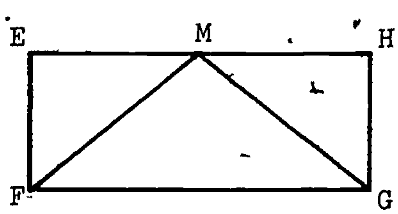
3. 

m $\overline{WX} \approx$ ________ cm

m $\overline{WZ} \approx$ ________ cm

m $\overline{XZ} \approx$ ________ cm

m $\overline{WY} \approx$ ________ cm

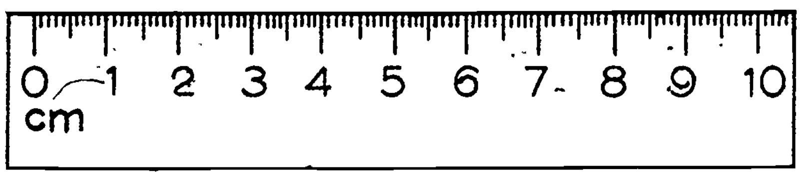
4. 

m $\overline{EF} \approx$ ________ cm

m $\overline{FG} \approx$ ________ cm

m $\overline{FM} \approx$ ________ cm

m $\overline{GM} \approx$ ________ cm


0 1 2 3 4 5 6 7 8 9 10
cm

Changing Units - Inches and Feet

You know that besides using inches as our unit, we also use the foot, yard, and mile as units. This is called the "British system of measurement."

There are also other units that go together with centimeters. Some of these are the millimeter, meter, and kilometer. This is called the "metric system of measurement."

Below is a list from smaller units to larger units for both systems.

British	Abbreviations	Metric	Abbreviations
inch	in.	millimeter	mm
foot	ft.	centimeter	cm
yard	yd.	meter	m
mile	mi.	kilometer	km

In countries using the British system you need to know how to change from inches to feet, feet to miles, miles to yards and so on.

In countries where the metric system is used you need to be able to change from centimeters to meters, meters to kilometers, and so on. You will also need to be able to change systems when studying science.

If someday you travel to a foreign country, say Mexico, you will find road signs marked in kilometers and speed limits in kilometers per hour. But American speedometers are marked in miles. Engineers often work in the metric system but in order to get their designs made they have to change to inches. For these reasons you will need to be able to change from one system to the other.

Class Discussion

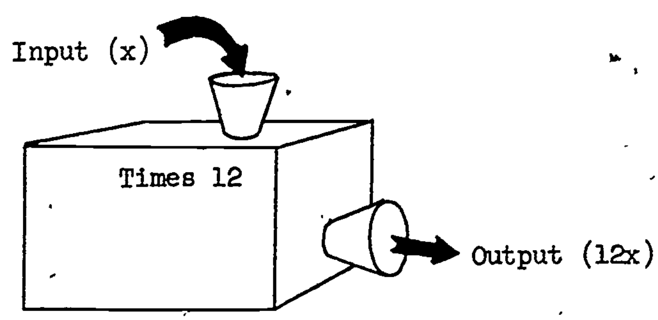
You know that if you measure something 1 foot long its length can be written as 12 inches. 2 feet can be written as 24 inches and so on.

1	foot	=	1 · 12 inches	
2	feet	=	2 · 12 inches	(24 inches)
3	feet	=	3 · 12 inches	(36 inches)
4	feet	=	4 · 12 inches	(48 inches)
x	feet	=	x · 12 inches	

and so on.

To change from feet to inches you multiply by ______________.

Suppose we have a "times 12" function machine like this:

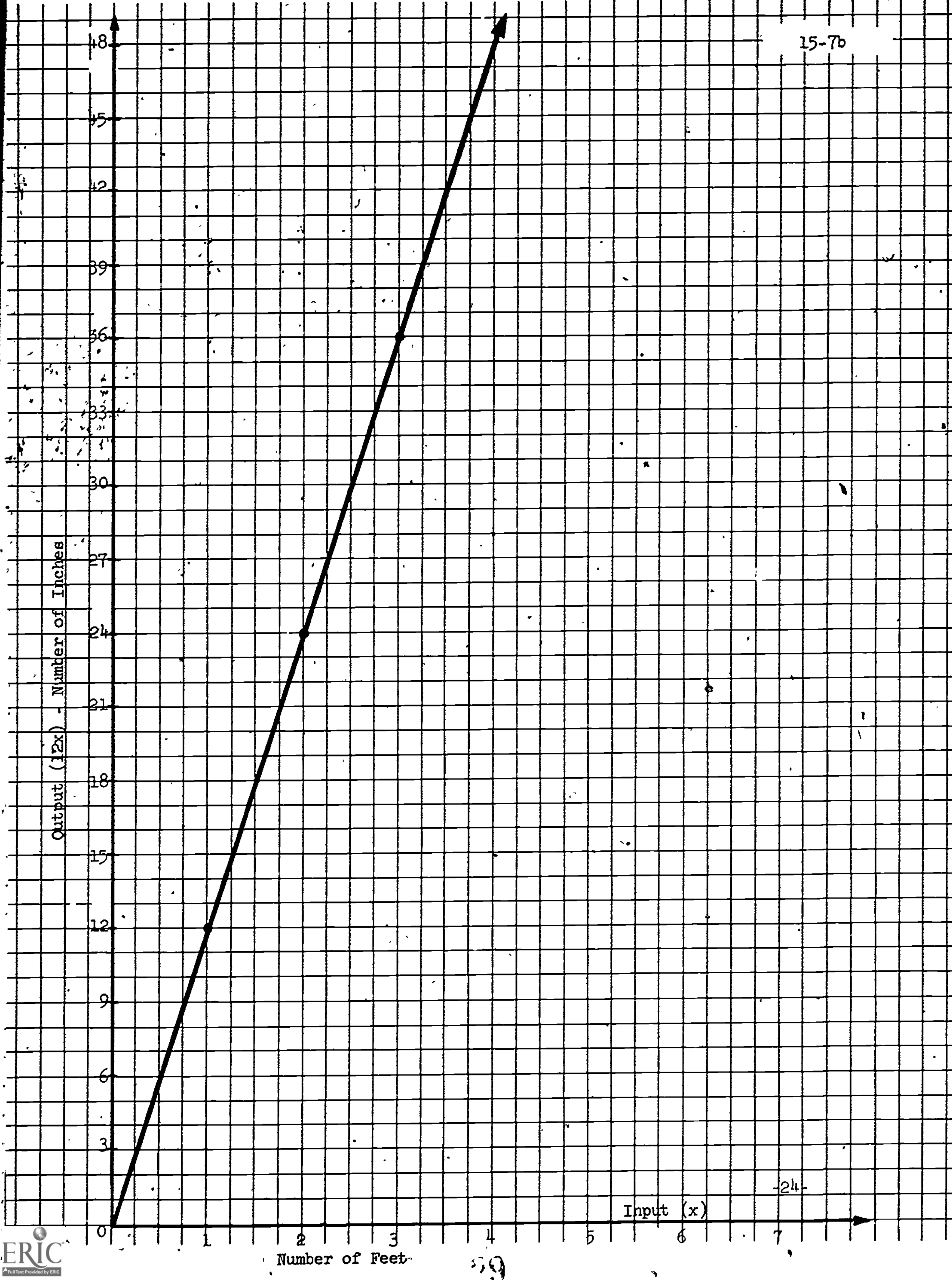


If we assign the number of feet to x and input x to our machine, it will output the number of inches that tells the same length.

Write this function in arrow notation: ______________________

This function will change from the foot unit to the inch unit for us. It will make our work easier and faster if we use the graph of this function on the next page.

It doesn't make sense to talk about negative length so we need only the upper right section of the graph.


Output (12x) - Number of Inches
48
45
42
39
36
33
30
27
24
21
18
15
12
9
6
3
0
1
2
3
4
5
6
7
Input (x)
Number of Feet

We can check our graph by trying a few conversions we already know. Read the graph for a length of 3 feet. Do you get 36 inches? ______

What do you get for 2 feet?

2 feet = ________ inches.

What do you get for $3\frac{1}{2}$ feet?

$3\frac{1}{2}$ feet = ________ inches.

What do you get for $1\frac{3}{4}$ feet?

$1\frac{3}{4}$ feet = ________ inches.

Suppose we want to change 9 inches to feet. This is the same as asking, "What input gives an output of 9 ?"

9 inches = ________ feet.

Did you get $\frac{3}{4}$ feet?

Fill these blanks.

18 inches = ________ feet

27 inches = ________ feet

33 inches = ________ feet

48 inches = ________ feet

45 inches = ________ feet

3 inches = ________ feet

Check your answers with your classmates.

You multiply the number of feet by ________ to change to number of inches.

You multiply the number of inches by ________ to change to number of feet.

Exercises

1. Use the graph on Page 15-7b to change these measurements from units of feet to units of inches.

   (a) $2\frac{1}{4}$ feet = ________ inches

   (b) $3\frac{1}{2}$ feet = ________ inches

   (c) $\frac{3}{4}$ feet = ________ inches

   (d) $1\frac{1}{4}$ feet = ________ inches

   (e) 4 feet = ________ inches

2. Use the graph on Page 15-7b to change these measurements from units of inches to units of feet.

   (a) 18 inches = ________ feet

   (b) 45 inches = ________ feet

   (c) 33 inches = ________ feet

   (d) 21 inches = ________ feet

   (e) 15 inches = ________ feet

Changing Units - Feet and Yards

Again we can use the graph of a function to change units from yards to feet.

Class Discussion

You know that 1 yard is 3 feet long. Fill the blanks below to help you find the "yards to feet" function.

1 yard = $\underline{1 \cdot 3}$ feet

2 yards = $\underline{2 \cdot 3}$ feet

3 yards = ________ feet

4 yards = ________ feet

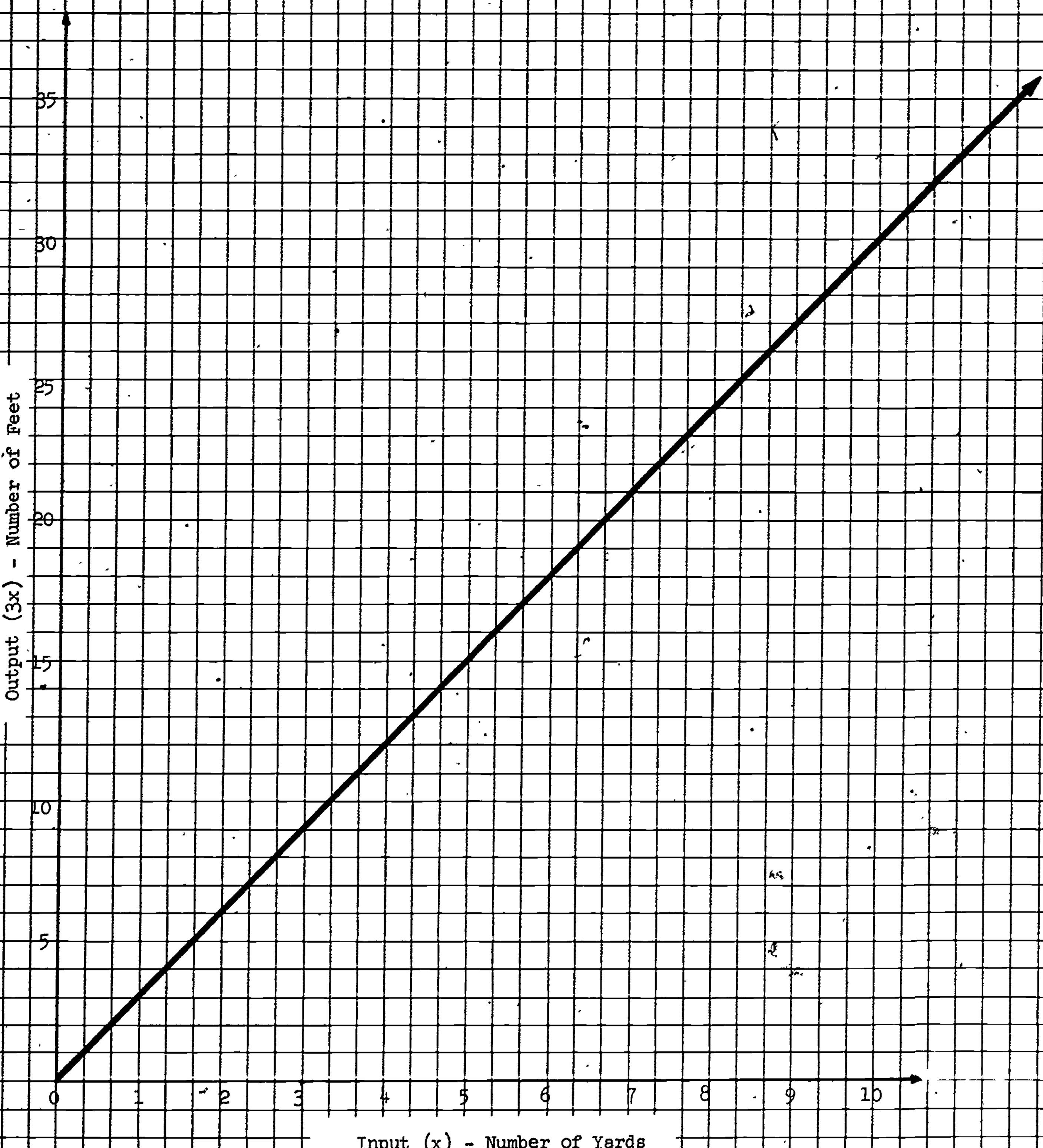
5 yards = ________ feet

x yards = ________ feet

Write the "yards to feet" function in arrow notation:

$f: x \longrightarrow$ __________

You have seen this function before. On the next page is its graph.


Output (3x) - Number of Feet
35
30
25
20
15
10
5
0
1
2
3
4
5
6
7
8
9
10
Input (x) - Number of Yards

It is handy to divide each yard (on the input axis) into thirds.

We can use this graph to change from yards to feet or feet to yards. To change 9 feet to yards we are again asking the question, "What input gives an output of 9 ?"

Exercises

1. Use the graph on Page 15-8a to answer these questions.

   (a) 4 yards = _______ feet

   (b) $2\frac{1}{3}$ yards = _______ feet

   (c) $7\frac{2}{3}$ yards = _______ feet

   (d) 9 yards = _______ feet

   (e) _______ yards = 27 feet

   (f) _______ yards = 24 feet

   (g) _______ yards = 17 feet

   (h) _______ yards = 19 feet

   (i) _______ yards = 7 feet

   (j) _______ yards = 25 feet

   (k) _______ yards = 2 feet

   (l) _______ yards = 1 foot

2. (a) You multiply the number of yards by _______________ to change to number of feet.

   (b) You multiply the number of feet by _______________ to change to number of yards.

Class Discussion

Suppose you wanted to change from yards to inches. You can do this in 2 steps:

Step 1. Change from yards to feet.

Step 2. Change from feet to inches.

1. Use the graphs on Pages 15-7b and 15-8a to make this conversion:

$$1\frac{1}{3} \text{ yards} = \_\_\_\_\_\_ \text{ feet} = \_\_\_\_\_\_ \text{ inches.}$$

We could go the other way as well. To change from inches to yards:

Step 1. Change from inches to feet.

Step 2. Change from feet to yards.

2. Use the graphs to make this conversion:

24 inches = ______ feet = ______ yards.

There are some difficulties that can come up by using graphs to make "double" conversions like those above. To see the trouble we can get into, try this conversion using the graphs.

33 inches = ______ feet = ______ yards.

It will be easier to do this problem using fractions.

You multiply the number of inches by ______ to change to number of feet. So

$$33 \text{ inches} = 33 \cdot \_\_\_\_\_\_ \text{ feet.}$$

You multiply the number of feet by ______ to change to number of yards. So

$$33 \cdot \frac{1}{12} \text{ feet} = 33 \cdot \frac{1}{12} \cdot \_\_\_\_\_\_ \text{ yards.}$$

Here is the problem we need to solve,

$$33 \cdot \frac{1}{12} \cdot \frac{1}{3}$$

We can change the order of multiplication and it will be easier to work the problem.

$$33 \cdot \frac{1}{3} \cdot \frac{1}{12}$$

$$= \underline{\qquad\qquad} \cdot \frac{1}{12}$$

$$= \frac{11}{12}$$

So:

$$33 \text{ inches } = \frac{11}{12} \text{ yards.}$$

Exercises

Follow these steps to change these measurements from inches to yards.

Step 1. Multiply by $\frac{1}{12}$ (change to feet).

Step 2. Multiply by $\frac{1}{3}$ (change to yards).

Change the order of multiplication whenever it makes the work easier. The first problem is done for you.

1. Change 28 inches into yards.

$28 \cdot \frac{1}{12}$ (change to feet)

$28 \cdot \frac{1}{12} \cdot \frac{1}{3}$ (change to yards)

$= \frac{28}{36}$ (multiplication)

$= \frac{7}{9}$ (simplifying)

28 inches = $\underline{\frac{7}{9}}$ yards.

2. Change 27 inches into yards.

27 inches = ________ yards.

3. Change 45 inches into yards.

45 inches = ________ yards.

4. Change 60 inches into yards.

60 inches = ________ yards.

Changing Units in the Metric System

In Lesson 15-7 there was this list of metric units from smallest to largest. (There are other units but these are the most common.)

Units	Abbreviations
millimeter	mm
centimeter	cm
meter	m
kilometer	km

These units are related by powers of ten. That is:

10 millimeters = 1 centimeter
100 centimeters = 1 meter
1000 meters = 1 kilometer

Because these units are related by powers of ten we use decimals instead of fractions to write these measurements. This makes conversion from one unit to another in the metric system very easy. All we have to do is multiply or divide by powers of ten. This is especially easy if you write these numbers in scientific notation.

Example 1.

$1 \text{ cm} = 1.0 \times 10^1 \text{ mm} = 10 \text{ mm}$
$2 \text{ cm} = 2.0 \times 10^1 \text{ mm} = 20 \text{ mm}$
$3 \text{ cm} = 3.0 \times 10^1 \text{ mm} = 30 \text{ mm}$
and so on.

Example 2.

$1 \text{ m} = 1.0 \times 10^2 \text{ cm} = 100 \text{ cm}$
$2 \text{ m} = 2.0 \times 10^2 \text{ cm} = 200 \text{ cm}$
$3 \text{ m} = 3.0 \times 10^2 \text{ cm} = 300 \text{ cm}$
and so on.

Example 3.

$1 \text{ km} = 1.0 \times 10^3 \text{ m} = 1000 \text{ m}$
$2 \text{ km} = 2.0 \times 10^3 \text{ m} = 2000 \text{ m}$
$3 \text{ km} = 3.0 \times 10^3 \text{ m} = 3000 \text{ m}$
and so on.

Look at the centimeter ruler on Page 15-9b. Notice that a centimeter unit is subdivided into 10 parts. Each of these small segments is a millimeter.

Class Discussion

1. How many centimeters long is the ruler on Page 15-9b? ____________

2. How many millimeters are there in one centimeter? ____________

3. How many of your rulers will you have to lay end to end to make a segment one meter long? ____________

4. Take your ruler to the chalkboard and mark off a segment that is one meter long.

5. How many millimeters are there in one meter? ____________

6. You multiply the number of millimeters by ______________ to change to number of meters. (Write your answer in exponential form.)

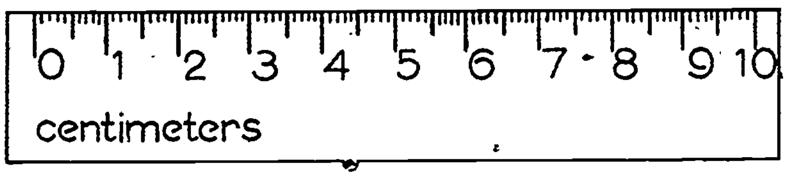
7. (a) You multiply number of meters by ________ to change to number of centimeters.
   (b) Write this answer in exponential form. ____________

8. (a) You multiply number of centimeters by ________ to change to number of meters.
   (b) Write this answer in exponential form. ____________

9. (a) In Example 3 on Page 15-9 you saw that we multiply the number of kilometers by ________ to change to number of meters.
   (b) You multiply the number of meters by ________ to change to number of kilometers.
   (c) Write the answer to Question (b) in exponential form.

   ____

10. (a) When you multiply by 10 the decimal point moves 1 place to the ____________.
    (b) When you multiply by $\frac{1}{10}$ the decimal point moves 1 place to the ____________.



Each centimeter is divided into 10 millimeters.

Exercises

1. Change these units from centimeters to millimeters by multiplying by 10^1. The first problem is done for you.

(a) 38.6 cm

$= 38.6 \times 10^1$ mm

$= 386$ mm

(d) 14.23 cm

______ mm

(b) 43.86 cm

______ mm

(e) 31.71 cm

______ mm

(c) 82.91 cm

______ mm

(f) 2.05 cm

______ mm

2. Change these units from millimeters to centimeters by multiplying by 10^{-1}. The first problem is done for you.

(a) 172.5 mm

$= 172.5 \times 10^{-1}$ cm

$= 17.25$ cm

(d) 1437.2 mm

______ cm

(b) 12.6 mm

______ cm

(e) 227 mm

______ cm

(c) 2.8 mm

______ cm

(f) .9 mm

______ cm

3. Change these units from meters to centimeters by multiplying by 10^2. The first one is done for you.

(a) 1.7 m

$= 1.7 \times 10^2$ cm

$= 170$ cm

(d) .361 m

_____________ cm

(b) 3.41 m

_____________ cm

(e) 12.5 m

_____________ cm

(c) 8.2 m

_____________ cm

(f) .028 m

_____________ cm

4. Change these units from centimeters to meters by multiplying by 10^{-2}. The first one is done for you.

(a) 85 cm

$= 85 \times 10^{-2}$ m

$= .85$ m

(d) 47 cm

_____________ cm

(b) 237 cm

_____________ m

(e) 198 cm

_____________ m

(c) 548.1 cm

_____________ m

(f) 336 cm

_____________ m

5. Change these units from kilometers to meters by multiplying by 10^3. The first one is done for you.

(a) 2.8 km

$= 2.8 \times 10^3$ m

$= 2800$ m

(d) 4.27 km

____________ m

(b) 7.6 km

____________ m

(e) 3.8 km

____________ m

(c) .65 km

____________ m

(f) 27.2 km

____________ m

6. Change these units from meters to kilometers by multiplying by 10^{-3}. The first one is done for you.

(a) 3728.7 m

$= 3728.7 \times 10^{-3}$ km

$= 3.7287$ km

(d) 281.4 m

____________ km

(b) 4021 m

____________ km

(e) 8729.5 m

____________ km

(c) 1931.9 m

____________ km

(f) 10,000 m

____________ km

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7. Change these to the units shown below.

(a) Change 2.3 m to mm. ________________ mm

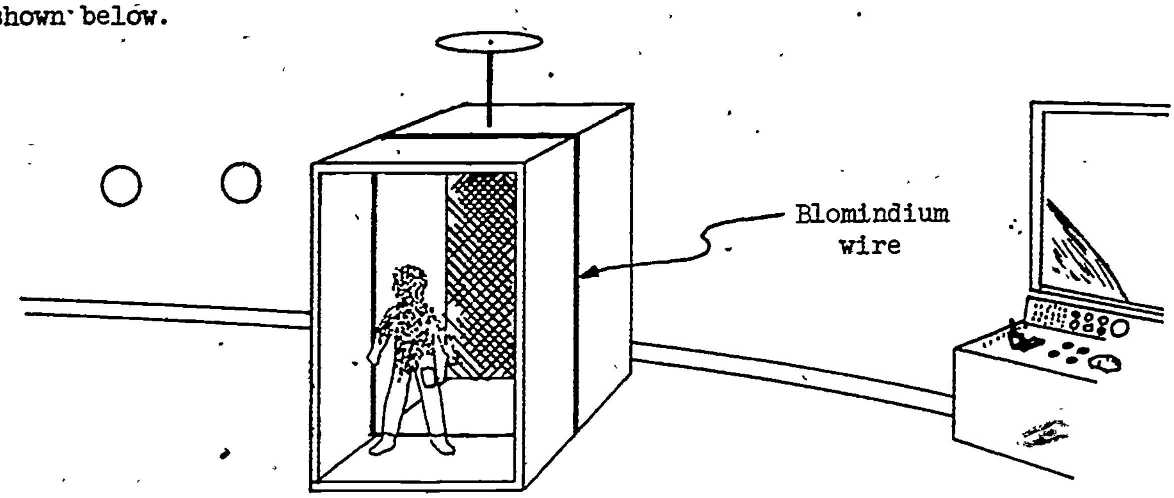
(b) Change 14,729 cm to km. ________________ km

(c) Change 1 km to mm. ________________ mm

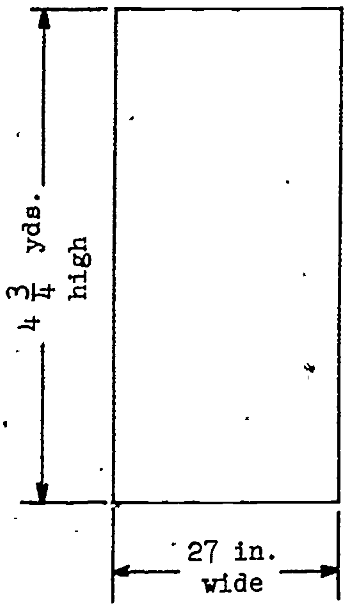
(d) Change 43,028 mm to m. ________________ m

Teletransporter Problem

The teletransporter on the space ship "Riteon V" requires a wire made of the rare metal "Blomindium". This wire runs up both sides and across the ceiling and floor of the activation chamber. A drawing of this chamber is shown below.



Here are the measurements of the wire.



Class Discussion

A United States supplier sells "Blomindium" wire at \$ 3,764.23 per foot, so we must be very careful to find its length in feet. In order to do this we need to change the dimensions to feet.

1. We multiply the number of yards by ____________ to change the length to number of feet. Do this multiplication.

2. The chamber is ____________ feet high.

3. We multiply the number of inches by ____________ to change the width to number of feet. Do this multiplication.

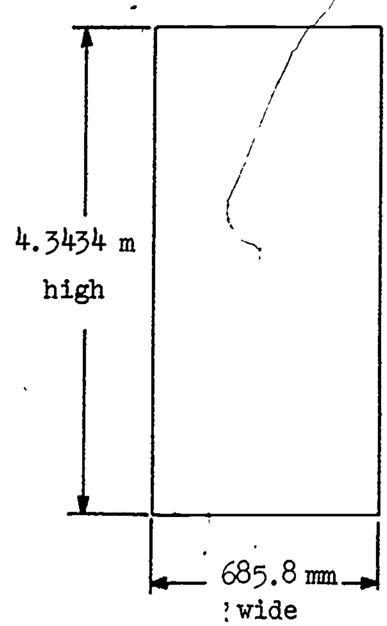
4. The chamber is ____________ feet wide.

Now we must add these dimensions and multiply their sum by 2 in order to find the length of the wire.

5. $2\,(14\frac{1}{4}\text{ ft.} + 2\frac{1}{4}\text{ ft.}) = 2 \cdot$ ____________ ft.

The wire is ____________ feet long. The wire costs \$ 124,219.59 from this supplier. (You can check this if you want.)

Suppose a European supplier sells Blomindium for \$ 126.30 per centimeter and we were given the dimensions of the chamber in these metric units.



Let's find out if we can save money by buying from the European supplier.

6. To change the height from number of meters to number of centimeters we multiply by ____________.

7. The chamber is ____________ cm high. (Hint: move the decimal point.)

8. To change the width from number of millimeters to number of centimeters we multiply by ____________.

9. The chamber is ____________ cm wide.

10. Add the height and width.

height	________	cm
+ width	________	cm
sum	________	cm

11. Multiply this sum by 2.

$2 \cdot$ ________ cm = ________ cm

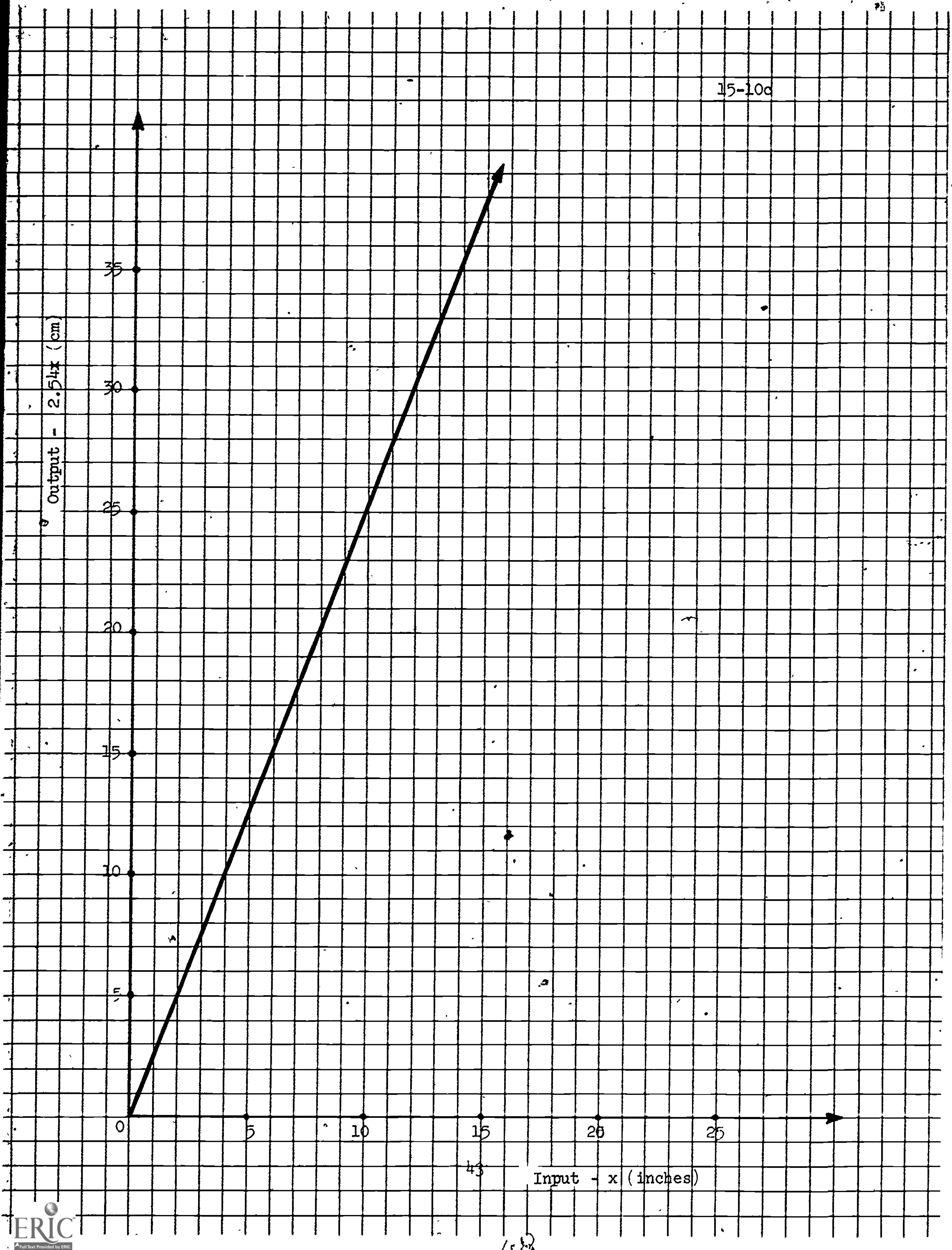
This is the length of the wire in centimeters. To find the cost from the European supplier we multiply \$ 126.30 times the length of the wire. This cost is \$ 127,037.59. We can save \$ 2,818.00 by buying the wire from the American supplier.

Metric units are related to each other by powers of ten. Many times as in this exercise it is necessary to change from units of the British system to units of the metric system.

When you compared your centimeter ruler with your inch ruler you found that there were about 2.5 centimeters in one inch. As a matter of fact there are exactly 2.54 centimeters in one inch. In other words, we can multiply number of inches by 2.54 to get number of centimeters. This function can be written

$$g: x \longrightarrow 2.54x .$$

The input to this function is the number of inches and its output is the number of centimeters. The graph of this function is on the next page.



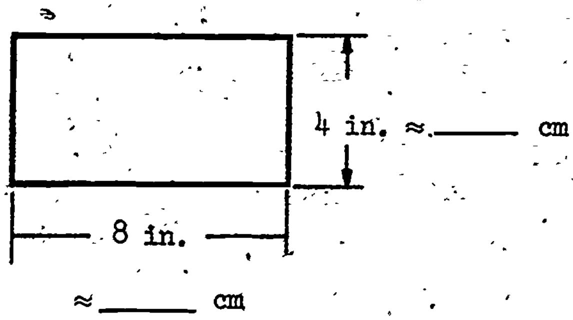
Output - 2.54x (cm)
35
30
25
20
15
10
5
0
5
10
15
20
25
Input - x (inches)

Exercises

Use the graph of $g: x \longrightarrow 2.54x$ on Page 15-10c to work these problems.

1. Change these dimensions to centimeters.

(a)



(b) Find the perimeter in centimeters of the rectangle in Problem (a).

$P = 2(\ell + w)$

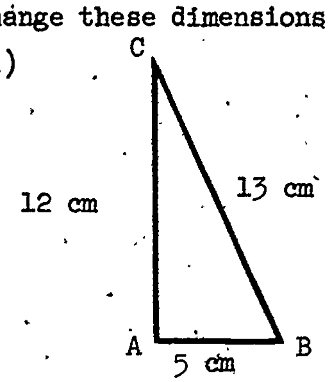
$P \approx 2(\_\_\_\_ + \_\_\_\_)$

$P \approx 2(\_\_\_\_\_\_)$

$P \approx \_\_\_\_\_\_\_\_$ cm

2. Change these dimensions to inches.

(a)



$m\,\overline{AB} \approx$ ________ inches

$m\,\overline{AC} \approx$ ________ inches

$m\,\overline{BC} \approx$ ________ inches

(b) Find the perimeter in inches of the triangle in Problem 2(a).

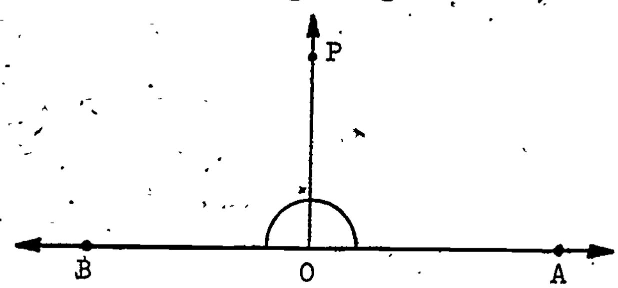
$P = m\,\overline{AB} + m\,\overline{AC} + m\,\overline{BC}$

$P \approx \_\_\_\_\_\_ + \_\_\_\_\_\_ + \_\_\_\_\_\_$

$P \approx \_\_\_\_\_\_\_\_$ inches

Angle Measurement - The Protractor

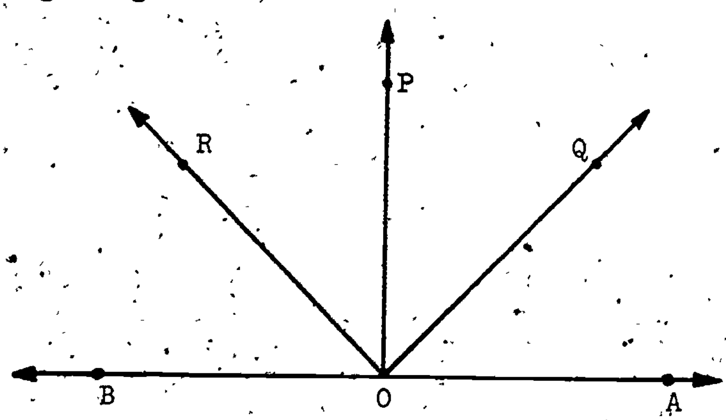
The measure, in degrees, of each of the right angles below is 90. (The ray $\overrightarrow{OP}$ bisects the straight angle AOB.)



The symbol for degree is a small circle written above and to the right of the measure of an angle. Thus,

"90 degrees" is written "90°".

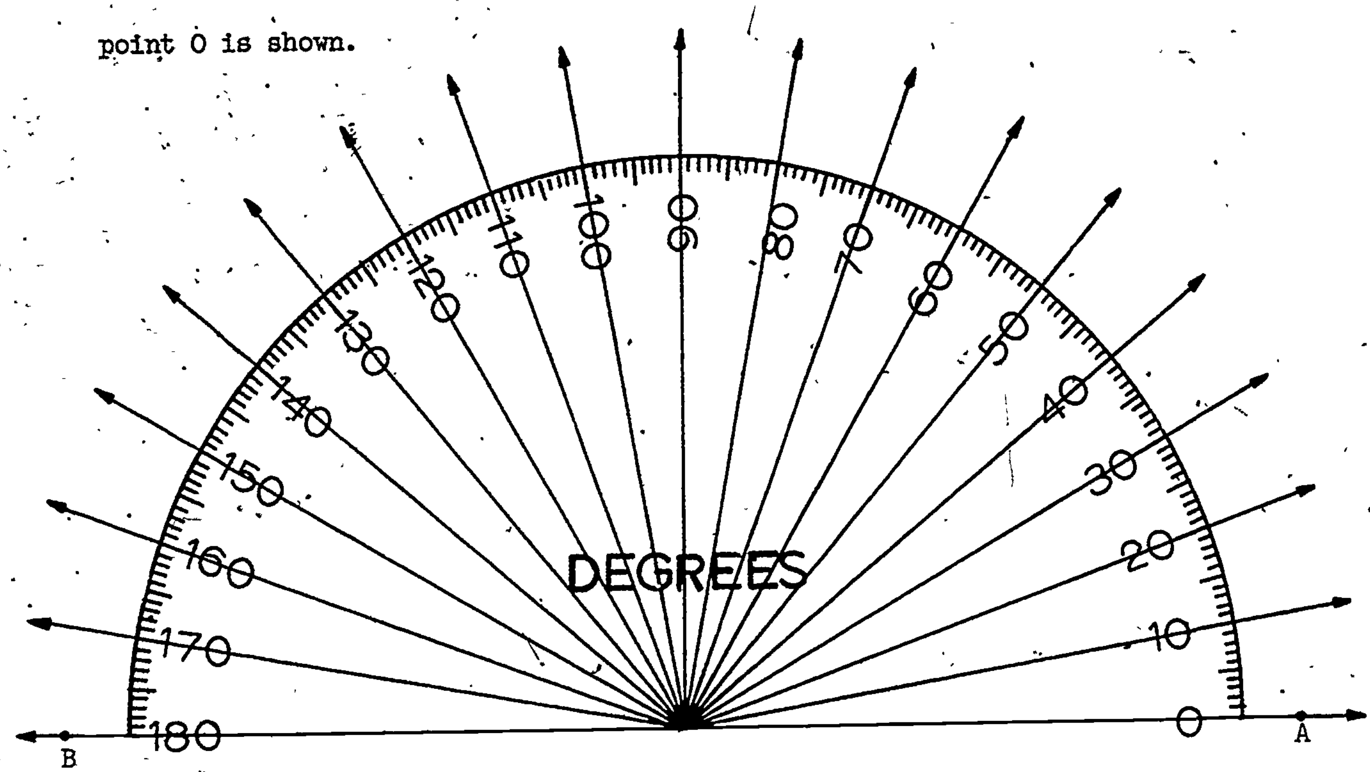
In the figure below, ray $\overrightarrow{OQ}$ bisects the right angle AOP and ray $\overrightarrow{OR}$ bisects the right angle POB.



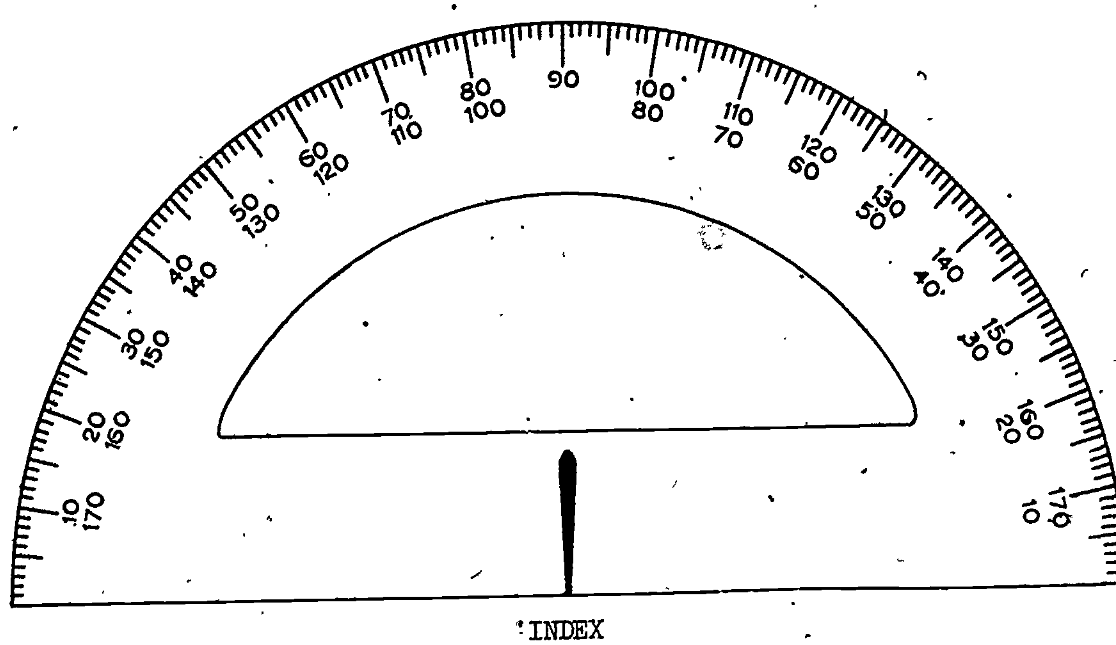
The measure of each of the angles ∠ AOQ, ∠ QOP, ∠ POR, and ∠ ROB is 45. Here we have 4 angles together forming the straight angle AOB.

Now imagine 180 congruent angles together forming the straight angle AOB. Each of these angles would have a measurement of one degree. Suppose that a half-circle is drawn from A to B with its center at point O. Then the rays of the 180 angles would intersect the half-circle at equally spaced points.

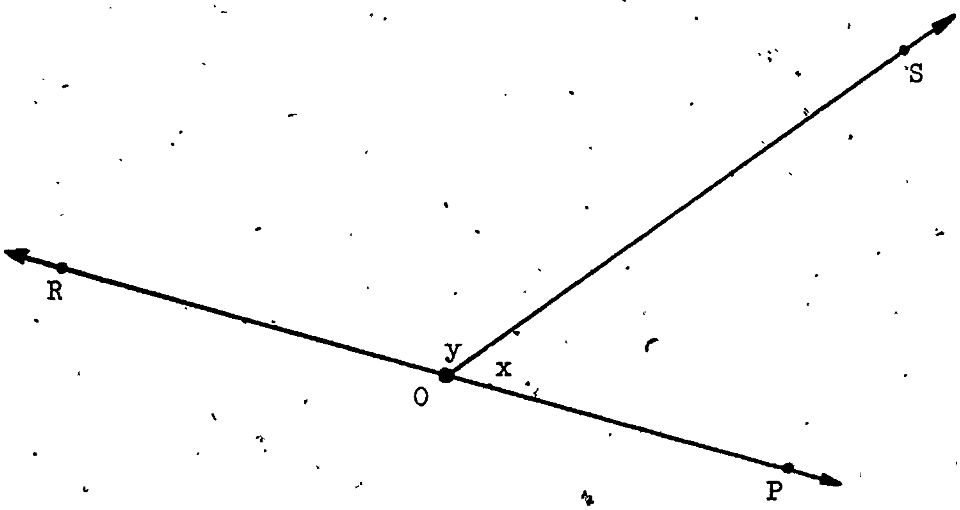
The picture below shows these points. Only every tenth ray from point O is shown.



This looks a lot like the protractor pictured below.



Class Discussion



Use your protractor to measure $\angle x$. Follow these steps.

1. Look at your protractor carefully. At the point that is the center of the half circle there is either an arrow, or a cross, or a ~~small hole.~~ This point is called the <u>index</u> of the protractor. (See the picture on Page 15-11a.)

2. Place the bottom edge of the protractor along ray $\overrightarrow{OP}$ so that the <u>index</u> is over the vertex (point O).

3. Notice that there are two scales on the protractor. Start at ray $\overrightarrow{OP}$ and use the scale that "counts" by tens up to ray $\overrightarrow{OS}$. You should find the number 50 at the point where $\overrightarrow{OS}$ intersects the half circle. The measure of $\angle x \approx$ ________ degrees.

You can measure $\angle y$ without moving your protractor. Follow these steps.

1. Check to see that the edge of the protractor is still along $\overrightarrow{OP}$ and that the index is still at point O. Notice that the edge is also along $\overrightarrow{OR}$.

2. Start at $\overrightarrow{OR}$ and use the scale that "counts" by tens up to $\overrightarrow{OS}$. Did you use the same scale you used before? ____________

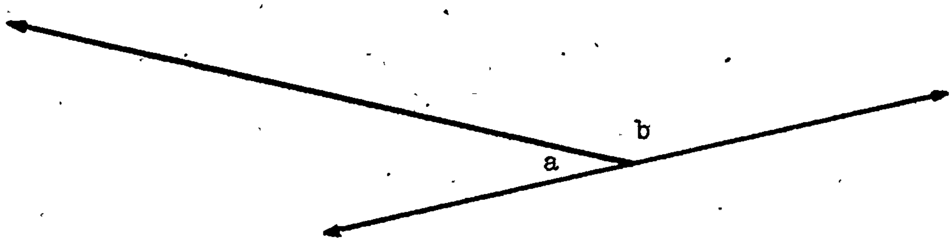
3. $m \angle y \approx$ ________ degrees.
4. $m \angle x + m \angle y \approx$ ________ degrees.

If the sum of the measures of two angles is 180, the angles are <u>supplementary</u>.

Exercises

1. Use your protractor to find the measures of each of the angles below.

(a)

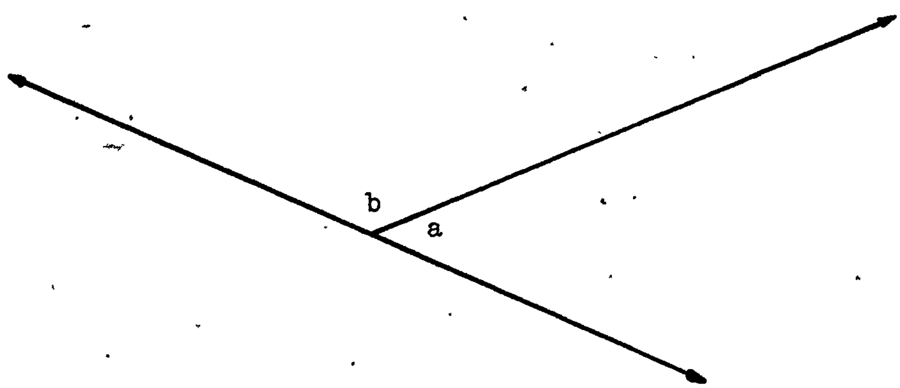


m∠a ≈ ________

m∠b ≈ ________

∠a and ∠b are called ______________ angles.

(b)

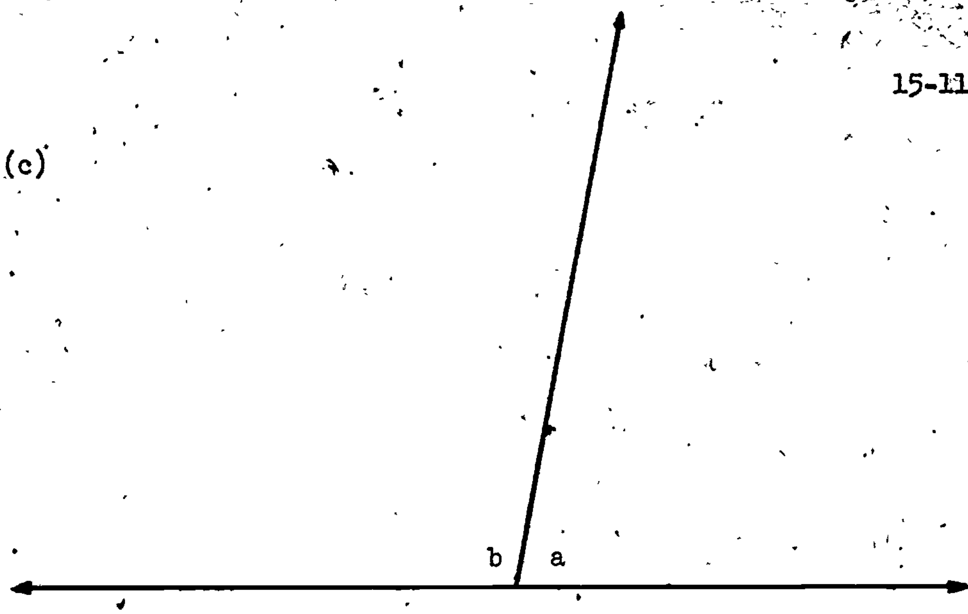


m∠a ≈ ________

m∠b ≈ ________

∠a and ∠b are called ______________ angles.

(c)



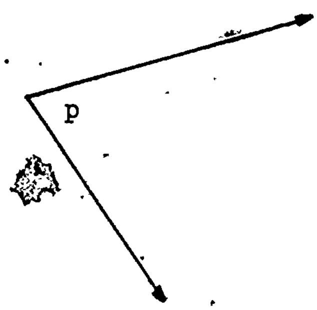
m ∠ a ≈ ________

m ∠ b ≈ ________

∠ a and ∠ b are called ________________ angles.

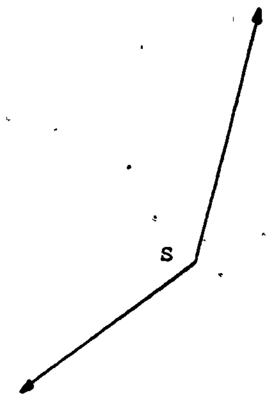
2. Sometimes the sides of a drawing of an angle are not long enough to intersect the half circle of the protractor. Since sides of an angle are really rays they can be extended. Use the straight edge of your protractor to draw longer sides on these angles and then measure them.

(a)



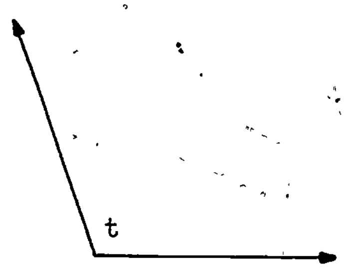
m ∠ p ≈ ________

(b)



m ∠ s ≈ ________

(c)

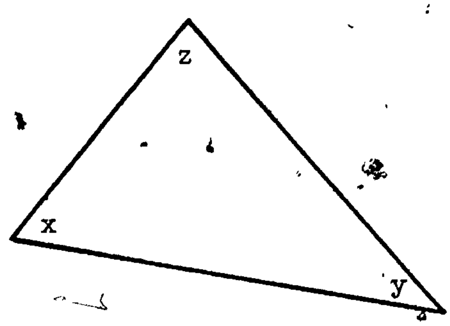


m ∠ t ≈ ________

(d) Which pair of these angles seem to be supplementary?

∠ ____ and ∠ ____

3.



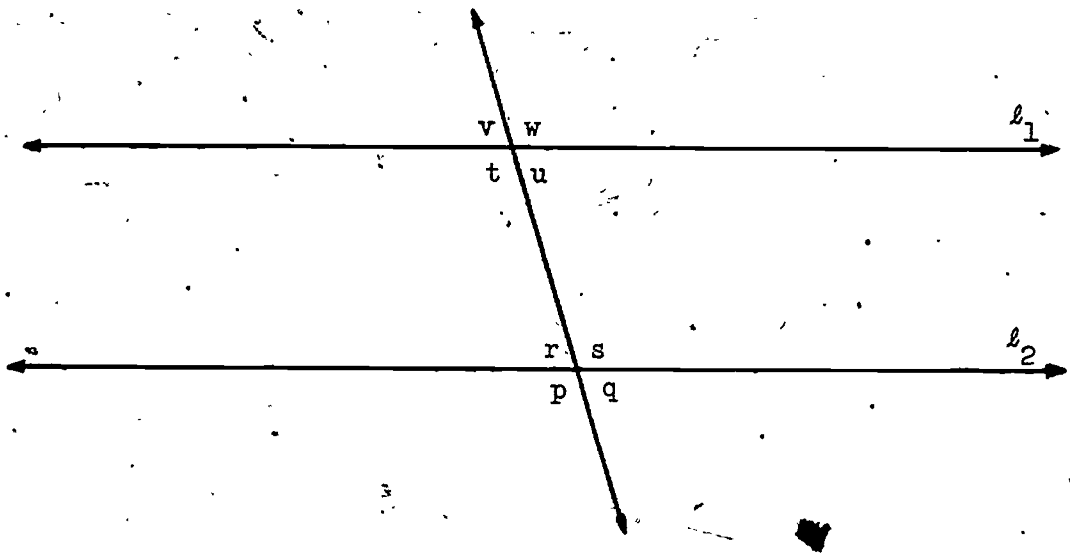
(a) Measure ∠ x with your protractor. m ∠ x ≈ ________.

(b) Measure ∠ y with your protractor. m ∠ y ≈ ________.

(c) Find the measure of ∠ z without measuring.

m ∠ z ≈ ________.

4. ℓ_1 and ℓ_2 are parallel.



(a) Measure $\angle r$ with your protractor. $m\angle r \approx$ __________.

(b) Find the measures of these angles without measuring them.

$m\angle s \approx$ __________ $m\angle q \approx$ __________

$m\angle t \approx$ __________ $m\angle u \approx$ __________

$m\angle w \approx$ __________ $m\angle v \approx$ __________

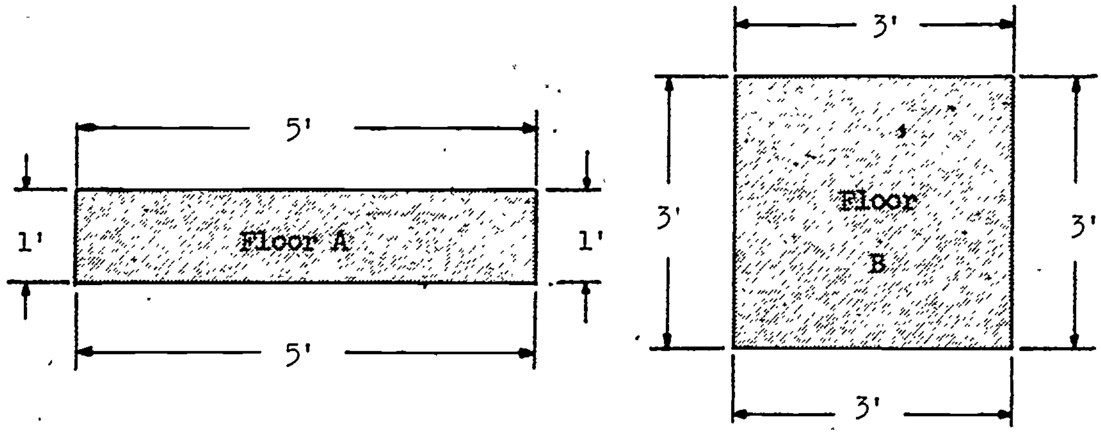
$m\angle p \approx$ __________

Measurement of a Surface Region - Rectangle

So far we have been talking about measurement of lengths: lengths of segments, sides of triangles, and so on. Suppose we want to know the size of a surface region. The length around a surface region won't always give us an idea of its size. It's possible to have differently shaped regions that have the same perimeter but are not the same size.

Class Discussion

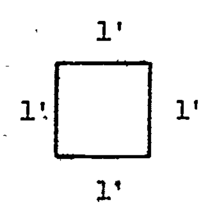
Suppose we want to tile the two floors shown below.



1. (a) What is the perimeter of "Floor A" ? ________ ft.

   (b) What is the perimeter of "Floor B" ? ________ ft.

The tiles we are going to use are 1 ft. by 1 ft. square as shown below.



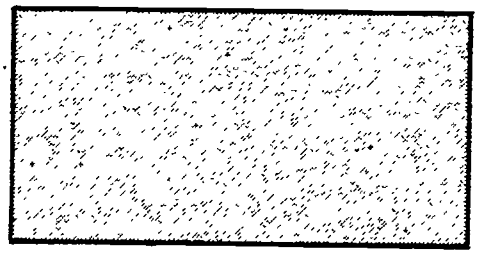
2. (a) How many tiles are needed to cover "Floor A" ? ____________

   (b) How many tiles are needed to cover "Floor B" ? ____________

The perimeters of these two floors are the same and yet it takes less tile to cover "Floor A" than it does to cover "Floor B". In this example knowing the perimeter doesn't help us tell which floor requires more tiles. We need a new kind of measurement for covering problems like tiling, carpeting, painting and so on. This kind of measurement is called area.

Area is the measure of a surface region.

Look at the rectangular region below.

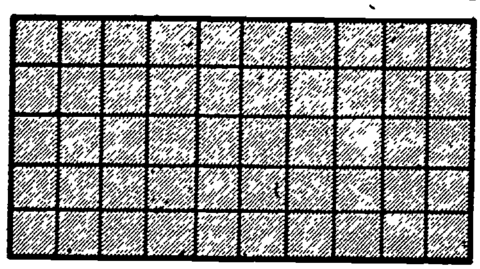


height = 5"

base = 10"

Instead of length and width, we will call one side the base and the side perpendicular to it the height. Suppose we want to cover the rectangle with square tiles that are 1 inch long on each side. (A square is a rectangle that has all four sides the same length.)

We can divide the rectangle into 1-inch squares and count them.



height = 5"

base = 10"

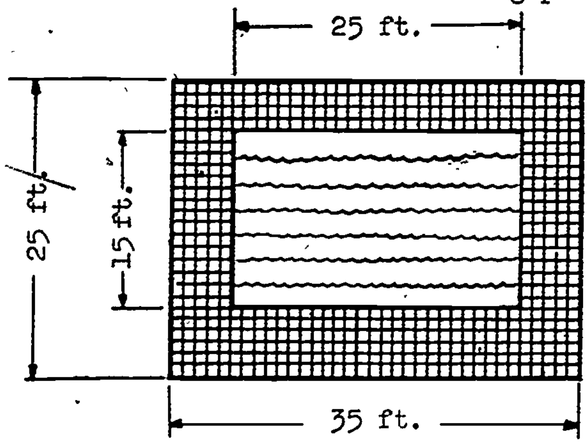
If we do this, we find we need 50 tiles. You know that multiplying the length of the base times the length of the height also gives 50 .

To find the area (abbreviated A) of a rectangle we simply multiply base (abbreviated b) times height (abbreviated h).

$$A = b \cdot h$$

Since we think of area as the number of square regions that it takes to cover the larger region, our units for measuring surface regions are square units. In the example above we covered the rectangle with square inches as the unit.

Suppose we want to tile around a swimming pool with these dimensions.



It is easy to find the number of tiles needed by finding the area of the large rectangle and subtracting the area of the pool.

1. A (of large rectangle) = ____ · ____

2. A (of pool) = ____ · ____

3. A (of tiled surface) = ____ - ____

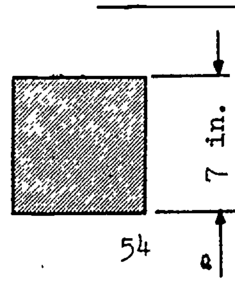
4. A (of tiled surface) = ________

If the tiles are each 1 square foot, how many do we need? __________

A rectangle with all its sides the same length is called a ____________. To find its area we use the length of the side (s) as a factor twice, that is, $A_{(square)} = s \cdot s$. Write this in exponential notation:

$A_{(square)} =$ ____________.

The area of the square below is ______________ square inches.



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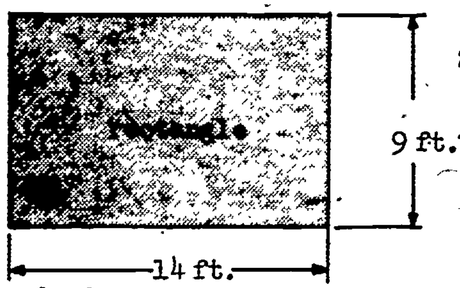
Exercises

Find the areas of these shaded regions.

1.

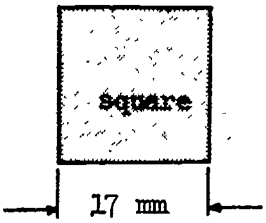


$A = b \cdot h$

$A = \_\_\_\_ \cdot \_\_\_\_$

$A = \_\_\_\_\_\_$ sq. ft.

2.

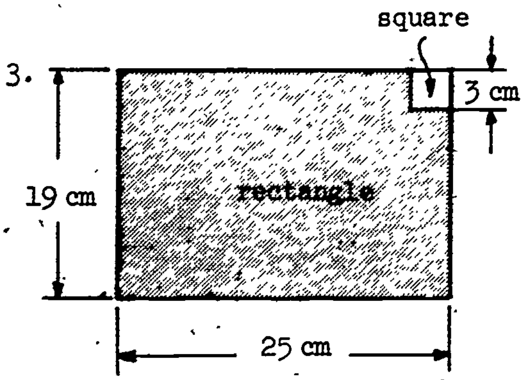


$A = s^2$

$A = \_\_\_\_^2$

$A = \_\_\_\_\_\_$ sq. mm

3.



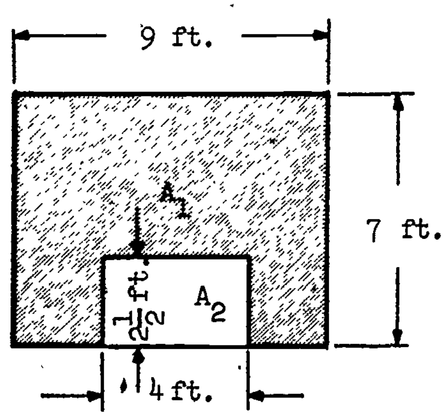
$A_{(shaded)} = A_{(rect.)} - A_{(sq)}$

$A_{(shaded)} = b \cdot h - s^2$

$A_{(shaded)} = \_\_\_\_ - \_\_\_\_$

$A_{(shaded)} = \_\_\_\_\_\_$ sq cm

4.



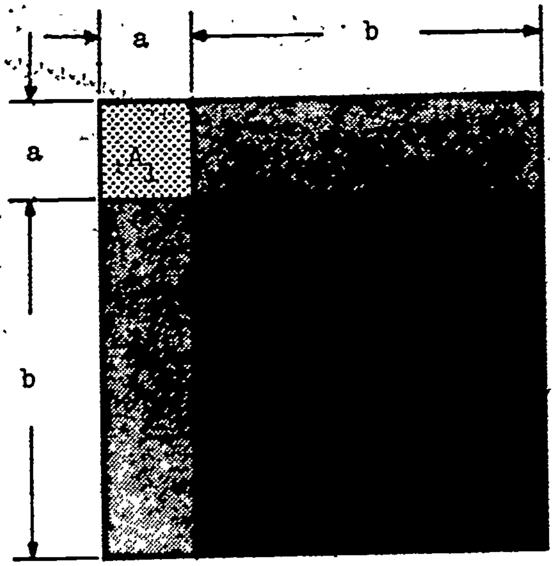
$A_{(shaded)} = A_1 - A_2$

$A_{(shaded)} = \_\_\_\_ - \_\_\_\_$

$A_{(shaded)} = \_\_\_\_\_\_$ sq. ft.

BRAINBOOSTER.

5.



A_1 = ____________

A_2 = ____________

A_3 = ____________

A_4 = ____________

Total Area = $A_1 + A_2 + A_3 + A_4$

Total Area = ____________ square units

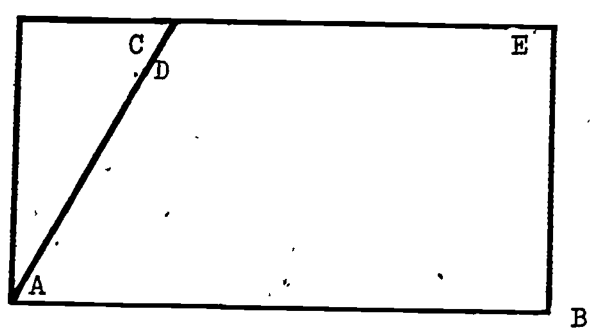
Measurement of a Surface Region - Parallelograms and Triangles

Class Discussion

In Chapter 11 you learned that a 4-sided figure with opposite pairs of sides parallel is called a ________________. You also learned that the perpendicular distance between two parallel lines is the same no matter where this distance is measured. This is the distance we have called the height of a rectangle.

Get out Page 15-13a. The two parallelograms are just alike. Cut out parallelogram P.

1. $\overline{AB} \parallel$ __________.
2. The distance between $\overline{AB}$ and $\overline{DC}$ is the length of the segment __________. On the drawing, label this segment "h" for height.
3. Cut out $\triangle$ BEC .
4. Place the triangle so that point C lies on point D like this.



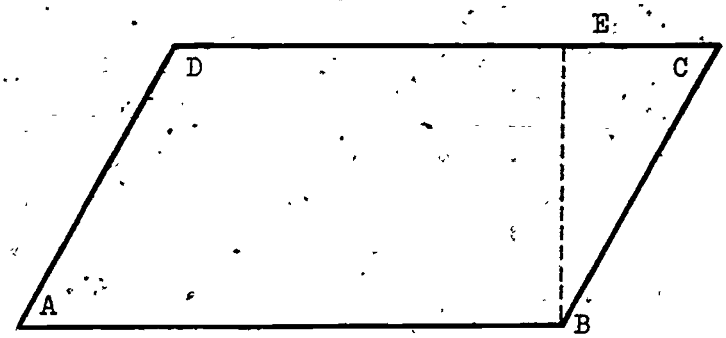
5. What kind of figure is this? ________________

Now look at parallelogram Q . Compare the rectangle you made with parallelogram Q to answer these questions.

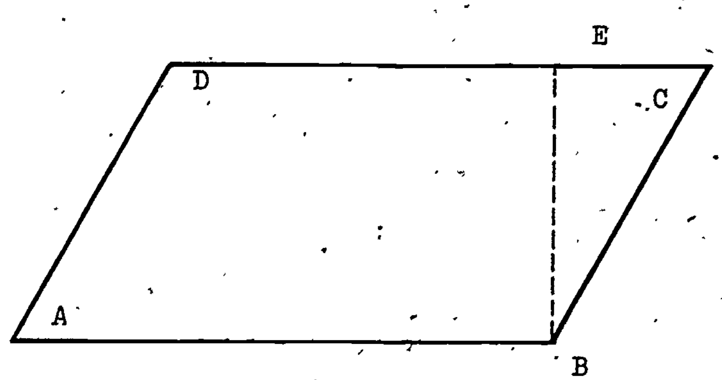
6. The base of the rectangle is segment __________.
7. The base of the rectangle has the same length as segment __________ of parallelogram Q .
8. The height of the rectangle has the same length as segment __________ in parallelogram Q . Label this segment "h" .
9. Is the area of parallelogram Q the same as the area of the rectangle? __________

Work Sheet

Parallelogram P



Parallelogram Q



From your answers above you have found that the area of a parallelogram is the length of the base times the height. The height is the perpendicular distance between opposite sides.

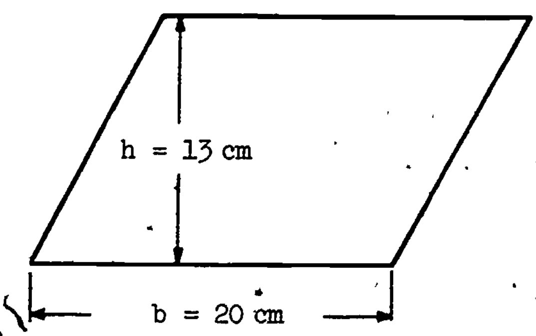
$$A = b \cdot h$$

Notice that this is exactly the same as for the rectangle. All you have to remember is that the height is always the perpendicular distance between parallel sides.

Exercises

Find the areas of these parallelograms.

1.

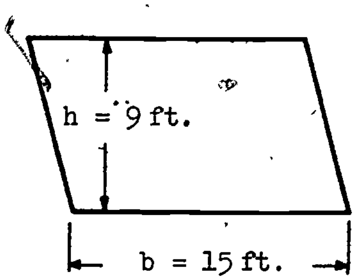


$A = b \cdot h$

A = _____ · _____

A = __________ sq cm

2.

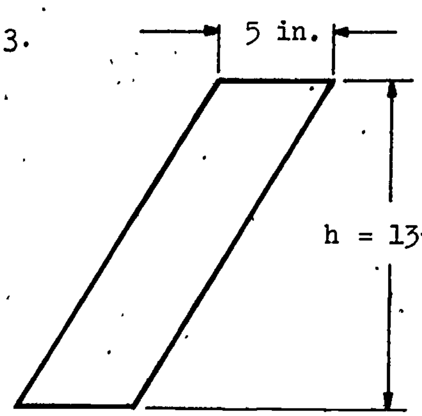


$A = b \cdot h$

A = _____ · _____

A = __________ sq. ft.

3.

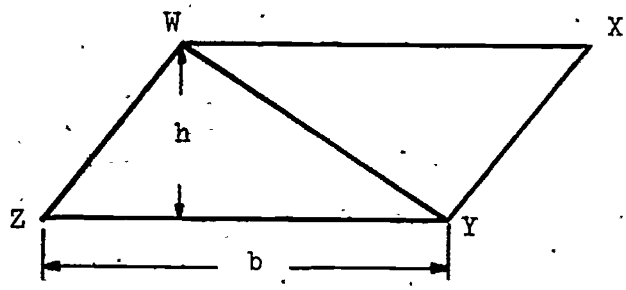


$A = b \cdot h$

A = _____ · _____

A = __________ sq. in.

Class Discussion



In parallelogram WXYZ the diagonal $\overline{WY}$ forms two congruent triangles. Write this congruence.

$\triangle$ _______ $\cong$ $\triangle$ _______

Since these triangles are congruent, $\overline{WY}$ separates the surface region inside the parallelogram into two parts of equal size. Each of these parts has an area that is $\frac{1}{2}$ the area of the parallelogram.

The area of WXYZ is:

$$A = b \cdot h$$

Write the area of $\triangle$ WYZ .

$$A = \_\_\_\_ \cdot b \cdot h$$

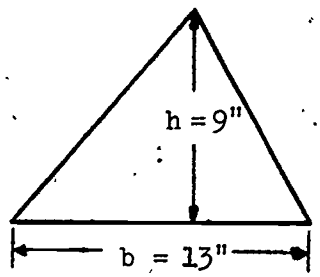
Write the area of $\triangle$ YWX.

$$A = \_\_\_\_\_\_\_\_$$

Exercises

1. Use $A = \frac{1}{2} b \cdot h$ to find the areas of the following triangles.

(a)

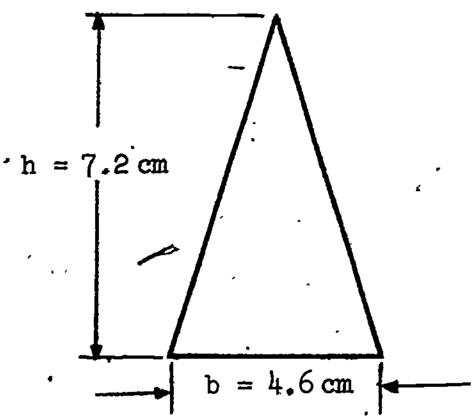


$A = \frac{1}{2} b \cdot h$

$A = \frac{1}{2} \cdot$ ____ $\cdot$ ____

$A =$ __________ sq. in.

(b)

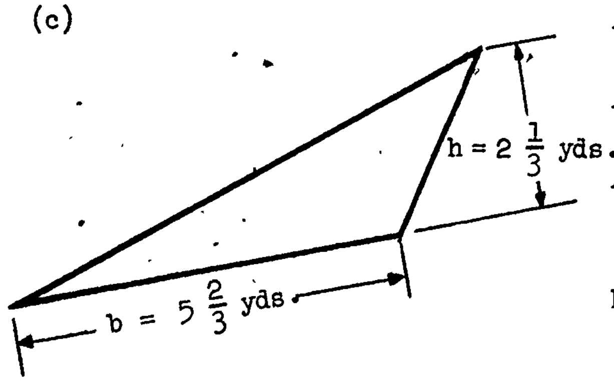


$A = \frac{1}{2} b \cdot h$

$A = \frac{1}{2} \cdot$ ____ $\cdot$ ____

$A =$ __________ sq cm

(c)



$A = \frac{1}{2} b \cdot h$

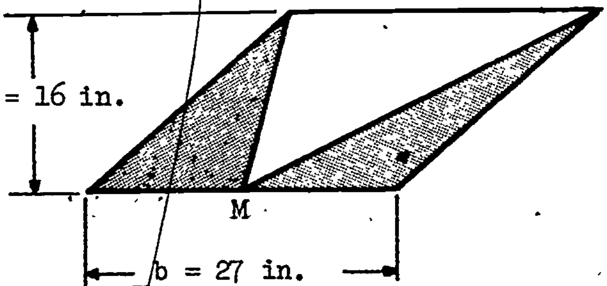
$A = \frac{1}{2} \cdot$ ____ $\cdot$ ____

$A =$ __________ sq. yds.

(<u>Hint</u>: Use your tables to help you with this arithmetic.)

2. Find the area of the shaded part.

(a)



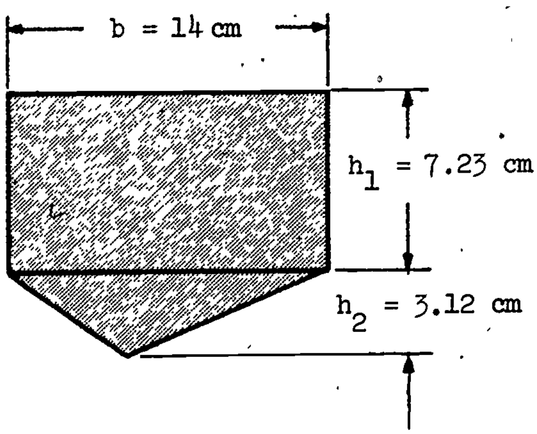
(M is the midpoint of the base.)

$A_{(shaded)} = A_{(parallelogram)} - A_{(unshaded)}$

$A_{(shaded)}$ = ______________ - ______________

$A_{(shaded)}$ = ____________________ sq. in.

(b)



$A_{(shaded)} = A_{(rectangle)} + A_{(triangle)}$

$A_{(shaded)}$ = ______________ + ______________

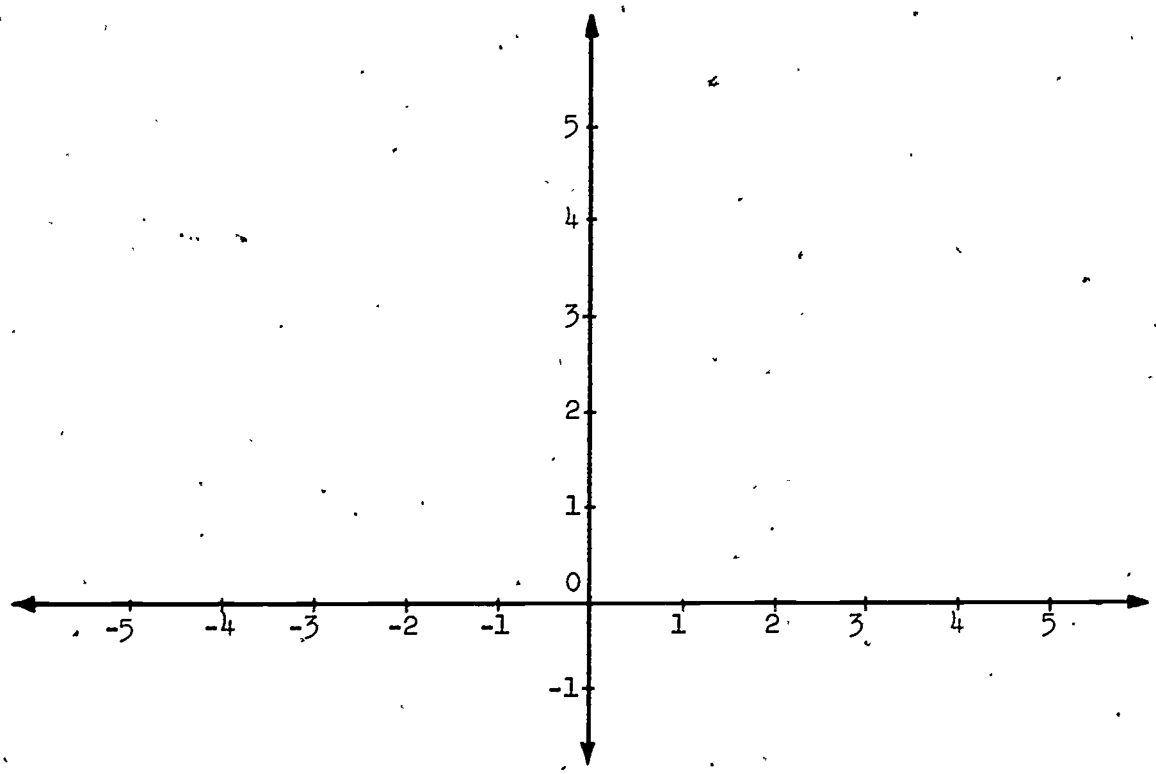
$A_{(shaded)}$ = ____________________ sq cm

Exact Lengths

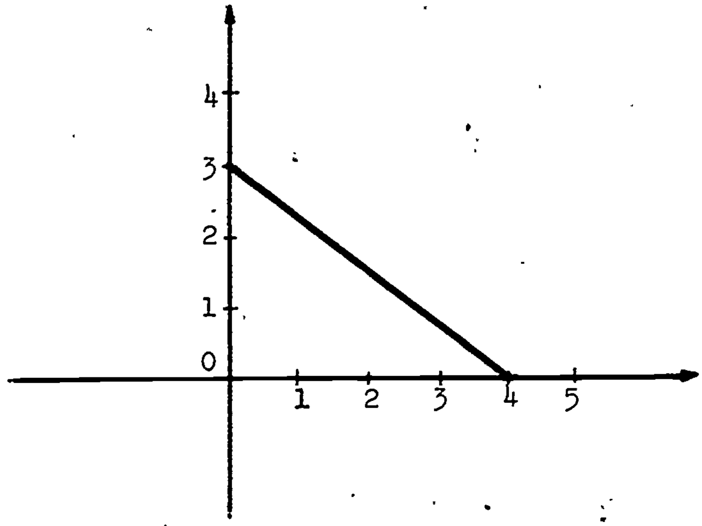
Throughout this chapter we have said over and over again that all measurement is approximate. What we mean by this is that we cannot measure real things exactly using real rulers or real protractors or any other real measuring instrument. Mathematicians leave the world of real things up to the scientists and engineers to worry about. Instead the mathematician deals with ideas only, and ideas can be exact. You have been asked repeatedly to associate numbers with points on the number line. In your imagination you have named a particular point on the number line "one" and a particular point "2" and so on. You have even named the point for $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, etc. But these are ideas, not things, and ideas can be exact. You can imagine a segment that is exactly 1 inch or 2 inches long even though you have seen that segments cannot be measured exactly using real rulers.

Class Discussion

Suppose we play around with this idea of exactness a little while. Think about this coordinate plane.

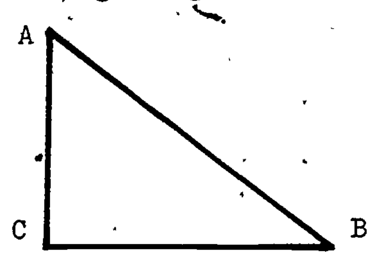


We have picked out the one point labeled 1, and the one point labeled 2, the one point labeled 3, and so on. Draw a segment from 3 on the vertical (output) axis to 4 on the horizontal (input) axis.

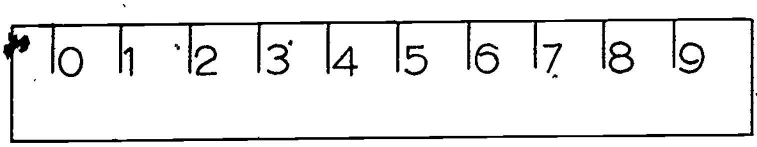


Would you say that the segment you drew has an exact length even if you can't measure it exactly? ____________

In mathematics we deal with ideas and we would like for our ideas to be exact. So, let's see if we can figure out what the exact length of our segment is. We know that we can't do this by measuring because it is approximate. If we imagine a right triangle whose dimensions are exactly as shown below, maybe we can calculate the exact length of the segment opposite the right angle.



Let's make a ruler using the same unit as used on the axis of our coordinate plane.



Measure the segment $\overline{AB}$ using a copy of this ruler. What is its approximate length? ____________

Suppose we use the "trial and error" method to find a way of calculating this length without measuring. We can test our results using our measured length even though it is approximate.

If we add the length of the other two sides of the triangle what do we get? ____________ Is 7 at all close to the measured length of 5 ? ____________ Does adding these lengths seem to work? ____________

Then let's try something else.

What do we get if we multiply the length of the other two sides. ____________ Does 12 come close to our measured length of 5 ? ____________ This doesn't seem to work either. Try subtracting and dividing the lengths of the sides. Do these methods seem to work? ____________

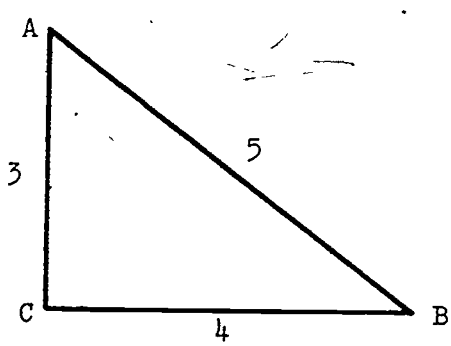
A long time ago a very smart Greek mathematician named Pythagoras worked on this problem for many years. Luckily he finally found the answer:

(1) he found $3^2 = 9$

(2) and then found $4^2 = 16$

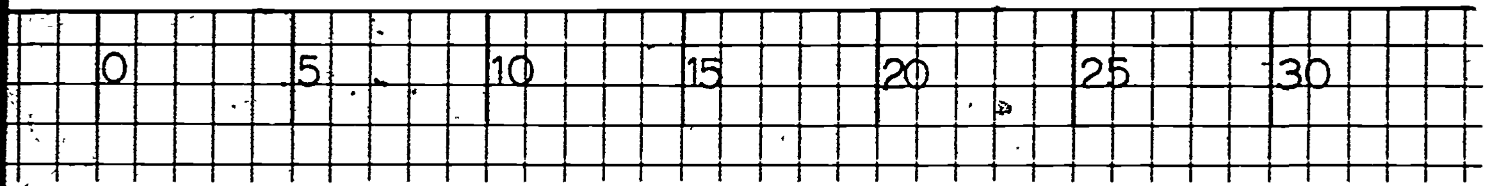
(3) and then added $3^2 + 4^2 = 25$.

His answer was the same number as if he squared the measured length of this segment! Try this to see if it works.



Does $3^2 + 4^2 = 5^2$?

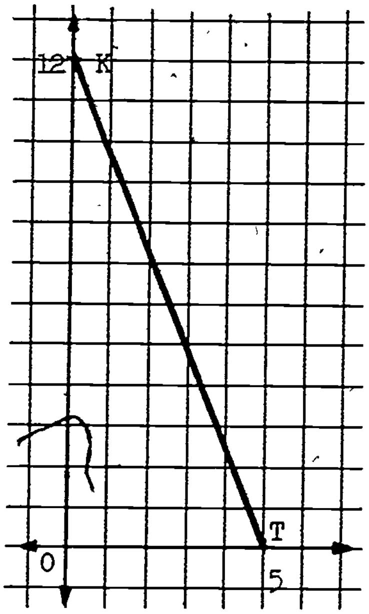
That is, does $(9 + 16) = 25$? ____________



Exercises

Test Pythagoras' idea on these right triangles. Cut off the ruler at the top of this page and use it to do the measuring.

1.



(a) $12^2 + 5^2 =$ ____ + ____

$12^2 + 5^2 =$ ________

(b) Use the ruler above.

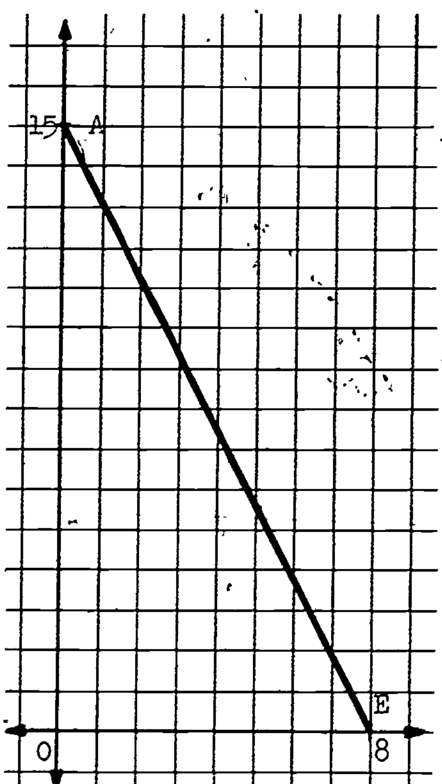
$m\ \overline{KT} \approx$ ________

$(m\ \overline{KT})^2 \approx$ ________

(c) Are your answers to parts (a) and (b) the same?

____

2.



(a) $15^2 + 8^2 =$ ____ + ____

$15^2 + 8^2 =$ ________

(b) Use the ruler.

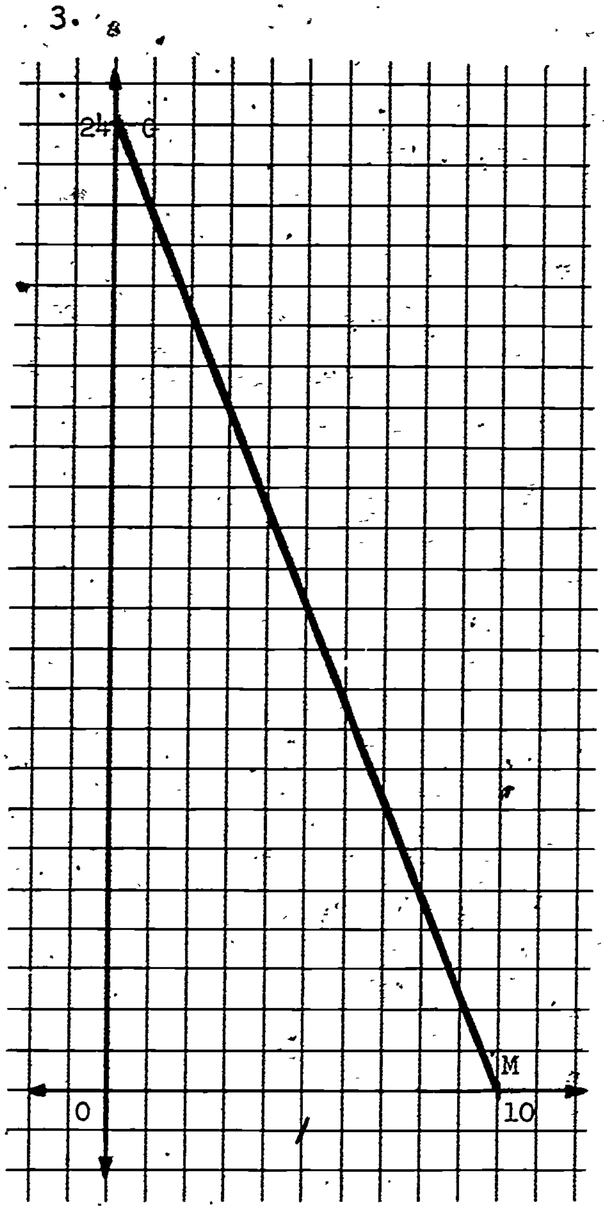
$m\ \overline{AE} \approx$ ________

$(m\ \overline{AE})^2 \approx$ ________

(c) Are your answers to parts (a) and (b) the same?

____

3.



(a) $24^2 + 10^2 =$ _____ + _____

$24^2 + 10^2 =$ __________

(b) Use the ruler.

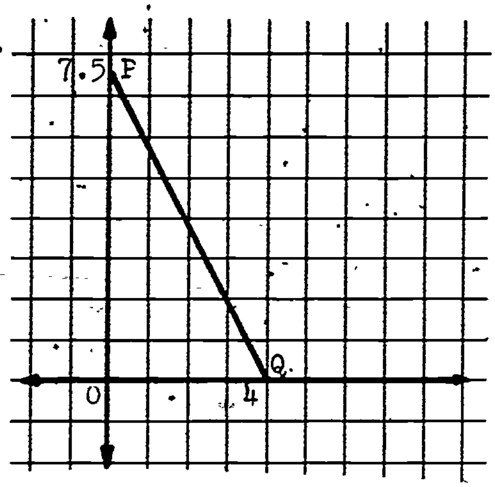
m $\overline{GM} \approx$ __________

$(m\ \overline{GM})^2 \approx$ __________

(c) Are your answers to parts (a) and (b) the same?

____

4.



(a) $7.5^2 + 4^2 =$ _____ + _____

$7.5^2 + 4^2 =$ __________

(b) Use the ruler.

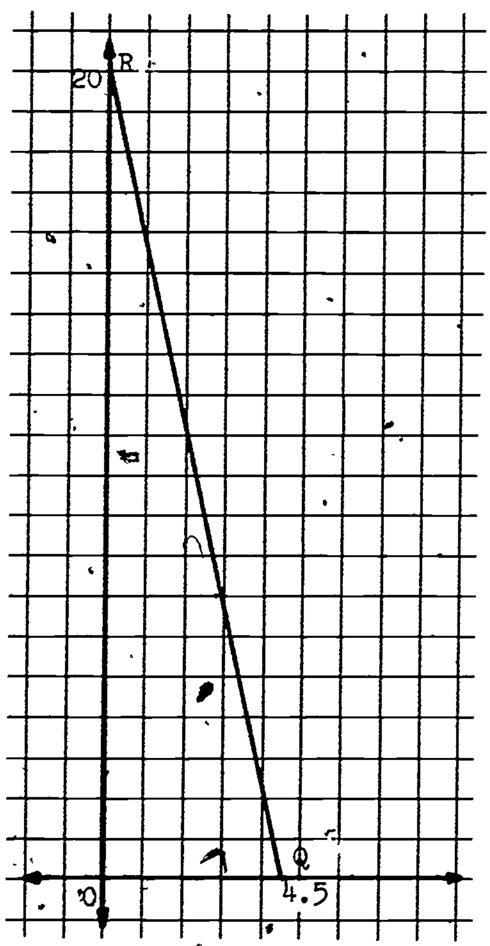
$m\ \overline{PQ} \approx$ __________

$(m\ \overline{PQ})^2 \approx$ __________

(c) Are your answers to parts (a) and (b) the same?

____

5.



(a) $4.5^2 + 20^2 =$ _____ + _____

$4.5^2 + 20^2 =$ __________

(b) Use the ruler.

$m\ \overline{RQ} \approx$ __________

$(m\ \overline{RQ})^2 \approx$ __________

(c) Are your answers to parts (a) and (b) the same?

____

Pre-Test Exercises

1. (Section 15-3.)

   Rewrite these fractions as mixed numbers.

   (a) $\frac{7}{3}$ = ________

   (b) $\frac{10}{7}$ = ________

   (c) $\frac{17}{8}$ = ________

2. (Section 15-3.)

   Rewrite these mixed numbers as fractions.

   (a) $2\frac{1}{6}$ = ________

   (b) $3\frac{5}{8}$ = ________

   (c) $1\frac{3}{7}$ = ________

3. (Section 15-3.)

   Add these mixed numbers and simplify the answer.

   (a) $1\frac{5}{8} + 5\frac{3}{8}$ = ________

   (b) $2\frac{4}{9} + 8\frac{3}{9}$ = ________

   (c) $2\frac{2}{3} + 2\frac{2}{5}$ = ________

   (d) $5\frac{1}{4} + 1\frac{1}{6}$ = ________

4. (Section 15-3.)

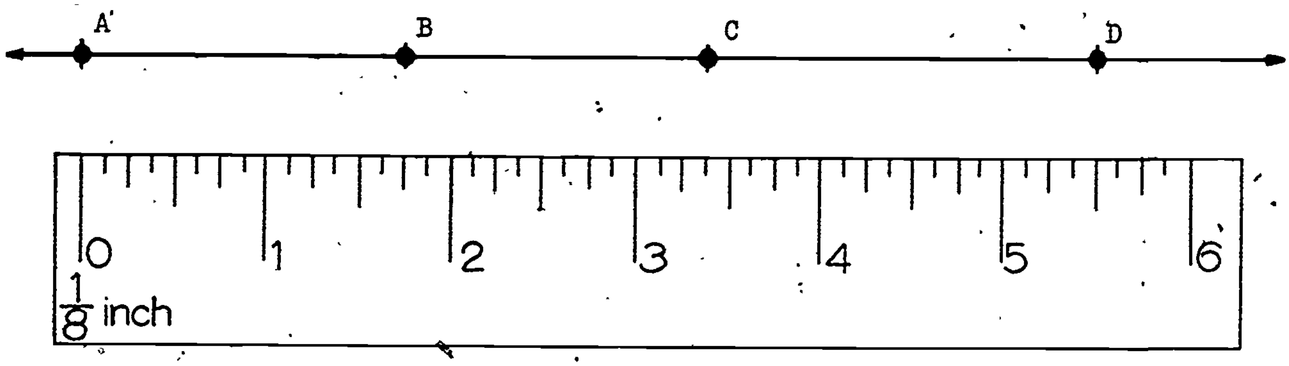
   Multiply these numbers and simplify your answers.

   (a) $1\frac{1}{7} \times 5$ = ________

   (b) $3\frac{5}{8} \times 6$ = ________

5. (Section 15-4.)

Use the picture below to answer these questions.



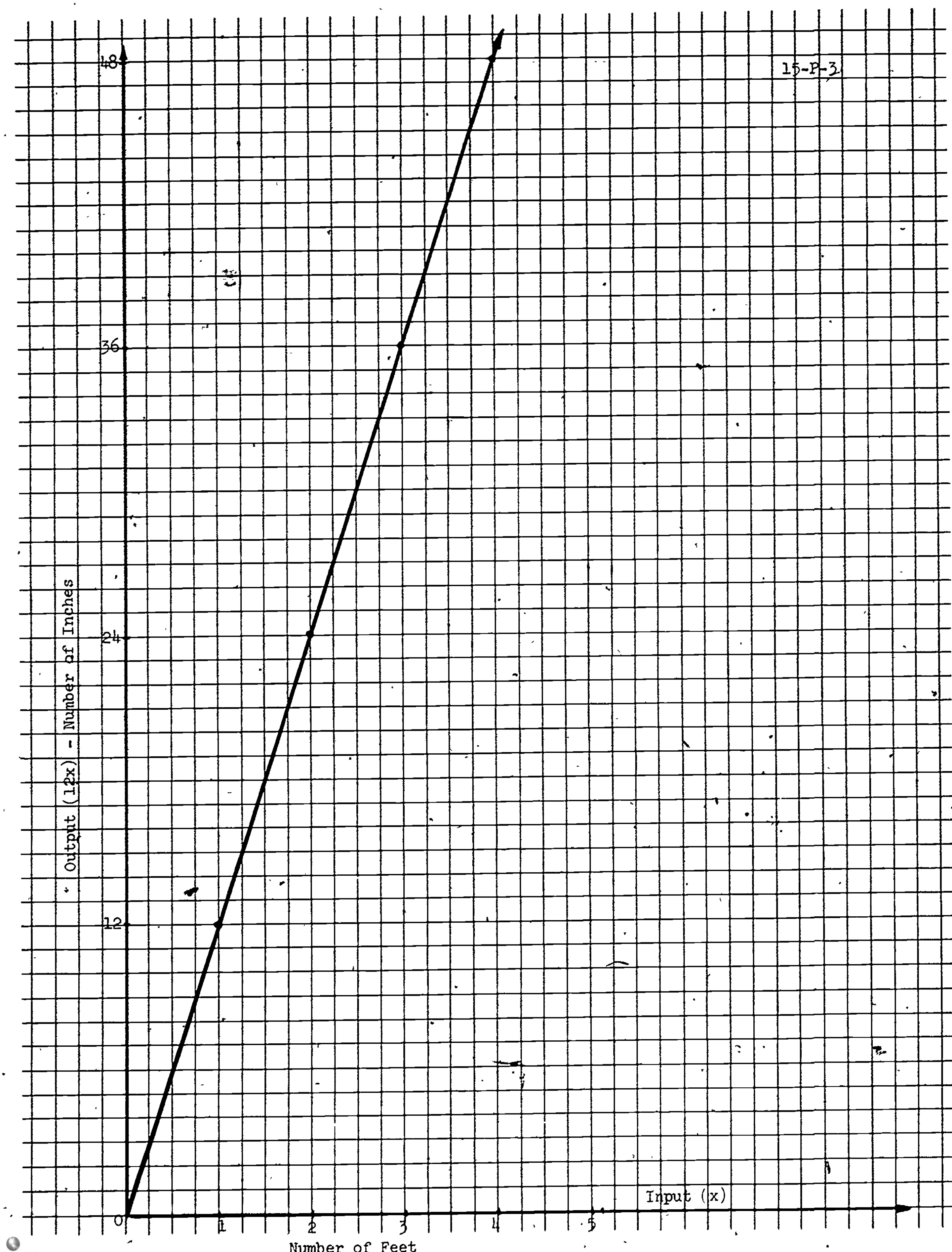
(a) m $\overline{AB}$ ≈ ____________

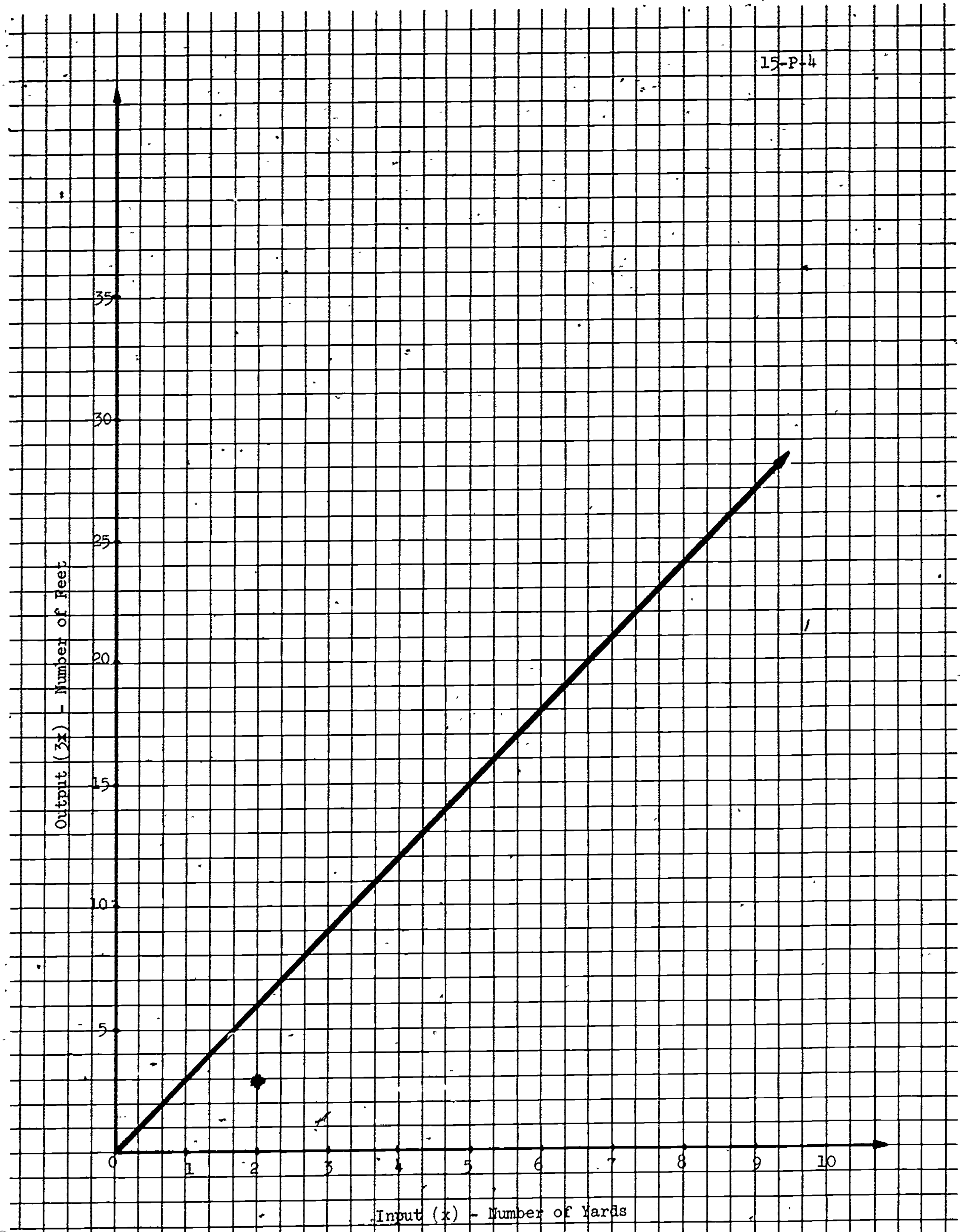
(b) m $\overline{AC}$ ≈ ____________

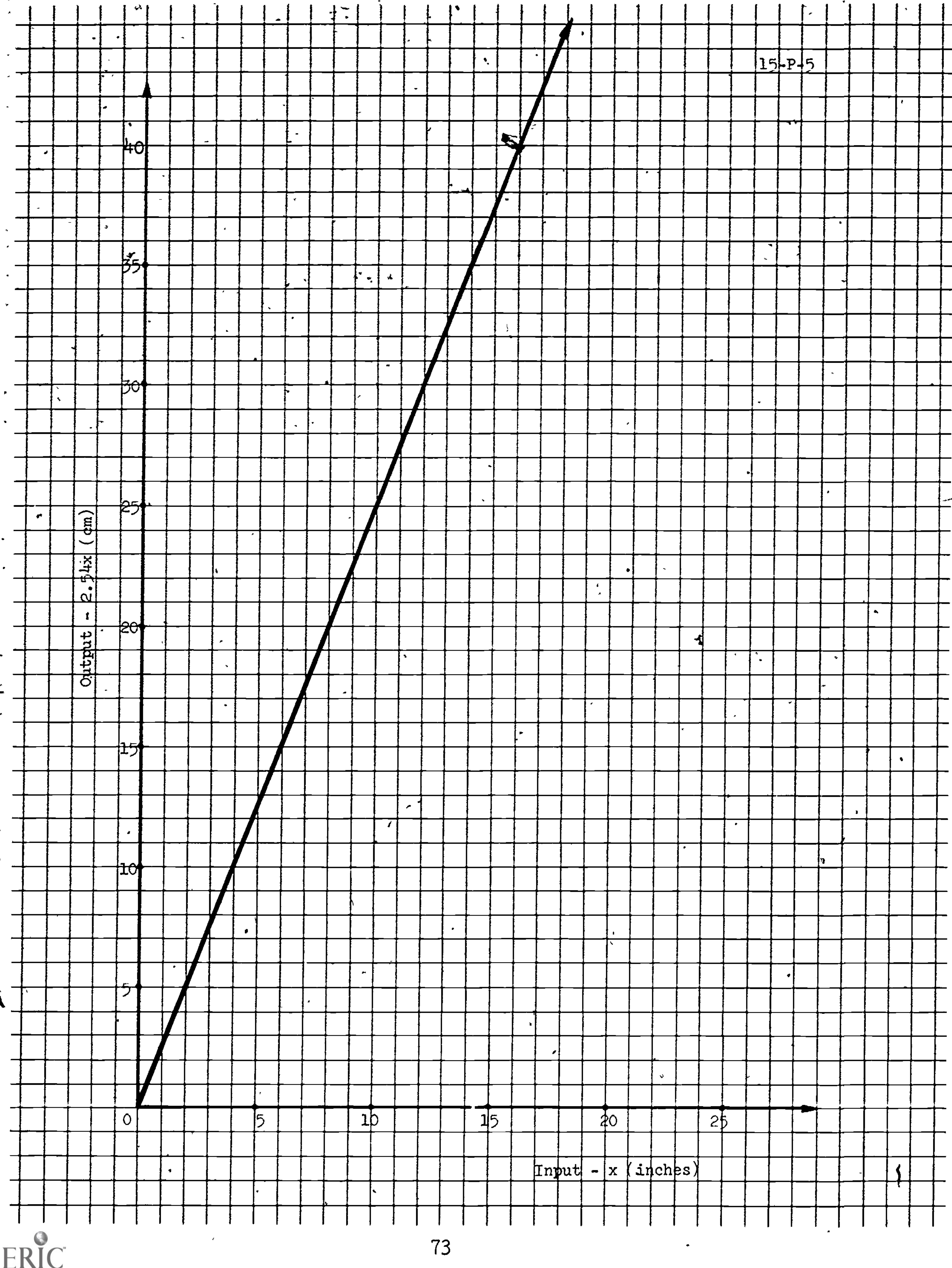
(c) m $\overline{BC}$ ≈ ____________

(d) m $\overline{BD}$ ≈ ____________

(e) m $\overline{CD}$ ≈ ____________


15-P-3
Output (12x) - Number of Inches
48
36
24
12
0
1
2
3
4
5
Input (x)
Number of Feet


Output (3x) - Number of Feet
35
30
25
20
15
10
5
0
1
2
3
4
5
6
7
8
9
10
Input (x) - Number of Yards


15-P-5
Output - 2.54x (cm)
40
35
30
25
20
15
10
5
0
5
10
15
20
25
Input - x (inches)

6. (Sections 15-7, 15-8, 15-10.)

Select the correct graphs (from the previous 3 pages) to make these conversions.

(a) $2\frac{1}{2}$ feet = ________ inches

(b) 21 inches = ________ feet

(c) 39 inches = ________ feet

(d) 15 inches ≈ ________ cm

(e) 17 feet = ________ yards

(f) $9\frac{2}{3}$ yards = ________ feet

(g) 48 inches = ________ yards

7. (Section 15-9.)

Use this list of metric units and powers of ten to make these conversions.

10 mm = 1 cm
100 cm = 1 m
1000 m = 1 km

(a) 3.9 cm = ________ mm

(b) 400 cm = ________ m

(c) 2.6 m = ________ cm

(d) 427 mm = ________ cm

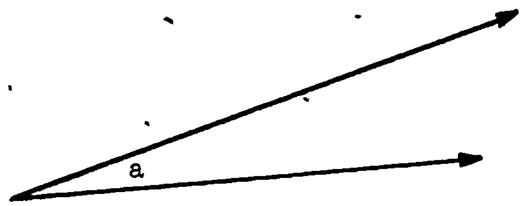
(e) 1.3 km = ________ m

(f) 2938 mm = ________ m

8. (Section 15-11.)

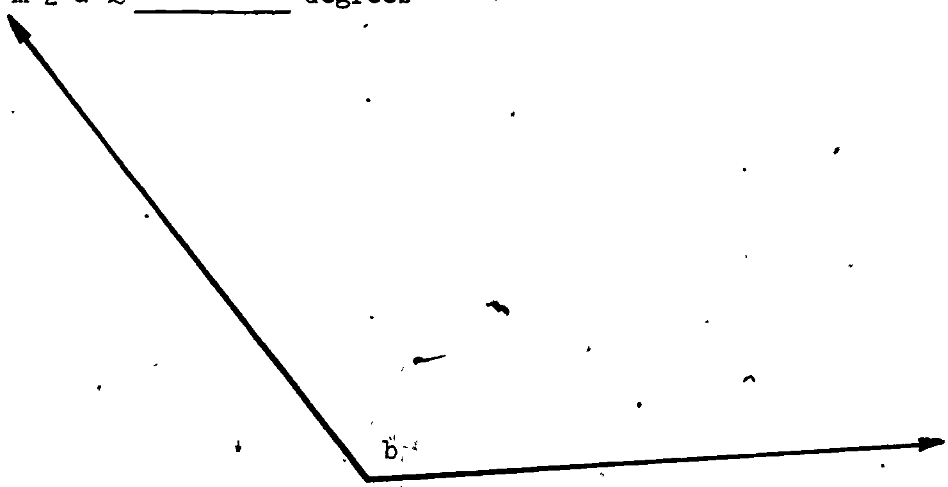
Use a protractor to measure these angles.

(a)



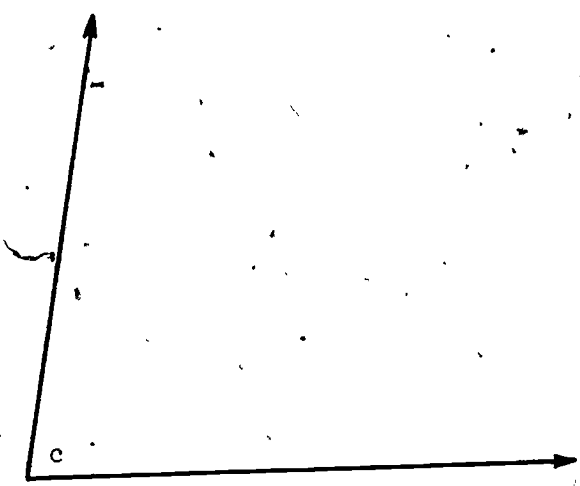
m ∠ a ≈ ________ degrees

(b)



m ∠ b ≈ ________ degrees

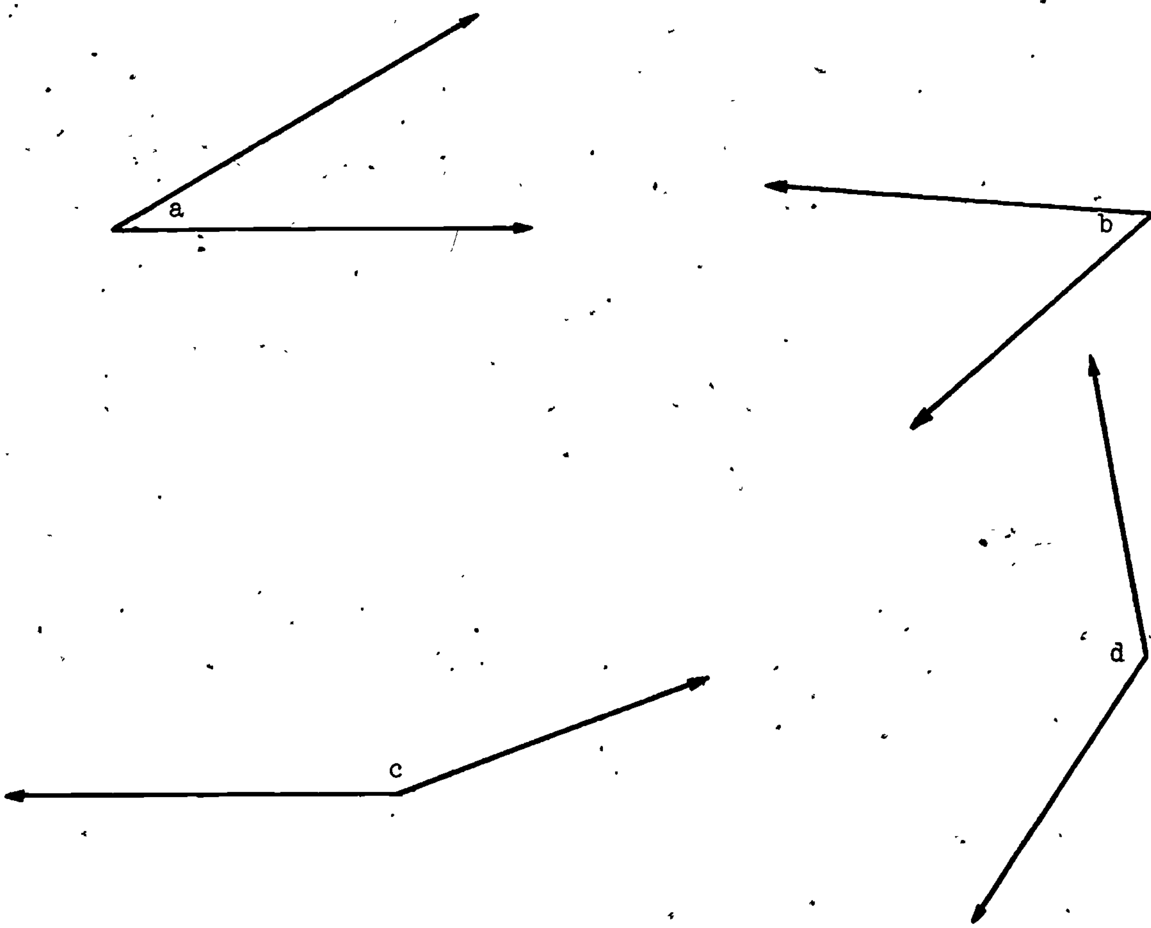
(c)



m ∠ c ≈ ________ degrees

9. (Section 15-11.)

Measure these angles to find which pair is supplementary.



∠ ______ and ∠ ______ are supplementary.

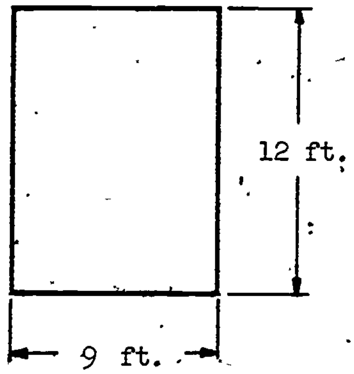
10. (Sections 15-12, 15-13.)

Find these areas. Use the correct equation from the list below.

$$A = b \cdot h$$

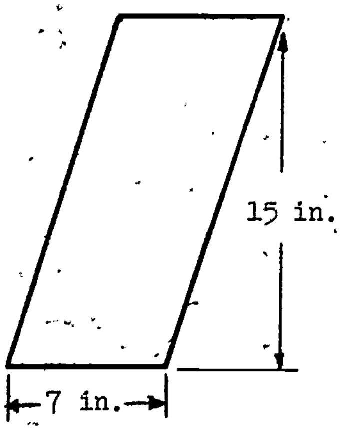
$$A = \frac{1}{2} b \cdot h$$

(a)



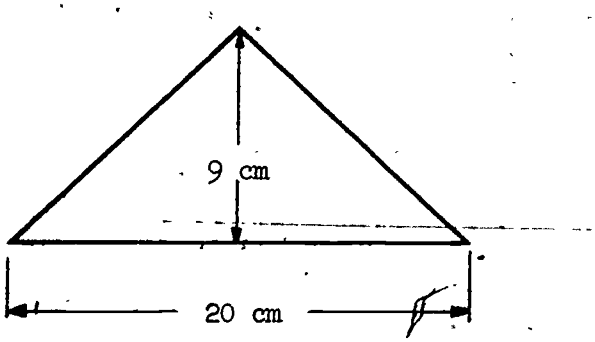
A = ________ sq. ft.

(b)



A = ________ sq. in.

(c)



A = ________ sq cm

1. Rewrite these fractions as mixed numbers.

   (a) $\frac{9}{7}$ = ________

   (b) $\frac{14}{9}$ = ________

2. Rewrite these mixed numbers as fractions.

   (a) $2\frac{1}{3}$ = ________

   (b) $1\frac{3}{7}$ = ________

3. Add these mixed numbers and simplify the answer.

   (a) $2\frac{3}{8} + 1\frac{7}{8}$ = ________

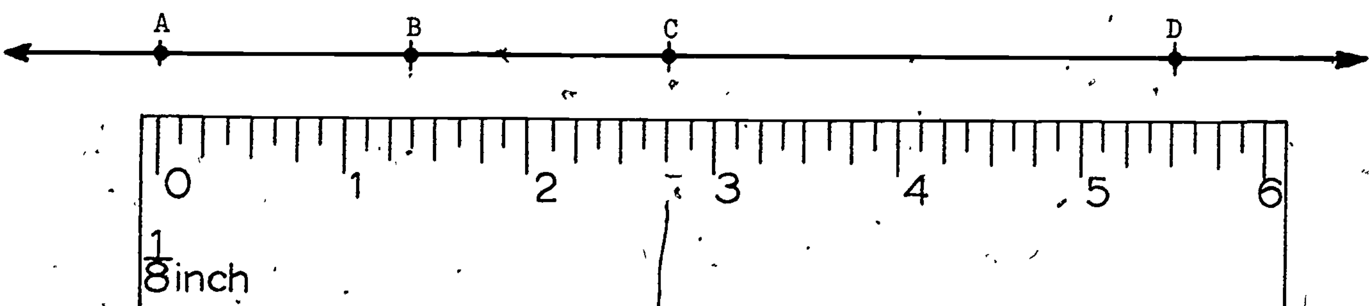
   (b) $3\frac{2}{9} + 4\frac{5}{9}$ = ________

4. Multiply these numbers and simplify your answers.

   (a) $2\frac{2}{7} \times 2$ = ________

   (b) $3\frac{5}{9} \times 5$ = ________

5. Use the picture below to answer these questions.



   (a) m $\overline{AB} \approx$ ________

   (b) m $\overline{AC} \approx$ ________

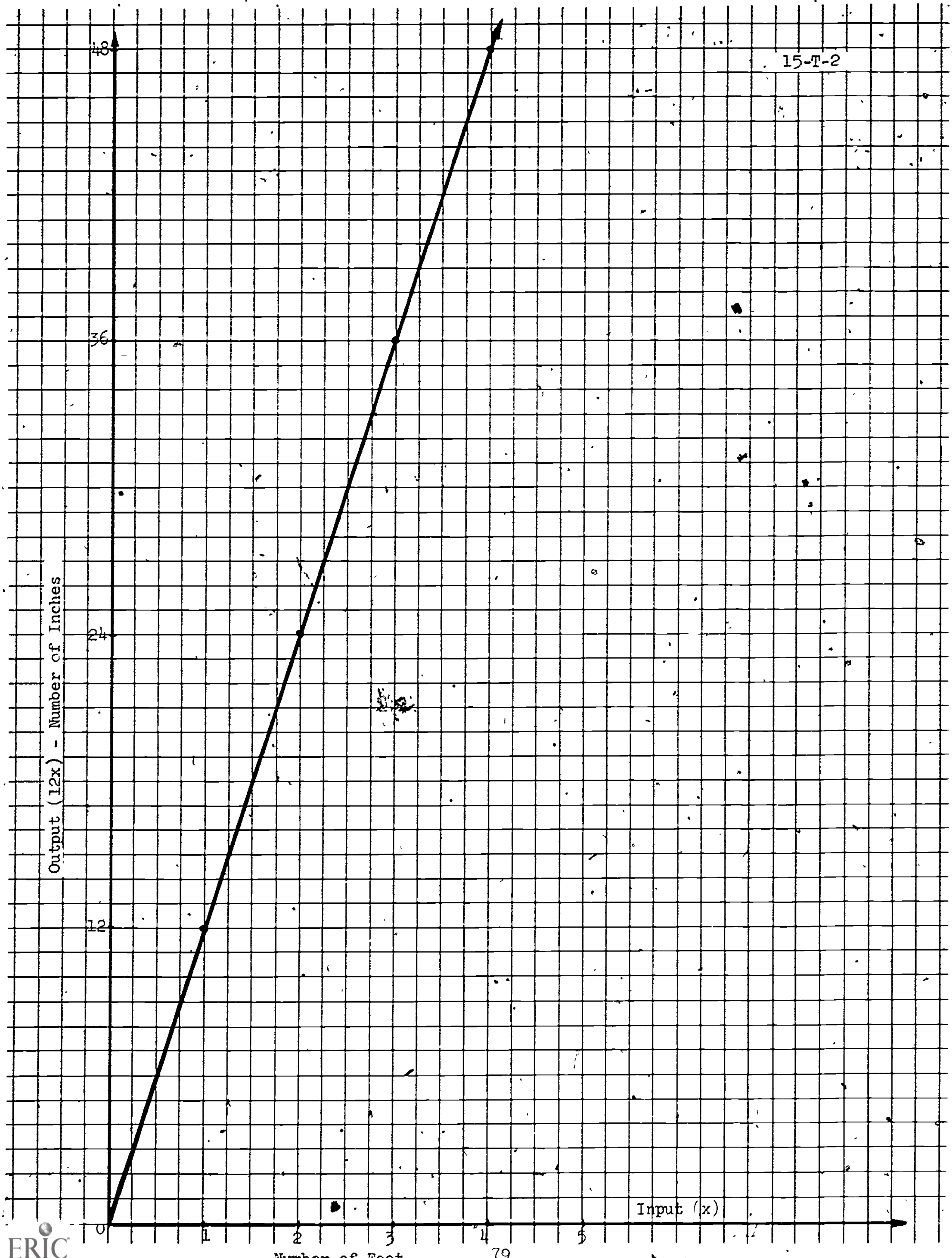
   (c) m $\overline{BC} \approx$ ________

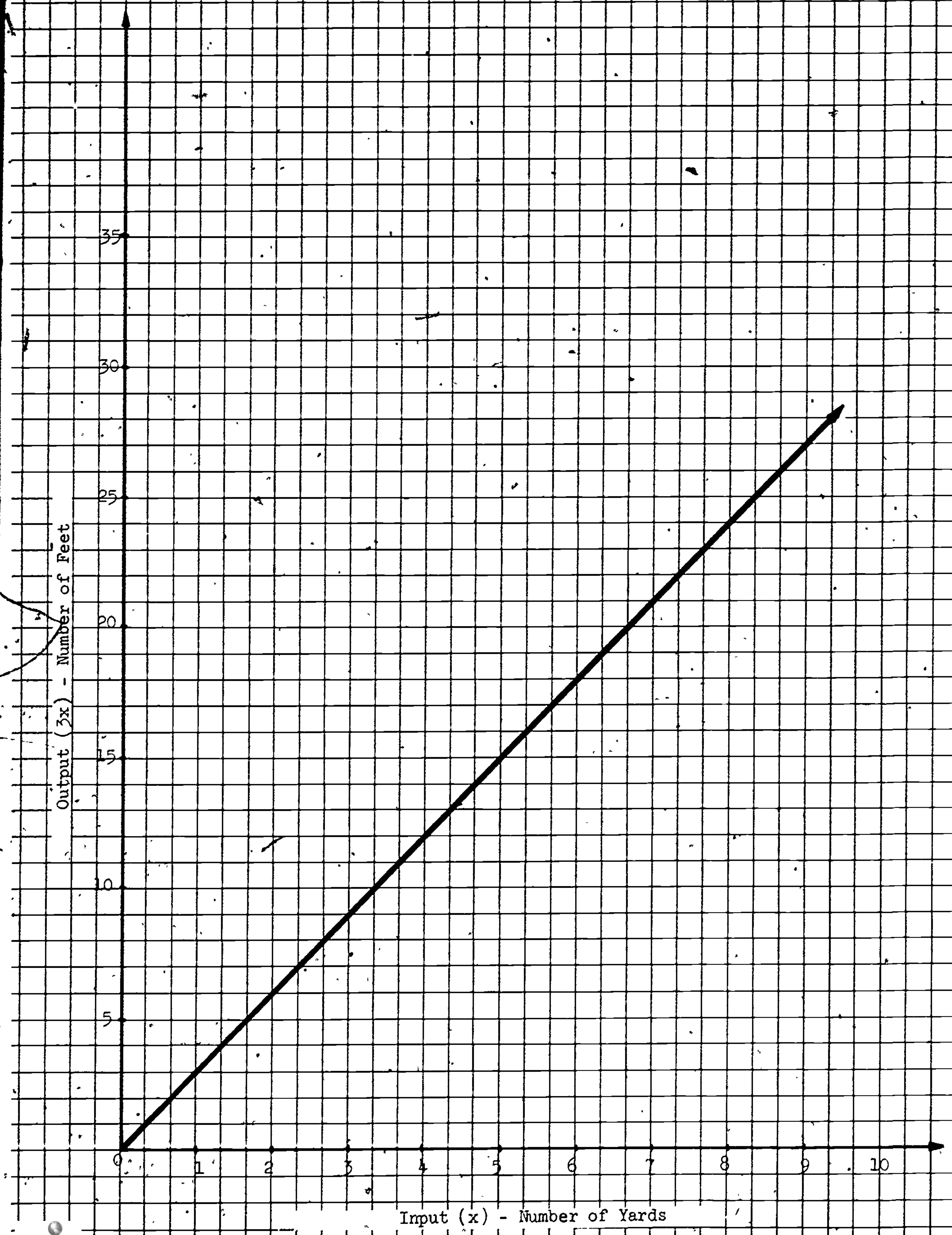
   (d) m $\overline{CD} \approx$ ________

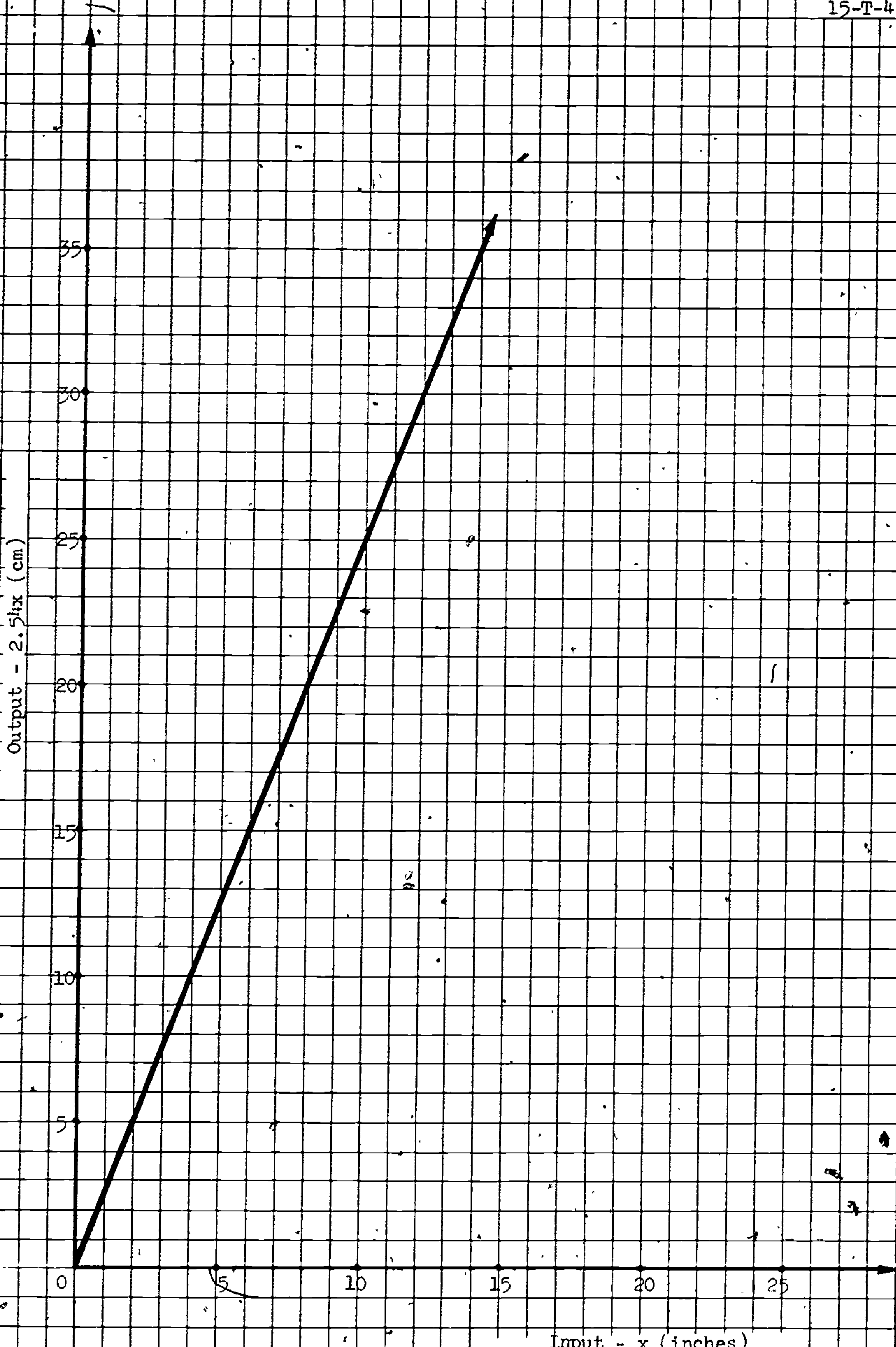
   (e) m $\overline{BD} \approx$ ________

   (f) m $\overline{AD} \approx$ ________

15-T-2


48
36
24
12
0
Output (12x) - Number of Inches
1
2
3
4
5
Input (x)
Number of Feet


Output (3x) - Number of Feet
35
30
25
20
15
10
5
0
1
2
3
4
5
6
7
8
9
10
Input (x) - Number of Yards



Output - 2.54x (cm)
35
30
25
20
15
10
5
0
5
10
15
20
25
Input - x (inches)

6. Use the correct graphs from the previous 3 pages to make these conversions.

   (a) $3\frac{3}{4}$ feet = __________ inches

   (b) 33 inches = __________ feet

   (c) 12 inches ≈ __________ cm

   (d) $5\frac{2}{3}$ yards = __________ feet

   (e) 19 feet = __________ yards

   (f) 24 inches = __________ yards

7. Use this list of metric units to make these conversions.

   10 mm = 1 cm
   100 cm = 1 m
   1000 m = 1 km

   (a) 4.2 cm = __________ mm

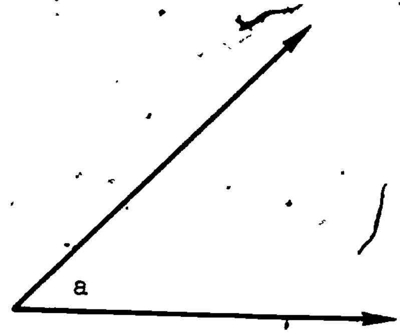
   (b) 525 cm = __________ m

   (c) 1.3 m = __________ cm

   (d) 7.3 km = __________ m

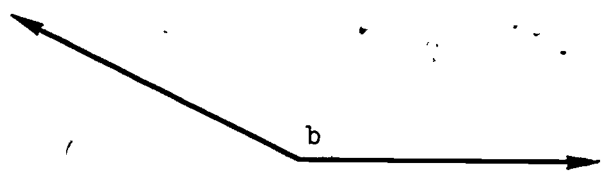
8. Use a protractor to measure these angles.

(a)



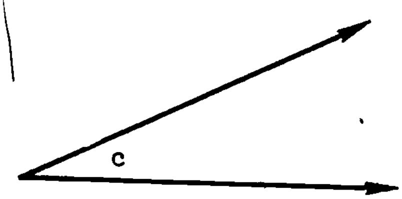
m ∠ a ≈ ________ degrees

(b)



m ∠ b ≈ ________ degrees

(c)



m ∠ c ≈ ________ degrees

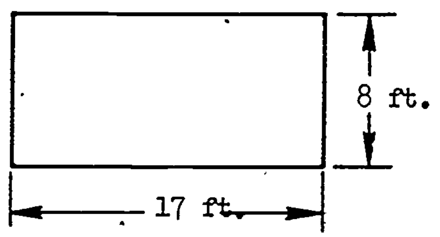
(d) ∠ ________ and ∠ ________ are supplementary.

9. Choose the correct equation from this list and find these areas.

$$A = b \cdot h$$

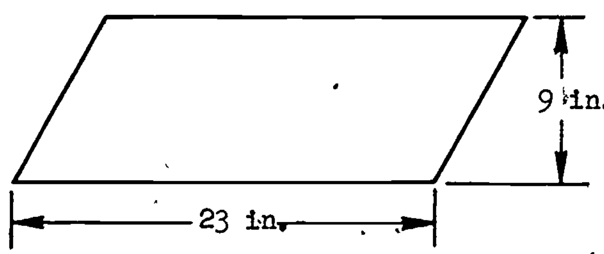
$$A = \frac{1}{2}b \cdot h$$

(a)



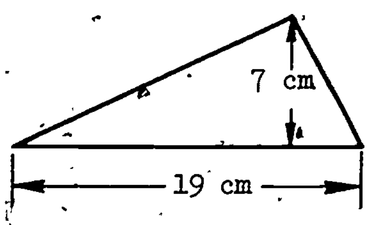
A = ________ sq. ft.

(b)



A = ________ sq. in.

(c)



A = ________ sq. cm

Check Your Memory: Self-Test

1. (Section 10-9.)

Write each number in decimal form.

(a) $\frac{3}{4}$ = ________

(b) $\frac{5}{8}$ = ________

(c) $\frac{9}{10}$ = ________

(d) $\frac{8}{5}$ = ________

(e) $\frac{89}{10}$ = ________

2. (Section 10-12.)

Divide.

(a) $\frac{42}{.3}$ = ________

(b) $\frac{7.8}{2}$ = ________

(c) $\frac{.155}{.5}$ = ________

(d) $\frac{.111}{3}$ = ________

(e) $\frac{49}{4.9}$ = ________

3. (Section 13-5.)

Find the least common multiple of each pair of numbers.

(a) 25 and 30 ________

(b) 15 and 24 ________

(c) 8 and 64 ________

(d) 8 and 11 ________

(e) 12 and 16 ________

4. (Section 13-6.)

Add. (Use the space at the right for your work.)

(a) $\frac{8}{7} + \frac{1}{2}$ = ________

(b) $\frac{4}{5} + \frac{2}{3}$ = ________

(c) $\frac{5}{9} + \frac{1}{3}$ = ________

(d) $\frac{5}{2} + \frac{-3}{4}$ = ________

(e) $\frac{6}{5} + \frac{1}{10}$ = ________

Now check your answers on the next page. If you do not have them all right, go back and read the section again.

Answers to Check Your Memory: Self-Test

1. (a) .75
   (b) .625
   (c) .9
   (d) 1.6
   (e) 8.9

2. (a) 140
   (b) 3.9
   (c) .31
   (d) .037
   (e) 10

3. (a) 150
   (b) 120
   (c) 64
   (d) 88
   (e) 48

4. (a) $\frac{23}{14}$
   (b) $\frac{22}{15}$
   (c) $\frac{8}{9}$
   (d) $\frac{7}{4}$
   (e) $\frac{13}{10}$

Chapter 16

REAL NUMBERS

Chapter 16

THE REAL NUMBERS

In Chapter 5, Integers, you saw that to find answers to subtraction problems like $3 - 5$, $2 - 7$, and $5 - 8$ we had to <u>invent</u> new numbers. We called these new numbers opposites.

$$3 - 5 = \text{opp } 2$$

$$2 - 7 = \text{opp } 5$$

$$5 - 8 = \text{opp } 3$$

and so on.

The whole numbers together with their opposites are called the integers.

But using only the integers we were still not able to find answers to all division problems like 3 divided by 6, 2 divided by 5, 7 divided by 4, and so on. Again we had to invent new numbers. Since division expressions did not always name integers, we wrote these new numbers as fractions.

$$3 \text{ divided by } 6 = \frac{3}{6}$$

$$2 \text{ divided by } 5 = \frac{2}{5}$$

$$7 \text{ divided by } 4 = \frac{7}{4}$$

and so on.

All the numbers - including the integers - which can be expressed as the quotient of an integer divided by a counting number are called the rational numbers.

Suppose we want to find the length of the side of a square whose area is 3. To answer this kind of question we will again need to invent some new numbers. These new numbers will be useful in answering other kinds of questions as well.

1. Complete this list by squaring these integers. (Squaring an integer means to raise it to the second power.) You can use your multiplication tables to help you.

$1^2 = 1$	$9^2 =$ ____
$2^2 = 4$	$10^2 =$ ____
$3^2 = 9$	$11^2 =$ ____
$4^2 =$ ____	$12^2 =$ ____
$5^2 =$ ____	$13^2 =$ ____
$6^2 =$ ____	$14^2 =$ ____
$7^2 =$ ____	$15^2 =$ ____
$8^2 =$ ____	$16^2 =$ ____

The numbers 1 , 4 , 9 , 16 , 25 , and so on, are all <u>squares of integers</u>. They are called the <u>perfect squares</u>.

Suppose you start with one of the perfect squares and try to find the integer that was squared to get it.

Start with 49 . The question we are asking is, "What integer multiplied by itself is 49 ?" Another way to ask this question is, "What is the <u>square root</u> of 49 ?" The symbol for square root is $\sqrt{\ }$.

<u>Example 1</u>. $\sqrt{49} = 7$ because $7^2 = 49$.

<u>Example 2</u>. $\sqrt{9} = 3$ because $3^2 = 9$.

The symbol $\sqrt{\ }$ is called a <u>radical</u>.

2. Complete this list.

$1^2 = 1$ so $\sqrt{1} = 1$

$2^2 = 4$ so $\sqrt{4} =$ ______

$3^2 = 9$ so $\sqrt{9} =$ ______

$4^2 = 16$ so $\sqrt{16} =$ ______

$5^2 = 25$ so $\sqrt{25} =$ ______

$6^2 = 36$ so $\sqrt{36} =$ ______

$7^2 = 49$ so $\sqrt{49} =$ ______

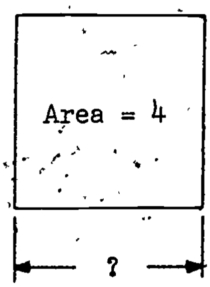
$8^2 = 64$ so $\sqrt{64} =$ ______

$9^2 = 81$ so $\sqrt{81} =$ ______

$10^2 = 100$ so $\sqrt{100} =$ ______

Only the <u>perfect squares</u> have square roots that are integers.

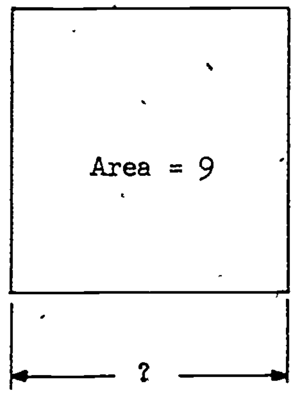
Now let's ask this question. How long is the side of a square with an area of 4 ?



Again we are asking the question, "What number multiplied by itself is 4 , or what is $\sqrt{4}$?"

$\sqrt{4} = 2$ because $2^2 = 4$. But the fact that $\sqrt{4}$ and 2 are <u>equal</u> means that $\sqrt{4}$ and 2 are different names for the same number.

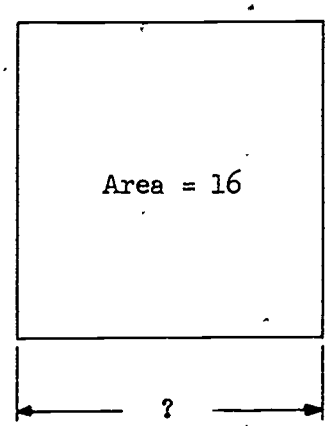
3. Here is a square with an area of 9 .



(a) What is the length of the side of the square? $\sqrt{\ \ }$ ______

(b) What is the integer name for this length? __________

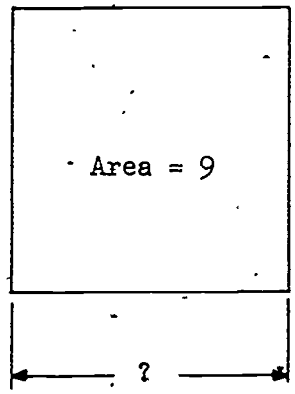
4. Here is a square with an area of 16 .



(a) What is the length of the side of the square? $\sqrt{\ \ }$ ______

(b) What is the integer name for this length? __________

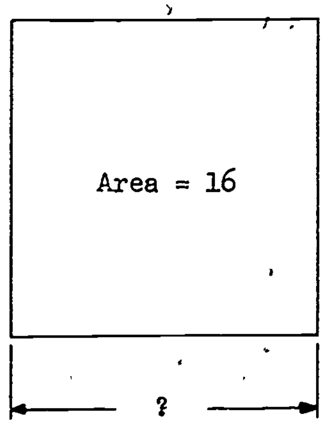
3. Here is a square with an area of 9 .



(a) What is the length of the side of the square? $\sqrt{}$ ______

(b) What is the integer name for this length? ______

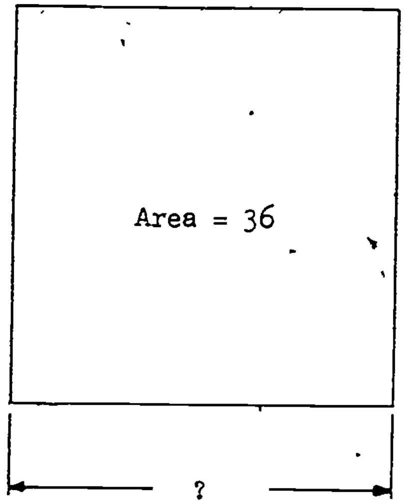
4. Here is a square with an area of 16 .



(a) What is the length of the side of the square? $\sqrt{}$ ______

(b) What is the integer name for this length? ______

5. Here is a square with an area of 36 .



(a) What is the length of the side of this square? $\sqrt{}$ ______

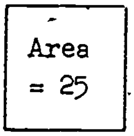
(b) What is the integer name for this length? ______

Exercises

What are the lengths of the sides of these squares. Use your multiplication tables to help you.

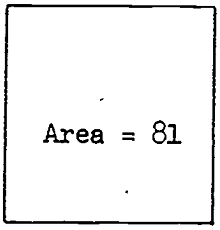
1. (a) Length of side = $\sqrt{\quad}$ ______

   (b) Integer name for length of side is ______ .



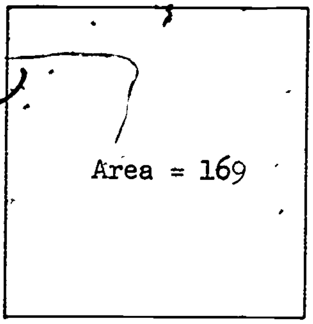
2. (a) Length of side = $\sqrt{\quad}$ ______

   (b) Integer name for length of side is ______ .



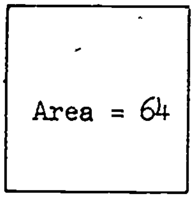
3. (a) Length of side = $\sqrt{\quad}$ ______

   (b) Integer name for length of side is ______ .



4. (a) Length of side = $\sqrt{\quad}$ ______

   (b) Integer name for length of side is ______ .



More about Squares and Square Roots

2 and $\sqrt{4}$ are different names for the same number.

$$2^2 = 4$$

$$\text{so: } (\sqrt{4})^2 = 4.$$

Class Discussion

1. (a) What is another name for 3 ?

$3 = \sqrt{}$ ______

(b) $3^2 = 9$

so $(\sqrt{9})^2 =$ ______

2. (a) What is another name for 5 ?

$5 = \sqrt{}$ ______

(b) $5^2 =$ ______

so $(\sqrt{25})^2 =$ ______

3. (a) What is another name for 6 ?

$6 = \sqrt{}$ ______

(b) $6^2 =$ ______

so $(\sqrt{})^2 = 36$

4. (a) What is another name for 7 ?

$7 = \sqrt{}$ ______

(b) $7^2 =$ ______

so $(\sqrt{49})^2 =$ ______

Some square roots do not have integer names.

5. (a) The $(\sqrt{3})^2 = 3$.
Can you think of an integer whose square is 3 ? ______

(b) Is there an integer name for $\sqrt{3}$? ______

6. (a) Is $(\sqrt{6})^2$ equal to 6 ? ____________

   (b) Can you think of an integer whose square is 6 ? ________

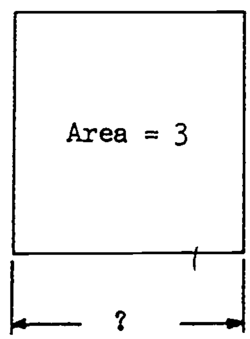
   (c) Is there an integer name for $\sqrt{6}$? ________

7. (a) Is $(\sqrt{8})^2$ equal to 8 ? ____________

   (b) Can you think of an integer whose square is 8 ? ________

   (c) Is there an integer name for $\sqrt{8}$? ________

8. Here is a square with area 3 .

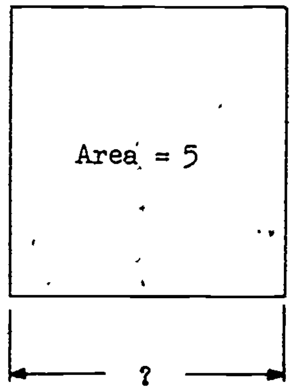


   (a) What is the length of its side? $\sqrt{\quad}$ ______

   (b) Is there an integer name for this length? ________

   (c) $(\sqrt{3})^2 =$ ________

9. Here is a square with area 5 .

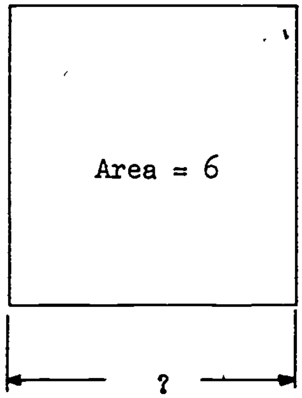


(a) What is the length of its side? $\sqrt{\ }$ ____

(b) Is there an integer name for this length? ________

(c) $(\sqrt{5})^2 =$ ____

10. Here is a square with area 6 .

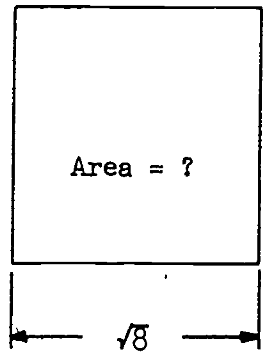


(a) What is the length of its side? $\sqrt{\ }$ ____

(b) Is there an integer name for this length? ________

(c) $(\sqrt{6})^2 =$ ____

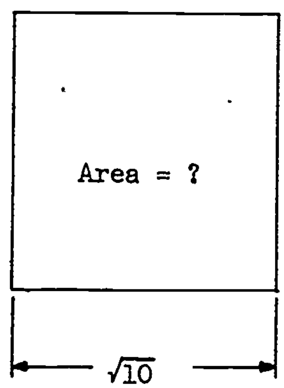
11. The length of the side of this square is $\sqrt{8}$.



(a) Is $(\sqrt{8})^2$ equal to 8 ? ________________

(b) What is the area of the square? ________

12. The length of the side of this square is $\sqrt{10}$.



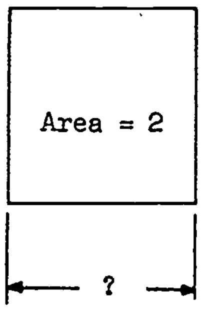
(a) $(\sqrt{10})^2 =$ ____

(b) The area of the square is ________ .

Exercises

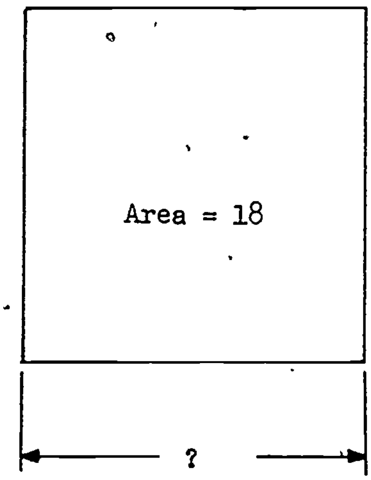
1. Find the length of the side of each of these squares.

(a)



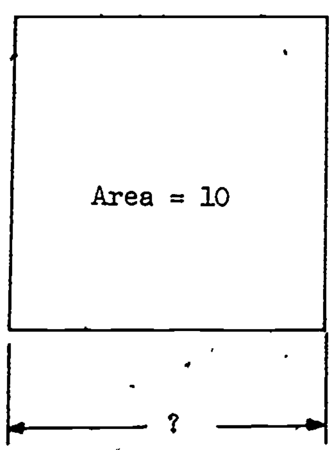
length of side = $\sqrt{}$ ____

(c)



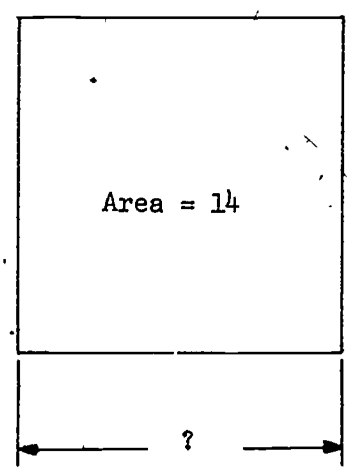
length of side = $\sqrt{}$ ____

(b)



length of side = $\sqrt{}$ ____

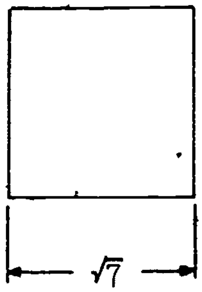
(d)



length of side = $\sqrt{}$ ____

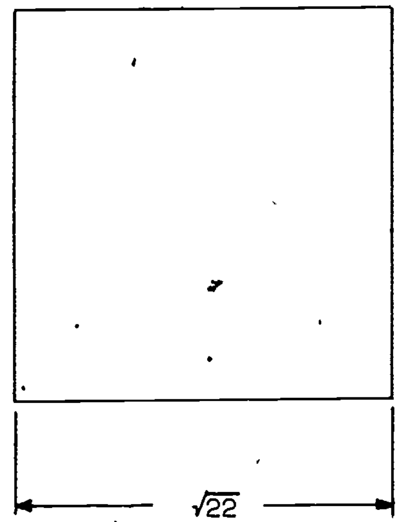
2. Find the area of each of these squares.

(a)



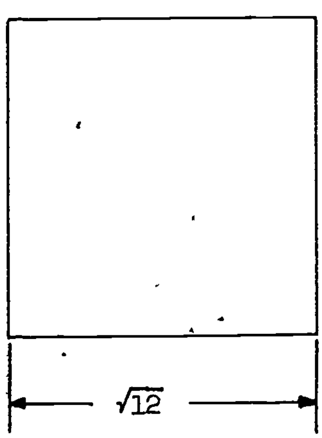
Area = ________

(c)



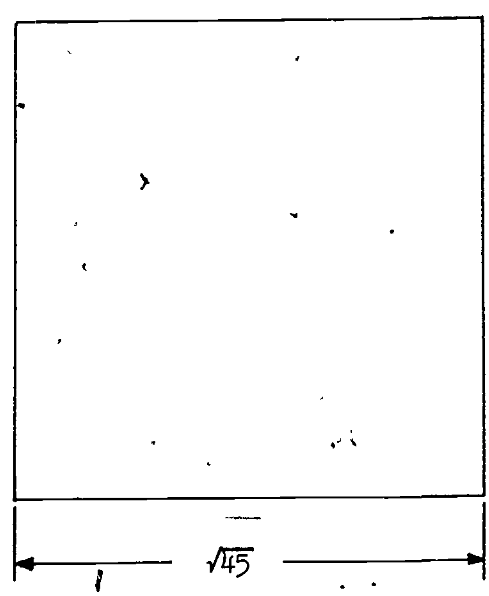
Area = ________

(b)



Area = ________

(d)



Area = ________

3. Square these numbers.

(a) $(\sqrt{5})^2$ = ________

(b) $(\sqrt{11})^2$ = ________

(c) $(\sqrt{9})^2$ = ________

(d) $(\sqrt{16})^2$ = ________

(e) $(\sqrt{55})^2$ = ________

(f) $(\sqrt{60})^2$ = ________

Irrational Numbers

Here is a list of perfect squares.

1
4
9
16
25
36
49
64
81
100
121
and so on.

The perfect squares are the only numbers that have integer square roots.

$\sqrt{1} = 1$

$\sqrt{4} = 2$

$\sqrt{9} = 3$

$\sqrt{16} = 4$

$\sqrt{25} = 5$

$\sqrt{36} = 6$

$\sqrt{49} = 7$

and so on.

The integers between the perfect squares, (2 , 3 , 5 , 6 , 7 , 8 , 10 , 11 etc.), have square roots that are not integers.

These square roots are some of the numbers that are called irrational numbers. Irrational means not rational.

These numbers are irrational	These numbers are rational
	$\sqrt{1}$
$\sqrt{2}$	
$\sqrt{3}$	
	$\sqrt{4}$
$\sqrt{5}$	
$\sqrt{6}$	
$\sqrt{7}$	
$\sqrt{8}$	
	$\sqrt{9}$
$\sqrt{10}$	
$\sqrt{11}$	
$\sqrt{12}$	
$\sqrt{13}$	
$\sqrt{14}$	
$\sqrt{15}$	
	$\sqrt{16}$
$\sqrt{17}$	
$\sqrt{18}$	
$\sqrt{19}$	
$\sqrt{20}$	
$\sqrt{21}$	
$\sqrt{22}$	
$\sqrt{23}$	
$\sqrt{24}$	
	$\sqrt{25}$
and so on.	and so on.

Class Discussion

1. Complete the following table. The first two are done for you.

Number	Write R if Rational or I if Irrational.	If Rational write the integer name.
$\sqrt{16}$	R	4
$\sqrt{5}$	I	___
$\sqrt{7}$	___	___
$\sqrt{8}$	___	___
$\sqrt{36}$	___	___
$\sqrt{14}$	___	___
$\sqrt{24}$	___	___
$\sqrt{49}$	___	___
$\sqrt{40}$	___	___
$\sqrt{81}$	___	___
$\sqrt{26}$	___	___
$\sqrt{100}$	___	___
$\sqrt{64}$	___	___
$\sqrt{38}$	___	___
$\sqrt{121}$	___	___
$\sqrt{18}$	___	___
$\sqrt{25}$	___	___

On the number line:

$\sqrt{4}$ is to the left of $\sqrt{9}$ because 2 is to the left of 3

$\sqrt{9}$ is to the left of $\sqrt{16}$ because 3 is to the left of 4

$\sqrt{16}$ is to the left of $\sqrt{25}$ because 4 is to the left of 5

$\sqrt{25}$ is to the left of $\sqrt{36}$ because 5 is to the left of 6

and so on.

Use the list on page 16-3a to help you with these questions.

2. (a) On the number line is $\sqrt{4}$ to the left or right of $\sqrt{6}$? _____

   (b) On the number line is 2 to the left or right of $\sqrt{6}$? _____

   (c) 2 $\underset{(>,\ <)}{\underline{\qquad}}$ $\sqrt{6}$

   (d) On the number line is $\sqrt{6}$ to the left or right of $\sqrt{9}$? _____

   (e) On the number line is $\sqrt{6}$ to the left or right of 3 ? _____

   (f) $\sqrt{6}$ $\underset{(>,\ <)}{\underline{\qquad}}$ 3

   (g) What integers is $\sqrt{6}$ between? _____ and _____ .

3. On the number line:

   (a) Is $\sqrt{16}$ to the left or right of $\sqrt{20}$? _____

   (b) Is 4 to the left or right of $\sqrt{20}$? _____

   (c) 4 $\underset{(>,\ <)}{\underline{\qquad}}$ $\sqrt{20}$

   (d) What is the next integer to the right of $\sqrt{20}$? _____

   (e) $\sqrt{20}$ $\underset{(>,\ <)}{\underline{\qquad}}$ 5 .

   (f) What integers is $\sqrt{20}$ between? _____ and _____ .

4. (a) What integer is to the left of $\sqrt{12}$? _____

(b) What integer is to the right of $\sqrt{12}$? _____

(c) $\sqrt{12}$ is between _____ and _____ . (What integers?)

5. $\sqrt{30}$ is between _____ and _____ . (What integers?)

Exercises

1. Complete the following table.

Number	Write R if Rational or I if Irrational.	If Rational write the integer name.
$\sqrt{6}$	_____	_____
$\sqrt{10}$	_____	_____
$\sqrt{49}$	_____	_____
$\sqrt{60}$	_____	_____
$\sqrt{121}$	_____	_____
$\sqrt{75}$	_____	_____
$\sqrt{1}$	_____	_____
$\sqrt{50}$	_____	_____

2. Between what two integers is each of these numbers?

(a) $\sqrt{5}$ is between _____ and _____ .

(b) $\sqrt{14}$ is between _____ and _____ .

(c) $\sqrt{40}$ is between _____ and _____ .

(d) $\sqrt{2}$ is between _____ and _____ .

(e) $\sqrt{3}$ is between _____ and _____ .

The Right Triangle

As we have done before we will locate these new numbers on the number line. The right triangle will help us do this.

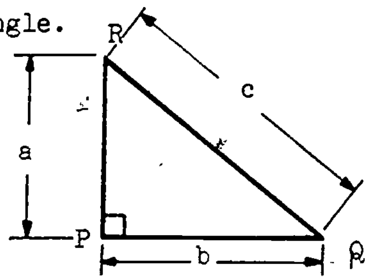
For right triangles in the last chapter we did these things.

1. We squared the length of each of the shorter sides.
2. We added these squares.
3. This sum was the same number as the square of the length of the long side.

This works for all right triangles.

Class Discussion

The longest side of a right triangle is called the hypotenuse (pronounced hi-pot´-uh-noose). The hypotenuse is always opposite to the ________ angle.



In triangle PQR above, $\overline{RQ}$ is the hypotenuse. It is "c" units long.

How long is side $\overline{PR}$? __________ units

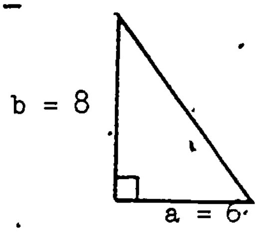
How long is side $\overline{PQ}$? __________ units

For all right triangles the relationships of the lengths of the sides is:

$$a^2 + b^2 = c^2$$

where c is the length of the hypotenuse.

Example 1.



For the right triangle above $a^2 =$ _____ and $b^2 =$ _____.

$$a^2 + b^2 = c^2$$

$$\_\_\_ + \_\_\_ = c^2$$

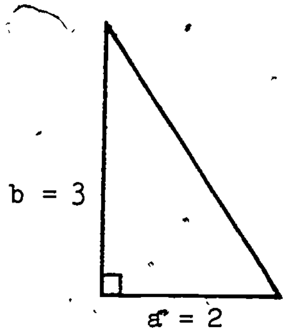
$$\_\_\_\_\_\_\_\_ = c^2$$

Since $c^2 = 100$ we can find the length of c. This is the number whose square is 100. We write:

$$c = \sqrt{100}$$

So: $c =$ _____

Example 2.



For the right triangle above $a^2 =$ _____ and $b^2 =$ _____.

$$a^2 + b^2 = c^2$$

$$\_\_\_ + \_\_\_ = c^2$$

$$\_\_\_\_\_\_\_\_ = c^2$$

There is no integer whose square is 13 so we simply write

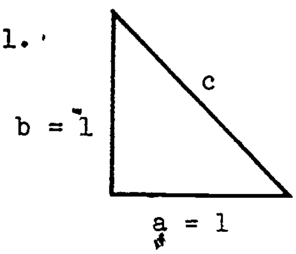
$$c = \sqrt{13}.$$

Our answer is $\sqrt{13}$ units for the length of the hypotenuse. This number is irrational.

Exercises

Find the length of the hypotenuse of each of these right triangles. For the triangles where these lengths are irrational leave your answer as a radical.

1.



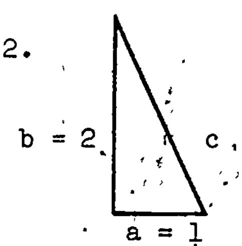
$a^2 + b^2 = c^2$

____ + ____ = c^2

_____ = c^2

_____ = c

2.



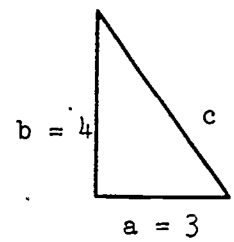
$a^2 + b^2 = c^2$

____ + ____ = c^2

_____ = c^2

_____ = c

3.



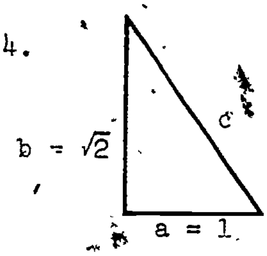
$a^2 + b^2 = c^2$

____ + ____ = c^2

_____ = c^2

_____ = c

4.



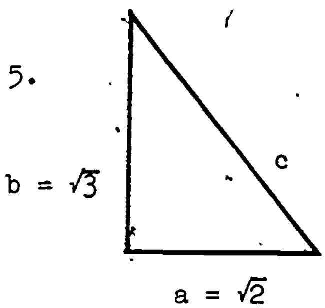
$a^2 + b^2 = c^2$

____ + ____ = c^2

_____ = c^2

_____ = c

5.



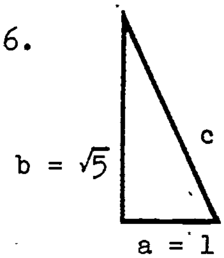
$a^2 + b^2 = c^2$

____ + ____ $= c^2$

______ $= c^2$

______ $= c$

6.



$a^2 + b^2 = c^2$

____ + ____ $= c^2$

______ $= c^2$

______ $= c$

The Real Number Line

In Chapter 5 you learned how to locate the integers on the number line. When you did this there were many points that were not yet named.

In Chapter 6 you learned how to locate the rational numbers on the number line. Perhaps you thought that the number line was "filled" up with the rational numbers. At the time of Pythagoras most people believed that all the points on the line could be named by rational numbers. Pythagoras discovered that this was not true. There would be many, many points left unnamed even if we could put all the rational numbers on the line. The points "left over" are named by the irrational numbers. Naming these points "fills up" the number line. We say the line is "complete". All the rational numbers together with all the irrational numbers name all the points on the line. We call the rationals together with the irrationals the real numbers.

You may think that all numbers are "real", but this is not so. There are some other kinds of numbers that behave in unusual ways. These strange numbers are useful for some things, and they are not called real numbers.

We can locate some of the irrational numbers by using the right triangle.

Class Discussion

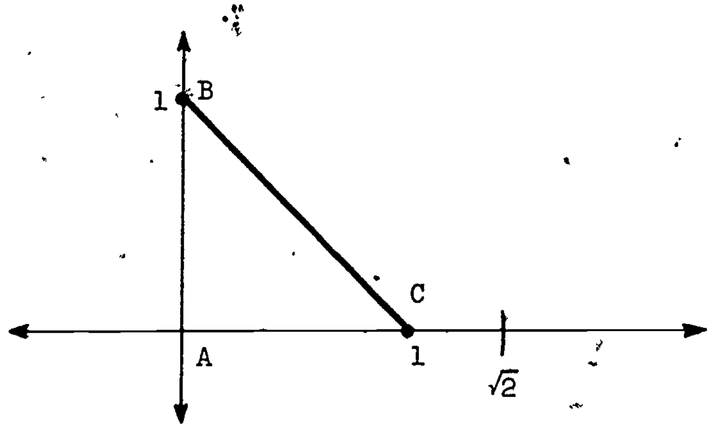
Get out the worksheet on page 16-5d. Use your compass and straightedge. Follow these steps to locate $\sqrt{2}$.

1. Label the origin point A .
2. On the worksheet the point (0,1) is labeled B .
3. The point (1,0) is labeled C .
4. Draw $\overline{BC}$.
5. Triangle ABC is a ________ triangle.
6. The hypotenuse of this right triangle is segment ________.
7. How long is $\overline{AB}$? ________ unit
8. How long is $\overline{AC}$? ________ unit
9. $(m\ \overline{BC})^2 = 1^2 + 1^2$

   $(m\ \overline{BC})^2 =$ ________.

   $m\ \overline{BC} =$ ________.
10. Place the needle point of your compass at point B and the pencil point at point C .
11. Without changing your compass setting, place the needle point at point A . Swing an arc that crosses the horizontal axis to the right of point C . Label this point $\sqrt{2}$.

    Your completed construction should look like this.



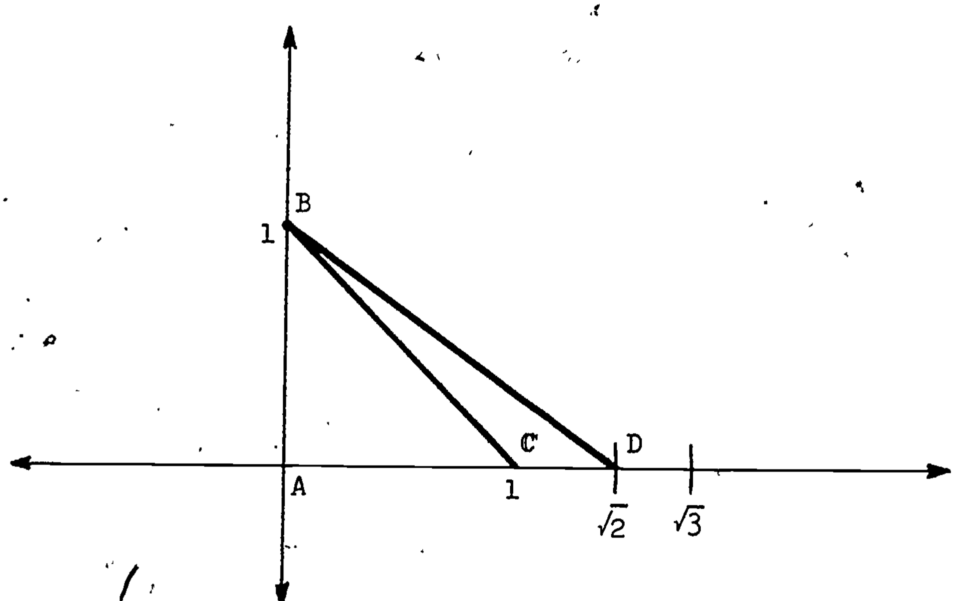
Follow these steps to locate $\sqrt{3}$.

1. Label $(\sqrt{2},0)$ point D .
2. Draw $\overline{BD}$.
3. How long is $\overline{AD}$? ____
4. How long is $\overline{AB}$? ____
5. $(m\ \overline{BD})^2 = 1^2 + (\sqrt{2})^2$

   $(m\ \overline{BD})^2 =$ ________

   $m\ \overline{BD} =$ ________
6. Place the needle point of your compass at point B and the pencil point at point D .
7. Without changing your compass setting, place the needle point at point A . Swing an arc that crosses the horizontal axis to the right of point D . Label this point $\sqrt{3}$.

Your completed construction should now look like this.



We can use the same procedure to locate $\sqrt{4}$ which you know is 2 . This will be a good check on our method.

1. Label $(\sqrt{3},0)$ point E .

2. Draw $\overline{BE}$.

3. How long is $\overline{AE}$? ________

4. How long is $\overline{AB}$? ________

5. $(m\ \overline{BE})^2 = 1^2 + (\sqrt{3})^2$

   $(m\ \overline{BE})^2 =$ __ + __

   $(m\ \overline{BE})^2 =$ ________

   $m\ \overline{BE} =$ ________

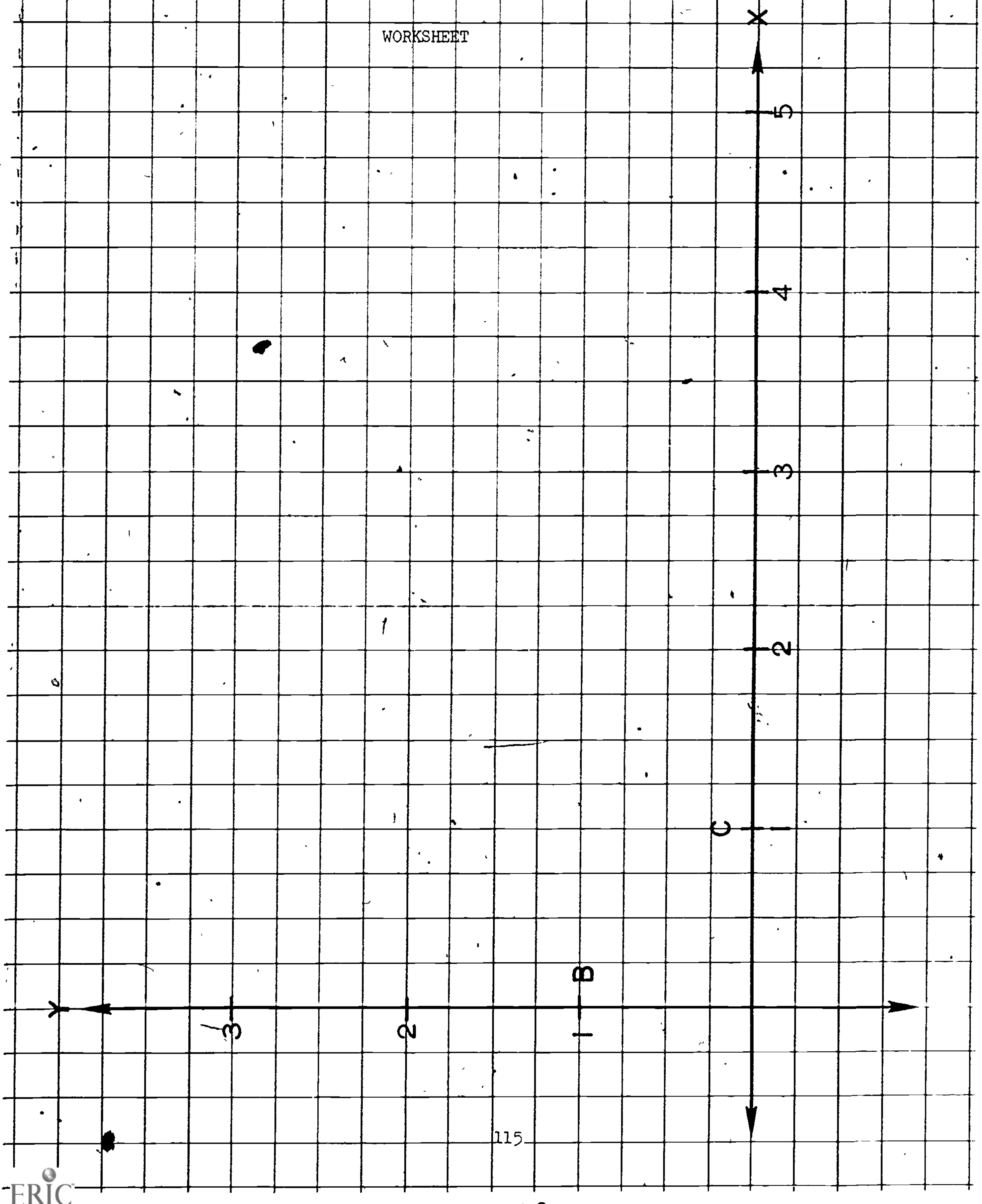
6. Place the needle point of your compass at point B and the pencil point at E .

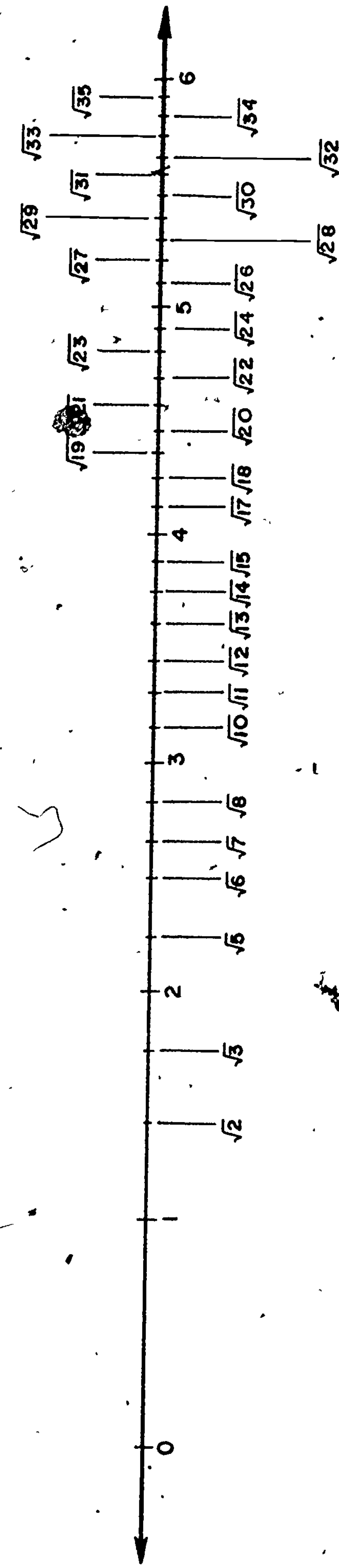
7. Without changing the setting, place the needle of your compass at point A and swing an arc to the right of point E . This is the location of $\sqrt{4}$. What integer is located at this point?

   ____

By continuing with this process we can locate $\sqrt{5}$, $\sqrt{6}$, $\sqrt{7}$, $\sqrt{8}$ and so on on the number line. On page 16-5e you will find a number line with many of these irrational numbers marked.

WORKSHEET




0
1
√2
√3
2
√5
√6
√7
√8
3
√10
√11
√12
√13
√14
√15
4
√17
√18
√19
√20
√21
√22
√23
√24
5
√26
√27
√28
√29
√30
√31
√32
√33
√34
√35
6

Exercises

1. Find the lengths of the diagonals of these figures.

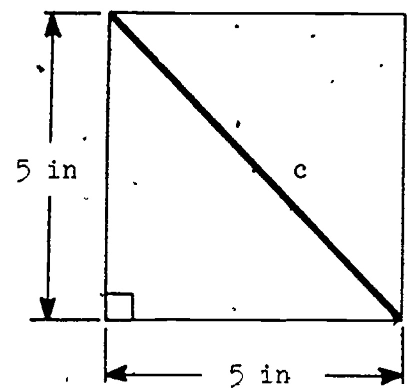
(a)

$a^2 + b^2 = c^2$

____ + ____ = c^2

____ = c^2

____ = c



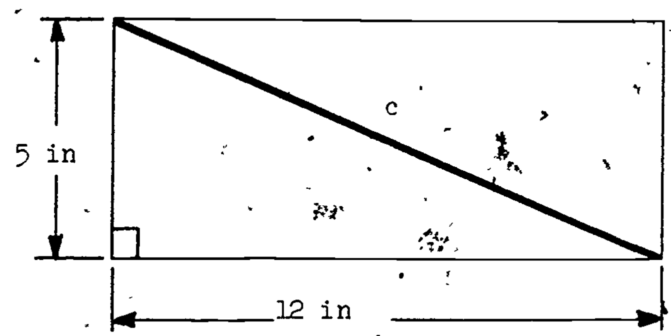
(b)

$a^2 + b^2 = c^2$

____ + ____ = c^2

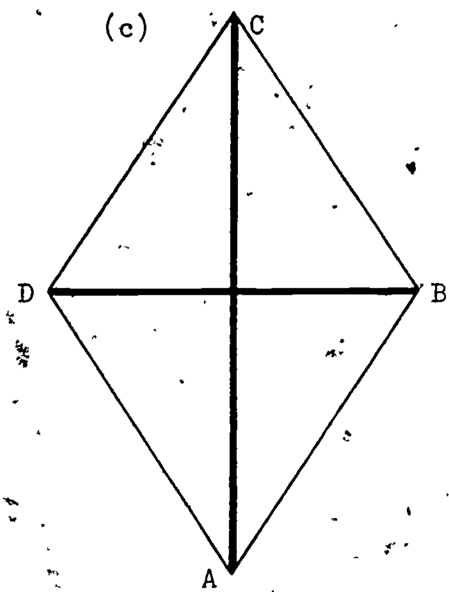
____ = c^2

____ = c



Hint: This is rational.

(c)



ABCD is a rhombus.

Diagonal $\overline{AC}$ is 12 in. long.

Diagonal $\overline{DB}$ is 8 in. long.

Find the length of the side of the rhombus.

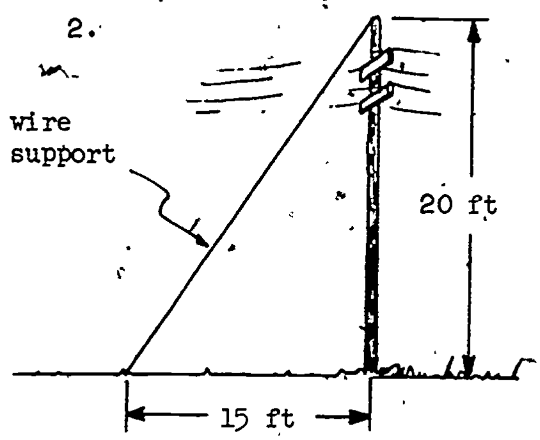
$a^2 + b^2 = c^2$

____ + ____ = c^2

____ = c^2

____ = c

2.



This telephone pole has a wire support from the top of the pole to a point on the ground 15 feet from the base of the pole. How long is the wire?

$a^2 + b^2 = c^2$

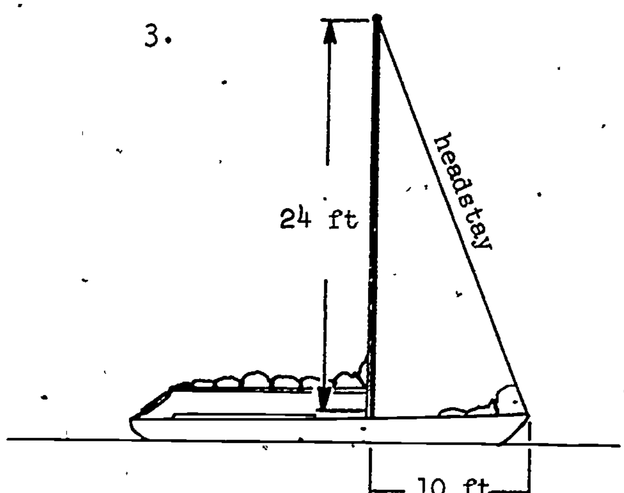
$\_\_\_\_ + \_\_\_\_ = c^2$

$\_\_\_\_ = c^2$

$\_\_\_\_ = c$

Hint: This number is rational. See if you can give its integer name.

3.



This sailboat has a 24 foot mast located 10 feet from the bow. The headstay is a wire that runs from the top of the mast to the bow. How long is this headstay?

$a^2 + b^2 = c^2$

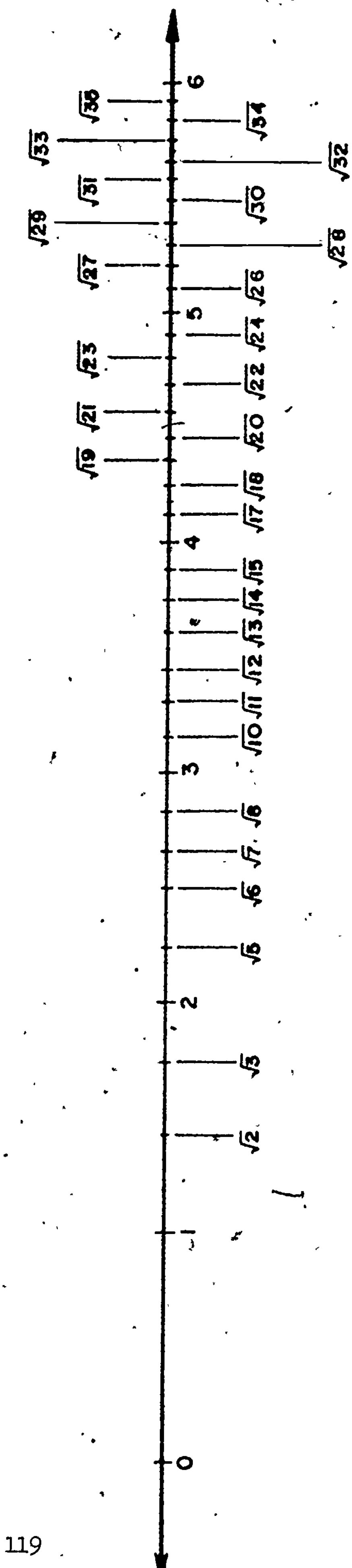
$\_\_\_\_ + \_\_\_\_ = c^2$

$\_\_\_\_ = c^2$

$\_\_\_\_ = c$

Hint: This number is rational. Square 26 and see if this is c^2.

Addition and Multiplication of Irrational Numbers



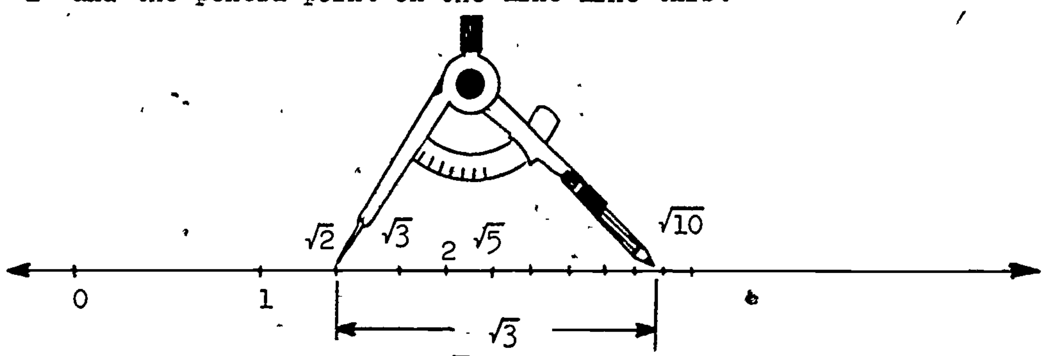
Addition and Multiplication of Irrational Numbers

Class Discussion

We can use the number line on page 16-6 to find out how to add irrational numbers.

You have seen how to add integers and rationals using arrows. You can use your compass as a "portable arrow" if we agree that the pencil point is the arrow head and the needle point is the arrow tail.

Suppose we first add $\sqrt{2} + \sqrt{3}$ on the number line on page 16-6. Place the needle point of your compass at 0 and the pencil point at $\sqrt{3}$. Carefully pick up your compass and place the needle point at $\sqrt{2}$ and the pencil point on the line like this.



Notice that you do not come to $\sqrt{5}$. In fact the point you come to is not even named. We simply name this point $(\sqrt{2} + \sqrt{3})$. This is a little less than $\sqrt{10}$.

Try adding $\sqrt{2} + \sqrt{5}$ using your compass and the number line.

Does $\sqrt{2} + \sqrt{5} = \sqrt{7}$? __________

Is the point you come to named? __________

$\sqrt{2} + \sqrt{5}$ is between what two square roots? ____ and ____ .

Let's see what we can find out when we add a square root to itself. Use your compass and the number line to add $\sqrt{2} + \sqrt{2}$.

This time you come to a point that is named.

$$\sqrt{2} + \sqrt{2} = \_\_\_\_\_\_\_\_$$

Add these irrational numbers.

$$\sqrt{3} + \sqrt{3} = \_\_\_\_\_\_\_\_$$

$$\sqrt{5} + \sqrt{5} = \_\_\_\_\_\_\_\_$$

$$\sqrt{6} + \sqrt{6} = \_\_\_\_\_\_\_\_$$

Do all of these come out to points that are named? ________

You have often thought of repeated addition as multiplication. In other words $\sqrt{2} + \sqrt{2} = 2 \cdot \sqrt{2}$.

Write these as multiplication problems.

$$\sqrt{3} + \sqrt{3} = \_\_\_\_ \cdot \_\_\_\_$$

$$\sqrt{5} + \sqrt{5} = \_\_\_\_ \cdot \_\_\_\_$$

$$\sqrt{6} + \sqrt{6} = \_\_\_\_ \cdot \_\_\_\_$$

You know that 2 can also be written as $\sqrt{4}$.

So: $\sqrt{2} + \sqrt{2} = \sqrt{4} \cdot \sqrt{2}$

When you added these on the number line you came to $\sqrt{8}$.

So: $\sqrt{4} \cdot \sqrt{2} = \sqrt{4 \cdot 2} = \sqrt{8}$.

Look at your answer to $\sqrt{3} + \sqrt{3}$. From your answer we can see that:

$$\sqrt{3} + \sqrt{3} = 2 \cdot \sqrt{3}$$
$$\sqrt{3} + \sqrt{3} = \sqrt{4} \cdot \sqrt{3}$$
$$\sqrt{3} + \sqrt{3} = \sqrt{4 \cdot 3}$$
$$\sqrt{3} + \sqrt{3} = \sqrt{12}$$

Also

$$\sqrt{5} + \sqrt{5} = 2 \cdot \sqrt{5}$$
$$\sqrt{5} + \sqrt{5} = \sqrt{4} \cdot \sqrt{5}$$
$$\sqrt{5} + \sqrt{5} = \sqrt{4 \cdot 5}$$
$$\sqrt{5} + \sqrt{5} = \sqrt{20}$$

and

$$\sqrt{6} + \sqrt{6} = 2 \cdot \sqrt{6}$$
$$\sqrt{6} + \sqrt{6} = \sqrt{4} \cdot \sqrt{6}$$
$$\sqrt{6} + \sqrt{6} = \sqrt{4 \cdot 6}$$
$$\sqrt{6} + \sqrt{6} = \sqrt{24}$$

There are three important facts that we can see from what we have just done.

(1) If a is a positive integer,

$$\begin{aligned}\sqrt{a} + \sqrt{a} &= 2 \cdot \sqrt{a} \\ &= \sqrt{4} \cdot \sqrt{a} \\ &= \sqrt{4 \cdot a}\end{aligned}$$

Because $4 \cdot a$ must be an integer; $\sqrt{4 \cdot a}$ must be the <u>square</u> <u>root</u> of some <u>integer</u>.

(2) If a and b are different positive integers, $\sqrt{a} + \sqrt{b}$ is <u>not</u> equal to $\sqrt{a + b}$. If $\sqrt{a}$ and $\sqrt{b}$ are irrational, their sum can only be written in the form $\sqrt{a} + \sqrt{b}$.

(3) If a and b are positive integers, then $\sqrt{a} \cdot \sqrt{b} = \sqrt{a \cdot b}$. You can see this if you replace a and b with perfect squares.

$$\sqrt{4} \cdot \sqrt{9} = 2 \cdot 3 = 6 \quad \text{and} \quad \sqrt{4 \cdot 9} = \sqrt{36} = 6$$

1. Find this product.

$$\sqrt{7} \cdot \sqrt{11} = \sqrt{\underline{7} \cdot \underline{11}}$$

$$\sqrt{7} \cdot \sqrt{11} = \underline{\sqrt{\quad\quad}}$$

2. Find this product.

$$\sqrt{5} \cdot \sqrt{7} = \sqrt{5 \cdot \underline{\quad}}$$

$$= \underline{\sqrt{\quad\quad}}$$

3. Find this product.

$$\sqrt{6} \cdot \sqrt{8} = \sqrt{\underline{\quad} \cdot \underline{\quad}}$$

$$= \underline{\sqrt{\quad\quad}}$$

Exercises

1. Write yes if the sum is named on the number line on page 16-6 and no if it is not. Use your compass to help you.

(a) $\sqrt{8} + \sqrt{11}$ ______

(b) $\sqrt{6} + \sqrt{6}$ ______

(c) $\sqrt{12} + \sqrt{14}$ ______

(d) $\sqrt{5} + \sqrt{5}$ ______

(e) $\sqrt{3} + \sqrt{3} + \sqrt{3}$ ______

2. Write these sums. Use the number line on page 16-6 and your compass to help you.

(a) $\sqrt{6} + \sqrt{6} =$ ______

(b) $\sqrt{7} + \sqrt{7} =$ ______

(c) $\sqrt{3} + \sqrt{3} + \sqrt{3} =$ ______

(d) $\sqrt{8} + \sqrt{8} =$ ______

(e) $\sqrt{5} + \sqrt{5} =$ ______

3. Find these products.

(a) $\sqrt{2} \cdot \sqrt{7} =$ ______

(b) $\sqrt{5} \cdot \sqrt{10} =$ ______

(c) $\sqrt{3} \cdot \sqrt{20} =$ ______

(d) $\sqrt{3} \cdot \sqrt{7} =$ ______

(e) $\sqrt{6} \cdot \sqrt{4} =$ ______

(f) $\sqrt{5} \cdot \sqrt{8} =$ ______

(g) $\sqrt{8} \cdot \sqrt{7} =$ ______

(h) $\sqrt{9} \cdot \sqrt{3} =$ ______

Simplifying Radicals

Class Discussion

In the last lesson you learned that if "a" and "b" are positive integers then

$$\sqrt{a} \cdot \sqrt{b} = \sqrt{a \cdot b} .$$

You saw this when you multiplied $\sqrt{4} \cdot \sqrt{3} = \sqrt{4 \cdot 3} = \sqrt{12}$.

Suppose we work backward.

$$\sqrt{12} = \sqrt{4 \cdot 3} = \sqrt{4} \cdot \sqrt{3} = 2\sqrt{3}$$

$2\sqrt{3}$ is considered a simplified form of $\sqrt{12}$.

Here is another example. We will simplify $\sqrt{32}$.

$$\sqrt{32} = \sqrt{16 \cdot 2}$$

$$\sqrt{32} = \sqrt{16} \cdot \sqrt{2}$$

But $\sqrt{16}$ has an integer name. What is this name? ________

So: $\sqrt{32} = 4\sqrt{2}$

In this example we will simplify $\sqrt{27}$.

$$\sqrt{27} = \sqrt{9 \cdot 3}$$

$$\sqrt{27} = \sqrt{9} \cdot \sqrt{3}$$

$$\sqrt{27} = 3 \cdot \sqrt{3}$$

A square root can be simplified if it has a perfect square as a factor. Remember that 4 , 9 , 16 , 25 , 36 , 49 , 64 , 81 , 100 , etc., are the perfect squares.

To simplify $\sqrt{18}$ first factor 18 into a perfect square times some other integer.

$$\sqrt{18} = \sqrt{\_\_\_ \cdot \_\_\_}$$

Write this as the product of two radicals (square roots).

$$\sqrt{18} = \sqrt{\_\_\_} \cdot \sqrt{\_\_\_}$$

Now write this as the product of an integer and an irrational number.

$$\sqrt{18} = \_\_\_ \cdot \sqrt{\_\_\_}$$

The simplified form of $\sqrt{18}$ is $3\sqrt{2}$.

Exercises

1. Simplify these irrational numbers by filling the blanks.

(a) $\sqrt{27} = \sqrt{\_\_\_ \cdot 3}$

$\sqrt{27} = \sqrt{\_\_\_} \cdot \sqrt{3}$

$\sqrt{27} = \_\_\_ \cdot \sqrt{3}$

(b) $\sqrt{28} = \sqrt{\_\_\_ \cdot 7}$

$\sqrt{28} = \sqrt{\_\_\_} \cdot \sqrt{7}$

$\sqrt{28} = \_\_\_ \cdot \sqrt{7}$

(c) $\sqrt{50} = \sqrt{\_\_\_ \cdot 2}$

$\sqrt{50} = \sqrt{\_\_\_} \cdot \sqrt{2}$

$\sqrt{50} = \_\_\_ \cdot \sqrt{2}$

(d) $\sqrt{45} = \sqrt{\_\_\_ \cdot 5}$

$\sqrt{45} = \sqrt{\_\_\_} \cdot \sqrt{5}$

$\sqrt{45} = \_\_\_ \cdot \sqrt{5}$

(e) $\sqrt{54} = \sqrt{\_\_\_ \cdot 6}$

$\sqrt{54} = \sqrt{\_\_\_} \cdot \sqrt{6}$

$\sqrt{54} = \_\_\_ \cdot \sqrt{6}$

(f) $\sqrt{40} = \sqrt{\_\_\_ \cdot 10}$

$\sqrt{40} = \sqrt{\_\_\_} \cdot \sqrt{10}$

$\sqrt{40} = \_\_\_ \cdot \sqrt{10}$

(g) $\sqrt{75} = \sqrt{\_\_\_ \cdot 3}$

$\sqrt{75} = \sqrt{\_\_\_} \cdot \sqrt{3}$

$\sqrt{75} = \_\_\_ \cdot \sqrt{3}$

(h) $\sqrt{48} = \sqrt{\_\_\_ \cdot 3}$

$\sqrt{48} = \sqrt{\_\_\_} \cdot \sqrt{3}$

$\sqrt{48} = \_\_\_ \cdot \sqrt{3}$

2. Multiply these irrational numbers. The first one is done for you.

(a) $\sqrt{3} \cdot \sqrt{5} = \sqrt{3 \cdot 5}$

$\sqrt{3} \cdot \sqrt{5} = \sqrt{15}$

(b) $\sqrt{6} \cdot \sqrt{7} = \sqrt{\_\_\_ \cdot \_\_\_}$

$\sqrt{6} \cdot \sqrt{7} = \_\_\_\_\_\_$

(c) $\sqrt{8} \cdot \sqrt{11} = \sqrt{\_\_\_ \cdot \_\_\_}$

$\sqrt{8} \cdot \sqrt{11} = \_\_\_\_\_\_$

(d) $\sqrt{10} \cdot \sqrt{3} = \sqrt{\_\_\_ \cdot \_\_\_}$

$\sqrt{10} \cdot \sqrt{3} = \_\_\_\_\_\_$

(e) $\sqrt{2} \cdot \sqrt{8} = \sqrt{\_\_\_ \cdot \_\_\_}$

$\sqrt{2} \cdot \sqrt{8} = \_\_\_\_\_\_$

Give the integer name for this number. ______

(f) $\sqrt{3} \cdot \sqrt{12} = \sqrt{\_\_\_ \cdot \_\_\_}$

$\sqrt{3} \cdot \sqrt{12} = \_\_\_\_\_\_$

Give the integer name for this number. ______

(g) $\sqrt{2} \cdot \sqrt{32} = \sqrt{\_\_\_ \cdot \_\_\_}$

$\sqrt{2} \cdot \sqrt{32} = \_\_\_\_\_\_$

Give the integer name for this number. ______

Subtraction of Real Numbers

We have always subtracted in the same way. With the integers we learned to rewrite the problem as an addition problem. We added the opposite of the subtrahend. We subtracted rational numbers in the same way. Since the real numbers include the rational numbers which also include the integers we must not change the rules for subtraction. We subtract the irrational numbers in the usual way.

In order to do this we need to locate the opposites of the irrational numbers on our number line.

Class Discussion

Take page 16-8a out of your notebook.

1. Place the needle point of your compass at zero and the pencil point at $\sqrt{2}$.
2. Without changing the setting of your compass swing an arc that crosses the line to the left of zero. This is the location of $^{-}\sqrt{2}$.
3. Repeat this process to locate $^{-}\sqrt{3}$, $^{-}\sqrt{5}$, $^{-}\sqrt{6}$, $^{-}\sqrt{7}$, $^{-}\sqrt{8}$, $^{-}\sqrt{10}$.
4. Page 16-8b is a picture of your number line. Locate and label the following points on this picture. Use your compass. Remember to rewrite the problem as addition.

(a) Point A

$\sqrt{5} - \sqrt{2}$

= ____ + ____

(b) Point B

$\sqrt{10} - \sqrt{3}$

= ____ + ____

(c) Point C

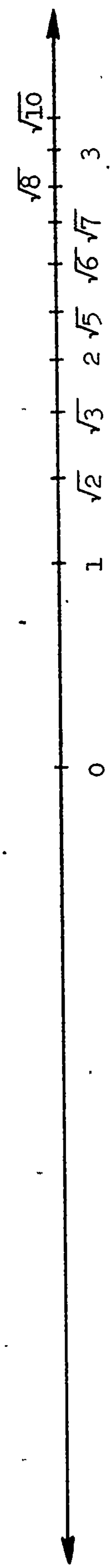
$\sqrt{7} - \sqrt{10}$

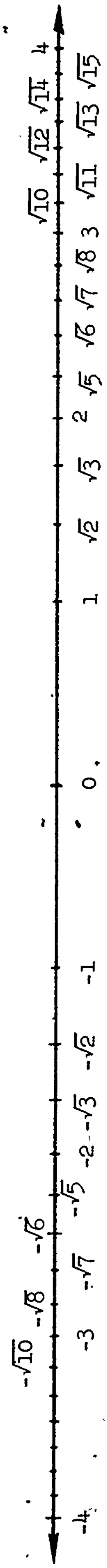
= ____ + ____

(d) Point D

$\sqrt{2} - \sqrt{6}$

= ____ + ____


0
1
$\sqrt{2}$
$\sqrt{3}$
2
$\sqrt{5}$
$\sqrt{6}$
$\sqrt{7}$
$\sqrt{8}$
3
$\sqrt{10}$


-4
-3
-√10
-√8
-√7
-√6
-√5
-2
-√3
-√2
-1
0
1
√2
√3
2
√5
√6
√7
√8
3
√10
√11
√12
√13
√14
√15
4

Notice that the points you marked have not been named. These points represent new irrational numbers whose names are simply $(\sqrt{5} - \sqrt{2})$, $(\sqrt{10} - \sqrt{3})$, $(\sqrt{7} - \sqrt{10})$, $(\sqrt{2} - \sqrt{6})$, etc.
From this you can see that we have not named all the points for the irrational numbers. In fact, this would be impossible.

5. The $\sqrt{4}$ is rational. As you know it is 2 . On the number line what is the opposite of $\sqrt{4}$? $^-\sqrt{4}$ = ______

6. $\sqrt{4} = 2$ because $2^2 = 4$. In step (5) you found that $^-\sqrt{4} = {}^-2$. What is $({}^-2)^2$? That is $({}^-2) \cdot ({}^-2)$ = ______ so, $({}^-\sqrt{4})^2 = 4$. (Remember that the product of two negative numbers is positive.)

7. From step (6) you see that there are two numbers whose square is 4 . $2^2 = 4$ and $({}^-2)^2 = 4$. You have found that 4 has two square roots.

8. (a) Does $({}^-3)^2 = 9$? ______

   (b) What are the two square roots of 9 ? ______ and ______

9. What are the two square roots of 16 ? ______ and ______
   of 25 ? ______ and ______

All positive real numbers have two square roots. But what about the negative real numbers? Let's see if we can find a number whose square is $^-4$. Two reasonable choices for this number $\sqrt{^-4}$ are either 2 or $^-2$. But this cannot be true because $2^2 = 4$ and $({}^-2)^2 = 4$. There is no real number whose square is $^-4$. In fact there are no real numbers that are the square roots of any of the negative real numbers.

$\sqrt{^-4}$ is not real

$\sqrt{^-5}$ is not real

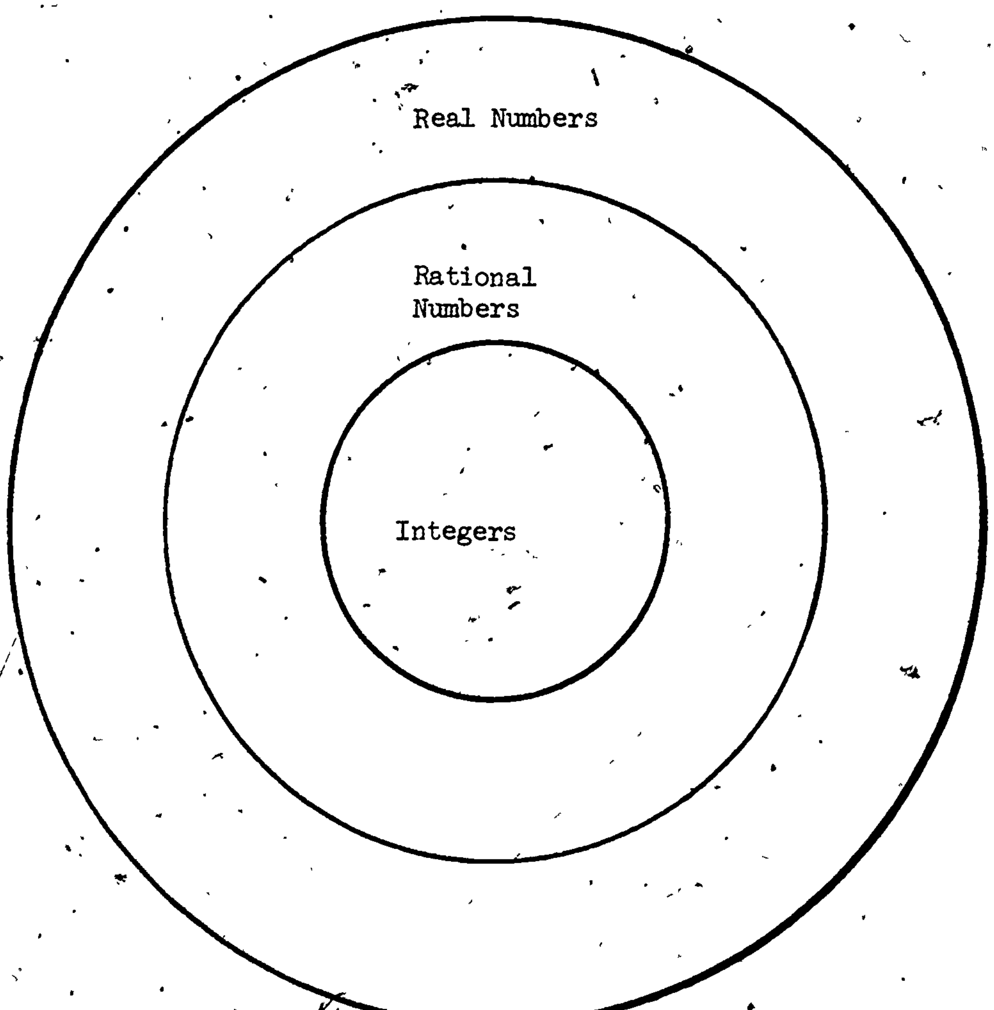
$\sqrt{^-9}$ is not real

$\sqrt{^-12}$ is not real

Exercises

Below is a list of numbers. Under this list is a diagram that shows that integers are also rational numbers and real numbers, and that rational numbers are also real numbers. Sort out the numbers in the list by writing them in the correct circle.

$\frac{2}{3}$	$\frac{1}{2}$	$^{-}\sqrt{9}$
5	$\sqrt{8}$	$\sqrt{2} - \sqrt{10}$
$\frac{3}{8}$	$\sqrt{5}$	$\frac{5}{32}$
$\frac{16}{4}$	36	$\frac{3}{^{-}4}$
$^{-}27$	$\sqrt{9}$	



Reciprocals of Real Numbers

You have seen that division means to multiply by the reciprocal of the divisor. In order to divide square roots we will need to locate their reciprocals on the number line.

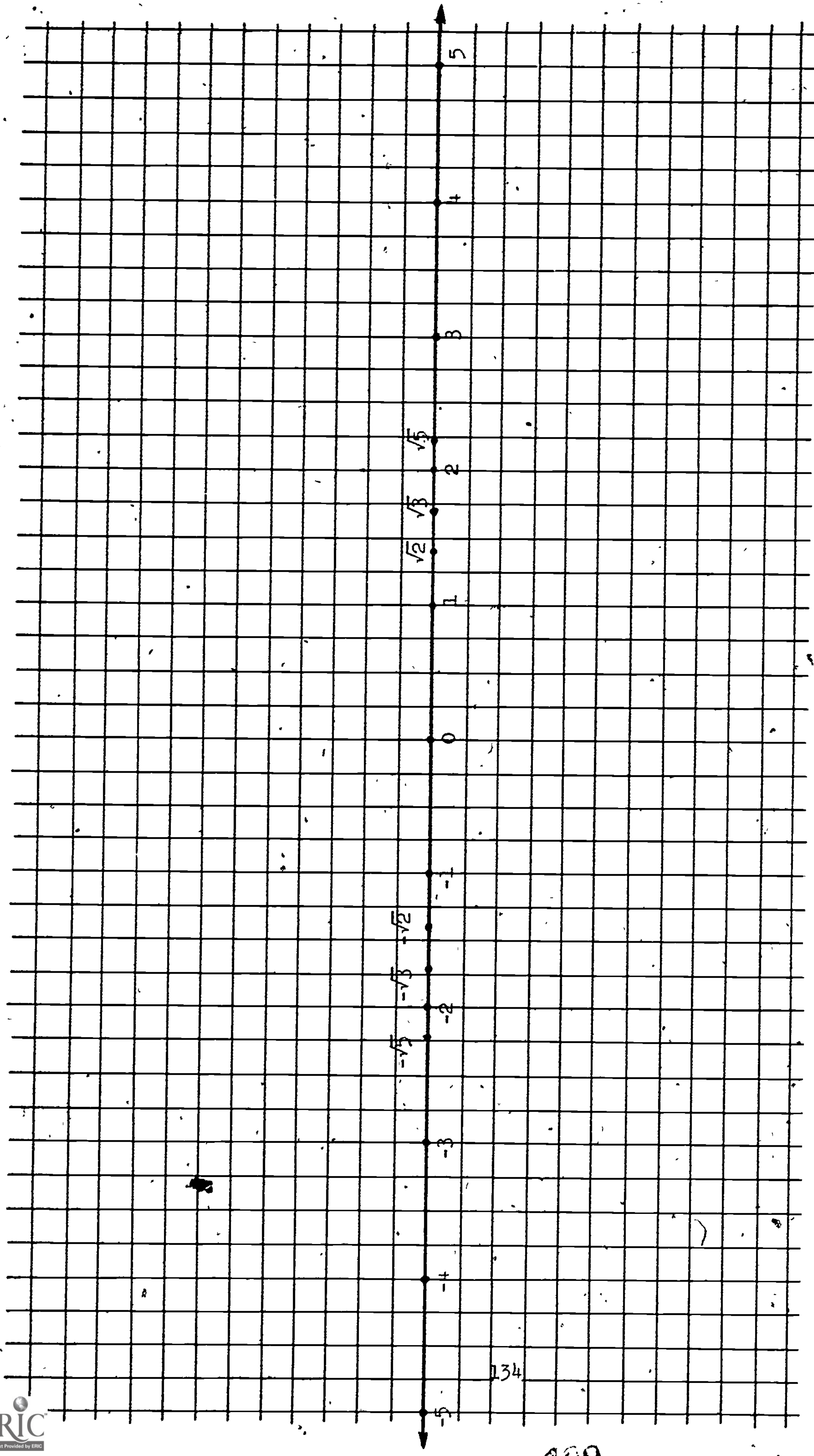
Class Discussion

On page 16-9a there is a picture of a real number line. We will locate some of the reciprocals of irrational numbers on this line.

You know that the reciprocal of 2 is $\frac{1}{2}$. In the same way the reciprocal of $\sqrt{2}$ is $\frac{1}{\sqrt{2}}$. Since rational numbers are also real numbers the product of reciprocals must still be 1. So

$$\sqrt{2} \cdot \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{\sqrt{2}} = 1 .$$

Our problem is to find the location of $\frac{1}{\sqrt{2}}$ on the number line. We can use the fact that $\frac{\sqrt{2}}{\sqrt{2}} = 1$ to help us.


5
4
3
√5
2
√3
√2
1
0
-1
-√2
-√3
-2
-√5
-3
-4
-5

Does it change a number if we multiply it by one? ________

Suppose we multiply $\frac{1}{\sqrt{2}}$ by $\frac{\sqrt{2}}{\sqrt{2}}$. Will this change the number? _____

We multiply these irrational numbers in the same way we multiply rational numbers.

That is:

$$\frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{\sqrt{2} \cdot \sqrt{2}}$$

The integer name for $\sqrt{2} \cdot \sqrt{2}$ is ________.

So:

$$\frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

This tells us that $\frac{1}{\sqrt{2}}$ is located half way between zero and $\sqrt{2}$. All we have to do is bisect this segment. Use your compass and straightedge to find the midpoint of this segment on page 16-6a. Mark this midpoint $\frac{1}{\sqrt{2}}$.

Now let's locate $\frac{1}{\sqrt{3}}$ which is the reciprocal of $\sqrt{3}$. Again we can use multiplication by one to help us.

$$\frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{\sqrt{3} \cdot \sqrt{3}}$$

$$\sqrt{3} \cdot \sqrt{3} = \_\_\_\_\_\_.$$

So:

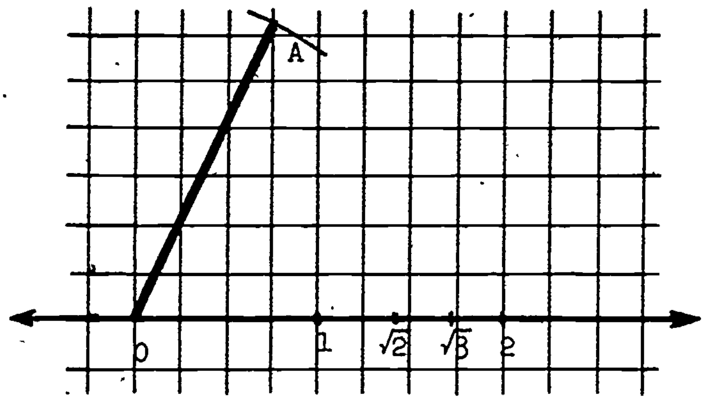
$$\frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$

$\frac{\sqrt{3}}{3}$ is $\frac{1}{3}$ the distance from zero to $\sqrt{3}$.

In Chapter 12 (Similarity) you saw that you can divide any segment into any number of parts using parallel lines.

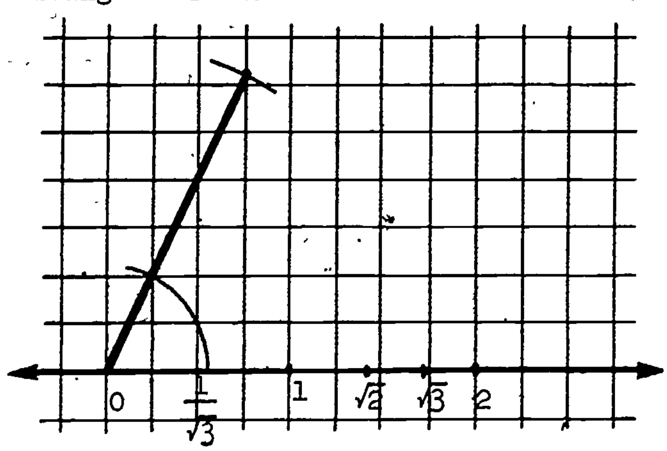
1. On the number line (p. 16-9a) set the needle point of your compass at zero and the pencil point at $\sqrt{3}$.

2. Count over 3 lines past zero. Swing an arc that intersects this line. Label this intersection point A and draw the segment from zero to A like this.



The vertical lines on the grid are parallel and equally spaced.

3. The segment from zero to A is divided into 3 equal parts by these parallel lines. Set your compass at zero and at the point where the segment crosses the first vertical line. Swing an arc down to the number line like this.



We have just located the reciprocal of $\sqrt{3}$.

Since $$\frac{1}{\sqrt{5}} = \frac{1}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{\sqrt{5}}{5}$$

we could locate $\frac{1}{\sqrt{5}}$ on the number line by a similar process to finding $\frac{1}{\sqrt{3}}$. In fact we could locate the reciprocals of all the square roots of integers by this method.

The process of rewriting $\frac{1}{\sqrt{5}}$ as $\frac{\sqrt{5}}{5}$ is called <u>rationalizing the denominator</u>.

We can use multiplication by 1 to rationalize the denominator of any ratio of square roots. Here are several examples.

$$\frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$\frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$

$$\frac{1}{\sqrt{7}} \cdot \frac{\sqrt{7}}{\sqrt{7}} = \frac{\sqrt{7}}{7}$$

$$\frac{\sqrt{5}}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{15}}{3}$$

$$\frac{\sqrt{8}}{\sqrt{7}} \cdot \frac{\sqrt{7}}{\sqrt{7}} = \frac{\sqrt{56}}{7}$$

etc.

Exercises

Follow these steps to locate the reciprocal of $\sqrt{5}$ on the number line at the bottom of this page.

1. Rationalize $\frac{1}{\sqrt{5}}$

$$\frac{1}{\sqrt{5}} = \frac{1}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \underline{\qquad}.$$

2. How many equal parts is the segment from 0 to $\sqrt{5}$ to be divided into? __________.

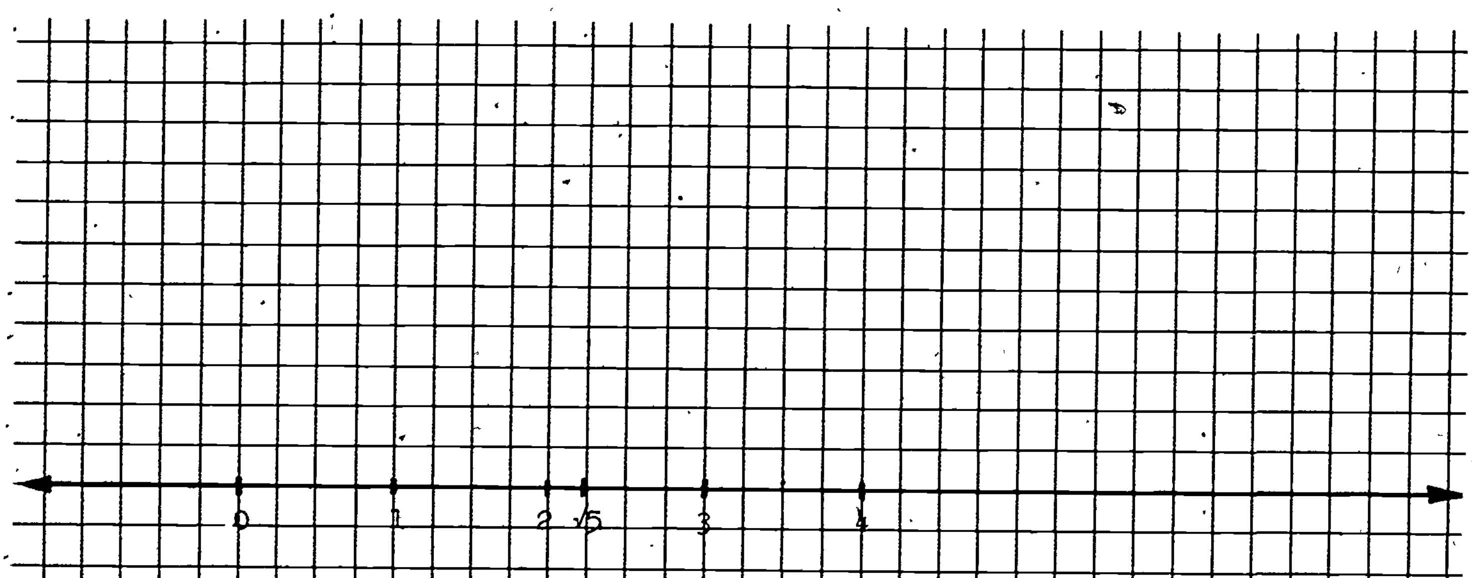
3. Set your compass at zero and $\sqrt{5}$ on the number line below.

4. Count over 5 lines from zero and swing an arc that intersects this line. (Do not change the compass setting.) Label this point P.

5. Draw the segment from zero to P.

6. The vertical lines of the grid divide this segment into fifths. Set your compass to $\frac{1}{5}$ the length of the segment from zero to P.

7. Mark off this length from zero on the number line. Label this point $\frac{1}{\sqrt{5}}$.



8. Rationalize the denominator in each fraction.

(a) $\frac{1}{\sqrt{7}}$ = ________________

(b) $\frac{\sqrt{3}}{\sqrt{2}}$ = ________________

(c) $\frac{2}{\sqrt{3}}$ = ________________ (Caution: The numerator here is an <u>integer</u>.)

(d) $\frac{5}{\sqrt{6}}$ = ________________

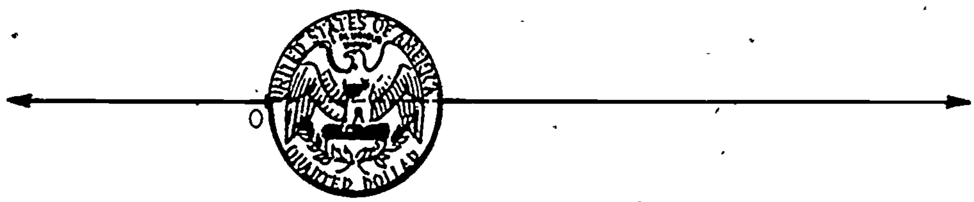
(e) $\frac{\sqrt{6}}{\sqrt{5}}$ = ________________

(f) $\frac{4}{\sqrt{7}}$ = ________________

π and the Circumference of a Circle

Class Discussion

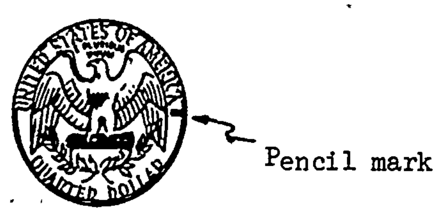
Get out a coin and carefully lay it on the number line at the bottom of this page so that one edge is at zero and the line passes under the center of the coin like this.



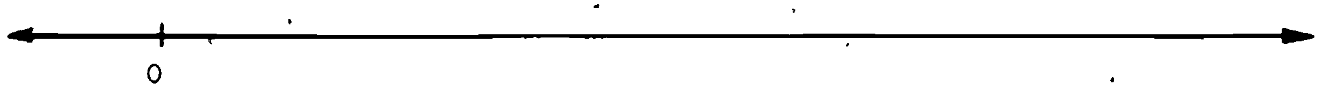
Place the needle point of your compass on the number line and against the edge of the coin. Punch a tiny hole in your paper. We will use the distance from zero to the hole as the unit for the number line, so mark the point at the hole and label it 1 .

Carefully set your compass at zero and 1 and use it to locate the points for 2 , 3 , and 4 . Label these points.

Make a small pencil mark on the coin on the edge of the coin like this.



Place the coin on edge on the number line so that the mark is located at zero. Roll the coin along the line without slipping until the mark again touches the line. Mark this point on the line. This point is a little to the right of ________.



Check with your classmates who used a different kind of coin. Did they come to about the same number on their number line? ____

You probably know that the distance across a circle through the center is called the <u>diameter</u> of the circle. The symbol for diameter is D . The distance from the center to the circle is called the <u>radius</u>. It is $\frac{1}{2}$ the length of the diameter. The symbol for radius is r . The length of the circle (distance around it) is called the <u>circumference</u> of the circle. The symbol for circumference is C .

On the number line on page 16-10 you used the ____________ of the coin as your unit. The circumference of the circle is a little more than ____________ times longer than the diameter. This is true for <u>all</u> circles no matter how long their diameter is.

The number you came to in your experiment with the coin is actually irrational. Its name is π (pronounced pie). From what you learned in your experiment we can write

$$c = \pi \cdot D$$

(circumference = π times diameter)

The number π is very close to the rational numbers 3.14 and $\frac{22}{7}$. For computing the circumference of a circle you can use 3.14 or $\frac{22}{7}$ as an approximation for π . To get a <u>rough</u> idea of the circumference you can simply multiply 3 times the diameter.

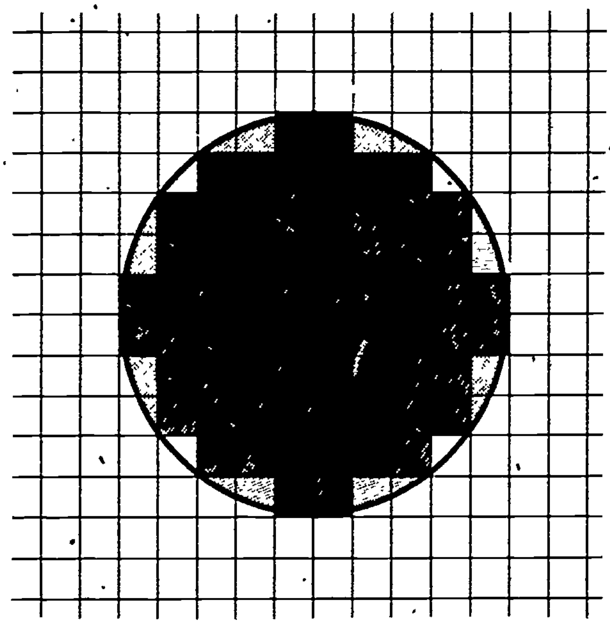
Exercises

For circles with the diameters given below first multiply by 3 to get a rough estimate of the circumference and then multiply by 3.14 to get a closer approximation of the circumference. The first one is done for you.

	D (diameter)	Rough estimate of C	Close approximation of C
1.	4 ft.	12 ft.	12.56 ft.
2.	5 in.	____ in.	____ in.
3.	8 cm	____ cm	____ cm
4.	7 yds.	____ yds.	____ yds.
5.	2.6 in.	____ in.	____ in.
6.	3.2 in.	____ in.	____ in.

Area of a Circle

Look at this circle.



The radius of the circle is 5 units long.

Each of the squares is a unit square. You can count these unit squares and find out about what the area of the circle is.

Class Discussion

Count the unit squares that are shaded dark. How many of these squares are there? _____ Write this number in the table on the next page.

Look at one of the regions that is shaded light. The circle cuts through 2 unit squares. If you put the two parts together you get about one square. So each lightly shaded region has an area of about one square. How many of these regions are there? _________ So about how many unit squares does this make? _________ Write this answer in the table.

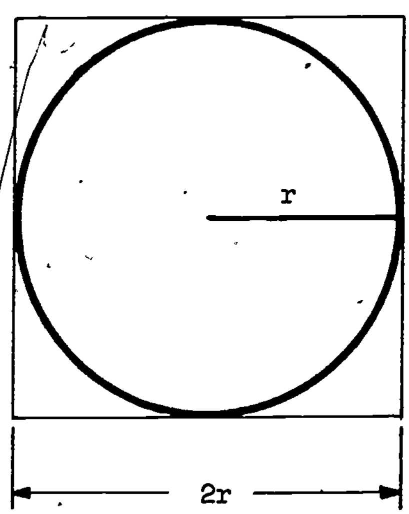
Now look at the unshaded regions. Each of the unshaded regions is about $\frac{1}{2}$ of a unit square. How many of these regions are there? ________ So about how many unit squares does this make? ________ Write this answer in the table.

Number of dark squares is	____
Number of light squares is about	____
Number of unshaded squares is about	____
The area of the circle is about	____

Add the number of squares in the table to find the approximate area of the circle.

Now let's see if we can find a way to figure out the area of the circle without counting the squares.

Here is a picture of the same circle. This time we have put the circle inside of a large square.



The length of the side of the square is 2 times the radius of the circle.

Area of square = $(2r)^2$

Area of square = $2r \cdot 2r$

Area of square = $2 \cdot 2 \cdot r \cdot r$

Area of square = $4r^2$

You can see that the area of this square is 4 times the square of the radius of the circle.

Remember that the radius of the circle is 5 units long. Find the area of this square.

Area of this square = $4r^2$

Area of this square = $4 \cdot (\quad)^2$

Area of this square = $4 \cdot$ ____

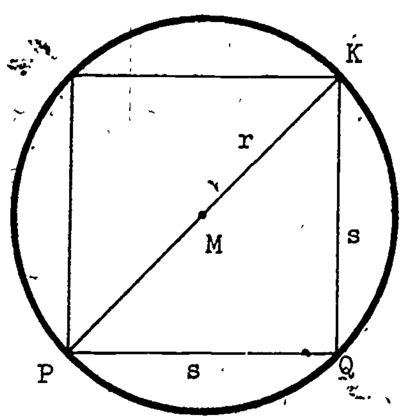
Area of this square = ____ square units

Remember that the area of the circle is about 78 square units.

Is the area of this square more or less than the area of the circle? ________

Let's try a smaller square.

Here is a picture of the same circle with a square inside.

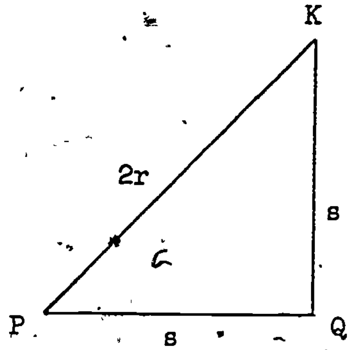


M is the center of the circle. $\overline{PK}$ is the diagonal of the square.

$\overline{KM}$ is the radius of the circle, so diagonal $\overline{PK}$ is 2 times the length of the radius of the circle.

$$m\ \overline{PK} = 2r$$

Here is triangle PQK taken out of the circle.



Triangle PQK is part of the square so $\overline{PQ}$ and $\overline{QK}$ are the same length. We don't yet know the length of $\overline{PQ}$ and $\overline{QK}$ so we use "s" for this length. "s" is the length of the side of the square so the area of the square is s^2. This is what we want to find.

The triangle is a right triangle.

So:

$$s^2 + s^2 = (2r)^2$$

$$s^2 + s^2 = 2r \cdot 2r$$

$$s^2 + s^2 = 2 \cdot 2 \cdot r \cdot r$$

$$s^2 + s^2 = 4r^2$$

$$2s^2 = 4r^2$$

$$\frac{1}{2} \cdot 2 \cdot s^2 = \frac{1}{2} \cdot 4 \cdot r^2$$

$$s^2 = 2r^2$$

But s^2 is the area of the square,

so: area of this square $= 2r^2$

The area of this square is 2 times the square of the radius of the circle.

Remember that the radius of the circle is 5 units long. Find the area of this square.

Area of this square = $2r^2$

Area of this square = $2 \cdot$ __( )^2__

Area of this square = $2 \cdot$ ____

Area of this square = _______ square units

Remember that the area of the circle is about 78 square units.

Is the area of this square more or less than the area of the circle? _______

Here is what we have learned so far:

(1) If we use $4r^2$ to find the area of a circle the answer is too big.

(2) If we use $2r^2$ to find the area of a circle the answer is too small.

so (3) The area of the circle is between $2r^2$ and $4r^2$.

Lets try $3r^2$ and see if we come close to the area of the circle. Is the area of the circle equalto $3r^2$?

$3r^2 = 3 \cdot$ __( )^2__

$= 3 \cdot$ _____

= _______ square units

Is your answer close to the 78 square units you got by counting? _______

Is your answer too big or too small? _______

Is your answer very much too small? _______

The area of a circle is a little bit more than $3r^2$. What number do you know, that is used with circles, and is a little more than 3 ? _______

Area of a circle = πr^2

Use this formula to find the area of the circle with radius 5 units. (use $\pi \approx 3.14$)

$$A_{(circle)} = \pi r^2$$

$$A_{(circle)} \approx 3.14 \cdot (\_\_)^2$$

$$A_{(circle)} \approx 3.14 \cdot \_\_\_\_$$

$$A_{(circle)} \approx \_\_\_\_ \text{ square units}$$

This answer is very close to the approximate area of 78 square units that you got by counting.

Example 1. Suppose we find the area of a circle whose radius is 4 inches. For computation we will use $\pi \approx 3.14$.

$$A = \pi r^2$$

$$A \approx 3.14 \cdot (4)^2$$

$$A \approx 3.14 \cdot (16)$$

$$A \approx 50.24 \text{ sq. in.}$$

Example 2. Find the area of a circle whose radius is 7 cm. (use $\pi \approx \frac{22}{7}$)

$$A = \pi r^2$$

$$A \approx \frac{22}{7} \cdot (7)^2$$

$$A \approx \frac{22 \cdot 7 \cdot 7}{7}$$

$$A \approx 22 \cdot \frac{7}{7} \cdot 7 \quad \text{(Remember } \frac{7}{7} = 1 \text{ .)}$$

so: $A \approx 22 \cdot 7$

$$A \approx 154 \text{ square cm}$$

Exercises

Use the examples on page 16-11e to work these problems.

1. Find the area of a circle whose radius is 6 feet. (use $\pi \approx 3.14$)

$$A = \pi r^2$$

$A \approx$ ________ sq. ft.

2. Find the area of a circle whose radius is 14 centimeters. (use $\pi \approx \frac{22}{7}$)

$$A = \pi r^2$$

$A \approx$ ________ sq. cm.

3. Find the area of a circle whose radius is 12 inches. (use $\pi \approx 3.14$)

$$A = \pi r^2$$

$A \approx$ ________ sq. in.

4. (a) Find the radius of a circle whose diameter is 42 inches.

$$r = \frac{1}{2} D$$

r = ____ inches.

(b) Find the area of this circle. (use $\pi \approx \frac{22}{7}$)

A ≈ ______ sq. in.

Pre-Test Exercises

1. (Section 16-3.)

Draw a circle around the square roots that are integers.

(a) $\sqrt{5}$ (e) $\sqrt{49}$

(b) $\sqrt{16}$ (f) $\sqrt{12}$

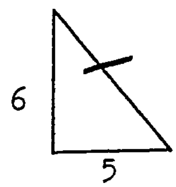
(c) $\sqrt{1}$ (g) $\sqrt{20}$

(d) $\sqrt{8}$ (h) $\sqrt{36}$

2. (Section 16-4.)

Find the length of the hypotenuse of each of these right triangles.

(a)



$a^2 + b^2 = c^2$

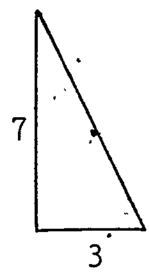
$\_\_\_^2 + \_\_\_^2 = c^2$

$\_\_\_ + \_\_\_ = c^2$

$\_\_\_ = c^2$

$\_\_\_ = c$

(b)



$a^2 + b^2 = c^2$

$\_\_\_^2 + \_\_\_^2 = c^2$

$\_\_\_ + \_\_\_ = c^2$

$\_\_\_ = c^2$

$\_\_\_ = c$

3. (Section 16-6.)

Circle the correct answer.

$\sqrt{5} + \sqrt{11} \doteq ?$

(a) $\sqrt{55}$

(b) $\sqrt{5 + 11}$

(c) $\sqrt{16}$

(d) $\sqrt{100}$

(e) none of these

4. (Section 16-6.)

Circle the correct answer.

$\sqrt{5} \cdot \sqrt{11} = ?$

(a) $\sqrt{55}$

(b) $\sqrt{5 + 11}$

(c) $\sqrt{16}$

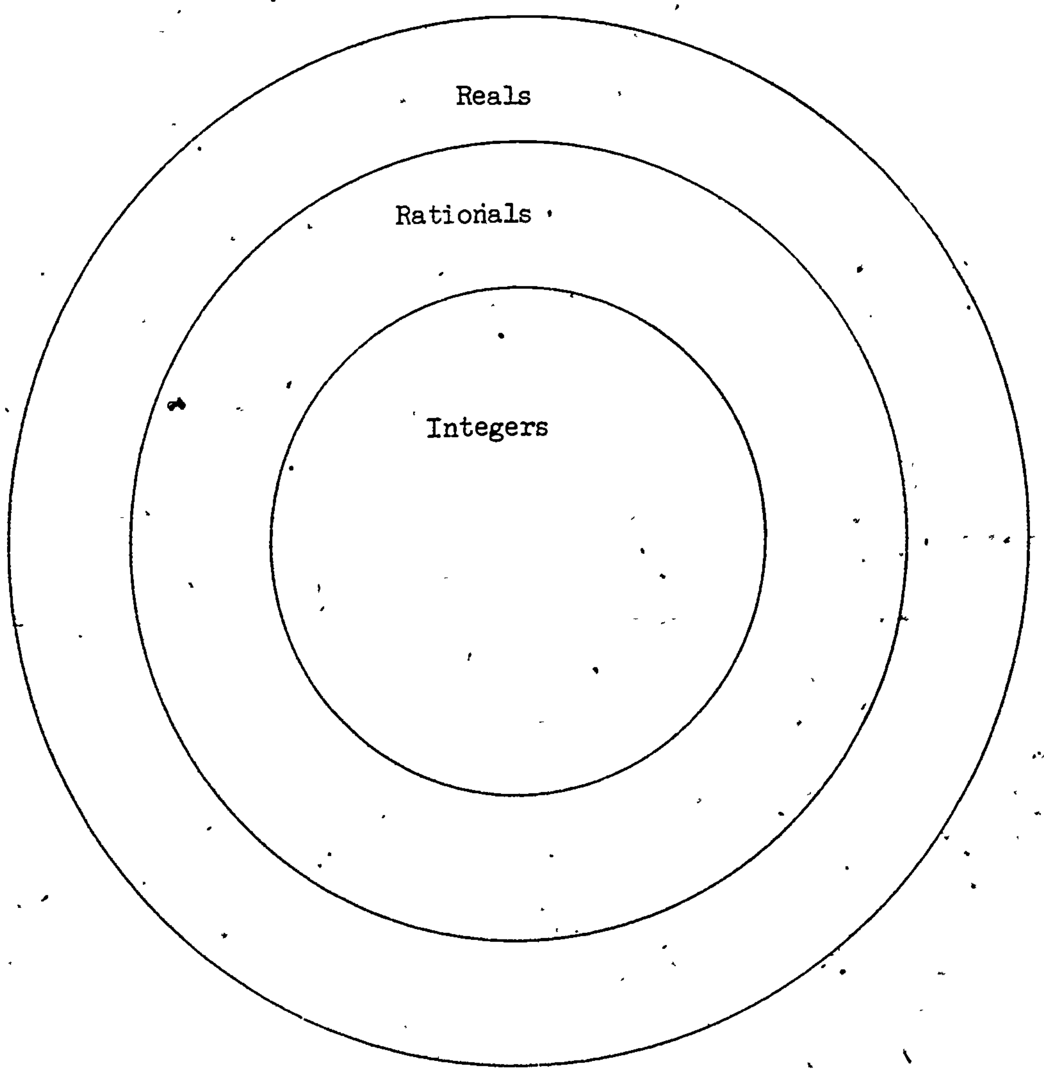
(d) $\sqrt{100}$

(e) none of these

5. (Section 16-8.)

Sort out the numbers in this list by writing them in the correct circle.

π	$\sqrt{8}$	$^{-}17$
$\frac{4}{5}$	$^{-}\sqrt{8}$	$\sqrt{5} + \sqrt{10}$
241	$\sqrt{25}$	



6. (Section 16-9.)

Rationalize these denominators

(a) $\frac{\sqrt{5}}{\sqrt{2}} =$

(b) $\frac{1}{\sqrt{7}} =$

7. (Section 16-7.)

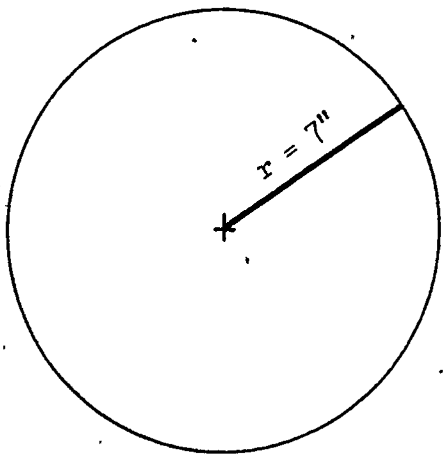
Simplify by factoring out a perfect square.

(a) $\sqrt{20}$

(b) $\sqrt{75}$

8. (Section 16-11.)

Find the area of this circle. (use $\pi \approx \frac{22}{7}$)



$A = \pi \cdot r^2$

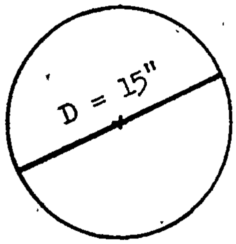
$A \approx$ ____ $\cdot$ ____^{2}

$A \approx$ ____ $\cdot$ ____

$A \approx$ ________ sq. in.

9. (Section 16-10.)

Find the circumference of this circle. (use $\pi \approx 3.14$)



$C = \pi \cdot D$

$C \approx$ ____ $\cdot$ ____

$C \approx$ ________ in.

10. (Section 16-10.)

What is the radius of a circle whose diameter is 32 ft?

________________.

Test

1. Draw a circle around the square roots that are integers.

   (a) $\sqrt{37}$

   (b) $\sqrt{64}$

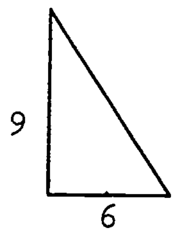
   (c) $\sqrt{18}$

   (d) $\sqrt{12}$

   (e) $\sqrt{49}$

   (f) $\sqrt{54}$

2. Find the length of the hypotenuse of this right triangle.



$$a^2 + b^2 = c^2$$

$$\underline{\qquad}^2 + \underline{\qquad}^2 = c^2$$

$$\underline{\qquad} + \underline{\qquad} = c^2$$

$$\underline{\qquad} = c^2$$

$$\underline{\qquad} = c$$

3. Circle the correct answer.

$$\sqrt{7} + \sqrt{10} = ?$$

   (a) $\sqrt{70}$

   (b) $\sqrt{7 + 10}$

   (c) $\sqrt{17}$

   (d) $\sqrt{14}$

   (e) none of these

4. Circle the correct answer.

$\sqrt{7} \cdot \sqrt{10} = ?$

(a) $\sqrt{17}$

(b) $\sqrt{42}$

(c) $\sqrt{70}$

(d) $\sqrt{7 + 10}$

(e) none of these

5. Simplify by factoring out a perfect square.

(a) $\sqrt{50}$ = ________

(b) $\sqrt{8}$ = ________

(c) $\sqrt{18}$ = ________

(d) $\sqrt{12}$ = ________

6. Rationalize these denominators.

(a) $\frac{\sqrt{7}}{\sqrt{3}}$ = ________

(b) $\frac{\sqrt{2}}{\sqrt{5}}$ = ________

(c) $\frac{3}{\sqrt{7}}$ = ________

(d) $\frac{5}{\sqrt{6}}$ = ________

7. If the diameter of a circle is 26 ft., what is the radius?

____

8. If the radius of a circle is 8 in., what is the diameter?

____

9. What is the area of a circle whose radius is 9 feet?
(use $\pi \approx 3.14$)

$A = \pi \cdot r^2$

$A \approx \_\_ \cdot \_\_^2$

$A \approx \_\_ \cdot \_\_$

$A \approx$ ________ square feet

10. What is the circumference of a circle whose diameter is 35 inches?
(use $\pi \approx \frac{22}{7}$)

$C = \pi \cdot D$

$C \approx \_\_ \cdot \_\_$

$C \approx$ ________ inches

Check Your Memory: Self-Test

1. (Section 15-9)

Fill the blanks.

(a) 15 m is the same as ______ mm

(b) 10 m is the same as ______ km

(c) 140 cm ~~is~~ the same as ______ mm

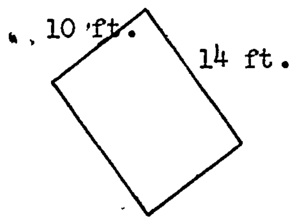
(d) 582 cm is the same as ______ m

(e) 2488 m is the same as ______ km

2. (Section 15-12)

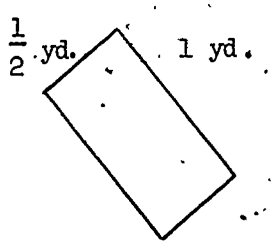
Find the area of the following.

(a)



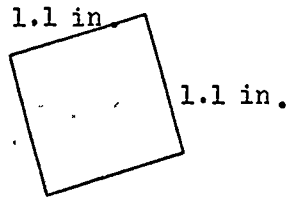
A: ______ sq. ft.

(b)



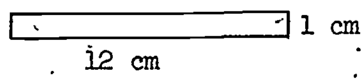
A: ______ sq. yd.

(c)



A: ______ sq. in.

(d)



A: ______ square cm

3. (Section 13-10)

Write the following using scientific notation.

(a) 965.013 = ________

(b) .00003 = ________

(c) 49.736 = ________

(d) .14826 = ________

(e) .0005368 = ________

4. (Section 13-8 and 13-9)

Multiply or divide. Write your answer using an exponent.

(a) $10^5 \times 10^{-3}$ = ________

(b) $1000 \times 10,000$ = ________

(c) $\frac{10000}{100}$ = ________

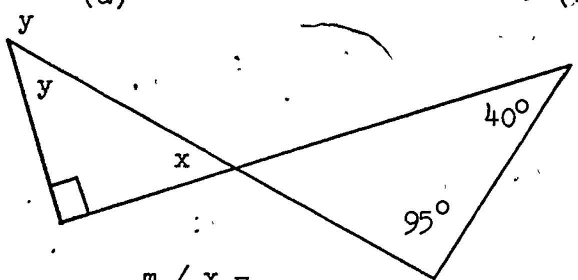
(d) $\frac{10^3}{10^{-2}}$ = ________

(e) $.00001 \times .0001$ = ________

5. (Sections 11-7 and 11-8)

Give the measures of $\angle x$ and $\angle y$ in each problem.

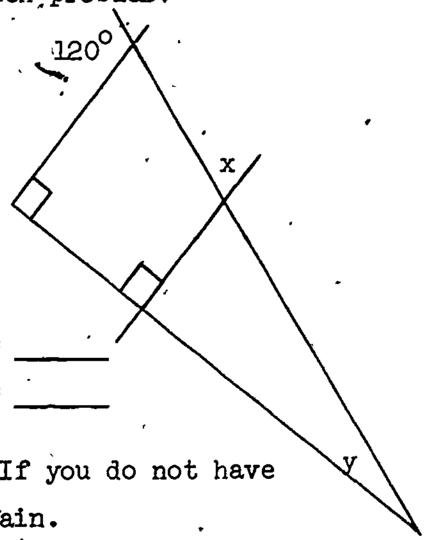
(a)



$m\angle x$ = ______

$m\angle y$ = ______

(b)



$m\angle x$ = ______

$m\angle y$ = ______

Now check your answers on the next page. If you do not have them all right, go back and read the section again.

Answers to Check Your Memory: Self-Test

1. (a) 15000

   (b) $\frac{1}{100}$ or .01

   (c) 1400

   (d) 5.82

   (e) 2.488

2. (a) 140 sq. ft.

   (b) $\frac{1}{2}$ sq. yd.

   (c) 1.21 sq. in.

   (d) 12 square cm

3. (a) 9.65013×10^2

   (b) 3×10^{-5}

   (c) 4.9736×10^1

   (d) 1.4826×10^{-1}

   (e) 5.368×10^{-4}

4. (a) 10^2

   (b) 10^7

   (c) 10^2

   (d) 10^5

   (e) 10^{-9}

5. (a) $m \angle x = 45$

   $m \angle y = 45$

   (b) $m \angle x = 60$

   $m \angle y = 30$