DOCUMENT RESUME

ED 110 361 95 SO 008 437

AUTHOR Greer-Wootten, Bryn; And Others
TITLE A Bibliography of Statistical Applications in Geography, Technical Paper No. 9.
INSTITUTION Association of American Geographers, Washington, D.C. Commission on College Geography.
SPONS AGENCY National Science Foundation, Washington, D.C.
PUB DATE 72
NOTE 99p.

EDRS PRICE MF-$0.76 HC-$4.43 PLUS POSTAGE
DESCRIPTORS *Bibliographies; College Instruction; *Educational Research; Educational Resources; Geographic Concepts; *Geography; *Geography Instruction; Higher Education; Instructional Materials; *Statistical Studies; Statistics

ABSTRACT

Included in this bibliography are resource materials available to both college instructors and students on statistical applications in geographic research. Two stages of statistical development are treated in the bibliography. They are 1) descriptive statistics, in which the sample is the focus of interest, and 2) analytical statistics, in which the population is the primary interest. For each of the sections, a short introductory statement is made concerning the general nature of problems investigated using that technique, where applicable. The bibliography treats 34 categories of geographic statistical concepts, such as measurement, set theory, geographic data, geography matrix, computer applications, sample designs and methods, descriptive statistics, index construction, analysis of variance, geostatistics, point pattern analysis, among others. Not included in the bibliography are references to sample space, expected values, random variables, population and sampling distributions. Entries are listed alphabetically by author, and include the title, source, number of pages, date, and place of publication. (Author/JR)

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A BIBLIOGRAPHY OF STATISTICAL APPLICATIONS IN GEOGRAPHY

by

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TECHNICAL PAPER NO. 9

Library of Congress Catalog Card Number 74-189379

Supported by a grant from the National Science Foundation

FOREWORD

The Technical Papers are explanatory manuals for the use of both instructors and students. They are expository presentations of available information on each subject designed to encourage innovation in teaching methods and materials. These Technical Papers are developed, printed, and distributed by the Commission on College Geography under the auspices of the Association of American Geographers with National Science Foundation support. The ideas presented in these papers do not necessarily imply endorsement by the AAG. Single copies are mailed free of charge to all AAG members.

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PREFACE

In developing this Bibliography, I have become aware of the tremendous explosion of statistical applications in almost all branches of geographic research. An original core listing expanded to almost four times its original size, reflecting an attempt on my part to include references from areas of research which I normally did not cover in teaching, as well as more recent work. The result has been a rather extensive coverage of many more topics than could be covered in the time available for a two-semester course. Perhaps the first twenty topics listed, together with simple correlation and regression, could be handled in such a time period. The remaining topics would then form background for advanced undergraduate or graduate courses.

A caveat should be entered on the utility of such a bibliography. In my experience, any listing of reference material is of limited value unless it is closely tied to a lecture series, *and* to exercises related to a particular topic. At various check-points in the development of the course it also seems valuable to ask for reading reports on a group of references, "bad" as well as "good," to validate student progress. In the early rush to print quantitatively oriented articles, it is now clear that some major problems in the application of statistical techniques to spatial data were not recognized. I feel it is important for the student to realize this, and therefore many of these early articles are included here. If these criticisms are placed in the correct perspective of the development of the use of such methods, as in Sections 1 - 3 in the Bibliography, then some degree of maturity may have been attained in this branch of the subject.

The original set of references was drawn up for use in a course entitled "Introduction to Geographic Theory and Quantitative Methods," to which students at McGill University, the University of California at Berkeley, and the Swiss Federal Institute of Technology at Zurich have been subjected over the years. For encouraging reactions to that earlier course outline, I am grateful to T. Lloyd, L. J. King, L. A. Brown, R. A. Murdie, M. F. Dacey, P. R. Gould, and G. Rushton. For help with the revision, I would like to thank several colleagues at York University. I. F. Owens, E. J. Spence, C. D. Morley, G. B. Norcliffe, and D. R. Ingram. Finally, I owe a great debt to Gillian Gilmour, not only for constructive criticism of this final version, but also for valuable aid in the development of that earlier set of materials, and to Les King, for his expert assistance in editing and revising an earlier draft.

INTRODUCTION: THE NATURE OF THE BIBLIOGRAPHY

In a recent review article, Krumbein (1968) recognizes three stages of statistical development: 1) descriptive statistics, in which the sample is the focus of interest, 2) analytical statistics, in which the population is the primary interest; and 3) the application of stochastic process models. In these terms, only the first two stages are treated in this Bibliography. Additionally, it is very important to realize that statistical techniques are only tools to the further understanding of substantive problems. The first three introductory sections of the Bibliography underline this statement.

Although we are primarily concerned with the application of various statistical techniques in geography, these efforts must be based securely on an understanding of probability theory. Notions of a sample space, expected values, random variables, population and sampling distributions, etc. are today usually learned by students before they register for any course in statistical methods within the geography department. They are therefore not treated in this Bibliography, although several general texts are listed in Section 4. (The reader is referred to the account of probability, and especially population distributions, given by Krumbein and Graybill, pp. 89–115, referenced in that section.) Probability concepts are also basic to an understanding of sampling theory (Section 11). Since the applicability of many statistical techniques depends intimately on the nature of the sample and on assumptions about the universe from which it was drawn, Section 11 assumes a major place in the Bibliography. The problems and benefits of sampling theory are generally neglected in geographic research, yet the applications of all the more powerful parametric tests depend on a sound sample design.

While a glance at the list of section headings will enable the reader to see what has been defined as "statistical applications" in this Bibliography, a statement concerning what has not been included may also be valuable, at least in indicating other major areas of vital research where quantitative techniques are important. As mentioned above, stochastic models have generally been excluded. The interested reader is referred to a long list of publications by Dacey on point processes as one avenue of approach in this field. (Contributions made by Dacey in the study of point patterns are reviewed by King, 1969, pp. 32–59 and 226–230, referenced in Section 4.) Simulation models such as those used in studying the diffusion of innovations (Hägerstrand, 1967, Brown, 1968), probability models using a Bayesian approach (Curry, 1966), and Markov Chain models (Brown, 1970) are similarly excluded. Optimization techniques are not covered, and the interested reader is referred to an introductory article on the use of linear programming (Cox, 1965), the review of dynamic programming (MacKinnon, 1970), and a recent book which emphasizes combinatorial programming (Scott, 1971). The Theory of Games is excluded (for example, Isard *et al*, 1970), as well as important advances in our knowledge of the relationships between topology and geography in the application of graph theory. (For an application to transportation networks, see Werner, 1968; and for fusion of this approach with probabilistic models, see a study of river networks, Shreve, 1967.) Finally, concerning behavioral approaches to geographic research, there is only limited reference in the Bibliography to some of the scaling problems involved in space preference measures. For some such work the reader is referred to the excellent review by Craik (1968), and for a discussion of one technique based on personal construct theory to Golant and Burton (1970).

References for Introduction

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ORGANIZATION OF THE BIBLIOGRAPHY

For each of the Sections, a short introductory statement is made concerning the general nature of problems investigated using that technique, where applicable. Some reference is also made to those articles that are relevant in my experience. The introduction is not aimed at being a statistical primer but is rather concerned with the concepts involved.

The following list of references will, in most cases, not deal entirely with the section topic; this applies in particular to the more recent articles in which statistical tests are often incorporated into the research design. In some cases where books or monographs are referenced, the relevant page numbers are indexed.

The heading SEE ALSO refers to articles that have been referenced in sections prior to the one in question. The notation used is as follows: Section number. Author (Date).

The heading REFERENCE indicates that some of the books or articles listed in the Sections of Reference (4) or Review Articles (5) have material on the topic in hand. The notation used is. Author (pages). From Section 4 the following texts are used: Gregory (1968), King (1969), and Krumbein and Graybill (1965); while the following review articles are listed where relevant: Barry (1963), Chorley (1966), Hart and Salisbury (1965), and Strahler (1954).

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SECTION 1

DEVELOPMENTS IN GEOGRAPHIC METHODOLOGY

The use of rigorous scientific methods of research with a statistical foundation commenced in human geography in North America on a large scale scarcely more than fifteen years ago. Although there had been some quantitative work earlier, for example centrography and social physics (see Section 19), the value of a statistical approach was first clearly stated in the articles by McCarty (1956) and Garrison (1956).

Some of the readings are "position papers" at various points in time; compare Ackerman (1958) and Kohn (1970). Others deal more explicitly with the evolution of the quantitative approach (LaValle *et al.*, 1967) and especially its relevance for the development of geographic theory (Burton, 1963). The relationships between traditional (qualitative) and modern geography are discussed by Spate (1960) and Mackin (1963). The vitality of this debate, especially among schools of geography outside of the United States, is reflected in the recent book edited by French and Racine (1971). Even so, the inadequacies of older methods had been clearly demonstrated more than a decade before (McCarty and Salisbury, 1961).

Apart from individual leaders generating change within the discipline, geography has shared with many other physical and social sciences an increased amount of information available for research. This information explosion has forced researchers to inquire into the organization of spatial data (see Section 8), as well as to make use of modern technology in storing and manipulating data (Section 10). A beneficial, and entirely natural, outcome of these external influences is seen in our ability to test hypotheses in a manner almost inconceivable ten years ago.

While this general change in geographic methodology contains a set of techniques that must be mastered in order to understand much of the literature today, the need for clarity and simplicity in the conceptual underpinnings of the discipline has been well demonstrated. The article by Thomas (1964), originally prepared for a high school course, provides an excellent example of this most important by-product of changes in methodology.

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SECTION 2

MODELS IN GEOGRAPHIC RESEARCH

As an activity, model building has become an increasingly important aspect of geographic research, concommitant with developments of a more technical nature outlined in the preceding section. Although *models* can be defined in many ways, a common feature is that they abstract from reality certain elements that are amenable to analysis. The way in which this transformation is carried out, and indeed the choice of which factors to include in the model, naturally rest upon substantive interpretations made by the researcher with respect to the problem in hand.

The process could be carried out in a qualitative manner, seeking to express likely relationships between factors on the basis of logical reasoning; such an activity is likely to lead to a diagrammatic representation of the relationships, as a conceptual model. Alternatively, the relationships could be specified 1) mathematically (leading to a mathematical model), 2) by constructing a smaller or larger physical representation (scale or iconic model), or 3) by transfer to another medium, for example substituting electrical impulses for flow phenomena (analogue model). A statistical model is a particular form of mathematical model. It is a mathematical expression, with variables, parameters, and constants, and in addition it contains one or more random components. In geography, statistical models are usually emphasized since much research is empirical in nature, and the random components are due in large part to sampling variability and measurement error.

The readings reflect two major influences upon the increased use of models in geography. First, the desires of researchers to understand the linkages that exist between elements of geographic systems are represented in a series of review articles in a seminal volume edited by Chorley and Haggett (1967). The second influence has emanated from outside the discipline, as other natural and social sciences overlap at the frontiers of geographic endeavor; see Ackerman (1963), the NAS/NRC publication *The Science of Geography* (1965), and Taaffe (1970). In addition, both Haggett (1965) and King (1966) place model building approaches into a larger substantive framework. Duncan, Cuzzort, and Duncan (1961) were among the first to point out many problems in forming models using areally based data, particularly the aggregation factor (scale problem) and its effects on explanation at different levels of inquiry (Dogan and Rokkan, 1969).

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SECTION 3

GEOGRAPHIC THEORY

The most important aspect of changes in geographic methodology has been a revitalized interest in the role of theory in the discipline. Although earlier debate over determinism and possibilism (Lukermann, 1965) can be viewed as one example of prior concern over theory, the methods used to test general propositions before the mid-1950's were idiographic in their orientation. Schaeffer's (1953) denunciation of this underlying weakness laid the foundations for more concise methods of explanation, but the development of statistical models in geography progressed at such a rate that explanation based on purely methodological grounds was no longer acceptable. Harvey's (1969) major review of this process indicates clearly that the explicit desire for more adequate explanation is at the same time a concerted effort in the development of theory, even though the latter may be largely derivative—the spatial equivalent of essentially temporally-based theories in other natural and social sciences.

It seems that there are three major fronts on which the difficult problems of theory construction in geography are being broached. First, there is an extension of Schaeffer's contribution with much greater emphasis on the philosophical foundation of the discipline. In particular, the dual activity of explanation and prediction is being subject to much debate, see the important contributions by Harvey, Olsson (1969, 1970), and Golledge and Amadeo (1968). A second approach is seen in the concern over relations between geography and geometry (Bunge, 1962). In this area, Nystuen (1963) has derived some basic elements of geographic space, while Tobler (1963) has identified the fundamental role of map transformations. The third attack is based upon stochastic process models, this appears to have particular relevance in physical geography (Curry, 1964, Scheidegger and Langbein, 1966).

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SECTION 4

REFERENCE

Section 4A contains geographic textbook references. Gregory's book (1968) originally appeared at a time when courses in quantitative methods were being introduced into undergraduate curricula and there were no prerequisites for such courses. Today, most undergraduates are required to register for an introductory statistics course prior to training in geographic methods. King's review of contributions in this area (1969) needs such a background. From the point of view of student response, Krumbein and Graybill (1965) has been found most satisfactory because it bridges the gap between methodological and substantive concerns in a comprehensive manner. A different approach is seen in Yeates' subject-matter oriented text (1968).

Two important collections of articles also are indexed in Section 4A. The Berry and Marble reader (1968) brings together a number of references, most of which have been published before. The two volumes on *Quantitative Geography*, edited by Garrison and Marble (1967), contain symposium articles which are referenced separately in the Bibliography. Also, as indicated earlier, relevant pages from Gregory, King, and Krumbein and Graybill are referenced at the end of sections.

More general works are listed in the second part, Section 4B. The following are important as background references in statistical methods: Blalock (1960); Coleman (1964); Cooley and Lohnes (1962, 1970); Freund (1967); Fryer (1966); Guenther (1965); Huntsberger (1961); Kendall and Stuart (1963, 1966, 1967); Morrison (1967); Seal (1964); Snedecor (1956).

A. *Spatial Analysis*

BERRY, B. J. L. and D. F. MARBLE (eds.). *Spatial Analysis.* Englewood Cliffs, N.J.: Prentice-Hall, 1968.

COLE, J. P. and C. A. M. KING. *Quantitative Geography.* London: Wiley, 1968, 692 pp.

GARRISON, W. L. and D. F. MARBLE (eds.). *Quantitative Geography. Part I: Economic and Cultural Topics.* (Studies in Geography No. 13). Evanston, Ill.: Northwestern University, 1967, 287 pp.

GARRISON, W. L. and D. F. MARBLE (eds.). *Quantitative Geography. Part II. Physical and Cartographic Topics.* (Studies in Geography No. 14). Evanston, Ill.: Northwestern University, 1967, 323 pp.

GREGORY, S. *Statistical Methods and the Geographer.* London: Longmans. 1968 (2nd ed.), 277 pp.

KING, L. J. *Statistical Analysis in Geography.* Englewood Cliffs, N.J.: Prentice-Hall, 1969.

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MILLER, R. L. and J. S. KAHN. *Statistical Analysis in the Geological Sciences.* New York: Wiley, 1962, 483 pp.

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B. *General References*

ACKOFF, R. L. *The Design of Social Research.* Chicago: University of Chicago Press, 1953.

ACKOFF, R. L. *Scientific Method.* New York Wiley, 1962.

BLALOCK, H. M. *Social Statistics.* New York: McGraw-Hill, 1960.

BLALOCK, H. M. *Causal Inferences in Nonexperimental Research.* Chapel Hill, N.C.: University of North Carolina Press, 1964.

BLALOCK, H. M. and A. B. BLALOCK (eds.). *Methodology in Social Research.* New York: McGraw-Hill, 1968.

BRAITHWAITE, R. B. *Scientific Explanation.* New York: Harper, 1960.

CATTELL, R. B. "Factor Analysis: An Introduction to Essentials I and II," *Biometrics*, Vol. 21, 1965, pp. 190–215 and 405–435.

CHARLESWORTH, J. C. (ed.). *Mathematics and the Social Sciences.* Philadelphia: American Academy of Political and Social Science Monograph, 1963.

COCHRAN, W. G. *Sampling Techniques.* New York: Wiley, 1963, (2nd Ed.).

COLEMAN, J. S. *Introduction to Mathematical Sociology.* New York: Free Press, 1964.

COOLEY, W. W. and P. R. LOHNES. *Multivariate Procedures for the Behavioral Sciences.* New York: Wiley, 1962.

COOLEY, W. W. and P. R. LOHNES. *Multivariate Data Analysis.* New York: Wiley, 1970, 339 pp.

DAVIS, J. A. *Elementary Survey Analysis.* Englewood Cliffs, N.J.: Prentice-Hall, 1971.

DRAPER, N. R. and H. SMITH. *Applied Regression Analysis.* New York: Wiley, 1966.

EZEKIEL, M. and K. A. FOX. *Methods of Correlation and Regression Analysis.* New York: Wiley, 1959 (3rd ed.).

FREUND, J. E. *Modern Elementary Statistics.* Englewood Cliffs, N.J.: Prentice-Hall, 1967 (3rd ed.).

FRYER, H. C. *Concepts and Methods of Experimental Statistics.* Boston: Allyn and Bacon, Inc., 1966.

FULLER, L. E. *Basic Matrix Theory.* Englewood Cliffs, N.J.: Prentice-Hall, 1962.

GLASER, B. and A. STRAUSS. *The Discovery of Grounded Theory.* Chicago: Aldine, 1967.

GOLDBERG, S. *Probability: an Introduction.* Englewood Cliffs, N.J.. Prentice-Hall, 1960.

GUENTHER, W. C. *Concepts of Statistical Inference.* New York. McGraw-Hill, 1965.

GUMBEL, E. J. *Statistics of Extremes.* New York. Columbia University Press, 1958.

HARMAN, H. H. *Modern Factor Analysis.* Chicago University of Chicago Press, 1960.

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HORST, P. *Factor Analysis of Data Matrices.* New York. Holt, Rinehart and Winston, 1965.
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SECTION 5

REVIEW ARTICLES

The articles in this section vary greatly in scope and purpose, but an attempt has been made to cover most areas of interest to geographers. Basic explanations of the use of statistical methods in geomorphology, for example, are illustrated in the articles by Chorley (1966) and Strahler (1954). These, plus the references by Barry (1963) and Hart and Salisbury (1965) have been indexed by page number in relevant sections in the Bibliography. In terms of this listing of reviews, the range extends from such introductory statements to fairly advanced surveys and critiques, such as Curry (1967), Strahler (1964), and King (1969).

AMOROCHO, J. and W. E. HART. "A Critique of Current Methods in Hydrological Systems Investigation," *Transactions, American Geophysical Union*, Vol. 45, 1964, pp. 307–321.

BARGER, G. L. (ed.). *Climatology at Work; Measurements, Methods and Machines.* (U.S. Department of Commerce Weather Bureau). Washington, D.C.: USGPO, 1960. (Sec. on "Climatological Statistics," pp. 46–67).

BARRY, R. G. "An Introduction to Numerical and Mechanical Techniques," in Monkhouse, F. J. and A. R. Wilkinson (eds.), *Maps and Diagrams: Their Compilation and Construction.* London: Methuen, 1963, pp. 385–423.

CHORLEY, R. J. "Application of Statistical Methods to Geomorphology," in Dury, G. H. (ed.), *Essays in Geomorphology.* London: Heineman, 1966, pp. 275–387.

CONRAD, V. and L. W. POLLAK. *Methods in Climatology.* Cambridge, Mass.: Harvard University Press, 1956.

CURRY, L. "Quantitative Geography; 1967," *The Canadian Geographer*, Vol. 11, 1967, pp. 265–279.

HARRISON, C. M. "Recent Approaches to the Description and Analysis of Vegetation," *Transactions of the Institute of British Geographers*, No. 52, 1971, pp. 113–127.

HART, J. F. and N. E. SALISBURY. "Population Changes in Middle Western Villages: A Statistical Approach," *Annals*, AAG, Vol. 55, 1965, pp. 140–160.

KING, L. J. "The Analysis of Spatial Form and its Relation to Geographic Research," *Annals*, AAG, Vol. 59, 1969, pp. 573–595.

KRUMBEIN, W. C. "Statistical Models in Sedimentology," *Sedimentology*, Vol. 10, 1968, pp. 7–23.

PANOFSKY, H. A. and G. W. BRIER. *Some Applications of Statistics to Meteorology.* University Park, Penn.: The Pennsylvania State University, 1965.

STRAHLER, A. N. "Statistical Analysis in Geomorphic Research," *Journal of Geology*, Vol. 62, 1954, pp. 1–25.
STRAHLER, A. N. "Quantitative Slope Analysis." *Bulletin of the Geological Society of America*, Vol. 67, 1956, pp. 571–596.
STRAHLER, A. N. "Quantitative Geomorphology of Drainage Basins and Channel Networks," in Chow, V. T. (ed.), *Handbook of Applied Hydrology* New York: McGraw-Hill, 1964, pp. 4-39.
TIMMS, D. W. G. "Quantitative Techniques in Urban Geography," in Chorley, R. J. and P. Haggett (eds.), *Frontiers in Geographical Teaching* London. Methuen, 1965, pp. 239–265.

SEE ALSO: Section 1. Chorley (1965); Gould (1969).
Section 2. Haggett (1965); King (1966).

SECTION 6

MEASUREMENT

Since statistics may be defined as a branch of applied mathematics that concentrates on the development of procedures for describing and reasoning from observations, the ways in which such observations are recorded or coded are clearly basic to any analysis. Indeed, different ways of assigning numbers or symbols to specify variations in characteristics of a set of objects (the process of measurement) implies that different procedures will be applicable to certain situations. The kind of measurement achieved is a function of the rules under which the numbers or symbols were assigned, and these operations, in turn, define and limit the manipulations permissible in handling the data. The manipulations and operations must be those of the numerical structure to which the measurement is isomorphic.

Four measurement scales are recognized: nominal, ordinal, interval, and ratio. A hierarchy of levels of measurement is represented here. Nominal scales are the weakest type of measurement and, in a sense, render least information about the observations, whereas the ratio scale is the highest level of measurement, isomorphic to the numerical structure of arithmetic. In addition, each higher level of measurement incorporates all the attributes of lower levels, for example, ordinal scales have all the characteristics of nominal scales (equivalence relations among members of a designated sub-class) as well as their own properties ("greater than" or "less than" relations).

In the nominal or classificatory scale, the assignment process is carried out with the purpose of designating sub-classes which represent unique characteristics. Ordinal scales (ranking) have the additional purpose of identifying ordered relations of some characteristic. The order itself has unspecified intervals, i.e., the magnitude of differences between ordered categories cannot be established. In interval scales, the ordered relations refer to values and are based on arbitrarily assigned and equal intervals, but with an arbitrary zero point (e.g. temperature on Fahrenheit or Centigrade scales). Thus the focus is on differences between values on a scale, and the normal rules of arithmetic can be applied to such intervals. Finally, the ratio scale incorporates all other properties of lower levels, and has an absolute zero point, permitting all arithmetic manipulations.

Some scales appear to be intermediate between these four types. One example might be the Likert scale, which is an attitudinal measure, eliciting responses to a question on the basis of five to seven classes, from "strongly disagree" to "strongly agree." In this case, an ordering has been imposed on what is probably an underlying continuous variable, so that the scale seems to lie between ordinal and interval scales as defined above. Indeed, many problems in geographic research using a behavioral approach are linked to the scaling question. Gould (1969) and Rushton (1969) discuss techniques for the conversion of non-metric information to a metric base. Other readings (for example, Hodge, 1963, and Krumbein, 1958) clearly indicate the implications of different scales for subsequent analysis. The use of dimensionless measures is described by Schumm (1956).

GOULD, P. R. "Problems of Space Preference Measures and Relationships," *Geographical Analysis,* Vol. 1, 1969, pp. 31–44.

HODGE, G. "The Use and Mis-Use of Measurement Scales in City Planning," *Journal of the American Institute of Planners,* Vol. 29, 1963, pp. 112–121.

KRUMBEIN, W. C. "Measurement and Error in Regional Stratigraphic Analysis," *Journal of Sedimentary Petrology,* Vol. 28, 1958, pp. 175–185.

KRUMBEIN, W. C. "The Geological 'Population' as a Framework for Analyzing Numerical Data in Geology," *The Liverpool and Manchester Geological Society Journal,* Vol. 2, 1960, pp. 341–368.

LABOVITZ, S. "Some Observations on Measurement and Statistics," *Social Forces,* Vol. 46, 1967, pp. 151–160.

RUSHTON, G. "The Scaling of Locational Preferences," in Cox, K. R. and R. G. Golledge (eds.), *Behavioral Problems in Geography: A Symposium.* (Studies in Geography No. 17). Evanston, Ill.: Northwestern University, 1969, pp. 197–227.

SCHUMM, S. A. "Evolution of Drainage System and Slopes in Badlands at Perth Amboy, New Jersey," *Bulletin of the Geological Society of America,* Vol. 67, 1956, pp. 597–646.

STRAHLER, A. N. "Dimensional Analysis Applied to Fluvially Eroded Landforms," *Bulletin of the Geological Society of America,* Vol. 69, 1958, pp. 279–300.

SEE ALSO: Section 1. Coppock and Johnson (1962); Garrison (1956).

REFERENCE: Section 4A. King (8–13; 230–231); Krumbein and Graybill (34–38; 44–53).
Section 5. Chorley (285–292).

SECTION 7

SET THEORY

Set theory is an integral part of finite mathematics and can be used as a foundation for the development of concepts appropriate to probability theory. If this material is not covered in prerequisite courses for the student, it can be introduced in an informative context as the logical basis of the regionalization problem in geography (see Golledge and Amadeo, 1966). References cited in Section 4B dealing with finite mathematics (especially Kemeny, Snell, and Thompson, 1965, and Robinson, 1969) and probability theory (especially Goldberg, 1960, and Mosteller, Rourke, and Thomas, 1961) exhibit the transition from set theory to probability concepts in an excellent manner for those geography courses in which it is necessary to establish the bases of all statistical reasoning.

COLE, J. P. *Set Theory and Geography.* (Bulletin of Quantitative Data for Geographers No. 2). Nottingham, U. K.: Department of Geography, Nottingham University, 1966.

GOLLEDGE, R. and D. M. AMADEO. "Some Introductory Notes on Regional Division and Set Theory," *The Professional Geographer*, Vol. 18, 1966, pp. 14–19.

HAMILL, L. "A Note on Tree Diagrams, Set Theory and Symbolic Logic," *The Professional Geographer*, Vol. 18, 1966, pp. 224–226.

SHEAR, J. A. "A Set-Theoretic View of the Köppen Dry Climates," *Annals*, AAG, Vol. 56, 1966, pp. 508–515.

WARNTZ, W. "Some Elementary and Literal Notions About Geographical Regionalization and Extended Venn Diagrams," in Bunge, W. *et al.*, *The Philosophy of Maps.* (Michigan Inter-University Community of Mathematical Geographers. Discussion Paper No. 12). Ann Arbor, Mich.: Department of Geography, University of Michigan, 1968, pp. 7–30.

SECTION 8

GEOGRAPHIC DATA

Sets of observations on variables of particular interest to geographers are ordered in a manner peculiar to the discipline, any unit of observation not only has a value on the particular attribute in question and a temporal characteristic but also has the property of geographic location. This is usually expressed as the geographers' primary concern with maps, but the exact specification of the locational coordinates presents some difficulties (Tobler, 1963).

Ideally, data should be referenced by point locations but practically, some aggregation factor is applied to produce a set of quadrats (cells). Since aggregation is a continuous function and statements made at different scales of aggregation can differ, a basic problem in geographic research is revealed. The use of point locations—the process of "geocoding" (Tomlinson, 1967)—in recent census operations has opened up new avenues of research (see the excellent series of publications by the U.S. Bureau of the Census). Forbes and Robertson (1967) report on the use of quadrats in census operations. A more general approach is given by Haggett, Chorley, and Stoddart (1965).

Census bureaus are the primary sources of information used in most economic-urban geographic research. It is therefore very important to understand the rationale behind the types of information collected (Fay and Klove, 1970), since these data may ~~be used~~ as approximations (operational definitions) of conceptual factors. Aggregations of unit characteristics, such as occupational groupings or industrial classifications, should be considered (Sweet, 1970) as well as the problem of using census-defined unit areas (see Morrill, 1969, and Abler, 1970). Potentially important new sources of information are being developed using airborne sensors (Cooke and Harris, 1970; Moore and Wellar, 1969).

The greater availability of all types of information amenable to spatial analysis has forced the geographer to consider seriously all aspects of collection, storage, and manipulation of data in a systems framework, using the computer (Section 10). The series of reports by Dueker (1966) addresses this area of overall data organization.

ABLER, R. "Zip-Code Areas as Statistical Regions," *The Professional Geographer,* Vol. 22, 1970, pp. 270–274.

COOKE, R. U. and D. R. HARRIS. "Remote Sensing of the Terrestrial Environment—Principles and Progress," *Transactions of the Institute of British Geographers,* No. 50, 1970, pp. 1–23.

DUEKER, K. J. *Spatial Data Systems: Organization of Spatial Data.* (Urban and Transportation Information Systems, Technical Report No. 4). Evanston, Ill.: Department of Geography, Northwestern University, 1966.

DUEKER, K. J. *Spatial Data Systems: Systems Considerations.* (Urban and Transportation Information Systems, Technical Report No. 5). Evanston, Ill.: Department of Geography, Northwestern University, 1966.

DUEKER, K. J. *Spatial Data Systems: Special Topics.* (Urban and Transportation Systems, Technical Report No. 6). Evanston, Ill.: Department of Geography, Northwestern University, 1966.

FAY, W. T. and R. C. KLOVE. "The 1970 Census," *The Professional Geographer,* Vol. 22, 1970, pp. 284–289.

FORBES, J. and I. M. L. ROBERTSON. "Population Enumeration on a Grid.

Square Basis: The Census of Scotland, a Test Case," *The Cartographic Journal*, Vol. 4, 1967, pp. 29–37.

HAGGETT, P., R. J. CHORLEY, and D. R. STODDART. "Scale Standards and Geographic Research: A New Measure of Areal Magnitude," *Nature*, Vol. 205, 1965, pp. 844–847.

HOFSTEN, E. Von. "Population Registers and Computers. New Possibilities for the Production of Demographic Data," *Revue de l'Institut International de Statistique*, Vol. 2, 1966, pp. 186–194.

LLOYD, P. E. "A Method for Conducting Rapid Reconnaissance Surveys of Manufacturing in Australian Cities," *Australian Geographical Studies*, Vol. 3, 1965, pp. 31–38.

MOORE, E. G. and B. S. WELLAR. "Urban Data Collection by Airborne Sensor," *Journal of the American Institute of Planners*, Vol. 35, 1969, pp. 35–43.

MORRILL, R. L. "Statistical Regions for the United States," *Geographical Review*, Vol. 59, 1969, pp. 612–614.

RICKERT, J. E. (ed.). *Urban and Regional Information Systems: Federal Activities and Specialized Programs*. Kent, Ohio: Kent State University, 1969, 277 pp.

ROBERTSON, I. M. L. "The Census and Research: Ideals and Realities," *Transactions of the Institute of British Geographers*, No. 48, 1969, pp. 173–187.

SWEET, D. C. "An Industrial Development Screening Matrix," *The Professional Geographer*, Vol. 22, 1970, pp. 124–127.

TOBLER, W. R. "Geographical Ordering of Information," *The Canadian Geographer*, Vol. 7, 1963, pp. 203–205.

TOMLINSON, R. F. *An Introduction to the Geo-Information System of the Canada Land Inventory*. Ottawa: Department of Forestry and Rural Development, 1967, 23 pp.

TOMLINSON, R. F. (ed.). *Environment Information Systems*. Proceedings of the UNESCO/IGU First Symposium on Geographical Information Systems, Ottawa, 1970. (A Publication of the International Geographical Union Commission on Geographical Data Sensing and Processing.) 1971, 161 pp.

U.S. BUREAU of the CENSUS. *Census Use Study: The DIME Geocoding System*. (Report No. 4). Washington, D.C.: USGPO, 1970, 46 pp.

U.S. BUREAU of the CENSUS. *Census Use Study: Family Health Survey*. (Report No. 6). Washington, D.C.: USGPO, 1969, 41 pp.

U.S. BUREAU of the CENSUS. *Census Use Study: Health Information System*. (Report No. 7). Washington, D.C.: USGPO, 1969, 67 pp.

U.S. BUREAU of the CENSUS. *Census Use Study: Data Uses in Urban Planning*. (Report No. 9). Washington, D.C.: USGPO, 1970, 28 pp.

U.S. BUREAU of the CENSUS. *Census Use Study: Geocoding with ADMATCH, A Los Angeles Area Experience*. (Report No. 14). Washington, D.C.: USGPO, 1971, 23 pp.

WELLAR, B. S. "Recent Developments in Urban Data Generation via Remote Sensing Techniques," in Rickert, J. E. (ed.), *Urban and Regional Information*

Systems., Federal Activities and Specialized Programs. Kent, Ohio. Kent State University, 1969, pp. 258–277.

REFERENCE: Section 4A. Krumbein and Graybill (33–34).

SECTION 9

THE GEOGRAPHIC MATRIX

Once the information required for any problem has been collected, how can one organize it most effectively for analytical purposes? A matrix format is commonly employed, especially since the data is most efficiently stored in this form for computer analysis (Section 10). Two types of geographic data, with different matrix forms, can be recognized: an *attribute* matrix, of order *n*-places by *m*-attributes or characteristics of the places; and an *interaction* matrix, of order *n*-places by *n*-places. Attribute matrices are most commonly employed in the literature since they refer to easily available data in comparison to the inter-place flows (of information, goods, money, people, etc.) indexed by the interaction matrix (Smith, 1970).

Many of the regular statistical procedures can be related effectively to this matrix organization, for example, descriptive statistics (Section 12) can be viewed as the outcome of manipulations on any one of the columns of the matrix, while correlation (Section 21) is the covariance of two or more columns. Since any column is a means in addition to the map of expressing a spatial distribution, the relevance of a matrix format is evident. Similarly, the grouping of rows of the matrix can be likened to the process of classification/regionalization (Section 32).

Apart from this operational utility, the matrix approach has certain benefits of a conceptual nature. For example, Berry (1964) was able to categorize much of the work in regional geography using a matrix organization, a later extension of this argument called for a synthesis of formal and functional regionalization schemes (Berry, 1968). The conceptual framework employed in this latter case was field theory, originally developed in psychology. Some difficulties in translating these ideas in a geographic context are outlined by Greer-Wootten (1971).

The readings also include a reference to the use of matrices from an analytical point of view (Gould, 1967), although matrix algebra itself is a foundation for much work in multivariate statistics (Section 24). Note also that Haggett and Chorley (1967) have proposed an alternative form of the geographic matrix to make it more relevant for model building the axes of their matrix are locational relativity and topological-geometric form.

BERRY, B. J. L. "Approaches to Regional Analysis. A Synthesis," *Annals*, AAG, Vol. 54, 1964, pp. 2–11.

BERRY, B. J. L. "A Synthesis of Formal and Functional Regions Using a General Field Theory of Spatial Behavior," in Berry, B. J. L. and D. F. Marble (eds.), *Spatial Analysis.* Englewood Cliffs, N.J.: Prentice-Hall, 1968, pp. 419–428.
GOULD, P. R. "On the Geographic Interpretation of Eigenvalues," *Transactions of the Institute of British Geographers,* No. 42, 1967, pp. 53–86.
GREER-WOOTTEN, B. "Some Reflections on Systems Analysis in Geographic Research," in French, H. M. and J-B. Racine (eds.), *Quantitative and Qualitative Geography. La Nécessité d'un Dialogue.* (Occasional Papers, Department of Geography, No. 1). Ottawa: University of Ottawa Press, 1971, pp. 151–174.
HAGGETT, P. and R. J. CHORLEY. "Models, Paradigms and the New Geography," in Chorley, R. J. and P. Haggett (eds.), *Models in Geography.* London: Methuen, 1967, pp. 19–41.
SMITH, R. H. T. "Concepts and Methods in Commodity Flow Analysis," *Economic Geography,* Vol. 46, 1970, pp. 404–416.
TEITZ, M. B. "Land Use Data Collection Systems: Some Problems of Unification," *Papers of the Regional Science Association,* Vol. 17, 1966, pp. 179–194.

REFERENCE: Section 4A. Krumbein and Graybill (53–56).

SECTION 10

COMPUTER APPLICATIONS

The uses of punched cards as a means of recording and storing data are well known (Melton, 1958). Benefits are evident in terms of permanence and reproducibility of original records and for speed, neatness, flexibility, and accuracy in manipulations. The general utility of computers in geographic research is documented by Pitts (1962) Kao (1963), and Gould (1970), while Haggett (1969) describes some of the implications for research. The effects on ordering information (Section 8) have already been noted (see also Hägerstrand, 1967, and Nordbeck, 1962).

There are two major applications of computers in geography: 1) as an aid in the rapid portrayal of information—in the use of the on-line printer for mapping purposes, for example, such as the SYMAP system (Rosing, 1969; Massey, 1970; Douglas, 1971) or the incremental plotter for line patterns (Kern and Rushton, 1969; Monmonier, 1970); and 2) for analytical purposes—in correlation studies, for example (Monmonier, 1971). In the latter case, it is worth noting that much of the current research using multivariate techniques would not be possible without access to high-speed computers—for example, factor-analytic studies of large data matrices (Sections 24, 25, 26) and the lengthy iterative process of classification (Section 32).

Trend surface analysis (Section 30) shows the direct link between analytical and graphic uses of the computer.

Three departments of geography have produced useful sets of computer programs: Iowa (Wittick, 1968), Northwestern (Marble, 1967), and Michigan (Tobler, 1970). In addition, there are two sources of on-going research and publication: the State Geological Survey associated with the University of Kansas (*Computer Contributions*), and the Department of Geography of the University of Nottingham, U.K. (*Computer Applications in the Natural and Social Sciences*).

In Section 4B, McCracken (1965) and Organick (1966) present the basic elements of FORTRAN programming, while Veldman (1967) lists many programs relevant for multivariate analysis.

CRAIG, W. J. "A Useful Computer Function to Measure Distances," *The Professional Geographer*, Vol. 23, 1971, pp. 157–159.

DOUGLAS, D. H. "Map Making with the Electronic Digital Computer," in French, H.M. and J-B. Racine (eds.), *Quantitative and Qualitative Geography. La Nécessité d'un Dialogue.* (Occasional Papers, Department of Geography, No. 1). Ottawa, Canada: University of Ottawa Press, 1971, pp. 97–114.

FAIMAN, M. and J. NIEVERGELT (eds.). *Pertinent Concepts in Computer Graphics*, Urbana, Ill.: University of Illinois Press, 1969.

GOSENFELD, N. and T. S. LOEBER. "A Computer-Based Approach to Planning in Underdeveloped Areas," *The Professional Geographer*, Vol. 17, 1965, pp. 5–10

GOULD, P. R. "Computers and Spatial Analysis: Extensions of Geographic Research," *Geoforum*, Vol. 1, 1970, pp. 53–69.

HÄGERSTRAND, T. "The Computer and the Geographer," *Transactions of the Institute of British Geographers*, No. 42, 1967, pp. 1–19.

HAGGETT, P. "On Geographical Research in a Computer Environment," *Geographical Journal*, Vol. 135, 1969, pp. 497–507.

HARBAUGH, J. W. and D. F. MERRIAM. *Computer Applications in Stratigraphic Analysis*. New York: Wiley, 1968. ("Introduction," pp. 2–51).

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SEE ALSO: Section 8. Hofsten (1966); Tomlinson (1967).

REFERENCE: Section 5. Barry (387–389).

SECTION 11

SAMPLE DESIGNS AND METHODS

The application of statistical methods in geography is intimately related to sampling theory, which deals with the problem of estimating characteristics of a population from values obtained from measurement of a sample. A population, or universe, is any class of objects or events arbitrarily defined on the basis of its unique and observable characteristics, while a sample is a collection of objects selected to represent a population. In certain field situations, and particularly in geomorphology, the conceptual population may not be available for sampling, therefore a target population must be defined (Krumbein, 1960; Chorley, 1966; Griffiths and Ondrick, 1968). In such cases, it is clear that any statements made about population characteristics (parameters) from summary sample values (statistics) will be affected by the degree of correspondence between target and conceptual populations.

In contrast, the situation in most economic-urban geographic research that uses areas (usually defined by the census) as sampling units, is one in which little, if any, selection process is applied. The conceptual population is then argued to be all such sets of areas, at such a stage of economic development, etc., that might have existed in the past or would exist in the future, as well as those existing in the present! If the sampling unit is defined as a coordinate point location or a quadrat (or similar geometric unit–see Matern, 1960; Holmes, 1970), the population characteristic of interest is usually some continuous feature such as land use, and the procedures of areal or location sampling are appropriate (Berry and Baker, 1968; Hadfield and Oorzeske, 1966; Holmes, 1967).

Sampling is usually carried out with two purposes in mind. 1) to estimate population parameters, and 2) to test a statistical hypothesis about the population. The *standard error* is used in computations for both goals and is thus an important concept. Imagine that a sample of quadrats is selected for measuring some attribute, say the proportion of land under wheat. From the resulting set of proportions, a sample distribution can be drawn up, usually graphically in the form of a frequency diagram or histogram (Section 12). An indication of the average proportion of land under wheat can be obtained by computing the arithmetic mean of the set of observations (Section 12). The question that then arises is. to what extent does the sample mean (a statistic) differ from the true population mean (parameter)? Sampling theory provides an answer to this question by the following argument. the single sample taken is but one of the theoretically infinite number of such samples, and for each of these samples an arithmetic mean could also have been computed. The complete set of sample means could be arranged in a frequency diagram, resulting in what is known as a *sampling distribution.* The standard deviation (Section 12) of a sampling distribution is known as the standard error.

If the frequencies are viewed as probabilities of occurrence of values of the sample mean, it is known that the shape of the sampling distribution has a particular form – the normal curve, a continuous probability function (Section 15). Indeed, many sample statistics have a normal distribution even though they may be derived from parent populations that are not normally distributed. Herein lies the value of a correctly conceived and executed sample, since the full power of parametric statistics can then be applied. For example, it is possible to place confidence limits around the particular sample mean obtained, so that, at a specified level of probability, the researcher may state that the true population mean lies in such an interval. It seems that few geographers have utilized sampling theory to the fullest in making estimates of population parameters. Rushton (1966) illustrates the procedures.

The selection procedures used to draw a sample are extremely important. Most statistical tests require that the sampling units were originally assigned some probability of selection – the general case of probability sampling. If these probabilities of selection were equal for all units, random sampling is employed. Non-probability sampling (purposive or quota samples) may be necessary in certain research areas in which there is virtually no knowledge of population characteristics, and some non-parametric tests may be applicable in such situations (see Section 18).

Sample designs can be viewed as sets of selection procedures, following certain rules (sampling plan) established by the researcher. For example, if it is clear that a population characteristic, such as slope angle, varies directly with a known factor, such as lithology, then stratification can be applied (Wood, 1955). The sampling fraction may or may not be equal for the different strata (a function of within-strata variability), but the design can only help to reduce the standard error of sample statistics, i.e. to increase the precision. Note that the other primary factor influencing precision, the size of the sample, would also be decided upon as an integral part of the sample design, usually as a cost factor. Furthermore, in order to estimate the population mean in this case, a weighted average of the strata sample means would have to be computed. The lesson is that each sample design, and there are many possibilities for invention in this respect, will require different means of estimating population parameters.

The major use of sampling in geographic research has been to test hypotheses (see Section 14), and this goal clearly places sample design into a larger context. The possibility of using certain tests depends intimately on selection procedures used and on certain assumptions about the nature of the parent population from which the sample was drawn It is clear that sample design and statistical design (the type of analysis to be carried out) should be incorporated into an experimental design that ensures that the ultimate objectives of the research will be attained (Krumbein and Miller, 1953; Krumbein, 1955, Haggett. 1964).

It seems that there are three main situations in which sampling is particularly relevant in geographic research. First. many studies in economic and urban geography (in which sampling units are discrete entities such as manufacturing plants or households) can draw upon the substantial literature of sampling theory developed by sociologists or economists. The use of standard references, such as Yates (1960) or Kish (1965), paying particular attention to areally-based sampling designs, appears to suffice in most cases. An interesting design, illustrating the strong relationship between problem specification and subsequent stages of research, is seen in the work of Hess, Riedel, and Fitzpatrick (1961). Most designs in this area would be randomized.

In contrast, when dealing with continuously distributed phenomena, such as land use, spatial sampling plans are called for. Berry and Baker (1968) have shown that a systematic type of design (based on a grid scheme, but not regular in terms of point locations within a cell) is the most efficient in such circumstances. In sampling from spatially continuous phenomena, a problem exists in that the values attributed to any one sample point are related to those of all surrounding points (the spatial autocorrelation factor, see Section 29). Holmes (1970) indicates clearly how this affects the failure of random selection procedures in location sampling. Line sampling (the use of traverses) is also employed in some cases (Latham, 1963, Haggett and Board, 1964).

Finally, we might note that sampling procedures used in geomorphology represent perhaps the highest degree of attainment in terms of specific designs related to specific problem areas. Griffiths and Ondrick (1968) illustrate the different approaches, as well as introducing methods of ~~testing~~ whether a sample distribution approximates an assumed parent population distribution (Section 15). Note that the population distributions of many geomorphic characteristics are known (see especially Krumbein and Graybill, 1965). Researchers in this area have also investigated extensively the problem of operator variance (errors due to different researchers measuring the same phenomena) in experimental settings (Griffiths and Rosenfeld, 1954). Chorley (1958) has described this effect in morphometric analysis.

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Section 6. Krumbein (1960).

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Section 5. Barry (385–387); Chorley (292–307); Hart and Salisbury (149–150); Strahler, 1954 (2–4).

SECTION 12

DESCRIPTIVE STATISTICS

Some overall characteristics of a body of data can be represented by summary measures, which are based upon manipulations of values of one attribute. For this reason, these measures are also called univariate statistics. If the measurement procedure resulted in nominal or ordinal scales, tabular frequency counts are usually made which can be represented graphically (bar-diagrams, pictographs, etc.). With interval or ratio scales there is a continuum of such values so that it is necessary to form groups or classes to make up a frequency diagram or histogram. Absolute or relative frequencies can be represented, and these values can also be summed across the classes to form cumulative frequency distributions, or ogives.

Two main types of summary measures are calculated. 1) Measures of central tendency, which describe the clustering of values about certain points, are the first type. For nominally scaled data, the modal class (containing most observations) can be obtained by inspection. The mode is the most frequent value. In ranked data, the median is an appropriate measure; it is the middle ranked value, so that exactly half the observations are located above or below this point. The arithmetic mean (sum of values divided by the number of observations) is a balancing point on the continuum of values for interval or ratio scales. 2) Measures of dispersion, which describe the spread of observations about the central value, are the second type. The range indicates the complete span, being the difference between maximum and minimum values. Deviations of individual values from the mean are used to form the most important measures of distributions. If these deviations are summed, the result will naturally be zero. This sum of deviations is known as the first moment about the mean of a distribution ($\mu_1 = \Sigma(X_i - \bar{X})^1/N = 0$). The second moment ($\mu_2 = \Sigma(X_i - \bar{X})^2/N$) is the mean of the squared deviations about the mean, and is defined as the variance. The square root of this quantity is the standard deviation.

Many empirical frequency distributions have similar characteristics, so that we may refer any sample distribution to some theoretical population distribution. The normal distribution is the most important population distribution in statistical analysis; and in this case, for example, the mode, median, and mean have the same location. Departures from the symmetrical (about the mean) bell-shaped curve in terms of the shape of the distribution are called degrees of skewness. For example, if the median is greater than the mean, the distribution is said to be negatively skewed. Skewness may be measured by the third moment about the mean, while the fourth moment indexes kurtosis—the degree to which the distribution is strongly peaked (leptokurtic) or relatively flat (platykurtic) compared to the normal.

A direct link with cartography is seen in the graphic representation of data values, and several references deal with the problem of specifying the most efficient class interval (for example, Armstrong, 1969; Jenks, 1963; Mackay, 1955; Scripter, 1970). Mapping can also be carried out in terms of class intervals derived from the standard deviation of the sample distribution (Yeates, 1965). The use of standard descriptive statistics in climatology is seen in the articles by Portig (1965) and Sumner (1953), while a measure of relative variability for different attributes (the coefficient of variation) is employed by Fuchs (1960).

Cumulative frequency distributions are illustrated in Berry's (1961) study of the rank-size rule. Alexandersson (1956) also used this type of graphic representation in an early study of city classifications (see also Morrissett, 1958). Some of the indices derived in descriptive studies (Section 13) are based on deviations from the mean, and similar measures have been employed in classifications (e.g., Nelson, 1955). Note that the validity of this approach depends on the sample distribution having characteristics of a normal curve. Descriptive measures are also computed in morphometric analysis (Clarke, 1966).

Finally, it should be noted that these summary statistics treat the sets of values in a linear fashion, disregarding the locative elements. Much of the conventional statistical analysis, however, can be faulted in the same manner. Tobler (1966) discusses the problems resulting from the two-dimensional aspect of geographic data when measures refer more properly to one dimension (such as time, for example). Some research has treated the problem of descriptive statistics for areal distributions (Section 19).

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SEE ALSO: Section 2. Duncan, Cuzzort, and Duncan (1961).

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Section 5. Barry (390–400); Chorley (308–325); Strahler, 1954 (4–6).

SECTION 13

INDEX CONSTRUCTION

Many descriptive indices have been derived in geographic research in order to summarize particular facets of a problem. Some are related to the procedures described in the preceding section; Lewis' (1966) level-of-living index is based upon the mean and standard deviation, for example, while McEvoy (1968) used ranking techniques. One of the better known indices was computed by Weaver (cited in Section 12) to describe the mixing of crop types in counties. It is also based on ranking and a comparison of actual proportions with ideal sets, by means of deviation scores. The method can be applied to any situation where relative proportions are organized in discrete groups; for example, industry groups (Johnson and Teufner, 1968). It has generated much criticism, however, both on substantive and methodological grounds (see Hoag, 1969; L. J. Johnson, 1969).

Some indices attempt to compare a region's share in some attribute; for example, employment in a type of manufacturing industry with its share of some basic aggregate. Location quotients and various coefficients of localization attempt to describe this concept (Britton, 1965). Similar reasoning applies to several indices of segregation developed in social geography (Clarke, 1971; Timms, 1965; Duncan and Duncan, 1955). Cumulative frequency distributions are used to define Lorenz curves for computing indices of diversification or concentration (Berry, 1959; Conkling, 1963; Kuklinski, 1965; Krumme, 1969; I. Johnson, 1967). Many problem areas require indices that have not been employed before. One example of this is Wong's (1969) coefficient of choice-perception.

A major difficulty with these descriptive measures is that they often do not lead anywhere, they are specific to a problem and cannot be generalized. Methodologically, they suffer because the sampling distributions of such measures are not known; it therefore is not possible to associate any probability statements with particular values. Two exceptions to this rule exist. First, the Gini coefficient, derived from Lorenz curve analysis, is related to a statistic that has a normal distribution (see King, p. 115). Second, a measure of regional homogeneity, originally produced by Sherr (1966) has been reworked by Radhakrishna and Subba (1968) who show that it is related to the chi-square distribution (Section 16).

In the list of references, some work on operational definitions of difficult concepts is also presented; for example, discussions of measures of shape (Boyce and Clark, 1964; Lee and Sallee, 1970), and of the definition of drainage basin axis (Abrahams, 1970; Ongley, 1968).

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SECTION 14

TESTING STATISTICAL HYPOTHESES: GENERAL CONSIDERATIONS

In order to reach an objective decision about research hypotheses, a set of rules is needed. The decision will be based on the outcomes of the sample and the risks the investigator is willing to take in making an incorrect decision. The particular sample of values is, of course, only one of a theoretically infinite number of samples of the same size from the same population. The complete set comprises the sample space, and in testing situations, this space is partitioned into two regions: a region of acceptance, and a region of rejection (or critical region). The decision to accept or reject is always made with respect to the null hypothesis (H_0: the hypothesis of "no differences"). If it is rejected, the alternate hypothesis (H_1) may be accepted. H_1 may be regarded as the operational statement of the research hypothesis.

Two types of error are possible for the decision taken. If H_0 is actually true and it is rejected, then a Type I error has been made. This is generally referred to as α, the coefficient of risk, significance level or size of the critical region. A Type II error (β) occurs when H_0 is accepted when in fact it is false. The power of a test is defined as $(1-\beta)$: the probability of rejecting H_0 when it is false. Obviously, the Type II error should be minimized as much as possible. The power is related to the type of statistical test chosen, but generally it increases with a larger sample size. Alternatively, the size of the sample can be determined by a consideration of the desired power of subsequent tests, indicating once again the linkages between all stages of experimental design.

The rather formal procedures for carrying out tests of hypotheses can be listed in a number of stages:

(1) state the null hypothesis (H_0), and the alternate hypothesis;
(2) choose a statistical test of H_0, stating the assumptions of using the related statistical model;
(3) specify a significance level (α);
(4) establish the relevant test statistic, which involves finding the appropriate sampling distribution;
(5) define the region of rejection;
(6) compute the value of the test statistic, using the sample values. If the computed value is greater than the tabulated value for the degrees of freedom associated with the test (a concept related to sample size), then the null hypothesis can be rejected. One can then make inferences about the nature of the relationships implied in the research hypothesis for the population, keeping in mind the chosen significance level. Note that any inferences must be based on substantive reasoning with respect to the theoretical framework of the problem.

In addition to the normal distribution, there are three sampling distributions (based on samples from the normal density) which are used primarily in test situations: (Student's) t, (Fisher's) F, and the chi-square (χ^2) distribution. The t-distribution is used in tests where sample means for two regions (or two subsets of unit areas, or two groups in a sample of households, etc.) are being compared. The null hypothesis in such a case would be stated as H_0: $\mu_1 = \mu_2$; i.e. there is no difference between the population means in the two regions. The null hypothesis is always stated in terms of population parameters. This test will have little value if the variances in the two regions are significantly different, since extreme values seriously affect the effectiveness of the arithmetic mean as a measure of central tendency. Usually, therefore, one tests for differences in the variances of the regions first of all, and the F-distribution is used in this case.

Examples of the application of these tests in the literature are given in Mackay (1967), Rushton (1966), Rushton, Golledge, and Clark (1967), and Swan (1970). The importance of a sound sampling basis for these procedures and the overall relevance of statistical testing in geographic research are underlined by Gould's article (1970). Other specific examples of tests are given in Sections 16 and 17. The procedures outlined here are applicable, however, for any work in which questions of significance (differences, relationships, etc.) are involved, as in correlation or regression studies (Sections 21 and 27).

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SECTION 15

TESTING THE FORM OF DISTRIBUTIONS

As indicated in the section on sampling, for many statistical tests the sample must have been drawn at random from a population with a known probability distribution, often the normal distribution. A number of procedures are available that test whether the set of sample values obtained approximates an expected or theoretical distribution. Some of these are basically graphic and descriptive, they are referenced in this section, although a more formal statement of tests of hypotheses was presented in Section 14. One example of graphical methods is seen in extreme value analysis, especially important in climatology and hydrology (Gumbel, 1967, Court, 1953, Dury, 1964) where "log-Gumbel" probability paper is used (see also Gumbel, 1958 in Section 4B).

Tests of normality are most common in the literature. Graphical procedures are also involved here, arithmetic or logarithmic probability paper is employed, and cumulative relative frequencies are plotted against class mid-points. The closer the resulting set of points to a straight line, the closer the distribution is to normal (Strahler, 1954, Chorley, 1966). An improved version of this method is to construct a fractile diagram, which essentially places a confidence band around the theoretical straight line (Thomas, 1967, Gibson, 1970, Tiedemann, 1968). Other tests are numerical, based on the concepts of statistical inference (see Section 14). A common test relates the observed frequencies per class with those expected if the sample was drawn from a normal distribution. The theoretical frequencies are obtained from tabulated probabilities of a standard normal curve (with zero mean and unit variance), and the relevant test statistic is chi-square (Maxwell, 1967). Note that all these methods depend on some prior definition of class intervals—usually an arbitrary process. A test that circumvents this difficulty has been derived by Snedecor, it uses moment measures of the distribution and is based on the t-statistic (Section 14). Berry and Tennant (1965) illustrate the use of this test. Note that if the results of tests indicate that the values are not drawn from a normal population, the researcher may choose to transform the data (commonly using logarithms) or to use non-parametric methods of analysis (Tanner, 1959).

Other references cited in this section are concerned with tests of the goodness-of fit of sample distributions with those expected from some theoretical model—for example, interactance models (Mackay, 1958). The distribution of distances moved in migration, or distances separating marriage partners, are also expected to take on certain forms, and samples can be tested (Morrill and Pitts, 1967; Moore, 1971). Dacey (1964) tested the outcomes of a probability model, using a modified form of the Poisson distribution, a discrete probability function, to generate expected frequencies of points per quadrat (see also Section 20). Finally, we might note that it is not uncommon that a given set of sample values approximates closely several related population distributions. Quandt (1964) has addressed the problem that then arises, how to choose the best fitting distribution.

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REFERENCE: Section 5. Chorley (325–331), Hart and Salisbury (152), Strahler, 1954 (6–9).

SECTION 16

OTHER CHI-SQUARE TESTS

The chi-square statistic is often used in testing the outcomes of theoretical models where observed and expected frequencies are involved. Besides fitting samples to theoretical distributions (as described in Section 15), probability models can also be tested in this manner (Brush and Gauthier, 1968). If the F-test of the equality of variances in two regions, for example, is extended to the case of *k* regions or groups, a chi-square statistic is involved. The test is known as Bartlett's test of the homogeneity of variance, and is a necessary assumption in the analysis of variance (Section 17) and covariance (Section 28). The articles by Maxwell (1967) and King (1961) include this test.

More commonly, chi-square tests are used in non-parametric situations (Section 18) when data are measured on a nominal scale. Two (or more) classifications of unit areas are thus involved, and the test seeks to establish whether or not the classifications are independent. The first step is to establish a contingency table (of *r*-rows by *c*-columns) by forming a cross-classification of the areas. If the two classifications are independent, then the expected cell frequencies are a product of the respective row and column probabilities, this statement has a theoretical basis in the concept of a joint probability function. These expected frequencies are computed and compared to the observed frequencies in a particular way, and summed to produce a value of chi-square. This computed value is compared to the tabulated chi-square, with $(r-1) \times (c-1)$ degrees of freedom, enabling one to accept or reject H_0.

This method is employed often in questionnaire analysis (see Baumann, 1969.) A related statistic, *C*, the contingency coefficient which varies between 0 and 1, is often computed at the same time (Carey, Macomber, and Greenberg, 1968; Friendly, 1965), as well as other measures of association such as lambda, gamma, etc. Problems of interpretation arise when the number of cells with expected frequencies of less than 5 is large, and when the contingency table is larger than 2 X 2. The first difficulty is usually resolved by reclassifying the data, while the second can be eased appreciably because of the additivity property of chi-square. Maxwell (1961), cited in Section 4B, shows how the total chi-square value can be partitioned as a series of 2 X 2 tables as a result of this property. Ray (1965) illustrates this method.

The general utility of the chi-square distribution in regional studies has been discussed by Mackay (1958) and Zobler (1957). A novel use of the revealed probability levels associated with different values of chi-square is presented by Kellman and Adams (1970) in their constellation diagrams of the linkages between categorized variables.

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Section 5. Barry (413–415); Hart and Salisbury (129–133; 135–136); Strahler, 1954 (9–10).

SECTION 17

ANALYSIS OF VARIANCE

The analysis of variance, in its simplest form, essentially extends the t-test of significant differences in means for two regions to the k-region case. The null hypothesis is then written as $H_0: \mu_1 = \mu_2 = \ldots = \mu_k$, and the test statistic used is based on the F-distribution. The analysis introduces an important concept in statistical inference—the partition of total variance into component parts. In the single-factor case, the two components are within-group (or region) and between-group variance. Note that there is then assumed to be no interaction between the groups.

The method assumes that the samples are drawn from normally distributed populations, and that within-group variance does not differ significantly from region to region. If the first assumption is not met, transformations can be applied to the sample values (Krumbein and Miller, 1954). The second assumption (homoscedasticity) is treated by Bartlett's test (Section 16); if it is fulfilled, the only way that a significant variation can exist from region to region is if the group means lie at different elevations. An example of a single-factor model is presented by Knos (1962) in his study of urban sectoral variations in land values. The validity of regional divisions can also be examined in this manner (Zobler, 1958; Laut, 1967; see also Section 33), as well as hierarchical classifications of settlement as expected in central place analysis (Mayfield, 1967). The technique also has considerable utility in examining sources of variation (Davis, 1971).

In addition, the analysis of variance can be extended to include other factors. For example, Murdie (1969) constructed a 6-sector X 6-zone grid for metropolitan Toronto in order to examine concentric and zonal models of spatial variation in urban social structure. The sector X zones (1st order) interaction effect could then be interpreted as indicative of the notion of nucleations within the overall patterns. This type of two-factor model has also been used by Timms (1971) and discussed by Johnston (1970).

It is clear that the number of factors could be increased, but difficulties in interpreting 1st-order, 2nd-order (etc.) interactions also increase. The value of these more complex models is great, however, as long as a firm grasp on the theoretical implications is maintained. Boyce's (1965) analysis of urban travel patterns is exemplary in this respect. A comprehensive review of different models is presented by Krumbein and Miller (1954), and the obvious relevance of the analysis for questions of experimental design is discussed by Krumbein (1953). Operator variance can also be evaluated in this context (Hill, 1968).

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Section 12. Andrews (1963).
Section 15. Maxwell (1967).
Section 16. Kellman and Adams (1970).

REFERENCE: Section 4A. Gregory (133–139); King (78–81); Krumbein and Graybill (191–221).
Section 5. Barry (410–413); Chorley (335–340); Hart and Salisbury (155–156); Strahler, 1954 (16–17).

SECTION 18

NON-PARAMETRIC STATISTICS

When measurement is carried out using nominal or ordinal scales, most parametric tests cannot be applied, so that a number of tests have been devised for this situation. Since no assumptions are made about the nature of the population distribution from which the sample was drawn, these methods are also called "distribution-free." Non-parametric methods are particularly useful when sample size is small (Haggett, 1961); in fact, regular statistical methods cannot be used with small samples unless the population distribution is known exactly. If all the assumptions of parametric methods are fulfilled and if the measurement is of the required level, the non-parametric tests are wasteful of data. The degree of this difference is expressed by the power efficiency of the non-parametric test, and the distinctions can be virtually eliminated by increasing sample size.

There are many tests in this category: Siegel, 1956 (cited in Section 4B) covers most methods. Useful reviews of the techniques are presented by Keeping (1967) and French (1971). Most parametric tests have non-parametric equivalents; for example, in testing for differences between measures of central tendency in two regions, the non-parametric test involves the median. Analysis of variance for the *k*-region case (the Kruskall-Wallis test) is illustrated by Fenwick (1965). Measures of association include contingency analysis using the chi-square statistic (Section 16) for nominal data, while correlation coefficients (Section 21) can be computed using ranked data. For example, there are two coefficients of simple correlation: Spearman's rho (Bucklin, 1966; Sternstein, 1962; Wood, 1967) and Kentall's tau (Cox, 1969; Golledge, Rushton, and Clark, 1966). A coefficient of multiple correlation is also available: Kendall's coefficient of concordance (ω).

The Kolmogorov-Smirnov test has been employed often in distribution-fitting (Section 15) when the sample size is small (Dacey, 1968). An associated statistic, D_{max}, is also very useful for those cases in contingency analysis where chi-square cannot be used because the number of cells with expected frequencies of less than 5 is large, and where reclassification would remove much detail in the classification. As in all K-S tests, the cumulative proportions are used, and the maximum difference in the two sets (D_{max}) is known to be distributed as chi-square. Johnston (1966) illustrates these procedures.

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SEE ALSO: Section 13. Amemiya (1964), McEvoy (1968); Zaidi (1968).
Section 14. Mackay (1967).
Section 15. Tanner (1959).
Section 16. Griffiths (1971).

REFERENCE: Section 4A: Gregory (181–183); King (84).

SECTION 19

GEOSTATISTICS

The term geostatistics is used to refer to two main channels of research into methods of applying linear statistics to areal distributions. The first approach, centrography, has the longest tradition, centers of gravity of the populations of the USA and the USSR attracted much interest in the first two decades of this century (Sviatlovsky and Eells, 1937). Bachi's work (1957, 1963) revitalized the subject and added important measures of dispersion (the standard distance, a bi-variate equivalent to the standard deviation) to the existing measures of central tendency. In the latter case, there was some confusion over the arithmetic-mean center and the median center (Hart, 1954), which was settled in the debate over the "point of minimum aggregate travel" (Porter, 1963, 1964; Court, 1964).

The second tradition is embedded in attempts to utilize physical laws in the study of social phenomena, evidenced in gravity models and in social physics. The latter research is associated with the work of Stewart and Warntz (1958, 1959) and Warntz's subsequent development of macrogeography (1965, 1967). The main goal of macrogeographic analysis is to examine the role of distance in understanding regularities in aggregates of socio-economic data, and to this end potential models are employed. Neft (1966) has summarized much of this work and integrated it with the measures derived in centrography. Besides descriptive statistics, Neft has also discussed measures of areal association.

Geostatistical methods are not reported frequently in the literature, although Wolpert (1967) has emphasized the value of the median center and model center of migration fields in developing appropriate parameters for the analysis of spatial flow phenomena. Finally, the solution of the Weberian "location-triangle" problem in economic geography utilizes similar approaches to those employed in geostatistics; see, for example, the contributions by Kuhn and Kuenne (1962) and Cooper (1967).

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SECTION 20

POINT PATTERN ANALYSIS

The spatial distribution of points in a study area is analyzed by a set of techniques that were originally developed in plant ecology (Greig-Smith, 1964). The general format of these methods is that the observed set is compared to the theoretical set of points that would be generated by one of a number of probability processes. For example, the Poisson distribution can be used to generate an expected point set that is randomly distributed, while the negative binomial function is thought to generate a clustered set. The technical details of applying these functions can be appreciated by reference to Harvey (1967). A useful listing of most of these probability models, with comments on parameter estimation and the availability of published tables, is given by McConnell (1966).

The primary question in this research area is: to what extent can the observed set of points be described as regular, random, or clustered? There are two approaches to answering this question: 1) quadrat counts and 2) point-to-point distance measurements. In the first case, a grid is laid over the study area, and the frequency of cells that contain 0, 1, 2, ... points is tabulated (Getis, 1964; Harvey, 1966). This observed frequency distribution can then be compared to a theoretical one, derived perhaps from the Poisson process, in which case randomness would be tested. Under certain assumptions concerning the independence of point allocation to cells, a chi-square test is applicable. We may note two problems in this method: *a*) the parameters used in the probability functions generally involve some estimate of the density of points per unit area; and *b*) the quadrat itself can be of varying size, so that different conclusions may be reached about the randomness of the distribution. In the first case, it is clear that an adequate specification of the study area boundary is necessary (Hsu and Tiedemann, 1968), although recent work in pattern recognition (Hudson, 1969) may identify sub-areas within a larger region in which the measures can be applied.

Distance measures have been developed from nearest-neighbor statistics used in ecology (Clark and Evans, 1954) and generalized as order-neighbor distances by Dacey (see Dacey and Tung, 1962, who also describe the regional-neighbor approach). The method involves computation of the mean distance and associated variances for each order, and comparison of these to expected distances. For example, under the assumption that the first-order distances are drawn from a normal population, a density dependent expected mean can be derived. Randomness can then be tested using the standard normal curve. Alternatively, a ratio of observed and expected mean distances can be computed (R: the nearest neighbor measure). The range of R is from 0, indicating complete clustering, to 2.14 for a hexagonal, most regular, pattern. If $R = 1$, the distribution is said to be random. This measure is illustrated in the studies by King (1962) and Barr *et al.* (1971).

Only a sample of the literature is given in the listing; see King's survey of the field. Although settlement patterns have been the main focus of empirical studies, it is clear that any distribution that can be represented as a set of points can be evaluated using these methods; see, for example, the case for drumlins, Smalley and Unwin (1968) and Trenhaile (1971). The implicit hope that analysis of point patterns would lead to greater understanding of processes generating the patterns has not been fulfilled. Indeed, Harvey's work (1966) indicates that any attempt to understand process from form analysis is likely to be self-defeating. In this sense, then, the great amount of research effort expended in the analysis of point patterns has not had any positive substantive feedback for the discipline. On the other hand, Dacey's contribution to spatial stochastic process modeling is exemplary in pointing up the evolution of geography toward a theoretical discipline.

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SEE ALSO: Section 11. Morrison (1970).
Section 17. Mayfield (1967).
Section 18. Dacey (1968).

REFERENCE: Section 4A. King (87–92; 98–109).

SECTION 21

CORRELATION

In correlation studies the emphasis is on the interdependence between two or more variables, with no implications of functional relationships. In terms of the matrix organization of geographic data, the researcher would be concerned with comparing two or more columns–examining the covariations of two or more spatial distributions. It is worth noting that this measure can be derived directly from probability theory, covariance has a specific interpretation in this respect. If two random variables, say X and Y, are independent, then the variance of their sum is equal to the sum of the respective variances: $Var(X + Y) = Var(X) + Var(Y)$. This expression is modified by the addition of the covariance term $(Cov(X,Y))$ for the case of dependent random variables. Note that the covariance of a variable with itself is the variance. $Cov(X,X) = \sigma_X^2$. The population correlation (ρ = rho) between X and Y is defined as $\rho = Cov(Z_X, Z_Y)$. where Z_X and Z_Y are the standardized variables for X and Y: eg. $Z_x = (X_i - \mu_x) / \sigma_x$. Alternatively, one can write $\rho = Cov(X,Y) / \sigma_x \sigma_y$.

The sample correlation coefficient (r) expresses the degree of association between two variables, and it has a range from −1 (a perfect negative or inverse relation) to +1 (perfect positive or direct association). The coefficient, then, expresses both magnitude and direction of association. On the assumption of a random sample from a bivariate normal population distribution, the research hypothesis of a significant association may be tested (usually, Ho . $\rho = 0$) by means of a t-test. It is also possible to test for significant differences between the correlation coefficients of two regions (Ho : $\rho_1 = \rho_2$) by means of a transformation and reference to the standard normal distribution; see Thomas, 1962 (cited in Section 27), who uses the test for different time periods. The square of the correlation coefficient, r^2, is known as the coefficient of determination, and it measures in a general sense the proportion of total variance that the two variables have in common. It is more correctly derived as a particular ratio of components of variance in regression problems (Section 27), but in empirical studies it is usually reported as a percentage, or as the level of explained variation.

Thus far only the bivariate situation has been described. With more than two variables, two alternative approaches are possible. First, the researcher may wish to compute all possible pair-wise correlations from a set of m attributes. In this way, a new matrix of intercorrelations (order $m \times m$) can be derived for further analysis, as by means of factor analysis (Section 24); see, for example, Berry (1963) and Norman (1969). Second, a multiple correlation analysis may be carried out. In this case, the coefficient of multiple correlation (R) indicates the degree to which two or more variables relate to a third. For example, $R_{Y.XZ}$ is the multiple correlation coefficient of X and Z with Y. R^2 is defined as the coefficient of multiple determination.

In multiple correlation studies it is also instructive to measure the associations between any two variables, with the influence of other variables held constant statistically. This is achieved by the partial correlation coefficients, which are usually denoted as follows: if three variables are involved (eg. $R_{Y.XZ}$), then there are three partial correlation coefficients: $r_{yx.z}$, $r_{yz.x}$ and $r_{xz.y}$. The first, indicating the association between X and Y with the influence of Z held constant, may be different even in sign from r_{yx}, the simple correlation coefficient. Note that partial correlation coefficients have been used in two major contexts—in causal analysis (Section 23) and in step-wise regression procedures (Aangeenbrug, 1968; see also Section 27). A standard text on correlation and regression methods is Ezekiel and Fox, 1959 (cited in Section 4B).

In geographic applications it seems that most studies do not necessarily fulfill the assumptions for the testing of hypotheses. Instead of using a complete inferential framework, then, correlation studies appear to be mostly descriptive; for example, in comparing different measures of manufacturing (Alexander and Lindberg, 1961; Leigh, 1969; Wong, 1968). Also, it is useful to examine correlations to further understanding of any regression analysis that is subsequently carried out (Ambrose, 1970). A more explicit representation of this approach is seen in the diagraming of correlation bonds (Melton, 1958). Smith (1965) illustrates an interesting application of correlation analysis. The matrix is transposed (reversal of rows and columns) and correlations computed between areal units (towns in the study) in order to classify them by means of cluster analysis (Section 32).

Difficulties in applying this technique in geographic research stem from the different sizes of areal units, as might be expected from previous statements made about the influence of the aggregation effect. One solution–weighting the calculations by the areas involved–was proposed by Robinson (1956). Thomas and Anderson (1965) present an inferential framework on more general grounds than those used by Robinson, but there are still a number of assumptions involved that inhibit a general solution. More importantly, the shape and orientation of two-dimensional collection units cannot be handled by such methods. Curry (1966) raised this question and generated much research on the way in which administrative units effectively filter out characteristic frequencies in the data. Spatial filtering methods attack this problem directly (MacDougall, 1970), and references in Sections 22, 29, 30, and 31 are all pertinent to the problem.

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SEE ALSO: Section 1. McCarty and Salisbury (1961).
Section 10. Monmonier (1971).
Section 13. Conkling (1963).
Section 15. Gibson (1970).
Section 17. Caine (1968).

REFERENCE: Section 4A. Gregory (167–181); King (130–135; 141–152); Krumbein and Graybill (234–239; 295–299).
Section 5. Barry (415–417; 420–421); Chorley (349–356); Hart and Salisbury (150–152; 155); Strahler, 1954 (23–24).

SECTION 22

ECOLOGICAL CORRELATION

Data on individuals, aggregated into a set of basic areal units such as counties, form the basic input for many geographic studies. Variations between areal units, analyzed in a number of ways, can then be used to make inferences about area structure or the structure of characteristics of areas (Beshers, 1960). This would appear to be a primary level of description (no matter how advanced the techniques employed) concerning spatial distributions or arrangements, and it naturally does not allow the researcher to make any statements about processes thought to lead to the observed patterns. Process, in most studies, is interpreted as the outcomes of sets of decisions, made by individuals or groups, with differentiated locational impacts on an existing spatial distribution.

The question that arises at this juncture is: can inferences about individual-level behavior (≡ process) be made from observed variation in terms of unit areas? This is the problem of ecological correlation. The recent volume edited by Dogan and Rokkan (1969) is particularly recommended for a series of articles on aspects of the problem (Alker, 1969; Valkonen, 1969). Certain assumptions are necessary to allow the transfer of inferences across the scale difference, but Valkonen illustrates how this appears to be especially a function of areal unit size—the larger the area the higher the ecological variation and the higher the ecological correlation compared to individual correlation. Clearly, the effect of different-sized units is methodologically related to the problem of modifiable units (Section 21) and to filtering techniques used in trend analysis (Tobler, 1969, cited in Section 30) and in space series analysis (Casetti, 1966, cited in Section 31). Substantive interpretations, however, differ considerably—at least as reported in the literature, A careful sorting out of theoretical and methodological elements is demanded in reading this literature.

In order to test adequately the applicability of this idea, a strategy is needed whereby individual data, referenced by locational coordinates, is available (a rare situation) and different aggregation factors can be applied to it. Goheen, 1970 (cited in Section 26) has presented a series of tables of factor structures (Section 24) for the city of Toronto over four time periods (pp. 139, 158, 174, 207), based on individual and areal aggregations of the same basic information. At the small scale used in his study, Goheen concluded that there was little difference in the results, i.e. ecological and individual correlations were relatively congruent.

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SEE ALSO: Section 21. Curry (1966).

SECTION 23

CAUSAL ANALYSIS

In empirical studies, the control of independent variables possible in laboratory situations (holding other factors constant) is not feasible, so that a particular methodology to establish causal connections between factors has been derived, largely due to the work of Blalock, 1964 (cited in Section 4B). The technique is based upon the partial correlation coefficients; for example, when a causal link between factors X and Y is expected and $r_{xy.z} = 0$, the product $r_{xz} \cdot r_{yz}$ can be used to predict r_{xy} as an expected simple correlation. This expected correlation can then be compared to the computed coefficient, providing an indication of the adequacy of the causal model. The lack of any causal connection between factors is expressed by $r_{xy} = 0$, and this can be compared directly to the observed correlation.

Cox (1968) has illustrated this technique in a study of suburban voting behavior, in which the inadequacies of an initial model are treated by the development of a better causal situation with different linkages. Criticisms of this article point out two major difficulties in applying the method in geographic research: 1) Taylor (1969) indicates that the researcher must apply extreme caution in the statement of relevant causal linkages; and 2) Kasperson (1969) shows that the transition from theoretical background to empirical operationalization has to be carefully evaluated.

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KASPERSON, R. E. "On Suburbia and Voting Behavior," *Annals,* AAG, Vol. 59, 1969, pp. 405–411.

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SECTION 24

FACTOR ANALYSIS AND PRINCIPAL COMPONENTS ANALYSIS: AN OVERVIEW

A variety of methods can be described under the general heading of factor analysis. They are all concerned with the relationships that exist between sets of variates; in other words, interdependencies among a set of attributes are the major concern. This has already been noted in the case of correlation studies, where an underlying concept was the extent to which two or more variables shared a common amount of the total variance between them. Factor analytic techniques seek a further answer to questions raised in this context: given a large number of variables, is it possible to describe composite variables, fewer in number, perhaps uncorrelated with each other, that summarize the known degree of redundancy that exists in the larger set? Factor analysis does provide an adequate answer to this question and is one of a number of techniques usually grouped together in multivariate analysis. The sampling assumptions are rather more restrictive for these analyses, and geographic research has tended to shy away from any inferential questions in this context. Thus factor analytic studies tend to be largely descriptive of regional structure.

Two forms of analysis dominate the field: principal components (Section 25) and factor analysis (Section 26). The differences between the two approaches have been adequately stated by King in his text, but they are rarely made explicit in the literature. Indeed, the particular model used is not always specified, and on occasion the possible options for factor analysis (e.g. rotation) are applied incorrectly to the principal components model. In the latter case, it seems clear that the researcher works from data towards the specification of a theoretical structure for the domain he is studying. The analysis itself is simply a mathematical orthogonalization (a process of making new variables independent of each other) of the existing set of attributes, and in the execution of the method the researcher may be lucky to give some empirical meaning to the derived mathematical artifacts or components. In contrast, a theoretical model should be tested by factor analysis: does the model agree with the data? If so, estimates of the parameters can be made.

Even with these strict differences between the two models, it is clear that some prior evaluation of the likely outcome of applying any technique should be made. In this case, relevant questions are: how many factors can be expected; what types of factors, seen as combinations of the original variables; how should the original attributes be defined in order to answer the research questions posed; what sort of relationships can be expected among the new factors; and, what are the most meaningful communality estimates? It is particularly the last two questions that serve to differentiate the two models. In principal components analysis, only orthogonal components are produced and there would seem to be little point in requiring non-orthogonal solutions. As we shall see, this is not the case in factor analysis, where oblique solutions may be more relevant, theoretically. Communality estimates refer to the extent that the particular factor solution accounts for the variance of any original variate. Operationally, these values are placed in the principal diagonal of the intercorrelation matrix, which is the stage at which analysis usually commences. In the principal components method, 1.0 is the element value. In other words, the variance is emphasized. (Since the attributes are usually standardized before the intercorrelations are computed, they have zero mean and unit variance.) Factor analytic models have a different orientation, and communality estimates used in geographic research often use R^2, the coefficient of multiple determination of the variable in question with all others in the set. Thus, these latter models stress the covariance aspects of the problem.

The steps in the analysis may be outlined in brief for the principal components solution, and then the differences for the factor analytic model can be stated: As we have already indicated, the attribute matrix (order n places by m attributes) is usually standardized, and the matrix of zero-order correlations is computed ($m \times m$). Analysis of the latter matrix proceeds by deriving the associated characteristic equation (a polynomial of the same order as the matrix) whose roots define the latent values or eigenvalues. For each eigenvalue, an eigenvector can be computed and when standardized the matrix ($m \times p$) describing factor structure can be derived (the standardized eigenvector associated with eigenvalue k is entered into the matrix as column k). In the case of principal components, $p = m$ in the complete solution. More commonly, however, p is taken as some number of factors less than the total number, and criteria for doing this are inexact. Included are either those factors with eigenvalues greater than 1.0 or those that account for more than 5% of the total variance (measured as the proportion of any eigenvalue to the total variance, which in this case equals the sum of the eigenvalues). The elements of the columns of the factor matrix, often called factor loadings, can be used for making a geographic interpretation of the component. This final stage of the analysis is usually aided considerably by computing the component scores—the location of each of the individual unit areas on the new variables, computed as a direct linear function. When these scores are mapped, it is usually possible to give some empirical interpretation to the component. Finally, we should note that the factors are extracted sequentially so that the first one accounts for as much of the variance as possible; the second then accounts for as much of the residual variance as possible and in addition is orthogonal to the first; and so on, for remaining factors.

Factor analytic models differ from the principal components solution in several respects. The statement of the model itself implies a radically different philosophy. Factor analysis tests a model that is based on the idea of partitioning the total variation into component parts: the common factor variance (the variance of Factor I + variance of Factor II + . . . etc.), the error variance (usually associated with measurement), and non-error variance specific to variables, excluded from the factor structure, or unique variance. In terms of an understanding of theoretical factors underlying some set of attributes, this would appear to be a realistic approach. This partition of the total variance also explains why the question of communality estimates is so important in factor analysis. It also explains why factor scores are only estimates of scale scores, compared to component scores. Finally, the idea of rotation of the factors is feasible and is theoretically of value in factor analysis. Rotation is carried out by changing the factor loadings so that some criterion, such as simple structure, is achieved or a best fit from a theoretical point of view is obtained. Note that the final communalities are not affected by rotation, although eigenvalues are.

The references include several articles on the utility of factor analytic models, especially with reference to the social geography of the city (factorial ecology); see, especially, Berry, 1971, Rees, 1971; Janson, 1969. A general review of multivariate analysis is presented by Thompson (1970), while applications of this approach are given for physical geography by McCammon (1966) and Mather and Doornkamp (1970). A general problem is that factor analysis assumes that relationships between variables are linear, and thus their effects are additive. Many correlation and regression studies, particularly of residential structure of urban areas, have shown that non-linear relationships are more likely. The relative value of using factor analysis against traditional methods such as correlation can then be addressed (Meyer, 1971). Evidently, many of the conclusions in such an inquiry will relate to the level of generality that the researcher wishes to achieve.

In Section 4B, the following texts are useful for factor analysis: Harman (1960); Lawley and Maxwell (1963); Horst (1965); and the two articles by Cattell (1965), particularly for the relative advantages of oblique and orthogonal solutions. Matrix algebra is treated in Fuller (1962), Hohn (1964), and Horst (1963), also cited in that section.

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SECTION 25

PRINCIPAL COMPONENTS ANALYSIS

Principal components analyses of the attribute matrix, defined as in Section 24, have been carried out for a number of purposes. An emphasis on structure, by examining the loadings matrix and associated mapped patterns of component scores, is illustrated in the studies by Blaikie (1971), Carey (1966) and Robson (1969). Another common procedure is to utilize the component scores as input to classification or regionalization schemes (Section 32). Moser and Scott (1961) and Yamaguchi (1969) present typologies of urban centers, while Ahmad (1965) and King (1966) are concerned also with the regional aspects of urban system classification. The value of the approach for the study of economic regions is clearly stated by Brown and Trott (1968). It is also possible to re-analyze the scores obtained from an analysis, looking for higher order component solutions. This is sometimes accomplished by rotating the axes to an oblique solution, and then forming an orthogonal solution for the correlated scores. An alternative method is to analyze only a sub-set of the original number of unit areas. The study by Jones and Jones (1970) represents this approach, since they performed a principal components analysis on a sub-set of the towns used by Hadden and Borgatta, 1965 (cited in Section 26).

The techniques as described thus far concentrate on forming composite variables from the matrix of inter-correlations; i.e. the method essentially analyzes differences between the columns of the matrix. The general name for this approach is R-mode analysis. It is possible, however, to carry out the analysis for the rows of the matrix, in which case a Q-mode analysis applies. The result is "composites of unit areas," or rough indications of types of area, such as farm types (Henshall and King, 1966). It is also possible to use other measurement scales in factor analytic studies. The presence or absence (i.e. nominal scaling) of attributes, coded as a series of 1–0 measures, can be analyzed directly (see Berry, Barnum, and Tennant, 1962, cited in Section 26) or through an inter-correlation matrix for such binary data, based on the phi-coefficient (Henshall, 1966). A similar phi-matrix was used by Garrison and Marble (1964) in their study of grouping tendencies among transportation nodes, since it was derived from the 1–0 connection matrix. Interaction matrices with elements representing actual flows between places can also be analyzed (Simmons, 1970) to produce common sets of origins and destinations.

One under-utilized value of principal components analysis is the very fact that the resulting scores for the set of areas are independent of each other. As will be seen from the discussion of independent variables in regression analysis in Section 27, the use of component scores would alleviate many problems of multicollinearity in general linear models. This approach is illustrated in the studies by Wong (1963) and Riddell (1970).

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REFERENCE: Section 4A. King (166–184).

SECTION 26

FACTOR ANALYTIC STUDIES

It is clear that many similar studies to those reported in the preceding section are cited here. For example, the use of factor scores as basic input for regionalization is described by Berry (1965) and Hadden and Borgatta (1965). Indeed, most of the studies are descriptive, rather than fulfilling the role that factor analytic models are presumed to have in the testing of theory. However, a fairly wide range of problem areas is represented, so that the general features of the analysis can be related to the interests that any student might have. The use of factor scores in general linear models is represented by the studies by Hartshorn (1971) and Lowry (1970). While these studies employ the factors as independent variables, it is obvious that they could be treated as dependent variables. One particularly interesting form of the linear model (trend-surface analysis—see Section 30) has been employed to partition the total variations of the factor over space into regional and local components (Goheen, 1970). Interaction matrices have also been analyzed using factor analysis, and if the results are then used in regionalization schemes a set of functional regions is obtained (Goddard, 1970; Illeris and Pedersen, 1968).

One exception to the generally descriptive use of factor analysis in geographic research is seen in the study by Jeffrey, Casetti, and King (1969). Individual time-series (Section 31) were available for each city in a sample of midwestern metropolitan areas. The research hypothesis was that the set of series contained three levels of variation: 1) factors operating throughout the system; 2) factors common to cities in predetermined groups (on the basis of similarities in cyclical behavior); and 3) a factor unique to each city. Bi-factor theory is applicable in such a context, and for the unemployment series used in the study the hypothesis was substantiated. It is apparent that studies of this nature are important in testing evolving geographic theory, which must incorporate multivariate components of spatial problems. This study underlines the fact that analytical models of the types described in this and the prior section could be extremely valuable in such contexts, since the concept of parsimony in empirical research situations is incorporated into techniques used for testing theory.

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Section 21. Berry (1963); Horton (1968).

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SECTION 27

REGRESSION

In contrast to correlation studies, functional relationships form the focus of interest in regression analysis. The most general statement of such a relationship is $Y = f(X)$, the variable Y is said to be dependent on variations in X. Y is known as the dependent or response variable, while X is an independent (or control, predictor) variable. The importance of a strong theoretical framework for posing regression questions must be realized at the outset, for example, what is the form of the relationship, what are the natural process mechanisms at work, what are possible increments of Y given unit increases in X, what is the value of Y when $X = 0$? The first question raises the distinction between linear and non-linear models.

The simple (i.e. bivariate) linear model is usually written $Y = \alpha + \beta X + \epsilon$. α and β are constants of regression. α is the value of Y when $X = 0$, the Y-intercept. β is the regression coefficient, indicating the slope of the line, a direct relationship being indexed by a positive value for β, while a negative value shows an inverse relationship. The error terms (ϵ) represent that part of the variability in Y not accounted for by the linear relationship. It is assumed that these deviations are independent and normally distributed, with zero mean and unknown variance. In geographic research the mapped pattern of such residuals from regression may indicate that the expected random pattern is not found, so that this assumption in the model is not fulfilled Non-linear relationships are often expressed in a linear form by means of transforming the original values for either X or Y or both, usually by logarithms For example, an exponential relationship (semi-logarithmic) can be written as $\log Y_i = \log a + \log b\ X + \epsilon$, while a power-(or logarithmic) function $(Y = aX^b)$ can be expressed as $\log Y = \log a + b \log X + \epsilon$. If these simple transformations do not suffice, than a polynomial regression may be carried out (Section 30).

Whatever the chosen form of relationship, the paired observations for each areal unit can be represented in a graphical format as a scatter diagram, usually with Y plotted on the ordinate and X on the abscissa. As indicated above, the association between X and Y is written as a mathematical expression, so that in the elemental form this appears to be a case of fitting a curve to the set of points. At such a stage, then, there are no inferential questions involved, but in choosing one of the infinite number of possible lines that could be drawn through the set the concept of an averaged relationship is employed. It is assumed that there is a sample distribution of Y values for each X, which is usually regarded as fixed, for example, for the fixed x_i there is a range of values of Y_i, and the observed value (y_i) is regarded as randomly drawn from that distribution. The set of Y_i distributions are assumed to be equally variable for the set of X values. For any X we assume that differences in the Y values are due to sampling error, and seek to reduce the effect of this variation by taking the mean of Y *given* X. In this sense the method used results in an averaged regression line. For a full discussion of the assumptions in regression analysis, see Poole and O'Farrell (1971).

The notions described above can be realized with the criterion that the deviations of points from the regression line should be minimized, this is achieved by the method of least squares. The sum of squares of the deviations (error terms) is used rather than the absolute value sum because it can be differentiated to find its minimum. The method also results in the maximum likelihood estimates for the regression constants, and it is easily extended to the non-linear case. The two normal equations produced are solved for *a* and *b*, substituting these values in the equation results in a set of estimated values of the dependent variable (Y_e) which can be compared to the observed values (Y_i). For worked examples, see King (Reference, Section 4A, pp. 121–122; 135–139).

The concept of partitioning the total variation of the Y values into component parts is again used in regression analysis. The total sum of squares, $\Sigma(Y_i - \bar{Y})^2$, for the dependent variable can be thought of as attributable entirely to random effects when treated by itself. However, in regression analysis, some of this variation is due to the known relationship between Y and X, and an expression for the sum of squares due to regression (explained variation) is derived, $\Sigma(Y_e - \bar{Y})^2$. The difference between total and regression sums of squares, then, is the unexplained variation, $\Sigma(Y_i - Y_e)^2$.

Two related questions may be raised at this point: 1) how important is the relationship between X and Y, and 2) how good is the fit of the line? The first question is treated by forming the ratio of the regression sum of squares to the total sum of squares, which is the coefficient of determination (r^2) described in Section 21. The square root of this ratio is, of course, the correlation coefficient. We note here that the regression and correlation coefficients always have the same sign. The second question really asks how large, on the average, are the deviations? The standard error of estimate answers this question, providing an indication of the variability of the scatter of points about the regression line and allowing the construction of confidence intervals or error bands. A standard error for the regression coefficient is also derived, and can be used to test the research hypothesis that the slope of the line is significant (Ho : $\beta = 0$). Naturally, a properly designed sample is necessary for the application of any inferential tests.

The simple linear model generally has limited value in most geographic research situations, unless it is suspected that one factor is the main determinant of observed variations in the dependent variable. The general linear model is more commonly employed, and it may be written as follows: $Y = \alpha + \sum_{i=1}^{m} \beta_i X_i + \epsilon$, where ϵ is defined as for the simple case. For example, with three independent variables, the model is $Y = \alpha + \sum_{i=1}^{3} \beta_i X_i + \epsilon = a + b_{Y1.23}X_1 + b_{Y2.13}X_2 + b_{Y3.12}X_3 + \epsilon$. It is clear that the regression coefficients are expressed in a different manner from the simple model; for this analysis they are known as partial regression coefficients. The concept of a partial correlation coefficient was introduced in Section 21. Similar reasoning applies to partial regression coefficients—the rate of change in Y for a unit change in any independent variable is computed, holding constant the effects of the other independent variables statistically. As they stand, partial regression coefficients cannot be compared since their values are affected by the particular metric employed. Normally, then, the researcher computes standardized regression coefficients; for example, the first partial regression coefficient in the three variable case above can be standardized as a beta coefficient, $\beta_1 = b_{Y1.23}(S_1/S_X)$. (See King, p. 140.) Beta values may be compared directly in order to evaluate the relative importance of each independent variable.

Least squares methods are also employed in fitting a plane (for the two independent variable case) or a hyperplane (otherwise) to the scatter of points. In addition, most modern computer programs use matrix solutions of the sets of normal equations; the texts by King and by Krumbein and Graybill have good discussions of these methods. The overall significance of the multiple regression can be tested using an analysis of variance approach, and significance tests (using the *t*-statistic) are available for individual partial regression coefficients, as well as for differences between any two coefficients. Some of the problems associated with this technique have been discussed previously; for example, the question of modifiable units in Section 21. Spatial autocorrelation is taken up in Section 29, and the inclusion of regional effects, i.e. groupings of units, is described in Section 28 (Analysis of Covariance). A further problem is that the model assumes independence in the predictor variables. Lack of independence is known as the problem of multicollinearity, and we have already noted that one value of a principal components or factor analysis of a set of variables is that a number of uncorrelated variables result.

The listing of references clearly shows the extent of the application of regression techniques in geography. Almost every sub-field including cultural geography (Sopher, 1968) is represented. Besides any particular methodological inquiry, then, the references can be used to examine theoretical constructs. For example, the role of the distance factor in gravity models is illustrated by Helvig's (1964) study of truck movements, while there are many other references to the frictional effects of distance in the cited articles dealing with population density and land value patterns in cities, or in the field of internal migration. It is often useful to follow through an author's use of several techniques, placing them into perspective (Blaikie, 1971). The use of regression techniques in fitting model parameters has already been described in Section 15, but Casetti, King, and Jeffrey (1971) provide a further example. The partial correlation coefficients are also employed in step-wise regression procedures, in which a sub-set of the total number of independent variables is chosen in order of importance of explaining the variability in the dependent variable. This method is illustrated in the studies by Reed (1967), Brunn and Hoffman (1970), and Olsson (1965).

The mapping of residuals from regression was originally suggested by Thomas (1960) as a means for evaluating other potential independent variables which had not been included in the original model formulation. As suggested above, a non-random pattern of residuals really means that the assumptions of the technique are not fulfilled, and it may indicate that non-linear relationships are present, or that the model itself should have an autoregressive structure built into it. Certainly, any application of inferential tests would be inappropriate. Residual maps are still employed frequently in the literature, especially in a standardized form (Logan, 1964; Mueller, 1970).

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Section 11. Griffiths and Ondrick (1968); Haggett (1964).
Section 12. Yeates (1965).
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Section 15. Berry and Tennant (1965); Mackay (1958); Moore (1971); Thomas (1967).
Section 16. Brush and Gauthier (1968); Ray (1965); Szabo (1965).
Section 20. Trenhaile (1971).
Section 21. Curry (1966).
Section 25. Logan (1970); McConnell and Horton (1969); Reynolds and Archer (1969); Riddell (1970); Schwind (1971); Wong (1963).
Section 26. Berry, Barnum and Tennant (1962); Bourne (1967); Cox (1969); Goheen (1970); Hartshorn (1971); Hodge (1965); Kissling (1967); Lowry (1970); Simmons (1964).

REFERENCE: Section 4A. Gregory (185–208); King (118–129; 135–141; 154–158; 162–163); Krumbein and Graybill (223–247; 277–295).
Section 5. Barry (417–419); Chorley (340–348; 366–377); Hart and Salisbury (153–155); Strahler, 1954 (18–23).

SECTION 28

ANALYSIS OF COVARIANCE

In analysis of variance models (Section 17) the groups may not be randomized in terms of the criterion variable, and if this is the case pre-treatment differences are said to exist. The original use of the analysis of covariance was to take out such pre-treatment differences. As employed in geographic research, however, a different interpretation is made. For example, Thomas (1960) studied the differential growth of suburban areas in Chicago, and it appeared that suburbs located in certain sectors had higher growth rates than those in other sectors. A sectoral effect could then be hypothesized, and the inclusion of this into the model should increase the level of explanation. Regional effects are obviously of the same genre. Note that in essence this factor is a nominal scale, and as such it cannot be included in ordinary regression models. The use of dummy variates, where a unit area would have a value of 1 if it was in the sector or region represented by the variable, and 0 otherwise, is one way in which attempts have been made to account for the effect. There are, however, certain technical difficulties associated with the use of dummy variates, see especially Lansing and Morgan, 1971 (pp. 314–343).

The analysis of covariance is more effective in such situations. It combines the analysis of variance and regression techniques, and is thus a rather powerful inferential model. To compensate for this for the applied researcher, however, the assumptions that must be fulfilled are rather restrictive. The analysis tests for the influence of groups (sectors, regions) on the level of explanation in the total regression model. Two assumptions must be fulfilled before the test can be made: 1) the variances in the dependent variable do not differ from group to group (Bartlett's test, Section 16); 2) the group regression lines are parallel (Ho : $\beta_1 = \beta_2 = \ldots = \beta_k$, for k groups), for which the test statistic is the F-distribution. If these assumptions are fulfilled, the only way in which the group regressions can differ is by having Y-intercepts at different elevations; this is tested by an F-statistic with reference to a common regression line. If the null hypothesis is rejected, a regional effect can be inferred, in the light of acknowledged functional relationships.

The analysis of covariance would appear to be an important method whereby regional effects can be evaluated (King, 1961; Kariel, 1963). It is clear, however, that the usual caveat of correctly designed sampling frameworks is needed in order to satisfy the assumptions.

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Section 27. Bogue and Harris (1954); Kariel (1963); Thomas (1960).

REFERENCE: Section 4A. King (163–164).
Section 5. Hart and Salisbury (156–157).

SECTION 29

SPATIAL AUTOCORRELATION

If the assumptions of the general linear model described in Section 27 are fulfilled, then the expected spatial pattern of residuals from regression (the error terms in the model–assumed to be independent and normally distributed) is a random one. Any lack of randomness is called a spatial autocorrelation effect. The techniques used to study this phenomenon need apply not only to residuals, however. The distribution of any one of the original values may be affected by its neighboring points in any direction. In this sense, the geographic problem differs considerably from the usual serial correlations met with in analyzing data organized along one dimension, i.e. time-series (Section 31) where the value at a point is dependent only on previous values (see particularly Curry, 1970).

In this section, methods which are not directly based on concepts from time-series analysis are referenced, although it should be noted that there are many linkages between these methods and those employed in the analysis of trend-surfaces (Section 30) and spatial series (Section 31). The original work in this area was by Geary (1954), who was concerned with the non-randomness of data values for neighboring counties as well as the possible effects on regression analysis. In order to estimate this effect, Geary devised a contiguity ratio (*C*), which, as the name suggests, incorporated the number of connections between any county and others in the study area. If C = 1, the distribution is regarded as random (i.e. no autocorrelation), and a sampling theory based on the normal distribution was derived. A distinction is made between a randomization approach, in which the set of areal units is regarded as the universe, and the normal approach in which the units are assumed to be a random sample from a parent population that is normally distributed. This distinction is carried over to more recent work in the differences between non-free and free sampling methods and theory.

Dacey (1968) generalized Geary's work with special reference to the regression residuals problem. Over- and under-predictions were categorized into Black and White, the probabilities of BB, BW, and WW joins were evaluated, and contiguity could then be tested with reference to the standard normal curve. The arrangement of points per cell, as used in point pattern analysis (Section 20), can also be evaluated for randomness, using a non-parametric test. Recent work by Cliff and Ord has considerably extended this earlier research, placing the results into a larger inferential framework and deriving the sampling distribution of a spatial autocorrelation coefficient under different sampling schemes. Interesting by-products of their efforts are 1) the possibility of employing the statistic in regionalization schemes, and 2) that size of areal units might be specified in order to minimize the dependence of data values on neighboring points.

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SECTION 30

TREND SURFACE ANALYSIS

A particularly interesting form of the general linear model described in Section 27 has been developed largely by geologists. Krumbein and Graybill (1965) specify the model as follows:

$$T(U_i, V_j) = \tau(U_i, V_j) + \epsilon_{ij}, \text{ where } T(U_i, V_j) \text{ is}$$

the observed value of a mapped variable at orthogonal grid locations U_i and V_j, $\tau(U_i,V_j)$ is the trend surface component, and ϵ_{ij} represents a random error term. The latter two terms are also called regional and local components in the literature—once again, a partition of variation.

Thus, we are dealing with a multiple regression problem, with two independent variables. Polynomial regression is often employed to estimate the coefficients (Tobler, 1964). The first-order polynomial surface is simply the linear trend ($T(U_i,V_j) = \beta_{00} + \beta_{10}U_i + \beta_{01}V_j + \epsilon_{ij}$), while the second-order (quadratic) surface would then be written as

$$T(U_i,V_j) = \beta_{00} + \beta_{10}U_i + \beta_{01}V_j + \beta_{20}U_i^2 + \beta_{11}U_iV_j + \beta_{02}V_j^2 + \epsilon_{ij}.$$

Note that the computation of the β-coefficients is considerably eased if the data are referenced by means of equally-spaced intervals, in which case tabled values of orthogonal polynomials can be used. If cyclic fluctuations in the trend are suspected, double Fourier series can also be computed directly in a similar manner (Harbaugh and Sackin, 1968, see also Section 31). With irregularly spaced data values, least squares methods are employed to estimate the coefficient values. Many of the listed references are for computer programs which are based on least squares; see the review by Harbaugh and Merriam (1968).

Applications of this technique have increased greatly in geography in recent years (Chorley and Haggett, 1965, Norcliffe, 1969). For example, trend surfaces have been computed for residuals from regression (Fairbairn and Robinson, 1967) and used in the comparison of intra-regional structures (Haggett, 1967; but compare Macomber, 1971). In physical geography the study of erosion surfaces has been resurrected by trend analysis (See King, 1969; Rodda, 1970, Smith *et al.*, 1969, Thornes and Jones, 1969, and the critical review by Tarrant, 1970). The relationships between trend analysis and smoothing (filtering) functions, and their use in map comparisons, are well explained by Tobler (1969).

Several problems are apparent. First, the trend will, of course, be computed only for that set of locations included in the study area. Selection criteria for the latter are therefore most important. Second, there are difficulties in establishing the significance of a trend at order k, say, compared to order $k + 1$ (Chayes, 1970). The usual procedure is to form a ratio of explained variance to total variance (reduction in total sum of squares), perhaps testing for significance using the F-distribution (Unwin, 1970). Howarth's (1967) experiments with random data, however, indicate that this procedure may be misleading for lower-order surfaces (up to the cubic).

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Section 5. Barry (422); Chorley (357–364).

SECTION 31

TEMPORAL AND SPATIAL SERIES

The analysis of a series of events ordered in one dimension, usually time, has long attracted the attention of econometricians, statisticians, and mathematicians. In the latter case, oscillations are treated as periodic functions and there is little attention to the error component in a time series, such as that for precipitation over a year. Statistical considerations are important when prediction is involved and error estimates have to be made. A typical procedure in time-series analysis would be to remove the trend, (e.g. by regression methods—Barrett, 1966), establish any seasonal or cyclic effect, and then analyze the residual error terms. Runs tests, based on a series of + and – terms with respect to the median, can also be carried out for randomness. Thus, the concept of a partition of the total variation of a set of values is also applied here.

The cyclic component is usually analyzed by harmonic or Fourier series, a mathematical expression consisting of terms containing sines and cosines. The assumption that is generally made in this case is that the series is stationary—statistical properties of the distribution (moment measures and autocorrelation functions) constant throughout the range of the data. Single Fourier series are used to describe periodic time series, the shape of the curve depending on the number of terms used and the values of the coefficients in the terms. The method appears to be particularly suitable in precipitation climatology (Horn and Bryson, 1960; Sabbagh and Bryson, 1962). Bryson and Dutton (1967) summarize the value of the approach, at first major irregularities are removed by smoothing the series, using the technique of moving averages. The remaining finite set of averages can be completely described by a harmonic function, and the number of cycles determines the order of the series. In many cases the second harmonic suffices in terms of description, i.e. semi-annual cycles are evidently most common in this branch of climatology. Maps can then be made of the phase angle and amplitude of each harmonic, and these can be used in regional studies. Note that knowledge of cyclical behavior, a prerequisite to meaningful application of the methods, is well advanced in climatology compared to many other branches of geography.

In two dimensions, as noted before, this simple approach has to be modified to account for directional influences on the values at any point. Trend can be handled by methods described in the preceding section, and double Fourier series can be employed for two-dimensional periodic phenomena. Casetti (1966) illustrates this approach in terms of the effects of different sized areal units acting as filters in producing different harmonics, and the implications of this for correlation and regression analysis. Granger (1969) discusses the relationships between time- and space-series Recent research efforts have been expended in applying more advanced techniques to geographic problems. Interest has centered upon the use of spectral analysis (Rayner, 1967, 1971; Bassett and Tinline, 1970), which is concerned with the identification of amplitudes and frequencies of component cycles making up the periodic proportion of the series A measure of correspondence between two spatial series (coherence) can be computed for each band. The value of this approach is that coherence can then be looked at for varying scales (Rayner, 1971), which would be extremely beneficial in terms of problems faced in the usual geographic applications of correlation analysis.

While it is clear that climatological research is particularly likely to benefit from applications of spectral techniques, since physical processes can be directly inferred, other branches of geography are likely to be influenced by the approach. For example, central place theory indicates a certain periodicity in the spacing of settlements. Tobler (1969) has examined the spectrum of population densities along U.S. 40 from Baltimore to San Francisco in this context. The diffusion of an innovation (agricultural subsidies), as modelled by Hägerstrand, has been studied by Barton and Tobler (1971) by means of an optical analogue to estimate changes in spectral densities over time. The researchers' potential ability to specify processes operating at different scales is a likely immense benefit of this approach, although the necessity of a strong theoretical framework for any empirical study is also underlined.

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REFERENCE: Section 4A. Gregory (209–224); King (222–226).
Section 5. Barry (401–404).

SECTION 32

CLASSIFICATION

The purpose of classification is to produce groups or clusters of unit areas in which the within-group distance or variance is minimized, and accordingly between-group variance is maximized. Dimensional analysis is employed to determine the distances separating areas in the space defined by the attributes under study. The most efficient technique would utilize orthogonal axes to define the space, since the theorem of Pythagoras can then be called upon to define distances. Most of the multivariate classifications accordingly are based upon the factor scores from prior analysis, since the new variables have standardized scales and are made orthogonal to each other. The distance measurements can be made in a space of any dimensions, and they are used to index similarity between areas; units located closer to each other are more similar. A distance matrix (of order n, say, the number of areas, Xn) which is square and symmetric can thus be derived, and the grouping routine operates on it in an hierarchical manner. At step one, there will be n single member groups. The matrix is searched to find the smallest distance, and the respective row and column pertaining to that observation are combined, producing an n-1 X n-1 matrix. The steps are repeated until there is only one group, containing all the unit areas. The within-group variance, equal to zero at the first step, successively increases, while the between-group variance is decreased.

The method described is only one of a number of alternative algorithms available for classification (Lankford, 1969). It is, however, one of the most-commonly reported in the literature, largely because of the influence of Berry whose seminal paper in 1961 reinvigorated long-standing interest in geography in the allied problem of regionalization (Grigg, 1965). The methodology is dependent on techniques of numerical taxonomy (see Sokal and Sneath, 1963, cited in Section 4B), and is fully described by Berry (1967). The types of grouping schemes resulting from this analysis are dependent on the nature of the input information. If an attribute matrix is used, the methods will produce regional types of the formal kind, in the absence of any strong contiguity in the original data. The solution can be made into a set of non-overlapping regions by the addition of contiguity or compactness constraints on the allocation of unit areas to existing groups. Alternatively, a set of functional regions is produced using as input an interaction matrix. Spence and Taylor (1970) provide an overview of the various alternatives.

A fair degree of subjectivity is evident in the methods reported in the literature (Johnston, 1968). There is, for example, a choice to be made in the coefficient of association that is used, as well as the type of algorithm. A difficult problem to resolve is: how many groups should be included in the final solution? Discriminant analysis (Section 33) can provide some aid in this respect. Johnston (1970) has also stressed the importance of a hypothesis-testing framework for any classification, since only in this way can inductive generalizations be made, which in turn will help to advance the theoretical framework of the discipline.

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Section 13. Dawson (1970).
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Section 25. Ahmad (1965); King (1966); Moser and Scott (1961); Munton and Norris (1969); Spence (1968).
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REFERENCE: Section 4A. King (194–204).

SECTION 32

DISCRIMINANT ANALYSIS

Discriminant analysis is employed for a set of observations which are already classified in some manner. Although the technique was originally developed to allocate new observations to a set of pre-established classes on the basis of certain characteristics, its most common use in geographic research today is as an aid in classification (King, 1970). At any stage in the grouping process described in the preceding section it is possible to compute the linear discriminant functions which are linearly related to the factor scores used as input to the algorithm. The coefficients of these functions are determined in such a way that discrimination between the groups is maximized. Thus the method has a strong similarity to principal components analysis, and the researcher is able to interpret the bases of the classification (Casetti, 1964). Multiple discriminant iterations are used in the Casetti studies to force classifications to optimal solutions, with criteria that are the object of classification itself, i.e. that the within-group variance is minimized and between-group variance maximized. Casetti also presents tests using chi-square statistics for determining the quality of a classification.

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Section 27. Kaiser, Massie, Weiss, and Smith (1968).

REFERENCE: Section 4A. King (204–215); Krumbein and Graybill (359–367).

SECTION 34

CANONICAL CORRELATION

Canonical correlation is the most general form of correlation analysis. It seeks to maximize the covariance or correlation between two sets of original variables, say X and Y, by computing new variates, say U_k and V_k, which are linear combinations of X and Y and are maximally correlated:

$$U_k = \alpha_k X \text{ and } V_k = \beta_k Y, \text{ where } \alpha_k \text{ and } \beta_k$$

are the coefficients of the resulting canonical vectors. It can readily be appreciated that this procedure has much in common with principal components analysis. Indeed, interpretation proceeds in an analogous fashion. The coefficients are like the loadings for different components, and the strength and sign can be used to indicate which of the original variables are to be considered, and the direction of their association. As indicated, the first pair of canonical vectors extracted has the highest correlation, and subsequent pairs not only report the maximum amount of correlation of the residual variance but are also made orthogonal to the first pair. The researcher is therefore able to describe the independent ways in which the relationships are specified between the two sets.

Although the technique was originally devised to account for any two sets of variables, the applications in geography are usually two sets of factor scores, i.e. the intercorrelations for each set are zero. For example, Berry (1966) compares the factor structure of Indian districts (derived from the attribute matrix) with the structure of flows between trade blocks (from a dyadic formulation of a set of interaction matrices). Gauthier (1968) compared levels of economic development with changes in the transportation surface using canonical correlation. An interesting recent development has seen the technique applied to the analysis of trend surfaces (Lee, 1969; Monmonier, 1970).

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