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ABSTRACT

The purpose of this study was to design a prototype written test of cognitive development based on the model of Piaget. Although the results were not consistent with Piaget's theory, test development was only partially successful. Specifically, validation of the scales was excellent or poor depending on the perspective. Analysis of the item difficulties within scales was excellent for the exclusion and combination scales and poor for the proportion scale. In fact, the prediction for the exclusion and combination scales was "perfect". Such results lend support to Piaget's contention of the invariant sequence of logical development for those logical structures tested. An analysis of those subjects' responses and reasoning on the formal items revealed that most had rotely learned how to solve proportion problems without understanding the underlying structural process. Such evidence would invalidate the contention that the "low" difficulties on the concrete items was due to the "high" readability of the items. Thus, it would seem that from a within scale analysis, the written test was generally successful, the criterion of cognitive level existence is still subject to question, and a different type of formal proportion item must be devised to alleviate the effect of specific past learning without understanding. (Author/DEP)

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PREFACE

Any study using a Piagetian methodology demands extremely sensitive and capable experimenters. The principal investigator was fortunate in having two co-experimenters who put long hours in intensive training and execution of the study. Special thanks are extended to Marion Homer and Victoria Knipper for their help and thoughtful suggestions.

Dale Clifford, Administrative Assistant, Chemistry Department, University of Dayton, provided supplies and guidance as to the proper chemicals to be used.

Statistical help was provided by Dr. Clinton I. Chase, Director of the Bureau of Educational Studies and Testing, Indiana University, Bloomington, and Dr. Richard J. Hofmann, Assistant Professor, Department of Education Psychology, Miami University.

Finally, the fine cooperation of the students, faculty, and administrators of the Centerville City Schools, Centerville, Ohio, is greatly appreciated.

DEVELOPMENT OF A WRITTEN TEST BASED UPON THE MODEL OF PIAGET

ABSTRACT

A group administered written test of cognitive development was constructed and partially validated. Consisting of 36 open-ended items, each logically equivalent to specific Piaget tasks, the test, and three Piaget tasks, were presented in a one-to-one situation. Subjects were classified according to Piaget's hierarchy of logical thought development. Convergent and discrimmant validation of the scales was mixed, while the item hierachies for two of the three written scales was "perfect." An approach to future Piaget studies was suggested and illustrated.

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CHAPTER 1

PROBLEM AND OBJECTIVES

PURPOSE

The purpose of the present project was to construct and validate a group administered written test that would assess the same intellectual constructs as those assessed by specific Piaget individually administered tasks.

BACKGROUND

Traditional Intellectual Assessment

Traditionally, assessment of intellectual constructs has been based on the work of Binet, with two methodological approaches: individual (i.e., SBIS, WPPSI, WISC, WAIS, etc.) or group (i.e., CTMM, OTIS, Henmon-Nelson, Lorge - Thorndike, etc.) administered tests. Within this tradition, an individual's intellectual assessment and subsequent categorization has been dependent on the mastery of specific information and on his position relative to a norm group within the normal curve model of probability. For example, if an individual did not know the specific fact that the Koran is the Islamic holy book, or that the Apocrypha were the disputed books in the Bible, he would not receive credit toward a classification of his intellectual prowess for those specific items. Because of its reliance on the knowledge of specific facts, this type of test generally has not proven to be adequate in assessing intellectual construct development and, in reality, has caused many problems of interpretation within the school situation. Many school personnel, not adequately trained in the construction and interpretation of such tests, have taken the resultant absolute number (i.e., IQ of 110) and based judgments of an individual's development on that numerical score. While the score is indeed an indication of an individual's intelligence relative to a norm group within the normal curve model of probability (a fact which many school personnel are not aware of or do not know the meaning of), it tells nothing about what constructs an individual has developed, what intellectual operations he is capable of, etc.; and the number (score) itself, as implied above, is dependent upon what facts an individual has learned, not upon what mental operations he can perform.

Piaget

Piaget has used a variation of the individual testing situation and has attempted to assess intellectual constructs which do not depend upon specific learnings or upon how an individual performs relative to a norm group within the normal curve; rather his worked has focused on assessing constructs that are necessary for adequate interaction with the world, generally not teachable, and develop in individuals at different times. He contends that intellectual development gradually develops throughout life, with the major changes occurring between birth and fifteen-sixteen years of age. The developmental changes are continuous; however, there are certain characteristics of intellectual functioning that remain relatively stable for certain durations of time. These relatively stable durations of development are called periods and represent qualitatively different levels of intellectual ability. Four such periods have been identified, the last two of which are the periods of concrete operations (7 - 11 years) and formal operations (11 + years). Within each of these two periods two subperiods have been distinguished: an initial subperiod where the intellectual characteristics of the overall period are manifested but inconsistently and intellectual regression to an earlier period tends to occur, and an ending subperiod, where the period characteristics are consistently manifested, little intellectual regression to an earlier period occurs, and the characteristics of the initial subperiod of the next period are sporadically manifested (Inhelder and Piaget, 1958). Essentially, it is contended that intellectual development occurs as an ordinal scale through which everyone progresses. An individual at a particular period would manifest the characteristics of that period in most of his behavior.

In order to test the concept that there are qualitatively different periods of intellectual development, Piaget and his collaborators have devised specific types of tasks which generally involve scientific problems presented in one-to-one situations. From an individual's performance in working with the tasks, he is classified as being in one of the periods.

Present Needs

Replications and entensions of Piaget's work have generally supported his contentions relative to the ordinal development of intellectual constructs and the adequacy of his approach in assessing intellectual development. Unfortunately, both the individual method within the Binet tradition and Piaget's use of the individual method are difficult to use, as well as very time consuming. While much information can be obtained about one person, the method of assessment is relatively inefficient. On the other hand, traditional group administered intelligence tests, which are more efficient, generally cannot explain why a specific response is given. A Piaget based group-administered written test of cognitive development would combine the efficiency of the later with results that could specifically indicate the intellectual skills mastered. Such a test would be invaluable to the classroom teacher, as it would give him not just a number indicating relative past learning (traditional intelligence), but a specific classification telling him just what constructs a child has already mastered and what operations that child is capable of performing, thereby assisting him in determining a level at which to pitch instruction. With today's educational innovations (open space, individualized instruction, etc.), such an instrument is surely needed to replace the rapidly-outmoded traditional IQ test.

REVIEW OF LITERATURE

Previous attempts at constructing a Piaget based written test of cognitive development have been partially successful. Singh (1970) and Dodwell (1961) have used written tests to assess the Piaget constructs of area, length, volume, and number conservation with some success. However, only Singh (1970) reports eliciting the subjects' reasons for a response to a specific item and his use of a subject's reasoning was limited to the pilot study for increasing the reliability of the written test.

Peel (1959), Case and Collinson (1962), Lunzer (1960, 1965), O'Brien and Shapiro (1968), O'Brien, Shapiro, and Reali (1971), Shapiro and O'Brien (1970), Keats (1955), Longeot (1962, 1964), and Gray (1973) report Piaget type logical operations assessed via written tests. The congruence between the written test items and the logical operations of specific Piaget tasks has covered the continuum from assumed congruence (Peel, 1959; Case and Collinson, 1962) to planned congruence (Lunzer, 1965; Gray, 1973). Except for the Gray and Lunzer work, no known attempt has been made to construct the written test items logically equivalent to specific Piaget tasks and validate the written items with a comparison of subjects' performances on the Piaget tasks. Because Piaget's entire theory is based on logical operations, any set of written items which would validly assess Piaget-type constructs must be constructed logically equivalent to specific Piaget tasks.

SIGNIFICANCE OF PROJECT

Several contributions to education have been made through the project. First, the generalizability of Piaget's theory was tested by interpreting the results of a written test within the Piagetian framework.

Second, the written test was a prototype Piaget-based written test of cognitive development. Further refinements of such a test could be used in assessing developmental level within today's innovative approaches to instruction. Since most of these approaches heavily rely on student assessment a written test of cognitive development would definitely be a boom, particularly if a series of such tests, each demanding the same cognitive mastery, were to be developed in different content areas. Such a series of tests would have the advantage over present-day tests by being able to more accurately determine the cognitive level of a student within a specific content domain which would obviously facilitate instruction and learning.

Third, as implied above, the written test was a prototype criterion-referenced test (Nitko, 1970), and, as such, indicated the types of cognitive skills individuals with specific scores demonstrated. For example, a score of Formal Operational I on the Exclusion scale, meant that the subject did, among other things, affirm correct cause and effect statements and did not deny incorrect cause and effect statements.

Finally, the written test was a measure of intellectual development not based upon the normal curve. It was a device based on the actual development of children and not an artificial statistical convention.

OBJECTIVES

1. The generalizability of Piaget's theory to written tasks which demanded the same logical operations for successful mastery as specific Piaget tasks was evaluated.

2. A model of a group-administered written test of cognitive development was evaluated as to its reliability and validity.

PROJECT RESTRICTIONS

There were three restrictions to the study. First, the design of the written test was based on the initial work of Longeot (1962, 1964) and a revision of that work by Gray (1970). Longeot used a combination of open-ended and multiple-choice questions. Questions corresponding to the present work's Exclusion and Proportion scales were multiple-choice, while questions corresponding to the combination scale were open-ended. Gray (1970) revised the Longeot scales so that they were all multiple-choice and the logic of each question referred to a specific stage-related logic of a specific Piaget task. Although the results, (Gray 1970) were encouraging, it was suggested that all of the items be open-ended and the written test, in initial testing, be given individually and that a subject's reasoning, along with his answers to the questions, be elicited. The presented study was an implementation of those suggestions.

Second, the relationship between the Piaget classification, physical tasks and written test, were not related to traditional measures of intellectual functioning. It was felt that the written test should be validated against the Piaget tasks before it was related to the traditional system. In other words, the Piaget model should be internally consistent before it is related to other models.

Finally, a corollary to the second restriction is that there was no necessity for relating the Piaget model to the traditional model. Each makes different assumptions about intellectual functioning in general, the distribution of intelligence, its measurement, and the criteria indicating its existence.[1] Although there are some similarities in measurement and criteria for existence (Gray, 1971), the models are so different that it is absurd to tactly acknowledge the validity of the traditional model by relating the new measures to it.

HYPOTHESES

1. For each scale of logical thought development, there will be no significant positive relationship ($p < 0.05$) between the written items and the corresponding Piaget tasks.

For each scale that the preceding hypothesis was rejected, the following hypotheses were tested.

2. Measures of the same scale of logical thought development will correlate higher with each other than they will with measures of different scales of logical thought development involving separate methods.

3. Measures of the same scale of logical thought development will correlate higher with each other than they will with measures of different scales of logical thought development involving the same method.

4. Measures of different scales of logical thought development will have the same pattern of interrelationships with each other across heterotrait-monomethod and heterotrait-heteromethod triangles.

The following hypothesis was tested for all three scales, regardless of the outcome of Hypothesis 1.

5. For each scale of logical thought development on the written test, there will be no significant positive relationship ($p < 0.05$) between the theoretically predicted item difficulty sequence and the empirical item difficulty sequence.

6. For the Piaget tasks and the written scales separately, there will be no significant difference ($p < 0.05$) between the experimenters or between the order (Piaget-Written, Written-Piaget) in which the tasks were presented.

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1 Gray, W. M., Piaget's Criterion-Referenced Measurement. In preparation.

CHAPTER II

METHOD OF INVESTIGATION

SAMPLE

Subjects were drawn from a middle to upper-middle class, predominately white suburb of Dayton, Ohio. A total of 168 subjects were randomly drawn from ages 9 - 15 with 24 subjects per age (See Appendix A for complete sampling procedure).

PROCEDURES

For each age group, one-half of the subjects were given the Piaget tasks of Oscillation of a Pendulum, Equilibrium in the Balance, and Combinations of Colored and Colorless Chemical Bodies in a one-to-one situation. Presentation of the tasks was made by one of three trained experimenters. Task sequence was randomized across subjects (See Appendix C for details). Experimenters presented the tasks within the framework provided by Inhelder and Piaget (1958) (See Appendix D for procedural details). All verbalizations were audio recorded and each subject's performance on each task was rated on a checksheet derived from Inhelder and Piaget (1958) (See Appendix E, Section 1). Within one week after the Piaget tasks presentation, subjects were given the written Test of Logical Thinking (TOLT) (See Appendix F, Section 1) in a one-to-one situation by a different experimenter. After each response, a subject was asked the reasoning behind his answer; and all verbalizations were audio recorded. The remaining subjects, one-half of each age group, were given the same Piaget tasks and written test in the manner described above, but were first given the written test, then the Piaget tasks.

DATA ANALYSIS

Piaget Tasks

For each Piaget task, subjects were classified according to the criteria presented by Inhelder and Piaget (1958). Possible classifications were preoperational, "beginning" concrete operational, "complete" concrete operational, "beginning" formal operational, or "complete" formal operational. Appendix E, Section 1 contains the substage performances for each task; and Appendix E, Section 2 the criteria for a subject performance classification.

Written Test

The written test contained three scales, each corresponding to one of the three Piaget tasks. Each scale was divided into four subscales logically equivalent to the logic manifested by subjects in "beginning" concrete operations, "complete" concrete operations, "beginning" formal operations, or "complete" formal operations when they attempt to solve the corresponding Piaget task. Appendix F, Section 1 is a copy of the test and Section 2 is a description of the item logic. The logic of the corresponding Piaget tasks is found in Appendix D, Section 4. For each scale there were twelve items, three items per subscale, for a total of 36 items. All items were open-ended and had a Dale-Chall readability score ranging from below fourth grade level to ninth and tenth grade level (See Table 1). The six ninth and tenth grade level items

TABLE 1

WRITTEN ITEM READABILITY

SUBSCALE	SCALE					
	Exclusion		Proportion		Combination	
	Item	Readability	Item	Readability	Item	Readability
Concrete I	1	4th	13	9th-10th	25	4th
	2	4th	14	9th-10th	26	4th
	3	4th	15	9th-10th	27	4th
Concrete II	4	4th	16	9th-10th	28	4th
	5	4th	17	9th-10th	29	4th
	6	4th	18	9th-10th	30	5th-6th
Formal I	7	4th	19	4th	31	4th
	8	4th	20	5th-6th	32	4th
	9	5th-6th	21	4th	33	4th
Formal II	10	4th	22	4th	34	4th
	11	4th	23	4th	35	4th
	12	4th	24	4th	36	4th

were the concrete proportion items and were rated as such because the word "complete" was not on the Dale-Chall list of 3,000 familar words. On all items, if a subject had difficulty with a word, the experimenter explained the word and the concrete proportion items were verbally explained in greater detail than the question itself presented.

For each item a subject was scored correct or incorrect according to his response and reasoning on the item. Reasoning criteria involved using the correct logic associated with the developmental level of the item (See Inhelder and Piaget, 1958; Gray, 1970; Appendices D, Section 4; E; and F of this report for a discussion of the necessary reasoning and specific item logic). On each scale, to be classified at a specific developmental level, a subject's answer pattern must have conformed to the following criteria: (a) at least two-thirds (2 out of 3) of the questions representing the specific subscale must have been answered correctly, and (b) at least two-thirds (2 out of 3) of the questions for each preceding subscale must have been answered correctly. If a subject's answer pattern did not fulfill the criteria, his classification was based on his scale response pattern, his reasoning on the "deviant" subscale items, and his response pattern on all three scales. Table 2 is a percentage summary of the "deviant" patterns. The proportion scale "deviant" patterns are considered in more detail in Chapter IV.

TABLE 2

WRITTEN ITEM RESPONSE PATT[illegible] MEETING CLASSIFICATION CR[illegible]A

Age	SCALE						Total		
	Exclusion		Proportion		Combination				
	Male	Female	Male	Female	Male	Female	Male	Female	Total
9	6.67 (1/15)		26.67 (4/15)		6.67 (1/15)	—	13.33 (6/45)		8.33 (6/72)
10	7.69 (1/13)		30.77 (4/13)	9.09 (1/11)			12.82 (5/39)	3.03 (1/33)	8.33 (6/72)
11	8.33 (1/12)	16.67 (2/12)	16.67 (2/12)	58.33 (7/12)	8.33 (1/12)		11.11 (4/36)	25.00 (9/36)	18.06 (13/72)
12		21.43 (3/14)	40.00 (4/10)	14.29 (2/14)			13.33 (4/30)	11.90 (5/42)	12.50 (9/72)
13	6.67 (1/15)	33.33 (3/9)	13.33 (2/15)	11.11 (1/9)	6.67 (1/15)	11.11 (1/9)	8.89 (4/45)	13.52 (5/27)	12.50 (9/72)
14		7.14 (1/14)	20.00 (2/10)	21.43 3/14	10.00 (1/10)		10.00 (3/30)	9.52 (4/42)	9.72 (7/72)
15	7.69 (1/13)		23.08 (3/13)	27.27 (3/11)	15.38 (2/13)		15.38 (6/39)	9.09 (3/33)	12.50 (9/72)
Total	5.68 (5/88)	11.25 (9/80)	23.86 (21/88)	21.25 (17/80)	6.82 (6/88)	1.25 (1/80)	12.12 (32/264)	11.25 (27/240)	11.71 (59/504)
	8.33 (14/168)		22.62 (38/168)		4.17 (7/168)				

NOTE: Main entries are percents.

CHAPTER III

RESULTS

Piaget Tasks - Written Scales Correspondence

Hypotheses 1 - 4 were formal statements of Campbell and Fiske's (1959) criteria [illegible] evaluating a multitrait-multimethod correlation matrix for convergent and discriminant validity. Table 3 presents the multitrait - multimethod values for the total sample. All entries were significantly greater than zero ($p < 0.005$, df = 166, one-tail) except the validity value for Exclusion - Pendulum, which was significant at $p < 0.05$. KR_{20} estimates of written

TABLE 3

MULTITRAIT - MULTIMETHOD CORRELATIONS: TOTAL

		Piaget Pen	Piaget Bal	Piaget Chem	Written Excl	Written Prop	Written Comb
Piaget	Pen						
	Bal	0.497					
	Chem	0.617	0.643				
Written	Excl	0.185*	0.355	0.261	(0.882)		
	Prop	0.368	0.421	0.477	0.602	(0.910)	
	Comb	0.359	0.418	0.471	0.488	0.660	(0.870)

NOTE - Underlined entries are convergent validities. Entries enclosed by solid lines are heterotrait-monomethod values and entries enclosed by broken lines are heterotrait-heteromethod values. Written scale KR_{20} estimates of reliability are enclosed in parentheses and total test KR_{20} was 0.938. Arrows indicate high-low direction of correlation pattern.

* $p < 0.05$, all other entries $p < 0.005$ (df = 166, one-tail).

test reliability and the three scales were quite substantial, ranging from 0.87 for the combination scale to 0.938 for total test, indicating a great deal of internal consistency, within each scale, and between scales or sample heterogeneity. This between scale consistency is also illustrated in the medium written monomethod values 0.488 - 0.66) and the same consistency is evident in the Piaget monomethod triangle. The other evidence of convergent and discriminant validity of the developmental scales was mixed.

Hypothesis 1 was rejected for each scale, but none of the validity values completely satisfied Campbell and Fiske's remaining convergent validity criterion: along with being significantly greater than zero, each validity value must also be large enough to warrant further investigation of validity. The exclusion-pendulum value was not large, absolutely, or in comparison with the other validity values, and in no case did it support Hypothesis 2 or 3.

Hypotheses 2 and 3 focused on the discriminant validity of each logical developmental scale. Values for proportion-balance and chemical-combination validities tended to support Hypotheses 2, but not 3 (See Tables 3 and 6). Only with the exclusion-pendulum validity value were the results clear and there in a direction opposite the hypotheses.

Hypothesis 4 was generally supported. All heterotrait triangles except the Piaget monomethod had the same high-to-low pattern of correlations, although the difference between some values was quite small indicating a reversal in pattern was possible. Clearly, the evidence for discriminability of each logical developmental scale from the others is unclear at best.

Sex differences were generally small and tended to follow the total sample pattern, although males had higher validity values (See Tables 4 and 5). Hypotheses 1 was rejected ($p < 0.005$; one-tail) for all validity values, for both sexes, except the exclusion-pendulum for females. Support for Hypotheses 2 and 3 for both sexes and each validity value could be considered the "same" except for the combination-chemical value (See Table 6). Intrasex correlational patterns coincided with those of the total sample for males, but the females were different for the lower-left Written-Piaget heterotrait-heteromethod triangle. The slight change in the Piaget monomethod triangle was trivial. A further breakdown of the sample (Age x Sex) was performed, but little in the way of usable data was obtained. Less than 25% of the validity values were significantly greater than zero and there was substantial variability within ages across ages within tasks (See Tables 7, 8, and 9).

TABLE 4

MULTITRAIT-MULTIMETHOD CORRELATIONS: MALE *

		Piaget			Written		
		Pen	Bal	Chem	Excl	Prop	Comb
Piaget	Pen						
	Bal	0.415					
	Chem	0.614	0.631				
Written	Excl	0.297	0.471	0.359	(0.896)		
	Prop	0.459	0.495	0.565	0.543	(0.930)	
	Comb	0.368	0.510	0.493	0.516	0.628	(0.872)

NOTE - Underlined entries are convergent validities. Entries enclosed by solid lines are heterotrait-monomethod values and entries enclosed by broken lines are heterotrait-heteromethod values. Written scale KR_{20} estimates of reliability are enclosed in paretheses and total test KR_{20} was 0.944. Arrows indicate high-low direction of correlation pattern.

* $p < 0.005$ (df = 86, one-tail) for all entries.

TABLE 5

MULTITRAIT-MULTIMETHOD CORRELATIONS: FEMALE

		Piaget			Written		
		Pen	Bal	Chem	Excl	Prop	Comb
Piaget	Pen						
	Bal	0.626**					
	Chem	0.620**	0.668**				
Written	Excl	<u>0.022</u>	0.227*	0.125	(0.866)		
	Prop	0.239*	<u>0.332**</u>	0.356**	0.681**	(0.881)	
	Comb	0.344**	0.306**	<u>0.441**</u>	0.448**	0.701**	(0.868)

NOTE - Underlined entries are convergent validities. Entries enclosed by solid lines are heterotrait-monomethod values and entries enclosed by broken lines are heterotrait-heteromethod values. Written scale KR_{20} estimates of reliability are enclosed in parentheses and total test KR_{20} was 0.931. Arrows indicate high-low direction of correlation pattern.

* $p < 0.05$
** $p \leq 0.005$ (df = 78, one-tail).

TABLE 6

SUPPORT FOR HYPOTHESES TWO AND THREE

SEX	HYPOTHESIS					
	TWO			THREE		
	Excl.-Pen.	Prop.-Bal.	Comb.-Chem.	Excl.-Pen.	Prop.-Bal.	Comb.-Chem.
Male	0/4	2/4	2/4	0/4	1/4	0/4
Female	0/4	3/4	4/4	0/4	0/4	0/4
Total	0/4	3/4	3/4	0/4	0/4	0/4

NOTE - Numerator indicates support of hypothesis and denominator indicates total possible support.

TABLE 7

CLASSIFICATION SUMMARY : TOTAL

Age	n	Piaget						Written					
		Pendulum		Balance		Chemical		Exclusion		Proportion		Combination	
		$\bar{x}$	s	$\bar{x}$	s	$\bar{x}$	s	$\bar{x}$	s	$\bar{x}$	s	$\bar{x}$	s
9	24	2.33	0.82	2.13	0.61	2.42	0.78	3.08	1.10	1.67	0.92	2.96	1.12
10	24	3.04	0.86	2.50	0.66	2.88	0.61	3.75	1.22	2.17	1.34	3.58	1.02
11	24	3.17	1.05	2.88	0.68	2.96	0.75	3.71	1.20	3.08	1.21	3.88	0.80
12	24	3.50	1.35	2.96	0.62	3.67	0.87	3.92	0.97	3.13	1.42	4.08	0.88
13	24	3.88	0.99	3.33	0.64	3.75	0.85	3.83	1.13	3.50	1.35	4.21	0.78
14	24	4.13	0.68	3.50	0.59	3.79	0.59	4.38	1.13	3.96	1.23	4.58	0.58
15	24	4.08	0.83	3.46	0.78	4.29	0.86	4.21	0.98	4.13	0.74	4.46	0.59
Total	168	3.45	1.12	2.96	0.80	3.39	0.97	3.84	1.15	3.09	1.44	3.96	0.98

TABLE 8

CLASSIFICATION SUMMARY: MALES

Age	n	Piaget						Written					
		Pendulum		Balance		Chemical		Exclusion		Proportion		Combination	
		$\bar{x}$	s	$\bar{x}$	s	$\bar{x}$	s	$\bar{x}$	s	$\bar{x}$	s	$\bar{x}$	s
9	15	2.27	0.70	2.20	0.68	2.47	0.74	2.93	0.96	1.67	0.90	3.00	1.20
10	13	2.92	1.12	2.46	0.78	2.84	0.69	3.77	1.36	2.15	1.34	3.69	0.95
11	12	3.17	1.19	2.83	0.72	2.83	0.83	3.67	1.37	3.08	1.44	3.75	0.87
12	10	3.70	1.42	3.20	0.42	3.70	0.95	3.80	1.03	3.30	1.57	4.10	1.10
13	15	3.73	1.09	3.40	0.51	3.53	0.83	3.73	1.16	3.47	1.51	4.20	0.86
14	10	4.10	0.74	3.70	0.68	4.00	0.67	4.50	1.08	3.90	1.45	4.70	0.48
15	13	4.23	0.73	3.62	0.77	4.54	0.66	3.92	1.12	4.08	0.76	4.31	0.63
Total	88	3.40	1.19	3.02	0.84	3.38	1.02	3.72	1.20	3.03	1.51	3.92	1.02

TABLE 9

CLASSIFICATION SUMMARY: FEMALES

Age	n	Piaget						Written					
		Pendulum		Balance		Chemical		Exclusion		Proportion		Combination	
		$\overline{x}$	s	$\overline{x}$	s	$\overline{x}$	s	$\overline{x}$	s	$\overline{x}$	s	$\overline{x}$	s
9	9	2.44	1.01	2.00	0.50	2.33	0.87	3.33	1.32	1.67	1.00	2.89	1.05
10	11	3.18	0.40	2.55	0.52	2.91	0.54	3.73	1.10	2.18	1.40	3.45	1.13
11	12	3.17	0.94	2.92	0.67	3.08	0.67	3.75	1.06	3.08	1.00	4.00	0.74
12	14	3.35	1.34	2.79	0.70	3.64	0.84	4.00	0.96	3.00	1.36	4.07	0.73
13	9	4.11	0.78	3.22	0.83	4.11	0.78	4.00	1.12	3.56	1.13	4.22	0.67
14	14	4.14	0.66	3.36	0.50	3.64	0.50	4.29	1.20	4.00	1.11	4.50	0.65
15	11	3.91	0.94	3.27	0.79	4.00	1.00	4.55	0.69	4.18	0.75	4.64	0.50
Total	80	3.50	1.04	2.90	0.76	3.41	0.91	3.98	1.09	3.15	1.37	4.01	0.93

Item Difficulty Sequences: Predicted and Empirical

Hypothesis 5 was rejected for the exclusion (Tau = 0.919, $p < 0.001$) and combination (Tau = 0.905, $p < 0.001$) scales but not for the proportion scale. Further comparison of the difficulty sequences of the three scales provided substantial support for Piaget's theory of logical thought development and excellent validation for two-thirds of the scales. For the exclusion and combination scales, the empirical item difficulties were exactly what was expected. That is, all "beginning concrete" (CI) items were the easiest, then the "complete concrete" (CII) items, then the "beginning formal" (FI) items, and finally the "complete concrete" (FII) items. Within the exclusion scale (See Table 10) there was very little difference in difficulty (except for item four) between CI and CII items but a substantial difficulty difference between concrete and formal items and some difference between FI and FII items. It appears that the exclusion items differentiated between concrete and formal subjects, partially differentiated subjects within formal operations and could not differentiate within concrete operations. The same interpretation holds for the combination scale, except the combination items were better able to differentiate between the formal operational subscales. Item difficulties for the proportion scale did not follow any consistent pattern, other than the FII items were expected to be the most difficult and they were the easiest, being missed by 26 people more than the next easiest item.

Experimenter and Task Presentation Order Effect

Two 2 x 3 x 3 ANOVA'S with repeated measures on the third factor were used to determine experimenter effect and order of task presentation effect. Tables 13 and 14 are the summary tables, corrected for unequal cell n's [2], for the Piaget Tasks and Written Scales, respectively. In both tables, the within subjects between tasks F - value was significant (F = 11.73, df = 5/307, $p < 0.001$, Piaget tasks; F = 23.88, df = 5/307, $p < 0.001$, written scales). Analyses of Tables 7 and 11 indicate that the differences were between the balance and other Piaget tasks, and between the proportion and other written scales. Such discrepancies among the different scales are not unusual (see Lovell, 1971; Gray, 1970) nor unexpected considering the irregularity of mental growth, especially during the ages encompassed by the sample.

The experimenter effect for the written scales was significant (F = 3.35, df = 2, 162, $p < 0.05$). Although such effect has been reported previously, (Bittner and Shinedling, 1968), the present results should not be interpreted too literally. Because of scheduling problems, one E did not test any 13, 14, or 15 year old subjects. Consequently, the significant E effect may reflect a difference in subject performances across ages, and not necessarily a difference

---

[2] Dr. Richard J. Hofmann derived the ANOVA formulas for use with unequal cell size.

TABLE 10

EMPIRICAL VS. HYPOTHETICAL SEQUENCE OF WRITTEN ITEMS: EXCLUSION

Sequence	QUESTION												TAU
	3	2	1	6	5	4	7	8	9	12	10	11	
Empirical	1 (94)	2 (93)	3 (92)	4 (91)	5 (90)	6 (77)	7.5 (56)	7.5 (56)	9 (53)	10 (45)	11.5 (43)	11.5 (43)	0.9186*
Hypothetical	2 CI	2 CI	2 CI	5 CII	5 CII	5 CII	8 FI	8 EI	8 FI	11 FII	11 FII	11 FII	

NOTE - Main entries are the ranks of the individual items for the specific sequences. Entries enclosed in parentheses are the item difficulties for each question. Alphabetic entries are the predicted subperiods of each question.

*$p < 0.001$

TABLE 11

EMPIRICAL VS. HYPOTHETICAL SEQUENCE OF WRITTEN ITEMS: PROPORTION

Sequence	QUESTION												TAU
	19	21	18	14	16	15	17	20	13	23	22	24	
Empirical	1 (73)	2 (64)	3 (62)	4 (55)	5 (54)	6 (53)	7 (52)	8 (49)	9 (46)	10 (20)	11 (17)	12 (18)	0.268
Hypothetical	8 FI	8 FI	5 CII	2 CI	5 CII	2 CI	5 CII	8 FI	2 CI	11 FII	11 FII	11 FII	

NOTE - Main entries are the ranks of the individual items for the specific sequences. Entries enclosed in parentheses are the item difficulties for each question. Alphabetic entries are the predicted subperiods of each question.

TALBE 12

EMPIRICAL VS. HYPOTHETICAL SEQUENCE OF WRITTEN ITEMS: COMBINATION

Sequence	QUESTION												TAU
	26	27	25	28	29	30	32	33	31	36	35	34	
Empirical	1 (99)	2 (98)	3 (95)	4 (93)	5 (92)	6 (80)	7 (71)	8 (70)	9 (69)	10 (39)	11 (38)	12 (36)	0.9045 *
Hypothetical	2 CI	2 CI	2 CI	5 CII	5 CII	5 CII	8 FI	8 FI	8 FI	11 FII	11 FII	11 FII	

NOTE - Main entires are the ranks of the individual items for the specific sequences. Entries enclosed in parentheses are the item difficulties for each question. Alphabetic entries are the predicted subperiods of each question.

* $p < 0.001$

in presentation; especially since the written test was strictly standardized across E's.

TABLE 13

ANALYSIS OF VARIANCE: PIAGET TASKS

Source	df	SS	MS	F
Between Subjects	167	337.51		
PW-WP	1	0.87	0.87	0.44
E	2	9.27	4.64	2.32
PW-WP x E	2	3.13	1.57	0.79
Error	162	324.23	2.00	
Within Subjects	337	159.33		
Piaget Tasks	5	23.46	4.59	11.73*
PW-WP-Piaget Tasks	5	2.87	0.57	1.43
E x Piaget Tasks	10	6.97	0.70	1.75
E x Piaget Tasks x PW-WP	10	1.88	0.19	0.475
Error	307	124.15	0.40	

* $p < 0.001$

TABLE 14

ANALYSIS OF VARIANCE: WRITTEN SCALES

Source	df	SS	MS	F
Between Subjects	167	520.02		
PW-WP	1	2.03	2.03	0.67
E	2	20.25	10.13	3.35*
PW-WP x E	2	8.12	4.06	1.34
Error	162	489.69	3.02	
Within Subjects	337	159.33		
Written Scales	5	75.25	15.05	23.88**
PW-WP-Written Scales	5	2.03	0.41	0.65
E x Written Scales	10	11.85	1.19	1.89
E x Written Scales x PW-WP	10	2.98	0.30	0.48
Error	307	193.22	0.63	

*$p < 0.05$

**$p < 0.001$

CHAPTER IV

CONCLUSIONS

The purpose of the study was to design a prototype written test of cognitive development based on the model of Piaget. Although the results were not inconsistent with Piaget's theory, test development was only partially successful. Specifically, validation of the scales was excellent or poor depending on the perspective. Analysis of the item difficulties within scales was excellent for the exclusion and combination scales and poor for the proportion scale. Infact, the prediction for the exclusion and combination scales was "perfect". Such results lend support to Piaget's contention of the invariant sequence of logical development for those logical structures tested. The poor results on the proportion scale appeared to come from many subjects' inability to multiply the two asymmetrical-transitive series, even after the E's extensively explained what was wanted, and the same subjects correctly solving the formal proportion problems. An analysis of those subjects' responses and reasonings on the formal items revealed that most had rotely learned how to solve proportion problems without understanding the underlying structural process. This is in direct contradiction to Brainerd's (1973) contention that a subject's judgements and not his reasoning is the appropriate criteria for judging cognitive level. Such evidence also would invalidate the contention that the "low" difficulties on the concrete items was due to the "high" readability of the items. Thus, it would seem that from a within scale analysis, the written test was generally successful, the criterion of cognitive level existence is still subject to question, and a different type of formal proportion item must be devised to alleviate the effect of specific past learning without understanding.

Evidence of convergent and discriminant validity of the scales was mixed. For the males and total sample, the convergent validities were significant, but not substantial, while for the females only the convergent validities for proportion-balance and combination-chemicals were significant and none were substantial. Only the pattern of intercorrelations within heterotrait triangles, within and across methods, provided any support for discriminability between the different developmental logical structures. Lack of discriminability would seem to support the contentions that concrete operations are semi-integrated and formal operations are an integrated whole bound by their lattice and group properties (Inhelder and Piaget, 1958; Piaget, 1957). This is evident in the significant and generally substantial entries in the monomethod triangles and the significant entries in the heteromethod triangles (See Tables 3, 4, and 5). Lack of convergent and discriminant validity also may have been a manifestation of horizontal décalage (Flavell, 1963).

In summary, a written test based on Piaget's periods of cognitive development is necessary and definitely constructible. Sequencing of items within a developmental scale of logic and validating the item scores with subject reasonings was demonstrated while validating scale scores against appropriate Piaget tasks was partially successful. Future studies should separately classify the data according to a subject's response and his reasoning, and then determine the relationship among the different modes of classification. Table 15 illustrates the suggested analysis. Such an analysis would begin to answer

TABLE 15

SUGGESTED MULTITRAIT-MULTIRESPONSE - MULTIMETHOD ANALYSIS OF FUTURE PIAGETIAN VALIDATION STUDIES

			PIAGET				WRITTEN			
			Task 1		Task 2		Scale 1		Scale 2	
			Res.	Rea.	Res.	Rea.	Res.	Rea.	Res.	Rea.
PIAGET	Task 1	Res.								
		Rea.								
	Task 2	Res.								
		Rea.								
WRITTEN	Scale 1	Res.								
		Rea.								
	Scale 2	Res.								
		Rea.								

Res. = Response, Rea. = Reasoning

the question about the appropriate criterion of cognitive structure existence and provide a more appropriate means of validating the written scales along with providing clearer data with respect to the phenomena of horizontal decalage.

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APPENDIX A

SAMPLING PROCEDURE

An alphabetical listing of the students in the Centerville, Ohio, City School District was obtained and the birth dates of all students were searched for those born between October 1 and April 30 for the years 1957 - 1964. Subjects' ages were rounded off to the nearest whole year (See Table A-1) and then categorized by the "whole age" figure. Approximately 2200 students met the birth date criteria. Parents of the subjects were then asked if they objected to their child taking part in a study about abstract reasoning, (See Appendix B). Those subjects whose parents objected were removed from the total sample.

TABLE A-1

SUBJECT SELECTION AGE CRITERIA

Age	Actual Age Range*	Birth Date Range
9	8:6 - 9:6	10/1/63 - 3/31/64
10	9:6 - 10:6	10/1/62 - 3/31/63
11	10:6 - 11:6	10/1/61 - 3/31/62
12	11:6 - 12:6	10/1/60 - 3/31/61
13	12:6 - 13:6	10/1/59 - 3/31/60
14	13:6 - 14:6	10/1/58 - 3/31/59
15	14:6 - 15:6	10/1/57 - 3/31/58

* Ages: Months

Within each age of the remaining subjects, a random sample of 24 was selected regardless of sex (See Table A-2).

At the time a subject was first tested, the nature of the tasks were explained and the subject was asked if he had any objections to be tested. If any objection was raised the subject was returned to school and another subject was selected from the specific age population sample of the objecting subject. Participation by all subjects was voluntary.

TABLE A-2

AGE x SEX BREAKDOWN OF SUBJECTS

Age / Sex	9	10	11	12	13	14	15	Total
Female	9	11	12	14	9	14	11	80
Male	15	13	12	10	15	10	13	88
	24	24	24	24	24	24	24	168
Total Sample Population	280	299	300	308	317	308	288	2100

APPENDIX B

PARENTS' PERMISSION LETTER

Dear Parent:

Your child has been randomly selected for possible inclusion in a research study during the school year 1972-1973. The study is an attempt to devise a new type of evaluation of student's abilities. Essentially, it incorporates the developmental ideas of the Swiss psychologist, Jean Piaget, into a written standardized test designed to measure children's levels of abstraction.

If included, your child would be involved for a total of two hours, with a maximum of one hour at a time. Results are to be kept confidential and in no way will be included in a child's permanent record.

If you do not wish your child to participate, please fill in the form below and return it to Stan Moreland, Assistant Superintendent for Curriculum, Centerville City School District, Virginia Avenue; Centerville, OH 45459, by November 7, 1972.

Sincerely,

Stan Moreland
Assistant Superintendent for Curriculum

I do not wish my child ______________________

Name of Child

to participate in the study.

____

Signature of Parent

APPENDIX C

PIAGET TASKS PRESENTATION SEQUENCES

Each subject was randomly assigned a specific task presentation sequence for the Piaget tasks. For each age group there were four subjects who received the same Piaget task presentation sequence and there were six possible sequences.

TABLE C-1

PIAGET TASKS PRESENTATION SEQUENCES

PBC*,	PCB,	CPB,	CBP,	BCP,	BPC

* P = Pendulum, B = Balance, C = Chemicals

One-half of each set of four subjects received the Piaget tasks first and the written tasks second, and vice versa. Thus, there were twenty-four subjects per age twelve given the Piaget tasks first and twelve given the written test first.

TABLE C-2

SUBJECT DISTRIBUTION: AGE x PIAGET TASK SEQUENCE x WRITTEN TEST (W) - PIAGET TASKS (P) SEQUENCE

Piaget Tasks	AGE														
	9		10		11		12		13		14		15		
Sequence[1]	PW[2]	WP	PW	WP	PW	WP	PW	WP	PW	WP	PW	WP	PW	WP	Total
PBC	2	2	2	2	2	2	2	2	2	2	2	2	2	2	28
PCB	2	2	2	2	2	2	2	2	2	2	2	2	2	2	28
CPB	2	2	2	2	2	2	2	2	2	2	2	2	2	2	28
CBP	2	2	2	2	2	2	2	2	2	2	2	2	2	2	28
BCP	2	2	2	2	2	2	2	2	2	2	2	2	2	2	28
BPC	2	2	2	2	2	2	2	2	2	2	2	2	2	2	28
Total	12	12	12	12	12	12	12	12	12	12	12	12	12	12	168

[1] P = Pendulum, B = Balance, C = Chemicals

[2] PW = Piaget Tasks first, Written test second
WP = Written test first, Piaget Tasks second

APPENDIX D

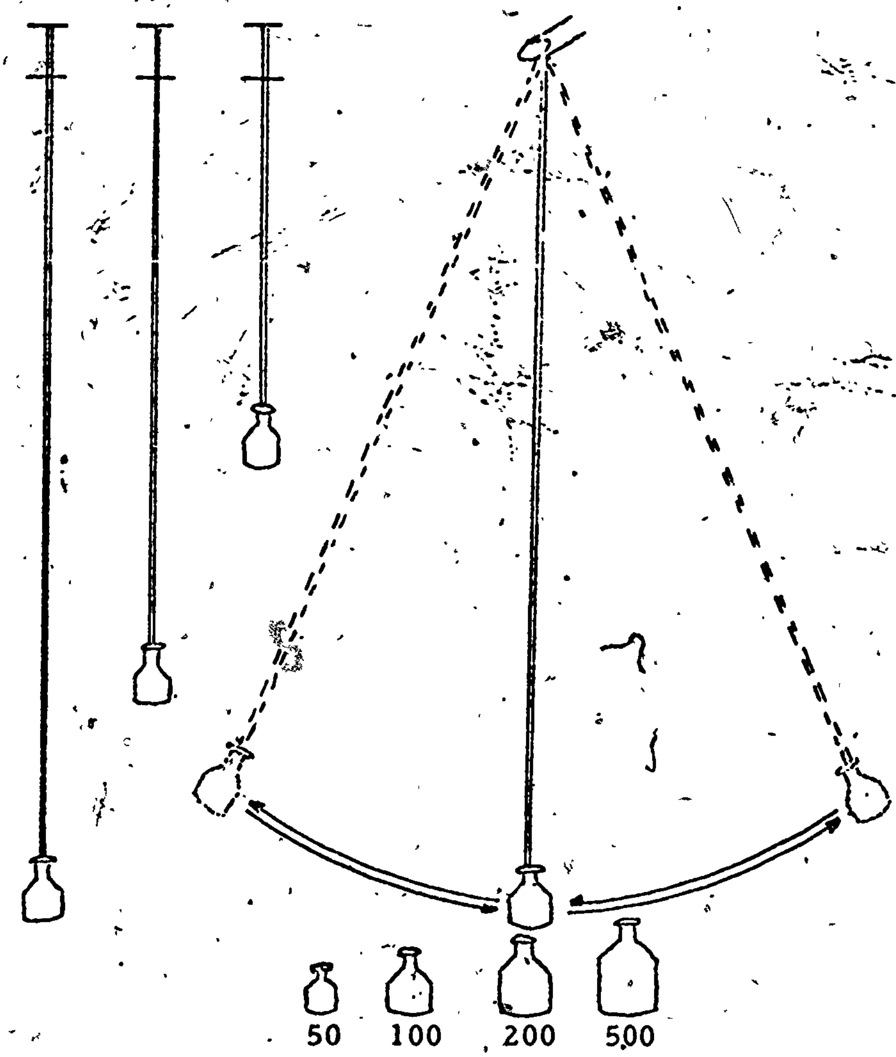
PROCEDURES FOR ADMINISTERING THE PIAGET TASKS

SECTION 1

OSCILLATION OF A PENDULUM

Present the subject with a simple pendulum in the form of a 100-gram weight tied halfway up the length of a string. The subject has the means to vary: the length of the string, the weight (1-, 2-, 5-, 10-, 20-, 50-, 100-, and 500-gram weights), the height of the dropping point, and the impetus imparted to the weight.

Demonstrate the pendulum; have the subject try it; then ask the subject how it (the pendulum) could be made to go faster or slower. The subject must demonstrate the solution and provide a satisfactory explanation for the proposed solution. Paper and pencil should be available if the subject wishes to note which combinations have been attempted.

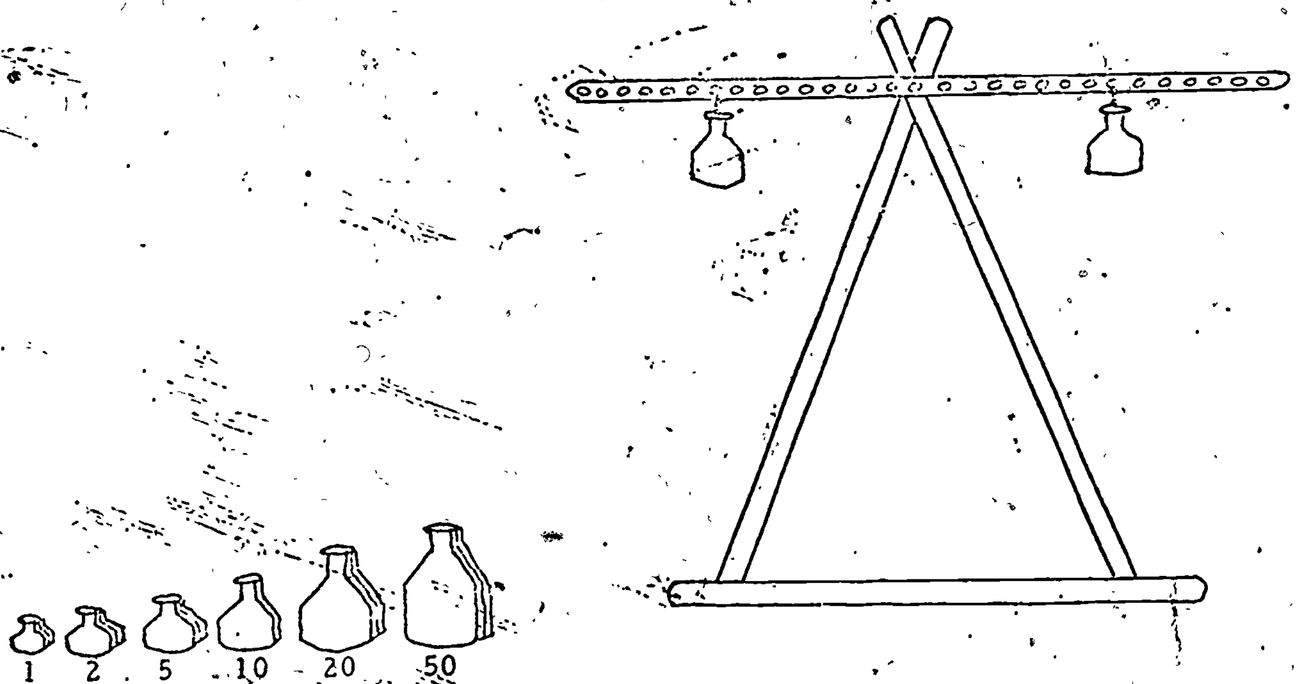


SECTION 2

EQUILIBRIUM IN THE BALANCE

Present the subject with a simple seesaw balance. Point out that the arm of the balance is level (horizontal) -- gesture with hand. Also present three series of weights (1-, 2-, 5-, 10-, 20-, and 50-gram weights) with attached hooks. Hand a 10-gram weight at hole number seven (counting from the fulcrum) and ask the subject to make the arm level (horizontal). As the subject is attempting to make the arm level, inquire as to why he did what he did. After the present problem has been completed, proceed to some of the following problems:

10 gram at 9 and 10 gram at opposite 14
5 gram at 10 and 1 gram at opposite 13
5 gram at 10 and 1 gram at same 13
50 gram at 15
20 gram at 5 and 50 gram at opposite 12
2 gram at 1 and 2 gram at opposite 5

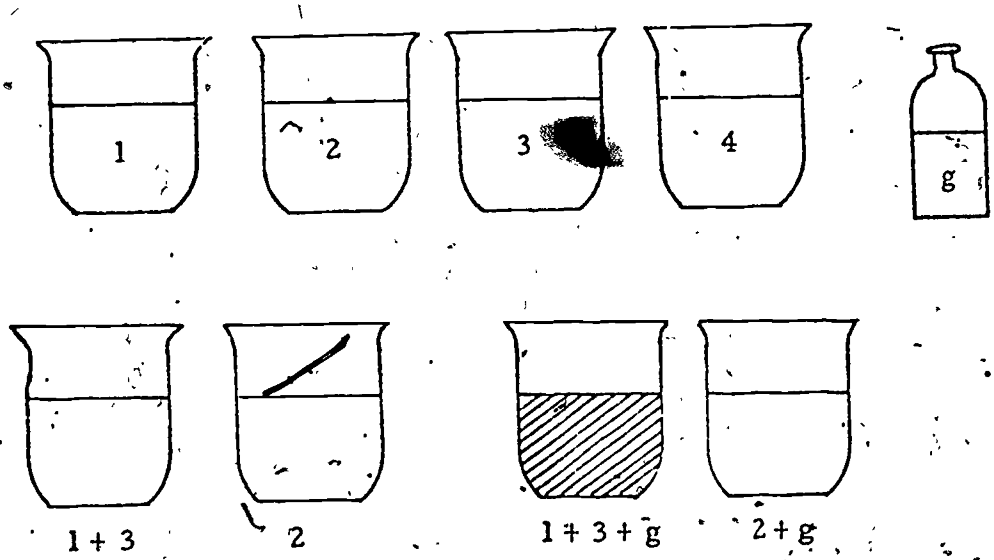


SECTION 3

COMBINATIONS OF COLORED AND COLORLESS CHEMICAL BODIES

Present the subject with four 100 ml beakers, each containing 50 ml of colorless, odorless liquids. The beakers are number 1, 2, 3, and 4; and the contents, unknown to the subject, are as follows: (1) dilute sulphuric acid (0.005 M H_2SO_4), (2) distilled water, (3) potassium iodate (0.01 M KIO_3), and (4) sodium thiosulphate (0.5 M $Na_2S_2O_3$ with 158 g/2 ℓ + 21.2 g Na_2CO_3). Also give the subject a small bottle, labeled g, also colorless and odorless, which contains potassium iodide (0.1 M KI 33.2 g/2 ℓ) and an eyedropper.

Next, present the subject with two 100 ml beakers, one containing a 50 ml solution of dilute sulphuric acid and potassium iodate (1 and 3), the other containing 50 ml of distilled water (2). Both of the solutions are colorless and odorless, and the contents are unknown to the subject. Add a few drops of potassium iodide (g) to each beaker and note the changes (1 and 3 and g will produce a yellow color; 2 and g will remain unchanged). After the subject has noted the changes, ask him to produce the yellow color, using 1, 2, 3, 4, and g in any way he wishes. Provide him with additional 100 ml beakers in which to make his combinations. Paper and pencil should be available if the subject should wish to note which combinations have been attempted.



SECTION 4

PIAGET TASKS LOGIC

OSCILLATION OF A PENDULUM

Subject was presented with the material to construct a simple pendulum: string, weights, and an apparatus on which the string was to be attached. The problem was to discover the factor that determines the frequency of the oscillations. Possible factors include (1) length of the string, (2) weight of suspended object, (3) height of dropping point, and (4) force of push given to weight. Subject had to isolate the possible factors, determine that string length is inversely related to oscillation frequency, and exclude the other three factors as causes of change in oscillation frequency. Logically:

p = string length modification
$\bar{p}$ = absence of string length modification
q = weight modification
$\bar{q}$ = absence of weight modification
r = dropping height modification
$\bar{r}$ = absence of dropping height modification
s = impetus modification
$\bar{s}$ = absence of impetus modification
x = oscillation frequency modification
$\bar{x}$ = absence of oscillation frequency modification

When the problem is correctly solved the following logical combinations are asserted to be true and the remaining possible combinations (not listed) are asserted to be false.

$$(p \wedge q \wedge r \wedge s \wedge x) \vee (p \wedge q \wedge r \wedge \bar{s} \wedge x) \vee$$
$$(p \wedge q \wedge \bar{r} \wedge s \wedge x) \vee (p \wedge q \wedge \bar{r} \wedge \bar{s} \wedge x) \vee$$
$$(p \wedge \bar{q} \wedge r \wedge s \wedge x) \vee (p \wedge \bar{q} \wedge r \wedge \bar{s} \wedge x) \vee$$
$$(p \wedge \bar{q} \wedge \bar{r} \wedge s \wedge x) \vee (p \wedge \bar{q} \wedge \bar{r} \wedge \bar{s} \wedge x) \vee$$
$$(\bar{p} \wedge q \wedge r \wedge s \wedge \bar{x}) \vee (\bar{p} \wedge q \wedge r \wedge \bar{s} \wedge \bar{x}) \vee$$
$$(\bar{p} \wedge q \wedge \bar{r} \wedge s \wedge \bar{x}) \vee (\bar{p} \wedge q \wedge \bar{r} \wedge \bar{s} \wedge \bar{x}) \vee$$
$$(\bar{p} \wedge \bar{q} \wedge r \wedge s \wedge \bar{x}) \vee (\bar{p} \wedge \bar{q} \wedge r \wedge \bar{s} \wedge \bar{x}) \vee$$
$$(\bar{p} \wedge \bar{q} \wedge \bar{r} \wedge s \wedge \bar{x}) \vee (\bar{p} \wedge \bar{q} \wedge \bar{r} \wedge \bar{s} \wedge \bar{x}) =$$
$$(p \rightleftarrows x) \wedge (q * x) \wedge (r * x) \wedge (s * x) =$$
$$p[q \vee r \vee s] \rightleftarrows x$$

EQUILIBRIUM IN THE BALANCE

Subject was presented with a simple seesaw balance and a series of weights. A weight or weights were placed on one arm and the subject had to balance the arms in any way that he saw fit. When the problem(s) is correctly solved the following logical proportions are "used". If two weights, W and W', are balanced at their corresponding distances from the axis, L and L', the amount of work, WH and W'H', needed to move the weights to the heights H and H' corresponding to the distances L and L' are equal. Consequently, a "double" inverse proportion exists:

$$\frac{W}{W'} = \frac{L'}{L} = \frac{H'}{H}$$

Discovering the above-noted "law" presupposes that the subject understands the construction of proportions, specifically:

$$\frac{W}{W'} = \frac{L'}{L}$$

$$\frac{W}{W'} = \frac{H'}{H}$$

$$\frac{L'}{L} = \frac{H'}{H}$$

Logically:

p = weight increase on one balance arm
$\bar{p}$ = weight decrease on one balance arm
p' = weight increase on other balance arm
$\bar{p}'$ = weight decrease on other balance arm
q = distance increase on one balance arm
$\bar{q}$ = distance decrease on one balance arm
q' = distance increase on other balance arm
$\bar{q}'$ = distance decrease on other balance arm

Using the I (Identity) N (Negation) R (Reciprocal) C (Correlative) group of transformations (Inhelder and Piaget, 1958; Piaget, 1957; Gray, 1970; Boyle, 1969) the following may be constructed.

$$\begin{aligned}
I\ (p \wedge q) &= p \wedge q \equiv \bar{p}' \wedge \bar{q}' \\
N\ (p \wedge q) &= \bar{p} \vee \bar{q} \equiv p' \vee q' \\
R\ (p \wedge q) &= \bar{p} \wedge \bar{q} \equiv p' \wedge q' \\
C\ (p \wedge q) &= p \vee q \equiv \bar{p}' \vee \bar{q}'
\end{aligned} \tag{1}$$

However, the system of transformation equations in (1) is equivalent to the proportionality:

$$\frac{p \wedge q}{\bar{p} \wedge \bar{q}} = \frac{p \vee q}{\bar{p} \vee \bar{q}} \equiv \frac{I}{R} = \frac{C}{N} \equiv \frac{\bar{p}' \wedge \bar{q}'}{p' \wedge q'} = \frac{\bar{p}' \vee \bar{q}'}{p' \vee q'} \tag{2}$$

Using the following rules a system of single propositions may be derived from the double propositional system. The four operations α, β, ρ, and σ are proportional if the following exist

$$\frac{\alpha}{\beta} = \frac{\rho}{\sigma} \tag{3}$$

$$\alpha \wedge \sigma = \beta \wedge \rho \tag{4}$$

$$\alpha \vee \sigma = \beta \vee \rho \tag{5}$$

and if equations (4) and (5) can be transposed by transforming $(\vee x)$ into $(\wedge \bar{x})$ or $(\wedge x)$ into $(\vee \bar{x})$. Then from equation (4) the following may be generated:

$$\alpha \vee \bar{\beta} = \rho \vee \bar{\sigma} \tag{6}$$

$$\alpha \vee \bar{\rho} = \beta \vee \bar{\sigma} \tag{7}$$

$$\bar{\alpha} \vee \beta = \bar{\rho} \vee \sigma \tag{8}$$

$$\bar{\alpha} \vee \rho = \bar{\beta} \vee \sigma \tag{9}$$

and from equation (5):

$$\alpha \wedge \bar{\beta} = \rho \vee \bar{\sigma} \quad (10)$$

$$\alpha \wedge \bar{\rho} = \beta \wedge \bar{\sigma} \quad (11)$$

$$\bar{\alpha} \wedge \beta = \bar{\rho} \wedge \sigma \quad (12)$$

$$\bar{\alpha} \wedge \rho = \bar{\beta} \wedge \sigma \quad (13)$$

It then can be observed that the four operations of α, β, ρ, σ can be considered to be an expression of the INRC group which always satisfies equations 3-13. For example:

$$\frac{I}{R} = \frac{C}{N} \text{ where } I = p \wedge q, \text{ then } \frac{p \wedge q}{\bar{p} \wedge \bar{q}} = \frac{p \vee q}{\bar{p} \vee \bar{q}} \equiv \frac{I}{R} = \frac{C}{N} \quad (14)$$

or where $I = p \vee q$, then

$$\frac{p \vee q}{\bar{p} \vee \bar{q}} = \frac{p \wedge q}{\bar{p} \wedge \bar{q}} \equiv \frac{I}{R} = \frac{C}{N} \quad (15)$$

The system of proportions may be extended to entities among which the INRC relationships do not hold, if the condition of group transformations is applied: i.e., $(\vee x)$ may be added to α if $(\wedge \bar{x})$ is added to σ when x has no part in common with α, and $(\vee x)$ may be eliminated in α if $(\wedge \bar{x})$ may be eliminated in σ, provided that x is wholly a part of α. Thus from

$$\frac{p \wedge q}{\bar{p} \wedge \bar{q}} = \frac{p \vee q}{\bar{p} \vee \bar{q}} \equiv \frac{I}{R} = \frac{C}{N} \quad (16)$$

$\frac{p}{\bar{q}} = \frac{q}{\bar{p}}$ can be derived while from

$$\frac{p \vee q}{p \wedge q} = \frac{\bar{p} \vee \bar{q}}{\bar{p} \wedge \bar{q}} \equiv \frac{I}{C} = \frac{R}{N} \quad (17)$$

$\frac{q}{p} = \frac{\bar{p}}{\bar{q}}$ is generated. Checking back to the definitions of the variables it can be seen that distance increase on one arm (q) can be compensated in a reciprocal fashion by a weight decrease on the same arm ($\bar{p}$) or it can be totally reversed by decreasing the distance ($\bar{q}$), and the weight increase (p) has the same effect as the increased distance. Using the equivalent operations on the other arm of the balance, equations (18) and (19) are respectively equivalent to (16) and (17).

$$\frac{\bar{p}' \wedge \bar{q}'}{p' \wedge q'} = \frac{\bar{p}' \vee \bar{q}'}{p' \vee q'} \equiv \frac{I}{R} = \frac{C}{N}$$

$$\frac{\bar{p}'}{q'} = \frac{\bar{q}'}{p'} \text{ or } \frac{\bar{q}'}{p'} = \frac{\bar{p}'}{q'} \qquad (18)$$

$$\frac{\bar{p}' \vee \bar{q}'}{p' \vee q'} = \frac{\bar{p}' \wedge \bar{q}'}{p' \wedge q'} \equiv \frac{I}{R} = \frac{C}{N}$$

$$\frac{\bar{p}'}{q'} = \frac{\bar{q}'}{p'} \text{ or } \frac{\bar{q}'}{p'} = \frac{\bar{p}'}{q'} \qquad (19)$$

Reciprocal proportions may be deduced in the same manner:

$$\frac{\alpha}{\beta} = R\frac{\rho}{\sigma} \qquad (20)$$

$$\alpha \wedge \sigma = R\ (\beta \wedge \rho) \qquad (21)$$

$$\alpha \vee \sigma = R\ (\beta \vee \rho) \qquad (22)$$

or

$$\sigma \wedge C\beta = R(\rho \wedge C\sigma) \qquad (23)$$

since $p \wedge \bar{q} = R\ (\bar{p} \wedge q)$ or $p \wedge q = R\ (\bar{p} \wedge \bar{q})$

or . . . .

Once an individual has acquired the double and/or single proportional systems, then numerical values may be inserted for the single values, such that a system of metrical proportions may be generated.

$$\frac{p}{q} = \frac{\bar{q}}{\bar{p}} \equiv \frac{I}{C} = \frac{R}{N}, \tag{24}$$

numerically corresponds to

$$\frac{n \cdot x}{n \cdot y} = \frac{n \div y}{n \div x} \tag{25}$$

$$\frac{2 \cdot 4}{2 \cdot 8} = \frac{2 \div 8}{2 \div 4} \tag{26}$$

and

$$\frac{p}{q} = R\,\frac{\bar{p}}{\bar{q}} \tag{27}$$

numerically corresponds to

$$\frac{n \cdot x}{n \cdot y} = \frac{x \div n}{y \div n} \tag{28}$$

$$\frac{2 \cdot 4}{2 \cdot 8} = \frac{4 \div 2}{8 \div 2} \tag{29}$$

all of which can be substituted for the initial qualitative propositions defined earlier.

COMBINATIONS OF COLORED AND COLORLESS CHEMICAL BODIES

Subject was presented with four similar beakers containing colorless, odorless liquids which were perceptually identical. The contents of the different beakers were as follows: (1) dilute sulphuric acid (0.005M H_2SO_4), (2) distilled water, (3) potassium iodate (0.01M KIO_3), (4) sodium thiosulphate (0.5M $Na_2S_2O_3$ with 158g/2ℓ + 21.2g Na_2CO_3). There also was an eyedropper bottle (g) containing potassium iodide (0.1M KI with 33.2g/2ℓ). The combination of 1, 3, and g yields a yellow color, while four bleaches or prevents the yellow color. Two is neutral.

Next, the subject was presented with two beakers, one containing 1 and 3 (unknown to the subject), the other containing 2 (unknown to the subject). The experimenter then added several drops of g to the two beakers and asked the subject to note the difference. Subject was then asked to produce the yellow color using 1, 2, 3, 4, and g in anyway. Logically, success is dependent upon the generation of a complete n-by-n set of combinations for elements 1, 2, 3, and 4 with g added to each combination and the realization the yellow color was a result of a combination (1, 3, and g) of liquids and not the result of only one liquid.

p = presence of 1
$\bar{p}$ = absence of 1
q = presence of 2
$\bar{q}$ = absence of 2
r = presence of 3
$\bar{r}$ = absence of 3
s = presence of 4
$\bar{s}$ = absence of 4
t = presence of g
$\bar{t}$ = absence of g
x = presence of yellow color
$\bar{x}$ = absence of yellow color

Subject must realize that complete affirmation with 2 exists:

$$(q \wedge x) \vee (q \wedge \bar{x}) \vee (\bar{q} \wedge x) \vee (\bar{q} \wedge \bar{x}) = (q * x)$$

and reciprocal exclusion or incompatibility with 4 occurs.

$$(s \wedge \bar{x}) \vee (\bar{s} \wedge x) = s \; w \; x$$

or

$$(s \wedge \bar{x}) \vee (\bar{s} \wedge x) \vee (\bar{s} \wedge \bar{x}) = s/x$$

Finally, the subject must understand that the yellow color is the result of the combination of 1, 3, and g only, even though 1, 2, 3, and g will also produce yellow.

$$x \rightleftarrows (p \wedge r \wedge t)$$

APPENDIX E

CHECKSHEET OF DEVELOPMENTAL LEVEL PERFORMANCES ON THE PIAGET TASKS

Section 1: Checksheet
Section 2: Classification Criteria

NAME: ______________________ ID: ______________________

SEX: __________ SEQUENCE: ______________

SCHOOL: ______________________ AGE: ______________________

E: ______________________

OSCILLATION OF A PENDULUM

L

Preoperations

- ______ Explanations imply that impetus provided by the subject is the cause of variations in oscillation frequency.
- ______ Influenced and misled by expectations of oscillation change.
- ______ Influenced and misled by expectations of experimental success.
- ______ Cannot dissociate own actions from motion of pendulum.
- ______ Cannot give objective account of the experiment (i.e.: incorrect oscillation count; incorrect judgment of oscillatory speed, etc.).
- ______ Cannot give consistent explanations which are mutually noncontradictory.
- ______ Constantly interferes with pendulum's motion.

"Beginning" concrete Operations

- ______ Serially orders string lengths.
- ______ Correctly counts number of oscillations.
- ______ Notices changes in oscillation frequency.
- ______ Serially orders height of dropping points.
- ______ Discovers inverse relationship between length of string and oscillation frequency.
- ______ Oscillation frequency due to more than one variable.
- ______ Consistently varies several variables simultaneously.

"Complete" concrete Operations

- ______ Serially orders weights.
- ______ Nonsystematically determines inverse correspondence between string length and oscillation frequency.
- ______ Varies several variables simultaneously (i.e.: weight and impetus; length and height, etc.).
- ______ Cannot separate effects of weight and string length.

Transitional	____	Deliberately varies variables simultaneously.
	____	Does not vary particular variable under consideration.
"Beginning" formal Operations	____	Isolates variables given combinations in which only one variable is varied.
	____	Cannot produce combinations in which one variable varies and all others remain constant.
	____	Nonsystematically varies one variable.
	____	Establishes cause of oscillation frequency and makes true implications.
	____	Cannot establish the noncausal variables and deny the false implications.
	____	Hesitation in giving answer.
	____	Complex responses (i. e.: string length, maybe weight, etc.).
"Complete" formal Operations	____	Varies single variable, holding "all other variables equal."
	____	Systematic approach to problem.
	____	Excludes noncausal variables and denies the false implications.
	____	Isolates all variables.
	____	Simplicity of correct responses.

EQUILIBRIUM IN THE BALANCE

Preoperations

_____ Intrudes in the working of the balance.

_____ Does not distinguish own actions from actions of balance.

_____ Tries to repeat with new weights what had just been accomplished with others, not considering weight differences.

_____ Use of terms (i.e., heavy, light, etc.) without use of comparison term.

_____ Removal of weight from "light" arm.

_____ Equilibrium does not imply the equality of weights (i.e.: all weights on one arm; gross differences between weight on each arm).

_____ Addition of weights to "heavy" arm.

_____ States that heavy side moves upward.

_____ States that light side moves downward.

_____ Symmetry (weight on each arm) because "it looks good".

_____ Symmetry of weight placement (extremes of arms -- 7 - 9).

_____ Generally unconcerned with distances of weights from axis.

_____ Does not look for any equality or coordination between the distances and weights.

_____ Weights not removed with deliberate aim of equalizing the balance.

_____ Removes weight to try something new and different.

_____ Notices weights should be on both arms to achieve equilibrium.

_____ Weights placed on both arms should be approximately equal for equilibrium.

_____ Symmetry of weight placement near axis (1-3).

_____ Search for correct solution for equilibrium involves successive corrections of previous actions.

_____ Notices role of distance when shown example.

_____ Chance discovery of the role played by changes in distance.

"Beginning" formal Operations

_____ Serially orders weights.

_____ Correctly compares one set of weights with another set.

_____ Makes use of the transitiveness of the relations of equality or inequality of weights.

_____ Serially orders distances.

_____ Distances are added and made symmetrical.

_____ Discovers symmetry of distances relative to the axis.

_____ Emphasis is on substitutions (additions or subtractions) of weights, especially with unequal weights.

_____ Unequal weights and distances are not balanced.

_____ Trial-and-error (empirical) discovery of equilibrium between smaller weight at greater distance and greater weight at smaller distance.

_____ Cannot invert discovered relationships (i.e., do on opposite arm what was done on the other).

"Complete" formal Operations

_____ Quantifies weights (i.e., B = 2A).

_____ Quantifies distances (counts the holes) from the axis.

_____ Unequal weights and distances solved by qualitative correspondences (i.e., heavier it is, closer to the middle, etc.).

_____ Point of reference for qualitative correspondence is axis.

"Beginning" formal Operations

_____ Simultaneous suspension of unequal weights at unequal distances produces discovery of law of compensations.

_____ Uses reciprocity (distance compensates weights) in explanations.

_____ Cannot give causal explanation of phenomena.

_____ Systematic in approach.

_____ Uses metrical proportions.

"Complete" formal Operations

_____ Successive and alternate suspensions of weights turn subject's attention to the inclination of the arms.

_____ Searches for an explanation of the phenomena using alternate suspensions.

_____ Causal explanation of phenomena (in terms of work).

_____ Indicates heights that weights are lifted to in explanation of phenomena.

_____ "Explains" concept of work and the energy needed to raise a weight on an arm a specific vertical distance.

COMBINATIONS OF COLORED AND COLORLESS CHEMICAL BODIES

Preoperations

- _____ Random association of two elements at a time.
- _____ Explanations based on simple phenomenalism (i.e., "It comes, it goes.").
- _____ Production and disappearance of color dependent on a "human" quality (animism) assigned to the color.

"Beginning" concrete Operations

- _____ Spontaneous and systematic assocation (multiplication) of g separately with 1, 2, 3, and 4.
- _____ Combines 1, 2, 3, and 4 with g in one beaker.
- _____ Does not go beyond multiplication of 1, 2, 3, and 4 by g unless prompted.
- _____ Order of mixing elements produces yellow color.
- _____ Inverts previous order of mixing elements to produce color.
- _____ Hypotheses are purely quantitative (i.e., "Too much water, not enough water, etc.").
- _____ Aware that 4 bleaches color.
- _____ Does not exclude 4 from combinations after he knows that it bleaches color.
- _____ Specific element produces color.

"Complete" concrete Operations

- _____ Nonsystematic n-by-n combinations.
- _____ Combinations involve tentative empirical efforts (trial-and-error).
- _____ Spontaneous generation of two-by-two or three-by-three combinations after the initial attempts of associating g separately with 1, 2, 3, and 4.

"Beginning" formal Operations

- _____ Systematic n-by-n combinations.
- _____ Combination of elements produces color.
- _____ Searches for other combinations, other than 1 + 3 + g, which may produce the yellow color.

	______	4 is proven as a color bleach.
	______	2 is empirically judged neutral.
"Complete" formal Operations	______	Combinations attempted in an extremely systematic manner.
	______	Proofs of combinations are extremely systematic.
	______	2 is systematically judged as neutral.
	______	Approach is geared toward proof.
	______	Increased speed in proofs.

SECTION 2

CLASSIFICATION CRITERIA

The checksheet in Section 1 of this appendix is what was used in evaluating each student's performance on the three Piaget tasks. Designations of the developmental levels were not included in the checksheets used.

Classification of a subject depended on the pattern of performances he exhibited. Generally, a subject was classified as being in the most sophisticated developmental level in which he manifested most of the characteristics. Exceptions to this general rule were made if a subject exhibited THE KEY performance for a specific level and virtually none of the other characteristics of that level. In such cases the subject was classified as being in the period in which he manifested THE KEY characteristic. Examples of such characteristics are presented in Table E-1.

TABLE E-1

KEY CLASSIFICATION PERFORMANCES

Task[1] and Subperiod[2]	Characteristic
P FI	Establishes cause of oscillation frequency and makes true implications.
P FII	Excludes non-causal variables and denies the false implications
B CII	Unequal weights and distances solved by qaulitative correspondences (i.e., heavier it is, closer to the middle, etc.)
B FI	Uses metrical proportions
B FII	Causal explanation of phenomena (in terms of work).
B FII	"Explains" concept of work and the energy needed to raise a weight on an arm a specific vertical distance.
C CII	Nonsystematic n-by-n combinations.
C FI	Systematic n-by-n combinations.
C FI	Combination of elements produces color.

[1] P - Pendulum, B = Balance, C = Chemical

[2] CII = "Complete" concrete Operations, FI = "Beginning" Formal Operations, FII = "Complete" Formal Operations

APPENDIX F

TEST OF LOGICAL THINKING

Section 1: TOLT

Section 2: Item Logic

Date: ____________ ID: ____________

Sequence: ____________

Test Of Logical Thinking

Name: ______________________________

Age: ______________________________

Sex: ______________________________

School: ______________________________

E: ______________________________

Exclusion: ______________________________

Proportion: ______________________________

Combination: ______________________________

Directions

You are going to do some problems which are easy, if you think about them. Some present some facts and you have to make a conclusion. Some ask you to complete drawings. Some will seem like arithmetic problems, but you do not need to do any arithmetic to answer them. Others ask you to put things together in different ways. There are 36 problems. PLEASE BE SURE TO ANSWER ALL OF THEM.

Below are three examples. Read them and decide on the correct answer for each one.

Example 1

All of the following sentences are true. What must happen for Ed to like Susan?

John likes Mary, Bill likes Ann, and Ed likes Susan.

John likes Mary, Bill does not like Ann, and Ed likes Susan.

John does not like Mary, Bill likes Ann, and Ed does not like Susan.

The correct answer is "John likes Mary."

Example 2

Chuck and Jim are playing a card game called "Battle." At the beginning of the game, Chuck and Jim each have 26 cards. In the 26 cards that Chuck has, there are 3 kings; and in the 26 cards that Jim has, there is 1 king. Each player will turn over one card at the same time. Who has more of a chance of turning over a king in the first "battle"?

The correct answer is "Chuck."

Example 3

Don, Chip, Bill, and Paul are going to ride on bumper cars. There are only two seats in each car. Each boy wants to ride with every other boy.

Write all of the possible two-man teams that can be formed.

There should be six teams listed: Don and Chip, Don and Bill, Don and Paul, Chip and Bill, Chip and Paul, and Bill and Paul.

There is no time limit; however, work as rapidly as you can.

PLEASE ANSWER EVERY QUESTION

---

1. George is taller than Bill.
Bill is taller than Harold.
Harold is taller than Carl.

Is George taller than Carl?

2. Paul is heavier than Ken.
Ken is heavier than John.
John is heavier than Ron.

Is Paul heavier than Ron?

3. Mike is bigger than Al.
Al is bigger than Sam.
Sam is bigger than Tom.

Is Mike bigger than Tom?

4. Bob is the richest of four men; Jim, the next richest; Lloyd, the next richest; and Tim, the next richest. The richest man owns the smallest car; the next richest man, the next smallest car, and so on.

Who owns the smallest car?

5. Joe is the fastest of four men; Bill, the next fastest; Ken, the next fastest; and Dave, the next fastest. The fastest man has the smallest feet; the next fastest man, the next smallest feet; and so on.

Who has the second smallest feet?

6. Gene is the best of four baseball players; Alan, the next best; Walt, the next best, and Rich, the next best. The best player is the shortest, the next best player, the next shortest, and so on.

Who is the third shortest?

7. All of the following sentences are true. What must happen for the husband to live?

The maid likes her job, the wife faints, the cook runs out the door, and the husband lives.

The maid likes her job, the wife does not faint, the cook does not run out the door, and the husband lives.

The maid does not like her job, the wife faints, the cook does not run out the door, and the husband does not live.

8. All of the following sentences are true. What must happen for John to talk to Luke?

Paul sings, Jill screams, the police are correct, and John talks to Luke.

Paul does not sing, Jill does not scream, the police are correct, and John talks to Luke.

Paul sings, Jill does not scream, the police are not correct, and John does not talk to Luke.

9. All of the following sentences are true. What must happen for Sam to be on vacation?

John is going with his friends, Tom is walking through a village, Bob is not going fishing, and Sam is on vacation.

John is not going with his friends, Tom is walking through a village, Bob is going fishing, and Sam is on vacation.

John is going with his friends, Tom is not walking through a village, Bob is not going fishing, and Sam is not on vacation.

10. All of the following sentences are true. What must happen for Alvin to go to the store?

Mary comes home, it rains, Mike is not here, and Alvin does not go to the store.

Mary does not come home, it does not rain, Mike is here, and Alvin does not go to the store.

Mary does not come home, it rains, Mike is not here, and Alvin goes to the store.

Mary comes home, it does not rain, Mike is here, and Alvin goes to the store.

11. All of the following sentences are true. What must happen for John to live in the city?

The police are correct, the newspaper is not reporting true news, the thief walks, and John does not live in the city.

The police are not correct, the newspaper is not reporting true news, the thief walks, and John lives in the city.

The police are correct, the newspaper is reporting true news, the thief does not walk, and John lives in the city.

The police are not correct, the newspaper is reporting true news, the thief does not walk, and John does not live in the city.

12. All of the following sentences are true. What must happen for there to be good weather?

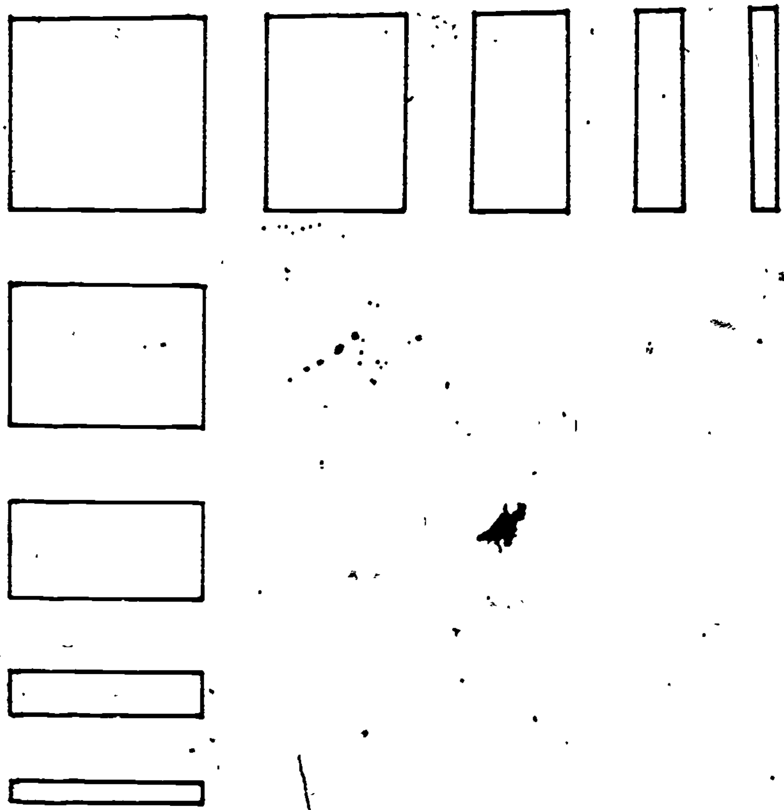
Charlie is swimming, Dave is not boating, Ken is playing in the sand, and there is good weather.

Charlie is not swimming, Dave is boating, Ken is not playing in the sand, and there is not good weather.

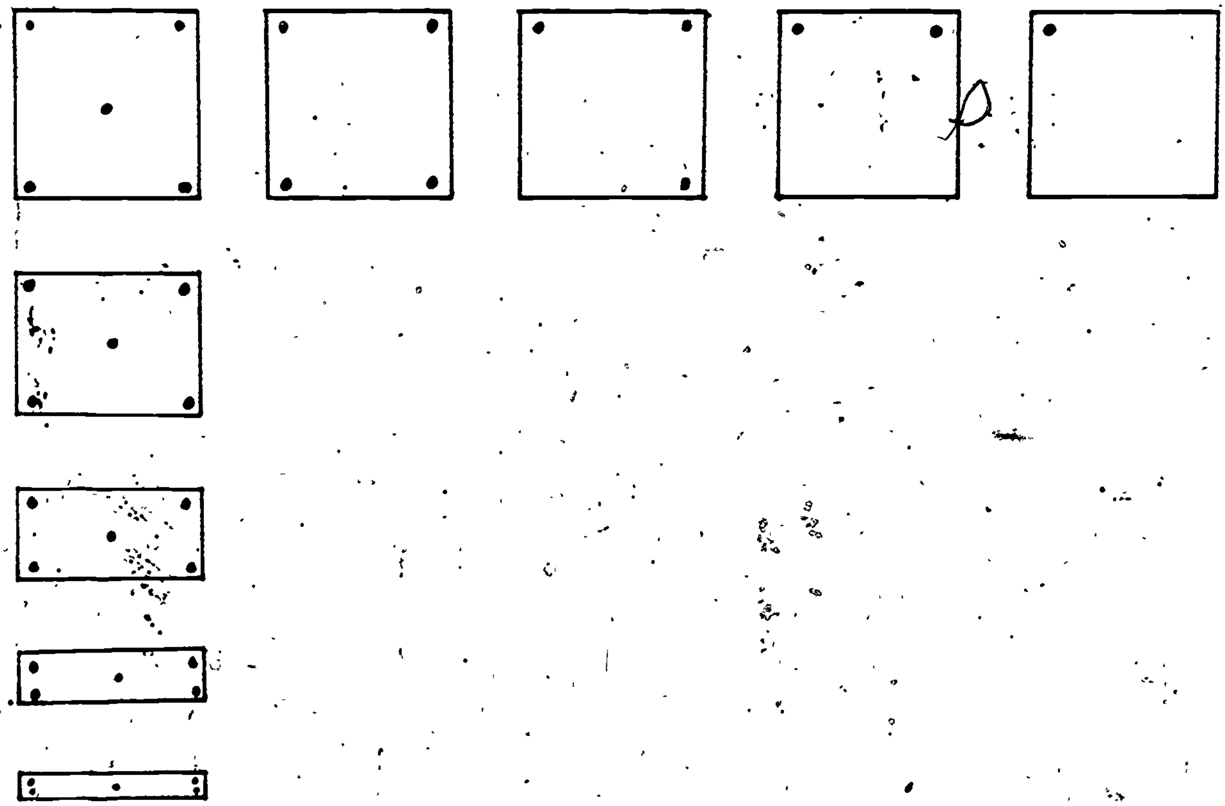
Charlie is not swimming, Dave is boating, Ken is not playing in the sand, and there is good weather.

Charlie is swimming, Dave is not boating, Ken is playing in the sand, and there is not good weather.

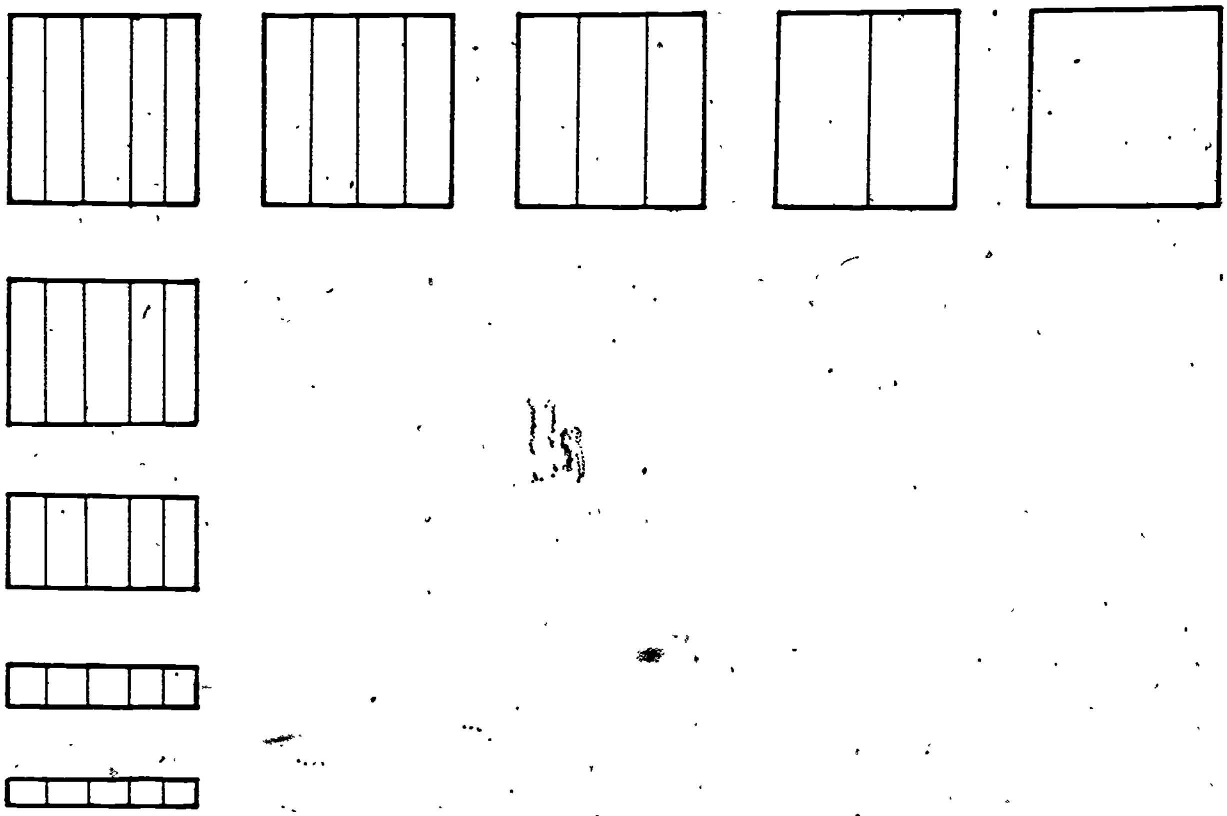
13. Complete the following drawing.



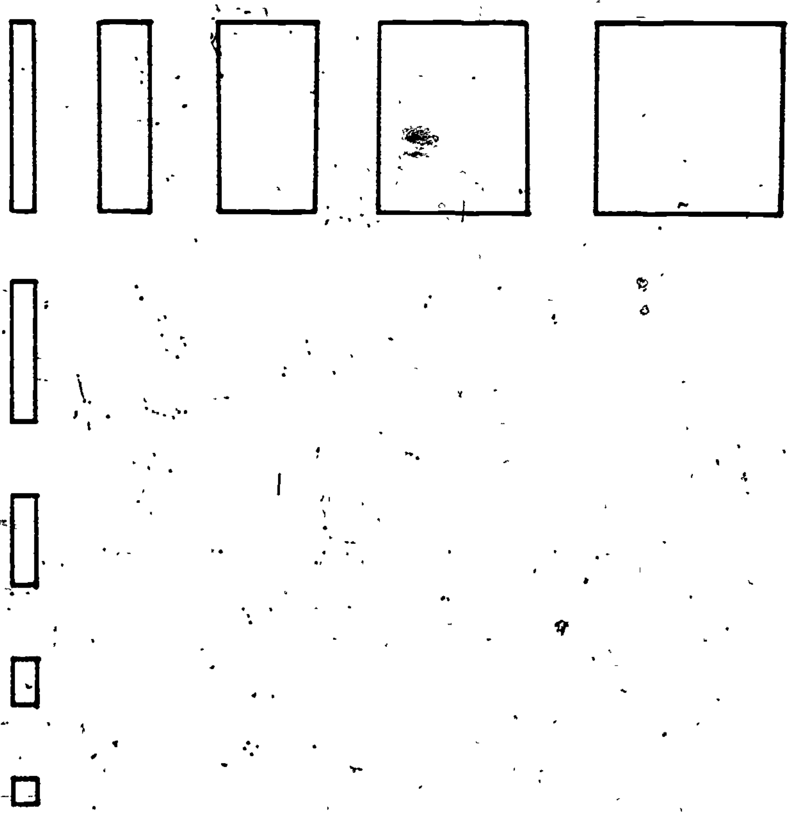
14. Complete the following drawing.



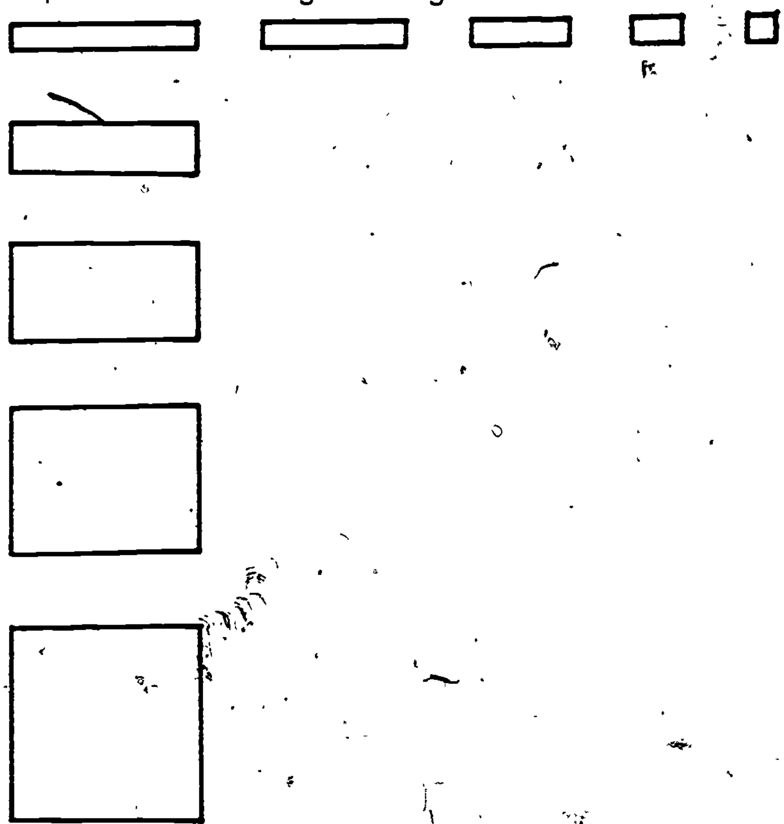
15. Complete the following drawing.



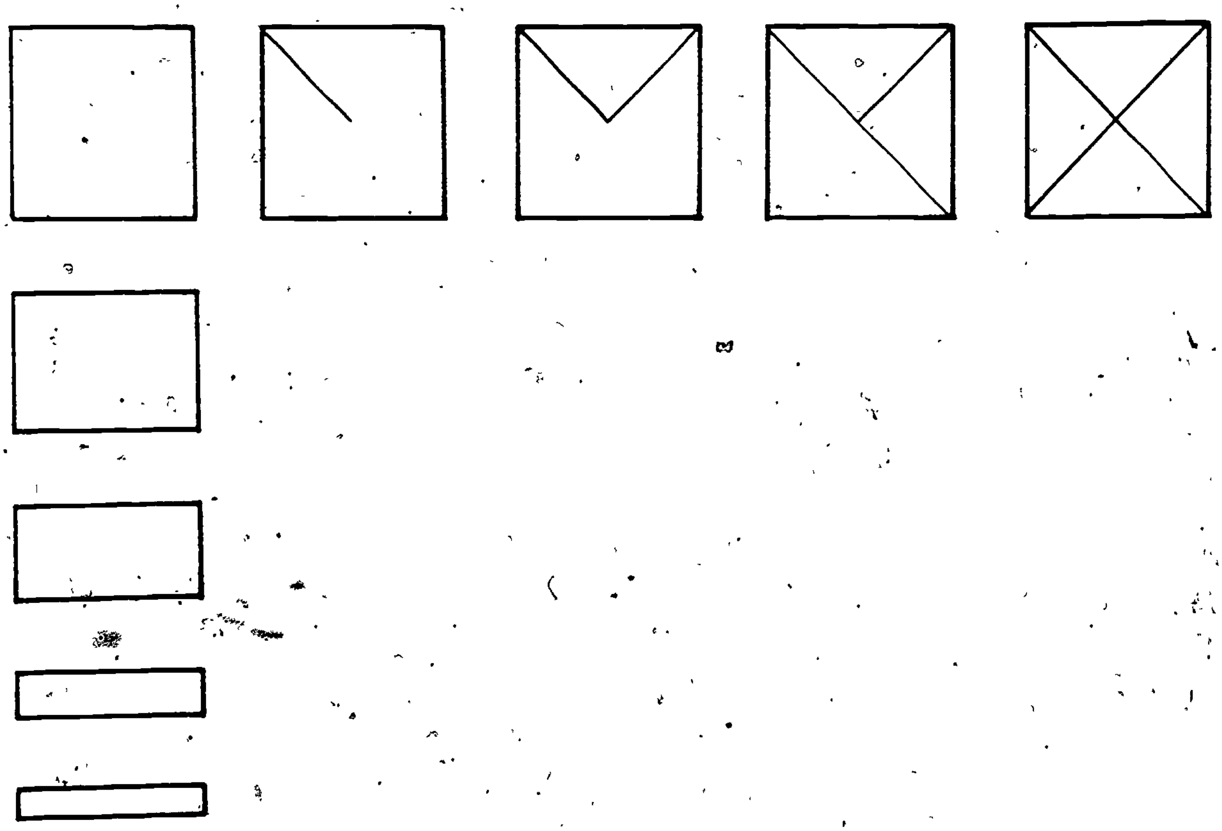
16. Complete the following drawing.



17. Complete the following drawing.



18. Complete the following drawing.



19. Two groups of children are going swimming. Teachers are going with them and will watch them. The first group is made up of 12 children and 2 teachers. The second group is made up of 18 children and 3 teachers.

In which group is each teacher in charge of the fewest children?

20. Fred buys 3 tickets in a raffle, and a total of 75 tickets are sold. Bob buys 2 tickets in another raffle, and a total of 50 tickets are sold.

Who has the best chance of winning his raffle?

21. During recess, two separate groups of children play ball. The first group is made up of 30 children and 6 balls. The second group is made up of 20 children and 4 balls.

Which group of children is it best to join if one wishes to catch a ball most often?

22. John and Chip each buy a bag of candy. In John's bag, there are 12 pieces of hard candy and 20 mints. In Chip's bag, there are 9 pieces of hard candy and 15 mints.

Who has the best chance of grabbing a piece of hard candy when he takes a piece of candy from his bag?

23. In a garage, there are 9 red cars and 12 blue cars. In a second garage, there are 15 red cars and 20 blue cars. A car is driven from each garage at the same time.

From which garage does one have the best chance of seeing a red car driven first?

24. At quitting time, the workers of a factory leave through two doors. At door one, 12 women and 18 men will leave. At door two, 18 women and 27 men will leave. One person leaves each door at the same time.

From which door does one have the best chance of seeing a woman leave first?

25. A coach has a choice of four players. He wants to give each player the same chance of making the team. He lets each player play for one week.

How many weeks will the coach need if each player is to have the same chance of making the team?

26. A boy has a choice of six fishing poles. He wants to try out each pole before he buys any one of them. He may try out only one pole a day.

How many days will he need to try out all of the poles?

27. A girl wants to buy a skirt. She has found eight skirts that she likes and would like to try them on to see which one looks the best on her. She can only try on one at a time. It takes her one minute to try on a skirt.

How many minutes will it take her to try on all of the skirts?

28. A group of friends decide to go dancing. There are three men (Al, Bob, and Chuck) and three women (Louise, Marsha, and Nancy). Each man wants to dance with each woman.

Write all of the possible man-woman couples of dancers there could be if each man danced with each woman.

29. A baseball manager has three pitchers (Sam, Tom, and George) and two catchers (Bill and Frank). The manager wants to find the best pair of pitcher and catcher.

Write all of the possible pairs of pitcher and catcher there could be if each pitcher threw to each catcher.

30. In a Chinese restaurant, the menu has two columns listing the things that can be ordered. Column A has duck, fish, peanuts, and rice. Column B has apples, bread, chicken, and ham. Only one pair of items, one from Column A and one from Column B, can be chosen at any one time.

Write all of the possible pairs of food that could be made if each food in Column A was chosen with each food in Column B.

31. Six boys (Andy, Charlie, David, Mike, Paul, and Sam) are going to play tennis. Each boy wants to play every other boy in a game.

Write all of the possible games that could be played if each boy played every other boy.

32. Seven men (Jim, Ken, Leo, Mel, Ned, Paul, and Tom) are going to race each other. Only two men can race at a time. Each man wants to race every other man.

Write all of the possible races that could be run if each man raced every other man.

33. Eight football teams are going to play each other. Each team will play every other team.

Write all of the possible games that could be played if each team played every other team.

34. A boy goes to an ice cream store and asks for four different ice cream sodas (chocolate, lemon, strawberry, and vanilla). They are served one at a time. The next day, he asks for the same sodas, but in a different order.

Write all of the possible ways that the sodas could be served to the boy.

35. In a four-car race, the order of finish is as follows: Chevrolet, Dodge, Ford, and Plymouth. They could have finished in any order.

Write all of the possible ways that the cars could finish if enough races were run

36. Four companies (Chrysler, Delco, Frigidaire, and Nabisco) are going to have offices on the first four floors of a new building. Each company may choose any of the floors for its offices. No two companies can be on the same floor.

Write all of the possible ways that the companies' offices could be arranged on the floors.

SECTION 2

ITEM LOGIC

Question	Logical Structure
Ex. 1	Exclusion - Formal I

$$p \wedge q \wedge x$$
$$p \wedge \bar{q} \wedge x.$$
$$\bar{p} \wedge q \wedge \bar{x}$$
$$\therefore p\,[q] \rightleftarrows x$$

Ex. 2	Proportion - Formal I

$$3/26 > 1/26$$

Ex. 3	Combination - Formal I

	D	C	B	P
D				
C	CD			
B	BD	BC		
P	PD	PC	PB	

1 - 3 Exclusion - Concrete I (Serial Ordering)

Each item consisted of a four entity ordered series. Subject had to understand that the entities were presented in decreasing order and conclude that the first was "greater" than the last.

1.

$$G > B$$
$$B > H$$
$$H > C$$
$$\therefore G > C$$

2.

$$P > K$$
$$K > J$$
$$J > R$$
$$\therefore P > R$$

3. $M > A$
$A > S$
$S > T$

$M > T$

4 - Exclusion - Concrete II (Biunivocal Inverse Correspondence)

Each item consisted of two, four entity ordered series, one increasing the other decreasing. Subject had to understand the correspondence between the entities of each series and determine which entity in the decreasing series corresponded to a specific entity in the increasing series.

4. $B > J > L > T$
$\updownarrow \quad \updownarrow \quad \updownarrow \quad \updownarrow$
$C_1 < C_2 < C_3 < C_4$

C_1 B
$\updownarrow$
C_1

5. $J > B > K > D$
$\updownarrow \quad \updownarrow \quad \updownarrow \quad \updownarrow$
$F_1 < F_2 < F_3 < F_4$

F_2 B
$\updownarrow$
F_2

6. $G > A > W > R$
$\updownarrow \quad \updownarrow \quad \updownarrow \quad \updownarrow$
$B_1 < B_2 < B_3 < B_4$

B_3 W
$\updownarrow$
B_3

7 - 9 Exclusion - Formal I (Establish Correct Implication)

Each item consisted of three true sentences, each containing four facts. Subject had to determine which of the first three facts consistently co-occurred with the fourth one across all sentences concluding that (1) the occurrence of a specific fact implied the occurrence of the fourth, (2) the nonoccurrence of that fact implied the nonoccurrence of the fourth, (3) and vice versa.

7.

$p \wedge q \wedge r \wedge x$

$p \wedge \bar{q} \wedge \bar{r} \wedge x$

$\bar{p} \wedge q \wedge \bar{r} \wedge \bar{x}$

$\therefore p\ [q \vee r] \rightleftarrows x$

8.

$p \wedge q \wedge r \wedge x$

$\bar{p} \wedge \bar{q} \wedge r \wedge x$

$p \wedge \bar{q} \wedge \bar{r} \wedge \bar{x}$

$\therefore r\ [p \vee q] \rightleftarrows x$

9.

$p \wedge q \wedge \bar{r} \wedge x$

$\bar{p} \wedge q \wedge r \wedge x$

$p \wedge \bar{q} \wedge \bar{r} \wedge x$

$\therefore q\ [p \vee r] \rightleftarrows x$

10 - 12 Exclusion - Formal II (Exclude Incorrect Implications)

Each item consisted of four true sentences, each containing four facts. Subject had to determine which of the first three facts consistently co-occurred with the fourth one across all sentences, concluding that none of the first three facts implied the occurrence of the fourth and vice versa.

10.

$p \wedge q \wedge \bar{r} \wedge \bar{x}$

$\bar{p} \wedge \bar{q} \wedge r \wedge \bar{x}$

$\bar{p} \wedge q \wedge \bar{r} \wedge x$

$p \wedge \bar{q} \wedge r \wedge x$

$\therefore [p \vee q \vee r] \ast x$

11.

$p \wedge \bar{q} \wedge r \wedge \bar{x}$

$\bar{p} \wedge \bar{q} \wedge r \wedge x$

$p \wedge q \wedge \bar{r} \wedge x$

$\bar{p} \wedge q \wedge \bar{r} \wedge \bar{x}$

$[p \vee q \vee r] * x$

12.

$p \wedge \bar{q} \wedge r \wedge x$

$\bar{p} \wedge q \wedge \bar{r} \wedge \bar{x}$

$\bar{p} \wedge q \wedge \bar{r} \wedge x$

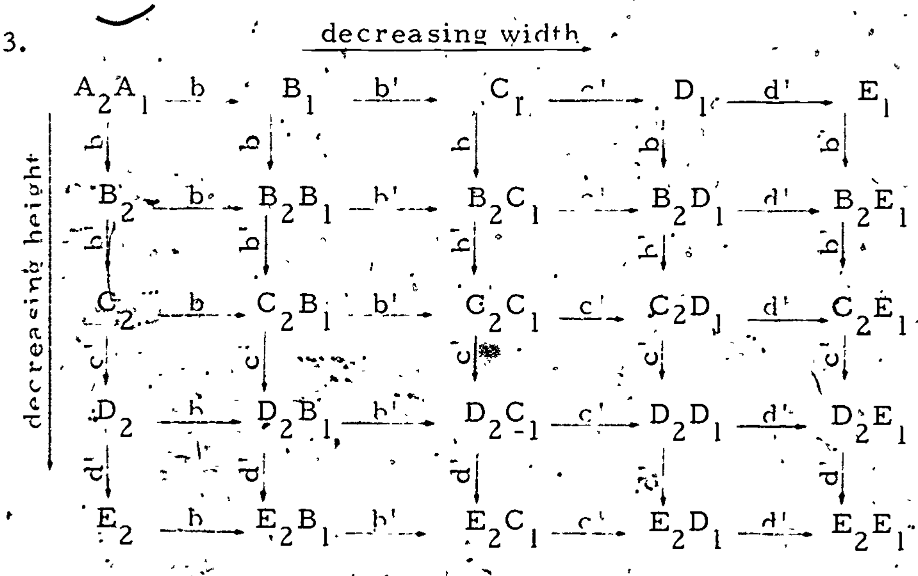
$p \vee \bar{q} \vee r \vee \bar{x}$

$[p \vee q \vee r] * x$

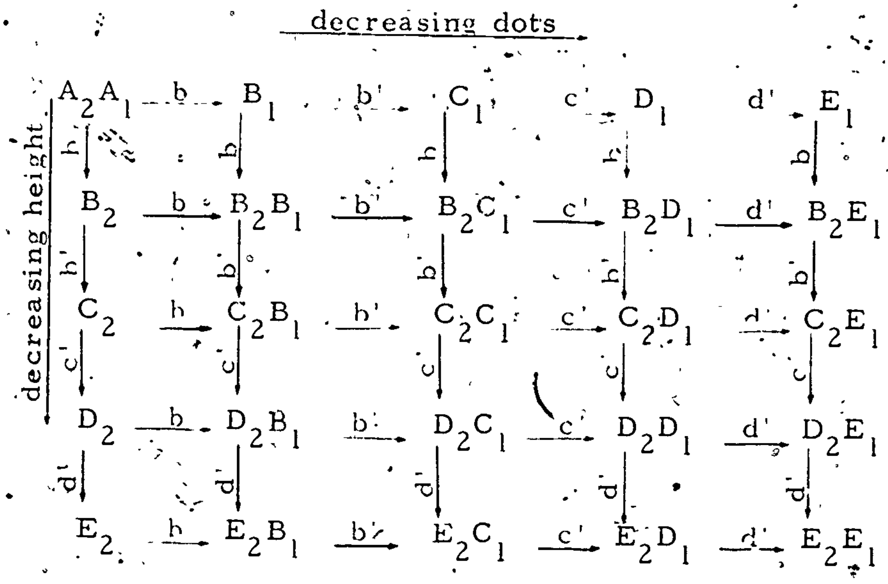
13 - 15 Proportion - Concrete I (Biunivocal Multiplication of Relations)

Each item consisted of a row and a column of five geometric figures in which the row and column were separate ordered series that decreased starting with the figure they both had in common. Subject had to determine how to coordinate the two series and draw sixteen figures completing a matrix of twenty-five figures characterized by the one-to-one multiplication of the two series.

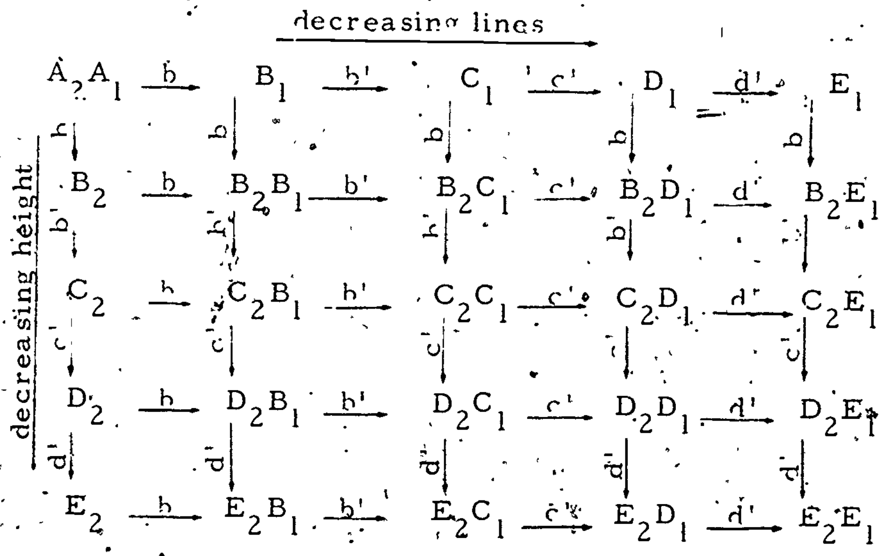
13.



14.


decreasing dots
decreasing height
A2A1 b B1 b' C1 c' D1 d' E1
B2 b B2B1 b' B2C1 c' B2D1 d' B2E1
C2 b C2B1 b' C2C1 c' C2D1 d' C2E1
D2 b D2B1 b' D2C1 c' D2D1 d' D2E1
E2 b E2B1 b' E2C1 c' E2D1 d' E2E1

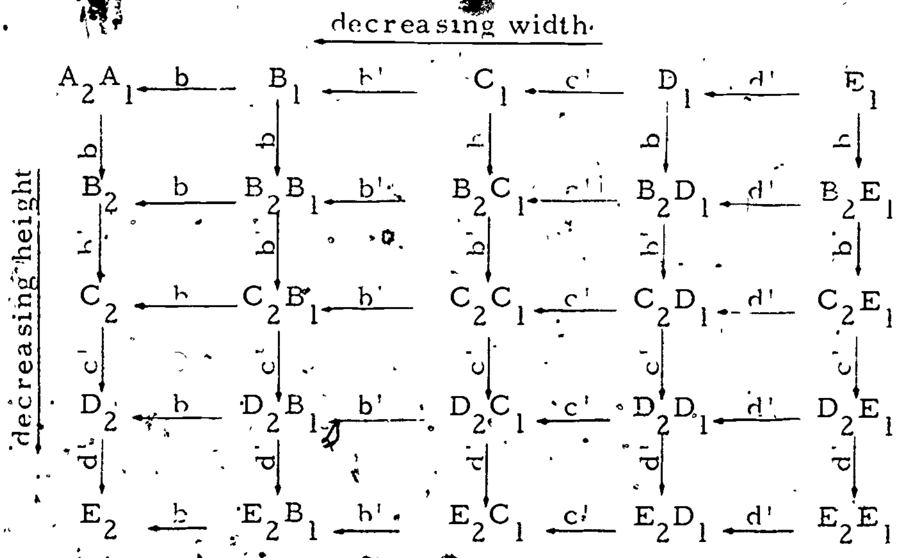
15.


decreasing lines
decreasing height
A2A1 b B1 b' C1 c' D1 d' E1
B2 b B2B1 b' B2C1 c' B2D1 d' B2E1
C2 b C2B1 b' C2C1 c' C2D1 d' C2E1
D2 b D2B1 b' D2C1 c' D2D1 d' D2E1
E2 b E2B1 b' E2C1 c' E2D1 d' E2E1

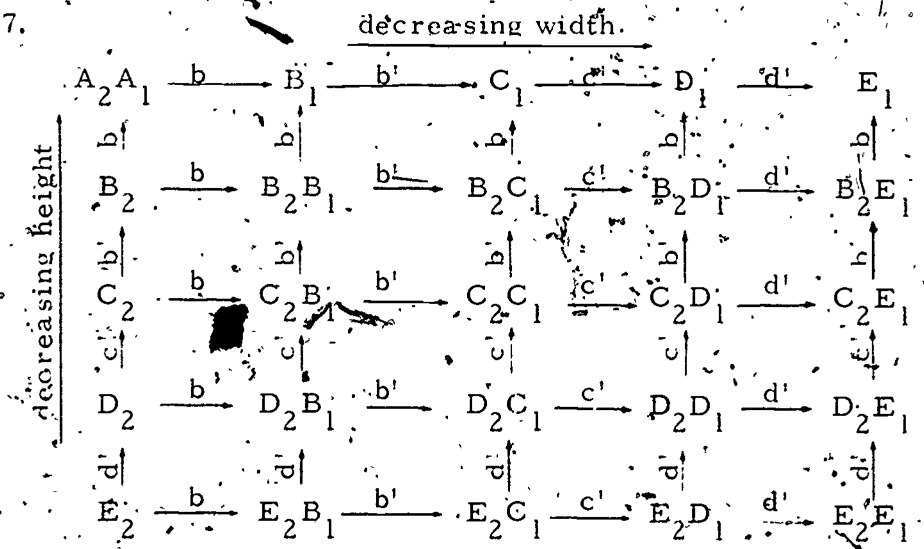
16 - 18 Proportion - Concrete II (Biunivocal Multiplication of Relations of Inverse Correspondences)

Each item consisted of a row and a column of five geometric figures in which the row and column were separate ordered series, one increasing the other decreasing, beginning with the figure they had in common. Subject had to determine how to coordinate the two series and draw sixteen figures completing a matrix of twenty-five figures characterized by the one-to-one multiplication of the two series.

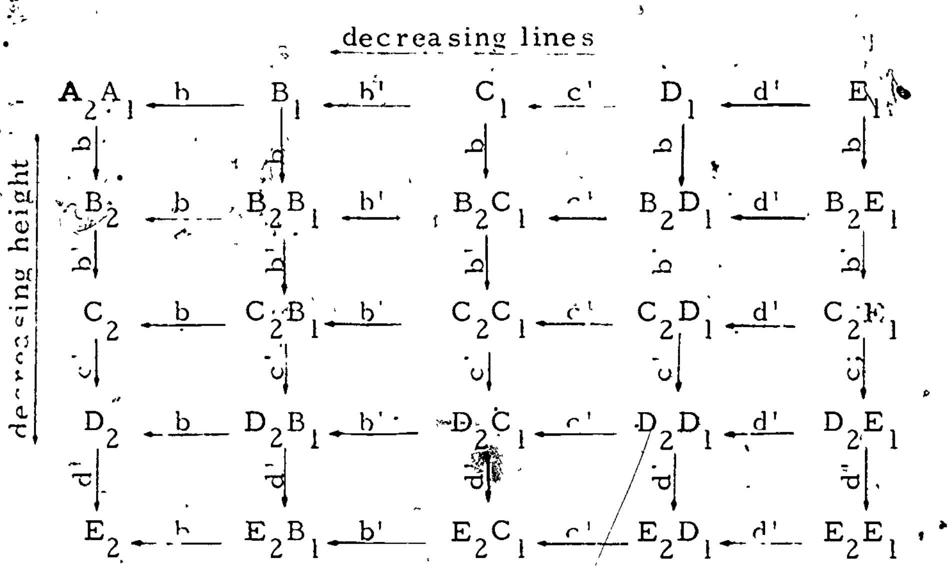
16.



17.



18.



19 - 21 Proportion Formal I ("Simple" Proportion)

Each item consisted of two sets of numbers. Subject had to construct the two correct porportions, reduce each to its lowest terms (all numerators reduced to 1), and conclude that the two proportions were equal.

GCD = Greatest Common Divisor
LCM = Least Common Multiple

19. $\frac{GCD}{X} = \frac{Y}{LCM \div 2}$ reduced to $\frac{GCD \div GCD}{X \div GCD} = \frac{Y \div Y}{(LCM \div 2) \div Y}$

20. $\frac{Y}{LCM \div 2} = \frac{GCD}{X}$ reduced to $\frac{Y \div Y}{(LCM \div 2) \div Y} = \frac{GCD \div GCD}{X \div GCD}$

21. $\frac{Y}{LCM \div 2} = \frac{GCD}{X}$ reduced to $\frac{Y \div Y}{(LCM \div 2) \div Y} = \frac{GCD \div GCD}{X \div GCD}$

22 - 24 Proportion Formal II ("Complex" Proportion)

Each item consisted of two sets of numbers. Subject had to construct the two correct proportions, reduce each to its lowest terms, and conclude that the two proportions were equal.

22.

$A + B = LCM \div 3$

$$\frac{A}{LCM \div 3} = \frac{GCD^2}{C + D} \text{ reduced to } \frac{A \div A}{(LCM \div 3) \div A} = \frac{GCD^2 \div GCD}{(C + D) \div GCD}$$

23.

$C + D = LCM \div 3$

$$\frac{GCD^2}{A + B} = \frac{C}{LCM \div 3} \text{ reduced to } \frac{GCD^2 \div GCD}{(A + B) \div GCD} = \frac{C \div C}{(LCM \div 3) \div C}$$

24.

$C + D = LCM \div 4$

$$\frac{GCD \cdot 4}{(GCD \cdot 4) + B} = \frac{C}{LCM \div 4} \text{ reduced to } \frac{(GCD \cdot 4) \div 6}{[(GCD \cdot 4) + B] \div 6} = \frac{C \div 9}{(LCM \div 4) \div 9}$$

25 - 27 Combination - Concrete I (One-to-Many Multiplication)

Each item consisted of a set of multiple entities and a second set consisting of one entity. Subject had to match to the set of one to every member of the multimember set and determine the total number of matches.

25.

	P_1	P_2	P_3	P_4	
1 WK	WP_1	WP_2	WP_3	WP_4	= 4 weeks

26.	FP_1	FP_2	FP_3	FP_4	FP_5	FP_6	
1 day	dF_1	dF_2	dF_3	dF_4	dF_5	dF_6	= 6 days

27.	S_1	S_2	S_3	S_4	S_5	S_6	S_7	S_8	
1 min.	MS_1	MS_2	MS_3	MS_4	MS_5	MS_6	MS_7	MS_8	8 minutes

28 - 30 - Combination - Concrete II (One-to-one Multiplication)

Each item consisted of two sets of multiple entities. Subject had to match each member of each set with each member of the other set.

28.

		Set 1		
		A	B	C
Set 2	L	LA	LB	LC
	M	MA	MB	MC
	N	NA	NB	NC

29.

		Set 1		
		S	T	G
Set 2	B	BS	BT	BG
	F	FS	FT	FG

30.

		Set A			
		D	F	P	R
Set B	A	AD	AF	AP	AR
	B	BD	BF	BP	BR
	C	CD	CF	CP	CR
	H	HD	HF	HP	HR

31 - 33 Combination - Formal I (Combination)

Each item consisted of a set of multiple entities. Subject had to construct all possible nonredundant pairs from the set.

31.

	A	C	D	M	P	S
A						
C	CA					
D	DA	DC				
M	MA	MC	MD			
P	PA	PC	PD	PM		
S	SA	SC	SD	SM	SP	

32.

	J	K	L	M	N	P	T
J							
K	KJ						
L	LJ	LK					
M	MJ	MK	ML				
N	NJ	NK	NL	NM			
P	PJ	PK	PL	PM	PN		
T	TJ	TK	TL	TM	TN	TP	

33.

	1	2	3	4	5	6	7	8
1								
2	21							
3	31	32						
4	41	42	43					
5	51	52	53	54				
6	61	62	63	64	65			
7	71	72	73	74	75	76		
8	81	82	83	84	85	86	87	

34 - 36 Combination - Formal II (Permutation)

Each item consisted of a set of four entities. Subject had to construct all possible permutations of the four entities.

34.	CLSV	LCSV	SCLV	VCLS
	CLVS	LCVS	SCVL	VCSL
	CVLS	LVCS	SVCL	VSCL
	CVSL	LVSC	SVLC	VSLC
	CSVL	LSVC	SLVC	VLSC
	CSLV	LSCV	SLCV	VLCS

35.	CDFP	DCFP	FCDP	PCDF
	CDPF	DCPF	FCPD	PCFD
	CPDF	DPCF	FPCD	PFCD
	CPFD	DPFC	FPDC	PFDC
	CFPD	DFPC	FDPC	PDFC
	CFDP	DFCP	FDCP	PDCF

36.	CDFN	DCFN	FCDN	NCDF
	CDNF	DCNF	FCND	NCFD
	CNDF	DNCF	FNCD	NFCD
	CNFD	DNFC	FNDC	NFDC
	CFND	DFNC	FDNC	NDFC
	CFDN	DFCN	FDCN	NDCF