DOCUMENT RESUME

ED 109 149 95 TM 004 590

AUTHOR Hooper, Frank H.; And Others
TITLE A Cross Sectional Investigation of Children's Classificatory Abilities. Technical Report No. 295.
INSTITUTION Wisconsin Univ., Madison. Research and Development Center for Cognitiv Learning.
SPONS AGENCY National Inst. of Education (DHEW), Washington, D.C.
REPORT NO TR-295
PUB DATE Feb 74
CONTRACT NE-C-00-3-0065
NOTE 99p.

EDRS PRICE MF-$0.76 HC-$4.43 PLUS POSTAGE
DESCRIPTORS Children; *Classification; *Cognitive Ability; Concept Formation; *Cross Sectional Studies; *Elementary Education; Individual Differences; Logical Thinking; Matrices; Object Manipulation; Preschool Education; Statistical Analysis; Task Analysis; *Task Performance
IDENTIFIERS Piagetian Tasks

ABSTRACT

A series of Piagetian concrete operations period tasks dealing with classificatory concepts was administered to 280 children (40 subjects from each of seven levels--preschool, kindergarten, and first, second, third, fourth, and sixth grades). Significant main effects for age were found for all the tasks. Few significant sex differences were observed. Scaling analyses of the pass/fail patterns indicated a generally reliable (reproducibility coefficients of .91-.93) order of difficulty. Factor analysis of a subset of the tasks indicated that three principal components were necessary to account for 85 percent of the performance variance. Thus, a nonunitary structure was found for the children's performances across the present task array and age range. In general agreement with the original contentions of Inhelder and Piaget, it was concluded that the child's understanding of the logic inherent in class inclusion relationships evolves gradually and is contingent upon the previous mastery of certain less complex classificatory skills. (Author/DEP)

A CROSS-SECTIONAL INVESTIGATION OF CHILDREN'S CLASSIFICATORY ABILITIES

RESEARCH
COGNITIVE LEARNING
DEVELOPMENT

WISCONSIN RESEARCH AND DEVELOPMENT

CENTER FOR COGNITIVE LEARNING

Technical Report No. 295

A CROSS-SECTIONAL INVESTIGATION OF CHILDREN'S CLASSIFICATORY ABILITIES

by

Frank H. Hooper, Thomas S. Sipple,
Jane A. Goldman, and Spencer S. Swinton

Report from the Project on
Children's Learning and Development

Frank H. Hooper
Principal Investigator

Wisconsin Research and Development
Center for Cognitive Learning
The University of Wisconsin
Madison, Wisconsin

February 1974

Published by the Wisconsin Research and Development Center for Cognitive Learning, supported in part as a research and development center by funds from the National Institute of Education, Department of Health, Education, and Welfare. The opinions expressed herein do not necessarily reflect the position or policy of the National Institute of Education and no official endorsement by that agency should be inferred.

Center Contract No. NE-C-00-3-0065

Statement of Focus

Individually Guided Education (IGE) is a new comprehensive system of elementary education. The following components of the IGE system are in varying stages of development and implementation: a new organization for instruction and related administrative arrangements; a model of instructional programing for the individual student; and curriculum components in prereading, reading, mathematics, motivation, and environmental education. The development of other curriculum components, of a system for managing instruction by computer, and of instructional strategies is needed to complete the system. Continuing programmatic research is required to provide a sound knowledge base for the components under development and for improved second generation components. Finally, systematic implementation is essential so that the products will function properly in the IGE schools.

The Center plans and carries out the research, development, and implementation components of its IGE program in this sequence: (1) identify the needs and delimit the component problem area; (2) assess the possible constraints—financial resources and availability of staff; (3) formulate general plans and specific procedures for solving the problems; (4) secure and allocate human and material resources to carry out the plans; (5) provide for effective communication among personnel and efficient management of activities and resources; and (6) evaluate the effectiveness of each activity and its contribution to the total program and correct any difficulties through feedback mechanisms and appropriate management techniques.

A self-renewing system of elementary education is projected in each participating elementary school, i.e., one which is less dependent on external sources for direction and is more responsive to the needs of the children attending each particular school. In the IGE schools, Center-developed and other curriculum products compatible with the Center's instructional programing model will lead to higher student achievement and self-direction in learning and in conduct and also to higher morale and job satisfaction among educational personnel. Each developmental product makes its unique contribution to IGE as it is implemented in the schools. The various research components add to the knowledge of Center practitioners, developers, and theorists.

Acknowledgments

The authors wish to acknowledge the cooperation and assistance provided by Mr. Donovan L. Richards, Assistant Superintendent for Instructional Services; Charles Bruce, Joseph Checota, Martin Kerwin, and Wayne J. Simon, elementary school principals; and the staff and students of the Douglas, Lincoln, Webster, and Schurz elementary schools of the Watertown, Wisconsin, school system. We also wish to acknowledge the assistance and cooperation of Mrs. Donna Schmutzler, Director of the Gingerbread Preschool, for providing the younger subjects. In addition, we thank Thomas Toniolo and David Weier for assisting in administration of the tasks and Patrick Lane for providing computer programing services.

Table of Contents

List of Tables

List of Figures

Abstract

A series of Piagetian concrete operations period tasks dealing with classificatory concepts was administered to 280 children (40 subjects from each of seven levels--preschool, kindergarten, and first, second, third, fourth, and sixth grades). As anticipated, significant main effects for age were found for all the tasks. Few significant sex differences were observed. Scaling analyses of the pass/fail patterns indicated a generally reliable (reproducibility coefficients of .91-.93) order of difficulty, i.e., free sorting < double series matrix reproduction < cross class matrix reproduction < cross class matrix transposition < "Some-All" understanding < double series transposition < producing three exhaustive sortings < class inclusion understanding < combinatorial reasoning. Factor analysis of a subset of the tasks indicated that three principal components were necessary to account for 85% of the performance variance. Thus, a nonunitary structure was found for the children's performances across the present task array and age range. In general agreement with the original contentions of Inhelder and Piaget, it was concluded that the child's understanding of the logic inherent in class inclusion relationships evolves gradually and is contingent upon the previous mastery of certain less complex classificatory skills.

Secondary analyses of the multiplicative matrix tasks supported the earlier findings that the reproduction instructional set was less difficult than the transposition instructional set. In contrast to the previous research of MacKay, the intermatrix comparisons indicated the following orders of difficulty for the preschool to second-grade subject subsample: (a) reproduction task--cross class > double series > class/series and (b) transposition task--cross class = double series > class/series. However, major methodological deficiencies in the present matrix transposition task formats seriously question the task comparabilities and therefore preclude any definitive generalizations.

I
Introductory Considerations

Class and relations concepts as basic categories of logico-mathematical thought represent an area of primary emphasis in the Piagetian model of cognitive development. In the case of the concrete operational period of middle childhood, this emphasis follows from a consideration of the logical groupings or groupements proposed by Piaget. These superordinate organizations are viewed as the structural units which subsume and interrelate all logical reasoning during the middle childhood period, i.e., approximately 5 to 11 years of age. Four of the groupements are concerned with logical classes, and four complementary cases deal with logical relations. The most important of these include primary addition of classes (as evidenced, for example, in the class inclusion problem), bi-univocal multiplication of classes (as shown in matrix completion or class intersection problems), addition of asymmetrical relations (as evidenced in seriation and transitive inference tasks, e.g., the conclusion that A must be less than C on some dimension, if shown that A < B and B < C), and bi-univocal multiplication of relations (as assessed, for example, in the serial correspondence and double seriation matrix tasks). As may be expected from a stage theory such as Piaget's, these logical groupements are theoretically predicted to emerge in parallel or concomitant fashion during the concrete operations period (cf. Flavell, 1970; Hooper, 1973; Wohlwill, 1963). Thus one would expect developmental synchrony to be shown in children's classificatory and relationality concept task performances within a general period. Inhelder and Piaget (1964) present evidence showing generally simultaneous acquisition patterns for classification and relationality (seriation) concepts. Each of these concept domains was seen to be mastered in a three-stage sequence--(I) preoperational, (II) transitional, and (III) operational classification and seriation--over the approximate age range of 4 to 10 years. Tasks derived from the additive groupings were found to be of approximately the same difficulty as those based upon the multiplicative groupings.

Classes as objects of interest to the developmental psychologist may be viewed in two mutually complementary ways. As Flavell (1970) has pointed out.

> The entities called "classes" have a dual status in human cognition. On the one hand, they are usually regarded as essential ingredients of the thinking process itself. Particular classes are constantly being retrieved or constructed by the individual and pressed into service as conceptual vehicles or instruments. On the other hand, classes can also constitute abstract objects of thought. The individual may, in addition to utilizing particular classes in his everyday thinking, have explicit or implicit knowledge of the logical properties of classes and classification systems in general. The distinction here is between classes considered as cognitive tools and classes taken as the elements of a subject matter or body of knowledge--between the classes the subject knows and what he knows about classes [p. 991].

It is classificatory knowledge in the sense of general logical systems and associated properties, rather than classes as isolated conceptual units, which constitutes one of the more unique contributions of the Genevan investigators. Two criterial properties of classes are emphasized by the the Piagetians: class intension and class extension. Class intension, the qualitative aspect of classificatory logic, refers to the array of properties that are unique to all members of a given class and thus serve to distinguish that class category from all others. In a sorting task utilizing geometric forms, these intensive properties could typically include the attributes of shape, color, and size. Extensive properties refer to the <u>quantitative</u> aspects of classificatory logic, i.e., the differential magnitude relationships implied in superordinate/subordinate class systems. This is usually assessed by presenting the subject with a mixed stimulus array (in an abstract verbal or concrete perceptible presentation format) that contains a hierarchy of classes and subclasses. Questions dealing with the relative magnitudes of class exemplars present and focusing upon such quantitative terms as <u>some</u>, <u>all</u>, and <u>none</u> are then asked.

In terms of a mature classificatory logic, the the class properties of intension and extension cannot be viewed in isolation from each other. It is the <u>coordination</u> of these intensive and extensive properties that defines the presence of logical classificatory abilities for the concrete operations period. As Inhelder and Piaget (1964) state,

> We will therefore pose the following as criteria for the operational existence of classes: (1) The subject can give an intensive definition of a class in terms of a more general class and one or more specific differences. (2) He can handle their extension in accordance with the structure of inclusion, as shown in the mastery of the quantifiers "all," "some," "one," and "none" [p. 7].

2

II
Previous Research

The initial results reported by Inhelder and Piaget (1964) and Piaget (1965) concerning the child's abilities to deal with the special properties of classes within an overall logical system have been generally substantiated in subsequent replication attempts (e.g., Elkind, 1961; Hood, 1962; Kofsky, 1966; Lovell, Mitchell, & Everett, 1962; Siegel & Kresh, 1971; and Smedslund, 1964). These studies dealt with the abilities associated with the additive properties of classes and subclasses as indexed, for example, in the "Some-All" and class inclusion tasks; the ability to form and recombine class-subclass relationships, e.g., horizontal reclassification and hierarchial classification skills; and the ability to view objects as potential members of mulciple class categories and to "multiply" classes as measured in the matrix type task (cf. Flavell, 1970). It should be pointed out that these secondary studies, despite certain methodological reservations (cf. Klausmeier & Hooper, 1974), are superior in general assessment design to the original Genevan research. Most of the secondary replication endeavors have administered all of various classification tasks to the particular age-graded subsamples of children examined. This was not true of the original Piagetian assessments, and any primary assertions regarding concept task acquisition sequences, relative task difficulties, and developmental interdependencies are accordingly open to question.

Kofsky (1963, 1966) has provided an excellent theoretical discussion of classification skills from the Piagetian orientation. She summarized the views of Inhelder and Piaget (1964) regarding the component behaviors found in the preoperational and concrete operational periods which culminate in class inclusion skills as follows:

> On the basis of their hypotheses, development appears to proceed in 11 partially ordered steps. They contend that classification begins when the child groups together two objects that are equivalent because they look alike in some way (resemblance sorting). As the child grows he learns to extend the scope of his grouping from two; to more than two (consistent sorting), to all the objects that could be considered equivalent in some respect (exhaustive sorting). The child also learns which are acceptable categories for grouping. Physical proximity becomes a less favored means of categorizing since the resulting groupings are transitory (conservation). Experience in constructing one class at a time prepares the child for forming successive and simultaneous classifications and for understanding class inclusion. Slowly the child begins to recognize that objects do not belong exclusively in different categories (multiple-class membership), and he actively tries out different groupings of objects, choosing first one and then another single attribute as a focus for grouping (horizontal classification). As his logical abilities develop, his method of choosing criteria becomes more complex. He chooses single attributes to construct successive classes (hierarchical classification). His use of combinatorial structure (Inhelder & Piaget, 1958) enables him to form classes that stand in an inclusion relationship to each other [1966, p. 192].

Kofsky designed 11 experimental tasks to assess the hypothetical classificatory acquisition sequence proposed by Inhelder and Piaget (1964) and administered these tasks, in a cross-sectional assessment design, to groups of children 4 to 9 years of age. The children's performances indicated that six levels of classificatory logic were present in the following order of increasing task difficulty:

Level 1--resemblance sorting and consistent sorting, i.e., the ability to match and sort objects on the basis of perceptible attributes;

Level 2--exhaustive sorting, in which all blocks sharing a common attribute were separated from a mixed array, and an understanding of "some and all" relationships (i.e., after presenting an array of nine blocks which consisted of four blue-squares and two blue triangles, and three red triangles, the child was asked a series of questions such as "Are all the triangles red?");

Level 3--a knowledge of multiple class membership in a task setting which included triangular-shaped blocks which varied in two sizes and two color dimensions, and an understanding that the overall number of objects in two subclasses equals the number in the superordinate class;

Level 4--conservation of classes in which the child had to continue to associate a nonsense syllable label with a specific geometric form in spite of irrelevant transformations, conservation of a class hierarchy (i.e., with an array of two blue and six red square blocks the child is asked, "If I took away all the reds, are there just blues left, just squares left, or both blues and squares?"), and horizontal reclassification in which an array of triangle and square shaped blocks in four colors were sorted and resorted according to the differing potential criteria;

Level 5--class inclusion skills which were assessed with the same stimulus array as the "some and all" task, and asked the child questions such as "Are there more triangles or blues?"; and

Level 6--hierarchical classification skills in which the child had to demonstrate that in an array of four red and three blue triangle-shaped blocks, all the blocks shared one attribute (shape) but that any one of the blocks had an additional attribute (color) shared by only some of the blocks in view.

Thus, Kofsky found a general sequence of classificatory skill acquisition across the 4- to 9-year-old age range. There was evidence, however, that alternative task mastery sequences were present since only 27% of the subjects (51% when three tasks were deleted from the analysis) passed all tasks up to a point in the predicted order of difficulty and failed the remainder. Despite this reservation, Kofsky's results (see also Allen, 1970) indicate that classificatory competence is certainly not a discontinuous "all or none" phenomenon when viewed within the usual age confines of the concrete operations period. The most difficult tasks, class inclusion and hierarchical classification, were respectively passed by only 60% and 40% of the 9-year-old subjects.

The important problem area of developmental synchrony or concurrence as postulated by the structural characteristics of the logical groupements of the middle childhood years has received limited attention in the secondary Piagetian research literature (Brainerd, 1972; Klausmeier & Hooper, 1973). As mentioned above, Piaget maintains that additive and multiplicative conceptual abilities, within and across the class and relationality domains, are parallel acquisitions. Kofsky's (1966) sequence of classificatory task mastery would appear to argue against this viewpoint. Differential task difficulties for partial versus complete multiplication of classes (the former was easier) were found by Findlay (1971). Some studies which have compared class inclusion and multiplicative classificatory task performances have found the latter to be of lesser difficulty (e.g., Kofsky, 1966) while some have found it to be of greater difficulty (e.g., Smedslund, 1964); others have found a decidedly equivocal picture (e.g., Wohlwill, 1968). A recent methodological analysis of the source of children's class inclusion errors (Brainerd & Kaszor, 1974) has shown that perceptual set factors (cf. Ahr & Youniss, 1970; Wohlwill, 1968) and misinterpretation of the criterion questions (cf. Klahr & Wallace, 1972) are not significant determinants of class inclusion performance. Class inclusion understanding was found to be a rather late-appearing concrete operational acquisition.

Relatively few studies (considering the overall impressive volume of Piagetian inspired research) have attempted a direct empirical assessment of class and relation concept correspondence for children in the concrete operations period. Those that have examined children's performances on more than two relationality or classification task domains (e.g., Brainerd, 1972; Chittenden, 1964; Lagattuta, 1970; Lovell et al., 1962; MacKay, Fraser, & Ross, 1970; Nassefat, 1963; Shantz, 1967; Smedslund, 1964; Wohlwill, 1968; Wohlwill, Devoe, & Fusaro, 1971) have a mixed but generally nonconfirmatory picture insofar as developmental synchrony is concerned. In those cases where moderately high degrees of interrelationship were found, as in the multiplicative class and relations tasks used by Lovell et al. (1962), Shantz (1967), and Smedslund (1964) as well as in the original performance similarities reported by Inhelder and Piaget (1964), the inferred ability commonalities may be confounded to an unknown extent by shared method variance (cf. Shantz. 1967, pp. 133-135).

Smedslund (1964) found that out of a total sample of 160 children ranging in age from 4 years 3 months to 11 years 4 months, only one subject failed a class inclusion task and passed measures of multiple classification and multiple relationality. In contrast, 21.9% of the subjects passed class inclusion and failed multiplicative classification; 24.4% passed class inclusion and failed relationality. Although comparisons were not presented for separate age groups, the multiplicative class and relation tasks were of approximately equal difficulty for the overall sample, i.e., 81% of the children either passed both tasks or failed both tasks. Shantz (1967) compared the performances of children 7 1/2, 9 1/2, and 11 1/2 years old on multiplication of classes (assessed by the Raven Coloured Progressive Matrices Test), multiplication of relations (assessed by using the "diagonals" of 4 x 4 matrices based on various continuous dimensions), and multiplication of infralogical spatial relations (assessed in an adaptation of Piaget's landscape task). Significant rank order correlations between the multiplicative class and relational matrix tasks were found for the two older subsamples.

In another cross-sectional assessment design study, Lagattuta (1970) examined children's abilities to deal with unidimensional classification and seriation (relationality) and matrix format multiplicative classification and seriation tasks. It was found that a child first develops (5 1/2 years of age) the ability to classify a simple arrangement; somewhat later (5 1/2 - 6 1/2 years) the child can successfully deal with a multiple classificatory matrix. Concurrent with this latter acquisition, simple serial skills develop (6 1/2 - 8 1/2 years of age), while the ability to successfully order a serial matrix was shown by the older subjects (8 1/2 years) only. It was tentatively concluded, in apparent contrast to Inhelder and Piaget (1964), that classificatory skills develop independently of and prior to seriation skills.

MacKay et al. (1970), drawing upon the earlier work of Bruner and Kenney (1966) and Inhelder and Piaget (1964), compared the relative difficulties of multiple classification, multiple seriation, and combined class/series matrix tasks for groups of children 5 to 8 years of age. Each child performed on one of the tasks and was required to reproduce and to transpose the presented matrix. The comparisons regarding multiple classification and seriation were derived from an initial experiment involving 90 children, while a second experiment assessed performance of an additional group of 48 children on the class/series matrix. As anticipated, performances on all the tasks improved significantly over the age interval assessed. Matrix reproduction was easier than transposition for the multiple seriation and multiple class/series cases, and this was most notable for the younger (5- to 7-year-old) subjects. Combining data from the two samples, it was shown that reproduction of a multiple seriation matrix was more difficult than reproduction of a class matrix which was, in turn, more difficult than the class/series case. Transposition of the seriation matrix was of greater difficulty than either of the other matrices. Transposition of the class and combined class/series task was of approximately equal difficulty. It was concluded that:

1. The ability to construct a matrix composed of discrete categories is developmentally an earlier acquisition than the ability to construct one composed of relational variables.

2. A matrix composed of discrete categories in both directions is of equivalent difficulty to one constructed of discrete categories in one direction and a relational variable in the other.

3. A matrix composed of discrete categories is no more easily reproduced than it is transposed, while matrices where either one or both variables are continuous are more easily reproduced than transposed.

4. The great majority of children under each condition who reproduce the matrix do so as it is presented [MacKay et al., 1970, p. 795].

Relatively few studies have employed longitudinal assessment designs in the investigation of Piagetian classificatory skills. Dudek and Dyer (1972) examined children's performances on a variety of Piagetian tasks from the kindergarten to third-grade years. Utilizing the stage scoring system of Laurendeau and Pinard (1962), they found that 71% of the children at the kindergarten level demonstrated stage 1B functioning for the class inclusion task. At the first-grade level approximately equal numbers of children were found to be at stages 1B, 2B, and 3B. The second- and third-grade assessment intervals indicated 59% and 82% of the children, respectively, to be functioning at the highest stage 3B level for the class inclusion task. In a continuing longitudinal study, Stephens (1972) has assessed the reasoning, moral judgment, and moral conduct of normals (I.Q. 90-110) and retardates (I.Q. 50-75) aged 6 to 20 years. The normal subjects' average mental age (as determined by the Wechsler Intelligence Scales) for criterial performance on a class intersection task was 6 years. Adequate performance on the majority of the class inclusion tasks, in contrast, was not achieved until mental ages of 7 to 8 years were attained. The most difficult class inclusion task (a format consisting of four cards, each of which pictured a duck, and three additional cards, each of which pictured another bird--"Are there more ducks or birds on the table?") indicated a corresponding mental age of 16 years.

Wohlwill et al. (1971) included spontaneous grouping/classification, class intersection, and class inclusion tasks in an investigation of concept development in children aged 5 to 8 years. There were three assessment points over an 18-month time interval. Over this age range and time interval, children's spontaneous grouping on a block placement task showed an increased reliance upon categorical rules and relationships. On the class intersection task, significant increases in the number of correct choices and the number of dimensions verbalized in describing the stimulus arrays were found. For the class inclusion tasks, in contrast, there was a notable lack of consistent older versus younger subject performance differences over the three test occasions. Pictorial presentation formats were consistently more difficult than verbally presented class inclusion tasks (see also Wohlwill, 1968).

III
The Present Investigation

The present investigation is concerned with the classificatory abilities of 5- to 12-year-old children. The classificatory ability domain was assessed by tasks measuring sorting, "Some-All" relationships, multiplicative (cross) classification, and class inclusion. In an attempt to replicate the findings of MacKay (1972) and MacKay et al. (1970), a measure of multiple relationality (double seriation) and a combined class/series matrix were included. In addition, a measure of combinatorial reasoning was administered to all but the youngest group of children. The primary objectives of this study were (1) to examine the development of representative classificatory skills from 5 to 12 years of age, (2) to examine the relative task difficulties and associated task interrelationships over this age range, (3) to examine the dimensional structure of the present Piagetian task series, and (4) to examine the relative difficulties within the matrix task formats (i.e., reproduction versus transposition) and across the various matrix types (i.e., cross classification versus double seriation) to assess the reliability of the results of MacKay et al. (1970).

Method

Subjects

Subjects for the study were 280 school children from the Watertown, Wisconsin, school district. This school district has a population of approximately 21,000 persons and encompasses (1) the city of Watertown, a prosperous semi-rural community 40 miles northwest of Milwaukee, and (2) the surrounding farming area. Forty Ss were drawn from each of seven grade levels: preschool, kindergarten, and first, second, third, fourth, and sixth grades. Distribution of the subject population by age and sex is given in Table I. The preschool subsample consisted of children enrolled in a private nursery school in Watertown. The remainder of the sample consisted of children enrolled in the Watertown public school system. At the preschool level, children were selected randomly from the entire population of the school. From each other grade level, the Ss were randomly selected from those children returning parental consent slips.

Procedure

Eight tasks were adapted to assess the development of classification and related skills.[1] These tasks were divided into four main groups: Group I (Free Sort, Dichotomies), Group II (Some-All, Class Inclusion), Group III (Cross Classification Matrix, Classification Seriation Matrix, Double Seriation Matrix), and Group IV (Combinatorial Reasoning). The order of presentation of these groups was randomized for each S. Within Group I, the order was fixed: the Free Sort task always preceded the Dichotomies. Within Group II as well as within Group III, the order of task presentation was randomized. One half of the children at the preschool, kindergarten, and second, fourth,

---

[1]The eight tasks used in the study were pilot tested with a sample of 23 children ranging in age-grade level from preschool to eighth grade. This provided essential familiarization experiences for the project assessment personnel. Based on this pilot study, decisions were made regarding which grade levels to include in the final study and which tasks to present at each grade level. In addition, certain procedural modifications were indicated for each of the respective task formats.

Table 1

DISTRIBUTION OF THE SUBJECT POPULATION BY GRADE, MEAN AGE, AND SEX

Grade	Subjects	Males	Females	Mean Age	Range
Pre	40	23	17	5-0	4-1 to 5-9
K	40	23	17	6-3	5-6 to 6-9
1	40	21	19	7-3	6-8 to 8-1
2	40	23	17	8-2	7-9 to 8-9
3	40	16	24	9-2	8-9 to 9-9
4	40	15	25	10-3	9-6 to 11-9
6	40	25	15	12-2	11-7 to 13-0

and sixth grade levels received the entire Piagetian task series first and a task based on the conceptual learning and development (CLD) model second. For the remaining subjects, this order was reversed.[2] Subjects from the kindergarten and first, second, third, fourth, and sixth grade levels received all the tasks in the battery. Because pilot data indicated that preschool children did not understand the instructions for the combinatorial reasoning task, the preschool Ss did not receive that task.

As a rule, Ss received the entire battery of tasks in one sitting. However, due to scheduling interruptions, it was necessary for a number of Ss to receive the battery in two sittings. The administration time for the battery ranged from 22 minutes to 60 minutes. The Es were 3 males and 1 female, all in their twenties.

Assessment Tasks

The materials and procedures used in the eight assessment tasks are summarized below. Complete instructions for these tasks are presented in Appendix A; scoring sheets are given in Appendix B.

1. Free Sorting (adapted from Kamii & Peper, 1969).

Material.[3]

3 small red circle blocks
3 small blue circle blocks
3 small red square blocks
3 small blue square blocks
2 large red circle blocks
3 large blue circle blocks
3 large red square blocks
2 large blue square blocks

[2]It should be pointed out that the present study served as a pilot to a larger-scale longitudinal investigation (Hooper & Klausmeier, 1973) relating a more comprehensive array of Piagetian tasks to a series of tasks based on the model of conceptual learning and development.

[3]Throughout the descriptions of materials, "small" (circle, square, triangle) refers to a block with a circumference of 204 millimeters and a thickness of 10 millimeters. "Large" (circle, square, triangle) refers to a block with a circumference of 308 millimeters and a thickness of 10 millimeters.

Procedure. The blocks were placed before S in a scrambled fashion so that the different classes were scattered throughout the arrangement. E made inquiries about the blocks until S had verbalized all relevant distinguishing attributes. This done, E instructed to "put together all the blocks that go with each other." When finished, S was asked why he grouped them as he did. Scores on the task ranged from 0-3. A subject was considered to have passed this task if his groupings were based upon size or shape or color (or any combination of them) as criteria; thus, scores ranging from 1 to 3 were considered passing level on this task.

2. Dichotomies (adapted from Kamii & Peper, 1969).

Materials. Stimuli were the same as in the free free sorting task, supplemented with two flat open boxes, each about 9 inches x 12 inches. One box was placed on each side of the group of mixed blocks.

Procedure. For the first dichotomy, S was instructed to divide all the blocks into two bunches by placing one kind in each box. When S had finished, he was asked to explain the way he divided the blocks. For the second and third dichotomies, this procedure was repeated; each time E asked S to divide the blocks in a different way than he had done before. If during these three dichotomous sorts, S had dichotomized the blocks according to two of the three accepted criteria (size, shape, and color), he was presented with a fourth sorting opportunity. Scores on this task ranged from 0 to 3. A subject was considered to have passed if he divided the blocks by size, by shape, and by color. Therefore, a score of 3 was designated a pass for this task.

3. Some-All (adapted from Kofsky, 1966).

Materials.

- 3 small red triangle blocks
- 2 small blue triangle blocks
- 4 small blue square blocks

Procedure. The stimuli, all oriented in the same direction, were placed on the table. The red blocks were mixed in with the blue as were the triangles with the squares. Prior to the trial questioning, warm-up inquiries were made so that S verbalized all attributes distinctive to the classes. The four trial questions were then presented.

1. "Look at all the red blocks. . . .
   Are all the red blocks triangles?"
2. "Look at all the triangle blocks. . . .
   Are all the triangle blocks red?"
3. "Look at all the square blocks. . . .
   Are all the square blocks blue?"
4. "Look at all the blue blocks. . . .
   Are all the blue blocks squares?"

These questions were asked in random order. For the last two questions presented, S was asked to justify his responses. Throughout this task S was not permitted to manipulate the blocks. Scores ranged from 0 to 4. To pass this task, a subject was required to answer all four trial questions correctly.

4. Class-Inclusion (adapted from Kofsky, 1966).

Materials.

- 3 small red triangle blocks
- 2 small blue triangle blocks
- 4 small blue square blocks
- 3 small yellow circles
- 2 small blue circles

Procedure. This task consisted of five trials presented in a fixed order. Stimuli for the first trial were three small red triangle blocks and two small blue triangle blocks. The question was, "Are there more triangle blocks or more red blocks?" Stimuli for the second trial were three small yellow circle blocks and two small blue circle blocks. The question was, "Are there more blue blocks or more circle blocks?" Stimuli for the third, fourth, and fifth trials were three small red triangle blocks, two small blue triangle blocks, and four small blue square blocks. The questions were, (3) "Are there more triangle blocks or more red blocks?" (4) "Are there more blue blocks or more square blocks?" and (5) "Are there more blue blocks or more triangle blocks?" For each trial, the blocks were placed such that the class attributes followed no apparent order. For trials 1, 2 and 3, E labeled the classes and asked S to count the members in each. Following S's answer to each of the last three trials, E asked for a justification. The score range was from 0 to 5. To pass, a subject was required to answer all the trial questions correctly.

5. Matrices (adapted from MacKay et al., 1970).

Materials. Each matrix was arranged on a square wooden board sectioned so as to produce nine individual squares, each 110 x 110 millimeters.

a. Cross Classification Matrix
Stimuli were three small square blocks, three small circle blocks, and three small triangle blocks. One block of each shape was red, one was yellow, and one was blue. The blocks were arrayed on the color and shape dimensions.

b. Classification Seriation Matrix
Stimuli were nine cylinder blocks, each having a diameter of 65 millimeters: three were 100 millimeters high, three were 75 millimeters high, and three were 50 millimeters high. One block of each height was red, one was yellow, and one was blue. The blocks were arrayed on the color and height dimensions.

c. Double Seriation Matrix
Stimuli were nine cylinder blocks; three were 100 millimeters high, three were 75 millimeters high, and three were 50 millimeters high. One block of each height had a diameter of 100 millimeters, one was 65 millimeters in diameter, and one was 35 millimeters in diameter. All the cylinders were blue. The blocks were arrayed on the width and height dimensions.

Procedure. The three matrix tasks were presented in random order. For each matrix, Ss were asked to perform, in a fixed order, the tasks of replacement, reproduction, and transposition. Instructions were identical for each matrix.

a. Replacement.
E removed first one, then two, and finally three (diagonally placed) blocks from the matrix, and each time S was asked to put them back where they were before E removed them.

b. Reproduction.
E removed all the blocks from the matrix, and S was asked to put them back so the board looked just the same as it did before E removed them.

c. Transposition.
E removed all the blocks from the matrix and then placed the block that had originally occupied the lower left-hand square on the upper left-hand square. S was then asked to place the blocks on the board so they made a "pattern like they did before."

Scores on each reproduction and each transposition matrix subtask ranged from 0 to 4. For each reproduction and transposition subtask, the child was given a point score of 0, 1, or 2 on each dimension. Identical placement of columns was assigned 2 points, as was the identical placement of rows. Reversed (or interchanged) placement of columns was assigned 1 point, as was the reversed placement of rows. Inconsistent placement of columns was assigned a point score of 0, as was the inconsistent placement of rows. In order to pass the cross-classification matrix reproduction subtask, a subject was required to classify one dimension in one direction and the other dimension in the other direction. In order to pass the classification seriation matrix reproduction subtask, a subject was required to classify the color dimension in one direction and to seriate the height dimension in the other direction. In order to pass the double seriation matrix reproduction subtask, a subject was required to seriate one dimension in one direction and the other dimension in the other direction. In order to pass each of the transposition subtasks, a subject was required to fulfill the same criteria as for the reproduction cases without moving the replaced block.

6. Combinatorial Reasoning (adapted from Goodnow, 1962)

Materials. MLM-65 counting chips, each chip the size of a nickel: 10 red, 10 green, 10 yellow, 10 blue, 10 white, 10 orange, 10 brown, 10 light blue.

Procedure. E explained that in this game S was to use the chips to make pairs of different colors. Using three groups of chips (each group a different color), E explained and demonstrated the two rules of the game: (1) each pair must have two colors and (2) each new pair must be a combination of colors not yet formed. E then placed on the table four different color piles of chips and asked S to form as many pairs as possible using these colors. When S finished, the chips were returned to their piles and S was asked to do the same with chips of six different colors. Subjects who satisfied the criteria of forming 12 or more correct pairs, and who had no more than four repeated pairs, were given two new colors and asked to continue making as many pairs as possible. Scores ranged from 0 to 28. The total correct pairs formed were calculated by subtracting the number of repeated pairs from the number of distinct pairs a subject formed. A subject was considered to have passed if the total correct pairs equaled 28, and a systematic approach (cf. Goodnow, 1962, pp. 6-7 and 15-17) was used.

Results

Initial Considerations

Initial considerations concern the evaluation of certain order of presentation effects. As indicated previously, one half of the preschool, kindergarten, and second-, fourth-, and sixth-grade subjects received the entire Piagetian task series first and the concept task series based upon the CLD model second. For the remaining subjects, this order of presentation was reversed. Insofar as the Piagetian task array is concerned, there was a notable absence of any significant presentation order effects with the exception of two cases. In the free sorting task, subjects who initially received the Piagetian task battery significantly outperformed (mean score = 2.38) the comparison group (mean score = 2.06). Conversely, subjects who initially received the CLD model "equilateral triangle" concept task were superior (mean score = 2.03) to the comparison group (mean score = 1.58) on the dichotomies task. Comparison of the scores of those subjects who received the "Some-All" task before the class inclusion task ($N = 139$) to the scores of subjects who received these tasks in reverse order ($N = 141$) failed to reveal any significant differences on the respective tasks. In similar fashion, a post facto comparison of the average scores of subjects on the three matrix tasks failed to indicate any significant differences due to the six possible different presentation orders for either the reproduction or transposition measurement formats.

Primary Results

The general performance patterns of the individual age-grade subsamples and the overall composite sample are presented in Tables 2 and 3 (means and standard deviations) and in Tables 4 and 5 and Figures 1 to 7 (percentages of passing subjects). Insofar as the mean scores are concerned, factorial variance analyses indicated significant grade level main effects for all 11 measures (see Table 6). Significant main effects for the sex factor were found for the cross classification reproduction and combinatorial reasoning tasks. F values which approached significance were observed for the dichotomies and cross classification transposition tasks. In all these instances the female subjects' scores exceeded those of their male counterparts. The age-grade level x sex interaction effect was significant for the "Some-All" relationship task and approached significance for the combinatorial reasoning task. In the former case post hoc comparisons indicated marginal female subject superiority at the preschool level, i.e., $t = 1.993$, $df = 38$, $p < .10$ (two-tailed probability value), and a variable pattern of nonsignificant sex differences at the remaining age-grade levels, i.e., male superiority at the kindergarten and second and fourth grade levels and higher female subject scores at the first, third, and sixth grade levels. In the combinatorial reasoning case, while female superiority was shown at all age-grade levels except kindergarten, it was most notable at the third and fourth grade levels, i.e., $t = 2.262$, $df = 38$, $p < .05$ and $t = 1.708$, $df = 38$, $p < .10$, respectively.

Subsequent data analyses and associated discussion will be concerned with the children's performances on the basis of dichotomous pass/fail criteria (Tables 4 and 5 and Figures 1 to 7). As anticipated, χ^2 comparisons indicated significant improvements in criterial performances across the present age-grade range (i.e., all χ^2 values exceeded 22.46, $df = 6$, $p < .001$). Comparisons of male versus female performances for the overall combined sample were nonsignificant except for cases of female superiority for the production of three dichotomies ($\chi^2 = 7.308$, $df = 1$, $p < .01$) and cross classification matrix reproduction ($\chi^2 = 5.767$, $df = 1$, $p < .02$) tasks. These patterns are in essential agreement with the parametric analyses of variance reported above.

Inspection of the trends presented in Figures 1 to 7 permits certain task-specific generalizations. Initially considering the overall composite sample of 280 children, the relative difficulty of the Piagetian task array may be characterized in terms of seven approximate ability groupings. These are presented in Table 7.

As Table 7 makes clear, there is a considerable range of task difficulty in the present measurement series. Figure 1 indicates that the free sorting task was mastered by 75% or more of the children at the youngest age-grade level. A similar pattern is shown for the case of producing a single dichotomy. Eighty percent of the second-grade children could produce a second dichotomy, however, the 75% subject passing criterion was not attained for the third dichotomy case until the fourth to sixth grade levels (the remaining analyses concerning the dichotomies task will deal exclusively with this latter third dichotomy case).

The percentages of subjects who passed the "Some-All," class inclusion, and combinatorial reasoning tasks are presented in Figure 2. As anticipated from the results of previous investigations (e.g., Inhelder &

Table 2

MEANS AND STANDARD DEVIATIONS OF THE ASSESSMENT TASKS FOR EACH OF THE SEVEN GRADE LEVELS[a]

Grade	(N)	Free Sort	Dichotomies	Some-All	Class Inclusion	Combinatorial Reasoning
Pre						
Males	23	1.61 (1.16)	.83 (1.23)	2.96 (.93)	2.65 (1.11)	
Females	17	1.65 (1.27)	1.00 (1.17)	3.47 (.51)	2.71 (1.53)	
Total	40	1.63 (1.19)	.90 (1.19)	3.18 (.81)	2.68 (1.29)	
K						
Males	23	2.52 (.73)	1.35 (1.34)	3.65 (.57)	3.22 (1.13)	7.70 (5.66)
Females	17	2.47 (.80)	1.06 (1.39)	3.41 (.62)	2.76 (.90)	6.71 (5.29)
Total	40	2.50 (.75)	1.23 (1.35)	3.55 (.60)	3.03 (1.05)	7.28 (5.46)
1						
Males	21	2.48 (.68)	1.24 (1.45)	3.48 (.68)	3.14 (1.15)	10.24 (6.61)
Females	19	2.58 (.84)	1.47 (1.35)	3.68 (.48)	3.63 (1.21)	14.68 (8.76)
Total	40	2.53 (.75)	1.35 (1.39)	3.58 (.59)	3.38 (1.19)	12.35 (7.93)
2						
Males	23	2.13 (.81)	1.65 (1.43)	3.74 (.54)	3.65 (.98)	18.61 (8.88)
Females	17	2.47 (.80)	2.41 (1.18)	3.53 (.72)	3.53 (1.23)	16.71 (8.87)
Total	40	2.28 (.82)	1.98 (1.37)	3.65 (.62)	3.60 (1.08)	17.80 (8.81)
3						
Males	16	2.25 (.86)	2.00 (1.41)	3.63 (.62)	3.38 (1.09)	18.19 (9.68)
Females	24	2.46 (.59)	2.21 (1.25)	3.88 (.34)	3.92 (1.28)	24.21 (6.72)
Total	40	2.38 (.70)	2.13 (1.30)	3.78 (.48)	3.70 (1.22)	21.80 (8.46)
4						
Males	15	2.20 (.77)	1.93 (1.33)	3.93 (.26)	4.33 (.82)	19.67 (9.60)
Females	25	2.36 (.76)	2.76 (.66)	3.88 (.44)	4.40 (.76)	24.16 (6.57)
Total	40	2.30 (.76)	2.45 (1.04)	3.90 (.38)	4.38 (.77)	22.48 (8.03)
6						
Males	25	2.40 (.65)	2.44 (1.12)	3.84 (.47)	4.40 (.82)	26.36 (4.08)
Females	15	2.40 (.83)	2.53 (1.06)	3.93 (.26)	4.53 (.92)	27.20 (1.08)
Total	40	2.40 (.71)	2.48 (1.09)	3.88 (.40)	4.45 (.85)	26.68 (3.29)
Overall Sample						
Males	146	2.23 (.86)	1.62 (1.40)	3.59 (.68)	3.52 (1.17)	16.80 (9.91)[b]
Females	134	2.35 (.87)	1.97 (1.31)	3.70 (.52)	3.69 (1.29)	19.40 (9.60)[b]
Total	280	2.29 (.87)	1.79 (1.37)	3.65 (.61)	3.60 (1.23)	18.07 (9.85)[b]

[a]Standard deviations are given in parentheses.
[b]Data not for all 280 subjects.

Table 3

MEANS AND STANDARD DEVIATIONS OF THE MATRIX ASSESSMENT TASKS FOR EACH OF THE SEVEN GRADE LEVELS[a]

		Cross Classification		Classification Seriation		Double Seriation	
Grade	(N)	Reproduction	Transposition	Reproduction	Transposition	Reproduction	Transposition
Pre-							
Males	23	1.96 (1.46)	1.39 (1.41)	3.09 (1.41)	2.13 (1.55)	2.91 (1.56)	1.96 (1.61)
Females	17	2.29 (1.53)	2.35 (1.32)	3.41 (.80)	2.65 (1.66)	3.18 (1.59)	2.29 (1.31)
Total	40	2.10 (1.48)	1.80 (1.44)	3.23 (1.20)	2.35 (1.59)	3.03 (1.56)	2.10 (1.48)
K							
Males	23	2.70 (1.26)	2.22 (1.73)	3.83 (.49)	2.48 (1.44)	3.39 (1.12)	1.91 (1.73)
Females	17	2.71 (1.31)	2.35 (1.37)	3.47 (1.07)	3.00 (1.46)	2.94 (1.60)	2.24 (1.60)
Total	40	2.70 (1.26)	2.28 (1.57)	3.68 (.80)	2.70 (1.45)	3.20 (1.34)	2.05 (1.66)
1							
Males	21	3.19 (1.08)	2.95 (1.40)	3.43 (1.08)	3.52 (.81)	3.90 (.44)	3.29 (1.15)
Females	19	3.84 (.50)	2.84 (1.30)	3.84 (.37)	3.53 (1.02)	3.84 (.69)	2.79 (1.47)
Total	40	3.50 (.91)	2.90 (1.34)	3.63 (.84)	3.53 (.91)	3.88 (.58)	3.05 (1.32)
2							
Males	23	3.09 (1.35)	2.57 (1.34)	3.96 (.21)	3.48 (1.12)	3.74 (.92)	3.70 (.93)
Females	17	3.94 (.24)	3.12 (1.32)	3.82 (.39)	2.88 (1.27)	3.76 (.97)	3.06 (1.39)
Total	40	3.45 (1.11)	2.80 (1.34)	3.90 (.30)	3.23 (1.21)	3.75 (.93)	3.43 (1.17)
3							
Males	16	3.19 (.91)	2.88 (1.31)	3.75 (.58)	3.31 (1.14)	4.00 (.00)	3.69 (.70)
Females	24	3.58 (.78)	3.42 (.78)	3.92 (.28)	3.63 (.65)	3.96 (.20)	3.88 (.45)
Total	40	3.43 (.84)	3.20 (1.04)	3.85 (.43)	3.50 (.88)	3.98 (.16)	3.80 (.56)
4							
Males	15	3.67 (.72)	3.47 (.64)	3.73 (.46)	3.53 (.64)	4.00 (.00)	3.53 (.92)
Females	25	3.52 (.77)	3.46 (.77)	4.00 (.00)	3.60 (.91)	4.00 (.00)	3.80 (.58)
Total	40	3.58 (.75)	3.48 (.72)	3.[illegible] (.30)	3.58 (.81)	4.00 (.00)	3.70 (.72)
6							
Males	25	3.64 (.70)	3.56 (.71)	3.96 (.20)	3.60 (.58)	3.68 (1.11)	4.00 (.00)
Females	15	3.80 (.41)	3.27 (.88)	3.93 (.26)	3.47 (1.06)	4.00 (.00)	3.60 (1.12)
Total	40	3.70 (.61)	3.45 (.78)	3.95 (.22)	3.55 (.78)	3.80 (.88)	3.85 (.70)
Overall Sample							
Males	146	3.03 (1.24)	2.68 (1.45)	3.67 (.81)	3.12 (1.23)	3.62 (1.02)	3.16 (1.41)
Females	134	3.40 (1.03)	3.02 (1.17)	3.79 (1.56)	3.29 (1.18)	3.70 (.97)	3.16 (1.30)
Total	280	3.21 (1.16)	2.84 (1.33)	3.73 (.71)	3.21 (1.21)	3.66 (1.00)	3.14 (1.35)

[a]Standard deviations are given in parentheses.

13

Table 4

PERCENTAGE OF SUBJECTS PASSING THE ASSESSMENT TASKS AT EACH OF THE SEVEN GRADE LEVELS[a]

Grade	(N)	Free Sort	Dichotomies	Some-All	Class Inclusion	Combinatorial Reasoning
Pre						
Males	23	78.3	17.4	30.4	4.3	
Females	17	76.5	17.6	47.1	11.8	
Total	40	77.5	17.5	37.5	7.5	
K						
Males	23	100	30.4	69.6	13.0	0
Females	17	100	29.4	47.1	0	0
Total	40	100	30.0	60.0	7.5	0
1						
Males	21	100	38.1	57.1	9.5	0
Females	19	94.7	31.6	68.4	31.6	0
Total	40	97.5	35.0	62.5	20.0	0
2						
Males	23	95.7	47.8	78.3	21.7	22.0
Females	17	100	76.5	64.7	23.5	5.9
Total	40	97.5	60.0	72.5	22.5	15.0
3						
Males	16	100	62.5	68.8	18.8	12.5
Females	25	100	66.7	87.5	45.8	20.8
Total	40	100	65.0	80.0	35.0	17.5
4						
Males	15	100	53.3	93.3	46.7	13.3
Females	25	100	84.0	92.0	56.0	24.0
Total	40	100	72.5	92.5	52.5	20.0
6						
Males	25	100	76.0	88.0	60.0	48.0
Females	15	100	80.0	93.0	73.3	40.0
Total	40	100	77.5	90.0	65.0	45.0
Overall Sample						
Males	146	95.9	45.9	68.6	24.7	17.0[b]
Females	134	96.3	56.7	73.1	35.8	15.3[b]
Total	280	96.0	51.1	70.7	30.0	16.3[b]

[b] Data not for all 280 subjects.

Table 5

PERCENTAGE OF SUBJECTS PASSING THE MATRIX ASSESSMENT TASKS AT EACH OF THE SEVEN GRADE LEVELS

Grade	(N)	Cross Classification Reproduction	Cross Classification Transposition	Classification Seriation Reproduction	Classification Seriation Transposition	Double Seriation Reproduction	Double Seriation Transposition
Pre							
Males	23	39.1	26.1	82.6	47.8	60.9	30.4
Females	17	47.1	47.1	82.4	58.8	76.5	29.4
Total	40	42.5	35.0	82.5	52.5	67.5	30.0
K							
Males	23	56.5	47.8	95.7	56.5	73.9	30.4
Females	17	52.9	41.2	88.2	70.6	64.7	35.3
Total	40	55.0	45.0	92.5	62.5	70.0	32.5
1							
Males	21	61.9	61.9	90.5	81.0	95.2	66.7
Females	19	94.7	63.2	100	89.5	94.7	52.6
Total	40	77.5	62.5	95.0	85.0	95.0	60.0
2							
Males	23	69.6	65.2	100	82.6	91.3	87.0
Females	17	100	76.5	100	70.6	94.1	58.8
Total	40	82.5	70.0	100	77.5	92.5	75.0
3							
Males	16	81.3	75.0	93.8	81.3	100	81.3
Females	24	83.3	91.7	100	91.7	95.8	91.7
Total	40	82.5	85.0	97.5	87.5	97.5	87.5
4							
Males	15	93.3	93.3	100	93.3	100	73.3
Females	25	96.0	92.0	100	92.0	100	88.0
Total	40	95.0	92.5	100	92.5	100	82.5
6							
Males	25	96.0	100	100	96.0	92.0	100
Females	15	100	100	100	93.3	100	86.7
Total	40	97.5	100	100	95.0	95.0	95.0
Overall Sample							
Males	146	69.9	65.8	94.5	76.0	86.3	66.4
Females	134	82.8	74.6	96.3	82.1	90.3	65.7
Total	280	76.1	70.0	95.3	78.9	88.2	66.1

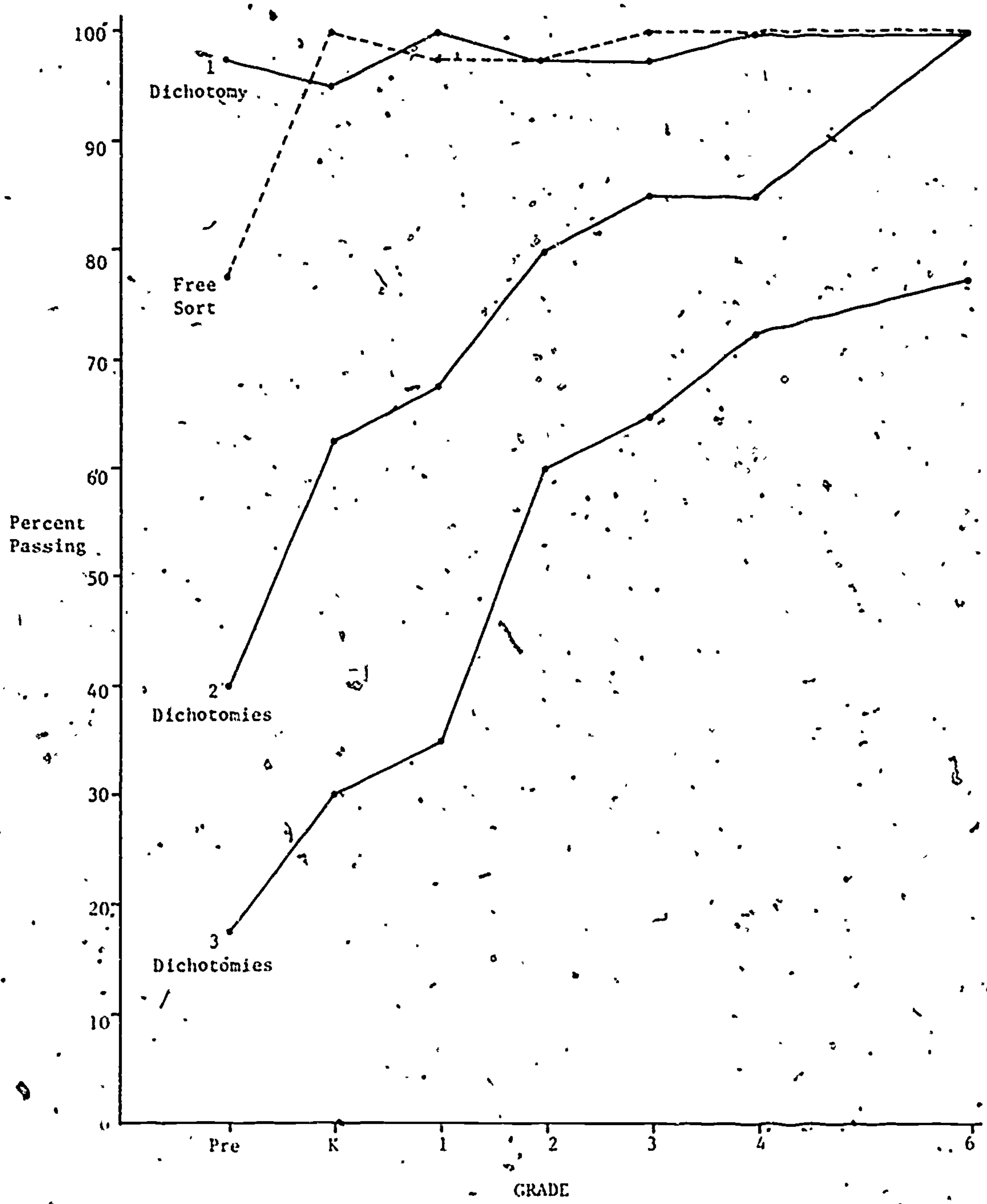


Figure 1. Percentage of subjects passing the free sort and the dichotomies tasks at each of the seven grade levels.

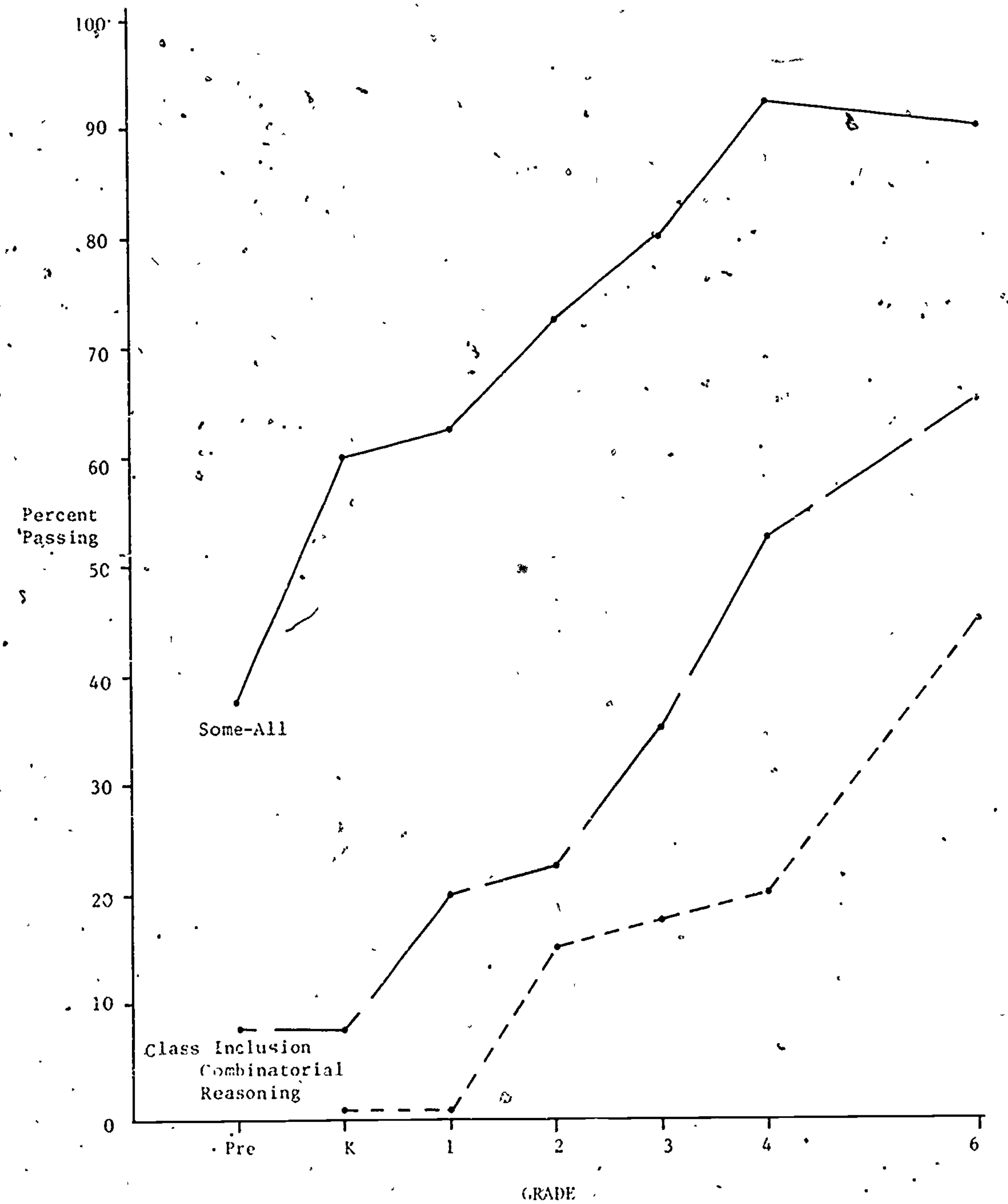


Figure 2. Percentage of subjects passing the Some-All, class inclusion, and combinatorial reasoning tasks at each of the seven grade levels.

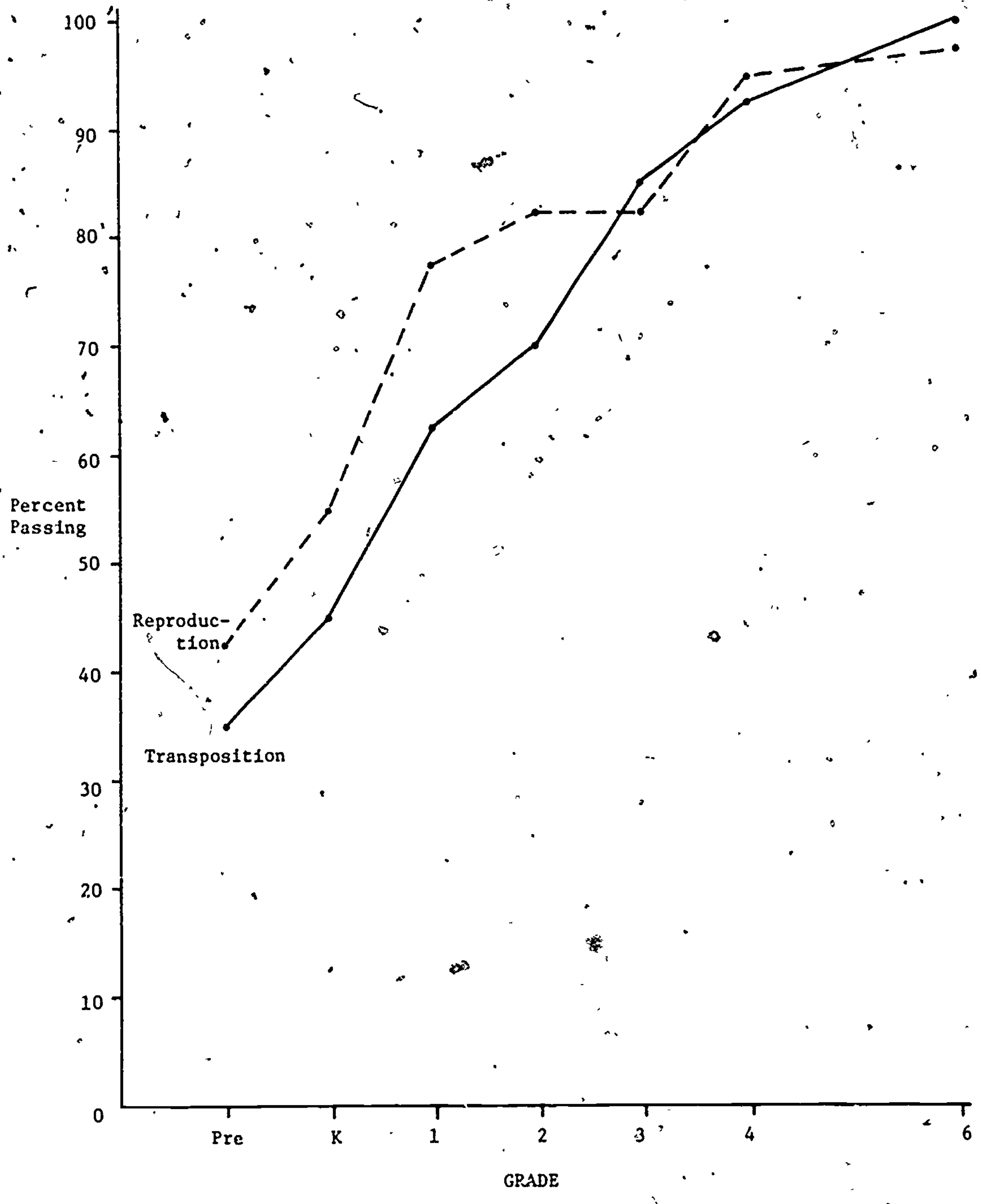


Figure 3. Percentage of subjects passing the cross classification matrix reproduction and transposition subtasks at each of the seven grade levels.

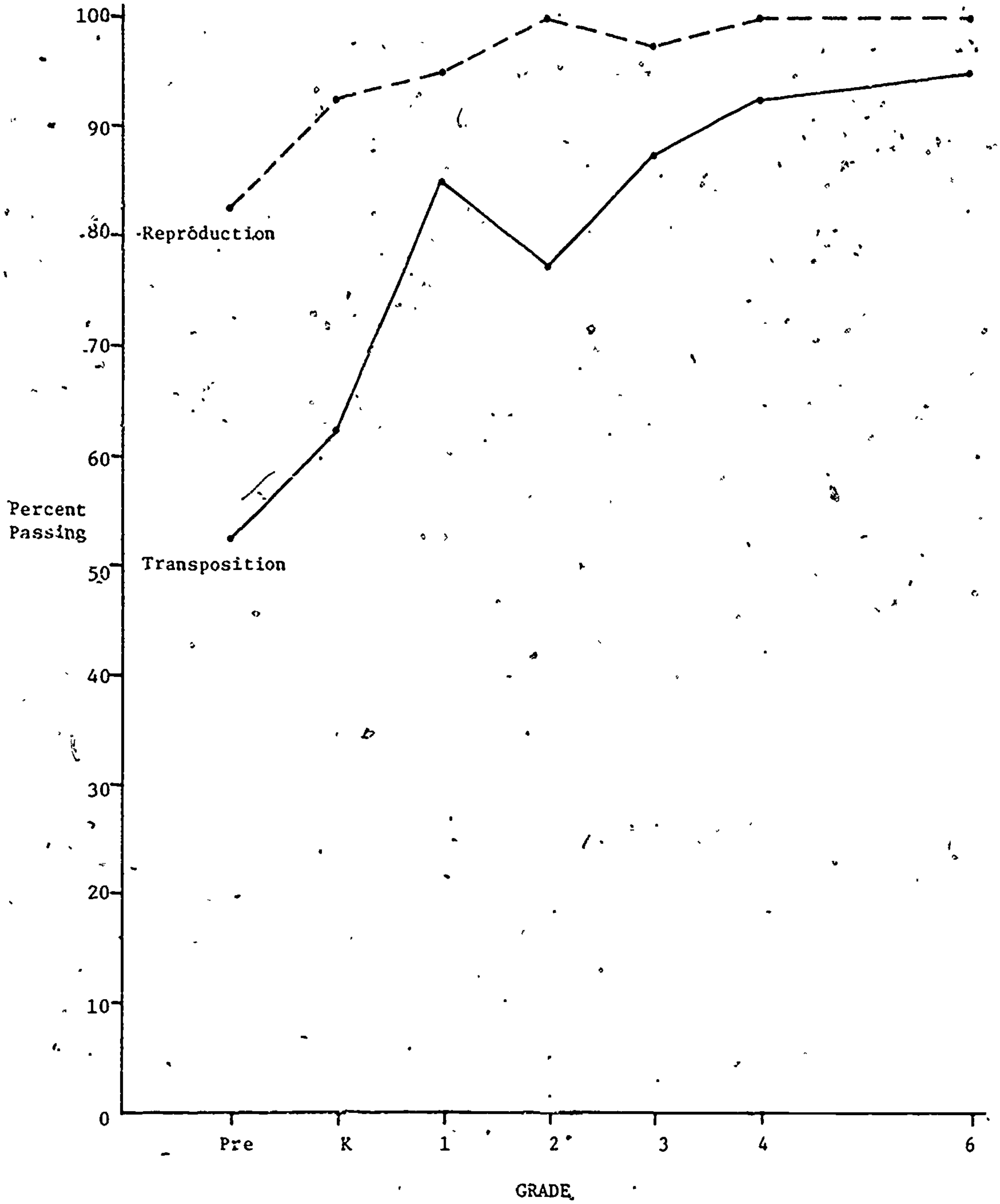


Figure 4. Percentage of subjects passing the classification seriation matrix reproduction and transposition subtasks at each of the seven grade levels.

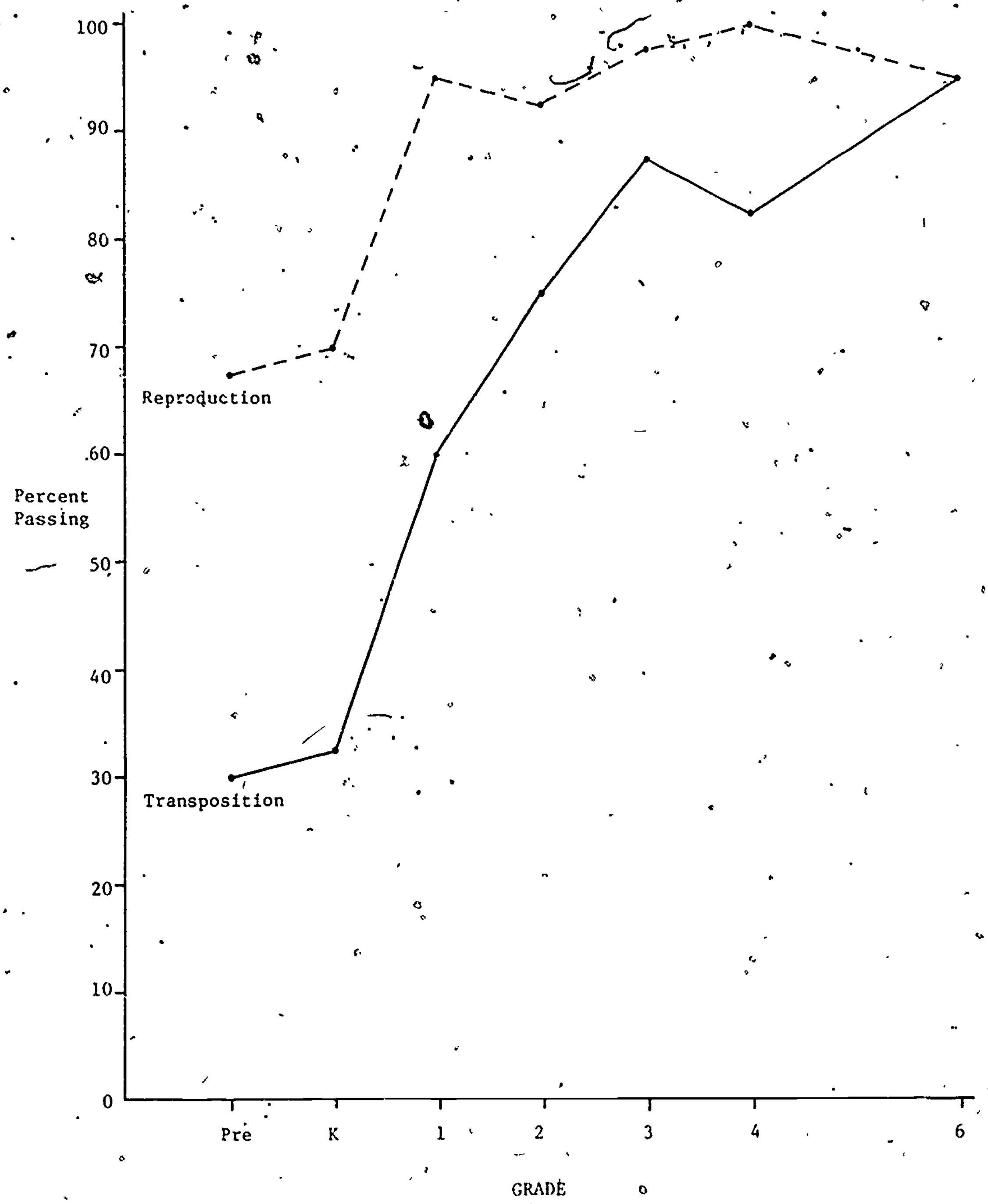


Figure 5. Percentage of subjects passing the double seriation matrix reproduction and transposition subtasks at each of the seven grade levels.

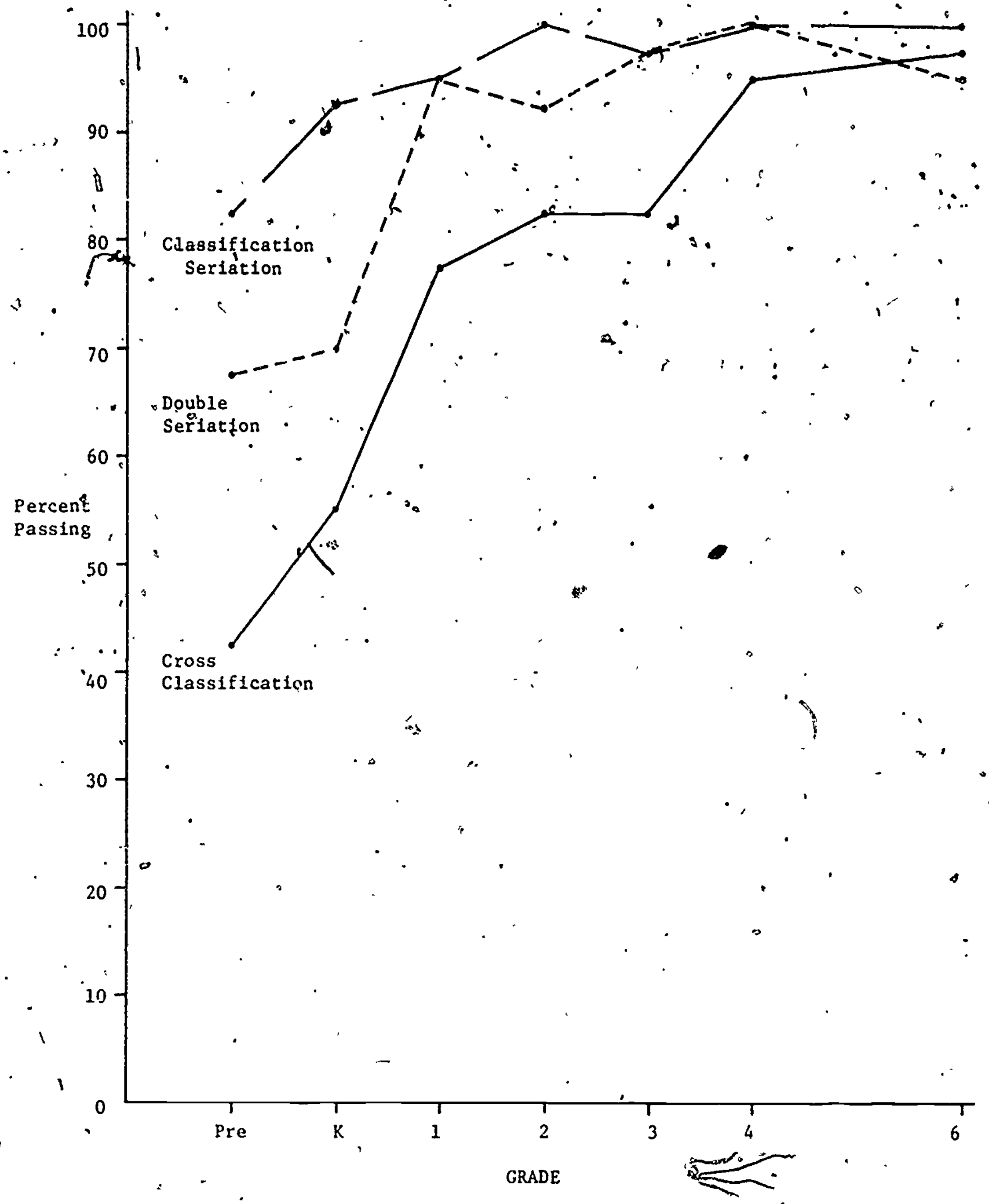


Figure 6. Percentage of subjects passing the matrices reproduction subtasks at each of the seven grade levels.

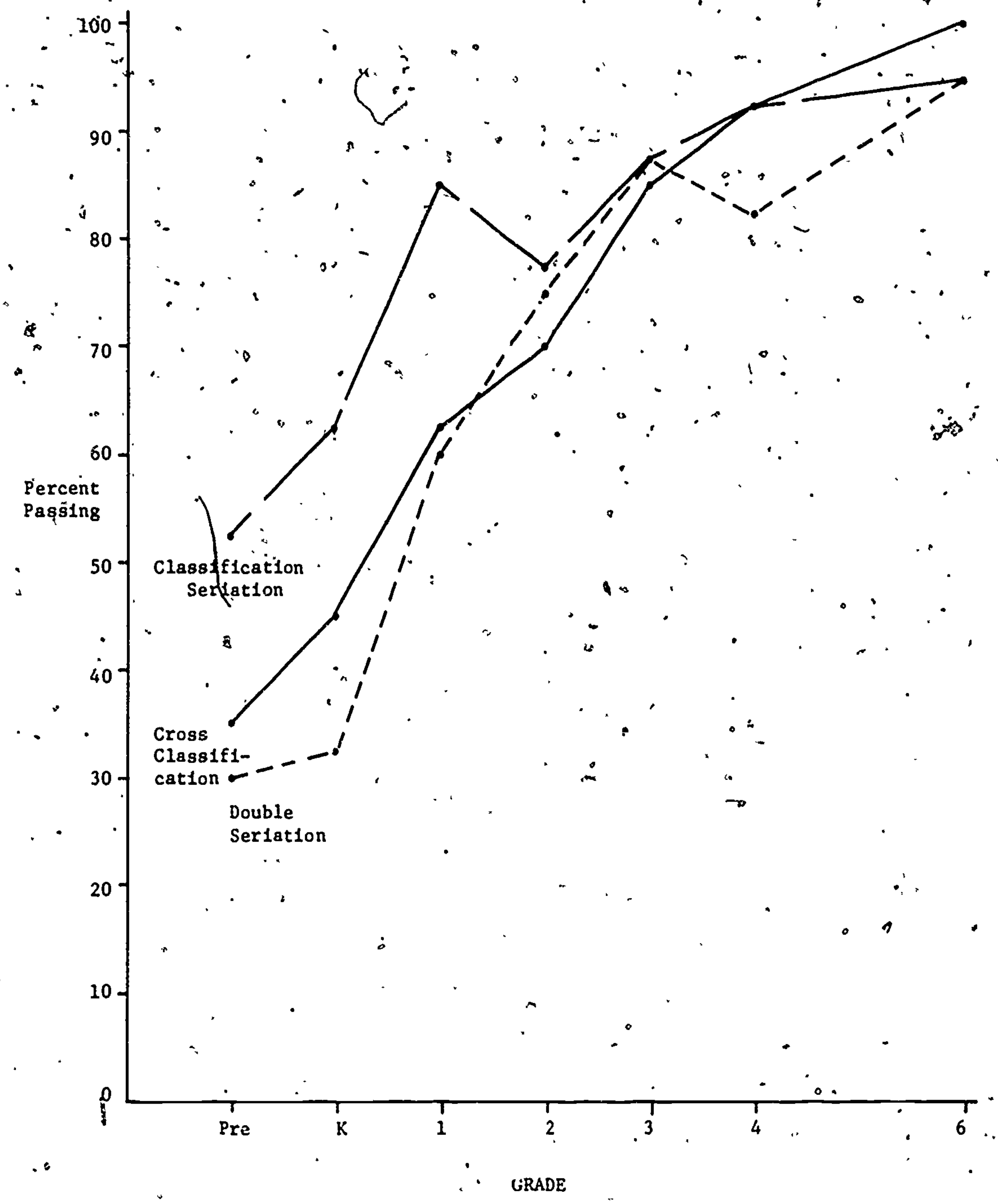


Figure 7. Percentage of subjects passing the matrices transposition subtasks at each of the seven grade levels.

Table 6

FACTORIAL VARIANCE ANALYSIS SUMMARY[a]

	Age			Sex			Age x Sex			
Variable[b]	Mean Sum of SQ	F Value	Probability	Mean SQ	F Value	Probability	Mean SQ	F Value	Probability	Error Mean SQ
1 FS	3.741	5.385	<.0001	.882	1.270	<.261	.175	.252	<.958	.695
2 3D	15.707	10.965	<.0001	5.492	3.523	<.062	1.453	.932	<.472	1.558
3 SA	2.464	7.732	<.0001	.443	1.390	<.240	.713	2.235	<.040	.319
4 CI	17.133	14.663	<.0001	.694	.594	<.444	1.149	.983	<.437	1.168
5 CCR	13.689	13.126	<.0001	7.269	6.970	<.009	1.193	1.144	<.337	1.043
6 CCT	15.406	10.597	<.0001	4.579	3.150	<.077	1.895	1.304	<.256	1.454
7 CSR	2.598	5.844	<.0001	.601	1.352	<.246	.747	1.682	<.126	.445
8 CST	9.545	7.463	<.0001	.669	.523	<.470	1.536	1.201	<.306	1.279
9 DSR	6.024	6.706	<.0001	.002	.003	<.959	.613	.683	<.664	.898
10 DST	24.082	17.993	<.0001	.258	.193	<.661	1.799	1.344	<.238	1.338
11 CR	2053.487	40.395	<.0001	265.141	5.216	<.023	104.183	2.049	<.073	50.836

[a]Degrees of freedom: age = 5, sex = 1, age x sex interaction = 5, and error = 228.

[b]FS - Free sorting
3D - Dichotomies
SA - "Some-All"
CI - Class inclusion
CCR - Cross class reproduction
CCT - Cross class transposition
CSR - Class series reproduction
CST - Class series transposition
DSR - Double series reproduction
DST - Double series transposition
CR - Combinatorial reasoning

Piaget, 1964; Kofsky, 1966), the "Some-All" task was of significantly lesser difficulty than the class inclusion measure. For the "Some-All" task the 75% subject passing level was reached for the second- and third-grade subsamples, this level was not even attained for the older fourth- and sixth-grade subsamples on the class inclusion task. Combinatorial reasoning as assessed in the child's systematic pair-wise combinations of colors is clearly the most difficult task in the present array. As expected for a measure related to formal operations period functioning, fewer than 50% of the sixth-grade subjects could successfully deal with these task demands.

A case of intermediate relative task difficulty is shown for the various matrix tasks (see Figures 3, 4, 5, 6, and 7). Seventy-five percent of the first-grade children could successfully reproduce the cross classification matrix, and 95% of this age-grade group passed the class/series and double series comparative cases. For the matrix transposition tasks the 75% criterion was reached in the first grade for the class/series case and in the second grade for the cross classification and double seriation matrices (see Figure 7). As expected (cf. MacKay et al., 1970), the reproduction task requirement was less difficult than the transposition instructional set for the double seriation and class/series matrix cases (Figures 4 and 5), and these differences were most notable at the younger age-grade levels. A more extensive analysis of the matrix task performances is presented in the final portion of this Results section.

Table 7

SUMMARY OF SEVEN ABILITY GROUPINGS

	Ability Grouping	% of Children Passing Task (N=280)
I	Producing one dichotomy	98.2
	Free sorting	96.7
	Reproducing the class/series matrix	95.3
	Reproducing the double series matrix	88.2
II	Transposing the class/series matrix	78.9
	Reproducing the cross class matrix	76.1
	Producing two dichotomies	74.3
III	Understanding "Some-All" relationships	70.7
	Transposing the cross class matrix	70.0
	Transposing the double series matrix	66.1
IV	Producing three dichotomies	51.1
V	Understanding class inclusion relationships	30.0
VI	Combinatorial reasoning (not given to preschool Ss, N=240)	16.3

Interrelationships Among the Tasks. The various pass/fail interrelationships within the present task array are presented in Tables 8 to 14 for the separate age-trade subsamples and in Tables 15 to 17 for the overall subject sample. Inspection of these tables reveals a number of task relationship patterns and task performance dependencies or intertask putative acquisition sequences. These issues lead to two associated analysis questions: (1) the question of intertask dimensionality and (2) the question of across-task scalability in terms of relative item difficulties.

The Dimensionality of the Tasks. Having established that there exist strong relationships among the 11 Piagetian tasks, we turn to the investigation of the dimensionality of these relationships. The task of assessing the dimensionality of pass/fail data has captured the interest of researchers from Thurstone through Guttman to Bentler. The techniques of the first two are utilized in the present discussion. One approach to studying relationships among binary items is that of computing tetrachoric correlation coefficients and employing principal components analysis to the resulting estimates of parameters of the assumed underlying multinormal distribution.

For the 11 Piagetian tasks, the following matrix of tetrachoric correlations was computed by means of Saunder's method, using a computer program written by Froemel (1970). The combinatorial reasoning task was scored "fail" for all preschool children to give a matrix on the complete sample of 280 children. The tetrachoric correlations among the 11 tasks are presented in Table 18. These correlation values were derived from the pass/fail patterns presented in Tables 15 to 17.

The tetrachoric coefficient, like most coefficients of association for 2 x 2 tables, gives a value of 1 when one cell is empty. In particular, r_{tet} for free sort versus combinatorial reasoning, based on the entries, is unity:

		CR 0	CR 1
FS	1	230	39
FS	0	11	0

Considering first the six matrix tasks, which represent more than half the tasks in the battery and thus are likely to determine the principal components of the entire set, we see that both stimulus (cross class, class/series, and double series) and mode (reproduction versus transposition) contribute to the relations among the matrices. The median of the three correlations within reproduction across matrix type is .670; within transformation the median is .689; within matrix type the median is .742. In contrast, the median of the six correlations having neither matrix type nor operation in common is .54.

A principal components analysis of the six matrix tasks led to a first principal component that accounts for 70.3% of the total variance, indicating the high association among the six tasks. Loadings on this component of variance range from .80 for cross class reproduction to .89 for class/series reproduction. The second principal component accounts for only an additional 10.6% of the variance and separates the reproduction tasks, with positive loadings from .02 for class/series reproduction to .48 for double series reproduction, from the transposition tasks, with negative loadings from -.03 for double series transposition to -.50 for class/series transposition. Table 19 illustrates both the high degree of unidimensionality of the tasks and the relatively smaller but predictable loading of the two types of tasks on the second factor of reproduction versus transposition method variance.

Although we see the matrix tasks to be essentially unidimensional, the important question for the current study is the degree to which this dimension is embedded in the structure of the entire Piagetian battery.

The complete set of 11 tasks was thus subjected to a principal components analysis, resulting in three principal components accounting for 57.0%, 15.2%, and 8.5% of the variance, respectively. Communalities based on these three components ranged from a high of .94 for free sort and .98 for combinatorial reasoning (the least and most difficult tasks, respectively), which formed a doublet factor because of their tetrachoric correlation of 1.0, to a low of .58 for "Some-All." All of the matrix tasks had relatively high loadings on the first principal component, with class inclusion and combinatorial reasoning also loading significantly on this dimension.

The second principal component was determined by the free sort-combinatorial reasoning dimension, with small negative loadings on the matrix tasks, while the third component again distinguished between reproduction and transposition matrix tasks, as had the second in the previous analysis, but had its highest loadings on dichotomies and "Some-All." Thus it would seem that while there exist relations among this set of tasks,

Table 8

CONTINGENCY TABLE OF FREQUENCIES AND PROPORTIONS PASSING THE ASSESSMENT TASKS--PRESCHOOL SUBSAMPLE[a]

	FS	3D	SA	CI	CCR	CCT	CSR	CST	DSR	DST
FS	31 .77	6 .19	12 .39	3 .10	12 .39	11 .35	25 .81	15 .48	21 .68	9 .29
3D	6 .86	7 .18	6 .86	3 .43	5 .71	4 .57	7 1.00	5 .71	7 1.00	3 .43
SA	12 .80	6 .40	15 .38	2 .13	6 .40	5 .33	14 .93	8 .53	10 .67	3 .20
CI	3 1.00	3 1.00	2 .67	3 .08	3 1.00	3 1.00	3 1.00	3 1.00	3 1.00	2 .67
CCR	12 .71	5 .29	6 .35	3 .18	17 .43	10 .59	15 .88	10 .59	13 .76	8 .47
CCT	11 .79	4 .29	5 .36	3 .21	10 .71	14 .35	14 1.00	12 .86	12 .86	8 .57
CSR	25 .76	7 .21	14 .42	3 .09	15 .45	14 .42	33 .83	21 .64	26 .79	12 .36
CST	15 .71	5 .24	8 .38	3 .14	10 .48	12 .57	21 1.00	21 .52	17 .81	11 .52
DSR	21 .78	7 .26	10 .37	3 .11	13 .48	12 .44	26 .96	17 .63	27 .67	11 .41
DST	9 .75	3 .25	3 .25	2 .17	8 .67	8 .67	12 1.00	11 .92	11 .92	12 .30

[a]Key:

	FS
FS	31 .77

31 of the 40 Ss passed FS, yielding a proportion of .77.

	CI
FS	3 .10

3 of the 31 Ss passing FS also passed CI, yielding a proportion of .10.

	DSR
DSR	27 .67

27 of the 40 Ss passed DSR, yielding a proportion of .67.

	CCR
DSR	13 .48

13 of the 27 Ss passing DSR, also passed CCR, yielding a proportion of .48.

Task abbreviations are explained in Table 6.

Table 9

CONTINGENCY TABLE OF FREQUENCIES AND PROPORTIONS PASSING THE ASSESSMENT TASKS--KINDERGARTEN SUBSAMPLE[a]

	FS	3D	SA	CI	CCR	CCT	CSR	CST	DSR	DST	CR
FS	40 1.00	12 .30	24 .60	3 .08	22 .55	18 .45	37 .93	25 .63	28 .70	13 .33	0 .00
3D	12 1.00	12 .30	9 .75	0 .00	8 .67	7 .58	11 .92	7 .58	12 1.00	3 .25	0 .00
SA	24 1.00	9 .38	24 .60	3 .13	12 .50	11 .46	23 .96	15 .63	16 .67	8 .33	0 .00
CI	3 1.00	0 .00	3 1.00	3 .08	3 1.00	3 1.00	3 1.00	3 1.00	2 .67	2 .67	0 .00
CCR	22 1.00	8 .36	12 .55	3 .14	22 .55	14 .64	21 .95	16 .73	20 .91	9 .41	0 .00
CCT	18 1.00	7 .39	11 .61	3 .17	14 .78	18 .45	16 .89	14 .78	14 .78	9 .50	0 .00
CSR	37 1.00	11 .30	23 .62	3 .08	21 .57	16 .43	37 .93	24 .65	27 .73	13 .35	0 .00
CST	25 1.00	7 .28	15 .60	3 .12	16 .64	14 .56	24 .96	25 .63	17 .68	11 .44	0 .00
DSR	28 1.00	12 .43	16 .57	2 .07	20 .71	14 .50	27 .96	17 .61	28 .70	12 .43	0 .00
DST	13 1.00	3 .23	8 .62	2 .15	9 .69	9 .69	13 1.00	11 .85	12 .92	13 .32	0 .00
CR	0 .00	0 .00	0 .00	0 .00	0 .00	0 .00	0 .00	0 .00	0 .00	0 .00	0 .00

[a]See key, Table 8; task abbreviations are explained in Table 6.

Table 10

CONTINGENCY TABLE OF FREQUENCIES AND PROPORTIONS PASSING THE ASSESSMENT TASKS-- FIRST-GRADE SUBSAMPLE[a]

	FS	3D	SA	CI	CCR	CCT	CSR	CST	DSR	DST	CR
FS	39 .98	13 .33	24 .62	8 .21	30 .77	25 .64	37 .95	34 .87	37 .95	24 .62	0 .00
3D	13 .93	14 .35	11 .79	4 .29	11 .79	12 .86	14 1.00	13 .93	14 1.00	8 .57	0 .00
SA	24 .96	11 .44	25 .63	7 .28	20 .80	17 .68	24 .96	22 .88	24 .96	14 .56	0 .00
CI	8 1.00	4 .50	7 .88	8 .20	6 .75	5 .63	8 1.00	8 1.00	8 1.00	5 .63	0 .00
CCR	30 .97	11 .35	20 .65	6 .19	31 .77	23 .74	31 1.00	28 .90	30 .97	20 .65	0 .00
CCT	25 1.00	12 .48	17 .68	5 .20	23 .92	25 .63	25 1.00	25 1.00	24 .96	17 .68	0 .00
CSR	37 .97	14 .37	24 .63	8 .21	31 .82	25 .66	38 .95	34 .89	37 .97	24 .63	0 .00
CST	34 1.00	13 .38	22 .65	8 .24	28 .82	25 .74	34 1.00	34 .85	33 .97	23 .68	0 .00
DSR	37 .97	14 .37	24 .63	8 .21	30 .79	24 .63	37 .97	33 .87	38 .95	24 .63	0 .00
DST	24 1.00	8 .33	14 .58	5 .21	20 .83	17 .71	24 1.00	23 .96	24 1.00	24 .60	0 .00
CR	0 .00	0 .00	0 .00	0 .00	0 .00	0 .00	0 .00	0 .00	0 .00	0 .00	0 .00

[a]See key, Table 8; task abbreviations are explained in Table 6.

Table 11

CONTINGENCY TABLE OF FREQUENCIES AND PROPORTIONS PASSING THE ASSESSMENT TASKS-- SECOND-GRADE SUBSAMPLE[a]

	FS	3D	SA	CI	CCR	CCT	CSR	CST	DSR	LST	CR
FS	39 .98	24 .62	28 .72	9 .23	32 .82	28 .72	39 1.00	30 .77	36 .92	30 .77	6 .15
3D	24 1.00	24 .60	20 .83	6 .25	22 .92	19 .79	24 1.00	20 .83	23 .96	19 .79	5 .21
SA	28 .97	20 .69	29 .73	7 .24	25 .86	21 .72	29 1.00	23 .79	28 .97	25 .86	6 .21
CI	9 1.00	6 .67	7 .78	9 .22	7 .78	8 .89	9 1.00	8 .89	9 1.00	8 .89	2 .22
CCR	32 .97	22 .67	25 .76	7 .21	33 .83	24 .73	33 1.00	25 .76	32 .97	25 .76	5 .15
CCT	28 1.00	19 .68	21 .75	8 .29	24 .86	28 .70	28 1.00	26 .93	27 .96	24 .86	4 .14
CSR	39 .98	24 .60	29 .73	9 .23	33 .83	28 .70	40 1.00	31 .78	37 .93	30 .75	6 .15
CST	30 .97	20 .65	23 .74	8 .26	25 .81	26 .84	31 1.00	31 .77	29 .94	25 .81	4 .13
DSR	36 .97	23 .62	28 .76	9 .24	32 .86	27 .73	37 1.00	29 .78	37 .93	30 .81	6 .16
LST	30 1.00	19 .63	25 .83	8 .27	25 .83	24 .80	30 1.00	25 .83	30 1.00	30 .75	6 .20
CR	6 1.00	5 .83	6 1.00	2 .33	5 .83	4 .67	6 1.00	4 .67	6 1.00	6 1.00	6 .15

[a]See key, Table 8; task abbreviations are explained in Table 6.

Table 12

CONTINGENCY TABLE OF FREQUENCIES AND PROPORTIONS PASSING THE ASSESSMENT TASKS--THIRD-GRADE SUBSAMPLE[a]

	FS	3D	SA	CI	CCR	CCT	CSR	CST	DSR	DST	CR
FS	40 1.00	26 .65	32 .80	14 .35	33 .83	34 .85	39 .98	35 .88	39 .98	35 .88	7 .18
3D	26 1.00	26 .65	29 .73	10 .38	24 .92	21 .81	25 .96	24 .92	26 1.00	24 .92	4 .15
SA	32 1.00	19 .59	32 .80	13 .41	26 .81	28 .88	31 .97	29 .91	31 .97	29 .91	6 .19
CI	14 1.00	10 .71	13 .93	14 .35	13 .93	13 .93	14 1.00	13 .93	14 1.00	12 .86	4 .29
CCR	33 1.00	24 .73	26 .79	13 .39	33 .83	29 .88	33 1.00	31 .94	33 1.00	30 .91	6 .18
CCT	34 1.00	21 .62	28 .82	13 .38	29 .85	34 .85	34 1.00	29 .85	33 .97	29 .85	7 .21
CSR	39 1.00	25 .64	31 .79	14 .36	33 .85	34 .87	39 .98	34 .87	38 .97	34 .87	7 .18
CST	35 1.00	24 .69	29 .83	13 .37	31 .89	29 .83	34 .97	35 .88	35 1.00	32 .91	6 .17
LSR	39 1.00	26 .67	31 .79	14 .36	33 .85	33 .85	38 .97	35 .90	39 .98	34 .87	6 .15
DST	35 1.00	24 .69	29 .83	12 .34	30 .86	29 .83	34 .97	32 .91	34 97	35 .88	6 .17
CR	7 1.00	4 .57	6 .86	4 .57	6 .86	7 1.00	7 1.00	6 .86	6 .86	6 .86	7 .18

[a]See key, Table 8; task abbreviations are explained in Table 6.

Table 13

CONTINGENCY TABLE OF FREQUENCIES AND PROPORTIONS PASSING

THE ASSESSMENT TASKS--

FOURTH-GRADE SUBSAMPLE[a]

	FS	3D	SA	CI	CCR	CCT	CSR	CST	DSR	DST	CR
FS	40 1.00	29 .73	37 .93	21 .53	38 .95	37 .93	40 1.00	37 .93	40 1.00	33 .83	8 .20
3D	29 1.00	29 .73	27 .93	18 .62	28 .97	26 .90	29 1.00	27 .93	29 1.00	23 .79	7 .24
SA	37 1.00	27 .73	37 .93	18 .49	35 .95	34 .92	37 1.00	34 .92	37 1.00	30 .81	8 .22
CI	21 1.00	18 .86	18 .86	21 .53	21 1.00	21 1.00	21 1.00	21 1.00	21 1.00	17 .81	5 .24
CCR	38 1.00	28 74	35 .92	21 .55	38 .95	35 .92	38 1.00	36 .95	38 1.00	31 .82	8 .21
CCT	37 1.00	26 .70	34 .92	21 .57	35 .95	37 .93	37 1.00	36 .97	37 1.00	31 .84	8 .22
CSR	40 1.00	29 .73	37 .93	21 .53	38 .95	37 .93	40 1.00	37 .93	40 1.00	33 .83	8 .20
CST	37 1.00	27 .73	34 .92	21 .57	36 .97	36 .97	37 1.00	37 .93	37 1.00	31 .84	8 .22
DSR	40 1.00	29 73	37 .53	21 .53	38 .95	37 .93	40 1.00	37 .93	40 1.00	33 .83	8 .20
DST	33 1.00	23 .70	30 .91	17 .52	31 .94	31 .94	33 1.00	31 .94	33 1.00	33 .83	7 .21
CR	8 1.00	7 .88	8 1.00	5 .63	8 1.00	8 1.00	8 1.00	8 1.00	8 1.00	7 .88	8 .20

[a]See key, Table 8; task abbreviations are explained in Table 6.

Table 14

CONTINGENCY TABLE OF FREQUENCIES AND PROPORTIONS PASSING THE ASSESSMENT TASKS--SIXTH-GRADE SUBSAMPLE[a]

	FS	3D	SA	CI	CCR	CCT	CSR	CST	DSR	DST	CR
FS	40 1.00	31 .78	36 .90	26 .65	39 .98	40 1.00	40 1.00	38 .95	38 .95	38 .95	18 .45
3D	31 1.00	31 .77	28 .90	20 .65	30 .97	31 1.00	31 1.00	29 .94	31 .97	29 .94	15 .48
SA	36 1.00	28 .78	36 .90	25 .69	35 .97	36 1.00	36 1.00	34 .94	34 .94	34 .94	18 .50
CI	26 1.00	20 .77	25 .96	25 .65	25 .96	26 1.00	26 1.00	24 .92	25 .96	25 .96	13 .50
CCR	39 1.00	30 .77	35 .90	25 .64	39 .98	39 1.00	39 1.00	37 .95	37 .95	37 .95	17 .44
CCT	40 1.00	31 .78	36 .90	26 .65	39 .98	40 1.00	40 1.00	38 .95	38 .95	38 .95	18 .45
CSR	40 1.00	31 .78	36 .90	26 .65	39 .98	40 1.00	40 1.00	38 .95	38 .95	38 .95	18 .45
CST	38 1.00	29 .76	34 .89	24 .63	37 .97	38 1.00	38 1.00	38 .95	36 .95	37 .97	16 .42
DSR	38 1.00	30 .79	34 .89	25 .66	37 .97	38 1.00	38 1.00	36 .95	38 .95	36 .95	16 .42
DST	38 1.00	29 .76	34 .89	25 .66	37 .97	38 1.00	38 1.00	37 .97	36 .95	38 .95	16 .42
CR	18 1.00	15 .83	18 1.00	13 .72	17 .94	18 1.00	18 1.00	16 .89	16 .89	16 .89	18 .45

[a] See key, Table 8; task abbreviations are explained in Table 6.

Table 15

CONTINGENCY TABLE OF FREQUENCIES AND PROPORTIONS PASSING THE ASSESSMENT TASKS--ALL GRADES[a]

	FS	3D	SA	CI	CCR	CCT	CSR	CST	DSR	DST
FS	269 .96	141 .52	193 .72	84 .31	206 .77	193 .71	257 .96	214 .80	239 .89	182 .68
3D	141 .99	143 .51	120 .84	61 .43	128 .90	120 .84	141 .99	125 .87	141 .99	109 .76
SA	193 .97	120 .61	198 .71	75 .38	159 .75	152 .77	194 .98	165 .83	180 .91	143 .72
CI	84 1.00	61 .73	75 .89	84 .30	78 .93	79 .94	84 1.00	80 .95	82 .98	71 .84
CCR	206 .97	128 .60	159 .75	78 .37	213 .76	174 .82	210 .99	183 .86	203 .95	160 .75
CCT	193 .98	120 .61	152 .78	79 .40	174 .89	196 .70	194 .99	180 .92	185 .94	156 .80
CSR	257 .96	141 .53	194 .73	84 .31	210 .79	194 .73	267 .95	219 .89	243 .91	184 .69
CST	214 .97	125 .51	165 .75	80 .36	183 .83	180 .81	219 .99	221 .79	204 .92	170 .77
DSR	239 .97	141 .57	180 .73	82 .33	203 .82	185 .75	243 .98	204 .83	247 .88	180 .73
DST	182 .98	109 .59	143 .77	71 .38	160 .86	156 .84	184 .99	170 .92	180 .97	185 .66

[a]See key, Table 8; task abbreviations are explained in Table 6.

Table 16

FREQUENCY AND PROPORTION PASSING THE COMBINATORIAL REASONING TASK AND EACH OTHER ASSESSMENT TASK COMBINING KINDERGARTEN, FIRST, SECOND, THIRD, FOURTH, AND SIXTH GRADES[a]

	FS	3D	SA	CI	CCR	CCT	CSR	CST	DSR	DST
N Passing CR										
N Passing Other Task	39	31	38	24	36	37	39	34	36	35
Proportion	1.00	.79	.97	.62	.92	.95	1.00	.87	.92	.90

[a]Task abbreviations are explained in Table 6.

Table 17

FREQUENCY AND PROPORTION PASSING EACH ASSESSMENT TASK PLUS THE COMBINATORIAL REASONING TASK, COMBINING KINDERGARTEN, FIRST, SECOND, THIRD, FOURTH, AND SIXTH GRADES[a]

	FS	3D	SA	CI	CCR	CCT	CSR	CST	DSR	DST
N Passing Task	238	136	183	81	196	182	234	200	220	173
N Passing CR	39	31	38	24	36	37	39	34	36	35
Proportion	.16	.23	.21	.30	.18	.20	.17	.17	.16	.22

[a]Task abbreviations are explained in Table 6.

Table 18

MATRIX OF TETRACHORIC CORRELATIONS[a]

	FS	3D	SA	CI	CCR	CCT	CSR	CST	DSR	DST	CR
FS	1.000										
3D	.664	1.000									
SA	.505	.595	1.000								
CI	1.000	.573	.663	1.000							
CCR	.302	.678	.358	.709	1.000						
CCT	.743	.614	.481	.829	.802	1.000					
CSR	.363	.702	.713	1.000	.849	.872	1.000				
CST	.380	.496	.398	.773	.646	.844	.923	1.000			
DSR	.498	.907	.382	.770	.828	.713	.916	.634	1.000		
DST	.696	.440	.425	.594	.671	.762	.928	.814	.875	1.000	
CR	1.000	.634	.901	.657	.625	.810	1.000	.325	.261	.651	1.000

[a]Task abbreviations are explained in Table 6.

Table 19

LOADINGS ON PRINCIPAL COMPONENTS OF THE SIX MATRIX TASKS (N=280)

Variables[a]	PC1	PC2
CCR	.80	.30
CCT	.84	-.25
CSR	.89	.02
CST	.82	-.50
DSR	.82	.48
DST	.86	-.03
Eigen Value	4.22	.64

[a]Task abbreviations are explained in Table 6.

the data do not form a unidimensional pattern; this is partly because of the difficulty factor introduced by the extreme range from free sort to combinatorial reasoning but more importantly because, while class inclusion seems to relate to general competence on the matrix tasks, dichotomies and "Some-All" are more strongly related to the reproduction-transposition dimension, the status of which is unclear in Piagetian theory.

Applying a varimax rotation procedure to the loadings resulted in the values given in Table 20.

The major effect of this column-variance maximizing rotation on the original principal component loadings was to emphasize the separation of the matrix tasks from the others by placing the second factor directly through the free sort-combinatorial reasoning doublet, thus increasing the loadings of all non-matrix tasks on this factor.

To the extent, then, that principal components analysis of tetrachoric correlations is a technique robust with respect to the problems of the analysis of binary data with a wide range of item difficulties, it seems that these tasks as presently constructed contain several performance dimensions. Thus, further development is needed if it is desired to discover invariant sequences other than those predictable from item difficulty alone.

While the problem of spuriously high relations leading to doublet factors may be solved relatively easily by omitting tasks of extreme easiness, such as free sort, the large number of matrix tasks seem to create two "matrix" factors which remain apparent when these tasks are combined with the others. To overcome this artifactual structure, cross classification and double seriation reproduction were selected as single measures of their respective constructs and an analysis was performed on a subset of six measures, none of which had tetrachoric intercorrelations of unity.

Deleting free sort, transposition matrix tasks, and the classification seriation reproduction task and applying the principal components with roots greater than 1, accounting for 55.9% and 19.4% of the total variance, respectively. A third component accounted for 9.4% of the variance. Loadings of the six variables on the first two components are given in Table 21.

The loadings on the first component exhibit a fairly narrow range, from .68 to .82, suggesting nearly equal amounts of common variance for the six tasks. The second seems to pick up a contrast between combinatorial reasoning and "Some-All" on the one hand and the matrix tasks on the other. While eliminating the extremely easy tasks and matrix transposition resulted in a set for which the main dimension loads nearly equally on all tasks, the loadings are not impressively high, suggesting that while the tasks have a significant amount of shared variance, partly because all are age-related, unique variances remain high, and taken as a set, they do not represent a reliable measure of a single continuum. This, of course, does not demonstrate that the underlying structures do not lie on a single dimension; it does suggest that further attention to reducing sources of performance variance in tasks embodying Piagetian concepts is needed if a simple structure is to be uncovered.

Guttman Scale Analysis. Several applications of Guttman scaling (Guttman, 1950) have been made to the investigation of the sequence of acquisition of Piagetian tasks (Kofsky, 1966; Wohlwill, 1960); the results have been unclear. The technique was applied to the present data for purposes of comparison with earlier studies.

Table 20

VARIMAX ROTATION OF FACTOR PATTERN FOR 11 VARIABLES

Variables[a]	Principal Components		
	PC1	PC2	PC3
FS	.16	.96	.04
3D	.24	.23	.86
SA	.07	.51	.56
CI	.57	.72	.16
CCR	.65	.06	.51
CCT	.74	.37	.23
CSR	.75	.47	.32
CST	.89	.10	.05
DSR	.65	.02	.63
DST	.80	.23	.18
CR	.19	.94	.26
Percent of Total Variance	34.8	27.8	18.1

[a]Task abbreviations are explained in Table 6.

Table 21

PRINCIPAL COMPONENT LOADINGS OF A SUBSET OF SIX VARIABLES

Variables[a]	Principal Components	
	PC1	PC2
3D	.82	-.18
SA	.68	.56
CI	.78	.08
CCR	.76	-.36
DSR	.75	-.58
CR	.68	.59
Eigen Value	3.35	1.17

[a]Task abbreviations are explained in Table 6.

Before examining the results of a scalogram analysis of these data, it is essential to clarify the point that Guttman's coefficient of reproducibility (CR) does not itself determine the dimensionality of a set of items. The index, $1 - \frac{E}{NK}$, where N = number of subjects, K = number of items, and E = errors (departures from ideal patterns), was proposed only as a measure of the degree to which knowledge of a subject's total score enables one to reproduce his pattern of responses to each item. Clearly, if items vary sufficiently in their average age of attainment, the coefficient will be high, even if there exists no single underlying learning hierarchy relating them. For example, most American children learn addition before learning French, but a large number of French children demonstrate that there is nothing necessary about this sequence. Some of the confusion surrounding the interpretation of Guttman scales stems from overgeneralizing a familiar introductory example to scalogram analysis. While measurement on a ruler leads to perfect scales and is unidimensional, the converse, that a high coefficient of reproducibility implies an underlying "ruler," is not true. Items such as, (1) "is over three feet tall," (2) "weighs over 70 pounds," and (3) "solves formal reasoning tasks," should lead to a good scale, but this example is not so subject to overgeneralization. Merely showing that two phenomena are age-related does not demonstrate that a single process explains their interrelations.

If the probability of passing these three items in a population is .9, .5 and .1, and the items are otherwise independent, the probabilities of observing each possible pattern will be:

	A	B	C	Probability
	.9	.5	.1	
Scale Patterns	1	1	1	.045
	1	1	0	.405
	1	0	0	.405
	0	0	0	.045
				.90
Non-Scale Patterns	1	0	1	.045
	0	1	0	.045
	0	1	1	.005
	0	0	1	.005
				.10

In this situation, the expected number of errors for 100 subjects is 10, CR = .97, and to demonstrate, at the .05 level, significantly fewer errors than predicted by chance, the score patterns would have to exhibit no more than 3 errors, or CR = .99. If the items were all near .5 in difficulty, however, a value of CR of .86 (fewer than 44 errors) would suffice to demonstrate more than chance reproducibility. Thus no "standard" level of coefficient of reproducibility exists.

Investigations of dimensionality require that tasks representing each structure be made as similar in difficulty as available performance variance permits; this is necessary if there is to be any possibility of obtaining evidence for numbers of ideal patterns significantly exceeding the chance level derivable from products of individual item difficulties, assumed to be independent.

A scalogram analysis program (Werner & Morrison, 1967) applied to the complete set of 11 Piagetian tasks for 200 subjects (grades K-4) yielded a CR of .91; only 67 of t[illegible] subjects exhibited ideal scale pattern[illegible] was assumed that the failure of these items to approach a perfect scale was due to the inclusion of the classification seriation matrix tasks in the set, since Piagetian theory would predict developmental synchrony for the structures underlying these tasks and the other matrix tasks while investigations of their performance difficulty in the current and previous studies (MacKay et al., 1970) have provided conflicting evidence as to relative difficulties.

In addition to the classification seriation tasks, the free sort task was omitted from the second analysis since its easiness (.95) made it inappropriate for scaling. The set of remaining items approached the various criteria for scalability; most "errors" were due to the fact that "Some-All" was very near in difficulty to the two cross classification tasks but not strongly related to them.

Finally, by deleting the "Some-All" task, an acceptable 7-item Guttman scale was obtained, with 56% of the patterns corresponding to ideal patterns and CR = .93; CR = .85 was expected due to chance, using Sagi's (1959) procedure. More impressively, Loevinger's (1945) method of counting errors led to 204 errors, as compared to 538 expected. This discrepancy amounts to nearly 25 error standard deviations, suggesting that the probability of chance occurrence, had these items been selected _a priori_, is negligible. However, since the result was obtained by deleting four items, it is best regarded as tentative until replicated.

Table 22 presents the frequencies of the ideal scale types and the most frequent non-scale types for each Guttman score.

While the ideal pattern _4_ was obtained more frequently than the Guttman model would predict, reflecting the higher interrelationship of the matrix tasks to each other, partly due to shared method variance, patterns of failing one or all of the matrix tasks in the order predicted by their relative difficulties were rare as compared to expectation. This suggests that future analyses using this technique might profitably drop the reproduction-transposition distinction and determine which single tasks best operationalize cross classification and double seriation skills. While in the present data deleting transposition tasks would have improved the scalability of the remaining items more than deleting reproduction tasks would have, this is largely due to the relative easiness of the reproduction tasks. The question of which tasks best operationalize the underlying constructs (most probably not the matrix tasks) is not strictly a question for data analysis. Thus, while the scale analysis leads to a scale of quite high reproducibility, the inclusion of the four matrix tasks seems to have affected the content to the extent of possibly distorting the underlying relationships. By deleting the "Some-All" task, scalability was achieved at the cost of losing all comparability with Kofsky's scaling study, since in creating a scale from her data, she deleted class inclusion, the only other identical task in the two studies. These results highlight a difficulty with the Guttman technique. If a set of items do form a scale, the investigator may take a subject's total score as an indicator of which particular items were passed. If the set requires deletion of items of theoretical interest to create a scale, simple replication of the scalability of the remaining set will not result in information about the dimensionality of the domain. Scaling operations

Table 22

FREQUENCIES OF IDEAL SCALE TYPES AND THE MOST FREQUENT NON-SCALE PATTERNS FOR EACH GUTTMAN SCORE[a]

CR	CI	3D	DST	CCT	CCR	DSR	Guttman Score	Frequency	Ideal Expected Frequency
1	1	1	1	1	1	1	7	30	30
1	1	0	1	1	1	1	7	8	
0	1	1	1	1	1	1	6	20	21
0	1	0	1	1	1	1	6	4	
0	0	1	1	1	1	1	5	8	12
0	0	1	0	1	1	1	5	6	
0	0	0	1	1	1	1	4	16	11
1	0	0	1	1	1	1	4	5	
0	0	0	0	1	1	1	3	4	23
0	0	0	0	0	1	1	2	7	26
0	0	1	0	0	1	1	2	5	
0	0	0	0	0	0	1	1	9	15
0	0	0	1	0	0	1	1	7	
0	0	0	0	0	0	0	0	17	62

[a]Task abbreviations are explained in Table 6.

would seem to be most usefully performed on sets of items that have been independently shown to be unidimensional, and as factor analysis suggested, the inclusion of the matrix tasks seems to have affected the dimensionality of the current items.

Additional Matrix Task Analyses

The purpose of this final results summary is to compare the present performances on the various matrix subtasks to the earlier findings of MacKay and his associates (MacKay, 1972; MacKay et al., 1970). Considering first the data concerning the children's replacement responses (possible score range of 0 to 3), very few subjects (9.3%) committed replacement errors on any of the matrices. No errors were found in the older subsamples, and 19 of the overall 26 cases were found in the preschool and kindergarten groups. More importantly for the comparisons to follow, there were few differences among the replacement errors for the three matrix types, i.e., 12, 8, and 14 instances for the cross classification, class/series, and double seriation matrix cases, respectively.

As indicated previously, there was a significant increase in the number of subjects passing each of the matrix subtasks across the present age-grade range. Comparisons across a more restricted age range comparable to that found in the MacKay et al., (1970) study, i.e., preschool (mean age, 5 years) to second grade (mean age, 8 years 2 months) were also carried out. For each reproduction and transposition subtask there was a significant increase in the proportion of successful subjects, i.e., all χ^2 values exceeded 9.40, $df = 3$, $p < .025$. This is essentially similar to the findings of the earlier investigation.

In the MacKay et al. (1970) study, transposition of the matrices which involved a continuous dimension (double seriation) was a significantly more difficult task than reproduction (see however, MacKay, 1972, for a case of equivalent transposition-reproduction difficulty in a sample of severely subnormal adults). Examination of the present matrix pass/fail frequencies reveals a concordant pattern of relative task difficulties. χ^2 comparisons of the number of successful subjects on the reproduction subtask versus the transposition subtask of the double seriation matrix showed the former to be significantly easier at the preschool (27 vs. 12), kindergarten (28 vs. 13), and first grade (38 vs. 24) levels. Combining the preschool through second-grade subsamples ($N = 160$), 130 children passed the reproduction case and 79 passed the transposition subtask ($\chi^2 = 35.88$, $df = 1$, p .001). Similar comparisons for the class/series matrix also revealed reproduction to be the easier task, i.e., the frequency of passing subjects for reproduction versus transposition for the preschool to second-grade subsamples was 33 versus 21, 37 versus 25, 38 versus 34, and 40 versus 31; the total frequencies were 148 versus 111

($\chi^2 = 27.73$, $df = 1$, $p < .001$) for the combined younger subject subsamples. In contrast, there was no significant difference between the number of successful reproduction versus transposition cases for the cross classification matrix at any of the separate age-grade levels, although the comparison for the 160 children in the composite younger group--103 passed the reproduction subtask and 85 passed the transposition subtask ($\chi^2 = 4.18$, $df = 1$, $p < .05$)--was marginally significant. In the preschool through second-grade composite sample, combining all the reproduction and transposition cases for the three matrices indicated that the former task was significantly easier, i.e., $N = 480$ with 381 versus 275 passing cases ($\chi^2 = 54.09$, $df = 3$, $p < .001$).[4]

A more direct comparison of the relative difficulty of the reproduction versus the transposition matrix subtasks is shown in Table 23. (Note that for Tables 23, 24, and 25 the comparisons within grade levels are binomial tests with one-tailed probabilities for Table 23 and two-tailed probabilities for Tables 24 and 25. The composite subsample comparisons are McNemar Tests for the Significance of Changes with associated χ^2 values and one-tailed probabilities for Table 23 and two-tailed probabilities for Tables 24 and 25.) All of the within-grade subsample comparisons on double seriation significantly favor the easier reproduction task, and only the first-grade subsample comparison fails to indicate a similar significant relative difficulty pattern for the class/series matrix. For the cross classification matrix, only the first-grade subsample and the composite preschool to second-grade sample comparison reach significance. One may conclude, therefore, that matrix reproduction is significantly easier than matrix transposition, and this is particularly true for the double seriation and class/series matrices.

In considering the relative difficulty of the three basic matrix types, we shall examine the reproduction and transposition cases separately. The relevant comparisons for the reproduction case are presented in Table 24. Considering initially the three matrices together, Cochran Q values for the number of passing subjects were 17.04 (preschool), 18.00 (kindergarten), 10.89 (first grade), and 9.25 (second grade), indicating significant differences across the matrix reproduction subtasks (all probabilities less than .01). Reproduction of the cross classification matrix was significantly more difficult than the counterpart class/series case at all of the younger age-grade levels and in terms of the composite sample. A similar case of relatively greater task difficulty for cross classification compared to double seriation is also shown, i.e., only the second-grade comparison fails to reach an acceptable significance level. Finally, the double seriation reproduction task appears to be of significantly greater difficulty than the class/series reproduction case, and this is most notable at the preschool and kindergarten age-grade levels where a sufficient degree of inter-task variability (absence of ceiling effects) permits direct comparisons. Thus, the relative task difficulties for the three matrix reproduction cases are as follows: cross classification > double seriation > class/series.

In the matrix transposition task case, the relative difficulties are somewhat less distinct (see Table 25). Considering initially the three matrices together, Cochran Q values for the number of passing subjects were 7.88 (preschool), 9.91 (kindergarten), and 9.94 (first grade), indicating significant differences across the matrix transposition subtasks (all probabilities less than .05). The double seriation transposition task is clearly more difficult than the class/series matrix case. In addition, the cross classification transposition task is also of greater difficulty than the class/series counterpart, at least insofar as the preschool, first grade, and overall composite sample comparisons are concerned. There is obviously very little difference in the relative transposition task difficulties for the cross classification and the double seriation matrices. Thus, the relative task difficulties for the three matrix transposition cases are as follows: cross classification = double seriation > class/series.

Returning to the reproduction tasks again, Table 26 presents the number and percentage of children who passed the reproduction tasks and who exactly reproduced the various matrices as originally presented to them (see scoring procedures in the Methods section). The percentages for the various composite sample totals, i.e., 78.6%, 80.4%, and 98.5% for the cross classification, class/series, and double seriation matrix cases, respectively, closely parallel the earlier results of MacKay et al., (1970, pp. 793-794).

---

[4] It is recognized that the use of the χ^2 statistic may be inappropriate in the present context since the repeated measurement design does not typically denote independent observations. However, the lack of presentation order effects and the similar task contrast patterns found for the related sample inference tests reported in this study would seem to discount the importance of these reservations.

Table 23

COMPARISON OF THE RELATIVE DIFFICULTIES OF THE REPRODUCTION VERSUS TRANSPOSITION MATRIX TASKS

CCT x CCR

Group	CCR	CCT +	CCT −	
Preschool	+	10	7	
	−	4	19	
Kindergarten	+	14	8	
	−	4	14	
First Grade	+	23	8	b
	−	2	7	
Second Grade	+	24	9	
	−	4	3	
Combined	+	71	32	
	−	14	43	

$\chi^2 = 6.283$ $p < .01$

CST x CSR

Group	CSR	CST +	CST −	
Preschool	+	21	12	a
	−	0	7	
Kindergarten	+	24	13	a
	−	1	2	
First Grade	+	34	4	
	−	0	2	
Second Grade	+	31	9	a
	−	0	0	
Combined	+	113	34	
	−	2	11	

$\chi^2 = 26.694$ $p < .001$

DST x DSR

Group	DSR	DST +	DST −	
Preschool	+	11	16	a
	−	1	12	
Kindergarten	+	12	16	a
	−	1	11	
First Grade	+	24	14	a
	−	0	2	
Second Grade	+	30	7	a
	−	0	3	
Combined	+	77	53	
	−	2	28	

$\chi^2 = 45.455$ $p < .001$

[a] $p < .01$ (one-tailed).
[b] $p < .05$ (one-tailed).

Table 24

COMPARISON OF THE RELATIVE DIFFICULTIES OF THE REPRODUCTION MATRIX TASKS

		CCR x DST				CCR x CSR				CSR x DSR		
		DSR +	DSR −			CSR +	CSR −			DSR +	DSR −	
Preschool	CCR +	13	4	a	CCR +	15	2	a	CSR +	26	7	
	−	14	9		−	18	5		−	1	6	
Kindergarten	CCR +	20	2		CCR +	21	1	a	CSR +	27	10	a
	−	8	10		−	16	2		−	1	2	
First Grade	CCR +	30	1	a	CCR +	31	0	b	CSR +	37	1	
	−	8	1		−	7	2		−	1	1	
Second Grade	CCR +	32	1		CCR +	33	0	b	CSR +	37	3	
	−	5	2		−	7	0		−	0	0	
Combined	CCR +	95	8		CCR +	100	3		CSR +	127	21	
	−	35	22		−	48	9		−	3	9	
		$\chi^2 = 15.721$ $p < .001$				$\chi^2 = 37.961$ $p < .001$				$\chi^2 = 12.042$ $p < .001$		

[a] $p < .05$ (two-tailed).

[b] $p < .01$ (two-tailed).

Table 25

COMPARISON OF THE RELATIVE DIFFICULTIES OF THE TRANSPOSITION MATRIX TASKS

		CCT x DST: DST +	CCT x DST: DST -		CCT x CST: CST +	CCT x CST: CST -		CST x DST: DST +	CST x DST: DST -
Preschool	CCT +	8	6	CCT +	12	2	CST +	11	10[a]
	-	4	22	-	9	17	-	1	18
Kindergarten	CCT +	9	9	CCT +	14	4	CST +	11	14[a]
	-	4	18	-	11	11	-	2	13
First Grade	CCT +	17	8	CCT +	25	0	CST +	23	11[a]
	-	7	8	-	9	6	-	1	5
Second Grade	CCT +	24	4	CCT +	26	6	CST +	25	6
	-	6	6	-	5	7	-	5	4
Combined	CCT +	58	27	CCT +	77	8	CST +	70	41
	-	21	54	-	34	41	-	9	40
		$\chi^2 = .521$			$\chi^2 = 14.881$ $p < .001$			$\chi^2 = 19.220$ $p < .001$	

[a] $p < .01$ (two-tailed).

Table 26

NUMBER AND PERCENTAGE OF SUCCESSFUL SUBJECTS, ACROSS THE YOUNGER GRADE LEVELS, WHO EXACTLY REPRODUCED THE VARIOUS MATRICES

	Cross Classification			Classification Seriation			Double Seriation		
		Exactly Reproduced			Exactly Reproduced			Exactly Reproduced	
Grade	No. Passed	No.	% of Passing	No. Passed	No.	% of Passing	No. Passed	No.	% of Passing
Pre	17	9	52.94	33	22	66.7	27	27	100
K	22	13	59.09	37	32	87.49	28	28	100
1	31	29	93.55	38	29	76.32	38	36	94.74
2	33	30	90.91	40	36	90.0	37	37	100
Composite	103	81	78.64	148	119	80.41	130	128	98.46

Table 27

NUMBER OF SUBJECTS FAILING EACH DIMENSION OF EACH MATRIX SUBTASK FOR THE VARIOUS AGE-GRADE LEVELS

	Cross Classification				Classification Seriation				Double Seriation			
	Reproduction		Transposition		Reproduction		Transposition		Reproduction		Transposition	
Grade	color	shape	color	shape	color	height	color	height	diameter	height	diameter	height
Pre	21	11	19	19	3	7	8	19	9	11	11	28
K	13	7	18	13	1	3	9	13	4	12	17	24
1	7	2	10	8	1	2	1	6	1	2	7	8
2	6	3	10	5	0	0	4	8	2	3	5	8
3	5	2	4	3	1	0	3	3	1	0	2	3
4	2	0	3	0	0	0	2	2	0	0	3	5
6	0	1	0	0	0	0	1	2	2	2	1	2
All Grades	54	26	64	48	6	12	28	53	19	30	46	84

Table 27 presents the types of error responses (color vs. shape, color vs. height, and diameter vs. height) for the matrix performances. The predominant error category for the cross classificatory tasks is color misplacement, i.e., 67.50% and 57.14% of the reproduction and transposition error cases, respectively, for the overall combined sample. In contrast, for the class/series matrix tasks the present children made more errors on the height dimension (66.67% and 65.43%) than on the color dimension (33.33% and 34.57%). Misplacements based on height were also the predominant error category (contrasted with the width or diameter dimension) for the double seriation cases, i.e., 66.22% and 64.62% of the total sample error cases for the reproduction and transposition tasks, respectively. This latter tendency contrasts with the previous findings of MacKay (1972, p. 601) which indicated that correct responses on the double seriation transposition task were more likely to focus upon the height dimension than upon the diameter dimension.

IV
Discussion

The present research study investigated the classificatory abilities and related multiple relationality skills of 5- to 12-year-old children. The concept task series included sorting skills, understanding of "Some-All" and class inclusion relationships, and combinatorial reasoning in addition to reproduction and transposition of multiple class, class series, and double seriation matrices. As anticipated in view of the previous related classificatory concept assessment research there was a marked score improvement over the present age-grade range (see Tables 2 to 5, and Figures 1 to 7). Significant age-grade main effects were found for the mean trials (Table 6), and percentages of successful subjects for all the various task settings. There was a general absence of significant sex differences or sex-grade level interactions. Few of the major presentation order comparisons were significant.

The relative order of difficulty of the present Piagetian task array in terms of the total sample ($N = 280$) performances may be characterized as representing a series of underlying ability groupings (see Table 7). The scaling analyses generally substantiate this picture. For the entire series of 11 task settings, the obtained reproducibility coefficient of .91 with 67 (33.5%) of the subjects exhibiting perfect scale patterns suggests the presence of a quasiscale. More definitive conclusions are possible for the reduced 7-item array as evidenced in the performances of 200 subjects (kindergarten to fourth-grade subsample). In this instance the reproducibility coefficient of .93 with 56% of the subjects demonstrating ideal pass/fail patterns indicates a reasonably reliable scale. Considering these seven tasks and acknowledging the relatively low difficulty present in the free sorting task and assuming the "Some-All" task to be of approximately equivalent difficulty to cross classification transposition (free sorting and "Some-All" were not included in the secondary scaling analysis), the following task difficulty order appears: free sorting < double series reproduction < cross class reproduction < cross class transposition < "Some-All" understanding < double series transposition < producing three dichotomies < class inclusion understanding < combinatorial reasoning. In retrospect, one might speculate that the free sorting and matrix reproduction tasks actually represent preoperational abilities while combinatorial reasoning assesses a formal operations period ability. This would permit the concrete operations period "label" to be assigned to the intermediate difficulty level tasks (matrix transposition, "Some-All," sorting consistently and resorting exhaustively on three criterial attributes, and class inclusion). In general, the present item difficulty results are in close accord with the earlier findings of Kofsky (1966), i.e., the concordant difficulty patterns for the free sorting, "Some-All," dichotomies, and class inclusion tasks.

Although the present study was not designed to explicitly evaluate the structural postulate of Piagetian theory dealing with intra-stage correspondence, certain post hoc generalizations are possible. Piaget has consistently predicted developmental synchrony for performance on task settings derived from the classificatory and relational groupements--cf. Piaget (1965, pp. 240-243; 1970a, pp. 723-727; 1970b, pp. 24-27 and 65-66) and Inhelder and Piaget (1964, pp. 278-290). Thus one would expect the respective matrix tasks to be of equivalent difficulty (Hamel & Van Der Veer, 1972; Shantz, 1967; Smedslund, 1964). This was clearly not the case in most instances for the younger subjects in the present sample. Moreover, Piaget also contends that tasks derived from the additive groupements should be mastered at approximately the same time as those based upon the multiplicative cases, i.e., groupement I, primary addition of classes, and groupement III, bi-univocal multiplication of classes (cf. Flavell, 1963, pp. 173-179 and 190-193). To the extent that the present class inclusion and cross class matrix tasks represent these respective groupements, a notably disparate item difficulty pattern is evident. Class inclusion is clearly much more difficult than any of the matrix tasks. As the scaling analyses suggest, there is very little evidence for intra-stage correspondence in the present results except for the sixth-grade subsample. It may well be, as suggested by Flavell (1970, pp. 1037-1040), that the stage correspondence or concurrence postulate must undergo considerable revision in the light of recent nonconfirmatory empirical evidence.

The possibility that a single unitary factor or dimension (e.g., as represented by the concrete period groupement operations) underlies the present subjects' performances seems rather remote. As the results of the dimensionality analyses indicated, several performance dimensions are present in the composite Piagetian task series. Even when a subset of tasks was examined (see Table 21) two, and possibly three, principal components were necessary to account for 75.3% to 84.7% of the task variances. Thus, while the first component accounting for 55.9% of the total variance indicated uniformly consistent loadings for all tasks (i.e., .68 to .82), the remaining task-specific variances suggest that the present task array does not represent a single psychological dimension. Much of the observed common task variance may indeed be a result of shared method variance and this is, of course, most notable for the six matrix tasks. Taken in conjunction with the task difficulty patterns discussed above, the dimensionality analyses certainly indicate a number of distinct factors (all of which are related to chronological age) as operative in the present subjects' Piagetian task performances.

A relatively small number of previous investigations have applied factor analytic techniques to the analysis of concrete operational task interrelationships. The majority of these studies present factor patterns and task loadings which are similar to the present findings. Vernon (1965) administered a broad range of intellectual tests to samples of English and West Indian 11-year-old boys. The test series included two Piagetian task arrays (arithmetic-orientational and visualization-conservation). While both of the Piagetian task clusters loaded significantly (.70) on the first-order or "g" factor, there were also significant loadings on a perceptual factor for both subject subsamples and on a "practical" ability factor for the English subject subsample. A similar case of multiple factor loadings was found by O'Bryan and MacArthur (1967, 1969) for a series of 15 concrete operational task performances of 85 males (mean age, 8 1/2 years). Following oblique rotation, two separate factors were found which were said to correspond to the two basic forms of reversibility--inversion and reciprocity--postulated to underly children's understandings of class and relational concept tasks, respectively. Moreover, these oblique primary factors were not correlated with each other. A third noncorrelated factor was identified as "logical inclusion" and revealed a significant loading only for a class inclusion task such as that employed in the present assessments.

A recent longitudinal assessment of normal and retarded children's logical reasoning and moral judgment-conduct (Stephens, 1972; Stephens, Glass, McLaughlin, & Miller, 1969) has also indicated that a number of separate factors are necessary to account for the performance variability across a series of Piagetian measures. The total sample of 150 children and adolescents (age range, 6 years 10 months to 18 years on initial testing and 8 years 10 months to 20 years on second testing) received a series of Piagetian concrete and formal reasoning tasks, the age-appropriate Wechsler Intelligence Scales, and the Wide Range Achievement Tests (a total of 47 separate variables). Five factors were identified for the initial test scores, and seven interpretable factors resulted from the retesting scores. In both instances the Piagetian tasks loaded on factors distinct from those defining the WISC, WAIS, and achievement tests. In the initial assessment analysis, 23 of the Piagetian tasks showed significant positive loadings (i.e., exceeding .25) on a single "operativity" factor. In contrast, as in the O'Bryan and MacArthur (1969) study, a separate factor was defined by loadings from a hierarchical class inclusion task. The data from the second testing session indicated separate factors for conservation, spatial orientation, spatial imagery, formal reasoning, and class inclusion-combinatorial understanding. The second principal component found in the varimax rotation for all 11 tasks in the present study (see Table 20) also revealed high positive loadings for class inclusion (.72) and combinatorial reasoning (.94), thus offering further evidence for a separate class inclusion-combinatorial structure factor. This factor accounted for 27.8 percent of the variance compared to 34.8 percent for the first principal component identified by the uniformly consistent matrix task loadings.

Berzonsky (1971) administered tests of causal reasoning, concrete operations (seriation, conservation, and class inclusion), and formal reasoning (combinations and pendulum) to 42 male and 42 female first-grade children. Although the majority of the concrete period tasks were found to load on a single factor following varimax rotation (in this study the seriation task proved to be the exception), a total of five separate factors were identified. These results were interpreted as not supporting Piaget's claim that non-naturalistic (precausal) reasoning is related to a preoperational logical status. Berzonsky (1971) concluded, "While both abilities were clearly identified they bear little relationship to each other. The results are also at variance with the unitary nature of logical thinking postulated by Inhelder and Piaget (1958). Instead it is suggested that at least three relatively independent abilities are involved [p. 475]."

The one exception to this consistently nonunitary picture of Piagetian concept performances (including the present results) would appear to be a recent methodological analysis of the egocentrism construct by Rubin (1973). In this investigation, measures of cognitive, spatial, role-taking, and communicative egocentrism, and conservation (the Goldschmid-Bentler Concept Assessment Kit, Form A) were given to 10 boys and 10 girls from kindergarten and second, fourth, and sixth grades (total $N = 80$). Following varimax rotation, all of the tasks were found to load significantly on an initial "decentration" factor which accounted for 56.9% of the total variance.

Insofar as these dimensionality studies are concerned, it would appear that factors unique to Piagetian concept task requirements can be readily observed. This is especially true for heterogeneous task arrays which include non-Piagetian standardized tests (e.g., Stephens, 1972; Vernon, 1965) or wide age-range subject samples (e.g., Rubin, 1973; Stephens, 1972). Yet the probability of obtaining nonrelated Piagetian factors for restricted age samples (e.g., Berzonsky, 1971; O'Bryan & MacArthur, 1969) and wider age intervals (e.g., Stephens, 1972; the present results) appears equally likely.

Finally, certain conclusions regarding the children's matrix task performances are in order. The performance patterns of the present subject sample on the various matrix tasks are in some respects concordant with the earlier results of MacKay et al. (1970) but notably disparate in a number of important aspects. As was shown in the earlier study, children's overall understanding of the matrix task requirements improved markedly from 5 to 8 years of age. Also, the reproduction subtasks proved to be of significantly lesser difficulty than the transposition subtasks, and this was particularly evident for the two cases which dealt with a continuous dimension, i.e., the class/series and double seriation matrices (see Table 23). Moreover, the great majority of children who successfully reproduced the various matrices responded by presenting an identical reproduction (see Table 26); this is in essential agreement with previous findings of MacKay et al.

In decided contrast to these essential replications, the contentions that

> "The ability to construct a matrix composed of discrete categories (cross classification) is developmentally an earlier acquisition than the ability to construct one composed of relational variables (double seriation) . . . [and] . . . A matrix composed of discrete categories in both directions (cross) classification) is of equivalent difficulty to one constructed of discrete categories in one direction and a relational variable in the other (class/series case) [MacKay et al., 1970, p. 795]

are not borne out in the present result patterns. The overall relative matrix difficulties were cross classification > double seriation < class/series for the reproduction task requirement and cross classification = double seriation > class/series for the transposition task requirement (see Tables 24 and 25). In addition to these contrasts with the results of MacKay et al. (1970), the greater difficulty of cross classification compared to double seriation (reproduction case) and the equivalent difficulty demonstrated for the transposition cases fail to agree with Lagattuta's (1970) findings.

In the present investigation, explicit attention was directed toward an accurate replication of the MacKay et al (1970) investigation. This included consideration of the stimulus materials, instructional sets, and scoring procedures utilized in addition to the selection of closely comparable subject samples. The single major exception to this concerns the task materials used in the double seriation matrix tasks. In the earlier Bruner and Kenney, (1966) and MacKay et al. (1970) studies, clear plastic beakers and open-ended gray plastic cylinders, respectively, were used. The present study, in contrast, used solid wooden cylindrical blocks of comparable height and diameter dimensions. From an assessment task viewpoint, the two investigations are decidedly similar.

In terms of general research paradigms, the present study embodies an inherently superior within-subject assessment design, i.e., all the children received all the tasks, thus permitting direct comparisons of relative task difficulties. This superiority is particularly noteworthy in view of the fact that presentation order effects were notably absent. The present study included 280 subjects while the earlier investigations of MacKay et al. (1970) assessed 90 and 48 children, respectively. In addition to these considerations, the present investigation, in apparent contrast to the MacKay et al. (1970) study, distinguished between appropriate directional and nondirectional inference tests (i.e., the one-tailed probability values associated for the age-grade levels and the reproduction versus transposition subtask comparisons, and the two-tailed probability values for the various matrix type comparisons such as cross class versus double series).

The fact that the present results substantiate the pattern that reproduction tasks are significantly less difficult than transposition tasks as found by Bruner and Kenney (1966) and MacKay et al. (1970) presents some assurance that we are indeed dealing with similar behavioral phenomena. In this regard, while the present results concerning the comparative difficulty of the cross class and the double series matrix cases disagree rather sharply with MacKay et al. (1970), the data are in general accord with the original research of Bruner and Piaget. Bruner and Kenney (1966, p. 158) found that 60% of the 5-year-olds, 70% of the 6-year-olds, and 80% of the 7-year-olds could successfully reproduce the double series matrix. The comparison percentages for the present appropriate age-grade groups are 67.5%, 70%, and 95%, respectively. For the double seriation transposition case, the comparison percentage values are in less clear agreement, i.e., 0.0% for Bruner & Kenney vs. (30%), 28% (32%) (32%), and 80% (60.0%) for the 5-, 6-, and 7-year-old subjects. In similar fashion, although Inhelder and Piaget did not utilize any direct counterpart to the present reproduction and transposition subtasks, their contention that "children reach an operational level in the multiplication of series about the same period (7-8 years) as cross classification [p. 278]" agrees with the present case of equivalent difficulty for cross class and double series transposition.

These general performance similarities notwithstanding, there appear to be certain critical methodological deficiencies in the present matrix task formats. These deficiencies concern the supposedly equivalent task requirements for the cross class and double series matrix cases. One would presume that all task solution requirements and instructional sets would be comparable for the reproduction and transposition problems. Any performance differences between the cross class and double series matrices would thus be attributable to stimulus array differences stemming from the logical categories at issue, i.e., discrete class exemplars representing class intersections (e.g., red circles) versus ordered items representing continuous underlying dimensions. This may be an operationally valid judgment for the reproduction subtask, but it is clearly not true for the transposition cases.

In actuality, transposition, as Bruner & Kenney (1966) define the term, applies only to concept tasks such as multiple seriation in which the relevant relationships among the stimuli are asymmetrical. The concept of asymmetrical relations is a defining attribute of "transposition." In contrast, the defining relationship in a cross classification problem (matrix format or otherwise) is a symmetrical one, i.e., the relation of an exemplar to its class. Thus, the cross classification matrix is logically incapable of satisfying the minimum requirement for a transposition instructional set (C. J. Brainerd, personal communication, November 24, 1972).

As these logical considerations imply, the present cross class matrix transposition task may be "solved" in a number of ways so long as the color/form placements are consistent across the three columns/rows. The essential arbitrariness of the placements of the class dimensions permits a greater number of correct patterns than is true for the double series transposition counterpart. However, the present matrix formats simply do not permit any rigorous determination of the relative difficulties of multiple class versus multiple series concepts. What is required, of course, is a task format which would operationally equate the cross class and double series transposition problems. Consider the following argument:

We wish to determine if a child is capable of classifying a set of objects simultaneously on two dimensions. After carefully constructing three squares, three circles, and three triangles, and painting one of each shape red, yellow, and blue, we confront a child. What do we ask him to do? We may ask him to arrange them in a nice pattern on a three-by-three grid and observe whether he spontaneously builds one of the 72 arrangements consistent by rows and columns of the 5040 x 72 configurations available to him. While spontaneous sorting may give us information concerning his aesthetics, it is difficult to draw conclusions about his ability to cross classify from this task. We must make it clearer what is expected of him.

On the other hand, if we exhibit the objects already cross classified in one manner, remove them from the board, and ask him to put them back the way they were, we confound the task with visual and perhaps verbal memory. Displacing one object from one corner of the board to another and asking the subject to rebuild the transposed matrix adds a component of spatial ability to the memory task.

Spatial ability, the ability to rotate a visual image in short-term memory, is, however, something other than the ability to cross classify. The transposition task may conceivably be solved with reference to a visual image. Two of the eight acceptable solutions to the transposition task represent rigid transformations of the original pattern and do not provide unequivocal evidence that the subject possesses the ability to cross classify.

An appropriate task, in which success can be attributed to the ability to consistently sort on two dimensions, is involution of the matrix. After presenting the objects arranged in a cross classification, we remove them from the board and place one that had been in a corner in the center position. The child is asked to put the other objects on the board to make the same kind of pattern as before. None of the eight possible solutions are now reflections or rotations of the original visual pattern.

While cross classification can be tested with matrix stimuli in this manner, double seriation cannot be. A double seriation is a cross classification of ordered attributes in which the center cell is fixed and any of the eight possible arrangements is a rigid transformation of the initial array. Since any double seriation is a special case of a cross classification, the question of relative difficulty admits of only two possible answers if stimulus salience is held constant: (1) either double seriation is no different from any other cross classification or (2) the additional perceptual cue of seriated values of attributes will <u>enhance</u> performance.

Employing the Bruner and Kenney (1966) stimuli, cylinders varying on height and diameter, we may present in Task A a cross classification which is not a double seriation, e.g., placing the short, skinny cylinder in the middle cell. After removing the pieces from the board, the tall, fat cylinder is placed in the middle cell and the subject is asked to put the other objects on the board in the same kind of pattern as before. Although the cylinders are seriable, they have not been seriated, and this cross classification task may differ from the color-form task only in stimulus saliency. Task B is identical to Task A except for the placement of the cylinder intermediate in height and thickness in the middle cell, thus forcing any correct cross classification to be a double seriation. By counterbalancing Tasks A and B, the relative difficulty of the two tasks can be determined. We are here comparing the difficulty of producing a double classification with that of producing a double seriation, with stimuli and number of possible solutions held constant. It is clear that Task B cannot be more difficult than Task A. The only empirical question is whether B is significantly easier than A. The authors are presently conducting an investigation utilizing matrix task formats as described here in addition to the original cross class and double series cases devised by MacKay et al. (1970).

In conclusion, the following primary results have been shown in the present investigation:

1. As expected from previous research on children's classificatory concept acquisitions, there was a significant improvement across the age range (5 to 12 years) for all the tasks employed. Few significant sex differences were found.

2. Consideration of the pass/fail performance patterns and associated scaling analyses revealed a generally reliable order of task difficulty, i.e., free sorting < double series reproduction < cross class reproduction < cross class transposition < "Some-All" understanding < double series transposition <producing three dichotomies < class inclusion understanding < combinatorial reasoning. These cross-sectional findings <u>suggest</u> the possibility of a similar acquisition sequence for an individual child over the present age range.

3. The task dimensionality analyses indicated that several performance factors are present in the overall Piagetian task series. This suggests a relatively nonunitary picture for cognitive growth as assessed by classificatory tasks related to concrete operations period functioning.

4. The subsidiary matrix task analyses indicated major discrepancies with the previous research. As anticipated, the matrix reproduction task requirement was consistently less difficult than the matrix transposition task. The comparisons across the three matrix types indicated the following orders of difficulty for the younger subjects: (a) reproduction task--cross classification

> double seriation > classification seriation, and (b) transposition task--cross classification = double seriation > classification seriation. However, major conceptual and methodological deficiencies in the present matrix transposition task formats preclude any definitive generalizations at this point.

5. In essential agreement with the contentions of Inhelder and Piaget (1964) and Kofsky (1966), the child's understanding of the logic inherent in class inclusion relationships (combinatorial structure) evolves gradually and is contingent upon the earlier mastery of certain classificatory skills of lesser complexity.

References

Ahr, P. R., & Youniss, J. Reasons for failure on the class inclusion problem. Child Development, 1970, 41, 131-143.

Allen, L. R. Scalogram analysis of classification behavior. Journal of Research in Science Teaching, 1970, V(7), 43-45.

Berzonsky, M. D. Interdependence of Inhelder and Piaget's model of logical thinking. Developmental Psychology, 1971, 4, 469-476.

Brainerd, C. J. Structures of thought in middle-childhood: Recent research on Piaget's concrete-operational groupements. Paper presented at the Third Annual Meeting on Structural Learning, Philadelphia, March 1972.

Brainerd, C. J., & Kaszor, P. An analysis of two proposed sources of children's class inclusion errors. Developmental Psychology, 1974, 10, 633-643.

Bruner, J. S., & Kenney, H. J. On multiple ordering. In J. S. Bruner, R. R. Olver, P. M. Greenfield et al., Studies in cognitive growth. New York: Wiley, 1966, pp. 154-167.

Chittenden, E. A. The development of certain logical abilities and the child's concepts of substance and weight: An examination of Piaget's theory. Unpublished doctoral dissertation, Columbia University, 1964.

Dudek, S. Z., & Dyer, G. B. A longitudinal study of Piaget's developmental stages and the concept of regression. Journal of Personality Assessment, 1972, 36, 380-389.

Elkind, D. The development of the additive composition of classes in the child: Piaget replication study III. Journal of Genetic Psychology, 1961, 99, 51-57.

Findlay, A. D. An investigation of the emergence of ability for partial and complete multiplication of classes in young school children. British Journal of Educational Psychology, 1971, 41, 96-98.

Flavell, J. H. The Developmental Psychology of Jean Piaget. Princeton, N. J.: Van Nostrand Reinhold, 1963.

Flavell, J. H. Concept development. In P. H. Mussen (Ed.), Carmichael's manual of child psychology. Vol. 1. New York: Wiley, 1970, pp. 983-1059.

Froemel, E. C. A Fortran IV subroutine for computing the tetrachoric correlation coefficient. Research Memorandum, No. 15. Statistical Laboratory, Department of Education, University of Chicago, June 1970.

Goodnow, J. A test of milieu differences with some of Piaget's tasks. Psychological Monographs, 1962, 76, (36, Whole No. 555).

Guttman, L. The basis for scalogram analysis. In S. A. Stouffer et al., Measurement and prediction. Princeton, N. J.: Princeton University Press, 1950, pp. 60-90.

Hamel, B. R., & Van Der Veer, M. A. A. Structure d'ensemble, multiple classification, multiple seriation and amount of irrelevant information. British Journal of Educational Psychology, 1972, 42, pp. 319-325.

Hood, H. B. An experimental study of Piaget's theory of the development of number in children. British Journal of Psychology, 1962, 53, 273-286.

Hooper, F. H. Cognitive assessment across the life-span: Methodological implications of the organismic approach. In J. R. Nesselroade and H. W. Reese (Eds.), Life-span developmental psychology: Methodological issues. New York: Academic Press, 1973, pp. 299-316.

Hooper, F. H., & Klausmeier, H. J. Description and rationale for a longitudinal assessment of children's cognitive development and concept learning. Working Paper No. 113. Madison: Wisconsin

Research and Development Center for Cognitive Learning, 1973.

Inhelder, B., & Piaget, J. The growth of logical thinking from childhood to adolescence. New York: Basic Books, 1958.

Inhelder, B., & Piaget, J. The early growth of logic in the child: Classification and seriation. New York: Harper & Row, 1964.

Kamii, C. K., & Peper, R. A Piagetian method of evaluating preschool children's development in classification. Unpublished manuscript, Ypsilanti, Michigan, Public Schools, July 1969.

Klahr, D., & Wallace, J. G. Class inclusion processes. In S. Farnham: Diggory (Ed.), Information processing in children. New York: Academic Press, 1972, pp. 144-172.

Klausmeier, H.J., & Hooper, F.H. Conceptual development and instruction. In F. Kerlinger & J. Carroll (Eds.), Review of educational research. Vol. II. Itasia, Illinois: Peacock, 1974. Pp. 3-54.

Kofsky, E. Developmental scalogram analysis of classificatory behavior. Unpublished doctoral dissertation, University of Rochester, 1963.

Kofsky, E. A scalogram study of classificatory development. Child Development, 1966, 37, 191-204.

Lagattuta, J. A. A developmental analysis of serial and classificatory ordering along one and multiple dimensions. Unpublished master's thesis, The Catholic University of America, 1970.

Laurendeau, M., & Pinard, A. Causal thinking in children. New York: International University Press, 1962.

Loevinger, J. The technique of homogeneous tests compared with some aspects of "scale analysis" and factor analysis. Psychological Bulletin, 1945, 45, 507-529.

Lovell, K., Mitchell, B., & Everett, I. R. An experimental study of the growth of some logical structures. British Journal of Psychology, 1962, 53, 175-188.

MacKay, C. K. Ikonic imagery in the severely subnormal. British Journal of Psychology, 1972, 63, 597-603.

MacKay, C. K., Fraser, J., & Ross, I. Matrices, three by three: Classification and seriation. Child Development, 1970, 41, 787-798.

Nassefat, M. Etude quanititative sur l'evolution des operations intellectuelles. Neuchatel: Delachaux et Niestle, 1963.

O'Bryan, K. G., & MacArthur, R. S. A factor analytic study of Piagetian reversibility. Alberta Journal of Educational Research,

O'Bryan, K. G., & MacArthur, R. S. Reversibility, intelligence, and creativity in nine-year old boys. Child Development, 1969, 40, 33-45.

Piaget, J. The child's conception of number. New York: Norton, 1965.

Piaget, J. Piaget's theory. In P. H. Mussen (Ed.), Carmichael's manual of child psychology. New York: Wiley, 1970, pp. 703-732. (a)

Piaget, J. Structuralism. New York: Basic Books, 1970. (b)

Rubin, K. H. Egocentrism in childhood: A unitary construct? Child Development, 1973, 44, 102-110.

Sagi, P. C. A statistical test for the significance of a coefficient of reproducibility. Psychometrika, 1959, 24, 19-27.

Shantz, C. V. A developmental study of Piaget's theory of logical multiplication. The Merrill-Palmer Quarterly, 1967, 13, 121-137.

Shantz, C. V., & Sigel, I. E. Logical operations and concepts of conservation in children: A training study. Final Report, Office of Education Grant No. OEG-3-6 068463-1645. Detroit, Michigan: Merrill-Palmer Institute, 1967.

Siegel, A. W., & Kresh, E. Children's ability to operate within a matrix: A developmental study. Developmental Psychology, 1971, 4, 232-239.

Smedslund, J. Concrete reasoning: A study in intellectual development. Monograph for the Society of Research in Child Development, 1964, 29, 3-39.

Stephens, B. The development of reasoning, moral judgment, and moral conduct in retardates and normals: Phase II Interim Project Report. H.E.W. Research Grant No. 15-P-5512/3-02, Philadelphia, Pennsylvania: Temple University, 1972.

Stephens, B., Glass, G. V., McLaughlin, J. A., & Miller, C. K. The factorial structure of reasoning, moral judgment, and moral conduct. Paper presented at the meeting of the American Educational Research Association, Los Angeles, February 1969.

Vernon, P. E. Environmental handicaps and intellectual development. British Journal of Educational Psychology, 1965, 35, 9-20.

Werner, R., & Morrison, D. G. A Fortran program for Guttman and other scalogram analyses (GUTTSCAL NUC0015). Evanston, Illinois: Vogelbach Computing Center, Northwestern University, 1967.

Wohlwill, J. F. A study of the development of the number concept by scalogram analysis. Journal of Genetic Psychology, 1960, 97, 345-377.

Wohlwill, J. F. Piaget's system as a source of empirical research. Merrill-Palmer Quarterly, 1963, 9, 253-262.

Wohlwill, J. F. Response to class-inclusion questions for verbally and pictorially presented items. Child Development, 1968, 39, 449-465.

Wohlwill, J. F., Devoe, V., & Fusaro, L. Research on the development of concepts in early childhood. Final Report for National Science Foundation G-5855. University Park: Pennsylvania State University, 1971.

Appendix A
Instructions For Classification Tasks

GPO 807-868-7

GROUP 1

Materials:

3 small blue squares	3 small red squares
2 large blue squares	3 large red squares
3 small blue circles	3 small red circles
3 large blue circles	2 large red circles

Warm-Up

The examiner places all objects randomly on the table and says, "Here are some blocks for us to work with. Tell me what you see."

If the child does not spontaneously say, "red," "blue," "circle," "square," "big," "little," the examiner picks up two blocks at a time for contrast and asks the child, "Are these (just) the same? How are they different?" or "What's different about them?" The examiner continues this process until the child has been presented with contrasts representing each of the attributes (color, size, shape) that he did not say spontaneously. When the child correctly identifies an attribute, the examiner indicates that he is correct and repeats the name of the attribute. To conclude the warm-up, the examiner summarizes, "Good, you told me that there are squares and circles, that some are red and some are blue, and that some are big and some are small."

Criteria

Any child who cannot distinguish, as indicated by some form of labeling, between the attributes presented will be eliminated from the sample.

FREE SORT

The examiner places all objects randomly on the table and says, "Put together all the blocks that go with each other." (The examiner should not use the words alike or different.) When the child has finished, the examiner asks, "Why did you put the blocks together like this?"

The examiner may repeat the directions three to four times, but the child should not be pushed. If the child responds, the examiner should be certain to ask if the child is finished before asking for an explanation. If the child does not respond, the examiner should go on to the first dichotomy.

DICHOTOMOUS SORTING

Materials: Same as in the free sort tasks, plus 2 flat open boxes.

First Dichotomy

The examiner mixes all the objects randomly, puts two boxes in front of the child and says, "Here are two boxes. Now I want you to divide these blocks into two bunches. Put one kind in this box, and one kind in this box." When the child has finished, the examiner, pointing to one box and then the other, says, "Why did you put these in here and these in here?"

The examiner may repeat the directions three to four times, but the child should not be pushed. If the child makes any kind of dichotomy, go on to the second dichotomy. If the child does not respond at all, the examiner should go on to the next task.

Second Dichotomy

The objects are again mixed and the examiner says, "Last time you separated the two ______________ (colors, sizes, shapes). This time I want you to make two bunches in another way. Put one kind in here and one kind in here. Remember, do it in a different way." (If the child starts to repeat his first dichotomy, the examiner allows him to finish what he started.) When the child has finished, the examiner, pointing to one box and then the other says, "Why did you put these in here and these in here?"

The examiner may repeat the directions. If the child makes a second dichotomy of any kind (including a repeat of the first dichotomy), the examiner should go on to the third dichotomy. If the child does no[illegible] [illegible]pond, the examiner should go on to the next task.

Third Dichotomy

The objects are again mixed and the examiner says, "First you separated the two ______________ (colors, sizes, shapes), and then you separated the two ______________ (colors, sizes, shapes). This time I want you to make two bunches in even <u>another</u> way. Put one kind in here and one kind in here. Remember, do it in even a <u>different</u> way." (If the child repeats his first or second dichotomy, the examiner allows him to finish what he started.) When the child has finished, the examiner, pointing to one box and then the other says, "Why did you put these in here and these in here?"

At this point: (a) if the child has made two of the three possible dichotomies, the examiner should go on to a fourth dichotomy, repeating directions for the third dichotomy; (b) if the child has made only one of the dichotomies or has made all three dichotomies, the examiner should go on to the next task.

GROUP II

SOME-ALL

Materials: 4 blue squares

2 blue triangles

3 red triangles

Procedure:

The examiner places the blocks on the table in random order, the blocks all oriented in the same direction, and says, "Tell me what you see here." If the child does not spontaneously say, "triangle," "square," the examiner asks what shapes he sees. If the child does not spontaneously say "red," "blue," the examiner asks what colors he sees. The examiner says,

1. "Look at all the red blocks." When the child has finished looking the examiner continues. "Are all the red blocks triangles?"
2. "Look at all the triangle blocks . . . . Are all the triangle blocks red?"
3. "Look at all the square blocks . . . . Are all the square blocks blue?"
4. "Look at all the blue blocks . . . . Are all the blue blocks squares?"

The four questions are presented in random order. For the last two questions presented the examiner asks, "How do you know?"

CLASS INCLUSION

Materials: 3 red triangles — 3 yellow circles

2 blue triangles — 2 blue circles

4 blue squares

Procedure:

For each task the examiner places the blocks on the table in random order. The child may count the blocks but may not manipulate or group them.

1. Materials: 3 red triangles, 2 blue triangles.
   The examiner places the triangles, all oriented in the same direction, on the table and says, "Tell me what you see here. If the child does not spontaneously count the blocks, the examiner asks, "How many triangle blocks are there? How many blue blocks? How many red blocks?" The examiner then asks, "Are there more triangle blocks or more red blocks? . . . more triangle blocks or more red blocks?
2. Materials: 3 yellow circles, 2 blue circles.
   The examiner places the circles on the table and asks the child to count them (as in 1). The examiner then asks, "Are there more blue blocks or more circle blocks? . . . more blue blocks or more circle blocks?"
3. Materials: 3 red triangles, 2 blue triangles, 4 blue squares.
   The examiner places the objects, all oriented in the same direction, on the table and asks the child to count them

(as in 1). The examiner then asks, (a) "Are there more triangle blocks or more red blocks? . . . more triangle blocks or more red blocks? How do you know?" (b) "Are there more blue blocks or more square blocks? . . . more blue blocks or more square blocks? How do you know?" (c) "Are there more blue blocks or more triangle blocks? . . . more blue blocks or more triangle blocks? How do you know?"

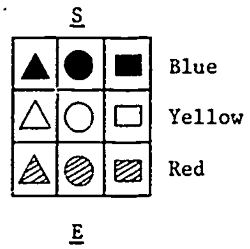
If in any case the subject does not respond, or responds in terms of the wrong attributes, the examiner may repeat the questioning once.

On the last three trials the examiner asks, "How do you know?"

GROUP III

CROSS CLASSIFICATION MATRIX

The examiner places the blocks on the board in the following positions:



From where the child is sitting, red is on top, yellow in the middle, and blue on the bottom. Squares are at his/her left, circles are in the center, and triangles are at the right.

Warm-Up

The examiner places the nine blocks on the table and says, "Tell me what you see here." If the child does not spontaneously give the attributes, "triangle," "circle," "square," the examiner asks what shapes he sees. If the child does not spontaneously give the attributes "red," "yellow," "blue," the examiner asks what colors he sees.

The examiner then says, "Now I am going to put the blocks on here in a special way." He places the blocks on the board saying, "Look very carefully at how the blocks go."

Replacement

The examiner says, "Now I am going to take away <u>one</u> block and I want you to put it back so that the board looks just the same as it does right now." These directions may be repeated in order to insure that the child realizes what is expected of him. If the child tends to watch the examiner while listening to the directions, he is asked to watch the board carefully before the examiner removes the block. The examiner then removes one block (randomly selected), puts it on the table near the bottom left of the board, and says, "Now you put this block back where it belongs." When the child has finished the examiner says, "Now I am going to take away <u>two</u> blocks and I want you to put them back so the board looks just the same as it does right now." The examiner then removes two adjacent blocks (randomly selected) and says, "Now you put these back where they belong." When the child has finished the examiner says, "Now I am going to take away <u>three</u> blocks and I want you to put them back so the board looks just the same as it does right now." The examiner then removes the blocks on one of the diagonals and says, "Now you put these back where they belong."

Reproduction

The examiner says, "This time I am going to take all of the blocks off the board and I want you to put them back so the board looks <u>just the same as it does right now</u>. Look at the board very carefully so you can remember just what it looks like." When the child has finished looking the examiner removes the blocks, mixes them, and says, "Now I want you to put the blocks back so that the board looks <u>just the same as it did before</u>."

Transposition

The examiner says, "This time I am going to take all of the blocks off of the board again. Then I am going to put one back, but it will be in a different place. I want you to put the blocks back so they make a pattern like this one. Now, look at the board again very carefully so you can remember just what it looks like." When the child has finished looking the examiner removes all of the blocks, mixes them, and places the bottom left-hand block (blue square) at the top left-hand corner. The examiner then says, "Now you put all these blocks back on the board so they make a pattern like they did before. Remember, this one must stay right here." If the child lifts the blue square, he is reminded that it is to stay in its place.

General

The child is always reassured about his performance whether he replaces all the objects correctly or not. If at any time the child replaces the blocks incorrectly, they are placed correctly by the examiner before proceeding to the next task. However, when this happens the examiner reassures the child that he is doing well and explains that he (the examiner) is arranging the blocks for the next game. After the warm-up, the examiner never mentions the color or shape of any of the blocks.

CLASSIFICATION SERIATION MATRIX

The examiner places the blocks on the board in the following positions:

S

B-1	Y-1	R-1
B-2	Y-2	R-2
B-3	Y-3	R-3

E

From where the child is sitting, the tall cylinders are on the top, the medium cylinders are in the middle, and the short cylinders are on the bottom. The red cylinders are at his/her left, yellows are in the center, and blues are at the right.

Warm-Up

The examiner places the nine blocks on the table and says, "Tell me what you see here." If the child does not spontaneously give the attributes, "red," "yellow," "blue," the examiner asks what colors he sees. If the child does not spontaneously mention the different heights, the examiner points to blocks differing in height (same color) and asks how they are different.

The examiner then says, "Now I am going to put the blocks on here in a special way." He places the blocks on the board saying, "Look very carefully at how the blocks go."

Replacement

The examiner says, "Now I am going to take away one block and I want you to put it back so that the board looks just the same as it does right now." These directions may be repeated in order to insure that the child realizes what is expected of him. If the child tends to watch the examiner while listening to the directions, he is asked to watch the board carefully before the examiner removes the block. The examiner then removes one block (randomly selected), puts it on the table near the bottom left of the board, and says, "Now you put this block back where it belongs." When the child has finished the examiner says, "Now I am going to take away two blocks and I want you to put them back so the board looks just the same as it does right now." The examiner then removes two adjacent blocks (randomly selected) and says, "Now you put these back where they belong." When the child has finished the examiner says, "Now I am going to take away three blocks and I want you to put them back so the board looks just the same as it does right now." The examiner then removes the blocks on one of the diagonals and says, "Now you put these back where they belong."

Reproduction

The examiner says, "This time I am going to take all of the blocks off the board and I want you to put them back so the board looks just the same as it does right now. Look at the board very carefully so you can remember just what it looks like." When the child has finished looking the examiner removes the blocks, mixes them, and says, "Now I want you to put the blocks back so that the board looks just the same as it did before."

Transposition

The examiner says, "This time I am going to take all of the blocks off of the board again. Then I am going to put one back, but it will be in a different place. I want you to put the blocks back so they make a pattern like this one. Now, look at the board again very carefully so you can remember just what it looks like." When the child has finished looking the examiner removes all of the blocks, mixes them, and places the bottom left-hand block (R-1) at the top left-hand corner. The examiner then says, "Now you put all these blocks back on the board so they make a pattern like they did before. Remember, this one must stay right here." If the child lifts the cylinder, he is reminded that it is to stay in its place.

General

The child is always reassured about his performance whether he replaces all the objects correctly or not. If at any time the child replaces the blocks incorrectly, they are placed correctly by the examiner before proceeding to the next task. However, when this happens the examiner reassures the child that he is doing well and explains that he (the examiner) is arranging the blocks for the next game. After the warm-up, the examiner never mentions the color or height of any of the blocks.

DOUBLE SERIATION MATRIX

The examiner places the blocks on the board in the following positions:

S

S-1	M-1	F-1
S-2	M-2	F-2
S-3	M-3	F-3

E

From where the child is sitting, the tall cylinders are on the top, the medium height cylinders are in the middle, and the short cylinders are on the bottom. Wide cylinders are at his/her left, the medium width cylinders are in the center, and the narrow cylinders are at the right.

Warm-Up

The examiner places the nine blocks on the table and says, "Tell me what you see here." If the child does not spontaneously mention the different heights, the examiner points to blocks differing in height (but not width) and asks how they are different. If the child does not spontaneously mention the different widths, the examiner points to blocks differing in width (but not height), and asks how they are different. (If the child hesitates, the examiner may provide him with the words "skinny" and "fat.")

The examiner then says, "Now I am going to put the blocks on here in a special way." He places the blocks on the board saying, "Look very carefully at how the blocks go."

Replacement

The examiner says, "Now I am going to take away one block and I want you to put it back so that the board looks just the same as it does right now." These directions may be repeated in order to insure that the child realizes what is expected of him. If the child tends to watch the examiner while listening to the directions, he is asked to watch the board carefully before the examiner removes the block. The examiner then removes one block (randomly selected), puts it on the table near the bottom left of the board, and says, "Now you put this block back where it belongs." When the child has finished the examiner says, "Now I am going to take away two blocks and I want you to put them back so the board looks just the same as it does right now." The examiner then removes two adjacent block (randomly selected) and says, "Now you put these back where they belong." When the child has finished the examiner says, "Now I am going to take away three blocks and I want you to put them back so the board looks just the same as it does right now." The examiner then removes the blocks on one of the diagonals and says, "Now you put these back where they belong."

Reproduction

The examiner says, "This time I am going to take all of the blocks off the board and I want you to put them back so the board looks just the same as it does right now. Look at the board very carefully so you can remember just what it looks like." When the child has finished looking the examiner removes the blocks, mixes them, and says, "Now I want you to put the blocks back so that the board looks just the same as it did before."

Transposition

The examiner says, "This time I am going to take all of the blocks off of the board again. Then I am going to put one back, but it will be in a different place. I want you to put the blocks back so <u>they make a pattern like this one</u>. Now, look at the board again very carefully so you can remember just what it looks like." When the child has finished looking the examiner removes all of the blocks, mixes them, and places the bottom left-hand block (F-1) at the top left-hand corner. The examiner then says, "Now you put all these blocks back on the board so they <u>make a pattern like they did before</u>. Remember, this one must stay right here:" If the child lifts block F-1, he is reminded that it is to stay in its place.

General

The child is always reassured about his performance whether he replaces all the objects correctly or not. If at any time the child replaces the blocks incorrectly, they are placed correctly by the examiner before proceeding to the next task. However, when this happens the examiner reassures the child that he is doing well and explains that he (the examiner) is arranging the blocks for the next game. After the warm-up, the examiner <u>never</u> mentions the height or width of any of the blocks.

GPO 807-265-5

GROUP IV

COMBINATORIAL REASONING

Materials: Counting chips: 10 red, 10 green, 10 yellow, 10 blue, 10 white, 10 orange, 10 brown, 10 light blue.

Two Colors

The examiner places on the table the pile of 10 red chips and next to these the pile of 10 green chips and explains, "Now we are going to work with chips of different colors. Here are some red chips and here are some green chips. With these red and green chips I can make a pair like this (RG), one red and one green."

Three Colors

The examiner then places on the table a pile of 10 yellow chips and says, "Let's use the yellow chips too. Now, I want you to make a pair of colors that is different from this pair. Put your pair right here under this (RG) one." If the child hesitates, the examiner repeats the directions. If the child still has difficulty, the examiner may repeat the directions a third time and help the child to make a second pair (either RY or GY).

When this is done the examiner says, "Good, you have two different pairs of colors; now, put together another pair of colors that you don't have yet. Put it here (under the two paris already completed)." The examiner may repeat the directions if necessary. If the child has particular difficulty, the examiner puts down one of whichever color is needed and says, "Put another color here next to the-----one. Put another color that is not-----(the color already paired with the color presented)."

Rules

The examiner explains, "This was practice to help you understand the game with the chips. There are two things to remember about using the chips. First, remember that each pair you make is to have two different colors. Does this pair (RR) count?" If the child says, "No," the examiner asks, "Why?" If the child says, "Yes," the examiner repeats the instructions and presents a second example (YY or GG). Once the child understands the rule, the examiner continues, "So, the first rule is that each pair has two colors. Second, remember that each time you are to put down a new (different) pair of colors. In this game this (GR) counts the same as this (RG) since they both use the same colors. Each time you put down a new pair of colors." The examiner then puts down another pair of colors that is the reverse of a pair already out and asks, "Does this count?" If the child says, "No," the examiner asks, "Why?" If the child says, "Yes," the examiner explains that X and Y are the same two colors as Y and X so they do not count as a new pair. The examiner then presents a second pair that is the reverse of a pair already on the table and discusses it as before. Once the child understands the rule, the examiner has him help put the chips back in their color piles.

Four Colors

When the chips are again in their color piles the examiner takes out the pile of 10 blue chips and places the four piles in a horizontal row about 15 inches in front cf the child and says, "Now, show me all the pairs you can make using these four colors. Remember, each pair must have two colors and each time you are to put down a new pair of colors." If the child seems to have difficulty starting the examiner may repeat the directions twice but may not manipulate the chips. If the subject pauses for a long time or says, "That's all," before completing all six possible pairs, the examiner says, "Look and see if you can find any more pairs." (This prompt may be given only once.) When the child has finished the examiner asks, "How did you decide to make this pair, then this pair . . . ?" The examiner then has the child help him put the chips back in their color piles.

Six Colors

The examiner then adds a pile of 10 white chips and a pile of 10 orange chips to the horizontal row of chips (they are placed at the end of the row to the child's right) and says, "Here are some white chips [illegible] e orange chips. Now there are six colors. I want you to show me again how many pairs you can make. Remember, each pair is to have two colors and each time you are to put down a new pair of colors." If the child appears to have difficulty starting, the examiner may repeat the directions twice but may not manipulate the chips. If the subject pauses for a long time or says, "That's all," before completing all 15 possible pairs, the examiner says, "Look and see if you can find any more pairs." (This prompt may be given only once.) When the child has finished the examiner asks, "How did you decide to make this pair, then this pair . . . ?"

Eight Colors

Leaving the chips as the child has arranged them on the table, the examiner then presents 10 brown chips and 10 light blue chips saying, "Now, here are some brown chips and some light blue chips. This time, I want you to show me how many more pairs you can make. Show me how many more pairs you can make with (now that you have) these new colors."

Appendix B
Scoring Sheets For Classification Tasks

Name ______________________

School ______________________

Grade ______________________

Birthday ______________________

Age ______________________

Sex ______________________

Examiner ______________________

Date ______________________

Order of Administration

_______ GROUP I

_______ Free Sort

_______ Dichotomies

_______ GROUP II

_______ Some-All

_______ Class Inclusion

_______ GROUP III

_______ Cross Classification Matrix

_______ Classification Seriation Matrix

_______ Double Seriation Matrix

_______ GROUP IV

_______ Combinatorial Reasoning

FREE SORT Time ____________

"Put together all the blocks that go with each other."

Approach:

1. Criteria
   a. keeps same
   b. shifts
   c. can't tell
2. Works quickly and efficiently
   a. yes
   b. no
   c. yes, then no
   d. no, then yes

Product:

1. No response
2. Graphic Sort
   a. no. of blocks used __________
   b. arrangement (pattern) __________
3. Non-Exhaustive Sort
   a. no. of blocks not sorted __________
   b. all blocks sorted, but __________ mistakes made
4. Exhaustive Sort

If 3 or 4, is the sort according to:

a. size
b. shape
c. color
d. size and shape
e. size and color
f. shape and color
g. size, shape, and color
h. other ______________________________

____

Justification:

"Why did you put the blocks together like this?"

1. No justification
2. Irrelevant justification ______________________________

   ____
3. Relevant justification
   a. size shape color
   b. other ______________________________

   ____

Time ____________

FIRST DICHOTOMY

Time ___________

"Divide all these blocks into two bunches. Put one kind in here and one kind in here."

Approach:

1. Criteria
   a. keeps same
   b. shifts
   c. can't tell
2. Works quickly and efficiently
   a. yes
   b. no
   c. yes, then no
   d. no, then yes

Product:

1. No dichotomy
   a. Graphic Sort
      (1) no. of blocks used __________
      (2) arrangement (describe) ____________________
   b. Non-Exhaustive Sort
      (1) no. of blocks not sorted __________
      (2) all blocks sorted, but __________ mistakes made
   c. Makes small grouping, but fails to dichotomize the groups according to a set criterion
   d. Equivalent Sort
   e. Other ____________________
2. Dichotomous Sort

Criterion:

(for 1b, 1c, 2)
   a. size
   b. shape
   c. color

Justification:

"Why did you put these in here and these in here?"

1. No justification
2. Irrelevant justification ____________________
3. Relevant justification
   a. size shape color
   b. other ____________________

Time ___________

GPO 807-241-4

SECOND DICHOTOMY Time ______

"Last time you separated the two ________ (sizes, shapes, colors). This time make two bunches in another way. Put one kind in here and one kind in here. Remember, do it in a different way."

Approach:

1. Criteria
   a. keeps same
   b. shifts
   c. can't tell

2. Works quickly and efficiently
   a. yes
   b. no
   c. yes, then no
   d. no, then yes

Product:

1. No dichotomy
   a. Graphic Sort
      (1) no. of blocks used ______
      (2) arrangement (describe) ______
   b. Non-Exhaustive Sort
      (1) no. of blocks not sorted ______
      (2) all blocks sorted, but ______ mistakes made
   c. Makes small grouping, but fails to dichotomize the groups according to a set crit·ion
   d. Equivalent Sort
   e. Other ______

2. Dichotomous Sort

Criterion:

(for 1b, 1c, 2)

a. size
b. shape
c. color

Justification:

"Why did you put these in here and these in here?"

1. No justification

2. Irrelevant justification ______

3. Relevant justification
   a. size shape color
   b. other ______

Time ______

THIRD DICHOTOMY Time ________

"First you separated the two __________(sizes, shapes, colors), and then you separated the two _________(sizes, shapes, colors). This time I want you to make two bunches in even another way. Put one kind in here and one kind in here. Remember, do it in even a different way."

Approach:

1. Criteria
   a. keeps same
   b. shifts
   c. can't tell
2. Works quickly and efficiently
   a. yes
   b. no
   c. yes, then no
   d. no, then yes

Product:

1. No dichotomy
   a. Graphic Sort
      (1) no. of blocks used ______
      (2) arrangement (describe) ____________________________
   b. Non-Exhaustive Sort
      (1) no. of blocks not sorted _____
      (2) all blocks sorted, but ______mistakes made
   c. Makes small grouping, but fails to dichotomize the groups according to a set criterion
   d. Equivalent Sort
   e. Other ____
2. Dichotomous Sort

Criterion:

(for 1b, 1c, 2)
   a. size
   b. shape
   c. color

Justification:

"Why did you put these in here and these in here?"

1. No justification
2. Irrelevant justification ____________________________________
   ____
3. Relevant justification
   a. size shape color
   b. other ____
   ____

Time ________

FOURTH DICHOTOMY   Time ________

"First you separated the two ________ (sizes, shapes, colors), and then you separated the two ________ (sizes, shapes, colors). This time I want you to make two bunches in even another way. Put one kind in here and one kind in here. Remember, do it in even a different way."

Approach:

_ 1. Criteria
   a. keeps same
   b. shifts
   c. can't tell
2. Works quickly and efficiently
   a. yes
   b. no
   c. yes, then no
   d. no, then yes

Product:

1. No dichotomy
   a. Graphic Sort
      (1) no. of blocks used ________
      (2) arrangement (describe) ________________________________
   b. Non-Exhaustive Sort
      (1) no. of blocks not sorted ________
      (2) all blocks sorted, but ________ mistakes made
   c. Makes small grouping, but fails to dichotomize the groups according to a set criterion
   d. Equivalent Sort
   e. Other ____

   ____
2. Dichotomous Sort

Criterion:

(for 1b, 1c, 2)

   a. size
   b. shape
   c. color

Justification:

"Why did you put these in here and these in here?"

1. No justification
2. Irrelevant justification ________________________________

   ____
3. Relevant justification
   a. size   shape   color
   b. other ____

   ____

Time ________

Time ________

SOME-ALL

Materials: 4 blue squares
2 blue triangles
3 red triangles

Randomize order, ask for justification on last two ("How do you know?")

________ 1. "Look at all the red blocks . . . . Are all the red blocks triangles?"

Answer: Yes* No

Justification:

________ 2. "Look at all the triangle blocks . . . . Are all the triangle blocks red?"

Answer: Yes No*

Justification:

________ 3. "Look at all the square blocks . . . . Are all the square blocks blue?"

Answer: Yes* No

Justification:

________ 4. "Look at all the blue blocks . . . . Are all the blue blocks square?"

Answer: Yes No*

Justification:

Time ________

Time ________

CLASS INCLUSION

1. Materials: 3 red triangles, 2 blue triangles

   "Are there more triangle blocks or more red blocks?"

   Answer:
   a. more triangles*
   b. more reds
   c. other ________

2. Materials: 3 yellow circles, 2 blue circles

   "Are there more blue blocks or more circle blocks?"

   Answer:
   a. more circles*
   b. more blues
   c. other ________

3. Materials: 3 red triangles, 2 blue triangles, 4 blue squares

3a. "Are there more triangle blocks or more red blocks?"

   Answer:
   a. more triangles*
   b. more reds
   c. other ________

Justification ("How do you know?") ________

____

3b. "Are there more blue blocks or more square blocks?"

   Answer:
   a. more blues*
   b. more squares
   c. other ________

Justification ("How do you know?") ________

____

3c. "Are there more blue blocks or more triangle blocks?"

   Answer:
   a. more blues*
   b. more triangles
   c. other ________

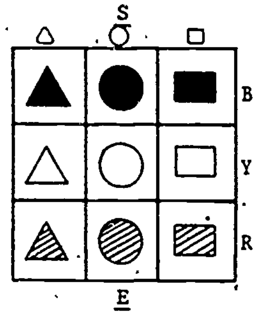
Justification ("How do you know?") ________

____

Time ________

CROSS CLASSIFICATION MATRIX

Time ____________



Replacement

One block

a. placed correctly

b. other ______________________________

____

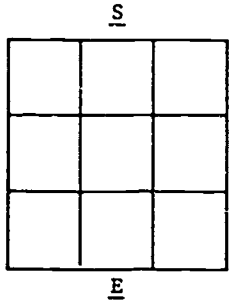
Two blocks

a. placed correctly

b. placed in reversed order

c. other ______________________________

____

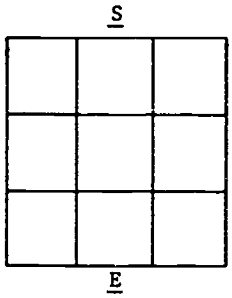


Three blocks

a. placed correctly

b. other ______________________________

____



Reproduction

Color

a. identical

b. transposed (indicate)

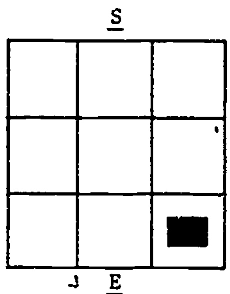
c. inconsistent (draw)

Shape

a. identical

b. transposed (indicate)

c. inconsistent (draw)



Transposition

Time ____________

CLASSIFICATION SERIATION MATRIX

Time ________________

Replacement

One block

- a. placed correctly
- b. other ________________

____

Two blocks

- a. placed correctly
- b. placed in reversed order
- c. other ________________

____

S

B	Y	R	
B-1	Y-1	R-1	1
B-2	Y-2	R-2	2
B-3	Y-3	R-3	3

E

Three blocks

- a. placed correctly
- b. other ________________

____

S

E

Reproduction

Color

- a. identical
- b. transposed (indicate)
- c. inconsistent (draw)

Height

- a. identical
- b. transposed (indicate)
- c. inconsistent (draw)

S

E

Transposition

S

		R-1

E

Time ________________

DOUBLE SERIATION MATRIX

Time ____________

Replacement

One block
- a. placed correctly
- b. other ______________________________

____

S

S	M	F	
S-1	M-1	F-1	1
S-2	M-2	F-2	2
S-3	M-3	F-3	3

E

Two blocks
- a. placed correctly
- b. placed in reversed order
- c. other ______________________________

____

S

E

Three blocks
- a. placed correctly
- b. other ______________________________

____

S

E

Reproduction

Width
- a. identical
- b. transposed (indicate)
- c. inconsistent (draw)

Height
- a. identical
- b. transposed (indicate)
- c. inconsistent (draw)

S

		F-1

E

Transposition

Time ____________

GPO 807-269-3

COMBINATORIAL REASONING

"Show me all the pairs you can make using these ____ colors. Remember, each pair must have two colors and each time you are to put down a new pair of colors."

Four Colors

____RG ____GY
____RY ____GB
____RB ____YB

____ number correct pairs
____ number repeats

Approach:
1. X & remaining colors, Y & remaining colors, etc.
2. Makes sure each color there 3 times. X + 3 (all other colors), Y + 3 (all other colors), etc., resulting in redundant pairs.
3. Random pairs, no apparent system.
4. Starts ____ then ____.
5. Other ____

Justification: "How did you decide to make this pair, then this pair . . . ?"

Six Colors

____RG ____GY ____YB ____BW
____RY ____GB ____YW ____BO
____RB ____GW ____YO ____WO
____RW ____GO
____RO

____ number correct pairs
____ number repeats

Approach
1. X & remaining colors, Y & remaining colors, etc.
2. Makes sure each color there 5 times, X + 5 (all other colors), Y + 5 (all other colors), etc., resulting in redundant pairing.
3. Random pairs, no apparent system.
4. Starts ____ then ____.
5. Other ____

Justification: ____

97

Eight Colors

_____BrR	_____BrW	_____LbR	_____LbW
_____BrG	_____BrO	_____LbG	_____LbO
_____BrY	_____BrLb	_____LbY	
_____BrB		_____LbB	

_______________ number correct pairs
_______________ number repeats - new pairs
_______________ number repeats - old pairs

Approach:

1. X & remaining colors, Y & remaining colors, etc.
2. Makes sure each there 7 times, X + 7(all other colors), Y + 7 (all other colors), etc., resulting in redondant pairs.
3. Random pairs, no apparent system.
4. Starts ________ then ________.
5. Other ______________________________

Justification: ______________________________

GPO 807-269-2

National Evaluation Committee

Helen Bain
Past President
National Education Association

Lyle E. Bourne, Jr.
Institute for the Study of Intellectual Behavior
University of Colorado

Sue Buel
Dissemination and Installation Services
Northwest Regional Educational Laboratory

Francis S. Chase
Professor Emeritus
University of Chicago

George E. Dickson
College of Education
University of Toledo

Chester W. Harris
Graduate School of Education
University of California

Hugh J. Scott
Consultant
National Evaluation Committee

H. Craig Sipe
Department of Instruction
State University of New York

G. Wesley Sowards
Dean of Education
Florida International University

Joanna Williams
Professor of Psychology and Education
Columbia University

Executive Committee

William R. Bush
Director, Program Planning and Management
Deputy Director, R & D Center

M. Vere DeVault
Professor
School of Education

Herbert J. Klausmeier
Principal Investigator
R & D Center

Joel R. Levin
Principal Investigator
R & D Center

Donald N. McIsaac
Associate Dean, School of Education
University of Wisconsin

Richard A. Rossmiller, Committee Chairman
Director
R & D Center

Len VanEss
Associate Vice Chancellor
University of Wisconsin–Madison

Dan Woolpert
Director, Management Systems
R & D Center

Faculty of Principal Investigators

Vernon L. Allen
Professor
Psychology

B. Dean Bowles
Associate Professor
Educational Administration

Frank H. Farley
Associate Professor
Educational Psychology

Marvin J. Fruth
Associate Professor
Educational Administration

John G. Harvey
Associate Professor
Mathematics

Frank H. Hooper
Associate Professor
Child Development

Herbert J. Klausmeier
V. A. C. Henmon Professor
Educational Psychology

Gisela Labouvie
Assistant Professor
Educational Psychology

Joel R. Levin
Associate Professor
Educational Psychology

L. Joseph Lins
Professor
Institutional Studies

James Lipham
Professor
Educational Administration

Wayne Otto
Professor
Curriculum and Instruction

Robert Petzold
Professor
Curriculum and Instruction

Thomas A. Romberg
Associate Professor
Curriculum and Instruction

Dennis W. Spuck
Assistant Professor
Educational Administration

Richard L. Venezky
Associate Professor
Computer Science

Larry M. Wilder
Assistant Professor
Communication Arts