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ABSTRACT

This technical report describes the operation of a field plotter and the kinds of geographic problems that the instrument can simulate. The field plotter is an easily operated analog computer which employs the electrically conductive sheet analog to simulate a wide range of physical and human-related spatial phenorena. The diffusion patterns of current in the sheet are mapped directly by marking and measuring voltage with a pencil probe. The rsulting map of electrical potential is a direct analog to the spatial distribution of response to a simulated geographic phenomenon. After a brief introduction, chapter two discusses the geographic concepts of location, distribution, and spatial processes and rates the manner in which the field plotter describes these concepts. Chapter three examines the technical theory behind the field plotter. The fourth chapter presents the operations of the field plotter. Chapter five examines six specific applications of the instrument to geographic problems and models. The sixth chapter describes how field plotter techniques can be applied to wider geographic problems. (Author/DE)

GEOGRAPHIC TECH

Living Maps of the Field Plotter

Analog Simulation of Selected Geographic Phenomena

ROBERT E. NUNLEY

COMMISSION ON COLLEGE GEOGRAPHY
TECHNICAL PAPER NO. 4

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LIVING MAPS OF THE FIELD PLOTTER

Analog Simulation of Selected Geographic Phenomena

by

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Commission on College Geography

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FOREWORD

The Technical Papers are explanatory manuals for the use of both instructors and students. They are expository presentations of available information on each subject designed to encourage innovation in teaching methods and materials. These Technical Papers are developed, printed, and distributed by the Commission on College Geography under the auspices of the Association of American Geographers with National Science Foundation support. The ideas presented in these papers do not necessarily imply endorsement by the AAG. Single copies are mailed free of charge to all AAG members.

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I. INTRODUCTION

The present technical report is designed to serve both those who do not have a field plotter to use and those who do. For readers who do not presently have such a device, the report describes the kinds of geographic problems the instrument can simulate and how it operates. Such readers probably should look through the illustrations and captions, and then read through the various sections in the order in which they are presented, paying little attention to the detailed instructions in sections IV and V. The report should help such readers decide the degree to which field plotters are applicable to their own teaching and research.

The second function is as a users manual for those who do have a field plotter. Users may find it best to go rapidly through sections II and III and begin careful study with section IV, Field Plotter Operation. Having read section IV, the user should then return to the earlier sections before proceeding to section V. The present report should constitute an adequate self-instruction guide in basic field plotting techniques. Command of these techniques will allow them to be combined and expanded for application to a seemingly endless variety of geographic problems.

The field plotter is an easily operated analog computer which employs the electrically conductive sheet analog to simulate a wide range of physical and human-related spatial phenomena. The device controls multiple current inputs to a conductive paper sheet. The diffusion patterns of current in the sheet are mapped directly by marking and measuring voltage with a pencil probe. The resulting map of electrical potential is a direct analog to the spatial distribution of response to a simulated geographic phenomenon.

The advent of a new type of field plotter makes practical the use of an imaginative method for illustrating elementary and advanced geographic concepts (figure 1). An hour of self-instruction enables the average freshman to understand and begin use of the field plotter. After 10 hours of study, the student has command of a wide range of modeling techniques sufficient for applying geographic concepts to the solution of local, regional, or large-scale problems, and is capable of simulating or analyzing spatial processes presented in standard textbooks. Thus, it is an ideal instrument for use in the laboratory or in directed study, as well as in the classroom.

Figure 1. A Field Plotter in Operation.

The author has employed field plotting techniques in both beginning and advanced classes over the last 7 years. During the first 3 years only 1 lecture per semester was devoted to the use of a rudimentary device to illustrate the gravity (attraction) and interaction potential concepts, and flow phenomena. Student interest led to the development of an improved instrument which was easier to use, but this instrument was too delicate and required, as did its predecessor, some knowledge of electronics. Anderson Enterprises of Eudora, Kansas, agreed to refine and custom-build this instrument.

Student interest increased, but the new device still lacked the necessary simplicity, flexibility, and ease of operation. Design problems were then discussed in detail at the Lawrence, Kansas, office of Interpretation Systems, Incorporated. ISI agreed to redesign and manufacture such a device, and the result has been a field plotter that can be easily operated with no knowledge of electronics, yields highly reproducible results, and is tamper-proof, portable, and self-contained.[1] In addition, it has reduced model construction and solution time by 75 to 95 percent.

Ten instruments were purchased from ISI in the latter part of the fall semester, 1969. Student response was overwhelmingly favorable. Each student completed at least one full study program which was self-designed and self-executed. The high student acceptance and interest suggested the development of a course based primarily on field plotter applications.

The present technical paper is designed to serve as a self-instruction guide to, and standard reference for, field plotter modeling. An early version of this guide was used during the spring semester, 1970, at the University of Kansas in a small honors section of the basic course, Fundamentals of Geography. In groups of 7, students devoted one 3-hour period a week to laboratory work, and one additional 3-hour period to lecture and discussion. Each student adopted a research topic of his choice. The course was designed to teach the application of basic geographic concepts and to develop the present technical paper. The present report will be used as the basic laboratory manual for a large regular section of our basic course beginning in the fall of 1970.

Field plotters are now being employed in a number of colleges and universities. A separate volume with supplementary techniques and more fully developed applications should be made available in 1971. Another volume, built around the High School Geography Project's *Geography in an Urban Age*, is in the planning stage. Contributions to either volume would be welcome.

1. Student model (FP-9) and precision research model (FP-15) field plotters available from Interpretation Systems, Incorporated, P.O. Box 1007, Lawrence, Kansas 66044, at $495 and $985, respectively.

II. GEOGRAPHIC CONCEPTS

Geographers commonly study the location, distribution, and spatial processes of phenomena on or near the surface of the earth. And there is a geometry (the spherical surface of the earth) common to all geographic study, as well as common influences that affect the location, distribution, and spatial interaction of most phenomena. The field plotter permits the description and analysis of many of these influences.

THE SINGLE POINT

One of the most elementary approaches in geography is the study of a single point. It can be described by coordinates of latitude and longitude, and its geographic significance normally can be expressed in terms of how it affects (or is affected by) the surrounding area. If the point or node of influence is a television transmission station, the area to which the signal is transmitted may be considered the reception region, a nodal region delimiting the area directly affected by the station. The point may be a local farmers market, and the nature of the market as well as the produce sold are affected by the agricultural practices of the nearby farmers. The location of all the farms supplying the market delimits (however roughly) the region that most directly affects the market. Further geographic impact could be expressed in terms of the varying quality of reception throughout the television region, or variations in the kinds of agricultural products originating from different parts of the region supplying the farmers market.

The geographic expression of quite dissimilar phenomena may be very similar to the station and the market. A rock dropped into the center of a large pond will create waves that radiate from it just as the television signal radiates from the transmission station. The distance at which the wave is no longer discernible may be taken to be the limit of the nodal region affected by the rock. Within the region one could differentiate between the nearby zones of high wave crests and the more distant, lower wave crests. The wave region is also more similar to the agricultural region than is immediately apparent. There is a tendency for different kinds of agricultural goods to be grown at different distances from a marketing center. Truck gardening tends to be more dominant near the town than it is at the edge of the agricultural region.

If a *uniform plane* is assumed, the *analogy* between the impact of a television station and a rock dropped into a pond is even closer. The uniform plane would mean no mountains or other topographic variations in the case of the television station, it would mean a calm lake with enough depth not to influence the wave propagation in the case of the rock. In the agricultural case, the uniform plane is even more involved; it would mean uniform topography, soils, transportation facilities, rainfall, and (if one wanted to be detailed) a host of other factors.

A LIMITED NUMBER OF POINTS

A more complex and typical subject of geographic study is the impact of location, distribution, and spatial interaction variables of two or more points. The added complexity of several television stations or several farmers markets heightens the difficulty of investigating their individual, mutual, and total influences in the nodal region. This difficulty in interpretation is further compounded by the normal variation in their spacing and size.

Curiously enough, however, the water wave analogy still holds. Dropping several rocks at the same time will have the effect of producing complex patterns that suggest, or perhaps simulate perfectly, the complex patterns resulting from several stations or several markets. To the degree that the analogy holds, altering the spacing and/or size of the rocks will better simulate the phenomena of interest.

Particularly when multiple points are employed there is a problem with scaling factors. It may be that to simulate a doubling in the height of a transmission tower one would have to increase by 50 percent the weight of the rock. In order to scale the height of a wave to correspond with the change in agricultural land use it might be necessary to multiply the wave height by some constant or take the logarithm of the distance from the agricultural markets.

LINES AND AREAS

The discussion thus far describes many geographic processes, but the description of some others requires a continuous series of points, or a line. For example, the impact of an international boundary on economic conditions requires that a line be considered. Thus, the percentage of the population involved in smuggling w probably decrease with distance away from the border. Here the uniform plan assumption means that the population is evenly distributed and transportation and accessibility are equal in all directions from the border.

Such a situation could be simulated by dropping a log the shape of the political boundary into the pond. Again, the height of the waves would decrease away from the line of impact and, perhaps, would thus simulate very closely the decrease in percentage f the population involved in smuggling. Of course, it may be necessary to work out some scaling factors to get a good simulation. It may be that the mathematics underlying the two phenomena are so different that no worthwhile simulation is possible. It is more likely, however, that the underlying mathematics required to describe the two processes are quite similar, and effective simulation would be possible.

Areas such as the built-up portion of a growing city have a strong impact on the value of land outside the corporate limits. The value of land will tend to decrease with distance away from the city limits. By reproducing the shape of the built-up area boundary with a log frame and dropping the frame into the pond so that all parts of it hit the water at the same time, a series of waves would be set up and the height of the waves would decrease with distance away from the logs. Perhaps a better simulation would be achieved by putting a floor underneath the log frame. One of the two analogies would probably hold.

A LARGE NUMBER OF POINTS

Still another case is one in which there is a large number of points to be considered. To describe, for example, the "central places" and their trade regions for all the market centers of a state would be difficult. Even if the spacing were regular and the size constant it would be quite a task to simulate them by simultaneously dropping one rock for each center and measuring the wave heights of such a complex pattern.

An iterative process, however, could greatly simplify the problem. It would be possible, using such an iterative process, to first locate the point of impact and size of each rock. Then only those rocks would need to be dropped simultaneously that lie within the area of influence of a given rock. In this way, a composite regional pattern could be constructed by superimposing the subregion simulations.

OTHER VARIATIONS

There are other interesting ways to illustrate further the analogy of water waves to geographic phenomena. One could show that the uniform-plane assumption could be relaxed by putting wave barriers in the pond and by using winds to expedite wave propagation. By changing the analogy somewhat, it would be possible to show how the wave motion could be used to simulate the diffusion of an innovation through an area, or to simulate human settlement of a previously uninhabited region. The two-dimensional case of a pond could be changed to one-dimensional through the use of a canal [illegible] could then be used to simulate transportation phenomena.

THREE BASIC GEOGRAPHIC CONCEPTS

The principal lesson to be drawn from the wave analogy is that nearly all geographic processes can be described in terms of propagation of influence from discrete sources (1), as modified by the properties of areas surrounding these sources (2), to produce varying effects throughout the area (3). The three basic concepts in the above sentence are difficult to state in rigorous terms, and the reader's indulgence is requested. The *first* is the influence(s) that relate(s) some point(s), line(s), area(s), or combinations thereof to all or part of the surrounding area and/or other areas. The *second* concept is the characteristics of the surrounding area and/or other areas that act as barriers or expediters to the influence(s). The *third* concept is that of variation in the resulting effect on all or part of the surrounding area and/or other areas. Each concept, to be intelligible, needs to be considered separately.

1. The influence(s) that relate(s) some point(s), line(s), or area(s) or some combination thereof to all or part of the surrounding area and/or other areas is the primary of the three concepts. If every aspect of a phenomenon is passive, it can have no geographic significance – it can have no effect on anything located elsewhere. If any aspect of a phenomenon exerts an influence, then that phenomenon possesses geographic significance – it must have some effect on something elsewhere. The phenomenon that has geographic significance may still be insignificant in any practical problem. The task is to define for any given problem the principal aspects of phenomena under study that exert influence at a distance. For the television transmitter, the dominant influence is the video waves; for the farmers market, it is the flow of agricultural goods and people.

2. Once the influence(s) is (are) identified, the characteristics of the surrounding area and/or other areas that act as barriers or expediters must be studied. Distance by itself is always something of a barrier to any influence, and it, therefore, is the basic ingredient in the uniform-plane assumption, which is the geographer's way of saying "other things being equal." But in real life the uniform plane never exists: therefore, the principal barriers and expediters should be isolated and studied. Curvature of the earth and topographic obstacles would probably be the most common barriers to television stations. Topographic obstacles and wet areas would probably be the most general barriers for agricultural products. Relay towers would appear to be the greatest expediters of television transmission, while roads and fertile soils are probably the most significant expediters of agricultural products. Once the barriers and expediters have been simulated, it is desirable to return to the uniform-plane assumption to gain a grasp of the fundamental pattern.

3. Wherever there is motion, there are barriers and expediters; therefore, variation will occur in the resulting impact of the motion on all or part of the surrounding area and any other area affected. The variation may be expressed in terms of potential.

Potential is here defined in terms of mathematics and physics – any of certain functions from which the intensity (or, in some cases, the velocity) at any point in a field may be readily calculated. If the motion and the barriers and expediters are known then it is possible, at least theoretically, to compute the potential. For even simple problems, however, an analytical solution is difficult. For any set of motions, barriers, and expediters simulated on the field plotter, a potential surface is produced that is accurate with a negligible error factor. That is, if the influence(s), barriers, and expediters are properly simulated, then the potential (spatial variation in impact) will be accurate.

Needless to say, not all geographic phenomena can be simulated effectively on the field plotter. Many geographic phenomena are too complex. Some are understood only in a stochastic (probabilistic) format. Still others are so poorly understood that much detailed work must be done before effective simulation will be possible. Nevertheless, many geographic phenomena can be simulated effectively; and for those that cannot at the present time, the field plotter provides some rough measure at least for trying to isolate key variables and then making first approximations. The field plotter results, then, may be interpreted as a probability function instead of a deterministic solution to many problems.

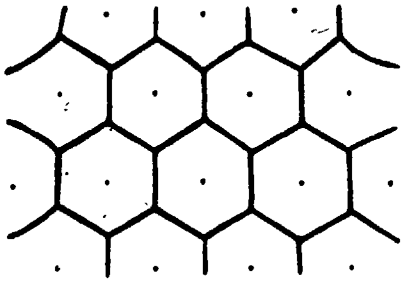
A brief summary of field plotter applications would be that the instrument may be used to study some aspects of most geographic problems, most aspects of a great many, and all aspects of some. First, the parameters of any problem need to be grouped into the three concepts of motion, barriers or expediters, and potential (either intensity or velocity). Then, within any one concept the parameters may be treated individually or collapsed into a smaller number; often into a single parameter.

One of the most effective uses of the field plotter is to bridge the gap between theory and reality. Consider, for example, the hexagon which is used to describe theoretical trade areas. Figure 2 depicts a series of field plotter outputs that begins with the simple theoretical pattern described by Christaller, progressively modified until empirically derived trade areas are simulated.

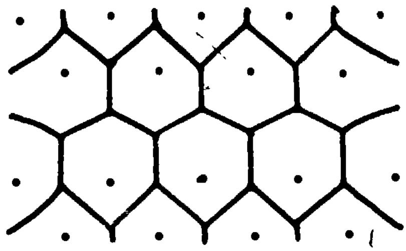
This series belongs to the iterative technique of solving a large number of points as discussed above. It also illustrates the three basic geographic concepts. The inputs of figure 2-a replicate the regular spacing and uniform size of towns as the primary concept; the influence being considered is the spatial dominance of each town. The only resistance used to consider the second concept is uniform; therefore, the uniform plane is being assumed. The third concept, potential, is expressed in the lines that separate the regions of dominant influence of each town.

The constraint of regular spacing of towns is relaxed while the size is held constant in figures 2-b and 2-c. The uniform plane (the second concept) is retained. Thus, the change in locations of the sources of influence result in the observable changes in potential (regions of dominant influence). The regions of 2-b are irregular hexagons that result from a slight change in town spacing. The regions of 2-c range from pentagons to octagons since the town spacing is much altered.

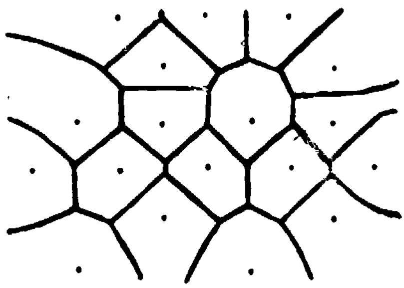
The constraint of uniform size is relaxed while the spacing is held constant in figure 2-d. In the center is the classical hexagon that results from the towns being the same size. If the towns on the upper half of the illustration are considered to be smaller than the central town, then the straight lines of the hexagon degenerate to a series of connected arcs lying further from the central town. Thus, the region of dominant influence of the central town is extended because of its larger size, and the shape of the trade area is altered. In the lower half of the illustration, the towns are considered even smaller than the towns of the upper half. Consequently, the region



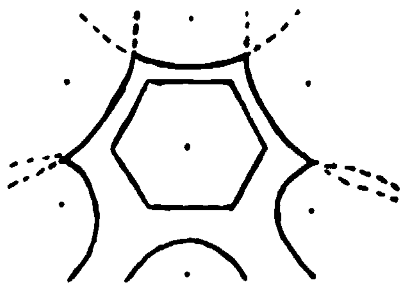
a. Regular Spacing and Uniform Size



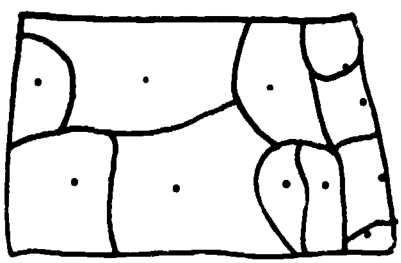
b Slightly Irregular Spacing



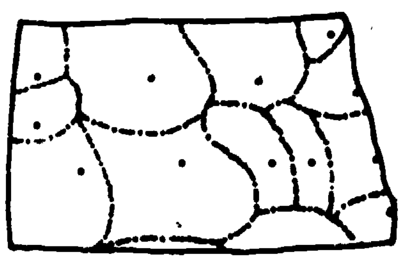
c. More Irregular Spacing



d. Varying Size



e. Realistic Spacing and Size



f. Trade Areas of North Dakota

Figure 2. From the Classical Hexagon to Observable Trade Areas

dominated by the central city extends between the small towns, leaving them dominant over a much reduced trade area.

Both size and spacing constraints are relaxed in figure 2-e. The spacing was determined by the location of towns in North Dakota; the strengths of the inputs were made proportional to the populations of the towns. The resulting simulated trade areas, even though the uniform plane is assumed, bear a close resemblance to the empirically derived trade areas displayed in 2-f. Towns outside the state, and not considered in the field plotter simulation, account for most of the variation. The goal, however, is not a perfect simulation but rather a clear demonstration of how the theoretical hexagon can be related to the trade areas that can be observed around any market area.

The series could be extended along several lines very profitably. The uniform-plane assumption could be relaxed by considering the highway network of the state. One could consider a road even in the case of 2-a, -b, -c, or -d, to illustrate changes in the theoretical models as well as in the final simulation.

A truly dynamic series can also be simulated. Stage one could be 2-a. In stage two, one could arbitrarily increase the value of one central point in such a way as to increase the size of its trade area. In stage three, the value of the central town could be raised in proportion to the change in size of its trade area that occurred in stage two, and the trade areas could be recomputed. In each successive stage the change in value for the central town could be made in proportion to the change in trade area resulting from the preceding stage. Similar simulations could be undertaken with higher or lower initial values in order to speed up or slow down the growth process. Starting two or more towns growing during the same period adds variety to the series. Increasing the value of each city by some function of the change in trade area, rather than directly proportional to the change, adds another dimension. The rules can be changed either to more closely approximate a known reality or simply to ascertain the consequences of changing one parameter of the model.

Let us review briefly the material presented thus far and then look at the material to follow. Four kinds of phenomena suitable for geographic study on the field plotter were presented. From the four, three basic geographic concepts – influence, resistance, and potential – were drawn. The hexagon series was used to illustrate the three concepts. Further illustration will be provided in section V by four series of experiments, one each for the four kinds of phenomena. Two additional series of experiments in section V deal with Models of Analysis and Models of Synthesis, employing techniques from the first four series. To better understand these experiments, however, you should consider section III, Field Plotter Concepts, and section IV, Field Plotter Operation.

III. FIELD PLOTTER CONCEPTS

The reader need not completely understand the following background theory for him to be able to use the field plotter. This section is designed to add depth to his understanding of the principles involved and the basis for use of field plotters in geography, as well as in other disciplines dealing with two-dimensional space.

FIELD PLOTTING TECHNIQUES IN PERSPECTIVE

The field plotter will simulate a wide range of phenomena in addition to those encountered in geography. It has been applied to problems in aerodynamics, heat flow, ground water distribution, petroleum reservoir engineering, and electrostatics, to name only a few. In general, the field plotter can model two-dimensional physical distributions described by Poisson's or Laplace's equation.[2] These equations, although seldom encountered by the non-mathematician, are as basic and universal in their application as the more familiar equations of motion or gravitation. They describe the static or equilibrium distribution of energy or matter in a very large class of physical processes. Although distributions related to climate, vegetation, and human activity are much more complex than most physics phenomena, nonetheless good approximations to their description have been achieved using the electrically conductive sheet analog.[3] Why then, haven't field plotting techniques been widely used in the field of geography?

Many factors have contributed to the neglect of this technique by geographers. Principal among these has been the lack of suitable instrumentation. Until very recently, all available field plotter instruments were the product of the analog computer era preceding the present digital computer age. These devices were generally custom-built and required considerable electronic sophistication on the part of the user. In addition, they were used by specialized researchers in non-geographic areas and suffered from the limitations of vacuum-tube technology. With the advent of solid state electronics less attention was devoted to analog computer methods, in particular the specialized area of field plotting techniques. Comprehensive treatment was given the conductive sheet analog in a book published in 1959.[4] That source pointed out another limitation in research applications of field plotting, namely the lack of a suitable isotropic conductive sheet material

To this author's knowledge there has been neither a good conductive sheet nor adequate instrumentation available until just this year. Further, there has been little attention given during the last decade to the conductive sheet analog except the limited use of the earlier type of field plotter instrument. The recent development of an excellent conductive paper (very uniform in resistance) and a redesigned field plotter utilizing up-to-date electronics represent an advance in the state of the art. Thus, geography as well as other scientific areas can potentially benefit from an enhanced capability in field plotting techniques.

This capability can be viewed as a valuable adjunct to digital computer simulation of geographic phenomena. The mathematics required for formulation of all but the simplest of spatial distributions are extremely difficult. If a formulation is achieved,

2. For further treatment see Appendix
3. Mackay, pp 167-174
   Chorley and Haggett, pp 746-756
   Nunley, et al. pp. 151-163
4. Karplus and Soroka. 1959.

numerical approximation and solution by computer may be equally difficult, time-consuming, or costly. For many of these problems the field plotter can readily provide a solution. Another use of field plotter analogs is in generating good starting values for digital computer models that require a great deal of time in converging upon a solution. Thus, the field plotter can be used to advantage in saving time and expense, and without mathematical sophistication. Finally, and most important for geographers with or without access to a digital computer, it provides in a single instrument a valuable tool that will allow modeling of a wide variety of real problems and the teaching of many geographic concepts with greater ease and deeper student understanding.

PRINCIPLE OF ANALOGY

In the preceding sections we have been applying what is called the principle of analogy, that is the representation of one physical process by another. In the case of the water waves analogy, we appealed to the intuitively evident similarities between the height and distribution of wave crests and the regional patterns of farmers markets and television stations. The analogous nature of these processes is founded on more than just intuitive similarities. The general way in which we view the physical world is expressed in terms of basic principles, such as the conservation of energy or matter. In the same fashion, there is a general method of describing any phenomena according to the manner in which energy or matter is generated (introduced into the system), stored, dissipated, or transferred (distributed). The physical process (heat, electricity, etc.) operating in the system is specified by two types of variables. The *vector* or *flow variable* indicates the field or direction of matter or energy transfer. A *streamline* graphically depicts the direction of flow; the spacing of streamlines indicates the concentration of flow. The *potential* or *scalar difference variable* describes the magnitude and space distribution of field intensity. It is represented by an *isopotential* which on the conductive sheet is a line connecting points of equal field intensity. The *properties* of a system are characterized by its matter or energy storage and dissipation parameters. These determine the way in which matter or energy is transferred or distributed.

The electric analog terms for field or system variables are *current* and *voltage*. *Resistance* is the property of the conductive sheet which determines the relationships between current distribution and electric field intensity (as measured by voltage). For a common household plumbing system the analog to current would be the water flow rate past a given point in a water pipe. Voltage is similar to the pressure required to push the water through the pipe system. A water valve or the frictional resistance of the inside surface of the pipe has the same effect as electrical resistance. By increasing the water pressure (by a pump) the water flow rate (current) is increased. Similarly, the greater the resistance to flow of water, the more pressure is required to maintain a specified flow rate.

ELECTRICALLY CONDUCTIVE SHEET

To employ the field plotter effectively, the user should have a basic understanding of current (I), voltage (V), and resistance (R). Their relationship is expressed by Ohm's Law in the following equation:

$$R = V/I$$

Ohm found experimentally that if the potential difference (voltage) between two ends of a wire was increased, the current through the wire increased so that the ratio of V to I remained a constant, R, as expressed by the above equation. Resistance is a constant for a particular material and is measured in volts per ampere (1 volt/ampere = 1 ohm). Current is expressed as the time rate of flow of electric charge, in amperes (1 ampere = 1 coulomb/second); and voltage in volts – one volt is the work required to move one coulomb of charge (6×10^{18} electrons) through 1 ohm resistance in one second. For the sake of brevity, the term current will also be used in the sense of current density, a vector indicating the direction of electron flow and the current per unit cross-sectional area. Another convenient measure of resistance is resistivity, expressed in ohms per square of conductive sheet material. It is defined as the resistance measured between two parallel sides of a square sheet and is independent of the size of the square.

Ohm's Law may also be written as:

$$V = IR$$

This formulation is more appropriate to a discussion of field plotting techniques. The modeling of a problem is accomplished by applying current to selected points or areas of the conductive sheet. By varying the input current to simulate the relative values of the various problem elements, the resulting current field distribution can be obtained directly by measuring voltage at a series of points throughout the sheet. (Voltage is measured as the difference in potential between some reference point and that point to which the voltage measuring probe is applied.) A second modeling technique requires varying the resistance of the sheet. Cutting the sheet forms a barrier of infinite resistance to the flow of electrons. Applying silver or carbon paint or aluminum tape decreases the resistance, and electron flow is expedited. Whether current inputs are varied or the sheet resistance altered, the effect is measured in the same way – by mapping the resulting voltage pattern.

To better develop an intuitive familiarity with the conductive sheet field variables and its resistance property, consider the series of models in figures 3 through 8. The rectangular strip of conductive sheet in figure 3 has an electrical current entering the left edge, flowing along the strip, and leaving at the right edge. Highly conductive (very low resistance) aluminum tape (or silver paint) has been placed across both ends of the sheet; these form terminal strips. Since they have nearly zero resistance to the flow of electrons, there is almost no potential difference between any two points on the terminal strip. Thus, all points on the terminal will have nearly the same voltage with respect to any point on the conductive sheet.

Because of this property, current applied to one of these strips will form a line current source. Current will radiate uniformly at right angles to this line and will flow to the opposite edge where it exits. This edge is the electrical *ground* of the sheet. The purpose of "grounding" is, as the term implies, to establish a voltage reference – an absolute minimum level to which all voltages are referred. It also serves to remove electrons from the sheet. If the sheet were not grounded, there could be no flow of electrons.

The series of lines parallel to the ends of the sheet are called *isopotentials* because they connect points of the same voltage (the potential difference between a point on the sheet and ground). Isopotentials can be compared to contours on a topographic map. The left edge of the sheet is similar to the crest of a hill; and the grounded edge, the base of the hill. The voltage between the two edges is analogous

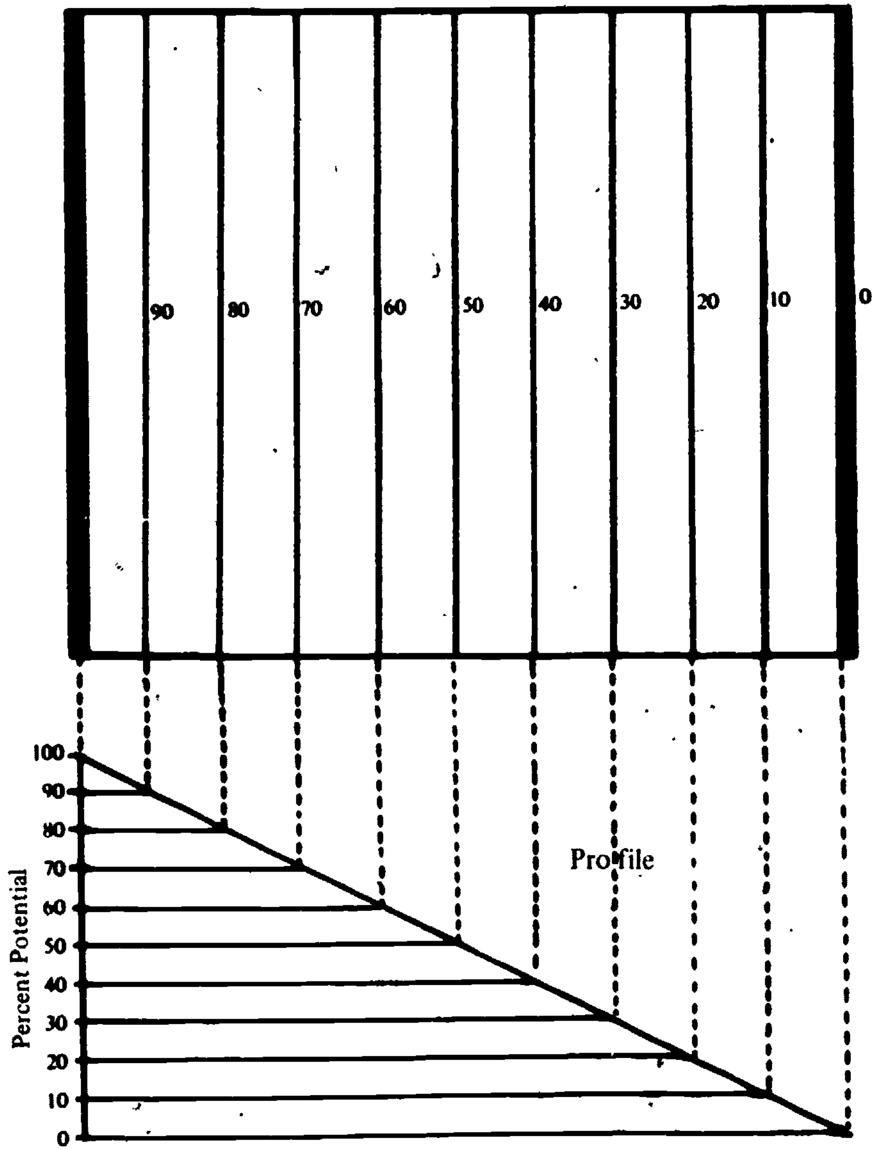


Figure 3. Rectangle Model, Line Input, Line Ground

to the relief or difference in elevation between the hill base and the hilltop. To understand why voltage (referred to ground) decreases from left to right, recall Ohm's Law. Voltage is equal to the product of current and resistance. Resistance is directly proportional to the length of the sheet. The resistance encountered by electrons flowing through the 1-centimeter length of sheet is one-tenth the resistance encountered in a 10-centimeter length. For the model we have been discussing, the current was held constant. With current unchanging along the strip, voltage varies as resistance, or the distance from ground. The voltage between a point 1 centimeter from ground is one-half the voltage of a point 2 centimeters from ground. The change in voltage is the same across any 1 centimeter along the sheet.

It is useful to express any sheet voltage as a percentage of the edge-to-edge voltage. Thus, the left edge has a 100-percent voltage value, the middle of the sheet is marked by a 50-percent isopotential and the grounded edge a zero-percent voltage. The spacing of isopotentials with a contour interval of 10 percent is a constant (the length of the sheet divided by 10). The slope of the voltage profile as depicted in figure 3 is linear, with a negative slope of 1 percent (edge-to-edge) voltage change for each 1 percent of sheet length change in distance from ground.

The relationships just discussed are valid for a conductive sheet of any size if it is rectangular, has a line constant current source and line ground along two opposite edges, and is of uniform resistivity. A change in sheet geometry; a point, area, or curved line current source; or local variation in resistivity will yield different current and potential distributions. Figure 4 is a model identical to the preceding except in the change from a line current source to a point current source centered on the left sheet boundary. Current radiates from the point of input as do rays from a point light source (current is of course confined to the surface of the paper, and the half plane to the right of the input). The sheet is bounded by cuts on all sides except the grounded right edge. Electrons cannot flow across these cuts, only parallel or away from them, so that electrons that begin parallel to the left side of the paper must "turn" away from the left side and flow parallel to the top and bottom cut boundaries. Electrons starting perpendicular to the left edge, flow in a straight line (along the midline of the sheet) to the center of the grounded edge. The graphical depiction of electron paths is accomplished by drawing lines from the input to ground, perpendicular to isopotential lines. These streamlines are shown as dashed lines in figure 4.

There are some noteworthy differences between the line and point source models. The potential distributions are obviously different in appearance; they also differ in the rate of decrease of voltage (slope) from the input to ground as shown by the profile in figure 4. Voltage at the midpoint of the sheet was 50 percent for the line source model. For this model it is less than 40 percent. Since the resistivity and size of the sheet and the magnitude of current input are the same in both models, the difference can be attributed to the effects of a point source. Viewing the area bounded by two streamlines as a tube of flow, the principles of conservation and continuity require the same total number of electrons to pass each point along the tube; regardless of tube width (distance between streamlines). Thus, where it widens, flow is less dense. This brings us to the concept of current density.

If we were to approach the input pin in figure 4 from the right, we would find current density (the total number of electrons flowing past a point along the tube divided by the tube width at that point) increasing at a mounting rate until it achieves a maximum density in the immediate vicinity of the input. Since voltage (electric field intensity) is the product of current (density) and resistance (resistivity),

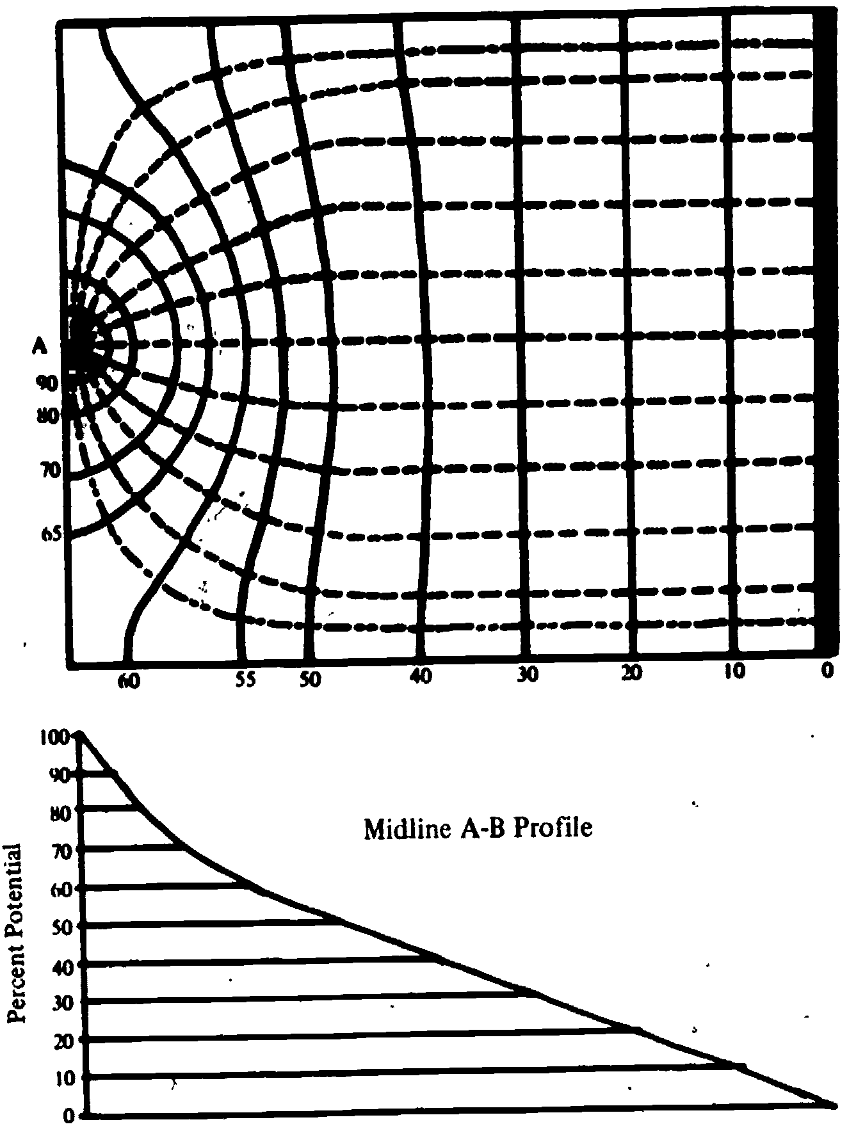


Figure 4. Rectangle Model, Point Input, Line Ground

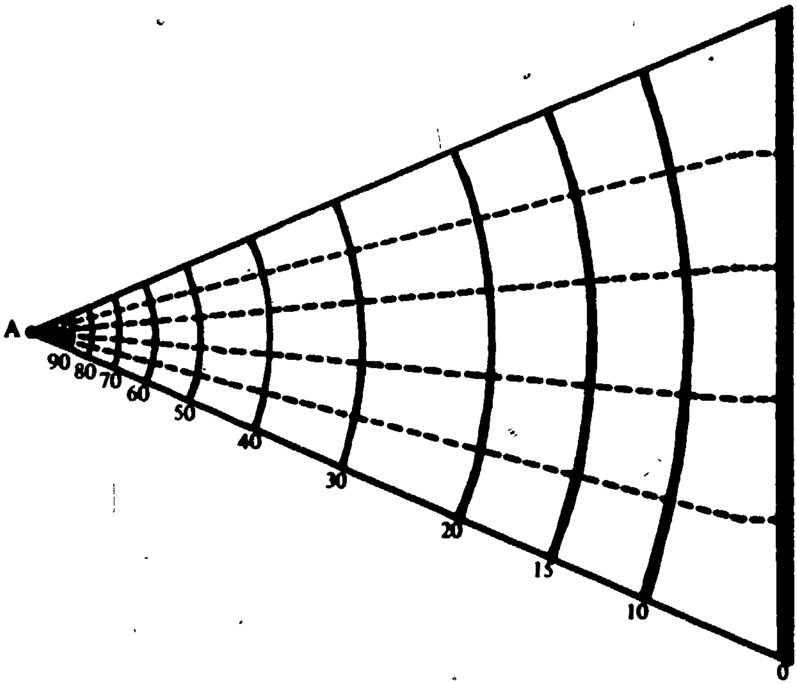
voltage also increases at an increasing rate. Compare the median profiles of the two models in figures 3 and 4 to see this effect. In the line source model, current density is the same throughout the sheet. Another effect of the point source is the lengthening of paths electrons must follow to reach ground. Streamlines in the line source model were straight and parallel to the top and bottom boundaries; thus, electrons flowed a minimal distance to ground. Resistance is proportional to path length, so that a net displacement of electrons toward ground in the vicinity of the point input is met with greater resistance than in the line source model. Again, an increase in voltage is the result.

Figure 5 is a model obtained by cutting the former model from each corner of the grounded side to the middle of the left side. The result is a triangle with a grounded base and a point source input at the apex. The amount of input current is the same as in the two preceding models. However, the current density has been altered by the triangular geometry, streamlines are radial, and the arc length distance between them is constant for a given radial distance from the input and a fixed angular separation. The resulting potential distribution is a series of arcs with a profile marked by a constant rate of decrease in slope from input to ground. Compare the triangle median profile with that of the point input rectangular model. It can be seen that the slope becomes linear to the right of the midpoint in the latter, whereas in the triangular model the slope continues to change past the midpoint.

The significance of the triangular geometry is that it approximates the sector of a circle which conforms to the radial flow pattern of a point input. As a result, current density is constant and uniform across any arc. Similarly, the line source and line ground on two opposite edges of a rectangular sheet yield a uniform current (constant current density) distribution over the entire sheet. The combination of a point source and rectangular geometry produce a current distribution intermediate between the two. The model in figure 6 represents a combination of the triangular and rectangular geometries, a point source and line ground. It is of interest because the "integrity" of the two geometries is preserved as they share a common element — the base of the triangle. Another example of a combination geometry is shown in figure 7. The illustrated potential pattern is symmetric about the common base of the two triangles. This symmetry is an expression of the conservation law governing the diffusion of electrons. That is, the same number of electrons must flow past each point and its mirror-image counter point; the same total number of electrons radiate from the point input as converge upon the point ground. The median profile shown in figure 7 reflects this symmetry. Notice that the midpoint is marked by the 50-percent potential level.

The discussion up to this point has been centered on the overall geometry of the sheet. The sheet in figure 8 is the result of modifying the internal geometry of the model in figure 6. A barrier to electron flow has been constructed by cutting out a region in the center of the sheet. Current flow is split by the triangular portion of the barrier and is divided equally between the two channels on either side of the barrier Current density in these channels becomes twice that of the sheet without the barrier. (The width of the rectangular part of the barrier is one-half the width of the sheet.) As a result of this increase in current density and the lengthened electron paths, the voltage is higher in the channels than it was for the corresponding areas of the model in figure 6.

Streamlines have been sketched (streamlines are *always* perpendicular to isopotentials), using the isopotentials as guides. These help to elucidate the potential pattern to the right of the barrier. Electrons flowing out of the channel tend to



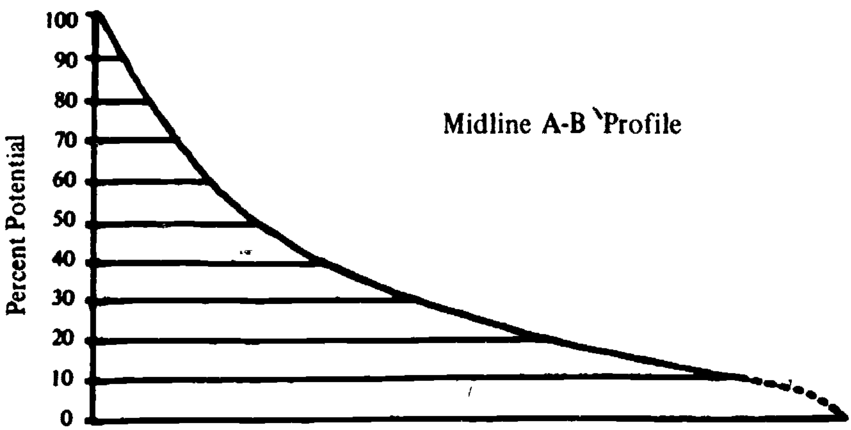
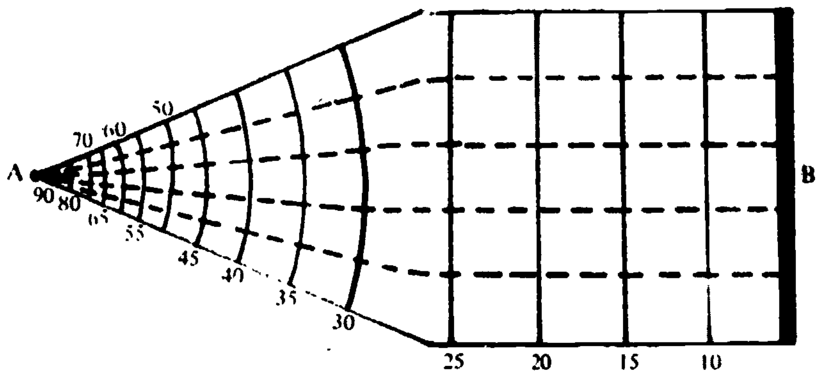


Figure 5. Triangle Model, Point Input, Line Ground



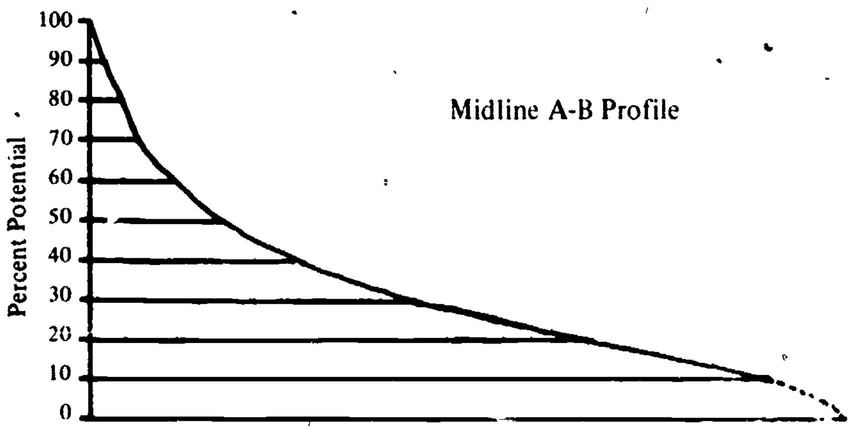
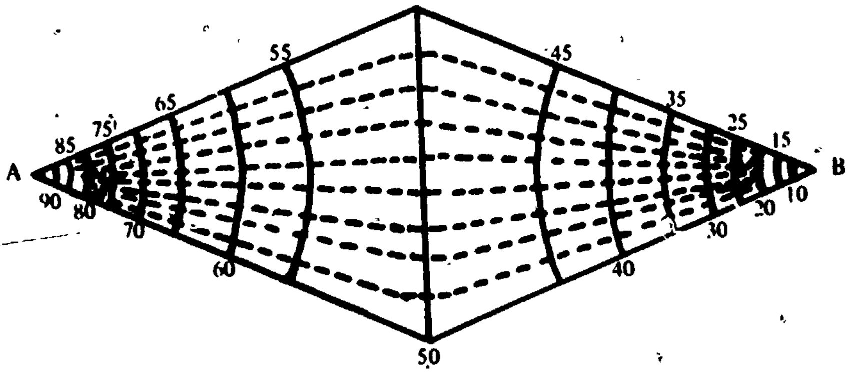


Figure 6. Triangle-Rectangle Model, Point Input, Line Ground



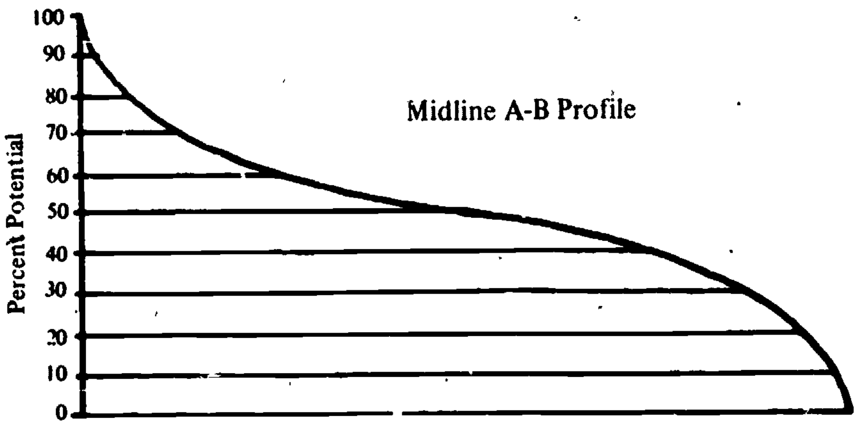
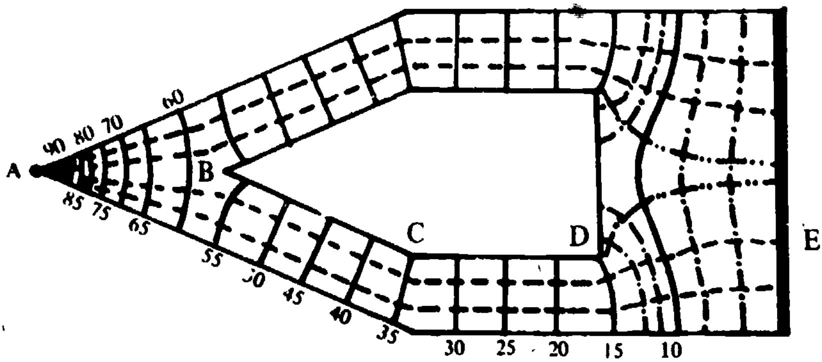


Figure 7. Two-Triangle Model, Point Input, Point Ground



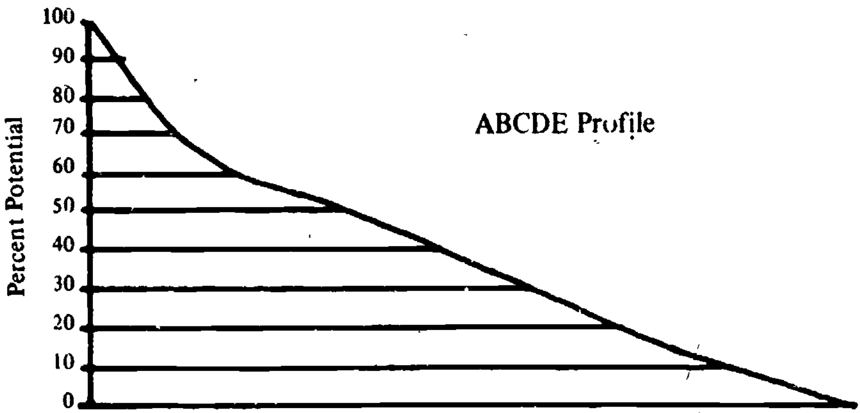


Figure 8. Barrier Model, Point Input, Line Ground

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follow the shortest path to the grounded right edge. Electrons closest to the bottom sheet edge flow directly to ground. In fact, examination of the tubes of flow formed by adjacent streamlines indicates that two-thirds of the flow in each channel reaches the portion of ground within a channel width distance from the bottom edge. The other one-third of the channel flow diffuses into the center half of the sheet so that streamlines from both channels converge on the median line. Very few electrons flow parallel to the right edge of the barrier, thus a low current density "dead zone" is found immediately adjacent to the barrier. If the sheet were extended sufficiently to the right, flow from the two channels would fully converge with isopotentials and streamlines straightening to form a uniform rectangular grid.

Summarizing the concepts presented in this section, we have examined the relationships between the three electrical variables of the conductive sheet. Also, the effect of overall and internal sheet geometry on these variables has been discussed. Streamlines, tubes of flow, and isopotentials were found to conform to some basic principles. Streamlines are *always* perpendicular to isopotential contours, just as in the case of curving parallels and meridians on the surface of a globe. The area between adjacent streamlines forms a tube of flow. Principles of conservation and continuity require that the same number of electrons must pass each point along the tube. Thus, variation in tube width reflects variation in current density. A constant contour interval in isopotentials, e.g., steps of 10 percent, will divide the tube of flow into areas with the same total number of electrons. These equal value areas (*not* necessarily equal area) are called *curvilinear squares*. They are areas bounded by adjacent streamlines and adjacent isopotentials. By arbitrary designation of one curvilinear square along the tube as a standard for comparison, the areas of all other curvilinear squares can be compared to the area of the reference square. Since all have the same total number of electrons flowing through them, comparison of their areas yields a direct measure of variation in current (or human population, habitat, etc.) density.

A cut in the paper forms an absolute barrier to electron flow. Streamlines never terminate on a cut, rather they tend to parallel or turn away from a cut boundary. On the other hand, isopotentials in the vicinity of a cut (ungrounded) boundary always intersect a cut at right angles since they are perpendicular to streamlines and streamlines are parallel to cuts. A very low resistivity, high conductivity strip of aluminum tape, silver or copper paint will form an isopotential barrier. Because the high conductive area has nearly no resistance, there is very little potential difference between any two points. The whole area is at the same potential or is isopotential. Isopotential contours do not intersect each other: thus, isopotential lines tend to parallel this type boundary. Streamlines do intersect these areas and at right angles. The application of current or grounding of a high conductivity area causes the current source or ground geometry to conform to the geometry of that area.

Figure 9 is a composite of the profiles of figures 3 through 7, corresponding to the first five of the six models discussed in this section. As can be seen, no general "exact description" is apparent that encompasses all five profiles, except that their slopes from input to ground are all negative and can be controlled and to some extent predicted. However, an individual model or model type profile can be mathematically defined. Profiles do provide a powerful tool for the comparison, analysis, and interpretation of conductive sheet analogs. The profiles presented in figure 9 were all along the midline of the models which facilitated comparison. In general, a profile can be constructed between any two points on the sheet as will be demonstrated in section V. Its utility can be compared with that of the common topographic map profile used in terrain analysis. Its use in the analog context will for most applications require a transformation (change in scaling of the ordinate and/or abscissa axes) of the profile from the conductive sheet coordinates and potential values to the real world system coordinates and phenomenal response values.

Understanding of the concepts of this section should allow efficient and effective use of the field plotting technique. The next section integrates the geographic and conductive sheet analog presented in this and the preceding sections, adding the instrument operation procedures necessary to performing the experiments in section V.

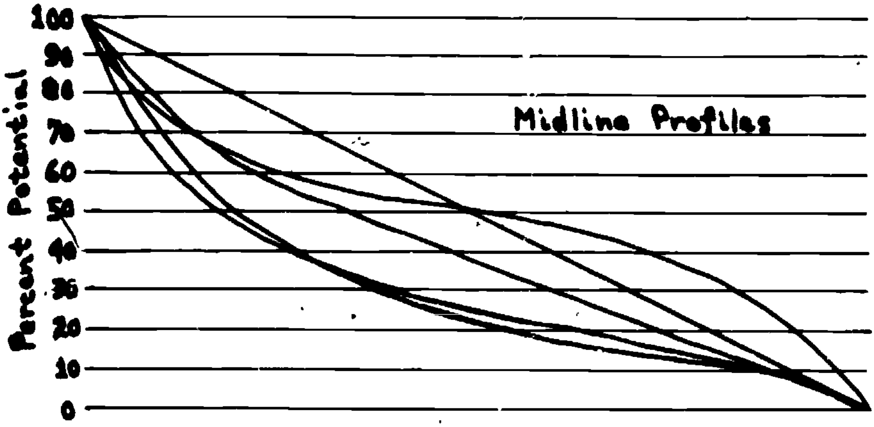

Figure 9. Composite of Profiles From Figures 3 Through 7

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IV. FIELD PLOTTER OPERATION

The field plotter is an instrument that can be operated after only a few minutes of basic instruction. If you do not have a field plotter, you may want to read through this section and use the photographs of a field plotter to provide you with the information necessary to understand the instructions; by doing so you will develop a feel for operational procedures. If the lack of a field plotter is a serious obstacle to understanding the instructions, then go directly to section V. If you have a field plotter, you should work through this section step by step to make sure that you are using it properly.

MODELING MATERIALS

There are some basic materials needed to operate a field plotter. The most important is the conductive paper. In a previous article we recommended Teledeltos paper.[5] We also pointed out several disadvantages of this paper, which is non-uniform in resistivity and has high contact resistance, both of which tend to lower experimental precision. A new conductive paper was recently found which yields distortion-free, high-precision results superior to all other conductive materials tested.[6] To date, we have analyzed over 900 different models on this paper and have yet to find any serious irregularities.

Materials which will allow the creation of very high conductivity (low resistivity) in selected areas will also be necessary. Metallic tape with a conductive adhesive is very useful, and silver or copper conductive paint may also be employed. The tape can be applied quickly and easily, can be cut to any shape, and provides reproducible results with high precision from model to model. Silver paint is an even better conductor, but several hours are required for it to dry to its maximum conductivity.[7] For some applications, taping a highly flexible, small-diameter solder wire onto the sheet is also useful. A carbon conductive paint is also available which provides a conductivity between that of aluminum tape and that of the conductive paper.

Finally, several other tools will facilitate performance of the exercises. A nearly transparent tracing paper the size of a conductive sheet will be needed; linear and semi-log graph paper will be useful for profiles; a straight-edge or T-square is also useful, but not necessary; an X-acto knife or single-edge razor blade will allow clean cutting of the conductive sheet; and a pair of scissors, a small paint brush, and a white marking pencil are also useful.[8] These graphics supplies may be obtained from a drafting supply store. Sewing pattern transfer ("tracing") paper facilitates transfer of a map onto the conductive paper. The model construction materials can be obtained from the sources listed below.[9]

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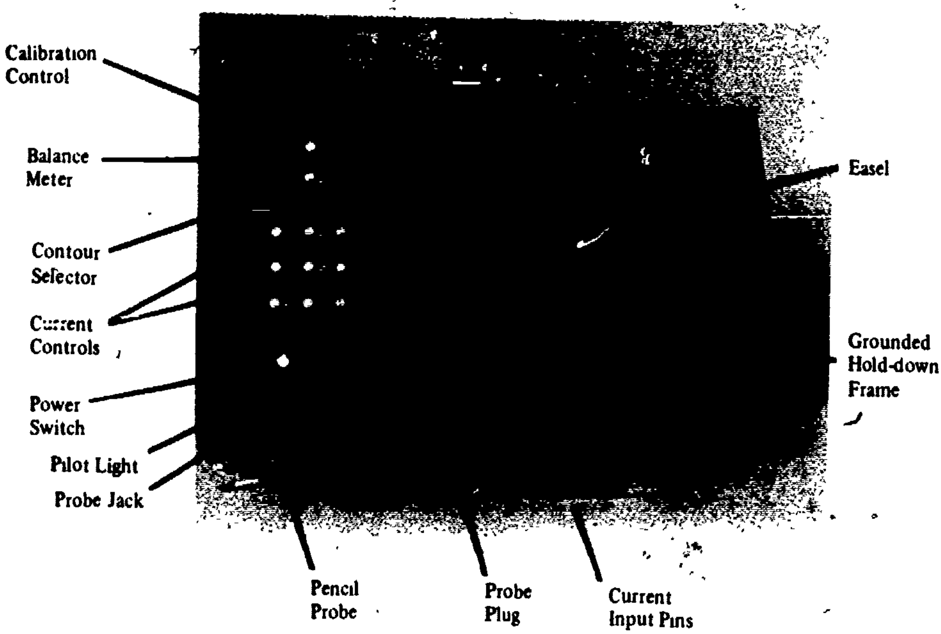
5. Nunley, et al., op. cit. pp. 151-163.

6. Manufactured to ISI specifications and available through the company in packages of 100, 13" x 16" sheets, precut to fit Model FP-9 or FP-15.

7. Copper conductive paint is very difficult to apply uniformly, and difficult to keep properly thinned, due to constant evaporation of the solvent base.

8. Eagle-Prismacolor soft marking pencils are available in a variety of colors.

9. Silver and copper conductive paints are available through C.G. Electronics, Division of Hydrometals, Inc., Rockford, Illinois. Aluminum tape and carbon conductive paint are available through ISI or Emerson-Cuming, Canton, Massachusetts, and Dynaloy, Inc., 408 Adams Street, Newark, N.J., respectively.



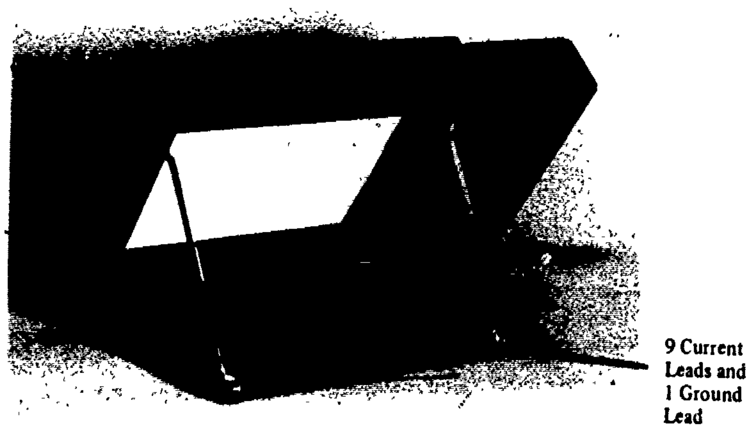


Figure 10. Front and Rear Views of the Field Plotter

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FIELD PLOTTER OPERATING INSTRUCTIONS

The pictures in figure 10 show the front and rear views of a student model field plotter. Although other field plotters may vary in detail, the operating principles are common to all types. If you have one other than the one on which this report is based, compare it with the photographs and descriptions and make the necessary changes in the operating instructions.

Begin by laying on the easel an electrically conductive sheet of paper, with the black side of the sheet facing you; lower and latch the hold-down frame. Position the number 1 current input pin over the center of the paper sheet and push the point through the paper into the easel corkboard until the head of the pin is in firm contact with the sheet – do not twist the pin, which might destroy the thin layer of carbon on the paper.

The support panel latches located at the top of the easel should now be released and the rear panel lowered to support the unit in a working position. This exposes the numbered current source connectors and power line cord. Clip the number 1 connector to the input pin protruding from the rear of the easel and plug the power line cord into a standard 117 VAC, 60-cycle outlet. Next, place the power switch in the "ON" position and insert the probe plug into the probe jack.

As can be seen in the closeup view of the control panel in figure 11, the current source controls are numbered 1 through 9. Control number 1 determines the current fed into the sheet through input pin 1, etc. Each control knob has 10 major divisions which are further subdivided into 10 parts. A setting of 10 will cause the maximum amount of current to be input to the sheet. Generally, each current control will be set so that the proper relative values of the various inputs will be maintained. Control 1 is arbitrarily set at a value of 10. Since in this example there is only one input, all controls other than 1 are set to zero (extreme counterclockwise position).

Current is now radiating from the center of the sheet and diffusing toward all four sides of the grounded easel boundary. To better understand what is now happening in the conductive sheet consider the water wave analogy presented in section II. A pebble thrown into a pool of water will create a series of concentric waves. The height of each wave crest will decrease as it moves away from the center. This decrease is caused by the dissipation of energy and the dilution or reduction of energy intensity as the wave encompasses an increasingly greater area. If, instead of a pebble, we employed a mechanical wave generator that repeatedly hit the water at a steady rate, a series of standing waves would result. The height of wave crests would decrease as the square of distance away from the source. That is, the height of crests would decrease by a given amount the first inch away from the center, it would drop about twice that amount across the second inch and four times that amount across the third inch away from the input, etc. until the waves completely disappeared.

Returning to the conductive sheet model, the steady current radiating in all directions from the point source, similarly creates standing waves. These can be located and measured by use of the probe, contour selector, and balance meter. In electrical terms, these standing waves are called isopotentials. "Iso" means the same; potential is the electrical analog of wave crest height. Potential is, more specifically, the measure of electrical field intensity. Thus, the line connecting points of the same intensity is an isopotential. Since the conductive sheet is not perfectly conducting, that is, it offers resistance to the flow of electricity, the electrical potential decreases away from the input pin in the same manner as wave crest height in the water wave analogy.

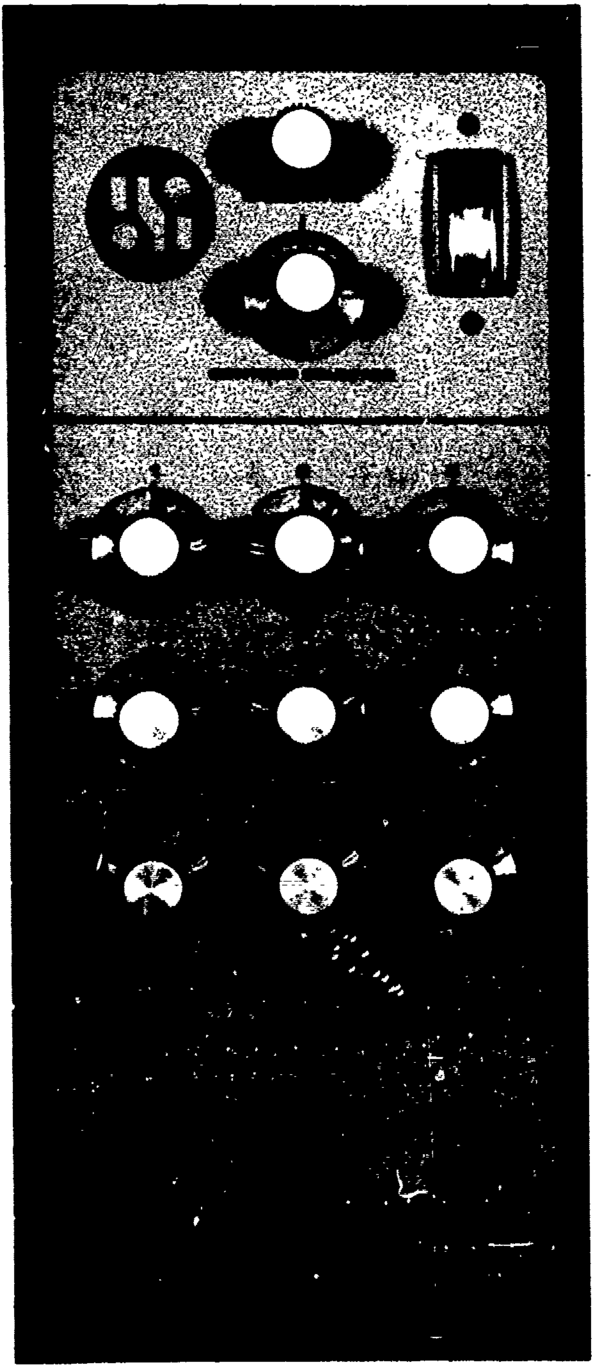

Figure 11. Closeup View of he Control Panel

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Directing your attention to the control panel, let us now prepare to map the isopotential contours. The calibration control establishes the input pin as the reference point (of highest potential) for all other points on the sheet. That is, all potential measurements are expressed as percentages of the potential at the reference point. This reference value then is selected as the 100-percent contour, or the 10 setting on the contour selector. The pencil probe has a dual purpose; to determine the location of a potential contour, and to mark this contour on the conducting sheet.

Pick up the pencil probe, making sure that the contour selector is set on 10, and place the point of the probe on the side of the input pin head. With the left hand turn the calibration control knob until the needle indicator on the balance meter is centered (nulled). Turning the calibration control clockwise will cause the needle to deflect downward; a counterclockwise movement will have a contrary effect. The instrument has now been calibrated.

We are ready to begin plotting isopotential contours. Set the contour selector knob on 5 or the 50-percent level and place the point of the pencil probe on the sheet near the source. Lightly move the probe point across the sheet until the meter indicates a balance, and there make a light pencil mark. (Do not perforate the paper or otherwise scratch the conductive coating on the paper.) Now imagine a circle centered at the source pin whose circumference passes through the pencil mark. Move the probe in a short arc and again lightly move the pencil toward and away from the source until the meter balances. Again, make a light mark, and repeat the probing process until you have gone a full circle. By now a set of points all at the 50-percent potential level have been marked. A line connecting them is the 50-percent isopotential or contour. The probing technique is depicted graphically in figure 12. For greatest accuracy, touch the sheet only with the probe point. Touching the sheet with your hand during probing may affect the measurements.

Set the contour selector successively at 4,3,2, and 1 as well as at the 60- and 70-percent levels and draw contours for these levels in a similar fashion. A complete map would be represented by an infinite number of these contour lines, but for practical purposes, 5 to 10 such contours will generally suffice in delineating the potential distribution. In general the contour intervals should be maintained at some constant difference such as 10 percent (one major scale division) to facilitate interpretation.

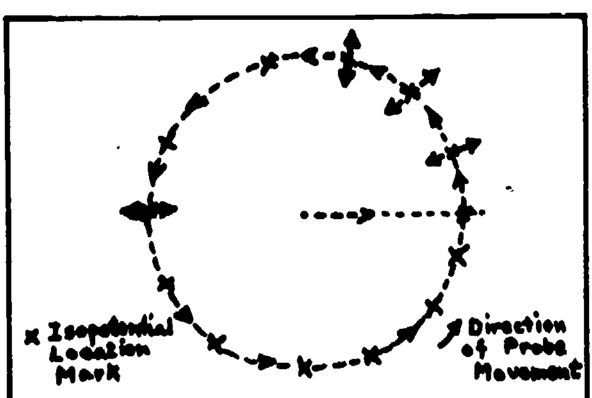


Figure 12. Probing Technique

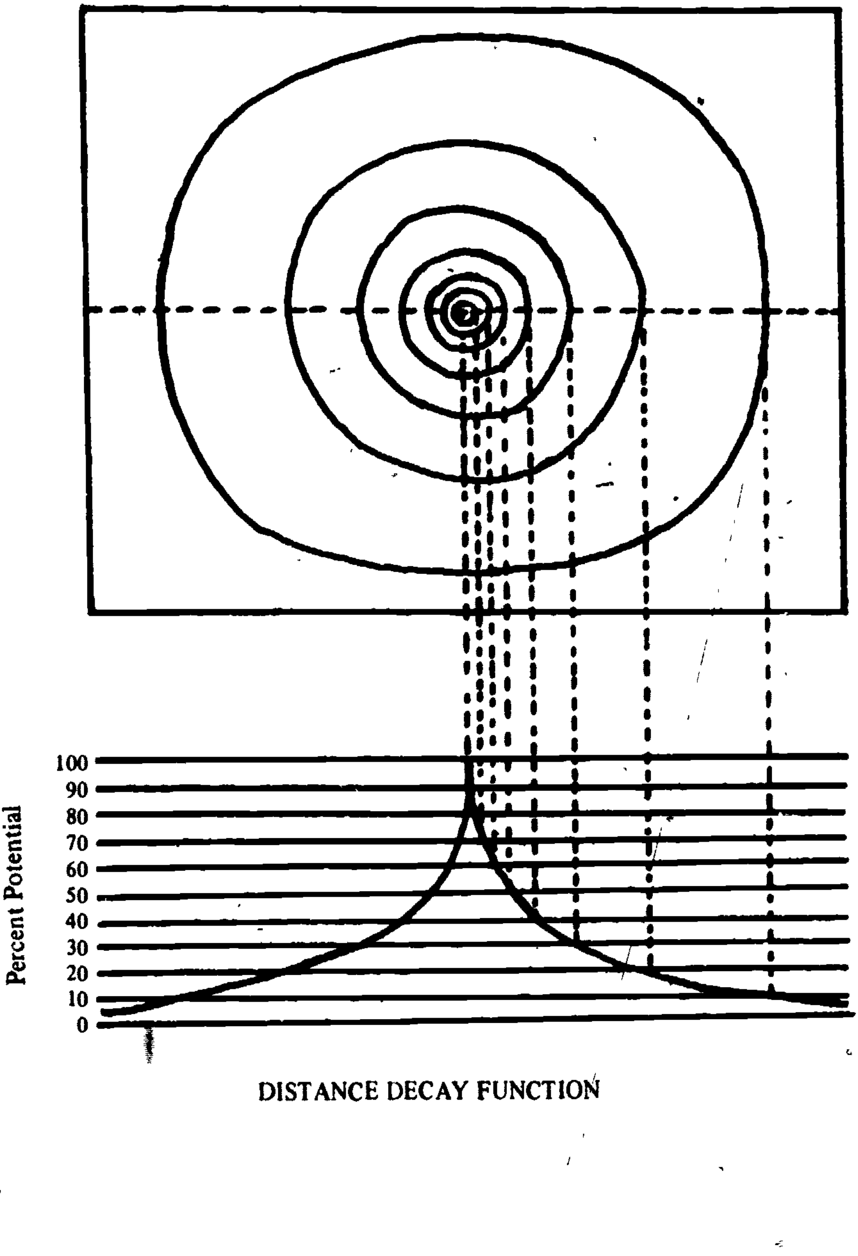


Figure 13. Concentric Zonal Model

You can readily verify the approximate logarithmic decrease of potential with increasing distance from the source. Draw a profile of *potential* versus *distance from the input pin* as shown in figure 13. Construction of the profile is a two-stage procedure. First, lay the edge of a piece of graduated (square-grid) paper along the line between the input point and the mid-point of the right conductive sheet side. Align the origin (corner) of the graph paper with the input point. Mark the intersections of contours with the graph paper edge. Next, divide and label the adjacent edge of the graph paper from zero at the origin, in steps of 10, to 100 (percent). Now plot the contour values corresponding to the markings of contour intersections. A line connecting the plotted points will be a graph or profile of potential versus distance from the input point. To obtain a profile from the input to the left conductive sheet side, repeat the two-stage process.

Looking now at the map just constructed you see the concentric zonal model. This model has been applied to the description of the variations in land uses and functional zones surrounding a city. Land uses such as the central business district, wholesale suppliers, industrial plants, agriculture, etc. are often differentiated into annular zones based on the transportation distances and land requirements. The model has also been used to represent urban growth and in urban population potential studies.

INTERACTION AND GRAVITY POTENTIAL MODELS

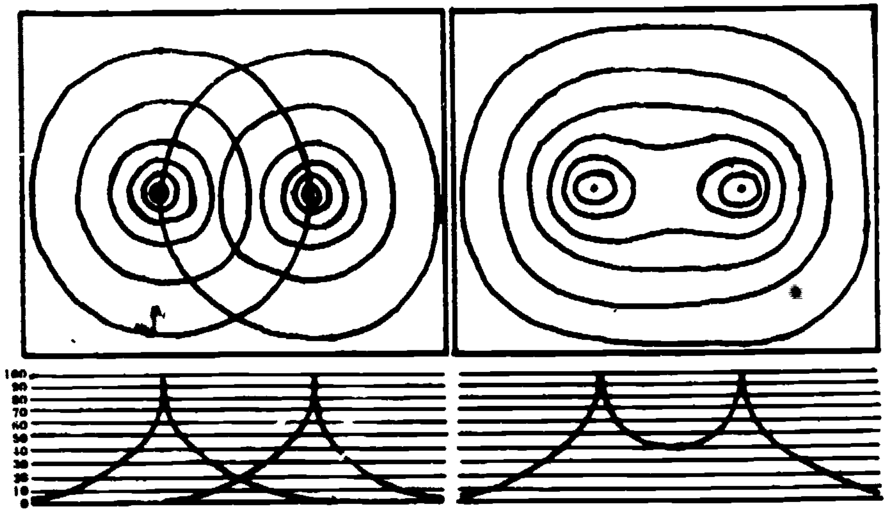
Let us examine another important concept, one generally known as the gravity model. This analog describes the interactions of concentrations of human activity as behaving in direct proportion to their mass (size, density) and inversely proportional to the square of the distances separating them. This is a simple statement of Newton's law of gravitation.

Consider two shopping centers, or a central city business district surrounded by outlying shopping centers. The ability of each of these to attract retail trade is dependent upon its size, parking space, and diversity of services and goods. The attractive potential of each of these can be viewed as decreasing with distance in a manner analogous to the concentric zonal model.

Figure 14 shows the potential maps and graphs of two shopping centers of equal drawing power. As one would expect, the halfway point between the two represents the distance at which their attraction is equal. A potential customer located at this break or crossover point would be equally likely to shop in either. However, if one shopping center is twice as large as the other, as in figure 15, the area of dominance of the larger one is extended, so that the break point is much closer to the smaller one.

The attraction potential maps shown in figures 14-a and 15-a can easily be reproduced on the field plotter by two input pins in the conductive sheet. For figure 14 both inputs would have the same current control settings; e.g., both could have a 10 current control setting. They should be calibrated together – both set on 10 simultaneously. However, the attraction-potential maps would be constructed independently. That is, the current control of one input would be set on zero while the attraction potential of the other input was mapped. To simulate figure 15-a, the current control setting of the larger shopping center would be twice that for the smaller.

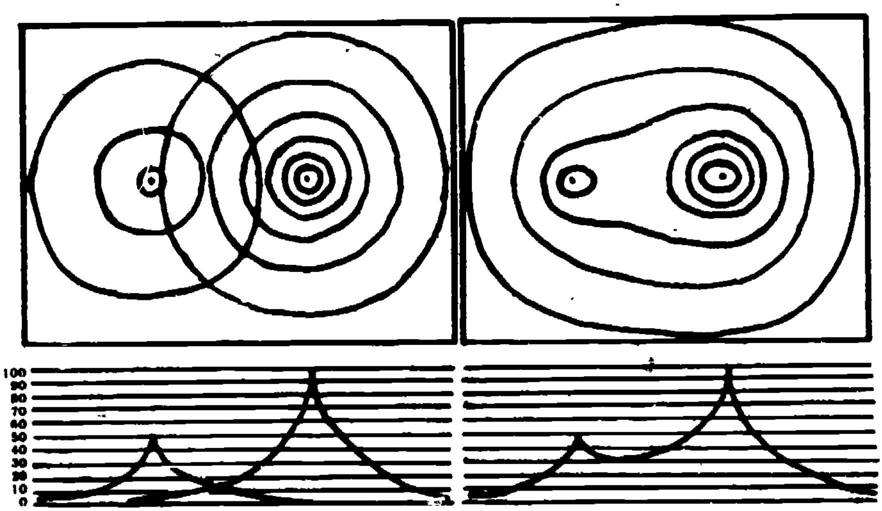
If we were to plot potential with both inputs turned on simultaneously we would draw a map as shown in figures 14-b and 15-b. These maps could also be graphically constructed by adding at each point in figure 14-a or figure 15-a the potentials due to



a. Attraction Potential

b. Interaction Potential

Figure 14. Two Towns of Equal Size



a. Attraction Potential

b. Interaction Potential

Figure 15. Two Towns, One Twice as Large as the Other

the attraction of each shopping center. This additive property of potentials brings us to the definition of another very useful model.

Imagine a metropolitan complex as being composed of a number of concentrations of people or establishments, each interacting with the other. These interactions take the form of flow of people, information, and goods. The result of these interactions is regional growth and the development of areas between these nuclei. As previously discussed, the additive property of potentials allows us to model these interactions by defining interaction potential as the sum of all attraction potentials bearing upon a location.

POPULATION POTENTIALS FOR TWO TOWNS

To demonstrate the application of the concepts just discussed and to more fully exercise the field plotter we shall construct a series of models of two towns, each illustrating a basic modeling technique: these are shown in figures 16 and 17.

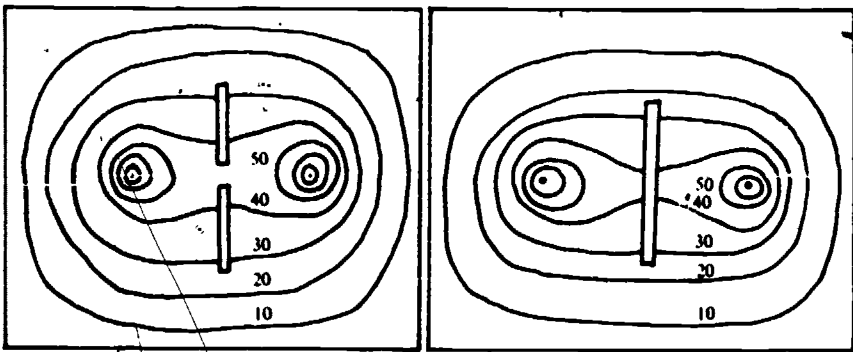
To begin, place a sheet of conductive paper in the field plotter. Insert two input pins (1 and 2) centered vertically and placed one-third of the total width in from each boundary as in figure 16. Next, cut a narrow slit in the conductive sheet as shown in figure 16-a. The resulting cutout area represents a canyon spanned by a bridge (i.e., electrons – people, goods, etc. – cannot cross the cut barriers); the paper is left intact between the two cutouts to simulate a bridge. Clip the 1 and 2 current leads onto the protruding pin shafts. Turn the 1 and 2 current controls to a setting of 10, and the contour selector to 10. Touch the probe to the head of input pin 1 and adjust the calibration control until the balance meter needle is centered. Now touch the head of input 2 with the probe. If the balance meter needle does not move or deflects downward, the model is properly calibrated. If the needle moves up, turn the calibration control until a balance is indicated. The model should now be calibrated. Using the potential pattern shown in figure 15-b as a guide, probe the model, locating and tracing the 10-, 20-, 30-, . . . and 70-percent contours.

The map you have plotted represents the interaction potential model shown in figure 14-b as modified by the cuts in the conductive sheet. Where the cuts have been made, there is an absolute barrier to the flow of electrons between the cities. As a result, the interaction potential of the towns is significantly reduced. The flow of people, goods, etc., is channeled across the bridge or around the canyon: thus the contours are stretched toward the bridge.

Figure 16-b represents the effect of a bridge washout on the interaction potential (or population potential of the area between the towns). To duplicate this model, cut out the bridge, recalibrate, and replot the map.

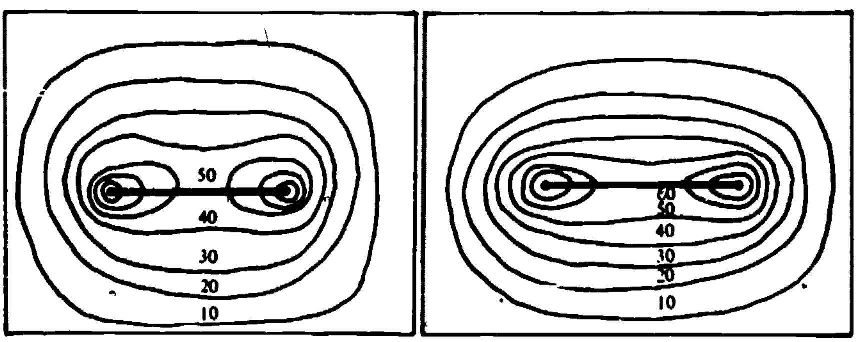
To model the concept of a current expediter such as a road, remove the two-town barrier model from the field plotter and replace it with a new sheet of paper. Mark the location (the same as before) of the input pins lightly with a pencil. Next apply a thin strip of carbon conductive paint with a small brush, in a line connecting the two input points, as shown in figure 17-a. Allow 5 minutes for the paint to dry, insert the pins and set the 1 and 2 current controls, calibrate and plot as before. The painted strip, representing a road between the towns, conducts electrons much better than the unpainted sheet. Thus, interaction of the towns is expedited by the presence of the road. Compare this model with figure 14-b.

To simulate, say, the effect of building a much improved highway between the two towns, apply two more layers of carbon paint (allowing the first layer to dry before painting on the second). The resulting strip is much more highly conductive than a single-layer strip. Recalibrating and replotting will produce the output shown



a. Canyon and Bridge b. Absolute Barrier

Figure 16. Two Towns and a Barrier



a. Road–One Coat of Paint b. Road–Three Coats of Paint

Figure 17. Two Towns and a Road

in figure 17-b. Notice that the higher potential level contours have moved closer to the road. This represents the development of a higher density corridor between the towns.

The influences being simulated for two cities can be observed in the real world. An Apollo photograph of the Dallas-Ft. Worth metropolitan region depicts the two cities growing together along field lines (figure 18). Compare the lines of growth in the photographs with figures 14 through 17. As a special challenge you may wish to attempt a field plotter simulation of this photograph. If you try it and meet only partial success, don't be too disappointed; after you have completed section V come back and try it again. If you do succeed to a fair degree, then try to improve on your performance after you have performed the experiments to follow.

SCALING

One last aspect of field plotter operation – scaling – needs clarification. Scaling has to do with the manipulation of data being prepared for input as well as manipulation of output data. Concerning inputs, two factors must be kept in mind – first, the inputs are relative, not absolute, and second, the largest input should be scaled to a current control setting value of at least 6.5. The value of 6.5. or greater assures that the field plotter will be in its optimum operating range; that is, where it will produce the most consistent results. This means that the highest value of a set of inputs should be set at whatever value between 7 and 10 facilitates setting the proper relative values of the other inputs. If the first attempt at scaling yields a value for the largest input that falls in the 6.5-10 range, no further adjustments are required. If, however, the largest input value is between 5 and 6.5, all inputs should be multiplied by 1.5; between 3.3 and 5.0, all the inputs should be doubled; between 2.2 and 3.3, they should be tripled, etc. In summary, survey the real-world values of all inputs and find a convenient factor to multiply (or divide) them by to bring the largest value into the operating range. Remember to recalibrate after each change in inputs because *only when the unit is calibrated* can the numerical value be interpreted quantitatively. You may verify the relative nature of inputs by increasing them or decreasing them all by the same percentage, remaining in the operating range (the principal input above 6.5), recalibrating, and probing the output. Results will be identical.

Inputs may be transformed by operating on input values before current control settings are made. For example, to model a particular process it might be more meaningful to base current control settings on the square of an input parameter than on the parameter itself. If two parameter values were 5 and 9 respectively, the squared input values would then be 25 and 81. A transformation such as this would obviously be useful if it were desirable to model a process where more emphasis is placed on larger parameter values than would be expected solely from arithmetic considerations.

Conversely, it may be desirable to transform output potential contour values. An example of a transformation of this nature would be to take the square root of output potential contour values. Such a transformation would tend to increase the spacing of the higher value contours, and move together the contours at lower potentials. The 100-, 90-, 80-, 70-, 60-, 50-, 40-, 30-, 20-, 10-percent contours in the transformed model would be the original 100-, 81-, 64-, 49-, 36-, 25-, 16-, 9-, 4-, 1-percent potential contours. The result of applying such a transformation to the concentric zonal model of figure 13 is shown in figure 19. It can be seen that a nearly

Figure 18. Dallas-Ft. Worth, Texas, Metropolitan Area As Seen From Apollo VI, April 13, 1968

uniform separation (linear relationship) now exists between contours in the 20- to 60-percent range, and that contours are more widely separated in the high-potential areas and more closely spaced in the low-potential areas.

A transformation of either input parameters or output potential contour levels can be very useful in describing certain phenomena which defy description in the absence of any such transformation. Although transformations are not frequently needed (depending on the class of phenomena modeled), their proper use can provide valuable insights into the underlying relationships governing the behavior of certain phenomena. If it were found, for example, that the growth of cities is more properly described by applying either an input or output transformation, then the very nature of the required transformation would yield valuable information with regard to the factors that influence growth.

This concludes the material that has been found most necessary for persons to understand the experiments that follow. Return to this section if you experience any serious difficulty. Also, if you scanned sections II and III, you may wish to go back and review them before proceeding to section V.

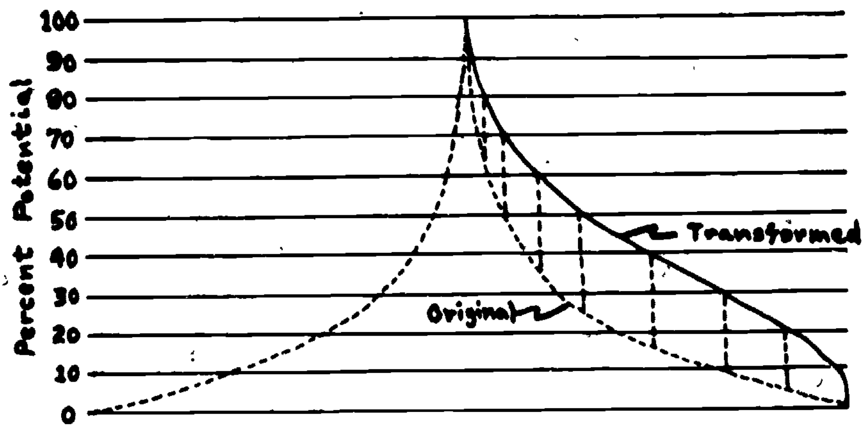


Figure 19. Transformation of the Concentric Zonal Model

V. MODELING TECHNIQUES

INTRODUCTION

The preceding sections have set forth the basic information necessary for understanding and operating the analog field plotter. In the interest of maintaining maximum flexibility in presentation, the following experiments are discussed only in the detail necessary to give a grasp of the application of the modeling techniques. A complete background for each of the models is beyond the scope of the present paper. Such background material is readily available in the geographic literature for most of the models.

The experiments are arranged and presented in such a way as to lead the reader from simple to complex models. The various techniques are introduced where they can be most easily understood. They were selected and arranged according to a design for teaching the principles of modeling, not for teaching the content or significance of the models themselves. The experiments presented here are not exhaustive, but are meant to provide a basis for the disign of models of interest to the reader. Most college freshmen have been able to use the field plotter creatively and efficiently once they have conducted and/or studied the following experiments.

It is not necessary for each student to work each experiment. We have had good s from assigning experiments to groups of two or three students. Subsequently, the various teams are brought together to discuss their work with the entire laboratory section. Thus it is possible for a given student to gain a thorough understanding of each exercise while actually working only 10 percent to 15 percent of them. We have found it desirable, however, to have each student do the first experiment. Where most students are conducting the same experiment, time can be saved and identical results can be assured by replacing some of the detailed instructions with standardized cardboard templates on which are printed the exact location of the inputs.

Three groups of models are presented. The first and largest group consists of conceptual and theoretical models; they illustrate selected concepts and principles of field plotter operation using hypothetical study areas. This first model group is divided into four series: (1) single-point inputs that simulate attraction (or gravity) potential models; (2) multiple-point inputs that describe interaction potential models; (3) line inputs that describe flow models; and (4) iterative inputs that allow the construction of hexagonal models. The second group, series 5, deals with models of analysis; they illustrate modeling with known input parameters. The third group, series 6, deals with models of synthesis; these illustrate the methods for varying the input parameters to simulate a known distribution.

The organization of this section is designed to allow rapid review of each series of experiments. For each series, first read the initial introductory paragraph. Next, review quickly the model illustrations and the accompanying figure titles. Then proceed more slowly through these illustrations, reading the legend below each figure. After the figures have been so reviewed, you may either proceed to the next series or read the detailed instructions on the page facing each illustration.

A. CONCEPTUAL AND THEORETICAL MODELS (SERIES 1-4)

Series 1: *Single-Point Inputs -- Attraction-Potential Models*

The first series is designed to familiarize you with models based on a single internal input. This is the basic pattern to which most of the subsequent work will be related. You will begin simply, and gradually learn ways in which the basic model can be altered to more closely simulate real-world phenomena. You will consider the effects of boundary shapes, partially grounded boundaries, barriers and roads. Each step will lead you to understand the operation of the model and its geographic interpretations.

The series includes:

a. Grounded Rectangular Boundary
b. Grounded Circular Boundary
c. Grounded Irregular Boundary
d. Rectangular Boundary With Three Sides Grounded
e. Rectangular Boundary With Two Sides Grounded
f. Rectangular Boundary With One Side Grounded
g. Ungrounded Rectangular Boundary With One Interior Point Grounded
h. Absolute Barrier in Northwest
i. Absolute Barriers in Northwest and Northeast
j. Partial Barriers With an Intervening Pass
k. Absolute and Partial Barriers and an Intervening Pass
l. Three Low-Resistance Roads Not to Ground
m. Three Roads of Different Grounding and Conductivity
n. Eight Low-Resistance Roads to Ground
o. Eight Higher Conductivity Roads to Ground
p. Eight Low-Resistance Roads Not to Ground
q. Eight Higher Conductivity Roads Not to Ground
r. Eight Radial Roads Straddling Partial Grounds
s. One Internal Input Not Centrally Located

Experiment 1-a. Grounded Rectangular Boundary

Purpose – to illustrate the simplest single point input model. This model serves as a departure for all other models.

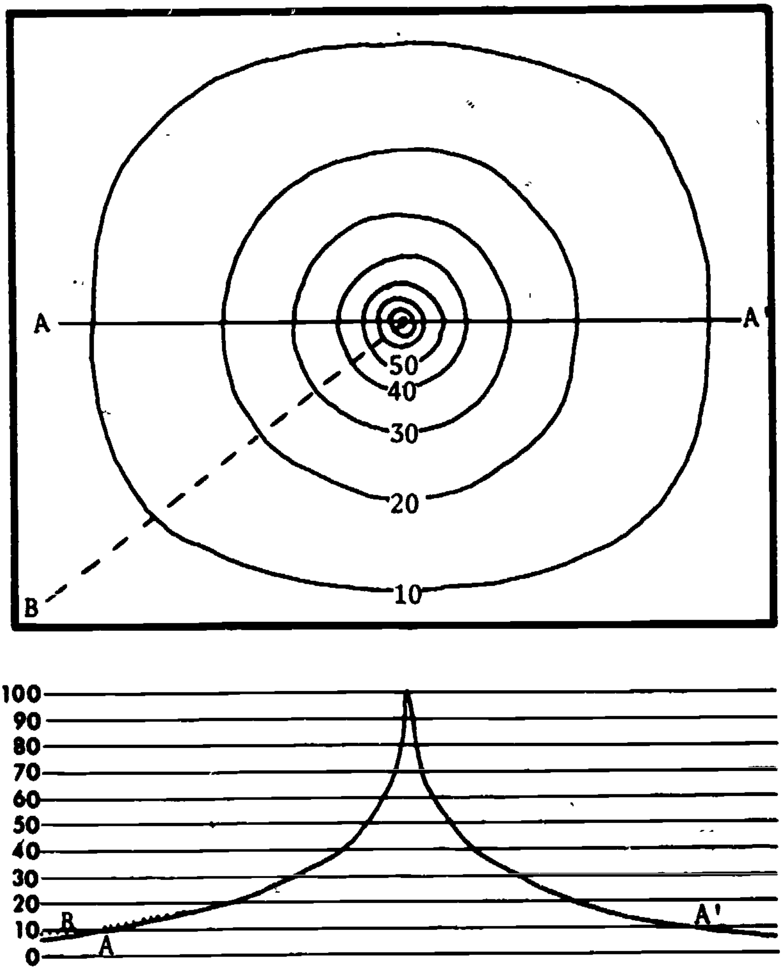
Modeling Procedures:

1. Follow carefully the instructions for setting up and operating the field plotter in section IV. Review those instructions if necessary.
2. Center a sheet of conductive paper on the easel and clamp the hold-down frame in place.
3. Delimit the working area of the paper. Using a wax pencil or chalk, lightly draw a border on the paper using the inside edge of the hold-down frame as a guide.
4. To locate the center of the paper, draw a line near the center of the paper along a straight edge running diagonally from one corner of the hold-down frame to the opposite corner. Repeat with the other pair of corners. The point where the two diagonals cross is the center.
5. Place an input pin in the center of the paper, clip a numbered current lead to the pin shaft, and turn the corresponding current control to a setting of 10.0.
6. Calibrate.
7. Probe the contours with values ranging from 70 to 10 percent at intervals of 10 (i.e., 70, 60, 50, ..., 10).

Output Description:

1. Remove the input pin and draw a continuous horizontal line from the middle of the left side to the middle of the right side, running through the center of the model. Label it A-A'.
2. Draw a dashed line from the center of the model to the lower left corner. Label the line B at the corner.
3. Think of the model as a map with north at the top and east to the right.
4. Plot a solid-line profile from east to west along A-A'. Note the slope of the profile. It is symmetrical about the center and decreases at a logarithmic rate away from the center.
5. Plot a dashed profile from the center to the southwest corner, B. Compare it with the cross-section A-A'. It varies from A-A' only near the boundary. The difference between the two profiles reflects the impact of a grounded rectangular boundary.

Interpretation of Results. The output of this model is the basic pattern of the gravity or attraction potential model. It may illustrate both the way in which a node affects the surrounding region and the way in which any point in a region affects a node.



Experiment 1-a. Grounded Rectangular Boundary

Current radiates from the center to the rectangular grounded boundary. The potential is highest near the input and decreases (at a decreasing rate) away from the input. The contours near the center of the model form concentric circles while near the margins they tend to conform to the shape of the grounded boundaries. Note on the profile that the east-west slope (A-A') is almost identical to the slope down to the southwest corner (B). This is the basic pattern of the gravity or attraction potential model; it is fundamental when relating any or all points in nodal region to the node.

Experiment 1-b. Grounded Circular Boundary (Uniform plane)

Purpose – to illustrate the effect of a circular grounded boundary on the attraction-potential model. This is the model most commonly used in standard geographic references to attraction potential models (gravity models).

Modeling Procedures:

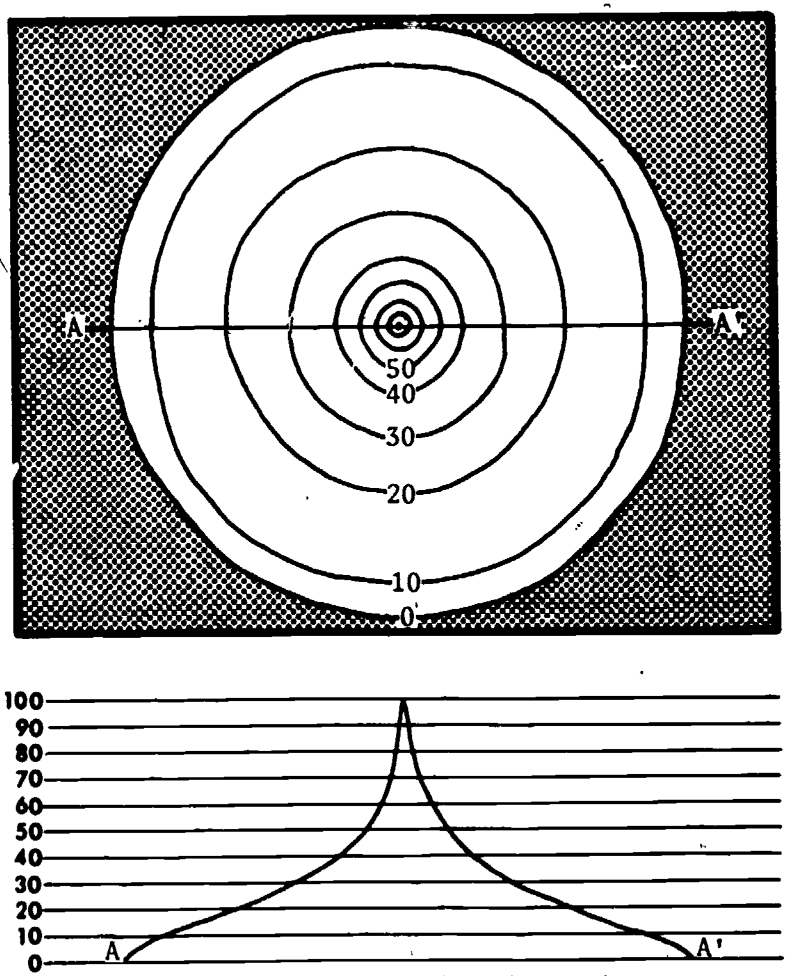
1. Follow modeling procedures 1 through 4 of experiment 1-a.
2. Draw a circle. Use a compass or short piece of string and draw about the center a circle that falls just short of the north and south borders (hold-down frame edges).
3. Ground the circle. Lay a piece of flexible wire around the edge of the circle and secure it to the conductive paper with a continuous strip of narrow masking tape. Make sure that the wire is lying flat on the paper to ensure good electrical contact. Wrap one end of the wire around input pin "G," insert the pin through the easel at a convenient point *outside*, the circle, and attach the ground current lead to the ground pin shaft protruding from the rear of the easel.
4. Follow modeling procedures 5, 6, and 7 of experiment 1-a.

Output Description:

1. Follow output description procedures 1, 3, and 4 of experiment 1-a.
2. Study the profile and note that the profile A-A' describes the slope of the curve in any direction from the center. If this symmetry is not obvious to you, plot another profile through the center.
3. Compare the output with that of experiment 1-a. Note that the boundary geometry is the only significant change.

Interpretation of Results. The output is the circular case of the basic pattern of the attraction potential model. It relates any and all points of a circular nodal region to the node. With the proper transformation of contours it describes the Burgess theory of concentric zonal differentiation of urban land uses. The model simulates perfectly the uniform (accessibility) plane assumptions of such theories since the electrons flow with equal regularity in all directions from the center.

Further Notes. This model can also be obtained by painting the circle with silver paint, inserting the ground pin into the wet paint, and letting the paint dry overnight. Combining silver paint with wire held down by thumb tacks insures excellent results.



Experiment 1-b. Grounded Circular Boundary

Current diffuses uniformly to all points on the grounded boundary. Any half of the model forms a mirror image of the opposite half. The model demonstrates variations in land values within a region dominated by a central place. With some modification, it demonstrates the Burgess theory of differentiation of land use. A uniform plane is being simulated perfectly since the circular boundary forces the electrons (goods, services, people, etc.) to *flow* uniformly in all directions. Recall the triangular geometry of figure 5 in section III. Compare the slopes of the two profiles; notice how closely the triangular model approximates the sector of a circle.

Experiment 1-c. Grounded Irregular Boundary

Purpose – to illustrate the attraction potential model in cases where the boundary geometry is not as simple as a rectangle or circle. This model is a more realistic application of the attraction-potential model.

Modeling Procedures:

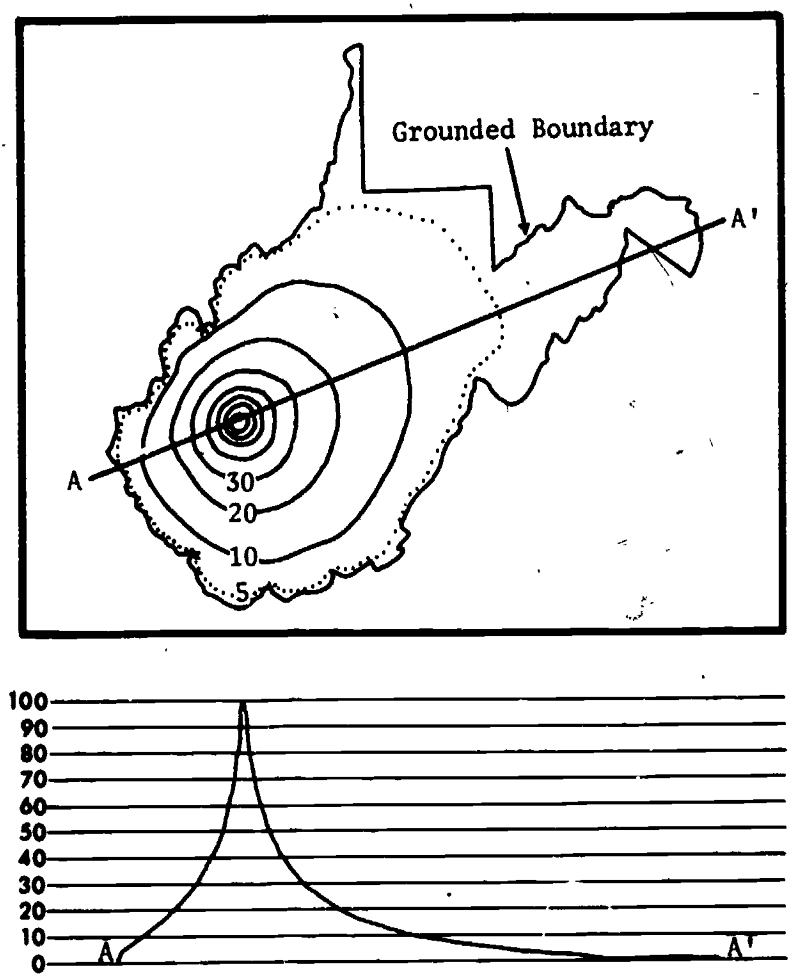
1. Select a state (or other political unit) in which you are particularly interested. Find a map of the area at a scale that barely permits the boundaries to fit within the hold-down frame.
2. To draw the outline of the state (or other political unit) on the conductive paper, place a piece of conductive paper underneath the map with the black side toward the map. Go over the political boundary with a stylus or pencil, and the outline will be embossed lightly on the conductive paper.
3. Ground the boundary. Use wire or silver paint, as in modeling procedure 3 of experiment 1-b.
4. Make an input at the location of the capital city (or political center). Make sure that the distance to the nearest part of the ground is at least one-half inch; if necessary, move the city or the boundary. The value of the input can be variously scaled according to the population, daily newspaper circulation, etc., of the political center.
5. Probe the model with a contour interval of 10 percent from 10 to the highest measurable value. Probe and draw supplementary contours in dashes where needed to enhance pattern definition.

Output Description:

1. Plot a profile. Select a straight line that passes through the input and some other interesting part of the output. Plot the profile along that line.
2. If you can't visualize how a profile would look elsewhere on the output, draw other profiles until you can visualize the entire surface. For guidance, it would help to study profiles of other experiments in this series.

Interpretation of Results. The output may be interpreted as a measure of the degree of interest in the routine political proceedings in the political center. Another interpretation might be the degree to which the citizen is politically aware. Yet another application might be the regional impact of a daily newspaper published in the political center.

Further Notes. Consider other possible phenomena that might be described realistically by the model.



Experiment 1-c. Grounded Irregular Boundary

Resistance is less and current density is greater in areas between the input and the nearby sections of the sheet boundary than in areas between the input and remote parts of the boundary. Consequently, the potential pattern portrayed by the contours is irregular. The outline of West Virginia is uniformly grounded, forming an absorptive boundary which lowers the values near it and permits no influence to or from outside areas. Charleston is the only input. The model may be interpreted as a first approximation to the degree of interest in the general, routine proceedings of the state legislature. Most of the interest is in the state capital or nearby, and decreases rapidly with distance.

Experiment 1-d. Rectangular Boundary With Three Sides Grounded

Purpose – to illustrate what happens to the fundamental model of attraction potential when one of the sides (the north) is not grounded. Changing but a single boundary makes possible a "feel" for the impact of such a change.

Modeling Procedures:

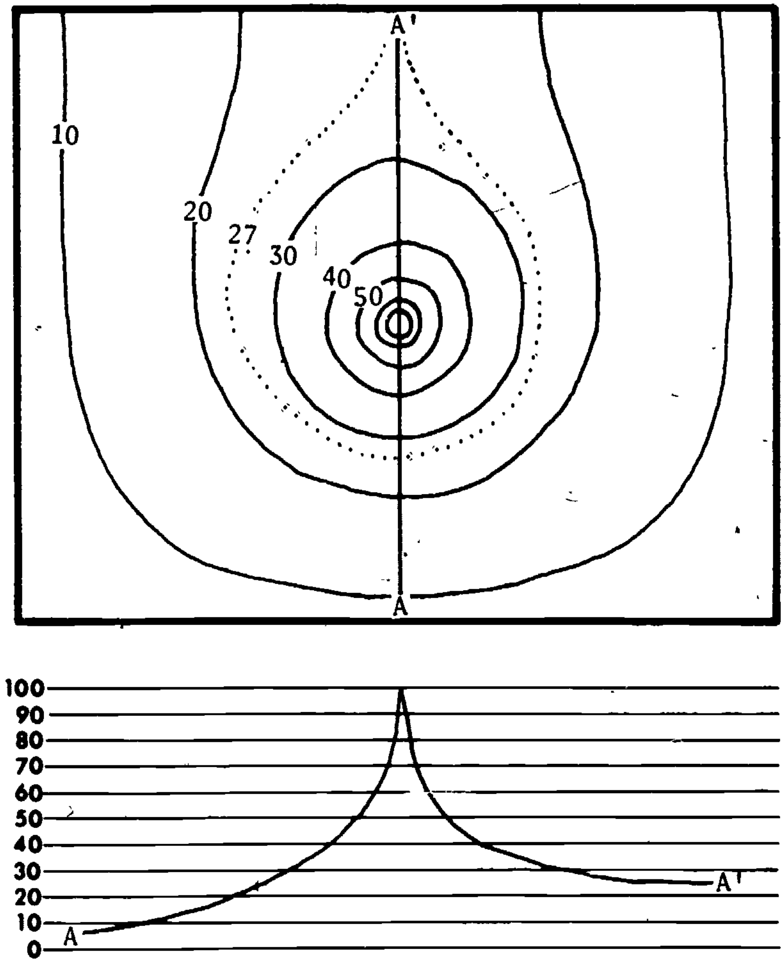
1. Follow modeling procedures 1 through 5 of experiment 1-a.
2. Cut the conductive paper along the north boundary. Use a sharp knife and make a clean cut, with the conductive paper clamped onto the easel; be careful not to cut the easel corkboard. If necessary, raise the hold-down frame, lift the north edge of the paper and cut the boundary with a pair of scissors.
3. Follow modeling procedures 6 and 7 of experiment 1-a.
4. Draw a supplementary, dashed-line contour corresponding to the lowest potential value to be found along the northern boundary. Touch the probe to the middle of the north side, turn the contour selector until a null is achieved, record the potential value of this position. Move the probe a short distance to the right and to the left. If the null meter needle deflects upward in both directions, you have found the point of minimum potential value along the northern boundary. Probe this value contour throughout the whole model.

Output Description:

1. Plot a profile along a north-south line through the input point. Label the south terminus A and the north terminus A'. Compare the profile slopes north and south of the input. The difference reflects the imposed boundary condition.
2. Note that a profile east-west through the center of this model would not differ greatly from the east-west profile of experiment 1-a. If this is not obvious, draw an east-west profile.

Interpretation of Results. Geographic spatial diffusion theory elucidates two boundary types – reflective and absorptive with opposite effects. A reflective boundary acts as a mirror repelling the influence that would extend beyond the boundary of an area back into the area, thus raising values near the boundary; the absorptive boundary merely stops the propagation of the influence. The northern boundary can be interpreted as a reflective boundary and the other boundaries as absorptive.

Further Notes. The next two experiments elaborate on the above interpretation.



Experiment 1-d. Rectangular Boundary With Three Sides Grounded

The north boundary is cut to form a barrier to electron flow. Electrons that flow north from the input pin must sweep either east or west to reach a grounded side. The *entire* model may be interpreted as one city and its surrounding region that form the *south half* of a larger symmetrical model of two identical cities on a uniform plane. Thus the model represents the interaction potential of the two cities, which is the sum of their *attraction* potentials. Point A' on the 27-percent contour marks the breakpoint of the influence between the two cities, and the low point on a ridge of high values connecting the two cities.

Experiment 1-e. Rectangular Boundary With Two Sides Grounded

Purpose – to further illustrate what happens to the fundamental model of attraction potential when two sides (the north and west) are ungrounded, and to show how it can be interpreted as an interaction-potential model.

Modeling Procedures:

1. Follow modeling procedures 1 through 5 of experiment 1-a.

2. Cut the conductive paper along the north and west boundaries, following the instructions of modeling procedure 2 of experiment 1-d. Pin down the northwest corner to hold the sheet on the easel.

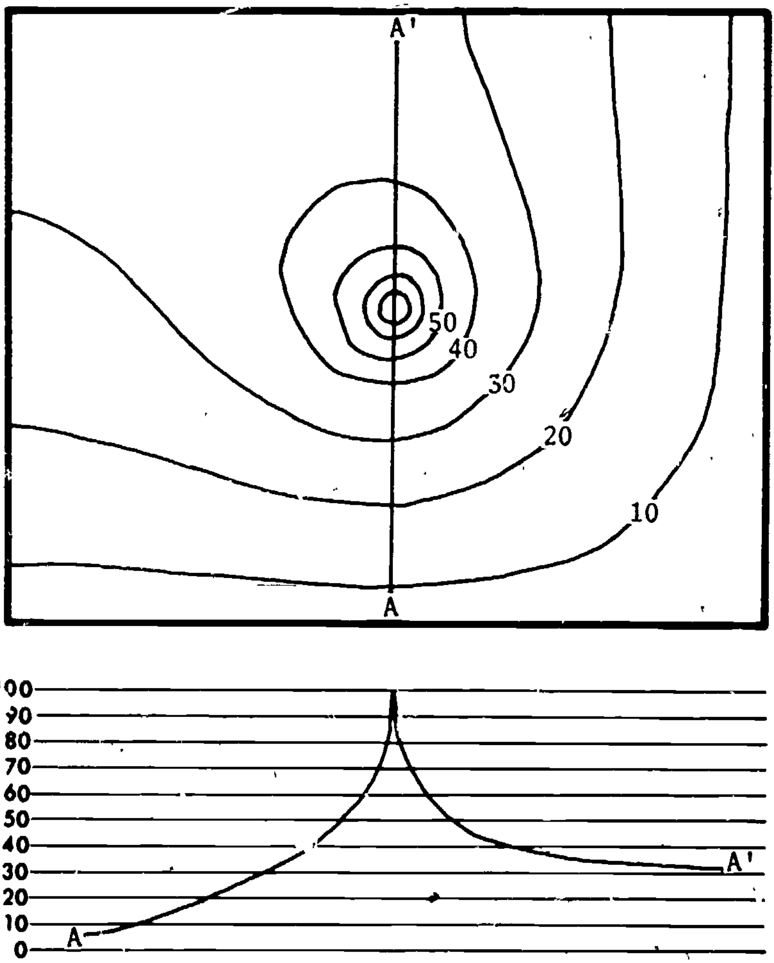
3. Follow modeling procedures 6 and 7 of experiment 1-a.

Output Description:

1. Follow output description procedure 1 of experiment 1-d.

2. Compare the potential pattern and profile with those of 1-d. Note that the "plateau" from the center of the model to A' is higher and flatter in experiment 1-d. Note also that the plateau of this model extends to the northwest.

Interpretation of Results. Continuing with the interpretation of experiment 1-d, the reflective boundaries formed by the north and west sheet edges act as mirrors that reproduce, in areas to the north and west, reflections of this model's potential pattern. This reflected potential pattern can be viewed as the interaction potential map of three cities – the sum of the attraction potentials (as in experiment 1-a) for a city to the north, a city to the west, and the one you have modeled. If one assumed that rural land values on land of similar quality are a function of proximity to cities, then the interaction potential could be interpreted as indicating rural land values. The highest intercity values occur along the ridge of high potential extending to the northwest. That area is closest to all three cities; thus it has good accessibility to markets in all three.



Experiment 1-e. Rectangular Boundary With Two Sides Grounded

The west and north boundaries are cut, thus deflecting electron flow; electrons flowing west must turn south and those flowing north must turn to the eastern grounded side. This model can be interpreted as one-third of a larger model of three identical cities (one north and one west), all on a uniform plane. Higher rural land values stretch northwest from the city because of that area's greater proximity to all three cities. A farmer in the southeast does not benefit from higher land values, when the other two cities are considered, because of his nearness to but one city and thus higher transportation costs or a smaller market for his produce.

Experiment 1-f. Rectangular Boundary With One Side Grounded

Purpose – to illustrate attraction (interaction) models with all but one side ungrounded.

Modeling Procedures.

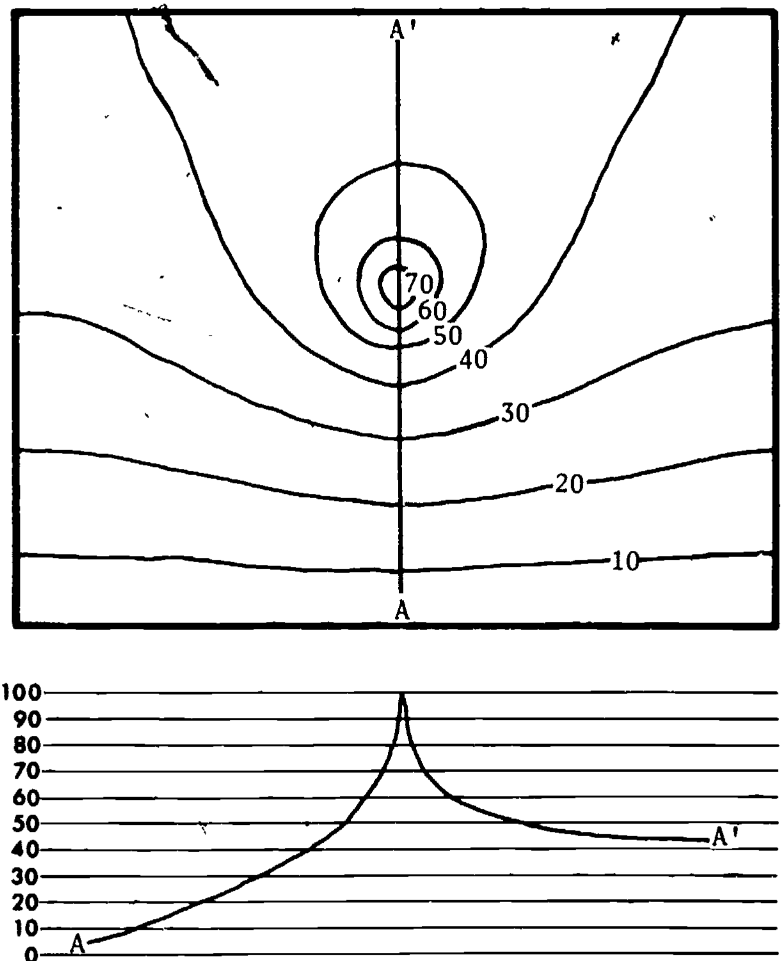
1. Follow modeling procedures 1 through 5 of experiment 1-a.
2. Cut the boundaries on the north, east, and west sides. Follow modeling procedure 2 of experiment 1-e.
3. Follow modeling procedures 6 and 7 of experiment 1-a.

Output Description.

1. Follow output description procedure 1 of experiment 1-d.
2. Compare the map and profile with those of 1-d and 1-e. Note that the plateau of high values now trends due north, and that the values are higher than in the previous experiments.

Interpretation of Results. Continuing with the interpretations of experiments 1-d and 1-e, the reflective boundaries now dominate the pattern and the absorptive boundary to the south begs explanation. No influence from the south is permitted beyond the boundary and so all of the area in the region with a potential lower than 30 percent is found in the south half. This area is remote from the mainstream of economic and social activity.

Further Notes. An entirely different interpretation could be variable resistance. One could assume that it is a simple attraction potential model of one city. The south half of the map is less influenced (greater resistance) by the city than the northern half (less resistance).



Experiment 1-f. Rectangular Boundary With One Side Grounded

Electron flow in all directions from the input must terminate on the grounded south side. The model may now be interpreted as one of four cities (one north, one west, and one east of the city indicated by the model input). It is assumed that each of the other three cities is the same size and is located in the same relative position in regions of the same size. Note on the cross-section the ridge of high land values extending toward the north. If the three neighboring cities were located centrally in regions of the same size, then the one to the north would be closer to the one shown than either of the other two, thus, the higher the land values between them.

Experiment 1-g. Ungrounded Rectangular Boundary With One Interior Point Grounded

Purpose – to introduce the concept of interior point grounding.

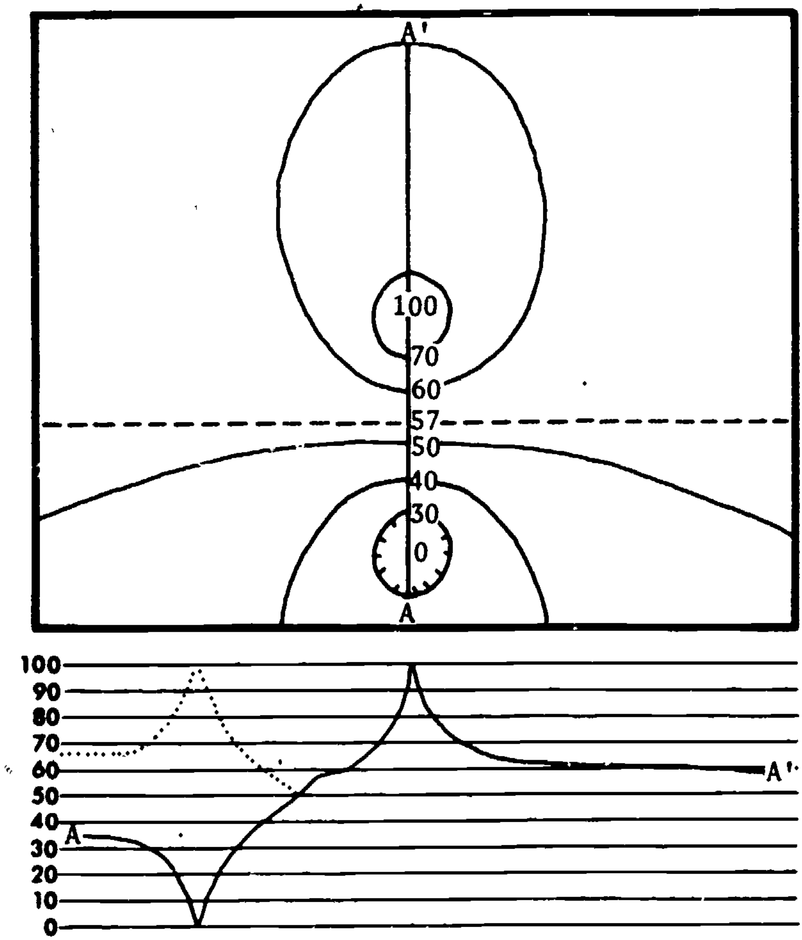
Modeling Procedures:

1. Follow modeling procedures 1 through 5 of experiment 1-a.
2. Lift the hold-down frame and leave it in the fully raised position during this experiment.
3. Locate the middle of the bottom side of the conductive paper. Insert the pin marked "G" at a point up 2 inches from the bottom-edge midpoint. Clip the ground lead to the ground pin shaft.
4. Calibrate.
5. Probe the 70-, 60-, 50-, 40-, and 30-percent contours.
6. By trial-and-error probing locate and note the value of the contour that follows a straight line between the input and ground pins.

Output Description:

1. Remove the two pins and draw a line from the middle of the bottom edge through the two pin locations to the middle of the top edge. Label this line A-A'.
2. Plot a potential profile along A-A'. Note the symmetry of the profile about the 50-percent potential level.
3. On the same grid, replot the portion of the profile below 50-percent, by replacing the plotted values by their complements; i.e., replot 40 as 60, 30 as 70, and zero as 100. The resulting dashed-line graph should be the mirror image of the corresponding original part of the profile.

Interpretation of Results. Construction of a mirror image of the potential profile below the 50-percent level allows the interpretation of the two points, as cities, with the slope of the profile indicating the degree of their relative trade dominance. When the profile slope is almost vertical, the dominance is almost complete; when the slope is horizontal, their relative dominance is negligible. The input pin has a larger trade area than does the ground pin because of their relative positions on the sheet. If the paper were cut to the known shape of the actual trade areas, the boundary would take on more meaning than does the rectangular boundary of this model. Another interpretation of the potential contours is that they represent successive stages of urban growth. The city located at the current input is growing faster and increases its dominant trade area at the expense of the other city. The dominant trade area of the southern city shrinks with time until it is wholly enclosed by that of its neighbor.



Experiment 1-g. Ungrounded Rectangular Boundary With One Interior Point Grounded

Electron flow emanates in all directions from the input pin in the center, but it must converge upon the grounded point in the south. One geographic interpretation of the resulting contours is that they are boundaries of trade dominance between two cities, derived by subtracting from 100, values below the 50-percent level on the profile and replotting them. The case of rapid growth of the northern city relative to the southern city would allow the interpretation of the 50, 40, and 30 contours as approximations to the successive locations of the lines of expansion indicating the area-of-trade dominance of the more dynamic city. The trade area of the southern city becomes restricted to the encircling 30-percent contour.

Experiment 1-h. Absolute Barrier in Northwest

Purpose – to illustrate the impact of a cut (ungrounded) barrier on the basic attraction potential model. Such a barrier relaxes the uniform-plane assumption inherent in the previous experiments.

Modeling Procedures:

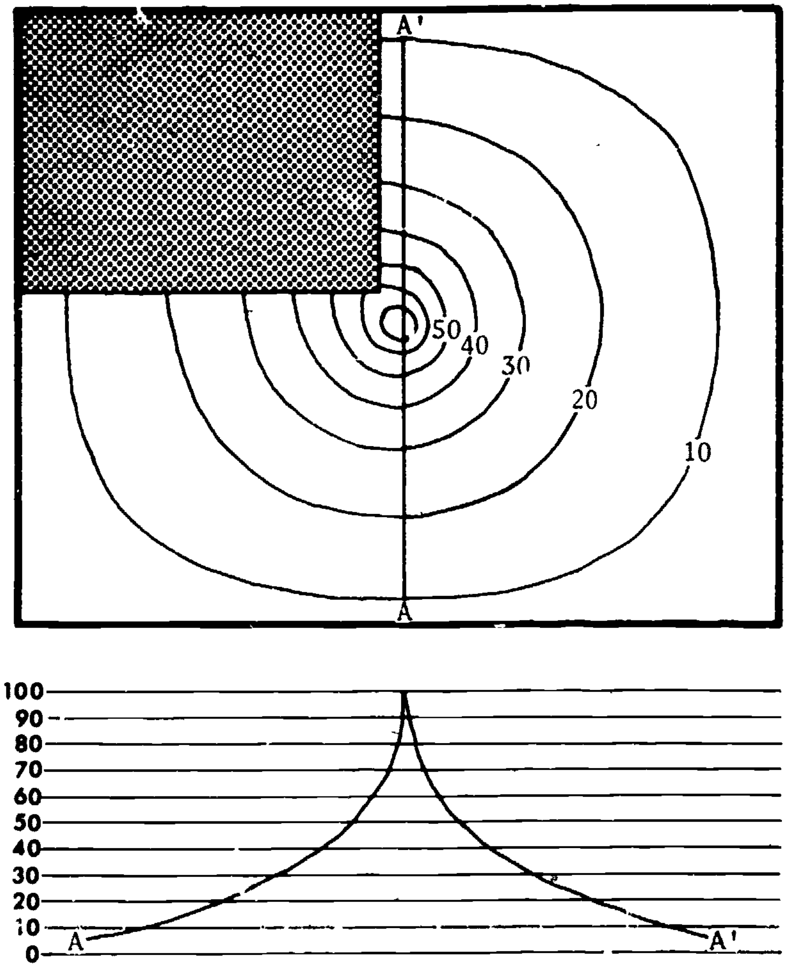
1. Follow modeling procedures 1 through 5 of experiment 1-a.
2. Draw a line from the input pin due north to the edge of the hold-down frame and another line due west to the edge. Also draw lines parallel to these, one line ½ inch west of the north-running line and another ½ inch north of the line running west. Cut the conductive paper along the edge of the rectangle defined by the second set of lines and the north and west hold-down frame edges. Remove the rectangular paper cutout.
3. Follow modeling procedures 6 and 7 of experiment 1-a.

Output Description:

1. Remove the input pin and extend the north-south line from the point of input to the south border. Label this line A-A'.
2. Draw a profile along A-A'. Note the flattening slope (lower rate of decrease) of the portion of the profile north of the input.

Interpretation of Results. The absolute barrier is preventing electrons from flowing in a northwesterly direction. It is also acting as a reflective boundary as described in experiments 1-d, -e, and -f. Consequently, there is a tendency for higher potential values to be found in the vicinity of such absolute barriers. One geographic interpretation of the output is the growth pattern of a town at the edge of a square lake. (The shape of the lake shoreline could have been cut more realistically.) The contours represent probable stages in the growth of the city. The city spread from the original site (enclosed by the 70 percent contour) in all directions, but preferentially grew toward the lake (60 percent contour) early in its development, continuing to the present. The greater relative attractive influence of the lake is clearly observable on the profile, but is not overwhelmingly dominant. Thus, this model is more like a city on the Great Lakes than a city on the Florida coast.

Further Notes. The absolute barrier could be grounded along its margins, and the opposite effect would be produced. The absolute barrier would become an absorptive barrier, reducing potential values near it (in comparison with the ungrounded barrier or uniform-plane models).



Experiment 1-h. Absolute Barrier in Northwest

Electrons cannot flow into the northwest quadrant because the conductive paper has been removed. The uniform plane assumption has been relaxed somewhat Since current cannot flow into the cutout area, it flows along and away from the cut, it does not diffuse uniformly. Note the tendency for higher value contours (particularly the 60-percent isopotential) to be drawn towards the barrier, the lower contours to be extended further along the barrier Notice that all contours intersect ungrounded barriers at right angles (this is always true) Conversely, contours tend to parallel grounded boundaries

Experiment 1-i Absolute Barriers in Northwest and Northeast

Purpose – to illustrate the difference in the potential distance-decay function when electron diffusion in one part of a region is radial and in another part it is restricted to a channel (linear flow).

Modeling Procedures:

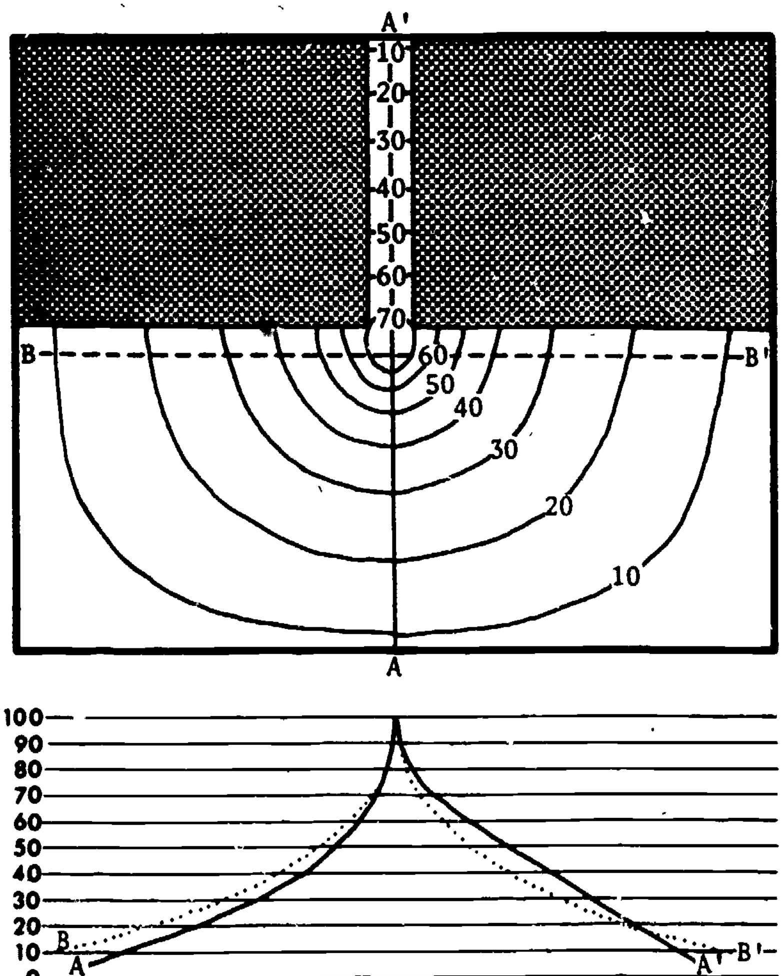
1. Follow modeling procedures 1 through 5 of experiment 1 a.
2. Follow modeling procedure 2 of experiment 1-h.
3. Remove a cutout from the northeast quadrant the same size as the northwest cutout in 1-h. Repeat modeling procedure 2 of experiment 1-h, substituting the word "east" for "west."
4. Follow modeling procedures 6 and 7 of experiment 1-a.

Output Description:

1. Follow output description procedure 1 of experiment 1-h.
2. Label as B-B' the east-west line through the input point.
3. Plot a solid line profile along A-A', and plot the B-B' profile as a dashed line. Note the constant (linear) profile slope of the channel to the north of the input pin. Compare it to the steeper sloped (logarithmic) profile to the south of the input.

Interpretation of Results. Electrons flowing from the input along the narrow strip are constrained to flow northward. There is no variation in current density or electron-path length: thus, voltage is a linear function of distance. Electrons flowing to the south diffuse radially from the input. Current density decreases toward the east, west, and south boundaries: thus, voltage drops in a non-linear, logarithmic fashion with distance. The narrow strip could be interpreted as a road passing through an uninhabitable region.

Further Notes. The results of this experiment suggest that there are two components to the distance parameters of geographic problems. The first has to do with the underlying geometry of the area under study and is dealt with in this experiment. The second component has to do with the phenomenon under study, as it (or some characteristic of it) diffuses through the area.



Experiment 1-1. Absolute Barriers in Northwest and Northeast

Electrons diffuse uniformly through the south half of the model area, but they are channeled into a narrow strip north of the input. The result is a straight arithmetic drop in potential along the strip, as indicated by the solid line on the right half of A-A' on the profile. The left half of A-A' depicts the logarithmic distance decay function. Note that the potential drop along the edge of the barriers, B-B', lies intermediate between the logarithmic and the linear decreases of profile A-A'. The model suggests that the logarithmic potential decrease is strictly appropriate only where the uniform plane is assumed.

Experiment 1-j. Partial Barriers With an Intervening Pass

Purpose – to illustrate one method of developing partial barriers. A partial barrier acts somewhat as a reflective barrier.

Modeling Procedures.

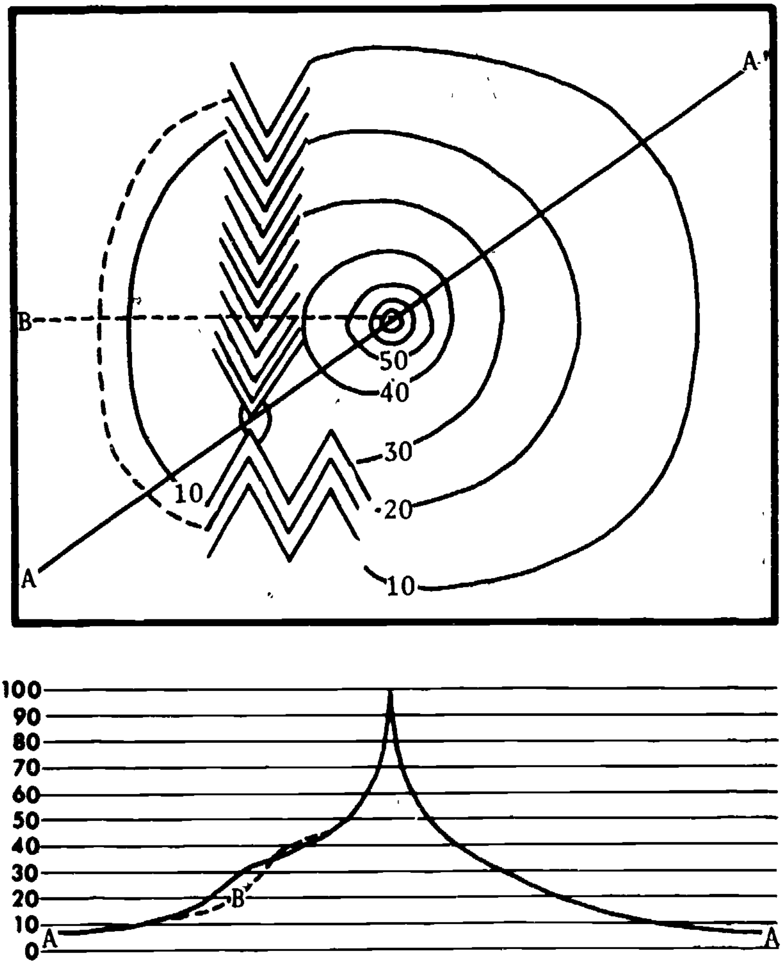
1. Follow modeling procedures 1 through 5 of experiment 1-a.
2. Draw a north-south line 3 inches west of the central input. Centered on this line draw a series of chevrons, ¼ inch to ½ inch apart, and let them extend 1 inch east and west from their common center line in such a way that the length of each chevron "arm" is 2". (Look at the illustration.) Leave a ¾-inch wide pass southwest of input pin. Next, draw a second, parallel line of chevrons below the pass and 3 inches to right.
3. Now cut the chevrons you have traced in step 2. The final result will be a single partial barrier north of the pass and a double partial barrier south of the pass.
4. Follow modeling procedures 6 and 7 of experiment 1-a.
5. Supplement the contour map by drawing the 5-percent isopotential as a dashed line.

Output Description:

1. Remove the input pin. Draw a diagonal line from southwest to northeast through the "pass" and center point. Label this line A-A'. (See the illustration.) Draw a dashed line from the center point to the mid-point of the west boundary. Label this line B.
2. Plot a solid-line profile along A-A' and a dashed-line profile from the center point to B.

Interpretation of Results. The partial barrier is made up of a series of absolute barriers. Movement of electrons along the axis of the chevrons is prohibited, but electrons can flow between any two chevron cuts. The additional length of the path between chevrons increases the total resistance encountered by electron flow in relation to the resistance of the shorter distance directly across the chevron. This increased electron path length is a direct measure of the effectiveness of the barrier. The barrier is partially reflective because it tends to increase potential values to the east of the chevrons.

Further Notes. Another way of making partial barriers is to punch holes in the paper in such a way as to lengthen the path of electron flow. The holes have the advantage that they can be made in such a way as to have the same effect in any direction; they have the disadvantage of being more time consuming in both design and construction.



Experiment 1-j. Partial Barriers With an Intervening Pass

The uniform-plane assumption is further modified in this model by a series of chevron cuts (V-hash marks) to simulate a partial barrier (e.g. mountains or swamps). Electrons flowing west must pass through the chevron channels (thus traveling twice as far) or go through the pass to get west of the partial barrier. The 10-percent contour affords a comparison between the values of potentials east of the partial barrier and those west of the barrier Note that the double barrier south of the pass is a more effective barrier than the single one north of the pass. This is due in part to its double size and in part to its orientation in relation to the input.

Experiment 1-k. Absolute and Partial Barriers and an Intervening Pass

Purpose – to illustrate one method of determining the relative impact of partial and absolute barriers. A partial barrier of the preceding model is made an absolute barrier and the resulting difference in output provides a relative measure of the partial barrier's effect.

Modeling Procedures:

1. Follow modeling procedures of experiment 1-j.

2. Draw a line starting on the eastern side of the northernmost chevron and ¼ inch east of where its arm (cut) terminates, thence south, along the eastern edge of the partial barrier to a point ¼ inch east of the southeastern tip of the first chevron north of the pass, thence ¼ inch west to the chevron arm tip. See the illustration of this line.

3. Cut the line. The partial barrier is now an absolute, reflective barrier.

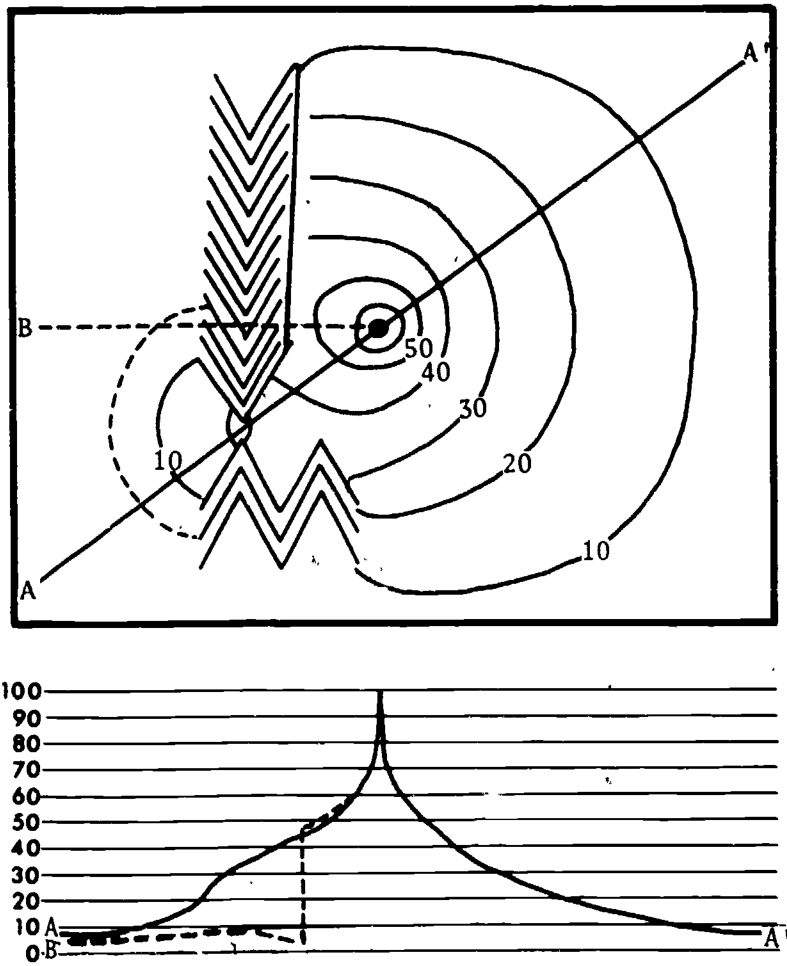
4. Follow modeling procedures 6 and 7 of experiment 1-a, using a different color of wax pencil or dashed lines to differentiate the output from that of model 1-j.

Output Description:

Follow the output description procedures of experiment 1-j.

Interpretation of Results. Two observations are clear First, the partial barrier did act to reduce potential values west of the chevron. Second, the influence of the pass in model 1-j was obscured by the effect of the partial barrier, but the impact of the pass is clear in this model. The two isopotentials immediately on either side of the pass indicate the focusing effect of the pass upon electron flow. This is directly analogous to the movements of people, and the population density of settlements in areas near a pass.

Further Notes. The relative reflectivity of the absolute and partial boundaries should be noted, since there is considerable change in potential values east of the barrier zone. A partial ground, introduced in model 1-r will illustrate how the absolute boundary could be made absorptive to avoid the increase in potential caused by reflection.



Experiment 1-k. Absolute and Partial Barriers and an Intervening Pass

When the partial barrier north of the pass is made an absolute barrier by a cut along its entire east side, the current is forced through the pass to reach the west side. The profiles indicate clearly the relative impact of the change. The 10-percent contour and the supplementary (broken) contour reflect the accessibility afforded by the pass. Note the slight reversal in the B profile, where the potential increases just west of the chevron marks after falling at the absolute barrier. This increase, of course, is owing to its proximity to the pass.

Experiment 1-1. Three Low-Resistance Roads Not to Ground

Purpose – to introduce the concept of roads as a further means of relaxing the uniform-plane assumption. This model is the first of a series of seven that illustrate ways to consider different kinds of roads and their impact.

Modeling Procedures:

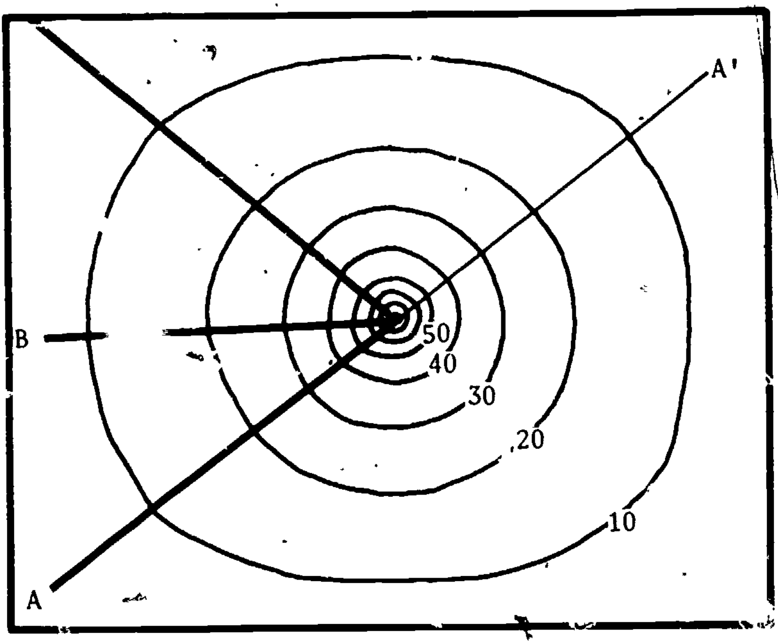
1. Follow modeling procedures 1 through 5 of experiment 1-a.
2. Apply one thin, uniform coat of carbon paint along a narrow 1/8-inch strip from the input pin to the lower left corner of the hold-down frame, another to the upper left corner, and one midway between the previous two. Stop painting all strips ½ inch short of the hold-down frame.
3. Allow 4 or 5 minutes for the paint to dry.
4. Follow modeling procedures 6 and 7 of experiment 1-a.

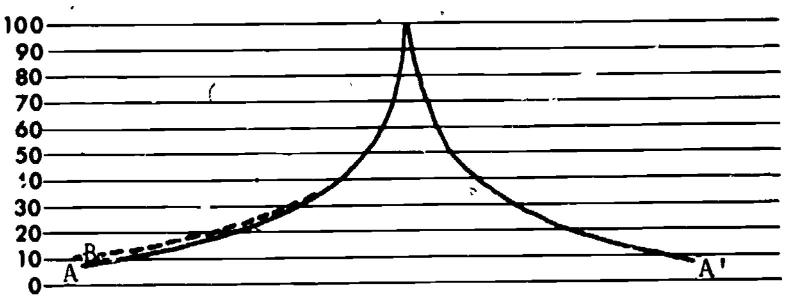
Output Description:

1. Remove the input pin and draw a line between the southwest and northeast corners. Label it A in the southwest and A' in the northeast.
2. Draw a dashed line due west from the input pin and label it B near the hold-down frame.
3. Plot a solid-line profile from southwest to northeast along A-A'. Note the symmetry of the profile about the input. The roads have little effect.
4. Verify that the profile from the center to B is virtually the same as the profile from the center to A.

Interpretation of Results. Roads simulated by one thin coat of carbon paint have little effect on the overall potential pattern. Their principal influence is in producing a slightly flatter profile slope. This is caused because the roads are of less resistance than the unpainted paper, and electron flow is expedited.

Further Notes. This model serves as a basis for comparison with the following six experiments.





Experiment 1-1. Three Low-Resistance Roads Not to Ground

Further modification of the uniform plane is possible. A single layer of carbon paint along a strip simulates a road, it expedites the flow of current since its conductivity is greater than that of the paper.

The three roads increase the potential in the western half of the model as compared to the eastern half. The profile indicates, however, that the shape of the curve is not markedly changed. Moreover, the profile (A-A') of the long road to the corner does not have a slope much different from the slope along the profile of the road running west (to B).

Experiment 1-m. Three Roads of Different Grounding and Conductivity

Purpose – to illustrate what happens to the preceding model when two of the roads are extended to the grounded edge of the sheet and each of the three has a different number of coats of carbon paint.

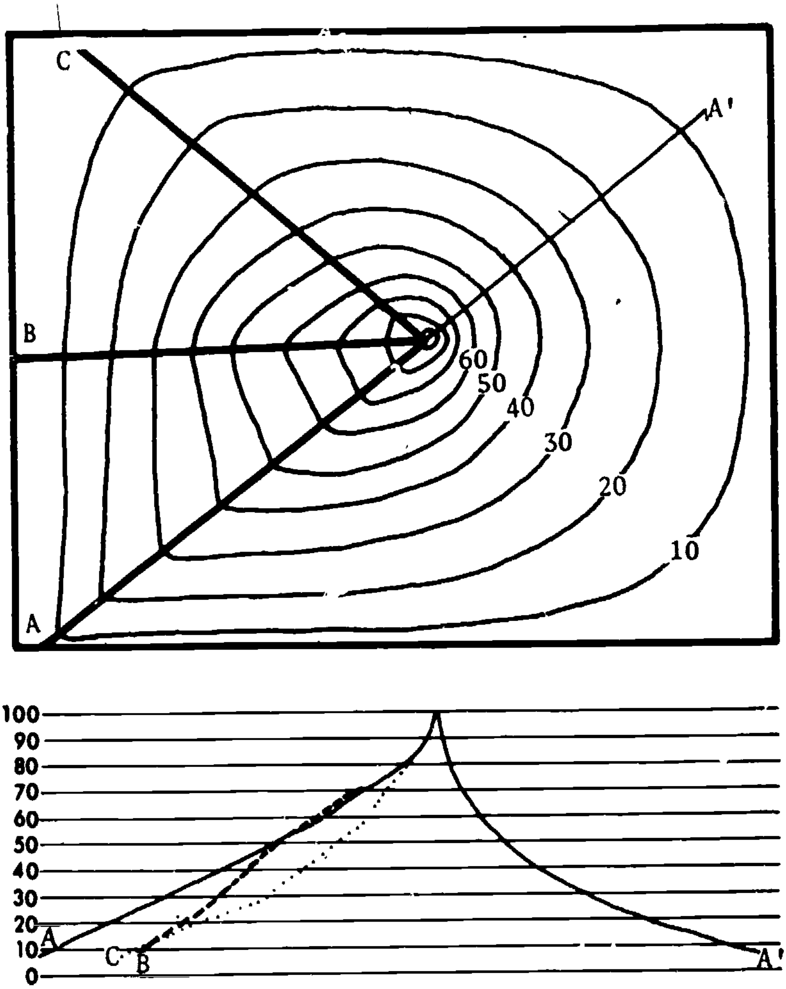
Modeling Procedures

1. Follow modeling procedures 1, 2, and 3 of experiment 1-l.
2. Lift the hold-down frame and extend the west road and the southwest road to the edge of the paper. Apply one layer of carbon paint to these extensions.
3. Paint a second coat on the entire west road and the entire southwest road. (All the way to the edge of the paper.)
4. Allowing 5 minutes for drying, apply a third coat to the southwest road.
5. Follow modeling procedures 6 and 7 of experiment 1-a.
6. Also probe the 80-percent contour.

Output Description

1. Follow output description procedures 1 and 2 of experiment 1-l.
2. Draw a dotted line from the input to the northwest, and label it "C" at the corner.
3. Plot a solid-line profile along A-A', a dashed-line profile from the input to B, and a dotted line from the input to C. Compare them.

Interpretation of Results. The combination of extending roads to the grounded boundary and varying their resistance by varying the number of coats of carbon paint, provides considerable flexibility in modeling different types of roads. The three-coat, southwest road is the best expediter of flow; it has virtually the same effect as the narrow strip of model 1-c – a linear decrease in potential with distance. The one-coat, northwest road (not to ground) has very little influence on the potential pattern. It could represent a rural, secondary road. The two-coat west road to ground has an impact intermediate between that of the southwest and northwest roads. The geographic importance of this modeling technique is that it offers the capability to examine the relative influences of proposed or actual roads, whose relative properties can be tailored to a particular situation.



Experiment 1-m. Three Roads of Different Grounding and Conductivity

Three different road conductivity values are employed. The road to the northwest is the same as in the previous model – one coat of carbon paint and ungrounded. The road west (to B) has two coats and extends to ground. Painting a strip to ground expedites further the flow of electrons. The road to the southwest had three coats, and also extends to ground. They simulate, say, a county road, a state highway, and a U.S. highway. More current is flowing along the road to the southwest than elsewhere, creating an almost perfectly linear potential profile slope (A-A').

Experiment 1-n. Eight Low-Resistance Roads to Ground

Purpose – to illustrate the impact of extending the radial network of roads to the grounded boundary.

eling Procedures:

1. Follow modeling procedures 1 through 4 of experiment 1-a.

2. Draw a line due north and south through the center of the model, another due east and west, and then two more lines, each equidistant between the first pair of lines. Note that the diagonal lines do *not* run to the corners of the hold-down frame.

3. Raise the hold-down frame and uniformly apply one thin, narrow (1/8-inch) coat of carbon paint along each line, being sure to extend the painted line to the edge of the paper.

4. Follow modeling procedures 5, 6, and 7 of experiment 1-a.

5. Probe for the 80-percent contour, and if detectable, the 90-percent contour as well.

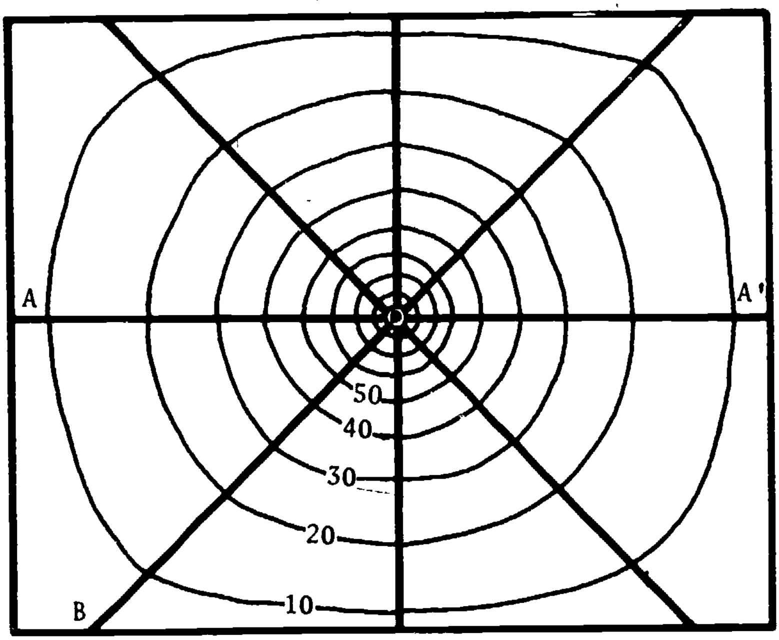
Output Description:

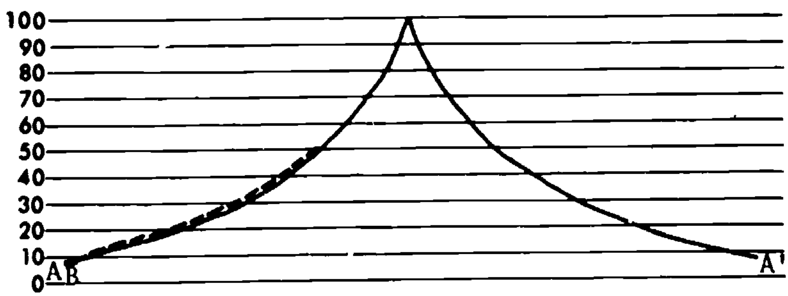
1. Follow output description procedures 1 through 5 of experiment 1-a.

2. Compare the results with the profile in 1-a. Note that the slope is flatter, and that higher potential values occur near the input.

Interpretation of Results. The radiating network of roads simulates the attraction potential of a city (the area of which is represented by the central point) and a large surrounding region with no cities or towns, such as those that occur in the drier areas of the interior southwestern United States.

Further Notes. An interesting variation on this model is to stop some or all of the diagonal (or other) roads short of the grounded edge. The result is a dramatic change from the simple uniformity of the potential pattern.





Experiment 1-n. Eight Low-Resistance Roads to Ground

As seen in experiment 1-m, roads that run to the grounded boundary are especially good conductors. Areas enclosed by the 80- to 90-percent contours that were too small to be plotted (too near the edge of the head of the input pin) on the models without roads can now be plotted. Electrons flow along the routes and radiate from them enroute, similar to the movement of traffic along the main highway leading from a city Note on the profile the lower rate of slope decrease (flattening) for this model compared to previous models.

Experiment 1-o. Eight Higher Conductivity Roads to Ground

Purpose – to illustrate the consequences of increasing the conductivity of a road extending to a grounded boundary. Additional layers of carbon paint even further expedite the flow of electrons.

Modeling Procedures:

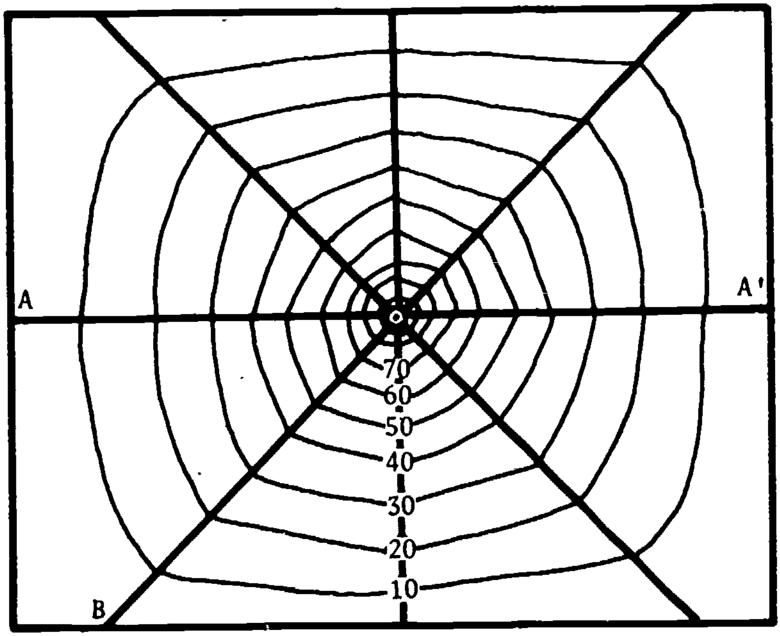
1. Follow modeling procedures 1, 2, and 3 of experiment 1-n.
2. Apply three more uniform coats of carbon paint to each road, allowing 4 or 5 minutes for drying between coats. Try to keep the edges of the roads as straight as possible.
3. Follow modeling procedures 4 and 5 of experiment 1-n.

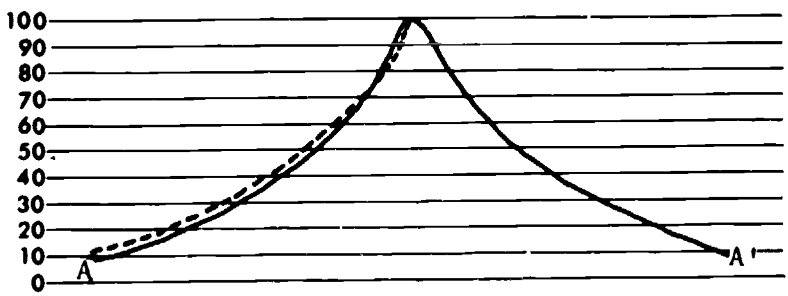
Output Description:

1. Follow the output procedures of experiment 1-n.
2. Compare the results with model 1-n; note that this model's profile slope is flatter and that its central potential values are higher.
3. Note also that the isopotentials are tending to form straight lines, particularly near the center.

Interpretation of Results. One geographic interpretation of the results would be that this model simulates the attraction potential of a city such as the one described in 1-n, but with better transportation to its surrounding region. Thus, the influence of the city drops off less rapidly with distance from the urban center. Contours near the center form nearly straight lines between roads. This reflects the relatively greater importance of a small change in distance from the center than the same change in distance in outlying areas (were contours curve more between roads).

Further Notes. Again, varying this model by stopping some roads short of the ground or leaving some with only one or two coats of paint provides insights into a more realistic model of an actual transportation network. Irregularities that tend to develop in this model can be corrected by assuring that carbon paint is applied uniformly, by waiting a full 5 minutes before applying another coat of paint, and by applying small amounts of additional paint where output measurement indicates that too little has been applied.





Experiment 1-o. Eight Higher Conductivity Roads to Ground

Additional layers of carbon paint result in increased conductivity of the roads relative to that of the paper. The profile slopes are more linear, indicating the stronger one-dimensional flow along the highway in contrast to the near two-dimensional diffusion in the intervening areas. The higher value contours tend to form straight lines between the roads, compared to the gentle arcs of the contours in model 1-n with lower conductivity roads. One model interpretation would be that the increase in potential values away from the input represents higher land values than would be expected in a city without good radial transportation routes

Experiment 1-p. Eight Low-Resistance Roads Not to Ground

Purpose – to illustrate what happens when roads of experiment 1-n are not extended to the grounded boundary. Electron flow is not so expedited and, consequently, there is a smaller difference in electron flow along the roads and in the areas between roads.

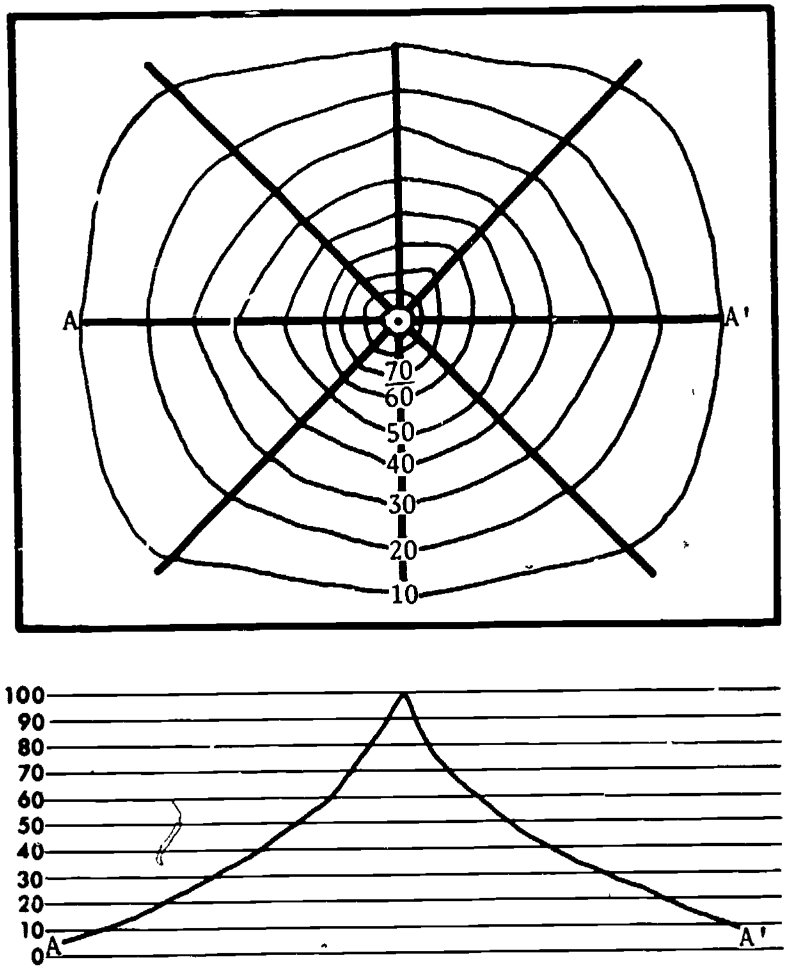
Modeling Procedures.

1. Follow modeling procedures 1 and 2 of experiment 1-n.
2. Apply uniformly one thin, narrow (1/8-inch) coat of carbon paint along each line, stopping ½ inch short of the hold-down frame.
3. Follow modeling procedures 4 and 5 of experiment 1-n.

Output Description:

Follow the output procedures of experiment 1-o.

Interpretation of Results. One way to interpret this model is that the area enclosed by the 60-percent contour is urban. The area outside the 60-percent contour is then the suburban commuting field. This interpretation is based on examination of the potential profile. Note that the profile slope from the urban center to the 60-percent level is steeper and distinct from the slope of the profile below 60 percent. Thus, a small change in distance from the center in the urban area is more significant than the same distance change in the suburban area.



Experiment 1-p. Eight Low-Resistance Roads Not to Ground

Roads that do not run to ground have a different effect. Electron flow along the road results in a greater difference between the potential of areas near the road and the potential of the remoter areas. Even the low-resistance roads not to ground take on characteristics of higher conductivity roads extended to the grounded boundary. The contours tend to form straight lines between roads and the slope of the profile flattens (decreases at a lower rate) as compared with that in experiment 1-o. This model can be interpreted as simulating the commuting field along radial highways in outlying areas near a city.

Experiment 1-q. Eight Higher Conductivity Roads Not to Ground.

Purpose – to illustrate the rather dramatic effects of higher conductivity roads that are not extended to the grounded boundary. The result is a reversal in trend of contours.

Modeling Procedures.

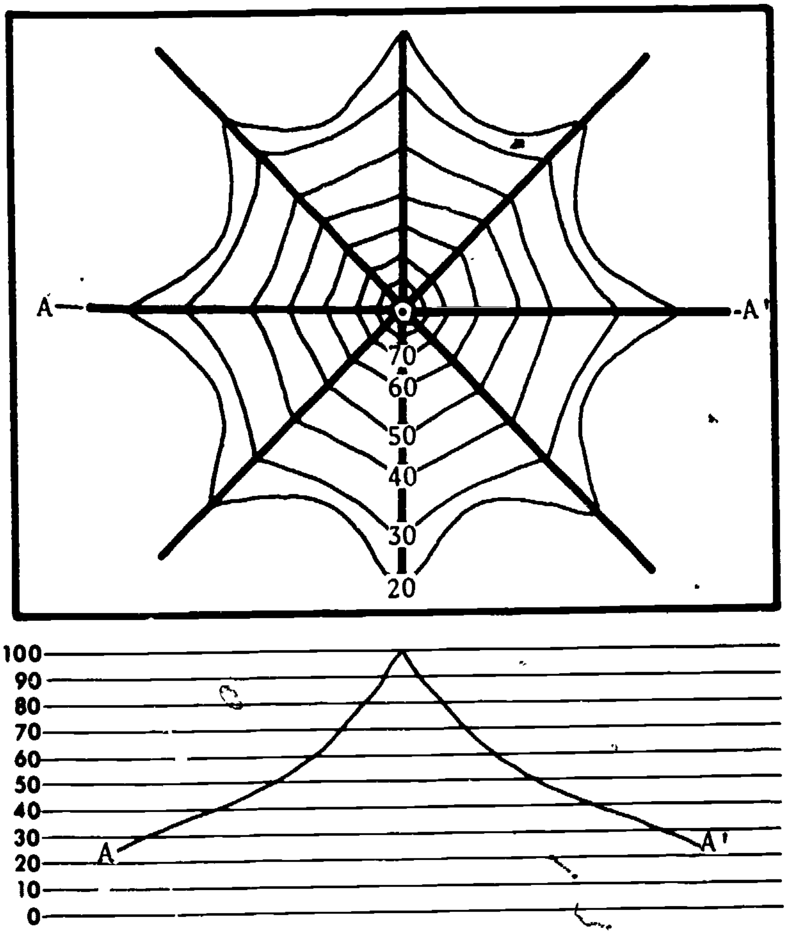
1. Follow modeling procedures 1 and 2 of experiment 1-p.
2. Apply uniformly five additional coats of paint, allowing 5 minutes drying time between coats.
3. Follow modeling procedures 4 and 5 of experiment 1-n.

Output Description:

Follow the output procedures of experiment 1-o.

Interpretation of Results. The spider-web potential pattern results from an increased difference between flow along roads and flow in areas between roads. The radial pattern of high-speed access roads allows commuters living close to these roads at greater distances from the urban center to reach work in the same time as those who live closer in, but who do not live as close to one of the major traffic arteries. The contours reflect this relationship between distance from a major road, distance from the urban center, and travel time to the urban center. Thus, a commuter living anywhere along a particular contour could reach the city in the same time as anyone living elsewhere along the same contour.

Further Notes. It is instructive to combine types of roads selected from experiments 1-n through 1-q in various ways to develop road network patterns to simulate a wide variety of urban phenomena.



Experiment 1-q. Eight Higher Conductivity Roads Not to Ground

Applying additional layers of conductive paint to roads on the previous model yields a spider-web potential pattern. The outlying contours are reversed inward, indicating the greater relative potentials along the roads than in the areas between the roads, where both lie at the same distance from the center of the city. The reversal of contours is less pronounced toward the center of the city, but none assume the normal outward curvature of contours as on preceding models. The overall potential, as indicated by the profile, decreases at a lower rate and thus is higher than other experiments of this series. This model can be interpreted as simulating the influence of radial highways on land values *within* the residential areas of a city.

Experiment 1-r. Eight Radial Roads Straddling Partial Grounds

Purpose – to introduce the concept of a pa   round and demonstrate its utility in simulating a traffic divide between two roads.

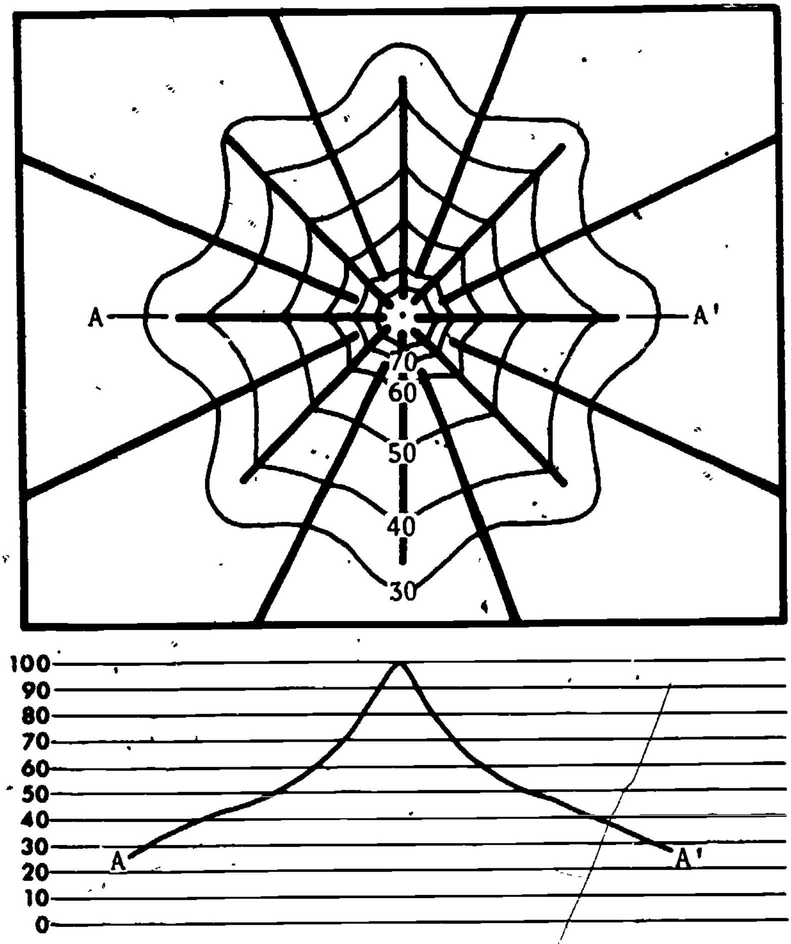
Modeling Procedures

1. Follow modeling procedures 1 and 2 of experiment 1-n
2. Apply uniformly one coat of carbon paint along each line for a distance of 5 inches from the center.
3. Represent traffic divides by drawing lines equidistant between roads, from the center to the edge of the conductive paper. Apply one coat of carbon paint to these lines also.
4. Follow modeling procedures 4 and 5 of experiment 1-n.
5. Add one layer of carbon paint to the roads, recalibrate, and probe. Notice the changes that occur.
6. Add three more coats to the road-divide lines, recalibrating and probing after each coat.

Output Description.

Follow the output procedures of experiment 1-o.

Interpretation of Results. The traffic divides provide an additional modeling technique for simulation of complex patterns. The divide may be placed equidistant between roads or along some natural obstacle that determines a road divide; it may also be placed according to empirically determined criteria.



Experiment 1-r. Eight Radial Roads Straddling Partial Grounds

The carbon conductive paint is here used to extend the ground as well as the input. The resulting partial ground simulates in this model the traffic divide between adjacent roads as a low activity line. The resulting impact is the reversal of all contours between the roads. By varying combinations of the number of coats of paint on the traffic divides and the number of coats of paint on the roads, the degree of contour reversal can be selectively or uniformly manipulated to produce a variety of related patterns. Such patterns approximate different conditions of accessibility and different land values within a city, as well as preferential urban expansion along main transportation arteries.

Experiment 1-s. One Internal Input Not Centrally Located

Purpose – to illustrate the impact of offsetting the input so that it is not centrally located. Such a change adds a new dimension to all the previous experiments and serves as a point of departure for the multiple-input models of the next series of experiments.

Modeling Procedures:

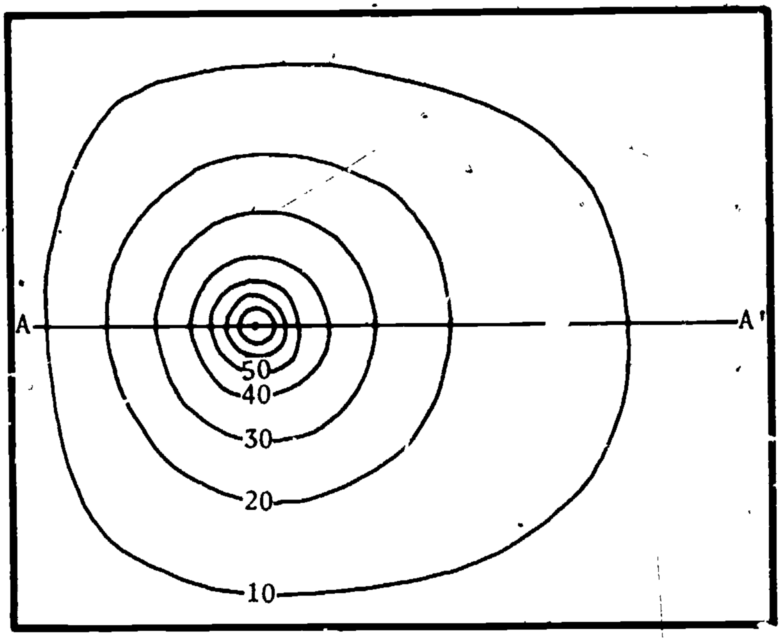
1. Follow modeling procedures 1 through 4 of experiment 1-a.
2. Locate a point 3 inches due west of the center, place an input pin at that point, clip a numbered lead to the pin shaft and turn the corresponding current control to a setting of 10.0.
3. Follow modeling procedures 6 and 7 of experiment 1-a.

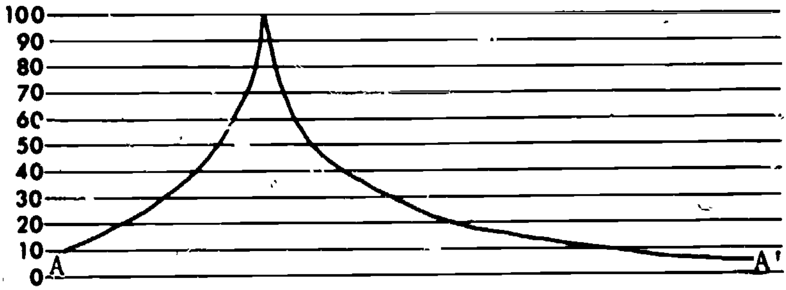
Output Description:

1. Follow output description procedures 1, 3, and 4 of experiment 1-a.
2. Note the long gentle profile to the east of the input and the steeper, shorter profile to the west. Note also that the profile is nearly symmetrical about the center above the 20-percent level and only slightly symmetrical at 20 percent. The 10-percent contour and profile, however, are greatly affected by offsetting the input.

Interpretation of Results. The output may be interpreted, as in the case of model 1-c, as the regional impact of a city daily newspaper published in a city where a larger competitor lies to the west and a more distant competitor to the east. The nearby area (enclosed by the 20-percent contour) is little affected by the competing newspapers. The newspaper circulation in more distant areas is affected, however, by competition.

Further Notes. Placing other single inputs at different locations near the boundary provides deeper understanding of the effect of their location on basic output patterns.





Experiment 1-s. One Internal Input Not Centrally Located

ll the previous models, except 1-c, were designed with one centrally located input in order to evaluate changes in other factors. In this model the input is offset and all four sides of the rectangular boundary are grounded. The slope of the curve is steepest in the direction of the closest boundary, and flattens toward the most distant boundary. Note that the lower value contours are reflected and compressed by the nearest grounded boundary, in contrast to the attraction of the ungrounded boundary in 1-h. Look back over the previous 18 models and verify that they are all special cases of the general case represented by the model in experiment 1-a.

Series 2: *Multiple-Point Input Models*

Now that you have developed a feeling for the effects of various boundary conditions, barriers, and expediters, you should be able to understand better the interactive effects created by the use of multiple internal inputs. The use of multiple inputs introduces a greatly expanded class of problems that can be addressed on the field plotter. As in Series 1, we begin with a simple model and increase model complexity with each experiment of the series. In reviewing these models, you should attempt to visualize the impact that would be created if the barriers and variations in boundary conditions developed for the single-input case were also present in the multiple-input experiments.

The series includes:

a. Interaction Potential - Two Equal Inputs
b. Four Equal Inputs
c. Eight Equal Inputs
d. Two Unequal Inputs
e. Exterior and Interior Grounds

Experiment 2-a. Interaction Potential - Two Equal Inputs

Purpose - to introduce the concept of multiple inputs of the same magnitude, and to interpret the resulting interaction potential pattern.

Modeling Procedures.

1. Follow carefully the instructions for setting up and operating the field plotter in section IV. Review those instructions if necessary.

2. Center a sheet of conductive paper on the easel and clamp the hold-down frame in place.

3. Delimit the working area of the paper. Using a wax pencil or chalk, lightly draw a border on the paper using the inside edge of the hold-down frame as a guide.

4. Locate the center of the paper (see modeling procedure 4 of 1-a, if necessary).

5. Locate one input point 3 inches due west of the center and the other input 3 inches due east of the center.

6. Insert input pins at these points, clip a different numbered current lead to the shaft of each pin, and turn the corresponding current controls to a setting of 10.0. Set all other current controls to zero.

7. Calibrate.

8. Probe the contou[illegible] n values ranging from 70 percent to 10 percent at intervals of 10 (i.e., 70, 60, 50, . . . , 10).

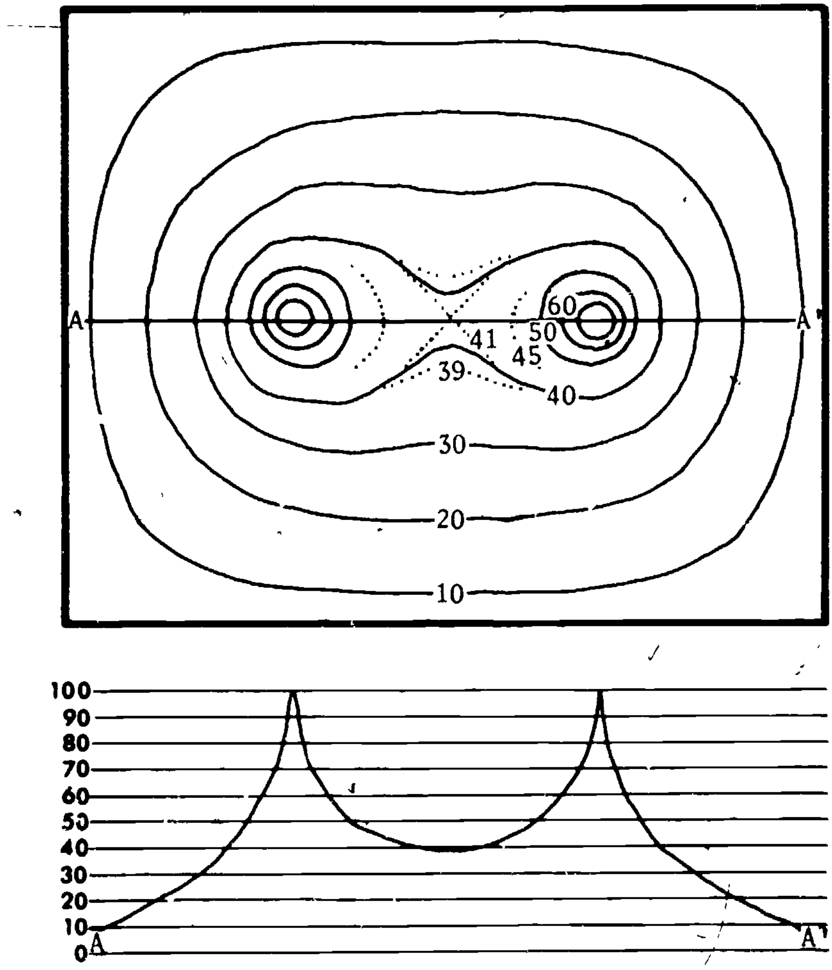
9. Probe supplementary contours near the center of the model and wherever contours are widely spaced.

Output Description

1. Remove the input pin and draw a horizontal line from the middle of the left side to the middle of the right side, running through the center of the model. Label it A-A'.

2. Plot a profile along A-A'. Note the slope of the profile, particularly the plateau in the center.

Interpretation of Results Interaction potential is an overall measure of relative accessibility of the two input locations. Points of equal accessibility are connected by a contour.



Experiment 2-a. Interaction Potential Two Equal Inputs

If model 1-s were reversed (with the input on the east side) and the potential value for each point in the original model added to potential value at the same points due to the eastern input, the result would be model 2-a. The east half of 2-a is similar (except for scale and orientation) to model 1-d. The breakpoint of relative influence is the low point on the plateau (portrayed by the profile) between the two cities. Some e living to one side of that breakpoint would tend to shop in the city lying on that side; the steeper the slope of the cross-section, the stronger the tendency.

Experiment 2-b. Four Equal Inputs

Purpose – to extend the concept of multiple inputs of the same magnitude

Modeling Procedures

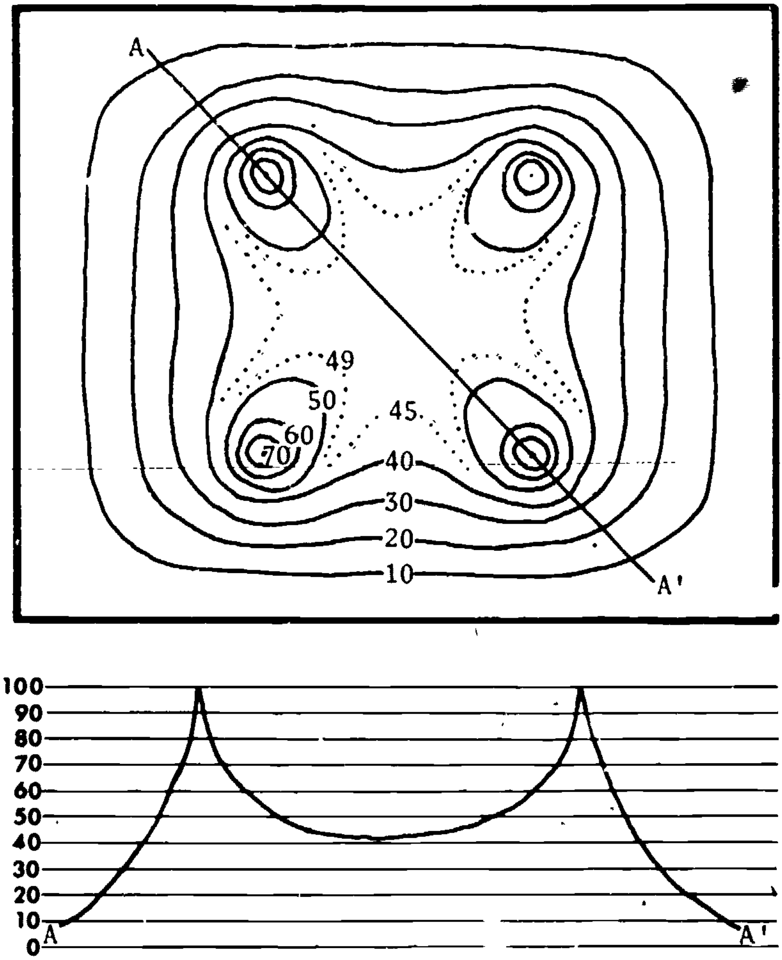
1. Follow modeling procedures 1 through 4 of experiment 2-a.
2. Locate and mark a point 3 inches due west of the center, another 3 inches due east, one 3 inches due north, and a final point 3 inches due south of the input.
3. Follow modeling procedures 6 through 8 of experiment 2-a.

Output Description:

1. Remove the input pin and draw a line through the northwest and southeast inputs. Label it A-A'.
2. Plot a profile along A-A'. Note the slope of the profile, especially the plateau in the center; it is higher than in model 2-a.

Interpretation of Results. The four inputs simulate, say, the interaction potential of four shopping centers. A person living within the 50-percent or higher, isopotential contours is most likely to trade at the shopping center enclosed by the same isopotential; the higher the value of the isopotential the greater that tendency. Someone living on the plateau but in an area whose potential is below the 50-percent level is well situated in terms of accessibility to any of the four shopping centers and is less predictable in his shopping habits. A person living in an area with a potential below 40 percent normally will shop at one of the nearest two shopping centers.

Further Notes. Moving the location of one or two of the inputs and probing the results gives an output that, when compared with this model, is very instructive.



Experiment 2-b. Four Equal Inputs

This model is a more complex extension of model 2-a. In the same fashion, it can be taken to represent the influence of four cities on the surrounding area. They interact to create a plateau of high potential. The model may be interpreted as four shopping centers which are equally accessible to centrally located (on the plateau) residents, but of limited accessibility to outlying residents. Note that the symmetry of the isopotentials is almost perfect, the greater the number of inputs the more important it is that the resistance paper be free of variation in resistance.

Experiment 2-c. Eight Equal Inputs

Purpose – to further illustrate multiple inputs and interaction potential.

Modeling Procedures:

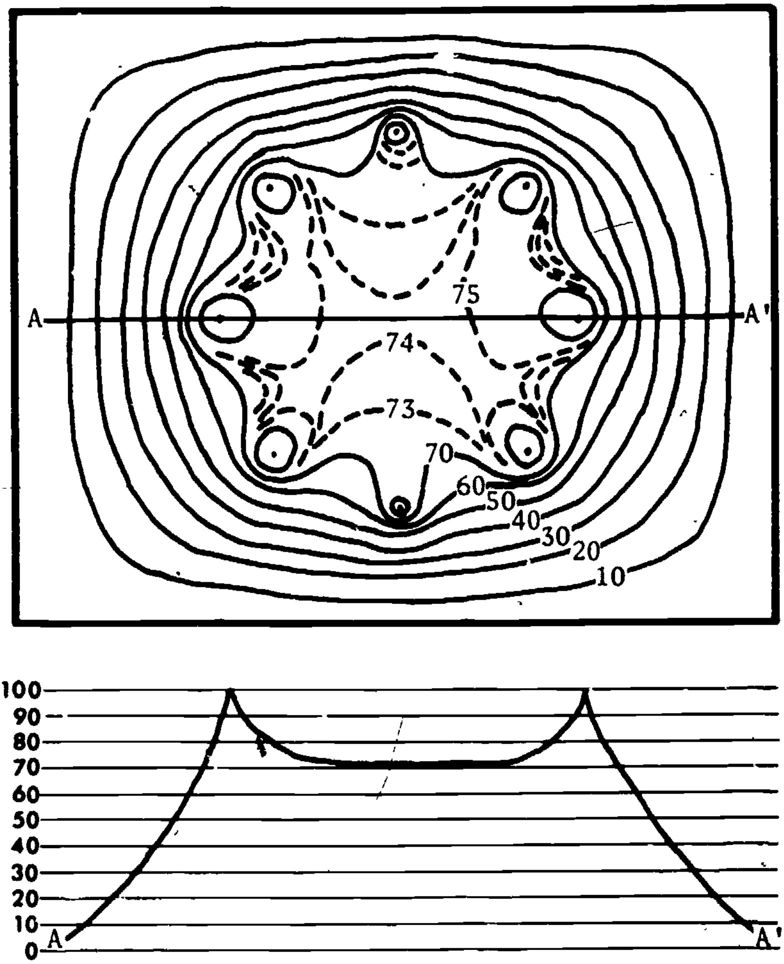
1. Follow modeling procedures 1 and 2 of experiment 2-a.
2. Using a compass, draw a circle (around the center of the paper) through the four input points of experiment 2-b. Mark the location (on the circle) of points halfway between each of the existing points. You should now have eight equally spaced input points on the circle.
3. Follow modeling procedures 6 and 8 of experiment 2-a.
4. Probe supplementary contours to highlight small potential changes on the plateau.

Output Description:

1. Follow output description procedures 1 and 2 of experiment 2-a.
2. Note that the whole plateau has a potential value over 70 percent.

Interpretation of Results. This model illustrates the high degree of accessibility that would exist in a city with eight shopping centers on the edge of the urban area. Each center does have its own area of dominance enclosed by the 80-percent isopotential contour. All eight together tend to attract trade from the central areas. This mutual attraction of trade is indicated by contours whose value is less than 80 percent.

Further notes. The supplementary contours on the plateau do not display perfect symmetry, owing in part to minor variations in the location and contact resistance of the input pins, and small variations in "equal" amounts of current fed into each pin. Nonetheless, the symmetry is very good.



Experiment 2-c. Eight Equal Inputs

This model is a more complex extension of models 2-a and 2-b. Here, however, the symmetry is less perfect owing to minor irregularities in the placement of inputs, small differences in current input values, and the gentler slope of the profile on the plateau. Note the higher elevation and flatness of the plateau as compared with that of 2-b Compare the shape of the 70-percent contour with that of the 30-percent contour on model 1-r, the two models could be combined to simulate the relative attractiveness of the downtown shopping district and outlying shopping centers.

Experiment 2-d. Two Unequal Inputs

Purpose – to illustrate multiple inputs of different magnitudes, and to describe a technique for locating the breakpoint of trade dominance between two cities of unequal size.

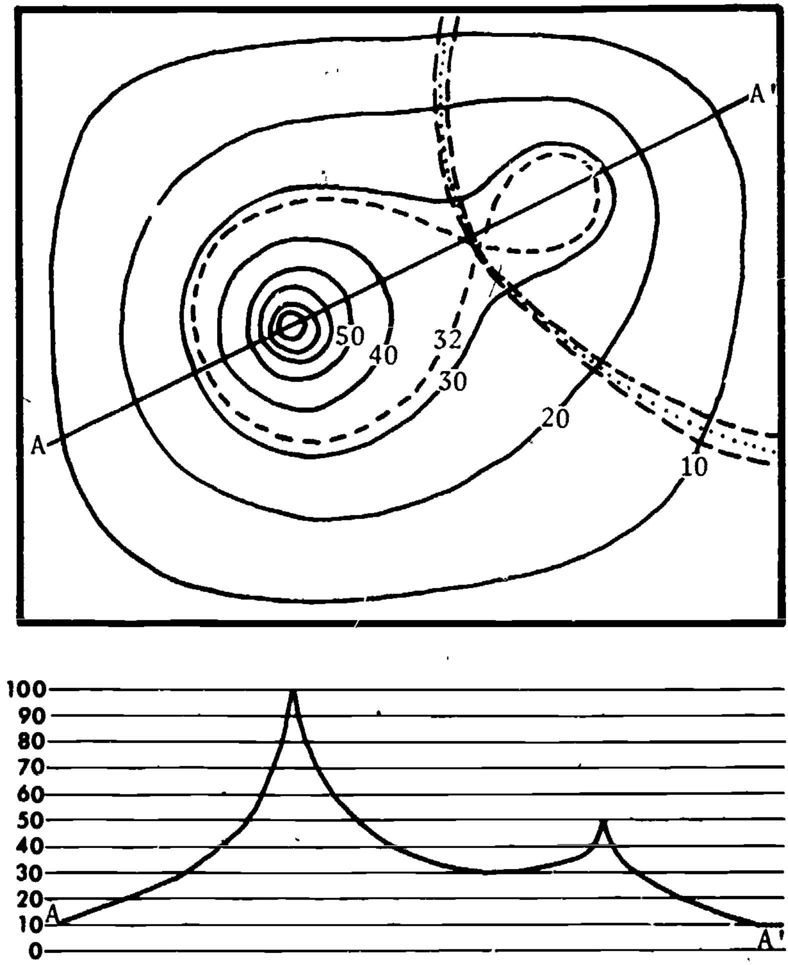
Modeling Procedures.

1. Follow modeling procedures 1 through 3 of experiment 2-a.
2. Locate a point 5.3 inches from the west margin and 6 inches from the north margin, and another point 3.2 inches from the north margin and 8.75 inches from the east margin.
3. Insert input pins at these two points, clip a numbered current lead to the shaft of each pin, and turn the corresponding current controls to 10.0 for the larger (western) town and 2.5 for the smaller (eastern) town.
4. Follow modeling procedures 7 and 8 of experiment 2-a.
5. To find the breakpoint, consult the illustration of model 2-d; notice the approximate location of the crossover point of the "self-crossing" contour. Probe in that vicinity for a point where the null meter indicator needle will deflect upward if the probe is moved toward either town, and will deflect downward if the probe is moved at right angles to a straight line between the two towns. Once you have found this point, turn the contour selector until a null is achieved, record this potential value, and probe the self-crossing supplementary contour.
6. To find the trade area boundary between the two towns, lift the hold-down frame, remove the current lead clipped to the small town input pin, and replace it with the ground lead. Position the probe on the breakpoint and turn the contour selector until a null results. Now, probe and draw the trade boundary at this contour selector setting.
7. Repeat step 6 with the larger town, pin grounded, and the smaller town, pin reconnected to its current lead.

Output Description:

1. Draw a line (from the left boundary to the right boundary) that passes through both towns and label it **A-A'**.
2. Plot a profile along **A-A'**. Note that the breakpoint between the two towns corresponds to the low point (on the profile).

Interpretation of Results. The amount of current simulates the size of a given town and can be weighed, if desired, by income or some other characteristic to achieve special results. The low point in the pass between two peaks represents a first approximation of the breakpoint of trade between two towns, and the line extended from that point approximates the boundary of the trade area.



Experiment 2-d. Two Unequal Inputs

The relative amounts of current input to the elements of interaction models can be adjusted to simulate the extent of the dominant influence of a city or shopping center. The dividing line between the trade areas of the two cities (or shopping centers) can be plotted using the technique described in 1-g, by alternately grounding one city while the other retains its input and probing the dividing line, beginning at the established breakpoint. The results are two dividing lines (shown on the model as dashed lines) between which the approximate trade boundary lies.

Experiment 2-e. Exterior and Interior Grounds

Purpose – to introduce the simultaneous use of an interior ground (sink) and grounding at the boundaries.

Modeling Procedures:

1. Follow modeling procedures 1 and 2 of experiment 2-b.

2. Locate a point 2-1/3 inches from the southeast town in the direction of the northwest town. Insert the ground pin at that point and clip the ground lead to the pin shaft.

3. Follow modeling procedures 7 and 8 of experiment 2-a.

4. Following modeling procedure 5 of experiment 2-d, locate the breakpoints between the northwest, northeast, and southwest inputs. Now probe the trade boundaries (shown as dotted-line, self-crossing contours in the illustration).

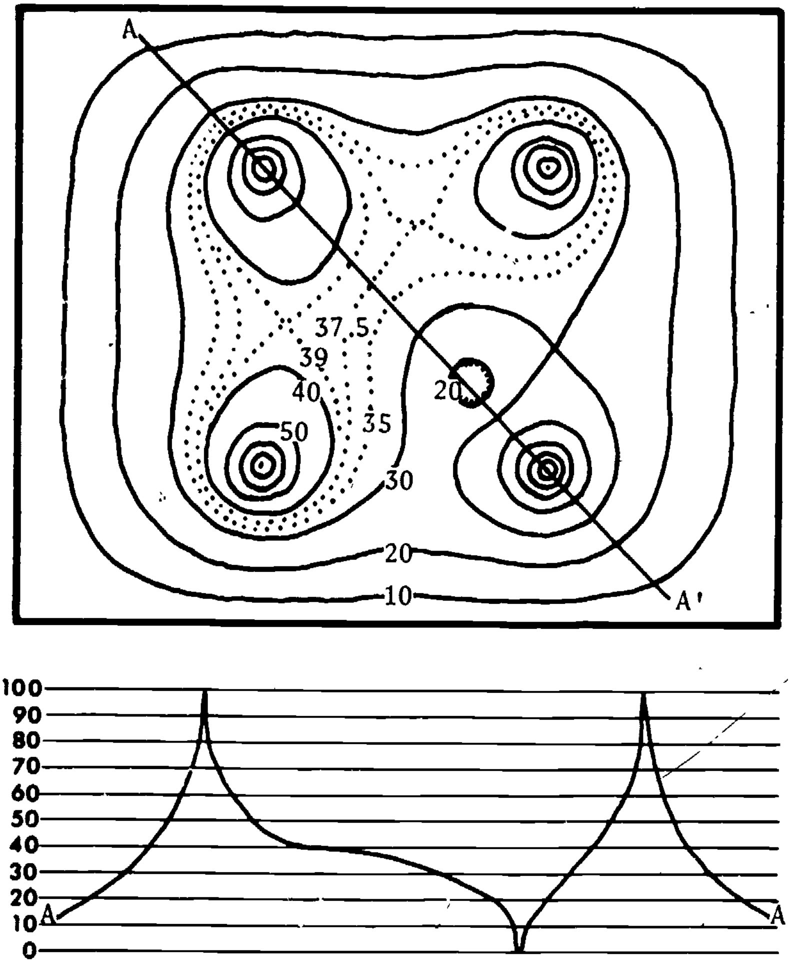
Output Description:

1. Remove the input pin and draw a line from boundary to boundary through the northwest and southeast towns. Label it A-A'.

2. Plot a profile along A-A'. Compare the profile with that of model 2-b.

Interpretation of Results. Continuing with the interpretation of model 2-b, this model simulates four shopping centers. The internal ground represents a burning city dump that retards residential development and discourages surface travel in the vicinity of it. The result is that the plateau of high potential is much eroded.

Further Notes. Several such interior grounds can be used by connecting wires from one grounded pin to pins inserted where other grounds are desired. (This technique may *not* be employed for current inputs.) The zero potential value of a ground makes it possible to accommodate as many ground pins as required.



Experiment 2-e. Exterior and Interior Grounds

An interior ground, when employed with a grounded boundary, simulates areas that not only do not attract but actively repel. A burning city dump prevents residential development in its immediate vicinity, and has a general negative impact on the interaction between residents of four nearby surrounding housing developments. The interior grounding may be at a point (by clipping the ground lead to an input pin) or of an area (by application of aluminum tape with a grounded pin stuck through the tape). An alternative interpretation of a grounded point or area would be as a nuisance industry.

Series 3. *Line Input - Flow Phenomena Models*

This series of experiments is designed to allow you to verify the relationships between streamlines and isopotentials, as discussed in the field plotter concepts section. These experiments introduce still another dimension in modeling technique that is possible with the field plotter–the input of current along a line at a boundary, rather that at an internal point. This technique can also be used in combination with single or multiple internal input models, as discussed previously.

The series includes:

a. Line Input and Line Ground on Opposite Boundaries
b. Streamline Model of Flow Against Ab olute Barriers
c. Velocity Model of Flow Against Barriers
d. Curvilinear Squares (Streamlines and Isopotentials)
e. Stream Velocity as Affected by Channel Width, Depth, and Gradient

Experiment 3-a. Line Input and Line Ground on Opposite Boundaries

Purpose – to introduce the concept of input of current along a line, in contrast with the point inputs employed in series 1 and 2, as well as to introduce a theoretical approach to flow phenomena, such as river flow.

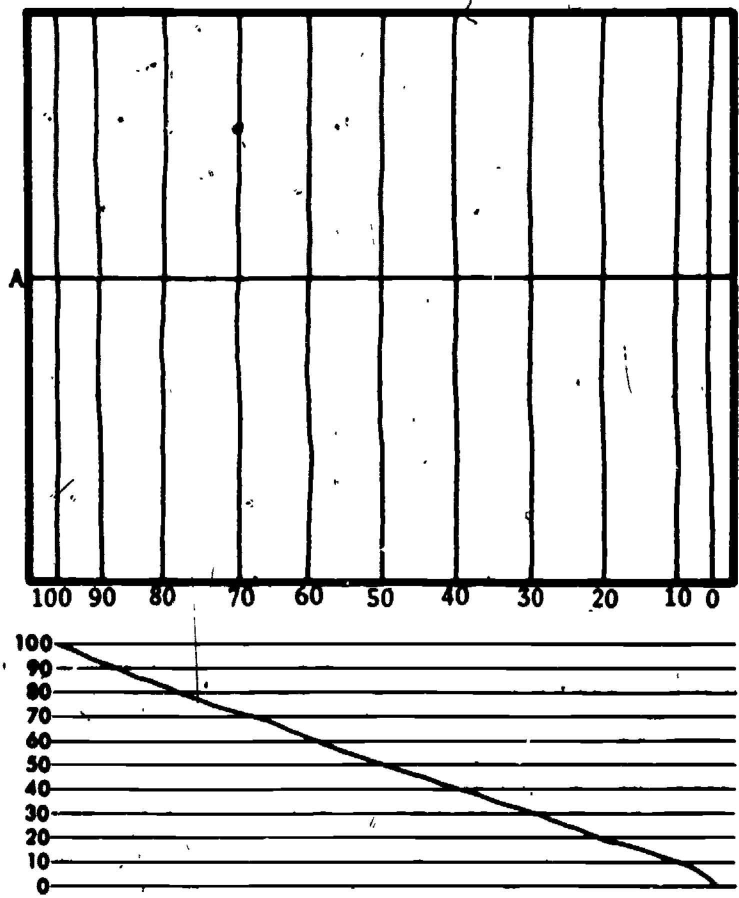
Modeling Procedures.

1. Cut a piece of conductive paper 10 inches by 15¾ inches in size. Place it on the field plotter easel with the right (east) edge extending ¼ inch under the right side of the hold-down frame
2. Place a thin strip of aluminum tape along the left (west) edge. (A narrow strip of silver paint can be used in place of aluminum tape)
3. Press the aluminum tape down tightly on the sheet and place an input pin through the center of it. Clip a current lead to the pin shaft, and turn the corresponding current control to a setting of 10·0.
4. Calibrate.
5. Probe the isopotentials (contours) of 100, 90, 80, 70, . . . 10.

Output Description.

1. Draw and east-west line through the middle of the sheet and label it A-A'.
2. Plot a profile along A-A'. Note the constant slope of the profile between 90 and 10. The ends of the profile are somewhat affected by the boundaries.

Interpretation of Results Electrons flowing from west to east can simulate other flow phenomena, such as river flow. One useful analogy is that between the relative spacing of the isopotentials and the velocity of flow. The width of the paper represents the width, depth, and variations in gradient of a stream. The time required for water to flow a given distance is indicated by the spacing of the isopotentials – the closer the isopotentials the shorter the time period required, the wider the spacing the longer the time period required. Since the contours are equally spaced (for a constant difference in potential values of adjacent contours) in this model, it simulates a constant rate of flow in a short section of a river.



Experiment 3-a. Line Input and Line Ground on Opposite Boundaries

Flow phenomena may be simulated by applying current to a highly conductive strip (silver paint or aluminum tape) along the west boundary of a rectangular sheet and by grounding a similar strip along its east boundary. The direction of electron flow is from west to east. The drop in potential is a linear decay function of distance. Thus, contours are equally spaced if there is a constant difference between potential values of adjacent contours. Generally, closely spaced contours are analogous to a more rapid flow of fluid, while widely spaced (relative to closely spaced contours on the same model) contours indicate slower movement. A geographic interpretation of 3-a might be water flow in a stream that is free of barriers and is of constant depth, width, and gradient.

Experiment 3-b. Streamline Model of Flow Against Absolute Barriers

Purpose – to introduce the concept of streamlines and apply them to a theoretical view of the path along which a given particle of water will flow, as well as to introduce the concept of barriers in flow models.

Modeling Procedures:

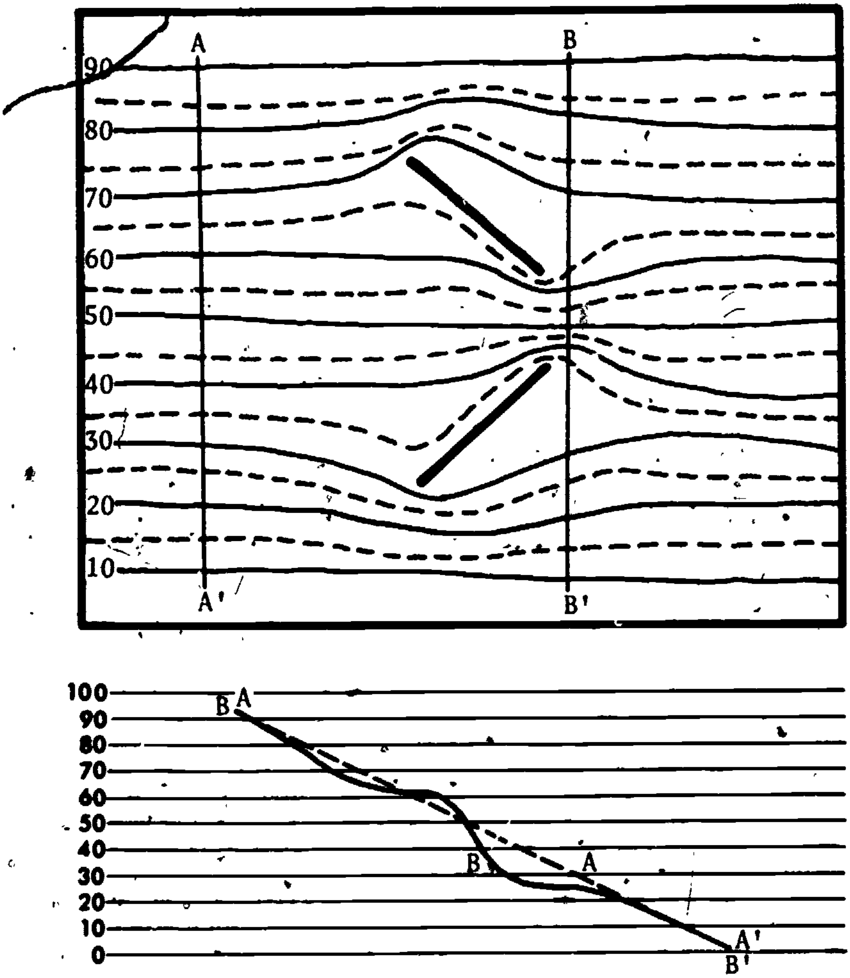
1. Cut a piece of conductive paper 10¾ inches by 15 inches in size. Place it on the field plotter easel with the bottom side extending ¾ inch under the bottom side of the hold-down frame.
2. Place a thin strip of aluminum tape along the top edge.
3. Follow modeling procedure 3 of experiment 3-a.
4. Toward the middle of the model draw two linear barriers on the approximate configuration shown in the illustration.
5. Cover the two barriers with a narrow (¼-inch strip) of aluminum tape (or silver paint).
6. Follow modeling procedures 4 and 5 of experiment 3-a.

Output Description:

Draw a north-south line approximately 2 inches to the left, and another ½ inch to the right of the two obstacles. Label them, respectively, A-A' and B-B'. Plot the profiles of these two lines. Note that profile A-A' is a linear function and profile B-B' is a highly varied, nonlinear function of distance.

Interpretation of Results. Continuing with the analogy of a river set forth in the preceding experiment, we now introduce the concept of a streamline – the path along which a given particle will flow. Model 3-a simulated the uniform flow of water from left to right. If that model had been constructed with a line input at the north boundary (as in this model) and a grounded south boundary, the contours would have extended from east to west, been equally spaced, and representated the direction and uniformity of flow. Thus, the contours of this model represent streamlines.

Further Notes. The next model is constructed in a manner similar to 3-a, so that not only direction of flow is evident (as shown in this model) but that flow velocity variations can be mapped.



Experiment 3-b. Streamline Model of Flow Against Absolute Barriers

A streamline model can be obtained by exciting the north boundary, grounding the south boundary, and covering the barriers with silver paint or aluminum tape. The result is a set of contours that represent streamlines along which water would flow. Note the reduced flow rate on the downstream side of the barriers. Water would tend to circulate behind the barriers rather than move directly downstream.

Experiment 3-c. Velocity Model of Flow Against Barriers

Purpose – to introduce the concepts of velocity of flow in relation to barriers, continuing the analogy of the two previous models.

Modeling Procedures:

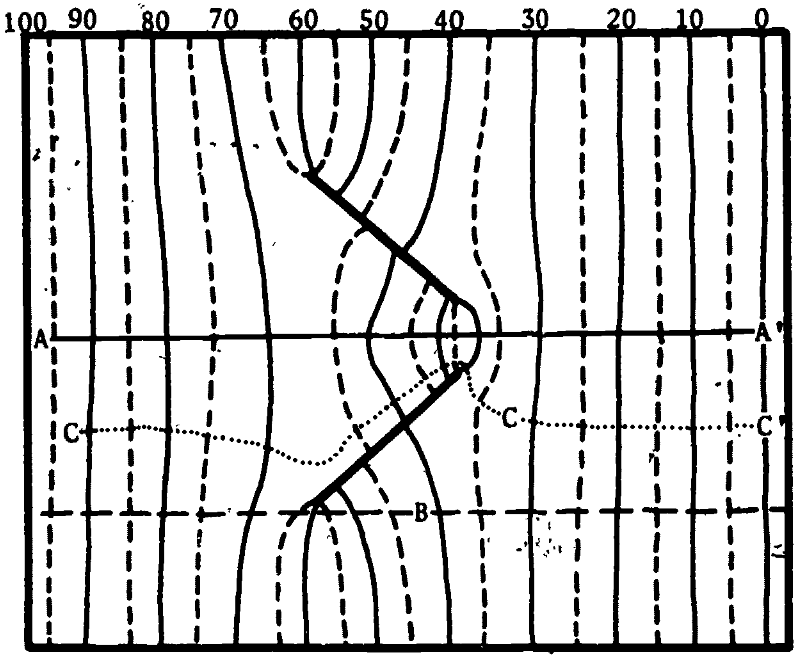
1. Follow modeling procedures 1, 2, and 3 of experiment 3-a, and 4 of 3-b.
2. Cut out the two obstacles, making them absolute barriers.
3. Follow modeling procedures 4 and 5 of experiment 3-a.

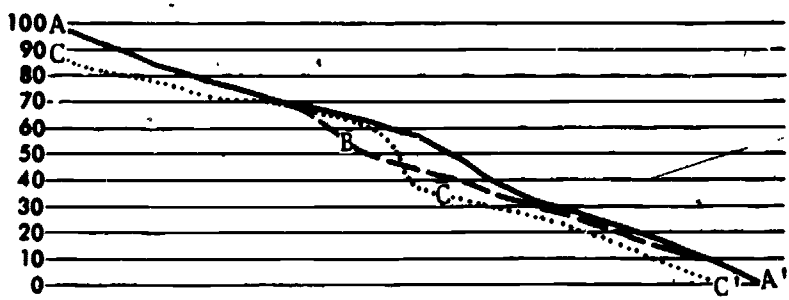
Output Description:

1. Follow output description procedures 1 and 2 of experiment 3-a.
2. Draw a horizontal line just below the bottom obstacle and label it B-B'. Draw another line along the lowest path shown in model 3-b that runs between barriers, and label it C-C'.
3. Plot a profile along both B-B' and C-C'. Note that the variation in velocity (rate of change of profile slope) is least for A-A' and greatest for C-C'.

Interpretation of Results. The effect of the barriers on flow velocity varies with location. The least change in velocity is along the edge of the stream. In the center of the river, the water is "backed up" by the obstacles; the rate of flow slows before reaching the narrow gap between the barriers. Water then speeds as it goes through the gap and, finally, resumes its normal flow speed downstream from the barriers. Water that flows nearest the obstacles is most affected – it slows for a longer stretch approaching the barriers and speeds up to a greater extent upon passing through the gap. Note that line B-B' permits one to interpret changes in velocity along a straight line but does not, as in the case of A-A' and C-C', directly indicate velocity along the variation of the flow path.

Further Notes. This model can be viewed as the inverse of the streamline model. It was accomplished by making all streamline boundaries into isopotential boundaries, and by changing all isopotential boundaries to streamline boundaries. This meant that the line input and line ground were switched from the north and south boundaries to the west and east boundaries, respectively. Also, the highly conductive (silver-painted or aluminum-taped) barriers of 3-b were cut out.





Experiment 3-c. Velocity Model of Flow Against Barriers

Two linear cutout barriers have a focusing effect upon flow. The A-A' profile reflects the variations in spacing of the contours and, thus, the variations in velocity of flow. Water slows in approaching the obstacles, speeds as it goes through the narrow gap, and resumes its regular velocity downstream. The probable course of water passing point C is indicated by the cross-hatched line C-C' (a streamline). Note, by examination of the C-C' profile, the greater variation in velocity than along either A-A' or B-B'.

Experiment 3-d. Curvilinear Squares (Streamlines and Isopotentials)

Purpose – to illustrate the way in which a streamline map and its inverse can be superimposed to produce a set of curvilinear squares that permit a measrue of water flow pressure.

Modeling Procedures:

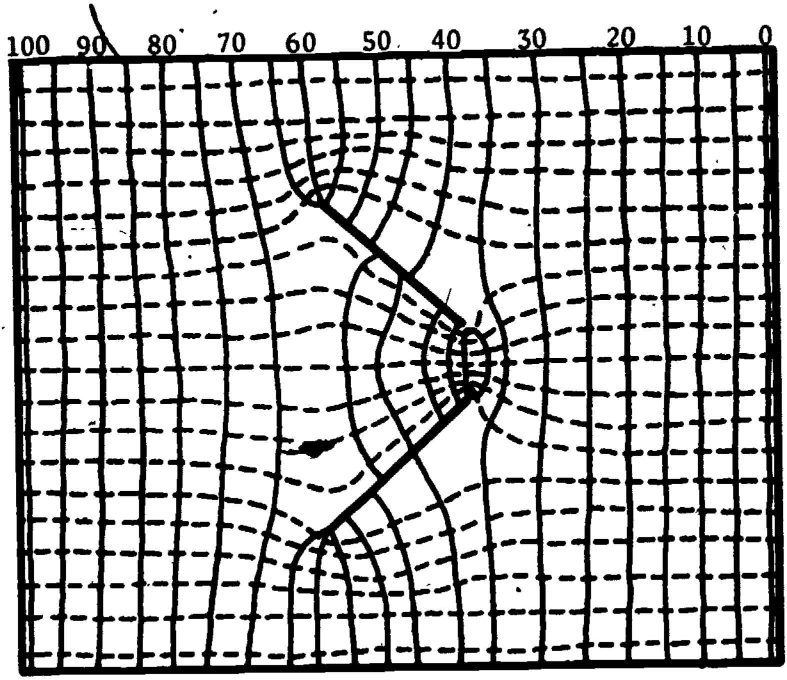
1. Follow all the modeling procedures of 3-b and 3-c.
2. Trace contours of model 3-c in solid lines; superimpose it on the sheet of model 3-b, aligning the barriers; and trace the streamlines as dashed lines.

Output Description:

Compute the approximate area of each curvilinear square. Use a fine-mesh, transparent grid or planimeter if available.

Interpretation of Results. Since the spacing of solid-line contours indicates the velocity of flow as developed by experiments 3-a and 3-c, and the dashed lines indicate the path of flow, it is possible to employ the two together to indicate water flow pressure. The isopotentials and streamlines always cross each other at right angles; moreover, the curvilinear squares formed by them are such that each square contains the same number of electrons (if both sets of contours are probed at equal increments of potential value). Density is a measure of number divided by area. Since the number (of electrons or whatever electrons represent by analogy) is constant, density varies inversely as the area of the curvilinear squares. That is, if the area of one square is one-half that of another, its density is twice that of the other. The river analog to electron density is water flow pressure. When water flows in the gap between the barriers, the curvilinear squares decrease in size (area) and indicate higher water flow pressure. The squares just downstream and behind the barriers are much larger, representing lower water pressure.

Further Notes. Perhaps similar models can be applied to simulate the phenomenon of movements of people and variations in population density as influenced by natural obstacles.



Experiment 3-d. Curvilinear Squares (Streamlines and Isopotentials)

When the streamlines of model 3-b are superimposed on the isopotential lines of 3-c, the result is a set of curvilinear squares. The solid lines are isopotential contours that indicate the velocity of flow, and the dashed streamlines indicate the path of flow. Note that the two always intersect at right angles. The area of squares formed by adjacent streamlines and isopotentials, when compared, are a direct measure of current density – the larger the area of a square, the lower the density. In a water stream, the interpretation of density would be water flow pressure.

Experiment 3-e. Stream Velocity as Affected by Channel Width, Depth, and Gradient

Purpose – to model simple, hypothetical cases of water flow in streams in order to better understand the effects of varying stream width, depth, and stream-bed gradient on water flow velocity.

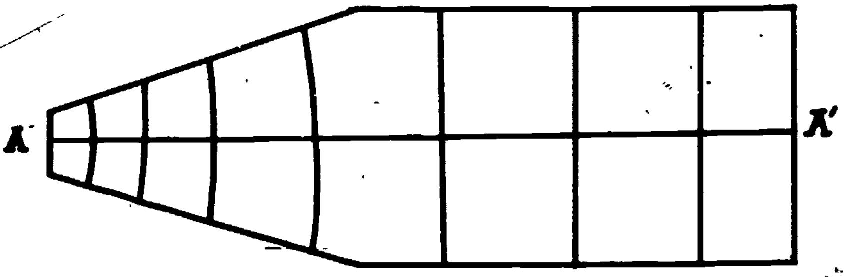
Modeling Procedures:

1. Draw on a sheet of conductive paper; a hypothetical stream trace such as those shown in the illustration. The length of the stream trace should be inversely proportional to the average stream gradient. For example, if a stream gradient (average slope of stream bed) of 10 feet per mile were to be represented by a 10-centimeter-long trace on the model, a gradient of 5 feet per mile would be modeled as a paper strip 20 centimeters long.
2. Next, vary the width of the paper strip along the stream trace, making the strip width directly proportional to the cross-sectional area of flow (the product of water depth and channel width) and/or inversely proportional to variations from the average stream gradient – inversely means that if the stream gradient steepens (doubles), the paper strip should become narrower (one-half as wide).
3. Form a line input (also for each stream tributary) at the upstream end of the paper strip, and a line ground at the opposite end of the strip. The line ground can be accomplished by clamping the strip end under the grounded hold-down frame. Current input pins can be inserted into aluminum tape applied across the main and tributary upstream ends.
4. Make the relative values of the current inputs in proportion to the fraction of main stream flow contributed by each tributary.
5. Follow modeling procedures 4 and 5 of experiment 3-a.

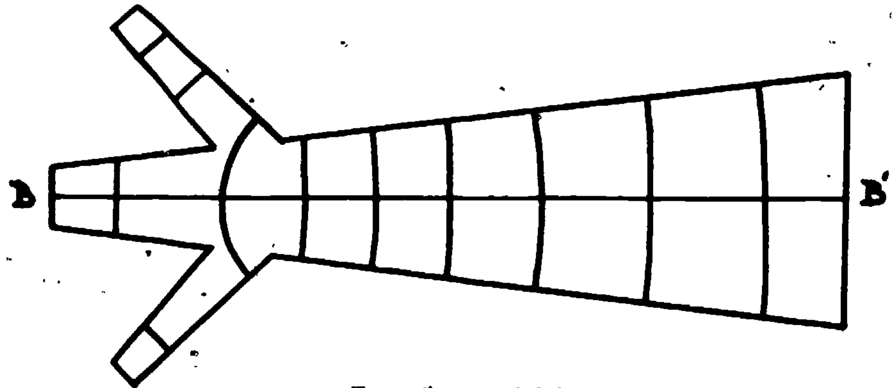
Output Description:

1. Follow output description procedures 1 and 2 of experiment 3-a.
2. Note the variation in slope of the profile along the length of the stream. The stream flow velocity is directly proportional to the *rate* of change of profile slope.

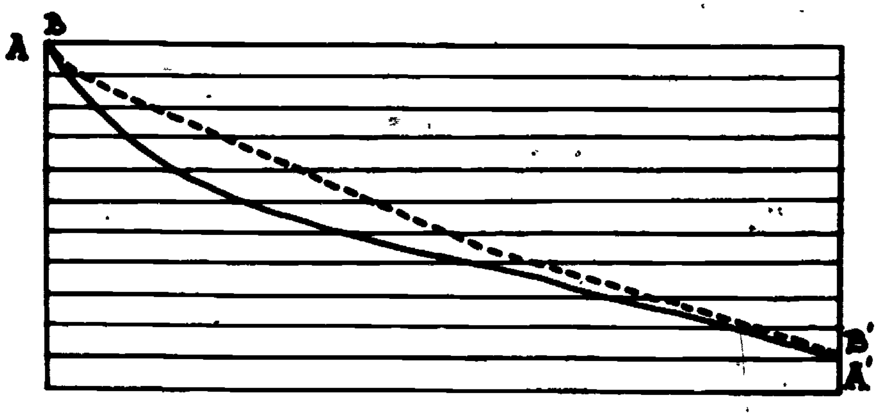
Interpretation of Results. A stream tends to widen and/or deepen as it flows from its source to its mouth. Stream gradient generally decreases with distance from the stream source. Both of these trends point to a wedge-shaped conductive sheet model, narrower at the source (or upstream end) and wider at the mouth (downstream). The slope of the stream model profile decreases downstream, and, consequently, stream velocity decreases (at the *rate* of change of profile slope).



A stream flowing onto a plain.



Two tributaries joining a main stream.



Experiment 3-e. Stream Velocity as Affected by Channel Width, Depth, and Gradient

This series of experiments is designed to provide for the consideration of many inputs on one model. The procedure is particularly appropriate for simulating hexagonal models of trade areas. The three major assumptions – uniform plane, regular spacing, and uniform size – are considered and then, one at a time, they are relaxed to approximate reality more closely. The series illustrates the way in which the field plotter can be used to bridge the gap between theoretical constructs and observable reality.

The series includes:

a. The General Case and Slight Variation in Spacing
b. More Variation in Spacing
c. Predominant Linear Alignment of Points
d. The General Case and Slight Variation in Size
e. More Variation in Relative Size of the Central Input
f. Greater Variation in Relative Size of the Central Input
g. Variations in Size of Several Inputs
h. Simulation of Trade Areas of North Dakota

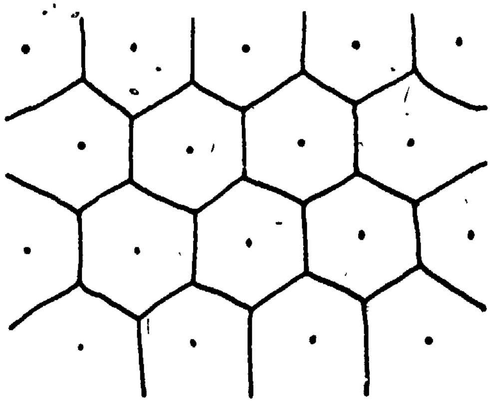
Experiment 4-a: The General Case and Slight Variation in Spacing

Purpose – to demonstrate iterative inputs and to introduce the general case of hexagonal models, set forth the assumptions to produce regular hexagons, and then slightly relax one of the assumptions.

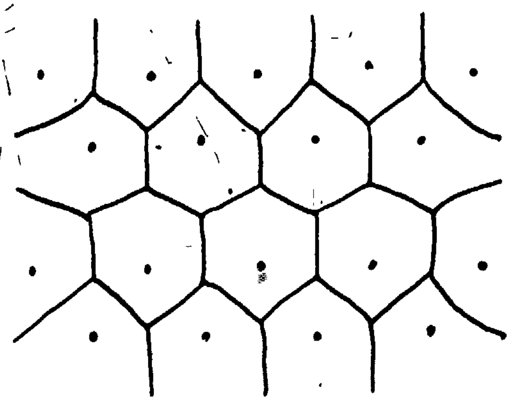
Modeling Procedures:

1. Locate 18 points in alternating rows of 5, 4, 5, and 4. Draw horizontal lines every 2 11/16 inches on a full sheet of conductive paper. On the top line and third line locate a point 1½ inches from the left margin; on the second and fourth lines locate points 4¾ inches from the left margin. Locate three other points (on each of the four lines), one point every 3¼ inches.
2. Place input pins in any two adjacent points. Clip a numbered lead to the shaft of one pin, and set the appropriate current control to 10.0. Clip the ground lead to the shaft of the other pin. Raise the hold-down frame.
3. Calibrate.
4. Leaving the hold-down frame in a raised position, probe the 50-percent line until it gets obviously closer to another pair of points than the pair being studied, or until it reaches the edge of the paper.
5. Move either the input or ground to a nearby point and repeat steps 3 and 4.
6. Repeat steps 3, 4, and 5 until all pairs have been studied.
7. Label the results 4-a-(1).
8. Move all the points on lines 2 and 4 up 1 inch and repeat steps 1 through 6. Label the results 4-a-(2).

Interpretation of Results. The resulting regular hexagons in the center depend upon three assumptions: first, a uniform plane; second, regular spacing; and third, inputs of uniform size. The uniform resistance of the paper simulates the first assumption, the regular spacing of the input points simulates the second, where the regular spacing required is that of pool balls in a rack or bowling pins. Using the same input for each and the 50-percent line as the breakpoint, simulates the inputs of uniform size. Note that size and spacing, taken together, are represented by pool balls in a triangular rack, and represent a closest packing principle – balls of that size could not be packed on any two-dimensional surface smaller than the triangular rack. Conversely, such close packing is uniform only because the balls are the same size. Moreover, no other spacing within the triangular rack is possible. Model 4-a-(1) simulates the hexagonal model of Christaller except for the areas near the boundary. Christaller assumed an "unbounded" uniform plane. Model 4-a-(2) illustrates how irregular hexagons result from even a slight alteration of the spacing while the uniform-plane and uniform-size assumptions are retained.



(1) General Case of the Hexagonal Model



(2) Alternate Rows Moved up One Inch

Experiment 4-a. The General Case and Slight Variation in Spacing

Three assumptions – the uniform plane, regular spacing, and uniform size of towns – are required to produce the classical hexagon pattern of trade areas. The rule of each assumption is made clearer by holding two of these three assumptions intact while the third is varied. The regular hexagons become irregular (2) with the slightest alteration of spacing. Bear in mind that 4-a-(2) is just a special case of 4-a-(1).

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Experiment 4-b. More Variation in Spacing

Purpose -to illustrate the effect of greater alteration of point spacing on the size and geometry of trade areas, retaining the uniform-plane and uniform-size assumptions.

Modeling Procedures:

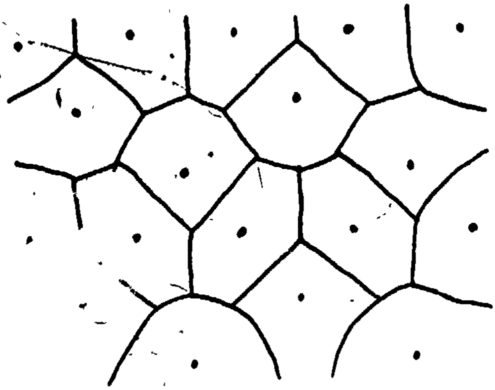
1. Follow modeling procedure 1 of experiment 4-a.
2. Move the first and third points (from the left of lines 2 and 4) down 1 inch, and move the second and fourth points on both lines up 1 inch.
3. Follow modeling procedures 2 through 6 of experiment 4-a.
4. Label the results 4-b-(1).
5. Move the first point (from the left end of line 1) to a place equidistant between the new positions of the first and third points of line 2; move the last point on line 1 down to a position the same distance to the right of the new position of the third point of line 2 as the other point just located lies to the left of it.
6. Follow modeling procedures 2 through 6 of experiment 4-a.
7. Label the results 4-b-(2).

Output Description:

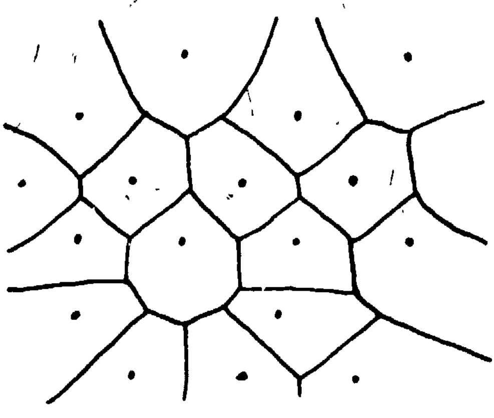
1. Note on model 4-b-(1) that although the hexagons are very irregular, all of the interior point trade areas are hexagonal. Note also that the area included within each interior hexagon remains relatively constant.
2. In model 4-b-(2), an octagon forms around the one input encircled by eight other inputs; two points have only five near neighbors and, consequently, their trade areas are pentagonal.

Interpretation of Results. Continuing with the interpretation of 4-a, trade areas develop forms other than hexagons when a town has fewer or more than six near neighbors. Nonetheless, even a highly irregular point spacing such as in 4-b-(2) has half of its interior trade areas in the shape of a hexagon. Furthermore, the area covered by each of the interior trade areas is more constant than might be expected.

Further Notes. It is very instructive to choose the size and shape of trade areas you might want to create; move the points to where you think you will get the expected results; and, finally, probe the model to see how closely you predicted the relationships.



(1) Highly Irregular Hexagons



(2) Irregular Pentagons, Hexagons, and Octagons

Experiment 4-b. More Variation in Spacing

Change the spacing of every other point as in 4-a-(2), except now alternately move points 1 inch down and 1 inch up. Highly irregular hexagons develop (1). By moving the two corner towns on the upper side, an octagon and two pentagons emerge (2). Hexagons still persist as the dominant trade area geometry in these two models and in the real world. The boundary conditions of this series have an interesting interpretation, but we will defer discussion until later.

Experiment 4-c. Predominant Linear Alignment of Points

Purpose – to illustrate the changes in size and shape of trade areas that result from linear arrangements of towns, even when the uniform-plane and uniform-size assumptions are retained.

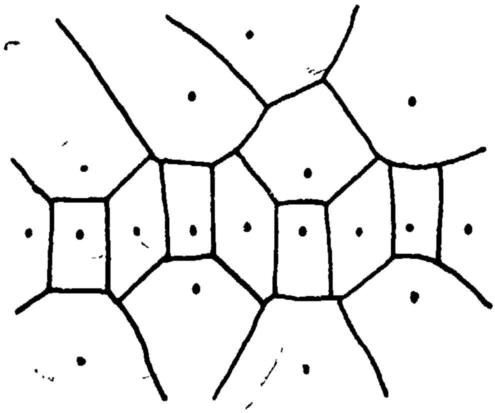
Modeling Procedures:

1. Beginning with the spacing developed in experiment 4-b-(1) move the first, second, fourth, and fifth points of line 1 to line 3, placing new points equidistant between each of the previously existing pairs of points on line 3.
2. Follow modeling procedures 2 through 6 of experiment 4-a.
3. Label the results 4-c-(1).
4. Beginning with the spacing of 4-c-(1), move the second-from-the-left point in the line of nine points to a position 3-7/8 inches north of (above) the center point of that line; move the second-from-the-right point to a position 3-7/8 inches below the center point.
5. Follow modeling procedures 2 through 6 of experiment 4-a.
6. Label the results 4-c-(2).

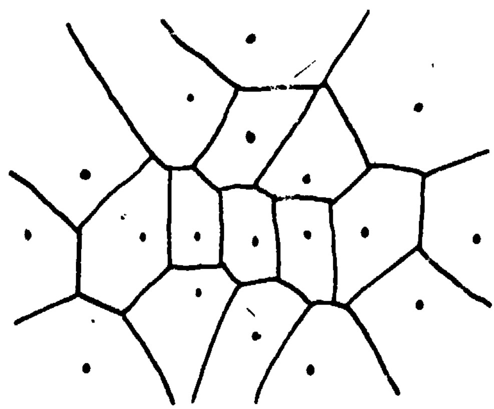
Output Descriptions:

1. Note the three rectangular trade areas of model 4-c-(1) that result from four neighboring inputs spaced in such a way that a line between two opposite pairs would form two lines that cross each other at right angles. Note also that the rectangles are separated from each other by hexagons. The smaller trade areas form a horizontal strip, since half of the points are on a single line.
2. The spacing of model 4-c-(2) provides the most clustered pattern in the present series. Half of its interior shapes are pentagons; the other half are hexagons.

Interpretation of Results. Hexagonal trade areas appear to emerge regardless of the kind of spacing; hexagonal forms are the only ones that have emerged in all spacing patterns used in this series so far. The kind of linear spacing of these two models is instructive but somewhat paradoxical. The uniform-plane assumption is retained in these models, but linear spacing tends to arise in the real world along a road or in a mountain valley, both of which are clearly not "uniform planes."



(1) Rectangles and Hexagons



(2) Pentagons and Hexagons

Experiment 4-c. Predominant Linear Alignment of Points

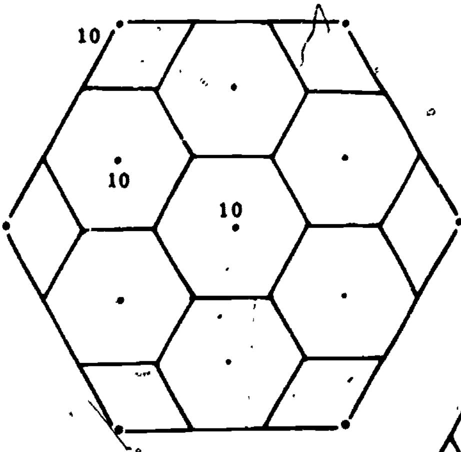
When the spacing is altered to simulate the general tendency of towns to devleop along a main road, rectangular trade areas develop (1). Towns usually are not evenly spaced along a road. Irregular spacing leads to the development of irregular pentagons and hexagons. Keep in mind, as you look at the above trade areas, that all towns are considered to be the same size. Further variation in spacing can result in a large variety of trade-area geometries.

Experiment 4-d. The General Case and Slight Variation in Size

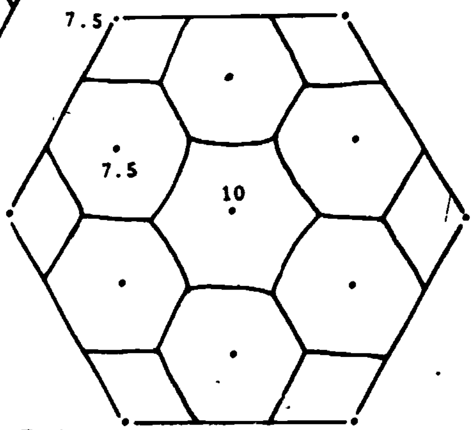
Purpose—to illustrate the kinds of changes that occur in size and shape of trade areas when the uniform-plane and regular-spacing assumptions are retained and the size of one town is varied, and to introduce a mathematical procedure for obtaining a more consistent technique for locating breakpoints.

Modeling Procedures:

1. Locate a point (input) at the center of the sheet. (See modeling procedure 2 of experiment 1-a, if necessary).
2. Using a protractor, draw an east-west line and a north-south line through the center. Draw lines that form 30-degree angles radiating from the center.
3. Locate a point 3-1/4 inches from the center on the north line. Locate a second point 5-3/4 inches from the center on the line 30 degrees to the right of the line running north. Continue in a clockwise direction locating points alternately 3-1/4 inches and 5-3/4 inches from the center. Draw a large hexagon by connecting the outer ring of points with a series of straight lines.
4. Follow procedures 2 through 6 of experiment 4-a, restricting all lines to be within the large hexagon. Label the results 4-d-(1).
5. Consider the central point to have a value of 10 while the rest of the inputs have a value of 7.5. A breakpoint between two such different sized points can be established by dividing the magnitude (input value) of the smaller into the magnitude of the larger, extracting the square root of the quotient and adding 1 to this square root. Next divide the sum thus calculated into the separation distance of the two points. The final quotient is the distance of the breakpoint from the smaller input on a line to the larger input. For this model $10 \div 7.5 = 1\text{-}1/3; \sqrt{1.333} = 1.15$; $1.15+1=2.15$, $3.25 \div 2.15=1.5$. Thus, the breakpoint lies 1-1/2 inches from each small town, on a line to the central town. Locate each breakpoint.
6. Insert an input pin at the center point, clip a numbered current lead to the shaft of the pin and turn the appropriate current control to a setting of 10.0. Insert the ground pin at one of the inner circle inputs and clip the ground lead to the shaft of the ground pin. Calibrate and place the probe on one of the breakpoints, turn the contour selector until a null is achieved, and maintain the null while probing the trade boundary.
7. Move the grounded pin to other cities on the inner circle and repeat procedure 6 until all the trade area boundaries are established. Map the trade area boundaries of all remaining inputs (outside the inner circle of points) by using the same procedure. $1 \div 1=1; \sqrt{1}=1$; $1+1=2$; $S/2= .5S$. (S=point separation distance.) Label the results 4-d-(2).



(1) The General Case of the Hexagonal Model



(2) 10:7.5 Town Size Ratio

Experiment 4-d. The General Case and Slight Variation in Size

The consequences of varying the size of towns while the uniform plane and the regular spacing are held constant can be illustrated by looking at a central town and its surrounding, regular-spaced neighbors (1) The six diamond-shaped segments represent part of the trade areas of towns located at the points of the outer hexagon. When the relative size of the central town is increased to a size of 10, as compared to 7.5 for its neighbors, the hexagon pattern is distorted. The central trade area is extended along each line that divides the trade areas of two smaller neighbors, increasing the central trade area by 12 percent.

Experiment 4-e. More Variation in Relative Size of the Central Input

Purpose- to illustrate the kinds of changes that occur in size and shape of trade areas as the size of the central town is further increased to twice and then four times the size of the other towns.

Modeling Procedures:

1. First, using procedures outlined in 4-d, set up and probe the trade area boundaries for a central input value twice as large as that of the other inputs. The square root of 2 is 1.41. Label the results 4-e-(1).

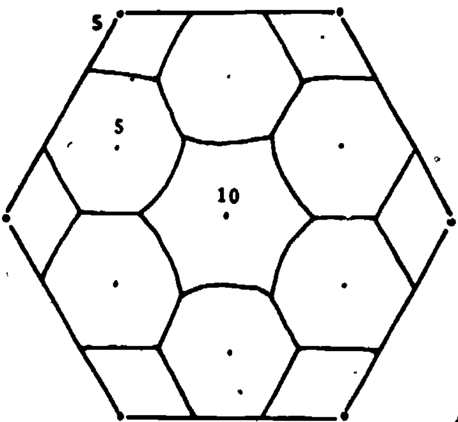
2. Set up and probe the trade area boundaries for a central input four times as large as the other inputs. Label the results 4-e-(2).

Interpretation of Results. By comparing the trade areas of the two models in 4-d and the two in 4-e, it is possible to gain considerable insight into the way a city could grow and extend its trade area or area of economic and social dominance. The regular hexagon of the initial model expanded into a hexagon made up of a series of arches when the size ratio was 4:?; the smaller towns yielded more area along the boundaries between them than they did between one of them and the growing central city. In the models of 4-e there was not such a noticeable change, but the pattern for 4-e (4-e-(2)) indicated a major change. The points of the expanding trade areas were penetrating almost to the trade boundaries that separated towns of the inner ring from towns of the outer ring. The trade areas of the inner ring took on the characteristics of the upper part of an ice cream cone. The trade area boundary between a given town from the inner circle and the two towns of the outer circle that are nearest it remains unchanged.

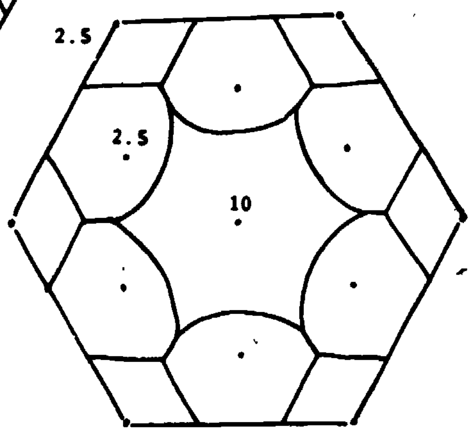
Further Notes. The formula for solving for the breakpoint, formalizing the step-by-step procedure described in experiment 4-d, is:

$$D_{x \leftarrow y} = \frac{d}{1 + \sqrt{P_x / P_y}}$$

where $D_{x \leftarrow y}$ is the distance the break-point is located from the smallest town (y) in the direction of the largest town (x), d is the total distance between the two towns, and P is the population.



(1) 2:1 Town Size Ratio



(2) 4:1 Town Size Ratio

Experiment 4-e. More Variation in Relative Size of the Central Input

When the ratio is increased to 2:1, the size of the trade area is increased another 12 percent (1). When the ratio is increased to 4:1, the size of the trade area increases another 60 percent, or a total 95 percent increase over its original size (2). Note that in both (1) and (2) the growing city has continued to extend its trade area at the uniform expense of its neighbors (each neighbor gives up the same amount of area).

Experiment 4-f. Greater Variation in Relative Size of the Central Input

Purpose- to illustrate the kinds of change that occur in the size and shape of trade areas as the size of the central town is yet further increased to 10 times and then 20 times the size of the other towns.

Modeling Procedures:

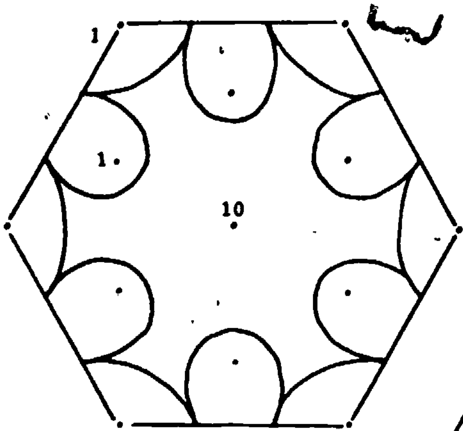
1. First, using procedures outlined in 4-d, set up and probe the trade area boundaries for a central input 10 times as large as the other inputs. Label the output 4-f-(1).

2. Set up and probe the trade area boundaries for a central input 20 times as large as the other inputs.

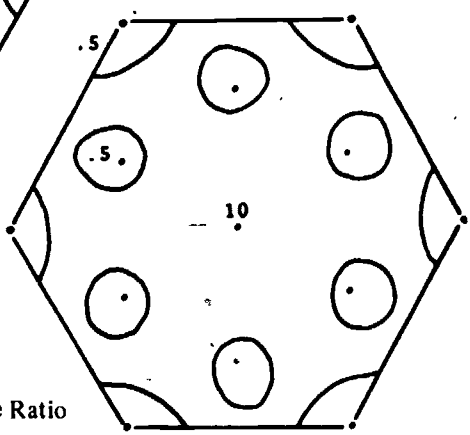
Interpretation of Results. The impact of the central town or towns on the outer circle became a significant factor well before the 10:1 ratio was established. By the time the growth had reached a 10:1 ratio, the trade boundaries of the towns of the outer circles had retreated from a sharp point to a gentle arc.

By the time the growth reaches a 20:1 ratio, the inner ring of towns is completely encircled; each one has a much reduced area of dominance, while the area of dominance of the central city lies well beyond their small areas. This means that most of the people living in very close to these small towns that are only 1/20th the size of the central city will, say, depend upon the local newspaper; but those who live some distance beyond the small town, on the side opposite the central city, will depend upon the newspaper of the central city and ignore the one published in the small town. Continuing with the newspaper analogy, the circulation of the central-city newspaper is extending further into areas formerly dominated by newspapers of the outer ring as well.

Further Notes. Another interpretation of the series (a static interpretation) based on 4-d-(1) could be that of different orders of services of one large city in relation to 12 smaller towns. Thus, for low-order goods (groceries) 4-d-(1) indicates the trade area of the larger city; 4-d-(2) would depict trade areas for higher and higher orders of service (specialty shopping, hospital care, etc.).



(1) 10:1 Town Size Ratio



(2) 20:1 Town Size Ratio

Experiment 4-f. Greater Variation in Relative Size of the Central Input

At a 10:1 ratio, the trade area of the growing town begins to surround the trade area of each near neighbor (1). When the relative size of the growing city reaches a 20:1 ratio in relation to its nearer neighbors, its trade area has completely enclosed the trade areas of its much smaller neighbors (2). Although not illustrated, at an approximate 5:1 ratio, the trade area of the central city impinges on the trade areas of the outer ring of cities (1) and (2).

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Experiment 4-g. Variations in Size of Several Inputs

Purpose– to illustrate the size and shape of trade areas that result from the uniform-plane and regular-spacing assumptions but with several different sizes of towns. In one set, 5 different sizes are employed for the 13 towns; in the other, 13 randomly selected sizes are employed.

Modeling Procedures:

1. Set up the basic pattern following modeling procedures 1, 2, and 3 of experiment 4-d.

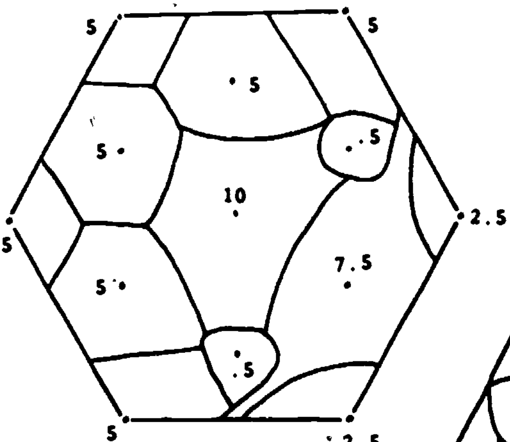
2. Assign sizes to each town according to the labels on 4-g-(1). Following modeling procedures of experiment 4-d, establish the breakpoints and 13 trade areas. Label the results 4-g-(1).

3. Set up the basic pattern. If you have access to a table of two-digit random numbers and know how to use it, assign a value of 10 to the central city and random sizes between 15 and 10 for the remainder. Otherwise, use the randomly generated values presented in model 4-g-(2). Establish the breakpoints and trade area boundaries. Label the results 4-g-(2).

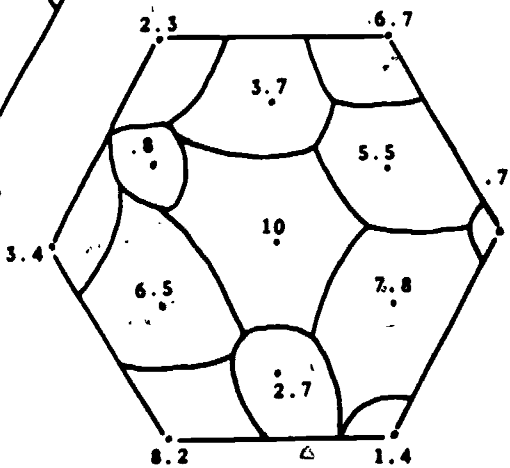
Interpretation of Results. The results much more closely approximate the shape of trade areas observable in the real world. One conclusion that can be made is that either shape or size of trade areas depend on both spacing and size of towns. Model 4-g varies both spacing and size simultaneously.

Further Notes. One of the most interesting applications of series such as 4-a through 4-g is to set up simulations of a self-perpetuating game. In a simple form such a series could begin with model 4-d-(1). Increase the size of the central city arbitrarily; measure the resulting increase in area and, in turn, increase the size of the central city in proportion to the increased trade area; and then continue to increase the trade area because of increased size of the town, and the size of the town because of the increased trade. An equilibrium condition is finally reached. Many variations of different degrees of complexity are possible.

A more involved form is encountered by starting two distant towns growing at the same time and at the same rate. They initially interact with nearby towns but, if there aren't too many intervening towns, they eventually interact with each other. A more interesting series is generated when they are started to grow at the same time but at different rates. Also, various predetermined changes can be made in terms of spacing and size that can be made into a set of rules. Adding random selection of the degree of change permitted in certain stages of a series makes it possible to have a tight set of rules but still get variations. With such variations it is possible to generate some surprisingly realistic and enlightening results.



(1) Assigned Sizes of Towns



(2) Randomly Derived Sizes of Towns

Experiment 4-g. Variations in Size of Several Inputs

When several sizes of towns are considered, while the uniform-plane and regular-spacing assumptions are retained, few straight-line boundaries remain. In model (1) towns are assigned values of 10, 7.5, 5, 2.5, and .5. In model (2) the towns were assigned values from a table of random numbers ranging from .5 to 10. The central town was intentionally given a value of 10. Straight lines have completely disappeared, making the model much closer to reality. These two models complete the series on varying size. The next model varies spacing and size simultaneously, while retaining the uniform-plane assumption.

Experiment 4-h. Simulation of Trade Areas of North Dakota

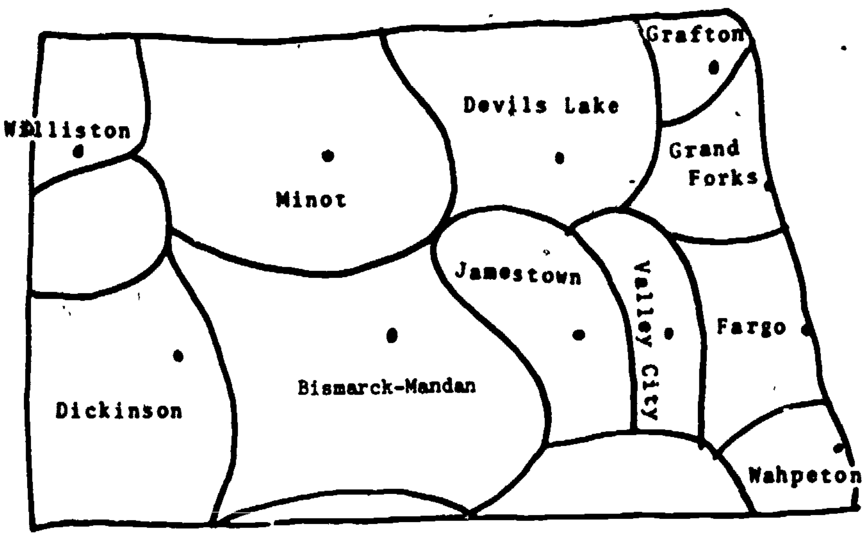
Purpose—to illustrate the simultaneous use of irregular spacing and varying size to simulate observable reality, and to use roads to relax the uniform-plane assumption.

Modeling Procedures:

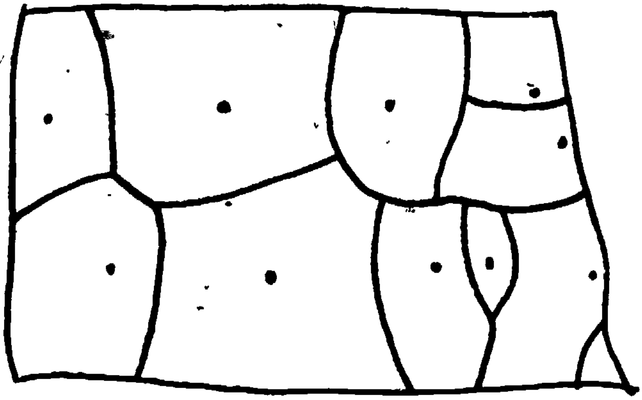
1. Draw a map of North Dakota the size of a sheet of conductive paper. Make inputs for the towns proportional to population as follows: Fargo, 9.9; Bismarck-Mandan, 6.6; Grand Forks, 6.0; Minot, 5.1; Jamestown, 2.7; Devils Lake, 2.7; Valley City, 2.0; Dickinson, 2.0; Williston, 2.0; Grafton, 1.2; Wahpeton, 1.2. Delimit the trade areas and label the results 4-h-(1).

2. Continue with 4-h-(1) by drawing the roads on that model and applying one layer of carbon paint to each road, and an additional two for the major highway—indicated by a double line on model 4-h-(2). Replot the trade areas.

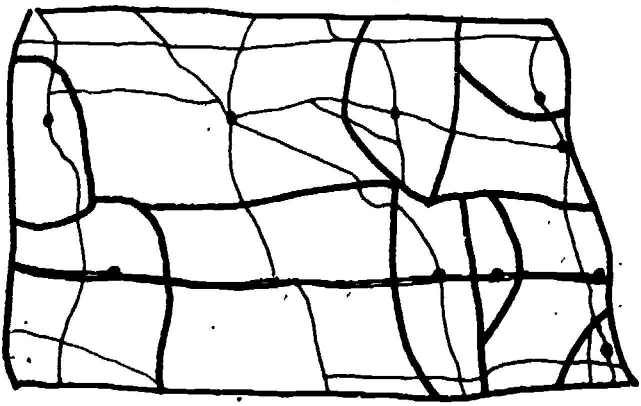
Interpretation of Results. Compare the results with the map below that was adapted from J. R. Borchert and R. B. Adams, *Trade Centers and Trade Areas of the Upper Midwest* (Upper Midwest Economic Study Urban Report Number 3, September 1963).



In addition to the three towns outside the state, other factors could be taken into consideration in this experiment to better simulate the observable trade areas. Topographic variation, employment phenomena, and school districts could provide a much improved simulation if they were considered.



(1) Spacing and Size of North Dakota Towns



(2) Trade Areas and Roads

Experiment 4-h. Simulation of Trade Areas of North Dakota

Both irregular spacing and varying size of towns are employed in model 4-h-(1), and the results closely approximate empirically derived trade areas. In model 4-h-(2) the uniform-plane assumption is relaxed somewhat by using carbon paint to simulate major roads. The major observable differences between the simulated trade areas and those empirically derived are owing to two towns in South Dakota and one in Montana that were not considered in the simulation models.

B. ANALYSIS MODELS (SERIES 5).

The four series thus far have dealt with conceptual and theoretical models, with the exception of experiment 4-h. The purpose of the present series is to illustrate the use of the field plotter to analyze geographic aspects of problems. Space does not permit the development of elaborate problems, but the various exercises can be combined to perform very elaborate experiments. The conductive paper can be obtained in rolls 31 inches wide and 20 feet long; consequently, it is possible to work very large and intricate problems. When such large models are employed, it is even possible to work on one part of a large problem at a time. In problems of analysis the field plotter is a fascinating instrument to work with, since it requires that an operator formulate his problem in precise terms, seek the necessary data, and seek further data to verify the results.

The series includes:

a. Four Cities, One Road, and a River
b. Four Cities, Two Roads, and a River
c. Attraction Potentials of Kansas City and St. Louis
d. Interaction Potential of Kansas City and St. Louis
e. Interaction Potential Considering Only Areas Along Major Highways
f. Interaction Potential of Supermarkets Considering a River and a Bridge
g. Trade Areas of Supermarkets
h. Trade Areas Considering Major Streets
i. Contours of Urban Land Use for Different Locational Constraints
j. Population Potential of the United States
k. Six Models of U.S. Population Potential
l. Great-Circle Routes on a Mercator Projection
m. Orthographic Map Projection
n. Glacier Flow in North America

Experiment 5-a. Four Cities, One Road, and a River

Purpose--to set forth a multiple-stage procedure for analyzing the influence of barriers and expediters on population potential.

Modeling Procedures:

1. Mount a sheet of conductive paper under the hold-down frame of the field plotter. Mark four points located approximately as those shown in the illustration. Insert current input pins at each of these points, and clip current leads to the shafts of the input pins (the number of the current lead and that of the input pin should be the same). Set the corresponding current controls to 10.0 for the northwest and southeast inputs (cities) and to 2.5 for the other two inputs. Make sure that the current controls not used are all set to zero.

2. Calibrate and probe the (population) potential pattern mapping contours whose values differ by 10 percent. Also probe a supplementary, 45-percent contour. Trace onto an overlay of transparent paper the population potential map you have just constructed. Label the tracing stage 1.

3. Apply a narrow strip of carbon paint to a line connecting the northwest and northeast cities. Allow 4 to 6 minutes for the paint to dry, recalibrate, and reprobe the model. Trace the new map and label the overlay stage 2.

4. Apply another layer of carbon paint and repeat the above procedure. Label the tracing of this map stage 3.

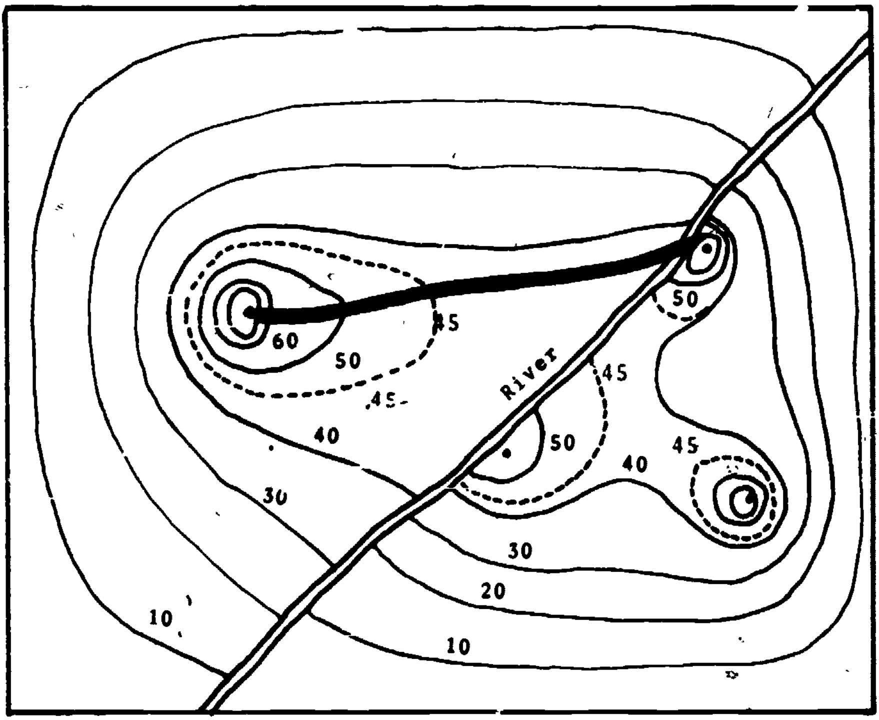
5. Draw a line approximating the trace of the river as shown in the illustration. Now cut the conductive sheet along this line being careful not to cut across the road. Repeat the calibration and probing procedures; the result is stage 4. Make a tracing.

Output Description:

1. Label each tracing clearly. Use a different color for each stage, draw all stages on one sheet of tracing paper (superimposed).

2. On the tracing of each stage draw arrows on each contour indicating the direction of shift of that contour from its position on the one being marked and its position in the next stage. At each arrow, indicate the number of fourths of inches required to move the contour to its new position in the following stage. Plot on a map of each stage the displacement distances for each contour, by drawing contours that connect points of equal displacement. Although the resulting map is not highly refined, it nonetheless serves to indicate the pattern of change in such a way that variations in densities are also readily observable.

3. A better, but more difficult, measure of change is obtained by overlaying a square grid and measuring the direction and magnitude of shift of a large number of contoured points, in going from stage to stage. Draw a map of at least one of the transitions between stages, plotting the "isodisplacement" contours using the more detailed measurements.



Experiment 5-a. Four Cities, One Road, and a River

In a developing country there are few paved roads. This model represents the population potential in such an area where there is one road from the large city in the northwest to a small town on the river (the northeastern town). The general potential north of the river is greater, owing to the road. The area south of the river reflects less interaction among the cities because of the absence of roads.

Experiment 5-b: Four Cities, Two Roads, and a River

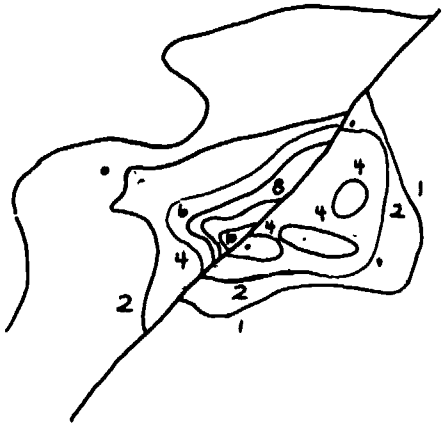
Purpose—to illustrate further the multiple-stage procedure for analyzing the influence of roads on population potential.

Modeling Procedures:

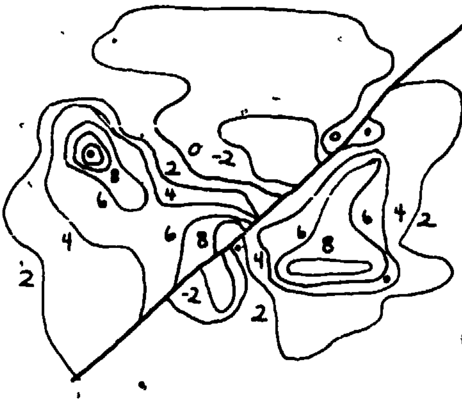
1. Follow modeling procedures 1 and 2 of experiment 5-a.
2. Place a short thin strip of aluminum tape across the river near the site of the central city. Press the tape tightly on either side of the river. (If no tape is available, use silver paint to jump electrical current across the river cutout.) You have simulated a bridge. Recalibrate, probe, and label the output stage 2.
3. Apply a narrow strip of carbon paint between the northwest and southeast cities, crossing the bridge and going through the central city. Allow 5 minutes for it to dry, recalibrate, probe, and label the map stage 3.

Output Description:

Follow output description procedures 1, 2, and 3. Illustrated below and to the left is a tracing that relates the relative displacement between stage 4 of 5-a and stage 3 of this experiment. To the right is a comparison of changes in potential values at discrete points.



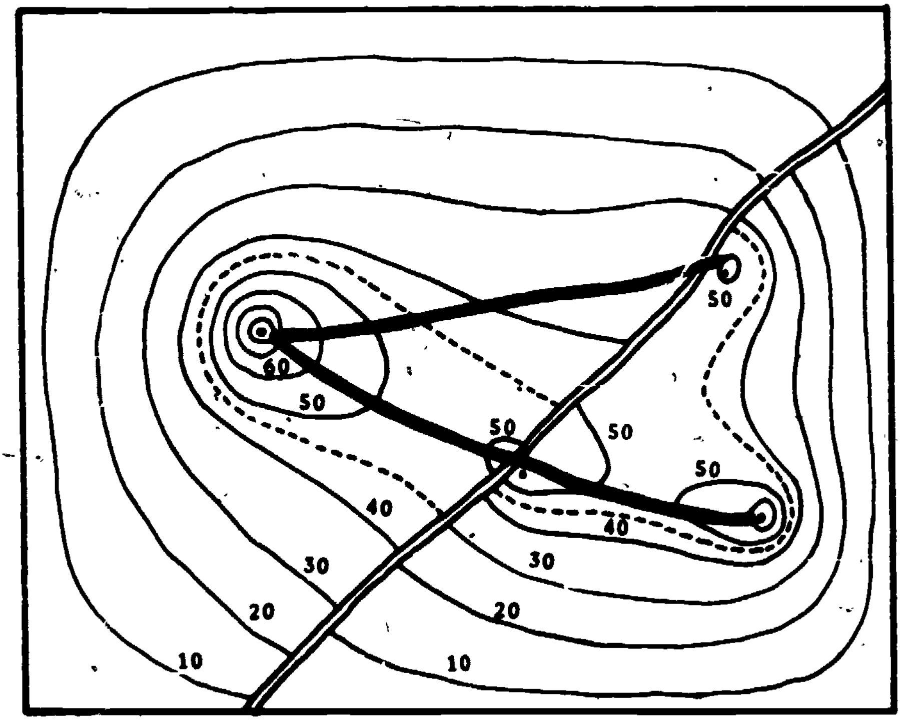
Contour Displacement From 5-a to 5-b



Potential Differences Between 5-a and 5-b at Fixed Points

Interpretation of Results. This model, when compared with 5-a, provides a preliminary analysis of the impact of an additional bridge and a road across the bridge connecting the two largest towns. Appropriate scaling of inputs and outputs may be required to permit their testing in the field. An alternative interpretation would be that stage 4 of 5-a represents the effect of washing out the bridge of 5-b (stage 2).

Further Notes. In general, a problem of analysis should be developed in stages. In this way, modification, alternative scaling, and comparison of results are greatly facilitated.



Experiment 5-b. Four Cities, Two Roads, and a River

Model 5-a is reproduced here with the exception that an additional bridge across the river is simulated. The general potential south of the river is much increased over model 5-a, since the two large cities are now connected by a road. Note that the 40-percent isopotential includes a much larger area southeast of the river. Note also that the 40-percent isopotential, that on model 5-a included all the area along the road from the large city in the northwest to the small town on the river, crosses that road about half-way on this model. The northeast town is now less favorably located.

Experiment 5-c. Attraction Potentials of Kansas City and St. Louis

Purpose– to illustrate the impact of boundary conditions on the attraction potentials of two cities.

Modeling Procedures:

1. Mount a full sheet of paper under the hold-down frame; draw the outline of the state of Missouri, as illustrated. Insert current input pins at the location of Kansas City and St. Louis and clip the numbered leads to pin shafts of the same number.

2. Turn the current control for St. Louis to 10 and the current control for Kansas City, and all other current controls, to zero. Calibrate and probe the isopotential contours. Now turn the current controls of Kansas City to 10 and St. Louis to zero. Recalibrate and probe the isopotential contours. Trace the state boundaries and contours and label the tracings stage 1.

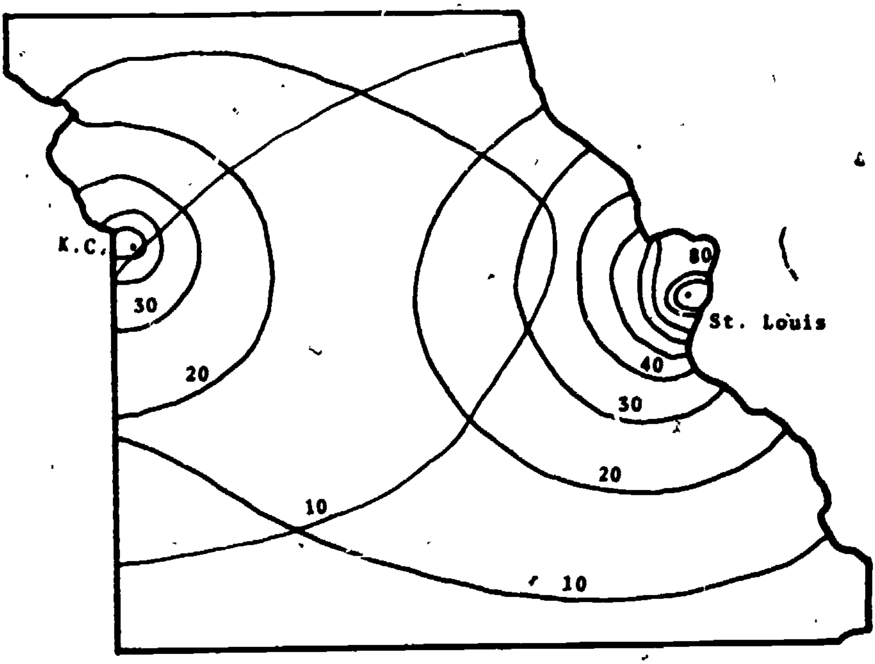
3. Cut the paper along the east and west boundaries of the state.

4. Repeat modeling procedure 2, but now label the tracing stage 2.

Output Description:

Follow output description procedures 1 and 2 of experiment 5-a.

Interpretation of Results. There are two comparisons that are very important. The first is the difference between stage 1 and stage 2, which is attributable to the change in boundary conditions. Study the differences carefully. Note that there is a stronger east-west component to the contour pattern of stage 2, owing to the reflective nature of the ungrounded east and west boundaries. Deciding in advance on boundary conditions should depend upon rational considerations as much as possible, but it is still a trial-and-error process. The second comparison is between the relative influences of Kansas City and St. Louis. In both stages 1 and 2, St. Louis emerges as the more dominant city.



Experiment 5-c. Attraction Potentials of Kansas City and St. Louis

The east and west Missouri state boundaries are ungrounded. The inputs are made in proportion to the urban population of each city. Each input potential pattern was mapped with the other input turned off. St. Louis attracts trade, people, etc. from the west more effectively than Kansas City attracts from the east. The reflective boundaries on the east and west, as well as the absorptive north and south boundaries, realistically impart an east-west orientation to the attraction potential of each city. A grounded rectangular boundary would overcome the east-west orientation if it were required.

Experiment 5-d. Interaction Potential of Kansas City and St. Louis

Purpose—to analyze the effects of boundary conditions on population potentials and trade areas.

Modeling Procedures:

1. Follow modeling procedure 1 of experiment 5-c.
2. Turn the current control for St. Louis to 10, and the one for Kansas City to 5; these represent the approximate population ratio of the two cities. Calibrate, probe, and trace the results, and label the tracing stage 1.
3. Cut the paper along the east and west boundaries.
4. Repeat modeling procedure 2, but now label the tracing stage 2.

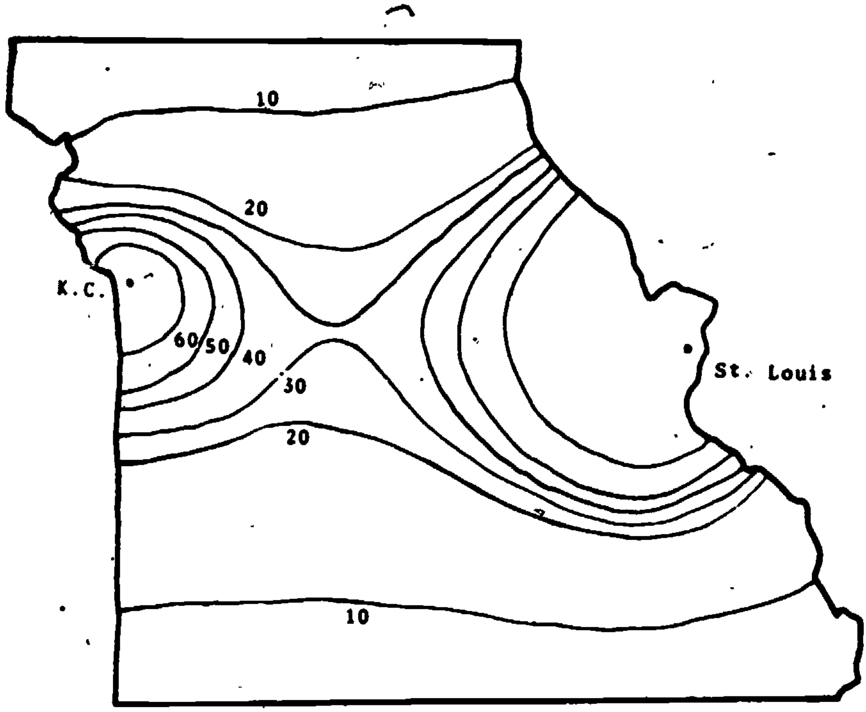
Output Description:

Follow output description procedures 1 and 2 of experiment 5-a.

Interpretation of Results. The state of Missouri is dominated by two cities. The difference between potential patterns of stage 1 and stage 2 is the stronger east-west orientation of stage 2. An intermediate stage could be obtained by cutting 1/2 inch slits every inch along the east and west boundaries. Varying the width of the slits creating partial boundaries in relation to the width of the intervening uncut areas would permit several gradations between stage 1 and stage 2.

Concerning the pattern of interaction potential in stage 2, the 10-percent contours are tending to parallel the grounded boundaries and intersect at right angles the east and west ungrounded boundaries. The 20-percent contour behaves in a similar way. These conditions describe the actual east-west orientation of the economic and cultural life of the state. The 30-percent line differentiates the areas of dominance of the two cities—where they come close to form a neck is the location of the breakpoint between the two. The area dominated by Kansas City is much smaller than that dominated by St. Louis, due to its smaller relative size and its relative location with respect to St. Louis and the state boundaries.

Further Notes. By now you have probably noticed that recalibration sometimes requires a very minor adjustment and sometimes a major one. The degree of adjustment required is a direct measure of the difference between two stages. It is instructive to note the change in the contour selector values (that are required to achieve the null).



Experiment 5-d. Interaction Potential of Kansas City and St. Louis

The model is constructed in the same way as model 5-c except that the inputs are turned on simultaneously. The resulting interaction potential pattern is the sum of the two attraction potentials bearing upon any locational point of the model. Kansas City dominates the western one-third of the state while St. Louis dominates the eastern two-thirds; these areas of dominance are enclosed by the 30-percent contours. The divide (breakpoint) between their areas of dominance is at the narrow neck of the 30-percent contours. Note that the transition from the 20-percent to 10-percent contours is across a broad area, indicating that their mutual influence is marginal and changes very little throughout the northern and southern parts of the state.

Experiment 5-e. Interaction Potential Considering Only Areas Along Major Highways

Purpose—to introduce another method of relaxing the uniform-plane assumption, as applied to interaction potentials.

Modeling Procedures:

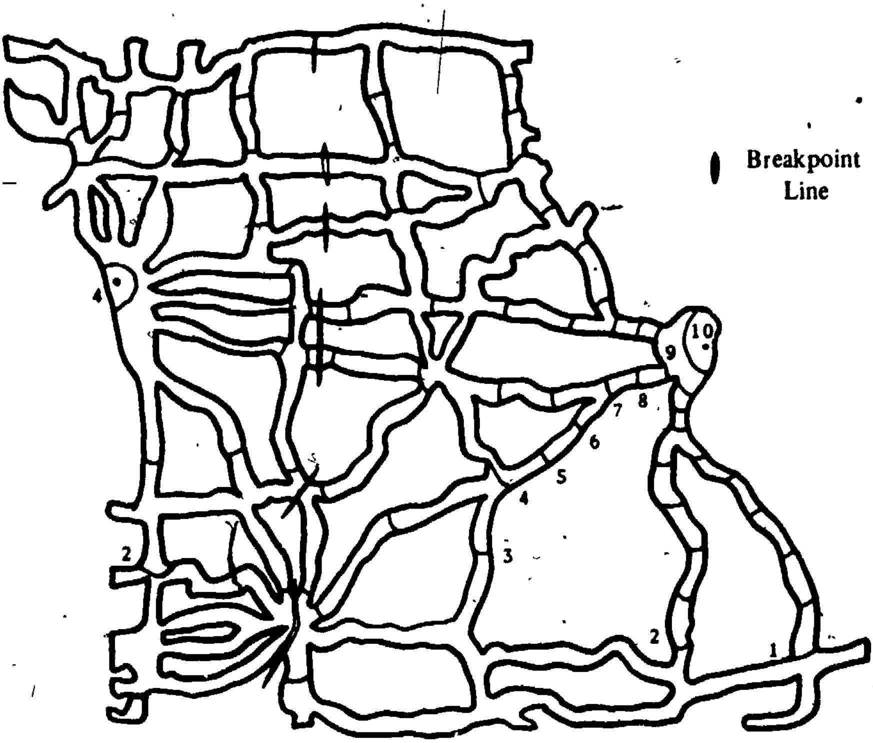
1. Follow modeling procedure 1 of experiment 5-c.
2. Draw the major highways of Missouri as depicted on model 5-e. Cut out the highway system by removing all the areas more than 1/4 inch from the roads.
3. Follow modeling procedure 2 of experiment 5-d.

Output Description:

1. The potential values change markedly when the sheet becomes a network of constant width strips. Here voltage decreases linearly, as contrasted with the logarithmic decrease of potential for the point input and a rectangular sheet. The very high values around St. Louis contrast strongly with the low values in the general vicinity of Kansas City.
2. Trace the results and compare them with model 5-d, stage 2.

Interpretations of Results. Since St. Louis is larger than Kansas City and since the road network in the vicinity of St. Louis is less developed (according to a road map), the interaction potential values are higher near and in the general vicinity of St. Louis. Whether or not this is true in fact is open to question. The breakpoints between the two cities occupy the same position as in stage 2 of model 5-d. (Verify this by superimposing the two maps.) Moreover, this model permits the establishing of breakpoints along each east-west highway. The breakpoint along the 20-percent contours is in the same area as on model 5-d. To be sure, the breakpoint is less meaningful the further it is from the line between the two cities. Consider, for example, the breakpoint near Springfield, where roads in the southwestern part of the state converge. The breakpoint occurs right at Springfield; but obviously a city the size of Springfield would have its own area of dominance except for the highest order of goods and services. For such higher order goods and services the model indicates that people of Springfield are as likely to go to St. Louis as Kansas City.

Further Notes. By applying one or more layers of conductive paint along the most important highways it would be possible to consider three or four highway types (four-lane, improved, etc.) according to the extent that they expedite traffic flow. Such a model would simulate reality more closely.



Experiment 5-e. Interaction Potential Considering Only Areas
Along Major Highways

This model is constructed like model 5-d, but the inputs are made simultaneously. Breakpoints on previous models could be made only at one point between two cities: with the road network of this model, several breakpoints can be established. Compare the trade areas established in this way with the technique used in model 5-d.

Experiment 5-f. Interaction Potential of Supermarkets Considering a River and a Bridge

Purpose—to apply the interaction potential concept at a detailed level of analysis.

Modeling Procedures:

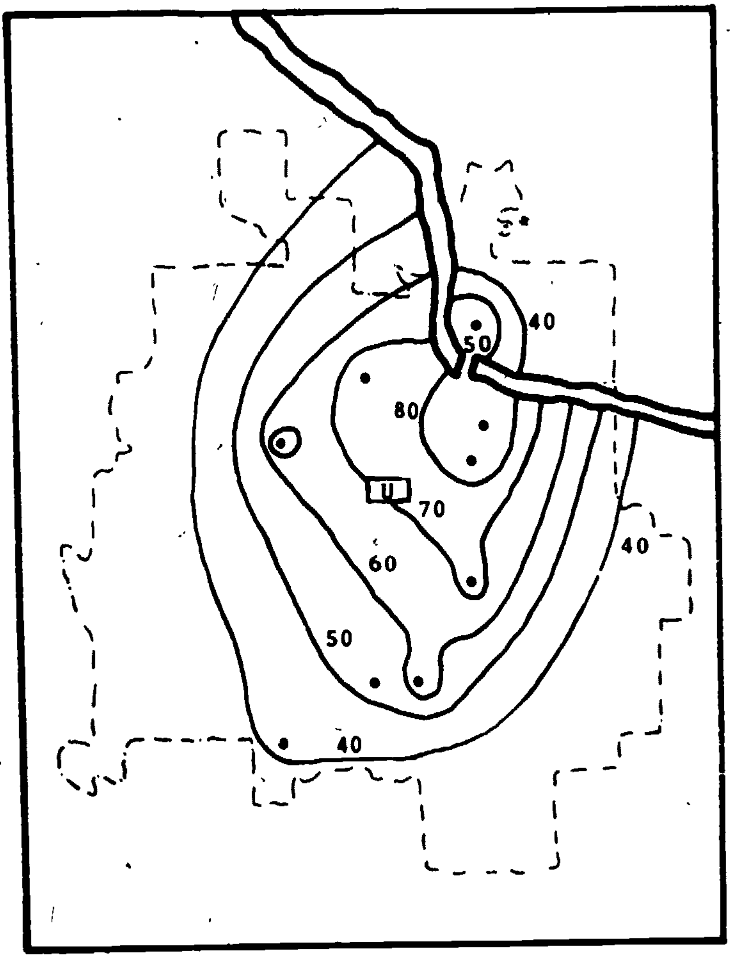
1. Draw a map of Lawrence, Kansas, from model 5-f, orienting it so that east is at the top of the easel (to accommodate better the shape of the paper and the grounding contacts of the hold-down frame). Locate the points representing the supermarkets and place inputs at those points.
2. Set all the current controls on 10; calibrate, probe, and trace the model or an overlay. Label it stage 1.
3. Cut out the river, leaving a narrow strip to simulate the bridge to North Lawrence.
4. Recalibrate, probe, and trace the model and label it stage 2.

Output Description:

If you have not already done so in one of the exercises in series 5, follow output description procedures 1 and 2 of experiment 5-a.

Interpretation of Results. The area of the nine supermarkets, although connected by a bridge, is made up of two areas—the large one south of the bridge which contains about 90 percent of the built-up area of the city, and the area north which contains the remaining 10 percent. The eight supermarkets to the south create an intricate potential pattern that is centered upon the downtown area. The one to the north has a much reduced potential when the river is considered. The only two supermarkets lying outside the 60-percent contour are the two newest and largest stores. They are located along the axis of greatest recent growth and projected growth.

Further Notes. The interaction potential could be made more realistic by weighting the value of each input according to its size relative to others. The size of parking lots, the square footage of the store, volume of sales, number of customers, the taxes paid, or cost of the building could all be suitable measures for more realistic simulation.



Experiment 5-f. Interaction Potential of Supermarkets Considering a River and a Bridge

The trade attractions of nine supermarkets in Lawrence, Kansas, are all considered simultaneously, providing their interaction potential (relative and mutual attractions of trade). Note that the potential of the area north of the river is reduced in comparison with the drawing power of other areas. The area enclosed by the 80-percent contour is the downtown shopping area. The two largest and newest stores, located in the southwest, are situated along the axis of city growth. The University of Kansas lies west and slightly south of downtown between the 70- and 80-percent contours.

Experiment 5-g. Trade Areas of Supermarkets

Purpose—to illustrate the application of the trade area concept to an analysis of local conditions.

Modeling Procedures:

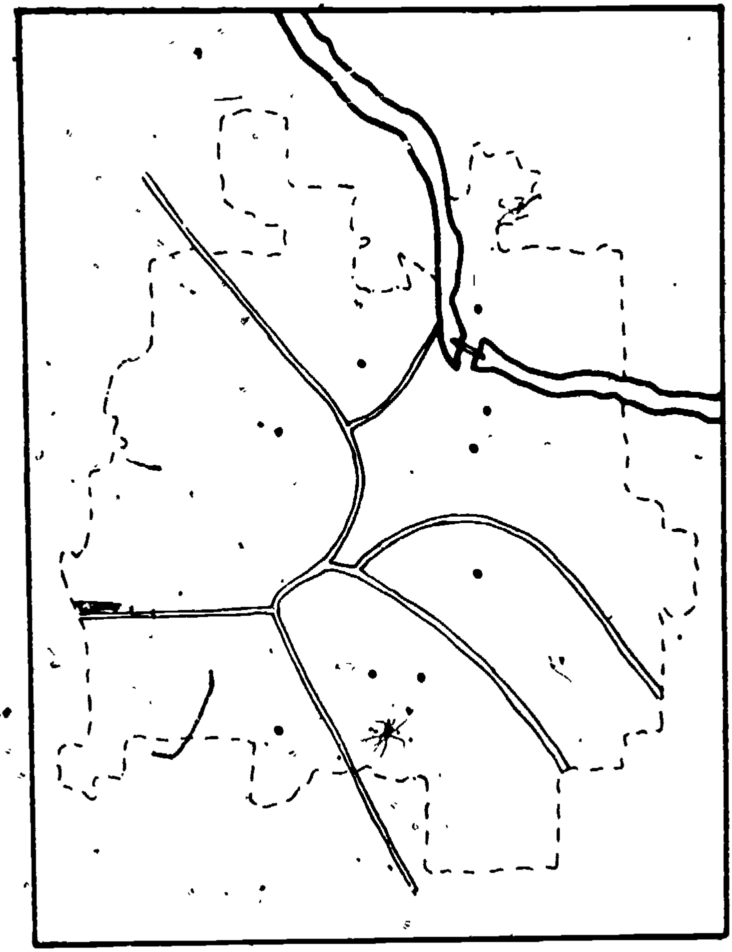
1. Follow modeling procedure 1 of experiment 5-f.
2. Considering both sets of two stores close together as simultaneous inputs (that is, leaving each of them at a value of 10 while checking the breakpoint with each neighboring set, one at a time) use the procedures established in experiment 2-e to locate the breakpoints and trade areas.

Output Description:

Trace the results on an overlay and label it model 5-g.

Interpretation of Results. The use of the trade area concept at the local level raises many questions. One is the problem of closely located stores. In this model, they were merged in that separate trade areas were not identified if two stores were very close; but they were considered separately in that each of the stores was considered in relation to its nearest neighbors. The result was an over-extension of the joint trade area to nearby stores and accounted for the predictions of shopping habits among the customers (mainly students) interviewed. An alternative would be to consider them as one input, but that leads to an underestimate of their trade area. Using both and taking the difference appears to offer the best simulation. Another problem is the way in which some supermarkets are related to city-wide shopping facilities—particularly some of the newer and larger supermarkets—while some serve as overgrown corner groceries that serve only a local segment of the city.

Further Notes. This experiment is one that most beginning students can understand readily and can gather data and verify results through field work. It is highly recommended for use with beginning undergraduates.



Experiment 5-g. Trade Areas of Supermarkets

Using techniques of establishing the breakpoints and trade areas described in experiment 2-d, theoretical trade areas were established considering only the river and bridge as departures from the uniform-plane assumption. The trade areas of two pairs of shopping centers (close together) were merged rather than differentiated. Field data were not available for comparison with the model results, but the shopping habits of a small number of students living throughout the town correlated well with the model trade areas.

Experiment 5-h. Trade Areas Considering Major Streets

Purpose—to illustrate further the application of the trade concept to an analysis of local conditions, including street patterns.

Modeling Procedures:

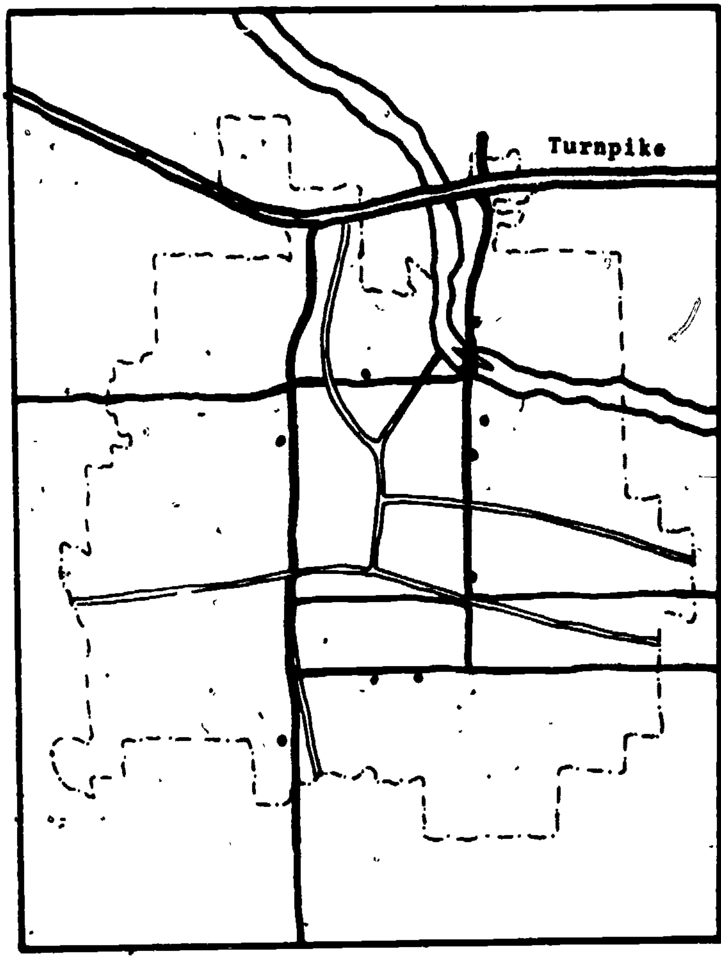
1. Follow the modeling procedures of experiment 5-g.
2. Draw the major street pattern, as illustrated, on the conductive sheet and apply one coat of carbon paint in a narrow strip along each street.
3. Recalibrate, probe, and trace the model.

Output Description:

Compare the two overlays of trade areas. Use procedures of 5-a to ascertain which areas were most affected by considering major streets.

Interpretation of Results. Modeling major streets reduced considerably the trade areas overestimated in 5-g. It makes possible the consideration of accessibility not just in terms of the two-dimensional plane that behaves according to traditional geometry but in terms of what has been called "Manhattan" geometry. That is, the square shape of our street pattern has a directional bias that tends to make squares out of circles—all points radially equidistant from a given location are not equidistant in travel time if travel is constrained to follow a square grid.

Further Notes. As in the previous model, it is possible to consider different relative input values for the supermarkets. It is possible also to take into account vari: in the quality and traffic capacity of roads. Traffic counts yield a good measure, street width, number of stop signs, and other such measures make it possible to justify the application of from one to five layers of carbon paint to get a better simulation. The traffic counts and description of the street itself can readily be obtained in a few hours of field work.



Experiment 5-h. Trade Areas Considering Major Streets

This model differs from model 5-g only in further relaxation of the uniform-plane assumption by considering the principal transportation routes. Carbon paint has been applied to these routes to expedite flow, making distance along the main streets more important than straight-line distance in any direction. The route between the interchanges of the Kansas Turnpike is cut on either side to act as an absolute barrier. However, the cuts along the turnpike stop at interchanges and where bridges cross over the turnpike.

Experiment 5-i. Contours of Urban Land Use for Different Locational Constraints

Purpose—to illustrate a method for analyzing contours of equal access to major avenues for given locational constraints.

Modeling Procedures:

1. Draw the trace of two roads, one 2 inches from the right edge and the other 2 inches from the bottom edge of a sheet of conductive paper. Mount the paper under the hold-down frame. Apply a layer of silver paint or aluminum tape to each road, being careful *not* to extend them to ground (stop them at least 2 inches short of the hold-down frame edges).

2. Place an input pin at the intersection of the two roads, connect a numbered lead to the shaft of the pin, and set the appropriate current control to 10.0. Calibrate.

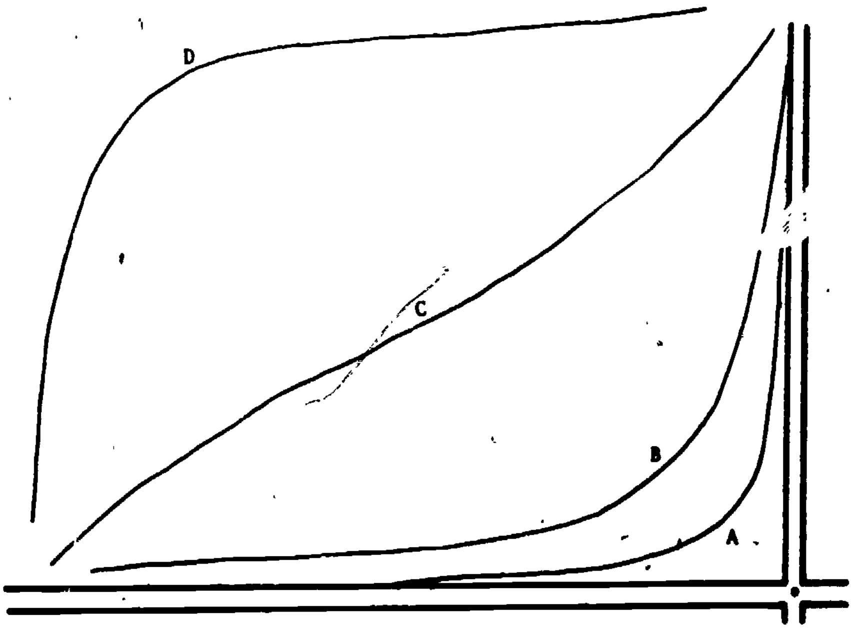
3. Place the probe 2 inches northwest of the intersection, turn the contour selector until a null reading is obtained on the meter, and follow that null to its furthest extent. Label it A.

4. Go another 2 inches (a total of 4 inches) northwest from the intersection, null and probe. Go about halfway between the ends of the two roads, null the meter, and probe the line. Label them B and C.

5. Go on nor' 'st to within 2 inches of the north edge of the frame, null the meter, and prob. the line. Label this contour D.

Interpretation of Results. The results simulate different locational constraints within a city. The model represents the northwest quarter of a small city, the development of which is controlled by its two main streets and its city limits. Both roads end at the city limits and are of a more highly conductive material than the carbon paint previously used to simulate roads. The notion here is that, for some purposes, the main avenues can be considered as near perfect conductors in relation to other areas within the city. Contour A represents high accessibility to both main streets as required for most commercial enterprises. Contour B in modeling procedure 4 outlines those areas with a lower demand for accessibility such as wholesale and manufacturing enterprises; the area within this zone has a high probability of being used for such activities. Lines C and D suggest locational constraints based on services provided by the city (fire department, school, water, sewers, natural gas, etc.); line D indicates development constraints that will tend to delimit the expansion of the city limits.

Further Notes. The road may be continued (to ground or nearly to ground) with carbon paint. The road may be made of silver near the center of the city, several layers of carbon paint in the areas of the city more removed from downtown, and only one layer of carbon paint beyond the city. Varying the east-west extent in relation to the north-south extent provides further variety.



Experiment 5-i. Contours of Urban Land Use for Different Locational Constraints

A problem type differing from those preceding is mapping contours of equal accessibility to the main roads and services of a small city. This model computes for a small city with an east-west spread greater than its north-south spread, the likely development of different land uses as a function of accessibility. This model displays only the northwest section of the city. Line A indicates the zone of development of commercial establishments that require close proximity to the main roads. At the other extreme, line D indicates the likely development of residential units that require accessibility only to the city's water and sewage disposal systems. Lines B and C represent intermediate constraints.

Experiment 5-j. Population Potential of the United States

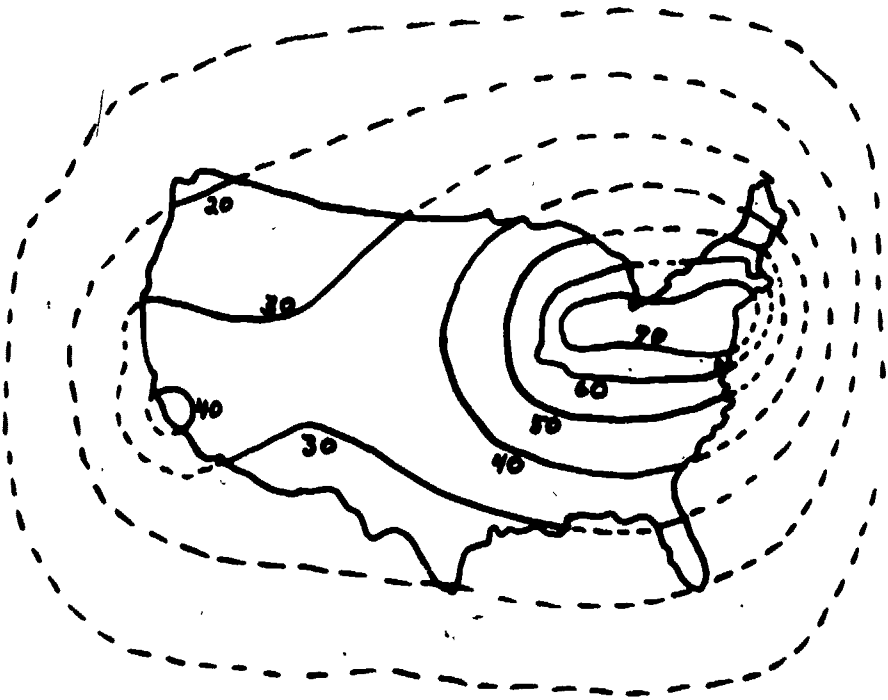
Purpose—to demonstrate a method of analyzing population potentials for large geographic areas.

Modeling Procedures:

1. Draw an outline map of the 48 coterminous United States on two sheets of paper, leaving at least 2 and no more than 4 inches between the map boundaries and the edges of the conductive paper. Clamp the paper under the hold-down frame.
2. Insert input pins at the locations of the nine largest SMSA's, using pin number 1 for New York (10.0); 2 for Los Angeles (6.7); 3 for Chicago (6.2); 4 for Philadelphia (4.3); 5 for Detroit (3.7), 6 for San Francisco (2.8); 7 for Boston (2.6); 8 for Pittsburgh (2.4); and 9 for St. Louis (2.1). Set the corresponding current controls to the proper relative values and based on population in millions (enclosed in parentheses after each city).
3. Calibrate and probe.

Interpretation of Results. This model permits analysis of the interaction potential of the nine largest U. S. metropolitan areas. Since regional population potential depends primarily on large cities, the model also represents the general pattern of the population potential of the 48 coterminous United States. Model contours are a measure of overall accessibility to these major cities and can be of significance to a number of general economic and social concerns. Further material on such models is available in *Macrogeography and Income Fronts* by Warntz. Population inputs can be multiplied by factors such as mean income, to model income potential; percentage of people in professional services, to map "brain" potential; or the percentage of people at or below the poverty level, to compute an urban poverty potential. Various boundary conditions can be employed to achieve special results. All the boundaries could be grounded to force zero values at the international boundaries, or all the ocean boundaries could be cut and the land boundaries with Canada and Mexico grounded, or vice versa, to achieve special effects. Mountains could be made barriers, and the interstate highway system or rail network could be modeled to relax the uniform-plane assumption. Airline routes could be modeled as well.

Further Notes. A more detailed simulation of maps by Warntz involves taking a section of the country and (with his isopotentials to form the boundaries of your study area) using the field plotter to simulate one section with the degree of detail that he achieved with over 3,000 inputs for the entire 48 coterminous states. It is even possible to use the output to define study areas even more detailed than Warntz achieved. In both cases, the procedure is to establish isopotentials of the amount necessary to get their proper values and then add inputs as necessary to gain the additional detail. Various boundary conditions, expediters, and barriers can also be employed.



Experiment 5-j. Population Potential of the United States

The interaction potential of the nine largest metropolitan areas (SMSA's) according to the 1960 census provides an interesting basic pattern of the 48 coterminous United States. The core of the northeast manufacturing belt is clearly outlined by the highest (70-percent) isopotential. The California sphere also emerges clearly, and the breakpoint between the spheres of influence of the west coast and northeast centers is indicated by the narrowing neck of the 30-percent isopotential. The pattern is quite similar to the basic patterns of population potentials and income potentials developed by William Warntz in *Macrogeography and Income Fronts* (Philadelphia: Regional Science Research Institute, 1965).

Experiment 5-k. Six Models of U. S. Population Potential

Purpose—to illustrate the emergence of an overall potential pattern by presenting the pattern based on one city and by comparing it with the patterns that result from adding other cities, one, and then two at a time.

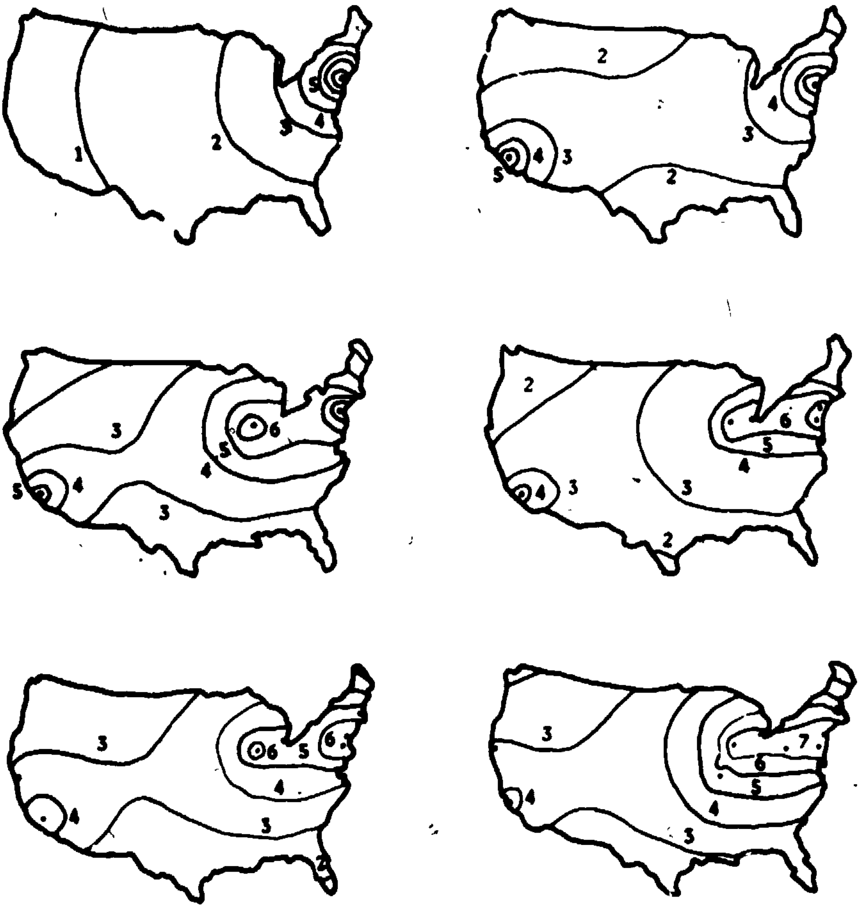
Modeling Procedures:

1. Follow all modeling procedures of experiment 5-i.
2. Make an overlay of the pattern and label it 9.
3. Turn the current control for the smallest town to zero, recalibrate, and draw the results on a separate sheet of tracing paper, labeling it with the number of inputs turned on (in this case, 8).
4. Repeat modeling procedure 3 until only input remains. Map this one also.

Output Description:

1. Arrange the model tracings, from one input to nine.
2. Trace the 30-percent contour of each stage on one overlay, and the 40-percent contour on another. Indicate the direction and magnitude of change in each of the two contours in going from one model to another.

Interpretation of Results. The pattern of potential is a measure of overall accessibility, such that being closer to large cities is given more value than being close to small cities. Consequently, the larger the city, the greater its impact on the overall potential pattern. Examining the various stages of the present experiment, it is possible to ascertain the tendency of the pattern of a few but increasing number of cities to converge on the general pattern of potential when all countries (3,200 inputs) are employed. When New York alone is considered, the results are isopotential contours that connect points equally accessible to New York; such a pattern may be of significance in connection with, say, stock-market operations. When Los Angeles is added, the basic opposition with New York interests is set forth; the pattern may be meaningful in terms of viewing stage outlines, roughly the major manufacturing belt within the 40-percent contour. Philadelphia and Detroit in stage 4 give the northeast an overwhelmingly 6 to 1 ratio of northeast to southwest influence (dominance), and the breakpoint of influence is less clear. San Francisco in stage 5 redresses the imbalance, more than enough to offset the inclusion of Boston. Stage 6 presents the pattern discussed in experiment 5-j. The basic pattern prevails throughout the series, and consideration of additional cities only changes details, not the general pattern. Note also that the basic pattern is made up of two distinct subpatterns (east and west); all but stage 5 (5 cities) also follow these.



Experiment 5-k. Six Models of U. S. Population Potential

By adding one city at a time for the first three models and two cities for the last three, it is possible to ascertain the impact of more cities on the overall pattern.

Experiment 5-1. Great-Circle Routes on a Mercator Projection

Purpose—to illustrate a method of computing, on a Mercator projection, great-circle routes between one established point and any other point on the surface of the globe (except for the very high latitudes).

Modeling Procedures:

1. Trace onto conductive paper the largest Mercator projection that fits on the sheet, leaving a 2-inch boundary on all sides of the map to minimize boundary conditions. Select the point you want to base the map on (say, Moscow). Apply a thin line of silver paint along the entire meridian passing through that point and the meridian halfway around the globe from that point. Place the ground pin through one of these meridians, connect the ground lead to the shank of the pin through one of these meridians, using small wire, jump the ground to the shank of a pin placed in the other meridian.

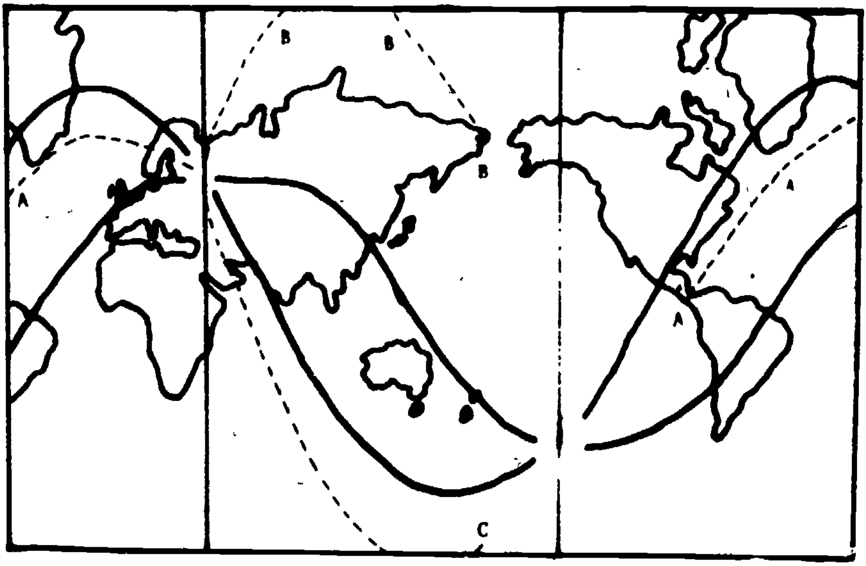
2. Stretch a piece of string tightly between one point and a point halfway around the globe in all directions from it. (Any path along a string thus positioned and stretched is a great circle). Select one or two great-circle routes thus derived that lie diagonally to the ones formed by the meridians, plot it (them) on the map, and apply a thin coat of silver paint leaving 1/2 inch of space between the ends of the great-circle routes and the meridians. Insert input pins in the center of the curved line segments between meridians, connect numbered leads to their shanks, and set the corresponding current controls to 10.0.

3. Calibrate with the hold-down frame in a raised position.

4. Place the probe on any point where you want to find the great-circle route from that point to the point on which the model is based, null the meter, and probe the route. Probe lines similar to paths A, B, and C on the model 5-1 illustration.

Interpretation of Results. The relation between a rhumb line (a line of constant compass bearing—which is a straight line on a Mercator projection) and a great-circle route (the shortest path between two points on the surface of a sphere) is clearly demonstrated. Between Moscow and the Panama Canal, a rhumb line would pass over the Iberian Peninsula but the great-circle route would arch through the Scandinavian Peninsula and pass north of the British Isles before passing along the coast of Greenland - North America (see path A). Path B portrays the poleward arc that indicates the shortest route between Moscow and the Bering Strait. Path C indicates how one flies southwest to get northwest more quickly.

Further Notes. Experimentation is underway in coating globes with conductive paint as well as constructing globes with Fuller's dymaxion triangles to solve similar problems.



Experiment 5-1. Great-Circle Routes on a Mercator Projection

A great-circle route (the shortest path between two points on the surface of a sphere) may be reproduced on the field plotter. The meridian passing through one point and the meridian halfway around the earth are both grounded. Next, two known great-circle routes are plotted and excited with current. Then the probe may be placed on any point and used to draw a great circle back to the original point. Thus, it is possible to establish the relationship between any great-circle route and the line of constant compass direction. This model was developed by M. A. Morgan in the chapter "Hardware Models in Geography" in R. J. Chorley and P. Haggett, *Models in Geography* (London: Methuen, 1967).

Experiment 5-m. Orthographic Map Projection

Purpose—to suggest methods of plotting certain map projections.

Modeling Procedures:

1. Draw a circle with a radius of 5 inches. Draw a horizontal line through the center to both edges of the circle (the equator) and a vertical line through the center of the circle (the principal meridian).

2. Plot the parallels by drawing lines parallel to the equator at a 1-inch spacing.

3. The process of plotting meridians is more involved. Apply a thin, narrow layer of silver paint along the prime meridian and along the outer circle, beginning ½ inch from the North and South Poles. Place an input pin in the middle of each of the two pole-to-pole arcs, attach numbered current leads to the shafts of the pins, and set the corresponding current controls to 10.0. Calibrate. The field plotter is now ready to plot meridians.

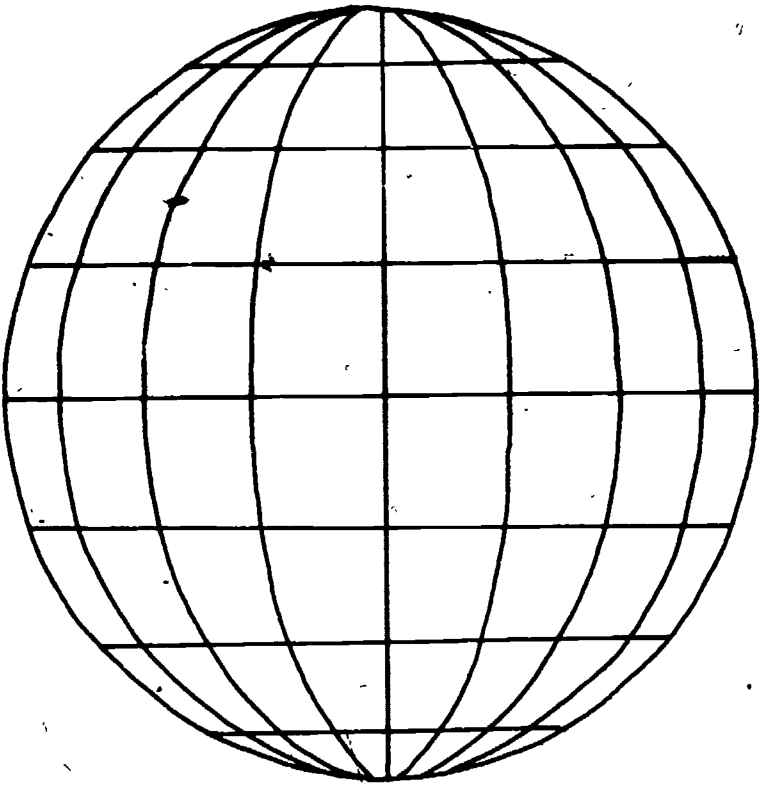
4. Along the equator, determine the spacing of the meridians, using trigonometric functions. The *first* meridian is located a distance from the principal one, computed as follows: 5 (the radius of the circle in inches) multiplied by the cosine of a quotient, which is found by multiplying the number of meridians to be plotted minus *one* and π(3.1416), and dividing that product by 10 (twice the number of meridians to be plotted). The distance of the *second* meridian is calculated the same way except the quotient is found by multiplying the number of meridians to be plotted minus *two*, . . .and the *fifth* is found by dividing the number of meridians minus *five*. The fifth one is 5 inches away; obviously it is the arc of the circle, since 5 minus 5 is zero, times π is zero, zero divided by 10 is zero, the cosine of zero is 1, and 1 times 5 is 5. Locate these points along the equator, set the probe on one point at a time; null the meter and probe the location of the meridian each time.

5. The formulae for the manipulation in procedure 4 can be stated mathematically. For the first distance,

$d_1 = r \cos\left(\frac{m(n-1)\pi}{2n}\right)$ where d = distance from the principal meridian, 1 = order of meridian, and n = total number of meridians to be drawn. For the second,

$d_2 = r \cos\left(\frac{(n-2)\pi}{2n}\right)$. In general $d_i = r \cos\left(\frac{(n-i)\pi}{2n}\right)$ where i is the order of meridian, ranging from 1 to n; $d_{n-2} = r \cos\left(\frac{2\pi}{2n}\right)$; $d_{n-1} = r \cos\left(\frac{\pi}{2n}\right)$; and $d_n = r \cos\left(\frac{0 \cdot \pi}{2n}\right) = r$.

Interpretation of Results. The resulting orthographic (ortho-true, graphic-view) projection depicts one-half of the earth's surface as it would be viewed from "infinity." Similar computations have been made for the stereographic, Mollweide's Homolographic, and a number of other projections. Nevertheless, it provides one of the simplest, most rapid techniques presently available for plotting such projections.



Experiment 5-m. Orthographic Map Projection

Different map projections can be constructed with the use of the field plotter. The procedure is somewhat cumbersome and requires some use of arithmetic, but it helps to visualize some characteristics of map projections and properties of maps in general. The example presented here is one of those found to be of more general interest to beginning students.

Experiment 5-n. Glacial Flow in North America

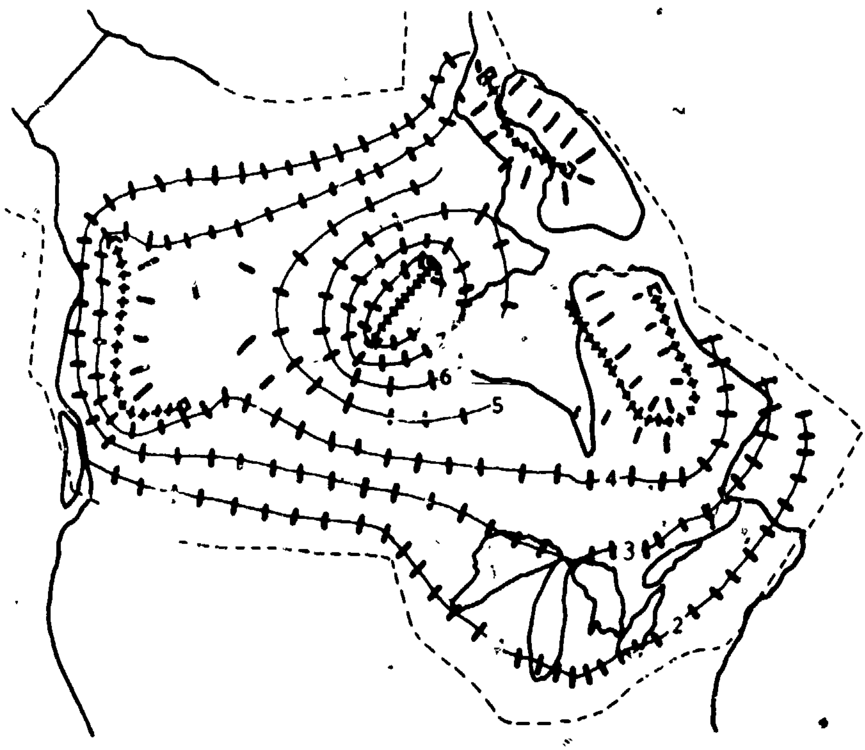
Purpose – to illustrate a technique for analyzing continental ice flow as well as a general technique for determining lines of ice flow.

Modeling Procedures:

1. Draw a map of the area under study (North America) on conductive paper. Run a ground with silver paint or aluminum tape along the outer extent of glacial flow. Use silver paint to outline the source regions of glaciers and excite each source region with the same amount of current (unless you have reason to evaluate source regions in terms of its relative quantitative contribution to glacial flow).
2. Calibrate and probe the interaction potential contours of the source regions, using supplementary contours where needed.
3. Draw short, heavy line segments along all contours, at right angles to the contours.

Interpretation of Results. The short, heavy line segments indicate the direction of flow of the ice. Where the lines run in a continuous and rather simple pattern, the output of the model may be considered a more reliable statement and should correspond well with evidence observable in the field. Where the short, heavy lines are sparser or form more intricate patterns, the output may be considered a less reliable statement. In such areas, field evidence indicates that the glacial flow itself was highly variable and not as simple in its flow pattern. Terrain obstacles can be modeled to simulate the topographic variations that affect glacial flow.

Further Notes. Additional experiments have been performed that dealt with valley glaciers as well as a continental glacier. Experimentation is also underway on segments of a glacier, particularly the active fronts of glacial advance and retreat. The scaling problems are being worked out for these small-scale models. One of the outstanding uses of the field plotter is to consider the sensitivity of a given input, and this experiment lends itself to that consideration. By varying the amount of input at each of the sources a given percentage, you can check the consequences of each change and determine which of the inputs is very sensitive in producing the pattern and which ones are not. As a rule, some are revealed to be considerably more sensitive than others.



Experiment 5-n. Glacial Flow in North America

One technique that can be employed in complex models of flow phenomena where the inverse is difficult to obtain is to plot the isopotential lines and then hand-plot the streamlines (lines of flow). In this model the isopotential lines represent the pressure in the ice, itself; and the short, heavy line segments indicate the directions at right angles to the isopotentials which, by definition, represent the ice flow lines. This model was developed by J. R. Mackay in "Glacier Flow and Analogue Simulation," 1965.

C. SYNTHESIS MODELS (SERIES 6)

One of the most difficult computational processes to attempt to handle is one in which there are inadequate data or theory to attack problems. Many times it is possible to take a known situation, simulate it by trial and error, and then analyze the procedures necessary to get a good simulation. In essence, one starts with the answer and, through the synthesis process, tries to develop an ability to ask the right questions. Synthesis by simulation has been called "a sandbox for scholars." The field plotter is well suited for many such problems. The present series represents only a few.

The series includes:

a. Wave Refraction Along an Irregular Shoreline
b. A Thunderstorm
c. A Weather Map – Simulation of Atmospheric Pressure in the Northern Hemisphere at the 500-Millibar Level

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Experiment 6-a. Wave Refraction Along an Irregular Shoreline

Purpose – to illustrate the control of isopotential patterns to simulate a known pattern of wave forms as they approach an irregular coastline.

Modeling Procedures:

1. Select an aerial photograph of an irregular shoreline on which the waveforms are clearly visible.

2. Trace the details of the shoreline and offshore islands or submerged bars on a piece of conductive paper. Apply a thin layer of silver paint along the shoreline of the mainland as well as the islands. Connect them to ground.

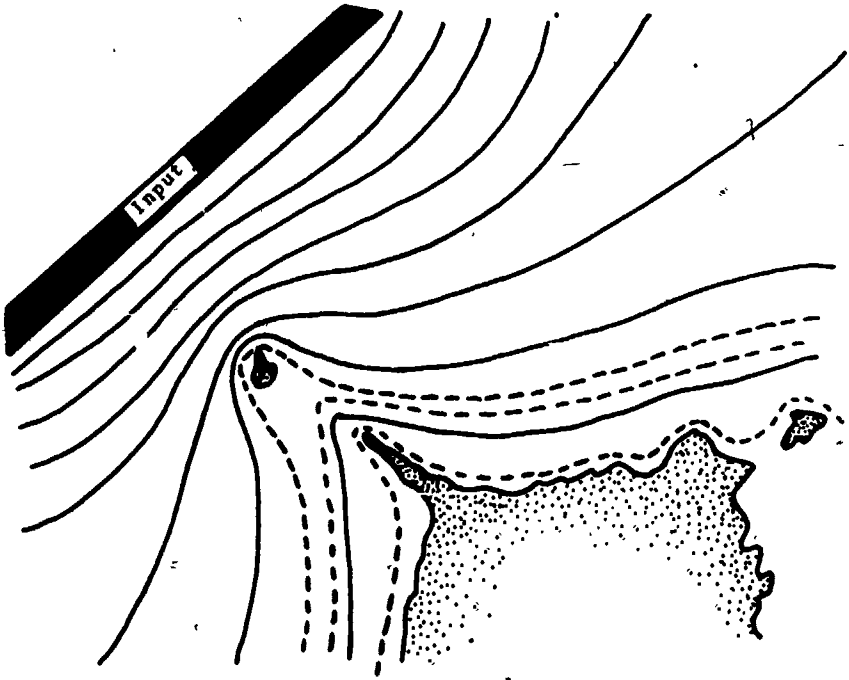
3. Trace onto the paper the line of wave crests furthest from the shoreline but still on the paper sheet. Apply a thin narrow layer of the silver paint along this line of waves and insert an input pin through its midpoint, connect a numbered lead to the shaft of the pin, and set the corresponding current control to 10.0.

4. With the hold-down frame in a raised position, calibrate and probe the model, using supplementary contours where needed to "fill in" the details of the pattern.

5. Compare the contour pattern with the actual, simulated pattern. Change the location of the input, grounding procedure, or apply carbon paint to simulate depth of water in ways that the comparison suggests, to better simulate the pattern. (It was found to be necessary to connect all the small islands leading out to the cliffed headland as if they were one continuous peninsula.) Boundary conditions, barriers, and expediters may also be added or deleted at any stage where such additions or deletions are suggested. In general, only one such alteration per stage should be made in order to permit a more complete evaluation of its impact.

6. Repeat procedure 5 until an "acceptable"simulation is achieved. Acceptable is defined in terms of the operator's objectives; often, however, more can be learned from a simple simulation with 4 or 5 stages than from a complex and intricate simulation of 25 stages, even though the latter may better simulate the known pattern.

Interpretation of Results. In models of synthesis the interpretation must go hand-in-hand with the performance of each modeling procedure and at each stage of the simulation. An overall interpretation of results and suggestions for further modification should also be made, however. In the present case, the most startling conclusion reached was that the island nearest the input had to be retained as a separate island in the simulation while the smaller islands between it and the mainland had to be simulated by grounding the strip that included them. The location of wavefronts and the general pattern were very close to the patterns on the photograph.



Experiment 6-a. Wave Refraction Along an Irregular Shoreline

Wave forms are refracted as they approach an irregular shoreline. Current enters from the strip input in the upper left, to simulating parallel bands of waves. The shoreline and offshore islands are grounded. The parallel bands tend to bend around the points of interference and conform to the configuration of the shoreline. The pattern being simulated was taken from an aerial photograph reproduced in Gilluly, Waters, and Woodford, *Principles of Geology* (San Francisco: W. H. Freeman and Co., 1958), p. 389. The peninsula shown here was a series of islands on the photograph, but trial-and-error methods indicated that the small islands behave more like a seawall, causeway, or peninsula insofar as wave refraction was concerned. Only the large island needed to be considered separately from the mainland.

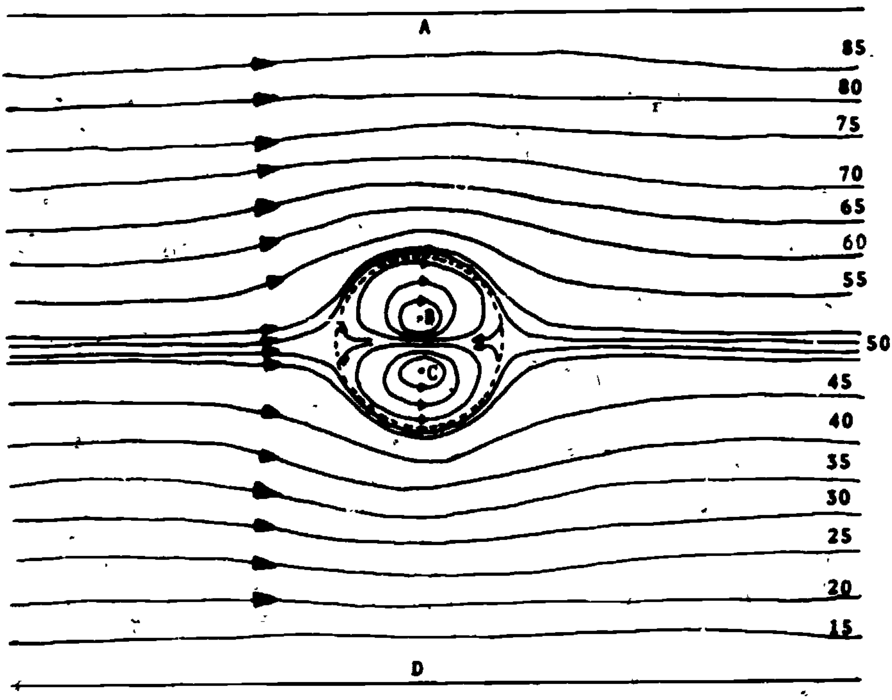
Experiment 6-b. A Thunderstorm

Purpose – to illustrate how a combination of line, boundary input and ground, and point input and ground can be employed to simulate a theoretical wind system of a thunderstorm within a general wind system.

Modeling Procedures:

1. Along the top and bottom of a sheet of conductive paper apply a narrow strip of silver paint or aluminum tape. Make the top line an input of 10.0 (current control setting) and the bottom line a ground. Label them A and D, respectively. Isopotentials between them would simulate the paths along which winds (east-west) in a general system would move.
2. From a midpoint between the input and ground strips place a point input (value 10.0) 1 inch toward the strip ground (D) and a point ground, 1 inch toward the strip input (A). Label them B and C, respectively. They represent the two vortices of a thunderstorm system.
3. Calibrate and probe the system. Begin with the vortices; probe them until elliptical patterns are obtained, and then probe the streamlines of the general wind system. Label the directions of winds as in the diagram of model 6-b.
4. Note the distance between B and C. Decrease the distance between B and C, recalibrate, and probe. Note the differences at each stage.
5. Rotate the two points B and C slightly, recalibrate, and probe. Rotate the two points through a few more stages, recalibrating and probing at each stage.

Interpretation of Results. The field plotter is being employed in this experiment to simulate a theoretical pattern. Observations have been made of the general wind system flowing around a thunderstorm as if it were a solid cylinder. Theoretically, to produce the effects of a solid cylinder, the storm must be made up of circular flows (one clockwise and one counterclockwise) around two vortices. Model 6-b, derived through trial-and-error methods, demonstrates clearly that such a cylindrical effect is possible. Subsequently, it was found that rotation of the two vortices produces a dissipation of the system. Observation has indicated that thunderstorms split and either form two thunderstorms or dissipate completely. Thus, there appears to be agreement between the model and real-system behavior.



Experiment 6-b. A Thunderstorm

The winds in a thunderstorm system appear to behave much like the isopotential lines of a field plotter model. Because of the geostrophic nature of winds, the lines of flux are often parallel to the isobars of atmospheric pressure. The result is that a map of the general wind system can be simulated with a strip input at the top of the model and a strip ground at the bottom. The thunderstorm within the general system can be simulated with point inputs and grounds representing vortices within the thunderstorm system. On a variation of this model the two point inputs can be rotated and the thunderstorm split, forming two separate systems, just as real thunderstorms split. The analogy is being subjected to further research.

Experiment 6-c. A Weather Map – Simulation of Atmospheric Pressure in the Northern Hemisphere at the 500-Millibar Level

Purpose – to illustrate the simulation of a complex system and to suggest some advanced modeling techniques. To work this exercise it will be necessary to have an assortment of resistors, a voltmeter, and some small current leads.

Modeling Procedures:

1. The overall model cannot be simulated on the FP-9; however, the problem can be set up on an FP-15 (15-input field plotter). The problem is reproduced here to demonstrate that it can be done and to justify its use (as well as the use of other models of atmospheric pressure at the 500-millibar level) as a basis for a study of areas smaller than the whole northern hemisphere.

2. Select a portion of a map of atmospheric pressure at the 500-millibar level and trace it onto a sheet of conductive paper. Apply a narrow layer of silver paint to the isobars of lowest and highest values.

3. Scale the low pressure cells by making the lowest pressure cell an input of 10, and the low pressure cell with the highest pressure a value of 2.5. Scale the other low pressure cells inversely between 2.5 and 10 and excite them with the appropriate amount of current. Excite isobars of the lowest value with current proportional to that used for the low pressure center.

4. Scale the high pressure center by grounding the highest value and using resistors (fixed or variable) to adjust the others. Select an arbitrary value for the high pressure center with the lowest pressure center grounded through a resistor with a value of, say, 500 ohms. Place the resistor between a pin inserted at that point and ground. Scale the other high pressure centers accordingly and place a resistor of the appropriate value between each input pin and ground. Likewise, place the appropriate value resistor along the highest high pressure isobar.

5. With the hold-down frame in a raised position, calibrate and probe the model. If the pattern of isobars can't be simulated, then alter the scaling (relative values) of the low pressure or high pressure cells, recalibrate, and probe the pattern system. It may be necessary to introduce intermediate-value low pressure or high pressure cells.

Interpretation of Results. Using additional resistors to create partial grounds provides additional flexibility in modeling. For those who have some experience in electronics, there are many such modifications that can be made. The person not experienced in electronics, however, is well advised to work within the techniques presented in preceding models. Such simulations of pressure systems permit one to compare the strength of the input necessary to produce the simulated pattern with the pressure measure of the cells and thus understand more fully the atmospheric processes for academic purposes or for prediction.

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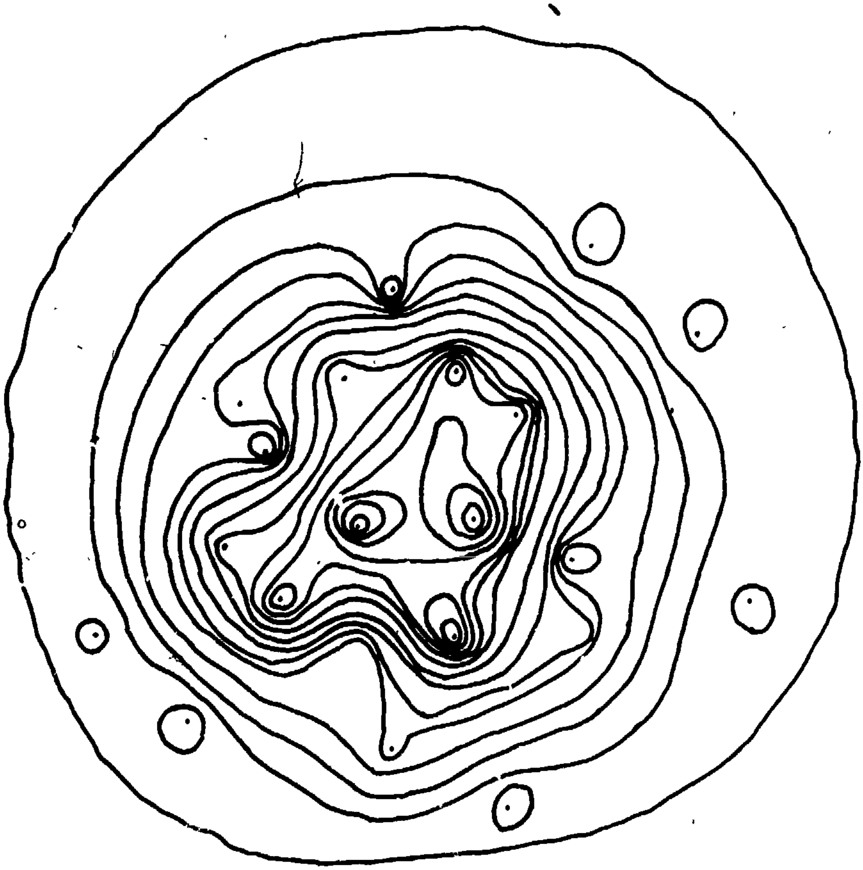

Experiment 6-c. A Weather Map – Simulation of Atmospheric Pressure in the Northern Hemisphere at the 500-Millibar Level

This model represents some of the advanced techniques that can be employed with the field plotter. The high pressures were simulated by point partial grounds (grounded through variable resistors), and the low pressures were simulated by normal current inputs. The equatorial belt of high pressure was simulated by an ordinary grounded circular boundary. The partial grounds, in effect, created negative inputs. Manipulation of the values of both types of point inputs permitted the simulation of the map with considerable detail and accuracy. Isopotential contours represent isobaric contours.

VI. SYNOPSIS OF MODEL ELEMENTS

The various model elements that have been introduced provide a potent set of tools for simulating geographic phenomena. They can be employed in various combinations to accommodate a wide variety of geographic problems.

In addition, some model elements were not illustrated because they require special equipment and technical ability to assure effective use. Such model elements extend the use of the field plotter, but they should be used with caution. They are mentioned here for those who might want to experiment with them.

A. BASIC MODEL ELEMENTS

1. Point Input—a pin placed directly on the paper and energized.
2. Line Input—a strip of silver paint (or aluminum tape with a conductive adhesive) with a pin stuck through it and energized. For best results the paint should be allowed to dry overnight and the tape should be thoroughly burnished.
3. Area Input—a layer of silver paint (or overlapping aluminum tape) with a pin stuck through it and energized.
4. Point Ground—a pin placed directly on the paper and connected to the ground lead. Small wire may be used to "jump" the ground to two or more point grounds.
5. Line Ground—the underneath side of the hold down frame when clamped down, or a strip of silver paint (or aluminum tape) with a pin stuck through it and connected to the ground lead.
6. Area Ground—a layer of silver paint (or overlapping aluminum tape) with a pin stuck through it and connected to a ground lead.
7. Absolute Barrier, Reflective—a clean, continuous cut in the conductive paper along the location of the barrier. The cut may be along a thin line, or it may be an area cut out of the paper.
8. Absolute Barrier, Absorptive—the same as "6. Area Ground". It may be used in addition to the conventional ground.
9. Partial Barrier—chevrons cut in the conductive paper so as to lengthen the path of electron flow. The strength of the barrier is proportional to the increased length of the path.
10. Highly Efficient Expediter—a strip or area of silver paint (or aluminum tape).
11. Low-Efficiency Expediter—a strip or area of carbon paint. The more layers one applies, the higher the efficiency. Each coat should be allowed 4 or 5 minutes to dry.
12. Point-to-Point Expediter, High-Efficiency—small wire held to the surface of the conductive paper only in selected points and bent away from the paper elsewhere. It simulates, say, airports. (The wire can also be placed in back of the model.)
13. Combination of Barriers—any simultaneous use of two or more model elements from 7, 8, or 9. The gap between two barriers simulates, say, a mountain pass.
14. Combination of Expediters—any simultaneous use of two or more model elements from 10, 11, or 12. An area of silver paint surrounded by an area of carbon paint simulates, say, a city and its suburban development.
15. Combination of Barriers and Expediters—any simultaneous use of model elements from 7, 8, or 9 with those from 10, 11, or 12. The gaps between linear barriers (from 7) strung end on end, with point-to-point expediters (from 12) touching at some of the gaps, simulate interchanges of an interstate highway system (where the expediter touches) and bridges across or passes under interstate highways (where the expediter doesn't touch).

B. ADVANCED MODEL ELEMENTS

1. Resisters–electrical components that induce additional resistance into a model. They can be fixed in value or variable. Employed with point-to-point expediters, for example, they can be used to simulate variations in flights scheduled between any two airports – the greater the number of flights the lower the value of resister employed.
2. Diodes–electrical components that permit flow of electrons in one direction only. Used in connection with expediters they permit modeling of one-way streets or the flow of rivers.
3. Resistance Networks–electrical systems connected to form surfaces built up of resisters (and diodes if desirable). They can be used to energize (or be energized by) the conductive paper. Initial inputs can go into the resistance network that simulate, say, the transport network inputs made to the conductive paper. Changes can be made in the resistance network and consequences of those changes measured on the resistance paper potential.

In general, the person who has a knowledge of electronics can conduct some interesting experiments with such advanced techniques. The 15 basic modeling elements, however, present seemingly endless capabilities, as of yet unexplored, that are easier to understand and use. They permit each individual to set up and work on problems he identifies as being meaningful. The 15 field plotter model elements present a challenge to all who would choose to master them and an intellectual reward to those who master them to any degree.

VII. SUMMARY

The preceding six sections present all the basic information and techniques necessary to use the field plotter effectively. No amount of information or techniques can replace the personal experience you gain on your own in using it to solve problems you find meaningful.

If you have not already done so, try to think of some ways you would like to employ the field plotter. Make a list of them. Select the one you think would be most feasible. Review the material you find most relevant to your project and make sure you understand it. Lay a strategy and begin.

Don't get too disappointed if you have to start over again. If the results are not what you want, then make a learning experience of it; try to ascertain why the model came out that way and use that understanding to plan a new approach. You will find that such unexpected results will diminish with experience.

There are many challenges presented by the field plotter. Whether the scaling factors for all or most of the models presented here can be refined to the point where simulation predictions can be attempted, has not yet been established. The possibility exists for modeling the entire complex of an urban area and its surrounding rural region. Likewise in problems of human migration, transportation, environmental pollution, climates, soils, and cartography there are many applications that need to be explored. The results of such exploration, hopefully, could become a meaningful and integral part of geography courses on a wide basis.

The field plotter and accessories suitable for wide use have become available only in 1970. Few geographers have worked with it although more than two dozen departments have recently acquired instruments. The field plotter has the exciting capability of tying together previously unrelated concepts and theories, thus moving geographers closer to some general theories that transcend many traditional subdivisions of discipline. Development of these new techniques will require the work of many in all geographic areas. Suggestions and contributions are welcome.

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APPENDIX – TECHNICAL NOTES

For those users who would wish to inquire into the mathematics and electronics underlying operation of the field plotter, the following technical notes are presented. However, an understanding of these notes is not necessary for use of the field plotter. Nevertheless, an enormous amount of work in establishing postulates and solving exceedingly complex mathematical relationships will be necessary to fully develop the art of applying the field plotter to geography into a mathematically based field-plotter science of geographic modeling. The material that follows outlines briefly the nature of the task.

"FIELD THEORY" AND GEOGRAPHY

Warntz (Geographical Review, 1964) summarized the basic concepts for considering potentials in geography. The next 10 paragraphs are primarily a summary of his summary, and the reader is referred to his article for more details. The following material defines potential and related concepts.

A potential function is a function from which the distribution and magnitude of phenomena at any point in a field may be derived. When an unbounded uniform plane is assumed, the potential, V, at a point, c, is a definite integral:

$$V_c = \int_A 1/r \, D \, dA,$$

where D is density of population of an infinitesimal element of area, dA; and r is distance of this element from point c. A value for the integral exists at each point on the surface (provided D is zero for sufficiently large distances from V_c), and a line connecting points of the same value on the surface is an isopotential line.

Potential functions are one of a general class of spatial distributions called field functions, and governed by field theory. By extension, it is possible to identify additional "field theory" quantities such as demographic force, demographic gradients, and demographic energy as complements to population densities and population potentials.

There have been many studies of these several related concepts. Reilly, in 1929, studied the intermediate area commanded by two competing market-center towns, and found that the breakpoint on the line between two towns, where the towns shared equally in the trade, occurred where

$$\frac{P_1}{r_1^2} = \frac{P_2}{r_2^2}$$

where P is the number of people in a town and r is the distance from that town along the line toward the other. Thus (P/r^2) is an *index of attraction*, expressed in units of persons per unit distance-squared. Zipf, in 1946, studied the exchange of information, commodities, and people between paired cities and observed that the number of exchanges varied directly as the product of the two populations divided by their distance apart and, thus, developed an *index of interactance*, $(P_1P_2)/r$, which is persons-squared per unit distance. Stewart studied university enrollment of in-state students (and their hometown distances from the university) in the United States, and observed that the total enrollment varied directly as the population of the state and inversely as the average distance of the state population from the

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campus; he, thus, developed a quantity (P/r) as an *index of population at a distance.* Stewart related these indices to Newton's law of gravitation They were related as follows:

The *demographic force of attraction* between two population groups is

$$F = K \frac{P_1P_2}{r^2}$$

and acts as a line joining them; their *demographic energy*, by virtue of the force field, is

$$E = K \frac{P_1P_2}{r}.$$

The *potentials of population* ar

$$V_1 = \frac{KP_2}{r} \text{ and } V_2 = \frac{KP_1}{r}.$$

Thus $E = P_1V_1 = P_2V_2$.

The symbol K represents the demographic gravitational constant and is left for empirical determination.

Thus, nearly complete "formal laws of demographic gravitation" can be stated that have application to the average (statistical) relations of people existing in a space continuum. Number of people replaces mass, with distance playing the same role in the social system that it does in the purely physical system. The *index of attraction* represents *demographic force;* the *index of interactance* becomes *demographic energy*, and the *index of influence of population at a distance* is called the *potential of population.* When a properly weighted population is substituted for mass, and with distance retaining its singular role both in physical and in social circumstance, then these relationships are conceptual tools for studying geographic processes – that is, spatial distributions – in economic and social systems.

To expand these concepts to include geographical structure or spatial form, geographical process, and diffusion, it is necessary to consider not just two concentrations of population but, rather, many concentrations distributed over the area. The value of interactive potential at a point is the sum of all the attractive (gravity) potential values bearing upon that point. This property of attractive potential is called the principle of superposition. If interactive potential values are determined for a large number of points throughout the plane, the overall geographical variation in this "field quantity" can be portrayed by means of isopotential contours. When the population distribution of an area is considered not as concentrated at points but as a continuous density distribution over the area, the total potential of population at any given point is computed as $\int (1/r)\, D\, dA$. In this context, gradients of potential (varying values of potential surface slope) have meaning.

The total resultant force of demographic gravitation contributed by the entire population and acting on unit population at any point on the surface is directed at right angles to the isopotential line through that point and has the value of 2V/2r, or the gradient along r, a line perpendicular to the potential contour. This quantity is the demographic force per unit population, it has the same measure as Reilly's *index of attraction* – persons per unit distance-squared. Also, it measures the intensity of the geographic force, unlike the Reilly case, where the force is defined *only* along a line connecting an isolated pair of population concentrations. Force, in units of persons per unit distance-squared, has the same relation to gradient as energy, in units of persons-squared per unit distance, has to potential. If on a map of potential of population a constant contour interval is maintained, the gradient varies inversely as the spacing of the contours. It is important to note that gradient as thus defined has the same units as density of population.

The value $\int (1/r)\, D\, dA$ cannot be computed directly on a digital computer, but a mechanical integration based on summations can yield approximations. The procedure is:

$$V_i = \frac{P_1}{r_{i1}} + \frac{P_2}{r_{i2}} + \frac{P_3}{r_{i3}} + \ldots + \frac{P_n}{r_{in}} = \sum_{j=1}^{n} \frac{P_j}{r_{ij}}$$

or

$$V_i = \frac{D_1 \Delta A_1}{r_{i1}} + \ldots = \sum_{j=1}^{n} \frac{D_j \Delta A_j}{r_{ij}}$$

An analogy with electrical systems can be utilized, and not just for descriptive purposes. In macrogeography the word "field" is used not as a vague attention-getting metaphor but always in the direct, operational sense. The mathematics of field theory for science in general apply, as well, to this particular branch of geography. It has become customary to refer to "gravity models" in social science, and demographic gravitation is regarded in terms of the gravitation of physical masses. But such a consideration does not suggest an easy or practicable analog solution to the problem of producing the potential for a population map. However, Coloumb, following Newton's work, showed electric and magnetic interactions were, like gravity, inversely proportional to the square of the interaction distance. Reliable analogies among the basic laws for all three different physical systems exist, and one set of mathematical expressions applies approximately to all.

The potential at any point in a field is equivalent to the "work" required to transport a unit of charge, population, mass, etc., between two points. One can convert the foregoing analogies to expressions of potential, since work equals $\int F dr$ (when the force is integrated over the path followed).

In the physical treatment of gravitation – a force of attraction – the potential (KM/r) is conventionally taken as *negative*. This is because energy [illegible] mechanical work, KM/r), would have to be drawn from a source outside the system, in order to move unit mass from within the system to outside the system. According to this convention, potential would increase with distance between attractive masses. In the social sciences, the convention used for demographic gravitation is reversed for convenience. Potential is considered as decreasing with distance from points of population concentration.

The contribution of Warntz, presented in the preceding paragraphs, set the stage for a much wider use of field theory in geography. The field plotter offers a means by which such a wider use may be realized. In order to work with the boundary conditions necessary to apply more widely such concepts advanced by Warntz and others, it is necessary to consider the equations of Laplace and Poisson. They are presented in the following material, first with a conceptual approach and, then, with a mathematical approach.

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LAPLACE AND POISSON EQUATIONS – A CONCEPTUAL APPROACH

There are two methods by which force, energy flux, or material flow can be applied or introduced to a physical system. One is the application (and removal) of energy along a system boundary, e.g., applying heat to the side of a block of material. The other is to introduce (or remove) energy inside a system's boundaries such as a radioactive heat source buried in the block. The Laplace equation applies to the first situation, and the Poisson equation to the second. These equations are usually written:

$$\nabla^2 V = \partial^2 V/\partial x^2 + \partial^2 V/\partial y^2 = 0 \qquad \text{Laplace's Equation}$$

$$\nabla^2 V = f(x,y) \qquad \text{Poisson's Equation}$$

The Cartesian (x, y) coordinates relate the two-dimensional distribution of system components and variables. V is the potential variable and is related to the vector variable by the following:

$$\text{Force, Flux, Current} = \nabla V = \frac{\partial V}{\partial x}\,\overline{u_x} + \frac{\partial V}{\partial y}\,\overline{u}$$

where $\overline{u_x}$ and $\overline{u_y}$ are unit vectors along the x and y axes.

This means that force or flux is equal to the rate of change in potential realized in going from point to point in the system. The function f (x, y) defines the distribution and strength of energy or matter sources within the system. When f is non-zero and equal to a constant – a number, independent of (x, y) – the internal sources are spread uniformly throughout the system. The Laplace equation is just the special case of the Poisson equation when f is equal to zero.

It is not necessary for one to be able to solve (or even formulate) these partial differential equations, mathematically, in order to use the field plotter. However, modeling a problem on the field plotter improves with a better understanding of the modeled conditions for which the field plotter will provide a solution.[10] Implicit in both equations is the assumption that the processes operating within and thro gh the modeled system is steady state or static. Both static and steady state processes are independent of time. An example of a steady state process is a mechanical wave (water) generator. By repetitively (at a constant rate) striking the water, a pattern of standing waves is established which does not change with time. The waves move, but their relative heights and relative spacings do not change with time. A gravity field between two stationary masses is an example of a static process. Here not only does the distribution pattern of the gravity field remain constant, but there is no movement (cyclic) as in the case of the steady state. When a process is described as static it connotes a constant, unchanging process. A steady state process, on the other hand, *can* change over short periods of time. the only requirement it has to satisfy is that over sufficiently long periods of time some repetitive *cycle* must be established. It is this cyclic pattern which is independent of time.

During the interval of time between start up and the appearance of a cyclic

10. The field plotter will *always* provide an accurate solution for the conductive sheet; however, that the modeled process is also described by the Laplace or the Poisson equation must be ascertained.

equilibrium (steady state) condition, the process can be viewed as in a *transient* condition. An example of this condition is the period of transition caused by a change from one constant rate of input to another, or perhaps the dying out of an influence. Furthermore, many geographic processes are dynamic in the transient sense — are changing in a nonperiodic, nonequilibrium, fashion. The field plotter applicable only to those processes for which the static or steady state assumption holds. This assumption is not unduly restrictive, however, since over sufficiently short periods of time a transient process changes very little and can be assumed constant or static. Thus, by modeling a dynamic, transient phenomena as a series of static "snapshots," the field plotter can, indeed, simulate the succession of distributions, each representing a stage in the change of the absolute or relative influence of a source, where properties of the region affecting the propagation of influence change over time.

Geographic processes are related to the flow or diffusion of information, goods, services, people, etc. The resulting spatial distribution patterns are usually expressed in terms of population density, such as the number of goods produced or sold per acre or per establishment, and the number of people, housing units, and vehicular traffic per square mile or per square kilometer. Variations in population density are commonly represented by contour maps. Each contour connects points of equal density just as isopotentials connect points of equal potential. These contour maps resemble topographic maps in that there are "hills" and "valleys" representing high and low density regions.

Observations of the behavior of diffusion processes have led to a numberof models that reasonably account for observed distribution patterns. These models include the previously mentioned gravity and interaction potential models. In addition, there are a variety of mechanistic diffusion models referred to as expansion diffusion, relocation diffusion, contagious and hierarchical diffusion models.[11] These are differentiated by the types of channels or carriers through or by which diffusion processes operate.

Although the mechanistic and analog type models differ in their approaches, they both employ some of the same concepts and yield similar results. For example, they both employ the wave analogy to describe diffusion processes such as the propagation of innovations or the growth of a city. The wave analogy has two aspects, the decrease of wave-energy density with some power of distance from the source and the decreasing height of waves (amplitude) with time (and distance). The steady state condition which can be modeled on the field plotter represents the superposition of successive (in time) waves in a standing wave pattern with the height of waves decreasing with distance from the source in some controllable fashion.

Both the mechanistic and analog type models take into account the influence of accessibility or availability and quality of transportation routes. Similarly, impediments to diffusion or barrier concepts (like streamline boundaries) are used to explain the channeling, redirection, or slowing of diffusion. By selective painting (with conductive paint) and cutting or shaping the conductive paper sheet used in the field plotter all of these influences can be simulated.

Further mention should be made of the types of problems to which the Laplace or Poisson equations apply. It was noted previously that the Laplace equation applies to systems that do not contain energy (or matter) sources. Energy is input instead at

11. For a more detailed treatment of these models, refer to Resource Paper No. 4, Commission on College Geography, Peter R. Gould, *Spatial Diffusion*.

the system's boundaries. The Poisson equation allows both internal and external (applied to a boundary) inputs. In either case, specifying the conditions along the system boundaries is of importance in properly determining the potential distribution. In general, the Laplace and Poisson equations apply to what are called boundary value problems. Since the process is steady state, thus independent of time, the initial conditions or initial variable values are not of concern. In general, there are the following types of boundary conditions:

- Streamline
- Isopotential
- Fixed Boundary
- Fixed Boundary Input

A streamline boundary terminates, guides, or is parallel to the field flow, flux, or current. On the conductive sheet a streamline boundary would be represented by a cut in the sheet. An isopotential boundary is represented by a highly conductive strip (for example, a strip of silver paint). Since electrons meet little resistance in flowing along or across such a strip the potential does not change. Thus, such a strip represents an isopotential, assuming the potential value of the field along its length. Some applications require boundary conditions other than those related to boundary shape (streamline and isopotential). These specify that the potential level or current value along or across a boundary be fixed. Current or potential need not be the same along the entire boundary, but merely a known or present distribution of values.

Although the analogous nature of various systems facilitates understanding, it is not sufficient to ensure the validity of using a particular analog to solve a specific problem. The basis for the valid use of analog techniques is that under special conditions the mathematical equations describing these different situations have exactly the same appearance. Of course, the symbols may be different, but because the mathematical form (e.g., Laplace or Poisson equation) is the same, one symbol may be substituted for another. This means that a problem in heat flow, for example, can be modeled on the electrically conductive sheet and the electrical potential distribution (problem solution) can be translated directly into a heat potential (temperature) distribution. The following table represents the analogous system variable and properties for different physical phenomena.

ANALOGOUS SYSTEM CHARACTERISTICS

	System Variables		System Properties		
Physical Process	Potential or Scalar Difference	Vector	Potential Energy Storage	Kinetic Energy Storage	Energy Dissipation
Heat Transfer	Temperature	Heat Flux	Thermal Capacity	---	Thermal Resistance
Fluid Flow	Pressure or Velocity Potential	Flow Rate	Compressibility	Inertia (density)	Viscosity
Electrostatics	Electric Potential	Flux	Dialectric	–	–
Electrodynamics	Voltage	Current	Capacitance	Inductance	Resistance
Electromagnetics	Electromagnetic (EM) Potential	Flux	Dialectric	Permeability	Conductivity
Magnetics	Potential Magnetomotive Force (MMF)	Flux	–	Permeability	Reluctance
Gravity	Gravitational Potential	Force	–	–	–
Mechanical Statics	Displacement	Force	Spring Constant	–	–
Mechanical Dynamics	Displacement or Velocity	Force	Spring Constant	Inertia (mass)	Viscous Damping
Elasticity	Strain	Stress	Young's Modulus	Inertia	Viscous Friction

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LAPLACE AND POISSON EQUATIONS - A MATHEMATICAL APPROACH

The distributions obtained on field plotters are, depending on the boundary conditions, solutions to either the Laplace equation or the Poisson equation. Correspondingly then, a mathematically rigorous description of models simulated on the field plotter should be possible. Although this is true in principle, there are but a few coordinate systems and geometries for which the Laplace and Poisson equations have an analytic solution. The usual mathematical approach is to use iterative or approximation techniques which, to be interpreted, are graphed, thereby giving the same sort of results generated directly by the field plotter.

The purpose of this section is to introduce approaches that can be used to solve the Laplace and the Poisson equations analytically. Since a substantial proportion of applied mathematics treats this subject, the following paragraphs can do no more than establish some perspective and a departure point for those who wish to go further.

In order to describe mathematically a physical process, it is first necessary to select a coordinate system. The coordinate system chosen does not change the physical results but can greatly facilitate the mathematical expression. For example, in two dimensional Cartesian coordinates (ordinarily x, y) the equation of a circle of radius a, is $x^2 + y^2 = a^2$, in polar coordinates (ordinarily r,θ), the equation of the same circle is $r=a$.

The differential operator, ∇^2, is called "Del-squared" or the "Laplacian-operator." As a physical operator it too can be represented in various coordinate systems. In two dimensional Cartesian coordinates it is given by:

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$$

In polar coordinates it is given by.

$$\nabla^2 = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2}{\partial \Theta^2}$$

A single-valued scalar function in two dimensional Cartesian coordinates can be written V (x, y) or in polar coordinates W (r,Θ) Note that these are different scalar functions even though they have the same physical realization.

Laplace's equation is written:

$$\nabla^2 V = 0$$

The solution to the Laplace equation is V by itself. Before one can solve the equation for V, however, it is necessary to state the *boundary conditions*

The boundary conditions required are (1) the size and geometry of the region over which V exists and (2) the "values" of V along the boundary of this region. There are two ways of specifying the "values" on the boundaries. The first is to give the scalar magnitude of V; all such specified problems are called Dirichlet problems. The second is to give the normal (perpendicular to the boundary) derivative of F; all such specified problems are called Neumann problems.

In addition, various combinations of Dirichlet and Neumann conditions can be specified - but not independently. There have been many cases of long-undetected wrong solutions to field problems that, although mathematically correct, proceeded from inadequately expressed boundary conditions.

A useful property of fields that obey the Laplace equation is that they have neither a maximum nor minimum *inside* the boundary. As an example of this non-extremum property of a solution to the Laplace equation, consider a region whose boundary has everywhere a value V = k, k being some constant value. We can immediately say that all points inside the region must have value k, otherwise there would be either a maximum or minimum.

Poisson's equation is written:

$$\nabla^2 V = f$$

where f is some scalar function defined over the region for which V is to be solved. Although the meaning of f can be interpreted many ways (which, of course, is responsible for the versatility of field plotters), consider it here as an excitation function or source function. Conceptually then, the Poisson equation governs those fields that have sources inside the boundaries, and the Laplace equation those that have no sources inside the boundary. Boundary condition considerations and types are the same for both the Poisson and Laplace equations.

The two most effective and powerful techniques for solving the Laplace and Poisson equations involve general mathematical statements of boundary conditions in terms of several (possibly infinite) parameters which are evaluated for particular cases. These techniques each produce general solutions characteristic of the geometry of the region over which a solution is sought. These techniques are based on *eigenfunctions* in the first case and *Green's function* in the second.

The *eigenfunction* approach is straightforward and can be used for solving the Laplace equation. The Green function approach is more elegant and powerful and is used for solving the Poisson equation. Incidentally, Green's function is almost always expressed in terms of the *eigenfunctions* used for the Laplace problem defined as a sourceless Poisson problem.

As the reader should now expect, the corresponding Laplace and Poisson equations are intimately connected for any particular problem.

For example, consider a region containing a finite number of point sources. This is, obviously, a Poisson problem. Imagine further, a small circle drawn about each point source. Now consider a new region whose outside boundary is the same as before but which does not include areas within these circles about the sources. This region contains no sources, therefore the Laplace equation applies, and likewise the region contains no maximum or minimum values. Finally, let the small circles shrink in size until they coincide with the point sources. We can now say that the maximum potential and the minimum potentials must occur at either the boundary or at the point sources. (This powerful result was the basis for the calibration circuit design in the ISI field plotter).

The general approach for analytic solution of fields, of course, involves specifying boundary conditions and then solving the Laplace or the Poisson equation. On the other hand, one can use electronic circuit theory and arrive at the same results. With either approach one does an equivalent amount of work. The general approach is more elegant, but the circuit approach affords a greater amount of physical insight. As an example, the problem of a single source at the center of a conducting disk is as follows:

Boundary conditions. (a) is the radius of the current source at the center of the disk with radius (b),

$\Psi(b) = 0$

$\Psi(a) = I_o R_t$ $\quad R_t$ is the resistance of the disk

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Laplace equation in cylindrical coordinates:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\Psi(r,\Theta)}{\partial r}\right) + \frac{1}{r^2}\frac{\partial\Psi(r,\Theta)}{\partial\Theta^2} = 0$$

The symmetry of the geometry requires that Ψ remain constant relative to changes in Θ, thereby making its derivative with respect to Θ equal to zero.

Thus:

$$\frac{1}{r}\frac{\partial}{\partial_r}\left(r\frac{\partial\Psi(r,\Theta)}{\partial r}\right) = 0$$

$$r\partial\frac{\Psi(r,\Theta)}{\partial r} = k \qquad k, \text{ a constant}$$

$$\Psi(r) = \Psi(r,\Theta) = \int\frac{k}{r}dr = k\ \ln Ar \qquad A, \text{ a constant}$$

Applying the boundary conditions:

$$\Psi(b) = 0 = k\ \ln Ab \Longrightarrow A = \frac{1}{b}$$

$$\Psi(a) = I_0R_t = k\ \ln\frac{a}{b} \Longrightarrow K = \frac{I_0R_t}{\ln a/b}$$

Thus, we have:

$$\Psi(r) = I_0R_t\frac{\ln r/b}{\ln a/b}$$

The circuit theory approach to the same problem states: By symmetry the current I_0 is distributed evenly across concentric circles, giving a density of

$$J_r = \frac{I_0}{2\pi rt}$$ where J_r is the radial and only non-zero component of the current density and t is the thickness of the sheet.

By the field form of Ohm's law the radial electric field is:

$$E_r = \rho J_r = \frac{\rho I_0}{2\pi rt}$$ where ρ is the volume resistivity

The potential at a radius r is then,

$$\Psi(r) = \Psi(a)\int_a^r\frac{\rho I_0}{2\pi rt}dr$$

$$= \Psi(a) - \frac{\rho I_0}{2\pi t}\ln\frac{r}{a}$$

Applying the boundary conditions:

$$\Psi(b) = 0,\ \Psi(a) = -\rho\frac{I_0}{a\pi t}\ln\frac{b}{a}$$

$$\therefore\ \Psi(r) = \frac{\rho I_0}{2\pi t}\left(-\ln\frac{r}{a} + \ln\frac{b}{a}\right)$$

$$\Psi(r) = -\frac{\rho I_0}{2\pi t}\ln\frac{r}{b}.$$

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The resistance from the inside to the outside of a conducting disk is:

$$R_t = \int_a^b \frac{\rho}{2\pi r t}\, dr = \frac{\rho}{2\pi t} \ln \frac{b}{a},$$

by substitution:

$$\Psi r = -\left(\frac{R_t}{\ln b/a}\right) I_0 \ln r/b$$

$$= R_t\, I_\theta \frac{\ln r/b}{\ln a/b}$$

the same answer as before.

Thus far, the Laplace and Poisson equations were formulated in two dimensions. In general they may be expressed in any number of dimensions, although for the most part physical analogs exist only in one, two, and three dimensions, and practical constraints make analog simulation difficult in three dimensions. In the case of two dimensions one can, alternatively, formulate field problems in terms of complex variables. This formulation is extremely powerful, since it leads to notational simplicity and allows the use of a number of complex variable techniques. The most important of these techniques is called *conformal mapping*. Conformal mapping allows the solution of a problem having a particular geometry to be transformed to a problem having a different geometry. This permits the analogous solution to the analog solution, so to speak.

SUMMARY OF TECHNICAL NOTES

The principal conclusion that can be drawn from the technical notes is that much difficult work remains to be done before it will be possible to describe mathematically the spatial processes being simulated by field plotters. Seeking an analytical solution to some of the simpler models employed in series 1 of the experiments is a difficult task for a student in his fifth semester of calculus and analytical geometry. An analytical solution for some of the more complex models is probably not presently within the capability of more than a few mathematicians and physicists, if indeed anyone is presently capable of providing such a solution. Apparently *Green's Function*, for example, is necessary; but it is used by only a few very advanced students of applied mathematics. No significant number of geographers will possess that degree of mathematical sophistication for many years to come. But we do possess a highly developed intuition for maps and spatial processes that can guide us in the use of analog equipment, and we do deal with problems that demand this kind of sophisticated treatment. Therein lies a major value of analog computers in general and the value of the field plotter specifically.

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