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ABSTRACT

This paper presents the results of the second class to participate in the fast-paced mathematics program begun by the Study of Mathematically Precocious Youth (SMPY) at Johns Hopkins University. Fast-paced mathematics classes were established to meet the needs of highly gifted junior high school students. Thirty-three students participated in the program. In 108 hours of instruction, 28 class members learned Algebra II and plane geometry at a high level of achievement. Twenty-three persons completed Algebra III. In addition, 14 boys successfully completed the 4 1/2 years of pre-calculus mathematics. These students self-paced themselves through their homework and preparation for class. Class success was based on: (1) identification of qualified students through appropriately difficult tests of mathematical and non-verbal reasoning; (2) a dynamic teacher who introduced challenging material at a rapid-fire pace; and (3) voluntary participation by students. It appears that once these considerations are met the academic and social aspects of such a program will proceed "naturally," as evidenced by these two classes. (Author/SD)

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Abstract

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William C. George and Susanne A. Denham[1]

Abstract

Fast-paced mathematics classes were established to meet the needs of highly gifted junior high school students. Thirty-three students participated in the program. In 108 hours of instruction 28 class members learned Algebra II and plane geometry at a high level of achievement. Twenty-three persons completed Algebra III. In addition, 14 boys successfully completed the four and one-half years of pre-calculus mathematics. These students self-paced themselves through their homework and preparation for class. Class success was based on: (1) identification of qualified students through appropriately difficult tests of mathematical and non-verbal reasoning; (2) a dynamic teacher who introduced challenging material at a rapid-fire pace; and (3) voluntary participation by students. It appears that once these considerations are met the academic and social aspects of such a program will proceed "naturally," as evidenced by these two classes.

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[1] Chapter VI of Daniel P. Keating (ed.), Intellectual talent: Research and development, a 18-chapter volume to be published in the summer or fall of 1975 by The Johns Hopkins University Press, Baltimore, Maryland 21218, in both paperbound and hard-cover editions. This book will be No. 2 of Studies of Intellectual Precocity.

Fast-Paced Mathematics: A Program in Curriculum Experimentation for the Mathematically Talented

William C. George and Susanne A. Denham[1]

One of the most successful innovations begun by the Study of Mathematically Precocious Youth (SMPY) is its program of special fast-paced mathematics classes. The first of these was described in detail by Fox (1974b[I:6]) in Volume I of this series.[2] In the summer of 1973, another such class was instituted, taught at The Johns Hopkins University by the same teacher, Mr. Joseph Wolfson. This chapter presents results of this second class to date and guidelines for the institution of such programs in school systems.[3] Thus, our purpose is to describe the workings of a fast-paced mathematics curriculum for young persons in the hope that the principles and practices developed will be implemented by others.

These programs were begun as experiments in fitting a curriculum to the needs of young students who reason extremely well mathematically. If students with ability <u>and</u> interest in mathematics were given the <u>opportunity</u> to learn as fast as they could, we reasoned, their achievements and satisfaction would probably be apparent (Fox 1974b[I:6]). Moreover, we wanted to show that accounting for individual differences in education, specifically of the gifted, would not be costly or administratively unmanageable. It was our hope that after the demonstration of the program's feasibility more fast-paced mathematics models could be established along similar lines in public school systems.

The First Model Program

The first class, begun in June of 1972, has since been concluded. It met and in some cases exceeded our expectations. We invited 29 top ex-sixth graders, one ex-eighth grader, and one ex-third grader from the Baltimore area; 21 of these accepted our invitation to take the course.[4] Nineteen of them persisted through the nine initial two-hour-per-week summer class meetings. One of these then left the program voluntarily; five others were asked to return to their school mathematics class. Three persons (one ex-seventh grade girl, one ex-eighth grade girl, and one ex-eighth grade boy) were added to the class in September. All 16 of these persons (nine boys and seven girls) completed Algebra I and Algebra II. Ten of the 16 students continued through Algebra III, trigonometry, and analytic geometry; eight of these ten also finished plane geometry during the summer of 1973, demonstrating that up to four and one-half years of pre-calculus mathematics can be learned well by some mathematically highly talented young students studying two hours on Saturday mornings for about 13 months. During the June 1972-September 1973 period, seven of the nine boys in the class skipped one or more grades. By fall of 1974, all seven enrolled in calculus or pre-calculus courses in high school. Five girls and one boy began independent work in plane geometry, one girl took a correspondence course in geometry, and a boy and a girl took a senior high plane geometry course. Several enrolled for credit in mathematics-related college courses, also.

As is evident, the first attempt at such a special class was highly successful. In less than 14 months, meeting one two-hour period per week, the eight top students completed material which would have taken five periods per week for four and one-half years in the regular school curriculum sequence. This gave them an invaluable head start toward higher learning and satisfied their desire to meet the challenge of mathematics, as well as saving both their parents and school system much money (for further details on this class, see Fox 1974b[I:6]).

Spurred on by this success, the staff of SMPY made plans to begin a similar class in the summer of 1973. This time, however, somewhat different selection procedures were employed. In January 1973 the second SMPY Talent Search was conducted (in conjunction with the Study of Verbally Gifted Youth's Talent Search—see chapter 7). Nine hundred and fifty-four Maryland seventh, eighth, and under-age ninth grade students who scored in the upper two percent on standardized mathematical or verbal reasoning aptitude tests were administered the Scholastic Aptitude Test (SAT), including both the Mathematics and the Verbal sections; 667 were in the mathematics talent search. From the results of this testing, it was decided that Baltimore County and Howard County students who had obtained at least a score of 500 on SAT-M <u>and</u> 400 on SAT-V would be eligible for a class to be conducted at The Johns Hopkins University. Scores on both SAT sections were used as criteria, since we had learned from the first special class that a certain minimum degree of verbal mastery was probably necessary to learn mathematics at a rapid-fire pace (Anastasi 1974 [I:5]; Fox 1974b [I:6]).

Thus, 85 students (not including 17 eligible students who had already participated in the first class or were participating in other special projects) were notified by mail of the opportunity to take part in a new fast-paced mathematics class. Steps 1-3 of table 6.1 summarize

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Insert Table 6.1 about here

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the details of the selection process. In order to qualify, the 41 students who expressed interest in the class were administered the 80 items in both forms of the Educational Testing Service's Cooperative Mathematics Algebra I test. As can be seen in table 6.1, the response rate for girls (35.3%) was much less than for boys (56.9%). Of the 41 students, only one boy did not qualify for entrance into the class; nine others, however, decided against coming, leaving 31 persons (28 ex-eighth graders, two ex-seventh graders, and one ex-sixth grader)[5] enrolled in the second class.

In June 1973 these 31 students began attending Algebra II classes conducted by Mr. Joseph Wolfson. Of the girls, 91.7% (all but one) had taken Algebra I already; of the boys, 75.8% had completed an Algebra I course. Those students who had not had Algebra I had been enrolled in courses diversely labeled as pre-algebra, general mathematics, and SSMCIS.[6]

Further Assessment of Participants

To learn more about each student so that appropriate counseling could be given, SMPY personnel administered a battery of cognitive and vocational interests tests. This included the Raven's Progressive Matrices, Standard and Advanced (SPM [Raven 1960] and APM [Raven 1965]); Sequential Tests of Educational Progress; Science (Sci., Form 1A); Revised Minnesota Paper Formboard Test (RMPFBT, Forms MA and MB);

Bennett's Mechanical Comprehension Test (MCT, Forms AA and CC); revised scales from Holland's Vocational Preference Inventory (VPI); the Strong-Campbell Interest Inventory (SCII); and the Allport-Vernon-Lindzey Study of Values (AVL). Table 6.2 shows, as of January 1974, means and standard

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Insert Table 6.2 about here

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deviations along with percentile ranks for appropriate norm groups for each cognitive test for the total group; for females and for males; for those who continued with the special class (Group C); and for those five students who chose to return to the regular mathematics program of the school (Group NC) at the end of the summer. As can be seen from the percentile ranks, these students score quite high on tests that are designed primarily for older persons. Their mathematical aptitude and concomitant skill in non-verbal reasoning are especially impressive for persons their age.

Cognitive Tests

Sex Differences on Cognitive Tests

Significance tests were computed between test score means of boys vs. girls (see table 6.2 for those means which showed a significant difference). Girls scored significantly lower than boys on the SAT-M and on the MCT(AA).

From these differences it appears that from the outset boys had more mathematical reasoning ability than girls, as evidenced by their SAT-M score, even though a greater percentage of girls than boys had taken Algebra I already. It seems that boys acquire some of their mathematical skills from sources outside the classroom (Keating 1974 [I:2]; Astin 1974[I:4]). Neither sex showed much variability in scores on the Raven's tests of non-verbal reasoning.

Cognitive Differences Between Group C and Group NC

At the .05 level, there were no statistically significant differences in scores of mathematical ability between the 28 persons (23 boys and five girls) choosing to continue (Group C) and the five (one boy and four girls) deciding not to continue (Group NC). The trend of the score differences, however, was consistent with the sex composition of the groups. There was little difference between the Algebra I scores and the non-verbal reasoning scores of the Raven's for these two groups. This seems to indicate that, within a homogeneous high-ability group, mathematical skill alone will not predict success well or insure continued interest in a fast-paced mathematics program. Those in Group NC left for various reasons, not necessarily because they lacked Algebra II proficiency. One girl, who left in the middle of the summer, had the lowest combined SAT-M + V score (M-510, V-430) in the program, but she had high expectations for success in mathematics. One boy did better than all of the other students who took the two Algebra I tests (total score 40 + 40). His total SAT-M + V score (M-570, V-530) ranked in the upper one-third of the class. After he ranked in the middle of the group on the first Algebra II test, given in August 1973, he dropped out.

It was hoped that the battery of tests given to these students would help shed light on the abilities necessary for success in such a fast-paced class. The relationship between these test scores and success on standardized mathematics tests will be discussed later in this chapter.

Results from the Strong-Campbell Interest Inventory are also of interest. Table 6.3 shows the means and standard deviations for Holland's

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Insert Table 6.3 about here

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six interest orientations (Holland 1973) and four pertinent Basic Interest Scales. As can be seen from these tables, the students, boys in particular, are investigative in outlook. By investigative Holland means inquisitive and scientifically oriented. (For more information on the interests and values of gifted youth, see Fox and Denham 1974 [I:8], and chapters 12 and 14 in this volume.) The total group also shows peaks on the science and mathematics basic interest scales. These findings make intuitive sense, but it is gratifying to see that the students are exhibiting on standardized tests interests congruent with our day-to-day observations.

Interest Inventories

Sex Differences on the SCII and VPI

Some differences between the sexes did emerge. One encouraging sign, however, is of a difference which did not occur. Boys were not significantly higher than girls on the investigative Holland scales. This finding may bode well for the progress of these girls. Girls may not choose the investigative scales first, but they still score highly on this scale (nearly as high as on the social scale), indicating their high interest orientation in this area. The boys were, however, far less social and artistic than the girls (a mean difference of 9.6 and 13.7 points); these findings agree with previous interest patterns noted in similar groups. The girls were significantly lower than the boys on science and mathematics basic interest scales ($p < .01$). Perhaps, then, their desire to learn in these fields is less all-consuming than that of the boys; or their environment may not have encouraged them to perceive these fields as particularly useful for their future occupations (Anastasi 1974 [1:5]). Furthermore, the girls were significantly higher than the boys on the social service interest scale ($p < .001$). The magnitude of this difference (16.6) may well be of practical educational significance.

Their highest interests, in social services and mathematics, may, in combination with their accompanying social-investigative orientation, lead them into the teaching field (probably in the social sciences), into medicine, into psychology or similar careers. On the other hand, the boys were far more scientifically oriented, pointing to possible careers as scientists, mathematicians, computer designers. In addition, table 3 shows that the boys differed greatly on the six Holland scales, whereas the girls were similar on all the Holland scales except the enterprising scale, which was considerably lower. This seems to indicate that boys are more definite in their likes and dislikes.

This distinction between investigative and social orientations is more pronounced when looking at rank order on the Holland scales. Table 6.4 shows these rank orderings Boys are more inclined to choose

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Insert Table 6.4 about here

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investigative (I) and realistic (R) scales; the social (S) scale is last in the order of importance for them. Conversely, the Strong-Campbell Basic Interest Scales indicate that the girls are most strongly oriented toward social activities. The interesting difference is that the close second choice of the girls is the investigative scale. This means that these girls are still more research-science-theory oriented than the general female population. Such a strong S-I value profile, along with considerable interest in mathematics, is encouraging when one considers how few women are actually in mathematics-science fields.

Holland's six VPI Occupational Scales, equivalent to the Strong-Campbell Holland Scales, were administered in the one-page form to the students. The girls had the investigative scale (I) on the VPI as first choice 25 percent of the time, versus the 17 percent expected by chance. The boys selected I as first choice 64 percent of the time. An earlier finding (Fox and Denham 1974[I:8]) was, however, corroborated: girls who were predominantly investigatively oriented did well in the class, while those who were artistically or socially inclined via these scales either did not continue in the fall (Group NC) or remained low in the class. On the other hand, almost equal numbers of highly investigative boys were successful and unsuccessful in Algebra II and Algebra III. Thus, it again appears that interest patterns must be taken into account in the organization of such classes, especially for female students (also noted by Fox 1974a).

Differences between the sexes and between Group C and Group NC on both the SCII and the VPI clearly indicate that an investigative orientation toward pursuing goals and choosing activities is helpful if one is to survive in an investigative environment. This finding would have been neatly predicted by Holland's (1973) theory.

This conclusion has far-reaching implications. Placing a socially (but not investigatively) orientedstudent in a highly investigative environment may not allow for the effective use of an individual's talents. It is worth considering whether social classroom environments should be constructed for the benefit of social, people-oriented types, and investigative environments should be engineered for those students who can benefit from them most. This would imply considerable segregation by sex (see chapter 12).

Differences Between Group C and Group NC on the SCII

Group NC, when compared with students who stayed in the program (Group C), were significantly lower (see table 6.3) on investigative and realistic orientation and scientific interest ($p < .01$). On the other hand, they were significantly higher ($p < .01$) on the Holland social scale. All of the students in Group NC except one were girls; apparently their lack of investigative orientation and scientific interests was such that they were poorly motivated to keep up with the work. Table 6.4 shows that Group NC ranked the investigative scale in the lower half; thus, social (S), conventional (C), and artistic (A) values predominated. For Group C the I-R profile is strong, with the A-S values ranking at the bottom. Social values are not the important attributes for success in this fast-paced class. It is again an inquisitive mind with a strong interest in mathematics that seems to be necessary to keep up in it. Those individuals whose values are investigative appear to have a better "fit" with this type of learning environment.

Values

The Allport-Vernon-Lindzey Study of Values (SV) was also administered to the group. This measure yields scores on six scales which are somewhat comparable to the Holland scales. This test was designed primarily for use with college students and graduates. The SV consists of two sets of questions asking the individual to choose preferences in relatively familiar situations. The six scales reflect relative value levels rather than absolute value levels. The average score for any individual's six values is 40. The six value scales are classified by Edward Spranger (1966) in the following manner: (1) the (T)heoretical individual is interested in scientific, intellectual, and philosophical pursuits; (2) the (E)conomic individual is interested in production, consumption, and wealth; (3) the (A)esthetic individual is noted for valuing grace, symmetry, and other artistic qualities; (4) the (S)ocial individual is interested in mankind and its welfare; (5) the (P)olitical individual is concerned with power; and (6) the (R)eligious individual is basically concerned with the mystic. Mean scores and standard deviations for the total class, boys, girls, those who continued and those who did not, are given in table 6.5.

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Insert Table 6.5 about here

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Table 6.5 shows that the boys have strong theoretical values. There is little differentiation in their point total between economic and political values. Both values are far higher for boys than for girls in the program. The boys have a higher social value (people oriented) and aesthetic value (sense of balance, symmetry) on the SV than on the comparable Holland scales.

Girls maintain a strong social value as their first choice. The aesthetic scale ranked higher than the theoretical scale on the SV. Girls were significantly lower on the theoretical value scale ($p < .001$) than the boys. On the SCII girls were not as investigative as boys but this difference was not significant. Girls placed investigative on the Holland Scale closely behind social; however, between the social and theoretical scales on SV there is a spread of 9.1 points. This difference between these two values is greater than the entire spread of points for all six Holland scales on the SCII. Boys are significantly lower ($p < .05$) on the religious value. On the SV, these highly selected girls (by cognitive test scores) look much more conventional than they do on the Holland scales.

The boys and girls in the Wolfson II class were noticeably different on some scales than the comparable high school norm groups as shown by table 6.6. and figures 6.1 and 6.2.

Insert Table 6.6 and Figures 6.1 and 6.2 about here.

For example, the boys were significantly higher ($p < .001$) on the theoretical (investigative, truth-seeking) value than a large population of high school boys (see table 6.6). They were, on the other hand, significantly lower ($p < .001$) than the norm group of boys on the religious (mystic) value. This combination of placing high priority on theoretical activities and less on religious aspects of life is found elsewhere in this study of gifted boys (Fox and Denham 1974 [I:8]) and creative persons (Hall and MacKinnon 1969; Helson and Crutchfield 1968).

The girls in the class were more aesthetic (beauty seeking, artistic; $p < .01$) and social (people-oriented; $p < .05$) than the norm population of high school girls. The norm group, on the other hand, was more economically oriented (concerned with production, consumption, and wealth; $p < .01$) than the girls in the fast-paced mathematics program.

Sex Differences on the SV

These girls and boys fit the general pattern of differential value orientation reported elsewhere (Allport, Vernon, and Lindzey 1970; Moshin 1950). That is, the boys are higher on theoretical, economic, and political (power-oriented) tendencies, while the girls favor aesthetic, social, and religious attitudes and pursuits more than the boys. The above finding is seen more explicitly when looking at the rank ordering of the SV values listed in table 6.7.

Insert Table 6.7 about here

The strong social-aesthetic value setting for girls supports the Holland VPI and the SCII interest scales. The boys rank theoretical qualities first and place social-aesthetic tendencies on the lower end of the scale. This seems to indicate an important difference. Boys, being more investigative and research minded, seem to be willing to work independently and on their own time. This is necessary for such a fast-paced mathematics class. Girls, even when strongly interested in mathematics, probably need the social environment of working as a group to maintain their success in a class. Again this lends credibility to the possibility of exploring interest-centered classes in addition to classes divided by sex, e.g., students all high on theoretical and low on social, or all high on social and low on theoretical.

Value Differences Between Groups C and NC

Group C was significantly higher than Group NC on the theoretical, political, and economic values, but far lower on the religious value, as seen by table 6.7. Their social and aesthetic values were not as strong as political and economic tendencies. Group NC, in contrast, tends to be strongly social with a high religious orientation. Economic values are last in their value ranking, as they were for the nine girls. This seems to indicate that those who chose not to continue in the fast-paced mathematics class were not as independently and theoretically oriented as those who did continue. Another factor that may be suggested by the data is that the strong social interest indicated in the SCII scales, as well as the AVL, may not be met by this type of class, even though the students express strong liking for mathematics. As in the previous analyses, however, the C-NC differences are almost wholly confounded with sex differences.

Success in the Program

Parental Variables

In addition it was hypothesized that parents might greatly affect their child's success or non-success. Parents who are more highly educated might value education more and be more facilitative of their child's progress. Table 6.8 shows that the modal educational level for

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6.8
Insert Table/ about here

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fathers of Group C was a master's degree, while Group NC's fathers' modal educational level was high school graduation. The bachelor's degree was the modal level of education for the mothers of those students who mastered the material the best (Group C). For the mothers of those students who chose to return to the regular classroom pace (Group NC), the modal educational level was high school graduation. Thus, it does appear that the majority of those students who chose to terminate their participation in the class did come from less well educated homes. This difference is probably sex-related to some extent. Five of the boys' parents in Group C had less than a high school education. None of these boys discontinued, however, while each girl whose parents had at best a high school education did not continue at this rigorous fast pace.

Self-report measures were also evaluated. From table/ 6.9 it is apparent that,

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6.9
Insert Table/ about here

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ent that, as perceived by the students, parents of girls and Group C had more influence (though not necessarily significantly) on their children with respect to the class. The parental influence, however, is found to be unrelated (r = -.12) to success in the class (a score of 28 on Form A or 29 on Form B of the Algebra III test, which was the median of the class). Possibly, the best students need the least "prodding."

So far we have described the philosophy behind such a special class, explained the selection procedure, and given detailed cognitive and interest profiles of the class. At this point, however, we must recount some aspects of the class that it is necessary to understand before undertaking such a project.

Teacher's Style and Ability

The teacher's style and ability are vital to the success of such a program. It is clear that without these prerequisites, the class cannot be the absorbing challenge that it has been for our students. The teacher we have employed so successfully in both our special classes does not rely on the usual pedagogical tools. He is a fast pacer-stimulator who races through the material at the pace of the quickest students, not the slowest. The other students are forced to stretch their minds and catch up--between classes, if necessary. With their talent, they can generally do so and enjoy it. It seems likely that college teachers would adapt more readily to this approach than secondary teachers who have been taught to teach to the slower members of the class. It is important to remember that the students self-pace themselves through their homework (Stanley 1973) and preparation for the class. By keeping up with their extensive assignments the students go over what they missed in class and study the finer points of the problem.

In addition to the fast pace set by the teacher, his style is qualitatively different from that of many mathematics teachers. The students work collectively and individually at the blackboard, with the teacher urging and congratulating, helping out a discouraged student here and there, but generally leaving them to master the work fast. Individuality is important and is encouraged.

During "lecture" periods the teacher is quite elaborative, explaining the more elegant tools of algebra with considerable finesse. His dramatic style is truly a "tour de force," worthy of emulation. The students are excited, effusive, but under his control. He is receptive to new ideas and novel approaches to solutions. Yet, no problem is allowed to dominate discussion for long. Spontaneous verbal interjections are not, however, suppressed.

In short, besides a keen mind and excellent knowledge of mathematics, the teacher of such a class must have a personality which is attractive to the students and is interwoven with a keen sense of humor. Also, genuine respect for one's students and their feelings is a must. These factors help establish a positive atmosphere in a fast-paced program. Each student must be imbued with enthusiasm, and the student must feel that the problems are elegant and meaningful.

The material covered in this unique manner included all of Algebra II and a large chunk of Algebra III, taught during one two-hour period per week from June to December, 1973. Results of standardized tests on this subject matter, given in August, November, and December, 1973, are summarized in table 6.10. As is obvious from the percentile ranks shown

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Insert Table 6.10 about here

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there, the students had mastered Algebra II well by November. Sex differences were marked on the August Algebra II test, the only test of those reported which was taken by the three girls who subsequently dropped out at the end of the summer (one other girl quit early), and were also pronounced on the Algebra III test.

Prerequisites for the Success of the Class and its Members

One question which crops up after we know who was successful in the class and who was not is this: what tests in our test battery are most predictive of this success?[7] The most predictive of these tests could then be used profitably in future screening sessions held by individual counselors, schools, and school systems. Table 6.11 shows product moment correlation coefficients between various cognitive and achievement tests taken by the students.

Insert Table 6.11 about here

It is apparent from table 6.11 that success in learning the Algebra II sequence of mathematics in this class was predicted best by the SAT-M ($p < .05$). Success in the more inclusive, high-powered Algebra III segment of the program was predicted more highly by the Algebra II ($p < .01$) and APM scores ($p < .05$). Thus, we see that for algebra success, abstract reasoning tests seem to have predictive validity. A large measure of non-verbal skill is necessary in order for students to tackle abstract material quickly and thoroughly. A firm understanding of Algebra I and Algebra II concepts in addition to abstract reasoning ability seem to be important factors for a student to understand Algebra III's extension of the abstract material presented previously. This information, however, suggests only the tentative nature of the conclusions regarding the predictors of success in such a class.

Before starting such a class, then, it is important to assess thoroughly the ability of the students in abstract reasoning, as well as their grounding in the prerequisites for the planned coursework. In addition, the coursework, like that described above, should be sufficiently challenging.

If individual school systems are going to develop similar classes, it is important, also, to assess the students' own feelings about their experiences in this fast-paced class. Prior to the inception of the class, which has been discussed previously, the students noted that they did enjoy mathematics. The majority of the potential pupils rated their liking for mathematics as "very high." Thus, one important prerequisite for success was present before the beginning of the class. Again, it is clear that investigative interests are linked to success in an investigative environment. The required interest level can be obtained through voluntary participation.

In August 1973, after eight weeks of classes, the students were asked to make several judgments on different aspects of the class. Compared with their regular mathematics classes, the majority of the students found the new class more productive, more fun, and more competitive (in order of decreasing frequency of response). Clearly, the class was a success from the point of view of the students. This finding should encourage educators interested in beginning such a class; for students of high caliber such an experience is rewarding.[8]

What particular aspects of the class are most appreciated? In the same set of August responses, the students rated the teacher's style highest, with the challenge of mathematics and their own feeling of accomplishment rated next highest, respectively. Another important factor was that they were encouraged to work on their own and think for themselves. Social aspects of the class were not rated as important for these students in Group C as opposed to Group NC. These ratings are very revealing: as we have mentioned before, the style of teaching in such a class emerges as very important. It seems that the ability of the teacher to motivate the students and make the curriculum alive and worthwhile is an essential part of the success of the class. Of course, Mr. Wolfson's effectiveness may be less with girls than an equally skilled, socially oriented woman might be.

In December of 1973 the students were asked to give information on their individual educational progress, again via questionnaire. The answers given then were illuminating. Over half were interested in gaining information and advice on taking further advanced coursework and possibly for going to college a year or two early. Five had already taken college courses, and two boys were taking calculus or precalculus courses in school in addition to the Saturday class.[9]

The successful academic progress of these students is obvious. Moreover, their zest for learning is apparent during class. As one college student observer[10] put it: "The first ten to fifteen minutes were devoted to settling down. Kids shuffled to seats, greeted one another, and collected homework papers. After that short adjustment period they were ready to concentrate on the day's work. The teacher allowed a high degree of openness in his classroom. Kids got up to leave the room for snacks and were allowed open-ended discussion of problems. The lesson was referred to as 'fun' by one student--the other student attitudes seemed to concur with this view. In this student-centered class the children were attentive and interested, volunteering to go to the board to demonstrate a point as needed. Several times solutions to problems were tried in several ways.

It was good to see the teacher encourage this type of thinking. Other times he let the class know they were jumping too far ahead in course content by putting off their advanced questions. It was impressive to see these children in action. Their high motivation was displayed by choosing this productive way to spend a Saturday morning."

Further testimony to the academic success of the class members can be noted: one member was the winner of the ninth-grade section of a local county-wide mathematics contest. Other local colleges have shown interest in this program. For instance, six members of the current mathematics class have taken a college course in computer science for credit. This is just another example of the ability of these students to grasp higher-level curriculum challenges.

Even within a highly select group such as this one, there is a great deal of differentiation (Stanley 1974). At the end of the Algebra III segment of the class it was decided to split the class into two sections. The majority had been able to keep up with the fast pace. Some of the students, however, needed more detail than the current teaching style was giving them. Therefore, when plane geometry was started, a second class section (N = 5) was begun in hopes that these talented youths could keep up with the rest of the class.

In the new section they were given more individual attention and a more detailed approach. Homework and self-pacing were still the key factors in this teaching style. One college student observer[11] described the new class thus: The teacher "seemed to have an uncanny control of the class and kept things moving at a fast clip by the attention she gave to the subject, drawing the pupils with her. She continually rewarded responses positively. She zeroed in for details. Some of her statements were as follows: 'Oh, that's terrific,' 'Do this for emphasis—this is important,' 'OK, how do I get from here to the conclusion?' She pretends to struggle with the problems with them, they get anxious and start thinking and produce the answers. When one girl worked out a solution another girl clapped spontaneously—that's how involved they were. The teacher enforced thinking and good working habits while problem solving. Some typical statements were: 'Think about your general line of attack,' 'Good habit to specify what you are given.' I feel she is skill building very effectively. Once she stopped writing on the board and gave attention to a boy who softly asked a question. Her teaching approach is not without humor, either, After a lengthy explanation and precise instruction, she said, turning toward the class, 'You have to put it down this way—I'd zap you if you didn't.' And zap them she did, for each one of those students walked out of there feeling sharper—and that is what really counts!"

In both sections of the class the concern is with the students. Both teaching styles have an important common feature: feedback. Questions are frequently asked and answers sought in order that the teacher may be sure that the class is following the materials. In both sections, the teachers use the blackboard and seek ingenious responses. The key to success here is to challenge the students to learn mathematics well, and in each section the approach is being fulfilled.

What of the social growth of these students for the duration of this special class? Anecdotal material gives evidence of this growth. Another college student observer[12] noted, "The physical appearance of the children as a class seemed similar to any regular class of this age with a few exceptions. No children were overly large or too maturely developed, but two boys stood out for their extreme smallness. They appeared to be younger than the majority of their classmates. Both were also valuable to the class progression. One responded maturely in his actions despite his size, contributing intelligent solutions and posing good questions. The other appeared to derive pleasure in producing witty remarks to make the others laugh. However, in this classroom situation, among their intellectual peers, both of these two boys were accepted and appreciated for their contributions."

At the board, age and sex appear to make very little difference. Members of the class frequently discuss the problems and ask the person next to them to explain the problem if they do not grasp the concept the first time. Quite frequently the two smallest boys in the class have the two tallest youths as their blackboard partners. All attack the problems with enthusiasm. Size or age does not seem to affect the enthusiasm of the individual or the verbal exchange. Various age groups work together toward goals or interests they have in common. This is an impressive observation when considering the age span of five years in the class. Their interest is in mathematics, and the common goal is to master the intricacies of mathematics.

While the students continuing through the summer and into the fall (Group C) did not rate social aspects of the class as important to them, it is obvious that they do get along well. As mentioned above, the youngest boys, whose size difference when compared with the older students is quite visible, fraternize freely and without reservation with the others. In general, however, the girls cluster together in the front of the room;

they do not seem to shun the male members of the class, though. One of the girls, however, is always in the middle of the boys. She works with them, discussing the problems and posing questions quite frequently. All in all, the classroom seems to differ little socially from a regular junior high school class, except perhaps for the obvious zest, freedom, and enthusiasm of the students.

Summary: Elements of a Successful Program

A retrospective look at the present special class reveals that in order to conduct such a class, careful attention must be paid to: (1) the identification of qualified, mathematically oriented students through appropriately difficult tests of mathematical and non-verbal reasoning and prerequisite achievement; (2) the selection of a dynamic, bright assertive teacher who can create an atmosphere of fun and productivity while introducing challenging materials; and (3) voluntary participation by the students. It appears that once these considerations are met the academic and social aspects of such a class will proceed "naturally," at least from the experience with these two groups. Some students will, however, need a somewhat slower and more detailed pace. These students will also thrive if given careful and prompt consideration.

If the above points are followed, then such a class would probably be successful, as these most certainly have been. Further specific guidelines, however, would probably be beneficial for individual school systems embarking upon such a course of action. As has been noted here many times before, the choice of an appropriate teacher is of paramount importance. This fact cannot be overemphasized.

Second, over and above the relatively simple task of identifying apt students through tests,[13] the scheduling adjustments for the students to participate in a fast-paced class must be made. This point has been viewed by some counselors and administrators as a great inconvenience. If seen in perspective, though, it need not be such a problem. Arranging a time and place for students to meet once or twice per week for class, and providing study periods during the rest of the week can be handled without much stress; flexibility is necessary, of course. If courses to be held during school hours were arranged at the beginning of an academic year, all students involved could be scheduled similarly, with their schedules planned around the special mathematics course. Other unusual courses taken by the students would, of necessity, have to be held during other periods. These scheduling hurdles have been surmounted without snags by the majority of our students' counselors. Ideally a mathematically highly talented student could progress through special classes in high-school mathematics and then advance into college courses on released time from his high school or at night, or by correspondence after he has completed his high school work (also see chapter 3).

Assuming that these three points (identification, teacher, and scheduling) are attended to at least on paper by school administrators, there remains an important problem to solve: funding. The felicitous solution to this possible roadblock is that, as we estimate it, the cost of such programs need not be exorbitant. For a large system-wide group of 60-100 pupils, a full-time teacher would travel daily to one or two junior high schools, each of which would have around 20 special mathematics pupils. Music and art teachers do this; teachers of the mathematically gifted could do this, as well. The cost per year for such a large, full-scale program would be somewhere in the range of $13,000-$16,000, depending on the teacher's salary, books, and clerical fees. A class of the 15-30 most mathematically able students in the school system could meet on Saturdays in a central location and cost the system less than $2000 for the instructor (at $50 per two-hour meeting). Mr. Wolfson's two Saturday programs were funded almost totally through parent contributions of three dollars per week.

The benefits of such a program far outweigh the costs, even monetarily. Obviously, talented students' time would be used more wisely, without the boredom which so often occurs in the regular classroom situation. Furthermore, the amount of money cited above would probably cover three years' worth of mathematical education, in one year, for many students. Thus the costs of the program would be amply justified, especially when one considers the impact it will have on students who will find mathematics more meaningful and challenging. The ablest and best-motivated of these will go on to careers in which their fine knowledge of mathematics will be highly useful.

Also, an attentive and interested student in the right educational environment will have more to contribute to his class and feel more satisfied as an individual. This consideration is especially important when the goal of education is to improve the individual and help him find a satisfactory place in society.

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Footnotes

[1] The authors would like to thank Dr. Julian C. Stanley for his helpful comments and encouragement in earlier drafts of this paper.

[2] In addition to the usual references, citations of chapters in Volume I of Studies of Intellectual Precocity will be as follows: [1:6]. The I indicates Volume I, and the 6 indicates the chapter number. [Editor]

[3] Appendix __.A updates the fast-math class progress through geometry, trigonometry, analytic geometry, and their course placement in the various school systems.

[4] Selection of the sixth graders was based on scores on the numerical, abstract reasoning, and/or verbal scales of The Psychological Corporation's Academic Promise Test; the eighth and the third graders were previously known to us.

[5] The sixth grader had not yet taken the SAT but was known to be highly talented; SMPY had previously arranged private tutoring for him.

[6] SSMCIS (Secondary School Mathematics Curriculum Improvement Study)

[7] By success we mean those people who were in the upper half of the distribution for a given cognitive or mathematical achievement test. This is not to infer that those whose scores are in the lower half of the distribution are doing poorly. When they are compared to a random school population taking the same courses their scores in many instances will put them in the top, and this is at a younger age.

[8] It is interesting to note that dropouts did not respond like non-dropouts in this August questionnaire.

[9] Three of the eight girls, as opposed to only two of 19 boys, wanted no information or advice on such educational advancement.

10 Members of a class on gifted children from a nearby college observed the special class on two Saturdays during January, 1974. Contributions quoted here are from Linda Bergman.

11 Contributions quoted here are from Paul Meyers.

12 Contributions quoted here are from Barbara McCloskey.

13 There has recently been a general retreat from testing in the schools. We feel it important to reiterate here that high-level, appropriately difficult tests are vital tools for the identification of students for such special programs. Moreover, standardized tests can accurately and convincingly show these students' progress in such programs.

Table 6.1: Selection of Wolfson II class

Step in Selection	Criteria	Total Number	Girls	Boys	Howard County		Baltimore County		Percent Proceeding to Next Step		
					Girls	Boys	Girls	Boys	Total	Girls	Boys
1. Initial invitation to join class	SAT-M ≥ 500 SAT-V ≥ 400	85	34	51	10	16	24	35	48.2	35.3	56.9
2. Algebra I tests taken	Alg.I ≥ 48	41	12	29	4	7	8	22	75.6	75.0	75.9
3. Entered class	Voluntary	31	9	22	3	6	6	16	83.9	55.6	95.4
4. Class after summer departures and additions *	Voluntary	28	5	23	2	6	3	17	100.0	100.0	100.0
5. Breaking into sections	Alg.III ≥ 26										
a. Section A (faster paced)		23	3	20	1	5	2	15			
b. Section B		5	2	3	1	1	1	2			

* Four girls and one boy dropped out of the special mathematics class in August 1973. Two boys (one ninth grader and one sixth grader) were added to the class in September. The percent proceeding from step 3 to 4 was calculated on totals before addition of two boys.

Table 6.2: Summary of Cognitive Test Results for All Students

	Total Group (N = 33)[a]				Boys (N = 24)			Girls (N = 9)			Group NC (N = 5)			Group C (N = 28)		
	N	Mean	S.D.	Percentile rank[b]	N	Mean	S.D.	N	Mean	S.D.	N	Mean	S.D.	N	Mean	S.D.
SAT-M [c]	33	580.9	49.5	86	24	593.3**	49.0	9	547.8	34.6	5	554.0	27.0	28	585.7	51.4
SAT-V	31	496.1	69.2	79	22	490.4	77.4	9	510.0	43.9	5	498.0	43.2	26	495.8	73.8
RPM SPM [d]	32	55.9	2.2	96	24	55.5	2.3	8	56.9	1.7	4	56.5	1.3	28	55.8	2.3
RPM APM	32	29.4	3.9	95+	24	28.9	4.2	8	30.8	1.9	4	29.0	4.1	28	29.4	3.9
Sci. IA [e]	30	46.7	7.7	78	22	47.7	7.1	8	44.0	9.2	4	45.8	10.0	26	46.8	7.6
MRPFBT MA [f]	32	46.8	7.6	34	23	46.7	8.3	9	47.0	6.0	5	50.4	5.9	27	46.1	7.8
MRPFBT MB	32	50.1	7.9	50	23	51.1	8.1	9	47.4	7.0	5	47.2	9.5	27	50.6	7.7
CMT I [g]	29	18.9	10.0	—	21	19.4	10.7	8	16.8	8.0	4	16.0	6.5	25	19.3	10.4
CMT II	29	29.4	6.6	--	21	30.0	7.1	8	27.8	4.9	4	30.8	4.4	25	29.2	6.9
MCT AA [h]	29	41.0	9.7	22	21	43.6*	9.5	8	34.3	6.4	4	35.3	2.5	25	42.0	9.9
MCT CC	28	32.4	10.9	33	21	33.8	10.8	7	26.7	9.2	3	25.0	10.5	25	33.2	10.6
Alg. IA + IB [i]	31	65.6	7.7	95	22	67.0	7.7	9	62.1	7.1	5	65.4	11.1	26	65.6	7.2

[a]This number includes all departures and additions to class as of January, 1974.

[b]Percentile ranks are shown for: 12th grade males (SAT-M, SAT-V), 20-year-olds (SPM and APM), college sophomores in spring (Sci. IA), engineering freshmen (RMPFBT and MCT), and eighth graders national (Alg. I); relevant norms are unavailable for CMT.

[c]Highest possible score on SAT-M and SAT-V, which test mathematical and verbal reasoning, respectively, is 800.

Table 6.2: Summary of Cognitive Test Results for All Students (Continued)

[d] Highest possible score on SPM, which tests non-verbal reasoning, is 60; for APM, it is 36.

[e] Highest possible score on Sci. IA is 75; this test is designed to measure college students' knowledge of general science.

[f] Highest possible score on either RMPFBT form, which measures spatial relations ability, is 64.

[g] Highest possible score on CMT I, vocabulary is 115; for CMT II, verbal analogies, it is 75.

[h] Highest possible score on both forms of MCT, which tests mechanical comprehension, is 60.

[i] Highest possible score of Alg. IA + IB combined is 80.

* $.01 < p < .05$

** $.005 < p < .01$

Table 6.3: Means and Standard Deviations of Holland Scales and Selected Basic Interest Scales from the Strong-Campbell Interest Inventory[a]

Scales[b]	Total Group[c] (N = 31) Mean	S.D.	Boys (N = 22) Mean	S.D.	Girls (N = 9) Mean	S.D.	Group NC (N = 5) Mean	S.D.	Group C (N = 26) Mean	S.D.
Holland:										
Investigative	54.9	7.1	56.2	5.4	51.8	9.7	47.2*	10.9	56.4	5.3
Realistic	49.7	8.8	50.0	8.6	49.2	9.8	42.2*	8.9	51.2	8.4
Conventional	48.3	9.0	48.3	8.8	48.3	10.0	52.2	12.4	47.5	8.4
Enterprising	45.7	8.5	45.7	8.6	45.7	8.5	45.2	9.5	45.8	8.6
Artistic	43.2	10.4	40.4*	9.7	50.0	9.3	48.2	6.8	43.2	5.0
Social	42.7	10.9	38.7**	9.2	52.4	8.4	53.2*	10.0	40.6	10.2
Basic Interest:										
Mathematics	58.0	5.6	59.4*	4.4	54.3	6.6	56.2	7.4	58.3	5.3
Science	57.3	8.4	59.9*	5.9	51.1	10.5	46.2**	10.6	59.5	6.2
Social Service	43.1	11.4	38.3**	7.8	54.9	10.5	49.4	11.9	41.9	11.4
Writing	42.4	10.8	41.0	10.3	46.1	12.0	46.0	11.9	41.8	11.0

[a]Mean scores are based on scales where a score of 60 or more is considered "high." Of course, these mean scores do not reflect the actual magnitude of some individual scores.

[b]Holland scales are based on Holland's (1973) theory, while interest scales are empirically devised.

[c]The N = 31 because two students in Group C were not administered the SCII.

*$.001 < p < .01$

**$p < .001$

Table 6.4: Rank Order of the Holland Scales from the Strong-Campbell Interest Inventory

Total Group (N = 31)	Boys (N = 22)	Girls (N = 9)	Group NC (N = 5)[a]	Group C (N = 26)[b]
I	I	S	S	I
R	R	I	C	R
C	C	A	A	C
E	E	R	I	E
A	A	C	E	A
S	S	E	R	S

Key:

I = Investigative

S = Social

A = Artistic

E = Enterprising

C = Conventional

R = Realistic.

[a] 1 boy and 4 girls.

[b] 21 boys and 5 girls.

Table 6.5: Means and Standard Deviations of the Six Values from the Allport-Vernon-Lindzey Study of Values (AVL)

Values	Total Group (N = 33)		Boys (N = 24)		Girls (N = 9)		Group NC (N = 5)		Group C (N = 28)	
	Mean	S. D.	Mean	S. D.	Mean	S. D.	Mean	S. D.	Mean	S. D.
Theoretical	46.2**	8.0	49.1***	6.7	38.5	5.4	40.4*	8.3	47.2	7.6
Economic	40.5	9.2	43.4***	8.7	32.8	5.7	33.6*	8.8	41.7	8.9
Aesthetic	35.6	9.5	32.6***	8.5	43.6	7.3	41.6	9.4	34.5	9.2
Social	41.1	7.8	38.7***	6.7	47.4	7.2	44.6	9.8	40.5	7.4
Political	41.8	5.8	43.3**	5.0	37.7	5.8	36.8*	8.0	42.7	4.9
Religious	33.9	11.9	31.6*	12.4	40.2	8.2	43.0*	5.1	32.3	12.1

*$.01 < p < .05$

**$.001 < p < .01$

***$p < .001$

Table 6.6: Means and Standard Deviations of the Six Values from the Allport-Vernon Lindzey Study of Values (SV)

	Boys				Girls			
	Class (N=24)		High School (N=5320)[a]		Class (N=9)		High School (N=7296)[a]	
Values	Mean	S.D.	Mean	S.D.	Mean	S.D.	Mean	S.D.
Theoretical	49.1***	6.7	43.3	6.4	38.3	5.4	37.0	6.9
Economic	43.4	8.7	42.8	6.9	32.8**	5.7	38.2	6.3
Aesthetic	32.6	8.5	35.1	7.7	43.6**	7.3	38.2	7.1
Social	38.7	6.7	37.0	6.2	47.4*	7.2	43.3	6.9
Political	43.3	5.0	43.2	5.9	37.7	5.8	39.0	5.9
Religious	31.6***	12.4	37.9	8.3	40.2	8.2	43.7	8.1

* $.01 < p < .05$

** $.001 < p < .01$

*** $p < .001$

[a] High school population is based on 5320 boys and 7296 girls in grades 10-12. These N's are found in the 1970 Allport-Vernon-Lindzey Manual for the Study of Values.

Table 6.7: Rank Order of the Six Values for the Allport-Vernon-Lindzey Study of Values (AVL)

Total Group (N = 33)	Boys (N = 24)	Girls (N = 9)	Group NC (N = 5)[a]	Group C (N =28)[b]
T	T	S	S	T
P	E	A	R	P
S	P	R	A	E
E	S	T	T	S
A	A	P	P	A
R	R	E	E	R

Key:

T = Theoretical S = Social

E = Economic P = Political

A = Aesthetic R = Religious

[a] 1 boy and 4 girls.

[b] 23 boys and 5 girls.

Table 6.8: Parents' Level of Education

Level of Education	Group C (N = 26)				Group NC (N = 5)			
	Father		Mother		Father		Mother	
	Number	Percent of Group	Number	Percent of Group	Number	Percent of Group	Number	Percent of Group
1	1	4	—	—	—	—	1	20
2	3	12	7	27	3	60	2	40
3	3	12	1	4	—	—	—	—
4	4	15	13	50	1	20	—	—
5	8	30	5	19	1	20	1	20
6	7	27	—	—	—	—	1	20

1 = less than high school

2 = high school

3 = some college, including A.A. degree

4 = Bachelor's degree

5 = Master's degree

6 = Doctor of Philosophy or Medical Doctor

Table 6.9: Means and Standard Deviations of Parental Influence Exerted on Students During Phases of Class[a]

	Group NC (N = 4)[b]		Group C (N = 26)		Group C Boys (N=21)		Group C Girls (N=5)	
Phase	Mean	S.D.	Mean	S.D.	Mean	S.D.	Mean	S.D.
Joining Class	3.0	0.71	3.5	1.47	3.5	1.53	3.8	1.17
Staying in Class	3.0	1.58	2.9	1.54	2.8	1.66	3.6	0.49
Monitoring Class	2.0*	0.71	3.0	1.00	2.9	1.04	3.6	0.49

*$p < .05$

[a] 5-point scale, where 5 = strongest influence

[b] 1 boy and 3 girls

Table 6.10: Results of Algebra II and Algebra III Tests[a]

	Total Group				Boys			Girls		
Test	N	Mean	S.D.	Percentile Rank[b]	N	Mean	S.D.	N	Mean	S.D.
First Algebra II (8/73)	30	27.2	4.4	80	22	28.1**	4.4	8	23.6	4.3
Second Algebra II (11/73)	28	34.0	3.5	97	23	34.4*	2.8	5	31.0	4.2
Algebra III (12/73)	28	28.7	4.8	81	23	29.8**	4.0	5	24.6	4.6

*$.01 < p < .05$

**$.001 < p < .01$

[a]Standardized tests used are Cooperative Achievement Mathematics Tests.

[b]Percentile ranks are shown for national high school norms (first and second Algebra II tests) and college liberal arts norms (Algebra III test).

Table 6.11: Product-Moment Correlations Between Test Scores for Various Tests[a] Given to the Wolfson II Class

	SPM (N=32)	APM (N=32)	SAT-M (N=33)	SAT-V (N=31)	Alg. 1A + 1B (N=31)	Alg. II(1) (N=31)	Alg. II(2) (N=28)	Alg. III (N=28)
SPM	—							
APM	.54**	—						
SAT-M	-.30	.33	—					
SAT-V	-.12	.06	.25	—				
Alg.IA + IB	.04	-.21	-.06	.18	—			
Alg. II(1)	-.18	.14	.07	.09	.39*	—		
Alg. II(2)	.24	.31	.43*	.00	.15	.39*	—	
Alg. III	.22	.42*	.35	-.18	.30	.31	.66**	—

* $.01 < p < .05$

** $p < .01$

[a] See table 6.2 for explanation of cognitive tests given in this table.

Figure 6.1: Mean Scores of Boys for the Six Values from the Allport-Vernon-Lindzey Study of Values

Figure 6.2: Mean Scores of Girls for the Six Values from the Allport-Vernon-Lindzey Study of Values

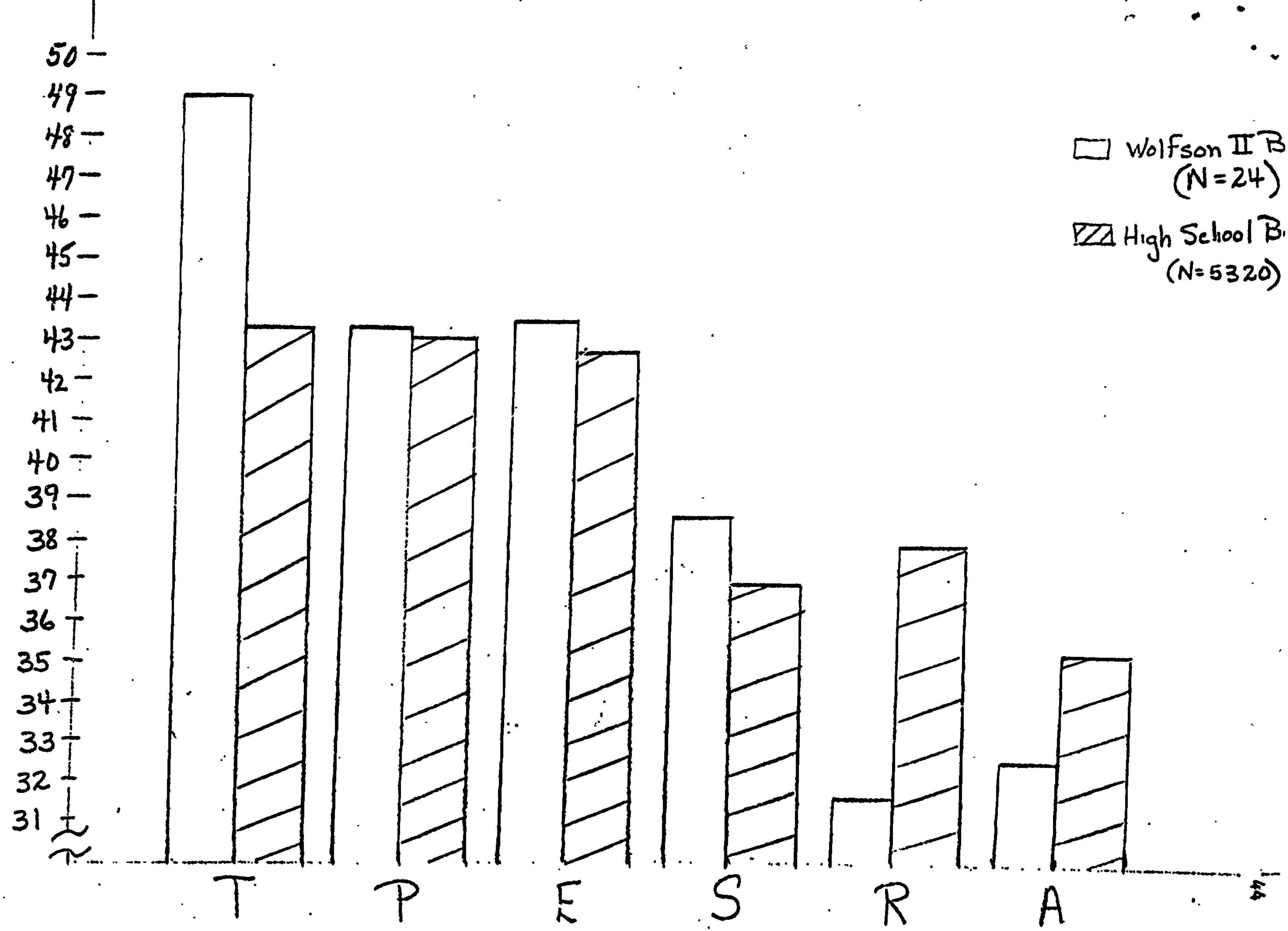

Fig. 1: Mean Scores for the Six Values from the Allport-Vernon-Lindzey Study of Values

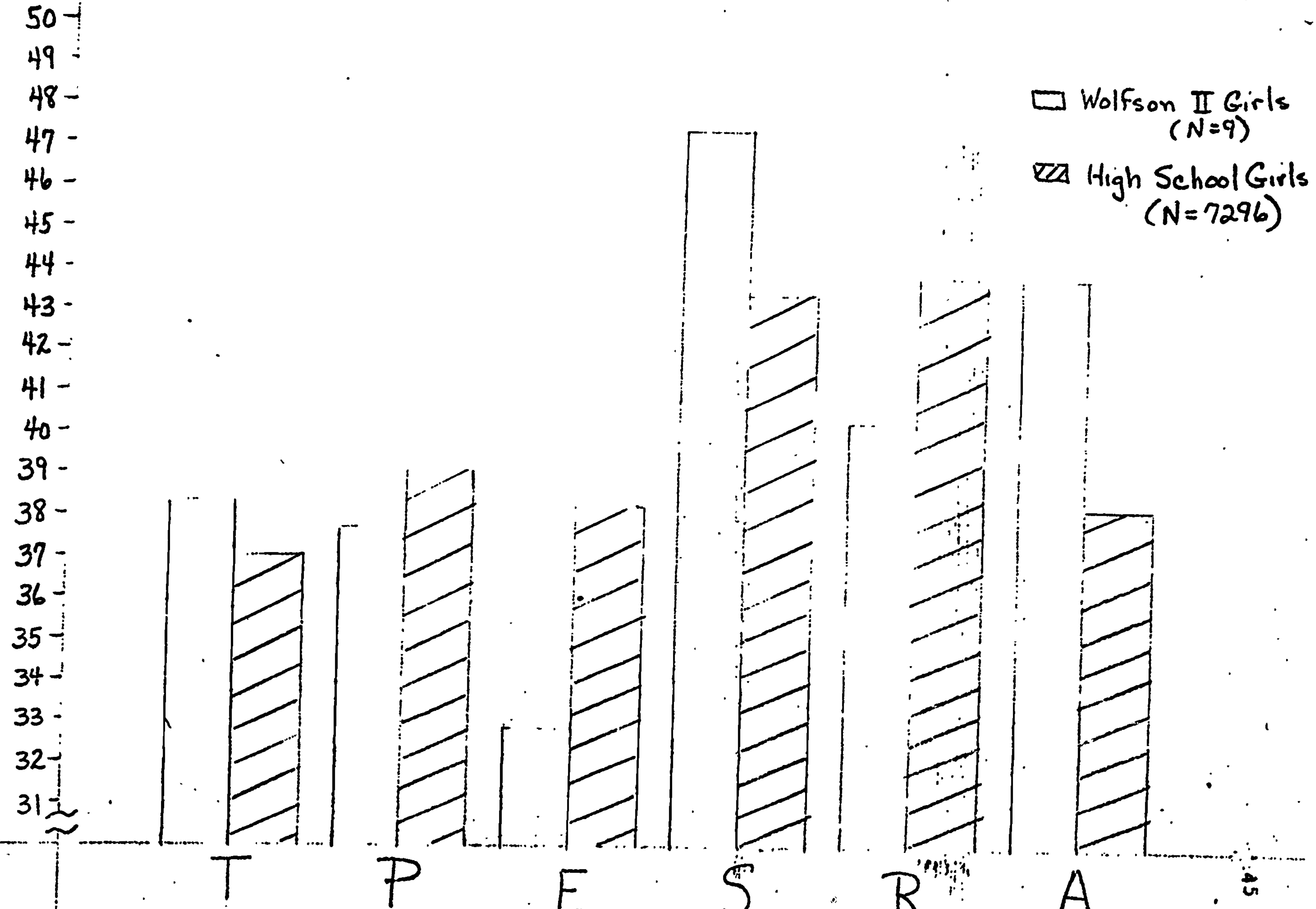

Fig. 2: Mean Scores for the Six Values from the Allport-Vernon-Lindzey Study of Values

Appendix 6.A1

Update on this Fast-Math Class

The success of fast-paced mathematics programs is demonstrated by the continued success of the Wolfson II class. Table 6.A1 shows the

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Insert Table 6.A1 about here

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progress of the program from plane geometry to the completion of the pre-calculus curriculum sequence.

As mentioned in chapter 6 the class was divided into sections, the faster-paced taught by Mr. Wolfson and the slower-paced taught by Miss Shuppert. These sections met once a week on Saturdays for 19 two-hour periods from 26 January 1974 until the Educational Testing Service Cooperative Mathematics Test in geometry was administered on 1 June 1974 and 22 June 1974. The classes covered about the same material but in a different manner. The smaller class (N = 5) was much more personalistic and group oriented. Math games were used to keep incentive high and make fast-math learning a stimulating experience. The fast-paced group (N = 23) was more individualistic, and new solutions were suggested in a much more theoretical manner. Both teachers, however, insisted on self-instruction through a properly paced homework process. Before-class preparation was a necessity. Neither class was slowed by those students who chose not to read and complete their homework.

The results of the class were indicative of the students' willingness to learn in this style. All 28 students scored at the 85th %ile or higher on the national high school norms as measured by this 80-minute, 80-item test. This means that in only 38 hours of instruction they exceeded the total score earned by 85% or more of the students who had studied for 170 or more 45-50 minute school periods.

Twenty members from the two groups were invited to continue through much of the summer of 1974 and complete their pre-calculus sequence by taking trigonometry and analytic geometry. In addition, two other highly talented boys were asked to join the class. The first, a seventh grader, had learned Algebra I, Algebra II, Algebra III, and plane geometry since December 1973 with the assistance of a tutor. The other boy, a member of the original Wolfson class (student five in Fox 1974b[I:6]), had just completed geometry in his high school. These two boys, along with 15 members (1 girl, 14 boys) of the invited group completed trigonometry in 16 hours. The mean score for the group on the 40-item, 40-minute ETS Cooperative Mathematics Test on trigonometry was 28. This is the 96th %ile of national high school norms. No student scored below the 76th %ile of national norms.

At this point, for various reasons, the last remaining girl in the class decided to leave the fast-math instruction program before completing analytic geometry. In the fall of 1974 she returned to the regular classroom and took the usual trigonometry and analytic geometry course that her high school offered.

<u>The Sixteen Who Completed the Pre-Calculus Sequence</u>

Sixteen boys continued the rapid-fire pace and completed analytic geometry in 14 hours of instruction. No one scored below the 75th %ile of national high school norms. In seven two-hour classes these students learned analytic geometry better than at least three-quarters of the high school population which takes analytic geometry five days a week for 90 periods. The mean score of this group on the Cooperative Mathematics Test in analytic geometry, 29, is the 95th %ile of national high school norms.

Of the 16 boys who completed analytic geometry in August of 1974, 14 had finished all the pre-calculus math curriculum in 108 hours. The fast-paced approach has repeatedly demonstrated its success. In September 1974 14 boys joined their regular high school calculus class. The other two boys at the strong encouragement of their school chose to join the regular trigonometry and analytic geometry class. As of January 1975, no student has reported encountering difficulty in the high school classroom. Two of these boys are eleventh graders, accelerated one and two years. They plan to enter a college or university in the fall of 1975. Ten of the boys are tenth graders, including one who is accelerated. Many of these students have expressed interest in beginning college a year early, in 1976-77. One of the two ninth graders skipped one grade and the other skipped three grades to reach the advanced courses at private schools in the Baltimore area. The ninth grader who is eleven has previously earned two "A's" for college credit in course work relating to computer science.

In September of 1974, nine of these highly able mathematical reasoners began a college calculus class taught for two hours each Saturday morning by Dr. Richard McCoart. The purpose of this program is to help prepare these young men for the College Entrance Examination Board's Advanced Placement Program (APP) examination in calculus at the higher (i.e., BC) Level. APP exams are offered each May. Quite a few high school programs are capable of preparing their students fairly well for the AB Level, but few have the student talent or instructional staff needed for the BC Level. By earning a 4 or 5 on the Calculus BC Level test each student will receive a year of college credits in mathematics, with the opportunity to take more advanced math courses in college. Many colleges will give a year of college credits for a 3, as well. A 4 or 5 on the AB Level will insure a student of at least four credits or one semester of calculus. Students who were in the Wolfson II class and are presently in the high school program only, have been encouragedto take at least the AB Level examination in May 1975.

Course Placement for the Remaining Fourteen

Taking calculus for college credit is the alternate route two students chose to solve possible scheduling problems created by limited course offerings in the junior high schools. The first, as a ninth grader in junior high school, registered for calculus at the local community college in the fall of 1974 after completing analytic geometry the previous summer. The second student chose to continue his mathematics program in college after completing plane geometry in June 1974. Having successfully completed a computer science course in the spring 1974 semester, this current eighth grader (1974-75) took a pre-calculus college course during the summer of 1974. The following semester he completed Calculus I and then registered for Calculus II in the spring 1975 semester.

The remaining twelve students (7 boys, 5 girls) completed Algebra II, plane geometry, and in seven cases Algebra III in a total of 39 two-hour meetings. Three of the tenth grade boys decided on doubling up on their mathematics during the 1974-75 school year to complete the pre-calculus sequence. At least one of these is planning to enter college in 1976-77. Eight of the remaining nine students (4 boys, 5 girls) are taking trigonometry and analytic geometry. The final student is entering a major university in 1975-76 and took Math Analysis I at the local community college in lieu of high-school mathematics during the spring semester of 1975. The final evaluation of success will have to wait until these students have completed the rest of their high school mathematics program.

The results of SMPY's curriculum experimentation programs encouraged Montgomery County and Charles County to each establish two fast-math classes on a county-wide basis for their highly talented mathematical reasoners in the fall of 1974. In addition, there are at least six other in-school classes based on the accelerative math model operating in the Baltimore-Washington area. For further information regarding the in-school model, see chapter 7 in this volume.

Appendix Table 6.A1: Update of Wolfson II from Plane Geometry

Step in Update	Criteria	Total Number	Girls	Boys	Howard County		Baltimore County		Percent Proceeding to Next Step		
					Girls	Boys	Girls	Boys	Total	Girls	Boys
1. Students taking plane geometry	Alg. III $\geq$ 26										
a. Section A (faster paced)		23	3	20	1	5	2	15	100	100	100
b. Section B.		5	2	3	1	1	1	2	100	100	100
2. Successful completion of plane geometry	Plane geometry $\geq$ 52										
a. Section A		23	3	20	1	5	2	15	78	67	80
b. Section B		5	2	3	1	1	1	2	40	0	67
3. Invited to take trigonometry[a]	Plane geometry $\geq$ 59	22	2	20	0	7	2	13	77	50	80
4. Entered trigonometry class	Voluntary	17	1	16	0	5	1	11	100	100	100
5. Successful completion of trigonometry	Trigonometry $\geq$ 20	17	1	16	0	5	1	11	94	0	100
6. Continued on to analytic geometry	Voluntary	16	0	16	0	5	0	11	100	—	100
7. Successful completion of analytic geometry	Analytic geometry $\geq$ 23	16	0	16	0	5	0	11	88	—	88

Appendix Table 6.A1: Update of Wolfson II from Plane Geometry (Cont.)

Step in Update	Criteria	Total Number	Girls	Boys	Howard County		Baltimore County		Percent Proceeding to Next Step		
					Girls	Boys	Girls	Boys	Total	Girls	Boys
8. Taking calculus during 1974-75 school year[b]	Voluntary	14	0	14	0	5	0	9	64	0	64
9. In McCoart Saturday morning college calculus class	Voluntary	9	0	9	0	4	0	5	89	0	89
10. In McCoart class until Christmas of 1974	Voluntary	8	0	8	0	3	0	5			

[a]Two new boys (a seventh grader and a ninth grader) were invited to join the new consolidated class in June 1974. The percent proceeding from step 2 to 3 was calculated before additon of two boys.

[b]This includes the regular high school program and/or college courses taken in lieu of high school calculus.