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ABSTRACT

A Piagetian Task Instrument (PTI) consisting of three tasks developed by Piaget was administered to 141 preservice elementary teachers enrolled in a mathematical class, 19 preservice secondary mathematics teachers, and 11 calculus honors students. This instrument was scored by methods similar to those of Schwebel (ED 063 896) and McKinnon and Renner, and was used to classify subjects as belonging to one of four stages (Concrete Substage II-A, Concrete Substage II-B, Formal Substage III-A, Formal Substage III-B). Frequency distributions of scores for the three groups were submitted to a chi-square analysis which confirmed (p less than .001) that the distribution of subjects among the stages varied with the group. Fifty-two percent of preservice elementary teachers were in concrete operational substages, while less than 5 percent of the preservice secondary mathematics teachers and none of the calculus students were in this stage. When SAT scores, available for 80 of the preservice elementary teachers, were correlated with PTI scores a significant coefficient (r = .68, p less than .01) was found. The author discusses related studies and implications for instruction. (SD)

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PIAGETIAN COGNITIVE DEVELOPMENT AMONG PROSPECTIVE TEACHERS

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Piaget has theorized that the cognitive growth of an individual occurs in approximately four stages, and not until one attains the final stage, formal operations, is he capable of understanding and assimilating many of the abstract concepts of mathematics. The notions of proportion and similarity, along with the ability to use the combinatorial method and propositional logic on verbal elements, do not appear well-formed until this stage (Inhelder and Piaget, 1958).

Lovell (1972) has reviewed most of the Piagetian research that is relevant to mathematical learning, and he concluded that "it is the development of the general ways of knowing that determines the manner in which taught material is understood (p. 171)." Beilin (1971), also discussing Piagetian theory, stated:

> No logical or mathematical learning is likely to occur, at least without great difficulty and tenuousness, if the concepts to be learned are far beyond the operational level of the child's available cognitions (p. 117).

The foregoing testament to the importance of readiness has serious implications for teachers of mathematics. If the assimilation of many mathematical concepts requires formal schemas and the student does not firmly possess such schemas, learning will be difficult, incomplete, and tenuous.

Therefore, instruction and content should be suited to the learner's level of cognitive development.

Although Piaget's theory has had some effect on elementary school curricula and methods, there have been few changes based on it at the secondary and college levels. It is tacitly assumed that most high-school and college students have attained the stage of formal operations. Consequently, they are treated as abstract verbal learners and expected to apprehend new concepts and propositions directly, without the aid of less abstract referents (Ausubel, 1966).

Several recent studies (Friot, 1970; McKinnon and Renner, 1971; Schwebel, 1973), however, indicate that a significant number of high-school and college students may not operate at the formal level. Treating these students as abstract verbal learners is failing to suit instruction and content to each learner's cognitive level. This failure possibly could account for much of the difficulty that so many students exhibit in learning mathematics. The present study was designed to explore this hypothesis by investigating the differences and similarities among selected groups of college students in stages of cognitive development as well as the relation between levels of cognitive development and general ability in mathematics. The four stages of cognitive development of interest were those that Piaget calls Concrete Substage II-A, Concrete Substage II-B, Formal Substage III-A, and Formal Substage III-B (Inhelder and Piaget, 1958). The specific hypotheses tested were:

Hypothesis 1: There is no significant difference in the proportions of subjects at each of the four stages of cognitive development, as determined by PTI scores, in the group of prospective elementary-school teachers, the group of honors calculus students, and the group of secondary-school mathematics student teachers.

Hypothesis 2: Within the group of prospective elementary-school teachers there is no significant relation between levels of cognitive development, as measured by PTI scores, and general ability in mathematics, as measured by SAT Mathematics scores.

METHOD

A Piagetian Task Instrument (PTI) consisting of Equilibrium in the Balance, Quantification of Probabilities, and Colorless Chemical Liquids (Inhelder & Piaget, 1958; Flavell, 1963) was administered by individual interview to determine stages of cognitive development. These particular tasks were used because they involve schemas that seem relevant to mathematical skills.

Equilibrium in the Balance was designed to determine if a subject had available, and could apply, the schema of metric proportions as embodied in the law of the lever. The apparatus for Equilibrium in the Balance consisted of a plastic equal-arm beam with equally-spaced pegs from which metal washers of equal weight could be suspended. The examiner first informed the subject that all the washers were the same weight, and demonstrated the balance by placing one washer on the fifth peg from the center on each arm. He next placed a group of washers on one peg and then asked the subject where to place a prescribed group of washers on one peg of the other arm in order to produce equilibrium. For example, the examiner asked, "If one washer is on this peg, where can you place two washers on the other side to balance the beam?" He did not mention the number of the peg, and no numerals were visible to the subject. On each trial, the subject was asked to justify his choices. If he made an incorrect choice, he was allowed to continue until he produced equilibrium. The criteria for determining a subject's stage of development on this task corresponded to those described by Inhelder and Piaget (1958), confirmed by Piaget (Lovell, 1971), and used by Lovell (1961) and Schwebel (1973). They are the following:

II-A The subject displays mostly intuitive behavior. He uses trial and error to determine the qualitative compensation of weight and distance from the fulcrum. He uses addition and subtraction to determine the location of washers. He makes no attempts to generalize his observations.

II-B The subject is aware that an increase in weight is compensated by a decrease in distance, that is, qualitative compensation; but does not use metric proportions. He makes correct choices in the cases of the 1:2 or 1:3 ratios, but in no others. He does not generalize and does not discover the product rule.

III-A The subject realizes the role of metrical correspondences and generalizes the proportion rule or the product rule. Although he uses a rule, he cannot explain it in mathematical terms, that is, in terms of inverse proportions.

III-B The subject knows at the start that proportions are involved, and makes correct choices with confidence. He explains the rule in terms of inverse proportions, the law of the lever, or torque.

Quantification of Probabilities was designed to determine if a subject could apply the schema of metric proportions to compare simple probabilities. The apparatus consisted of some clear marbles, some blue marbles, two small containers, and two metal cans. The examiner showed the subject the two containers which contained sets of marbles of different numerical composition. For example, one set would contain two blue and three clear marbles while the other would contain three blue and four clear marbles. The examiner then asked the subject, "Imagine that each set of marbles has been placed in its own metal can, both cans shaken well, and one marble poured from each can. From which can is there a better chance of rolling out a blue marble?" The subject was asked to explain his choice. The criteria for determining a subject's stage of cognitive development on this task were the following:

II-A The subject does not apply any probabilistic scheme. He predicts solely on the basis of intuition or nonquantitative aspects.

II-B The subject attempts to quantify probabilities, but predicts on the basis of absolute numbers of marbles. He does not use ratios.

III-A The subject quantifies probabilities in most cases and compares ratios, but he is not certain that this is a suitable general method.

III-B The subject quantifies probabilities in each trial and compares ratios. He is certain of the general suitability of this method. He insists that comparing ratios is the logical way to decide.

Colorless chemical Liquids was designed to determine if a subject could generate a complete system of possible combinations and could employ hypothetico-deductive reasoning to test the possible effects of a variable. The apparatus for Colorless Chemical Liquids consisted of five labeled flasks containing colorless, odorless liquids, several test tubes, and five eyedroppers. The flask labeled A contained dilute sulfuric acid, B contained distilled water, C contained sodium thiosulfate, D contained hydrogen peroxide, and G contained potassium iodide. The combination of liquids from A,D, and G produced a yellow colored solution. The presence of C prevented the color from forming. B, the water, is neutral. Thus, the colored solution could be produced only with A+D+G or A+D+G+B.

The examiner showed the subject two test tubes containing colorless liquids. The subject was unaware that one tube contained water and the other contained A+D. The examiner added a few drops of G to each and noted the reactions. In the tube containing A+D+G the solution turned yellow; no reaction was apparent in the other tube. The examiner then said, "Your task is to figure our how to reproduce the yellow solution. You may use these five liquids any way you wish, and do the mixing in these test tubes." Sixteen test tubes were available. As the subject proceeded, the examiner noted his actions and asked him to explain his procedure. The criteria for this task were:

II-A The subject tries, at most, each liquid with G and all with G. He does not attempt any other combinations unless prompted. His explanations are purely quantitative, that is, he believes the liquids act as they do because of the amounts used despite evidence to the contrary. If he does not produce the color with the one-by-G combinations, he suspects a trick and does not know any other possible strategies. He requires very much prompting.

II-B The subject tries almost all combinations, but is not systematic in a way that indicates he wishes to try all possible combinations. If he discovers the correct combination he is unsure that others could also produce the color. He displays little use of propositional logic. He requires prompting throughout.

III-A The subject uses a systematic method to test the possible combinations, and he realizes that the color results from a combination of liquids. He is aware of all possible combinations. He displays the use of some propositional logic, and requires only moderate prompting to determine the function of each liquid.

III-B The subject behaves very much like the III-A subject, but is more sure of himself. He uses deductive methods from the start and has an organized plan. He requires virtually no prompting to determine the role of each liquid. He has a clear aim toward proof from the start.

The scoring scheme devised for this study was very similar to those used by McKinnon and Renner (1971) and Schwebel (1973). Based on the criteria specified with each task description, the behavior on each task was classified as characteristic of one of four stages of cognitive development. A numerical score of 1, 2, 3, and 4 was assigned to classifications II-A, II-B, III-A, and III-B, respectively. The sum of the three task scores was the PTI score. This score was used as a measure of cognitive development for the correlational analysis, and also to determine each subject's overall stage of development according to the scale shown in Table 1.

Insert Table 1 about here

The sample consisted of selected students at The University of Texas at Austin. The group of prospective elementary-school teachers contained 136 females and five males enrolled in "Modern Topics in Elementary Mathematics", the two-semester mathematics course required of all elementary and special education majors. Their ages ranged from 18 to 28 years. Two had subject matter concentrations in mathematics. A total of 149 were interviewed, but eight were excluded because they stated they were not education majors.

The group of secondary-school mathematics student teachers contained 14 females and five males ranging in age from 20 to 48 years. All were engaged in student teaching in the Austin (Texas) public schools. Many had completed their course work in mathematics. Only three (males) had a second teaching field in a science.

The group of honors calculus students contained seven males and four females ranging in age from 18 to 22 years. Only two were majoring in mathematics, but their instructor stated that all were exceptionally able in mathematics.

RESULTS

Hypothesis 1 was tested by calculating the appropriate chi-square statistic according to the procedure described by Siegel (1956, p. 178). All subjects were classified as having attained one of the four stages of cognitive development as described above. Only two subjects were at stage II-A. Therefore, stages II-A and II-B were combined. Table 2 presents the statistics related to hypothesis 1.

Insert Table 2 about here

The chi-square value of 57.33 for four degrees of freedom presented in Table 2 indicates that hypothesis 1 could be rejected. There was a significant difference in the proportions of subjects at the different stages of cognitive development in the three groups. Indeed, the proportion (52%) of prospective elementary-school teachers classified as concrete operational was significantly different from the corresponding proportions of both secondary-school mathematics student teachers and the honors calculus students.

Also, only five percent of the prospective elementary-school teachers were at stage III-B, while 47 percent of the student teachers and 64 percent of the honors calculus students were at this stage. The proportions of subjects at the different stages in the group of student teachers and the group of honors calculus students were not significantly different $(P > .10)$.

Hypothesis 2 was tested by calculating the appropriate Pearson product-moment correlation coefficient. The scattergram and related data are presented in Figure 1.

Insert Figure 1 about here

The correlation coefficient of .68, which is significant beyond the .01 level, indicates that hypothesis 2 could be rejected. There was a significant relation between levels of cognitive development and general ability in mathematics within the group of prospective elementary-school teachers.

CONCLUSIONS

1. As a group the prospective elementary-school teachers differed significantly from both the secondary-school mathematics student teachers and the honors calculus students in cognitive development. This is clearly indicated by the frequencies shown in Table 2.

2. A substantial number of the prospective elementary-school teachers enrolled in "Modern Concepts in Elementary Mathematics" are not formal operational. This agrees with the findings of McKinnon and Renner (1971) and Schwebel (1973) concerning college students in general.

The sample examined seemed to justify the generalization contained in this conclusion. Although not random, it was considered representative of the population of prospective elementary-school teachers enrolled in "Modern Concepts in Elementary Mathematics." It contained all of the students in five of the 18 sections of the course. In the opinions of their instructors, who had taught several sections of the course prior to the spring of 1974, the sections were typically composed with respect to types of learning difficulties exhibited, ability, motivation, and sex ratios.

3. Among the prospective elementary-school teachers there was a substantial relation between cognitive development and general ability in mathematics as defined in this study. Among the 80 subjects (all female) for whom information was available, the correlation between PTI scores and SAT Mathematics scores was .68, which was significant beyond the .01 level. This conclusion disagrees with that of Schwebel (1973) who found no relation between cognitive development and SAT Total scores in his sample of college students. In the present study the correlation between PTI scores and SAT Total scores was .60. In theory SAT scores, at least SAT Mathematics scores, should correlate fairly well with performance on the Piagetian tasks normally used to distinguish concrete from formal thought capabilities. In their explication of the SAT, Donlon and Angoff (1971) state:

> In mathematical material the test has moved away from the curriculum-oriented type of item to items that depend more heavily on logical reasoning and on the perception of mathematical relationships (p. 16).

Possibly, the discrepancy between Schwebel's (1973) finding and that of the present study is related to the different sex ratios in the two samples examined. His sample contained 51 percent male subjects, but the sample with SAT scores available in the present study contained no male subjects. However, since Schwebel (1973) did not report data on the relation between cognitive development and SAT scores by sex, an explanation of the above disagreement in results must await further research.

In their sample of college freshmen, McKinnon and Renner (1971) found no correlation between scores on a PTI and scores on the ACT. Comparison of their result and conclusion 6 is difficult since reviews of the ACT and SAT suggest that the two examinations should not be considered equivalent measures of general mathematical ability (Wallace, 1972).

In summary, the principal conclusions of this study are that a substantial number of prospective elementary-school teachers at The University of Texas at Austin were found to be concrete operational, and their ability in mathematics was related to levels of cognitive development.

The results of this study suggest that

1. It is inappropriate to assume that most high-school and college mathematics students have attained a level of cognitive development that

justifies treating them as abstract verbal learners. Apparently many persons pass through adolescence without attaining the stage of formal operations.

2. The mathematical experiences provided for prospective elementary-school teachers should be examined to ensure that the content, methods, and achievement expectations are appropriate for the students' stages of cognitive development.

3. If it is assumed desirable that all students eventually attain the stage of formal operations to some degree, the mathematical experiences provided for prospective elementary-school teachers should aid and encourage the transition from concrete to formal operations. Indeed, all teachers whose students have reached adolescence should strive to effect this transition. The studies of McKinnon and Renner (1971) and Keasey (1972) were such attempts.

4. Teachers should ask questions on their tests that require explanations by the students and thereby furnish some written indication of the level of thought processes used. The information obtained may not be suitable for evaluating achievement, but it would be a useful diagnostic tool. Such questions would also encourage the student to reflect on his own thoughts.

The results of this study have provided empirical evidence that what was perhaps a suspicion among mathematics teachers of prospective elementary-school teachers is indeed a fact; that is, many of these students are not formal operational. Also, it was found that the cognitive development of these students is very likely related to their ability in mathematics. It would be premature and ungrounded, however, to conclude that these results account for most of these students' difficulties with mathematics. Learning mathematics is a complex process and involves more than can be explained solely by the developmental theory of Piaget. Only further research in this area, research fusing the most relevant aspects of all learning-related theories, can provide needed insight and answers.

TABLE 1

Stage of Development Scale

PTI Score	3,4	5,6,7	8,9,10	11,12
Stage	II-A	II-B	III-A	III-B

TABLE 2

Stages of Cognitive Development
in Three Groups of College Students
Observed Frequencies

	Prospective Elementary-School Teachers	Secondary-School Mathematics Student Teachers	Honors Calculus Students
Stage II-A or II-B	74	1	0
Stage III-A	60	9	4
Stage III-B	7	9	7

Chi-Square Analysis

Chi-Square = 57.33	N=171 with D.F. = 4	$P < .001$

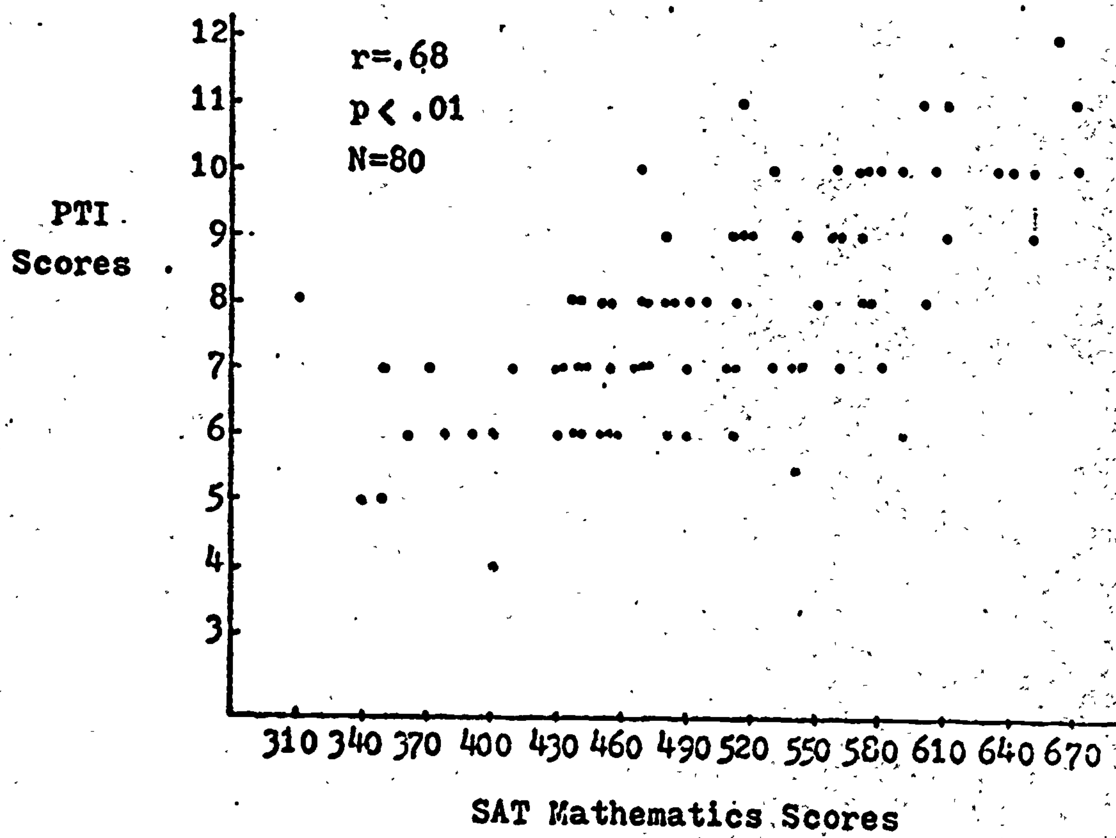
r=.68
p < .01
N=80
PTI Scores
12
11
10
9
8
7
6
5
4
3
310 340 370 400 430 460 490 520 550 580 610 640 670
SAT Mathematics Scores

Figure 1

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