DOCUMENT RESUME

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ABSTRACT

Two processes involved in reading mathematics are discussed in this document: eye movements and relating the mathematical idea to the appropriate word or symbol. Many kinds of eye movements are used in mathematics--around, top-to-bottom, bottom-to-top, diagonal, backward and forward, and follow the arrow. Examples of each kind are presented. The language of mathematics uses many kinds of words--those that are primarily mathematics words, words of everyday usage, and words that have several different meanings, some mathematical and some not. Many examples of each of these kinds of words are presented as incentives for reading and mathematics teachers to cooperatively explore techniques used in reading that may be adapted to mathematics. (TO)

Reading Mathematics

The purpose of this publication is to show how reading is used in teaching mathematics. The publication attempts to explain how eye movements influence the understanding of mathematics—that **many** *eye movements are necessary, not just the left-to-right ones involved in traditional reading.*

It will also explain how preconceived meanings of words in the student's mind must sometimes be overcome in order for mathematical meanings to be substituted.

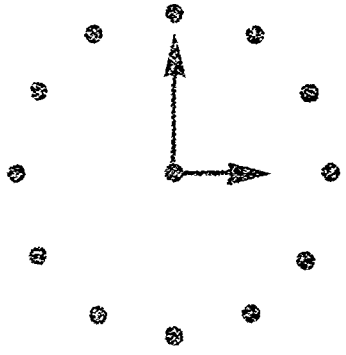
And last but not least, it will encourage teachers of different disciplines to work together in solving problems.

Office of Instructional Services
Georgia Department of Education
Atlanta, Georgia 30334
Jack P Nix, State Superintendent of Schools
1975

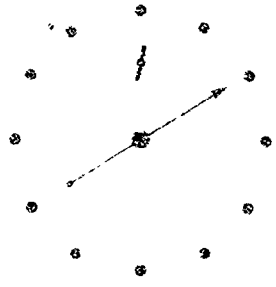
Section

Most of us read everyday, yet few of us are aware of the process involved—of what our eyes do as we read. Do your eyes move only back and forth across a page? Which way do your eyes move when you read a clock? Suppose you are using a stopwatch in experimenting with forces in high school science. What eye movements are used?

What time is it?

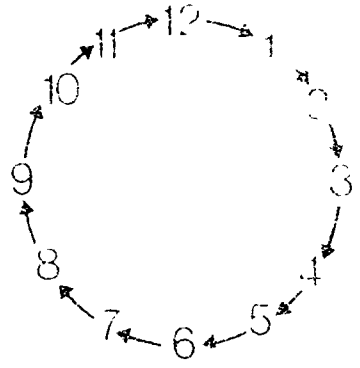


How long did it take?

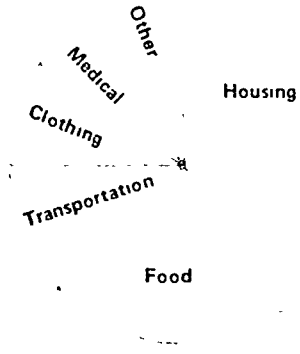


As you study the following examples of possible student activities take time to identify the eye movements involved in reading or performing the activities.

Beginning at nine, move your eyes with the arrows four spaces. What number do you see?

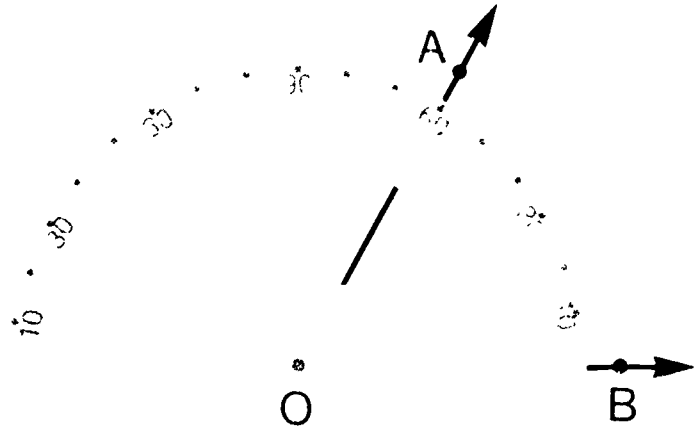


The Smith's spend more money on transportation than on what other expenses?

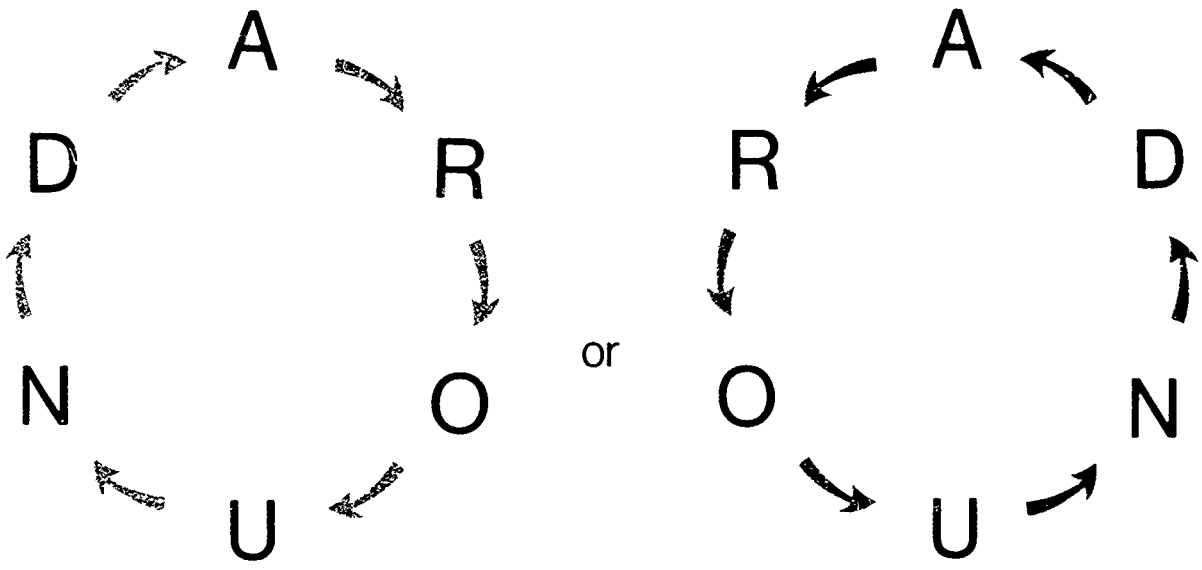


BUDGET

Find the measure of angle AOB.



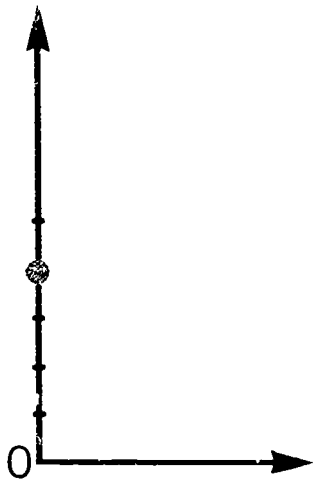
Notice that in the preceding exercises your eyes moved...



Add and check.

$$\begin{array}{rr} 6 & 7 \\ 3 & 9 \\ 2 & 4 \\ \hline & \end{array}$$

How many spaces are between zero and the dot?



Find the product.

$$\begin{array}{r} 43 \\ \times\ 2 \\ \hline \end{array}$$

List the set of fractions equivalent to one-half.

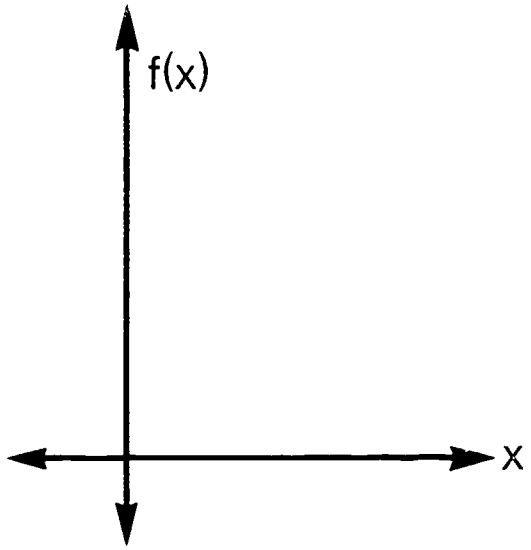
$$\frac{0}{2} \qquad \frac{1}{2} \qquad \frac{2}{2}$$

$$\frac{0}{4} \qquad \frac{1}{4} \qquad \frac{2}{4} \qquad \frac{3}{4} \qquad \frac{4}{4}$$

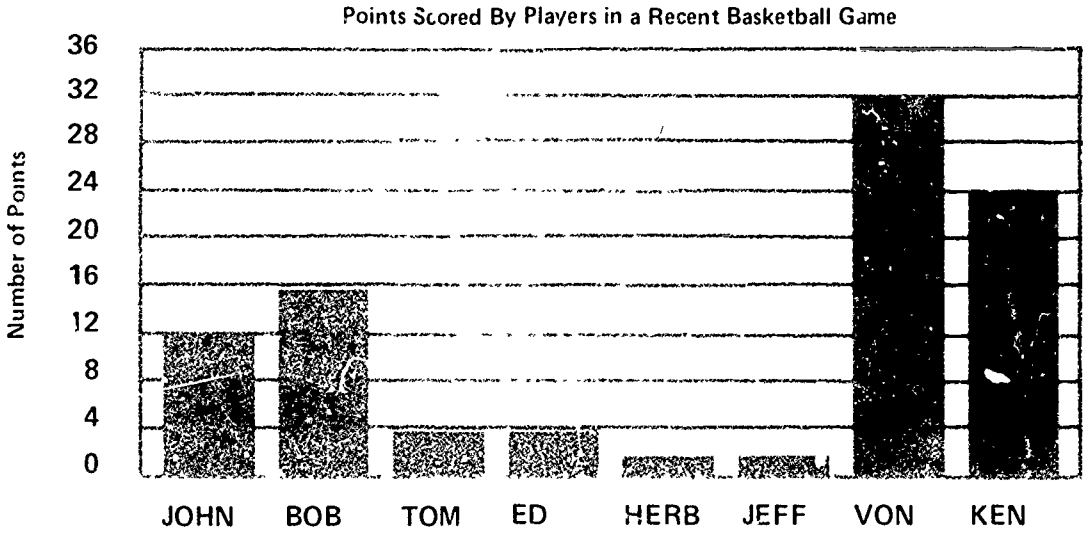
$$\frac{0}{8} \quad \frac{1}{8} \quad \frac{2}{8} \quad \frac{3}{8} \quad \frac{4}{8} \quad \frac{5}{8} \quad \frac{6}{8} \quad \frac{7}{8} \quad \frac{8}{8}$$

Complete the table and graph the function.

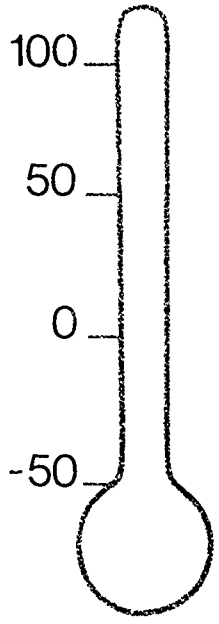
x	f(x)
0	1
1	3
2	5
3	7
4	
5	



How many points did Bob score in the basketball game?

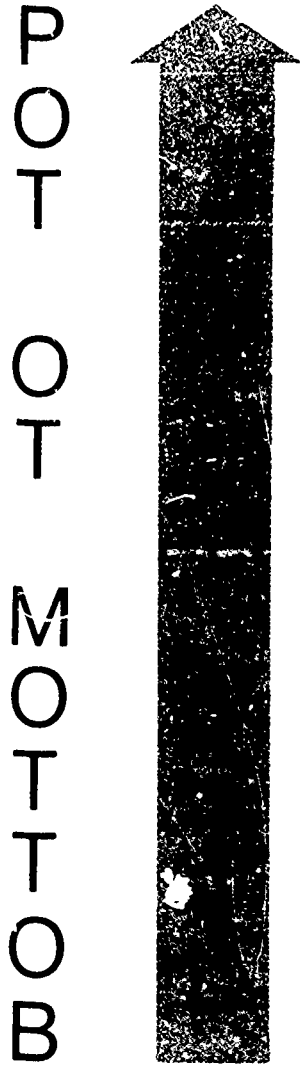


What is the temperature reading on this thermometer?

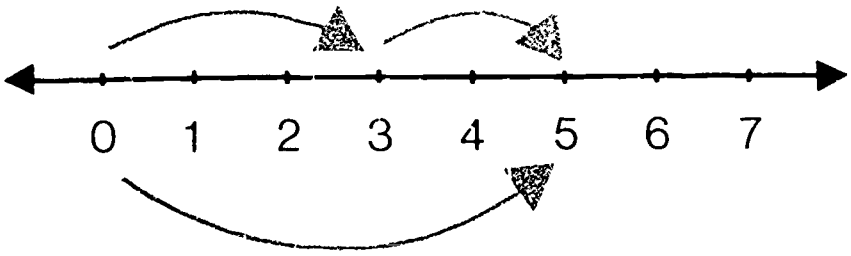


Notice that in the preceding examples your eyes moved...

or



Using the number line, find the sum of three and two



Numerator → 3

Denominator → 4

---

was born in
the month of

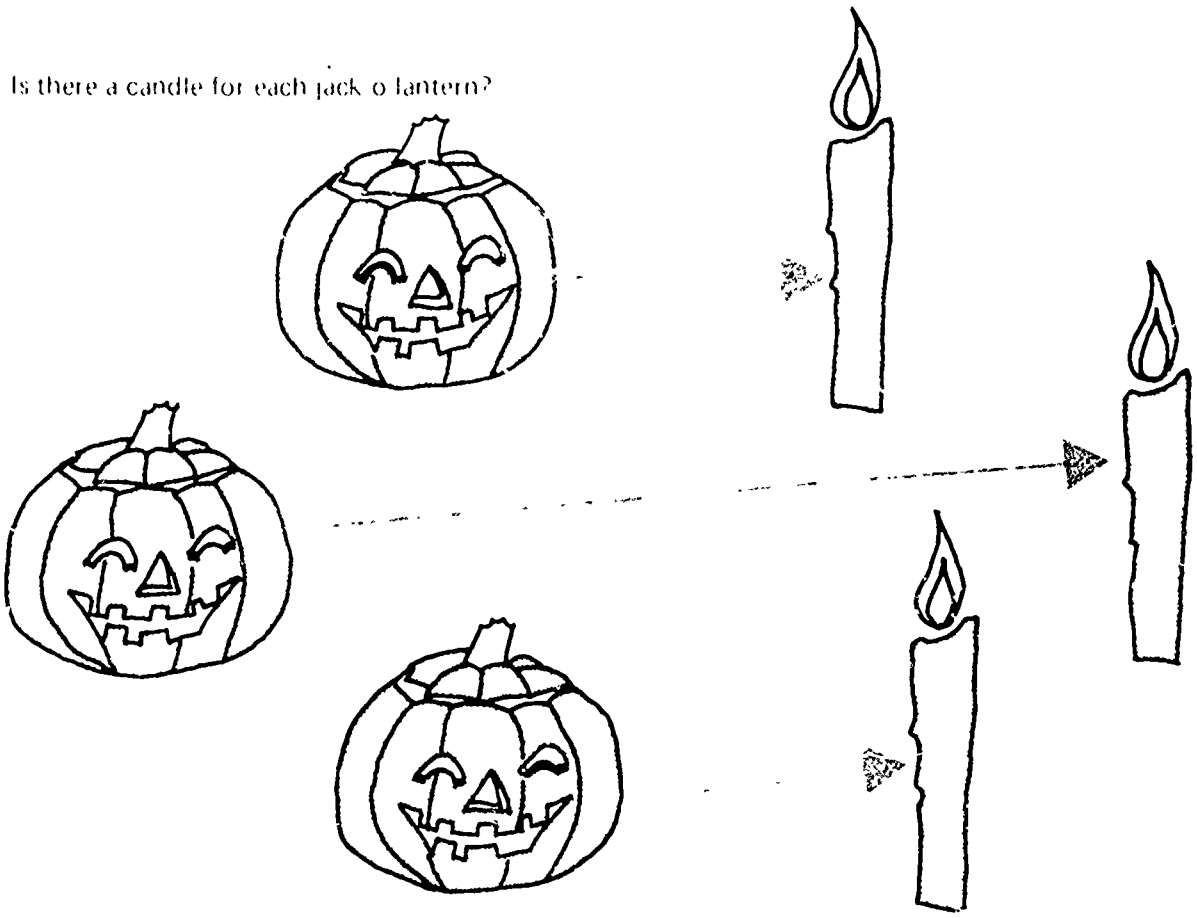
Jan
Bill → August
Joe
Gladys → September
Ann

---

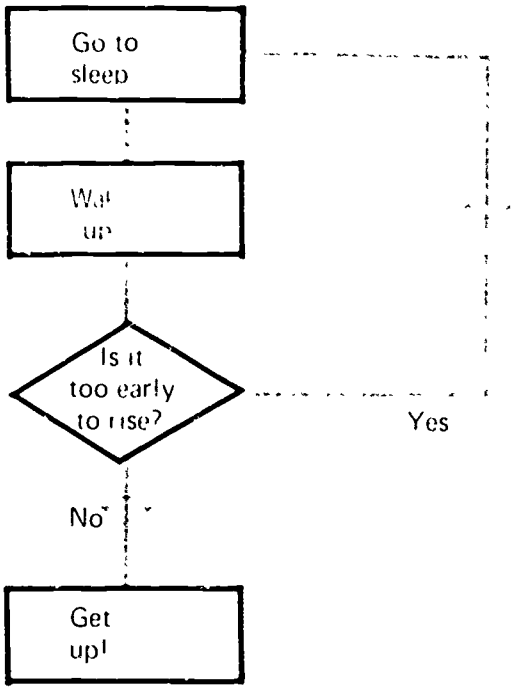
factor factor product

4 x 7 = 28

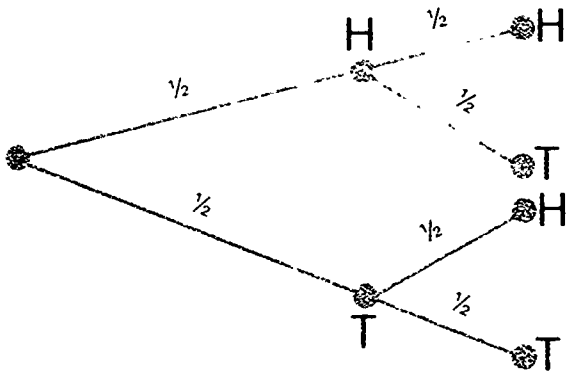
Is there a candle for each jack o lantern?



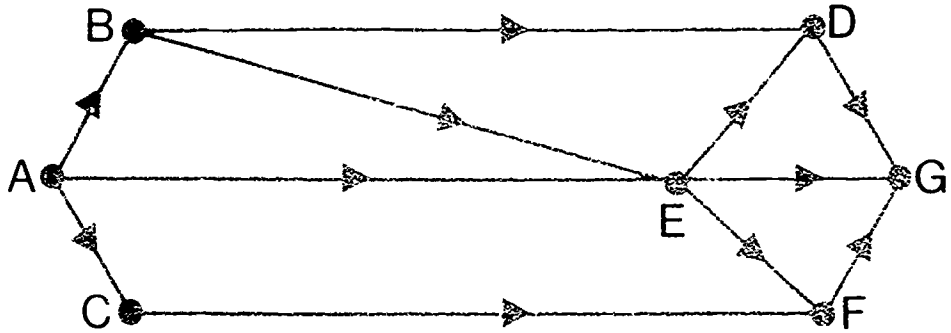
Read the flow chart.



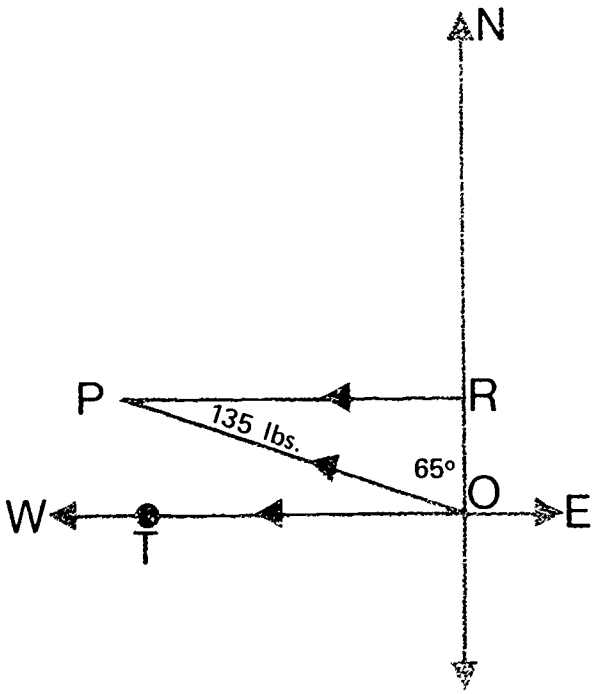
Using the diagram, determine the probability of a head followed by a tail after two coin tosses.



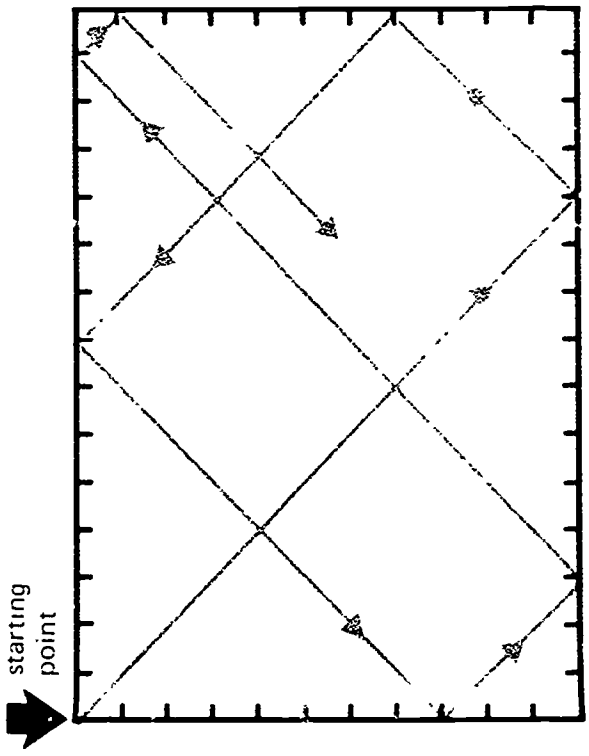
Determine how many different paths there are from A to D.



Determine vertical and horizontal components of the force illustrated in the diagram below.



The idealized path of a ball on a billiard table if the ball is projected at 45° is indicated below.



Notice that in the preceding exercises your eyes sometimes moved to...

F·O·L·L·O·W·T·H·E·A·R·R·O·W

Write as a fractional number.

$4\frac{3}{4}$

Find the quotient.

$2\overline{)8}$

Write as a product.

3^2

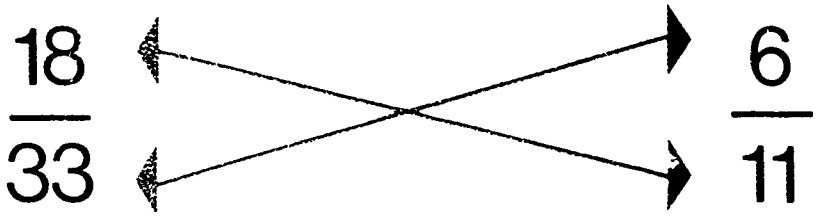
Write this base seven number in base ten.

24_{seven}

Find 'x' in this table.

X	1	2	3	4	5	···
1	1	2	3	4	5	
2	2	4	6	8	10	
3	3	6	9	12	15	
4	4	8	12	16	20	
5	5	10	15	20	25	
⋮						

Are the means equal to the extremes?



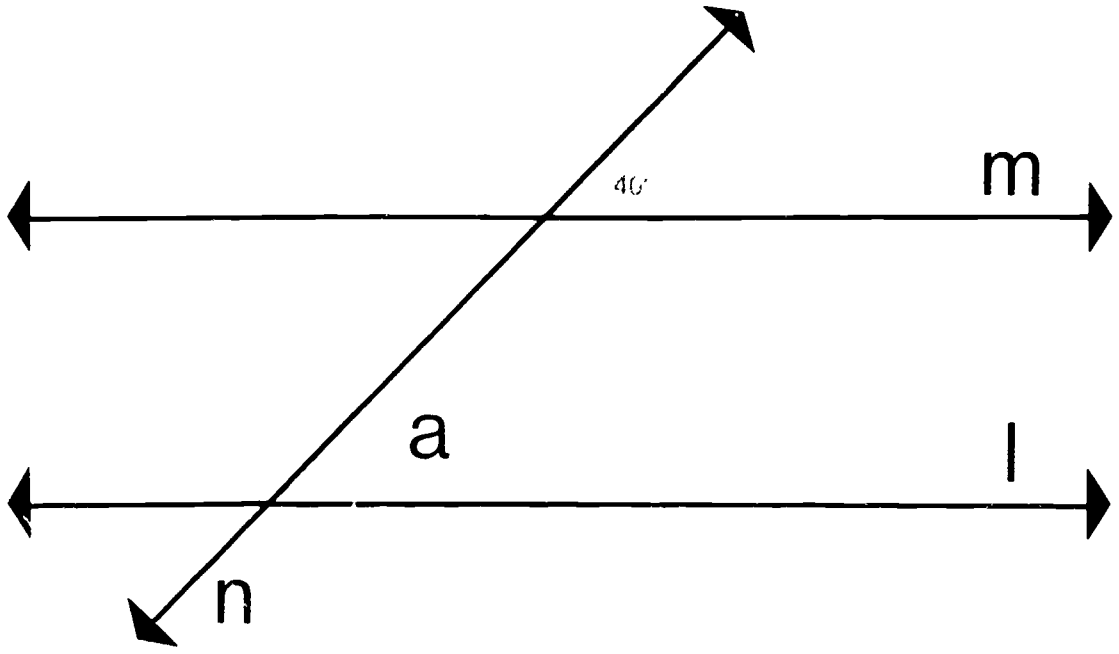
$18 \times 11 = ?$

$33 \times 6 = ?$

Is this an Abelian group?

	0	1	2	3
0	0	1	2	3
1	1	0	3	2
2	2	3	0	1
3	3	2	1	0

How many degrees in angle a?



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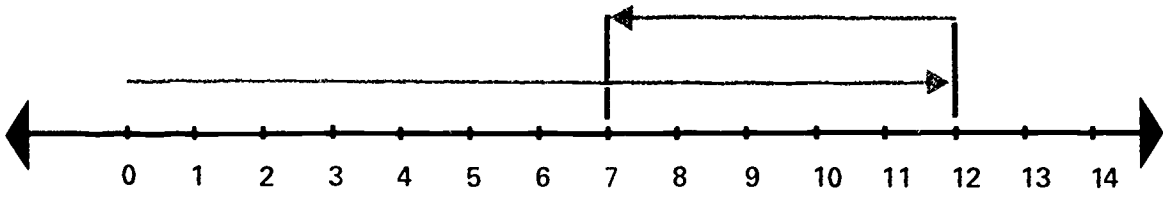
Notice that in the preceding exercises your eyes moved...

Put in the commas, then read aloud the number.

1 0 0 0 0 0 0

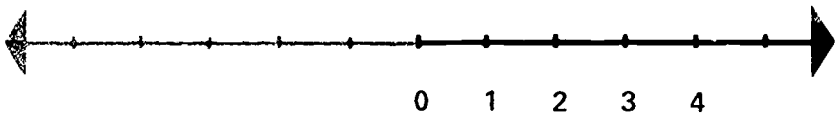
Use the number line to find 12 - 5.



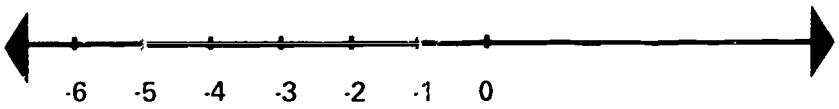
Write the words describing how you would read this problem.

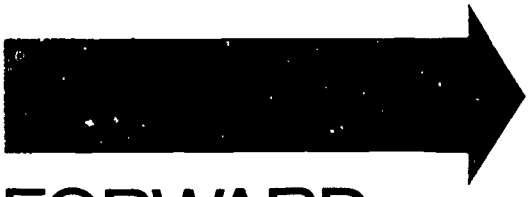
$$2\overline{)10}$$

Write -2, -3, -1 on the number line.

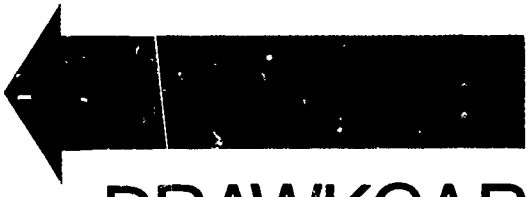


How many units are shown from -1 to -5?





FORWARD



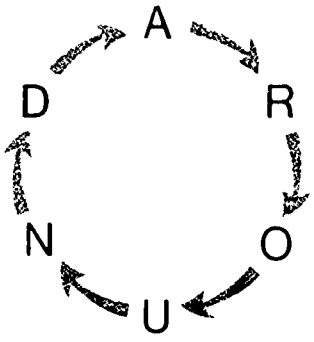
DRAWKCAB

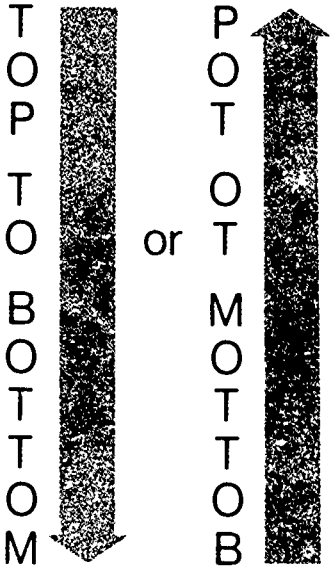
And so . . .

We read—and reread—mathematics . . .

LEFT-TO-RIGHT, sometimes,

but other times we read—





F O L L O W T H E A R R O W

You have experienced many kinds of eye movements used in mathematics . . . around, top to bottom, bottom to top, diagonal, backwards and forwards, and follow the arrow. Reading mathematics is not a simple process

Reading teachers in the primary grades spend much time helping children learn to follow the left to right ordering of words in ordinary reading Children need guidance in using appropriate eye move ments for varied reading situations.

Actually, any reader needs some explanation when he attempts to read words or symbols arranged in an unfamiliar way.

A fourth grade student needs to "sense" the ordering of $\frac{2}{3}$.

A high school student needs guidance in following the written form of matrix multiplication.

$$\begin{pmatrix} 3 & 5 & 2 \\ 2 & 3 & 5 \\ 5 & 2 & 3 \end{pmatrix} \times \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix} = \begin{pmatrix} (3\times1) + (5\times4) + (2\times7) & & \\ & & \\ & & \end{pmatrix}$$

Whatever the topic being taught in mathematics, there is probably some form of word or symbol ordering that could confuse some student.

So be prepared . . .

- to talk about the ordering of symbols and words with your students
- and be sensitive to difficulties.

Eye movement is one process involved in reading mathematics. Another important process is relating the mathematical idea to the appropriate word or symbol; written or spoken.

The language of mathematics uses many kinds of words.

- There are words that are primarily mathematics words.

- There are words of everyday usage.

  There are three boats docked at the wharf.

  How many plates are in the set of dishes?

- There are words that have several different meanings—some may be mathematical, some not.

  foot

  plane

  volume

  set

Have you ever thought about the conflicting ideas that some words communicate?

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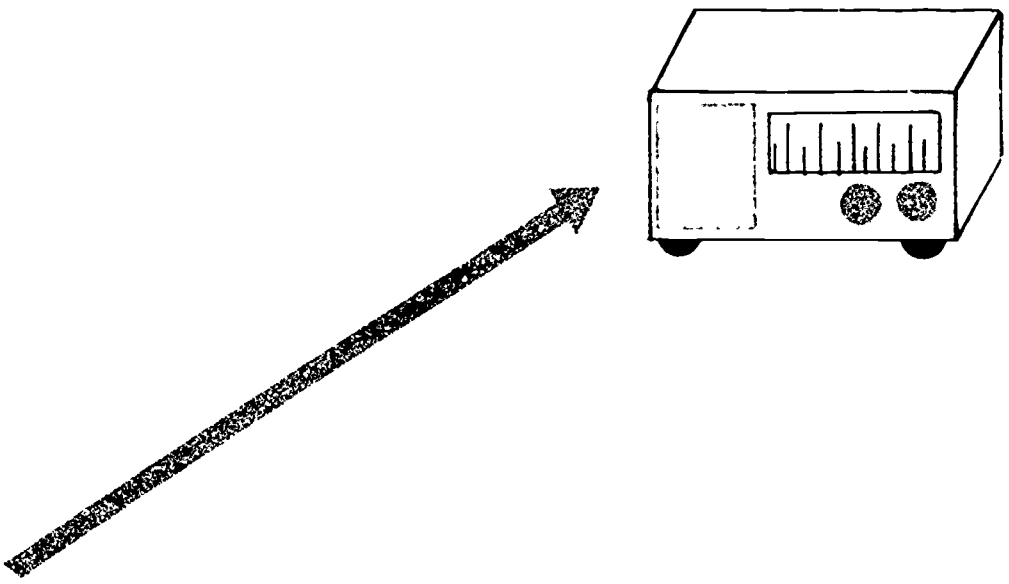
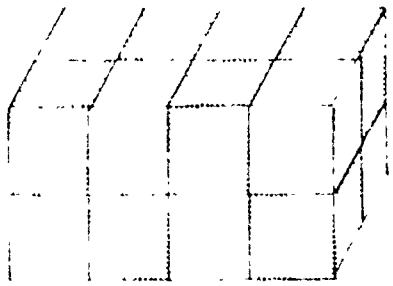
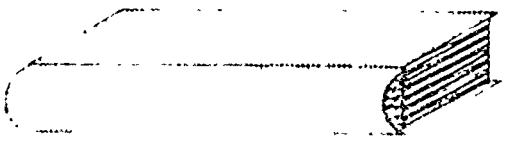


VOLUME

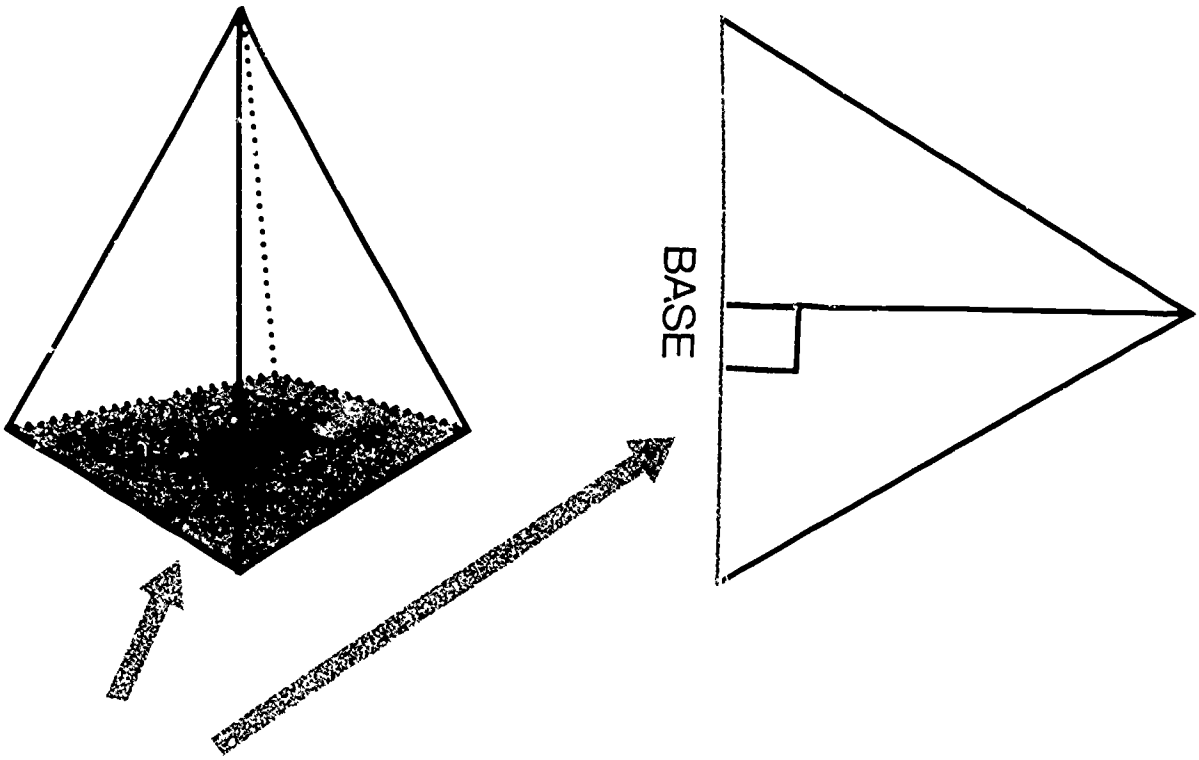




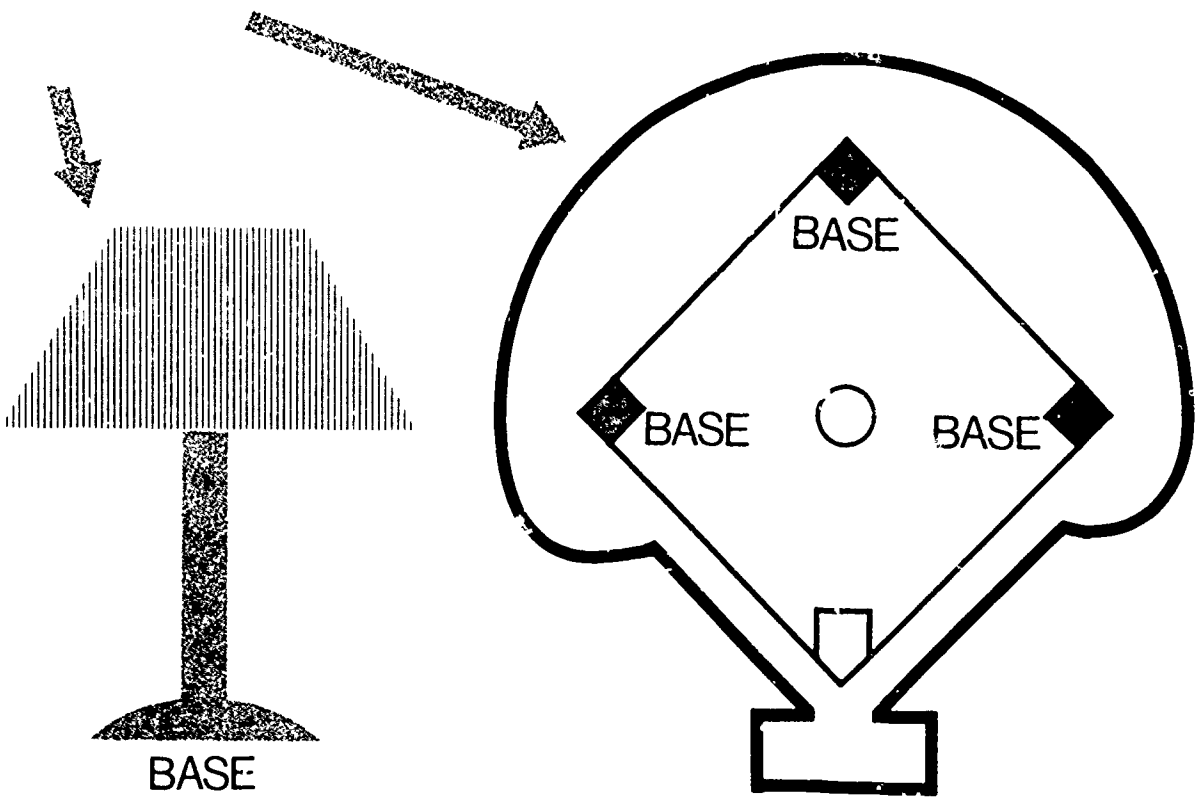
TIMES

X	1	2	3	...
1	1	2	3	
2	2	4	6	
3	3	6	9	
⋮				

25



BASE



Thou shalt not . . .

RULE

$$a \times b = b \times a$$

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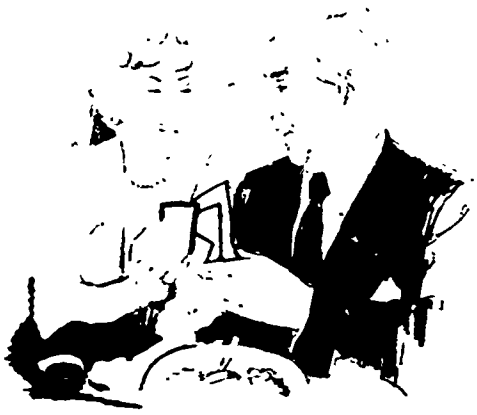


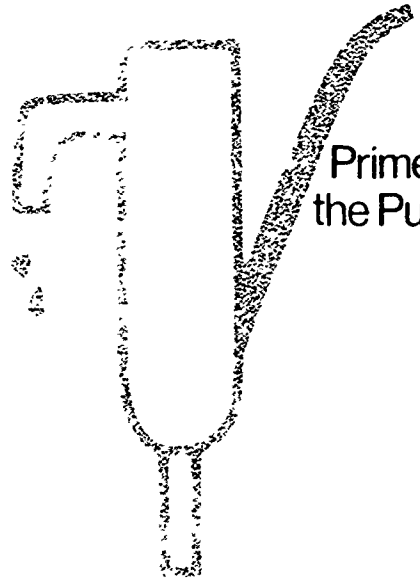
Prime of Life

Prime Beef

PRIME

5

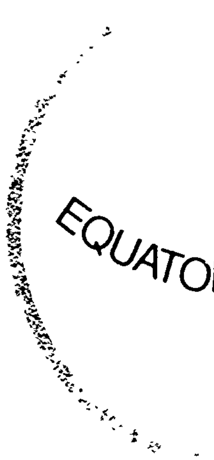
Prime Number



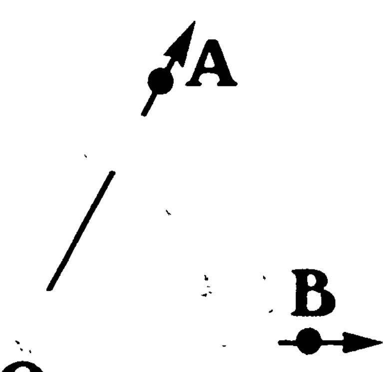
Prime the Pump

Prime Time

DEGREES



$$a^2 + ab + b^2$$



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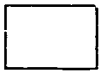
Many everyday words have several different meanings, for example, **prime** rib and **prime** the pump Mathematics is not unique in using words that have several different meanings.

But there IS something special about mathematics. Words with several meanings may be especially confusing to the mathematics student.

This confusion occurs because of the way our minds work. Our minds tend to recall what is most familiar and most recently used A student spends a small part of his day in mathematics class. Perhaps this is the reason he so often hears or reads a mathematics word but thinks of a non-mathematical meaning. For example:

like fractions		"Bleh! I hate fractions!"
foot	.	"The bottom of a leg."
absolute error	.	"A really wrong mistake."
volume	·	"The sound level of a radio."

Most ideas require more than one word or symbol to be communicated. In fact, many words and symbols must be used within a context to be understood.

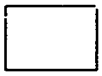


I caramel popcorn.

Find the fractions with denominators. $\frac{4}{5}, \frac{2}{3}, \frac{2}{5}$

Combine terms. $4x + 2y - 3x$

. . wow!



Wait I have an idea.

Super bad

8 2

2
3

4

.33

Consider the word

It's an oft-repeated word in everyday usage.

Bread butter . Let's make a decision then begin to work .

occurs throughout mathematics with varied meanings.

The sum of four three is seven The quotient of eight two is four . . . Tad has three bags of marbles there are five marbles in each bag. If Tad counts all his marbles he has 15 marbles. . . Vivian has 12 records Jody has nine records How many more records does Vivian have than Jody?

Did you notice how was used in an addition situation, a division situation, a multiplication situation and a subtraction situation?

has a precise mathematical meaning in

logic If p is true q is false, then (p q) is false.

set theory If $x \epsilon A$ $x \epsilon B$, then
$x \epsilon A$ B

algebra Find the integers such that $x < 7$ $x > -2$

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A word can communicate many different ideas. Context is often necessary to specify a particular meaning.

word → idea$_1$, idea$_2$, idea$_3$

But then again . . .

an idea can also be expressed with different words or symbols.

idea → word$_1$, word$_2$, symbol$_1$, symbol$_2$

EXAMPLES

multiplication	multiply 6 by 2 6 times 2 the product of 6 and 2 6 x 2 6 (2) 6 · 2
associativity	the associative property the associative principle the associative law the associative rule 2+ (4+5) = (2+4) +5 (ab) c=a (bc)

Hopefully, most students learn to cope with the relations between ideas and symbols. But some students become confused when the directions change from

Multiply	to	Do these multiplication problems
Use the associative property	to	Use the associative principle
$17 + 4 =$	to	$\begin{array}{r} 17 \\ \underline{+4} \end{array}$

It takes only a moment and a few words to help communication.

> "You know what a table is in your house, but now we are talking about a different kind of a table, an addition table."

> "The associative principle is sometimes called a rule, law or property."

Much activity goes on in the mind when "doing" mathematics. There is interplay among ideas, words, symbols and images.

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Section III

Perhaps one of the major goals of education is to help people develop a facility for solving real-world problems.

That is, given a situation, a person must first state the problem, then dip into a reservoir of tools to find the mathematics which best models the problem. The solution must then be evaluated in terms of its appropriateness to the situation. By a mathematical model we mean a statement of the problem in terms of mathematics.

An example of a potential mathematical situation might be that a class has decided to improve the attractiveness of its school yard.

Many problems could evolve from this situation. Here are a few possibilities.

1. Draw to scale the school yard as it is now.
   (Do you see that many decisions are subsumed in this statement of the problem? For instance, what area of the school yard should be considered? What is an efficient way to measure the part of the school yard under consideration? What unit will be used to draw the model to scale?)

2. Using the scale drawing of (1), redesign the existing yard.

3. Determine how many new plants should be bought.
   (Assume that seven is the number of shrubs that should be included in the new design and three is the number of healthy shrubs. One example of a mathematical model for this situation might be: $7-3=[\ ]$.)

4. What will be the total cost of redesigning the school yard?
   (Here the mathematical model would be an addition statement.)

5. How much money would the class have to raise to complete the school yard?

   (Here the model might be $[\ ]-\ .=$

   where represents the total cost, represents the amount of money on hand and represents the amount needed.)

Teaching mathematical modeling is not an easy task. However, we feel that students at all levels should experience many situations which lead to mathematical modeling.

We have tried in these pages to have you experience reading and doing mathematics. We considered the processes of eye movements and mental relationships used when reading mathematics We looked carefully and extensively at eye movement, vocabulary and problem solving.

We did not attempt to show the scope of the questions, suggested techniques and skills concerning reading mathematics. We did not give as many techniques and activities for helping students develop the eye movement and vocabulary appropriate for reading mathematics as you may have liked. We did not survey the diverse tools and methods of problem solving.

However, we hope you now are more aware of what is involved in reading mathematics. It is a complex process.

We hope these pages will be an incentive for reading teachers and mathematics teachers to cooperatively explore techniques used in reading and how they may be adapted in mathematics. If you are a teacher in a self-contained classroom, we hope there will be more interplay between what goes on in your reading time activity and mathematics time activity.

This is a beginning . . .

Resource List

Many times actual situations with which we are confronted require tools from the physical and social sciences in addition to—or rather than—mathematical tools. Consequently, cooperation among teachers of every academic area and at all levels of instruction becomes necessary.

A wealth of ideas for solving real life problems can be found in the following sources.

Biggs, Edith E., and MacLean, James R., *Freedom to Learn. An Active Learning Approach to Mathematics.* Reading, Mass.: Addison-Wesley Co., 1969.

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Kidd, Kenneth P., *et al, The Laboratory Approach to Mathematics* Chicago. Science Research Associates, Inc., 1970.

Mathematics for Georgia Schools, Vols. I and II. Atlanta. Georgia Department of Education.

Mosteller, Frederick, *et al, Statistics by Example,* Vols. I IV Reading, Mass.: Addison-Wesley Publishing Co., 1973.

Nuffield Mathematics Project teachers' guides. New York: John Wiley and Sons, Inc.

School Mathematics Study Group, *Secondary School Mathematics.* Pasadena, Cal.. A. C. Vroman, Inc., 1969.

Unified Science and Mathematics for Elementary Schools. Newton, Mass.. Education Development Center.

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