

DOCUMENT RESUME

FD 105 023

UD 015 013

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TITLE A General Economic Framework for Welfare Reform Analysis.
INSTITUTION Rand Corp., Santa Monica, Calif.
SPONS AGENCY Office of Economic Opportunity, Washington, D.C.
REPORT NO R-1346-OEO
PUB DATE Jul 73
NOTE 66p.

EDRS PRICE MF-\$0.76 HC-\$3.32 PLUS POSTAGE
DESCRIPTORS Economic Factors; *Economic Research; Employment Patterns; Family Attitudes; Family Income; Family Resources; *Finance Reform; Financial Policy; *Guaranteed Income; Heads of Households; Labor Force; Models; Policy Formation; *Public Policy; *Welfare

ABSTRACT

An analysis of the potential effects of different types of welfare reforms can proceed in one of two ways. One can explore the effects of a given plan in detail, or one can cull from the set of feasible alternative plans the major changes that these plans will bring about and then analyze the effect of these changes in a general framework. At their most fundamental levels, the evaluations of all alternative income maintenance schemes require accurate measures of family responses to changes in the market wage that household members receive if they choose to work, and the total income at the family's disposal. In the household production model the family is viewed as a firm engaged in the production of basic items of consumption usually called "household commodities". Families are assumed to produce household commodities by combining their own time with purchased goods and services. They obtain these goods and services in exchange for the time they allocate to market work and possibly in exchange for income from other (non-wage) source as well. The workings of this model are illustrated through the derivation of an equation that relates changes in husband's time in the home to changes in "factor prices"--husband's and wife's wages and prices of purchased market goods--and family full wealth. (Author/JM)

ED105023

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PREPARED FOR THE OFFICE OF ECONOMIC OPPORTUNITY

DENNIS N. DeTRAY R-1346-OEO JULY 1973

The research reported herein was performed pursuant to a grant from the Office of Economic Opportunity, Washington, D.C. Reports of The Rand Corporation do not necessarily reflect the opinions or policies of the sponsors of Rand research.

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UD 015013



PREFACE

This report is one of four written under Grant No. 90088-D-72-01 from the Office of Economic Opportunity. At its initiation the objective of the project was to develop and explore a general economic framework to assess the potential effects of various proposed welfare reform measures. When the grant began (July 1972) work was divided into three phases: (1) formulation of a conceptual framework, (2) specification of necessary econometric methodology, and (3) analysis and evaluation of data sources and future data needs. This report grew out of work done during the early phases of the project from July to October 1972. The intent at that time was to direct the general framework presented here toward analysis of several specific welfare reform plans; that is, this report was to serve as an introduction to the conceptual framework on which later work would be based. As the possibility for welfare reform became more remote toward the end of 1972, it was determined to alter the plan for the remaining research from one that emphasized conceptual questions to one that explored differences among existing empirical estimates of the amount of time people supply to market work, a key factor in Congressional debate on welfare reform. Even without the planned analyses of specific welfare programs, this report should still serve as a useful primer for those interested in applying the household production model to areas of family behavior that are likely to be affected by welfare reform, such as fertility, labor supply, or migration. It may also be a useful prelude to the reports written in part under this grant by James P. Smith (see below).

The other three reports are:

DaVanzo, J., D. N. De Tray, and D. Greenberg, *Estimating Labor Supply Responses: A Sensitivity Analysis*, R-1372-OEO, The Rand Corporation, forthcoming.

Smith, J. P., *Family Decisionmaking over the Life Cycle: Some Implications for Estimating Labor Supply*, R-1121-EDA/OEO, The Rand Corporation, forthcoming (a).

Smith, J. P., *Assets and Labor Supply over the Life Cycle*, The
Rand Corporation, forthcoming (b).

SUMMARY

An analysis of the potential effects of different types of welfare reforms can proceed in one of two ways. One can explore the effects of a given plan in detail, or one can cull from the set of feasible alternative plans the *major* changes that these plans will bring about and then analyze the effect of these changes in a general framework. The decision to take one approach over the other rests in part on the extent to which alternative plans require the same basic information to evaluate. At their most fundamental levels, the evaluations of all alternative income maintenance schemes require accurate measures of family responses to changes in two key variables--the market wage that household members receive if they choose to work, and the total income at the family's disposal. This report explores one economic framework for assessing these responses.

In the household production model the family is viewed as a firm engaged in the production of basic items of consumption usually called "household commodities." Families are assumed to produce household commodities by combining their own time with purchased goods and services. They obtain these goods and services in exchange for the time they allocate to market work and possibly in exchange for income from other (nonwage) sources as well. Because of the close link between the production and consumption of household commodities--the processes can, in fact, be considered one and the same--commodities are not traded in the marketplace and thus have no explicit market price. However, since each uses up a certain portion of the household's scarce resources, each has an implicit shadow price that consists of the marginal per unit commodity resource requirements valued at their opportunity cost (market wage, price of market goods and services, and so on).

The workings of this model are illustrated through the derivation of an equation that relates changes in husband's time in the home to changes in "factor prices"--husband's and wife's wages and the prices of purchased market goods--and family full wealth.

The discussion of the household production model and its application and extension by others suggests several key areas that require increased care and attention in future efforts to assess the potential effects of welfare reform. These are:

1. Most individuals affected by welfare reform do not live alone, but in multiperson households. A major hypothesis underlying the household production model is that people's decisions on consumption and work are based not only on their own characteristics but on those of other family members.

2. Household decisions are interrelated at any one point in time and over time. Under these conditions it is often difficult to sort out the direction of causation and the magnitude of the effect of a change in one variable on another. Unless economic models are correctly formulated to account for this joint determination of family behavior, and suitable econometric techniques are used to estimate responses, analysis may yield seriously misleading policy implications.

3. Most measures of wealth or income used in empirical studies--for example, observed assets--are endogenous to household decision-making and therefore may vary systematically over the life cycle in ways unrelated to household full wealth. If other family behavior, say hours worked, also varies systematically over time either for the same reasons that assets vary or for different reasons, we may be falsely attributing causation to an observed relationship between assets and hours worked.

4. Exploring labor supply responses of married women poses especially serious problems, one of which involves estimating the "opportunity cost" of market work. Observed differences between wives in the amount of market work they supply will depend not only on the different market wages that these women face, but on the different values they place on their time in the home as well. Therefore, to make accurate predictions of the response of wives to exogenous changes in their market wages (either actual or potential), one must first be able to estimate their value of time in nonmarket activities.

5. A wife's responses to a change in her wage rate may also vary with family composition. The presence of young children in the home

may make changes in a wife's hours worked in the market highly inelastic with respect to changes in her wage; when her children are older, her change in market hours may be much more responsive.

At a more general level, the model outlined in this report highlights several areas that many researchers have neglected when studying the effects of welfare reform. The most prominent is the lack of attention paid to the long-run effect of these programs. Although the distinction is seldom made explicitly, the assumptions underlying many previous studies of welfare reform imply that their concern lies with the short-run effects of that legislation. Although the short-run effects of welfare reform may be important, one should not use their parameters to gauge long-run effects. Researchers and policymakers alike must recognize the possibility that welfare reform may affect family behavior in ways other than the amount of time people allocate to market work. Fertility, marriage and marital stability, migration, and investments in adult and child education may all be affected by the tax and income effects of an income maintenance program.

ACKNOWLEDGEMENTS

The author would like to thank Larry Bacow, John Cogan, Julie DaVanzo, David Greenberg, Robert Roll, Charles Phelps, and James P. Smith for their helpful comments on earlier versions of this report. Any remaining errors are solely the author's responsibility. David Greenberg is responsible for both the substance and writing of Appendix A.

CONTENTS

PREFACE	11i
SUMMARY	v
ACKNOWLEDGMENTS	1x
 Section	
I. INTRODUCTION	1
II. THE CASE FOR A GENERAL FRAMEWORK	3
III. THE HOUSEHOLD PRODUCTION MODEL	7
IV. EXTENSIONS OF THE MODEL	19
Intertemporal Considerations	19
Environmental Variables	29
Corner Solutions	31
V. RESEARCH IMPLICATIONS	36
 Appendix	
A. TWO HYPOTHETICAL INCOME MAINTENANCE SYSTEMS	41
B. MATHEMATICAL MODEL	48
BIBLIOGRAPHY	55

I. INTRODUCTION

This report outlines and explores a broadly applicable and cohesive framework within which the potential effects of proposed welfare reforms can be studied. Justification for yet another model of how programs such as the 1971 Family Assistance Plan (FAP) will affect the fabric of American family life can be found in two major shortcomings of almost all previous economic research in this area. The first is the relatively narrow scope of most previous studies; many have concentrated exclusively on the so-called labor-leisure decision, with little regard for other important areas of household behavior that may be altered by the introduction of an income maintenance program (IMP).¹ The second concerns the lack of agreement on the potential effect of an IMP even among those studies that have been concerned only with the labor-leisure choice (Cain and Watts, forthcoming; DaVanzo, De Tray, and Greenberg, forthcoming).

With respect to the first of these shortcomings, evidence is beginning to accumulate to support the contention that programs such as FAP could alter much more than simply the amount of time that people want to work. These programs change people's wages and income, which in turn may affect such diverse spheres of family behavior as decisions on family size, marriage, migration, education, and consumption patterns. For example, an often-raised but seldom analyzed example of this is the provision that a household must include a child under 18 to qualify for any form of welfare payment. Indeed, although one of the major criticisms leveled at earlier welfare programs was that they encouraged families to break up or look as if they had broken up (because households with able-bodied males as heads were ineligible for welfare), no attention has been paid to the pro-natalist, pro-family bias of every proposed welfare reform that Congress has considered over the past several years. Clearly, any *general* framework for the

¹See, for example, the studies contained in Cain and Watts (forthcoming); for an exception, see Cain (1972).

analysis of the major effects of welfare reform on both individual families and society as a whole must be capable of dealing with such diverse topics as labor supply and completed fertility. The conceptual framework presented in this report may not tell policymakers whether a pro-natalist welfare reform program is better than ignoring the wants of children born to poor parents, but it should serve as a basis for deriving the information needed to make such a decision.

One reason for the unacceptably wide range of labor-leisure response estimates may be that previous labor supply studies explore labor force behavior in isolation from other family activities. Accounting for the simultaneous nature of household decisionmaking and especially for the complexity of nonmarket family behavior may help to narrow the range of estimated labor supply response parameters.

The report sets out a general framework for welfare reform analysis but stops short of undertaking the research agenda implicit in the discussion. It is intended as a guide for others who might undertake this task. Because of this perspective, the model presented here is more detailed than usual to facilitate future additions to, and alterations of, the underlying framework.

Section II briefly reviews the potential range of IMP programs; in it an attempt is made to justify the development of an economic framework that is applicable to all forms of welfare reform. Section III presents a general statement of the basic tenets of the model and a detailed derivation of its use to explore conceptually family responses to changes in their environments. The basic model has little that wedds it exclusively to problems of welfare reform analysis; it is, rather, a general approach to understanding and predicting family behavior. Its relevance to issues of welfare reform stems from the broad and pervasive potential effect of that legislation on family life both in and outside the market sector. Section IV contains a less rigorous coverage of various extensions of the model. Section V contains a summary of research recommendations. Appendix A, written by David Greenberg, sets out two hypothetical income maintenance programs, and Appendix B presents the details of the mathematical model.

II. THE CASE FOR A GENERAL FRAMEWORK

There are two potentially conflicting goals in the development of a framework for welfare reform analysis. Since there are several candidate programs, the framework should be general enough to analyze the full range of alternative plans; however, the framework should be sufficiently program-specific that the distinguishing features of the different programs are included in the analysis. How seriously these goals conflict depends on the different forms that future welfare programs may take. This section contains outlines of two hypothetical income maintenance programs whose features capture the major competing alternative welfare reform plans.¹ The information necessary to analyze the potential effects of each plan is compared to assess what proportion of that information is general to both plans.

Differences in potential income maintenance plans are often ones of degree rather than direction with at least one important exception. Certain of the programs considered by Congress have relied heavily on the "natural" work incentive inherent in a negative income tax type of program--that is, that earnings rise as market work increases. A second set of programs has discarded this incentive as insufficient and turned toward more direct work requirements. Such requirements and tests would necessitate job creation programs, wage subsidies, and the like.

For analytical purposes the most important features of a negative income tax plan (NIT) are the base level of support and the "tax rate" that the program imposes on earnings of recipients. Together these parameters determine a break-even income above which individuals or families are not eligible for welfare. In a politically feasible negative income tax plan, households may also have to meet other criteria to qualify for welfare payments. The most common of these are that a child reside in the household and that monetary assets be below some

¹These plans were developed by David Greenberg. Appendix A reproduces the relevant parts of Greenberg's work.

minimum level. How a negative income tax changes people's behavior depends on both the number and the form of all these criteria.

The main economic effects of a negative income tax plan are two-fold. The first effect stems from the change in household income that an NIT produces and the second from changes in the market wage rates that welfare recipients will receive if they work. To predict the effect that these changes may have on household behavior we must know:

- (1) How families "spend" increases in income. Classical labor theory predicts that some of this increased income will be used to replace family earnings--that is, family members will work less in the marketplace. However, a change in income may affect many other areas of family behavior including decisions on family size, marriage, and investments in education.
- (2) How families react to a change in the "price" of their time--that is, in their market wage. Again, although economists have traditionally concentrated on the division of time between market work and "leisure," a change in market wages may also affect how families allocate their time outside of work, and through this how many children they have, how they raise their children, and so on.

The major features of an income maintenance plan that incorporates a work requirement (WR) are more varied and less certain than the features of a pure NIT plan. In addition to a negative income tax component for those deemed unemployable, a program including a WR is likely also to include a public employment component, provisions for day care for children of working mothers, and possibly a wage subsidy component. The rationale for these features is that when a plan requires able-bodied people to work if they are to receive benefits, and there is unemployment in the private sector, it is likely that jobs will be made available through the public sector. However, to keep this public employment program as small as possible, steps would have to be taken to ensure that people would, whenever possible,

choose private employment over public employment--hence the need for a subsidy on wages earned in the private sector. It is also important to note that if policymakers are confident of their ability to separate employable individuals from nonemployable individuals, they may be much less concerned with the work disincentives of a high base level of support or a high marginal tax rate for the NIT part of the plan.

Ignoring the NIT component of this type of program, information is still needed on how a family spends an exogenous increase in its income, even though under a WR plan a household may *not* use the increased income to reduce market work. If policymakers are less concerned with work disincentives under a WR plan, then break-even incomes may be higher under that plan than under a pure NIT program. Therefore, assessing income effects accurately is more important under a WR plan than under a pure NIT plan since the net income transfer is apt to be highest for that type of program.

The second set of information required to assess the effect of an NIT plan concerns a family's reaction to changes in family members' market wages. Although it is less clear than in the case of income effects, the same set of information implied by point (2) above may be needed to assess the effects of a WR plan even though such a plan requires that at least certain family members work. The reason for this is the incorporation of a public employment program and a wage subsidy component into the WR plan. To predict the public employment needs or to set the level of wage subsidy will require the same basic response information as will estimates of work reduction brought about by an NIT plan. Coordinating the various components of the WR plan would be extremely difficult without accurate assessment of these effects, so they should significantly reduce transition costs stemming from the implementation of a WR plan.

To this point I have stressed the similarity of information required to analyze a pure NIT and a WR welfare reform plan; however, a work requirement provision does raise analytical problems separate and distinct from those encountered with a pure NIT plan. The first involves assessing the effectiveness of the policing mechanism for the work requirement. The *potential* market work disincentives of a

plan that incorporates a work requirement are apt to be much larger than those for a pure NIT plan because policymakers will be much less concerned with these disincentives given the work requirement.¹ If the work requirement provision of the WR plan were ineffectively policed, the reduction in market work brought about by a WR might be considerably larger than that of the pure NIT plan. The extent to which this is not the case will depend on the ability of the government to separate employable individuals from nonemployable individuals and to insure that employable individuals do in fact work. How difficult this is depends on how great the incentives are for welfare recipients to avoid the work requirement.

A second, and equally difficult, problem involves the public employment provision of the WR plan. The supply of individuals to a public employment program will depend on a complex set of interactions among the work requirements provision, the wage subsidy and day care provisions, and the characteristics of the public employment program itself. Some of this is general information--for example, information on the response of people's labor supply to changes in their wage rates. A significant portion is, however, unique to the public employment program. For example, the attractiveness of employment in the private marketplace relative to employment in a public employment program will depend on differences in wages, types of work, commuting distances to each type of work, job security, and so on.²

To develop an analytical framework that would supply information on all the above points is beyond the scope of this study. I have chosen to concentrate on the broader set of issues common to both types of programs. These include assessing the potential effect of wage changes on the amounts of time that household members allocate to various activities (particularly, but not exclusively, market work), assessing which areas of household behavior are particularly sensitive to changes in income, and attempting to measure how households respond to major changes in such areas as fertility and marriage.

¹See Appendix A for support for this statement.

²See Greenberg (1972) for an elaboration on this point.

III. THE HOUSEHOLD PRODUCTION MODEL¹

In the traditional economic model of the household emphasis is placed on the division of the household member's time between market work and "leisure" and on the amounts of goods and services that households choose to purchase. The model presented in this section differs from this traditional approach in that it recognizes that the word "leisure" may be a poor description of much of that time household members spend outside of market work. One result of this recognition is that the framework can more easily be extended to account for a far broader sphere of choice than simply decisions on which market goods and services to purchase.

In the household production model the family is viewed as a firm engaged in the production of basic items of consumption usually called "household commodities." Families are assumed to produce household commodities by combining their own time with purchased goods and services. They obtain these goods and services in exchange for the time they allocate to market work and possibly in exchange for income from other (nonwage) sources as well. Because of the close link between the production and consumption of household commodities--the processes can, in fact, be considered one and the same--commodities are not traded in the marketplace and thus have no explicit market price. However, since each uses up a certain portion of the household's scarce resources, each has an implicit shadow price that consists of the marginal per unit commodity resource requirements valued at their opportunity cost (market wage, price of market goods and services, and so on).

A simple illustration may be useful. When a couple has a child, that child can be thought of as one "input" into the household commodity from which enjoyment of one's own children flows. This commodity--call it child services for short--will require family

¹The seminal works on this approach are those of Becker (1965), Lancaster (1966), Mincer (1962), and Muth (1966).

expenditures on market goods and services (physician and hospital fees, for example) and with the birth of a child, it will require significant inputs of parent time, most often that of the mother. How much of this household commodity a family wants will depend on how expensive a unit of child services is relative to other commodities; this in turn will depend on the price of market goods and services that enter child services production, the price of family members' time, and how much of each of these inputs the production of a unit of child services requires. In addition, a family will have some flexibility in determining the relative proportion of each input used in producing its desired level of child services. Some families, particularly those in which the wife faces a high potential or actual market wage, will substitute "hired" time for "own" time by purchasing the services of a nanny or babysitter; in other families, the wife who works may augment her home time by working less in the market.

The core of the model is based on several simplifying assumptions. In particular, the different factors affecting households through time are collapsed into a single measure. An individual's wage, hours of market work, and wealth are implicitly his or her *lifetime* average market wage, market labor supply, and wealth. As demonstrated in Section IV, it is often necessary to relax this particular assumption to avoid misinterpreting analysis of cross-sectional data.

Some of the propositions offered here, especially those having to do with market labor supply, could have been derived from a more traditional labor economics approach. However, in the past the concept of a household commodity has proved a useful vehicle for discussing and analyzing intra-household time allocation as well as a fruitful source of specific hypotheses about family behavior.¹ Therefore, I will develop the model using the household-commodity terminology throughout and assume that the reader will recognize when this is "excess baggage" and when it is not.

After Becker (1965) and Lancaster (1966), I assume the household

¹See De Tray (1972a, 1973), Ghez (1970), Grossman (1972, 1973), Michael (1969), and Willis (1973) for examples.

acts as if it faces a single utility function¹ that has as arguments not market goods and services but more basic items of consumption-- household commodities. The household wishes to maximize

$$U = U(Z_1, Z_2, \dots, Z_n) \quad (1)$$

where U is household utility and the Z_i are consumption levels of various household commodities. A key feature of the Z_i is that they are produced by the household rather than purchased directly. That is:²

$$Z_i = g_i(X_i, M_i, F_i) \quad (2)$$

where g_i = production function for Z_i (assumed here to be homogeneous of degree 1).

X_i = input of purchased goods and services (X) into the i th production process.³

M_i = input of husband's time (M) into the i th production process.

F_i = input of wife's time (F) into the i th production process.

Households are constrained in their activities in at least two ways and must therefore exercise choice over the consumption levels of different Z . First, as in classical consumer demand theory, families cannot spend more on purchasing market goods and services than

¹One might think of this function as the result of unobserved and possibly implicit bargaining among family members on household priorities *prior* to decisions on labor supply, investments, and so forth.

²The form of these production functions implies that each production process is independent; that is, joint production is ruled out. For a detailed discussion of this topic, see Grossman (1971). Other factors may also affect these household production functions-- for example, age, education, and the location of the family. The role played by these "environmental" variables can be important (and often complex, see Michael, 1969); their introduction into the model will be postponed to a later section.

³For notational simplicity, I assume that there is only one market good and that it enters all production processes.

their total money income. If both husband and wife work and the price of $X = \sum X_i$ is treated as the numeraire¹ then,

$$X \leq L_m w_m + L_f w_f + V \quad (3)$$

where L_m = market labor supply of husband.

L_f = market labor supply of wife.

V = nonwage-related income.

w_j = market wage of the j th individual ($j = M, F$).

The household is also constrained in the amount of time it has available for market work and other activities. If T is total available time for one individual, then

$$\sum_i M_i + L_m = T \quad (4)$$

$$\sum_i F_i + L_f = T.$$

In other words, by definition, time spent in the labor force plus time spent in household production exhausts total available time.²

Since time enters both constraints, Equations (3) and (4) can be collapsed into a "full wealth" or total resources constraint:

$$R = X + w_m \sum_i M_i + w_f \sum_i F_i = (w_m + w_f)T + V, \quad (5)$$

where R represents the family's full wealth. Here "wealth" refers not simply to total money income assets but to a broader "potential" wealth concept. Full wealth may be thought of as the sum total of

¹That is, if the per unit price of X is p and the nominal wage rates for husband and wife are W_m and W_f , then $w_m = W_m/p$ and $w_f = W_f/p$, where w_m and w_f are the "real" wage rates of husband and wife. Note also that for simplicity the possibility of several different market goods entering the production functions is excluded in this formulation of the model.

²Lest the reader think that "leisure" has been entirely excluded from the model, the Z_i may include such commodities as "seeing a movie," "a pleasant walk," and the like.

all household resources, including the total value of time, whether or not that time is spent in the marketplace working. The distinction between monetary and full wealth is an important one because money income is not independent of the household allocation process and will vary as family members change the amount of time they spend at market work. The level of full wealth, however, does not vary from period to period except for exogenous changes in the household's economic condition brought about by unanticipated gifts, inheritances, wage changes, catastrophes, and, for our purposes, especially unanticipated changes in welfare laws. Equation (5) indicates that the household "spends" its full wealth in two ways. As in the traditional consumer choice model, part of full wealth is exchanged in the marketplace for purchased goods and services; the remainder is used to "buy" time of different household members from the marketplace, that is, from market work.

The household is assumed to maximize utility (Equation (1)) subject to both the technological constraints implied by Equation (2) and the full wealth constraint, Equation (5). The familiar first order utility maximization conditions are:

$$U_{X_1} = U_{X_2} = \dots = U_{X_n} = \frac{U_{M_1}}{w_m} = \dots = \frac{U_{M_n}}{w_m} = \dots = \frac{U_{F_1}}{w_f} = \dots = \frac{U_{F_n}}{w_f}, \quad (6)$$

where $U_{j_1} = (\partial U / \partial Z_1) (\partial Z_1 / \partial j_1) =$ the "derived" marginal utility of input j ($j = M, F, X$) when used in the production of commodity i ($i = 1, n$). These imply that, for the household to be in equilibrium, the increment to family utility generated by the last dollar spent on each input into each Z_1 must be equal.

We are often interested not just in utility maximization but in the derived demand for time and purchased goods implied by that maximization. For notational convenience, assume that the n household commodities can be collapsed into two composite commodities Z_1 and Z_2 .¹

¹Extending the model to include more commodities adds little of interest except to allow for the possibility of complementarity between some Z s.

By totally differentiating the first order conditions for a utility maximum and solving for changes in input levels (dM_1 , dF_1 , dX_1) in terms of changes in wage rates, one can derive the household's demand for time or goods.¹ To illustrate, changes in the demand for husband's time in the production of Z_1 can be expressed as:

$$\frac{dM_1}{M_1} = \frac{dZ_1}{Z_1} + s_{F_1} \sigma_{M_1 F_1} \frac{dw_f}{w_f} - (s_{F_1} \sigma_{M_1 F_1} + s_{X_1} \sigma_{M_1 X_1}) \frac{dw_m}{w_m}, \quad (7)$$

where $d(i)/(i)$ = percent change in i .

s_{F_1} = share of total cost of Z_1 accounted for by F_1
 ($s_{F_1} = (w_f F_1)/(w_f F_1 + w_m M_1 + X_1)$ with s_{X_1} similarly defined for X_1).

$\sigma_{M_1 F_1}$ = Allen partial elasticity of substitution² between husband's and wife's time in the production of Z_1 , and

$\sigma_{M_1 X_1}$ = Allen partial elasticity of substitution between M_1 and X_1 in the production of Z_1 .

In turn, the change-in-output term $(dZ_1)/(Z_1)$ can be expanded to yield:

¹See Appendix B for details of this and the following derivations.

²Allen (1967, p. 503). The partial elasticity of substitution between two factors of production (a and b) is defined as:

$$\sigma_{ab} = \sigma_{ba} = \frac{\frac{b}{a} \left(\frac{\partial a}{\partial b} \right)}{\frac{MP_a}{MP_b} \left(\frac{\partial (MP_b)}{\partial (MP_a)} \right)}$$

Intuitively one can think of it as a measure of the ease with which one factor of production substitutes for another.

$$\begin{aligned} \frac{dZ_1}{Z_1} = & k_2 \sigma_{1,2} (s_{F_2} - s_{F_1}) \frac{dw_f}{w_f} \\ & + k_2 \sigma_{1,2} (s_{M_2} - s_{M_1}) \frac{dw_m}{w_m} \\ & + \eta_1 \frac{dR}{R}, \end{aligned} \quad (8)$$

where k_2 is the share of full wealth, R , "spent" on Z_2 ; $\sigma_{1,2}$ is the elasticity of substitution between Z_1 and Z_2 in $U(Z_1, Z_2)$; and η_1 is the income (full wealth) elasticity of Z_1 .

Combining Equations (7) and (8) yields the total percentage change in demand for M_1 , given changes in w_m , w_f , and R .

$$\begin{aligned} \frac{dM_1}{M_1} = & [-(s_{F_1} \sigma_{M_1 F_1} + s_{X_1} \sigma_{M_1 X_1}) + k_2 \sigma_{1,2} (s_{M_2} - s_{M_1})] \frac{dw_m}{w_m} \\ & + [s_{F_1} \sigma_{M_1 F_1} + k_2 \sigma_{1,2} (s_{F_2} - s_{F_1})] \frac{dw_f}{w_f} \\ & + \eta_1 \frac{dR}{R}. \end{aligned} \quad (9)$$

Although a similar equation can be derived for M_2 , policymakers have traditionally been concerned not with the allocation of time to one particular family activity but with the allocation of time between market work and all other (nonmarket) activities (M_1 and M_2). If M represents total time spent in commodity production ($M = M_1 + M_2$), then the change in demand for husband's time in the home, $dM/M = (-[L_m/M] \cdot [dL_m/L_m])$, can be written as a weighted average of dM_1/M_1 and dM_2/M_2 ; that is:

$$\frac{dM}{M} = \frac{M_1}{M} \left(\frac{dM_1}{M_1} \right) + \frac{M_2}{M} \left(\frac{dM_2}{M_2} \right). \quad (10)$$

Combining derived demand equations for M_1 and M_2 , and simplifying,

$$\begin{aligned} \frac{dM}{M} = & -[(\sigma_{MF}^* + \sigma_{MX}^*) + \gamma(s_{M_2} - s_{M_1})^2] \frac{dw_m}{w_m} \\ & + [\sigma_{MF}^* - \gamma(s_{F_2} - s_{F_1})(s_{M_2} - s_{M_1})] \frac{dw_f}{w_f} \\ & + \eta_M^* \frac{dR}{R}, \end{aligned} \quad (11)$$

where:

$$\sigma_{MF}^* = \left[\left(\frac{M_1}{M} \right) s_{F_1} \sigma_{M_1 F_1} + \left(\frac{M_2}{M} \right) s_{F_2} \sigma_{M_2 F_2} \right]$$

$$\sigma_{MX}^* = \left[\left(\frac{M_1}{M} \right) s_{X_1} \sigma_{M_1 X_1} + \left(\frac{M_2}{M} \right) s_{X_2} \sigma_{M_2 X_2} \right]$$

$$\gamma = \frac{\sigma_{1,2} k_1 k_2}{k_M}$$

$$\eta_M^* = \frac{M_1}{M} \eta_1 + \frac{M_2}{M} \eta_2$$

and

k_i = fraction of full wealth "spent" on i ($i = Z_1, Z_2, M$).

The first term on the right hand side of Equation (11) states that the effect of, say, an increase in the husband's own wage rate (w_m) will depend on two factors: (1) the elasticity of substitution of husband's time in all household production with the two remaining factors ($\sigma_{MF}^* + \sigma_{MX}^*$) and (2) a substitution in consumption effect, $\gamma(s_{M_2} - s_{M_1})^2$. As is well known, all other factors taken together

must be a substitute for any given factor (Allen, 1967, p. 503) and, thus, the substitution in production term must be positive; the substitution in consumption term is also positive. The net effect, therefore, is unambiguous: An increase in his own wage (holding full wealth constant) will cause the husband to work less in household production and allocate more hours to labor force activities.

The effect of a change in the wife's wage on the allocation of husband's time is given by the second term in Equation (9). Unfortunately, neither component of this term is unambiguous in sign. The elasticity of substitution of husband's time for wife's time in all household production, σ_{MF}^* (again a "weighted" sum of the two individual cross elasticities, $\sigma_{M_1F_1}$ and $\sigma_{M_2F_2}$) is positive or negative as husband's time is, in net, a substitute or complement for wife's time in household production. Although ambiguous in sign, the substitution in consumption term $-\gamma(s_{F_2} - s_{F_1})(s_{M_2} - s_{M_1})$ does have the interesting property that its sign depends on the correlation between the shares of husband's and wife's time in the two production activities. If husband's and wife's time are *both* used most intensively in the *same* commodity, then the substitution in consumption effect tends to increase husband's time allocated to market work when the wife's wage rises; conversely, if the husband's time-intensive commodity is not the wife's time-intensive commodity, this effect will tend to draw husband's time out of the labor force and into commodity production.

The final term in Equation (11) represents the change in demand for husband's time that results from a change in full wealth, R . If no commodity is inferior, an increase in R will induce husbands to work less in the marketplace and more in household production. Since changes in full wealth can arise from several different sources, it is often useful to break R or (dR/R) into its constituent parts. Recall that

$$R = w_m T + w_f T + V \quad (12)$$

measures the household's full wealth. Changes in full wealth can,

therefore, occur from changes in the real wages of either husband or wife, or from a change in nonwage-related income.

Equation (12) seems to imply that the effect of a change in market wages on full wealth depends on total available time of the family members. However, for those hours allocated to nonmarket activities, an increase in market wages has two offsetting effects: On one hand, the value of that time to the household has increased; on the other, the cost of household commodities using that time has risen. It can be shown that these two effects exactly offset each other and, therefore, that an increase in market wages affects a family's full wealth only to the extent that husbands and wives work in the marketplace.¹ Therefore, dR/R can be written as:

$$\frac{dR}{R} = \left(\frac{e_m}{R}\right) \frac{dw_m}{w_m} + \left(\frac{e_f}{R}\right) \frac{dw_f}{w_f} + \left(\frac{V}{R}\right) \frac{dV}{V}, \quad (13)$$

where e_m = lifetime market earnings of husband ($=L_m \cdot w_m$)

e_f = lifetime market earnings of wife ($=L_f \cdot w_f$).

Changes in the demand for husband's time in household production (or equivalently, changes in time supplied to the labor force) stemming from changes in wages, prices, or different components of full wealth are conceptually accounted for by Equation (11).

The advantage of Equation (11) over other "demand" relationships derived from this model (for example, Equations (8) and (9)) is that the dependent variable, dM/M , is readily observable. Because labor economists have concentrated on the division of time between "work" and "leisure," most micro data sets contain some information on hours spent at market work and therefore, as a residual, on hours spent not

¹See Appendix B, p. 48. John Cogan has pointed out that this result is particularly important in empirical applications of this model. It indicates that were full wealth as defined in Equation (12) entered as an independent variable in a labor supply regression, the wage coefficient could no longer be viewed as the effect of a *compensated* change in market wages. For the details of this argument, see Cogan (1973).

working. But, as this model emphasizes, much production (in the utility-yielding sense) may occur outside the marketplace. Since there seems to be no *a priori* reason for assuming that intra-household time allocation is any more or less flexible than market-nonmarket time allocation, one should at least consider the prospect that government policies affecting people's wages and income may substantially change how people use their nonmarket time.

Changes in government policy that alter family members' time value or household income may affect their demand for child services and the inputs that enter child services "production." If the plan under consideration is a pure negative income tax program, then this model would predict the following: Since an NIT will raise household full wealth (R), families will consume more of all commodities that are not inferior; since an NIT will effectively reduce the market wages (w_m and w_f) of program participants who work outside the home,¹ the price of those household commodities that use relatively large amounts of family members' time will fall relative to other commodities. Child services would appear to have both characteristics²-- a normal good and produced through a relatively (wife's) time-intensive process. Since these income and price effects move in the same direction, the potential effects of an NIT on future fertility *could* be quite large, as much as one additional child for every two families with incomes below the poverty line.³

One can further complicate this model of child services production by recognizing that more than *numbers* of offspring may enter that production process. It has been argued elsewhere (De Tray, 1972a,

¹For families currently on welfare, an NIT may leave after-tax wages unchanged or, where current taxes are particularly high, even increase the effective market wage

²See various papers in the *Journal of Political Economy Supplement*, March/April 1973. My own work (De Tray, 1972a, 1973) also supports these contentions.

³Cain (1972, pp. 58-60), using admittedly very crude estimates of the necessary parameters, speculates that a typical income maintenance plan might increase desired fertility in the neighborhood of 15 percent (say from 3.7 to 4.3 children for families below the poverty line).

1973) that parents derive utility from present and future *characteristics* of the children (their expected levels of schooling, earning power, wealth, and so on) as well as from the children themselves. If child characteristics are primarily a function of parent investment in children, then an NIT may alter desired investment in children as well as desired numbers of children. The increase in wealth brought about by an NIT will likely increase the desired level of parent investment in children; however, an NIT may also affect the price of numbers of children relative to investment in children. If investing in children were goods-intensive relative to numbers of children, this latter effect would by itself cause parents to want more children, and invest less in each child. Since the final level of investment in children--whether parent or nonparent--may play a dominant role in determining a child's future economic prospects, these sorts of NIT effects should be seriously considered by policymakers concerned with long range welfare requirements.

This example illustrates that although the framework developed here is adequate for the analysis of market-nonmarket time allocation, its strength lies in its ability to shed light on how government welfare programs may affect nonmarket-related activities. In addition to desired fertility, this model is proving useful for analyzing the effect of wage and income changes on marriage and divorce patterns, on migration, and on the demand for health and education.¹

¹See, for example, Becker (1973), DaVanzo (1972b), and Grossman (1972).

IV. EXTENSIONS OF THE MODEL

To emphasize the basic structure of the model, the preceding section by assumption set aside a number of potentially important features of the household and household production. Among the more relevant of these for present purposes are intertemporal considerations, the effect of environmental variables on the household production process, and the effect of corner solutions (when an individual devotes no time to market work). In this section I consider the effect on the model of dropping these assumptions.¹

INTERTEMPORAL CONSIDERATIONS

How serious a shortcoming the one-period setting of the preceding model is depends on both the question being asked and the available data. Because past micro data sets have seldom been longitudinal, researchers have often turned to cross-sectional data to test hypotheses drawn from a one-period model. When this is the case, a number of interpretational problems may arise, not the least of which involves accounting for so-called life-cycle components of a person's behavior.

General Issues

Since Irving Fisher's classic work on the topic (Fisher, 1930), economists have known that intertemporal phenomena such as interest rates and time preferences will have a systematic effect on the current behavior of individuals. The two main Fisherian results can be summarized as follows. With neutral time preference,² a positive

¹As the reader will note, the following moves to a less rigorous (less mathematical) level than the previous discussion. Since the topics covered in this section have been rigorously introduced into the theory elsewhere, I felt it an unnecessary burden to the reader to introduce each of these concepts formally into the model.

²Formally, time preference is measured by the slope of the family's indifference curve between consumption now and consumption in

interest rate makes future consumption less expensive than current consumption, which causes consumers to reduce current consumption (that is, save) so that they can consume more in the future. Time preference for the present will encourage individuals to consume more now and less in the future. Although economists have been aware of these results for some time, it was not until quite recently that they recognized the full implications for cross-sectional regressions on labor supply.

Rigorous models of life-cycle consumption and labor force behavior can be found in a number of sources (see especially Becker, 1969a; Ghez, 1970; Smith, 1972, forthcoming), and only the main results and implications will be given here. The usual assumptions underlying these models concern the knowledge that consumers have of future wages, prices, wealth, and so on and generally take the form of extending the traditional full information assumption of one-period models to all future periods.¹ The force of the argument is that what people do now depends not only on the current prices, constraints, wages, and so on, but on the past and expected future levels of these variables as well. Technically, consumers are assumed to equate the discounted value of the marginal utility generated by a dollar's expenditure over all periods of his or her expected lifetime as well as over all inputs in any one period. As in the one-period model, this results in derived demand equations for each of the various inputs, but now for *each* period of the life cycle as well.

the future when equal amounts of consumption occur in both periods. For neutral time preference, that slope is -1, meaning that if the cost of a unit of consumption were the same in both periods, the level of consumption would also be the same in both periods.

¹Other assumptions underlying the particular model used here (Smith, forthcoming) are that the parameters of the production and utility functions do not vary over time; that is, the household's ability to substitute consumption from one period to another is the same for any two periods, and substitution in production among the various inputs is the same for all periods in the life cycle. In certain circumstances these assumptions are inappropriate--for example, in the case of the commodity "children." Elaboration of this problem is taken up below (p. 26).

Drawing from Smith's (forthcoming)¹ work on life cycle labor supply in a family context, and under the assumption that the household produces only one composite commodity, the demand for husband's time in commodity production in period t can be written as follows:

$$\frac{dM_t}{M_t} = -(s_{M_t} \sigma_c + s_{F_t} \sigma_{MF} + s_{X_t} \sigma_{MX}) \frac{dw_{Mt}}{w_{Mt}} + s_{F_t} (\sigma_{MF} - \sigma_c) \frac{dw_{ft}}{w_{ft}} + \sigma_c (r - \alpha),$$

where σ_c = intertemporal elasticity of substitution in consumption.²

r = rate of interest.

α = rate of time preference.

This equation can be interpreted similarly to the one-period derived demand function for husband's time in the home (Equation (11)). It implies that the predicted change in demand for husband's time in the home from one period to the next depends on the change in the husband's market wage and the change in the wife's market wage between the two periods, the rate of interest, and the family's time preference for current versus future consumption.³

The effects on husband's time in the home of the two wage terms are similar to those of Equation (11). A rising path for husband's

¹See also Smith (1972).

²The intertemporal elasticity of substitution in consumption is the ease with which families can substitute consumption in one period for consumption in another--that is,

$$\sigma_c = \frac{\frac{Z_i}{Z_j} \frac{dZ_i}{dZ_j}}{\frac{U_i}{U_j} \frac{dU_i}{dU_j}},$$

where Z_i and Z_j are commodity consumption in periods i and j, and U_i and U_j are the marginal utilities of consumption in each period.

³And the given parameters of the utility and production functions underlying this equation. As before, the price of market goods and services has been taken as the numeraire in this model.

wages will induce him to work less now and more in subsequent periods than he would have worked with a constant or falling future wage path. If husband's and wife's time are substitutes in commodity production, then the effect of a change in the wife's future wage will depend on the degree of that substitutability relative to the household's ability to substitute commodity consumption from one period to another-- that is, on the relative magnitudes of σ_{MF} and σ_c . If, for example, families find it relatively easy to substitute husband's time for wife's time in the production of household commodities but relatively difficult to shift commodity consumption from one period to another, then an increase in his wife's wages will induce a husband to withdraw time from the market in order to participate more heavily in non-market activities.

The effect of the two pure time parameters, r and α , is that postulated by Fisher. If the interest rate exceeds the family's time preference for the present, the household will, to the extent possible (given the magnitude of σ_c), substitute away from current consumption in favor of future consumption.

Assets and "Income Effects"

One example of the importance of life-cycle considerations for those interested in estimating the potential effect of welfare reform on market work effort is given in a forthcoming Rand report by Smith. As Smith points out, a frustrating aspect of empirical labor supply studies is the difficulty their authors have in arriving at a supply of labor curve that exhibits either the necessary positive slope even at the lowest end of the wage range or, more crucially, the positive pure substitution effect predicted by economic theory.¹ The blame

¹Put another way, the gross estimated effect of an increase in the observed market wage rate is often a decrease in the number of hours an individual allocates to the market; for the pure substitution effect to be of the right sign, the nonwage-related income effect must be negative and larger in absolute value than the wage coefficient. But this income coefficient is often positive, let alone sufficiently negative to offset the wage term (see, for example, DaVanzo, De Tray, and Greenberg, forthcoming).

for this poor showing is often laid at the feet of those who collected the data, on the data (not enough low-wage people in the sample, for example), or on the systematic effect of differences in tastes across individuals (Graenberg and Kusters, 1970).

Smith argues that the problem may lie in the measure of other income, wealth, or assets that is used to estimate the effect of a change in nonwage-related income on the quantity of labor an individual wants to supply. Nonwage-related income in any period is integrally associated with the level of assets a family holds in that period. Before causation can be attributed to an observed link between assets and labor supply, we must first determine how households arrive at their desired levels of asset holdings.

Smith's simplest case will suffice to illustrate the main thrust of his argument. In the standard Fisherian model of consumption, a consumer with neutral time preference and a constant wage rate over time, and who faces a zero interest rate, will spread his consumption evenly over his lifetime.¹ Since the production-consumption of commodities in the home requires time, a constant commodity consumption path implies a constant derived demand for time in the home and therefore a constant number of hours worked in the marketplace over the life cycle.

If the same consumer now faces a positive interest rate (and still has neutral time preference),² he will reduce current consumption in order to save and consume more in the future. Since a reduction in current consumption also reduces the amount of time required for home production of commodities, at a given wage rate the consumer will now work more in the early periods of his life cycle. The opposite holds for future periods. Increased commodity consumption will increase the demand for time at home and reduce the amount of time spent working at a given wage.

¹This result depends on perfectly functioning capital markets.

²Strictly speaking, as long as the interest rate exceeds the rate of time preference for the present, consumers will have an incentive to save now in order to consume more in the future.

If families start with only wage incomes and wish to have zero assets at the end of their lives, the positive interest rate case discussed above is as illustrated in Fig. 1. Recall that with zero interest rates and neutral time preferences, a person consumes his or her earnings in every period, neither saving nor dissaving, with the level of consumption the same in every period. If this were the case, one could represent the paths of commodity consumption, wage rates, and hours worked in the market by a straight line with zero slope running from the initial time period to the end of life. Contrast this to an individual who faces a positive interest rate (or whose time preference for the present is less than the interest rate). As Fig. 1 illustrates, the desired consumption path rises over time since commodity consumption is cheaper in later periods. Hours of market work and therefore market earnings fall throughout this person's lifetime because the demand for time in the home for commodity production is increasing over time. To accommodate this path, given the constraint of zero beginning and ending assets, the individual saves in early periods of his life cycle and dissaves in later periods. The amount of savings in any period is the difference between total income (earnings from market work plus interest income on prior accumulated savings, if any) and commodity consumption expenditures in that period. Since net assets are simply accumulated savings or dissavings, they, too, will rise in early stages of the life cycle and fall during later periods as the middle diagram in the series indicates.

These diagrams illustrate that two individuals with the same real full wealth (no other assets at the beginning of the life cycle and the same wage path) may exhibit very different asset-labor supply relationships.¹ Furthermore, the exact nature of this relationship will

¹Smith, in his other examples, traces through the effect on desired asset levels and work effort over the life cycle of differences in the level of wage paths, differences in the rate of change of wages over time, and different starting asset (initial wealth) positions; in each case he shows that the observed asset-labor supply relationship depends, among other things, on where in their life cycle people are.

CC = Consumption
EE = Earnings
II = Total income

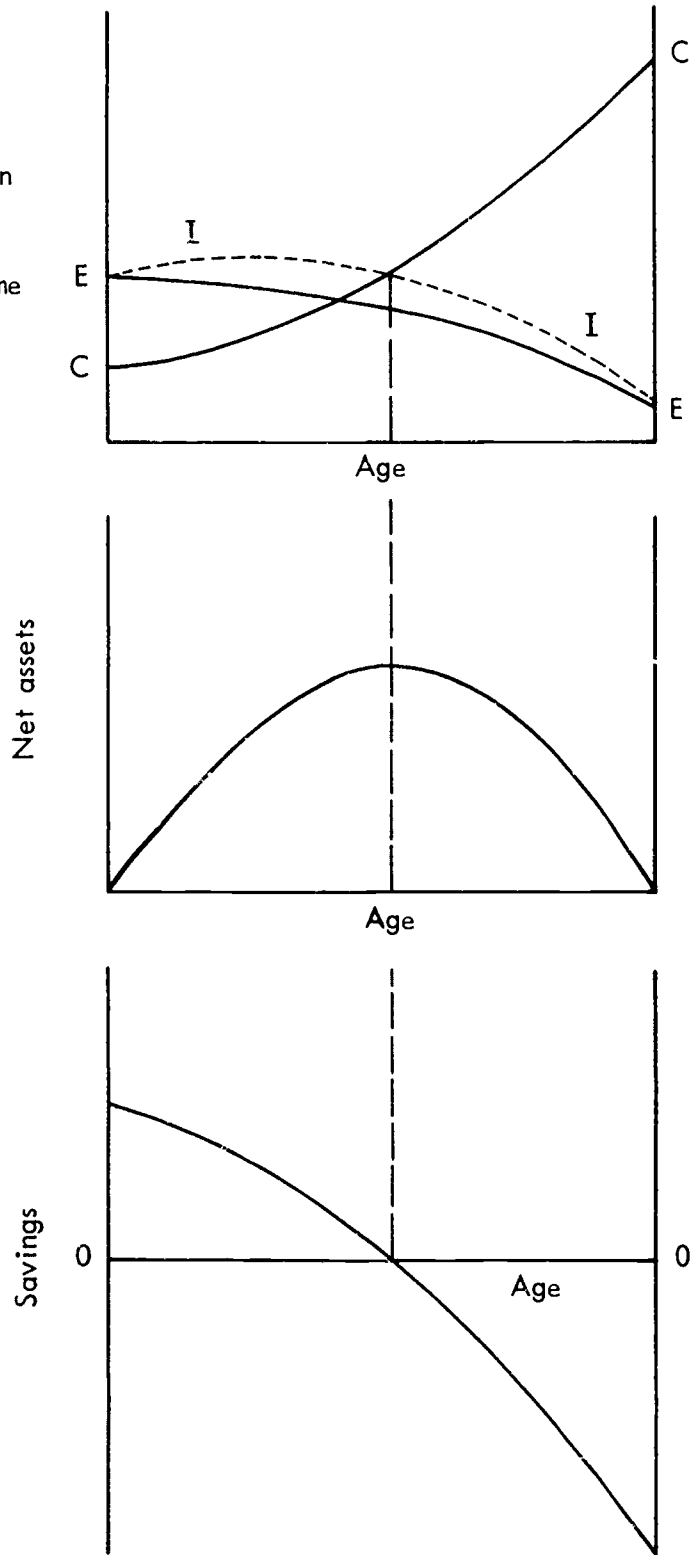


Fig.1 — Assets and labor supply

Source: Smith (forthcoming)



vary considerably depending on where each individual is in his life cycle. To illustrate, assume that the economy consists of two single-person households that differ only in their innate riskiness (one facing a zero interest rate, the other a positive interest rate, with all other characteristics the same), and that these two individuals are both the same age. If both were nearing the end of their (expected) lifetime, and nothing was known about their life-cycle histories, the traditional theoretical conclusion would be reached: Other things equal, the more nonwage-related income an individual has the less time he supplies to market work. However, if these same individuals were at relatively early stages in their life cycles, just the opposite would hold, since the individual with the positive level of assets would be working more than the person with zero assets.

As negative as this conclusion may seem for the future of labor supply studies, all is not lost. The main lesson is that *if* assets are to be used as a proxy for real full wealth in these studies, the sample used for regression estimation must be as free as possible of potentially important life-cycle effects. One way to accomplish this might be to choose the sample such that the individuals included all face the same or similar life-cycle wage paths, interest rates, and other factors that may affect intertemporal labor supply allocation; another is to "remove" the effects of differing life-cycle characteristics by regression techniques.¹

Fertility and Labor Supply

One simplifying assumption that Smith makes in setting out his theory of life-cycle behavior is that the underlying parameters of the commodity production functions are the same in each period. In essence, this is a reversion to the one-period approach in that the various substitution elasticities and so forth are lifetime averages of the values these parameters take in each period. In some cases this assumption may obscure important considerations for programs having the broad general effect of proposed welfare reforms. A case

¹See DaVanzo, De Tray, and Greenberg (forthcoming) for one attempt at this second solution.

in point is the production of the commodity "child services" over the life cycle.¹ One might reasonably argue, for example, that the degree to which husband's time or purchased goods and services substitute for wife's time in the production of child services depends on the age of the children²--other inputs may be relatively poor substitutes for mother's time when children are very young but quite good substitutes when children are teenagers.

The importance of this observation to studies of the effect of welfare reform relates to the short-run effects that such legislation might have on family behavior. Assume for simplicity that husband's time does not enter the production of child services. Then a household with one or more young children may face a very small elasticity of substitution between wife's time and other inputs (both time and purchased goods and services). Any change in the wife's market wage rate would therefore have relatively little effect on her division of time between household activities and labor force participation. A family with identical wealth and wage characteristics but with older children might act quite differently from those in the first household. With only older children in the home, the wife's response to a change in her wage rate might be relatively elastic. At least where short-run considerations are important, researchers must consider not only the number but also the age of children in each household for accurate estimates of labor supply responses.

When long-run considerations are of primary concern, the number, ages, and spacing of children are no longer predetermined and become objects of choice whose values may be significantly altered by welfare reform. This distinction between the short- and long-run implications

¹The term child services is used here to capture all the many consumer- and producer-durable aspects of having and rearing children. For more on the components of this commodity, see De Tray (1972a) and the references therein.

²See also Gronau (1972, p. 29) on this point. If interest lies only in the ultimate number of children that the household desires, then this issue is not a serious problem except that one must be careful not to take the readily observable female time intensity of young children as *prima facie* evidence that child services in general are female time intensive.

of welfare reform is often overlooked by both policymakers and researchers. Most studies of the effect of welfare reform implicitly assume either that the most important of these effects occur in the short run or that many family activities are rigidly defined by society and households exercise no choice over them. There is good evidence that the second assumption is not correct,¹ but very little information one way or the other on the potential long-run effects of the new welfare proposals.²

The distinction between short- and long-run effects of welfare reform is, in terms of estimation procedures, a distinction between predetermined and simultaneously determined variables. If interest lies primarily in the effect of a program during its first few years, one might reasonably assume that many characteristics of households--for example, family size and family composition--are beyond the scope of family choice and can, therefore, be treated as predetermined variables. It would then be necessary to ensure only that all variables (such as age and spacing of children) that may affect household production function parameters are included in the estimating equation. If, however, the goal is to assess the effect of a potential program on labor supply in the longer run, then treating such variables as family size and child spacing as predetermined would likely yield biased response estimates and quite possibly incorrect policy prescriptions. Here simultaneous equations estimation techniques would be required for estimates of families' responses to changes in wages, wealth, and so on that have desirable statistical properties.³

¹For some of this evidence as it relates to fertility decisions, see the March/April 1973 Supplement to the *Journal of Political Economy*.

²Two examples of attempts to explore the longer-run effects of welfare reform can be found in Cain (1972) and Reischauer (1971). Cain explores the potential effect of welfare reform on desired numbers of children, and Reischauer is concerned with the effect of welfare in general on black migration and family stability.

³For several attempts at this in the context of a developing nation, see Nerlove and Schultz (1970), DaVanzo (1972a), and Maurer, Ratajczak, and Schultz (1973). A more detailed discussion of estimation techniques including their "desirable properties" is given in DaVanzo and Greenberg (1973).

Choosing the Appropriate Model

At the beginning of this subsection I stated that the choice between the life-cycle and one-period framework depends on the particular characteristics of the problem being analyzed. Assessing the effects of an IMP offers an especially good example of how the issues at stake determine the framework. Smith (forthcoming) considers this problem in some detail, and so only the essentials of the exercise will be given here.

The key factor is the proportion of an individual's lifetime during which he or she is a recipient of IMP payments. If a family expects to receive welfare throughout most or all of its lifetime, the tax rate effect of an IMP will lower family members' lifetime wages but leave wage differences unaltered from one period to another. In this case the one-period model is the appropriate framework for analysis since the IMP can be viewed as affecting only lifetime averages.

At the other extreme, some families may expect to receive welfare in only one period (say one year) during their lifetimes. For this subset of the population, an IMP will leave lifetime averages unchanged but will significantly alter members' wages in one period relative to all other periods. Here the life-cycle model is the more appropriate framework since these families are confronted with a substantial change in their wage *paths* but little change in the level of those paths.

Smith's work is an attempt to develop a model appropriate for analyzing those families that fall between the two extreme examples. The result is a blending of the one-period and life-cycle models that specifically considers the proportion of the life cycle over which a family receives welfare payments. Although the full implications of this model have yet to be worked out, it will likely play an important role in future research on the effects of an IMP on family behavior.

ENVIRONMENTAL VARIABLES

Household production may be a function not only of the amount of inputs but also of the "environment" in which that production took place. Such environmental factors as location, climate, age, and,

possibly, education¹ act to shift household production functions and in that sense determine the effectiveness of direct inputs.

Including environmental variables in the analysis permits the systematic prediction of differences in demand for certain household commodities and, therefore, systematic differences in the demand for factors that enter the production of those commodities. Classical consumer demand theory would appeal to differences in tastes to explain these same observed results. By reducing the model's reliance on tastes as a reason for "unexplained" differences in behavior, the legitimate scope of positive economic analysis is considerably broadened.²

A complex and important example of the effect of an environmental variable on household behavior concerns the equivalent of entrepreneurial ability in the household production model. Just as firms with the same level of inputs may have different levels of outputs, so may families with similar levels of purchased inputs and home time produce significantly different levels of household commodities (even if tastes do not differ). One reason may be that households differ in their efficiency in household production. This effect can be given empirical content by relating it to levels of education of family members. The most extensive work in this area is that of Michael (1969). Assume that education of family members is an important determinant of household production efficiency and assume also that changes in education are factor neutral in their effect on the productivities of different inputs.³ Then Michael argues that increasing the level of education of household members will have much the same effect as increasing the nonwage-related component of household full wealth; that is, changes in years of schooling holding nominal full wealth (and all prices and

¹In general these factors are treated as exogenous to the family's decisionmaking process; the exception is education, which, over the long run, is subject to family choice.

²For many questions this distinction between environmental factors and taste variables is not important; however, the environmental approach has proved a useful tool for analyzing the effect of such factors as education on household consumption patterns. See Michael (1969).

³That is, an increase in education raises the marginal productivity of all inputs in household production by the same percentage.

wages)¹ constant should have the same effect on the family's demand for purchased goods and services as would a change in nonwage-related wealth if education levels did not change. Although Michael's empirical evidence does not offer overwhelming support for this hypothesis, it does in general support the underlying framework.

Although the types of environmental variables discussed so far may or may not be of major importance to studies of the potential effects of welfare reform, other environmental factors are likely to be critical to the analysis. One often discussed problem of the existing welfare structure and of proposed welfare reform is the differential urban-rural effect of those programs. On the average, urban residents seem to fare better than rural residents under current welfare regimes, but because of cost-of-living differences this situation would probably reverse with a uniform, federally administered welfare scheme. However, it is often very difficult to judge the relative position of urban and rural welfare recipients since the environments in which these two groups reside obviously differ considerably. Cost-of-living differences aside, it might well be that conditions prevailing in most rural areas allow rural families to maintain a given level of welfare (household commodity consumption in the terminology of this model) with fewer annual payment dollars than similar families living in urban areas.

CORNER SOLUTIONS

The one-period model of Section II and the intertemporal model presented earlier in this section explicitly assume an interior solution for all time allocation; that is, husband and wife both spend at least part of their time working in the marketplace. The value of

¹The effect of the factor neutrality assumption is to hold relative prices of household commodities constant. However, holding market wages constant and changing levels of education does raise certain conceptual questions for the traditional human capital earnings function. Given a correctly specified earnings function, if wages remain constant while education rises (and, of course, assuming education does increase market productivity), then some other determinant of wages must be falling.

this assumption is that a measure of the relative scarcity of a family member's time is readily available in the form of market wage that he or she forgoes in order to spend time at home. As useful as this assumption is, it unfortunately does not apply to a large and important segment of our population.

In any given year, the current market wage that many women could receive if they chose to work is apparently less than the value of their time in household production, since many women allocate no time at all to market work. Some women never participate in market work, implying that their lifetime market wage level is less than the lifetime value of their time in the home even when all time is allocated to household production. A crucial task now facing economists involves modifying the household production model and especially empirical applications of that approach to include individuals who do not work.

Several solutions for valuing the time of those who do not work have recently been suggested, but none has proved very satisfactory.¹ The most prevalent solution to date has been to attribute to those who do not work an imputed wage based on the earnings of people who do work. This procedure takes the form of estimating a wage function based on education, experience or age, and possibly other variables, and then calculating a wage for those who do not work by substituting their characteristics into that function. That approach ignores the fact that certain women with a given set of characteristics choose to work while others with apparently the same characteristics do not work.

At issue is the direction of the possible bias when wages are calculated by the above method. If other determinants of the labor supply decision are held constant, then in the framework proposed here a woman bases her decision whether to work on the relative value of her time in household production versus the market wage she could receive if she did work. Therefore, if two women are of the same age, education, and so forth, but one works and the other does not, we have no way of knowing whether the wage of the working woman is above or

¹The possible exception is the work of James Heckman (1972); however, since Heckman's work was still at a preliminary stage when this report was written, I do not consider it in detail here.

below the potential wage of the woman who does not work. For example, it is possible that a nonworking wife has a higher potential market wage *and* a higher value of marginal product in household production than a working wife, but that she is sufficiently more productive in nonmarket activities that she allocates no time to market work.

A considerably more sophisticated approach to this problem has been set out in a recent paper by Gronau (1973), who concentrates on the labor force participation decision (that is, whether to work at all), and attempts to estimate the degree to which the nonmarket value of time for women who do not work exceeds the market wage of women with the same market-wage-related characteristics. In one sense, as he himself recognizes, Gronau sidesteps the issue by assuming that the potential market wage of wives with a given set of wage-related characteristics is the same as the average wage received by women with those same characteristics who do work. In addition, as he again acknowledges, his analysis is weakened by his arbitrary division of wives' characteristics into those that affect both home productivity and potential market wage, and those that affect the former but not the latter. These reservations aside, however, Gronau's empirical results deserve serious consideration.

Two of Gronau's main findings may affect future assessments of the effect of welfare reform. The first concerns the value that housewives who do not work place on their time. Gronau found that on the average, the marginal value product of time in the home for nonworking wives may exceed the average market wage of wives with the same market-wage characteristics by as much as 12 to 15 percent.

Gronau's work also sheds some light on another central theme of this report--how children should enter the model. Gronau found not only that the age composition of children in the home is important in determining housewives' value of time, but also that the magnitude of this effect depends on the wives' education levels. Put another way, the marginal effect of an additional child of a given age differs considerably among wives with elementary, high school, and college educations. For example, having a child less than three years old in the home increases the value of time in the home for women with an

elementary education by around 5 percent, but of high school educated women by 11 percent, and of college graduates by almost a third (Gronau, 1973, p. S192).¹

In another study, Gronau (1972) considers a related set of problems that bear on the relationship between the average wage for women who do work and the average potential wage for all women. The main thesis of this second paper is that for subgroups of the population in which a significant portion of the members do not work, a serious "selectivity bias" is introduced into the analysis when the mean or median wage of those with a given set of market-related characteristics who work is used as an estimate of the median wage offered to all people in that group. This bias arises because the distribution of wage offers--the range of wages the market would be willing to pay women with a *given set* of market-wage-determining characteristics--is not observable.² What we do observe is the *truncated* wage offer distribution based on those women who in fact do work. Since the unobservable part of the distribution is presumably the lower tail (the women who choose not to work are those who received relatively low market wage offers and rejected them in favor of their value of time in the home), the mean wage offer of the truncated distribution will exceed that of the "true" mean wage offer (that for the whole distribution). Gronau shows that the extent of this bias is a function of the average level of participation within any group; thus, the median wage for

¹This interaction between the numbers of young children and the education level of the wife has been the subject of considerable research and speculation recently (Leibowitz, 1972; Smith, 1972; De Tray, 1972a, and others). The case is not proved one way or the other, but sufficient evidence does exist to support the contention that this children-education interaction may be one important determinant of female labor force behavior.

²The distribution of wage offers for women with a given set of characteristics (both market-related and nonmarket-related) results from Gronau's assumption that the wage a person receives has two components, one based on the mean wage for all people with the same characteristics and the other a normally distributed random element. Because of the random component in market wage offers and given that job search is costly, two identical individuals may face very different actual market wage offers.

prime age working men is probably a close approximation of the median wage offer for all men, but the median observed wage for secondary workers--women, the young, and the aged, for example--may considerably overstate the median wage for the entire wage distribution of these groups.¹

¹In an important application of his model, Gronau demonstrates that using observed wage differences may overestimate the sensitivity of labor force participation to changes in market wage rates. When the wage offer distribution shifts to the right (to a higher overall level) the observed mean wage of those who do work increases but by a smaller amount than the shift in the distribution. This implies that were one to look across education classes, the observed differences in average market wages would tend to *understate* the true differences in underlying wage offer curves and their mean values and thus *overstate* the actual sensitivity of participation to market-wage changes.

V. RESEARCH IMPLICATIONS

As Section II indicates, the main economic effects of proposed welfare reforms result from changes those reforms would bring about in families' wealth and in the wages family members could receive if they choose to work. To assess the potential effects of these changes we must accurately measure the effect of exogenous wealth and wage changes in household behavior. To this end, the following recommendations can be drawn from the preceding discussion:

1. Individuals affected by welfare reform do not necessarily live alone, but often in multi-person households. A major hypothesis underlying the household production model is that people's decisions on consumption and work are based both on their own characteristics and on those of other family members. These links between the behavior of household members are complex and as yet not well understood, but to ignore them when assessing the potential effects of welfare reform must put the validity of the resulting estimates in serious question. An obvious first step is to include both spouses' characteristics in regressions estimating the effect of different reforms on either spouse's behavior. Even if we are interested only in the effect of a program on, say, the labor supply of primary workers, the data set must include detailed and parallel information on both husband and wife.

2. Cross-sectional analysis of families runs the risk of confusing life-cycle phenomena with changes in families' behavior brought about by exogenous changes in their environment. In some cases, it may not be possible to remove all life-cycle considerations from the data, but the effect of these factors can be minimized in several ways. More detailed time series data on individual families should allow for much better control of life-cycle-related changes in people's behavior; until such data are available, variations caused by differences in age, in work histories brought about by different educational attainments, and so forth should be removed from cross-sectional household data. In some cases, removing time-related phenomena may require

instrumental variables estimation techniques; in other cases, stratifying the sample population by age may be all that is required.

3. Household decisions are interrelated both at a point in time and over time. It is, therefore, often difficult to sort out the direction of causation and the magnitude of the effect of a change in one variable on another. Unless economic models are correctly formulated to account for this joint determination of family behavior, and suitable econometric techniques are used to estimate responses, analysis may yield seriously misleading policy implications. This household model and its extensions can help to identify possible sources of so-called simultaneous bias and, therefore, to correct estimation techniques and to serve as a basis for specifying types of data that would lead to improved welfare reform analysis.

4. Estimating the effect of changes in the family's full wealth poses a difficult set of problems. Most measures of wealth or income used in empirical studies--for example, assets--are endogenous to household decisionmaking and may vary systematically over the life cycle in ways unrelated to household full wealth. If other family behavior, say hours worked, also varies systematically over time either for the same reasons that assets vary or for different reasons, we may be falsely attributing causation to an observed relationship between assets and hours worked. One possibility for solving this problem would be to analyze a subset of the population whose members face similar life-cycle patterns of wages and so on and who are at approximately the same point in their life cycles. In this case, variations across individuals in observed levels of assets should indicate actual variations in family full wealth. A second solution is to look elsewhere for a measure of household full wealth--for example, toward an estimated full wealth variable based on initial assets, educational levels, location, and other relevant variables. Both of these approaches may overtax the power of existing data sets but may still act as guides for future data collection.¹

¹For a preliminary empirical exploration of these issues see DaVanzo, De Tray, and Greenberg (forthcoming).

5. Exploring labor supply responses of married women has especially serious problems, not the least of which involves estimating the "opportunity cost" of market work. Observed differences between wives in the amount of market work they supply will depend not only on the different market wages that these women face, but on the different values they place on their time in the home as well. Therefore, in order to accurately predict the response of wives to exogenous changes in their market wages (either actual or potential), one must first be able to estimate their value of time in nonmarket activities. This task is complicated by the proposition that market wages and value of time in the home can both be altered by the household. Intertemporal household data and simultaneous equations estimation techniques may both be required to sort out cause and effect in these relationships.

6. A wife's responses to a change in her wage rate may also vary over time and with family composition. The presence of young children in the home may make changes in a wife's hours worked in the market highly inelastic with respect to changes in her wage; when her children are older, her change in market hours may be much more responsive. Therefore, even when only short-run considerations are of interest, one must include in the estimating equations not only the number of children a family has but their age distribution as well.

7. If a wife does have young children, her labor supply will depend in part on the price and availability of substitutes for her time in the home. A highly subsidized supply of day care services and facilities may have a considerable effect on the number of hours a woman wants to work. It may not be possible to estimate the extent of this effect given existing data on day care, but future data collection should include the necessary information as a priority item.

At a more general level, the model outlined in this report highlights several areas that researchers have neglected when studying the effects of welfare reform. The most prominent is the lack of attention to the long-run effects of these programs. The assumptions underlying many previous studies of welfare reform imply that their concern lies with the short-run effects of that legislation. Although these short-run effects may be important, one should not use short-run

parameters to gauge long-run effects. Further, researchers and policy-makers alike must recognize the possibility that welfare reform may affect family behavior in ways other than the amount of time people allocate to market work. Fertility, marriage and marital stability, migration, and investments in adult and child education may all be affected by the tax and income effects of an IMP.

In general, then, researchers must broaden their perspective of the effects of welfare reform, both temporally and in terms of the spheres of family behavior analyzed. The household production model and the econometric tools outlined here are a first step in that direction.

Appendix A
TWO HYPOTHETICAL INCOME MAINTENANCE SYSTEMS¹

THE BASIC SYSTEMS

Two basic determinants of the form taken by an income maintenance system are the system's component programs and whether eligibility for the various components is determined on the basis of employability tests. Candidate programs for inclusion within the system include a negative income tax program; a wage subsidy program; a public employment program; day-care services; and other income-in-kind programs, such as food stamps, subsidized housing, and subsidized medical care. An income maintenance system may be composed of one, several, or all of these.

Employability tests involve the establishment of administrative standards that are used to attempt to separate those who can work (or those who policymakers or administrators feel "should" work) from those who cannot work. The question of whether employability tests should be used to determine program eligibility is part of a larger issue: Should those whose health, age, or family responsibility do not preclude their working be the beneficiaries of income that is not received in exchange for hours of labor? Differences among policymakers over this issue is a major reason the almost universally discredited existing welfare system has not yet been superseded.

Because of the importance of this issue, it seems useful to stress employment requirements in one of the hypothetical income maintenance systems examined and not in the other. Once this approach is taken,

¹This appendix was prepared by David Greenberg. In it he sets out two basic income maintenance systems that attempt to capture the range of issues and the alternatives that will ultimately determine the overall character of a national federal income maintenance system. The word "systems," rather than programs, is used to imply that income maintenance may be provided through several component programs. Although hypothetical, the parameters used in the two systems are within a range that recent legislative history indicates is reasonable. For a summary of how Family Assistance Plan parameters have changed over time see De Tray (1972b).

the general outlines of the two systems become clear. It is likely, for example that in the absence of employment requirements and employability tests, the most important vehicle for income maintenance would be a negative income tax program; but in the presence of such requirements and tests, much greater use would be made of job creation programs. This conclusion follows from two important implications of administratively determining that a particular individual "should" work: (1) If the "employable" individual does not work, he should not be guaranteed an income. (2) If he is unable to secure conventional employment, the state should provide him with a job. In the presence of employability tests, consistency dictates that those who are not required to work be guaranteed an income under a negative income tax program, and those who are required to work be guaranteed a job under a public employment program. In the absence of employability tests, job creation programs are not required, and income maintenance can be provided entirely through a negative income tax program.

If participation in a negative income tax program is made contingent upon employability tests, it seems likely that in addition to negative income tax and public employment components, day-care and wage subsidy programs will also be incorporated into the income maintenance system. If mothers are expected to work, provisions must be made to care for their children. A wage subsidy program may be necessary if the public employment program pays sufficiently high wages to attract substantial numbers of persons employed in low wage, private sector jobs. In the absence of employability tests, the necessity for such programs is greatly diminished.

The two hypothetical income maintenance systems, A and B, are summarized in Table A-1. The table indicates that System A consists solely of a negative income tax plan. Participation in this program is not contingent upon employability tests. System B also uses a negative income tax program, but only those who "cannot" work may participate. To provide sufficient income for those categorized as "employable," System B depends upon a wage subsidy program, a public employment program, and day-care services. So far as one can judge from present conditions, the nation's future welfare system will

Table A-1

TWO HYPOTHETICAL INCOME
MAINTENANCE SYSTEMS

	System	
	A	B
Uses employability tests		x
Program components		
Negative income tax	x	x
Wage subsidy		x
Public employment program		x
Day care		x

probably more closely approximate System B than System A. Nevertheless, it is the "pure" negative income tax represented by System A that most previous analysis has focused upon. For this reason, System A appears very useful as a benchmark with which more complex, "mixed" systems might be compared.

MAJOR PROVISIONS OF THE COMPONENT PROGRAMS

Negative Income Tax Program

As can be seen from Table A-2, the two systems contain important differences in negative income tax program provisions.¹ For example, as already stressed, those who "should" work are precluded from participating in the negative income tax program under System B but not under System A. Therefore, there is more concern under System A than under System B with work disincentives associated with program guarantee levels and tax rates, and the tax rate and guarantee level are therefore both set lower under System A.² However, because the

¹Amounts in this and subsequent tables relate to 1972 wage and income levels.

²Both are also somewhat lower than the guarantee level and tax rates associated with actual IMP bills such as the 1972 H.R. 1, and they may represent lower limits. This appears particularly true of the tax rate on earnings; no one to my knowledge has seriously

Table A-2

MAJOR PROVISIONS OF THE NEGATIVE INCOME TAX COMPONENTS
OF SYSTEMS A AND B

Provision	System A	System B.
Coverage	Household income must be below the program breakeven point	Household income must be below the program breakeven point and head of household must be disabled or over 65 or a female with at least one dependent under three
Base level of support	\$700 for each of the first two adults in household; \$400 for each additional member	\$1200 for each of the first two adults in household; \$700 for each additional member
Tax rate	50% on wage income, 100% on all other taxable income, 10% on the value of net assets (other than personal effects and furnishings)	75% on wage income, 100% on all other taxable income, 10% on the value of net assets (other than personal effects and furnishings)

breakeven points and, hence, the income levels at which households are no longer covered are positively related to guarantee levels and negatively related to tax rates, breakeven points are similar for the two programs.

Public Employment

The major provisions of the public employment segment of System B are for the most part straightforward. They are set out in Table A-3. Restricting coverage to heads of households is intended to limit the number of program participants. The requirement that participants first be unemployed for three months is an attempt to limit the program to long-term unemployed. The intent of the furlough requirement

advocated a rate that falls below 50 percent. On the other hand, the System B guarantee level, which roughly approximates the federal poverty line, and tax rates are both higher than those associated with H.R. 1 and seem close to approximating politically feasible upper bounds.

Table A-3

MAJOR PROVISIONS OF THE PUBLIC EMPLOYMENT
COMPONENT OF SYSTEM B

Coverage	Heads of households, 16 to 65 years of age, who have been unemployed, registered with the employment service, and actively seeking employment for at least three months.
Base level of support	\$1.60 per hour for up to 9 hours per week. After 11 consecutive months on program a participant is required to take a 1 month furlough. During furlough month he will receive \$128.
Tax rate	0% on all income below an amount calculated by adding \$1200 for each of the first 2 adult members of the household and \$700 for each additional member and dividing the sum by .75; 25% on first \$1000 above that amount; 50% on second \$1000; 75% on third \$1000; and 100% on all income more than \$3000 in excess of the amount. The amount of the tax is not permitted to exceed the amount of income maintenance received by the household.

is to stimulate movement out of the program and back into the private sector. During the furlough month, the Employment Service will provide assistance in finding private sector employment. The tax on all income in excess of the System B negative income tax breakeven point is an attempt at compromise. Too high a tax rate would either discourage other household members from working or encourage family splitting. On the other hand, too low a tax rate would allow relatively high-income households to receive income maintenance payments. In the analysis of these programs, we will assume that, on the average, an hour of work in a private sector job that pays \$1.60 per hour is perceived by the employable poor as equivalent to an hour of work in a public employment job. This assumption implies, for example, that training received in public employment is thought to be no more valuable than on-the-job training received in the private sector for \$1.60 per hour.

Wage Subsidy

The distinguishing feature of a wage subsidy is that no income maintenance payments are received unless someone in the household

works. Two alternative wage subsidy programs are presented in Table A-4. The first alternative is the classical wage subsidy; the greater the number of hours worked, the larger the amount of the subsidy. An important feature of Alternative 1 is that, at the same number of hours worked, employment in any private sector job with a wage rate above \$1.20 per hour will result in higher household income than employment in a public employment job. The second alternative may be more politically feasible than the first. It is very similar to the System B negative income tax program, except that no subsidy is provided unless someone in the household works. Moreover, the household is penalized rather severely unless sufficient hours are worked to bring earnings up to the amount the head of the household would receive for a year's participation in the public employment program. Unlike

Table A-4

MAJOR PROVISIONS OF THE WAGE SUBSIDY COMPONENT OF SYSTEM B

<u>Alternative 1</u>	
Coverage	All private sector workers who earn less than \$2.00 per hour.
Base level of support	Participants would receive their wage rate, plus one-half the difference between \$2.00 and their hourly wage.
Tax rate	Same as for public employment (see Table A-3).

<u>Alternative 2</u>	
Coverage	All households with income below the System B negative income tax program breakeven point and with at least one member employed in either a private sector or public employment job.
Base level of support	Same as for System B negative income tax program (see Table A-2).
Tax rate	Same as for System B negative income tax program (see Table A-2), except that if actual earnings fall below \$3068--the annual "earnings" from a public employment job--the tax rate is applied against the latter amount.

Alternative 1, Alternative 2 permits participants in the public employment program, as well as low wage private sector workers, to receive a wage subsidy. Furthermore, the income received by households headed by persons categorized as employable would be much more closely geared to family size under the second alternative than under the first. This minimizes potential inequities between households that are eligible to participate in the negative income tax program and those that are not. On the other hand, Alternative 1 offers considerably greater inducement to work in the private sector than on a public employment job.

DAY CARE

Free day care will be provided for children under six who are in families participating in the public employment program or the wage subsidy program. Day care will be provided only during hours when all members of the household who are 16 years and older are at work, in school, or traveling to or from work or school. It will be assumed that parents consider the services provided by the day care program as equally beneficial to their children as those provided by a typical babysitter.

Appendix B¹
MATHEMATICAL MODEL

The simple model presented in Section II of the text can be summarized by the following four equations:²

$$U = U(Z_1, Z_2) \quad (B.1)$$

$$Z_1 = g_1(M_1, F_1, X_1) \quad (B.2)$$

$$Z_2 = g_2(M_2, F_2, X_2) \quad (E.3)$$

$$R = \pi_1 Z_1 + \pi_2 Z_2 = T(w_m + w_f) + V \quad (B.4)$$

where the price of $X(p)$ is taken as the numeraire. The household thus maximizes utility (Equation (B.1)) subject to one resource constraint, and two production or technological constraints. The solution to this problem is simplified if we approach it in two stages--first find the cost minimizing solutions for the production of Z_1 and Z_2 given prices (w_m , w_f and implicitly, p), and second maximize utility given for cost minimizing marginal shadow prices of Z_1 and Z_2 .

The first order conditions for a cost minimum for Z_1 may be written as

$$w_m = \pi_1 g_{M_1}$$

$$w_f = \pi_1 g_{F_1}$$

$$1 = \pi_1 g_{X_1}$$

$$Z_1 - g_1(M_1, F_1, X_1) = 0$$

¹I would like to thank John Cogan and James P. Smith for their help and comments on the derivations presented here. This appendix in part follows a similar model that Cogan developed before I began work on this study.

²All symbols are defined on p. 53.

where π_1 is the marginal cost of Z_1 . Totally differentiating these four equations and writing the results in Hessian form yields:¹

$$A = \begin{array}{c} \left| \begin{array}{cccc} 0 & g_{M_1} & g_{F_1} & g_{X_1} \\ g_{M_1} & g_{M_1 M_1} & g_{M_1 F_1} & g_{M_1 X_1} \\ g_{F_1} & g_{M_1 F_1} & g_{F_1 F_1} & g_{F_1 X_1} \\ g_{X_1} & g_{M_1 X_1} & g_{F_1 X_1} & g_{X_1 X_1} \end{array} \right| \begin{array}{c} \left| \frac{1}{\pi_1} d\pi_1 \right| \\ dM_1 \\ dF_1 \\ dX_1 \end{array} \left| \begin{array}{c} dZ_1 \\ \frac{1}{\pi_1} dw_m \\ \frac{1}{\pi_1} dw_f \\ 0 \end{array} \right| \end{array} .$$

Solving for changes in the household's desired level of husband's time in the home using Cramer's Rule,

$$\frac{dM_1}{M_1} = \frac{dZ_1}{M_1} \frac{A_{12}}{|A|} + \frac{1}{\pi_1 M_1} \frac{A_{22}}{|A|} dw_m + \frac{1}{\pi_1 M_1} \frac{A_{32}}{|A|} dw_f \quad (B.5)$$

where A_{ij} is the cofactor of the i,j th element of A and $|A|$ is the determinant of A . Holding prices constant, the first term on the right hand side becomes:

$$\frac{dM_1}{M_1} = \frac{dZ_1}{M_1} \frac{A_{12}}{|A|}$$

or

$$\frac{A_{12}}{|A|} = \frac{\partial M_1}{\partial Z_1}$$

where the partial derivative notation indicates that prices are held constant.

¹The "1" in g_1 has been suppressed for notational convenience.

The last two terms on the right hand side can easily be transformed into weighted Allen partial elasticities of substitution¹ (Allen, 1967, p. 503) so that the derived demand equation now reads

$$\frac{dM_1}{M_1} = \frac{dZ_1}{Z_1} \left(\frac{Z_1}{M_1} \frac{\partial M_1}{\partial Z_1} \right) + s_{F_1} \sigma_{M_1 F_1} \frac{dw_f}{w_f} + s_{M_1} \sigma_{M_1 M_1} \frac{dw_m}{w_m} . \quad (B.6)$$

Since $[(Z_1/M_1)(\partial M_1/\partial Z_1)]$ is unity for a homogeneous-of-degree-one production function,² and $s_{M_1} \sigma_{M_1 M_1} = -(s_{F_1} \sigma_{M_1 F_1} + s_{X_1} \sigma_{M_1 X_1})$ (Allen, 1967, p. 504), Equation (B.6) may be written as

$$\frac{dM_1}{M_1} = \frac{dZ_1}{Z_1} + s_{F_1} \sigma_{M_1 F_1} \frac{dw_f}{w_f} - (s_{F_1} \sigma_{M_1 F_1} + s_{X_1} \sigma_{M_1 X_1}) \frac{dw_m}{w_m} . \quad (B.7)$$

Utility maximization given the marginal cost of Z_1 implicit in the cost minimization solution yields the following familiar conditions:

$$U_1 = \lambda \pi_1$$

$$U_2 = \lambda \pi_2$$

$$R - (\pi_1 Z_1 + \pi_2 Z_2) = 0$$

¹For example,

$$\frac{1}{\pi_1 M_1} \frac{A_{32}}{|A|} dw_f = \left(\frac{Z_1}{M_1 F_1} \frac{A_{22}}{|A|} \right) \frac{F_1 w_f}{\pi_1 Z_1} \frac{dw_f}{w_f}$$

where the term in brackets on the right hand side is $s_{M_1 F_1}$ (see Allen, 1967, p. 504).

²The partial derivative $(\partial M_1/\partial Z_1)$ holds relative factor prices constant but not quantities of inputs; that is, the movement measured by this term $[(Z_1/M_1)(\partial M_1/\partial Z_1)]$ is along a ray emanating from the origin, and for homogeneous-of-degree-one production functions, a 1 percent increase in output requires a 1 percent increase in all inputs.

where U_1 and U_2 are the marginal utilities of Z_1 and Z_2 and λ is the marginal utility of wealth.

Writing the total differential of these conditions in Hessian form,

$$B = \begin{vmatrix} 0 & -\pi_1 & -\pi_2 \\ -\pi_1 & U_{11} & U_{12} \\ -\pi_2 & U_{12} & U_{22} \end{vmatrix} \begin{vmatrix} d\lambda \\ dZ_1 \\ dZ_2 \end{vmatrix} = \begin{vmatrix} -dR + Z_1 d\pi_1 + Z_2 d\pi_2 \\ \lambda d\pi_1 \\ \lambda d\pi_2 \end{vmatrix}.$$

Again this system may be solved for dZ_1/Z_1 using Cramer's Rule, to give

$$\frac{dZ_1}{Z_1} = \frac{1}{Z_1} (-dR + Z_1 d\pi_1 + Z_2 d\pi_2) \frac{B_{12}}{|B|} + \frac{\lambda d\pi_1}{Z_1} \frac{B_{22}}{|B|} + \frac{\lambda d\pi_2}{Z_1} \frac{B_{32}}{|B|}$$

or

$$\frac{dZ_1}{Z_1} = \frac{R}{Z_1} \frac{\partial Z_1}{\partial R} \left(\frac{dR}{R} - \frac{\pi_1 Z_1}{R} \frac{d\pi_1}{\pi_1} - \frac{\pi_2 Z_2}{R} \frac{d\pi_2}{\pi_2} \right) + k_{1\sigma_{11}} \frac{d\pi_1}{\pi_1} + k_{2\sigma_{12}} \frac{d\pi_2}{\pi_2} \quad (B.8)$$

where $k_i = (\pi_i Z_i)/R$.

From Equation (B.4)

$$\frac{dR}{R} = \frac{w_m^T}{R} \frac{dw_m}{w_m} + \frac{w_f^T}{R} \frac{dw_f}{w_f} + \frac{V}{R} \frac{dV}{V}$$

also,

$$\frac{\pi_i Z_i}{R} \frac{d\pi_i}{\pi_i} = k_i \left(s_{M_i} \frac{dw_m}{w_m} + s_{F_i} \frac{dw_f}{w_f} \right).$$

Therefore, the first term on the right hand side becomes

$$\begin{aligned} & \eta_1 \left[\left(\frac{w_m^T - w_m^M}{R} \right) \frac{dw_m}{w_m} + \left(\frac{w_f^T - w_f^F}{R} \right) \frac{dw_f}{w_f} + \left(\frac{V}{R} \right) \frac{dV}{V} \right] \\ &= \eta_1 \left[\left(\frac{e_m}{R} \right) \frac{dw_m}{w_m} + \left(\frac{e_f}{R} \right) \frac{dw_f}{w_f} + \left(\frac{V}{R} \right) \frac{dV}{V} \right] \end{aligned} \quad (\text{B.9})$$

where

$$M = M_1 + M_2$$

$$F = F_1 + F_2.$$

Since $k_1^{\sigma_{11}} = -k_2^{\sigma_{12}}$, Equation (B.8) may be rewritten using (B.9), as

$$\frac{dZ_1}{Z_1} = \eta_1 \left[\left(\frac{e_m}{R} \right) \frac{dw_m}{w_m} + \left(\frac{e_f}{R} \right) \frac{dw_f}{w_f} + \left(\frac{V}{R} \right) \frac{dV}{V} \right] + k_2^{\sigma_{12}} \left[\frac{d\pi_2}{\pi_2} - \frac{d\pi_1}{\pi_1} \right].$$

Note that when price effects are taken into account, η_1 applies not to total potential earnings but only to that fraction of time spent at market work. Expanding the price change terms,

$$\begin{aligned} \frac{dZ_1}{Z_1} &= \eta_1 \left[\left(\frac{e_m}{R} \right) \frac{dw_m}{w_m} + \left(\frac{e_f}{R} \right) \frac{dw_f}{w_f} + \left(\frac{V}{R} \right) \frac{dV}{V} \right] \\ &+ k_2^{\sigma_{12}} (s_{M_2} - s_{M_1}) \frac{dw_m}{w_m} + k_2^{\sigma_{12}} (s_{F_2} - s_{F_1}) \frac{dw_f}{w_f}. \end{aligned}$$

Definition of Symbols

u = household utility.

Z_1 = consumption of 1th household commodity.

g_1 = production function for Z_1 (assumed here to be homogeneous of degree 1).

X_1 = input of purchased goods and services into the 1th production process.

M_1 = input of husband's time into the 1th production process.

F_1 = input of wife's time into the 1th production process.

L_m = market labor supply of husband.

L_f = market labor supply of wife.

V = nonwage-related income.

w_j = market wage of the j th individual ($j = M, F$).

T = total available time.

R = family full wealth.

$d(i)/(i)$ = percent change in i .

s_{F_1} = share of total cost of Z_1 accounted for by F_1 ($s_{F_1} = (w_f F_1) / (w_f F_1 + w_m M_1 + X_1)$) with s_{X_1} similarly defined for X_1 .

$\sigma_{M_1 F_1}$ = Allen partial elasticity of substitution (Allen, 1967, p. 503) between husband's and wife's time in the production of Z_1 .

$\sigma_{M_1 X_1}$ = Allen partial elasticity of substitution between M_1 and X_1 in the production of Z_1 .

k_1 = share of full wealth spent on Z_1 .

σ_{12} = elasticity of substitution between Z_1 and Z_2 .

η_1 = full wealth elasticity of Z_1 .

BIBLIOGRAPHY

- Allen, R.D.G., *Mathematical Analysis for Economists*, St. Martin's Press, New York, 1967.
- Barth, M. C. and D. H. Greenberg, "Incentive Effects of Some Pure and Mixed Transfer Systems," *Journal of Human Resources*, Vol. VI, No. 2, Spring 1971.
- Bawden, D. L., G. Cain, and L. J. Hausman, "The Family Assistance Program: An Analysis and Evaluation," *Public Policy*, Vol. XIX, No. 2, Spring 1971, pp. 323-353.
- Becker, G. S., "A Theory of the Allocation of Time," *Economic Journal*, Vol. LXXV, September 1965, pp. 493-517.
- , *The Allocation of Time over the Life Cycle*, 1969 (mimeo.).
- , *Economic Theory*, Alfred A. Knopf, New York, 1971.
- , "A Theory of Marriage: Part I," *Journal of Political Economy*, Vol. 81, No. 4, July/August 1973.
- Cain, G. C., *The Effect of Income Maintenance Laws on Fertility in the United States*, 117-72, Institute for Research on Poverty, The University of Wisconsin, Madison, 1972.
- and H. Watts (eds.), *Income Maintenance and Labor Supply: Econometric Studies*, Markham Press, Chicago, forthcoming.
- Cogan, J., *Labor Supply and the Demand for Children*, Ph.D. dissertation, University of California, Los Angeles, 1973.
- DaVanzo, J., *The Determinants of Family Formation in Chile, 1970: An Econometric Study of Female Labor Force Participation, Marriage, and Fertility Decisions*, R-830-AID, The Rand Corporation, August 1972a.
- , *An Analytical Framework for Studying the Potential Effects of a. Income Maintenance Program on U.S. Interregional Migration*, R-1081-EDA, The Rand Corporation, December 1972b.
- , D. N. De Tray, and D. Greenberg, *Estimating Labor Supply Responses: A Sensitivity Analysis*, R-1373-OEO, The Rand Corporation, forthcoming.
- De Tray, D. N., *The Interaction Between Parent Investment in Children and Family Size*, R-1003-RF, The Rand Corporation, April 1972a.

- , *A Conceptual Basis for the Study of Welfare Reform Effects*, R-1066-DOC, The Rand Corporation, October 1972b.
- , "Child Quality and the Demand for Children," *Journal of Political Economy*, Vol. 81, No. 2, Supplement, March/April 1973.
- Fisher, I., *The Theory of Interest*, The Macmillan Company, New York, 1930.
- Ghez, G., *A Theory of Life Cycle Consumption*, Ph.D. dissertation, Columbia University, 1970.
- Greenberg, D. H., *Estimating the Supply Population for Public Employment Programs*, P-4780, The Rand Corporation, February 1972.
- and M. Kosters, *Income Guarantees and the Working Poor: The Effect of Income Maintenance Programs on Hours of Work of Male Family Heads*, R-579-OEO, The Rand Corporation, December 1970.
- Gronau, R., *The Wage Rates of Women - A Selectivity Bias*, National Bureau of Economic Research, September 1972 (mimeo.).
- , "The Effect of Children on the Housewives' Value of Time," *Journal of Political Economy*, Supplement, March/April 1973.
- Grossman, M., *The Economics of Joint Production in the Household*, Center for Mathematical Studies in Business and Economics, Report 7145, University of Chicago, September 1971.
- , "On the Concept of Health Capital and the Demand for Health," *Journal of Political Economy*, Vol. 80, No. 2, March/April 1972.
- , "Unemployment and Consumption: Note," *American Economic Review*, Vol. LXIII, No. 1, March 1973.
- Haveman, R. and R. Lampman, *Two Alternatives to FAP's Treatment of the Working Poor*, #94-71, Institute for Research on Poverty, University of Wisconsin, 1971 (mimeo.).
- Heckman, J., "Shadow Prices, Market Wages, and Labor Supply," *Econometrics* (forthcoming).
- Lancaster, K. J., "A New Approach to Consumer Theory," *Journal of Political Economy*, Vol. LXXV, April 1966, pp. 132-157.
- Leibowitz, Arleen, "Home Investments in Children," *Journal of Political Economy* (forthcoming).
- Maurer, K., R. Ratajczak, and T. P. Schultz, *Marriage, Fertility, and Labor Force Participation of Thai Women: An Econometric Study*, R-829-AID/RF, The Rand Corporation, April 1973.

- Michael, R. T., *The Effects of Education on Efficiency in Consumption*, Ph.D. dissertation, Columbia University, 1969.
- Mincer, J., "Labor Force Participation of Married Women," in H. Gregg Lewis, ed., *Aspects of Labor Economics, Universities-National Bureau Conference Series 14*, Princeton University Press, 1962.
- Muth, R. F., "Household Production and Consumer Demand Functions," *Econometrica*, July 1966.
- Nerlove, M. and S. J. Press, *Log-Linear and Logistic Models*, R-1306-EDA, The Rand Corporation, forthcoming.
- Nerlove, M. and T. P. Schultz, *Love and Life Between the Censuses: A Model of Family Decisionmaking in Puerto Rico, 1950-1960*, RM-6322-AID, The Rand Corporation, September 1970.
- Reischauer, R. D., *The Impact of the Welfare System on Black Migration and Marital Stability*, Ph.D. dissertation, Columbia University, 1971.
- Smith, J. P., *Labor Supply Decisions in the Life Cycle*, Ph.D. dissertation, University of Chicago, 1972.
- , *Family Decisionmaking over the Life Cycle: Some Implications for Estimating Labor Supply*, R-1121-EDA, The Rand Corporation, November 1973.
- , *Assets and Labor Supply over the Life Cycle*, The Rand Corporation, forthcoming.
- Watts, H. W., *Adjusted and Extended Preliminary Results from the Urban Graduated Work Incentive Experiment, #69-70*, Institute for Research on Poverty, University of Wisconsin, June 1970.
- Willis, R. J., "A New Approach to the Economic Theory of Fertility," *Journal of Political Economy*, Vol. 81, No. 2, Part II, March/April 1973.