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ABSTRACT

The standard one-period labor supply model that economists have used is in some ways an inadequate tool to evaluate a Family Assistance Plan (FAP). The principal difficulty is that an FAP will have important interperiod or life cycle effects. The pure life cycle model, an extension of the work of Becker and Ghez, is derived here without reference to its implications for an FAP. The timing of market participation is shown to depend upon the life cycle wage pattern of men and women, the rate of interest and the rate of time preference, and any age-related changes in the productivity of nonmarket uses of time. A comparison of the predictions of the pure life cycle and pure oneperiod models attempts to clarify circumstances under which the life cycle model should be used and those under which the single-period model is appropriate. The theoretical model is then used to predict and analyze the expected labor supply effects of an FAP. Finally, human capital investments are included in the model. The final section contains an empirical simulation of the predicted effects of an FAP on the hours behavior of men and women. These results suggest what the theory itself implied--a much larger effect of an FAP on the work behavior of married women than on the work behavior of male heads of households. (Author/JM)

FAMILY DECISIONMAKING OVER THE LIFE CYCLE: SOME IMPLICATIONS FOR ESTIMATING THE EFFECTS OF INCOME MAINTENANCE PROGRAMS

PREPARED FOR THE ECONOMIC DEVELOPMENT ADMINISTRATION AND THE OFFICE OF ECONOMIC OPPORTUNITY

JAMES P. SMITH

R-1121-EDA/OEO

NOVEMBER 1973

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Rand
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PREFACE

Over the past few years the Rand Corporation has been developing a framework to assess the effects of different income maintenance plans. This report is one of several that have stemmed from this effort. It was co-sponsored by the Office of Economic Opportunity under Grant 90088-D-72-01 and by the Economic Development Administration, Department of Commerce, under Grant OER388-G-71-11. It develops a new conceptual framework for analyzing the potential economic effects of income maintenance programs and is based on the author's Ph.D. dissertation. This model differs from previous efforts by economists in that it takes specific account of life-cycle variations in family earnings profiles. When family income varies from period to period, the traditional one-period model that economists have used to assess the effects of income maintenance plans may not be adequate. Although this model has not yet been applied directly to the assessment of different income maintenance plans, it should play an important role in future work in that area.

Other Rand studies published under these two grants and related to the present work are:

David H. Greenberg, *Problems of Specification and Measurement: The Labor Supply Function*, R-1085-EDA, December 1972.

Julie DaVanzo and David H. Greenberg, *Suggestions for Assessing Economic and Demographic Effects of Income Maintenance Programs*, R-1211-EDA, June 1973.

Dennis N. De Tray, *A General Economic Framework for Welfare Reform Analysis*, R-1346-OEO (forthcoming).

Julie DaVanzo, Dennis N. De Tray, and David H. Greenberg, *Estimating Labor Supply Functions: A Sensitivity Analysis*, R-1372-OEO (forthcoming).

T. P. Schultz, *The Estimation of Labor Supply Functions for Secondary Workers*, R-1265-NIH/EDA (forthcoming).

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SUMMARY

The standard one-period labor supply model that economists have used is in some ways an inadequate tool to evaluate a Family Assistance Plan (FAP). The principal difficulty is that an FAP will have important interperiod or life cycle effects. In the standard model, which contains just one time period, interperiod effects are ignored. The one-period model is appropriate only when the proposal being investigated does not alter the incentives to substitute economic activity between time periods. But an FAP will typically change an individual's wages by different percentage amounts at different points in his life cycle, providing him with incentives to alter the timing of his market participation. Observing the change in labor supplied in only one period can give a misleading indication of the *total* effect of an FAP. For the purpose of studying an FAP, a complete model of labor supply must incorporate its effects on the timing of market responses. Recent contributions by Ghez and Becker have permitted inclusion of the timing aspects in an economic model of choice. By extending Becker's original one-period model to a lifetime context, Ghez and Becker were able to place in sharp focus the previously neglected influence of cyclical, seasonal, and life cycle movements in wage rates and other variables.

The pure life cycle model, an extension of the work of Becker and Ghez, is derived here without reference to its implications for an FAP. The timing of market participation is shown to depend upon the life cycle wage pattern of men and women, the rate of interest and the rate of time preference, and any age-related changes in the productivity of nonmarket uses of time. A comparison of the predictions of the pure life cycle and pure one-period models attempts to clarify circumstances under which the life cycle model should be used and those under which the single-period model is appropriate.

The theoretical model is then used to predict and analyze the expected labor supply effects of an FAP. The effects of an FAP partly reflect life cycle considerations and partly the more standard one-period model. The appropriate model to use, a marriage of the two

pure special cases, shows that it is essential to identify those periods in the family life cycle when the family is eligible for benefits and those in which it is not. The use of the one-period model has probably led researchers to underestimate the magnitude of the labor market withdrawals in those years in which the family is eligible for benefits.

Finally, human capital investments are included in the model. This generalization leads to a number of predictions concerning which groups in society are most likely to have the largest labor supply reactions to an FAP. For example, economic theory suggests that young married women and individuals in older families will exhibit larger reductions in their market work than other groups in society.

The final section contains an empirical simulation of the predicted effects of an FAP on the hours behavior of men and women. These results suggest what the theory itself implied--a much larger effect of an FAP on the work behavior of married women than on the work behavior of male heads of households. For families where investments are important, the reduction in male market work becomes very small and one cannot exclude the possibility that their market hours will actually increase.

ACKNOWLEDGMENTS

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CONTENTS

I. NTRODUCTION

Perhaps the most important piece of social legislation to be considered recently by the Congress was the Family Assistance Plan (FAP) proposed by the Nixon Administration. Although this particular bill was defeated, it is likely that some form of FAP will be submitted to future legislatures. In some ways an FAP is a radical departure from existing public policy, and a large proportion of current policy research deals with the direct and indirect behavioral consequences of this and other income transfer legislation. One behavioral response that has received considerable attention is the labor supply effect. Because an FAP contains negative income tax elements, many felt that it might seriously disrupt work incentives and lead to a large reduction in the work effort of new welfare recipients. Since economists could offer a well-developed theory dealing with the labor s·pply aspects, it was natural that they would play a central role in designing and evaluating alternative proposals.

Economists offered the standard textbook model of an individual choosing between labor and leisure[1] in which an individual derives utility from the consumption of market goods and his own leisure. Conceptually, the opportunity cost of an hour of leisure is the amount of market goods forgone by consuming an additional hour of leisure; the price of leisure thus equals the real wage rate. Because there are income and substitution effects operating in conflicting directions, an increase in the real wage has an ambiguous effect on the number of hours worked. As long as leisure is a normal good, the larger income associated with a higher wage would tend to reduce market work. The higher wage has also made the consumption of leisure more expensive and, on this substitution effect, market hours will increase.

An FAP includes two dimensions that could alter an individual's labor supply: First, there is an income subsidy to recipients that

[1]The classic exposition of this model in terms of the income and substitution effects of a wage change is contained in Robbins (1930).

increases real wealth; and second, the program imposes a tax on market earnings. Since the FAP simultaneously increases real income and lowers the net wage received from working, economists using the standard model have predicted that enactment of such a proposal would lower the amount of market work of those individuals participating in the program.[1] The unambiguity in the theory has unfortunately not been matched by similar success in the empirical work already completed by economists. In their book summarizing a number of empirical studies of income maintenance programs, Cain and Watts emphasize the wide range of labor supply reactions estimated in these studies.[2] Many of the criticisms and suggestions for improving th se estimates have centered either on increasing the quality of the data or on using a more sophisticated econometric technique. There has been almost no questioning of the appropriateness of the theoretical model.

The standard model is in some ways an inadequate tool to evaluate an FAP and may give misleading or incorrect predictions on labor supply effects. One difficulty is that the standard model contains just one time period, but income transfer programs generally have important interperiod or life cycle effects. The one-period model is appropriate only when the proposal being investigated operates so that it does not alter the incentives to substitute economic activity between time periods. But an FAP will typically change an individual's wages by different percentage amounts at different points in his life cycle, providing him with incentives to alter the timing of his market participation. Observing the change in labor supplied in only one period may give a misleading indication of the *total* effect. To study an FAP, a complete model of labor supply should incorporate the effect on the timing of market responses. Recent contributions by Ghez and Becker[3] permit inclusion of timing in an economic model of choice. By

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[1] This statement is a comparison between an FAP and no welfare system, and not a comparison between an FAP and the existing welfare system.

[2] For a discussion and comparison of a number of labor supply studies by economists, see Cain and Watts (1973).

[3] See Ghez (1970) and Becker (1969).

extending Becker's original one-period model to a lifetime context, Ghez and Becker were able to place in focus the previously neglected influence of cyclical, seasonal, and life cycle movements in wage rates and other variables.

I first derive the pure life cycle model without reference to its implications for an FAP. This model is an extension of the work of Becker and Ghez. The timing of market participation is shown to depend upon the life cycle wage pattern of men and women, the rate of interest and the rate of time preference, and any age-related changes in the productivity of nonmarket uses of time. The predictions of the pure life cycle and pure one-period model are contrasted to clarify those circumstances under which the life cycle model should be used and those under which the single-period model is appropriate.

In Section III, I use the theoretical model of Section II to predict and analyze the expected labor supply effects of an FAP. The effect of an FAP partly reflects life cycle considerations and partly the more standard one-period model. The appropriate model to use is a marriage of the two pure special cases. Particular emphasis is given to those instances in which the predictions of this new model diverge from those obtainable in the model currently used.

This new model shows that one must identify those periods in the family life cycle when the family will be eligible for benefits and those in which it is not. In those years in which the family is eligible for benefits, the use of the one-period model has probably led researchers to underestimate the magnitude of the labor market withdrawals. Later in this section, I include human capital investments in the model, which leads to a number of predictions concerning which groups in society are most likely to have the largest labor supply reactions to an FAP. For example, economic theory suggests that young married women and old people will reduce their market work more than other groups in society.

Another aspect of the model is the use of the family context. An individual's decision about the number of hours to exchange for market dollars is often made in a family context.[1] Hence, the hours of work of any family member depend not only on his wage and other variables specific to him, but also on similar variables of other members and on those variables common to the family un[illegible] this model of the family, a fundamental assumption is that the [illegible]on of labor within a family is determined in large part by economic forces. Relative productivity differences of males and females in both the market and the household sectors supply the incentives for families to concentrate the time of a member in the sector of his comparative advantage.

Ignoring this family context could lead to serious errors in predicting the effects of an FAP. Current research has typically selected one family member and attempted to estimate the magnitude of the reduction in market work encouraged by changing the incentives that he alone faces. In a family that participates in an FAP, however, both the male and female wages are reduced simultaneously. The model developed in this study explicitly allows the work incentives of all members to change. Part of the incentive to substitute in favor of male time in the household is offset because the earnings females can receive by additional market work has been reduced. With this model, a strong *a priori* case can be made that the main avenue of the reduction in a family's work effort will be a large decrease in hours worked by female household members. Also, I show that an FAP generally will have a greater effect on the true wage of females than on the wage of males. Studies that have concentrated on the reallocation of market time of either males or females separately are giving policymakers seriously biased estimates of the labor supply effects of an FAP.

Section III also contains an empirical simulation based on some estimates of the life cycle labor supply equations. These estimates provide limits on the values of the household production and intertemporal utility functions. Based on these results, predicted responses to an FAP are given for males and females for eligible and noneligible periods.

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[1] Jacob Mincer pioneered in the treatment of issues of labor supply in a family context. See Mincer (1962); see also Cain (1966).

II. THE LIFE CYCLE MODEL

In deciding on the number of hours each member should supply to the market, a family is actually confronted with two problems. Given the long-run or permanent values of family wealth and the wages of the individual members, the family must determine the lifetime levels of market time of each of its members. In addition, since the family is faced with temporal variations in wages and other variables, a decision must be made concerning the optimal timing of hours of the individual members. The intensity of market participation is not constant over the cycle, because factors are present that change the demand for commodities and the marginal costs of household production.

In the framework of the one-period model, the variables that determine the levels of market participation are long-run or permanent measures of wage rates and wealth. This model is best suited to predict average lifetime participation rates. But individuals are also confronted with temporal variations in wage rates and other variables that could elicit timing responses about the long-run levels desired. This variation is outside the scope of the single-period model, since changes in economic conditions between time periods are by definition excluded. The distinction between the life cycle and single-period model is illustrated in Figure 1. Curves 11 and 22 are the life cycle market hours paths for two individuals (or groups of individuals). In any sample, we will simultaneously observe individuals at different points on a given "profile" of hours worked (for example, at points D or E on 11) and also individuals at the same point in the life cycle but on different profiles (for example, the change from D to F). Conceptually, then, we may divide the total sample variation in market hours into (1) the life cycle variation, which corresponds to movements along a given profile (from D to E and F to G); and (2) the permanent variation, which measures variation across different profiles (from D to F and E to G). The purpose of the one-period model is to explain the second component--variation reflecting permanent or average lifetime differences among individuals in the levels of their market participation.

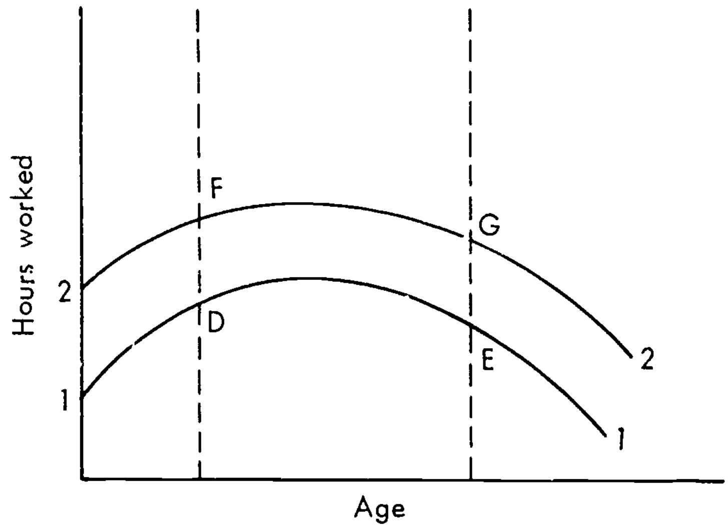

Fig. 1 — Comparison of one-period and life cycle models

The goal of the life cycle model is to account for the intertemporal dispersion about the average levels.

HOUSEHOLD PRODUCTION MODEL

In the derivation of labor supply, I shall be using the household production model popularized by Gary Becker. It provides a theoretical framework in which one may analyze family labor supply issues. The family is viewed as if it were a small firm producing its ultimate wants within the household. To satisfy these wants, the family (firm) purchases market goods and services as one input in the production process. Following tradition, the ultimate wants desired by the family are called "commodities," and the purchased market inputs are labeled "goods." The novelty and content of the household model come from the assumption that purchased market goods are not the sole inputs used by the family. Instead, households combine market goods with the time of various family members to achieve more basic desires. Families are therefore implicit demanders of their own scarce time resource. To illustrate: The commodity "enjoyment of a play" is ultimately consumed by directly purchasing such market goods as a theater ticket, travel to and from the theater, and babysitting services. Also, a considerable amount of time (which has alternative uses) of those family members involved is used up during the consumption activity and must properly be viewed as part of the full cost of consumption. Similarly, child services are consumed and produced by a vector of purchased medical and other services and with considerable inputs of male and, especially, female time.

This approach differs from the traditional one since the price of *any* activity now has two components--the money goods price and the time price. The time price has been generally neglected in the traditional approach, but both receive equal prominence in the theoretical structure of the new model. The relative empirical importance of the two components depends on their shares in the cost of producing an activity.

THE DERIVATION

In the remainder of this section, I develop a family life-cycle model from which derived demand equations are obtained giving the amount of time required in home production in every period for each family member. At any moment in time, the family must combine market goods and time to minimize the cost of obtaining the desired bundle of commodities. However, utility maximization requires more than simply consuming the optimal bundle of commodities in each period. The consuming unit must also allocate its consumption over time in a manner consistent with its tastes for commodities in future periods and the expected prices of these future commodities relative to present prices. Combining the intertemporal-utility maximization problem with that of the least-cost combinations of the inputs of time and goods to use in each period yields some interesting and testable predictions concerning the market-hours behavior of individual family members over time.[1]

Assume for simplicity that the intertemporal utility function of a family that has a horizon of n periods (equal to its life span) has constant intertemporal elasticity of substitution;[2] that is,

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[1]The following model was developed in Smith (1972). It relies on the work of Ghez and Becker (1972).

[2]One may be damned or saved by his assumptions of functional form. Two properties of the constant elasticities of substitution are unitary

$$(1) \qquad U = \left(\sum_{t=1}^{n} a_t \, Z_t^{\frac{\sigma_c - 1}{\sigma_c}} \right)^{\frac{\sigma_c}{\sigma_c - 1}},$$

where U is family utility, Z_t is the level of consumption of commodities in period t, time preference parameters are a, and σ_c is the intertemporal elasticity of substitution in consumption. Z is produced within the household by using purchased market goods (X_t) and time inputs of the husband (M_t) and wife (F_t):

$$(2) \qquad Z_t = B_t f(X_t, M_t, F_t),$$

where f is assumed to be homogeneous of degree one and B_t is a parameter that allows the efficiency of household production to change over time. The family is faced with both time and money constraints that can be written (using the price of market goods as numeraire) as

$$(3a) \qquad M_t + N_{mt} = F_t + N_{ft} = T, \quad t = 1, 2, \ldots, n$$

$$(3b) \qquad \sum_{t=1}^{n} \frac{X_t}{1 + r)^t} = \sum_{t=1}^{n} \frac{w_{mt} N_{mt} + w_{ft} N_{ft}}{(1 + r)^t} + A_0 .$$

The time constraint (3a) indicates that the total amount of time available to each family member (T, a given) in every period is absorbed either in the household production process or in hours at work (N_{mt} and N_{ft}).[1] Equation (3b) states that the discounted value of money expenditures on goods is equal to the discounted market earnings of

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wealth elasticities and weak separability. By weak separability, I mean that the marginal rate of substitution between commodities in any two periods is independent of the levels of consumption in all other periods.

[1]Throughout the derivation of the model, I assume that all solutions are interior ones.

both the husband and the wife and initial property wealth (A_0). The interest rate used in discounting is r, w_m is the husband's wage, and w_f is the wife's wage. The two constraints combine easily:

$$(4) \qquad R = \sum_{t=1}^{n} \frac{\pi_t Z_t}{(1 + r)^t} ,$$

where

$$R = T \sum_{t=1}^{n} (w_{mt} + w_{ft})(1 + r)^{-t} + A_0$$

is Becker's "full wealth" concept, and

$$\pi_t = (X_t + w_{mt} M_t + w_{ft} F_t)/Z_t$$

is the average or unit cost of production of Z_t. When π_t is minimized, it is independent of Z_t and therefore is the marginal cost or shadow price of Z_t.

By full or potential wealth I mean the maximum market earnings the family would receive if both the husband and the wife spent all their time in the market sector.[1] This potential wealth is spent in two ways. Some wealth is absorbed by purchasing market goods. The rest is implicitly spent by buying the time of the husband and wife from the market, for the purpose of home production. Although this latter expenditure will not appear in national income accounts, it is a real part of the cost of consumption. Notice that in computing the shadow price (π_t), the market goods component is treated symmetrically with the forgone earnings component of the husband ($w_{mt}\ M_t$) and the wife ($w_{ft}\ F_t$).

Equations (1), (2) and (4) constitute the complete structure of the model. It is assumed that the family desires to maximize lifetime

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[1]This definition of full wealth holds only if one ignores investment in human capital.

utility (1) subject to the production function (2) and the wealth constraint (4).

This problem is easily solved with a two-stage optimization procedure. First maximize utility (1) subject to the budget restraint (4) with the prices π_t taken as given. The result of this maximization is the demand function (or consumption function) for the basic commodity at each age (t):

$$(5) \qquad Z_t = R\left(P^{\sigma_c - 1}\right)\pi_t^{-\sigma_c}(1 + r)^{\sigma_c t} a_t^{\sigma_c} ,$$

where

$$P = \left[\sum_{t=1}^{n} \frac{\pi_t}{(1 + r)^t}^{1-\sigma_c} a_t^{\sigma_c}\right]^{\frac{1}{1-\sigma_c}}$$

is the lifetime "price index" of the basic commodity.[1]

Equation (5) states that family real consumption (Z_t) at age t will be greater: (1) the greater is its lifetime real income or wealth (R/P); (2) the smaller is the price (π_t) of the commodity at age t *relative* to its "lifetime price" (P); (3) the greater is the age (t) (assuming that the interest rate is positive); (4) the greater is the interest rate (r); and (5) the greater is the preference (a_t) for consuming at that age relative to other ages. The relevant wealth or income concept is that of lifetime real income rather than income at age t as conventionally measured; the relevant price is the price at age t relative to the "average" lifetime price (P).

Solving for the percentage change in consumption from one period to the next, we have

$$(6) \qquad \frac{dZ_t}{Z_t} = -\sigma_c \frac{d\pi_t}{\pi_t} + \sigma_c(r - \alpha) ,$$

where α is an index of time preference indicating whether the family

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[1]The derivation of this price index is shown in Appendix A.

has a time preference for the present ($\alpha > 0$), for the future ($\alpha < 0$), or a neutral time preference ($\alpha = 0$).[1]

Note that the full-wealth term (R/P) drops out when we consider changes in the levels of consumption over time. If people have unbiased expectations about future earnings, then the level of full wealth does not change. With these assumptions, a person's full wealth will not affect the change in consumption from one period to the next.

From Equation (6). if the interest rate is zero and the family has neutral time preference ($a_t = a_{t+1}$), equal consumption levels will occur if the prices in all periods are identical. *A fortiori*, increasing prices in the future implies $Z_{t+1} < Z_t$, or that commodity consumption will be declining in future periods. Because it lowers the relative price of future commodities, a positive interest rate will raise future consumption levels.

The second step in maximizing lifetime utility involves minimizing the price π_t (average and marginal cost of production of Z_t) at each age t. This minimization of the cost of production leads to the derived demand functions for the household time (M_t and F_t) of the husband and wife. The well-known properties of these functions can be stated in differential equation form.

With a relation from the theory of derived demand, at cost minimization the following holds for the inputs of the husband and wife, where σ_{ij} is the Allen partial elasticity of substitution between inputs i and j:[2]

$$(7) \qquad \frac{dM_t}{M_t} = \frac{dZ_t}{Z_t} - (S_{F_t}\sigma_{MF} + S_{X_t}\sigma_{MX})\frac{dw_{mt}}{w_{mt}} + S_{F_t}\sigma_{MF}\frac{dw_{ft}}{w_{ft}} - \frac{dB_t}{B_t} .$$

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[1] $\alpha = \log (a_t/a_{t-1})$ is the index of time preference. It is defined in the conventional sense of the slope of the indifference curves at equal consumption levels.

[2] A derivation is given in Allen (1967).

The percentage change in the price of a commodity (π_t) may be expressed as

$$(8) \qquad \frac{d\pi_t}{\pi_t} = S_{M_t}\left(\frac{dw_{mt}}{w_{mt}} - \frac{dB_t}{B_t}\right) + S_{F_t}\left(\frac{dw_{ft}}{w_{ft}} - \frac{dB_t}{B_t}\right) - S_{X_t}\frac{dB_t}{B_t}$$

$$\frac{d\pi_t}{\pi_t} = S_{M_t}\frac{dw_{mt}}{w_{mt}} + S_{F_t}\frac{dw_{ft}}{w_{ft}} - \frac{dB_t}{B_t} \quad .$$

According to Equation (8) any percentage change in the cost of household production may be expressed as a weighted average of the percentage changes in the *real* costs of inputs with the weights being the share of these inputs in the total cost of home production. A change in the real opportunity cost of an hour of home time is the difference between any changes in the market wage rate (w_t) and changes in the efficiency of an hour of time in home production (B_t). For example, if market wages of men and women increase by one percent and the efficiency of their time at home also increases by one percent, the real cost of another hour of time in the household sector remains unchanged. The additional market earnings forgone by spending another hour at home are offset by the additional home output secured. The input shares are used as weights in Equation (8) because they measure the importance of each input in the household sector.

Substituting (8) into (6) and then (6) into (7) yields the derived demand equations for husbands' and wives' home time and market goods.

$$(9) \qquad \frac{dM_t}{M_t} = -(S_{M_t}\sigma_c + S_{F_t}\sigma_{MF} + S_{X_t}\sigma_{MX})\frac{dw_{mt}}{w_{mt}}$$

$$+ S_{F_t}(\sigma_{MF} - \sigma_c)\frac{dw_{ft}}{w_{ft}} + \sigma_c(r - \alpha) + (\sigma_c - 1)\frac{dB_t}{B_t} \quad .$$

$$(10) \qquad \frac{dF_t}{F_t} = -(S_{F_t}\sigma_c + S_{M_t}\sigma_{MF} + S_{X_t}\sigma_{FX})\frac{dw_{ft}}{w_{ft}}$$

$$+ S_{M_t}(\sigma_{MF} - \sigma_c) \frac{dw_{mt}}{w_{mt}} + \sigma_c(r - \alpha) + (\sigma_c - 1) \frac{dB_t}{B_t} .$$

$$(11) \qquad \frac{dX_t}{X_t} = S_{M_t}(\sigma_{MX} - \sigma_c) \frac{dw_{mt}}{w_{mt}} + S_{F_t}(\sigma_{FX} - \sigma_c) \frac{dw_{ft}}{w_{ft}}$$

$$+ \sigma_c(r - \alpha) + (\sigma_c - 1) \frac{dB_t}{B_t} .$$

Equations (9) and (10) indicate that the hours of work of each family member, given the parameters of the utility and production function, are determined by variations in the price of time of both members, the rate of interest, the time preference, and any changes during the life cycle in the efficiency of home production.

To illustrate: As the real wage of the wife increases over the life cycle, the amount of her time spent in the household will decline for two reasons. Because the price of one of the inputs is rising, the relative price of future commodities has risen. The resulting decline in future consumption will, on this "scale" effect, reduce the demand of wife's home time. The magnitude of this effect (represented by $S_{F_t} \sigma_c$) depends on the possibilities for intertemporal substitution (the larger σ_c, the more elastic is the demand curve for commodities) and the share in total costs of the wife's time. In addition to this intertemporal substitution between commodities, there is the possibility of substitution in the production process. As w_{ft} increases, the wife's time will be substituted against by the other two inputs. This effect ($S_{M_t} \sigma_{MF} + S_{X_t} \sigma_{FX}$) will also lead to a decline in the use of the wife's time as her real wage rises.[1] It follows that in those periods when the real wage of the wife is high, the model predicts that her hours of market work will also be high, other things equal. In contrast to

[1]We know that $S_{F_t} \sigma_{FF} + S_{M_t} \sigma_{MF} + S_{X_t} \sigma_{FX} = 0$, and σ_{FF} is necessarily less than zero. Hence ($S_{M_t} \sigma_{MF} + S_X \sigma_{FX}$) is positive. For a proof of these statements, see Allen (1967).

the traditional one-period labor-leisure choice, the sign of this effect is unambiguous. Since full wealth is fixed in this analysis, there are no income effects. It is the income effects in the static theory that give rise to the possibility of a negatively sloped supply curve of hours.

As the real wage of the husband varies over his lifetime, the effect on hours worked by his wife is again determined by the two avenues of substitution. Increases in the price of his time will also raise the price of future commodities and induce a fall in the use of all inputs including the time of his wife. However, in the production process, the relative price of the wife's time is declining, and F_t per unit of output will increase if the two time-inputs are substitutes ($\sigma_{MF} > 0$). Thus, the behavior of hours of work of the wife is ambiguous with respect to the wage of the husband. If commodity substitution swamps substitution in production ($\sigma_c > \sigma_{MF}$), the wife's market hours will increase as her husband's real wage rises. The roles of a positive interest rate and the degree of time preference are the standard Fisherian ones. A positive interest rate (by lowering discounted prices) and time preference for the future will increase future consumption and decrease hours of work of all family members.

The interpretation of the term (dB_t/B_t) is an interesting one. Since the type of technical change is of the Hicks neutral variety, a one percent improvement in efficiency will lower future prices by one percent and increase the amount consumed in the future. The effect on the use of inputs is uncertain because input requirements per unit of output have also declined by one percent. Whether time at home increases with an improvement in the efficiency of home time depends on whether the elasticity of demand for commodities is greater or less than one ($\sigma_c - 1 > 0$).

THE ADVANTAGES OF THE HOUSEHOLD MODEL

Do any of these predictions depend on the use of the household production framework? An alternative and more traditional model would let the level of utility in any period depend *directly* on goods purchased in the market and on the leisure time of the husband and the wife. In this latter approach, which I label the direct utility model, the intertemporal utility function would be an appropriately discounted sum of these single-period functions. Although the language used to describe the economic forces at work would differ, the predictions for the age and wage variables are identical in the household production and the direct utility models. In the direct utility model, for example, one would describe the increase in female hours worked that results from an interperiod increase in her wage as follows: (1) her leisure time is more expensive in that period relative to the cost of her leisure in other periods, and (2) her leisure time is more expensive relative to the cost in that period of her husband's leisure and market goods. In view of the similarity in predictions, what does one gain from using the less familiar language of the household model? The advocates[1] of the household model argue that their approach is superior because it generalizes the important role of the price of time. In the direct utility model, the only price altered directly by a change in the wage rate is the price of leisure. In the household model, the wage rate directly affects the price of a wide variety of activities in proportion to the importance of time in the activity. The gain from this generalization cannot be appreciated in the single-equation model that I have derived above. It is in a more complete simultaneous equation system that this advantage of the household model is apparent. For example, the number of children has usually been included as an explanatory variable in the empirical work estimating labor supply relations. But in the direct utility model, the justification for including children is somewhat *ad hoc* since leisure is the only activity in the theory that absorbs time. Children can easily be introduced in the household model simply by letting one of the desired commodities (Z) be child services and assuming that this Z is relatively time intensive for the wife.

The second advantage of the household model is that it permits one to include in the theory the effect on market work of variation in the

[1] A clear statement of the household model and a test of role of nonmarket efficiency is contained in Michael (1972).

nonmarket efficiency of time (B_t). In the direct utility model, this variation would appear as taste changes in the utility function, and it is difficult to relate variation in taste to measurable variables. But nonmarket efficiency may in principle be attributed to a number of individual characteristics--education, amount of home time experience, and duration of marriage. Of course, the process of refining empirical measures of B_t is only beginning, and it is certainly not going to be an easy task. As progress is made on this front, the value of the household model will increase.

THE SINGLE PERIOD MODEL--A COMPARISON

It is instructive to compare these life cycle demand equations with the more standard equations that one can derive from the one-period model.[1] I have shown in Appendix A that the change in the demand for the nonmarket time of men and women across families at any age t may be written:

$$\text{(12)} \qquad \frac{dM_t}{M_t} = \eta_t \frac{dA_0}{A_0}\frac{A_0}{R} + \left[\eta_t \frac{\overline{E}_M}{R} + S_{M_t}\sigma_{MM}\right]\frac{dw_{mt}}{w_{mt}} + \left[\eta_t \frac{\overline{E}_F}{R} + S_{F_t}\sigma_{MF}\right]\frac{dw_{ft}}{w_{ft}} \; ;$$

$$\text{(13)} \qquad \frac{dF_t}{F_t} = \eta_t \frac{dA_0}{A_0}\frac{A_0}{R} + \left[\eta_t \frac{\overline{E}_F}{R} + S_{F_t}\sigma_{FF}\right]\frac{dw_{ft}}{w_{ft}} + \left[\eta_t \frac{\overline{E}_M}{R} + S_{M_t}\sigma_{MF}\right]\frac{dw_{mt}}{w_{mt}} \; ,$$

where $\overline{E}_M$ and $\overline{E}_F$ are the discounted market earnings of males and females, and η_t is the full income elasticity of consumption of commodities at age t.

[1]In Appendix A, I derive the one-period model by assuming that the life cycle paths of wages and other variables are identical in shape within a given socioeconomic group. But among individuals within this group, the level of the wage profiles may vary. Therefore, if the wage of individual i exceeds the wage of individual j by λ percent at age t, i's wages will also exceeds j's by λ percent at all ages.

All other terms are defined as they were in the life cycle model. The economic forces operating in the one-period model may be illustrated by use of the demand equation for female time (13). According to Equation (13), differences in the level of female market time are caused by differences across families in initial assets (A_0) and age-specific wages of both spouses. As long as there is not complete home specialization by a household member, variation in initial assets involves only an income effect.[1] If all commodities are normal, then the sign of the asset term must be positive. An increase in initial assets due to increasing wealth will increase the demand for each commodity Z_t. To produce these additional commodities, the household will demand more time and goods. Therefore, a negative relation is predicted between *initial* assets and the level of female market time at each age.[2]

This asset term does not appear in the female life cycle demand equation (10). Variations in initial family assets levels correspond directly to variations in real wealth across families. These different levels of initial assets or real wealth should affect the level of market work at any age among families and thus the asset term is included in the one-period model (Equation 13). But different initial asset levels among families will not alter the choice of the amount of time in supply to the market at different ages. This is the reason for the absence of an asset term in the life cycle equation.[3]

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[1]This is not a harmless assumption. Robert J. Willis has shown that, in those families where women do not work, an increase in income will increase the shadow price of her time. The value of her time rises because of the increase in the ratio of other inputs (goods and male time) to her fixed household time. For a complete exposition of the model and an application to the economics of fertility, see Willis (1973).

[2]In most expositions of the one-period model, the initial assets term is usually labelled "other income." However, it is easy to convert the stock of assets A into a flow of nonlabor income I, since $I = r \cdot A$. The first term in Eq. (13) could be expressed as variation in other income flows rather than assets but the economics remains the same.

[3]For a treatment of the role of assets in a life cycle model, see Smith (forthcoming).

The coefficient of the female wage rate in the one-period model has the familiar substitution and income effects incorporated within it. It must be remembered that dw_{ft}/w_{ft} represents a quite different wage change in Equation (13) than in the life cycle equation. For the wage change in Equation (13) not to induce any change in the relative cost of time between time periods, the wage must change by the same percentage amount at each age.

An increase in the female wage rate by the same percent at each age will increase family real wealth in proportion to the share of the woman's lifetime market earnings in the family's total wealth $(\bar{E}_F/R)$. This rise in real wealth increases the demand for commodities (Z_t) and hence the derived demand for goods and time. The second part of the female coefficient $(s_F\sigma_{FF})$ is the substitution effect. With the higher female wage rate, families have an incentive to become less female time-intensive at home.[1] On this substitution effect, as male time and market goods are substituted for female time, the amount of female market work will increase at each age. The net effect on female market hours of an increase in the female wage rate depends on which of these two effects is stronger. If the income effect exceeds the substitution effect, age-specific female market hours will fall as female wages rise.

Income and substitution effects are also associated with a change in the male's wage. If male wages increase by the same percentage amount at each age, family income will rise in proportion to the share of the male's earnings in full wealth $(\bar{E}_M/R)$. This in turn increases the demand for commodities and all inputs, including the wife's home time. Because the male's time is more expensive, other inputs will be substituted for his home time. If the two time inputs are substitutes

[1] In a more general model, there would be another avenue of substitution open to the family--substitution between commodities in any time period. As the female wage rises, the household will substitute against female time-intensive commodities. This will also lead to an increase in female market time. Since I am interested only in the division between market and nonmarket time, it would be superfluous to complicate the model by allowing for more than one commodity per time period.

for one another ($\sigma_{MF} > 0$), both substitution and income effects operate to reduce female market time as the husband's wage increases. If the two inputs are complements ($\sigma_{MF} < 0$), then the wife's market hours will decrease only if the income effect is stronger than the substitution effect.

III. THE FAMILY ASSISTANCE PLAN MODEL

In this section, the theoretical framework developed in Section II is used to construct a model designed to capture the channels through which an FAP is likely to affect the number of hours worked by family members.[1] Economists have been using the one-period model to predict the labor supply effects of an FAP.[2] The conceptually appropriate framework is neither pure one-period nor pure life cycle, but rather one that contains elements of both models. Because an FAP alters the rate of exchange between time and market goods and increases a family's wealth, the one-period model incorporates some of the relevant economic factors at work. The life cycle aspect must also be included, because an FAP will change the cost of consuming at different ages, inducing families to reallocate their consumption and working patterns over time. This life cycle dimension had been ignored, which led to a number of errors involving the expected magnitude and direction of the labor supply effects.

The existing treatment of this policy issue has not recognized the importance of the family context. An FAP simultaneously alters the work incentives of all family members. However, researchers typically select one family member and attempt to estimate the magnitude of the reduction in market work that was encouraged by changing the incentives that person faces. The model developed here explicitly allows the work incentives of *all* family members to be changed by the FAP. It also permits study of the relative size of the reduced work effort by married men and women. With this model, a strong *a priori* case can be made that the largest labor force withdrawals will be those of married women, indicating a need to redirect empirical work, which formerly concentrated on male heads of households.

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[1] I would like to thank H. Gregg Lewis for his extensive comments that clarifed my thinking on many of the issues in this section.

[2] Two examples are Greenberg and Kosters (1970), and Green and Tella (1969).

Finally, investments in human capital are incorporated into the analysis. This extension raises three interrelated issues: (1) Are any of the predictions of the life cycle model substantially altered by including human capital investment? (2) Are any predictions changed or added concerning the labor supply effects of an FAP? and (3) Will an FAP affect the incentives to invest in human capital?

THE PROGRAM COMPONENTS

The program components associated with any specific FAP are generally complex, and income guarantees and marginal tax rates may vary substantially among households. A family's subsidy is influenced by its demographic characteristics (marital status, age, region, schooling, and family size), its willingness to work, and the form in which income is received. Also, FAPs have been tied to other social welfare programs such as food stamps, social security, and daycare services. This tends to affect both the level of support and the marginal tax rate on earnings. For analytical simplicity,[1] I reduce the FAP to its two simple but essential provisions: (1) Families with zero earnings receive an income payment of S dollars per year for each year they are eligible for benefits. (2) This payment is reduced by μ cents for each dollar of earnings in that year. The welfare payments (S_t) a family receives in year t are:[2]

$$S_t = S - \mu\,(E_{M_t} + E_{F_t})\ . \tag{14}$$

Obviously a family will not receive a subsidy if family earnings exceed S/μ. Many families may qualify for benefits only during some part of their life cycle and this will have important labor supply consequences. It is well established empirically that both individual and family

[1]For a discussion of these complexities, see De Tray (1972).

[2]In practice, most proposed forms of family assistance allow the family some earnings before they lose any of the subsidy. If Q is the maximum allowable earnings, the formula for computing benefits becomes $S_t = S - \mu(E_{M_t} + E_{F_t} - Q)$ if $E_{M_t} + E_{F_t} > Q$, $= S$ if $E_{M_t} + E_{F_t} < Q$.

earnings vary considerably with age. Consider a family that has the somewhat typical concave age-family earnings profile as seen in Fig. 2. This family will definitely be eligible for benefits between ages 0 and t_1, and after age t_2. But during some ages between t_1 and t_2 family earnings will be large enough so that participation in the program would no longer be beneficial.[1] It is analytically essential to distinguish between the periods when the family is eligible for payments and those when they are not.

The relatively flat age earning profiles observed for the less educated and the poor does not necessarily imply that these life cycle considerations are unimportant. Eligibility criteria other than income that are specific to a life cycle stage are often part of these programs. Moreover, the income relevant for eligibility is earned and not potential income. Many nonpoor will have low observed income at certain life cycle stages (college students, older workers near or after retirement). The observed income profiles for the poor are also averages over individuals and could suppress many instances of individuals in that group having profiles with more curvature. Finally, another characteristic of the poor is that there is income instability from one year to the next. Still, an unanswered question is the extent to which these life cycle factors will prove to be empirically important. The longitudinal data becoming available may be quite useful for this question. With earnings on individuals for many consecutive years, we can discover some of the characteristics of individuals who are likely to be temporary participants in the program.

Some income maintenance programs explicitly restrict participation to certain stages of the life cycle. The most obvious example is the social security program where age determines eligibility. Using a single-period framework is particularly inappropriate for this program. But other maintenance programs implicitly introduce age as a criterion for eligibility by restricting participation based on individual characteristics that are closely related to one's life cycle stage:

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[1]Families with incomes slightly higher than S/μ will also participate because they will react by lowering working hours, reducing their earnings below S/μ.

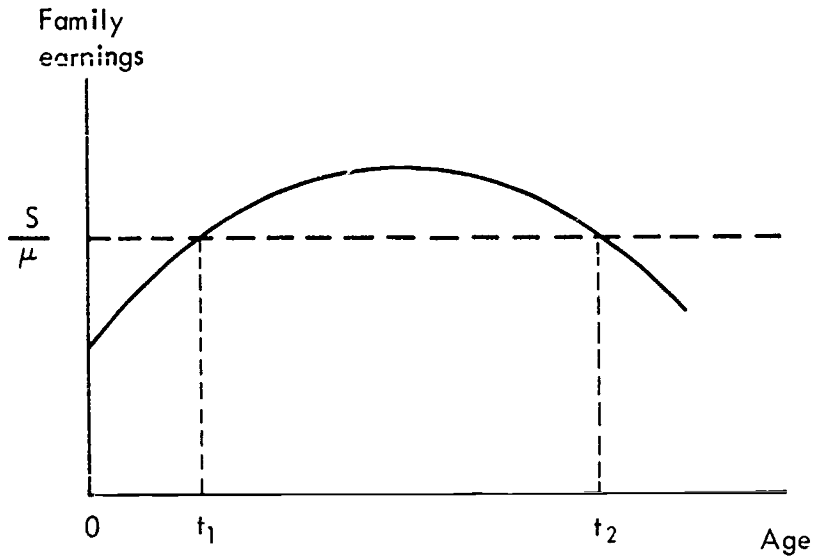

Fig. 2 -- Family Age-Earnings Profile

marital status, number or presence of children at home, school attendance or amount of net worth or assets. For example, in the FAP proposed by the Nixon Administration, to be eligible for benefits the family's net worth in certain forms could not exceed $1500. Even though age earnings profiles may be relatively flat for the poor, this is not in general true of the net worth age profiles.[1]

THE WEALTH EFFECT

In Appendix A, it is shown that for any family at age t, we may express the demand for male home time[2] as:

$$\text{(15)} \qquad \log (M_t) = \eta_t \log(\frac{R}{P}) + \sigma_c \log(P) + \eta_{mm} \log(w_{mt})$$

$$+ \eta_{mw} \log(w_{ft}) + \sigma_c (r - \alpha) t \ ,$$

where η_{mm} and η_{mw} are the elasticities of male home time with respect to the male and female wage. This particular formulation is useful

[1]For some documentary evidence on this, see Smith (forthcoming).

[2]Only the male equations are derived in the text. A simple relabelling of subscripts gives the female equations.

because it separates the determinants of demand into four categories: (1) a family's real wealth, (2) the price of consuming at one age relative to other ages, (3) the age-specific wages of men and women, and (4) the rate of interest and rate of time preference. An FAP is unlikely to alter interest rates or time preferences, but it will affect the other three categories. As long as payments are received at any time, the family's real wealth (R/P) will be increased. If a family is not eligible for a subsidy at every age, the cost of household production (P) will be lowered at ages when payments are received relative to other ages. Finally, in the periods of eligibility, the opportunity costs of time of both men and women (w_{mt} and w_{ft}) are lowered relative to market goods. Each factor may affect the supply of market hours of men and women, but it is convenient to have separate discussions of the effect of a change in real wealth (R/P) and all those changes attributed to substitution effects (either substitution in consumption between periods or substitution among factors of production).

If any year in which benefits are received, the undiscounted increase in real income is approximately the subsidy itself.[1]

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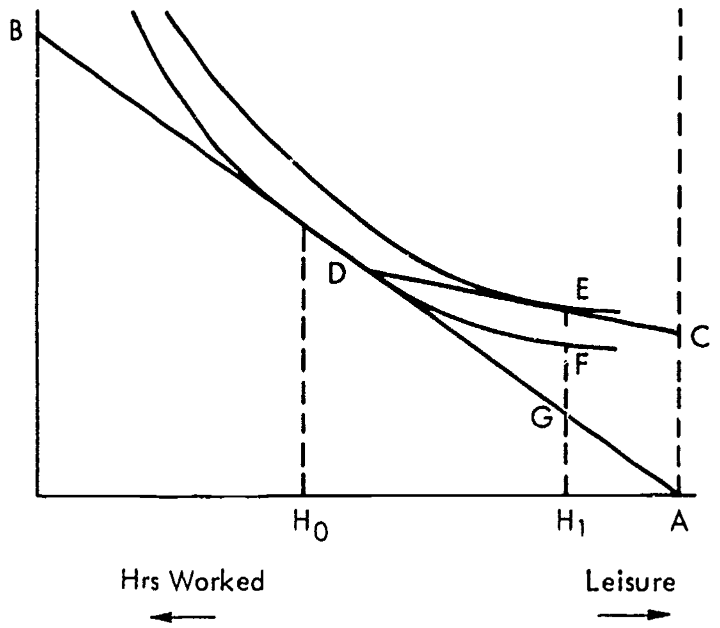
[1]Strictly speaking the subsidy overstates the increase in real income in that year. In the absence of the FAP, an individual faces budget constraint AB and elects to work h_0 hours. When the FAP is



(16) $$S_t = S - t(E_{M_t} + E_{F_t}) \; .$$

To evaluate the increase in lifetime wealth, it is necessary to know the ages when benefits are received and the subsidy at each age. Assume for simplicity that the family receives benefits only during time periods t_1 and t_2 and will not be eligible at any other age. The discounted value of these benefits (G) are

(17) $$G = \sum_{t=t_1}^{t_2} S_t \; (1 + r)^{-t} \; .$$

If $\bar{S}$ is the constant per-period subsidy that is equivalent in wealth terms to the flow of S_t over this time period, then

(18) $$G = \frac{\bar{S}}{r} \, \frac{1}{e^{rt_1}} \left(1 - \frac{1}{e^{rn}}\right) ,$$

where n is the number of periods in which benefits are received. The increase in lifetime wealth depends on a number of factors besides the subsidy. The increase in real wealth will be greater the earlier these benefits are received (the smaller t_1), the larger the number of time periods when benefits are received (n), and the larger the average per period subsidy ($\bar{S}$).

Even this simple representation illustrates some conceptual and statistical problems in the literature dealing with the wealth effect of an FAP. The difficulty involves discovering an empirical measure of wealth differences among individuals that corresponds to the change

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instituted, the relevant budget constraint becomes CDB and the individual now chooses to work h_1 hours. The amount of the subsidy he receives is EG. But if we took EG dollars away from this person he would actually be worse off (on a lower indifference curve) at h_1 hours than he was before the FAP. FE is the amount the subsidy overstates the true gain.

in wealth associated with an FAP. In the spirit of the one-period model, researchers are acting as if benefits are received at every age so that $\bar{S}/R$ would measure the change in wealth of participants. But this overestimates the additional wealth for families receiving benefits for some fraction of their life. Empirically, the income used to estimate the wealth effect is current period's nonearnings income or assets. Only if this income is assumed to be received at every age will percentage differences in this income among individuals correspond to percentage differences in wealth. But nonlabor income and assets are characterized by large transitory components. Even if one knew the time horizon appropriate to the statistical measures of income, one must still determine the subgroups in the population that differ in either n or t_1.

This analysis suggests that the usefulness of the controlled experimental data collected to study an FAP is limited. These experiments have been conducted for reasonably short periods of time. One of the most famous, the New Jersey experiment, lasted three years. For families whose participation in an FAP exceeds the time span of the experimental program, the hours response for the families in the experimental data is not applicable. The number of periods of eligibility is a critical variable in determining the hours response to an FAP because it determines in part the increase in real wealth and also the strength of the incentive to substitute consumption between time periods.

THE SUBSTITUTION EFFECT

By holding R/P constant in Equation (15), it is possible to isolate those labor supply reactions that are pure substitution effects. Two relative prices are altered by an FAP: (1) the cost of consuming in one time period relative to the cost in consuming in another, and (2) the cost in any period of using one input in household production relative to the cost of using other inputs. In any period when FAP benefits are received, the cost of household consumption (π_t) is lowered because the real cost of using male and female time in the household sector falls by μ percent (the implicit tax rate).[1] Since these prices are lowered only in the eligible periods, the interperiod price structure is altered. Assuming for simplicity that benefits are received only between time periods t_1 and t_2,[2] the change in the lifetime price index caused by the FAP is

$$(19) \qquad \frac{dP}{P} = -\left(\sum_{t=t_1}^{t_2} k_t S_{M_t} + k_t S_{F_t} \right) \mu ,$$

where k_t is the share of consumption of commodities in time period t in total full wealth.

As an approximation, we may write[3]

$$(20) \qquad \frac{dP}{P} = -\left((\bar{S}_M + \bar{S}_F) \sum_{t=t_1}^{t_2} k_t \right) \mu = -(\bar{S}_M + \bar{S}_F)\Omega\mu$$

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[1]This ignores human capital investments, which will be discussed later.

[2]This assumption is not critical to the analysis. It only simplifies the summation notation.

[3]Since $S_{M_t} = \bar{S}_M + \delta_t$, where $\bar{S}_{M_t}$ is the mean share over all time periods and δ_t is the deviation from the mean share in period t. The approximation involves

$\sum_{t=t_1}^{t_2} k_t \delta_t \stackrel{\sim}{=} 0$--that is, a weighted average of the deviations of the time shares from the mean lifetime share is approximately zero. Although this is strictly true only when the summation is over all periods, I am assuming that it is approximately true when the summation is confined to those periods in which maintenance is received. In any case, any error caused by this approximation is likely to be small.

where $\bar{S}_M$ and $\bar{S}_F$ are the average lifetime shares of male and female time in household production and $\sum_{t=t_1}^{t_2} k_t = \Omega$ is a measure of the fraction of life during which a family receives FAP benefits.[1]

At those ages when the family is not eligible for benefits, the wages of all family members are unaffected,[2] but the cost of consuming at these ages relative to ages when benefits are received is increased. During the noneligible ages, the percentage changes in the demand for male and female home time and market goods resulting from an FAP are

$$\text{(21)} \qquad \frac{dM_t}{M_t} = \frac{dF_t}{F_t} = \frac{dX_t}{X_t} = -\ (\sigma_c(\bar{S}_M + \bar{S}_F)\Omega)\mu\ .$$

A different set of demand equations is appropriate for those ages when the family is covered. In addition to the lower cost of consuming at these ages, there is a percentage reduction in the opportunity cost of male and female time of μ percent. The home time demand

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[1]This interpretation of Ω holds precisely if the shares are identical in each period. In that case

$$\sum_{t=t_1}^{t_2} k_t = nk_t = \frac{n}{N} = \Omega\ ,$$

where n is the number of time periods the family receives benefits and N is the total number of time periods in the lifetime. If the shares are not identical, $\sum_{t=t_1}^{t_2} k_t < \frac{n}{N}$ when the average share of total wealth consumed during the period of maintenance eligibility is less than the average lifetime share in all periods. But even in this case, Ω is a positive monotonic function of the number of periods of eligibility, and that is the interpretation I maintain in the text.

[2]This is not true if we consider the incentive an FAP gives to investments in human capital. This complication is considered below.

equations for the eligible ages are[1]

(22) $$\frac{dM_t}{M_t} = \left| \sigma_c[(\bar{S}_M + \bar{S}_F)(1 - \Omega)] + \bar{S}_{X_t}\sigma_{MX} \right| \mu \ ;$$

(23) $$\frac{dF_t}{F_t} = \left| \sigma_c[(\bar{S}_M + \bar{S}_F)(1 - \Omega)] + \bar{S}_{X_t}\sigma_{FX} \right| \mu \ ,$$

and the market goods demand function[2] is:

(24) $$\frac{dX_t}{X_t} = \left| (\bar{S}_M + \bar{S}_F)\ \sigma_c\ (1 - \Omega) + \bar{S}_{X_t}\sigma_{XX} \right| \mu \ .$$

These equations demonstrate the importance of distinguishing between the eligible and noneligible states of the life cycle. On the substitution effect, market work of men and women will actually increase during the noneligible periods. With the higher cost of household consumption in these periods the family has an incentive to reallocate some of its household production toward the eligible periods. During the nonbenefit ages this releases some time of men and women from the

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[1]Since both male and female wages fall by the same percentage (μ) we may write

$$\frac{dM_t}{M_t} = -[\sigma_c(\bar{S}_M + \bar{S}_F)\Omega - S_{M_t}(\sigma_{MM} - \sigma_c) - S_{F_t}(\sigma_{MF} - \sigma_c)]\mu$$

or

$$\frac{dM_t}{M_t} = \left| \sigma_c[(S_{M_t} + S_{F_t}) - (\bar{S}_M + \bar{S}_F)\Omega] + S_{X_t}\sigma_{MX} \right| \mu \ .$$

Assuming S_M and $\bar{S}_M$ are approximately equal, and similarly for S_F and $\bar{S}_F$,

$$\frac{dM_t}{M_t} = \left| \sigma_c[(\bar{S}_M + \bar{S}_F(1 - \Omega)] + S_{X_t}\sigma_{MX} \right| \mu \ .$$

By a similar proof, one may derive the female equation.

[2]Since market goods are treated as inputs into household production, their demand equations are derived in a manner identical to that of household time.

household sector, and their market work will tend to increase. Previous studies of the labor supply effects of FAPs have ignored this potential increase in market work at some ages. Because they have concentrated exclusively on the reductions in market work that are expected to occur during the eligibility period, the average lifetime reduction in market work has been overstated.

During the eligible periods, on the substitution effect, market hours of men and women should decline. The term $\sigma_c[(\bar{S}_M + \bar{S}_F)(1 - \Omega)]$ summarizes the interperiod substitution component. The family will attempt to produce more of its lifetime consumption in the periods of eligibility, increasing the demand for male and female time in the household sector. This effect will be stronger the more elastic (the larger σ_c) is the intertemporal demand elasticity, the larger the share in household consumption of male and female time combined $(\bar{S}_M + \bar{S}_F)$, and the fewer the number of periods of FAP eligibility $(1 - \Omega)$. Because household production costs are lowered by approximately the same percentage in all eligible periods, there is no incentive to reallocate consumption between these periods. At one extreme, if benefits are received at every age $(1 - \Omega = 0)$, the problem of interperiod substitution can be ignored. If a family receives benefits at only one age, $(1 - \Omega)$ is close to one and the interperiod effects could be great. As the number of eligible periods increases, the importance of this intertemporal substitution diminishes. The second term ($S_{X_t}\sigma_{MX}$ or $S_{X_t}\sigma_{FX}$) measures the incentive to substitute the time of men and women for market goods in the household production process. Because the price of both have fallen by μ percent, there is no incentive to substitute between the two time inputs. However, the family will attempt to substitute both male and female time for market purchased goods.[1]

[1] I am ignoring the possibility of complementarity between inputs. Theoretically, either σ_{MX} or σ_{FX}, but not both, may be negative.

MARKET GOODS CONSUMPTION

Policymakers may also be concerned with the consumption patterns of market purchased goods that result from the introduction of an FAP. Because of the larger real wealth, demand for market goods will increase in proportion to the income elasticities for these goods. For the two substitution effects, it is again necessary to distinguish between the eligible and noneligible periods. During the noneligible ages, the only price affected is the increased cost of consuming at these ages. On the substitution effect, the demand for market goods will fall in those periods when benefits are not received. In the periods when benefits are received, for market goods the two substitution effects are working in conflicting directions. The increased relative cost of market goods induces substitution toward the use of time in household production ($S_{X_t} \sigma_{XX}$ is necessarily negative). However, the cost of consuming at these ages relative to other ages has fallen, and this should lead to an increase in the use of all inputs including market goods consumption at these ages. For the substitution effect alone, one cannot determine whether the demand for consumption will fall or rise.

In the section that deals with empirical estimation of the effect of an FAP, I will show that substitution in production appears stronger than substitution in consumption between time periods. But the different incentives faced in the eligible and noneligible periods imply that the average reduction in consumption during the eligible periods will exceed the reduction in consumption in the other time periods and the age pattern of market goods consumption will be altered.

Another factor that may be important if participation in these programs lasts for only a short time is the cost of entry into and exit from the labor market.[1] These costs have been ignored in the analysis so far. They include the costs of obtaining another job at a future date and the loss of all benefits specific to a particular job. Workers whose wages include the return on past job-specific training cannot expect to receive this wage when they reenter the labor market. These

[1]This was brought to my attention by Yoram Ben-Porath.

factors will militate against large short-term responses, especially for males where the training components are more important.

A COMPARISON WITH THE EXISTING MODEL

This model may be contrasted with the one period model that currently dominates the literature. With the one-period model derived in Equation (A.10) in the appendix, the predicted effect of an FAP on male home hours would be

$$(25) \qquad \frac{dM}{M} = -S_M \sigma_{MM} \mu + \eta_t \frac{dR}{R} ,$$

where dR/R is the percentage increase in real icome resulting from the subsidy. Equation (25) states than an FAP reduces market work for two reasons: (1) Recipients' income has increased and, because of the standard income effect (η_t), they will purchase more leisure and reduce market work. (2) The cost of leisure or home hours has been reduced, which will lead, on the substitution effect ($S_M \sigma_{MM}$), to an increase in nonmarket hours demanded.[1] To estimate the effect of an FAP it is necessary to know the magnitude of both the income and the substitution effects.

The Family Context

There are two reasons why Equation (25) differs from the model derived in this study. Equation (8) ignores the life cycle dimension and some important aspects of the family context. It is generally acknowledged that the family context is necessary on econometric grounds. Because male and female wages are positively correlated and the female wage coefficient in the male labor supply function is negative, the estimated male wage coefficient will be biased downward if female wages

[1] This equation in a variety of forms has been derived in numerous places in the literature. My own derivation is contained in Appendix A.

are ignored.[1] Yet the importance of the family context cuts deeper. In some of the more prominent work in this field (Greenberg and Kosters; Green and Tella), the effect of an FAP on male hours worked is simulated by letting male wages decline by the marginal tax rate while holding female wages constant.[2] Since the wages of both spouses are affected, the incentive to substitute female time for male time is offset because the female wage also falls. The two wage coefficients should be added to obtain the net effect on male hours.

The family context is also useful because it permits us to investigate the hours behavior of family members simultaneously. I show that female market hours are likely to be affected by an FAP to a considerably greater extent than male market hours. Many studies have concentrated on male heads of households and concluded that the labor supply effects are not large. But from the viewpoint of the family unit, the total withdrawal of market hours by all family members must be considered.

The Role of The Life Cycle

A more important defect of Equation (25) is the neglect of the life cycle. The lower cost of consuming while a family receives welfare benefits strengthens the withdrawal of market hours caused by the substitution in household production. On this account, the one-period model underestimates the work reduction in the eligible period. The

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[1]Let the true male labor supply regression be

$$N_M = b_0 + b_1 w_m + b_2 w_f .$$

If a regression is run excluding the female wage, the estimated male wage coefficient (b) is

$$b_1' = b_1 + C_{w_m w_f} b_2 ,$$

where $C_{w_m w_f}$ is the coefficient in a regression of male wages on female wages. Since $C_{w_m w_f} > 0$ and $b_2 < 0$, $b_1' < b_1$.

[2]See Greenberg and Kosters (1970); Green and Tella (1969).

total lifetime effect of an FAP should also include the possibiliy of increased market participation at the other stages of the life cycle.

These life cycle arguments have important implications in the empirical testing of labor supply models in general and the effects of an FAP in particular. The wage change affects labor supply in two ways. An observed wage difference between two people in a sample could represent a permanent difference in the price of their time, and any measured difference in hours of work should be attributed to the standard income and substitution effects of the one-period model. However, the permanent wages of the two people may be identical, and the wage difference would be measuring the timing element, with the higher-wage person supplying more hours.

The design and structure that we impose on the data will to a great extent determine whether the permanent or the timing effect is captured. In my work on the life cycle model (Smith, 1972), the empirical strategy involved calculating the mean values of market time and wage rates at each age. The regressions were run using these mean values as units of observation, and the observed variation was the variation in wage rates about their average lifetime levels. Because of this design, I was able to interpret my results as a legitimate test of the life cycle model. However, if I had selected people between 35 and 40, grouped them by education levels, and run regressions across these groups, I would probably have isolated the one-period model since the remaining wage variation is more likely a permanent one. For a similar reason, the one-period model is more appropriate when the units of observation are geographical areas (Standard Metropolitan Statistical Areas).

Unfortunately, most studies have tested the one-period model (and the effect of an FAP) with observations on individuals in a cross section where the wage variation contains both life cycle and permanent elements. For example, suppose we observed two people with hourly wages of $3.00 and $3.20. Also, assume that the one with a $3.00 wage expects to receive the same wage for the remainder of his life, and the other expects a rising wage profile. If both worked the same number of hours, an empirical study would incorrectly conclude that an FAP would have little effect. The effect of the higher wage of the

$3.20 person is offset by his present wage, which is low relative to future wages. If, as is likely, people with higher wages also expect a more steeply sloped profile, the wage coefficient will be underestimated in cross sections.[1] Thus these cross-section studies will likely underestimate the market hours reduction due to an FAP.[2]

THE DIFFERENTIAL HOURS RESPONSE OF FAMILY MEMBERS

One interesting application of the theory concerns the relative magnitude of the labor market reductions of different family members. It is frequently asserted, for example, that an FAP will have a greater effect on the hours behavior of secondary workers (married women) than it will have on male heads of households, but there is little systematic investigation based on the underlying theory to justify this hypothesis. Ignoring income effects, Equations (21)-(24) describe the percentage changes in male and female household time. During the noneligibility periods, the model predicts an equal percentage reduction in home time for both spouses.[3] Because married women specialize more in the nonmarket sector, there will be a larger absolute and percentage increase in female market hours during the noneligible periods. The relative change in nonmarket hours during the eligible period is

$$\frac{dM_t}{M_t} - \frac{dF_t}{F_t} = s_X(\sigma_{MX} - \sigma_{FX}) . \quad (26)$$

[1] I owe this argument to James Heckman.

[2] This bias could be especially important when comparing interregional differences in estimated wage coefficients. The lower-wage regions are also the regions where in general the wage profiles are less steep. Because of the poor design of the experiment, the researcher should accept the incorrect hypothesis that regional differences are important.

[3] This conclusion will be altered when I introduce investments in human capital. Of course, one may construct models in which this is not the case. Many commodities could be included in each time period varying in both their relative input intensities and in the ease with which they substitute between periods. The model in the text illustrates how far a much simpler model goes.

There will be a larger percentage increase in home time demand for the time input that substitutes more easily for market goods. Since the prices of male and female time are being reduced by the same proportion, there is no incentive to substitute the time of one family member for that of another. But both time inputs will be substituted against market goods. *A priori* arguments on the relative magnitude of the ease of substitution of male and female time against market goods are not usually convincing, and empirical estimates must be relied upon. Empirical work that I will report later indicates that $\sigma_{MX} > \sigma_{FX}$ so that the percentage increase in male home time will be slightly larger than that for female home time.[1] But the interest of policy-makers usually centers on the relative percentage reductions in market hours. Males spend many fewer hours in the nonmarket sector so larger percentage increases in nonmarket hours for males could easily translate into smaller reductions for male market hours. By algebraic manipulation, one can show that the difference between the percentage changes in male and female market hours is[2]

$$(27) \qquad \frac{dN_{mt}}{N_{mt}} - \frac{dN_{ft}}{N_{ft}} = \left[\left(\frac{F_t}{N_{ft}} - \frac{M_t}{N_{mt}}\right)[(S_M + S_F)(1 - \Omega)\sigma_c]\right] + \left[S_X\left(\sigma_{FX}\frac{F_t}{N_{ft}} - \sigma_{MX}\frac{M_t}{N_{mt}}\right)\right].$$

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[1] Jacob Mincer in his pioneering work "Labor Force Participation of Married Women--A Study of Labor Supply," was one of the first to systematically bring into economic theorizing the case of market substitutes as a determinant of the supply elasticities of hours of work. Mincer argued that the supply curve for women was more elastic than that of men because there were better market substitutes for their home services. As I pointed out in the text, it is not necessary that $\sigma_{FX} > \sigma_{MX}$ for the *supply elasticities* of female time to exceed those of males. See Mincer (1962).

[2]

$$\frac{dN_m}{N_m} \equiv -\frac{M}{N_m}\frac{dM}{M} .$$

With this identity, substitute the percentage change in home time derived in Eq. (26).

Both terms in Equation (27) should be positive. The first term is necessarily positive as long as females specialize relatively more than males in nonmarket activities. On this substitution in consumption between time periods, an FAP induces a larger absolute and percentage decline in market work by women. The second term in Equation (27) is the substitution in production effect with the partial elasticities of substitution weighted by the degree of specialization in the nonmarket sectors. Even though goods substitute more easily against male time than against female time, the difference is not large enough to offset greater female home specialization. This second term is also likely to be positive, and the theory indicates a larger absolute and percentage reduction in female market work than in male market work as a consequence of an FAP.

INVESTMENTS IN HUMAN CAPITAL

A limitation of the life cycle model presented earlier is that it ignored investments in human capital. The pattern of observed wages is not exogenous but results from a process of accumulation of human capital over one's lifetime. In Appendix B, the work of other economists dealing with the optimal life cycle pattern of human capital investments is presented. Here I simply outline the principal conclusions of this model.

The arguments that follow are more general than human capital investments. The section applies to all nonpecuniary elements in wage rates that are not taxed. The human capital investments are singled out because they are typically an important component of these investments, and also they are known to vary in a systematic way over the life cycle.

According to human capital theory, the optimal production of human capital is expected to decline with age because the marginal return from producing another unit declines with age and the marginal cost is likely to increase.[1] The reduction in returns with age results principally from the finiteness of life; the older one is when the investments are made, the shorter the number of periods remaining in which to

[1] See Ben-Porath (1967).

recoup the costs. If human capital investments have positive net present values, postponing these investments for a year reduces the excess benefit received. This tendency for the level of investment to decline with age will be reinforced if the marginal cost of investment rises over time. Marginal costs are expected to rise because the rising wage over the life cycle makes human capital production more expensive.[1]

Given this investment profile of declining age and human capital, the amount of time spent will also decline with age because of both the reduced scale of investment at the older ages and the incentive to substitute away from time and in favor of goods purchased in the market in the production of human capital. Thus, both investment time and consumption time fall as the wage rate rises. The predictions of the model without human capital are reinforced by the decline in investment time; hours of market work will increase as the wage rate rises over the life cycle.[2]

Although the inclusion of human capital investments does not alter the predictions of the simpler life cycle model, it does suggest some additional implications concerning the labor supply effects of an FAP. Many human capital investments are self-financed and appear as forgone earnings. During those years in which an individual is investing in himself, his actual earnings are lower than his potential market earnings. As a simplification assume that all human capital investment costs are forgone earnings.[3] If E_t^P are the earnings an individual would receive if he did not invest at age t, and C_t are the dollar costs of

[1]The effect of the rising wage could be offset if human capital is an argument in the investment function. In the neutrality assumption put forth by Ben-Porath (1967), human capital had equiproportionate effects on the efficiency of market time and the ability to produce human capital. Thus, the effect of a higher wage would be negated by the increased efficiency of producing human capital and marginal cost would remain constant over the life cycle. In later work, Ben-Porath tested the neutrality assumption. He found that the rate of decline of investment with age was more rapid than one would suspect with neutrality and concluded that the marginal cost of investment increased over the life cycle. See Ben-Porath (1970).

[2]This is a point made by Ghez and Becker (1972).

[3]That is, only time enters the production of human capital.

his investments in that time period, his observed earnings (E_t^*) will be

(28) $$E_t^* = E_t^P - C_t \ .$$

Figure 3 shows a typical age profile for an individual's potential and observed earnings. Much of the early work of human capital theorists involved analyzing the properties of these age profiles. From the previous discussion, we know that the optimal life cycle investments path implies that human capital investment (C_t) will decline with age. Since the vertical distance between these two profiles measures the investment at that age, these two earnings profiles will converge as the individual ages. The potential earnings profile rises as long as investments take place, for this guarantees a growing human capital stock. Potential earnings will grow at a decreasing rate because

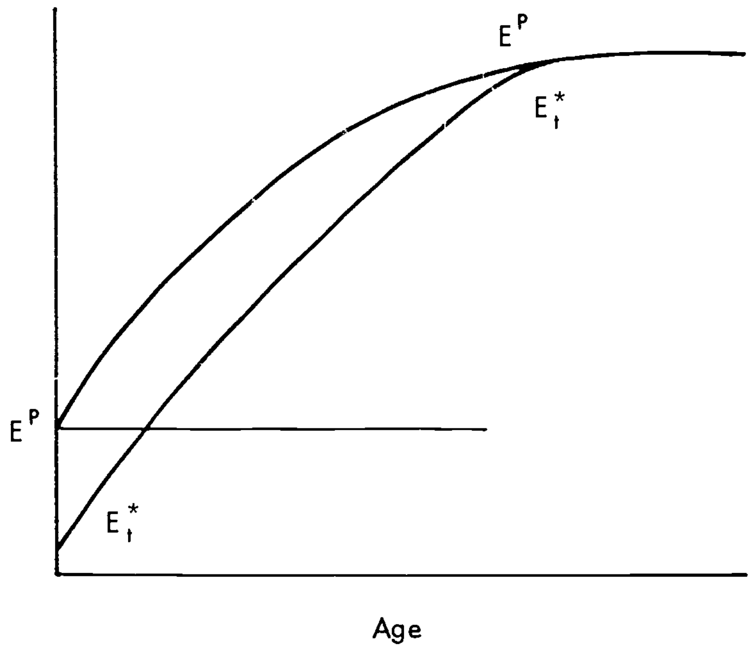

Fig. 3 — Potential and observed earnings

investments decline with age. Finally, the observed earnings profile in general will also grow with age at a decreasing rate.[1]

This discrepancy between observed and potential wages is important, because the potential wage is the opportunity cost of an individual's time and hence governs the allocation of that time among alternative uses. But only observed earnings are subject to the tax. The marginal tax rate of μ percent will lower observed earnings by μ percent, but potential earnings will fall by $\mu(1 - C_t^*)$ where C_t^* is the fraction of potential earnings absorbed by human capital investments.[2] Clearly, if individuals differ in C_t^*, their labor supply reactions to an FAP will differ as well. We can use the theory of the optimal life cycle path of human capital investment to identify the distribution of C_t^* over demographic subgroups in the population.[3] Since one determinant of the profitability of these investments is the expected length of stay in the labor force, men have a greater incentive than women to invest in market forms of human capital. The observed wage profile for men and women confirm this prediction because the female profile is much flatter than the male. Therefore, an FAP will not lower the true wages of males and females by the same percentage. Equations (21)-(24), which assumed a proportionate reduction in wages, are no longer appropriate. For example, if male wages fall by a constant fraction (λ) of female wages at every age in which benefits are received, the new equations would be[4]

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[1]Observed earnings will grow with age as long as investments continue and decrease with age. The profile will be concave as long as the rate of decline in investment with age is not too rapid. For a much more detailed analysis of these profiles, see Mincer (1972).

[2]Equation (28) may be rewritten $E_t^* = E_t^P(1 - C_t^*)$ where $C_t^* = C_t/E_t^P$ and $\mu E_t^* = \mu(1 - C_t^*) E_t^P$. This concept of the fraction of potential earnings invested was introduced by Jacob Mincer.

[3]In Mincer's (1972) terminology $C_t^* = k_t$. Mincer uses the age or experience distribution of k_t to explain log earnings age profiles of individuals.

[4]This assumes the male and female profiles are parallel. Obviously, this need not be the case.

$$(29) \qquad \frac{dM_t}{M_t} = \frac{dF_t}{F_t} = \frac{dX_t}{X_t} = -\left[(\lambda\bar{S}_M + \bar{S}_F)\sigma_c\right]\mu ,$$

$$(30) \qquad \frac{dM_t}{M_t} = \left[\sigma_c(1 - \Omega)(\lambda\bar{S}_M + \bar{S}_F) + S_{X_t}\sigma_{MX}\lambda - (1 - \lambda)\sigma_{MF}S_F\right]\mu ,$$

$$(31) \qquad \frac{dF_t}{F_t} = \left[\sigma_c(1 - \Omega)(\lambda\bar{S}_M + \bar{S}_F) + S_{X_t}\sigma_{FX} + (1 - \lambda)\sigma_{MF}S_M\right]\mu .$$

The qualitative predictions for the noneligible periods (Equation (29)) remain the same. However, in the eligible periods the family has an incentive to substitute female time for male time in home production because male wages do not fall as much as female wages; this tends to dampen the previously predicted increased demand for male home time and further increase the specialization of females in the household. The relative demand for the two inputs may be expressed as

$$(32) \qquad \frac{dM_t}{M_t} - \frac{dF_t}{F_t} = S_{X_t}(\sigma_{MX}\lambda - \sigma_{FX}) - (1 - \lambda)\sigma_{MF}(S_M + S_F) .$$

The conditions for the percentage increase in male time to exceed female home time are more stringent. Even if $\sigma_{MX} > \sigma_{FX}$, both terms in this expression are likely to be negative.[1] For example, if males invest twice as much as their wives, the partial elasticity of male time against market goods would have to be twice as large as the partial elasticity of female time against market goods. Therefore, in contrast to the model without human capital investments, the percentage increase in female nonmarket hours should exceed that of males. All the statements involving the greater female market hours reduction are reinforced. If λ is small enough, an FAP could lead to an increase in male hours worked during the eligible periods because the incentive to substitute female for male time in the household could swamp the other factors.

[1] The second term is necessarily negative.

A well-known implication of human capital theory is that investments will decline with age. In the eligible periods, the cost of household production will not fall by an equivalent amount at each age. Rather, the largest decreases will occur at the older ages where investments are less important. Consider two families that participate in an FAP, one family during ages 26-30 and the other during ages 56-60. The model predicts a greater reduction in male hours worked in the older family. λ serves as a negative index of human capital investment.[1] As λ increases, the reduction in male wages approaches that in female wages, and male home time increases. The principal avenue of male market withdrawal will be in the form of earlier retirement. For females, however, there will be a greater reduction in working time of the wife in the younger family,[2] where the relative reduction in female wages will be the greatest. At all ages in which benefits are received, the reduction in female market hours will be greater than that of males, and the relative reduction in female market hours will be greatest for the younger participants. Mincer has empirical evidence that C_t^* is approximately the same for males of different education groups at the same year of labor market experience, but it increases by education group at the same age.[3] If so, at a given age male market time should be reduced more for the less educated, and female time be reduced more for the more educated families.

The incentives to invest in human capital are also altered by an FAP. Each additional dollar of potential earnings used to finance self-investment increases the subsidy by μ cents. Because the fraction

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[1] $\dfrac{d\left(\dfrac{dM_t}{M_t}\right)}{d\lambda} = \sigma_c(1 - \Omega)S_M - S_F\sigma_{FF}$. This derivative is positive.

[2] $\dfrac{d\left(\dfrac{dF_t}{F_t}\right)}{d\lambda} = S_M(\sigma_c - \sigma_{MF}) - \Omega\sigma_c$. Since empirically $\sigma_c < \sigma_{MF}$, this derivative is negative.

[3] See Mincer (1972).

of total investment costs that are forgone earnings are probably greater for job investments than for school, these programs especially encourage job investments.[1] We should also expect some switching to the more time-intensive techniques of producing human capital and an increase in the proportion of specific job training financed by employees. If policymakers ignore these incentives to invest and use market earnings to estimate potential participants in these programs, the number of young families opting to receive benefits will be underestimated. Finally, this encouragement of job investment is more important for males. But hours spent investing on the job are reported as working hours. This makes even more plausible the possibility of observed job hours increasing for young married males, while the family is receiving income transfers.

A TENTATIVE SIMULATION

It is evident that predictions about the direction and importance of the labor supply effects require empirical estimates of the relative magnitude of the parameters of the household production and intertemporal utility functions. In some earlier work, I estimated the pure life cycle home time demand functions for white married men and women.[2] These results are summarized in Table 1. The demand equations for the pure life cycle model were shown to be

$$(33) \qquad \frac{dM_t}{M_t} = -\left(S_M\,\sigma_c + S_F\,\sigma_{MF} + S_X\,\sigma_{MX}\right)\frac{dW_{mt}}{W_{mt}} + S_F(\sigma_{MF} - \sigma_c)\frac{dW_{ft}}{W_{ft}} + \sigma_c(r - \alpha)$$

$$(34) \qquad \frac{dF_t}{F_t} = -\left(S_F\,\sigma_c + S_M\,\sigma_{MF} + S_X\,\sigma_{MX}\right)\frac{dW_{ft}}{W_{ft}}$$

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[1] If job training is completely general, all investment costs will appear as forgone earnings.

[2] For a more detailed description of the sample and the empirical procedure followed, see Smith (1972).

Table 1

LIFE CYCLE HOME TIME DEMAND EQUATIONS[a]

Dependent Variable	Independent Variables: Log Male Hourly Wage	Log Female Hourly Wage	Age	Number of Children Under Seven	Constant	R^2
Log male home time[b]	-.1065 (11.71)	.0283 (1.71)	.00007 (.49)	-.0158 (5.92)	9.31 (241.6)	.88
Log female home time	.0246 (2.78)	-.0852 (4.20)	.0007 (4.02)	.0364 (11.20)	9.31 (121.3)	.88

[a]t-values in parentheses.

[b]Home time defined as (8760 - Average Yearly Hours Worked).

Source: Smith (1972) based on 1967 Survey of Economic Opportunity data. Regressions covered white married families, spouse present, ages 22-64.

$$+ S_M(\sigma_{MF} - \sigma_c) \frac{dW_{mt}}{W_{mt}} + \sigma_c(r - \alpha) \; .$$

The empirical estimates were based on a sample of white married families, spouse present, using the 1967 SEO survey. This sample was stratified by the age of the husband, and within every age cell, mean values of all variables were calculated. In the absence of secular growth, the observed variation between age cells will correspond to the expected life cycle variation for a cohort if the cohort's expectation is unbiased on average. Of course, real wages have grown over time so that younger cohorts have a higher expected wealth. But if we assume further that real wages grow secularly at a constant rate, the estimated wage coefficient will be unbiased, but the age coefficient will be a biased estimate of the interest rate effect.[1] Thus using equations (33)-(34), aggregating at each husband's age and integrating yields

[1]Intuitively, if real wealth grows secularly at a constant rate, wealth becomes perfectly correlated with age, and all wealth effects are picked up by the age variable.

$$(35) \qquad \log M_t = a_0 + a_1 \log \overline{W}_{mt} + a_2 \log \overline{W}_{ft} + a_3 \text{ age}$$

$$(36) \qquad \log F_t = b_0 + b_1 \log \overline{W}_{ft} + b_2 \log \overline{W}_{mt} + b_3 \text{ age} ,$$

which are the equations estimated in Table 1.

These life cycle equations may be used to simulate the pure substitution component of an increase transfer.[1] One problem is that the effects of income transfer programs depend also upon the expected number of years a family will participate and the extent of differential investment profiles among family members. But by assuming extreme values, the effects of these programs may be simulated. These estimates are presented in Table 2.

First consider the case of no human capital investments ($\lambda = 1$). If the male and female wage coefficients are added in the estimated life cycle equations,

$$(36) \qquad \frac{dM_t}{M_t} = - (\overline{S}_M + \overline{S}_F)\ \sigma_c + S_X\ \sigma_{MX} = -.0782$$

$$(37) \qquad \frac{dF_t}{F_t} = - (\overline{S}_M + \overline{S}_F)\ \sigma_c - S_X\ \sigma_{FX} = - .0606 .$$

These equations are identical to the predicted effect of an income maintenance program derived in Equations (22) and (23) for the eligible periods if the family participates for only one year (Ω 0). The substitution effect will be strongest if a family participates for only one year, because the importance of the incentive to substitute between time periods diminishes as the number of periods of

[1]It is possible to calculate the substitution effect in the life cycle model only because I have assumed that a single cohort has unbiased expectations of future incomes so that real wealth remains constant through a cohort's life cycle experience. Differences in wealth between cohorts are in the age variable, and this wealth effect can not be disentangled from the interest rate and time preferences effects. The appealing feature of the life cycle equation is that the substitution effect can be isolated.

Table 2

ESTIMATED CHANGE IN MARKET WORK
(substitution effect only)

[illegible]	Investment Assumption	Eligible Period: Percent Change in Nonmarket Time: Maximum ($\Omega \simeq 0$)	Minimum ($\Omega \simeq 1$)	Eligible Period: Absolute Change in Market Work[a]: Maximum ($\Omega \simeq 0$)	Minimum ($\Omega \simeq 1$)	Noneligible Period Maximum Increase in Market Work
Males	$\lambda = 1$	3.91	1.89	-259	-125	134
Females	$\lambda = 1$	3.03	1.01	-248	-83	166
Males	$\lambda = 2/3$	2.14	.44	-141	-29	111
Females	$\lambda = 2/3$	3.44	1.76	-282	-139	[illegible]
Males	$\lambda = 1/2$	1.25	-.29	-83	18	100
Females	$\lambda = 1/2$	3.64	2.13	-298	-175	124
Males	$\lambda = 1/3$	.36	-1.02	-24	+67	89
Females	$\lambda = 1/3$	3.85	2.50	-316	-197	110

[a]Absolute changes evaluated at mean home hours 6612 for males and 8196 for females based on 1966 SEO data.

participation increases. Following this procedure, the first two lines in Table 2 state that for families in which human capital investment is not important, the maximum percentage increase in male and female nonmarket hours for a 50 percent marginal tax rate is 3.91 percent and 3.03 percent.[1] Based on the 1966 mean levels of nonmarket time of 6612 hours for males and 8196 for females, the absolute reduction in market hours are 269 for males and 248 for females. The larger percentage increase in male home time implies that market goods are a better substitute for male home time than for female home time, since

$$\frac{dM_t}{M_t} - \frac{dF_t}{F_t} = S_X(\sigma_{FX} - \sigma_{MX}) = -.076 .$$

[1]The use of maximum and minimum effects may be misleading. These estimated effects have variances so that the actual effect may exceed or fall below the "maximum" or "minimum" estimates in text.

In terms of the disruption of market activity, these estimates imply a 12 percent reduction in market hours for males and a 44 percent reduction in market hours for females. This considerably greater effect on females becomes even more dramatic since, on average, 50 percent of married women do not work at all during a year so that in terms of working women, this implies a reduction from 1128 to 632 hours worked (a decline of 486 hours). The minimum estimates in Table 2 are obtained by assuming that a family participates in the program at every age so that there is no interperiod substitution at all, but only substitution between inputs in production. Only the $s_X\sigma_{MX}$ and $S_X\sigma_{FX}$ components of the total substitution effect estimated in Equations (33)-(34) are relevant. We can obtain a lower bound on these since Equation (37) may be rewritten

$$\frac{dF_t}{F_t} = \sigma_c + S_X(\sigma_{FX} - \sigma_c) = .0606 .$$

The positive signs of the cross substitution effect in the life cycle equations imply that substitution in production between inputs exceeds substitution in consumption between time periods.[1] Therefore, σ_c must be smaller than .0606. It is also unlikely that the share of market goods in household production is less than 1/3. Based on these arguments, it is safe to state that $S_X\sigma_{MX} \geq .0378$ and $S_X\sigma_{FX} \geq .0202$. These values were used to estimate the minimum effect of a reduction in market work of 125 hours for males and 83 hours for females. The maximum increase in market work for the noneligible periods was calculated from Equation (21) using this upper bound for σ_c and assuming $\omega \approx 1$.[2] The estimates for the other investment assumptions were obtained using Equations (33)-(34) and putting in the appropriate value

[1] Ghez also found this for the life cycle market goods consumption equations, directly supporting the argument in the text.

[2] The greatest increase in market work in the noneligible period will occur when a family participates for all years but one.

of λ. For the maximum estimates it is necessary only to multiply the male coefficient by λ before summing the two wage coefficients.[1]

These results suggest what the theory itself implied--these programs will have a much greater effect on the work behavior of married women than on that of male heads of households. This is always true in terms of percentage withdrawal of market hours. Even for an investment parameter of 2/3, the max absolute withdrawal of market work of women is twice that of men. For families where investments are important, the reduction in male market work becomes very small, and we can not exclude the possibility that their market hours will actually increase.

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[1] For the minimum estimates, one must also know $\sigma_{MF}S_F$ and $\sigma_{MF}S_M$. But from the life cycle model

$$\sigma_{MF}S_F = S_F\,\sigma_c + .0283$$

$$\sigma_{MF}S_M = S_M\,\sigma_c + .0246$$

so that (if S_M and S_F are approximately 1/3)

$$\sigma_{MF}S_F \simeq .0485$$

$$\sigma_{MF}S_M \simeq .0448$$

IV. CONCLUSIONS

Until recently, economists have analyzed labor supply decisions in a one-period conceptual framework that ignores intertemporal variation in time worked. In this study I have developed a model that explicitly analyzes variations in labor supply that occur over a person's lifetime. A comparison of the standard one-period model and the life-cycle model shows that neither is appropriate for the analysis of all policy issues nor are all data sets appropriate for both models. In some situations the interperiod effects emphasized in the life-cycle model can be safely ignored; in other instances, it is these life-cycle elements that dominate behavior.

One of the major conclusions of this report is that for a significant part of the recipient population the one-period model may be inappropriate to analyze the potential effects of a Family Assistance Plan. Because participation depends on family characteristics that vary over the life cycle, many families will be eligible for program payments in certain periods during their life time and ineligible in other periods. For these families neither the one-period nor the life-cycle model is the appropriate framework; rather a blend of the two models is required. The appropriate combination of the one-period and the life-cycle models results in a number of new predictions about the labor supply effects of an FAP. Especially important is the mileage obtained for between-sex comparisons of changes in labor supply. For example, a strong *a priori* case can be made that the principal reduction in market work during eligible periods will take place among females. In contrast, for young males one might even expect an increase in market work. For all family members, there is the possibility of an *increase* in market work during those parts of the life-cycle when benefits are not received. Finally, the model predicts that an FAP will encourage investments in human capital.

Appendix A

THE ONE-PERIOD MODEL

Some of the differences between the life cycle and one-period models are apparent if the one-period model is developed formally. For any family at age t, we may express the demand for male home time[1] as:

$$\text{(A.1)} \qquad \log(M_t) = \eta_t \log(R) + (\sigma_c - \eta_t)\log(P) + \eta_{mm}\log(w_{mt}) + \eta_{mf}\log(w_{ft}) + \sigma_c(r - \alpha)t$$

where η_t is the full income elasticity of consumption of commodities at age t, and η_{mm} and η_{mf} are the elasticities of male home time with respect to husband's and wife's wages.[2]

Equation (A.1) shows that levels of market time among individuals at a given age t will differ because of variation in the family's nominal wealth (R), the lifetime cost or price (P) of home production, and the age-specific wages of husbands or wives (w_{mt} or w_{ft}). To distinguish between the life cycle and one-period models, assume that the life cycle paths of wages and other variables are identical in shape within a given socioeconomic group. But among individuals within any group, the level of the profiles may vary. Therefore, if the wage of individual i exceeds the wage of individual j by λ percent at

[1]Equation (A.1) is derived from Equations (5), (7), and (8) in the text, ignoring the B_t terms. First express Equation (5) in terms of percentage changes and substitute Equations (5) and (8) into Equation (7). Integration of Equation (7) yields Equation (1) in the text.

[2]Only the male equation is derived explicitly in the text. With the appropriate labeling of subscripts, the female equation is identical. The reader will notice that the assumption of unitary income elasticities used in the life cycle model has been dropped. The assumption was convenient in the life cycle model, since income effects did not appear.

age t, i's wages will also exceed j's by λ percent at all ages.[1] Nominal wealth was defined as:

$$R = T \sum_{t=1}^{n} \frac{w_{mt} + w_{ft}}{(1 + r)^t} + A_0 .$$

The percentage change in nominal wealth across families (with the age neutrality assumptions) is:

$$\text{(A.2)} \quad \frac{dR}{R} = \frac{T}{R} \frac{dw_{mt}}{w_{mt}} \left(\sum_{t=1}^{n} \frac{w_{mt}}{(1 + r)^t} \right) + \frac{T}{R} \frac{dw_{ft}}{w_{ft}} \left(\sum_{t=1}^{n} \frac{w_{ft}}{(1 + r)^t} \right) + \frac{dA_0}{A_0} \frac{A_0}{R}$$

or

$$\text{(A.3)} \quad \frac{dR}{R} = \frac{T^*}{R} \frac{dw_{mt}}{w_{mt}} \bar{w}_m + \frac{T^*}{R} \frac{dw_{ft}}{w_{ft}} \bar{w}_f + \frac{dA_0}{A_0} \frac{A_0}{R}$$

where T^* is the total lifetime period (equal to nT) and $\bar{w}_m$ and $\bar{w}_f$ are the average discounted lifetime wage rates of husbands and wives.[2] Equation (A.3) shows that the percentage change in nominal full income across families is a weighted average of the percentage changes in male

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[1] The motivation for this assumption is analytical simplicity. The model is considerably complicated if it is relaxed. Given that the aim of this appendix is to contrast the life cycle and one-period models as simply as possible, the added complexity of allowing λ to differ by age argues for the simple assumption used.

[2] The economic meaning of Eq. (A.3) is evident in the following simple example. Consider any two families, i and j. Let family j's nominal wealth exceed that of family i's for the following reasons: (1) Age-specific husband's wages are 2 percent higher in family j, (2) age-specific wife's wages are 1 percent higher in family j, (3) nonhuman sources of wealth are 5 percent higher in family j. Further, assume that the shares in nominal wealth of male human capital, female human capital, and nonhuman capital are .6, .3 and .1. In this case, the nominal wealth of family j will be 2 percent higher owing to the husband's human capital, .3 percent higher owing to the wife's human capital, and .5 percent higher owing to nonhuman capital. However, we know that family j's real wealth is not 2 percent higher than family i's, since the higher wages of husband and wife increase the cost of producing a given amount of household commodities for that family.

wages, female wages, and the initial asset position of the family. The weights are the shares in total full wealth of male human capital wealth ($\bar{w}_m T/R$), female human capital wealth ($\bar{w}_f T/R$), and all nonhuman forms of wealth (A_0/R).

To measure the differences in real wealth between families, it is necessary to deflate the change in nominal wealth by the increase in the price index (P). This price index was defined as:

$$P = \left[\Sigma \left(\frac{\pi_t}{(1 + r)^t} \right)^{1-\sigma_c} a_t^{\sigma_c} \right]^{\frac{1}{1-\sigma_c}} .$$

If the price (π_t) changes in any period, the resulting change in the price index is:

$$\text{(A.4)} \qquad \frac{dP}{P} = P^{\sigma_c - 1} a_t^{\sigma_c} \left(\frac{\pi_t}{(1 + r)^t} \right)^{1-\sigma_c} \frac{d\pi_t}{\pi_t} .$$

If we allow prices to change in all periods and express the change in π_t in terms of changes in male and female wages:

$$\text{(A.5)} \qquad \frac{dP}{P} = P^{\sigma_c - 1} \left[\frac{dw_{mt}}{w_{mt}} \sum_{t=1}^{n} a_t^{\sigma_c} \left(\frac{\pi_t}{(1 + r)^t} \right)^{1-\sigma_c} S_{M_t} \right.$$

$$\left. + \frac{dw_{ft}}{w_{ft}} \sum_{t=1}^{n} a_t^{\sigma_c} \left(\frac{\pi_t}{(1 + r)^t} \right)^{1-\sigma_c} S_{F_t} \right]$$

$$\text{(A.6)} \qquad \frac{dP}{P} = \frac{dw_{mt}}{w_{mt}} \sum_{t=1}^{n} k_t S_{M_t} + \frac{dw_{ft}}{w_{ft}} \sum_{t=1}^{n} k_t S_{F_t}$$

$$\text{(A.7)} \qquad \frac{dP}{P} = \frac{dw_{mt}}{w_{mt}} \bar{S}_M + \frac{dw_{ft}}{w_{ft}} \bar{S}_F ,$$

where k_t is the share of commodity consumption in period t in full income. Equation (A.7) demonstrates that the change in the price index has the useful interpretation of being a weighted average of the percentage changes in male and female wages, where the weights are the average shares of male and female time in producing a Z.[1] Using Equations (A.3) and (A.7), we may then derive the demand equations for husband's time across these families.

$$\text{(A.8)} \qquad \frac{dM_t}{M_t} = \eta_t \left(\frac{T^*}{R} \bar{w}_m \frac{dw_{mt}}{w_{mt}} + \frac{T^*}{R} \bar{w}_f \frac{dw_{ft}}{w_{ft}} + \frac{dA_0}{A_0} \frac{A_0}{R} \right)$$

$$+ (\sigma_c - \eta_t) \left(\bar{S}_M \frac{dw_{mt}}{w_{mt}} + \bar{S}_F \frac{dw_{ft}}{w_{ft}} \right)$$

$$+ \eta_{mm} w_{mt} \frac{dw_{mt}}{w_{mt}} + \eta_{mf} w_{ft} \frac{dw_{ft}}{w_{ft}} .$$

Because we are dealing with variations at a particular age, the interest-rate and time-preferences terms are eliminated. Since

$$\eta_{mm} = S_{M_t} (\sigma_{MM} - \sigma_c)$$

and

$$\eta_{mf} = S_{F_t} (\sigma_{MF} - \sigma_c) ,$$

we may rewrite Equation (A.8) as[2]

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[1] $\bar{S}_M$ and $\bar{S}_t$ are weighted averages of the time shares across commodities, where the weights are the shares of Z_t in full income.

[2] I have dropped the following term from the male equation $(\bar{S}_M - S_{M_t})\sigma_c$ on the assumption that $(\bar{S}_M - S_{M_t})$ is likely to be quite small in comparison with other terms. Strictly speaking, these shares are equal only if the Cobb-Douglas form is assumed.

$$\text{(A.9)} \qquad \frac{dM_t}{M_t} = \eta_t \frac{dA_0}{A_0} \frac{A_0}{R} + \left[\eta_t \left(\frac{T^*w_{mt}}{R} - \bar{S}_M\right) + S_{M_t}\sigma_{MM}\right] \frac{dw_{mt}}{w_{mt}}$$

$$+ \left[\eta_t \left(\frac{T^*w_{ft}}{R} - \bar{S}_F\right) + S_{F_t}\sigma_{MF}\right] \frac{dw_{ft}}{w_{ft}}$$

$$\text{(A.10)} \qquad \frac{dM_t}{M_t} = \eta_t \frac{dA_0}{A_0} \cdot \frac{A_0}{R} + \left[\eta_t \frac{\bar{E}_M}{R} + S_{M_t}\sigma_{MM}\right] \frac{dw_{mt}}{w_{mt}}$$

$$+ \left[\eta_t \frac{\bar{E}_F}{R} + S_{F_t}\sigma_{MF}\right] \frac{dw_{ft}}{w_{ft}} ,$$

where $\bar{E}_M/R$ and $\bar{E}_F/R$ are the ratios of the present values of lifetime earnings of males and females to full wealth.

Appendix B

INVESTMENTS IN HUMAN CAPITAL

A limitation of the life cycle model derived in this report is that the pattern of life cycle wages was assumed to be an exogenous datum to the individual. One of the earliest arguments advanced by human capital theorists was that a person's earnings at any age are the return on the human capital accumulated by the individual up to that age.[1] Indeed, an important part of life cycle decisionmaking involves the optimal level of resources (both time and goods) to devote at each age to augmenting an individual's capacity to produce and consume in the future. In this appendix, I will briefly summarize the work of other economists dealing with the optimal life cycle pattern of human capital investments. It is reassuring that the basic predictions of the earlier model are not invalidated when human capital investments are allowed. However, the inclusion of these investments adds some additional predictions concerning the pattern of labor supply effects of an FAP. These were discussed in the text.

A number of authors (especially Ben-Porath and Becker) have developed models describing the optimal life cycle path of human capital investments.[2] The following is a heuristic outline based on the treatment of Ghez and Becker of the basic structure of these models.[3] Each individual is assumed to own a stock of human capital (H_t) at each age. The wages (w_t) received in each period are proportional to this stock:

(B.1) $$w_t = \gamma H_t .$$

γ is the rental value of a unit of human capital. It is determined by the aggregate demand and supply curves for human capital in the

[1]This proposition is contained in Becker (1964) as well as numerous other sources.

[2]See Becker (1964), and Ben-Porath (1967).

[3]See Ghez and Becker (1972).

economy. It is assumed to be constant to the individual. One can enlarge his stock of human capital by devoting resources of goods (X_t^1) and time (M_t^1) to its production:

$$I_t = I_t(X_t^1, M_t^1) , \tag{B.2}$$

where I_t is the amount of human capital investment in period t.[1] Ignoring depreciation, the amount of human capital at age (t + 1) is

$$H_{t+1} = H_t + I_t . \tag{B.3}$$

This extension[2] alters the structure of the earlier model contained in Equations (1)-(3). The intertemporal utility function (Equation (1)) and the household production functions (Equation (2)) are assumed to remain unchanged.[3] But the time constraint (Equation (3a) should be expanded to include the three alternative uses of time-market work (N_{mt}), household production (M_t), and investments in human capital (M_t^1); and the expenditure constraint (Equation (3b)) should be expanded to include direct dollar expenditures on investments (X_t^1).

$$T = N_{mt} + M_t + M_t^1 \tag{B.4}$$

$$\sum_{t=1}^{n} \frac{X_t + X_t^1}{(1 + r)^t} = \sum_{t=1}^{n} \frac{W_t N_{mt}}{(1 + r)^t} + A_0 . \tag{B.5}$$

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[1]For simplicity, I have dropped the family context in this section so that only one time input enters the production function.

[2]In a more general model, human capital would be permitted to depreciate at rate δ_t so that $H_{t+1} = H_t(1 - \delta_t) + I_t$.

[3]Of course, this is not necessary. It is conceivable that the amount of human capital enters one's utility function directly--pride in one's educational achievements. Perhaps a more important consideration is that one's household or nonmarket efficiency (B_t) is related to H_t--that is, education enhances one's productivity in both the market and nonmarket sectors. For an excellent discussion of this latter possibility, see Michael (1972). In any case, I am ignoring these considerations.

If intertemporal utility (Equation (1)) is maximized subject to the production functions for human capital (Equation (B.2)) and household production (Equation (2)) and the time (Equation (B.4)) and expenditure constraints (Equation (B.5)), the necessary conditions for an optimum would be[1]

(B.6) $$\frac{\partial U}{\partial Z_t} = \lambda \frac{\pi_t}{(1 + r)^t}$$

(B.7) $$\pi_t = \frac{W_t}{MP_t} = \frac{1}{MP_x}$$

(B.8) $$\varepsilon_t = \sum_{j=0}^{n-t} \frac{\gamma \frac{\partial H_{t+j}}{\partial I_t} N_{m\,t+j}}{(1 + r)^j}$$

(B.9) $${}_t = \frac{w_t}{\frac{\partial I_t}{\partial M_t}} = \frac{1}{\frac{\partial I_t}{\partial X_t^1}} .$$

Equations (B.6) and (B.7) are identical to the solutions obtained in the simpler model. According to Equation (B.6) the marginal utility of consumption in any time period t should be proportional to the discounted cost (π_t) of producing consumption in that time period. And in Equation (B.7) the additional dollar cost of producing a unit of household consumption through expenditures of time equals the marginal cost through expenditures on goods. These two equations show that the predictions of the earlier model are not changed by allowing human capital to be in the model. That is, the effect of an increase in the wage rate is independent of the reason for the wage change.[2] An increase in the real wage makes consumption more expensive and also

[1]This solution is given by Becker and Ghez (1972), pp. 1-28. For detailed proofs, their work should be consulted.

[2]Ghez and Becker (1972) first reached this conclusion.

induces substitution in consumption in favor of goods and against time. As in the earlier model, this model predicts that time spent in household consumption will fall as the wage rate increases.

Equations (B.8) and (B.9) are the equilibrium conditions for the production of human capital. In Equation (B.8) the marginal cost of producing a unit of human capital in period t (ε_t) is equated to the discounted future returns accruing from this additional investment. $\gamma\left(\frac{\partial H_{t+j}}{\partial I_t}\right)$ measures the increase in the market wage in period t+j that is a consequence of an increase in investment in period t. Thus, the numerator in the right hand side of Equation (B.8) is the increase in market earnings in each future year that results from the investment in human capital.[1] In equation (B.9) the marginal costs of producing a unit of human capital from an additional dollar expenditure of time and goods are equated.

[1] It is not necessary to confine the gains from human capital investment to increased productivity in the market sector. More generally, the gains will include increases in nonmarket efficiency in producing both household consumption and human capital. These extensions are not considered in the model in the text.

GLOSSARY OF SYMBOLS

U	Family utility
Z_t	Level of consumption of commodities in period t
n	Number of periods in family's horizon (equal to lifespan)
a_t	Time preference parameter in family's utility function
α	Index of time preference in period t (equal to log (a_t/a_{t-1})
X_t	Total quantity of market goods purchased in period t
M_t, F_t	Amount of male (husband's) time and female (wife's) time spent in home production in period t
N_{mt}, N_{ft}	Amount of husband's time and wife's time spent at work in period t
r	Interest rate
A_t	Level of assets at end of period t
w_{mt}, w_{ft}	Husband's and wife's wage in period t
R	Family's level of full wealth
π_t	Shadow price of commodities in period t
P	Lifetime price index of commodities
σ_{ij}	Allen partial elasticity of substitution in home production between inputs i and j (i,j = m,f, and x for M_t, F_t, and X_t)
k_t	Share of full wealth accounted for by commodities consumed in period t
s_{it}	Share of total cost of commodities in period t accounted for by input i (i = m,f, or X for M_t, F_t, and X_t)
$\bar{s}_i$	Average value of s_{it} over lifetime weighted by k_t
η_t	Full-wealth elasticity of consumption of commodities in period t
$\bar{w}_m$, $\bar{w}_f$	Average discounted values of w_{mt} and w_{ft} over lifetime
E_{mt}, E_{ft}	Total earnings in period t by husband and wife
$\bar{E}_m$, $\bar{E}_f$	Present value of discounted lifetime earnings by husband and wife
T	Total time available per person per period

T^*	Total time available per person over lifetime (equal to nT)
η_{ij}	Elasticity of i with respect to j (i = m,f for M_t and F_t; j = m,f for w_m and w_f)
S	Maximum welfare subsidy
S_t	Welfare payments received in period t
μ	Marginal tax rate of an FAP
G	The wealth a family receives by participating in an FAP

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