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ABSTRACT

This is a collection of the last 12 units (intermediate-upper levels) of materials designed for the Conceptually Oriented Mathematics Program (COMP). The program is intended to diagnose student difficulty, provide a prescriptive program of improvement, and meet individual needs through small-group instruction. Content has been organized into 11 broad concept areas: sets, numerals, order, addition, subtraction, multiplication, division, functions and graphs, geometry, measurement, and number sentences and phrases. These areas are then fitted into 25 vertical levels, each level having two or more steps. The material actually contained here is a collection of lesson outlines covering most concept areas at most levels. Each lesson has the following format: concept area, behavioral objective, mathematical ideas, vocabulary, activities, references, and worksheets. Teaching aids are also suggested. (LS)

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CONCEPTUALLY ORIENTED MATHEMATICS PROGRAM [LEVEL 14-25]

A Title III Project

developed and produced

by

Columbia Public Schools

Columbia, Missouri

1973

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CONCEPTUALLY ORIENTED MATHEMATICS PROGRAM
(COMP)

The initials COMP call to mind the idea of something being "comprehensive." This definition is, indeed, quite fitting for our COMP (Conceptually Oriented Mathematics Program), provides a comprehensive program designed to meet individual needs in mathematics through small group instruction.

The Conceptually Oriented Mathematics Program, is a management program that is both diagnostic and prescriptive. Students are tested to determine their individual strengths and weaknesses and are grouped accordingly. The program provides continuous progress until the student is ready for algebra. The major areas of concern are today's subject-centered curriculum which fails to recognize individual differences of students and the self-contained classroom organization which encourages students to fit textbook molds. COMP is designed to adhere to the learning needs of the individual.

To support the classroom teacher in her role as classroom manager rather than lecturer the following materials were developed by selected classroom teachers in the Columbia Public Schools: A Scope and Sequence Chart, Teacher's Guide Books, Placement Tests, Post Test I and II, Individual Profile Sheets.

The Scope and Sequence Chart is the visual organization of the Conceptually Oriented Mathematics Program. Ten broad concept areas are developed vertically through twenty-five levels of complexity. Each level has been broken down into two or more steps. The inclusion of Step Z in a level provides topics for horizontal enrichment. The Scope and Sequence Chart serves to give the instructor a better understanding of the total program and as an outline to be followed in planning the instruction for a particular level. Instruction in each level may be planned around either horizontal or vertical progress through a given level.

The Teacher's Guide Books serve as a handbook for teachers. The instructions and suggestions in the manual were developed to supplement the teacher's knowledge of mathematics and to relieve her of some of the pressures involved in finding appropriate activities for concept development. It is the intent of the program to stimulate creativity in the classroom and to discourage rote learning. A complete lesson has been prepared for each concept presented. Each lesson has the following format:

The concept
The behavioral objective
The mathematical ideas
Vocabulary
Activities
Textbook References - Houghton Mifflin
Addison-Wesley
Worksheets
Other references: Film Strips
Commercial Tapes
Enrichment Books

Suggested activities for developing and reinforcing concepts are included. Textbook references include multiple grade levels for each reference and the number of the corresponding ditto master is in parenthesis following each listing of page numbers. While the student is progressing through a level the teacher is offering guidance and using innovative teaching aids suggested in the manual.

EVALUATION AND IMPLEMENTATION

I. Types of Tests

The diagnostic and evaluative part of this program is based on two types of tests:

A. Placement Tests were developed to determine the level of competency which a student has achieved in each of the ten concept areas. The placement tests consists of ten multiple choice test items for each stated behavioral objective in a given level. The placement test results are marked on the profile sheets. The results of the placement test will be the basis for forming the initial instructional groups. Tests for Levels 1-6 stress basic facts and will not have multiple choice answers.

B. Post Tests were developed to test the competency of the students after completing the work in a given level. The tests basically consist of five problems for each stated behavioral objective in a given level. Space is provided on the post test for the student to show his work. Students will be retaught those concepts in which they did not achieve 80 percent mastery for a given behavioral objective. Following this period of instruction, the second post test will be given over those concepts which have been retaught. It is recommended that a student does not begin a new level until his profile sheets indicates mastery of 80 percent of the behavioral objectives on the present level.

II. Placement

Begin testing those students new to the program at these recommended levels.

Grade 1	Level 1	(Optional)
Grade 2	Level 2	
Grade 3	Level 4	
Grade 4	Level 6	
Grade 5	Level 9	
Grade 6	Level 11	
Grade 7 & 8	Determined by the individual school.	

All students who pass their initial test should continue with the testing until the level of competency has been determined for each child.

Those students who were not proficient on the initial test should be given the preceding tests until their level of proficiency is determined.

III. Profile Sheets

A. Mark with a red X all concepts passed on the placement tests.

B. Mark with a blue X all concepts passed on Post Test I which is given after teaching the level.

C. Reteach any concepts in which the student did not achieve 80 per cent proficiency.

D. Mark with a blue X the results of Post Test II which is given after reteaching any concept not passed on Post Test I.

E. Indicate each years accomplishments by drawing a line after the last level completed that year and writing the year this work was completed.

F. The slash marks indicate that a concept was not taught in the level.

Students who have been in the program the previous year will be given the Placement Test for the last level completed the previous year. This test is given to determine the students who need review before continuing to the next level. The teacher determines the amount of time and the assignments needed for review. A Post Test should be given as soon as the teacher thinks the student is ready to proceed to a new level.

Instruction Materials and Manipulative Aids

The following materials are available in each building unless marked with an asterisk. These should be ordered through the Mathematics Office on the pink sheet available in your building.

I. Audio-Visual Equipment

Tape Recorders.
Audio-Pacers.
Tapes:

* A. Imperial International Learning Tapes: Primary
B. Imperial International Learning Tapes: Intermediate
C. COMP Tapes (project Tapes)
* D. Continuous Progress Laboratory--Series 500.
* E. Blank Tapes - for teacher made tapes

* Filmstrips - Curriculum series.

362 - Introduction to Plane Geometry
363 - Vocabulary: Lines and Angles I
364 - Vocabulary: Lines and Angles II Relationships
365 - Lines, Relationships, Direction
366 - Vocabulary - Triangles
367 - Polygons - Geometry
368 - Geometry Circles I
369 - Vocabulary Circles II
373 - Geometry in Art

Filmstrips - and Cassettes Forms We See

EF 1106 - 1 Circles
EF 1106 - 2 Triangles
EF 1106 - 3 Quadrilaterals
EF 1106 - 4 Polygons
EF 1106 - 5 Solid Figures

II. Teaching Aids.

Concept and Skill Cards.
Number Lines:

Demonstration size.
Transparent: centimeters
Individual.

Geometric Shapes (small)

* Geometric Shapes (large)

Counters.
Attribute Blocks.
Open-End Abacus.
Base Ten Blocks

II. Teaching Aids. (Continued)

Multi-Base Blocks.
Colored Centimeter Rods.
Primary rulers.
Meter stick.
Fractional Parts: Felt
Clock Faces.
Flannel Boards and Cut-Outs.
Cubit Foot.
Chalkboard Compass.
Cross Numbers I & II
Arithmablocks
Addition/Subtraction Math Path
Addition/Subtraction Bingo
Motivators - Number Sequence
Motivators - Number Puzzler
Mirror Cards
Sum Fun
Smarty
Geoboards
Winning Touch
Tell Time Quizmo
* Action Fractions Squares and Triangles
Math Match
Tangrams
Prime Drag
Liquid Measure Game

III. Reference Materials: Books.

A. Franklin Series.

1. Mirror Magic
2. Learn to Fold.
3. Paper and Pencil Geometry
4. Making and Using Nomographs.
5. Learning about Measurement.
6. Mathematics Around the Clock.
7. Patterns and Puzzles in Mathematics.
8. Probability: The Science of Chance.

B. Mathematics in the Making Series.

1. Patterns, Areas, and Perimeters.
2. Binary and Other Number Systems.
3. Looking at Solids.
4. Rotation and Angles.
5. Curves.
6. Scale Drawing and Surveying.
7. Transformations and Symmetry.
8. Networks.

C. Scholastic Self-Teaching Arithmetic Books.

D. Let's Play Games in Mathematics.

III. Reference Materials: Books: (Continued)

* E. Metric Measurement, Minneapolis Public Schools

* F. Introducing the Metric System, Educational Methods, Inc.

* G. Flowcharting, McQuigg and Harness, Houghton Mifflin

SCOPE AND SEQUENCE CHART

	SETS	NUMERAL	ORDER	ADDITION	SUBTRACTION	MULTIPLICATION	DIVISION	FUNCTION	GEOMETRY	MEASUREMENT
Level 1 Step A	Recognizing the members of a set	Cardinal number (0-6)							Identifying common geometric shapes	
Step B	Comparing sets	Cardinal number (7-10)	Comparing numbers 0-10 "<" "=" ">"	Sums to 6	Finding the missing addend to 6					
Step C	Separating sets (subsets)		Ordinals to 10							Money: penny nickel dime
Level 2 Step A		Place value 10-99	Ordered numbers 10-99	Sums 7-10	Finding the missing addend 7-10				Sets of points	Time: hour
Step B		Odd and even numbers	Comparing numbers 10-99 "<" "=" ">"	three one-digit addends						Linear measure: non-standard units
Level 3 Step A				Sums to 12	Finding the missing addend to 12			Introducing picture and bar graphs		Linear measure: inch
Step B		$\frac{1}{2}$, $\frac{1}{4}$ (region)		Expanded notation						Time: half-hour
Step C				Two digit addends (basic facts to 9)	Finding the missing addend with 2 digit numbers					Linear measure: half-inch

SCOPE AND SEQUENCE CHART

	SETS	NUMERAL	ORDER	ADDITION	SUBTRACTION	MULTIPLICATION	DIVISION	FUNCTION	GEOMETRY	MEASUREMENT
Level 4 Step A			Ordinals through 10 with words	Basic facts sums 13-15	Missing addends sums 13-15			Forming number pairs when rule is given	Line properties	Money: counting to 50 cents
Step B		Place value to 399	Ordinals 100-999	Word Problems facts 13-15	using basic			Determining the rule involving addition and subtraction		Liquid measure: cup, pint, quart
Level 5 Step A		Extending place value to 999	Comparing numbers 100-999 "<" "=" ">"	Basic facts sums 16-18	Missing addends sums 16-18	2 as a factor	2 as a factor			Money: making change up to 50 cents
Step B	Equivalent subsets	Recognizing and naming $\frac{1}{2}, \frac{1}{4}$ (sets)		Word Problems facts 16-18	using basic	5 as a factor	5 as a factor	Number plane		Time: quarter-hour
Level 6 Step A				Addends 2 or 3 digit no renaming	Missing addends 2 or 3 digit no renaming	3 as a factor	3 as a factor			Liquid measure: gallon half-gallon
Step B		Recognizing and naming $\frac{3}{4}, \frac{1}{3}, \frac{2}{3}$ (regions or sets)	Fractional numbers on number line	Renaming 2 and 3 two-digit addends	Renaming two-digit missing addends	4 as a factor	4 as a factor			Money: counting to $1.00
Step C		Properties of 0 and 1		Word Problems renaming		Word Problems (2-5 as factors)	Word problems (2-5 as factors)			Money: making change up to $1.00

SCOPE AND SEQUENCE CHART

	SETS	NUMERAL	ORDER	ADDITION	SUBTRACTION	MULTIPLICATION	DIVISION	FUNCTIONS AND GRAPHS	GEOMETRY	MEASUREMENT
Level 7 Step A	Commutative principle (addition)	1000-9999	Comparing numbers 0-9999 <, >, =	3 digit with renaming	3 digit with renaming	6-7 as factors	6-7 as factors		Rays, planes curves	
Step B	Associative principle (addition)			3 digit addends with renaming; Word Problems	3 digit addends with renaming; Word Problems	6-7 as factors; Word Problems	6-7 as factors; Word Problems		Angles Parallel lines	Linear Measure: $\frac{1}{4}$ inch
Level 8 Step A	Commutative principle (multiplication)					8-9 as factors	8-9 as factors		Parallelogram quadrilateral square rectangle triangle	Time: 5 minutes
Step B	Distributive principle common factors	Rational numbers - fractions $\overline{5}, \overline{6}, \overline{8}$	Comparing fractions $\overline{5}, \overline{6}, \overline{8}$ <, >, =			8-9 as factors; Word Problems	8-9 as factors; Word Problems		Area Measurement	Linear Measure: foot, yard Dozen
Step C		10,000 to 9,999,999	Comparing numbers 10,000 to 9,999,999 <, >, =			Multiples	Factors			
Level 9 Step A	Solving two-stage word problems	Even and odd numbers (2-n places)				2-digit factors by a 1-digit factor with or without renaming	2 digit product with 1 digit factor with or without remainder	Number pairs (involving all operations)	Circles radius diameter	Time: problem solving
Step B	Associative principle (multiplication)	Roman Numerals I to L				Word Problems with a 1-digit factor and 2 digit factor or product	Word Problems with a 1-digit factor and 2 digit factor or product	Naming number pairs on a graph	Recognizing spheres cylinders cubes	Money: (making change up to $10)

SCOPE AND SEQUENCE CHART

	SETS	NUMERAL	ORDER	ADDITION	SUBTRACTION	MULTIPLICATION	DIVISION	FUNCTIONS AND GRAPHS	GEOMETRY	MEASUREMENT
Level 10 Step A	Fractional numbers equaling 1	Equivalent fractions using identity element 1		Fractions with like denominators	Fractions with like denominators	3 or 4 digit factor by 1 digit factor	3 or 4 digit product by 1 digit factor with or without remainder		Symmetry	
Step B	Set notations "∪", " n()" "{}"	Place value through billions		Word Problems using fractions with like denominators		Word Problems for Step A			Properties of line segments: intersecting congruent perpendicular	Temperature degree
Step Z	Cross Product	Roman Numerals C, D, M		Reconstruction Problems						
Level 11 Step A	Intersection of sets "∩" Venn diagrams	Common factors		Fractions renaming sum or addend		Two 2-digit factors	3-digit product by 2-digit factor without remainders	Linear and bar graphs	Properties of planes	Average Range
Step B	Finite Infinite	Composite & prime numbers	Solution sets for inequalities			Word Problems for Step A		Function rules written in equation form		Weight: ounces pound ton
Level 12 Step A	Statements of disagreement	Mixed numerals	Comparing any fractional number <, >, =	Column 4-6 digits	4-6 place with renaming	3-digit factor by 2-digit factor	3-digit product by 2-digit factor with remainders	Naming number pairs with two operations	Properties of closed surfaces: pyramid cylinder cone	Linear measure: $\frac{1}{8}$ inch
Step B	Statements of deduction	Exponents		2-stage word problems using all 4 operations					Volume: cubic unit	Linear measure: centimeter
Step Z	Abstract Mathematical system	Base 3 and 4		Clock arithmetic						

SCOPE AND SEQUENCE CHART

	SETS	NUMERAL	ORDER	ADDITION	SUBTRACTION	MULTIPLICATION	DIVISION	FUNCTIONS AND GRAPHS	GEOMETRY	MEASUREMENT
Level 13 Step A	Naming the LCM using intersection			Fractions with unlike denominators	Fractions with unlike denominators		4 digit dividend with 2 digit divisor		Identifying pentagons and hexagons	Linear measure: (mm, cm, dm, m, km.)
Step B	Naming the GCF using intersection	Prime factorization with factor trees		Fractional numerals: one addend greater than one	Mixed numerals without regrouping	Word problems with n-digit factors	Applying the divisibility rules for 2, 3,4,5,6,9,10		Naming the perimeter of a polygon	Finding the median
Level 14 Step A				Denominate numbers	Denominate numbers	Whole number by a unit fraction	Multiplying by a reciprocal		Naming the line of symmetry	Using a protractor to measure angles
Step B	Comparing Cardinal number of 2 disjoint sets as a rate or ratio					Whole number by a fraction <1		Locating a point on a number plane using a number pair	Identifying similar and congruent shapes	
Step C						Both factors <1			Area of square, rectangle, parallelogram	
Step Z				← FLOW CHARTS →						
Level 15 Step A	Expressing Probability as a fractional number	Base 5		Mixed numerals with regrouping	Fraction from whole number	Whole number by a mixed numeral			Surface area and volume of a rectangular prism	
Step B		Decimals in tenths and hundredths		Decimal addends in tenths and hundredths	Decimals in tenths and hundredths	Decimal products to hundredths			Identifying and constructing arcs and chords	
Step Z				Addition in Base 5	Subtraction in Base 5					

	SETS	NUMERAL	ORDER	ADDITION	SUBTRACTION	MULTIPLICATION	DIVISION	FUNCTIONS AND GRAPHS	GEOMETRY	MEASUREMENT
l 16 Step A		Writing equations and inequalities for word problems	Rounding whole numbers		Mixed numerals with renaming	Mixed numeral times a mixed numeral	Mixed Numerals	Functions with fractions		
Step B	Graphing solution sets on the number line	Base eight					Five digit dividend with two place divisor (standard form)	Graphing function equations	Classifying polygons	
Step Z		Hebrew numerals (introduce)		Roman numerals		The lattice method				
l 17 Step A		Prime factorization by dividing				Clock arithmetic	Square root (estimating)	Using formulas (solve for unknown	Area of triangle	Metric system (units of weight)
Step B	LCM and GCF by Prime Factorization	Base two (introduce)		Base two		Base two			Pythagorean Theorem	
Step Z	Developing the idea of a "perfect" number			by casting out nines	Checking					
l 18 Step A		Decimals to thousandths (introduce)		Decimals to thousandths	Decimals to thousandths	By powers	of 10			
Step B		Introducing integers		Integers on number line	Integers on number line	Decimal fractions	Decimal fractions		Circumference of circle	
Step C		Renaming fractions as a percent				Finding the percent of a whole number			Area of a circle	
Step Z		Locating rational numbers on numberline				Profit and loss (percent)				

SCOPE AND SEQUENCE CHART

	SETS	NUMERAL	ORDER	ADDITION AND SUBTRACTION	MULTIPLICATION	DIVISION	FUNCTIONS AND GRAPHS	GEOMETRY	MEASUREMENT	NUMBER SENTENCES & PHRASES
Level 19 Step A	Extending set terminology ($\in, \notin, \phi, \subset, \not\subset$)	Converting terminating decimals to fractions	Rounding decimals	Properties (commutative, associative, distributive)		Decimals (rounding quotients)		Relating set operations to geometric figures	Measuring to the nearest unit	Simplifying expressions (whole numbers)
Step B		Converting fractions to terminating or repeating decimals	Ordering decimals			Relating fractions, decimals & percents				Simplifying expressions (fractional numbers)
Step Z		Converting repeating decimals to fractions							Finding measurement using greatest possible error	
Level 20 Step A			Comparing fractional numbers (using the comparison property)					Separating lines, planes and spaces	Metric system (units of capacity)	Solving proportions
Step B			Solving fractional inequalities (using the comparison property)					Related angles		Percent problems
Step C								Degree measure without a protractor		Word problems using proportion
Step Z		Base 12							Constructing circle and divided bar graphs	

SCOPE AND SEQUENCE CHART

	SETS	NUMERAL	ORDER	ADDITION AND SUBTRACTION	MULTIPLICATION	DIVISION	FUNCTIONS AND GRAPHS	GEOMETRY	MEASUREMENT	NUMBER SENTENCES & PHRASES
Level 21 Step A				Integers (without using number line)	Integers	Integers	Graphing integers on a number line using inequalities	Extending the classification of triangles	Statistical averages	Simplifying expressions using integers
Step B							Graphing ordered pairs of integers on a plane	Area of a trapezoid		Word problems (commission, interest, discount
Step Z		Roman Numerals				Euclidean Algorithm		Vectors	Frequency distributions	Solving consumer problems
Level 22 Step A		Negative rationals	Comparing rational numbers					Transversals and parallel lines		Number phrases
Step B		Opposites (rational numbers)	Using the symbols ≮, ≯, ≰, ≱					Ruler and compass constructions		Evaluating open number phrases
Step C		Absolute value		Rational numbers	Rational numbers	Rational numbers				Using replacement set to find solution set
Step D	Closure									Solving equations by inspection (rational numbers)
Step Z	Set-builder notation	Greatest Integer notation	Sets: Just for fun	Sets: beginning logic			Probability: simple experiments	Networks		

SCOPE AND SEQUENCE CHART

	SETS	NUMERAL	ORDER	ADDITION AND SUBTRACTION	MULTIPLICATION	DIVISION	FUNCTIONS AND GRAPHS	GEOMETRY	MEASUREMENT	NUMBER SENTENCES & PHRASES
Level 23 Step A					Using exponents	Using exponents		Congruent triangles (SAS, ASA, SSS)		Solving equations using addition property of equality
Step B		Negative and zero exponents			Using integral exponents	Using integral exponents		The 5 regular polyhedrons		Solving equations using multiplication property of equality
Step C		Scientific notation			Using scientific notation	Using scientific notation		Classifying prisms	Extending conversions within the metric system	Translating word sentences into number sentences
Step D		Significant digits						Surface area & volume of pyramids & prisms	Converting to metric units of length	Using equations in one variable
Step Z	Logic - Using truth tables	Fractional exponents		Using exponents			Probability: combined events	Euler's formula		Equations with absolute value
Level 24 Step A	Develop set of real numbers	Changing repeating decimals to fractions	Comparing real numbers	Rational approximations				Parts of cones, cylinders, & spheres	Metric units of area	
Step B			Density				Graphing real numbers	Surface area of cones, cylinders, & spheres	Metric units of volume	
Step C							Domain, range, & function	Volume of cones, cylinders, & spheres		Solving inequalities in 1 variable
Step Z	Logic - Extending operations on sets						Empirical probability	Point of symmetry		Compound inequalities

SCOPE AND SEQUENCE CHART

	SETS	NUMERAL	ORDER	ADDITION AND SUBTRACTION	MULTIPLICATION	DIVISION	FUNCTIONS AND GRAPHS	GEOMETRY	MEASUREMENT	NUMBER SENTENCES & PHRASES
Level 25 Step A		Reading radical expressions	Using order to estimate square roots					Solving right triangles		Solving equations in 2 variables
Step B		Square root by "divide & average" method					graphing linear equations in 2 variables		Metric units of mass and capacity	Solving inequalities in 2 variables
Step C		Square root by the Algorithm method					Graphing linear inequalities in 2 variables			
Step D		Square root using tables					Graphing systems of linear equations			
Step Z	Logic - Using quantifiers	Square root & the sum of odd numbers					Permutations and combinations	Trigonometry		Implication and proofs

TABLE OF CONTENTS

Level 14

Step A

Step B

Step C

Step Z

Level: 14 Step: A Concept: Addition

I. Concept:

Addition: Naming the sum of denominate addends.

II. Behavioral Objective:

The student given addends which are denominate numbers will be able to name them as a sum.

III. Mathematical Ideas:

A. Measurements can be expressed in more than one unit.

B. Denominate addends may be renamed as a sum.

IV. Vocabulary To Stress:

ton	second	century	centimeter
ounce	year	addend	meter

V. Activities:

A. H.M.T.E. 5, pp. 81-83.

B. Play store. Have children make up problems that the storekeeper might use. Call on a classmate to work the problem.

C. Enrichment: Multiplication and division of denominates with one factor a denominate number. AW4, pp. 154-155.

D. You may provide students with lined or squared paper from which they make their own conversion tables, as:

Pounds	1	2	3	4
Ounces	16	32	48	64

Meters	1	2	3	4
Centimeters	100	200	300	400

E. Make metric measure conversion tables. (As above)

Text References:

Book: 4

Addison-Wesley(1971, 1968) pp. 130-131

Book: 5

Houghton Mifflin(1967) pp. 56-59
Houghton Mifflin(1972) pp. 21, 52-54, 55, (15), 353, 337(part)
Addison-Wesley(1971, 1968) pp. 83, 109, 330, 336, 119

Level: 14 Step: A Concept: Addition

Text References: continued

Book: 6

Houghton Mifflin(1967)	pp.	26
Houghton Mifflin(1972)	pp.	20, 46, 47
Addison-Wesley(1971, 1968)	pp.	202, 333, (38), 268(part)

Other References:

Franklin Series, Learning About Measurement.

Level: 14 Step: A Concept: Addition

State the totals in the simplest way.

1. One gravel truck can hold 1 ton of gravel. Another truck can hold 2000 pounds of gravel. How many tons can both trucks hold?

2. During the summer Mrs. Smith froze 5 qt. 3 pt. of strawberries and 4 qt. 6 pt. of blackberries. How many berries did she freeze in all?

3. Linda's dog had 3 new puppies. One weighed 3 lb. 2 oz., another weighed 2 lb. 14 oz., and the last puppy weighed 2 lb. 10 oz. What was the total weight of the puppies?

4. Columbus discovered America in 1492. One hundred and fifteen years later the first permanent settlement was started. This was in 1607. One hundred and sixty-nine years later our country became independent. How many centuries and years was it between the time that Columbus discovered America and when America became independent?

5. Mrs. White went to the store. She bought 1 lb. 3 oz. of hamburger, 2 lb. 11 oz. of steak, and 1 lb. 6 oz. of chicken. How much did all of the meat weigh?

Level: 14 Step: A Addition

WORKSHEET

10 millimeters = 1 centimeter
10 centimeters = 1 decimeter
100 centimeters = 1 meter

1. Jane bought 2 meters of cloth to make a skirt and 25 decimeters of trimming for it. How many meters of cloth and trimming did she buy?

2. In the track meet, Jack jumped 2 meters, 3 centimeters. Mark's jump was 13 centimeters higher than Jack's. How high did Mark jump?

3. John ran 567 meters in the cross country race while Kevin ran 179 meters. What was the total distance the 2 boys ran?

4. A mosquito measures 2 millimeters in width. What would be the width of 151 mosquitoes?

5. We walked 546 meters to the park and 177 meters to Ann's house. What is the total distance we walked?

Level: 14 Step: A Concept: Subtraction

I. Concept:

Subtraction: Naming the missing addend in denominate numbers given the sum and one addend.

II. Behavioral Objective:

The student given a sum and one addend in denominate numbers will be able to name the second addend.

III. Mathematical Ideas:

A. Some subtraction of denominate numbers includes regrouping using measurement units.

B. Measurement units and their equivalents are used in everyday life.

IV. Vocabulary To Stress:

ton	ounce	year
second	century	addend

V. Activities:

A. Let the children make up a problem and then trade papers and work the problem.

* B. Give the children practice in renaming denominate numbers, as:
8 yd. 1 ft. = 7 yd. 4 ft.
19 lb. 6 oz. = 18 lb. 22 oz.

Text References:

Book: 4

Addison-Wesley(1971, 1968)		(19)

Book: 5

Houghton Mifflin(1967)	pp.	59(part)
Houghton Mifflin(1972)	pp.	54, (15), 336(part)
Addison-Wesley(1971, 1968)	pp.	109-111, 119

Book: 6

Houghton Mifflin(1967)	pp.	26
Addison-Wesley(1971, 1968)	pp.	202(part), 333, (38), 268(part)

Other References:

Franklin Series, Learning About Measurement.

* Suggested Introductory Activity.

Level: 14 Step: A Concept: Subtraction

WORKSHEET: Subtraction of denominate numbers

1. 6 lb . = 5 lbs. ______________ oz.
2. 12 ft. = 11 ft. ______________ in.
3. 12 ft. 6 in. = 11 ft. __________ in.
4. 5 lbs. 1 oz. = 4 lb. __________ oz.
5. 3 hrs. 5 min. = 2 hrs. ________ min.
6. 11 yrs. = 10 yrs. ____________ mos.
7. 8 yrs. 7 mo. = 7 yrs. ________ mos.
8. 5 min. 4 sec. = 4 min. ________ sec.
9. 2 tons = 1 ton ______________ lbs.
10. 2 tons 500 lbs. = 1 ton ________ lbs.
11. 13 min. = 12 min. ____________ sec.
12. 10 min. 11 sec. = 9 min. _______ sec.
13. 12 ft. 1 in. = 11 ft. ________ in.
14. 10 hrs. 3 min. = 9 hrs. _______ min.
15. 5 yds. 2 ft. = 4 yds. ________ ft.
16. 1 lbs. 5 oz. = ______________ oz.
17. 1 ton 400 lb. = ______________ lb.
18. 2 days 5 hrs. = 1 day _________ hrs.
19. 1 min. 2 sec. = ______________ sec.
20. 1 lb. 8 oz. = ______________ oz.

Level: 14 Step: A Concept: Subtraction

WORKSHEET: SUBTRACTION

Find the answers to the following problems.

1. If a gravel truck, holding 3 tons, dumped 500 pounds of it on our playground, how many tons and pounds would be left in the truck?

2. David bought a pound of candy at the store and ate 3 ounces of it on the way home. How many ounces of candy did he have left?

3. In the broad jump Randy jumped 5 feet 10 inches. Ann's best was 3 feet 11 inches. Randy could jump how much farther than Ann?

4. Steve's best time in the race was 5 minutes, 7 seconds. John ran it in 4 minutes, 8 seconds. John was how many seconds faster than Steve?

5. Can you subtract 5 hours, 23 minutes, 36 seconds from 6 hours 23 minutes, 13 seconds? If so, solve the problem.

Level: 14 Step: A Concept: Multiplication

I. Concept:

Multiplication: Multiplying a whole number by a unit fraction.

II. Behavioral Objective:

The student given a whole number and a unit fraction will be able to compute the product.

III. Mathematical Ideas:

A. Multiplication and division are inverse operations.

B. Multiplication is related to the joining of equivalent sets.

C. Division is related to the seperation of a set to form 2 or more equivalent subsets.

$$\frac{1}{5} \text{ of } 80 = 16$$
$$\frac{1}{5} \times 80 = 16$$
$$80 \div 5 = 16$$
$$\frac{80}{5} = 16$$

IV. Vocabulary To Stress:

of - times - equivalent sets

V. Activities:

A. Tic-Tack-Toe. Draw a tick-tack-toe figure on the blackboard. Each space should contain a whole number and a fractional number to be multiplied. (This can also be applied to whole numbers, fractions, addition, subtraction, or division.) Each pupil should have a different color of chalk. The pupils will take turns. The child who is X may choose a space and multiply the problem in that space. If it is correct he puts in an X; if not, 0 gets the square. The object is to get three X's or 0's in a row across, down, or diagonal.

Text References:

Book: 3

Houghton Mifflin(1967)	pp. 166-167, (42)
Houghton Mifflin(1972)	pp. 164, 65, (40)

Book: 4

Houghton Mifflin(1967)	pp. 153(part)
Houghton Mifflin(1972)	pp. 148-149, 253, 264, 265

Level: 14 Step: A Concept: Multiplication

Text References: continued

Book: 5

Houghton Mifflin(1967)	pp. 198-201, (48, 49), 204, (50), 210-211, (51), 308
Houghton Mifflin(1972)	pp. 128, 129, 141, 191, 136, 137, (34), 340(part), 132-135
Addison-Wesley(1971, 1968)	pp. 292-293, 295, 338

Book: 6

Houghton Mifflin(1967)	pp. 200, 201
Houghton Mifflin(1972)	pp. 253
Addison-Wesley(1971, 1968)	pp. 166-169, 171, 183

Book: 7

Addison-Wesley(1971)	pp. 231-233

Other References:

Imperial Tape #22 (Intermediate), first part.

Level: 14 Step: A Concept: Multiplication

WORKSHEET

(Multiplication of a whole number by a unit fraction.)

1. There were 90 people at the ball game. $\frac{1}{3}$ of them went home when it began to rain. How many went home when it began to rain?

2. There are 20 cookies on a plate. Jack ate $\frac{1}{5}$ of them. How many did Jack eat?

3. I had $180 in the bank. I spent $\frac{1}{6}$ of it. How much did I spend?

4. 50 children were in the orchestra. $\frac{1}{10}$ of them played violins. How many played violins?

5. 75 doughnuts were in the box. We took $\frac{1}{5}$ of them to a picnic. How many did we take to the picnic?

6. We went to camp for 2 weeks. It rained $\frac{1}{7}$ of the days. How many rainy days did we have?

7. How many cookies in $\frac{1}{3}$ of a dozen?

8. Dick missed $\frac{1}{4}$ of the 20 problems on the test. How many did he miss?

9. I had $570 in my checking account and spent $\frac{1}{5}$ of it on my vacation. How much did I spend?

10. How many ounces in $\frac{1}{4}$ pound?

Level: 14 Step: A Concept: Division

I. Concept:

Division: Dividing by using multiplication with reciprocal.

II. Behavioral Objective:

The student given a division problem with two fractional numbers will be able to name the reciprocal and multiply.

III. Mathematical Ideas:

A. Two numbers whose product is one are called reciprocals.

B. Multiplication and division are inverse operations.

IV. Vocabulary To Stress: reciprocal - inverse

V. Activities:

A. Teacher's edition Imperial Tape #23, (Intermediate).

B. Fraction game. Draw a football field. Make a set of cards with fraction examples on one side and answers on the others. Put the problem side up. Two people play. Taking turns a child draws a card and answers the question. If he is correct he moves forward 10 yards; if wrong 10 yards backwards. They start on the 50 yard line and go to the opposite goal. As in football they receive 6 points for crossing the goal line and one extra point for answering another problem right after scoring 6 points.

Text References:

Book: 4

Houghton Mifflin(1967) pp. 150-152

Book: 5

Houghton Mifflin(1967) pp. 308-310, 329, 341
Houghton Mifflin(1972) pp. 270-273, (67)

Book: 6

Houghton Mifflin(1967) pp. 252-255, (58), 256-259, (59), 264, 265(part)
Houghton Mifflin(1972) pp. 128, 129, (34), 272-275, 351
Addison-Wesley(1971, 1968) pp. 174, 185-187, (34), 188, (35), 189, (36, 37), 190, 332

Book: 7

Houghton Mifflin(1967) pp. 354-359, (54)
Houghton Mifflin(1972) pp. 212-214, 258-263, 427
Addison-Wesley(1971) pp. 241-247, (48) part)

Book: 8.

Houghton Mifflin(1972) pp. 144-147

Other References:
Modern Math Filmstrip #764
Imperial Tape #23 (Intermediate) (part of this)

Level: 14 Step: A Concept: Division

WORKSHEET: DIVISION

Find the correct answer to the following problems.

1. $5 \div \frac{1}{2} =$
2. $5 \times 2 =$
3. $1 \div \frac{1}{6} =$
4. $6 \div \frac{7}{8} =$
5. $\frac{1}{4} \div \frac{1}{7} =$
6. $\frac{1}{3} \div \frac{2}{3} =$
7. $\frac{10}{15} \div \frac{5}{3} =$
8. $\frac{6}{9} \div \frac{4}{6} =$
9. $\frac{2}{3} \div \frac{1}{4} =$
10. $\frac{2}{5} \div \frac{4}{5}$
11. $16 \div \frac{1}{4} =$
12. $4 \div \frac{1}{4} =$
13. $9 \div \frac{1}{3} =$
14. $3 \div \frac{1}{3} =$
15. $\frac{1}{3} \div \frac{1}{3} =$
16. $\frac{1}{4} \div \frac{1}{4} =$
17. $\frac{1}{3} \div \frac{1}{6} =$
18. $\frac{2}{5} \div \frac{1}{5} =$
19. $\frac{1}{10} \div \frac{1}{12} =$
20. $\frac{1}{5} \div \frac{3}{4} =$

Level: 14 Step: A Concept: Geometry

I. Concept:

Geometry: Naming the line of symmetry for symmetrical figures.

II. Behavioral Objective:

The student given a figure which is symmetric about a line segment will be able to identify the line of symmetry.

III. Mathematical Ideas:

A. A shape is symmetrical about a line that separates the shape into 2 matching halves.

B. A flip may be used to determine symmetry about a line.

IV. Vocabulary To Stress: symmetry - balance

V. Activities:

A. Have children cut out shapes and determine by folding which shapes are symmetrical.

B. Have children cut symmetrical designs using geometric shapes to paste on colored background.

Text References:

Book: 5

Houghton Mifflin(1967) pp. 282, 283, (70)
Houghton Mifflin(1972) pp. 292-295, (70)
Addison-Wesley(1971, 1968) pp. 342

Book: 6

Houghton Mifflin(1967) pp. 293
Hougthon Mifflin(1972) pp. 100, 101, 301
Addison-Wesley(1971, 1968) pp. 343

Book: 7

Houghton Mifflin(1972) pp. 399

Other References:

Mathematics in the Making, 7
Franklin Series: Mirror Magic
Mirror Cards
Curriculum Filmstrip, 373 - Geometry in Art
Geoboard card #13
Modern Math Filmstrip #739

Level: 14 Step: A Concept: Measurement

I. Concept:

Measurement: Using a protractor to measure angles.

II. Behavioral Objective:

The student given a protractor will be able to measure any given angle.

III. Mathematical Ideas:

A. The unit for measuring angles is a degree.

B. A protractor is used to measure angles.

IV. Vocabulary To Stress:

degree
protractor
angle
congruent
vertex

V. Activities:

A. Measure the degrees of each angle of a right triangle. Draw a large right triangle and cut it out. Color the two smaller angles with different colors. Tear off the two smaller angles and fit them into the right angle. What did you find out?

Text References:

Book: 5

Houghton Mifflin(1967)	pp. 70-71, (21), 72-73
Houghton Mifflin(1972)	pp. 96, 97
Addison-Wesley(1971, 1968)	pp. 90-91

Book: 6

Houghton Mifflin(1967)	pp. 71-71, 73, 268, 277, 278
Houghton Mifflin(1972)	pp. 96
Addison-Wesley(1971, 1968)	pp. 104-105

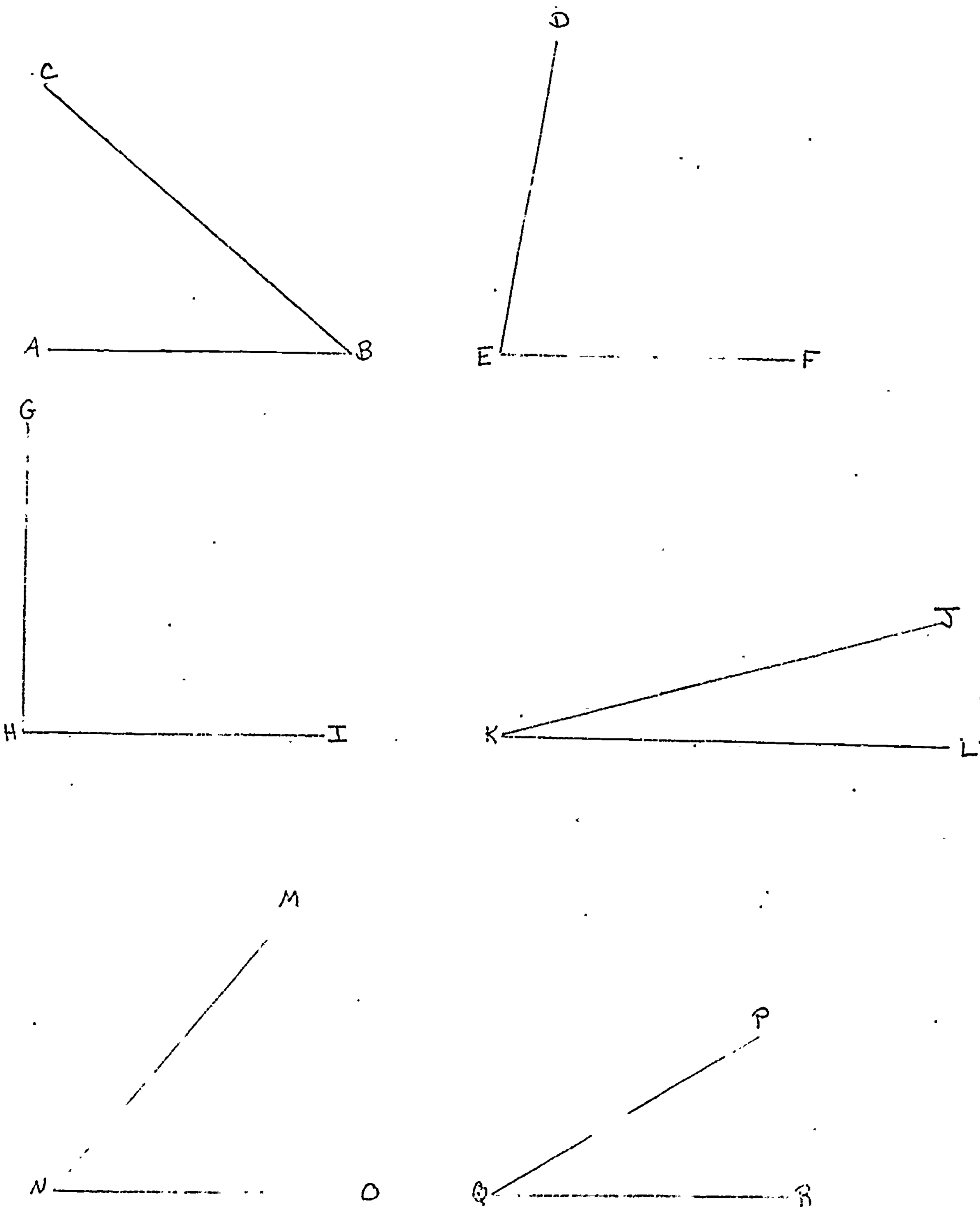
Book: 7

Houghton Mifflin(1967, 1970)	pp. 423-431, (67)
Houghton Mifflin(1972)	pp. 104
Addison-Wesley(1971)	pp. 282-285, (56)

Level: 14 Step: A Concept: Measurement

WORKSHEET: MEASUREMENT

Use protractor to measure each angle. Write answers at bottom of page.



1. m(<ABC) =
2. m(<DEF) =
3. m(<GHI) =
4. m(<JKL) =
5. m(<MNO) =
6. m(<PQR) =

Level: 14 . Step: B Concept: Sets

I. Concept:

Sets: Comparing the cardinal numbers of two disjoint sets as a ratio or rate.

II. Behavioral Objective:

The student given two disjoint sets will be able to compare their cardinal numbers and write the ratio or rate.

III. Mathematical Ideas:

A. A ratio is a fractional number used to compare the cardinal numbers of two sets.

B. Rate is the comparison of two unlike sets.

IV. Vocabulary To Stress: ratio - cardinal number

V. Activities:

A. Teacher's edition Imperial International Learning, Lesson #30.

B. Have children place on their desks pencils, eraser or crayons. Compare sets of two pencils to three pencils. Write the ratio as a fraction. Place two pencils with four crayons. Write the ratio as a fraction. Or, place two pennies with 1 pencil. Write the rate as a fraction, etc.

Text References:

Book: 5

Houghton Mifflin(1967)	pp. 116, 218-219, (55), 220, 221, 222, (56), 276, 279
Houghton Mifflin(1972)	pp. 224, 225, (56), 296-299, (72), (73)
Addison-Wesley(1971, 1968)	pp. 218-219, 137-139

Book: 6

Houghton Mifflin(1967)	pp. 198-199, (41), 200-203, (42), 286(part)
Houghton Mifflin(1972)	pp. 152, 153, 302, 303
Addison-Wesley(1971, 1968)	pp. 206-213

Book: 7

Houghton Mifflin(1967, 1970)	pp. 464-465
Houghton Mifflin(1972)	pp. 67, 70, 85, 142-143, 330-333
Addison-Wesley(1971)	pp. 343-347, (68)

Other References:

Imperial Tape #30 (Intermediate)
Modern Math Filmstrip #768

Level: 14 Step: B Concept: Multiplication

I. Concept:

Multiplication: Multiplying a whole number by a fractional number less than 1.

II. Behavioral Objective:

The student given a whole number and a fractional number less than 1 will be able to compute the product.

III. Mathematical Ideas:

A. Multiplication is related to the joining of equivalent sets.

B. A whole numeral can be written as a fractional numeral. $\left(5 = \frac{5}{1}\right)$

C. Multiply numerator times numerator and denominator times denominator.

IV. Vocabulary To Stress: denominator - numerator

V. Activities:

A. See Tic-Tack-Toe, Level 14-A Multiplication

Text References:

Book: 4

Houghton Mifflin(1967) pp. 152-153
Houghton Mifflin(1972) pp. 153

Book: 5

Houghton Mifflin(1967) pp. 206-207, (51), 292-293, (71, 72), 290, 291.
Houghton Mifflin(1972) pp. 253-255, (62)

Book: 6

Houghton Mifflin(1967) pp. 234-236, (51), 237
Houghton Mifflin(1972) pp. 253, 128, 131, 198, 349(part)
Addison-Wesley(1971, 1968) pp. 172-174

Book: 7

Houghton Mifflin(1972) pp. 423(part)

Other References:

Imperial Tape #22 (Intermediate) first part
Modern Math Filmstrip #763

Level: 14 Step: B Concept: Multiplication

WORKSHEET: MULTIPLICATION

$\frac{1}{4} \times \frac{12}{1} =$	$\frac{1}{5} \times \frac{20}{1} =$	$\frac{1}{6} \times 18 =$
$\frac{3}{4} \times \frac{12}{1} =$	$\frac{2}{5} \times \frac{20}{1} =$	$\frac{5}{6} \times 18 =$
$\frac{1}{7}$ of 21 =	$\frac{1}{4}$ of 24 =	$\frac{1}{8}$ of 64 =
$\frac{2}{7}$ of 21 =	$\frac{3}{4}$ of 24 =	$\frac{3}{8}$ of 64 =
$\frac{1}{9}$ of 27 =	$\frac{1}{12}$ of 24 =	$\frac{1}{13}$ of 26 =
$\frac{8}{9}$ pf 27 =	$\frac{11}{12}$ of 24 =	$\frac{11}{13}$ of 26 =
$\frac{1}{10}$ of 40 =	$\frac{1}{12}$ of 36 =	$\frac{1}{11}$ of 77 =
$\frac{9}{10}$ of 40 =	$\frac{5}{12}$ of 36 =	$\frac{2}{11}$ of 77 =
$\frac{1}{8}$ of 40 =	$\frac{1}{10}$ of 70 =	$\frac{1}{9}$ of 81 =
$\frac{5}{8}$ of 40 =	$\frac{7}{10}$ of 70 =	$\frac{2}{9}$ of 81 =

Level: 14 Step: B Concept: Functions and Graphs

I. Concept:

Functions and Graphs: Locating a point on a plane using a number pair.

II. Behavioral Objective:

The student given a number pair will be able to plot the point on a number plane.

III. Mathematical Ideas:

A. A point on a plane may be located by a number pair.

B. The first numeral in the number pair is found on the horizontal axis, the second on the vertical axis.

IV. Vocabulary To Stress: horizontal axis - vertical axis

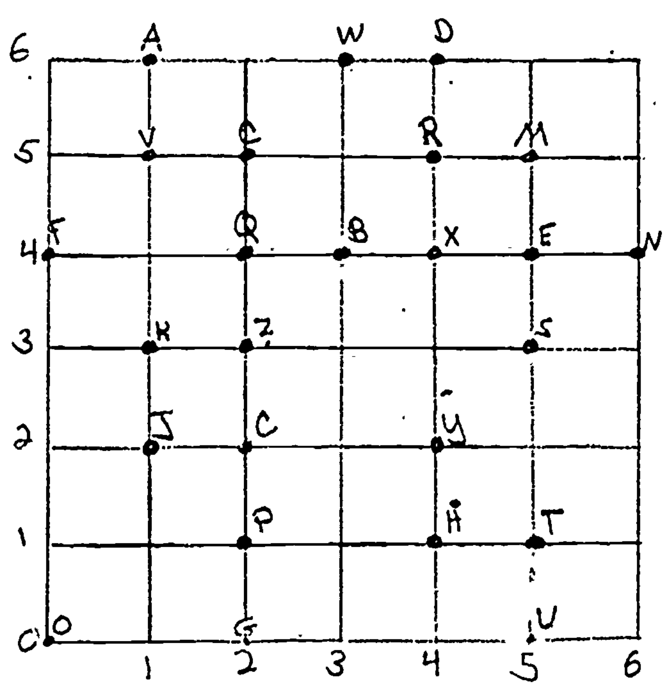
V. Activities:

* A. Put desks in rows and find Row 2 Seat 4, etc.

* B. Use latitude and longitude on maps to locate a ship lost at sea, etc.

C. Franklin Series: Making and Using Graphs, pp. 53-62.

D. Spelling Game (in groups). Make up a word and spell it with number pairs corresponding to points on a number plane as:



The first student to name the word wins.
OR
Have children name a word and first one to name number pairs wins.

Level: 14 Step: B Concept: Functions and Graphs

Text References:

Book: 4

Houghton Mifflin(1972) pp. 304, 305, 308, (75)

Book: 5

Houghton Mifflin(1967) pp. 88-91, 280-281, (25)
Houghton Mifflin(1972) pp. 294, 118, 119, 120, 121, (33)
Addison-Wesley(1971, 1968) pp. 306-309

Book: 6

Houghton Mifflin(1967) pp. 94
Houghton Mifflin(1972) pp. 232, 298
Addison-Wesley(1971, 1968) pp. 304-305

Book: 7

Houghton Mifflin(1972) pp. 53, 94-95
Addison-Wesley(1971) pp. 43-44

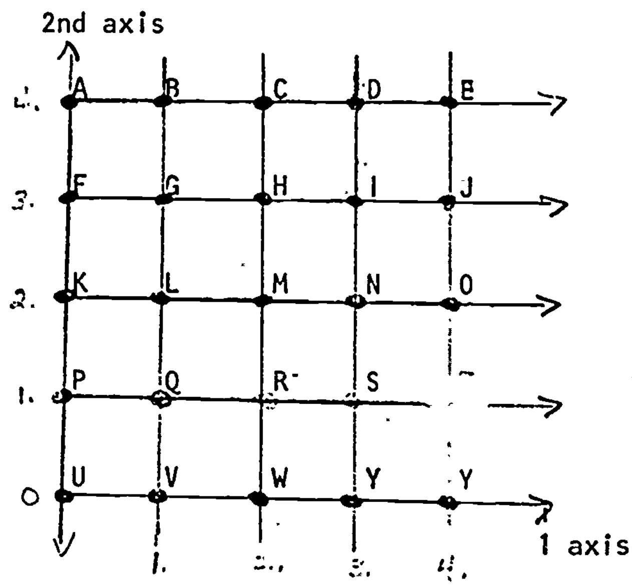
Other References:

Arithmetic Teacher, May 1969 - Plots, Dots and Profiles
Franklin Series Making and Using Graphs, pp. 63-64.
Let's Play Games in Mathematics 5 - #31
Geoboard Card #38
Modern Math Filmstrip #754
Modern Math Filmstrip #746

* Suggested Introductory Activities.

Level: 14 Step: B Concept: Functions and Graphs

WORKSHEET



Usint the diagram above, name the words.

1. (1, 0) (4, 4) (2, 1) (3, 0)
2. (2, 0) (4, 4) (1, 2) (1, 2)
3. (3, 4) (4, 2) (3, 2) (4, 4)
4. (2, 2) (0, 4) (4, 1) (2, 3)
5. (3, 3) (3, 1)
6. (0, 3) (0, 0) (3, 2)

Level: 14 Step: B Concept: Geometry

I. Concept:

Geometry: Identifying similar and congruent polygons.

II. Behavioral Objective:

The student, given drawings of 2 shapes, will be able to determine whether they are congruent or similar.

III. Mathematical Ideas:

A. Angles may be measured by using a protractor.

B. Two congruent shapes are similar because their corresponding angles are congruent and the ratio of the lengths of corresponding sides is 1 to 1.

C. Two congruent angles have the same size.

D. Slides, flips and turns may be used to match 2 congruent shapes.

E. Ratios may be used to compare lengths.

IV. Vocabulary To Stress: congruent - similar - ratio

V. Activities:

A. You might have students cut 3 cardboard strips 18 cm, 27 cm, and 36 cm long. Ask each student to form a triangle with his strips, then ask how they might form a triangle with sides $\frac{2}{3}$ as long as those of the first triangle. Lead them to see that they need strips 12 cm, 18 cm, and 24 cm long. Have them form the second triangle and compare the 2 triangles, which should have the same shape. You might ask the students to measure the angles of each triangle to check that corresponding angles are congruent.

B. Have the children construct a square or rectangle. Use protractor. They should then draw a diagonal. Cut along the diagonal. Compare the 2 triangles. They are congruent. Shear off equal amounts from each side to see similar triangle. Children would also enjoy making more congruent triangles. Pass out one triangle to each child. On a "go" signal have them find their partner (a child with a triangle congruent to their own).

Level: 14 Step: B Concept: Geometry

Text References:

Book: 4

Houghton Mifflin(1972)	pp. 100, 101

Book: 5

Houghton Mifflin(1967)	pp. 70, 71, (21), 72-74, 266-267, (67), 277, 278, 286, 287 (part)
Houghton Mifflin(1972)	pp. 297, 298, 287
Addison-Wesley(1971, 1968)	pp. 272, 273

Book: 6

Houghton Mifflin(1967)	pp. 82, 280
Houghton Mifflin(1972)	pp. 105, (27, 28), 302, 303

Book: 7

Houghton Mifflin(1972)	pp. 110-113, 278-279, 338
Addison-Wesley(1971)	pp. 274-275

Other References:

H.M. Geoboard cards #16 and #17

Level: 14 Step: C Concept: Multiplication

I. Concept:

<u>Multiplication</u>: Naming the product with both factors less than one.

II. Behavioral Objective:

The student given two fractional numerals less than one will be able to compute the product.

III. Mathematical Ideas:

A. A subset can be separated in the same way as a set

B. In multiplication with fractional numbers the product may be less than either factor.

C. Numerators are multiplied by numerators.

D. Denominators are multiplied by denominators.

IV. Vocabulary To Stress: subset - region

V. Activities:

A. Cube Toss: Prepare two wooden one inch cubes. Write a different fractional numeral on each face. The child throws the cubes, like dice and multiplies the two that come up on top. How many different problems are possible? One point for each correct answer.

B. Imperial Tape #22 (Intermediate) (2nd part)

Text References:

Book: 5

Houghton Mifflin(1967)	pp. 202-203, 210-211, (52), 212-213, (53), 294-295, (73), 296-298, (74), 299, 302-303, (75), 304-305, (76), 307, 312-313, 315
Houghton Mifflin(1972)	pp. 146, 147, 258-267
Addison-Wesley(1971, 1968)	pp. 291, 293, 295-298, (52), 300-301. 338

Book: 6

Houghton Mifflin(1967)	pp. 238-239, (52), 240-241, (53), 242-243, (54)
Houghton Mifflin(1972)	pp. 132-137, 349
Addison-Wesley(1971, 1968)	pp. 170-171, (52)

Book: 7

Houghton Mifflin(1967, 1970)	pp. 349-354, (53)
Houghton Mifflin(1972)	pp. 200-201, 204-205, 254, 423
Addison-Wesley(1971)	pp. 233-237, (45)

Other References:

Modern Math Filmstrip #763

Level: 14 Step: C Concept: Geometry

I. Concept:

Geometry: Naming the area of a square, rectangle, parallelogram.

II. Behavioral Objective:

The student, given the measure of the sides of a square, rectangle, or parallologram, can compute the area of the region.

III. Mathematical Ideas:

A. The plane area inside a closed curve is a region.

B. The measure of a rectangular region may be found by naming the product of the number of units of area along the length (l) and the number of units of area along the width (w) $A = l \times w$

C. The measure of a square region can be expressed as $A = S^2$

D. The area of a parallelogram can be expressed as $A = b \times h$

IV. Vocabulary To Stress:

area	height
rectangle	base
parallelogram	

V. Activities:

A. Have children cut out 1 inch squares and use them to find area of the top of a small book, box, etc.

B. Have children cut out 1 sq. in., 1 sq. ft., 1 sq. yd.

Text References:

Book: 5

Houghton Mifflin(1967)	pp. 114-115, 86, 273, (24, 34, 69)
Houghton Mifflin(1972)	pp. 110-111, 304-305, 309-311, (30 47)
Addison-Wesley(1971, 1968)	pp. 117, (31)

Book: 6

Houghton Mifflin(1967)	pp. 108, 142, 143, 269, 122
Houghton Mifflin(1972)	pp. 109, 306-309
Addison-Wesley(1971, 1968)	pp. 108, 109

Book: 7

Houghton Mifflin(1967, 1970)	pp. 437-443,)69)
Houghton Mifflin(1972)	pp. 116-117
Addison-Wesley(1971)	pp. 388-392, (80, 81)

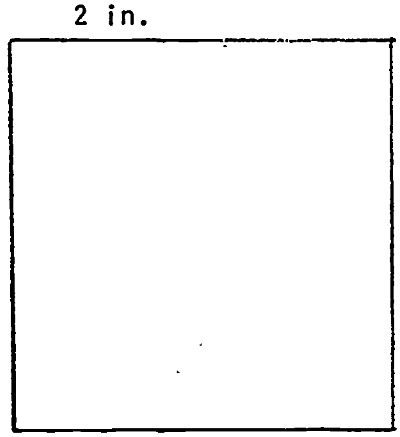
Other References:

Imperial Tape #37 (intermediate)
Geoboard cards #21, #22
Learning about Measurement (Franklin Series) pp. 23-37

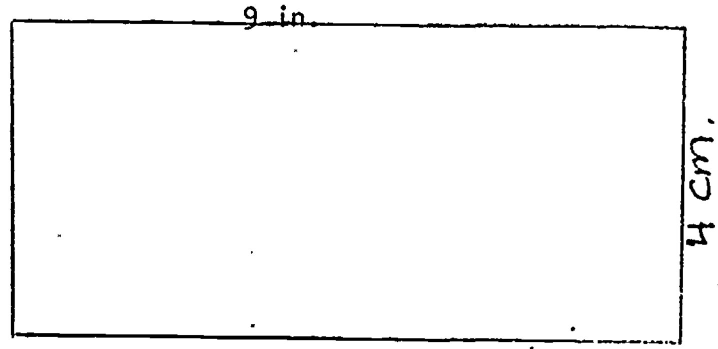
Level: 14 Step: C Concept: Geoemtry

Find the area of each. Be sure to label the answer in square units.

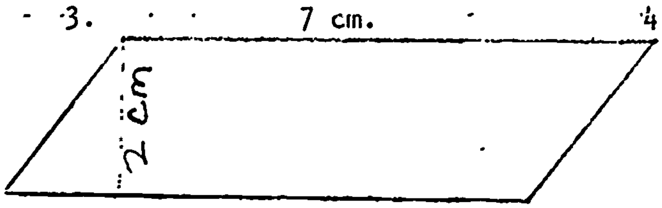
1.



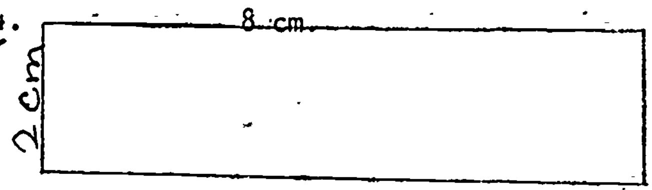
2.



3.

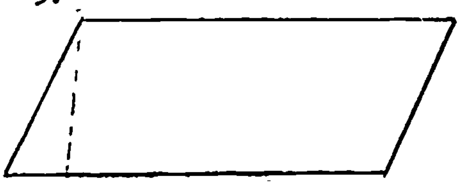


4.

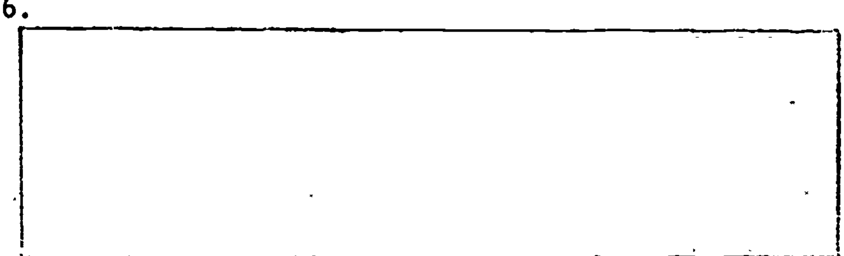


Measure these, <u>using metric ruler</u>, and find area:

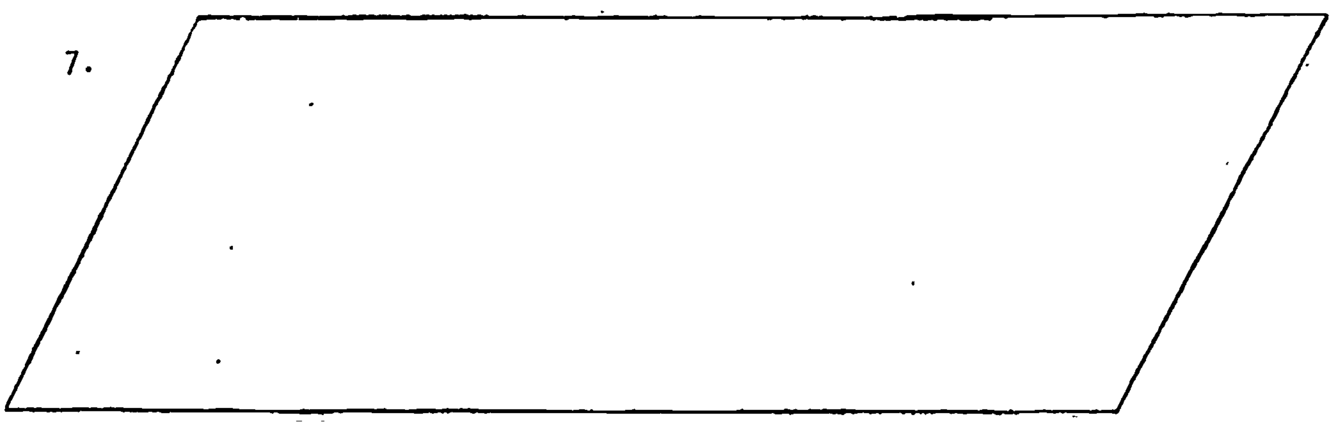
5.



6.



7.



<u>Extra credit!</u> Name the perimeter of each of the above figures:

1. 3. 5. 7.

2. 4. 6.

Level: 14 Step: Z Concept: Flow Charts

I. Concept:

Flow Charts: Using a flow chart to present a sequence of instructions.

II. Behavioral Objective:

The students can follow a sequence of written instructions presented in the form of a flow chart.

III. Mathematical Ideas:

A. A flow chart is a pictorial representation of instructions for problem solving.

B. Every step is indicated in its proper order.

C. Certain shapes indicate decisions to be made or instructions to be followed.

D. Lines with arrows are used to indicate direction of flow.

IV. Vocabulary To Stress: sequence - decision - input - output

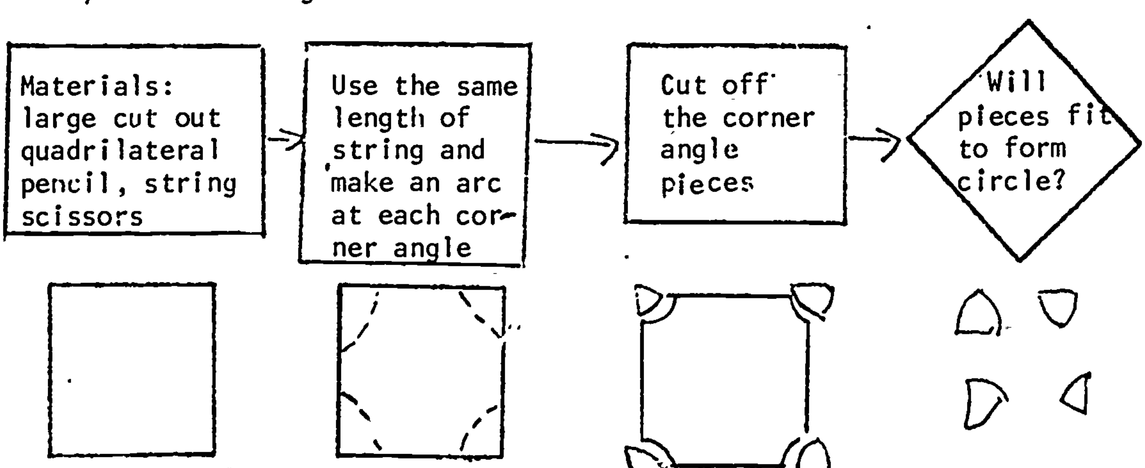
V. Activities:

* A. Houghton Mifflin (1972) Teachers' Ed. pp. 50-51. Book 5.

B. Have the students draw flow charts describing everyday activities, such as getting a drink of water, or sharpening a pencil. Have them exchange with a partner to test for accuracy and completeness.

C. Have students show steps of their flow charts on cards. Have the students exchange cards with a partner, cards out of order, and see who can place cards in correct order first.

D. For you to investigate:



E. Addends 46, 38 → Add the ones → Add all the tens → [ ]

$$\begin{array}{r} 46 \\ +\ 38 \\ \hline \end{array} \qquad \begin{array}{r} 46 \\ +\ 38 \\ \hline \end{array} \qquad \begin{array}{r} 46 \\ +\ 38 \\ \hline \end{array}$$

14-28

Level: 14 Step: Z Concept: Flow Charts

Text References:

Book: 4

Houghton Mifflin(1972) pp. 60, 216

Book: 5

Houghton Mifflin(1972) pp. 50-51, 230-232, 288-291, 307, 316

Book: 6

Houghton Mifflin(1972) pp. 104, 108, 158, 234-237, 294-297, 304

Book: 7

Houghton Mifflin(1972) pp. 223-229, 236-239, 242-244, 250

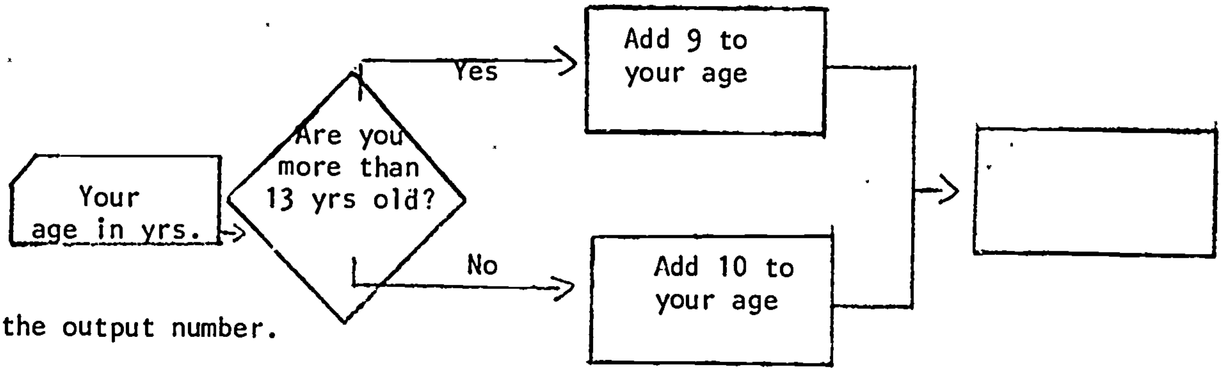
Other References:

Flowcharting by McQuigg-Harness

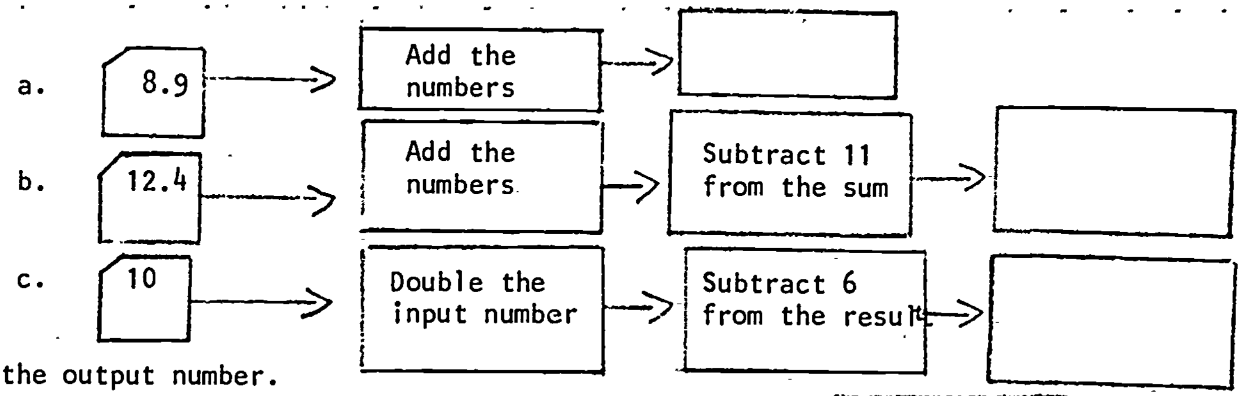
Level: 14 Step; Z Concept: Flow Charts

WORKSHEET

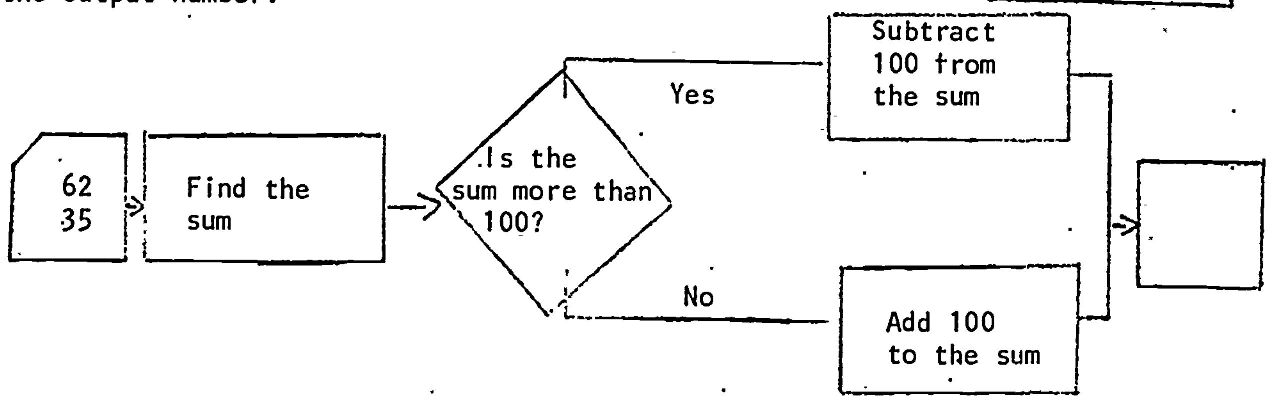
1. Find the output number.



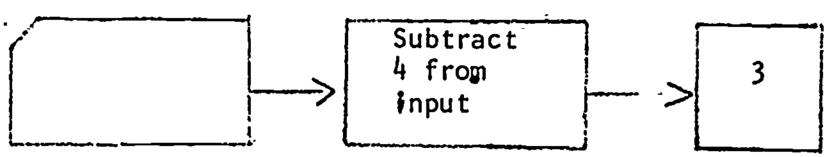
2. Find the output number.



3. Find the output number.

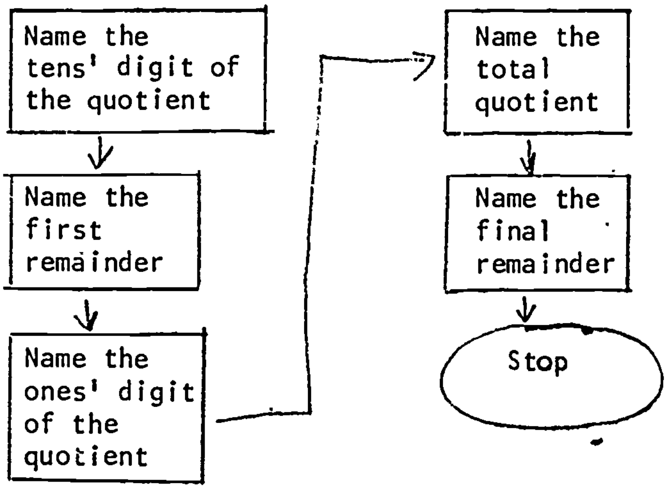


4. What is the input number?



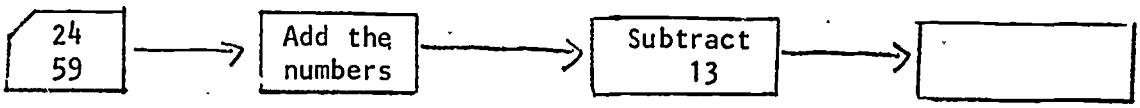
Level: 14 Step: Z Evaluation

1.

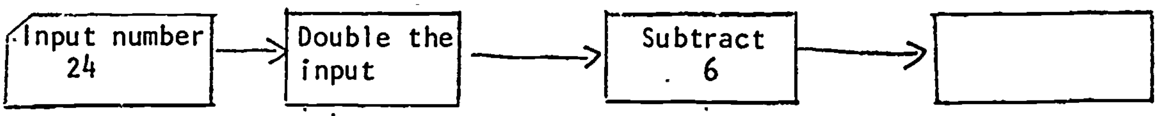


$8\overline{)497}$

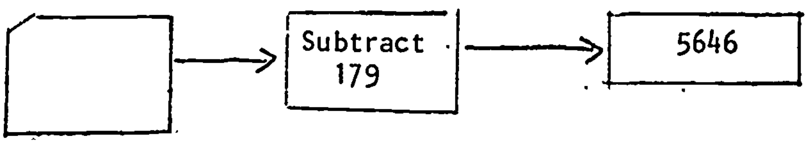
2. Find the output number.



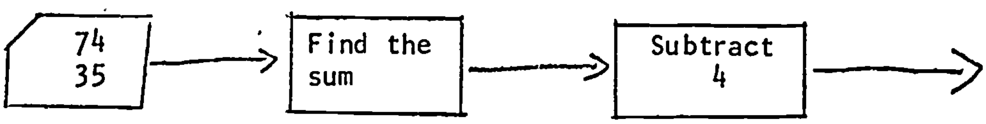
3. Find the output number



4. Find the input number.



5. Find the output number.



Level: 14 ANSWERS TO WORKSHEETS: LEVEL 14

14-3

1. 2 t
2. 3 gal. 1 qt. 1 pt
3. 8 lb. 10 oz.
4. 2 centuries 84 yr.
5. 5 lb. 4 oz.

14-4

Addition

1. 4 m. 5 decimeters
2. 2 m. 16 centimeters
3. 746 m.
4. 302 mm or 30 cm 2 mm
5. 723 m

14-6

Subtraction of denominate numbers.

1. 16 oz.
2. 12 in.
3. 18 in.
4. 17 oz.
5. 65 min.
6. 12 mo.
7. 19 mo.
8. 64 sec.
9. 2000 lbs.
10. 2,500 lbs.
11. 60 sec.
12. 71 sec.
13. 13 in.
14. 65 min.
15. 5 ft.
16. 21 oz.
17. 2400 hours
18. 29 hrs.
19. 62 sec.
20. 24 oz.

14-7

1. 5500 lb.
2. 13 oz.
3. 1 ft. 11 in.
4. 59 sec.
5. 59 min. 37 sec.

14-10

Multiplication-Whole number by unit fraction

1. 30 people
2. 4 cookies
3. $30
4. 5 children
5. 15 doughnuts
6. 2 days
7. 4 cookies
8. 5 problems
9. $114
10. 4 oz.

14-12

Division

1. 10
2. 10
3. 6
4. $\frac{48}{7}$
5. $\frac{7}{4}$
6. $\frac{1}{2}$
7. $\frac{2}{5}$
8. 1
9. $\frac{8}{3}$
10. $\frac{1}{2}$
11. 64
12. 16
13. 27
14. 9
15. 1
16. 1
17. 2
18. 2
19. $1\frac{1}{5}$
20. $\frac{4}{15}$

ANSWERS TO WORKSHEETS: LEVEL 14

14-15

Measurement - using protractor

1. 40^o
2. 80^o
3. 90^o
4. 15^o
5. 50^o
6. 30^o

14-18

Multiplication - whole number by fraction

3	4	3
9	8	15
3	6	8
6	18	24
3	2	2
24	22	22
4	3	7
36	15	14
5	7	9
25	49	18

14-21

Function & Graphs

1. very
2. well
3. done
4. math
5. is
6. fun

14-26

Geometry (area)

1. 4 sq. in
2. 36 sq. cm.
3. 14 sq. cm.
4. 16 sq. cm.
5. 10 sq. cm.
6. 33 sq. cm.
7. 75 sq. cm.

Extra Credit (perimeter)

1. 8 in.
2. 26 cm.
3. 19 cm.
4. 20 cm.
5. 14 cm.
6. 28 cm.
7. 42 cm.

ANSWERS TO ENRICHMENT (Step Z) WORKSHEET

14-29

1. Answers will vary
2. a. 17
   b. 5
   c. 14
3. 195
4. 7

14-30 Evaluation

1. 62 r 1
2. 70
3. 42
4. 5825
5. 105

Level: 14

ANSWER SHEET - POST TEST I

Step A

Addition-A

1. 18 gallon 1 quart
2. 15 lbs. 4 ozs.
3. 13 yds. 0 feet
4. 9 yrs. 3 mo.
5. 9 weeks 0 days

Subtraction-A

6. 7 wk. 1 day
7. 50 minutes
8. 2 gallon 3 qt.
9. 3 lbs. 14 oz.
10. 1820 lbs.

Multiplication-A

11. $\frac{3}{4}$
12. $\frac{6}{8}$ or $\frac{3}{4}$
13. $\frac{4}{7}$
14. $\frac{7}{12}$
15. $\frac{5}{9}$

Division-A

16. $\frac{6}{4}$ or $1\frac{1}{2}$
17. $\frac{1}{8}$
18. 5
19. $\frac{1}{2}$
20. $\frac{4}{21}$

Geometry-A

21. BD
22. EG
23. JL
24. MP
25. UX

Measurement-A

26. B
27. C
28. C
29. A
30. A

Step B

Sets-B

31. 6, per
32. $\frac{[illegible]}{11}$
33. rate, 12, per
34. rate, 6, per
35. $1.00

Multiplication-B

36. 6
37. 3 yds.
38. $5\frac{1}{4}$ or $5\frac{2}{8}$
39. $3\frac{1}{2}$
40. $5\frac{5}{8}$

Function-B

41. Must look on graph
42. Must look on graph
43. 3,5
44. F
45. 5,4

Geometry-B

46. congruent
47. AB, BC
48. 1,3
49. 1,4
50. 1,4

Level: 14 ANSWER SHEET - POST TEST I

Step C

Multiplication-C

51. $\frac{7}{16}$
52. $\frac{7}{18}$
53. $\frac{21}{126}$ or $\frac{1}{6}$
54. $\frac{60}{80}$ or $\frac{3}{4}$
55. $\frac{16}{36}$ or $\frac{4}{9}$

Geometry-C

56. 200 sq. cm.
57. 225 sq. cm.
58. 3000 sq. cm.
59. 2240 sq. ft.
60. 368 sq. in.

Level: 14

ANSWER SHEET - POST TEST II

Step A

Addition-A

1. 4 ton 400 lbs.
2. 8 yd. 0 ft. 4 in.
3. 9 hr. 0 hr.
4. 16 gal. 2 qt.
5. 17 hr. 51 min. 20 sec.

Subtraction-A

6. 1 yd. 2 ft.
7. 2 gal. 2 qt. 1 pt.
8. 1 yd. 0 ft. 8 in.
9. 2 wk. 2 day
10. 1 hr. 32 min.

Multiplication-A

11. $\frac{5}{13}$
12. $\frac{6}{15}$ or $\frac{2}{5}$
13. $\frac{10}{11}$
14. $\frac{9}{16}$
15. $\frac{5}{15}$ or $\frac{1}{3}$

Division-A

16. $\frac{2 \times 5}{20}$ or 5×4
17. $\frac{8}{5}$
18. $\frac{4}{15}$
19. $\frac{5}{7}$
20. $\frac{32}{35}$

Geometry-A

21. $\overline{BE}$
22. $\overline{IX}$
23. $\overline{LN}$
24. $\overline{VQ}$
25. $\overline{XZ}$

Measurement-A

26. 50°
27. 70°
28. 120°
29. 10°
30. 35°

Step B

Sets-B

31. $\frac{9}{11}$
32. rate, 9, per
33. \$1.80
34. rate, 70
35. $\frac{11}{21}$

Multiplication-B

36. $5\frac{5}{6}$
37. $4\frac{2}{4}$ or $4\frac{1}{2}$
38. 9
39. $12\frac{4}{5}$
40. 9

Function-B

41. 0,3
42. P
43. 3,4
44. R
45. 0

Geometry-B

46. congruent
47. yes
48. yes
49. 1,2
50. 1,3

Level: 14 ANSWER SHEET - POST TEST II

Step B

Multiplication-B

51. $\frac{3}{8}$
52. $\frac{25}{56}$
53. $\frac{8}{55}$
54. $\frac{20}{63}$
55. $\frac{28}{45}$

Step C

Geometry-C

56. 15 sq. cm.
57. 40 sq. in.
58. 72 sq. feet
59. 35 sq. ft.
60. 180 sq. in.

TABLE OF CONTENTS

Level 15

Step A

Step B

Step Z

Level: 15 Step: A Concept: Sets

I. Concept:

Sets: Expressing as a fractional number the probability that a particular event or set of events may occur, compared with the total number of related events.

II. Behavioral Objective:

The student given a particular event or set of events and the total number of related events will be able to state the probability as a fractional number.

III. Mathematical Ideas:

A. A fractional number is used to state a probability.

B. A probability cannot be less than 0 or greater than 1. If the probability is 0, the event cannot happen. If the probability is 1, the event must happen.

IV. Vocabulary To Stress: probability - related

V. Activities:

* A. Make and use spinners: Directions in Addison-Wesley 6, Teacher's Edition pp. 320.

B. Teachers Edition Imperial Tapes (Intermediate), Lesson #40.

Text References:

Book: 4

Houghton Mifflin(1972) pp. 150-151, 278, 279

Book: 5

Houghton Mifflin(1967) pp. 252-255
Houghton Mifflin(1972) pp. 204-207, (52)
Addison-Wesley(1971. 1968) pp. 345

Book: 6

Houghton Mifflin(1967) pp. 226-229, 250-251
Houghton Mifflin(1972) pp. 216, 278, 279, (67)
Addison-Wesley(1971, 1968) pp. 312-321

Book: 7

Houghton Mifflin(1972) pp. 318-325
Addison-Wesley(1971, 1968) pp. 89-101, (20, 21)

Other References:

Franklin Series: Probability
Imperial Tape #40 (Intermediate)
Modern Math Filmstrip #767

* Good introductory activity

Level: 15 Step: A Concept: Sets

WORKSHEET: SETS

1. In a box there are 12 candies; 5 mints, 4 caramels, and 3 lemon drops. If you reach in without looking, what is the probability of getting a mint? A caramel? A lemon drop? A chocolate bar?

2. Set C = {0, 2, 4, 6, 8, 10}
What is the probability of choosing a multiple of 2? Of choosing an odd number?

3. Set A = {11, 12, 13, 14, 15, 16, 17, 18, 19, 20}
What is the probability of choosing a multiple of 2? Of choosing 19? Of choosing a multiple of 3?

4. If John has 10 shirts, (1 red, 5 blue, 2 yellow, and 2 green), what is the probability that he will wear a white shirt? A blue shirt? A green shirt?

5. There are 15 horses in the race, 4 Arabians, 5 palominos, and 6 quarter horses. What is the probability that a quarter horse will win? That an Arabian or palomino will win?

Level: 15 Step: A Concept: Numeral

I. Concept:

<u>Numeral</u>: Using base 5 to teach place value.

II. Behavioral Objective:

The student given a numeral in base 5 or base 10 will be able to express the number in the other base.

III. Mathematical Ideas:

A. The position of a digit in any number determines its value.

B. 0-4 are the only symbols needed in a base 5 system.

C. 10_{five} represents one group of 5.

IV. Vocabulary To Stress: decimal system, place value

V. Activities:

* A. ESA Multibase Arithmetic Blocks, cards 1-5.

B. Teacher's Edition Imperial Tapes (Intermediate), p. 35.

C. Make a base 5 chart:

125	25	5	1
	1	3	3

<u>Step 1</u>: Put the base 5 numeral in the chart. (i.e.) 133 five. To change this base 5 number to base 10 multiply (25 x 1), (5 x 3), (1 x 3) and add all the products. (25 x 1) + (5 x 3) + (1 x 3) = 43 ten.

<u>Step 2</u>: Change base ten to base five. A base ten number will not fit into the base five chart.

Example:

$$\begin{array}{r} 4 \\ 23\overline{)123} \\ 100 \\ 5\overline{)\ 23} \quad 4 \\ 20 \\ \hline 3 \end{array}$$

125	25	5	1
	4	4	3

To check multiply as you learned in Step 1.

$(4 \times 25) + (4 \times 5) + (3 \times 1) = 123_{ten}$

$123_{ten} = 443_{five}$

Level: 15 Step: A Concept: Numeral

Text References:

Book: 5

Houghton Mifflin(1967) pp. 22-24
Houghton Mifflin(1972) pp. none

Book: 6

Houghton Mifflin(1967) pp. 266
Houghton Mifflin(1972) pp. none

Book: 7

Houghton Mifflin(1967) pp. 127-129

Other References:

Imperial Tape #35 (Intermediate)

* Suggested introductory activity.

Level: 15 Step: A Concept: Numeral

WORKSHEET: NUMERAL

I. Write the following base five numerals in base ten.

1. 21_{five} = ________ten
2. 13_{five} = ________ten
3. 10_{five} = ________ten
4. 20_{five} = ________ten
5. 14_{five} = ________ten
6. 41_{five} = ________ten
7. 44_{five} = ________ten
8. 100_{five} = ________ten
9. 102_{five} = ________ten
10. 110_{five} = ________ten
11. 144_{five} = ________ten
12. 200_{five} = ________ten

II. Write the following base ten numerals in base five.

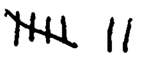
1. 5_{ten} = ________five
2. 8_{ten} = ________five
3. 10_{ten} = ________five
4. 15_{ten} = ________five
5. 24_{ten} = ________five
6. 25_{ten} = ________five
7. 28_{ten} = ________five
8. 30_{ten} = ________five
9. 33_{ten} = ________five
10. 100_{ten} = ________five

Level: 15 Step: A Concept: Numeral

WORKSHEET: NUMERAL

Write your answer for each exercise in the blank for the exercise.

Write the base five numeral for the number of tally marks in each set. Remember to write the word "five" to tell that the numeral shows grouping by fives.

1.  ______________

2.  ______________

Write the missing digits.

3. 43_{five} is ? fives, ? ones? 3. _____fives, _____ones

4. 34_{five} is ? fives, ? ones? 4. _____fives, _____ones

5. 20_{five} is ? fives, ? ones? 5. _____fives, _____ones

Write base ten numerals for the following:

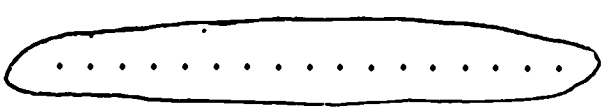
6. 30_{five} ______________________

7. 21_{five} ______________________

8. 100_{five} ______________________

9. 123_{five} ______________________

10. 2004_{five} ______________________



11. Find the base five numeral for the number of dots in the above set.

____

12. Write the base five numeral that means:

$1 \times (5 \times 5)$

____

Level: 15 Step: A Concept: Addition

I. Concept:

Addition: Adding mixed numerals and naming the sum in simplest form.

II. Behavioral Objective:

The students, given two mixed numerals, will be able to compute the sum and express it in simplest form.

III. Mathematical Ideas:

A. To add or subtract two fractional numbers with unlike denominators, we rename one or both fractions as equivalent fractions with the same denominator.

B. Equivalent fractions name the same fractional numbers.

C. A fractional number greater than 1 may be expressed as a mixed numeral or as an expanded numeral.

D. To rename a fractional number in simplest form, we divide the numerator and denominator by their greatest common factor.

IV. Vocabulary To Stress: equivalent - numerator - denominator - simplest form

V. Activities:

A. The students may practice naming equivalent fractions by playing a game of "Thumper", a rhythm game. Establish the rhythm with the students thumping their desks twice, clapping their hands twice, and snapping their fingers twice. Play passes around the class. Leader names a fraction. After each "Thump, thump" and "Clap, clap", a student in his turn on "Snap, snap" names an equivalent fraction. Play continues until a student names a nonequivalent fraction, repeats a fraction already named or breaks the rhythm. Then a new fraction is named and play resumes.

B. "Rename Me." In this game the leader gives a fractional number and each student gives another name for the number. Award one point for each correct response. The class could be divided into teams or could work for individual points if used with a small group.

Text References:

Book: 5

Houghton Mifflin(1967)	pp. 247
Houghton Mifflin(1972)	pp. 212-213, (54), 217, 218, 219, 346, 347
Addison-Wesley(1971)	pp. 246-249

Level: 15 Step: A Concept: Addition

Text References: continued

Book: 6

Houghton Mifflin(1967)	pp. 220-221
Houghton Mifflin(1972)	pp. 194-197, (48)
Addison-Wesley(1971, 1968)	pp. 150-153, (28), 162-163

Book: 7

Houghton Mifflin(1967, 1970)	pp. 345-349, (52) (part of this)
Houghton Mifflin(1972)	pp. 193, 422
Addison-Wesley(1971)	pp. 224-225, (43)

Other References:

Imperial Tapes (Intermediate) #20

Level: 15 Step: A Concept: Subtraction

I. Concept:

Subtraction: Subtracting a fractional number from a whole number.

II. Behavioral Objective:

The student given a whole number and a fractional number will be able to compute their difference.

III. Mathematical Ideas:

A. Any whole number may be written a an expanded numeral, as: $5 = 4 + \frac{3}{3}$

B. A fractional number greater than one may be expressed as an expanded numeral or mixed numeral.

IV. Vocabulary To Stress: expanded numeral - mixed numeral

V. Activities:

A. Use felt pieces on flannel board to show wholes and fractional parts.

Text References:

Book: 5

Houghton Mifflin(1967)	pp. 207-209
Houghton Mifflin(1972)	pp. 209-211, (53)
Addison-Wesley(1971, 1968)	pp. 250, 251, (47, 48)

This concept may be taught from the following worksheets since the textbooks combine concepts.

Level: 15 Step: A Concept: Subtraction

Find the difference.

1. $7 - 4\frac{1}{3}$

2. $8 - \frac{4}{5}$

3. $9 - \frac{1}{2}$

4. $10 - 4\frac{1}{5}$

5. $6 - \frac{7}{12}$

6. $54 - \frac{7}{8}$

7. $21 - \frac{1}{5}$

8. $23 - \frac{7}{8}$

9. $7 - \frac{2}{3} =$ ______

10. $9 - 6\frac{1}{3} =$ ______

11. $21 - 8\frac{4}{5} =$ ______

12. $18 - \frac{12}{13} =$ ______

13. $75 - 24\frac{17}{20} =$ ______

14. $95 - 76\frac{9}{11} =$ ______

15. $96 - 66\frac{19}{20} =$ ______

16. $87 - 16\frac{17}{20} =$ ______

17. $90 - 54\frac{7}{9} =$ ______

18. $105 - 96\frac{6}{7} =$ ______

19. $77 - 64\frac{1}{5} =$ ______

20. $99 - 63\frac{9}{25} =$ ______

Level: 15 Step: A Concept: Subtraction

WORKSHEET

1. The Smiths filled their gas tank with 10 gallons of gas. If they used $2\frac{1}{2}$ gallons of gas one day, how many gallons remain in the tank?

2. Mother had 4 quarts of milk in the refrigerator. Mark drank $1\frac{1}{2}$ quarts after school. How many quarts were left?

3. Mary bought 12 yards of ribbon for tying packages. She used $4\frac{6}{7}$ yards when wrapping her friend's package. How many yards did she have left?

4. The Girl Scouts entertained their mothers at a tea. They bought 6 pounds of cookies. When the tea was over they weighed the cookies and found they had $\frac{3}{4}$ of a pound left. How many pounds of cookies did they use?

5. Mother weighed 142 pounds before she joined Weight Watchers. She lost $12\frac{1}{2}$ pounds. What did she then weigh?

Level: 15 | Step: A | Concept: Multiplication

I. Concept:

Multiplication: Multiplying a whole number by a fractional number greater than 1.

II. Behavioral Objective:

The student given a mixed numeral and a whole number will be able to compute the product.

III. Mathematical Ideas:

A. The distributive property is used in multiplying a mixed numeral times a whole number.

$$2\frac{1}{4} \times 3 = \left(2 + \frac{1}{4}\right) \times 3 = (2 \times 3) + \left(\frac{1}{4} \times 3\right) = 6\frac{3}{4}$$

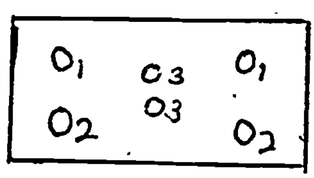
B. A mixed numeral can be renamed as a fractional numeral. $\left(6\frac{1}{3} = \frac{19}{3}\right)$

IV. Vocabulary To Stress: mixed numeral

V. Activities:

A A. See other multiplication of fractions activities.

B. Ring Toss: Using a big piece of cardboard, push 6 nails through the back side. Then label two of them as #1, 2 of them as #2, and two of them as #3. Make three sets of cards with multiplication of fractions. The problems in the 1's stack should be easier to work than those in the 2's or 3's stack. Divide the class into two teams. The children will toss jar rubber rings from a predecided line and try to hook them on the nail. When he hooks a nail he must stop throwing. He draws a card from the number pile (the same as the one he hooked). He then must write the problem on the board and work it. If he gets it right, his team gets the number of points that problem was worth. If he does the problem wrong his team must lose the number of points.

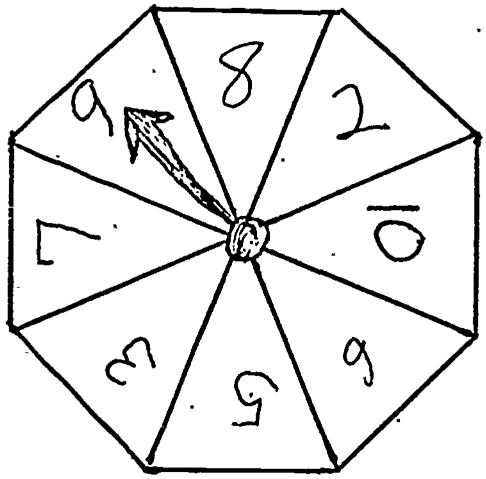


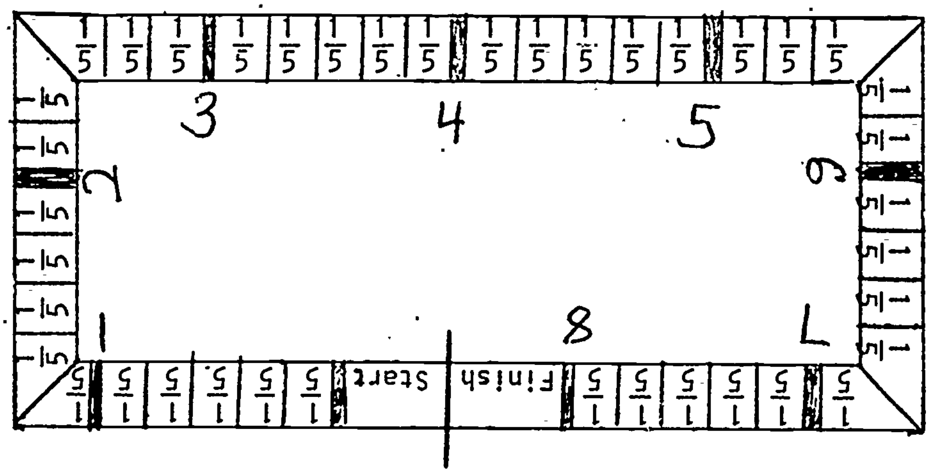
C. See Imperial Tapes (Intermediate), Teachers' Manual, pp. 50-51..

Level: 15 Step: A Concept: Multiplication

V. Activities: continued

D. Grand Prix. Make a large playing board and spinner as shown. Four players or teams may compete, each provided with a small car to be used as a marker. Take turns twirling the spinner to determine how far to advance. Teaching for $\frac{6}{5} = 1\frac{1}{5}$, $\frac{10}{5} = 2$, etc.





Text References:

Book: 5

Houghton Mifflin(1967) p. 326
Additon-Wesley(1971, 1968) pp. 243-244

Book: 6

Houghton Mifflin(1967) pp. 260, (60)
Houghton Mifflin(1972) p. 199
Addison-Wesley(1971, 1968) pp. 177, 176, 330, 331

Book: 7

Houghton Mifflin(1967, 1970) p. 353
Addison-Wesley(1971) pp. 237-239

Other References:

Imperial Tape, #24 (Intermediate)

Level: 15 Step: A Concept: Multiplication

WORKSHEET

Give answers in simplest form.

1. $7\frac{1}{8} \times 8$

2. $8\frac{1}{3} \times 3$

3. $5\frac{4}{5} \times 4$

4. $9\frac{2}{4} \times 6$

5. $2\frac{3}{7} \times 4$

6. $10\frac{1}{7} \times 7 =$

7. $5\frac{1}{9} \times 7 =$

8. $11\frac{2}{9} \times 9 =$

9. $12\frac{1}{4} \times 7 =$

10. $13\frac{1}{3} \times 9 =$

11. $15\frac{1}{4} \times 3 =$

12. $16\frac{1}{3} \times 2 =$

13. $8\frac{5}{7} \times 7 =$

14. $9\frac{5}{7} \times 7 =$

15. $8\frac{1}{9} \times 8 =$

16. $5\frac{7}{9} \times 9 =$

17. $6\frac{5}{6} \times 5 =$

18. $7\frac{1}{7} \times 7 =$

19. $16\frac{1}{4} \times 4 =$

20. $9\frac{1}{9} \times 9 =$

Level: 15 Step: A Concept: Geometry

I. Concept:

Geometry: Solving by formula the surface area or the volume of a rectangular prism.

II. Behavioral Objective:

The student given the dimensions of a rectangular prism will be able to compute the surface area or the volume of the rectangular prism.

III. Mathematical Ideas:

A. Area is the measure of a region.

B. Volume is the measure of space inside a closed surface.

C. Formula for area is A = l x w.

D. Formula for volume is V = l x w x h.

IV. Vocabulary To Stress:

area	square yard (sq. yd.)
volume	cubic foot (cu. ft.)
square foot (sq. ft.)	cubic yard (cu. yd.)

V. Activities:

A. Teacher's Edition Imperial Tape (Intermediate), Lesson 38.

B. Use sugar cubes to show l x w x h. Also use milk cartons to show l x w x h. Shown in: (Arithmetic Teacher, May 1969.)

C. Addison-Wesley T. Ed. 5, pp. 282, 283.

* D. Pupils make 1 inch cubes (see page 82, Houghton Mifflin 5th, 1967). Use cubes for filling small boxes to find volume. (Also pattern D, p. 113, Houghton Mifflin, 1972).

Text References:

Book: 5

Houghton Mifflin(1967)	pp. 87, (24), 260-261, (66), 270-274, (69)
Houghton Mifflin(1972)	pp. 308-311, (74), 110-113, (30), 116, 117, (32)
Addison-Wesley(1971, 1968)	pp. 282-287, 289

Level; 15 Step: A Concept: Geometry

Text References: continued

Book: 6

Houghton Mifflin(1967)	pp. 91-93, (22), 270-271, (62)
Houghton Mifflin(1972)	pp. 118-119
Addison-Wesley(1971, 1968)	pp. 262-265

Book: 7

Houghton Mifflin(1967, 1970)	pp. 454-457, (71)
Houghton Mifflin(1972)	pp. 118-119, 306
Addison-Wesley(1971)	pp. (14), 283-285, (37, 58), 132, 396-399, (83, 84)

Book: 8

Houghton Mifflin(1972)	pp. 118-119, 305

Other References:

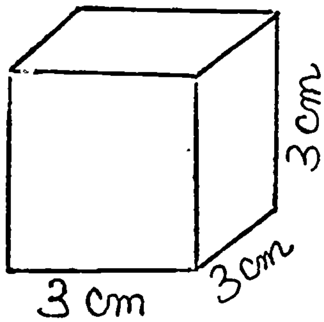
Imperial Tape #38 (Intermediate)
Franklin Series, Learning About Measurement, Volume: pp. 54-59.

Level: 15 Step: A Concept: Geometry

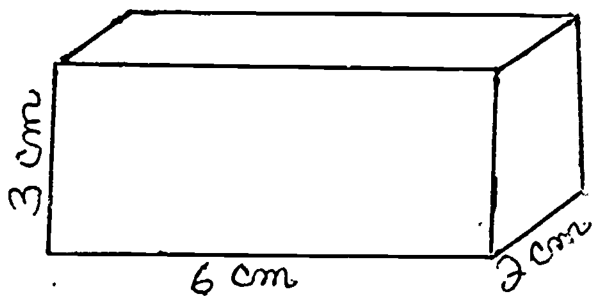
WORKSHEET

Find the surface area and volume of the following:

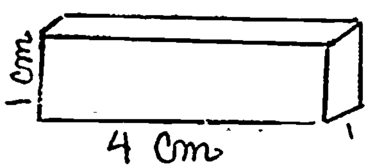
1.



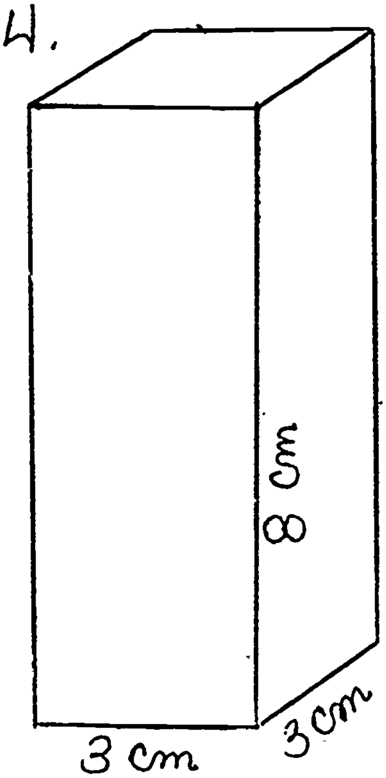
2.



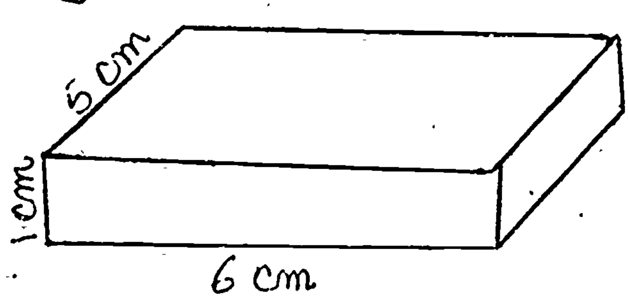
3.



4.



5.



1. ________ area

________ volume

2. ________ area

________ volume

3. ________ area

________ volume

4. ________ area

________ volume

5. ________ area

________ volume

Level: 15 Step: B Concept: Numeral

I. Concept:

Numeral: Naming a common fraction with 10 or 100 in the denominator as a decimal fraction.

II. Behavioral Objective:

The student given common fractions with denominators of 10 or 100, will be able to write the equivalent decimal fractions.

III. Mathematical Ideas:

The first place to the right of the decimal point is tenths while the second place to the right of the decimal point is hundredths.

IV. Vocabulary To Stress:

decimal point
tenths
hundredths
decimal fraction
mixed numeral

V. Activities:

A. Decimal Race. Make a collection of large flash cards with three or four decimal fractions on each one. This could be used as a race with the class divided into two teams. On the board draw a ladder with 10 steps. The team that answers ten cards correctly first wins. The teams take turns answering. The child must say the decimal that is the largest. If one student misses the pupil from the opposite team has a chance. If he misses it also, the correct answer is given and the next person takes his turn. This could be used for addition or subtraction of decimals.

B. Use first part of Imperial Tape (Intermediate) #25

C. Make a chart on the board or on cardboard. Have each child make one at his desk. Have them put decimal problems in this chart. Name the place value.

hundredths	tens	ones	decimal point	tenths	hundredths
1	6	8	.	2	3

Level: 15 Step: B Concept: Numeral

Text References:

Book: 5

Houghton Mifflin(1967)	pp. 330-333, 332, all; 333, #1-20
Houghton Mifflin(1972)	pp. 274, 318, 320
Addison-Wesley(1971, 1963)	pp. 256-260

Book: 6

Houghton Mifflin(1967)	pp. 298-299
Houghton Mifflin(1972)	pp. 142-143

Book: 7

Houghton Mifflin(1967, 1970)	pp. 372-375, (56)
Houghton Mifflin(1972)	pp. 14-15
Addison-Wesley(1971)	pp. 296-298

Other References:

Modern Math Filmstrip #759

Level: 15 Step: B Concept: Addition

I. Concept:

Addition: Naming the sum of 2 decimal addends in tenths and hundredths.

II. Behavioral Objective:

The student given decimal addends in tenths and hundredths will be able to compute the sum.

III. Mathematical Ideas:

A. The position of a digit in a number determines its value. (place value).

B. Addition with decimal fractions follows the same rules as for whole numbers.

IV. Vocabulary To Stress: tenths - hundredths

V. Activities:

A. Make addition tables with decimals. Have the children work them. Several different charts could be made.

+	.01	.02	.03	.04	
.10	.11	.12	.13	?	
.20	.21	?	?	.24	
.30					

Text References:

Book: 4

Houghton Mifflin(1972) pp. 320, 321, 322

Book: 5

Houghton Mifflin(1967) pp. 331, 334
Houghton Mifflin(1972) pp. 274, 278, 321, 322
Addison-Wesley(1971, 1968) pp. 261-262

Level: 15 Step: B Concept: Addition

Text References: continued

Book: 6

Houghton Mifflin(1967)	pp. 302-305. (69)
Houghton Mifflin(1972)	pp. 206-208, (51)
Addison-Wesley(1971, 1968)	pp. 222, (39, 40, 41), 223-225

Book: 7

Houghton Mifflin(1967, 1970)	pp. 379-382
Houghton Mifflin(1972)	pp. 151-152, 196-197, 218-219, 420, 422
Addison-Wesley(1971)	pp. 300-301

Other References:

Imperial Tape #25 (Intermediate) (part)

Level: 15 Step: B Concept: Subtraction

I. Concept:

Subtraction: Naming the missing addend in a decimal subtraction problem.

II. Behavioral Objective:

The student given a sum and an addend in tenths or hundredths will be able to subtract to find the missing addend.

III. Mathematical Ideas:

Subtraction with decimal fractions follows the same rule as subtraction for whole numbers.

IV. Vocabulary To Stress: tenths - hundredths

V. Activities:

A. Teacher's Edition Imperial Intermediate Tapes, pp. 52-53.

Text References:

Book: 4

Houghton Mifflin(1972) pp. 323(part)

Book: 5

Houghton Mifflin(1967) pp. 331, 334, 337
Houghton Mifflin(1972) pp. 275, 279, (68), 321, 322
Addison-Wesley(1971, 1968) pp. 261, (50), 263, (51), 264-265, 338

Book: 6

Houghton Mifflin(1967) pp. 302-305, (69)
Houghton Mifflin(1972) pp. 206-208, (51)
Addison-Wesley(1971, 1968) pp. 222, (39, 40, 41), 223-225

Book: 7

Houghton Mifflin(1967, 1970) pp. 379-383
Houghton Mifflin(1972) pp. 151-152, 196-197, 218-219, 420, 422
Addison-Wesley(1971) pp. 300-301

Other References:

Imperial Tape #25 (Intermediate)

Level: 15 Step: B Concept: Multiplication

I. Concept:

Multiplication: Multiplying decimal fractions with no more than 2 decimal places in the product.

II. Behavioral Objective:

The student given 2 decimal factors will be able to find the product.

III. Mathematical Ideas:

The number of places in the product to the right of the decimal point equals the sum of the places to the right of the decimal point in the factors.

IV. Vocabulary To Stress: decimal point

V. Activities:

A. Teacher's Edition Imperial Tapes, Lesson 26.

B. Math Games for Greater Achievement (4-9), page 123.

C. Write sentences on board as: .54 x 7 =
$1.25 x 25 =
Have children make a word problem to fit the equation and solve.

Text References:

Book: 5

Houghton Mifflin(1967)	pp. 335-336, 338-339(part)
Houghton Mifflin(1972)	pp. 279, 280, (69), 282-283, 323(part), (67)

Book: 6

Houghton Mifflin(1967)	pp. 306-307, (70)
Houghton Mifflin(1972)	pp. 210, (52, part)
Addison-Wesley(1971, 1968)	pp. 230-231, (42), 232-233, (43)

Book: 7

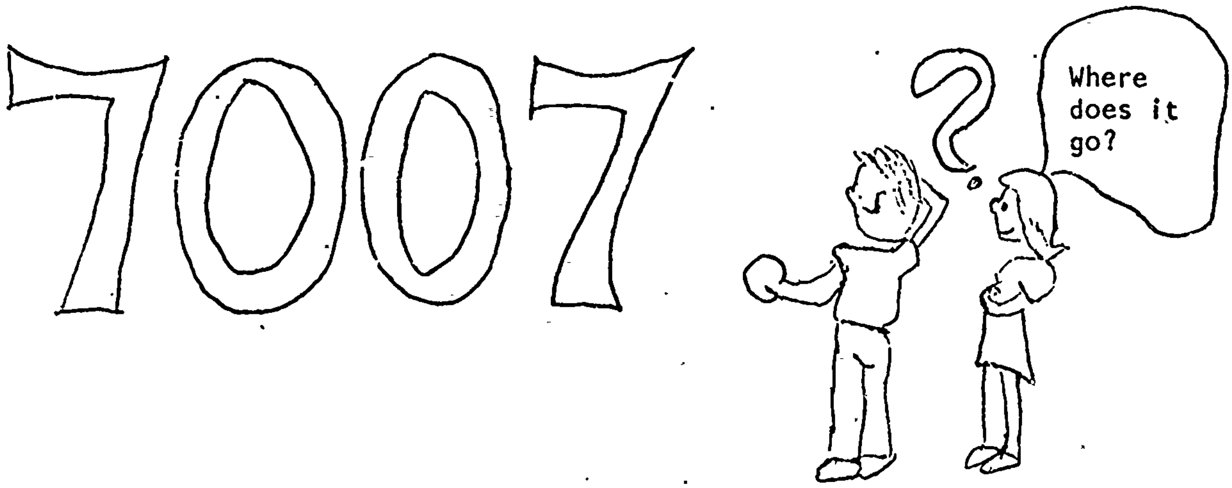
Houghton Mifflin(1967, 1970)	pp. 393-397
Houghton Mifflin(1972)	pp. 208, 210, 218-219, 256, 424
Addison-Wesley(1971, 1968)	pp. 304-306

Other References:

Imperial Tape #26 (Intermediate)
Modern Math Filmstrip . #766

Level: 15 Step: B Concept: Multiplication

MULTIPLICATION OF DECIMALS



1. $.9 \times .9$
2. $.8 \times .9$
3. $.11 \times 6$
4. $.6 \times .9$

5.

5. $.23 \times 4$
6. $.22 \times 9$
7. 3.28×7
8. 2.34×3

9. 5.26×3
10. 2.71×5
11. 2.72×8
12. 9.45×7

13. 4.7×5.6
14. 4.45×67
15. 4.7×3.3
16. 10.41×8

Level: 15 Step: B Concept: Geometry

I. Concept:

Geometry: Identifying and constructing arcs and chords.

II. Behavioral Objective:

The student given a circle will be able to identify and draw arcs and chords.

III. Mathematical Ideas:

A. Every point on a circle is a fixed distance (radius) from a particular point called the center.

B. A segment of the circumference of a circle is an arc.

C. A line segment joining 2 points of a circle is called a chord.

IV. Vocabulary To Stress:

radius	arc
compass	chord
diameter	circumference

V. Activities:

A. Teacher's Edition Imperial Tapes, lesson #36

B. Give the children several different sized radii and diameters and have them construct circles using a compass. See fun worksheet.

C. Have the children make designs in circles with a compass.

D. Review definitions for circumference, diameter, radius, semi-circle.

Text References:

Book: 5

Houghton Mifflin(1967)	pp. 76-77, 262-263
Houghton Mifflin(1972)	pp. 101, 288, (27)
Addison-Wesley(1971, 1968)	pp. 92-93 (all)

Book: 6

Houghton Mifflin(1967)	pp. 80
Houghton Mifflin(1972)	pp. 102-103
Addison-Wesley(1971, 1968)	pp. 250-251

Book: 7

Houghton Mifflin(1972)	pp. 288
Addison-Wesley(1971)	pp. 405

Other References:

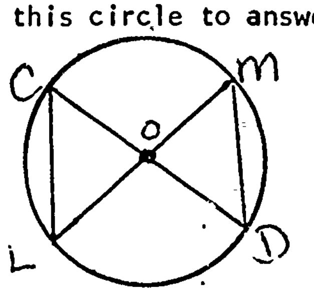
Tape #36; Mathematics in the Making 4; Mathematics in the Making 5; Curriculum Filmstrips 368, 369.
Modern Math Filmstrip #770 (part of it)
Forms We See — Filmstrip and tape EF 1106-1 Circles (Part of it)

Level: 15 Step: B Concept: Geoemtry

WORKSHEET: GEOMETRY

1. With your compass draw a circle with a radius of 2 inches. Be sure to mark a point for the center of your circle before you begin. Draw a diameter and label it DC.

2. Use this circle to answer the following questions:



Name 2 diameters. ______________________

____

What is $\overline{CL}$ called? ______________________

3. A line segment joining any 2 points on a circle is called a ______________________

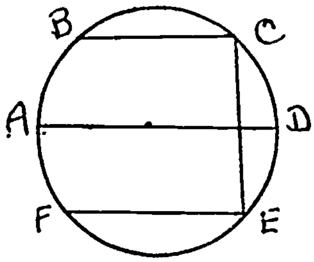
4. With your compass draw a circle using a 1 inch radius. Draw in and and label a radius.

5. A part of the circumference of a circle is called an ______________________

Level: 15 Step: B Concept: Geometry

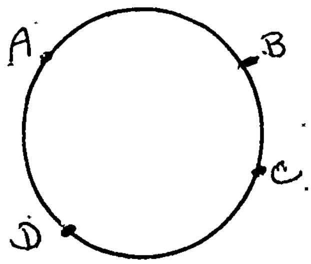
WORKSHEET

Identifying arcs and chords.



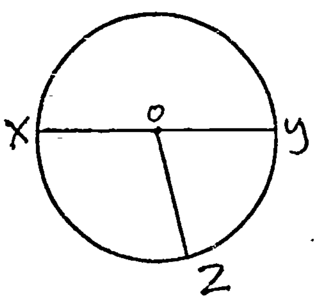
Name 4 chords

1. ________
2. ________
3. ________
4. ________



Name 4 arcs

5. ________
6. ________
7. ________
8. ________



Name the diameter

Name a radius

9. ________
10. ________

Definition Match

11. ________ A line segment that joins points of a circle

12. ________ A line segment from the center of the circle to any point on the circumference

13. ________ The distance around a circle is

14.. ________ A line segment which has end points in circle and which passes through the center is the

15. ________ The set of all points on a circle between two given points is an

a. circumference

b. diameter

c. chord

d. arc

e. radius

Level: 15 Step: Z Concept: Additon

I. Concept:

Addition: Naming the sum in base 5 of two addends in base 5.

II. Behavioral Objective:

The student given two numerals in base 5 will be able to write the sum in base 5.

III. Mathematical Ideas:

A. A digit can have face value, place value, and total value.

B. In each place the numeral cannot be greater than four.

IV. Vocabulary To Stress: place value - face value

V. Activities:

A. Let children experiment with base 5 blocks, as in Level 12 Step Z.

B. Have children complete addition table for base 5 as done for base 7 in teacher's edition Imperial Tape Lesson 35 (Intermediate).

C. Use abacus with four counters on each wire.

* D. ESA Multibase Arithmetic Blocks Cards 6A-8.

Test References:

Book: 7

Houghton Mifflin(1967) pp. 141, 144

Other References:

Imperial Tape #35 (Intermediate) (part)

* Suggested introductory activity.

Level: 15 Step: Z Concept: Addition

WORKSHEET: ADDITION

BASE FIVE

++	0	1	2	3	4
0	0	1	2	3	4
1	1	2	3	4	10
2	2	3	4	10	11
3	3	4	10	11	12
4	4	10	11	12	13

1. 14_{five} 1 set of ________ and ________ ones.

Add the following problems:

2. $3_{five} + 1_{five}$
3. $2_{five} + 2_{five}$
4. $2_{five} + 1_{five}$
5. $2_{five} + 3_{five}$
6. $3_{five} + 4_{five}$
7. $3_{five} + 3_{five}$
8. $4_{five} + 2_{five}$
9. $12_{five} + 4_{five}$
10. $11_{five} + 12_{five}$
11. $13_{five} + 10_{five}$
12. $21_{five} + 11_{five}$
13. $12_{five} + 13_{five}$
14. $123_{five} + 101_{five}$
15. $143_{five} + 103_{five}$

Level: 15 Step: Z Concept: Addition

EVALUATION: ADDITION

1. $\begin{array}{r} 31_{\text{five}} \\ +\ 12_{\text{five}} \\ \hline \end{array}$

2. $\begin{array}{r} 42_{\text{five}} \\ +\ 31_{\text{five}} \\ \hline \end{array}$

3. $\begin{array}{r} 33_{\text{five}} \\ +\ 33_{\text{five}} \\ \hline \end{array}$

4. $\begin{array}{r} 101_{\text{five}} \\ +\ 100_{\text{five}} \\ \hline \end{array}$

5. $\begin{array}{r} 324_{\text{five}} \\ +\ 423_{\text{five}} \\ \hline \end{array}$

6. $\begin{array}{r} 144_{\text{five}} \\ +\ 323_{\text{five}} \\ \hline \end{array}$

7. $\begin{array}{r} 3_{\text{five}} \\ +\ 24_{\text{five}} \\ \hline \end{array}$

8. $\begin{array}{r} 141_{\text{five}} \\ +\ 204_{\text{five}} \\ \hline \end{array}$

9. $\begin{array}{r} 321_{\text{five}} \\ +\ 231_{\text{five}} \\ \hline \end{array}$

10. $\begin{array}{r} 104_{\text{five}} \\ +\ 233_{\text{five}} \\ \hline \end{array}$

11. $\begin{array}{r} 122_{\text{five}} \\ +\ 303_{\text{five}} \\ \hline \end{array}$

12. $\begin{array}{r} 211_{\text{five}} \\ +\ 44_{\text{five}} \\ \hline \end{array}$

13. $\begin{array}{r} 303_{\text{five}} \\ +\ 42_{\text{five}} \\ \hline \end{array}$

14. $\begin{array}{r} 111_{\text{five}} \\ +\ 34_{\text{five}} \\ \hline \end{array}$

15. $\begin{array}{r} 1024_{\text{five}} \\ +\ 432_{\text{five}} \\ \hline \end{array}$

Level: 15 Step: Z Concept: Subtraction

I. Concept:

Subtraction: Naming the missing addend, given a sum and one addend in base 5 numerals.

II. Behavioral Objective:

The student given a sum and an addend in base 5 will be able to name the missing addend in base 5.

III. Mathematical Ideas:

A. A digit does have face value, place value and total value.

B. In each place the numeral cannot be greater than four.

C. Renaming one five as five ones.

* IV. Activities:

A. ESA Multibase Arithmetic Blocks Cards 9-10.

Text References:

Book: 7

Houghton Mifflin(1967) pp. 147, 149

* Suggested introductory activity.

Level: 15 Step: Z Concept: Subtraction

WORKSHEET: SUBTRACTION

Can you find a way to use the base-5 addition chart when subtracting?

+	0	1	2	3	4
0	0	1	2	3	4
1	1	2	3	4	10
2	2	3	4	10	11
3	3	4	10	11	12
4	4	10	11	12	13

1. 2_{five}
   $- 1_{five}$

2. 3_{five}
   $- 1_{five}$

3. 11_{five} = 11111 0
   $- 4_{five}$ = -1111

4. 13_{five} = 11111 000
   $- 3_{five}$ = - 000

5. 22_{five} = 1_{five}______ ones
   $- 14_{five}$ = 1_{five}______ ones

6. 33_{five} = 2_{five}______ ones
   $- 24_{five}$ = 2_{five}______ ones

7. 443_{five} = 43_{five}______ ones
   $- 204_{five}$ = 20_{five}______ ones

8. 42_{five}
   $- 33_{five}$

9. 32_{five}
   $- 23_{five}$

10. 12_{five}
    $- 3_{five}$

Level: 15 Step: Z Concept: Subtraction

EVALUATION: SUBTRACTION

Find the difference in the following problems:

1. 23_{five}
   $-\ 12_{five}$

2. 31_{five}
   $-\ 22_{five}$

3. 34_{five}
   $-\ 24_{five}$

4. 31_{five}
   $-\ 14_{five}$

5. 22_{five}
   $-\ 13_{five}$

6. 222_{five}
   $-\ 113_{five}$

7. 132_{five}
   $-\ 44_{five}$

8. 121_{five}
   $-\ 32_{five}$

9. 311_{five}
   $-\ 22_{five}$

10. 231_{five}
    $-\ 14_{five}$

11. 100_{five}
    $-\ 4_{five}$

12. 110_{five}
    $-\ 14_{five}$

13. 342_{five}
    $-\ 233_{five}$

14. 3412_{five}
    $-\ 1321_{five}$

15. 1140_{five}
    $-\ 421_{five}$

Level: 15 Answers to Worksheets

15-2

1. $\frac{5}{12}, \frac{4}{12}, \frac{3}{12}, 0$
2. $\frac{6}{6}, 0$
3. $\frac{5}{10}, \frac{1}{10}, \frac{3}{10}$
4. $0, \frac{5}{10}$ or $\frac{1}{2}, \frac{2}{10}$
5. $\frac{6}{15}, \frac{9}{15}$

15-5

I.
1. 11_{ten}
2. 8_{ten}
3. 5_{ten}
4. 10_{ten}
5. 9_{ten}
6. 21_{ten}
7. 24_{ten}
8. 25_{ten}
9. 27_{ten}
10. 30_{ten}
11. 49_{ten}
12. 50_{ten}

II.
1. 10_{five}
2. 13_{five}
3. 20_{five}
4. 30_{five}
5. 44_{five}
6. 100_{five}
7. 103_{five}
8. 110_{five}
9. 113_{five}
10. 400_{five}

15-6

1. 12_{five}
2. 23_{five}
3. 4 fives, 3 ones
4. 3 fives, 4 ones
5. 2 fives, 0 ones
6. 15_{ten}
7. 11_{ten}
8. 25_{ten}
9. 38_{ten}
10. 129_{ten}
11. 32_{five}
12. 100_{five}

15-10

1. $2\frac{2}{3}$
2. $7\frac{1}{5}$
3. $8\frac{1}{2}$
4. $5\frac{4}{5}$
5. $5\frac{5}{12}$
6. $53\frac{1}{8}$
7. $20\frac{4}{5}$
8. $22\frac{1}{8}$
9. $6\frac{1}{3}$
10. $2\frac{2}{3}$
11. $12\frac{1}{5}$
12. $17\frac{1}{13}$
13. $50\frac{3}{20}$
14. $18\frac{2}{11}$
15. $29\frac{1}{20}$
16. $70\frac{3}{20}$
17. $35\frac{2}{9}$
18. $8\frac{1}{7}$
19. $12\frac{4}{5}$
20. $35\frac{16}{25}$

15-11

1. $7\frac{1}{2}$ gal.
2. $2\frac{1}{2}$ qts.
3. $7\frac{1}{7}$ yds.
4. $5\frac{1}{4}$ lbs.
5. $129\frac{1}{2}$ lbs.

Level: 15 Answers to Worksheets

15-14 Mixed Numerals times a whole number.

1. 57
2. 25
3. $23\frac{1}{5}$
4. 57
5. $9\frac{5}{7}$
6. 71
7. $35\frac{7}{9}$
8. 101
9. $85\frac{3}{4}$
10. 120
11. $45\frac{3}{4}$
12. $32\frac{2}{3}$
13. 61
14. 68
15. $64\frac{8}{9}$
16. 52
17. $34\frac{1}{6}$
18. 50
19. 65
20. 82

15-17 Surface area and volume

1. 54 sq. cm. area
   27 cu. cm. volume
2. 72 sq. cm. area
   36 cu. cm. volume
3. 18 sq. cm. area
   4 cu. cm. volume
4. 114 sq. cm. area
   72 cu. cm. volume
5. 32 sq. cm. area
   30 cu. cm. volume

15-24

1. .81
2. .72
3. .66
4. .54
5. .92
6. 1.98
7. 22.96
8. 7.02
9. 15.78
10. 13.55
11. 21.76
12. 66.15
13. 26.32
14. 2968.15
15. 15.51
16. 83.28

15-26

2. $\overline{CD}$ $\overline{LM}$
   CL is called a chord.
3. chord
5. arc

15-27

1. BC
2. CE
3. AD
4. FE
5. AB
6. BC several
7. CD possible
8. DA answers
9. XOY
10. OZ or OX or OY
11. chord, c
12. radius, e
13. circumference, a
14. diameter, b
15. arc, d

Level: 15 Answers to Enrichment (Step Z)

15-29

1. five and 4 ones
2. 4_{five}
3. 4_{five}
4. 3_{five}
5. 10_{five}
6. 12_{five}
7. 11_{five}
8. 11_{five}
9. 21_{five}
10. 23_{five}
11. 23_{five}
12. 32_{five}
13. 30_{five}
14. 224_{five}
15. 301_{five}

15-32

1. 1_{five}
2. 2_{five}
3. 2_{five}
4. 10_{five}
5. 3_{five}
6. 4_{five}
7. 234_{five}
8. 4_{five}
9. 4_{five}
10. 4_{five}

Level: 15

EVALUATIONS (STEP Z)

15-30

1. 43_{five}
2. 123_{five}
3. 121_{five}
4. 201_{five}
5. 1302_{five}
6. 1022_{five}
7. 32_{five}
8. 400_{five}
9. 1102_{five}
10. 342_{five}
11. 430_{five}
12. 310_{five}
13. 400_{five}
14. 200_{five}
15. 2011_{five}

15-33

1. 11_{five}
2. 4_{five}
3. 10_{five}
4. 12_{five}
5. 4_{five}
6. 104_{five}
7. 33_{five}
8. 34_{five}
9. 234_{five}
10. 212_{five}
11. 41_{five}
12. 41_{five}
13. 104_{five}
14. 2041_{five}
15. 214_{five}

Level: 15 ANSWER SHEET - POST TEST I

Step A

Sets-A

1. $\frac{5}{10}$ or $\frac{1}{2}$, d.
2. $\frac{7}{10}$, a
3. $\frac{0}{10}$ or 0, a
4. $\frac{8}{10}$ or $\frac{4}{5}$, c
5. $\frac{10}{10}$ or 1, c

Number-A

6. 103_{five}
7. 30_{ten}
8. 39_{ten}
9. 24_{five}
10. 113_{five}

Addition-A

11. $7\frac{5}{6}$
12. $12\frac{5}{12}$
13. $16\frac{3}{6}$ or $16\frac{1}{2}$
14. $51\frac{11}{18}$
15. $72\frac{17}{22}$

Subtraction-A

16. $74\frac{6}{11}$
17. $4\frac{1}{4}$
18. $46\frac{11}{13}$
19. $54\frac{7}{11}$
20. $68\frac{6}{17}$

Multiplication-A

21. $10\frac{2}{3}$
22. 27
23. $14\frac{2}{9}$
24. $29\frac{1}{3}$
25. $1.47

Geometry-A

26. 54 cu. ft.
27. 90 sq. ft.
28. 60 cu. ft.
29. 126 cu. ft.
30. 142 sq. in.

Step B

Numeral-B

31. .54
32. 1.95
33. .6
34. .07
35. 1.58

Addition-B

36. 1.99 mi.
37. 8.44 lbs.
38. $2.63
39. 1.83 in.
40. .39

Subtraction-B

41. 8.62
42. 1.8 miles
43. 1.85
44. 46.32
45. .06

Multiplication-B

46. 50.75
47. 15.48
48. 99.6
49. 74.8
50. 59.22

Geometry-B

51. chord
52. diameter
53. AB
54. diameter, d
55. arc, b

Level: 15 ANSWER SHEET - POST TEST II

Step A

Sets-A

1. A
2. D
3. D
4. C
5. A

Number-A

6. 13_{ten}
7. 23_{ten}
8. 133_{five}
9. 8_{ten}
10. 123_{five}

Addition-A

11. $5\frac{5}{6}$
12. $11\frac{7}{15}$
13. $18\frac{5}{12}$
14. $10\frac{1}{2}$
15. $9\frac{1}{2}$

Subtraction-A

16. $63\frac{5}{7}$
17. $5\frac{1}{8}$
18. $20\frac{1}{5}$
19. $74\frac{11}{12}$
20. $140\frac{5}{16}$

Multiplication-A

21. $18\frac{2}{3}$
22. $40\frac{5}{9}$
23. $14\frac{8}{9}$
24. $26\frac{1}{3}$
25. $82\frac{1}{2}$

Geometry-A

26. 184 sq. in.
27. 160 cu. in.
28. 528 cu. in.
29. 448 sq. in.
30. 180 cu. ft.

Step B

Number-B

31. B
32. C
33. C
34. D
35. A

Addition-B

36. 1.97
37. 1.19
38. 1.59
39. 2.22
40. 6.25 ft.

Subtraction-B

41. .69 of an in.
42. 5.17
43. .29
44. 74.84
45. .41 in.

Multiplication-B

46. 8.72
47. 15.48
48. 19.71
49. 45.78
50. 54.64

Geometry-B

51. diameter, a
52. chord, b
53. arc, d
54. diameter
55. radius, c

TABLE OF CONTENTS

Level 16

Step A

Step B

Step Z

I. Concept:

Numeral: Writing equations and inequalities for word problems.

II. Behavioral Objective:

The student given a word problem will be able to write an equation or an inequality.

III. Mathematical Ideas:

A. A number sentence may be used to express a word problem.

B. A number sentence may be used to state the number operation corresponding to a particular set operation.

C. A number is a solution of a number sentence if its name makes the sentence true.

IV. Vocabulary To Stress: equation solution inequality

V. Activities:

A. Have students make up their own word problems. Let the class write the equations or inequalities and solve the problems.

B. Write two equivalent statements on cards, one per card as $\boxed{n + 4 = 11}$ and $\boxed{11 - 4 = n}$. Students can play a simple game called "Find My Mate". They sit in a circle holding 4 or 5 cards each. Each turns up a card and watches cards of other players and tries to find one equivalent to his. The first player to see one picks it up and keeps both cards. Leave unmatched cards face up as second cards are turned up. The game continues until all equivalent statements are paired. The player with the most cards wins.

C. Divide students into several groups. On the blackboard write as many sets of equations giving values to n as groups. One from each team comes to the board as you give the rule, such as add 3. You write "n = 4". Student writes "n + 3 = 7". You write "n = 7". Student writes "n + 3 = 10". The first one to complete his equation scores a point for his team.

Text References:

Book: 6

Houghton Mifflin(1967) pp. 133, 137
Addison-Wesley(1971, 1968) pp. 28, 30, 75, 338, 339, T339a, T339b, (18)

Book: 7

Houghton Mifflin(1967, 1970) pp. 48-49, 55-56, 74-75, 80-81, 83-84, 92-94
Houghton Mifflin(1972) pp. 43, 70, 211
Addison-Wesley(1971) pp. 67-71, 83-84, (18, 40, 50, 64)

Other References:

Imperial Tape 28, (Intermediate), "Writing Math Sentences". (Do not write on tape worksheets. Student uses own paper.)

1: 16
Step: A

WORKSHEET: EQUATIONS AND INEQUALITIES

Write the equation for the following word problems. Let n represent the unknown number. Then solve the equation.

1. There are 17 boys and 12 girls in the classroom. What is the total number of students in the classroom? ____________________

2. Five boys went fishing. Three of the boys caught four fish each. How many fish were caught? ____________________

3. Opera tickets cost $6.00 each. If you have $42.00 how many tickets can you buy? ____________________

4. Isabell receives $2.00 allowance each week. If she saves $\frac{1}{2}$ of her allowance each week, how much will she save in three weeks? ________

5. Mike and Joe are mowing lawns to make money to take on their vacation trip. On Monday they made $6.50; on Tuesday, $5.00; on Wednesday, $7.00; on Thursday, $4.00. How much money did they make in all? ____________________

6. Mary does baby sitting in the summer. She charges 75¢ an hour How many hours will she have to baby sit to make $10.00? ____________________

7. The product of 7 and a number is equal to 56. What is the number?

____

8. What number divided by 9 is equal to 7? ____________________

9. Twenty-three is equal to some number increased by five. What is the number?

____

Write and solve the sentences using inequalities:

10. Six less than the number is greater than 12. ____________________

11. Four times the number is less than 24. ____________________

12. Nine more than the number is less than 13. ____________________

Level: 16 Step: A Concept: Order

I. Concept:

Order: Rounding whole numbers to the nearest ten, hundred, and thousand.

II. Behavioral Objectives:

The student given a whole number will be able to round it to the nearest ten, hundred, or thousand.

III. Mathematical Ideas:

A. In rounding whole numbers replace with a zero each digit to the right of the digit in the desired place, e.g. 42 is 40 when rounding to nearest ten.

B. If the digit to the right of the digit in the desired place is five or more, round up; if less than five, leave unchanged.
e.g. 43 rounded to nearest 10 is 40.
46 rounded to nearest 10 is 50.

IV. Vocabulary To Stress:

round up round down

V. Activities:

*A. Provide number line activities to help in teaching rounding numbers. H.M. (1972) p. T57.

B. Review place value.

Text References:

Book: 6

Houghton Mifflin(1967)	pp.	21, 104-105, (24)
Houghton Mifflin(1972)	pp.	16-17, (6), 66.
Addison-Wesley(1971, 1968)	pp.	3

Book: 7

Houghton Mifflin(1972)	pp.	16
Addison-Wesley(1971)	pp.	14-15

Book: 8

Houghton Mifflin(1972)	pp.	33 (1-15)

Other References:

Modern Mathematics Filmstrip 750

* This activity could be used to help introduce the concept, using smaller numbers if [illegible].

Level: 16 Step: A Concept: Subtraction

I. Concept:

Subtraction: Subtracting mixed numerals with renaming.

II. Behavioral Objective:

The student given two mixed numerals will be able to compute the difference when it involves renaming.

III. Mathematical Ideas:

A. The mixed numeral may be renamed using the idea that $1 = \frac{n}{n}$. For example, $3\frac{1}{2}$ may be rewritten as $2\frac{3}{2}$.

B. From then on, subtraction is the same as previously learned.

IV. Vocabulary To Stress:

mixed numeral
expanded numeral
simplest form
equivalent

V. Activities:

A. Game "Action-Fraction". (In your building)

B. Matching equivalent mixed numerals. Cut 2-by-4 inch cards of tagboard. Rule each card 2" x 4" and write non-equivalent mixed numerals on either half, as shown. Design the cards so there will be four fractions in each equivalent set, 40 in all. Two to four players can draw 5 cards from the fraction dominoes that are face down on the table. Each child must match a numeral already played with an equivalent from his hand or draw another card from the pack. The first child to play all of his cards correctly wins. (Sample set - $2\frac{1}{4}$, $2\frac{2}{8}$, $1\frac{5}{4}$, $1\frac{10}{8}$)

Level 16 Step: A Concept: Subtraction

Text References:

Book: 5

Houghton Mifflin(1967)	pp. 248, (64), 250-251, (65), 256-257, 259.
Houghton Mifflin(1972)	pp. 210, 213, (54).
Addison-Wesley(1971, 1968)	pp. 250, (47), (48), (49)

Book: 6

Houghton Mifflin(1967)	pp. 223-224, (49)
Houghton Mifflin(1972)	pp. 200-203, (50)
Addison-Wesley(1971, 1968)	pp. 151, 162

Book: 7

Houghton Mifflin(1967, 1970)	pp. 347-348
Houghton Mifflin(1972)	pp. 194, 422
Addison-Wesley(1971)	pp. 225, (44)

Book: 8

Houghton Mifflin(1972)	pp. 128-129

Other References:

Imperial Tape #21 (Intermediate)

Level: 16 Step: A Concept: Multiplication

I. Concept:

Multiplication: Computing product of 2 mixed numerals.

II. Behavioral Objective:

The student given 2 mixed numerals will be able to compute the product.

III. Mathematical Ideas:

A. Mixed numerals can be changed to fractions greater than 1.

B. Multiply numerator times numerator and denominator times denominator.

IV. Vocabulary To Stress

mixed numeral fraction greater than 1

V. Activities:

A. Teacher's Edition Imperial Tape. Lesson 24 (Intermediate)

B. Race Track. Use a large piece of tagboard. Make a long and winding racetrack and divide it into many squares. On some squares put ! and on others put ?. Make up two sets of cards, one set for each symbol. Put a multiplication problem on each card and instructions such as move forward two, extra turn, etc. If the child gets the correct answer he follows the instructions on the card. If he answers wrong he can either be made to go back two spaces or to the last space he was on. Have students draw cars to go with the racetrack.

Text References:

Book: 5

Houghton Mifflin(1967)	p. 328

Book: 6

Houghton Mifflin(1967)	pp. 244, (56), 246-247, 261.
Houghton Mifflin(1972)	p. 260
Addison-Wesley(1971, 1968)	p. 177

Book: 7

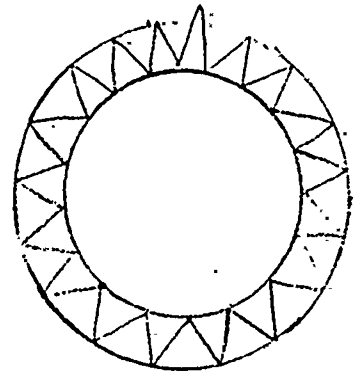
Houghton Mifflin(1967, 1970)	pp. 349-353, (53)
Houghton Mifflin(1972)	pp. 206-207, 210
Addison-Wesley(1971)	pp. 237-238, (46)

Book: 8

Houghton Mifflin(1972)	pp. 136-137

Other References:

Imperial Tape #24 (Intermediate)
Modern Mathematics Filmstrip #763(part of it)



1. $1\frac{7}{9} \times 3\frac{1}{5} =$

2. $6\frac{1}{2} \times 8\frac{1}{6} =$

3. $4\frac{1}{2} \times 2\frac{2}{3} =$

4. $1\frac{5}{8} \times 3\frac{2}{3} =$

5. $5\frac{1}{5} \times 4\frac{1}{4} =$

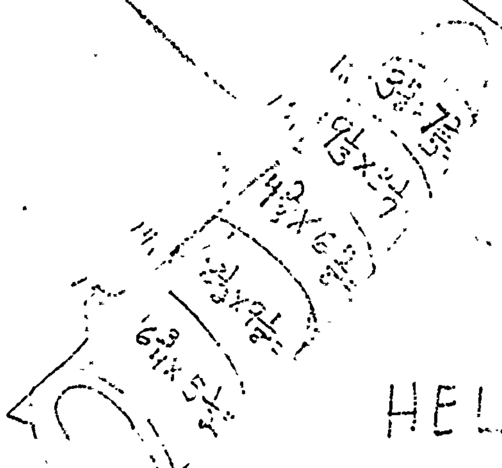
6. $6\frac{2}{3} \times 3\frac{1}{5} =$

7. $2\frac{4}{9} \times 2\frac{1}{5} =$

8. $2\frac{1}{5} \times 2\frac{3}{4} =$

9. $2\frac{1}{4} \times 3\frac{1}{3} =$

10. $4\frac{1}{5} \times 3\frac{1}{2} =$



HELP THE SPACESHIP TO THE MOON

Level: 16 Step: A Concept: Multiplication

WORKSHEET III - MULTIPLICATION

I. Change the following mixed numerals to a fraction greater than one.

A. $1\frac{1}{4}=$ ______ F. $2\frac{2}{7}=$ ______ K. $5\frac{2}{4}=$ ______

B. $2\frac{2}{3}=$ ______ G. $3\frac{3}{4}=$ ______ L. $7\frac{1}{5}=$ ______

C. $3\frac{1}{4}=$ ______ H. $8\frac{1}{2}=$ ______ M. $4\frac{3}{8}=$ ______

D. $5\frac{1}{3}=$ ______ I. $6\frac{2}{3}=$ ______ N. $6\frac{2}{5}=$ ______

E. $6\frac{2}{3}=$ ______ J. $9\frac{1}{4}=$ ______ O. $4\frac{2}{3}=$ ______

II. Multiply the following mixed numerals:

A. $2\frac{1}{2}\times 3\frac{1}{4}=$ ______ K. $2\frac{2}{7}\times 1\frac{1}{2}=$ ______

B. $5\frac{1}{6}\times 2\frac{2}{6}=$ ______ L. $1\frac{1}{4}\times 2\frac{1}{3}=$ ______

C. $1\frac{1}{4}\times 2\frac{1}{4}=$ ______ M. $3\frac{1}{5}\times 1\frac{1}{2}=$ ______

D. $1\frac{1}{5}\times 2\frac{1}{3}=$ ______ N. $4\frac{3}{8}\times 2\frac{1}{2}=$ ______

E. $4\frac{2}{3}\times 4\frac{3}{8}=$ ______ O. $1\frac{1}{4}\times 1\frac{1}{4}=$ ______

F. $1\frac{1}{4}\times 5\frac{2}{4}=$ ______ P. $2\frac{1}{3}\times 1\frac{1}{5}=$ ______

G. $2\frac{2}{3}\times 8\frac{1}{2}=$ ______ Q. $3\frac{1}{2}\times 1\frac{1}{3}=$ ______

H. $2\frac{2}{3}\times 3\frac{1}{4}=$ ______ R. $2\frac{1}{4}\times 1\frac{1}{2}=$ ______

I. $1\frac{1}{5}\times 3\frac{3}{4}=$ ______ S. $1\frac{2}{3}\times 2\frac{1}{2}=$ ______

J. $1\frac{1}{3}\times 6\frac{2}{3}=$ ______ T. $8\frac{1}{2}\times 1\frac{1}{2}=$ ______

Level: 16 Step: A Concept: Division

I. Concept:

Division: Dividing mixed numerals

II. Behavioral Objective:

The student given two mixed numerals will be able to compute the quotient.

III. Mathematical Ideas:

A. Express the mixed numerals as fractions.

B. From then on, division is the same procedure as previously learned.

C. A complex fraction is a fraction whose numerator and/or denominator is a fraction: $\frac{\frac{2}{3}}{\frac{4}{5}}$, $\frac{3}{\frac{7}{8}}$ etc.

IV. Vocabulary To Stress:

mixed numeral renaming reciprocal

V. Activities:

Play the game "Who Divides Fastest?". Use two teams. Two problems such as a) $4\frac{1}{2} \div 2\frac{1}{3}$, b) $2\frac{1}{3} \div 3\frac{1}{4}$ are put on the board. One member from each team comes to the board and does the first step--changing mixed numerals to fractions. The next member writes the fractions again, using the reciprocal. The next could multiply, then divide, then change to simplest form. The number of steps used depends upon the team. They might be able to divide and write in simplest form in one operation.

Test References:

Book: 6

Houghton Mifflin(1967) pp. 261, (60), (61-part of page)
Addison-Wesley(1971, 1968) pp. 188, 332

Book: 7

Houghton Mifflin(1967, 1970) pp. 357-359
Houghton Mifflin(1972) pp. 214, 427
Addison-Wesley(1971) pp. 244, 256, (48, 49)

Book: 8

Houghton Mifflin(1972) p. 147

Other References:

Modern Mathematics Filmstrip #764 (last part)

Level: 16 Step: A Concept: Division

WORKSHEET - DIVISION

I. Change the following mixed numerals to fractions.

A. $3\frac{2}{3}$ = ______	F. $2\frac{1}{3}$ = ______	K. $1\frac{1}{5}$ = ______
B. $2\frac{1}{2}$ = ______	G. $1\frac{1}{3}$ = ______	L. $2\frac{2}{3}$ = ______
C. $6\frac{2}{3}$ = ______	H. $2\frac{1}{4}$ = ______	M. $4\frac{2}{3}$ = ______
D. $1\frac{1}{4}$ = ______	I. $3\frac{1}{4}$ = ______	N. $4\frac{1}{4}$ = ______
E. $5\frac{1}{2}$ = ______	J. $7\frac{1}{2}$ = ______	O. $4\frac{1}{2}$ = ______

II. Write the fractions above and beside them write their reciprocals as shown in A.

A. $\frac{11}{3}$, $\frac{3}{11}$	F. ______	K. ______
B. ______	G. ______	L. ______
C. ______	H. ______	M. ______
D. ______	I. ______	N. ______
E. ______	J. ______	O. ______

III. Find the quotients:

A. $2\frac{1}{5} \div 3\frac{2}{3}$	F. $2\frac{1}{2} \div 2\frac{1}{3}$	K. $1\frac{1}{2} \div 1\frac{1}{5}$
B. $1\frac{1}{8} \div 2\frac{1}{2}$	G. $3\frac{1}{4} \div 1\frac{1}{3}$	L. $1\frac{2}{3} \div 2\frac{2}{3}$
C. $1\frac{1}{6} \div 6\frac{2}{3}$	H. $3\frac{1}{2} \div 2\frac{1}{4}$	M. $2\frac{2}{3} \div 4\frac{2}{3}$
D. $3\frac{1}{5} \div 1\frac{1}{4}$	I. $5\frac{1}{2} \div 3\frac{1}{4}$	N. $3\frac{1}{4} \div 4\frac{1}{4}$
E. $2\frac{1}{3} \div 5\frac{1}{2}$	J. $6\frac{1}{2} \div 7\frac{1}{2}$	O. $1\frac{1}{4} \div 4\frac{1}{2}$

Level: 16 Step: A Concept: Function

I. Concept:

Function: Naming the second member of an ordered pair of fractions in an equation with two variables.

II. Behavioral Objective:

The student given a fractional number and a function rule will be able to name the second member of the ordered pair.

III. Mathematical Ideas:

A function is a set of ordered pairs governed by a rule.

IV. Vocabulary To Stress:

function ordered pair

V. Activities:

* A. Game. One student begins writing on the chalkboard ordered pairs such as $(\frac{1}{4}, \frac{3}{4})$, $(\frac{2}{8}, \frac{6}{8})$, $(\frac{2}{4}, 1)$. When a student discovers the rule, he comes up and continues with another ordered pair. Other students follow as they discover the rule. H.M.T.E. 5, pp. 288-289.

B. Put television functions on chalkboard.

C. Play the game, "I'm thinking of a number". Give the fractional numeral and the rule. If you are thinking of $\frac{7}{8}$, say $\frac{1}{2}$ add $\frac{3}{8}$. The game could be changed to "I'm thinking of a Rule". Write the number pair on the chalkboard and have the student tell you the rule by which you named the second number. H.M.T.E. 5, (1967), p. 139.

Text References:

Book: 4

Houghton Mifflin(1967) pp. 313
Houghton Mifflin(1972) pp. 280

Book: 5

Houghton Mifflin(1967) pp. 240, 299
Houghton Mifflin(1972) pp. 202
Addison-Wesley(1971) pp. 299

Book: 6

Houghton Mifflin(1967) pp. 246
Addison-Wesley(1971, 1968) pp. 154, 178, 198

Level: 16 Step: A Concept: Function

Text References: continued

Book: 7

Houghton Mifflin(1972) p. 189
Addison-Wesley(1971) pp. 248-250

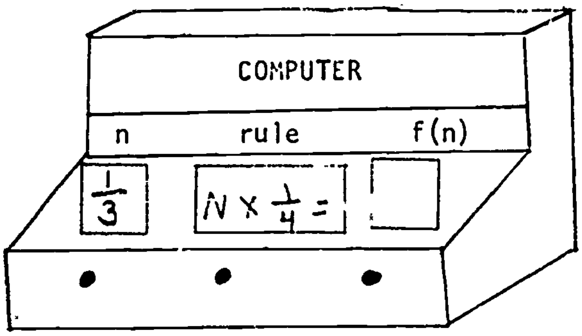
* This game could be used to help introduce the concept after the initial introduction.

Level: 16
Step: A

WORKSHEET: FUNCTION

1.

A.

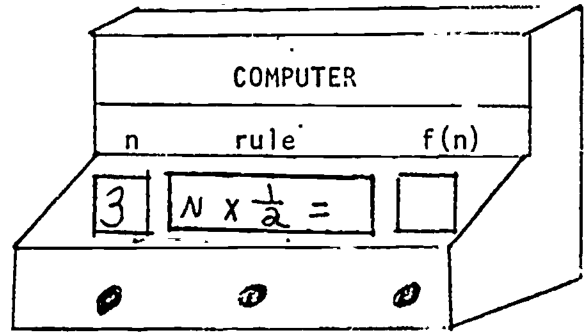


$n \times \frac{1}{4}$

n	f(n)
B. $\frac{1}{4}$	
C. $\frac{1}{5}$	
D. $\frac{2}{5}$	
E. $\frac{3}{6}$	

2.

A.

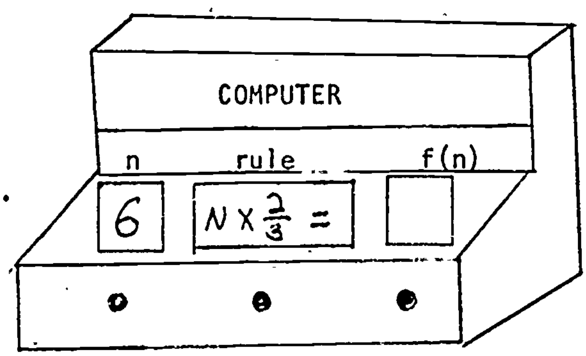


$n \times \frac{1}{2}$

n	f(n)
B. 10	
C. 12	
D. 20	
E. 14	

3.

A.

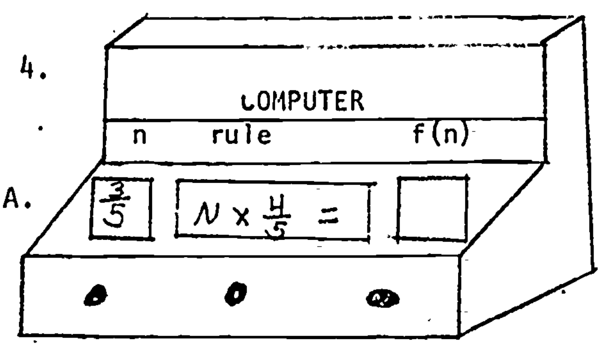


$n \times \frac{2}{3}$

n	f(n)
B. 9	
C. 21	
D. 5	
E. 7	

4.

A.



$n \times \frac{4}{5}$

n	f'n)
B. $\frac{4}{5}$	
C. $\frac{10}{12}$	
D. $\frac{3}{6}$	
E. $\frac{5}{9}$	

Level: 16
Step: A

WORKSHEET: FUNCTION

I. Name the rule for each row: Answer

1. $\left(\frac{1}{2},\frac{3}{4}\right),\left(\frac{1}{4},\frac{2}{4}\right),\left(\frac{1}{8},\frac{3}{8}\right),\left(\frac{2}{16},\frac{6}{16}\right),\left(\frac{3}{4},1\right)$ __________

2. $\left(\frac{1}{10},\frac{3}{10}\right),\left(\frac{3}{15},\frac{6}{15}\right),\left(\frac{2}{20},\frac{6}{20}\right),\left(\frac{3}{30},\frac{9}{30}\right),\left(\frac{3}{5},\frac{4}{5}\right)$ __________

3. $\left(2\frac{1}{8},3\frac{4}{8}\right),\left(4\frac{4}{8},5\frac{7}{8}\right),\left(6,7\frac{3}{8}\right),\left(4\frac{1}{8},5\frac{4}{8}\right),\left(1\frac{1}{8},2\frac{4}{8}\right)$ __________

4. $\left(\frac{1}{4},\frac{3}{4}\right),\left(\frac{3}{8},\frac{7}{8}\right),\left(\frac{2}{16},\frac{10}{16}\right),\left(\frac{3}{6},1\right),\left(\frac{1}{10},\frac{6}{10}\right)$ __________

5. $\left(\frac{3}{16},\frac{1}{16}\right),\left(\frac{5}{24},\frac{2}{24}\right),\left(\frac{6}{8},\frac{5}{8}\right),\left(\frac{8}{32},\frac{4}{32}\right),\left(\frac{5}{8},\frac{4}{8}\right)$ __________

II. Complete the set of ordered pairs:

RULE

1. $+\frac{1}{3}$ $\left\{\left(\frac{1}{6},\_\_\right),\left(\frac{1}{13},\_\_\right),\left(\frac{2}{9},\_\_\right),\left(\frac{3}{15},\_\_\right),\left(\frac{1}{3},\_\_\right)\right\}$

2. $-\frac{1}{2}$ $\left\{\left(\frac{5}{8},\_\_\right),\left(\frac{3}{4},\_\_\right),\left(\frac{8}{12},\_\_\right),\left(\frac{5}{6},\_\_\right),\left(\frac{7}{10},\_\_\right)\right\}$

3. $+\frac{1}{10}$ $\left\{\left(\frac{6}{10},\_\_\right),\left(\frac{3}{20},\_\_\right),\left(\frac{3}{100},\_\_\right),\left(\frac{7}{40},\_\_\right),\left(\frac{3}{70},\_\_\right)\right\}$

4. $+\frac{2}{6}$ $\left\{\left(\frac{1}{6},\_\_\right),\left(\frac{3}{18},\_\_\right),\left(\frac{1}{36},\_\_\right),\left(\frac{3}{12},\_\_\right),\left(\frac{8}{24},\_\_\right)\right\}$

5. $-\frac{1}{4}$ $\left\{\left(\frac{3}{4},\_\_\right),\left(\frac{3}{8},\_\_\right),\left(\frac{8}{16},\_\_\right),\left(\frac{5}{12},\_\_\right),\left(\frac{2}{4},\_\_\right)\right\}$

Level: 16
Step: A

WORKSHEET: FUNCTION

I. Find values for a, b, c, d in the following ordered pairs.

RULE

1. $+\frac{1}{4}$ $\left\{\left(\frac{1}{4},\frac{2}{4}\right),\left(\frac{1}{8},a\right),\left(\frac{1}{16},b\right),\left(\frac{2}{4},c\right)\right\}$
2. $+\frac{1}{6}$ $\left\{\left(\frac{1}{12},\frac{3}{12}\right),\left(\frac{2}{6},a\right),\left(\frac{3}{24},b\right),\left(\frac{2}{18},c\right)\right\}$
3. $-\frac{1}{8}$ $\left\{\left(\frac{1}{4},\frac{1}{8}\right),\left(\frac{1}{2},a\right),\left(\frac{6}{8},b\right),\left(\frac{3}{8},c\right)\right\}$
4. $+\frac{2}{5}$ $\left\{\left(\frac{1}{5},\frac{3}{5}\right),\left(\frac{1}{10},a\right),\left(\frac{3}{15},b\right),\left(\frac{4}{20},c\right)\right\}$
5. $+1\frac{1}{2}$ $\left\{\left(\frac{1}{4},1\frac{3}{4}\right),\left(\frac{2}{8},a\right),\left(\frac{1}{12},b\right),\left(3\frac{1}{10},c\right),\left(4\frac{1}{2},d\right)\right\}$
6. $-\frac{1}{3}$ $\left\{\left(\frac{5}{12},\frac{1}{12}\right),\left(\frac{8}{24},a\right),\left(\frac{6}{12},b\right),\left(\frac{2}{3},c\right)\right\}$

II. Name the rule

1. ______ $\left\{\left(\frac{3}{7},\frac{4}{7}\right),\left(\frac{2}{21},\frac{5}{21}\right),\left(\frac{2}{49},\frac{9}{49}\right),\left(\frac{1}{14},\frac{3}{14}\right),\left(\frac{3}{35},\frac{8}{35}\right)\right\}$
2. ______ $\left\{\left(\frac{4}{8},\frac{7}{8}\right),\left(\frac{1}{2},\frac{7}{8}\right),\left(\frac{1}{24},\frac{10}{24}\right),\left(\frac{1}{16},\frac{7}{16}\right),\left(\frac{2}{8},\frac{5}{8}\right)\right\}$

III. Complete the sets of ordered pairs.

RULE

1. $+\frac{1}{6}$ $\left\{\left(\frac{3}{12},\_\_\_\right),\left(\frac{3}{30},\_\_\_\right),\left(\frac{4}{6},\_\_\_\right),\left(\frac{2}{18},\_\_\_\right),\left(\frac{7}{42},\_\_\_\right)\right\}$
2. $-\frac{1}{3}$ $\left\{\left(\frac{2}{3},\_\_\_\right),\left(\frac{9}{24},\_\_\_\right),\left(\frac{4}{6},\_\_\_\right),\left(\frac{8}{18},\_\_\_\right),\left(\frac{7}{12},\_\_\_\right)\right\}$
3. $+\frac{1}{9}$ $\left\{\left(\frac{2}{3},\_\_\_\right),\left(\frac{6}{9},\_\_\_\right),\left(\frac{4}{36},\_\_\_\right),\left(\frac{7}{9},\_\_\_\right),\left(\frac{3}{18},\_\_\_\right)\right\}$

Level: 16 Step: B Concept: Sets

I.
I. Concept:

Sets: Graphing solution sets on the number line.

II. Behavioral Objective:

The student given the replacement set of whole numbers and the number sentence will be able to write and graph the solution set on the number line.

III. Mathematical Ideas:

A. A graph is a set of points on a number line or number plane.

B. If the replacement set is the set of whole numbers:

1. The solution set of the equation $n + 2 = 8$ may be graphed on the number line as the point 6, written $\{6\}$.

2. The solution set of the inequality $n<9$ may be graphed as the points 0, 1, 2, 3, 4, 5, 6, 7, 8, written $\{0,1,2,3,4,5,6,7,8\}$.

IV. Vocabulary To Stress:

graphing	replacement set	number line
solution sets	number sentence	

Text References:

Book: 6

Houghton Mifflin(1967)	pp. 138-139, (32), 161
Houghton Mifflin(1972)	pp. 224-225, (55)
Addison-Wesley(1971, 1968)	pp. 302-303

Book: 7

Houghton Mifflin(1967, 1970)	pp. 290-295, (44)
Houghton Mifflin(1972)	pp. 52

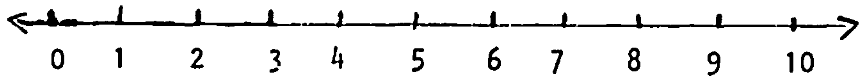
Other References:

The Arithmetic Teacher, October 1967, "Relations", pp. 473-475.
Modern Mathematics Filmstrip #771

Level: 16 Step: B Concept: Sets

WORKSHEET: SETS

I. Use the number line to name the solution set. The replacement set is {whole numbers}



A. $4 < n < 8 = \{\qquad\}$

B. $3 < n < 9 = \{\qquad\}$

C. $6 < n < 10 = \{\qquad\}$

D. $3 < n < 7 = \{\qquad\}$

E. $8 < n < 10 = \{\qquad\}$

F. $5 < n < 9 = \{\qquad\}$

G. $2 < n < 4 = \{\qquad\}$

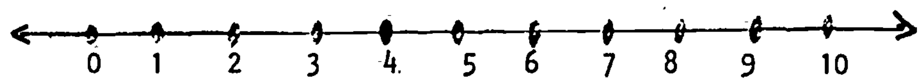
H. $5 < n < 8 = \{\qquad\}$

I. $4 < n < 9 = \{\qquad\}$

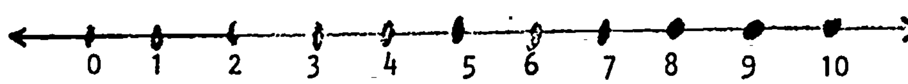
J. $2 < n < 6 = \{\qquad\}$

II. Graph the solution set for each sentence on the number line. The solution set is {whole numbers less than 10}.

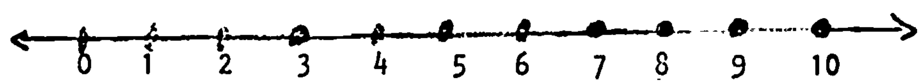
A. $n < 7$



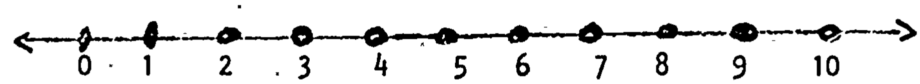
B. $n \times 5 = 10$



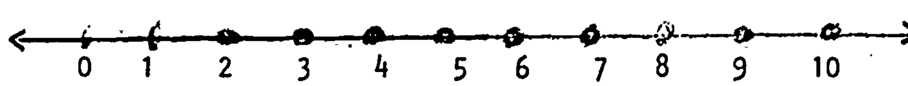
C. $n - 3 > 6$



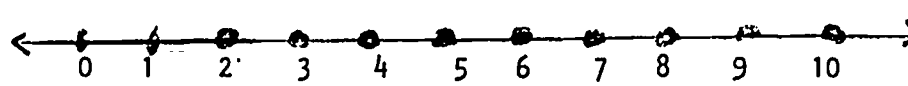
D. $6 < n < 10$



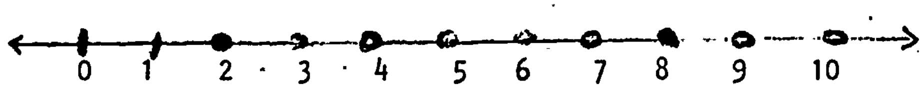
E. $6 + n = 10$



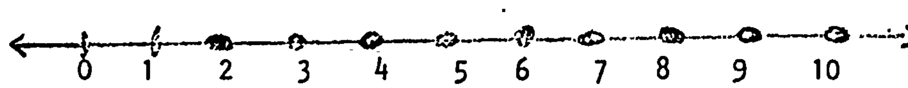
F. $8 - n < 4$



G. $10 > n > 6$



H. $n + 5 = 12$



Level: 16 Step: B Concept: Numeral

I. Concept:

Numeral: Comparing the base eight numeration system to the decimal system.

II. Behavioral Objective:

The student given either a base eight or a base ten numeral will be able to write the number in the base not given.

III. Mathematical Ideas:

A. The base eight system has only eight digits (0-7) to represent numbers.

B. The digits in any base-place system have three values: face, place, and total.

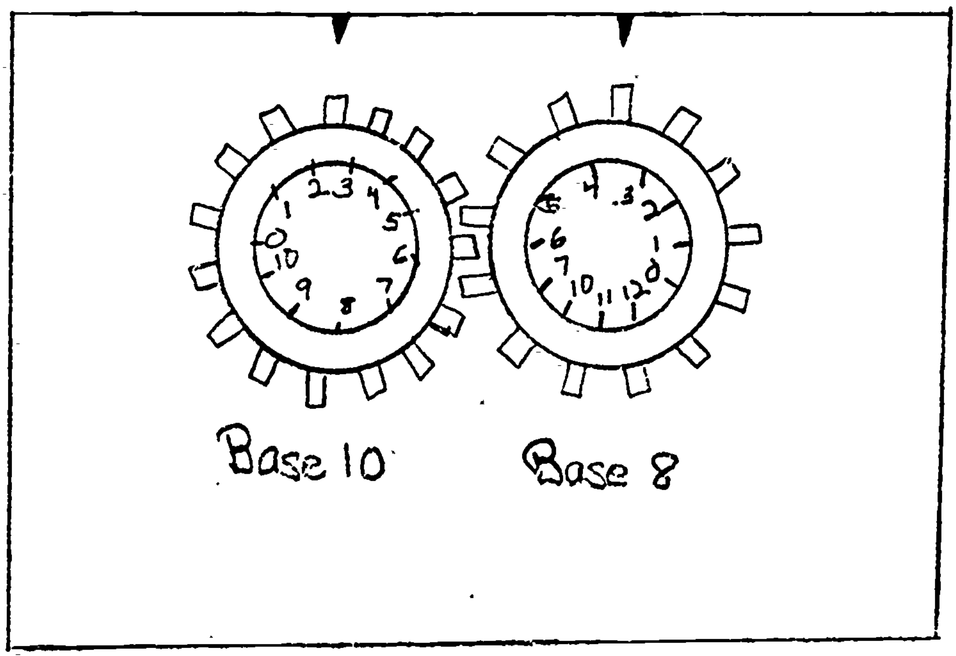
C. The notation for writing a base eight numeral is the subscript eight.

IV. Vocabulary To Stress

base eight	place	total
ten	face	value

V. Activities:

A. Base Wheels. The Arithmetic Teacher, Nov. 1970, pp. 619-621. Make discs of base eight and base ten numerals. Attach them to two gear type wheels mounted on a frame and backed by a chalk board so the problems can be written directly on the board. Start each disc at zero. Turn the left disc to a value of base ten and the corresponding value of base 8 will turn at the same time. The equivalent values in the two systems will be under the marker.



Activities: Continued

* B. Make a base 8 chart and put the base eight numeral in the base chart.
e.g. 36_{eight}

8^3	8^2	8^1	8^0
512	64	8	1
		3	6_{eight}

1. To change 36_{eight} to base ten follow this procedure:

   Multiply $(8 \times 3) + (1 \times 6)$ and add the products.
   $24 + 6 = 30_{ten}$

2. To change 124_{ten} to base eight follow this procedure; (Remember you cannot put a base ten number in a base eight chart.)

   Divide 124_{ten} by the largest multiple of eight less than 124_{ten} and so on until your remainder is less than eight.

   64)124 | 1
   64
   8) 60 | 7
   56
   4

512	64	8	1
	1	7	4_{eight}

   Multiply to check:

   $(64 \times 1) + (8 \times 7) + (4 \times 1) =$
   $64 + 56 + 4 = 124_{ten}$

   Using the base chart should make changing bases much easier for the children. It can be applied to any base for any numeral.

16-20

Level: 16 Step: B Concept: Numeral

Text References:

Book: 4

Houghton Mifflin(1967) pp. 214-217

Book: 5

Houghton Mifflin(1972) pp. 20

Book: 6

Houghton Mifflin(1967) pp. 24-25, 29
Houghton Mifflin(1972) pp. 22-23
Addison-Wesley(1971, 1968) pp. 14-15

Book: 7

Addison-Wesley(1971) pp. 23-25, (5)

Other References:

Modern Mathematics Filmstrip #736

* This activity will be very helpful when introducing the concept.

Level: 16 Step: B Concept: Numeral

WORKSHEET

8^3	8^2	8^1	8^0
512	64	8	1

I. Use the base 8 chart above and change the following base 8 numerals to base 10.

1. 46_{eight} = ________
2. 57_{eight} = ________
3. 75_{eight} = ________
4. 224_{eight} = ________
5. 135_{eight} = ________
6. 127_{eight} = ________
7. 86_{eight} = ________
8. 34_{eight} = ________
9. 63_{eight} = ________
10. 56_{eight} = ________

II. Use the chart above to change the following base 10 numerals to base 8.

1. 86_{ten} = ________
2. 75_{ten} = ________
3. 98_{ten} = ________
4. 129 ten = ________
5. 329_{ten} = ________
6. 569_{ten} = ________
7. 38_{ten} = ________
8. 92_{ten} = ________
9. 46_{ten} = ________
10. 163_{ten} = ________

Level: 16 Step: B Concept: Division

I. Concept:

Division: Finding the quotient using standard form when the dividend contains five digits and the divisor two digits.

II. Behavioral Objective:

The student given a division problem with five digits in the dividend and two digits in the divisor will be able to compute the answer using standard form.

III. Mathematical Ideas:

A. Division is the process of renaming a product and one factor in terms of the missing factor.

B. Multiplication and division are inverse operations, so students should be able to check their work by multiplying.

IV. Vocabulary To Stress:

dividend	quotient	opposite operation
divisor	remainder	estimate

V. Activities:

A. Game "It's in the Bag!" from Let's Play Games in Mathematics, Vol. 6, #34. (In your building). Use division problems such as you have had in this lesson.

B. Game "The Winning Touch". (In your building). Reviews Multiplication.

Text References:

Book: 6

Houghton Mifflin(1967)	pp. 116-117, (28)
Houghton Mifflin(1972)	pp. 89-90, (23), 340
Addison-Wesley(1971, 1968)	pp. 67-69, (19), 326

Book: 7

Houghton Mifflin(1967, 1970)	pp. 157-158, (25)
Houghton Mifflin(1972)	pp. 82-83, 185, 417
Addison-Wesley(1971)	pp. 65-66, (16)

Book: 8

Houghton Mifflin(1972)	pp. 49

Other References:

Imperial Tape (Intermediate) #14 (a portion of it)

Level: 16 Step: B Concept: Function

I. Concept:

Function: Mapping of one set into another set.

II. Behavioral Objective:

The student given function equations will be able to locate the ordered pairs on the number plane.

III. Mathematical Ideas:

A. Each point is associated with a unique number pair and for every number pair there is a unique point.

B. A number plane is formed by the union of two perpendicular number lines.

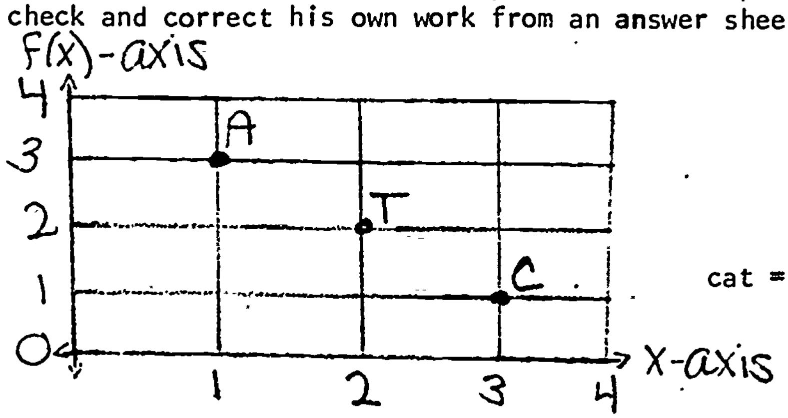
C. The ordered number pairs which satisfy a function equation may be located on a number plane.

IV. Vocabulary To Stress:

mapping	x-axis	origin
number plane	f(x)-axis	

V. Activities:

To review the number plane, give each child a ditto on which each letter of the alphabet has been assigned to a different point on a grid. Also include a list of words for the child to spell in ordered pairs. Allow the child to check and correct his own work from an answer sheet when he has finished.



(a small example showing only 3 letters)

cat = (3,1), (1,3), (2,2)

Text References:

Book: 6

Houghton Mifflin(1967)	pp. 94-95, 144-145, (34)
Houghton Mifflin(1972)	pp. 298
Addison-Wesley(1971)	pp. 304, (54), 305, (55), 306, (56), 307-309.

Book: 7

Houghton Mifflin(1972)	pp. 53, 94-95
Addison-Wesley(1971)	pp. 43-46, (8)

Level: 16 Step: B Concept: Geometry

I. Concept:

Geometry: Classifying polygons by the number of sides.

II. Behavioral Objective:

The student given a polygon of no more than eight sides will be able to name the polygon.

III. Mathematical Ideas:

A. A polygon is a simple closed curve formed by the union of line segments. The line segments are sides of the polygon and the common end points are vertices.

B. A diagonal of a polygon is a line segment which joins two vertices but which is not a side of the polygon.

IV. Vocabulary To Stress:

polygons	vertices
line segments	diagonal

V. Activities:

* A. Have samples of the different polygons in the classroom. Let the students see and feel them. After discussion have them make polygons from heavy paper or sytrofoam. These can be labeled and displayed with a math display or, if small, kept in the desk for reference.

B. Look in magazines and newspapers for shapes showing different polygons, e.g. highway signs, linoleum designs, buildings, etc.

Text References:

Book: 6

Houghton Mifflin(1967)	pp. 74, 76, 84, 279(part of page)
Houghton Mifflin(1972)	pp. 95, 293, (70)

Book: 7

Houghton Mifflin(1967, 1970)	pp. 412, 413, 417

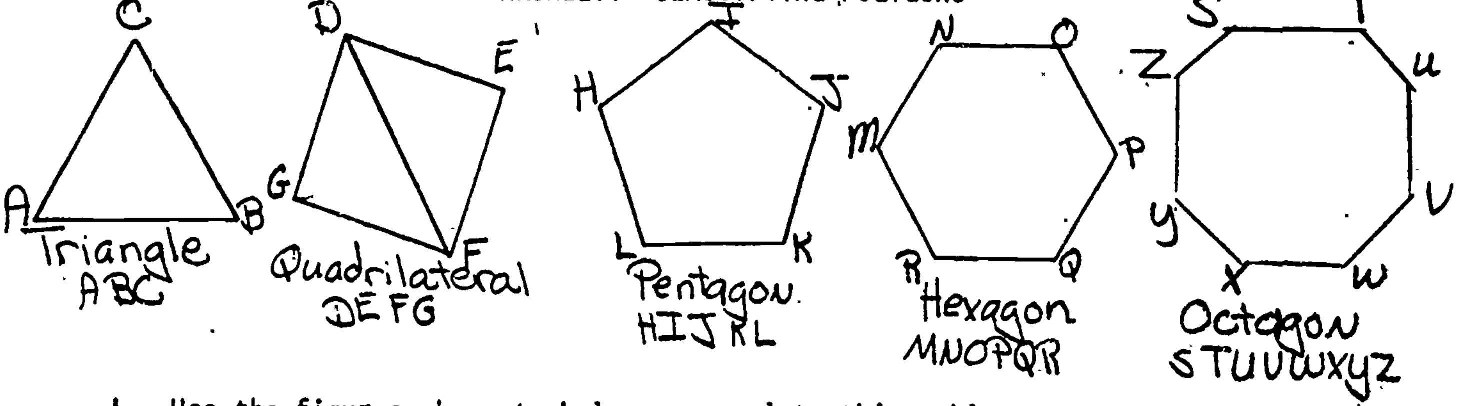
Other References:

The Franklin Series: Paper & Pencil Geometry, pp. 59-60.
Filmstrip & Tape--Forms We See. #EF1106-3, EF1106-4--part of each.
Curriculum Filmstrips-Polygons: Geometry #367.

* This activity may be used to help introduce the concept.

Level: 16 Step: B Concept: Geometry

WORKSHEET: CLASSIFYING POLYGONS



I. Use the figures above to help you complete this table.

Polygon	No. of sides	No. of vertices	No. of diagonals
a.	b.	5	c.
d.	e.	4	2
f	g.	8	h
i.	3	j.	k.
Hexagon	l.	m.	n.

II. Identify the following polygons using the descriptions below:

1. 3 sides and no diagonals __________
2. 6 sides __________
3. 8 vertices __________
4. 5 diagonals __________
5. 4 sides __________
6. 5 vertices __________
7. 9 diagonals __________
8. 8 sides __________
9. 4 vertices __________
10. 2 diagonals __________

16-26

Level: 16 Step: Z Concept: Numeral

I. Concept:

Numeral: Introducing the Hebrew numerals.

II. Behavioral Objective:

The student given the Hebrew and Hindu-Arabic numeral table of conversion and a numeral will be able to rename the numeral in the other system.

III. Mathematical Ideas:

A. A place-value numeration system is one in which the position of a digit in a numeral determines its value.

B. The Hebrew system is not a place-value system.

C. The zero is not needed in a non-place-value system.

IV. Vocabulary To Stress:

Hebrew
Hindu-Arabic
conversion table
place-value

V. Activities:

A. Let students write Hebrew numerals for classmates to change to our numerals. Each student should make his own key to check work of classmates.

B. Make a Hebrew calendar using the Hebrew numerals or names for the numerals.

Text References:

Book: 6

Houghton Mifflin(1967) pp. 23; 31

Level: 16 Step: Z Concept: Numeral

WORKSHEET

I. Write our numerals for the Hebrew numerals.

1. tzadee bays ________
2. shin tess ________
3. yood vov ________
4. koof nun ________
5. mem bays ________
6. kaf hay ________
7. lamed gimel ________
8. kaf hess ________
9. tov dalet ________
10. shin kaf alef ________
11. mem dalet ________
12. pay hay ________
13. tzadee tess ________
14. resh kaf zayin ________
15. yood dalet ________
16. ayin zayin ________

II. Write the Hebrew names for the numerals.

1. 75 ________________
2. 256 ________________
3. 83 ________________
4. 56 ________________
5. 97 ________________
6. 19 ________________
7. 23 ________________
8. 72 ________________
9. 332 ________________
10. 56 ________________
11. 49 ________________
12. 26 ________________
13. 62 ________________
14. 81 ________________
15. 202 ________________
16. 105 ________________

Evaluation - Level 16

Step: Z
Concept: Numeral

Score ____/10

Using your book, write the Hebrew numerals for our numerals.

1. 68	
2. 91	
3. 123	
4. 456	
5. 37	

Write our numerals for these Hebrew numerals.

6. kaf tess	
7. ayin hay	
8. resh lamed hess	
9. yood hess	
10. mem vov	

Level: 16 Step: Z Concept: Addition

I. Concept:

<u>Addition</u>: Finding the sum using the Roman numeral system.

II. Behavioral Objective:

The student given the addends in Roman numerals will be able to name the sum without the aid of a conversion table.

III. Mathematical Ideas:

A. The value of a Roman numeral is the sum of the values of the individual digits.

B. The Roman system uses both ten and five as bases.

C. A zero is not needed in the Roman numeral system.

IV. Vocabulary To Stress

Roman numerals individual digits non-place-value

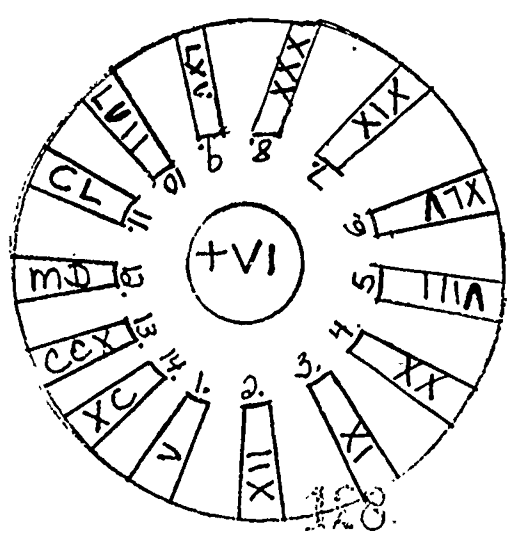
V. Activities:

A. Memorize the Roman numerals I, V, X, L, C, D, M.

* B. Have fun by adding, using Roman numerals. Show on the board that the value of a Roman numeral is the sum of the values of the individual digits.

C. Have students make a list of buildings, chapters in books, tombstones, statues, markers, etc., that have the dates written in Roman numerals. Also have the students bring a report of the same to share with the class.

D. Make a Pizza board addition exercise as follows: Around the edge of the board clip Roman Numerals as shown below. Affix one numeral in the center of the board to which the child will add to each of the others. Each answer may be written on the back of each individual numeral so the child may turn them over and check his answers when he finishes the complete board. Answers should be written with Roman numerals.



Level: 16 | Step: Z | Concept: Addition

Text References:

Book: 6

Houghton Mifflin(1967)	pp.	22
Houghton Mifflin(1972)	pp.	25, (8)

Book: 7

Houghton Mifflin(1967, 1970)	pp.	107-111
Addison-Wesley(1971)	pp.	31

Other References:

Bell, Stewart E., Mathematics in the Making 2.
World Book Encyclopedia (or any good encyclopedia).

* This activity could be used to introduce the concept.

Level: 16 Step: Z Concept: Addition

WORKSHEET II: ROMAN NUMERALS

I. Name the sums as Arabic Numerals.

A. XX + IX = ________

B. XL + IV = ________

C. LVIII + XL = ________

D. MMCD + XL = ________

E. D + CC = ________

F. XCVII + LX = ________

G. CD + CII = ________

H. LXII + XXIV = ________

I. XCV + XVII = ________

J. XL + XV = ________

K. LXIX + XCIV = ________

L. CV + CCVII = ________

M. XXVII + XIV = ________

N. XIV + XV = ________

O. MD + MM = ________

P. DII + XCIV = ________

II. Name the sums as Roman Numerals.

A. V + XL = ________

B. XX + VIII = ________

C. C + LXI = ________

D. C + DV = ________

E. VII + XXVII = ________

F. X + XL = ________

G. LXI + CC = ________

H. XL + L = ________

I. LXX + XV = ________

J. LXXXV + XLII = ________

K

K. XIV + XV = ________

L. XIII + XXVII = ________

M. CCLVI + LXVII = ________

N. LV + LXVI = ________

O. VIII + IX = ________

P. LIV + VII = ________

EVALUATION - Level 16

Step: Z
Concept: Addition

Score ____ / 10

Name the sums with Roman Numerals.

1. CCXX + XLV

2. XCVII + LXXXIX

3. XIX + LXV

4. L + XCVII

5. MII + DXLV

6. CM + CLVI

7. MMCCLXIV + XIX

8. XI + VII

9. VII + IX

10. C + CD

Level: 16 Step: Z Concept: Multiplication

I. Concept:

Multiplication: Using the lattice method to find the product.

II. Behavioral Objective:

The student given two factors will be able to find the product by using the lattice method.

III. Mathematical Ideas:

A. Mathematics is not an unchanging subject.

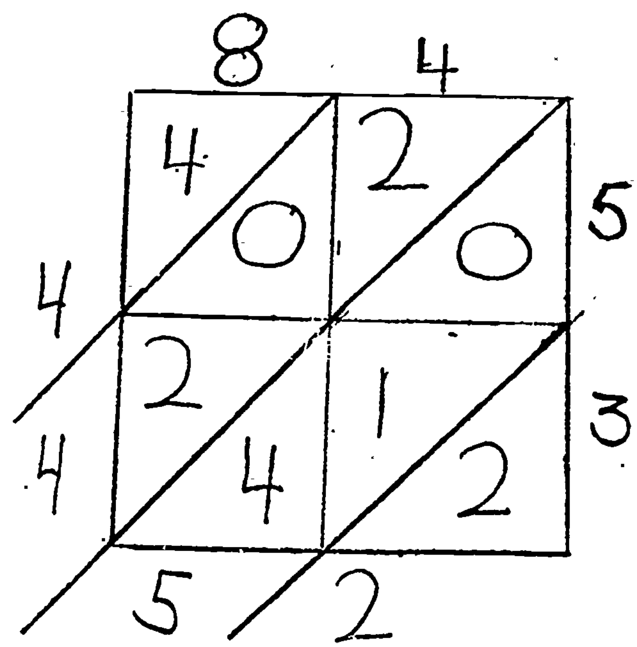
B. Present methods are more efficient in many respects, and so reflect the evolution of computational skills.

IV. Vocabulary To Stress:

lattice method mathematician multiplication

V. Activities:

A. Use this simple method of multiplying by two numbers before introducing work in the text. Draw a square on the board and divide it into 4 equal parts. Draw a diagonal line dividing each square into halves. Place the numbers of the multiplicand above each square and the numbers of the multiplier to the right of each square. Tens are recorded above and units below in each square. Next, add on the diagonal. The answer is 4452. Have students work on the board and at desks with the two digit multiplier. When they see how simple and how much fun it is to multiply by this method they will want to try 3 and possible 4 digit multipliers.



B. Research: Napier. See Patterns and Puzzles, pp. 86-89, Franklin Series for instructions how to make Napier's Bones.

Level: 16 Step: Z Concept: Multiplication

Text References:

Book: 6

Houghton Mifflin(1967) pp. 102-103
Houghton Mifflin(1972) pp. 61

Other References:

The Arithmetic Teacher, May 1969.
The Franklin Series, Patterns and Puzzles..
New Math for Adults Only, May & Moss, cpy., 1966.

Level: 16 Step: Z Concept: Multiplication

WORKSHEET: MULTIPLICATION

Complete the multiplications using the lattice method.

1.

		4	5	6	X
					7
					5

2.

		7	5	9	X
					8
					6

3.

		8	3	2	X
		4 / a	1 / 8	b / 2	6
		6 / 4	2 / c	1 / d	8
5	6	5	e	f	

4.

		6	3	9	X
		2 / a	1 / 2	b / 5	4
		c / 2	2 / d	e / 3	7
3	f	0	g	h	

5.

		5	7	6	X
		1 / a	2 / 1	b / 8	3
		c / 5	4 / d	4 / e	7
f	1	3	1	2	

6.

		7	8	4	X
		3 / 5	4 / 0	2 / 0	a
		4 / 9	c / 6	2 / 8	b
4	d	6	e	8	

EVALUATION - Level 16

Step: Z
Concept: Multiplication

Score ____ / 5

Use the lattice method to solve the following problems. Draw your own lattices for the last 3 problems.

1.

		2	3	9	X
					7
					5

2.

		4	5	6	X
					6
					7

3. 758X32

4. 683X98

5. 843X57

Level: 16

ANSWERS TO WORKSHEETS

16-2

1. $17 + 12 = n$, $n = 29$ students
2. $5 \times 4 = n$, $n = 20$ fish
3. $\$42.00 \div \$6.00 = n$, $n = 7$ tickets
4. $\left(\frac{1}{2} \times \$2.00\right) \times 3 = n$, $n = \$3.00$
5. $\$6.50 + \$5.80 + \$7.00 + \$4.00 = n$, $n = \$23.30$
6. $\$10.00 \div \$0.75 = n$, $n = 13\frac{1}{3}$ hours
7. $7 \times n = 56$, $n = 8$
8. $n \div 9 = 7$, $n = 63$
9. $n + 5 = 23$, $n = 18$
10. $n - 6 > 12$, $n = \{19, 20, 21, \ldots\}$
11. $4 \times n < 24$, $n = \{0, 1, 2, 3, 4, 5\}$
12. $n + 9 < 13$, $n = \{0, 1, 2, 3\}$

16-7

1. $5\frac{31}{45}$
2. $50\frac{1}{48}$
3. 12
4. $5\frac{23}{24}$
5. $22\frac{1}{10}$
6. $21\frac{1}{3}$
7. $5\frac{17}{45}$
8. $6\frac{1}{20}$
9. $7\frac{1}{2}$
10. $14\frac{7}{10}$
11. $28\frac{11}{30}$
12. $29\frac{1}{27}$
13. $30\frac{11}{12}$
14. $77\frac{9}{16}$
15. $37\frac{1}{8}$

16-8

I. A. $\frac{5}{4}$
B. $\frac{8}{3}$
C. $\frac{13}{4}$
D. $\frac{16}{3}$
E. $\frac{20}{3}$
F. $\frac{16}{7}$
G. $\frac{15}{4}$
H. $\frac{17}{2}$
I. $\frac{20}{3}$
J. $\frac{37}{4}$
K. $\frac{22}{4}$
L. $\frac{36}{5}$
M. $\frac{35}{8}$
N. $\frac{32}{5}$
O. $\frac{14}{3}$

II. A. $8\frac{1}{3}$
B. $12\frac{1}{18}$
C. $2\frac{13}{16}$
D. $2\frac{4}{5}$
E. $20\frac{5}{12}$
F. $6\frac{7}{8}$
G. $22\frac{2}{3}$
H. $8\frac{2}{3}$
I. $4\frac{1}{2}$
J. $8\frac{2}{9}$
K. $3\frac{3}{7}$
L. $2\frac{11}{12}$
M. $4\frac{4}{5}$
N. $10\frac{15}{16}$
O. $1\frac{9}{16}$
P. $2\frac{4}{5}$
Q. $4\frac{2}{3}$
R. $3\frac{3}{8}$
S. $4\frac{1}{6}$
T. $12\frac{3}{4}$

Level: 16

ANSWERS TO WORKSHEETS

16-10

I.
- A. $\frac{11}{3}$
- B. $\frac{5}{2}$
- C. $\frac{20}{3}$
- D. $\frac{5}{4}$
- E. $\frac{11}{2}$
- F. $\frac{7}{3}$
- G. $\frac{4}{3}$
- H. $\frac{9}{4}$
- I. $\frac{13}{4}$
- J. $\frac{15}{2}$
- K. $\frac{6}{5}$
- L. $\frac{8}{3}$
- M. $\frac{14}{3}$
- N. $\frac{17}{4}$
- O. $\frac{9}{2}$

II.
- A. $\frac{11}{3}, \frac{3}{11}$
- B. $\frac{5}{2}, \frac{2}{5}$
- C. $\frac{20}{3}, \frac{3}{20}$
- D. $\frac{5}{4}, \frac{4}{5}$
- E. $\frac{11}{2}, \frac{2}{11}$
- F. $\frac{7}{3}, \frac{3}{7}$
- G. $\frac{4}{3}, \frac{3}{4}$
- H. $\frac{9}{4}, \frac{4}{9}$
- I. $\frac{13}{4}, \frac{4}{13}$
- J. $\frac{15}{2}, \frac{2}{15}$
- K. $\frac{6}{5}, \frac{5}{6}$
- L. $\frac{8}{3}, \frac{3}{8}$
- M. $\frac{14}{3}, \frac{3}{14}$
- N. $\frac{17}{4}, \frac{4}{17}$
- O. $\frac{9}{2}, \frac{2}{9}$

III.
- A. $\frac{3}{5}$
- B. $\frac{9}{20}$
- C. $\frac{7}{40}$
- D. $2\frac{14}{25}$
- E. $\frac{14}{33}$
- F. $1\frac{1}{14}$
- G. $2\frac{7}{16}$
- H. $1\frac{5}{9}$
- I. $1\frac{9}{13}$
- J. $\frac{13}{15}$
- K. $1\frac{1}{4}$
- L. $\frac{5}{8}$
- M. $\frac{4}{7}$
- N. $\frac{13}{17}$
- O. $\frac{5}{18}$

16-13

1.
- A. $\frac{1}{12}$
- B. $\frac{1}{16}$
- C. $\frac{1}{20}$
- D. $\frac{1}{10}$
- E. $\frac{3}{24}$ or $\frac{1}{8}$

2.
- A. $\frac{3}{2}$
- B. 5
- C. 6
- D. 10
- E. 7

3.
- A. 4
- B. 6
- C. 14
- D. $\frac{10}{3}$
- E. $\frac{14}{3}$

4.
- A. $\frac{12}{25}$
- B. $\frac{16}{25}$
- C. $\frac{2}{3}$
- D. $\frac{2}{5}$
- E. $\frac{4}{9}$

Level: 16

ANSWERS TO WORKSHEETS

16-14

I. 1. $+\frac{1}{4}$

2. $+\frac{2}{10}$

3. $+1\frac{3}{8}$

4. $+\frac{2}{4}$

5. $-\frac{2}{16}$

II. 1. $\left(\frac{3}{6}\right)\left(\frac{7}{18}\right)\left(\frac{5}{9}\right)\left(\frac{8}{15}\right)\left(\frac{2}{3}\right)$

2. $\left(\frac{1}{8}\right)\left(\frac{1}{4}\right)\left(\frac{2}{12}\right)\left(\frac{2}{6}\right)\left(\frac{2}{10}\right)$

3. $\left(\frac{7}{10}\right)\left(\frac{5}{20}\right)\left(\frac{13}{100}\right)\left(\frac{11}{40}\right)\left(\frac{10}{70} \text{ or } \frac{1}{7}\right)$

4. $\left(\frac{3}{6}\right)\left(\frac{9}{18}\right)\left(\frac{13}{36}\right)\left(\frac{7}{12}\right)\left(\frac{16}{24}\right)$

5. $\left(\frac{2}{4}\right)\left(\frac{1}{8}\right)\left(\frac{4}{16}\right)\left(\frac{2}{12}\right)\left(\frac{1}{4}\right)$

16-15

I.

1.	$a = \frac{3}{8}$	$b = \frac{5}{16}$	$c = \frac{3}{4}$	
2.	$a = \frac{3}{6}$	$b = \frac{7}{24}$	$c = \frac{5}{18}$	
3.	$a = \frac{3}{8}$	$b = \frac{5}{8}$	$c = \frac{2}{8}$	
4.	$a = \frac{5}{10}$	$b = \frac{9}{15}$	$c = \frac{12}{20}$	
5.	$a = 1\frac{6}{8}$	$b = 1\frac{7}{12}$	$c = 4\frac{6}{10}$	$d = 6$
6.	$a = 0$	$b = \frac{2}{12}$	$c = \frac{1}{3}$	

II. 1. $+\frac{1}{7}$

2. $+\frac{3}{8}$

III. 1. $\left(\frac{5}{12}\right)\left(\frac{8}{30}\right)\left(\frac{5}{6}\right)\left(\frac{5}{18}\right)\left(\frac{14}{42}\right)$

2. $\left(\frac{1}{3}\right)\left(\frac{1}{24}\right)\left(\frac{2}{6}\right)\left(\frac{2}{18}\right)\left(\frac{3}{12}\right)$

3. $\left(\frac{7}{9}\right)\left(\frac{7}{9}\right)\left(\frac{8}{36}\right)\left(\frac{8}{9}\right)\left(\frac{5}{18}\right)$

Level: 16

ANSWERS TO WORKSHEETS

16-17

I. A. 5,6,7
B. 4,5,6,7,8
C. 7,8,9
D. 4,5,6
E. 9
F. 6,7,8
G. 3
H. 6,7
I. 5,6,7,8
J. 3,4,5

II. A. 0,1,2,3,4,5,6
B. 2
C. 10
D. 7,8,9
E. 4
F. 5,6,7,8
G. 7,8,9
H. 7

16-21

I. 1. 38_{ten}
2. 47_{ten}
3. 61_{ten}
4. 148_{ten}
5. 93_{ten}
6. 87_{ten}
7. 70_{ten}
8. 28_{ten}
9. 51_{ten}
10. 46_{ten}

II. 1. 126_{eight}
2. 113_{eight}
3. 142_{eight}
4. 201_{eight}
5. 511_{eight}
6. 1071_{eight}
7. 46_{eight}
8. 134_{eight}
9. 56_{eight}
10. 243_{eight}

16-25

I. a. pentagon
b. 5
c. 5
d. quadrilateral
e. 4
f. octagon
g. 8
h. 20
i. triangle
j. 3
k. 0
l. 6
m. 6
n. 9

II. 1. triangle
2. hexagon
3. octagon
4. pentagon
5. quadrilateral
6. pentagon
7. hexagon
8. octagon
9. quadrilateral
10. quadrilateral

Level: 16

ANSWERS TO ENRICHMENT (STEP Z) WORKSHEETS

16-27

I. 1. 92
2. 309
3. 16
4. 150
5. 42
6. 25
7. 33
8. 28
9. 404
10. 321
11. 44
12. 85
13. 99
14. 227
15. 14
16. 77

II. 1. ayin hay
2. resh nun vov
3. pay gimel
4. nun vov
5. tzadee zayin
6. yood tess
7. kaf gimel
8. ayin bays
9. shin lamed bays
10. nun vov
11. mem tess
12. kaf vov
13. sameh bays
14. pay alef
15. resh bays
16. koof hay

16-31

I. A. 29
B. 44
C. 98
D. 2440
E. 700
F. 157
G. 502
H. 86
I. 112
J. 55
K. 163
L. 312
M. 41
N. 29
O. 3500
P. 596

II. A. XLV
B. XXVIII
C. CLXI
D. DCV
E. XXXIV
F. L
G. CCLXI
H. XC
I. LXXXV
J. CXXVII
K. XXIX
L. XL
M. CCCXXIII
N. CXXI
O. XVII
P. LXI

16-35

1. 34,200
2. 65,274
3. a = 8
b = 1
c = 4
d = 6
e = 7
f = 6
4. a = 4, b = 3, c = 4, d = 1, e = 6, f = 0, g = 3, h = 3
5. a = 5, b = 1, c = 3, d = 9, e = 2, f = 2
6. a = 5
b = 7
c = 5
d = 4
e = 8

Level: 16
Step: Z

Answer Sheet - Evaluation

16-28

Numeral

1. sameh hess
2. tzadee alef
3. koof kaf gimel
4. tov nun vov
5. lamed zayin
6. 29
7. 75
8. 238
9. 18
10. 46

16-32

Addition

1. CCLXV
2. CLXXXVI
3. LXXXIV
4. CXLVII
5. MDXLVII
6. MLVI
7. MMCCLXXXIII
8. XVIII
9. XVI
10. D

16-36

Multiplication

1. 17,925
2. 30,552
3. 24,256
4. 66,934
5. 48,051

Level: 16 Answer Sheet - Post Test I

Step: A

Numeral

1. $50 \times (9 \times 7) = n$
2. $n + 4 < 10$
3. $(2 \times 295) + (6 \times 79) = n$
4. $15 \times n = 60$
5. $(9 \times n) + 8 = 80$

Order

6. 50
7. 300
8. 80
9. 2000
10. 800

Subtraction

11. $1\frac{5}{8}$
12. $2\frac{5}{8}$
13. $1\frac{2}{3}$
14. $2\frac{11}{12}$
15. $2\frac{11}{15}$

Multiplication

16. $10\frac{5}{12}$
17. $7\frac{11}{28}$
18. $3\frac{33}{40}$
19. $11\frac{2}{3}$
20. $11\frac{1}{3}$

Division

21. $1\frac{13}{15}$
22. $\frac{3}{4}$
23. 1
24. $1\frac{1}{3}$
25. $4\frac{9}{10}$

Functions and Graphs

26. $\frac{1}{12}$
27. $\frac{7}{10}$
28. $\frac{1}{4}$
29. $1\frac{2}{9}$
30. $\frac{7}{12}$

Sets

31. {4,5,6}
32. {3}
33. {6,7,8,9}
34. {0,1,2,3,4,5,6,7,8}
35. {21}

Numeral (base 8)

36. 11_{ten}
37. 30_{eight}
38. 8_{ten}
39. 17_{ten}
40. 176_{eight}

Step: B

Division

41. 498
42. 268
43. 832
44. 908 R. 2
45. 873 R. 2

Functions and Graphs

46. 5
47. 3
48. 1
49. 0
50. Graph should be correct

Geometry

51. Rhombus
52. Hexagon
53. Pentagon
54. Triangle
55. Octagon

Level: 16 Answer Sheet - Post Test II

Step: A

Numeral

1. $n \times 25 < 77$
2. $100 \times (5 + 4) = n$
3. $7 \times n > 30$
4. $n - (675 + 500) = 325$ (\$3.25)
5. $72 \div 4 = n$ or $72 \div n = 4$

Order

6. 40.
7. 200
8. 50
9. 4000
10. 600

Subtraction

11. $1\frac{3}{8}$
12. $2\frac{11}{12}$
13. $1\frac{2}{3}$
14. $3\frac{7}{10}$
15. $1\frac{5}{8}$

Multiplication

16. $30\frac{1}{3}$
17. $2\frac{4}{5}$
18. $3\frac{33}{64}$
19. $23\frac{5}{6}$
20. $2\frac{13}{18}$

Division

21. $\frac{36}{55}$
22. $1\frac{7}{8}$
23. $4\frac{4}{15}$
24. $\frac{63}{64}$
25. 1

Functions & Graphs

26. $\frac{3}{8}$
27. $\frac{7}{10}$
28. $\frac{1}{12}$
29. $\frac{1}{4}$
30. $\frac{5}{9}$

Step: B

Sets

31. {12, 13, 14}
32. {0, 1, 2, 3, 4, 5, 6}
33. {0, 1, 2, 3}
34. {6}
35. {0, 1, 2, 3, 4, 5, 6, 7, 8, 9}

Numeral

36. 36_{eight}
37. 12_{ten}
38. 204_{eight}
39. 33_{ten}
40. 55_{eight}

Division

41. 843
42. 756 R.3
43. 803
44. 573 R.2
45. 958

Functions & Graphs

46. 0
47. 2
48. 6
49. 10
50. Graphed correctly

Geometry

51. T
52. W
53. X
54. U
55. V

TABLE OF CONTENTS

Level 17

Step A

Step B

Step Z

Level: 17 Step: A Concept: Numeral

I. Concept:

Numeral: Introducing prime factorization by division.

II. Behavioral Objective:

The student given a composite number will be able to express it as the product of prime factors using the division method.

III. Mathematical Ideas:

A. A composite number may be expressed in only one way as the product of prime factors. This is called the Fundamental Theorem of Arithmetic.

B. A composite number is any whole number greater than one that is not prime.

C. The exponent is the number placed to the right of and above the symbol. It tells the number of times the base is used as a factor. Ex. $2^3 \times 5 = 2 \times 2 \times 2 \times 5 = 40$. Two is the base and three is the exponent.

IV. Vocabulary To Stress:

Fundamental Theorem of Arithmetic
prime factors
composite number
exponent

V. Activities:

A. Have the students work in pairs, either helping each other or competing with each other to see who can name the prime factorization of a number first. H.M.T.E. 6 (1972) p. T226.

B. Use the prime factors 2 and 3 an even number of times to name a composite number such as ($2^2 \times 3^2 = 36$).. Now use 2 and 3 an odd number of times, as ($2^3 \times 3 = 24$). H.M.T.E. 6 (1972) p. T227.

Text References:

Book: 6

Houghton Mifflin (1967) pp. 172.
Houghton Mifflin (1972) pp. 162, 344 (15-22)

Book: 7

Houghton Mifflin (1967, 1970) pp. 231-235, (35)
Hougnton Mifflin (1972) pp. 161, 227
Addison-Wesley (1971) pp. 176-178, (32)

Book: 8

Houghton Mifflin (1972) p. 160

Level: 17 Step: A Concept: Numeral

Other References:

Modern Mathematics Filmstrips #751, 761

Imperial Tape (Intermediate) #15 and 16. (Do not write on tape worksheet. Student uses own paper.)

Level: 17 Step: A Concept: Numeral

WORKSHEET

I. Complete the examples and name the factorizations.

A.
$$\begin{array}{r|l} 3) & 27 \\ 3) & a \\ & b \end{array}$$
27 = ________

B.
$$\begin{array}{r|l} 2) & 48 \\ 3) & a \\ 2) & b \\ 2) & c \\ & d \end{array}$$
48 = ________

C.
$$\begin{array}{r|l} 5) & 40 \\ 2) & a \\ 2) & b \\ 2) & c \end{array}$$
40 = ________

D.
$$\begin{array}{r|l} 3) & 72 \\ 3) & a \\ 2) & b \\ 2) & c \\ & d \end{array}$$
72 = ________

II. Use repeated division to name the prime factorization of each numeral below. Name the factorizations below each using exponents.

A.)50 B.)54 C.)84

D.)100 E.)150 F.)200

G.)132 H.)96 I.)36

Level: 17 Step: A Concept: Multiplication

I. Concept:

Multiplication: Solving multiplication problems using clock-arithmetic.

II. Behavioral Objective:

The student given two factors from a finite set will be able to name the product.

III. Mathematical Ideas:

A. The clock-arithmetic (modular) system has a finite number of elements.

B. The symbol $\overset{12}{=}$ indicates a clock equation with the twelve numbers 0-11 on the clock face.

C. A clock-arithmetic system is closed for multiplication.

D. Multiplication in a clock system is defined as repeated clock-wise movements of equal units.

E. The associative, commutative, and distributive properties apply to the operation of multiplication in clock-arithmetic.

F. The symbol $\overset{7}{=}$ indicates a clock equation with the seven numbers 0-6 on the clock face.

G. The numbers in a clock system are not ordered. Example:

$$6 + 3 \overset{12}{=} 9, \text{ but } 9 + 3 \overset{12}{=} 0$$

IV. Vocabulary To Stress:

modular system clock-arithmetic

V. Activities:

* A. It may help students to understand if you make clock faces out of cardboard (paper plates), strips of cardboard, and paper fasteners. Solve some of the problems using these clock faces and students may more easily understand this new concept. (Use Primary Clock for $\overset{12}{=}$.

B. Students may make up their own clock problems, put on the chalkboard, and explain them to the class or allow another student to solve them.

Level: 17 Step: A Concept: Multiplication

V. Activities: Continued.

C. Make up tables such as this.

Find the solution set of each of the following. The sentences on the left side are ordinary addition and multiplication sentences. Those on the right are 7-clock (or Mod 7) sentences.

ORDINARY ADDITION		ADDITION 7-CLOCK	
1. 7 + 3 = N	6. 9 + 2 = N	1. 7 + 3 = N	6. 9 + 2 = N
2. 8 + 2 = N	7. 3 + 6 = N	2. 8 + 2 = N	7. 3 + 6 = N
3. 5 + 4 = N	8. 5 + 2 = N	3. 5 + 4 = N	8. 5 + 2 = N
4. 6 + 5 = N	9. 3 + 3 = N	4. 6 + 5 = N	9. 3 + 3 = N
5. 7 + 2 = N	10. 1 + 7 = N	5. 7 + 2 = N	10. 1 + 7 = N
ORDINARY MULTIPLICATION		MULTIPLICATION 7-CLOCK	
1. 2 x 3 = N	6. 4 x 2 = N	1. 2 x 3 = N	6. 4 x 2 = N
2. 3 x 4 = N	7. 2 x 2 = N	2. 3 x 4 = N	7. 2 x 2 = N
3. 2 x 6 = N	8. 3 x 3 = N	3. 2 x 6 = N	8. 3 x 3 = N
4. 5 x 2 = N	9. 5 x 3 = N	4. 5 x 2 = N	9. 5 x 3 = N
5. 2 x 7 = N	10. 2 x 8 = N	5. 2 x 7 = N	10. 2 x 8 = N

Text References:

Book: 5

Houghton Mifflin (1967) pp. 188-189

Book: 6

Houghton Mifflin (1967) pp. 189-193
Houghton Mifflin (1972) pp. 181, 183-184, (46 - a portion of it)
Addison-Wesley (1971, 1968) pp. 90-91

Book: 7

Houghton Mifflin (1972) pp. 178-179, 180, 181

Other References:

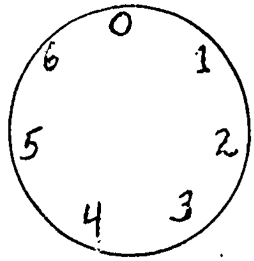
Franklin Mathematics Series, Mathematics Around the Clock

* This activity could be used to help introduce the concept.

17-6

Level: 17 Step: A Concept: Multiplication

WORKSHEET



I. If the equations below are correct write "Yes", if they are not correct write "No" and write the correct answer beside them.

A. $5 \times 6 \stackrel{?}{=} 2$ ______

B. $(2 \times 3) \times 6 \stackrel{?}{=} 1$ ______

C. $3 \times 6 \stackrel{?}{=} 3$ ______

D. $2 \times 8 \stackrel{?}{=} 1$ ______

E. $4 \times 3 \stackrel{?}{=} 5$ ______

F. $2 \times 5 \stackrel{?}{=} 2$ ______

G. $6 \times 2 \stackrel{?}{=} 4$ ______

H. $5 \times 2 \times 2 \stackrel{?}{=} 6$ ______

I. $2 \times (3 \times 2) \stackrel{?}{=} 3$ ______

J. $(2 \times 3) \times 2 \stackrel{?}{=} 5$ ______

K. $2 \times 2 \stackrel{?}{=} 6$ ______

L. $3 \times 5 \stackrel{?}{=} 2$ ______

M. $4 \times 2 \stackrel{?}{=} 1$ ______

N. $5 \times 3 \stackrel{?}{=} 3$ ______

II.

A. John left home at four o'clock and arrived at his friend's home at 1:00 ?. If his friend Joe made the same trip starting at the same time and it took him twice as long, what time did Joe arrive at the friend's home?

______

B. If Mary leaves home at $3 \times (2 + 7) \stackrel{?}{=}$ to catch a plane, at what time does she leave?

______

Level: 17 Step: A Concept: Division

I. Concept:

<u>Division</u>: Naming square roots.

II. Behavioral Objective:

The student given a number which is a perfect square will be able to name the square root.

III. Mathematical Ic

A. Naming the square root is the opposite of squaring.

B. The square root of a number is a number, which when multiplied by itself, produces the given number.

C. The sign $\sqrt{}$ means "the square root of".

D. For any non-negative rational number a, if a x a = b, $\sqrt{b} = a$.

IV. Vocabulary To Stress:

square root squaring exponent second power

V. Activities:

Students may work in pairs with one student squaring numbers he chooses and asking his partner to find the square root of these squares. H.M.T.E. (1972), p. T139.

Text References:

Book: 6

Houghton Mifflin (1967) pp. 123-124, 172, 274.
Houghton Mifflin (1972) pp. 88, (23), 341.

Book: 7

Houghton Mifflin (1972) pp. 168, 170, 229, 252, 425.

Level: 17 Step: A Concept: Function

I. Concept:

Function: Using formulas.

II. Behavioral Objective:

The student given a formula and replacement for all but one place-holder will be able to determine the unknown number.

III. Mathematical Ideas:

A. A formula is an open equation containing two or more place-holders.

B. A formula is a type of open number sentence.

C. Formulas are used to describe physical facts that are known to be true. Example: $A = \ell \times w$, A representing area;
ℓ representing length;
w representing width.

IV. Vocabulary To Stress:

formula replacement determine physical facts.

V. Activities:

A. Give the students a perimeter of 48 inches and have them draw as many rectangles as they can with this perimeter. After they have finished drawing the figures, ask them to find the area of each rectangle.

B. Students may be interested in preparing a "Distance and Rate Table" similar to the one in problem #4, page 142, H.M. (1967). These may be given to classmates to solve or as a special individual project for enrichment.

Text References:

Book: 6

Houghton Mifflin (1967) pp. 142, (33), 143
Houghton Mifflin (1972) pp. 45, 64-65, (17), 230, 231, 306.
Addison-Wesley (1971, 1968) p. 74

Other References:

Imperial Tape (Intermediate) #37

The Franklin Series, Learning About Measurement, pp. 26-36.

Level: 17 Step: A Concept: Funcuions & Graphs

WORKSHEET

I.

Using the formula A = l x w name the missing measures for each of the rectangles.

AREA	a sq. in.	144 sq. in.	168 sq. in.	d sq. in.	150 sq. in.	352 sq. in.
LENGTH	12 in.	b in.	14	22 in.	e in.	22 in.
WIDTH	8 in.	8 in.	c in.	6 in.	10 in.	f in.

a = ________ c = ________ e = ________

b = ________ d = ________ f = ________

II.

Using the formula P(perimeter) = 2 x (l + w), name the missing measure for each of the rectangles.

PERIMETER	a in.	24 in.	56 in.	d in.	96 in.	70 in.
LENGTH	9	b in.	10	14	e in.	15 in.
WIDTH	7	4	c	18	20	f in.

a = ________ c = ________ e = ________

b = ________ d = ________ f = ________

III. The formula to name a distance (D) traveled is D = R x T where R stands for Rate and T for Time. In this table rates are given in miles per hour, time in hours, and the distance in miles.

RATE (MPH)	4	b	50	70	e	60
TIME (HR.)	3	3	c	6	6	f
DISTANCE (M.)	a	15	100	d	300	360

Level: 17 Step: A Concept: Geometry

I. Concept:

<u>Geometry</u>: Naming the area of a triangle.

II. Behavioral Objective:

The student given the dimensions of a triangle will be able to compute its area.

III. Mathematical Ideas:

A. The set of all points inside a simple closed curve is called a region.

B. The measure of a region may be determined by comparing it with a standard unit region.

C. In a triangle, if b is the number of units along the base and h is the number of units along the height, $A = \frac{1}{2}(b \times h)$.

IV. Vocabulary To Stress:

region triangle area height base square units

V. Activities:

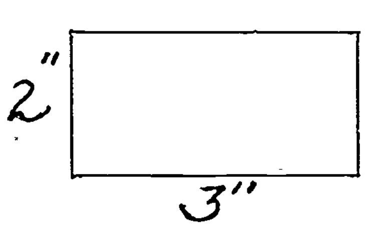
* A. Geoboards may be used for students to work in pairs to form shapes and name the areas. See card #25, 29, 31.

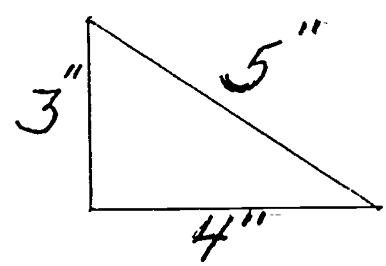
* B. Students could draw on graph paper triangles of various sizes and compute the area of each one.

C. Students who do not understand relationships between units should construct unit squares. Have them make a sq. in., a sq. ft., and a sq. yd. from heavy paper and compare them. H.M. (1972) T.E. p. T165.

D. Find the areas of the pairs of figures below and compare them. Then find the perimeters and compare them.

A.





AREA ______________ AREA ______________

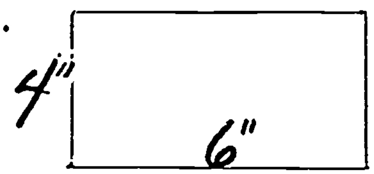
PERIMETER ______________ PERIMETER ______________

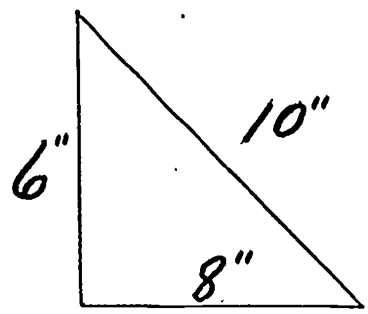
Level: 17 Step: A Concept: Geometry

V. Activities: Continued

D. Continued

B.





AREA ______________ AREA ______________

PERIMETER ______________ PERIMETER ______________

Make other comparisons such as these.

Text References:

Book: 5

Houghton Mifflin (1967) p. 115
Houghton Mifflin (1972) pp. 111, 304-305, 310-311.
Addison-Wesley (1971, 1968) pp. 276, 277

Book: 6

Houghton Mifflin (1967) pp. 79, 108, 269.
Houghton Mifflin (1972) pp. 108-109, (29 part of it), 309.
Addison-Wesley (1971, 1968) pp. 113

Book: 7

Houghton Mifflin (1967, 1970) pp. 441-443, (69).
Houghton Mifflin (1972) pp. 117
Addison-Wesley (1971) pp. 389, 391, 392, (80, 81).

Book: 8

Houghton Mifflin (1972) p. 53

Other References:

Franklin Series, <u>Learning About Measurement</u>, Area; pp. 20-25.

Mathematics in the Making I (The Bell Series) <u>Pattern</u>, <u>Area</u>, <u>and Perimeter</u> (In your building)

Curriculum Filmstrip #366 - <u>Triangles</u>

* These activities may be used to help introduce the concept.

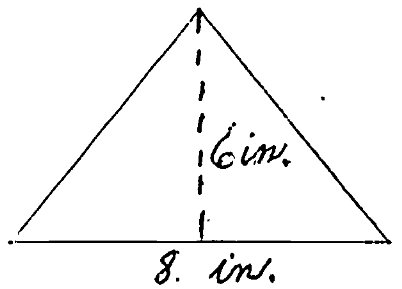
Level: 17 Step: A Concept: Geometry

WORKSHEET

I.

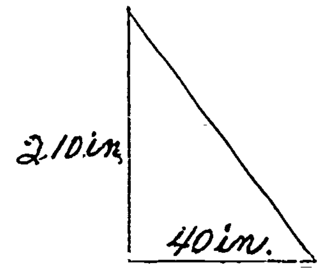
Find the areas of the following triangles. Express your answer in square units.

1.



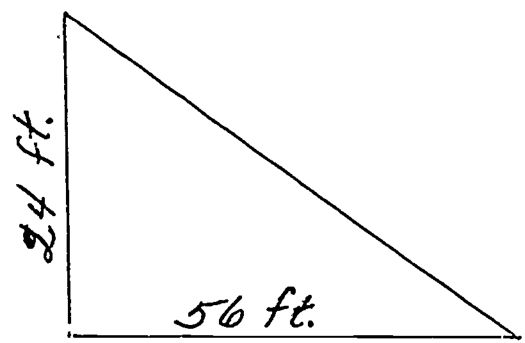
____

2.



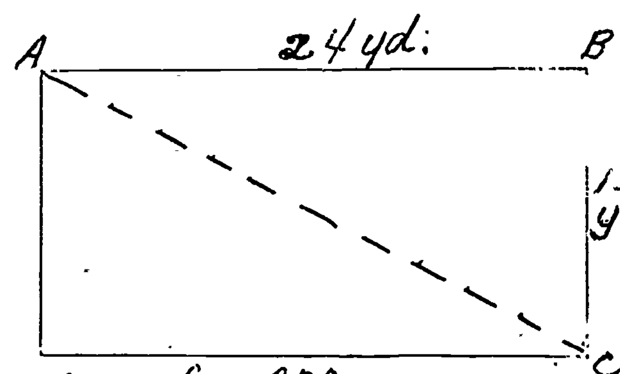
____

3.



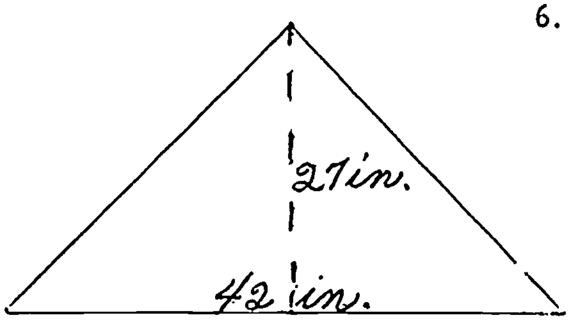
____

4.



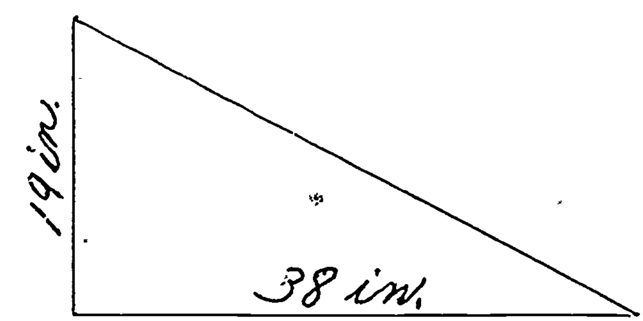
AREA of △ ABC = ________________

5.



____

6.



____

7. Using the formula $A = \frac{1}{2}(a \times b)$, find the areas using the dimensions given.

a = 20 in.	a = 66 ft.	a = 36 yd.	a = 58 cm.
b = 36 in.	b = 70 ft.	b = 32 yd.	b = 39 cm.
A = ________	A = ________	A = ________	A = ________

Level: 17 Step: A Concept: Measurement

I. Concept:

Measurement: Renaming the metric units of weight as other units of weight in the metric system.

II. Behavioral Objective:

The student given units of weight in the metric system will be able to rename them in other metric units of weight.

III. Mathematical Ideas:

A. The metric system is based on powers of 10.

B. A gram is a unit of weight.

C. 1000 grams is a kilogram.

D. A kilogram is a little more than two pounds.

E. A centigram is $\frac{1}{100}$ of a gram.

F. A milligram is $\frac{1}{1000}$ of a gram.

IV. Vocabulary To Stress:

gram, kilogram, centigram, milligram

prefixes: centi = $\frac{1}{100}$ milli = $\frac{1}{1000}$ kilo = 1000

V. Activities:

A. Complete tables such as the following: (H.M. 5(1972) T.E. p. 23)

GRAMS	KILOGRAMS	GRAMS
1000	1 kg.	0 gm.
1500	a	b
2100	c	d
2500	e	f
5470	g	h
8001	i	j

Level: 17 Step: A Concept: Measurement

A. Continued.

ANSWERS FOR TEACHER

	Kilograms	Grams
a-b	1	500
c-d	2	100
e-f	2	500
g-h	5	470
i-j	8	1

B. Make a bulletin board using the following facts.

Did you know?

A paper clip weighs approximately 1 gram.

The wire in a paperclip measures 1 millimeter in diameter.

A nickel weighs 5 grams.

A dime is 1 millimeter thick.

A cube of sugar weighs approximately 2 grams.

Eight common straight pins weigh approximately 1 gram.

from Arithmetic Teacher, April 1973

Text References:

Book: 5

Houghton Mifflin (1967) pp. 26, 27
Houghton Mifflin (1972) p. 23 (9-12)

Book: 6

Houghton Mifflin (1967) p. 27
Houghton Mifflin (1972) pp. 21, (7 -a portion of it), 336, 353 (tables in metric measure)

Book: 8

Houghton Mifflin (1972) p. 20

Other References:

The Arithmetic Teacher, April 1973.

Metric Measurement - Minneapolis Public Schools

* This activity could be used when introducing the con[illegible]t.

Level: 17 Step: A Concept: Measurement

WORKSHEET I

A gram is a unit of weight equal to the weight of one cubic centimeter of water. It is used to weigh light articles, especially drugs and medicines. An ounce equals a little more than 28 grams. A kilogram equals 1000 grams or about 2.2 pounds. The kilogram is used to compute most weights. A silver half-dollar of United States money weighs 12 $\frac{1}{2}$ grams. Forty dollars in silver weigh one kilogram.

Measures of Weight

A centigram (cg.) = $\frac{1}{100}$ gram

A milligram (mg.) = $\frac{1}{1000}$ gram

A kilogram = 1000 grams

Solve the following problems using the information given above.

1. 2 kilograms = ______________ grams.
2. 7000 grams = ______________ kilograms.
3. 50 centigrams = ______________ gram.
4. one-half gram = ______________ milligrams.
5. A kilogram is equal in weight to ______________ grams.
6. One gram = ______________ kilograms.
7. 1000 milligrams = one ______________.
8. 100 centigrams = one ______________.
9. 4 $\frac{1}{2}$ kilograms = ______________ grams.
10. 2 kilograms = about ______________ pounds.
11. 6 kilograms = ______________ grams.
12. 1 centigram = ______________ milligrams.
13. 40 centigrams = ______________ milligrams.
14. 3000 grams = ______________ kilograms.
15. one-half kilogram = ______________ grams.

Level: 17 Step: A Concept: Measurement

WORKSHEET II

A centigram (cg.) = $\frac{1}{100}$ gram (gm.)

A milligram (mg.) = $\frac{1}{1000}$ gram (gm.)

A kilogram (kg.) = 1000 grams (gm.)

Solve the following problems using the above table.

1. The prefix kilo means ________. Therefore, one kilogram is equal in weight to how many grams? ________.

2. One gram is equal to ________ of a kilogram.

3. 19,000 mg = ________ gm.

4. 2 kg = ________ gm.

5. 8000 gm = ________ kg.

6. 50 cg = ________ mg.

7. 2 cg = ________ mg.

8. 9 kg = ________ gm.

9. 500 mg = ________ gm.

10. 100 cg = ________ gm.

11. 4 kg = about ________ pounds.

12. 4000 mg = ________ gm.

13. 6 kg = 6000 ________

14. 200 cg = two ________

15. The prefix centi means ________

Level: 17 Step: A Concept: Measurement

WORKSHEET III

Measures of Weight

A centigram (cg.) = $\frac{1}{100}$ gram

A milligram (mg.) = $\frac{1}{1000}$ gram

A kilogram (kg.) = 1000 grams

Solve the following problems using the information given above.

1. 78 cg. = ______________ mg.
2. 376 g. = ______________ cg.
3. 432 cg. = ______________ g.
4. 4800 mg. = ______________ cg.
5. 340 cg. = ______________ mg.
6. 9 kg. = ______________ g.
7. 357,000 g. = ______________ kg.
8. 83 g. = ______________ mg.
9. 7 kg. = ______________ mg.
10. 73,000 mg. = ______________ g.
11. 5200 cg. = ______________ g.
12. 381 g. = ______________ mg.
13. 92 g. = ______________ cg.
14. 794,000 g. = ______________ kg.
15. 7,000,000 mg. = ______________ kg.
16. 45 kg. = ______________ cg.
17. 93 g. = ______________ cg.
18. 291 cg. = ______________ g.
19. 10 mg. = ______________ cg.
20. 100 cg. = ______________ g.
21. 2400 cg. = ______________ g.
22. 5 kg. = ______________ g.
23. 65,000 g. = ______________ kg.
24. 57 cg. = ______________ mg.
25. 4 g. = ______________ mg.

Level: 17 Step: 8 Concept: Sets

I. Concept:

Sets: Finding LCM and GCF by using prime factorization.

II. Behavioral Objective:

The student given two numbers will be able to find the LCM and the GCF by using prime factorization.

III. Mathematical Ideas:

A. The least common multiple of two or more numbers is the product of all the different prime factors of the numbers, each prime occurring in the product the greatest number of times that it appears in any of the prime factorizations of the numbers.

e.g. $12 = 2 \times 2 \times 3$

$20 = 2 \times 2 \times 5$

$LCM = 2 \times 2 \times 3 \times 5 = 60$

B. Any prime factors common to both members should be included only once as the 2 x 2 in the example above.

C. The greatest common factor of two or more numbers is the product of the common prime factors of the numbers.

e.g. $70 = 2 \times (5 \times 7)$

$105 = 3(5 \times 7)$

$GCF = 5 \times 7 = 35$

IV. Vocabulary To Stress:

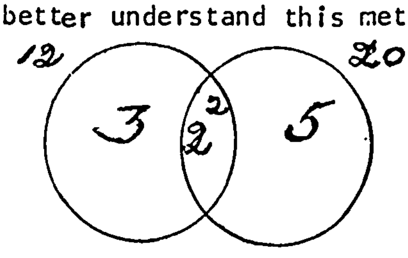
Least Common Multiple
Greatest Common Factor
Venn Diagram
prime factorization
intersection
union

V. Activities - Venn Diagrams

* A. Use Venn Diagrams to help children better understand this method of finding LCM. e.g.

$12 = 2 \times 2 \times 3$

$20 = 2 \times 2 \times 5$



$LCM = 2^2 \times 3 \times 5 = 60$

Level: 17 Step: B Concept: Sets

V. Activities (Continued)

* B. Help children understand that the LCM is the product of the prime factors in the union of the sets.

* C. The GCF also may be found by using a Venn Diagram as shown above. GCF = 2 x 2 = 4 or the product of the prime factors in the intersection of the sets.

Text References:

Book: 6

Houghton Mifflin (1967) pp. 173, 174, 175, (39)
Houghton Mifflin (1972) pp. 165, 168, 169, 344 (7-22)
Addison-Wesley (1971, 1968) pp. 86-89

Book: 7

Houghton Mifflin (1972) pp. 162, 165, 421

Book: 8

Houghton Mifflin (1972) pp. 163, 166-167, 422

* These activities will be helpful when introducing the concept.

Level: 17 Step: 8 Concept: Sets

WORKSHEET

I. Use prime factorization to name the greatest common factor.

A. 24 = 2 x 2 x 2 x 3

48 = 2 x 2 x 2 x 2 x 3

GCF = ___ x ___ x ___ x 3 = _____

B. 40 = 2 x 2 x 2 x 5

35 = 5 x 7

GCF = _____

C. 16 = 2 x 2 x 2 x 2

24 = 2 x 2 x 2 x 3

GCF = ___ x ___ x ___ x = _____

D. 54 = 2 x 3 x 3 x 3

42 = 2 x 3 x 7

GCF = _____

E. 18 = 2 x 3 x 3

24. 2 x 2 x 2 x 3

GCF = _____

F. 12 = 2 x 2 x 3

15 = 3 x 5

GCF = _____

G. 27 = 3 x 3 x 3

63 = 3 x 3 x 7

GCF = ___ x ___ = _____

H. 25 = 5 x 5

15 = 3 x 5

GCF = _____

I. 32 = 2 x 2 x 2 x 2 x 2

64.= 2 x 2 x 2 x 2 x 2 x 2

GCF = _____

J. 25 = 5 x 5

45 = 5 x 3 x 3

GCF = _____

II. Use prime factorization to name the least common multiple.

A. 27 = 3 x 3 x 3

36 = 2 x 2 x 3 x 3

LCM = _____

B. 14 = 2 x 7

28 = 2 x 2 x 7

LCM = _____

C. 42 = 2 x 3 x 7

21 = 3 x 7

LCM = _____

D. 15 = 3 x 5

10 = 2 x 5

LCM = _____

E. 33 = 3 x 11

66 = 2 x 3 x 11

LCM = _____

F. 4 = 2 x 2

18 = 2 x 3 x 3

LCM = _____

Level: 17 Step: B Concept: Numeral

I. Concept:

Numeral: Comparing the binary system of numeration with the decimal system.

II. Behavioral Objective:

The student given a numeral in either base 2 or base 10 will be able to express it in the other base.

III. Mathematical Ideas:

A. In base two, two digits are used: 0 and 1.

B. In any base-place system of numeration each digit has a face-value, a place value, and a total value.

C. The numeral names for two-digit numbers in the decimal system do not apply to other bases.

IV. Vocabulary To Stress:

binary base-place system compare

V. Activities:

If you do not have a binary abacus in the classroom, make one by driving nails through a board. The nails should be just long enough to hold 1 bead.

Take the classroom abacus and compare counting and adding by using each system.

Text References:

Book: 5

Houghton Mifflin (1972) p. 28

Book: 6

Houghton Mifflin (1967) pp. 184-185
Houghton Mifflin (1972) pp. 176-178, (45 part of it)

Book: 7

Houghton Mifflin (1967, 1970) pp. 115-120, (18)

Book: 8

Addison-Wesley (1971) pp. 15-18, (4 sec. 1, 2)

Other References:

Bell, Stuart E., Binary and Other Numeration Systems, Mathematics in the making.

Level: 17 Step: B Concept: Numeral

WORKSHEET

I. Name the binary numerals with decimal numerals.

A. 11_{two} = ______ D. 1110_{two} = ______ G. 1111_{two} = ______

B. 101_{two} = ______ E. 1010_{two} = ______ H. 10011_{two} = ______

C. 1001_{two} = ______ F. 10100_{two} = ______ I. 110_{two} = ______

II. Write the binary numeral.

A. 26_{ten} D. 17_{ten} G. 7_{ten} J. 13_{ten}

B. 32_{ten} E. 15_{ten} H. 9_{ten} K. 24_{ten}

C. 21_{ten} F. 11_{ten} I. 33_{ten} L. 41_{ten}

Put answers from problems above in table as shown.

	2^5	2^4	2^3	2^2	2^1	ones
	32	16	8	4	2	1
A		1	1	0	1	0
B						
C						
D						
E						
F						
G						
H						
I						
J						
K						
L						

Level: 17 Step: B Concept: Addition

I. Concept:

Addition: Applying the properties of addition in a decimal system to the binary system.

II. Behavioral Objective:

The student given two addends in base 2 will be able to name the sum in the binary system.

III. Mathematical Ideas:

A. It is easier to compute in base two because there are only 4 basic facts for each operation. The addition facts are:

$$0 + 0 = 0$$

$$0 + 1 = 1$$

$$1 + 0 = 1$$

$$1 + 1 = 10_{two}$$

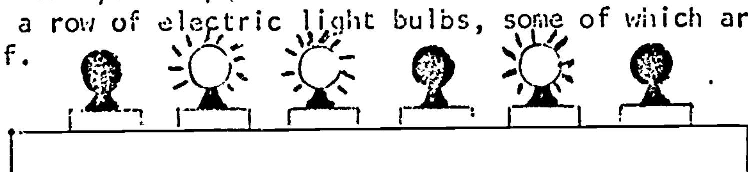
B. The associative, commutative, and identity properties of base 10 also apply to base 2.

IV. Vocabulary To Stress:

addends sum binary system binary operation associative

V. Activities:

A. As the binary system is used with most computers, this would be an ideal time to take students to see a computer work. Before taking the trip, discuss with the students the way a computer works. Computers are sometimes called electronic brains whose language is binary. Compare the values and transistors in a computer to a row of electric light bulbs, some of which are on and some off.



If you say "on" is one and "off" is zero, the number shown on the row of bulbs is 011010. A set of Christmas tree lights would work very well to give a demonstration.

Level: 17 Step: 6 Concept: Addition

V. Activities: (Continued)

* 8. Students may check their computation in base two by changing all the numbers involved into base 10. For example.

$$\begin{array}{rl} 111_{two} & = 7 \\ + \ 101_{two} & = 5 \\ \hline 1100_{two} & = 12 \end{array}$$

Does $1100_{two} = 12_{ten}$?
(Yes, so the results check.)

Text References:

Book: 5

Houghton Mifflin (1972) p. 28

Book: 6

Houghton Mifflin (1967) pp. 186-187
Houghton Mifflin (1972) pp. 179, (45-part), 345.

Book: 7

Houghton Mifflin (1967, 1970) pp. 120,144

Book: 8

Addison-Wesley (1971) pp. 19-21, (4, sec. 3).

Other References:

<u>The Arithmetic Teacher</u>, March 1970, "Another Use for Binary Numerals", pp. 225, 226.

Bell, Stewart E., <u>Mathematics in the Making 2</u>.

* This activity may be used when introducing the concept.

Level: 17 Step: B Concept: Numeral

WORKSHEET

I. Name the sums.

1. $110_{two} + 1_{two}$

2. $101_{two} + 1_{two}$

3. $1111_{two} + 101_{two}$

4. $1000_{two} + 111_{two}$

5. $1011_{two} + 11_{two}$

6. $10000_{two} + 11111_{two}$

7. $10101_{two} + 1010_{two}$

8. $111_{two} + 11_{two}$

9. $10110_{two} + 1001_{two}$

10. $1111_{two} + 1001_{two}$

II. Name the difference.

1. $1111_{two} - 100_{two}$

2. $1000_{two} - 11_{two}$

3. $10001_{two} - 101_{two}$

4. $1101_{two} - 101_{two}$

5. $1111_{two} - 1010_{two}$

6. $11111_{two} - 1001_{two}$

7. $10110101_{two} - 1011011_{two}$

8. $110001_{two} - 11111_{two}$

9. $1101_{two} - 10_{two}$

10. $101010_{two} - 10101_{two}$

Level: 17 Step: 8 Concept: Multiplication

I. Concept:

Multiplication: Applying the properties of multiplication in a Base 10 system to a Base 2 system.

II. Behavioral Objective:

The student given two factors in base 2 will be able to name the product.

III. Mathematical Ideas:

A. Base 2 has only four multiplication facts:

$0 \times 0 = 0$, $0 \times 1 = 0$, $1 \times 0 = 0$, $1 \times 1 = 1$

B. In multiplication, associative, commutative, distributive, and identity properties of base 10 also apply to base 2.

C. One is the multiplicative identity element. (If a denotes a whole number then $a \times 1 = a$ and $1 \times a = a$.)

IV. Vocabulary To Stress:

product associative commutative distributive identity

Text References:

Book: 6

Houghton Mifflin (1967) pp. 186-187
Houghton Mifflin (1972) p. 179

Book: 7

Houghton Mifflin (1967, 1970) p. 154

Book: 8

Addison-Wesley (1971) p. 21, (4, sec. 4)

Other References:

Bell, Stewart E., Binary and Other Numeration Systems.

Imperial Tape (Intermediate) #35, Number Systems with Bases Other Than 10.

Level: 17 Step: B Concept: Multiplication

WORKSHEET

I. Name the products in the binary system.

A. 1011 x 101	B. 1001 x 110	C. 1010 x 10	D. 10100 x 11	E. 101 x 10
F. 11 x 11	G. 1001 x 111	H. 10 x 10	I. 110 x 10	J. 1011 x 100
K. 11 x 10	L. 1011 x 11	M. 100 x 11	N. 1000 x 111	O. 110 x 100

II. Check your binary answers above by using decimal numerals as shown.

A.
```
  1011 = 11
 x 101 = x5
  1011  (55)
101100
110111 = (55)
```

B.

C.

D.

E.

F.

G.

H.

I.

J.

K.

L.

M.

N.

O.

Level: 17 Step: B Concept: Geometry

I. Concept:

Geometry: Using the theorem of Pythagoras, ($c^2 = a^2 + b^2$) the square of the hypotenuse of a right triangle is found by adding the squares of the other two sides.

II. Behavioral Objective:

The student given the linear measurements of a right triangle will be able to find the hypotenuse (c) by using the formula $c^2 = a^2 + b^2$.

III. Mathematical Ideas:

A. In a right triangle the side opposite the right angle is the hypotenuse.

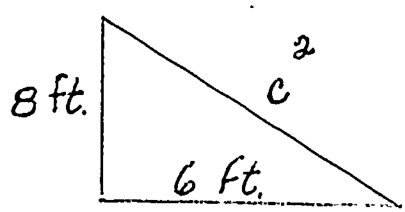
B. $c^2 = a^2 + b^2$ is the Theorem of Pythagoras.

IV. Vocabulary To Stress:

hypotenuse right triangle square of a measurement

V. Activities:

A. Either you or the students make up story problems that apply the Pythagorean Theorem, as: A ladder is set 6 feet from the house to reach a window 8 feet from the ground. How long is the ladder? (10 feet)

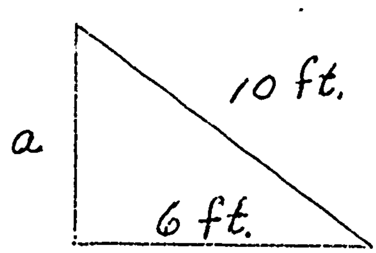


$8^2 + 6^2 = c^2$

$64 + 36 = c^2$

$\sqrt{100} = 10$ - length of ladder

B. The problem could be restated to read a 10 foot ladder is set 6 feet from the house to reach a window. How high is the window?



$a^2 + 6^2 = 10^2$

$a^2 + 36 = 100$

$a^2 = 64$

$\sqrt{64} = 8$ - height of window

C. Use Geoboard Card #40.

Level: 17 Step: B Concept: Geometry

Text References:

Book: 5

Houghton Mifflin (1967) pp. 268-269, (68), 281.
Houghton Mifflin (1972) p. 306.

Book: 6

Houghton Mifflin (1967) pp. 83, 85, 274, 294-295.
Houghton Mifflin (1972) pp. 110-111, (30), 303.
Addison-Wesley (1971, 1968) pp. 114-115.

Book: 7

Houghton Mifflin (1972) pp. 102-103
Addison-Wesley (1971) pp. 333-336, (82).

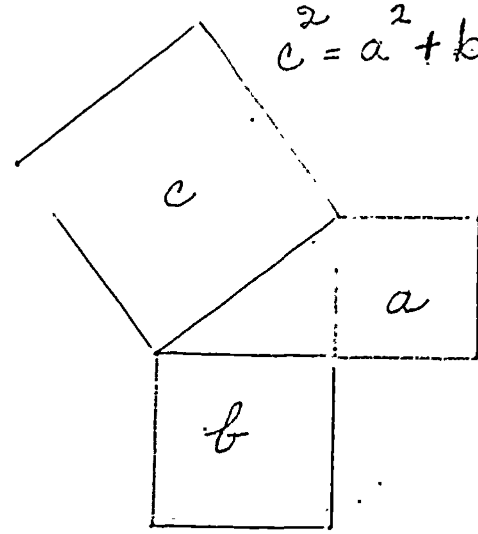
Level: 17 Step: 3 Concept: Geometry

WORKSHEET

I. Find the area of the missing square region, and then give the length of the side of that square.

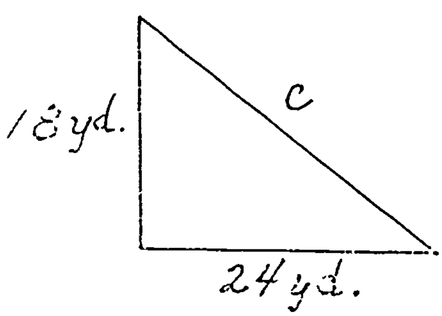
$c^2 = a^2 + b^2$

	a^2	b^2	c^2	Length
A.	49		625	b =
B.		576	676	a =
C.	9	16		c =
D.	25		169	b =
E.		64	100	a =
F.	144	256		c =

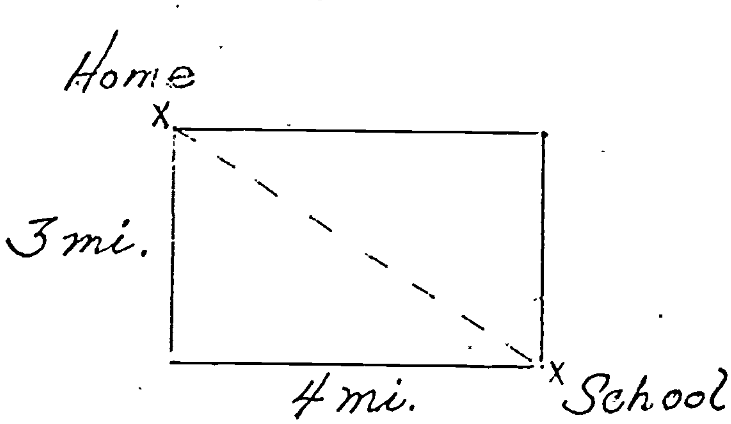


II. Use the Pythagoras Theorem to solve the following problems.

A. Mr. Jones bought a triangular field and wants to put a fence around it. Using the diagram below, find how many yards of wire he will need.



B. To get from home to school, Tom could go around the corner which would make a total of 7 miles. If he takes the short cut, how far will he go?



Level: 17 Step: Z Concept: Sets

I. Concept:

Sets: Developing the idea of a "perfect" number.

II. Behavioral Objective:

The student given a number will be able to determine if it is perfect.

III. Mathematical Ideas:

A. A number is perfect if the sum of its factors, not counting the number, is equal to the number itself. Example: The factors of 6 are 1, 2, 3, 6, and 1 + 2 + 3 = 6. Hence, 6 is a perfect number.

B. Six and twenty-eight are the only perfect numbers less than 100.

IV. Vocabulary To Stress:

factor product perfect number

V. Activities:

Worksheets and tests are not appropriate for this concept.

Text References:

Book: 6

Houghton Mifflin (1967) p. 177
Houghton Mifflin (1972) p. 163
Addison-Wesley (1971, 1968) p. 81

Book: 7

Houghton Mifflin (1967) p. 221
Addison-Wesley (1971) p. 173

Book: 8

Addison-Wesley (1971) p. 94

Other References:

Bell, Stewart E., Binary and Other Numeration Systems.

Level: 17 Step: Z Concept: Addition, Subtraction Multiplication, Division

I. Concept:

Addition, Subtraction, Multiplication, Division: Checking by "casting out nines".

II. Behavioral Objective:

The student given a problem to compute will be able to check the answers by casting out nines.

III. Mathematical Ideas:

A number is divisible by nine if the sum of the digits of its compact numeral is divisible by 9. Example: 864 is divisible by 9 because 8 + 6 + 4 = 18 which is divisible by 9.

IV. Vocabulary To Stress:

casting out nines subtracting compute sum of digits

Text References:

Book: 6

Houghton Mifflin (1967) p. 183
Houghton Mifflin (1972) p. 175, (44)

Level: 17 Step: Z Concept: Addition, Subtraction, Multiplication, Division

WORKSHEET I

Solve and check work of every other problem by casting out nines.

1. 4065 + 2164	2. 4698 + 3891	3. 4026 − 237	4. 7349 − 1876
5. 7068 + 1275	6. 8061 − 2152	7. 468 × 8	8. 976 × 9
9. 476 × 6	10. 489 × 7	11. 869 × 5	12. 976 × 8
13. 486 × 27	14. 698 × 39	15. 489 × 75	16. 578 × 68

II. Check the numbers that are divisible by nine.

1. 288 ____	5. 8946 ____	9. 368 ____	13. 849 ____
2. 144 ____	6. 342 ____	10. 468 ____	14. 2844 ____
3. 656 ____	7. 198 ____	11. 1233 ____	15. 2106 ____
4. 1062 ____	8. 1097 ____	12. 427 ____	16. 263 ____

ENRICHMENT EVALUATION - LEVEL 17

Step: Z
Concept: Addition, Subtraction, Multiplication

I. Solve and check your work (on this page) by casting out nines.

A. 468 + 579	B. 896 + 538	C. 789 + 387
D. 375 − 164	E. 871 − 186	F. 735 − 468
G. 972 x 6	H. 364 x 9	I. 732 x 8

II. Check the numbers that are divisible by 9.

A. 279 ____________	F. 270 ____________
B. 126 ____________	G. 183 ____________
C. 189 ____________	H. 432 ____________
D. 635 ____________	I. 728 ____________
E. 504 ____________	J. 621 ____________

Level: 17 ANSWERS TO WORKSHEETS

17-3

I. A. a = 9
b = 3
27 = 3 x 3 x 3 or 3^3

B. a = 24
b = 8
c = 4
d = 2
48 = 2 x 2 x 2 x 2 x 3 or
2^4 x 3

C. a = 8
b = 4
c = 2
40 = 2 x 2 x 2 x 5 or 2^3 x 5

D. a = 24
b = 8
c = 4
d = 2
72 = 2 x 2 x 2 x 3 x 3 or 2^3 x 3^2

II. A. 2 x 5 x 5 or 2 x 5^2

B. 2 x 3 x 3 x 3 or 2 x 3^3

C. 2 x 2 x 3 x 7 = 2^2 x 3 x 7

D. 2 x 2 x 5 x 5 = 2^2 x 5^2

E. 2 x 3 x 5 x 5 or 2 x 3 x 5^2

F. 2 x 2 x 2 x 5 x 5 = 2^3 x 5^2

G. 2 x 2 x 3 x 11 = 2^2 x 3 x 11

H. 2 x 2 x 2 x 2 x 2 x 3 = 2^5 x 3

I. 2 x 2 x 3 x 3 = 2^2 x 3^2

17-6

I. A. Yes
B. Yes
C. No - 4
D. No - 2
E. Yes
F. No - 3
G. No - 5
H. Yes
I. No - 5
J. Yes
K. No - 4
L No - 1
M. Yes
N. No - 1

II. A. 5
B. 6

17-9

I. a = 96 sq. in.
b = 16 in.
c = 12 in.
d = 132 sq. in.
e = 15 in.
f = 16 in.

II. a = 32 in.
b = 8 in
c = 18 in.
d = 64 in.
e = 28 in.
f = 20 in.

III. a = 12
b = 5
c = 2
d = 420
e = 50
f = 6

17-12

1. 24 sq. in.
2. 4200 sq. in.
3. 672 sq. ft.
4. 156 sq. yd.
5. 567 sq. in.
6. 341 sq. in.
7. 360 sq. in.
2310 sq. ft.
576 sq. yd.
1131 sq. cm.

Level: 17 ANSWERS TO WORKSHEETS: CONTINUED

17-15

1. 2000
2. 7
3. $\frac{50}{100}$ or $\frac{1}{2}$
4. 500
5. 1000
6. $\frac{1}{1000}$
7. gram
8. gram
9. 4500
10. four
11. 6000
12. 10
13. 400
14. 3
15. 500

17-16

1. 1000,1000
2. $\frac{1}{1000}$
3. 10
4. 2000
5. 6
6. 500
7. 20
8. 3000
9. $\frac{1}{2}$
10. one
11. 8
12. 4
13. grams
14. grams
15. $\frac{1}{100}$

17-17

1. 730
2. 37,500
3. 4.32
4. 430
5. 5400
6. 9000
7. 357
8. 63,000
9. 7,000,000
10. 73
11. 52
12. 301,000
13. 5200
14. 734
15. 7
16. 4,500,000
17. 3300
18. 2.91
19. one
20. one
21. 24
22. 5000
23. 65
24. 570
25. 4000

17-20

I. A. 2 x 2 x 2, 24
B. 35
C. 2 x 2 x 2 = 8
D. 6
E. 6
F. 3
G. 3 x 3 = 9
H. 5
I. 32
J. 5

II. A. 100
B. 28
C. 42
D. 30
E. 66
F. 36

17-22

I. A. 5_{ten}
B. 5_{ten}
C. 9_{ten}
D. 14_{ten}
E. 10_{ten}
F. 20_{ten}
G. 15_{ten}
H. 19_{ten}
I. 6_{ten}

II. A. 11010_{two}
B. 100000_{two}
C. 10101_{two}
D. 10001_{two}
E. 1111_{two}
F. 1011_{two}
G. 111_{two}
H. 1001_{two}
I. 100001_{two}
J. 1101_{two}
K. 11000_{two}
L. 101001_{two}

Level: 17 ANSWERS TO WORKSHEETS: CONTINUED

17-25

I.
1. 111_{two}
2. 110_{two}
3. 10100_{two}
4. 1111_{two}
5. 1110_{two}
6. 101111_{two}
7. 11111_{two}
8. 1010_{two}
9. 11111_{two}
10. 11000_{two}

II.
1. 1011_{two}
2. 101_{two}
3. 1100_{two}
4. 1000_{two}
5. 101_{two}
6. 10110_{two}
7. 1011010_{two}
8. 10010_{two}
9. 1011_{two}
10. 10101_{two}

17-27

I.
A. 110111_{two}
B. 110110_{two}
C. 10100_{two}
D. 111100_{two}
E. 1010_{two}
F. 1001_{two}
G. 111111_{two}
H. 100_{two}
I. 1100_{two}
J. 101100_{two}
K. 110_{two}
L. 100001_{two}
M. 1100_{two}
N. 111000_{two}
O. 11000_{two}

II.
A. $11 \times 5 = 55$
B. $9 \times 6 = 54$
C. $10 \times 2 = 20$
D. $20 \times 3 = 60$
E. $5 \times 2 = 10$
F. $3 \times 3 = 9$
G. $9 \times 7 = 63$
H. $2 \times 2 = 4$
I. $6 \times 2 = 12$
J. $11 \times 4 = 44$
K. $3 \times 2 = 6$
L. $11 \times 3 = 33$
M. $4 \times 3 = 12$
N. $8 \times 7 = 56$
O. $6 \times 4 = 24$

17-30

I.
A. 576
$b = 24$
B. 100
$a = 10$
C. 25
$d = 5$
D. 144
$b = 12$
E. 36
$a = 6$
F. 400
$c = 20$

II.
A. 72 yd.
B. 5 mi.

Level: 17

ANSWERS TO ENRICHMENT (STEP Z)

WORKSHEET

17-33

1. 4065 = 6 + 2164 = 4 6229 = 1	2. 4698 = 0 + 3891 = 3 8589 = 3	3. 4026 = 12 - 237 = 12 3789 = 0	4. 7349 = 23 - 1876 = 22 5473 = 1
5. 7068 = 3 + 1275 = 6 8343 = 0	6. 8061 = 15 - 2152 = 10 5909 = 5	7. 468 = 0 x 8 = 8 3744 = 0	8. 976 = 4 x 9 = 0 8784 = 0
9. 476 = 8 x 6 = 6 2856 = 3	10. 489 = 3 x 7 = 7 3423 = 3	11. 869 = 5 x 5 = 5 4345 = 7	12. 976 = 4 x 8 = 8 7808 = 5
13. 486 = 0 x 27 = 0 13122 = 0	14. 698 = 5 x 39 = 3 27222 = 6	15. 489 = 3 x 75 = 3 36675 = 0	16. 578 = 2 x 68 = 5 39304 = 1

II. Check the numbers that are divisible by nine.

1. 288 ✓	5. 8946 ✓	9. 368	13. 849
2. 144 ✓	6. 342 ✓	10. 468 ✓	14. 2844 ✓
3. 656	7. 198 ✓	11. 1233 ✓	15. 2106 ✓
4. 1062 ✓	8. 1097	12. 427	16. 263

EVALUATION

17-34

I. A. 1047 (3)
B. 1434 (3)
C. 1176 (6)
D. 211 (4)
E. 685 (1)
F. 267 (6)
G. 5832 (0)
H. 3276 (0)
I. 5856 (6)

II. A. ✓
B. ✓
C. ✓
D.
E. ✓
F. ✓
G.
H. ✓
I.
J. ✓

Level: 17 Answer Sheet - Post Test I

Step A

Numeral

1. $2 \times 3 \times 5^2$
2. $2 \times 3^2 \times 5$
3. $2^4 \times 3^2$
4. $3^2 \times 7$
5. $2^2 \times 3 \times 7$

Multiplication

6. 3
7. 8
8. 6
9. 5
10. 4

Division

11. 12
12. 11
13. 15
14. 13
15. 21

Functions & Graphs

16. 120 feet
17. 800 sq. ft.
18. 50 miles per hr.
19. 60 cu. ft.
20. 224 sq. ft.

Geometry

21. 160 sq. ft.
22. 96 sq. yd.
23. 154 sq. mi.
24. 72 sq. in.
25. 216 sq. ft.

Measurement

26. 4000
27. 500
28. 5
29. one
30. 2.2 pounds

Step B

Sets

31. $3 \times 5 \times 7$
32. 2×3
33. 3×5
34. 36
35. 7

Numeral

36. 15_{ten}
37. 5_{ten}
38. 23_{ten}
39. 101001_{two}
40. 10010_{two}

Addition

41. 100111_{two}
42. 1000_{two}
43. 1100_{two}
44. 100101_{two}
45. 10000_{two}

Multiplication

46. 100100_{two}
47. 1111_{two}
48. 10100_{two}
49. 10101_{two}
50. 1011111_{two}

Geometry

51. 10
52. 25
53. 5
54. 13
55. 15

Level: 17 Answer Sheet - Post Test II

Step A

Numeral

1. $2^2 \times 13$
2. $2^2 \times 5^2$
3. 2^5
4. $2^3 \times 3^2$
5. $2 \times 3 \times 11$

Multiplication

6. 3
7. 2
8. 3
9. 6
10. 8

Division

11. 13
12. 14
13. 32
14. 25
15. 110

Function & Graphs

16. 18 in.
17. 512 cu. in.
18. 24 ft.
19. 8 hours
20. 336 sq. ft.

Geometry

21. $A = \frac{1}{2} a \times b$
22. 77 sq. in.
23. 140 sq. ft.
24. 120 sq. in.
25. 36 sq. in.

Measurement

26. 6000
27. 10
28. 2
29. 5
30. 2

Step B

Sets

31. 72
32. 6
33. 12
34. 180
35. 5×7

Numeral

36. 12_{ten}
37. 2_{ten}
38. 16_{ten}
39. 110010_{two}
40. 111_{two}

Addition

41. 11001_{two}
42. 10_{two}
43. 1110_{two}
44. 11111_{two}
45. 10110_{two}

Multiplication

46. 0
47. 101000_{two}
48. 1110011_{two}
49. 11011_{two}
50. 1110100_{two}

Geometry

51. 8
52. 13
53. 25
54. 10
55. 25

TABLE OF CONTENTS

Level 18

Step A

Step B

Step C

Step Z

Level: 18 Step: A Concept: Numeral

I. Concept:

Numeral: Introducing the thousandths' place in decimal fractions.

II. Behavioral Objective:

The student given a fractional number having a denominator 1000 will be able to rewrite the number as a decimal fraction.

III. Mathematical Ideas:

A. The third place to the right of the decimal point has the place value of thousandths.

Example: $.001 = \frac{1}{1000} = \frac{1}{10^3}$

B. Equivalent decimal fractions name the same fractional number.

Example: $.3 = .30 = .300$

IV. Vocabulary To Stress:

decimals	hundredths	fractional number	denominator
tenths	thousandths	numerator	

V. Activities:

A. Have students bring baseball batting averages to school. Discuss these with the class.

B. Have students assist you in writing expanded numerals such as this example:

$$5436.489 = 5000 + 400 + 30 + 6 + \frac{4}{10} + \frac{8}{100} + \frac{9}{1000}$$
$$= 5000 + 400 + 30 + 6 + .4 + .08 + .009$$

C. Help students discover the rules for placement of decimal points when renaming numbers with denominators of 1000.

Example: $\frac{4}{1000} = .004$, $\frac{42}{1000} = .042$, $\frac{1436}{1000} = 1\frac{436}{1000} = 1.436$

Text References:

Book: 6

Houghton Mifflin (1967) pp. 300-301, (68)
Houghton Mifflin (1972) pp. 144-145, (38)
Addison-Wesley (1971, 1968) pp. 216-221

Book: 7

Houghton Mifflin (1967, 1970) pp. 372-376, (56)
Houghton Mifflin (1972) pp. 14-15
Addison-Wesley (1971) pp. 295-299, (57)

Other References:
Imperial Tape (Intermediate) #29
Modern Mathematics Filmstrip #759

Level: 18 Step: A Concept: Numeral

WORKSHEET

I. Rename the numbers in decimal form.

$\frac{13}{100}$ = ____	$\frac{19}{100}$ = ____	$\frac{82}{100}$ = ____	$\frac{32}{1000}$ = ____	$\frac{80}{1000}$ = ____
$\frac{20}{100}$ = ____	$\frac{27}{100}$ = ____	$\frac{97}{100}$ = ____	$\frac{75}{1000}$ = ____	$\frac{43}{1000}$ = ____
$\frac{45}{100}$ = ____	$\frac{32}{100}$ = ____	$\frac{28}{100}$ = ____	$\frac{16}{1000}$ = ____	$\frac{17}{1000}$ = ____
$\frac{13}{100}$ = ____	$\frac{16}{100}$ = ____	$\frac{65}{100}$ = ____	$\frac{76}{1000}$ = ____	$\frac{60}{1000}$ = ____

II. Rename the numbers as fractions with denominators of 100 or 1000.

.16 = ____	.42 = ____	.018 = ____	.63 = ____	.006 = ____
.18 = ____	.38 = ____	.015 = ____	.82 = ____	.001 = ____
.97 = ____	.73 = ____	.236 = ____	.136 = ____	.321 = ____
.13 = ____	.15 = ____	.140 = ____	.150 = ____	.46 = ____

III. Rename the decimal fractions as expanded numerals as shown:

$$.36 = \frac{3}{10} + \frac{6}{100}$$

A. .64 = ________	F. .25 = ________	K. .97 = ________
B. .91 = ________	G. .39 = ________	L. .406 = ________
C. .126 = ________	H. .132 = ________	M. .72 = ________
D. .368 = ________	I. .206 = ________	N. .65 = ________
E. .56 = ________	J. .48 = ________	O. .83 = ________

Level: 18 Step: A Concept: Addition

I. Concept:

Addition: Adding decimals to thousandths.

II. Behavioral Objective:

The student given a series of numbers (whole and decimals up to thousandths) will be able to write them vertically and add.

III. Mathematical Ideas:

A. Decimals are fractions with denominators 10, 100, 1000, and so on.

B. Decimals use the same place-value system as whole numbers.

C. Adding with decimal fractions follows the same procedure used in the corresponding operations with whole numbers.

D. Annexing zeros to a decimal will not change its value.

E. Addition of decimals should be developed by the use of fractions.

IV. Vocabulary To Stress:

tenths hundredths thousandths

V. Activities:

* A. Show the addition operation by replacing the given numerals with fractions.

B. Dictate a series of decimal numerals to add.

C. Students may complete number patterns for addition as shown below:

a.

					Total
+	3	.3	.03	.003	3.333
1.	7				
2.	8				
3.	6				
4.	9				

b.

					Total
+	7	.8	.06	.003	7.863
1.					
2.					
3.					
4.					

Use these numerals to complete pattern b, with student completing last column.

1. 8, .9, .05, .007, ____________
2. 4, .3, .06, .009, ____________
3. 5, .6, .08, .003, ____________
4. 7, .5, .01, .002, ____________

Level: 18 Step: A Concept: Addition

Text References:

Book: 5

Houghton Mifflin (1972) pp. 323, 325

Book: 6

Addison-Wesley (1971, 1968) pp. 222-223 Use just addition (39 rows 1 to 4), (40), 334.

(Also use worksheets in this Teacher's Guide.)

Book: 7

Houghton Mifflin (1967, 1970) pp. 379-383, (58)
Houghton Mifflin (1972) pp. 151-152, 196-197, 348, 420, 422
Addison-Wesley (1971) pp. 300-303, (58)

Book: 8

Addison-Wesley (1971) pp. 254, (53)

Other References:

Imperial Tape (Intermediate) #25

Modern Mathematics Filmstrip #765

* This activity may be used to help introduce the concept.

Level: 18 Step: A Concept: Addition

WORKSHEET

Arrange numbers vertically and add.

1. 4.6 + .38 + 3 + 2.06	2. 1.4 + 7 + .089 + 1.460	3. 16.1 + 8 + .283 + .01
4. 5.2 + 93 + .16 + .127	5. .97 + 6 + 1.3 + .276	6. .89 + 268 + .4 + .10
7. .086 + .124 + 6 + .12	8. .27 + .126 + 32.6 + 82	9. .765 + .2 + 17 + .86
10. 9.76 + 1.238 + 6 + .48	11. 9.768 + .83 + 4 + .6	12. .7 + .86 + .327 + 9
13. 26.09 + .273 + 14 + .3	14. 74.08 + .13 + .126 + 93	15. .4 + .6 + .3 + .11
16. .126 + .039 + .756 + .2	17. 8.26 + .127 + .13 + .2	18. .39 + .056 + 24.3 + 2

Level: 18 Step: A Concept: Addition

WORKSHEET

Arrange the following numbers vertically and add.

A. 46 + .2 + 4.6 + .061 =

B. 8 + 23.8 + .41 + .468 =

C. .271 + 6.8 + .3 + 4 =

D. 5.6 + 3 + .27 + .138 =

E. 146.8 + .21 + .16 + 42 =

F. .56 + 2.8 + .37 + .126 =

G. 82.1 + .368 + .2 + 5 =

H. 3.08 + 5 + .163 + .2 =

I. .26 + .2 + 18 + .127 =

J. 2 + 45 + .6 + .268 =

K. .16 + 32 + 1.8 + .276 =

L. 5.6 + .17 + .002 =

M. 6 + .8 + .126 + 1.3 =

N. .14 + 7.8 + .306 + 5 =

A.	B.
C.	D.
E.	F.
G.	H.
I.	J.
K.	L.
M.	N.

Level: 18 Step: A - Concept: Subtraction

I. Concept:

Subtraction: Subtracting decimals to thousandths.

II. Behavioral Objective:

The student given numbers written horizontally (whole and decimals up to thousandths) will be able to write them vertically and subtract.

III. Mathematical Ideas:

A. Decimals are fractions with denominators 10, 100, 1000, and so on.

B. Subtracting with decimal fractions follows the same procedure used in the corresponding operations with whole numbers.

C. Annexing zeros to a decimal will not change its value.

D. Subtraction of decimals should be developed by the use of fractions.

IV. Vocabulary To Stress:

tenths hundredths thousandths

V. Activities:

* A. Show the subtraction operation by replacing the given numerals with fractions.

B. Dictate a series of decimal numerals to subtract.

C. Collect from journals or textbooks examples of the uses of decimals.

Text References:

Book: 6

Addison-Wesley (1971, 1966) pp. 222-223 Use just subtraction (39 rows 5-8), 334

(Also use Worksheets in this Teacher's Guide)

Book: 7

Houghton Mifflin (1967, 1970) pp. 379-383, (58)
Houghton Mifflin (1972) pp. 151-152, 196-197, 348, 420, 422
Addison-Wesley (1971) pp. 300-303, (56)

Book: 8

Addison-Wesley (1971) p. 254, (54)

18-8

Level: 13 Step: A Concept: Subtraction

Other References:

Imperial Tape (Intermediate) #25

* This activity may be used to help introduce the concept.

Level: 18 Step: A Concept: Subtraction

WORKSHEET I

Write the following numbers vertically and subtract.

A. 15.8 - 2.75

B. 23.146 - 8.15

C. 8 - .568

D. .488 - .27

E. 2.56 - .238

F. 46.8 - 37.856

G. 24 - .386

H. 4.6 - 2.387

I. .39 - .126

J. 268.8 - 10.27

K. 25 - 8.39

L. .756 - .23

M. 9 - 1.3

N. 45.8 - .468

A.	B.
C.	D.
E.	F.
G.	H.
I.	J.
K.	L.
M.	N.

Level: 18 Step: A Concept: Subtraction

WORKSHEET 11

Write the numbers vertically below each one and subtract.

A. 48.2 - 16.008	B. 8.65 - .127	C. 2 - .316
D. 3 - .46	E. .568 - .47	F. 89.1 - .468
G. 36 - .482	H. .8 - .48	I. 37.1 - .82
J. 436.1 - .76	K. .6 - .128	L. 14 - 2.8
M. 7 - .469	N. 3.8 - 2.16	O. 56 - .3
P. .468 - .28	Q. 5.1 - .469	R. .83 - .658
S. 4.9 - .8	T. 36.3 - .48	U. 2 - .364

Level: 13 Step: A Concept: Multiplication & Division

I. Concept:

Multiplication and Division: Multiplying and dividing by powers of ten.

II. Behavioral Objective:

The student given a decimal or a whole number will be able to multiply or divide it by a power of ten simply by moving the decimal point.

III. Mathematical Ideas:

A. Multiplying by powers of ten moves the decimal point as many places to the right as there are zeros. 10 x 24 = 240, 100 x 24 = 2400, 1000 x .24 = 240

B. Dividing by powers of ten moves the decimal point as many places to the left as there are zeros in the divisor. 24 ÷ 10 = 2.4, 24 ÷ 100 = .24, 2.4 ÷ 1000 = .0024

C. A whole number is always followed by a decimal point although it is not shown.

IV. Vocabulary To Stress:

powers

V. Activities:

* A. Give students problems such as 10 x 36, 100 x 36, etc. to multiply orally. Add others such as 10 x 4.2, 100 x 4.2, 1000x 4.2, etc.

B. Make comparisons between the two operations. For example, 27 x 10 and 27 ÷ 10.

Text References:

Book: 5

Addison-Wesley (1971, 1968) pp. 50, (10), 51, 56 (part of page), 328 (sets 10 and 11)

Book: 6

Houghton Mifflin (1972) p. 211
Addison-Wesley (1971, 1968) pp. 230-231, (42), 334 (Set 39)
(Use Worksheets in this Guide also)

Book: 7

Houghton Mifflin (1972) pp. 129
Addison-Wesley (1971) pp. 310-313, (60)

* This activity may be helpful in introducing the concept.

Level: 18 Step: A Concept: Multiplication & Division

WORKSHEET I

Multiplication

	a.	b.	c.
1.	10 x .236 = ______	100 x .24 = ______	10 x 38.2 = ______
2.	48.6 x 100 = ______	6.5 x 10 = ______	55 x 10 = ______
3.	3.7 x 10 = ______	1000 x 2.3 = ______	6.6 x 100 = ______
4.	64 x 1000 = ______	.82 x 100 = ______	62.3 x 10 = ______
5.	9.4 x 10 = ______	.37 x 1000 = ______	1.5 x 10 = ______
6.	82 x 10 = ______	8.2 x 100 = ______	8.2 x 1000 = ______
7.	100 x .875 = ______	10 x 45.38 = ______	100 x .06 = ______

Division

	a.	b.	c.
1.	2.4 ÷ 10 = ______	.36 ÷ 100 = ______	126 ÷ 1000 = ______
2.	65 ÷ 10 = ______	8.3 ÷ 100 = ______	56 ÷ 1000 = ______
3.	39.6 ÷ 10 = ______	33.4 ÷ 100 = ______	2.7 ÷ 10 = ______
4.	.62 ÷ 100 = ______	.324 ÷ 10 = ______	262 ÷ 1000 = ______
5.	.875 ÷ 100 = ______	15 ÷ 10 = ______	15.5 ÷ 10 = ______

Level: 18 Step: A Concept: Multiplication & Division

WORKSHEET II

Multiply or divide as indicated.

1. $1000 \times \frac{1}{10} =$ ________________
2. $1000 \times .1 =$ ________________
3. $1000 \times .001 =$ ________________
4. $12.16 \times 10 =$ ________________
5. $100 \times 4.63 =$ ________________
6. $1000 \times .09 =$ ________________
7. $6.3 \div 10 =$ ________________
8. $12.4 \div 100 =$ ________________
9. $1000 \times .007 =$ ________________
10. $23.4 \div 10 =$ ________________
11. $.26 \div 100 =$ ________________
12. $86 \div 1000 =$ ________________
13. $2.8 \div 10 =$ ________________
14. $10 \times 75 =$ ________________
15. $100 \times 7.5 =$ ________________
16. $16 \div 10 =$ ________________
17. $100 \times .07 =$ ________________
18. $1000 \times .641 =$ ________________
19. $2.4 \div 10 =$ ________________
20. $36.5 \div 100 =$ ________________
21. $8.5 \times 10 =$ ________________
22. $.75 \times 1000 =$ ________________
23. $36 \div 1000 =$ ________________
24. $9.8 \times 100 =$ ________________
25. $75 \times 100 =$ ________________
26. $3.2 \times 10 =$ ________________
27. $3.2 \div 10 =$ ________________
28. $3.2 \div 100 =$ ________________
29. $10 \times 8.4 =$ ________________
30. $100 \times .27 =$ ________________

Level: 10 Step: 3 Concept: Numeral

I. Concept:

Numeral: Identifying integers on the number line.

II. Behavioral Objective:

The student given a number line will be able to identify points with positive and negative integers.

III. Mathematical Ideas:

A. The set of integers consists of the whole numbers together with their negatives.

B. Integers are sometimes called signed whole numbers.

C. Integers are numbers which tell us how much and in what direction from the starting point.

D. The starting point for naming integers is 0.

E. The plus (+) sign is used to denote positive integers.

F. The negative (-) sign is used to denote negative integers.

IV. Vocabulary To Stress:

integers negative positive

V. Activities:

A. Play the game integers. Make 2 sets of 12 cards each, one for positive integers and one for negative integers. Two players share the cards evenly except for 10 which are left on the table face down. Each player forms as many pairs of opposites as he can from the cards he has. The first player chooses a card from the stack on the table. If he can match it, he takes it. In this way all the cards will be paired. (H.M.T. Ed. (1972) p. T413.)

* B. Give examples of numbers less than zero such as temperatures below zero and below high water mark. Establish the fact that the + and - do not mean the same as the plus and minus signs. They are adjectives describing types of integers, + being positive, and - being negative.

C. If necessary, make additional charts similar to the ones in H.M. (1967) p. 334 and H.M. (1972) p. 326 for further practice.

Level: 13 Step: B Concept: Numeral

Text References:

Book: 6

Houghton Mifflin (1967) pp. 332-334
Houghton Mifflin (1972) p. 326
Addison-Wesley (1971, 1968) pp. 290-292

Book: 7

Houghton Mifflin (1967, 1970) pp. 500-503
Houghton Mifflin (1972) pp. 384-385
Addison-Wesley (1971) pp. 141-144

* This activity may be used to help introduce the concept.

Level: 18 Step: C Concept: Addition

I. Concept:

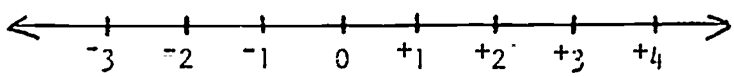
Addition: Adding integers on a number line.

II. Behavioral Objective:

The student given any two integers as addends will be able to name the sum.

III. Mathematical Ideas:

A. The set of integers are the whole numbers together with their negatives.



B. Addition of positive integers is shown by moves to the right on the number line and of negative integers by moves to the left.

C. Zero is neither negative nor positive.

IV. Vocabulary To Stress:

integer positive negative

V. Activities:

* A. Use the number line to show how addition of integers may be thought of in terms of moves on the number line. (H.M. T. Ed. (1967) p. T394)

B. Complete the grid by adding across and down.

$^{+}5$	$^{-}3$	a
$^{-}7$	$^{+}6$	b
c	d	e

a = $^{+}2$
b = $^{-}1$
c = $^{-}2$
d = $^{+}3$
e = $^{+}1$

C. Play Tic-Tac Integers. Make a 4 x 4 grid. One pupil starts the game by writing an integer in any cell. The second player writes an integer in a different cell. The object of the game is to put the last integer in a row, column, or diagonal so that the sum of all the integers in that row, column, or diagonal is zero as shown. Continue until all cells are filled.

ADD →
↓

$^{-}6$	$^{+}3$	$^{-}3$	$^{+}6$	= 0
	$^{+}4$			
	$^{-}10$	$^{+}7$		
	$^{+}3$		$^{-}5$	= 0
	= 0			

Level: 16 Step: B Concept: Addition

Text References:

Book: 6

Houghton Mifflin (1967) pp. 335-336, (77), 337, (78)
Houghton Mifflin (1972) pp. 327, (79), 328-329
Addison-Wesley (1971, 1968) pp. 293-295

Book: 7

Houghton Mifflin (1967, 1970) pp. 504-508
Houghton Mifflin (1972) pp. 386-389, 432
Addison-Wesley (1971) pp. 144-150, 292, [illegible], 299

Book: 8

Houghton Mifflin (1972) p. 226
Addison-Wesley (1971) pp. 54-55

Other References:

The Franklin Series, "Making and Using Graphs and Nomographs", pp. 45-46.

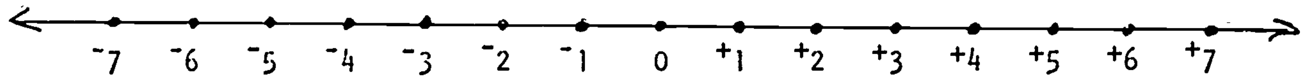
Sherzer, Laurence, "Adding integers using only the concepts of one-to-one correspondence and counting". The Arithmetic Teacher, May 1969.

* This activity may be used to help introduce the concept.

Level: 18 Step: B Concept: Addition

WORKSHEET

Use the number line to help you name the sums.



1. $^{+}2 + {}^{+}6 =$ ____________
2. $^{+}4 + {}^{-}6 =$ ____________
3. $^{+}7 + {}^{-}3 =$ ____________
4. $^{-}3 + {}^{-}2 =$ ____________
5. $0 + {}^{+}5 =$ ____________
6. $^{-}6 + {}^{+}3 =$ ____________
7. $^{-}4 + {}^{+}5 =$ ____________
8. $^{+}7 + {}^{-}6 =$ ____________
9. $^{-}3 + {}^{-}5 =$ ____________
10. $^{+}6 + {}^{+}2 =$ ____________
11. $^{+}7 + {}^{-}5 =$ ____________
12. $^{-}6 + {}^{+}7 =$ ____________
13. $^{-}2 + {}^{-}1 =$ ____________
14. $0 + {}^{-}1 =$ ____________
15. $^{+}5 + {}^{-}3 =$ ____________
16. $^{+}6 + {}^{+}3 =$ ____________
17. $^{-}6 + {}^{-}2 =$ ____________
18. $^{+}2 + {}^{+}2 =$ ____________
19. $^{-}6 + {}^{-}1 =$ ____________
20. $^{+}6 + {}^{-}1 =$ ____________
21. $^{-}4 + {}^{+}3 =$ ____________
22. $^{+}5 + {}^{-}2 =$ ____________
23. $^{-}4 + {}^{-}3 =$ ____________
24. $^{+}5 + {}^{+}1 =$ ____________
25. $^{+}5 + {}^{-}4 =$ ____________
26. $^{-}6 + {}^{+}1 =$ ____________

Level: 18 Step: B Concept: Subtraction

I. Concept:

<u>Subtraction</u>: Subtracting integers on a number line.

II. Behavioral Objective:

The student given the sum and one addend will be able to name the missing addend.

III. Mathematical Ideas:

A. The set of integers are the whole numbers together with their opposites.

B. Subtracting an integer is the same as adding its opposite.

Example: $^+2 - {}^+5 = {}^+2 + {}^-5 = {}^-3$

C. Two numbers whose sum is zero are opposites.

Example: $^+2 + {}^-2 = 0$

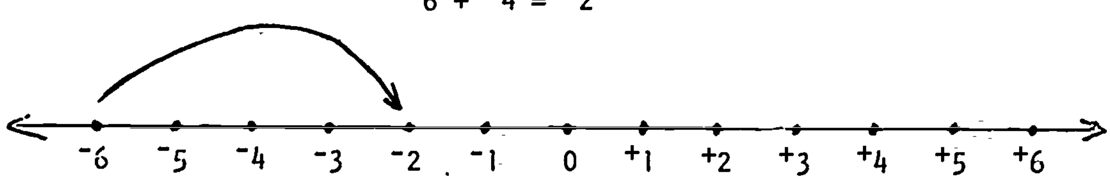
IV. Vocabulary To Stress:

integers opposites

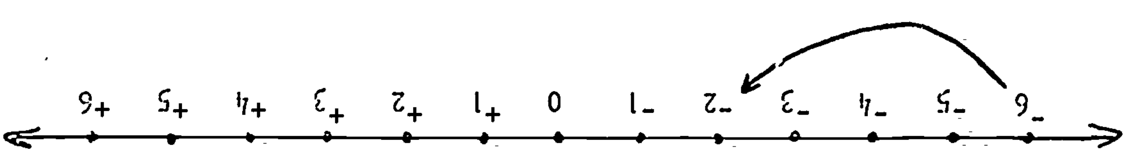
V. Activities:

A. Attach cardboard number lines to the board as shown to show that addition and subtraction are opposites.

$^-6 + {}^+4 = {}^-2$



$^-6 - {}^-4 = {}^-2$



B. Use a grid similar to the one used for adding integers only subtract as shown.

$$\begin{array}{r} ^-6 \\ -\ ^+4 \\ \hline \end{array}$$

$^-6$	$^+4$	a
$^-2$	$^+6$	b
c	d	e

a = $^-10$
b = $^-8$
c = $^-4$
d = $^-2$
e = $^-2$

18-20

Level: 18 Step: B Concept: Subtraction

Text References:

Book: 6

Houghton Mifflin (1967) pp. 338, (79), 339
Houghton Mifflin (1972) pp. 328-329
Addison-Wesley (1971, 1968) pp. 296-297

Book: 7

Houghton Mifflin (1967, 1970) pp. 509-512
Houghton Mifflin (1972) pp. 390-391, 432
Addison-Wesley (1971) pp. 151-153 (86)

Book: 8

Addison-Wesley (1971) p. 58

Other References:

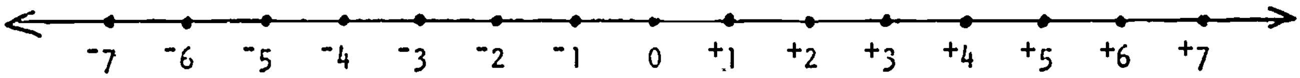
Milne, Esther, "Subtraction of Integers--Discovered Through A Game", The Arithmetic Teacher, February 1969, pp. 148-149.

The Franklin Series: "Making and Using Graphs and Nomographs". pp. 47-48

Level: 18 Step: B Concept: Subtraction

WORKSHEET

Use the number line to help you name the remainders.



1. $^{+}6 - ^{-}5 =$ ____________
2. $^{+}7 - ^{+}6 =$ ____________
3. $^{-}1 - ^{+}5 =$ ____________
4. $^{-}2 - ^{-}7 =$ ____________
5. $^{-}3 - ^{+}3 =$ ____________
6. $^{-}2 - ^{+}4 =$ ____________
7. $^{-}1 - ^{+}3 =$ ____________
8. $^{-}7 - ^{-}7 =$ ____________
9. $^{-}7 - ^{+}7 =$ ____________
10. $^{-}2 - ^{-}4 =$ ____________
11. $^{+}6 - ^{-}2 =$ ____________
12. $^{+}2 - ^{-}6 =$ ____________
13. $^{-}3 - ^{+}5 =$ ____________
14. $^{-}2 - ^{+}6 =$ ____________
15. $^{-}1 - ^{+}1 =$ ____________
16. $^{-}4 - ^{+}4 =$ ____________
17. $^{-}6 - ^{-}7 =$ ____________
18. $^{-}5 - ^{-}1 =$ ____________
19. $^{-}1 - ^{+}5 =$ ____________
20. $^{+}6 - ^{-}3 =$ ____________
21. $^{+}3 - ^{-}4 =$ ____________
22. $^{+}6 - ^{-}3 =$ ____________
23. $^{+}3 - ^{+}2 =$ ____________
24. $^{-}7 - ^{-}6 =$ ____________
25. $^{-}4 - ^{+}3 =$ ____________
26. $^{-}5 - ^{+}4 =$ ____________

Level: 18 Step: 3 Concept: Multiplication

I. Concept:

Multiplication: Multiplying decimal fractions with no more than 3 decimal places in the product.

II. Behavioral Objective:

The student given two factors whose product contains no more than three digits to the right of the decimal will be able to write the product.

III. Mathematical Ideas:

A. The number of decimal places in the product is found by adding the decimal places in each factor.

B. A decimal fraction is a base-place numeral for a fractional number.

C. The product of two factors less than one will be less than either factor.

IV. Vocabulary To Stress:

decimal fraction thousandths.

V. Activities:

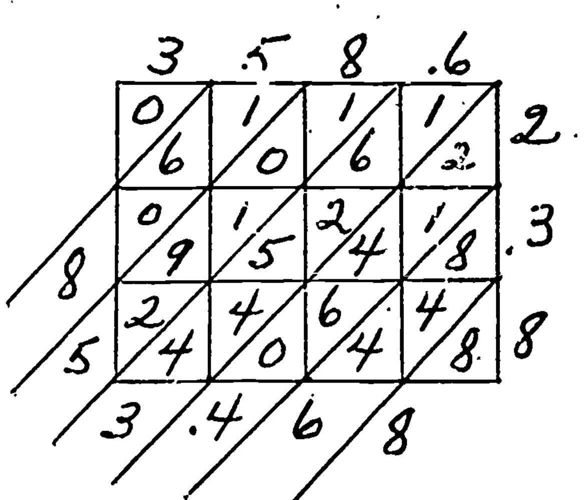
* A. Use examples such as the following:

$$4 \times .3 = 4 \times \frac{3}{10} = \frac{4 \times 3}{10} = 1.2$$

$$4 \times .03 = 4 \times \frac{3}{100} = \frac{4 \times 3}{100} = .12$$

$$4 \times .003 = 4 \times \frac{3}{1000} = \frac{4 \times 3}{1000} = .012$$

B. Lattice multiplication works well with decimal fractions. The decimal point is placed in the product just as it is in standard multiplication. (Imperial Tape Manual (Intermediate) #26).
For example: 358.6 x 2.38



Answer: 853.468

Level: 18 Step: B Concept: Multiplication

V. Activities: Continued

C. The two middle numerals in each column of the chart below are addends which give the sums at the top of each column. They are also factors which give the products found at the bottom of each column. Use subtraction, addition,or multiplication to find each missing numeral. (Imperial Tape Manual (Intermediate) #26)

SUMS	a	2.0	24.3	b	1.20
	.08	.8	24	.6	.40
	.6	c	d	.8	e
PRODUCTS	f	g	h	i	j

a = .68 c = 1.2 e = .8 g = .96 i = .48

b = 1.4 d = .3 f = .048 h = 7.2 j = .320

Text References:

Book: 5

Houghton Mifflin (1972) pp. 325, (78)

Book: 6

Houghton Mifflin (1967) pp. 306-307, (70), 308-310, (71)
Houghton Mifflin (1972) pp. 209, 210, (52), 263, 264, 265, 267
Addison-Wesley (1971, 1968) pp. 232-233, (43,44), 234, 235, 236, 248 (part of page)

Book: 7

Houghton Mifflin (1967, 1970) pp. 393-397, (62)
Houghton Mifflin (1972) pp. 208, 256, 350, 396, 424
Addison-Wesley (1971) pp. 304-309, (59), 335 (set 40)

Other References:

Imperial Tape (Intermediate) #26

Modern Mathematics Filmstrip #766

* This activity may be used to help introduce the concept.

Level: 18 Step: B Concept: Division

I. Concept:

Division: Dividing decimal fractions through thousandths.

II. Behavioral Objective:

The student given a decimal fraction as the dividend and a whole number or a decimal fraction as the divisor will be able to write the quotient through thousandths.

III. Mathematical Ideas:

A. Work with decimals proceeds in the same way as the work with whole numbers.

B. The value of the remainder changes as its place value changes.

C. If the divisor contains a fractional number, it can be changed to a whole number by multiplying both the divisor and the dividend by a power of ten.

IV. Vocabulary To Stress:

decimal fractions dividend divisor quotient power of ten

V. Activities:

A. Complete tables like the following for further practice. Make them as large as you desire.

* 1.

	÷	2	4	6	3	8
a.	2.4					
b.	.24					

2.

	÷	2	.2	4	.4	6	.6
a.	.48						
b.	.048						

Level: 18 Step: B Concept: Division

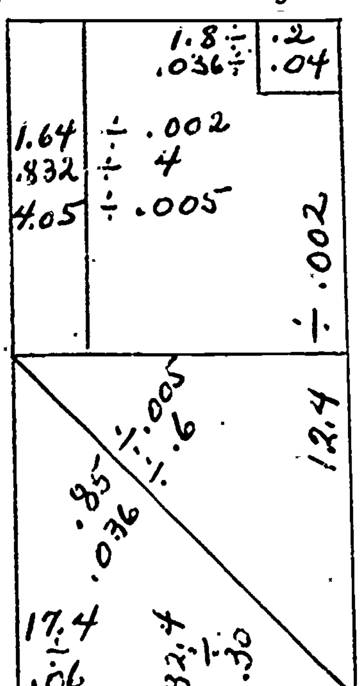
V. Activities: Continued

B. Distribute a copy of the puzzle and have students fill in the signs for the correct operations. (+, -, x, ÷) (Imperial Tape Manual (Intermediate) #27).

	2		4		6		
1.	(.60	+	.8)	÷	.2	=	7.0
3.	(.3		.30)		.06	=	.030
5.	(.006		.6)		.202	=	.404
	=		=		=		=
	.894		.40		.214		.614

ACROSS	DOWN
1. +, ÷ (given)	2. +, -
3. x, -	4. x, ÷
5. +, -	6. x, +

C. Follow the pattern below in making a puzzle for each student. Cut apart, have them fit together and solve the problems.



For The Teacher

1.8 ÷ .2 = 9
.036 ÷ .04 = .9
1.64 ÷ .002 = 820
.832 ÷ 4 = .208
4.05 ÷ .005 = 810
.85 ÷ .005 = 170
.036 ÷ .6 = .06
17.4 ÷ .06 = 290
12.4 ÷ .002 = 6200
32.4 ÷ .03 = 1080

Level: 18 Step: B Concept: Division

Text References:

Book: 5

Houghton Mifflin (1972) pp. 327, (79)

Book: 6

Houghton Mifflin (1967) pp. 311-313, (72), 314, (73), 315, 316.
Houghton Mifflin (1972) pp. 269-270, (64), 276-277, (66), 318, 319, 321, 352
Addison-Wesley (1971, 1968) pp. 237, (45, 46), 238-239, (47, 48), 240-244, (49), 248 (part of page)

Book: 7

Houghton Mifflin (1967, 1970) pp. 397-402, (63)
Houghton Mifflin (1972) pp. 264-267, 396, 428
Addison-Wesley (1971) pp. 319-322, (62), (63), 335

Book: 8

Addison-Wesley (1971) p. 260 (56, 57)

Other References:

Imperial Tape (Intermediate) #27

* Tables such as these may be used after the initial introduction has been given before making book assignments.

Level: 18 Step: B Concept: Geometry

I. Concept:

Geometry: Finding the circumference of a circle.

II. Behavioral Objective:

The student given the radius of a circle will be able to find the circumference of that circle.

III. Mathematical Ideas:

A. The radius of a circle is the distance from the center to any point on the circle.

B. The diameter of a circle is a chord that passes through the center of the circle and is twice the radius.

C. The ratio of the circumference to the diameter is a constant called π (pi), which is approximately equal to $3\frac{1}{7}$ or $\frac{22}{7}$.

D. The circumference of a circle is the distance around the circle. $C = \pi \times d$.

IV. Vocabulary To Stress:

radius (r) circumference (C) diameter (d) constant chord

V. Activities:

A. Have the students estimate and measure the circumference of a wall clock, the basketball ring, etc. Then have them measure the true radius and find the circumference.

B. Measure around the edge of a cardboard disk with a piece of string and compare its length to the length equal to the diameter. The student will discover that a little more than three times the length of the diameter will fit around the circle. H.M.T. Ed. (1972) Book 5, p. T159.

Text References:

Book: 6

Houghton Mifflin (1967) pp. 272
Houghton Mifflin (1972) pp. 102, 290, (69)
Addison-Wesley (1971, 1968) pp. 252-253

Book: 7

Houghton Mifflin (1967, 1970) pp. 444-448, (70-part of page)
Houghton Mifflin (1972) pp. 288
Addison-Wesley (1971) pp. 419-421, (67)

Book: 8

Addison-Wesley (1971) pp. 406, 410, (65)

18-28

Level: 18 Step: B Concept: Geometry

Other References:

Curriculum Filmstrips #368, 369

Forms We See Filmstrip & Tape #EF 1106-1(part of it)

Modern Mathematics Filmstrip #770

Level: 18 Step: B Concept: Geometry

WORKSHEET

I. Find the circumference to answer the questions below.

A. Mrs. Brown is making a round table cloth that has a diameter of 58 inches. How many inches of lace will she need to put around the edge of it?

____

B. What is the circumference of a merry-go-round that has a radius of 3 feet?

____

C. Mrs. Jones baked a pie that had a diameter of 8 inches. What was the circumference?

____

D. How many feet will a bicycle tire move in one revolution if it has a diameter of 28 inches?

____

II. Name the circumference of a circle with a diameter of:

A. 10" = ______________ F. 14 feet = ______________

B. 24" = ______________ G. 21" = ______________

C. 800 miles = ______________ H. 35 feet = ______________

D. 5 yards = ______________ I. 11" = ______________

E. 6 cm. = ______________ J. 7 miles = ______________

Level: 18 Step: C Concept: Numeral

I. Concept:

Numeral: Renaming a fraction as a percent and a percent as a fraction.

II. Behavioral Objective:

The student given a common fraction or a percent will be able to write the equivalent fraction in the form not given.

III. Mathematical Ideas:

A. A percent is a fraction in a special form.

B. A percent is a numeral for a fractional number.

C. One hundred percent is a numeral for 1.

$$\frac{100}{100} = 1,\ 100\% = 100 \times \frac{1}{100} = 1$$

D. A percent is a fraction with a denominator of 100.

Example:

$$25\% = \frac{25}{100}$$

IV. Vocabulary To Stress:

percent fractional number equivalent fractions

V. Activities:

A. To enforce the idea that many things change forms to be useful (e.g. water, ice, steam), ask the group to think of other things and use this discussion to show that percent is another way to express a fractional number.

B. Have students write sets of names for the more common fractions. (H.M. T. Ed. 1972, Book 6, p. T209)

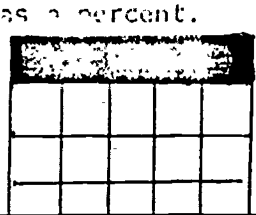
$$\left\{\frac{1}{2},\ \frac{2}{4},\ \frac{5}{10},\ \frac{50}{100},\ 50\%\right\}$$

$$\left\{\frac{1}{4},\ \frac{2}{8},\ \frac{5}{20},\ \frac{25}{100},\ 25\%\right\}$$

Reverse the process as:

$$50\%,\ \frac{50}{100},\ \frac{5}{10},\ \frac{1}{2},\ \text{etc.}$$

C. Have student shade fractional parts of the squares in a given figure and express it as a percent.



$\frac{1}{4} = 25\%$

Level: 13 Step: C Concept: Numeral

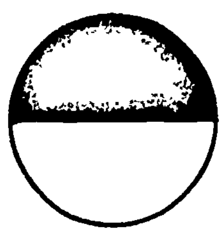
V. Activities: Continued

D. Display the information below in a table or graph. Convert the data to percent notation. (Addison-Wesley, Book 6, p. 282.)

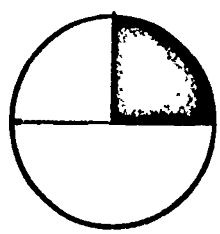
According to NASA, the weight of an artificial satellite is distributed approximately as follows:

Scientific payload		$\frac{1}{5}$
Weight of shell		$\frac{1}{4}$
Communication equipment		$\frac{3}{10}$
Power Supply	battery	$\frac{1}{8}$
	solar cells	$\frac{1}{8}$

E. Use pie-type illustrations to show percent.



$\frac{1}{2} = 50\%$



$\frac{1}{4} = 25\%$

Text References:

Book: 5

Houghton Mifflin (1972) pp. 329, 331, 343

Book: 6

Houghton Mifflin (1967) pp. 318, 320-321, (75)
Houghton Mifflin (1972) pp. 146-148
Addison-Wesley (1971, 1968) pp. 272-274, (50), 337 (top)

Book: 7

Houghton Mifflin (1972) pp. 138, 145
Addison-Wesley (1971) pp. 361-364, 374

Level: 10 Step: C Concept: Numeral

Text References (Continued)

Book: 8

Houghton Mifflin (1972) pp. 70-71
Addison-Wesley (1971) pp. 281-282, (63)

Other References:

Imperial Tape (Intermediate) #31, 32

Level: 18 Step: C Concept: Multiplication

I. Concept:

<u>Multiplication:</u> Naming a percent of a whole number.

II. Behavioral Objective:

The student given a whole number will be able to find a given percent of the number.

III. Mathematical Ideas:

A. A percent is a numeral for a fractional number whose denominator is 100.

B. The percent is the number of hundredths. It may be written:

%, per 100, for each 100, $\frac{1}{100}$. Thus $23\% = \frac{23}{100}$; $230\% = \frac{23}{10}$

C. Naming the percent of a number is the same as multiplying it by a fractional number. 25% of 200 means $.25 \times 200$ or $\frac{1}{4} \times 200$.

D. Multiplying with fractional numbers is associative.

IV. Vocabulary To Stress:

percent associative discount commission

V. Activities:

A. Discuss the uses of percent in the business world. Bring out the uses of percent in figuring sales tax, sale prices, etc.

B. Have students bring copies of local newspapers which advertise certain items at a discount. Have students figure the amount the items have been reduced and what they would have to pay for the item.

C. Use 3 x 5 inch cards to make eighteen 3-card sets such as:

$\boxed{\frac{1}{3}}$ - $\boxed{.333}$ - $\boxed{33\frac{1}{3}\%}$

Put all the cards together, deal out two cards to each player. The remaining cards form the extra pile. The object is to draw one card and if a **triple** is formed as shown above, the player lays his cards down. If not, he discards one card in the discard pile. He then draws two cards and the next player proceeds. Score 10 points for each triple and a minus 2 points for any cards left in the player's hand at the end of any number of specified rounds.

Level: 18 Step: C Concept: Multiplication

V. Activities: Continued

D. Fill in tables such as the following:

	X	70	60	90
50%	.50	35	30	45
20%	.20	a	b	c
30%	.30	d	e	f
40%	.40	g	h	i

a = 14
b = 12
c = 18
d = 21
e = 18
f = 27
g = 28
h = 24
i = 36

Text References:

Book: 6

Houghton Mifflin (1967) pp. 322-323, (76), (80)
Houghton Mifflin (1972) pp. 149, (39), 213, 214
Addison-Wesley (1971, 1968) pp. 275, 278-279, (51), 280, 288, 289, 337

Book: 7

Houghton Mifflin (1967, 1970) pp. 471
Houghton Mifflin (1972) pp. 131, 218, 323, 418, 425
Addison-Wesley (1971) pp. 362, (77)

Other References:

Imperial Tape (Intermediate) #32, 33

Level: 18 Step: C Concept: Multiplication

WORKSHEET

Solve the following problems.

1. Tom saved 60% of the $500 he made during the summer. How much did he save?

2. Susan received $18 for baby-sitting during the month of June. If she spent 80% of it for a new dress, how much did the dress cost?

3. About 55% of an 80 acre field was planted in corn. How many acres of corn did the farmer have?

4. 40% of the class attended the movie shown in the library. If there are 30 students in the class, how many saw the movie?

5. Tom runs 2 miles every morning to keep in good shape for the track meet. Harry runs only 25% of Tom's distance. How far does Harry run?

6. If Susan has saved 30% of the $9 she needs to buy a blouse, how much has she saved? How much does she still need?

7. Jack owes his father $25. If he has saved 60% of this amount, how much does he have to pay on his debt?

8. 80% of the 25 children in the fifth grade class are 10 years old. How many are 10 years old?

Level: 18 Step: C Concept: Geometry

I. Concept:

Geometry: Finding the area of a circle.

II. Behavioral Objective:

The student given the radius of a circle will be able to find the area.

III. Mathematical Ideas:

A. The radius of a circle is the distance from the center to any point on the circle.

B. The ratio of the measure of the circumference (C) to the measure of the diameter (d) is a constant. This number is named π (the Greek letter pi) and is approximately equal to $3\frac{1}{7}$ or $\frac{22}{7}$.

C. To find the area of a circle, multiply pi times the radius of the circle squared. $A = \pi \times r^2$.

IV. Vocabulary To Stress:

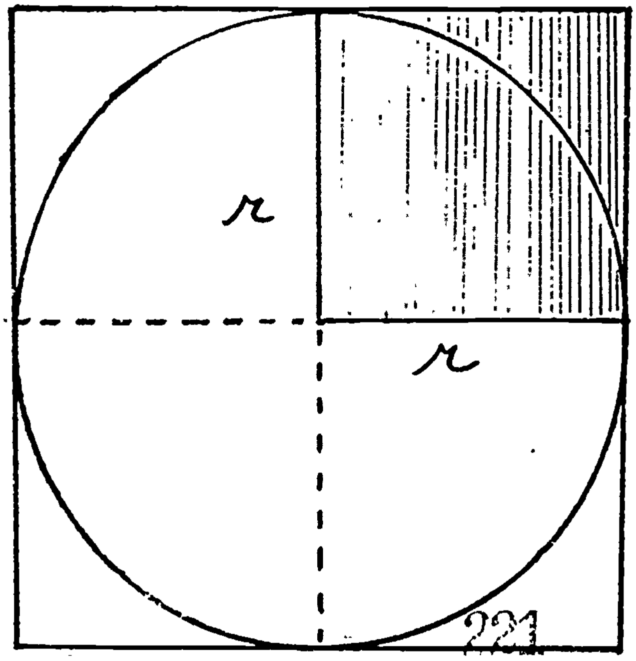
radius area pi (π) constant

V. Activities:

A. Curriculum Filmstrips # 368-369.

B. Have the students estimate the area of a large round table or other available items, then have them measure the true radius and find the area.

* C. Estimate the area of a circle by using a figure such as this. The area of a circle is a little less than 4 times the area of the shaded portion. ($4 \times r^2$) (Addison-Wesley, Book 6, p. 255)



Level: 18 Step: C Concept: Geometry

Text References:

Book: 6

Houghton Mifflin (1967) pp. 273, (63 - part of page)
Houghton Mifflin (1972) pp. 310-311, (75)
Addison-Wesley (1971, 1968) pp. 254-255

Book: 7

Houghton Mifflin (1967, 1970) pp. 446-448 (70 - part of page)
Houghton Mifflin (1972) pp. 299
Addison-Wesley (1971) pp. 423-425, (80)

Book: 8

Addison-Wesley (1971) pp. 411-412, (86)

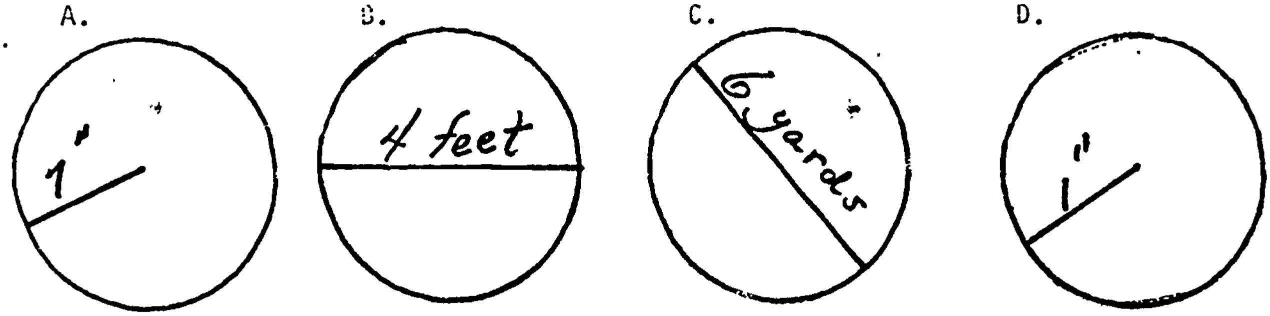
* This activity may be used to help introduce the concept.

Level: 18 Step: C Concept: Geometry

WORKSHEET

I. Using $\frac{22}{7}$ for π, find the areas of these circles.

A. B. C. D.



Area = ________ Area = ________ Area = ________ Area = ________

II. Solve the following problems using $\frac{22}{7}$ for π.

A. Find the area of a round table that has a diameter of 6 feet.

____

B. Jean made a hooked rug for a class project. If the rug had a diameter of 36 inches, what was its area?

____

C. What is the area of a flower bed that has a radius of 2 feet?

____

D. A circular mirror has a diameter of 14 inches. Find the area of the mirror.

____

E. A circular swimming pool is 70 feet in diameter. Find the area of the pool.

____

F. An antique plate has a radius of 4 inches. What is its area.

____

Level: 18 Step: Z Concept: Numeral

I. Concept:

Numeral: Naming and locating rational numbers on a number line.

II. Behavioral Objective:

The student given a number line will be able to name and locate the rational numbers.

III. Mathematical Ideas:

A. The set of rational numbers is the set of all numbers of the form $\frac{a}{b}$ where a and b are integers and $b \neq 0$.

B. Division by 0 is undefined.

C. The rational numbers are closed with respect to addition, subtraction, multiplication, and division.

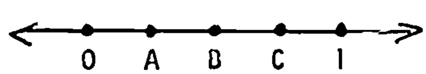
IV. Vocabulary To Stress:

rational numbers closed

V. Activities:

A. Have students show different numbers on the number line as whole numbers, integers, fractional numbers, then rational numbers.

B. 1) Give the correct point on the number line (A,B,C) for certain rational numbers as shown.



$\frac{1}{4}$ = point (A)

2) Give the rational number for point B. ($\frac{2}{4}$ or $\frac{1}{2}$)

Text References:

Book: 6

Houghton Mifflin (1967) pp. 340-341
Houghton Mifflin (1972) pp. 330-331, (80)
Addison-Wesley (1971, 1968) pp. 130, 131, 134-135

Book: 7

Houghton Mifflin (1967, 1970) pp. 331, (47)
Addison-Wesley (1971) pp. 198-200

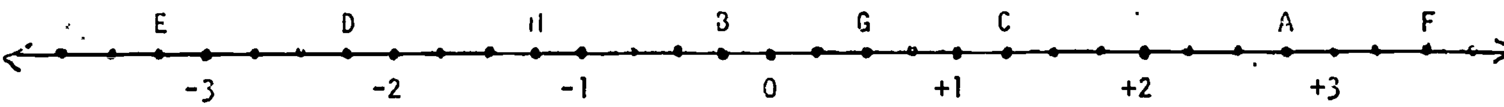
Other References:

Modern Mathematics Filmstrip #757

18-40

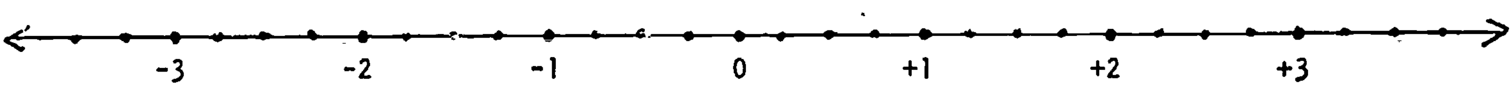
Level: 13 Step: Z Concept: Numeral

WORKSHEET



I. Name the rational number on the number line labeled:

1. A ________
2. B ________
3. C ________
4. D ________
5. E. ________
6. F ________
7. G ________
8. H ________

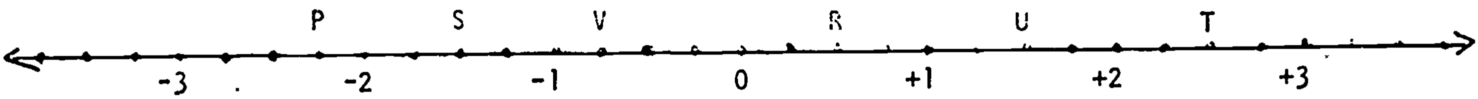


II. Find the rational numbers below on the number line above and label with the letters given.

1. $-\frac{3}{4}$ (A)
2. $+2\frac{1}{4}$ (B)
3. $-1\frac{1}{2}$ (C)
4. $-1\frac{1}{4}$ (D)
5. $+\frac{1}{4}$ (E)
6. $+3\frac{3}{4}$ (F)
7. $-2\frac{1}{4}$ (G)
8. $+1\frac{1}{4}$ (H)
9. $+2\frac{3}{4}$ (I)
10. $+\frac{1}{2}$ (J)

Level: 18 Step: Z Concept: Numeral

ENRICHMENT EVALUATION



I. Name the rational number on the number line above labeled

1. P __________

2. R __________

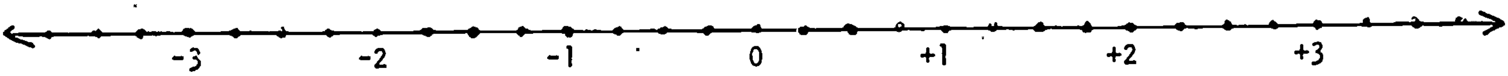
3. S __________

4. T __________

5. U __________

6. V __________

II. Label the rational number on the number line with the letter given.



1. $-1\frac{1}{4}$ (A)

2. $+2\frac{3}{4}$ (B)

3. $+\frac{1}{2}$ (C)

4. $-\frac{3}{4}$ (D)

5. $-1\frac{1}{2}$ (E)

6. $+2\frac{1}{4}$ (F)

III. Name the rational number in the form $\frac{a}{b}$.

1. $3 \div 4 =$ __________

2. $-2\frac{1}{2} =$ __________

3. $-.25 =$ __________

4. $33\frac{1}{3}\% =$ __________

5. $-3 \div 4 =$ __________

6. $2 \div 5 =$ __________

7. $+.60 =$ __________

8. $-4 \div 7 =$ __________

9. $-1\frac{1}{2} =$ __________

Level: 18 Step: Z Concept: Multiplication

I. Concept:

Multiplication: Using percents in solving problems dealing with profit and loss.

II. Behavioral Objective:

The student given the cost and selling price of an article will be able to find the profit or loss.

III. Mathematical Ideas:

A. A percent can indicate a reduction in the price of an item. (Discount)

B. A percent can be used to state a profit or loss.

C. A percent is a numeral for a fractional number whose denominator is 100.

IV. Vocabulary To Stress:

percent	loss	reduction
profit	discount	selling price

V. Activities:

A. Have students watch the advertisements in local papers for items being sold below cost. Have them find the selling price and/or cost.

B. Ask the students to name the price of a coat. Then ask what the sale price would be if the price were reduced by one fourth. (Ask other similar questions.)

Text References:

Book: 6

Houghton Mifflin (1967) pp. 324-325
Houghton Mifflin (1972) pp. 150-151, 343
Addison-Wesley (1971, 1968) pp. 2[illegible]1-265

Book: 7

Houghton Mifflin (1967, 1970) pp. 474-476

Other References:

Imperial Tape (Intermediate) #34

Modern Mathematics Filmstrip #769

Level: 18 Step: 2 Concept: Multiplication

WORKSHEET

Solve the problems.

1. A baseball bat marked at $6 is reduced by 20%. How much is the amount of reduction. How much does the bat sell for after the reduction?

   ______________ ______________

2. A book was marked at $5.50. If there was a 10% discount, what was the selling price of the book?

   ____

3. Mr. Jones bought a tractor for $500 and sold it at a discount of 30%. How much was the selling price.

   ____

4. Mrs. Johnson bought an antique china cabinet for $80. After repairing it, she sold it for a 20% profit. How much was the selling price?

   ____

5. If a storekeeper buys a shirt for $10 and sells it for $11, what is his percent of profit?

   ____

6. The original price of a game was $2.50. Jack bought it at a discount of 20%. How much did he pay for it?

   ____

7. Find the amount of discount for the following:

COST	% OF DISCOUNT	AMOUNT OF DISCOUNT
a. $12	30%	______________
b. $14	10%	______________
c. $25	20%	______________

Level: 18 | Step: Z | Concept: Multiplication

ENRICHMENT EVALUATION

I. Name the amounts to complete the chart.

Marked Price	$8	$12	$10	$15	$16	$20	$4	$6	$11	$13
Rate of reduction as a % of the marked price	20%	10%	30%	5%	10%	25%	20%	$33\frac{1}{3}\%$	50%	10%
Amount of reduction	$1.60	a	b	c	d	e	f	g	h	i

II. Solve the following problems.

A. A boy's suit was marked at $50. If there was a 20% discount, what was the selling price of the suit?

____

B. A baseball that usually sells for $4.50 is reduced 10%. What is the amount of reduction? How much does the baseball sell for after the reduction?

________________ ________________

C. If a dress cost the storekeeper $25 and was sold for $20, what was the loss? What was the percent of loss?

________________ ________________

18-2

I.

.18	.19	.82	.032	.080
.20	.27	.97	.075	.043
.45	.32	.28	.016	.017
.13	.16	.65	.076	.060

II.

$\frac{16}{100}$	$\frac{42}{100}$	$\frac{18}{1000}$	$\frac{63}{100}$	$\frac{6}{1000}$
$\frac{18}{100}$	$\frac{38}{100}$	$\frac{15}{1000}$	$\frac{82}{100}$	$\frac{1}{1000}$
$\frac{97}{100}$	$\frac{73}{100}$	$\frac{236}{1000}$	$\frac{136}{1000}$	$\frac{321}{1000}$
$\frac{13}{100}$	$\frac{15}{100}$	$\frac{140}{1000}$	$\frac{150}{1000}$	$\frac{46}{100}$

III.

A. $\frac{6}{10} + \frac{4}{100}$	F. $\frac{2}{10} + \frac{5}{100}$	K. $\frac{9}{10} + \frac{7}{100}$
B. $\frac{9}{10} + \frac{1}{100}$	G. $\frac{3}{10} + \frac{9}{100}$	L. $\frac{4}{10} + \frac{6}{1000}$
C. $\frac{1}{10} + \frac{2}{100} + \frac{6}{1000}$	H. $\frac{1}{10} + \frac{3}{100} + \frac{2}{1000}$	M. $\frac{7}{10} + \frac{2}{100}$
D. $\frac{3}{10} + \frac{6}{100} + \frac{8}{1000}$	I. $\frac{2}{10} + \frac{6}{1000}$	N. $\frac{6}{10} + \frac{5}{100}$
E. $\frac{5}{10} + \frac{6}{100}$	J. $\frac{4}{10} + \frac{8}{100}$	O. $\frac{8}{10} + \frac{3}{100}$

18-5

1. 10.04	7. 6.330	13. 40.663
2. 9.949	8. 114.996	14. 167.336
3. 24.393	9. 18.825	15. 1.41
4. 98.487	10. 17.478	16. 1.121
5. 8.546	11. 15.198	17. 8.717
6. 269.39	12. 10.887	18. 26.746

18-6

A. 50.881	F. 3.856	K. 34.236
B. 32.678	G. 87.668	L. 5.772
C. 11.371	H. 8.443	M. 8.226
D. 9.008	I. 18.587	N. 13.246
E. 189.17	J. 47.868	

Level: 18 ANSWERS TO WORKSHEETS: CONTINUED

18-9

A. 14.05	F. 8.944	K. 16.61
B. 14.996	G. 23.614	L. .526
C. 7.432	H. 2.213	M. 7.7
D. .198	I. .264	N. 45.332
E. 2.322	J. 258.53	

18-10

A. 32.192	H. .32	O. 55.7
B. 8.523	I. 36.28	P. .188
C. 1.684	J. 435.34	Q. 4.631
D. 2.54	K. .472	R. .172
E. .098	L. 11.2	S. 4.1
F. 88.632	M. 6.531	T. 35.82
G. 35.518	N. 1.64	U. 1.136

18-12

Multiplication

	(a)	(b)	(c)
1.	2.36	24	382
2.	4860	65	550
3.	37	2300	660
4.	64,000	82	623
5.	94	370	15
6.	820	820	8200
7.	87.5	453.8	6

Division

	(a)	(b)	(c)
1.	.24	.0036	.126
2.	6.5	.083	.056
3.	3.96	.384	.27
4.	.0062	.0324	.262
5.	.00875	1.5	1.55

18-13

1. 100	11. .0026	21. 85
2. 100	12. .086	22. 750
3. 1	13. .28	23. .036
4. 121.6	14. 750	24. 980
5. 463	15. 750	25. 7500
6. 90	16. 1.6	26. 32
7. .89	17. 7	27. .32
8. .124	18. 641	28. .032
9. 7	19. .24	29. 84
10. 2.34	20. .365	30. 27

18-18

1. $^{+}8$	10. $^{+}8$	19. $^{-}7$
2. $^{-}2$	11. $^{+}2$	20. $^{+}5$
3. $^{+}4$	12. $^{+}1$	21. $^{-}1$
4. $^{-}5$	13. $^{-}3$	22. $^{+}3$
5. $^{+}5$	14. $^{-}1$	23. $^{-}7$
6. $^{-}3$	15. $^{+}2$	24. $^{+}6$
7. $^{+}1$	16. $^{+}9$	25. $^{+}1$
8. $^{+}1$	17. $^{-}8$	26. $^{-}5$
9. $^{-}8$	18. $^{+}4$	

18-21

1. $^{+}11$	10. $^{+}2$	19. $^{-}6$
2. $^{+}1$	11. $^{+}8$	20. $^{+}9$
3. $^{-}6$	12. $^{+}8$	21. $^{+}7$
4. $^{+}5$	13. $^{-}8$	22. $^{+}9$
5. $^{-}6$	14. $^{-}8$	23. $^{+}1$
6. $^{-}6$	15. $^{-}2$	24. $^{-}1$
7. $^{-}4$	16. $^{-}8$	25. $^{-}7$
8. 0	17. $^{+}1$	26. $^{-}9$
9. $^{-}14$	18. $^{-}4$	

18-29

I. A. $182\frac{2}{7}$ or 183 inches

B. $18\frac{6}{7}$ feet

C. $25\frac{1}{7}$ inches

D. 7 feet 4 inches

II. A. $31\frac{3}{7}$ in.

B. $75\frac{3}{7}$ in.

C. $2514\frac{2}{7}$ mi.

D. $15\frac{5}{7}$ yds.

E. $18\frac{6}{7}$ cm.

F. 44 feet

G. 66 inches

H. 110 feet

I. $34\frac{4}{7}$ inches

J. 22 miles

18-35

1. \$300
2. \$14.40
3. 44
4. 12
5. $.50 = \frac{1}{2}$ mile
6. \$2.70, \$6.30
7. \$15
8. 20

Level: 18 ANSWERS TO WORKSHEETS: CONTINUED

18-38

I A. 154 sq. in.

B. $12\frac{4}{7}$ sq. ft.

C. $28\frac{2}{7}$ sq. yd.

D. $3\frac{1}{7}$ sq. in.

II. A. $28\frac{2}{7}$ sq. ft.

B. $1018\frac{2}{7}$ sq. in.

C. $12\frac{4}{7}$ sq. ft.

D. 154 sq. in.

E. 3850 sq. ft.

F. $50\frac{2}{7}$ sq. in.

Level: 18 ANSWERS TO ENRICHMENT (STEP Z)

WORKSHEETS

18-40

I. 1. $+ 2 \frac{3}{4}$

2. $- \frac{1}{4}$

3. $+ 1 \frac{1}{4}$

4. $- 2 \frac{1}{4}$

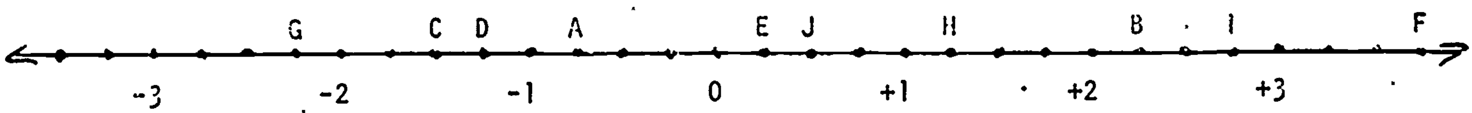
5. $- 3 \frac{1}{4}$

6. $+ 3 \frac{1}{2}$

7. $+ \frac{1}{2}$

8. $- 1 \frac{1}{4}$

II.



18-43

1. \$1.20, \$4.80
2. \$3.15
3. \$350
4. \$96
5. 10%
6. \$2
7. \$3.60, \$1.40, \$5

EVALUATIONS

18-41

<u>Numeral</u>

I. 1. $- 2 \frac{1}{4}$

2. $+ \frac{1}{2}$

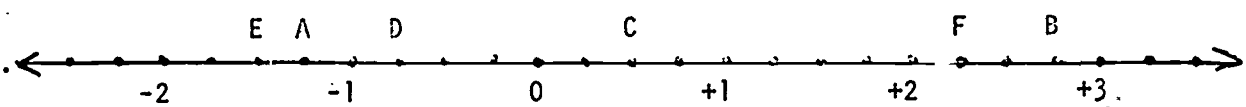
3. $- 1 \frac{1}{2}$

4. $+ 2 \frac{1}{2}$

5. $+ 1 \frac{1}{2}$

6. $- \frac{3}{4}$

II.



III. 1. $\frac{3}{4}$

2. $- \frac{5}{2}$

3. $- \frac{1}{4}$

4. $\frac{1}{3}$

5. $- \frac{3}{4}$

6. $\frac{2}{5}$

7. $+ \frac{3}{5}$

8. $- \frac{4}{7}$

9. $- \frac{3}{2}$

Level: 18 ANSWERS TO ENRICHMENT (STEP Z) CONTINUED

EVALUATIONS CONTINUED

18-44

Multiplication

I. a. $1.20	d. $1.60	g. $2.00
b. $3.00	e. $5.00	h. $5.50
c. $.75	f. $.80	i. $1.30

II. A. $40
B. $.45, $4.05
C. $5 loss, 20% loss

Level: 13 Answer Sheet - Post Test I

Step A

<u>Numeral</u>

1. .001
2. 3.428
3. .011
4. .003
5. .053

<u>Addition</u>

6. 20.905
7. 43.946
8. 26.031
9. 27.990
10. 162.251

<u>Subtraction</u>

11. 3.722
12. 4.815
13. .268
14. 25.7
15. 42.31

<u>Multiplication & Division</u>

16. 24
17. 3510
18. .75
19. .0234
20. 6250

<u>Numeral</u>

21. false
22. true
23. 9 degrees
24. T
25. J

<u>Addition</u>

26. 0
27. -1
28. +4
29. -9
30. -6

Step B

<u>Subtraction</u>

31. negative
32. -2
33. -3
34. +5
35. -10

<u>Multiplication</u>

36. product
37. 21.708
38. 7.680
39. $33.00
40. 3.8

<u>Division</u>

41. ten
42. quotient
43. 1.24
44. 960.9
45. 8.01

<u>Geometry</u>

46. circumference
47. $\pi \times d$
48. $12\frac{4}{7}$ ft.
49. 44 ft.
50. $47\frac{1}{7}$ ft.

Step C

<u>Numeral</u>

51. 25%
52. 100
53. 75%
54. $\frac{3}{20}$
55. $\frac{9}{20}$

<u>Multiplication</u>

56. 80
57. 140
58. 50%
59. $1.80
60. 98

<u>Geometry</u>

61. π
62. 7
63. $50\frac{2}{7}$ sq. ft.
64. 154 sq. ft.
65. $380\frac{2}{7}$ sq. ft.

Level: 18 Answer Sheet - Post Test II

Step A

Numeral

1. .352
2. 6.742
3. .003
4. thousandths
5. 1.122

Addition

6. 26.676
7. 40.508
8. 10.245
9. 70.790
10. 10.445

Subtraction

11. 23.615
12. .514
13. 15.662
14. $34.84
15. .24

Multiplication & Division

16. 420
17. .38
18. 2340
19. 346.60
20. .05638

Step B

Numeral

21. R
22. true
23. "
24. +5
25. 4 degrees

Addition

26. -4
27. -8
28. +5
29. positive
30. negative

Subtraction

31. true
32. false
33. -3
34. +1
35. +2

Multiplication

36. 55.941
37. 64.052
38. 32.566
39. 362.08
40. 72.616

Division

41. 6.3
42. 2.71
43. .134
44. 1180
45. 500

Circumference of a circle

46. $62 \frac{6}{7}$
47. $25 \frac{1}{7}$
48. 44 ft.
49. 154 in.
50. π (pi)

Step C

Numeral

51. 2
52. 34%
53. 60%
54. 10%
55. 80%

Multiplication

56. 45
57. 200
58. 75
59. 14.80
60. 85 cents

Geometry

61. $\pi \times r^2$
62. 6 feet
63. $113 \frac{1}{7}$ sq. ft.
64. $28 \frac{2}{7}$ sq. ft.
65. $78 \frac{4}{7}$ sq. in.

TABLE OF CONTENTS

Level 19

Step A

Step B

Step Z

Level: 19 Step: A Concept: Sets

I. Concept:

Sets: Extending set terminology.

II. Behavioral Objective:

The student given a series of sets or a Venn diagram will be able to make true statements by using the symbols ($\in$, $\notin$, $\emptyset$, $\not\subset$, $\subset$).

III. Mathematical Ideas:

A. $\in$ means "is a member or element of".

B. $\notin$ means "is not a member or element of".

C. $\not\subset$ means "is not a subset of".

D. $\emptyset$ means "the empty or null set".

E. $\subset$ means "is a subset of".

IV. Vocabulary To Stress:

element ($\in$)	negation symbol (/)
null ($\emptyset$)	Venn diagrams

V. Activities:

A. Use Venn Diagrams to show relationships requiring the symbols $\in$, $\notin$, $\emptyset$, $\not\subset$, $\subset$. Use the overhead or colored chalk to make Venn Diagrams.

B. Have student give special report on John Venn.

C. The students might make problems and Venn diagrams of their own.

Text References:

Book: 7

Houghton Mifflin (1967, 1970) pp. 6-14, (1,2)

Level: 19 Step: A Concept: Sets

Worksheet

I. $A = \{1, 3, 5, 7\}$ $C = \{x; w, m\}$

$B = \{4, x, 9, w, m\}$ $D = \{\text{odd numbers}\}$

Determine whether the following statements are true or false. Refer to the above sets.

1. $\{1, 5, 7\} \subset A$ ________
2. $\{w\} \in C$ ________
3. $w \in C$ ________
4. $C \subset B$ ________
5. $3 \notin D$ ________
6. $A \not\subset D$ ________
7. $x \notin A$ ________
8. $\emptyset \subset B$ ________
9. $\emptyset \not\subset D$ ________
10. $7 \in A$ ________

II. $X = \{a, b, c\}$ $Z = \{3, 6, 8, 11\}$

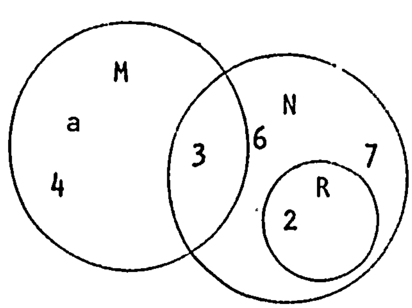
$Y = \{b, d, p, n\}$ $T = \{\text{even numbers}\}$

$P = \{\text{letters in the alphabet}\}$

Circle the symbol in the parentheses which will make the following statements true.

11. $d \ (\in, \notin, \subset) \ Y$
12. $\{a, c\} \ (\not\subset, \subset, \in) \ X$
13. $7 \ (\in, \notin, \subset) \ Z$
14. $\{3, 8\} \ (\in, \subset, \not\subset) \ T$
15. $\emptyset \ (\not\subset, \subset, \in) \ Y$
16. $X \ (\subset, \not\subset, \in) \ P$
17. $P \ (\subset, \not\subset, \in) \ T$
18. $a \ (\in, \notin, \subset) \ X$
19. $\{h\} \ (\subset, \not\subset, \in) \ P$
20. $\{6, 8\} \ (\in, \subset, \not\subset) \ X$

III. Refer to sets M, N, and R in the Venn diagram to answer 21-25.



21. $R \ (\not\subset, \subset, \in) \ N$
22. $\{4, 3\} \ (\subset, \not\subset, \in) \ M$
23. $2 \ (\subset, \in, \notin) \ N$
24. $6 \ (\subset, \in, \notin) \ M$
25. $\{6\} \ (\subset, \not\subset, \in) \ M$
26. $M \ (\subset, \not\subset, \in) \ N$

Level: 19 Step: A Concept: Numeral

I. Concept:

Numeral: Converting a terminating decimal numeral to a common fraction in simplest form.

II. Behavioral Objective:

The student given a decimal numeral will be able to convert it to a fraction in simplest form.

III. Mathematical Ideas:

Every terminating decimal numeral can be expressed as a fraction.

IV. Vocabulary To Stress:

numerator denominator simplest form

V. Activities:

A. Review place values.

B. Card game changing decimals to fractions. See directions in Teacher's Manual for Imperial Tapes (Intermediate), Lesson 32, p. 67.

Text References:

Book: 7

Houghton Mifflin (1967, 1970) pp. 386-389, (60).

Level: 19 Step: A Concept: Numeral

Worksheet

Write the following terminating decimals as fractions or mixed numerals in simplest form.

1.	.04	______	13.	.905	______
2.	1.1	______	14.	.0002	______
3.	2.12	______	15.	7.23	______
4.	.063	______	16.	23.625	______
5.	.052	______	17.	.0625	______
6.	3.25	______	18.	3.03	______
7.	.1111	______	19.	.44	______
8.	.006	______	20.	.045	______
9.	.32612	______	21.	6.94	______
10.	5.55	______	22.	.112	______
11.	.1006	______	23.	.088	______
12.	.90	______	24.	7.07	______

Level: 19 Step: A Concept: Order

I. Concept:

<u>Order</u>: Rounding decimal numerals.

II. Behavioral Objective:

The student given a decimal numeral will be able to round it to an indicated place value.

III. Mathematical Ideas:

A. If the digit to the right of the digit in the desired place is five or greater, round up; if less than five, leave unchanged.

B. In rounding decimals, the digits to the right of the digit in the desired place are dropped.

IV. Vocabulary To Stress:

approximation symbol ($\doteq$)

V. Activities:

A. Review place value.

B. Review rounding whole numbers.

C. Use number line to illustrate rounding.

Text References:

Book: 7

Houghton Mifflin (1967, 1970)	pp. 383-385 (59)
Houghton Mifflin (1972)	pp. 16-17, 136
Addison-Wesley (1971)	pp. 314-317

Book: 8

Houghton Mifflin (1972)	pp. 38, 77, 131
Addison-Wesley (1971)	pp. 262-263, (46)

Level: 19 Step: A Concept: Order

Worksheet

Round the following to the nearest (a) $\frac{1}{1000}$, (b) $\frac{1}{100}$, (c) $\frac{1}{10}$, (d) whole 'mber.

1. .5845	2. .9864	3. .6075	4. 5.9555
a)	a)	a)	a)
b)	b)	b)	b)
c)	c)	c)	c)
d)	d)	d)	d)
5. .2056	6. 7.8953	7. .68745	8. .98952
a)	a)	a)	a)
b)	b)	b)	b)
c)	c)	c)	c)
d)	d)	d)	d)
9. 555.5555	10. 26.9898	11. 1.4786	12. .06432
a)	a)	a)	a)
b)	b)	b)	b)
c)	c)	c)	c)
d)	d)	d)	d)

Level: 19 Step: A Concept: Addition & Subtraction Multiplication

I. Concept:

Addition & Subtraction. Multiplication: Identifying the commutative, associative and distributive properties.

II. Behavioral Objective:

The student given either the general form or a specific example will be able to identify the commutative, associative and the distributive properties.

III. Mathematical Ideas:

A. The commutative property:

1. Addition: If a and b denote rational numbers, then $a + b = b + a$

2. Multiplication: If a and b denote rational numbers, then $a \times b = b \times a$

B. The associative property:

1. Addition: If a, b and c denote rational numbers, then $(a + b) + c = a + (b + c)$

2. Multiplication: If a, b and c denote rational numbers, then $(a \times b) \times c = a \times (b \times c)$

C. The above properties do not apply to subtraction or division.

D. Distributive property:

1. With respect to addition: If a, b and c denote rational numbers, then $a(b + c) = (a \times b) + (a \times c)$

2. With respect to subtraction: If a, b and c denote rational numbers, then $a(b - c) = (a \times b) - (a \times c)$

IV. Vocabulary:

variable commutative associative distributive

Text References:

Book: 7

Houghton Mifflin (1967, 1970) pp. 40-43, 50-55, 64, 71-74, 89-100, 329-330, 506

Houghton Mifflin (1972)	pp. 30-33, 62-65, 190, 253, 382-383
Addison-Wesley (1971)	pp. 47-52, 145, 154, 224, 231, 237

Book: 8

Houghton Mifflin (1967, 1970)	pp. 45-46, 68-70
Houghton Mifflin (1972)	pp. 30-34, 222-223 (8,38)
Addison-Wesley (1971)	pp. 10-12, 36, 51,59, 130, 137

Level: 19 Step: A Concept: Addition & Subtraction Multiplication

Worksheet

Indicate whether the following are illustrations of the commutative, associative or distributive properties or none of these. Use C, A, D or N for your answers.

_____ 1. (7 x 5) + 4 = 4 + (7 x 5)

_____ 2. (3 + 4) + 5 = 3 + (4 + 5)

_____ 3. a - b = b - a

_____ 4. (a x b) x c = a x (b x c)

_____ 5. b x a = a x b

_____ 6. a(b + c) = (a x b) + (a x c)

_____ 7. 2(3 + 4) = (2 + 3) x (2 + 4)

_____ 8. 5 x 4 = 4 x 5

_____ 9. f(g + t) = (f x g) + (f x t)

_____ 10. (7 + 3) + 2 = 7 + (3 + 2)

_____ 11. 9 + 1 = 1 + 9

_____ 12. 9(10 + 11) = (9 x 10) + (9 x 11)

_____ 13. m x n = n x m

_____ 14. 18 ÷ 9 = 9 ÷ 18

_____ 15. (2 x 3) x 4 = 2 x (3 x 4)

_____ 16. 6 + (3 x 2) = (3 x 2) + 6

_____ 17. a(b - c) = (a x b) - (a x c)

_____ 18. a + b = b + a

_____ 19. b ÷ a = a ÷ b

_____ 20. (a + b) + c = a + (b + c)

_____ 21. (3 + 2) x (4 + 5) = (4 + 5) x (3 +2

_____ 22. (5 x 6) + (5 x 7) = 5(6 + 7)

_____ 23. (4 x 6) - 3 = 3 - (4 x 6)

_____ 24. (r + s) x (r + p) = r(s + p)

_____ 25. (6 + 7) x 3 = 3 x (6 + 7)

_____ 26. 5 x (8 x 9) = (5 x 8) x 9

_____ 27. 9 x (2 + 3) = (2 + 3) x 9

_____ 28. a(b + c) = (a + b) x (a + c)

_____ 29. (8 x 4) + (6 x 2) = (6 x 2) + (8 x4

_____ 30. x + y = y + x

Complete the following to illustrate the indicated property.

31. C: 4 + 7 = _____

32. A: (7 x 11) x 5 = _____

33. A: 4+ (3 + 2) = _____

34. C: c x d = _____

35. D: u(v + w) = _____

36. D: (5 x 6) + (5 x 7) = _____

37. C: 8 x 5 = _____

38. A: a + (m + n) = _____

39. D: 7(4 + 8) = _____

40. C: (4 x 3) + 8 = _____

Level: 19 Step: A Concept: Division

I. Concept:

Division: Extending the operation of division with decimals.

II. Behavioral Objective:

The student given two numbers with n number of decimal places will be able to find the quotient to the nearest tenth, hundredth, or thousandth.

III. Mathematical Ideas:

A. Multiplying by powers of ten shifts the decimal point to the right.

B. Annexing zeros at the end of a decimal numeral does not change its value.

C. To round quotient, carry out the quotient one place beyond the desired place. If this digit is 5 or greater, round up. If it is less than 5, leave the digit in desired place unchanged.

Text References:

Book: 7

Houghton Mifflin (1967, 1970)	pp. 397-402, (63).
Houghton Mifflin (1972)	pp. 264-266.
Addison-Wesley (1971)	pp. 319-322, (62, 63).

Book: 8

Houghton Mifflin (1972)	pp. 148-149. (36)
Addison-Wesley (1971)	pp. 260, 262-263, (56, 57).

Other References:

Imperial Tapes (Intermediate) Tape 27

Level: 19 Step: A Concept: Geometry

I. Concept:

Geometry: Relating set operations to geometric figures.

II. Behavioral Objective:

The student given a drawing will be able to name the intersection or union of points, rays, line segments, and planes.

III. Mathematical Ideas:

A. Operations of union and intersection can be applied to geometric figures.

B. If two planes in space intersect, their intersection is a line.

C. Two lines in space intersect in at most one point.

D. The union of two rays may be a line, an angle, or a ray.

E. A line which does not lie in a plane intersects the plane in at most one point.

IV. Vocabulary To Stress:

intersection	line	concurrent	coincident
union	segment	coplanar	
ray	plane	colinear	

V. Activities:

A. Review the symbols for the naming of point, lines, rays, line segments, planes and angles.

B. Review operations of union and intersection.

C. Review vocabulary and problems from Houghton Mifflin (1967, 1970) pp. 192-207.

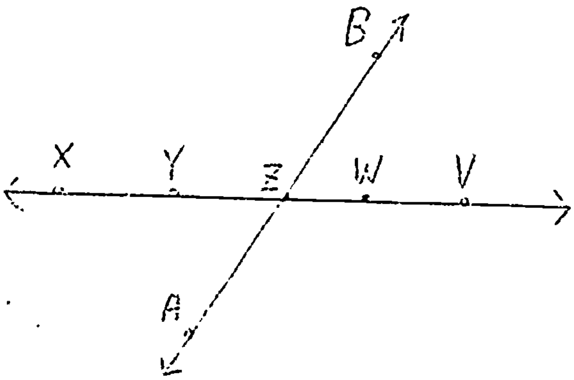
Text References:

Book: 7

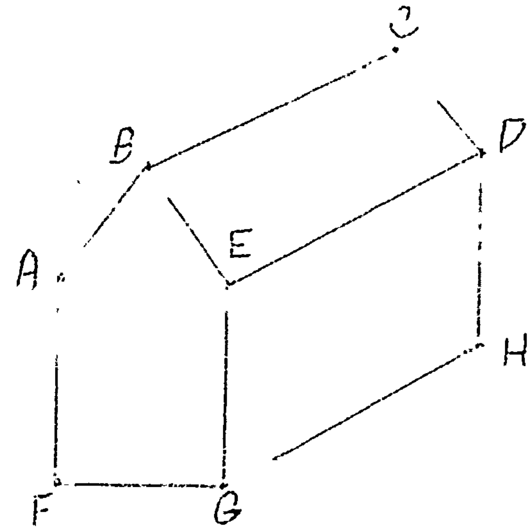
Houghton Mifflin (1967) pp. 192-207, 257-262, 272, (27, 28, 30, 31, 40)

Level: 19 Step: A Concept: Geometry

Worksheet



(Exercises 1-10)



(Exercises 11-15)

Use the above drawings to perform the indicated operations:

1. $\overrightarrow{ZA} \cup \overline{ZB}$ = ____________

2. $\overline{YZ} \cap \overline{YW}$ = ____________

3. $\overline{XZ} \cup \overline{ZW}$ = ____________

4. $\overline{YV} \cap \overline{ZW}$ = ____________

5. $\overleftrightarrow{WV} \cup \overline{XY}$ = ____________

6. $\overrightarrow{ZW} \cap \overline{WV}$ = ____________

7. $\overline{AB} \cap \overline{XW}$ = ____________

8. $\overrightarrow{BA} \cap \overrightarrow{AB}$ = ____________

9. $\overrightarrow{BA} \cap \overrightarrow{AB}$ = ____________

10. $\overline{XY} \cap \overline{WV}$ = ____________

11. Plane ABF $\cap$ Plane BCD = ____________

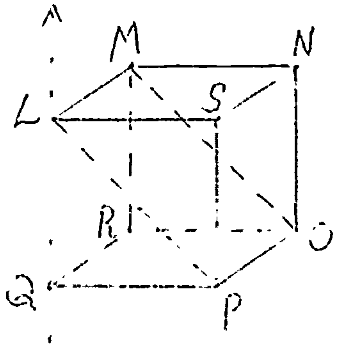
12. $\overleftrightarrow{ED} \cap \overleftrightarrow{DH} \cap \overleftrightarrow{DC}$ = ____________

13. Plane EDH $\cap$ Plane BCD = ____________

14. $\overleftrightarrow{BC} \cap \overrightarrow{ED}$ = ____________

15. Plane AFG $\cap$ Plane EGH $\cap$ Plane BED = ____________

Level: 19 Step: A Concept: Geometry



Worksheet

Use the figure at the left to perform the indicated operations:

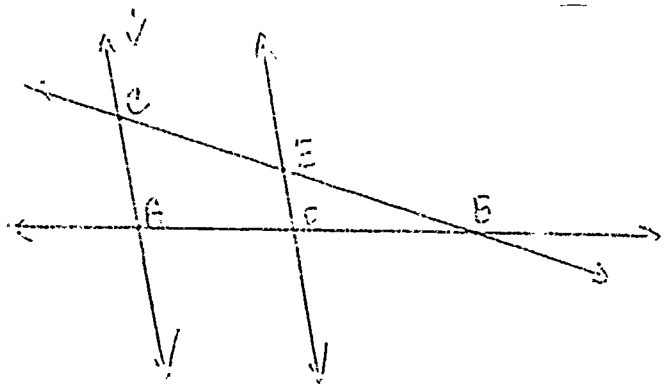
1. $\overleftrightarrow{LQ} \cap$ plane ROP = ____________

2. Plane SNO $\cap$ plane RQP = ____________

3. Plane LMN $\cap$ Plane ROP = ____________

4. $\overline{SN} \cap \overline{NO}$ = ____________

5. Plane MOP $\cap$ LQR = ____________



Use the figure at the left to perform the indicated operations:

6. $\overleftrightarrow{DE} \cap \overleftrightarrow{AB}$ = ____________

7. $\overrightarrow{BC} \cap \overrightarrow{DE}$ = ____________

8. $\overline{AC} \cap \overline{DE}$ = ____________

9. $\overleftrightarrow{AD} \cap \overrightarrow{DB}$ = ____________

10. $\overline{AD} \cap \overline{DB}$ = ____________

11. $\overrightarrow{BA} \cap \overrightarrow{BC}$ = ____________

12. $\overline{AD} \cap \overline{CE}$ = ____________

13. $\overrightarrow{BC} \cap \overrightarrow{EB}$ = ____________

14. $\overrightarrow{BA} \cup \overrightarrow{BC}$ = ____________

15. $\overline{CE} \cup \overline{EB}$ = ____________

16. $\overline{AB} \cup \overline{AD}$ = ____________

17. $\overrightarrow{BA} \cup \overline{AD}$ = ____________

18. $\overrightarrow{DB} \cup \overrightarrow{DE}$ = ____________

19. $\overrightarrow{EB} \cup \overrightarrow{EC}$ = ____________

20. $\overleftrightarrow{AB} \cap \overline{BD}$ = ____________

21. $\overrightarrow{EC} \cap \overrightarrow{DA}$ = ____________

22. $\overrightarrow{EC} \cup \overline{BE}$ = ____________

23. $\{B\} \cup \overline{CB}$ = ____________

24. $\overline{AD} \cap \overleftrightarrow{CB}$ = ____________

25. $\overline{ED} \cup \overline{DB} \cup \overline{BE}$ = ____________

Level: 19 Step: A Concept: Measurement

I. Concept:

Measurement: Measuring to the nearest unit.

II. Behavioral Objective:

The student given line segments will be able to measure them to the nearest 1", $\frac{1}{2}$", $\frac{1}{4}$" or $\frac{1}{8}$".

III. Mathematical Ideas:

A. Every measurement is an approximation.

B. The smaller unit of measure used, the more precise the measurement.

C. Answers should indicate the unit of measure being used.

Examples: (a) A segment one inch in length measured to the nearest $\frac{1}{4}$" would be expressed as $\frac{4}{4}$" or 1 $\frac{0}{4}$"

(b) A segment 1 $\frac{1}{2}$" in length measured to the nearest $\frac{1}{4}$" would be expressed as $\frac{6}{4}$" or 1 $\frac{2}{4}$"

(c) A segment 2 $\frac{1}{16}$" in length measured to the nearest $\frac{1}{4}$" would be expressed as $\frac{8}{4}$" or 2 $\frac{0}{4}$"

IV. Vocabulary To Stress:

approximation precision

V. Activities:

Measure distances on a map to the nearest $\frac{1}{8}$",or $\frac{1}{16}$" and use the results to make a mileage chart.

Text References:

Book: 7

Houghton Mifflin (1967, 1970) pp. 417-423, (66)
Houghton Mifflin (1972) p. 17
Addison-Wesley (1971) pp. 278-281 (55)

Book: 8

Houghton Mifflin (1972) pp. 19
Addison-Wesley (1971) pp. 196-197

Level: 19 Step: A Concept: Number Sentences and Phrases

I. Concept:

Number Sentences and Phrases: Simplifying expressions involving whole numbers.

II. Behavioral Objective:

The student given an expression involving whole numbers, with or without grouping symbols, will be able to simplify the expression by using the order of operations.

III. Mathematical Ideas:

A. Perform operations within grouping symbols first.

B. When one pair of grouping symbols is enclosed in another, perform the operation enclosed in the inner symbol first.

C. In the absence of grouping symbols, the order of operations is:

(1) Multiplications and divisions in order from left to right, and then

(2) additions and subtractions in order from left to right.

Example: $18 - \underbrace{6 \div 3} \times 2 + 1$

$18 - \underbrace{2 \times 2} + 1$

$18 - 4 + 1$

$14 + 1 = 15$

IV. Vocabulary To Stress:

brackets	parenthesis	expression
braces	fraction bar	simplify

Text References:

Book: 7

Houghton Mifflin (1967, 1970) pp. 81-84, (13)

Other References:

Patterns and Puzzles in Mathematics. (Franklin Series) p. 93.

Level: 19 Step: A Concept: Number Sentences and phrases

Worksheet

Simplify the following expressions.

1. $(16 + 4) \div \left[(8 \times 2) \div 4\right] =$ ________

2. $\left\{\left(\frac{100 - 36}{2 \times 4}\right) \times 3\right\} - 7 =$ ________

3. $\left[\{(1776 - 324) \times 5\} - 5\right] \times 100 =$ ________

4. $80 \div 8 - 80 \div 10 =$ ________

5. $\left[(5 \times 20) + (6 \div 3)\right] - 51 =$ ________

6. $\left[30 \div (5 \times 2)\right] \div 3 =$ ________

7. $\left[(4 \times 2) + (6 \times 10) - (3 \times 20) + 56\right] \div 8 =$ ________

8. $10 \div 1 \times 5 =$ ________

9. $\left[(4 \times 5) - (36 \div 9)\right] \div 8 =$ ________

10. $63 \div 7 - 6 \div 3 =$ ________

Level: 19 | Step: A | Concept: Number sentences and phrases

Worksheet

Simplify the following expressions.

1. $\{(14 + 4) \div (2 \times 3)\} - \frac{6 \times 2}{12 - 8} =$

2. $24 - 4 \div 2 \times 2 =$

3. $125 - 100 \div (5 \times 5) =$

4. $125 - 100 \div 5 \times 5 =$

5. $\{(8 \times 5) - [30 - (5 \times 2)]\} \div 2 =$

6. $42 - (3 \times 4) \div \frac{5 + 3}{2 \times 2} =$

7. $60 - 10 \div 2 + 3 =$

8. $27 \div [(5 \times 9) \div 15] \times 9 =$

9. $\frac{5 \times 6}{21 - 16} - \left(\frac{9 + 3}{8 \div 2}\right) \times 2 =$

10. $[(3 \times 5) - (36 \div 4)] \div 3 \times 2 =$

Level: 19 Step: B Concept: Numeral

I. Concept:

Numeral: Converting a fraction to a terminating or repeating decimal.

II. Behavioral Objective:

The student given a fraction will be able to convert it to either a repeating or a terminating decimal.

III. Mathematical Ideas:

A. Every fraction can be expressed as either a repeating or a terminating decimal.

B. If the prime factorization of the denominator of a fraction in lowest terms contains only 2's and/or 5's, the fraction may be expressed as a terminating decimal.

C. If the prime factorization contains any other factor (i.e. 3, 7, 11, 17, etc.) the fraction may be expressed as a repeating decimal.

D. The bar drawn over the repeating digits is called a vinculum.

E. The repeating block of digits is called a repetend.

IV. Vocabulary To Stress:

repeating decimal terminating decimal

V. Activities:

Card game changing fractions to decimals. See directions in Teacher's Manual for Imperial Tapes (Intermediate), Lesson 32, p. 67.

Text References:

Book: 7

Houghton Mifflin (1967, 1970)	pp. 389-393, (60, 61)
Houghton Mifflin (1972)	pp. 136-137
Addison-Wesley (1971)	pp. 328-332, (65, 66)

Book: 8

Houghton Mifflin (1967, 1970)	pp. 224-228
Houghton Mifflin (1972)	pp. 78-81
Addison-Wesley (1971)	pp. 277-279, (62)

Level: 19 Step: B Concept: Order

I. Concept:

Order: Arranging decimals in order.

II. Behavioral Objective:

The student given a series of decimal numerals will be able to arrange them in order of increasing or decreasing size.

III. Mathematical Ideas:

A. Comparison property (Trichotomy property): If a, b, c, and d denote whole numbers such that if $b \neq 0$ and $d \neq 0$, then one and only one of the following statements is true:

$$\frac{a}{b} < \frac{c}{d}, \quad \frac{a}{b} = \frac{c}{d}, \quad \frac{a}{b} > \frac{c}{d}$$

B. Annexing zeros at the end of a decimal numeral does not change its value.

C. Decimal numbers can be compared by annexing zeros to terminating decimals or continuing the repeating digits in repeating decimals so that they contain the same number of decimal places. This is analogous to finding common denominators of fractions.

IV. Activities:

A. Review the use of the vinculum.

B. Review place value.

C. Use number line to show decimal order.

Text References:

Book: 7

Houghton Mifflin (1967, 1970) pp. 376-379, 393, (57)
Addison-Wesley (1971) pp. 333-335

Book: 8

Houghton Mifflin (1972) p. 81

19-20

Level: 19 Step: B Concept: Order

Worksheet

Arrange the following decimal numerals in order from the smallest to the largest.

1. $.38,\ .3\overline{80},\ .3799,\ .37\overline{8}$

2. $2.2\overline{9},\ 2.3,\ 2.\overline{29},\ 2.\overline{3}$

3. $.624,\ .\overline{6},\ \frac{5}{8},\ .6249$

4. $4.08,\ 4.079,\ 4.0\overline{7},\ 4.16$

5. $\frac{2}{3},\ .6,\ .67,\ .\overline{65}$

6. $9.99,\ 9.\overline{89},\ 9.98,\ 9\frac{9}{10}$

7. $4.073,\ 4.0\overline{47},\ 4.0\overline{6},\ 4.101$

8. 6.159, 6.0999, 6.1, 6.025

9. $\frac{3}{4},\ .\overline{7},\ .749,\ .751$

10. 1.2453, 1.2534, 1.2435, 1.2345

11. $.\overline{8},\ \frac{7}{8},\ .\overline{87},\ .877$

12. $1.11,\ 1.101,\ 1.\overline{1},\ 1.099$

13. $\frac{1}{2},\ .\overline{50},\ .505,\ .055$

14. $67.\overline{76},\ 67.67,\ 67.\overline{607},\ 67.\overline{7}$

15. 1.034, 1.304, 1.043, 1.031

Level: 19 Step: B Concept: Division

I. Concept:

Division: Relating fractions, decimals, and percents.

II. Behavioral Objective:

The student given a number written as a fraction, a decimal or a percent will be able to write it in the forms not given.

III. Mathematical Ideas:

A. Every fraction can be expressed as either a repeating or a terminating decimal.

B. n% stands for $\frac{n}{100}$, or $n \times \frac{1}{100}$, or $n \times .01$.

IV. Activities:

Card game relating fractions, decimals, and percents in Teacher's Manual for Imperial Tapes (Intermediate), Lesson 32, p. 67, 68.

Text References:

Book: 7

Houghton Mifflin (1967, 1970)	pp. 468-471, (73)
Houghton Mifflin (1972)	pp. 130,135, 139, 145, 419
Addison-Wesley (1971)	pp. 358-363, (73, 74)

Book: 8

Houghton Mifflin (1972)	pp. 70, 71, 190, 409, 417.
Addison-Wesley (1971)	pp. 281-282

Other References:

Imperial Tapes (Intermediate), Lesson 32. Tape, games and puzzle.

Level: 19 Step: B Concept: Number Sentences and Phrases

I. Concept:

Number Sentences and Phrases: Simplifying expressions involving fractional numbers.

II. Behavioral Objective:

The student given an expression involving fractional numbers, with or without grouping symbols, will be able to simplify the expression by using the order of operations.

III. Mathematical Ideas:

A. Perform operations within grouping symbols first.

B. When one pair of grouping symbols is enclosed in another, perform the operation enclosed in the inner symbol first.

C. In the absence of grouping symbols, the order of operations is:

(1) Multiplications and divisions in order from left to right, and then

(2) additions and subtractions in order from left to right.

Example: $4\frac{2}{3} - \underbrace{\frac{5}{8} \div \frac{3}{4}} \times \frac{1}{2} + \frac{5}{8}$

$$4\frac{2}{3} - \underbrace{\frac{5}{6} \times \frac{1}{2}} + \frac{5}{8}$$

$$\underbrace{4\frac{2}{3} - \frac{5}{12}} + \frac{5}{8}$$

$$4\frac{1}{4} + \frac{5}{8} = 4\frac{7}{8}$$

IV. Vocabulary To Stress:

brackets	parenthesis	expression
braces	fraction bar	simplify

V. Activities:

Note to Teachers: This behavioral objective is put in this level mainly as a review of the operations with fractions.

Text References:

Book: 7

Houghton Mifflin (1967, 1970) pp. 81-82 (rules), 340-341, 344-345, 349, 354, 358-359 (53)

Level: 19 Step: B Concept: Number Sentences and Phrases

Worksheet

Simplify the following expressions: (Show work below each problem.)

1. $\left(11 + 1\frac{1}{10}\right) - \left(11 \times 1\frac{1}{10}\right) =$

2. $\frac{4}{7} \div \frac{2}{7} - \frac{2}{3} =$

3. $\left(\frac{5}{6} + \frac{2}{3}\right) \div \frac{3}{2} =$

4. $8\frac{2}{3} - \left[7 + \left(\frac{4}{5} \times \frac{2}{3}\right)\right] =$

5. $\left(\frac{5}{8} - \frac{1}{2}\right) \times \frac{2}{3} =$

6. $19 \div \left(\frac{3}{4} + \frac{1}{5}\right) =$

7. $\frac{7}{8} \times \frac{2}{3} - \frac{7}{12} =$

8. $\left(6 \times 2\frac{1}{3}\right) - 1\frac{1}{4}$

9. $\left(1\frac{5}{6} + 2\frac{1}{12}\right) \div \frac{1}{12} =$

10. $6 - \left(\frac{3}{5} \times 1\frac{1}{2}\right) =$

Level: 19 Step: B Concept: Number Sentences and Phrases

Worksheet

Simplify the following expressions. Show work below each problem.

1. $\frac{3}{10} \times \left(1 - \frac{3}{8}\right) =$

2. $\left(2\frac{2}{3} - \frac{5}{12}\right) \div \frac{3}{5} =$

3. $\frac{7}{12} + \frac{5}{8} \times \frac{2}{3} =$

4. $\left(1\frac{5}{6} \div \frac{2}{3}\right) \times \left(\frac{3}{7} - \frac{3}{14}\right) =$

5. $1 - \frac{3}{4} \div \frac{5}{8} \times \frac{2}{3} + \frac{1}{5} =$

6. $7\frac{1}{5} - \left[3 + \left(\frac{4}{5} \times \frac{1}{2}\right)\right] =$

7. $\left[\left(1\frac{4}{7} + 3\frac{3}{14}\right) \div \frac{3}{7}\right] - \frac{1}{6} =$

8. $\frac{1}{2} + \frac{1}{2} - \frac{1}{2} \times \frac{1}{2} \div \frac{1}{2} =$

9. $\left[\frac{4}{5} - \left(1\frac{1}{2} \div \frac{3}{4} - 1\frac{1}{2}\right)\right] \times \frac{2}{3} =$

10. $1\frac{2}{3} - \frac{3}{8} \div \frac{1}{4} + \frac{1}{4} \times 2\frac{2}{3} =$

Level: 19 Step: Z Concept: Numeral

I. Concept:

Numeral: Converting repeating decimals to fractions.

II. Behavioral Objective:

The student given a repeating decimal will be able to change it to a fraction in simplest form.

III. Mathematical Ideas:

A. Every terminating or repeating decimal numeral names some rational number.

B. The bar drawn over the repeating digits is called a vinculum.

C. The repeating block of digits is called a repetend.

IV. Vocabulary To Stress:

repeating decimal terminating decimal

V. Activities:

The following method may be used to change a repeating decimal to a fraction.

Example: Show that $.\overline{81}$ names a rational number.

Let $n = .\overline{81} = .81\overline{81}$

and $100 \times n = 100 \times .81\overline{81}$

$$\begin{array}{rl} \text{then } 100n = & 81.\overline{81} \\ \text{(subtract) } n = & .\overline{81} \\ \hline 99n = & 81 \\ n = & \frac{81}{99} = \frac{9}{11} \end{array}$$

Therefore: $.\overline{81} = \frac{9}{11}$

Refer to level 24 concept-numeral and Modern Mathematics (H.M. Book 8, 1967), p. 230-231, examples 1, 2, 3.

Text References:

Book: 8

Houghton Mifflin (1967, 1970) pp. 230-231
Houghton Mifflin (1972) pp. 261, 400
Addison-Wesley (1971) pp. 280, 336-338

19-26

Level: 19 Step: Z Concept: Numeral

Worksheet

Change the following repeating decimals to mixed numerals or fractions in simplest form.

1. $.58\overline{3}$ 1. ______________

2. $.\overline{09}$ 2. ______________

3. $.8\overline{3}$ 3. ______________

4. $.\overline{3}$ 4. ______________

5. $.\overline{1}$ 5. ______________

6. $5.\overline{12}$ 6. ______________

7. $.\overline{6}$ 7. ______________

8. $3.\overline{426}$ 8. ______________

9. $.\overline{45}$ 9. ______________

10. $1.2\overline{56}$ 10. ______________

Level: 19 Step: Z Concept: Numeral

ENRICHMENT EVALUATION

Change the following decimal numerals to mixed numerals or fractions in simplest form.

Work Space Answer

1. $1.\overline{12}$ 1. ____________

2. $.\overline{312}$ 2. ____________

3. $5.\overline{4}$ 3. ____________

4. $1.\overline{618}$ 4. ____________

5. $.\overline{1465}$ 5. ____________

Level: 19 Step: Z Concept: Measurement

I. Concept:

Measurement: Finding the greatest possible error.

II. Behavioral Objective:

The student given the length of a segment to the nearest unit will be able, by using greatest possible error, to determine the smallest and largest length the segment may have.

III. Mathematical Ideas:

A. Every measurement is an approximation.

B. The greatest possible error is one-half the unit of measure used.

C. The smaller the unit of measure used, the more precise the measurement and the smaller the greatest possible error.

IV. Vocabulary To Stress:

greatest possible error precision

Text References:

Book: 7

Houghton Mifflin (1967, 1970)	pp. 419-423
Addison-Wesley (1971)	pp. 278-281(55)

Book: 8

Houghton Mifflin (1967, 1970)	pp. 202-207, (32)
Houghton Mifflin (1972)	p. 23
Addison-Wesley (1971)	pp. 196-198, (38)

Level: 19 Step: Z Concept: Measurement

ENRICHMENT EVALUATION

1. Given a measurement of $1\frac{1}{2}$", the greatest possible error is

_______________.

2. Which of the following inequalities describes the measurement of $\overline{AB}$ correctly?

A ——— $1\frac{1}{2}$" ——— B

____

A) $1\frac{1}{4} \leq 1\frac{1}{2} < 1\frac{3}{4}$ B) $1\frac{1}{4} < 1\frac{1}{2} < 1\frac{3}{4}$ C) $1\frac{3}{8} \leq 1\frac{1}{2} < 1\frac{5}{8}$

3. Given a measurement of 2" the greatest possible error is

_______________.

4. Which of the following inequalities describes the measurement of $\overline{CD}$ correctly?

C ——— 2" ——— D

____

A) $1\frac{3}{4} \leq 2 < 2\frac{1}{2}$ B) $1\frac{1}{2} \leq 2 < 2\frac{1}{2}$ C) $1\frac{3}{4} \leq 2 < 2\frac{1}{4}$

5. Which of the following inequalities describes the measurement of $\overline{MN}$ correctly?

M ——— $\frac{3}{4}$" ——— N

____

A) $\frac{1}{2} \leq \frac{3}{4} < 1$ B) $\frac{3}{8} \leq \frac{3}{4} < 1\frac{1}{8}$ C) $\frac{5}{8} \leq \frac{3}{4} < \frac{7}{8}$

Level: 19

ANSWERS TO WORKSHEETS

19-2

1. T	8. T	15. $\subset$	21. $\subset$
2. F	9. F	16. $\subseteq$	22. $\subseteq$
3. T	10. T	17. $\not\subseteq$	23. $\in$
4. T	11. $\notin$	18. $\in$	24. $\notin$
5. F	12. $\subseteq$	19. $\subset$	25. $\not\subset$
6. F	13. $\not\subset$	20. $\not\subseteq$	26. $\notin$
7. T	14. $\not\subset$		

19-4

1. $\frac{1}{25}$	7. $\frac{1111}{10,000}$	13. $\frac{181}{200}$	19. $\frac{11}{25}$
2. $1\frac{1}{10}$	8. $\frac{3}{500}$	14. $\frac{1}{5000}$	20. $\frac{9}{200}$
3. $2\frac{3}{25}$	9. $\frac{8,153}{25,000}$	15. $7\frac{23}{100}$	21. $6\frac{47}{50}$
4. $\frac{63}{1000}$	10. $5\frac{11}{20}$	16. $23\frac{5}{8}$	22. $\frac{14}{125}$
5. $\frac{13}{250}$	11. $\frac{503}{5000}$	17. $\frac{1}{16}$	23. $\frac{11}{125}$
6. $3\frac{1}{4}$	12. $\frac{9}{10}$	18. $3\frac{3}{100}$	24. $7\frac{7}{100}$

19-6

1. a) .585	2. a) .986	3. a) .608	4. a) 5.956
b) .58	b) .99	b) .61	b) 5.96
c) .6	c) 1.0	c) .6	c) 6.0
d) 1	d) 1	d) 1	d) 6
5. a) .206	6. a) 7.895	7. a) .687	8. a) .990
b) .21	b) 7.90	b) .69	b) .99
c) .2	c) 7.9	c) .7	c) 1.0
d) 0	d) 8	d) 1	d) 1
9. a) 555.556	10. a) 26.990	11. a) 1.479	12. a) .064
b) 555.56	b) 26.99	b) 1.48	b) .06
c) 555.6	c) 27.0	c) 1.5	c) .1
d) 556	d) 27	d) 1	d) 0

Level: 19

ANSWERS TO WORKSHEETS

19-9

1. C
2. A
3. N
4. A
5. C
6. D
7. N
8. C
9. D
10. A
11. C
12. D
13. C
14. N
15. A
16. C
17. D
18. C
19. N
20. A
21. C
22. D
23. N
24. N
25. C
26. A
27. C
28. N
29. C
30. C
31. $7 + 4$
32. $7 \times (11 \times 5)$
33. $(4 + 3) + 2$
34. $d \times c$
35. $(u \times v) + (u \times w)$
36. $5(6 + 7)$
37. 5×8
38. $(a + m) + n$
39. $(7 \times 4) + (7 \times 8)$
40. $8 + (4 \times 3)$

19-12

1. $\overrightarrow{BA}$ or $\overrightarrow{BZ}$
2. $\overline{YZ}$ or $\overline{ZY}$
3. $\overline{XW}$ or $\overline{WX}$
4. $\overline{ZW}$ or $\overline{WZ}$
5. $\overleftrightarrow{XV}$ or $\overleftrightarrow{VX}$ (or any 2 points on the line can be substituted for those given.)
6. $\overline{WV}$ or $\overline{VW}$
7. $\{Z\}$
8. $\overline{AB}$ or $\overline{BA}$
9. $\overleftrightarrow{AB}$ or $\overleftrightarrow{BA}$ (or any 2 points on the line can be substituted for those given.)
10. ϕ
11. $\overleftrightarrow{EB}$
12. $\{D\}$
13. $\overleftrightarrow{ED}$
14. ϕ
15. $\{E\}$

19-13

1. $\{Q\}$
2. $\overleftrightarrow{PO}$ or $\overleftrightarrow{OP}$
3. ϕ
4. $\{N\}$
5. $\overleftrightarrow{ML}$ or $\overleftrightarrow{LM}$
6. $\{D\}$
7. $\{E\}$
8. ϕ
9. $\overrightarrow{DB}$
10. $\{D\}$
11. $\{B\}$
12. ϕ
13. $\overrightarrow{EB}$
14. $\angle CBA$
15. $\overline{CB}$
16. $\overline{AB}$
17. $\overrightarrow{BA}$
18. $\angle EDB$
19. $\overleftrightarrow{CB}$
20. $\overline{BD}$
21. ϕ
22. $\overrightarrow{BC}$
23. $\overline{CB}$
24. ϕ
25. Δ DBE

19-16

1. 5
2. 17
3. 725,500
4. 2
5. 51
6. 1
7. 8
8. 50
9. 2
10. 7

19-32

Level: 19

ANSWERS TO WORKSHEETS

19-17

1. 0
2. 20
3. 121
4. 25
5. 10
6. 36
7. 58
8. 81
9. 0
10. 4

19-20

1. $.37\overline{8}$, .3799, .38, $.3\overline{80}$
2. $2.\overline{29}$, $2.2\overline{9}$, 2.3, $2.\overline{3}$
3. .624, .6249, $\frac{5}{8}$, $.\overline{6}$
4. $4.0\overline{7}$, 4.079, 4.08, 4.16
5. .6, $.\overline{65}$, $\frac{2}{3}$, .67
6. $9.\overline{89}$, $9\frac{9}{10}$, 9.98, 9.99
7. $4.0\overline{47}$, $4.0\overline{6}$, 4.073, 4.101
8. 6.025, 6.0999, 6.1, 6.159
9. .749, $\frac{3}{4}$, .751, $.\overline{7}$
10. 1.2345, 1.2435, 1.2453, 1.2534
11. $\frac{7}{8}$, .877, $.\overline{87}$, $.\overline{8}$
12. 1.099, 1.101, 1.11, $1.\overline{1}$
13. .055, $\frac{1}{2}$, .505, $.\overline{50}$
14. $67.\overline{607}$, 67.67, $67.\overline{76}$, $67.\overline{7}$
15. 1.031, 1.034, 1.043, 1.304

19-23

1. 0
2. $\frac{4}{3}$ or $1\frac{1}{3}$
3. 1
4. $\frac{17}{15}$ or $1\frac{2}{15}$
5. $\frac{1}{12}$
6. 20
7. 0
8. $\frac{51}{4}$ or $12\frac{3}{4}$
9. 47
10. $\frac{51}{10}$ or $5\frac{1}{10}$

19-24

1. $\frac{3}{16}$
2. $3\frac{3}{4}$
3. 1
4. $\frac{33}{56}$
5. $\frac{2}{5}$
6. $3\frac{4}{5}$
7. 11
8. $\frac{1}{2}$
9. $\frac{1}{5}$
10. $\frac{5}{6}$

Level: 19

ANSWERS TO ENRICHMENT (STEP Z)

WORKSHEETS

19-26

1. $\frac{7}{12}$
2. $\frac{1}{11}$
3. $\frac{5}{6}$
4. $\frac{1}{3}$
5. $\frac{1}{9}$
6. $5\frac{4}{33}$
7. $\frac{2}{3}$
8. $3\frac{142}{333}$
9. $\frac{5}{11}$
10. $1\frac{127}{495}$

EVALUATIONS

19-27

1. $1\frac{4}{33}$
2. $\frac{104}{333}$
3. $5\frac{4}{9}$
4. $\frac{539}{333}$ or $1\frac{206}{333}$
5. $\frac{1465}{9999}$

19-29

1. $\frac{1}{4}$"
2. a
3. $\frac{1}{2}$"
4. b
5. c

Level: 19 ANSWERS - Post Test I

Step A

Sets

1. $\not\subset$
2. [illegible]
3. $\subset$
4. $\not\subseteq$
5. $\subset$

Numeral

6. $\frac{3}{5}$
7. $\frac{1}{25}$
8. $\frac{41}{500}$
9. $\frac{4}{25}$
10. $\frac{56}{125}$

Order

11. 4.6
12. 1.737
13. 23
14. .80
15. 8.1

Add., Sub., Mult.

16. c distributive
17. a commutative
18. d none of these
19. b associative
20. d none of these

Division

21. .9
22. .20
23. .86
24. .034
25. .487

Geometry

26. c
27. b
28. d
29. b
30. a

Measurement

31. $3\frac{0}{2}$"
32. $3\frac{1}{4}$"
33. $3\frac{2}{8}$"
34. $3\frac{0}{2}$"
35. $2\frac{3}{4}$"

Number Sentences & Phrases

36. 2
37. 64
38. 3
39. 2
40. 8

Step B

Numeral

41. .125
42. 1.4
43. $.1\overline{3}$
44. $.58\overline{3}$
45. $.\overline{1}$

Order

46. .09, .1, .101, $.\overline{1}$
47. .625, $.6\overline{3}$, $.\overline{64}$, 1.6
48. .003, .03, $.\overline{03}$, .303
49. 9.89, 9.899, 9.9, 9.98
50. .0731, .074, .08, .0807

Division

51. $\frac{16}{25}$, 64%
52. $\frac{9}{100}$, .09
53. .8 or .80, 80%
54. $\frac{5}{4}$, or $1\frac{1}{4}$, 125%
55. $\frac{37}{100}$, .37

Number Sentences & Phrases

56. $\frac{5}{12}$
57. 5
58. 3
59. 1
60. $\frac{4}{3}$ or $1\frac{1}{3}$

Level: 19 ANSWERS - Post Test II

Step A

<u>Sets</u>

1. b
2. a
3. b
4. a
5. c

<u>Numeral</u>

6. $\frac{13}{50}$
7. $\frac{3}{40}$
8. $\frac{3}{8}$
9. $\frac{3}{50}$
10. $\frac{7}{10}$

<u>Order</u>

11. .1
12. 6
13. 7.898
14. 5.96
15. .100

<u>Add., Sub., Mult.</u>

16. b
17. d
18. a
19. b
20. c

<u>Division</u>

21. .5
22. .16
23. .38
24. .061
25. .084

<u>Geometry</u>

26. c
27. a
28. b
29. c
30. a

<u>Measurement</u>

31. 3"
32. $2\frac{1}{2}$"
33. $2\frac{3}{4}$"
34. $2\frac{5}{8}$"
35. $1\frac{0}{4}$"

<u>Number Sentences & Phrases</u>

36. 5
37. 87
38. 0
39. 36
40. 8

Step B

<u>Numeral</u>

41. .5625
42. $.\overline{18}$
43. .7
44. .625
45. $.1\overline{6}$

<u>Order</u>

46. 7.1, $7.\overline{1}$, 7.12, 7.2
47. 1.01, 1.1, $1.\overline{1}$, 1.2
48. 3.8251, 3.826, 3.83, $3.\overline{83}$
49. .04, .3911, .4, .401
50. $.5\overline{12}$, .5136, .514, .516

<u>Division</u>

51. $\frac{7}{25}$, 28%
52. $\frac{11}{100}$, .11
53. .15, 15%
54. $\frac{3}{8}$, 37.5% or $37\frac{1}{2}$%
55. $\frac{13}{50}$, .26

<u>Number Sentences & Phrases</u>

56. 1
57. $\frac{1}{2}$
58. $1\frac{1}{2}$
59. $\frac{1}{6}$
60. $\frac{5}{8}$

TABLE OF CONTENTS

LEVEL 20

Step A

Step B

Step C

Step Z

Level: 20 Step: A Concept: Order

I. Concept:

<u>Order:</u> Comparing fractional numbers using the comparison property.

II. Behavioral Objective:

The student given two fractional numbers will be able to compare them by using the comparison property.

III. Mathematical Ideas:

A. Comparison property (Trichotomy property): If a, b, c, and d denote whole numbers such that if $b \neq 0$ and $d \neq 0$, then one and only one of the following statements is true:

$$\frac{a}{b} < \frac{c}{d}, \qquad \frac{a}{b} = \frac{c}{d}, \qquad \frac{a}{b} > \frac{c}{d}.$$

B. By finding common denominators and then comparing the numerators, $a \cdot d$ is related to $b \cdot c$ in the same manner that $\frac{a}{b}$ is related to $\frac{c}{d}$. Thus,

$$\frac{a}{b} < \frac{c}{d} \text{ if and only if } ad < bc. \ (b, d \neq 0)$$

$$\frac{a}{b} > \frac{c}{d} \text{ if and only if } ad > bc. \ (b, d \neq 0)$$

$$\frac{a}{b} = \frac{c}{d} \text{ if and only if } ad = bc. \ (b, d \neq 0)$$

IV. Vocabulary To Stress:

comparison property for rational numbers

V. Activities:

Review symbols for greater than, less than, and equal to.

Text References:

Book: 6

Addison-Wesley (1971) pp. 132-133, 328

Book: 7

Houghton Mifflin (1967, 1970) pp. 359-363, (55)
Houghton Mifflin (1972) p. 148
Addison-Wesley (1971) pp. 198-207, 210, 349, (37, 38)

Book: 8

Addison-Wesley (1971) pp. 125-127, (25)

Level: 20 Step: A Concept: Geometry

I. Concept:

Geometry: Separating lines, planes, and spaces.

II. Behavioral Objective:

The student given a drawing will be able to identify points and lines in half-lines, half-planes and half-spaces.

III. Mathematical Ideas:

A. A point on a line separates the line into two half-lines.

B. A line separates a plane into two half-planes.

C. A plane separates space into two half-spaces.

IV. Vocabulary To Stress:

separation	half-lines	half-spaces	boundary (edge)
opposite rays	half-planes	origin	

V. Activities:

A. See-through geometric figures (plastic boxes) can be used to illustrate the vocabulary.

B. Review of vocabulary on pp. 197-198 in Houghton Mifflin (1967, 1970) may be necessary since these words are used in some of the problems.

Text References:

Book: 7

Houghton Mifflin (1967, 1970) pp. 252-257, 262-267, (39, 41)
Addison-Wesley (1971) pp. 128-130

Book: 8

Addison-Wesley (1971) p. 177

Level: 20 Step: A Concept: Measurement

I. Concept:

Measurement: Renaming metric units of capacity as other units of capacity in the metric system.

II. Behavioral Objective:

The student given units of capacity in the metric system will be able to rename them in other metric units of capacity.

III. Mathematical Ideas:

A. The metric system follows the pattern of the decimal numeral system. Any metric unit can be changed to other metric units by either multiplying or dividing by powers of 10.

B. The liter is the basic unit of capacity (the amount the container will hold). It is a little larger than the liquid quart.

C. 1000 liters is a kiloliter.

D. One liter is equal to 100 centiliters.

E. One liter is equal to 1000 milliliters.

F. One centiliter is equal to 10 milliliters.

G. One milliliter is equal to one cubic centimeter (cc).

IV. Vocabulary To Stress:

liter (l)	prefixes: centi means $\frac{1}{100}$
kiloliter (kl)	
centiliter (cl)	milli means $\frac{1}{1000}$
milliliter (ml)	
cubic centimeter (cc)	kilo means 1000

V. Activities:

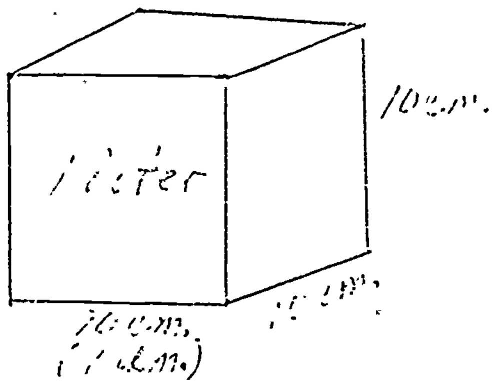
Note to the teacher: The following information may be helpful in teaching units of capacity. It is quoted from "Introducing the Metric System" programmed for Coronet Instructional Films by Educational Methods, Incorporated, p. 52.

"Volume is the measure of the space bounded by any closed solid figure. In referring to the inside volume of a container, we often use the term capacity. Capacity is the amount the container will hold.

The liter (rhymes with "meter") is the basic unit of capacity in the metric system. It is used to measure liquids such as milk and gasoline. The scientists who designed the metric system decided how much a liter should be by constructing a cubic container with inside dimensions of exactly 1 decimeter (10 cm) on all sides. When the container was filled with pure water of a certain temperature, the amount of liquid it held was called 1 liter."

Level: 20 Step: A Concept: Measurement

V. Activities: Continued



The volume of the container is 10 x 10 x 10 or 1000 cubic centimeters. Since 1 liter = 1000 cubic centimeters, and 1 liter = 1000 milliliters, then 1 cc = 1 ml. A milliliter or a cubic centimeter would hold about as much liquid as a thimble.

Text References:

Book: 7

Houghton Mifflin (1972) pp. 20-22
Addison-Wesley (1971) pp. 324, 327

Book: 8

Houghton Mifflin (1967, 1970) pp. 192-195
Addison-Wesley (1971) pp. 256-257

Other References:

<u>Introducing The Metric System</u>, Educational Methods, Inc. for Coronet Instructional Films

<u>Metric Measurement</u>, Minneapolis Public Schools

<u>The Arithmetic Teacher</u>, April & May, 1973

Level: 20 Step: A Concept: Measurement

WORKSHEET

1. 1 kiloliter = ____________ liters.

2. 100 centiliters = 1 ____________.

3. 6000 liters = ____________ kiloliters.

4. 2 liters = ____________ milliliters.

5. ____________ liters = 25 kiloliters.

6. 400 centiliters = ____________ milliliters.

7. 400 centiliters = ____________ liters.

8. 3000 milliliters = ____________ liters.

9. 50 centiliters = ____________ liters.

10. A ____________ is a little larger than a quart.

Since 1 kiloliter (abbreviation kl.) = 1000 liters (l.), then 2.3 kl. = 2.3 x 1000 l. or 2300 liters.

Express each of the following as liters:

11. 1.730 kl. = ____________ l.

12. 2.050 kl. = ____________ l.

13. .6034 kl. = ____________ l.

14. .0052 kl. = ____________ l.

Fill in the following blanks:

	liter	centiliter	milliliter
Example	2.5	250	2500
15.		47.6	
16.			6541
17.	.976		
18.		1.08	

Level: 20 Step: A Conccept: Measurement

WORKSHEET

Fill in the blanks:

1. The prefix centi means __________. Therefore, one liter is equal to ______________ centiliters.
2. The prefix milli means __________. Therefore, ____________ milliliters = 1 liter.
3. The prefix kilo means ____________. Therefore, ____________ liters = 1 kiloliter.
4. 17,000 ml. = ____________ l.
5. 2 cl. = ____________ ml.
6. 500 ml. = ____________ l.
7. 7 kl = ____________ l.
8. 7 l. = ____________ cl.
9. 7 cl. = ____________ ml.
10. 1500 l. is equal to 1 kl. and ____________ l.

Complete the following chart.

	Liters =	Kiloliters	+	Liters
Example	2362 =	2	+	362
11.	1756 =		+	
12.	5100 =		+	
13.	4768 =		+	
14.	6903 =		+	
15.	84,321 =		+	

16. One cubic centimeter is equal to 1 milliliter.

If a doctor needed 500 cc. for a blood transfusion, what part of a liter would this be?

____

17. 10 cc. = ____________ ml.
18. 1000 cc. = ____________ l.

Level: 20 Step: A Concept: Measurement

WORKSHEET

Convert the following metric units of capacity.

1. 78 cl. = ______________ ml.

2. 376 l. = ______________ cl.

3. 432 cl. = ______________ l.

4. 4800 ml. = ______________ cl.

5. 340 cl. = ______________ ml.

6. 9 kl. = ______________ l.

7. 357,000 l. = ______________ kl.

8. 83 l. = ______________ ml.

9. 7 kl. = ______________ ml.

10. 73,000 ml. = ______________ l.

11. 5200 cl. = ______________ l.

12. 381 l. = ______________ ml.

13. 92 l. = ______________ cl.

14. 794,000 l. = ______________ kl.

15. 3,000,000 ml. = ______________ kl.

16. 45 kl. = ______________ cl.

17. 92 l. = ______________ cl.

18. 291 cl. = ______________ l.

19. 10 ml. = ______________ cl.

20. 400 ml. = ______________ l.

21. 2400 cl. = ______________ l.

22. 5 kl. = ______________ l.

23. 6500 l. = ______________ kl.

24. 57 cl. = ______________ ml.

25. 4 l. = ______________ ml.

Fill in the following chart.

kiloliter	liter	centiliter	milliliter
26.		3000	
27.	2.34		
28.			856,000
29. .007685			
30.			111.1

Level: 20 Step: A Concept: Number Sentences & Phrases

I. Concept:

Number Sentences and Phrases: Solving proportions.

II. Behavioral Objective:

The student given a proportion with one variable will be able to solve for the value of the variable.

III. Mathematical Ideas:

A. A ratio is a fractional number used to compare the cardinal number of two sets of like elements.

B. If two ratios are equal, the statement is called a proportion.

C. If two ratios are equal, the product of the means is equal to the product of the extremes.

If $\frac{a}{b} = \frac{c}{d}$ $(b, d \neq 0)$, then $ad = bc$.

IV. Vocabulary To Stress:

proportion ratio means extremes

V. Activities:

A good review of the meaning of ratio is important before solving proportions. Ratio is introduced in Level 14.

Text References:

Book: 7

Houghton Mifflin (1967, 1970) pp. 464-468, 475, (72)
Houghton Mifflin (1972) pp. 142, 317, 330-335, 430 (64)
Addison-Wesley (1971) pp. 348-349, 351-357, (69, 70)

Book: 8

Houghton Mifflin (1972) pp. 322, 416, 430 (57)
Addison-Wesley (1971) pp. 152-154, (25, 34)

Other References:

Imperial Tapes (Intermediate), Lesson 30 and Tape.

Level: 20 Step: A Concept: Number Sentences & Phrases

WORKSHEET

ANSWERS

Solve for N in the following proportions:

1. $\frac{28}{21} = \frac{160}{N}$

2. $\frac{21}{39} = \frac{N}{130}$

3. $\frac{N}{30} = \frac{\frac{3}{10}}{9}$

4. $\frac{N}{27} = \frac{100}{180}$

5. $\frac{18}{7} = \frac{N}{3}$

6. $\frac{60}{90} = \frac{8}{N}$

7. $\frac{14}{4} = \frac{63}{N}$

8. $\frac{1200}{N} = \frac{2}{5}$

9. $\frac{7\frac{1}{2}}{24} = \frac{N}{16}$

10. $\frac{4.5}{6} = \frac{N}{4}$

11. $\frac{2.7}{6} = \frac{18}{N}$

12. $\frac{\frac{3}{2}}{\frac{7}{4}} = \frac{N}{14}$

13. $\frac{1\frac{1}{4}}{10} = \frac{N}{8}$

14. $\frac{N}{3.2} = \frac{5}{8}$

15. $\frac{N}{.3} = \frac{32}{6}$

16. $\frac{.003}{.06} = \frac{N}{80}$

17. $\frac{N}{4.5} = \frac{7}{9}$

18. $\frac{N}{1.2} = \frac{3}{2}$

1. ________
2. ________
3. ________
4. ________
5. ________
6. ________
7. ________
8. ________
9. ________
10. ________
11. ________
12. ________
13. ________
14. ________
15. ________
16. ________
17. ________
18. ________

Level: 20 Step: B Concept: Order

I. Concept:

Order: Solving an inequali+, involving fractional numbers and one variable.

II. Behavioral Objective:

The student given an inequality involving fractional numbers and one variable will be able to determine the set of whole number values for the variable.

III. Mathematical Ideas:

A. Comparison property (Trichotomy property): If a, b, c, and d denote whole numbers such that if $b \neq 0$ and $d \neq 0$, then one and only one of the following statements is true:

$$\frac{a}{b} < \frac{c}{d}, \quad \frac{a}{b} = \frac{c}{d}, \quad \frac{a}{b} > \frac{c}{d}.$$

B. By finding common denominators and then comparing the numerators, $a \cdot d$ is related to $c \cdot b$ in the same manner that $\frac{a}{b}$ is related to $\frac{c}{d}$. Thus,

$$\frac{a}{b} < \frac{c}{d} \text{ if and only if } ad < cd; \quad (b, d \neq 0)$$

$$\frac{a}{b} > \frac{c}{d} \text{ if and only if } ad > cd. \quad (b, d \neq 0)$$

C. By definition, division is the inverse operation of multiplication. This definition does not hold when the divisor is zero and therefore division by zero is undefined.

Example (1): If $\frac{6}{0} = n$, then $6 = n \times 0$
In this example there is no replacement for n which will make the statement true.

(2): If $\frac{0}{0} = n$, then $0 = n \times 0$
In this example any replacement for n will satisfy the statement.

Text References:

Book: 7

Houghton Mifflin (1967, 1970) pp. 359-364

Level: 20 Step: B Concept: Order

WORKSHEET

Given the following inequalities, find all the whole number values for n.

1. $\frac{2}{5} > \frac{n}{6}$ { ________ }

2. $\frac{n}{2} < \frac{3}{4}$ { ________ }

3. $\frac{8}{3} < \frac{6}{n}$ { ________ }

4. $\frac{5}{6} > \frac{9}{n}$ { ________ }

5. $\frac{1}{n} > \frac{1}{3}$ { ________ }

6. $\frac{n}{4} > \frac{7}{9}$ { ________ }

7. $\frac{n}{9} < \frac{6}{16}$ { ________ }

8. $\frac{25}{32} > \frac{n}{10}$ { ________ }

9. $\frac{n}{9} > \frac{6}{16}$ { ________ }

10. $\frac{2n}{3} < \frac{3}{4}$ { ________ }

11. $\frac{9}{40} > \frac{2}{3n}$ { ________ }

12. $\frac{3}{8} < \frac{2}{n}$ { ________ }

13. $\frac{6}{11} < \frac{n}{9}$ { ________ }

14. $\frac{12}{17} > \frac{n}{3}$ { ________ }

15. $\frac{2n}{7} > \frac{4}{5}$ { ________ }

16. $\frac{1}{4} > \frac{n}{3}$ { ________ }

20-12

Level: 20 Step: B Concept: Order

WORKSHEET

Find all the whole number values for n:

1. $\frac{n}{6} < \frac{4}{8}$ { ________ }

2. $\frac{7}{n} < \frac{2}{8}$ { ________ }

3. $\frac{2\frac{1}{2}}{7} > \frac{5}{n}$ { ________ }

4. $\frac{9}{10} > \frac{18}{n}$ { ________ }

5. $\frac{3}{4} < \frac{n}{8}$ { ________ }

6. $\frac{2}{3} > \frac{n}{8}$ { ________ }

7. $\frac{3}{2} < \frac{n}{36}$ { ________ }

8. $\frac{1}{4} < \frac{n}{9}$ { ________ }

9. $\frac{n}{6} < \frac{3}{10}$ { ________ }

10. $\frac{9}{4} > \frac{n}{2}$ { ________ }

11. $\frac{1}{3} > \frac{2n}{9}$ { ________ }

12. $\frac{5}{6} < \frac{4}{n}$ { ________ }

13. $\frac{7}{9} > \frac{n}{4}$ { ________ }

14. $\frac{11}{17} < \frac{n}{4}$ { ________ }

Level: 20 Step: B Concept: Geometry

I. Concept:

Geometry: Related Angles.

II. Behavioral Objective:

The student given a drawing will be able to identify the adjacent, vertical, and straight angles and name points or segments in their interior or exterior.

III. Mathematical Ideas:

A. An angle is the union of two rays with a common endpoint.

B. An angle divides a plane into three disjoint subsets of points.

C. Adjacent angles are two angles having a common vertex and a common side but whose interiors have no points in common.

D. Vertical angles are two nonadjacent angles formed by two intersecting lines.

E. Vertical angles are congruent.

F. A straight angle is an angle whose sides form a straight line.

IV. Vocabulary To Stress:

interior	adjacent angles	straight angles
exterior	vertical angles	

V. Activities:

A. Use geo-board for demonstration.

B. Review symbols for line, segment and angle.

Text References:

Book: 7

Houghton Mifflin (1967, 1970) pp. 267-273, (42)
Addison-Wesley (1971) pp. 267-268

Book: 8

Addison-Wesley (1971) pp. 178-179, 183, 189-190

Level: 20 Step: B Concept: Geometry

WORKSHEET

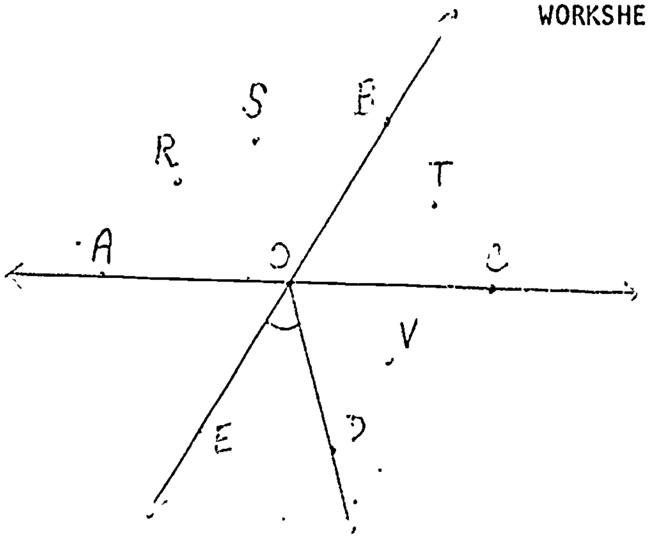


Figure 1

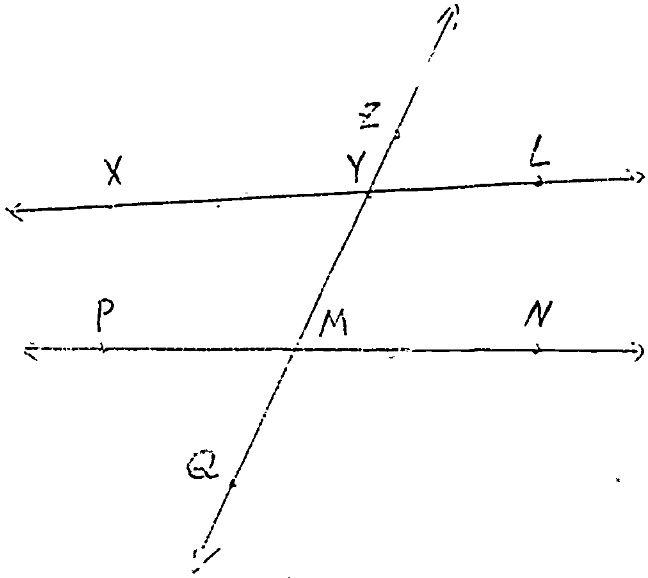


Figure 2

Refer to Figure 1 to answer questions 1-10.

1. All the points that lie in the interior of ∠ AOB are ________________.
2. All the points that lie in the exterior of ∠ AOB are ________________.
3. Two angles adjacent to ∠ EOD are ________________.
4. An angle that is vertical to ∠ AOE is ________________.
5. All the points that lie in the interior of ∠ EOC are ________________.
6. A segment lying wholly within the interior of ∠ AOB is ________________.
7. All the points lying in the exterior of ∠ EOC are ________________.
8. ∠ AOB and ∠ EOC are called ________________ angles.
9. ∠ EOB and ∠ AOC are called ________________ angles.
10. ∠ DOC and ∠ COB are called ________________ angles.

Refer to Figure 2 to answer questions 11-15.

11. ∠ XYZ and ∠ ________________ are vertical angles.
12. Two angles adjacent to ∠ PMY are ∠ YMN and ∠ ________________.
13. ∠ PMQ and ∠ YMN are ________________ angles.
14. ∠ ZYL and ∠ LYM are ________________ angles.
15. ∠ NMQ is vertical to ∠ ________________.

Level: 20 Step: B Concept: Number Sentences & Phrases

I. Concept:

Number Sentences & Phrases: Solving percent problems.

II. Behavioral Objective:

The student given a percent problem will be able to solve it by writing either a proportion or a number sentence.

III. Mathematical Ideas:

A. A percent may be written as a ratio in which the second term is 100.

B. If two ratios are equal, the statement is called a proportion.

C. Basic property of fractions: If a, b, and c denote whole numbers, and if $b \neq 0$ and $c \neq 0$, then:

$$\frac{a}{b} = \frac{a \times c}{b \times c} \text{ and } \frac{a \div c}{b \div c} = \frac{a}{b}$$

D. If two ratios are equal, the product of the means is equal to the product of the extremes.

$$\text{If } \frac{a}{b} = \frac{c}{d} \quad (b,\ d \neq 0), \text{ then } \frac{a \times d}{b \times d} = \frac{b \times c}{b \times d}$$

and thus $ad = bc$.

IV. Vocabulary To Stress:

ratio proportion extremes means

V. Activities:

Text References:

Book: 7

Houghton Mifflin (1967, 1970) pp. 468-475, (74)
Houghton Mifflin (1972) pp. 130-131, 145, 268-269, 418, 425
Addison-Wesley (1971) pp. 358-362, (73)

Book: 8

Houghton Mifflin (1972) pp. 71, 150
Addison-Wesley (1971) p. 284

20-16

Level: 20 Step: B Concept: Number Sentences & Phrases

WORKSHEET

In problems 1-3, solve by writing a proportion and also by writing a number sentence. Show your work.

	Proportion	Number Sentence
1. 75% of 250 is ________.		
2. 14 is ________ % of 20.		
3. 12 is 40% of ________.		

Solve the following by writing either a proportion or a number sentence. Show your proportion or number sentence.

4. 90 is ________ % of 1800.
5. 3% of 200 = ________.
6. 3 is ________ % of 5.
7. of ________ is 4.
8. $33\frac{1}{3}$ % of 768 is ________.
9. 6 is 12% of ________.
10. 95% of 5300 is ________.
11. 56 is ________ % of 80.
12. 40 is 50% of ________.
13. 6% of 325 is ________.
14. 36 is ________ % of 45.
15. 40% of 65 is ________.
16. 39 is ________ % of 52.
17. 18 is 15% of ________.
18. 20% of 735 is ________.
19. 200% of 735 is ________.
20. 50% of 62 = ________.

Level: 20 Step: C Concept: Geometry

I. Concept:

Geometry: Determining degree measurement without a protractor.

II. Behavioral Objective:

The student given sufficient angle measurement in a drawing will be able to find the measurement of the other angles without a protractor.

III. Mathematical Ideas:

A. The sum of the measures of supplementary angles is 180°.

B. The sum of the measures of the angles of a triangle is 180°.

C. The sum of the measures of complementary angles is 90°.

D. Vertical angles are congruent.

IV. Vocabulary To Stress:

supplementary angles complementary angles vertical angle

V. Activities:

Tear or cut the vertices of a triangle and arrange them as a straight angle to show that the sum of the degree measures of a triangle is 180°.

Text References:

Book: 7

Houghton Mifflin (1967, 1970) pp. 425, 428, 430-431
Houghton Mifflin (1972) p. 276
Addison-Wesley (1971) pp. 267, 278 (79-part)

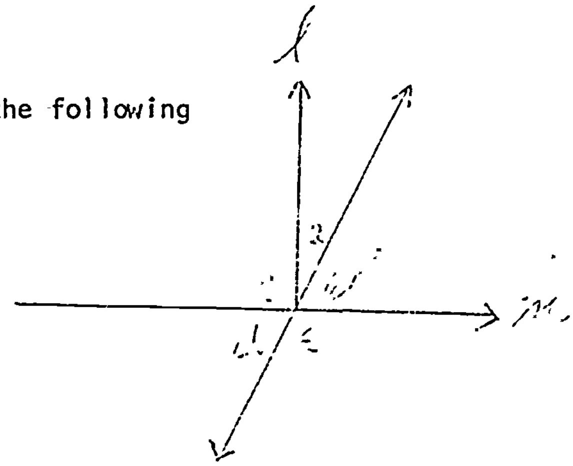
Book: 8

Addison-Wesley (1971) pp. 301-303 (69)

Level: 20 Step: C Concept: Geometry

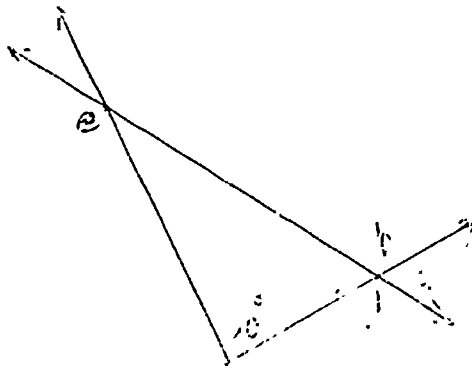
WORKSHEET

Refer to the diagram at the right to answer the following questions. ℓ is perpendicular to m.

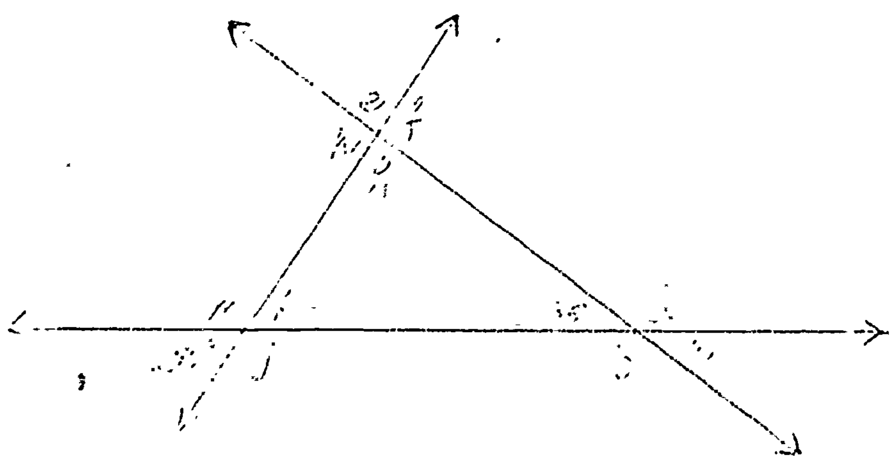


1. ∠ a measures ________ degrees.
2. ∠ c measures ________ degrees.
3. ∠ d measures ________ degrees.
4. ∠ e measures ________ degrees.

In the diagram at the right,



5. ∠ c measures ________ degrees.
6. ∠ d measures ________ degrees.
7. ∠ e measures ________ degrees.
8. ∠ a measures ________ degrees.

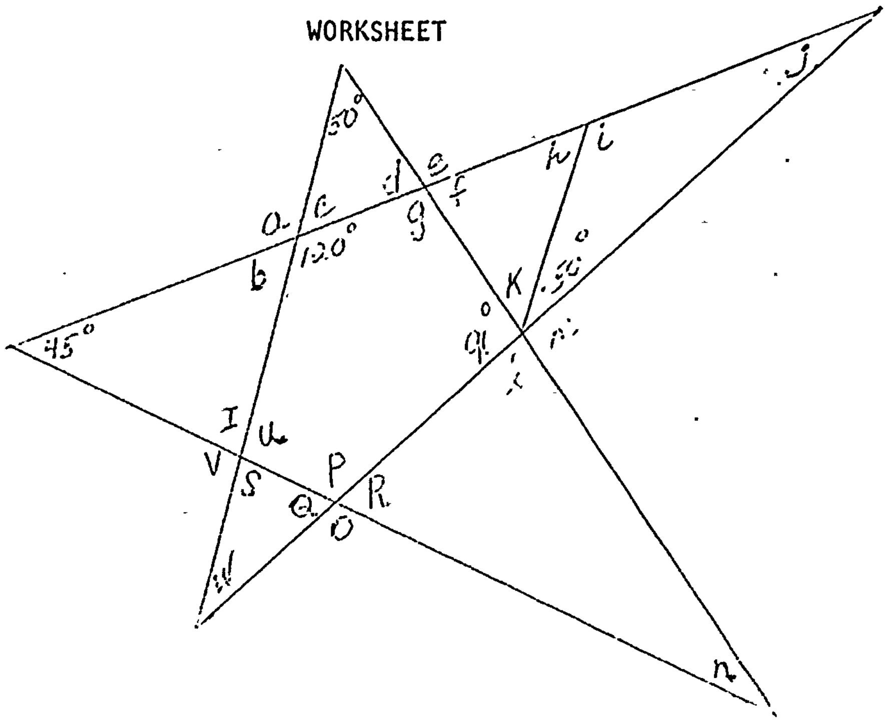


Refer to the diagram above and give the degree measure of each of the following angles.

9. ∠ m measures ________ degrees.
10. ∠ b measures ________ degrees.
11. ∠ e measures ________ degrees.
12. ∠ c measures ________ degrees.
13. ∠ d measures ________ degrees.
14. ∠ k measures ________ degrees.
15. ∠ g measures ________ degrees.
16. ∠ h measures ________ degrees.
17. ∠ j measures ________ degrees.
18. ∠ f measures ________ degrees.

Level: 20 Step: C Concept: Geometry

WORKSHEET



I. Use what you know about complementary angles, supplementary angles, vertical angles, and the sum of the angles of a triangle to find the degree measure of the following angles.

a. ____	f. ____	j. ____	o. ____	t. ____
b. ____	g. ____	k. ____	p. ____	u. ____
c. ____	h. ____	l. ____	q. ____	v. ____
d. ____	i. ____	m. ____	r. ____	w. ____
e. ____	j. ____	n. ____	s. ____	

II.

1. What is the sum of the degree measures of the angles of the pentagon in the center of the figure? __________

2. If you draw the diagonals from one vertex of the pentagon, you would have ________ triangles?

3. What should be the total number of degrees in the sum of the angles of these triangles? ________ Check this with your answer in 1. They should be the same.

4. What is the sum of the degree measures of the four angles of any quadrilateral in the drawing? __________

Level: 20 Step: C Concept: Number Sentences and Phrases

I. Concept:

Number Sentences and Phrases: Solving word problems using proportion.

II. Behavioral Objective:

The student given a word problem will be able to write a proportion and solve it.

III. Mathematical Ideas:

A. If two ratios are equal, the statement is called a proportion.

B. Basic property of fractions: If a, b, and c denote whole numbers and if $b \neq 0$ and $c \neq 0$ then $\frac{a}{b} = \frac{a \times c}{b \times c}$ and $\frac{a \div c}{b \div c} = \frac{a}{b}$

C. If two ratios are equal, the product of the means is equal to the product of the extremes, If $\frac{a}{b} = \frac{c}{d}$ (b, $d \neq 0$), then $\frac{a \times d}{b \times d} = \frac{b \times c}{b \times d}$ and thus ad = bc.

D. On a scale drawing all distances are proportional to the corresponding distances on the object being represented.

IV. Vocabulary To Stress:

ratio proportion means extremes

V. Activities:

Note to teacher: The intention of this behavioral objective is to cover various types of problems that can be solved using proportions such as scale drawings, rate-type (distance-time, cost-pound etc.), batting average and some percent problems. Special types of percent problems such as commission, interest and discount will be covered in level 21.

Text References:

Book: 7

Houghton Mifflin (1967, 1970) pp. 464-468 (72)
Houghton Mifflin (1972) pp. 145, 339-343
Addison-Wesley (1971) pp. 351-357 (70) (77)

Book: 8

Houghton Mifflin (1967, 1970) pp. 359-362 (60)
Houghton Mifflin (1972) pp. 323, 325, 330-331
Addison-Wesley (1971) pp. 153-158 (34,35)

Level: 20 Step: C Concept: Number Sentences and Phrases

WORKSHEET

Use a proportion to solve the following word problems:

1. In a class the ratio of the number of boys to the number of girls is 3 to 2. If there are 14 girls in the class, how many boys are in the class?

2. If two pieces of pastry cost 15 cents, find the price of a dozen.

3. The Browns traveled 352 miles in 5.5 hours. Traveling at the same rate of speed, how far can they go in 10 hours?

4. If Jerry's batting average is .300, how many hits did he make if he was 60 times at bat? (hint: change batting average to a fraction)

5. A picture is 5 inches long and 3 inches wide. If Joe wants to enlarge it so that the length is 20 inches, what will be its width?

6. In one week an organization raised $80. This was 40% of its quota. How much was its quota?

7. Jim spelled 85% of the 40 words on his spelling test correctly. How many words did he spell correctly?

8. Mary bought a 12 oz. box of candy. This was what percent of a pound?

9. An ocean liner is 990 feet long. How long is a model of the ship made to a scale of 1" = 100'?

10. The scale on a map is 1" = 20 miles. If the distance between two cities on the map is $12\frac{1}{2}$ inches, what is the actual distance in miles between the two cities?

11. A recipe calls for 4 cups of flour to 6 tablespoons of shortening. How many tablespoons of shortening are needed when 6 cups of flour are used?

12. If 10 oranges cost $.78, how many oranges can you buy for $3.12?

Level: 20 Step: Z Concept: Numeral

I. Concept:

Numeral: To convert a decimal numeral to a base 12 numeral and a base 12 numeral to a decimal numeral.

II. Behavioral Objective:

The student given a decimal numeral or a base 12 numeral will be able to convert it to the one not given.

III. Mathematical Ideas:

The position of a digit in a place value numeration system determines its value. The place value of each position is a power of the base of the system.

IV. Vocabulary To Stress:

T denotes "10" and E denotes "11" in the duodecimal (base 12) system.

Text References:

Book: 7

Houghton Mifflin (1967, 1970) pp. 127-129

Book: 8

Addison-Wesley (1971) p. 24

Level: 20 Step: Z Concept: Numeral

WORKSHEET

The base 12 place values are:

	248,832	20,736	1,728	144	12	1
exponent form	12^5	12^4	12^3	12^2	12	1

12

I. Change the following base 12 numerals to base 10 numerals by first writing in expanded form:

Expanded form base 10

1. 63_{12} = ____________ = ________
2. 654_{12} = ____________ = ________
3. $18T_{12}$ = ____________ = ________
4. $TE9_{12}$ = ____________ = ________
5. 19_{12} = ____________ = ________
6. 702_{12} = ____________ = ________
7. $1ET06_{12}$ = ____________ = ________
8. $TTEE_{12}$ = ____________ = ________
9. $10T6_{12}$ = ____________ = ________
10. $2T8_{12}$ = ____________ = ________

II. Change the following base 10 numerals to base 12 numerals:

1. 34 = ____________
2. 20,000 = ____________
3. 241,000 = ____________
4. 1,967 = ____________
5. 674 = ____________
6. 243 = ____________
7. 508 = ____________
8. 47 = ____________
9. 2,145 = ____________
10. 226,210 = ____________

Level: 20 Step: Z Concept: Numeral

ENRICHMENT EVALUATION

The base 12 place value are:

	248,832	20,736	1,728	144	12	1
Exponent form	12^5	12^4	12^3	12^2	12	1

12

I. Change the following base 12 numerals to base 10 numerals by first writing in expanded form:

Expanded form base 10

1. 125_{12} = ______________________ = ____________
2. $4TE_{12}$ = ______________________ = ____________
3. $3E5_{12}$ = ______________________ = ____________
4. 971_{12} = ______________________ = ____________
5. $1E5T_{12}$ = ______________________ = ____________

II. Change the following base 10 numerals to base 12 numerals:

1. 3309 = ______________
2. 1848 = ______________
3. 2305 = ______________
4. 8712 = ______________
5. 236,471 = ______________

Level: 20 Step: Z Concept: Measurement

I. Concept:

Measurement: Constructing circle and divided-bar graphs.

II. Behavioral Objective:

The student given sufficient data will be able to construct or interpret a circle and a divided bar graph.

III. Mathematical Ideas:

A. The area that represents each part must bear the same ratio to the area of the whole graph as the size of the part bears to the size of the total data.

B. The degree measure of a circular arc depends on the degree-measure of its central angle.

IV. Vocabulary To Stress:

central angle arc (major & minor) sector statistics data

Text References:

Book: 7

Houghton Mifflin (1967, 1970) pp. 481-484, (76)
Houghton Mifflin (1972) pp. 146-147 (30)

Book: 8

Houghton Mifflin (1972) pp. 72
Addison-Wesley (1971) pp. 232-235 (52)

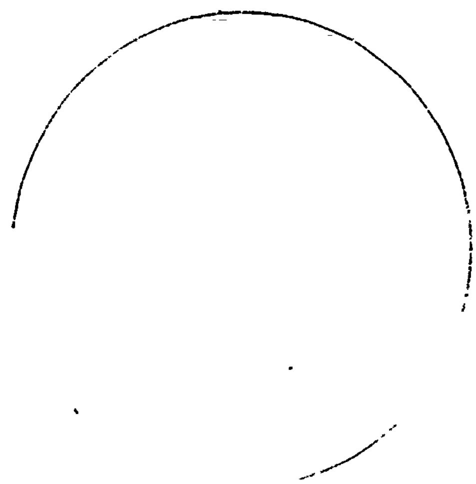
Other References:

Imperial Tapes (Intermediate), Tape 39

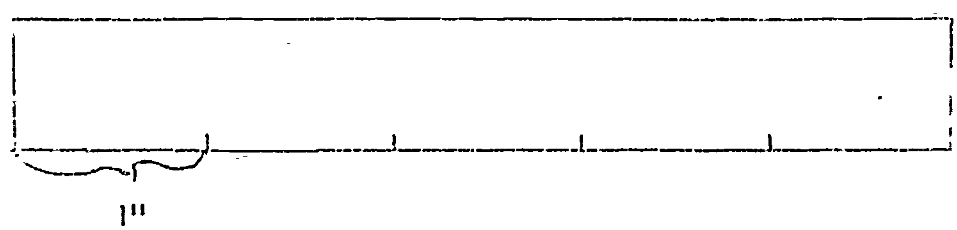
20-26

Level: 20 Step: Z Concept: Measurement

ENRICHMENT EVALUATION



1. Jim uses his allowance in the following way: $\frac{1}{12}$ for savings, $\frac{1}{4}$ for movies, $\frac{1}{8}$ for food, $\frac{3}{8}$ for hobbies, and $\frac{1}{6}$ for school supplies. Make a circle graph to show how Jim uses his money. Indicate the percents and the degrees.



2. In the election for class president Liza received 24 votes, Kim received 15 votes, Randy received 12 votes, and Bob received 9 votes. Make a divided-bar graph to show what percent of the votes each one received. Indicate the measurements and the percents.

Level: 20 ANSWERS TO WORKSHEETS

20-5

1. 1000
2. liter
3. 6
4. 2000
5. 25,000
6. 4000
7. 4
8. 3
9. $\frac{1}{2}$
10. liter
11. 1730
12. 2050
13. 603.4
14. 5.2
15. .4
16. 6.541, 654.1
17. 97.6, 976
18. .0108, 10.8

20-6

1. $\frac{1}{100}$, 100
2. $\frac{1}{1000}$, 1000
3. 1000, 1000
4. 17
5. 20
6. $\frac{1}{2}$
7. 7000
8. 700
9. 70
10. 500
11. 1 and 756
12. 5 and 100
13. 4 and 768
14. 6 and 903
15. 84 and 321
16. $\frac{1}{2}$
17. 10
18. 1

20-7

1. 780
2. 37,600
3. 4.32
4. 480
5. 3400
6. 9000
7. 357
8. 83,000
9. 7,000,000
10. 73
11. 52
12. 381,000
13. 9200
14. 794
15. 3
16. 4,500,000
17. 9,200
18. 2.91
19. 1
20. .4
21. 24
22. 5000
23. 6.5
24. 570
25. 4000

	kiloliter	liter	centiliter	milliliter
26.	.03	30	3000	30,000
27.	.00234	2.34	234	2340
28.	.856	856	85,600	856,000
29.	.007685	7.685	768.5	7685
30.	.0001111	.1111	11.11	111.1

20-9

1. 120
2. 70
3. 1
4. 15
5. $7\frac{5}{7}$
6. 12
7. 18
8. 3000
9. 5
10. 3
11. 40
12. 12
13. 1
14. 2
15. 1.6
16. 4
17. 3.5
18. 1.8

Level: 20 ANSWERS TO WORKSHEETS

20-11

1.	{0,1,2}	6.	{4,5,6...}	11.	{3,4,5...}
2.	{0,1}	7.	{0,1,2,3}	12.	{1,2,3,4,5}
3.	{1,2}	8.	{0,1,2,3,4,5,6,7}	13.	{5,6,7...}
4.	{11,12,13...}	9.	{4,5,6...}	14.	{0,1,2}
5.	{1,2}	10.	{0,1}	15.	{3,4,5...}
				16.	{0}

20-12

1.	{0,1,2}	6.	{0,1,2,3,4,5}	11.	{0,1}
2.	{29,30,31...}	7.	{55,56,57...}	12.	{1,2,3,4}
3.	{15,16,17...}	8.	{3,4,5...}	13.	{0,1,2,3}
4.	{21,22,23...}	9.	{0,1}	14.	{3,4,5}
5.	{7,8,9...}	10.	{0,1,2,3,4}		

20-14

1.	R,S	6.	$\overline{RS}$	11.	$\angle$ LYM
2.	E,D,V,C,T	7.	A,R,S,B,T	12.	$\angle$ PMQ
3.	$\angle$ DOC, $\angle$ EOA	8.	vertical or opposite	13.	vertical or opposite
4.	$\angle$ BOC	9.	straight	14.	adjacent
5.	(D,V)	10.	adjacent	15.	$\angle$ PMY

20-16

1.	187.5 or $187\frac{1}{2}$	6.	60%	11.	70%	16.	75%
2.	70%	7.	80	12.	80	17.	120
3.	30	8.	256	13.	19.5	18.	147
4.	5%	9.	50	14.	80%	19.	1470
5.	6	10.	5035	15.	26	20.	31

20-18

1.	30	6.	125	11.	95	16.	85
2.	90	7.	145	12.	35	17.	130
3.	60	8.	55	13.	145	18.	85
4.	120	9.	130	14.	50		
5.	55	10.	145	15.	95		

Level: 20 ANSWERS TO WORKSHEETS

20-19

I.

a. 120	g. 110	m. 90	s. 75
b. 60	h. 50	n. 25	t. 75
c. 60	i. 130	o. 115	u. 105
d. 70	j. 20	p. 115	v. 105
e. 110	k. 60	q. 65	w. 40
f. 70	l. 90	r. 65	

II.

1. 540
2. 3
3. 540
4. 360

20-21

1. 21	5. 12	9. 9.9 or $9\frac{9}{10}$ in.
2. 90	6. $200	10. 250 miles
3. 640	7. 34	11. 9
4. 18	8. 75%	12. 40

Level: 20

ANSWERS TO ENRICHMENT

WORKSHEETS

20-23

I.

1. $(6 \times 12) + (3 \times 1) = 75$
2. $(6 \times 12^2) + (5 \times 12) + (4 \times 1) = 928$
3. $(1 \times 12^2) + (8 \times 12) + (T \times 1) = 250$
4. $(T \times 12^2) + (E \times 12) + (9 \times 1) = 1581$
5. $(1 \times 12) + (9 \times 1) = 21$
6. $(7 \times 12) + (2 \times 12) = 1010$
7. $(1 \times 12^4) + (E \times 12^3) + (T \times 12^2) + (6 \times 1) = 41{,}190$
8. $(T \times 12^3) + (T \times 12^2) + (E \times 12) + (E \times 1) = 18{,}863$
9. $(1 \times 12^3) + (T \times 12) + (6 \times 1) = 1854$
10. $(2 \times 12^2) + (T \times 12) + (8 \times 1) = 416$

II.

1. $2T_{12}$
2. $E6T8_{12}$
3. $E7574_{12}$
4. $117E_{12}$
5. 482_{12}
6. 183_{12}
7. 364_{12}
8. $3E_{12}$
9. $12T9_{12}$
10. $TTTTT_{12}$

EVALUATION

20-24

I.

1. $(1 \times 12^2) + (2 \times 12) + (5 \times 1) = 173$
2. $(4 \times 12^2) + (T \times 12) + (E \times 1) = 707$
3. $(3 \times 12^2) + (E \times 12) + (5 \times 1) = 569$
4. $(9 \times 12^2) + (7 \times 12) + (1 \times 1) = 1381$
5. $(1 \times 12^3) + (E \times 12^2) + (5 \times 12) + (T \times 1) = 3382$

Level: 20 EVALUATION

20-24 (cont.)

II.

1. $1TE9_{12}$
2. $10T0_{12}$
3. 1401_{12}
4. 5060_{12}
5. $E4T1E_{12}$

20-26

1. Savings 30°, $8\frac{1}{3}\%$

   Movies 90°, 25%

   Food 45°, $12\frac{1}{2}\%$

   Hobbies 135°, $37\frac{1}{2}\%$

   School supplies 60°, $16\frac{2}{3}\%$

2. Liza: 40%, 2"

   Kim: 25%, $1\frac{1}{4}$"

   Randy: 20%, 1"

   Bob: 15%, $\frac{3}{4}$"

Level: 20 ANSWERS - POST TEST I

Step A

Order

1. $<$
2. $>$
3. $>$
4. $<$
5. $=$

Geometry

6. b
7. c
8. b
9. b
10. c
11. a
12. d
13. a
14. a
15. b

Measurement

16. 3000
17. 200
18. 500
19. 6
20. 7

Number Sentences & Phrases

21. {14}
22. {120}
23. $\{\frac{21}{2}$ or $10\frac{1}{2}\}$
24. {2,8}
25. {5}

Step B

Order

26. {3, 4, 5...}
27. {0, 1, 2, 3}
28. {1, 2}
29. {10, 11, 12,...}
30. {0, 1, 2, 3, 4}

Geometry

31. a
32. b
33. c
34. a
35. c
36. b
37. a
38. c
39. a
40. b

Number Sentences & Phrases

41. 135 (135.00)
42. 60%
43. 25%
44. 51 (51.00)
45. 215

Geometry

46. 60°
47. 90°
48. 120°
49. 55°
50. 125°
51. 145°
52. 85°
53. 145°
54. 50°
55. 95°

Step C

Number Sentences & Phrases

56. 15 apples
57. 68%
58. 150 miles
59. 7 inches
60. 182 boys

Level: 20 ANSWERS - POST TEST II

Step A

Order

1. >
2. =
3. <
4. =
5. >

Geometry

6. a
7. c
8. b
9. a
10. a
11. b
12. a
13. c
14. d
15. c

Measurement

16. 4
17. 600
18. 4000
19. 5
20. 800

Number Sentences & Phrases

21. $\{\frac{40}{3}$ or $13\frac{1}{3}\}$
22. {5}
23. {69}
24. {58}
25. {4}

Step B

Order

26. {1, 2}
27. {0, 1, 2, 3, 4}
28. {9, 10, 11...}
29. {8, 9, 10...}
30. {0, 1, 2, 3}

Geometry

31. c
32. b
33. b
34. c
35. c
36. a
37. b
38. b
39. c
40. c

Number Sentences & Phrases

41. 19.6
42. 5%
43. 144
44. 20%
45. 476

Step C

Geometry

46. 57°
47. 90°
48. 123°
49. 45°
50. 105°
51. 150°
52. 30°
53. 62°
54. 28°
55. 62°

Number Sentences & Phrases

56. 5 hours
57. 60 boys
58. 11 doz.
59. 6 inches
60. $2.

TABLE OF CONTENTS

Level 21

Step A

Step B

Step Z

Level: 21 Step: A Concept: Addition & Subtraction

I. Concept:

Adding and subtracting integers without using the number line.

II. Behavioral Objective:

The student given two or more integers will be able to find their sum or difference without the use of the number line.

III. Mathematical Ideas:

A. The sign of an integer indicates direction. The number indicates the absolute value (distance from zero on the number line).

B. The sum of two positive integers is a positive integer.

C. The sum of two negative integers is a negative integer.

D. The sum of two integers with unlike signs is given the sign of the integer with the greater absolute value.

E. The sum of every integer and its opposite is zero.

F. If a and b are integers, then the difference a - b is equal to the sum of a and the opposite of b.

IV. Vocabulary To Stress:

opposites absolute value

Text References:

Book: 7

Houghton Mifflin (1967, 1970) pp. 504-513, (78, 79)
Houghton Mifflin (1972) pp. 386-391, 432 (72, 73)
Addison-Wesley (1971) pp. 144-153, (26, 27)

Book: 8

Houghton Mifflin (1972) pp. 228-230, 382, 426 (39,70)
Addison-Wesley (1971) pp. 52-58 (12, 13)

Level: 21 Step: A Concept: Multiplication

I. Concept:

Multiplication: Multiplying integers.

II. Behavioral Objective:

The student given two integers will be able to find their product.

III. Mathematical Ideas:

A. The product of two positive integers is a positive integer.

B. The product of two negative integers is a positive integer.

C. The product of a positive integer and a negative integer is a negative integer.

D. Multiplicative property of $^{-}1$: The product of an integer and $^{-}1$ is the opposite of the given integer.

IV. Vocabulary To Stress:

opposite

V. Activities:

See article "Charged Particles: A Model for Teaching Operations With Directed Numbers", from The Arithmetic Teacher, May 1969, pp. 349-353.

Book: 7

Houghton Mifflin (1967, 1970)	pp. 513-517, (80-use after division)
Houghton Mifflin (1972)	pp. 398-399, 432 (75)
Addison-Wesley (1971)	pp. 154-158, (28-use after division)

Book: 8

Houghton Mifflin (1972)	pp. 238-239, 383, 427 (42, 71)
Addison-Wesley (1971)	pp. 59-63, (14, sec. 1)

Level: 21 Step: A Concept: Division

I. Concept:

<u>Division</u>: Dividing integers

II. Behavioral Objective:

The student given two integers will be able to find their quotient.

III. Mathematical Ideas:

A. Multiplication and division are inverse operations.

B. The quotient of two positive integers is a positive integer.

C. The quotient of two negative integers is a positive integer.

D. The quotient of a positive and a negative integer is a negative integer.

Text References:

Book: 7

Houghton Mifflin (1967, 1970) pp. 518-521
Houghton Mifflin (1972) pp. 400-401, 432 (76)
Addison-Wesley (1971) p. 157

Book: 8

Houghton Mifflin (1972) pp. 242, 383, 427 (42, 71)
Addison-Wesley (1971) p. 63, (14, sec. 2)

Level: 21 Step: A Concept: Graphs

I. Concept:

Graphs: Graphing integer inequalities on a number line.

II. Behavioral Objective:

The student given a compound inequality involving integers and one variable will be able to graph the set of integers on the number line.

III. Mathematical Ideas:

A. Every negative integer is less than zero or any positive integer.

B. Given two integers on a number line, the graph of the lesser lies to the left of the graph of the greater.

IV. Vocabulary To Stress:

opposites	$\leq$ means less than or equal to
compound inequality	$\geq$ means greater than or equal to

V. Activities:

A. This exercise can be used for reviewing integers on a number line.

Have the students draw 15 mile markers in one row and put L in the second mile marker. As the teacher reads A through M orally the students will put the letter in the correct milepost. When all markers are filled, have a student read the letters in order.

a. A trip from L to B is denoted by +3
b. A trip from A to L is denoted by -7.
c. A to M, +4
d. M to X, -6
e. X to P, -3
f. A to R, -6
g. B to Z, +5
h. P to C, +11
i. R to S, -2
j. M to F, -2
k. L to H, +10
l. H to T, -4
m. Q to V, -8

Lesson: 21 Step: A Concept: Graphs

V. Activities: Continued

B. Draw 12 mile markers and mark S in the 9th marker.

a. S to P, +2
b. P to R, -5
c. T to P, +7
d. S to M, +3
e. R to A, -4
f. C to M, +5
g. T to B, -3
h. R to L, +4
i. L to X, -7
j. D to Z, -3

ANSWERS: A. S-L-R-P-B-V-X-T-A-Z-F-H-M-Q-C
B. B-A-X-T-Z-R-C-D-S-L-P-M

Text References:

Book: 7

Houghton Mifflin (1967, 1970) pp. 500-504
Addison-Wesley (1971) pp. 158-160 (29-use after Step B)

Book: 8

Houghton Mifflin (1972) p. 240
Addison-Wesley (1971) p. 76

Level: 21 Step: A Concept: Graphs

WORKSHEET

On the number line, graph the set of integers that will make the inequality a true statement.

1. $-3 \leq n \leq 1$

-5 -4 -3 -2 -1 0 +1 +2 +3 +4 +5

2. $-2 < n < 3$

-5 -4 -3 -2 -1 0 +1 +2 +3 +4 +5

3. $-2 \leq n < 4$

-5 -4 -3 -2 -1 0 +1 +2 +3 +4 +5

4. $2 \geq n > -3$

-5 -4 -3 -2 -1 0 +1 +2 +3 +4 +5

5. $-5 \leq n \leq -2$

-5 -4 -3 -2 -1 0 +1 +2 +3 +4 +5

6. $-5 < n < 0$

-5 -4 -3 -2 -1 0 +1 +2 +3 +4 +5

7. $4 > n > -4$

-5 -4 -3 -2 -1 0 +1 +2 +3 +4 +5

8. $-1 \leq n \leq 5$

-5 -4 -3 -2 -1 0 +1 +2 +3 +4 +5

9. $-4 \leq n \leq 0$

-5 -4 -3 -2 -1 0 +1 +2 +3 +4 +5

10. $-1 \leq n < 1$

-5 -4 -3 -2 -1 0 +1 +2 +3 +4 +5

Level: 21 . Step: A Concept: Geometry

I. Concept:

Geometry: Extending the classification of triangles.

II. Behavioral Objective:

The student given the length of the sides or the degree measure of the angles of a triangle will be able to classify the triangle as (1) scalene, equilateral or isosceles and (2) obtuse, acute or right.

III. Mathematical Ideas:

Triangles can be classified by the lengths of the sides or the degree measure of the angles.

IV. Vocabulary To Stress:

scalene	acute	equilateral
obtuse	isosceles	right

V. Activities:

Use geo-board to illustrate types of triangles.

Tangrams--see references.

Text References:

Book: 7

Houghton Mifflin (1967, 1970)	pp. 413, 425, 427
Houghton Mifflin (1972)	pp. 279-280
Addison Wesley (1971)	pp. 269-271, 273

Book: 8

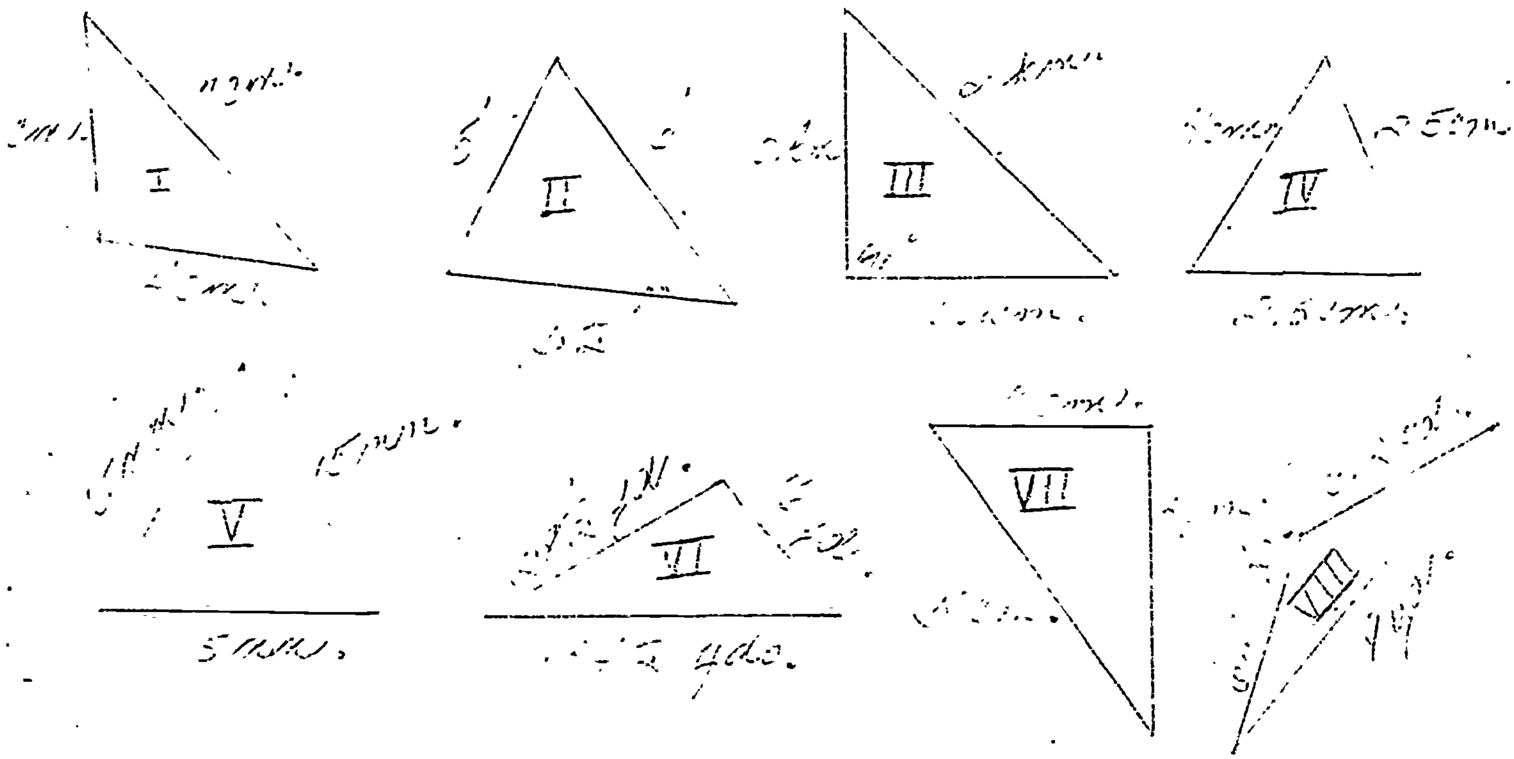
Houghton Mifflin (1972) p. 285

Other References:

Patterns & Puzzles in Mathematics, Franklin Series, pp. 15-18, 60-61.

Level: 21 Step: A Concept: Geometry

WORKSHEET



Classify the above triangles by answering both a and b for each figure.

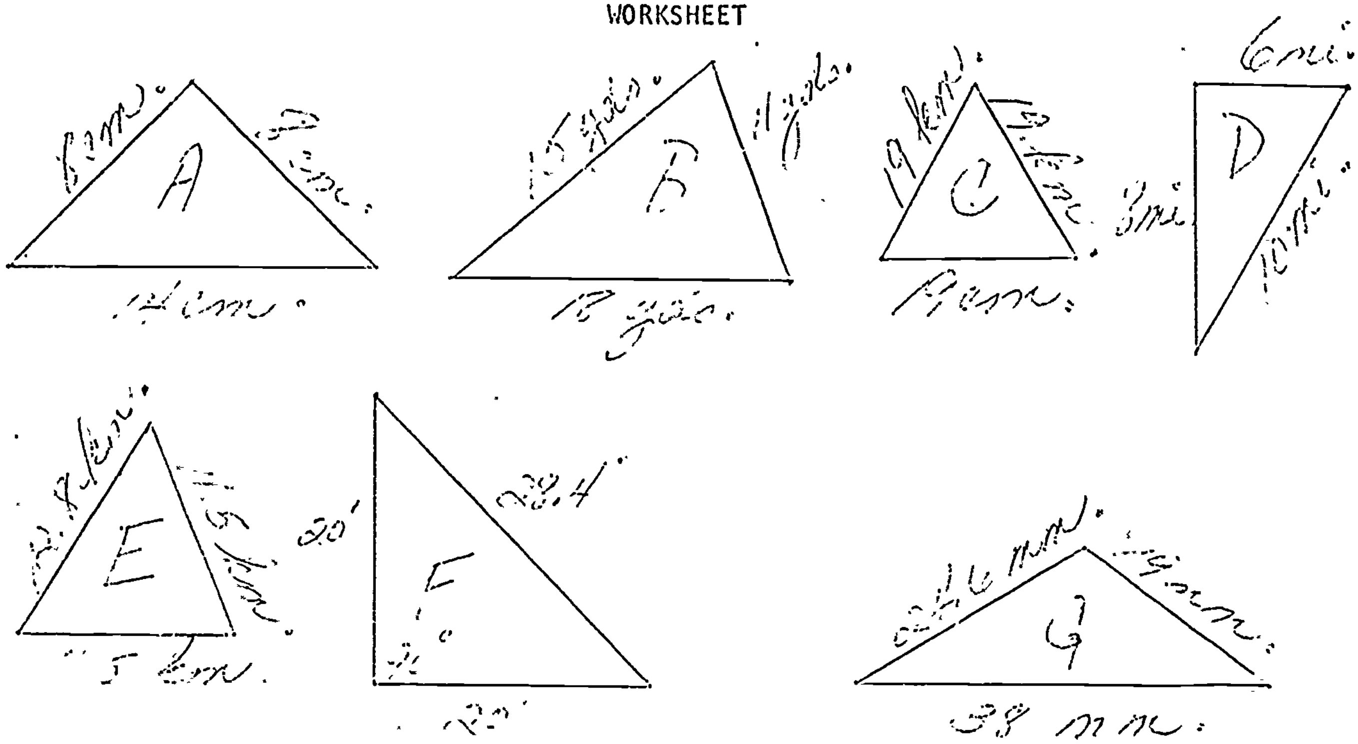
(a) Is the triangle acute, obtuse or right?

(b) Is the triangle isosceles, scalene or equilateral?

1. Figure I (a) ____________
   (b) ____________
2. Figure II (a) ____________
   (b) ____________
3. Figure III (a) ____________
   (b) ____________
4. Figure IV (a) ____________
   (b) ____________
5. Figure V (a) ____________
   (b) ____________
6. Figure VI (a) ____________
   (b) ____________
7. Figure VII (a) ____________
   (b) ____________
8. Figure VIII (a) ____________
   (b) ____________

Level: 21 Step: A Concept: Geometry

WORKSHEET



Use the letter to make the following statements true. If no triangle will satisfy the conditions given, write "none" in the blank. Use each answer once.

1. Triangle ________ is both obtuse and scalene.
2. Triangle ________ is both right and isosceles.
3. Triangle ________ is both acute and equilateral.
4. Triangle ________ is both obtuse and isosceles.
5. Triangle ________ is both right and equilateral.
6. Triangle ________ is both acute and scalene.
7. Triangle ________ is both acute and isosceles.
8. Triangle ________ is both right and scalene.

21-10

Level: 21 Step: A Concept: Measurement

I. Concept:

Measurement: Determining statistical averages.

II. Behavioral Objective:

The student given statistical data in the form of tables or sets of numbers will be able to determine the mean, median and mode.

III. Mathematical Ideas:

A. The mean (generally called "average") is the sum of all the items in the data divided by the total number of items.

B. When a set contains an odd number of items (arranged in increasing or decreasing order), the median is the middle item. When a set contains an even number of items (arranged in increasing or decreasing order), the median is the mean of the two middle items.

C. The mode is the item in a set of data appearing with the greatest frequency.

IV. Vocabulary To Stress:

statistical data mean median mode

V. Activities:

Have students find the mean, median and mode by collecting statistical information such as (1) daily temperatures for a week or longer, (2) weights or heights of students in the class, or (3) class test scores.

Text References:

Book: 6

Houghton Mifflin (1972) pp. 74-78

Book: 7

Houghton Mifflin (1967, 1970) pp. 490-494, (77)
Houghton Mifflin (1972) pp. 87, 221, 236-239 (45)
Addison-Wesley (1971) pp. 106-107, (23)

Book: 8

Houghton Mifflin (1972) pp. 174-177 (33)
Addison-Wesley (1971) pp. 229-231, (51, Sec. 1)

Level: 21 Step: A Concept: Number sentences & Phrases

I. Concept:

Number Sentences and Phrases: Simplifying expressions using integers.

II. Behavioral Objective:

The student given an expression with three or more integers, with or without grouping symbols, will be able to simplify the expression by using the order of operations.

III. Mathematical Ideas:

A. Simplify within the grouping symbols first.

B. The order of operations in the absence of appropriate grouping symbols: multiplication and divisions in order from left to right; then additions and subtractions in order from left to right.

IV. Vocabulary To Stress:

brackets	parentheses	expression
braces	fraction bar	simplify

Text References:

Book: 7

Houghton Mifflin (1967, 1970) pp. 508, 517, 520.
Houghton Mifflin (1972) p. 432

Book: 8

Addison-Wesley (1971) pp. 66, (15)

21-12

Level: 21 Step: A Concept: Number Sentences & Phrases

WORKSHEET

Simplify the following expressions:

1. $(^{-}2)^2 + {^{-}4} \times {^{-}1} =$ ________

2. $^{-}9 - 8 + 17 =$ ________

3. $24 - (^{-}9 \div {^{-}3}) =$ ________

4. $^{-}7 \div \left[(3 \cdot {^{-}2}) + {^{-}1}\right] =$ ________

5. $\dfrac{^{-}3 + (10 \div {^{-}5})}{^{-}4 + 5} =$ ________

6. $18 - \left[(3 + {^{-}2}) \cdot {^{-}3}\right] =$ ________

7. $12 - (4 \cdot {^{-}2}) =$ ________

8. $^{-}3 \cdot (3 - {^{-}3}) =$ ________

9. $\left[4 \times (6 - {^{-}2})\right] \div 8 =$ ________

10. $(^{-}4 + 7) \div 3 =$ ________

11. $^{-}4 \cdot (8 \div {^{-}2}) =$ ________

12. $10 - \left[(4 - {^{-}6}) \cdot {^{-}2}\right] =$ ________

13. $(5 \cdot {^{-}2}) \div (3 - {^{-}2}) =$ ________

14. $4 - 7 + 6 \times {^{-}3} =$ ________

15. $(7 - {^{-}7}) \div (3 - {^{-}4}) =$ ________

16. $^{-}15 \div \left[3 - {^{-}2}\right] =$ ________

17. $9 \times {^{-}3} - 4 \div {^{-}2} =$ ________

18. $25 - \left[(^{-}5 \cdot {^{-}2}) - 1\right] =$ ________

19. $(^{-}11 \cdot {^{-}2}) \div (^{-}6 + {^{-}5}) =$ ________

20. $\dfrac{^{-}2 + (6 \div {^{-}3})}{5 - 9} =$ ________

Level: 21 | Step: A | Concept: Number sentences & Phrases

WORKSHEET

Simplify the following expressions:

1. $^{-}4 \cdot (7 - 11) =$ ________

2. $35 - (^{-}21 \div 3) =$ ________

3. $^{-}48 \div (4 \times 2 \times {}^{-}3) =$ ________

4. $16 - (3 - 11) =$ ________

5. $14 + (^{-}3 \times 4) - {}^{-}6 =$ ________

6. $10 - (8 \cdot {}^{-}2) =$ ________

7. $^{-}5 \cdot (4 - 7) =$ ________

8. $(^{-}3)^2 \cdot (^{-}8 + {}^{-}1) =$ ________

9. $(3 \cdot {}^{-}7) \div (3 \cdot {}^{-}1) =$ ________

10. $\dfrac{^{-}6 + (8 \div {}^{-}2)}{3 - 8} =$ ________

11. $(4 \cdot {}^{-}8) \div (4 \cdot {}^{-}4) =$ ________

12. $\left[5 \times (4 - {}^{-}2)\right] \div {}^{-}2 =$ ________

13. $\left[(3 \times {}^{-}4) \div {}^{-}6\right] \times {}^{-}12 =$ ________

14. $^{-}16 - 2 \times {}^{-}3 + {}^{-}4 \times {}^{-}2 =$ ________

15. $(^{-}19 + 19) \cdot (^{-}6 \div 2) =$ ________

16. $\left[8 + (^{-}3 \cdot 2)\right] \div 2 =$ ________

17. $24 \div {}^{-}3 + 2 \times {}^{-}2 =$ ________

18. $(^{-}3 \times 7) - (22 \div {}^{-}11) =$ ________

19. $14 - (^{-}8 \div 4) =$ ________

20. $\left[(9 \div {}^{-}3) + (4 \times {}^{-}6)\right] \div 9 =$ ________

Level: 21 Step: B Concept: Graphs

I. Concept:

Graphs: Graphing ordered pairs of integers on a plane.

II. Behavioral Objective:

The student given ordered pairs of integers will be able to graph them on a coordinate plane.

III. Mathematical Ideas:

A. An ordered pair is required to graph one point on a plane.

B. The signs of the ordered pairs are as follows:

Quadrant I (+,+) Quadrant III (-,-)
Quadrant II (-,+) Quadrant IV (+,-)

IV. Vocabulary To Stress:

horizontal	x-axis	quadrants
vertical	y-axis	origin

V. Activities:

A. Graph pictures from coordinates of points.

B. Use a city map to locate streets.

Text References:

Book: 7

Houghton Mifflin (1972) pp. 392-393 (74)
Addison-Wesley (1971) pp. 161-164, (29)

Book: 8

Houghton Mifflin (1967, 1970) pp. 438-443, (72)
Houghton Mifflin (1972) pp. 232-233 (40, 74, 77)
Addison-Wesley (1971) pp. 75-78, (19)

Other References:

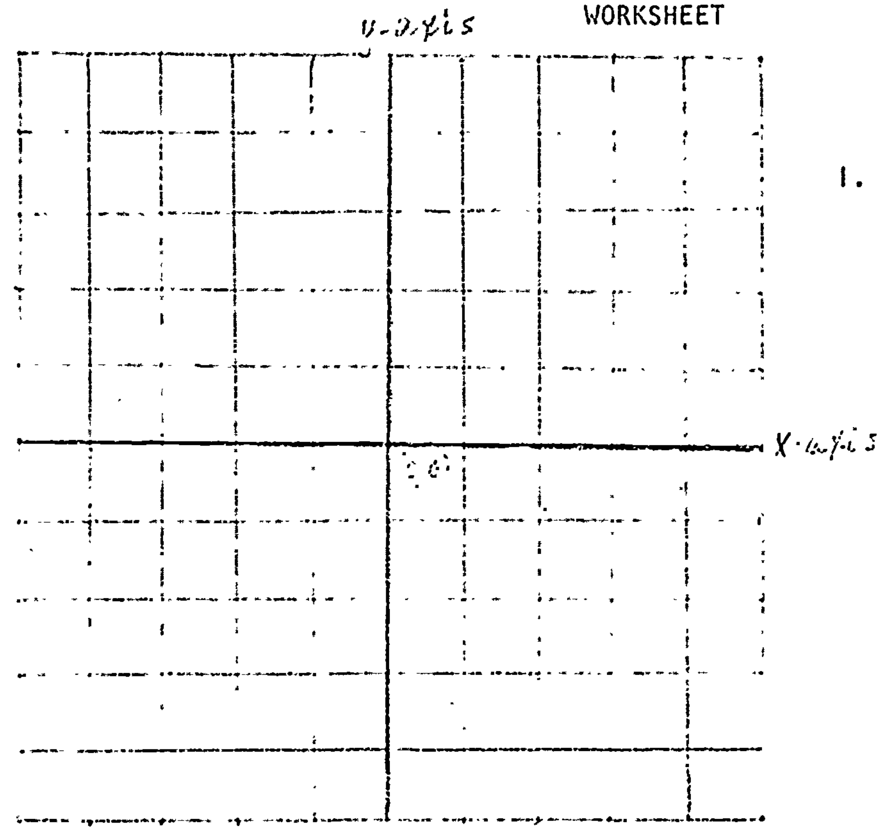
"Graphing Pictures", by Oliver E. Eason (J. Weston Walch).

"Making and Using Graphs and Nomographs" (Franklin Mathematics Series) pp. 53-70.

Level: 21 Step: B Concept: Graphs

WORKSHEET

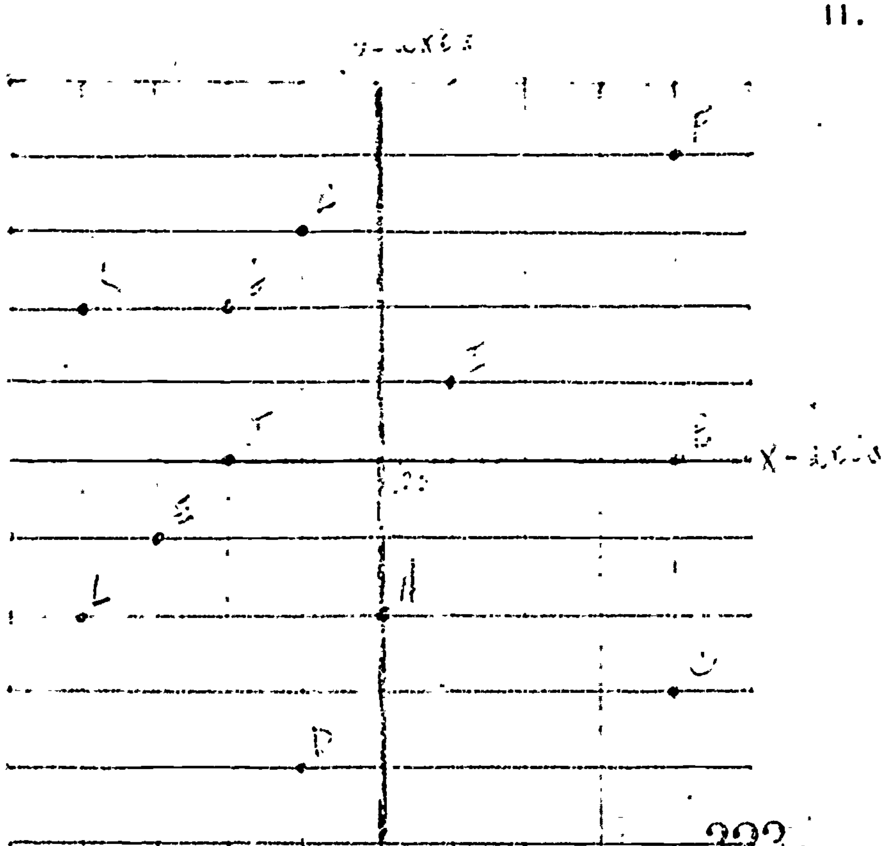
I



I. Graph the ordered pair on the number plane. Use the letter to label the point.

Q (4,3)	V (3,0)
R (⁻1, ⁻1)	W (⁻3,5)
S (3, ⁻2)	X (1, ⁻4)
T (⁻4,3)	Y (⁻2,0)
U (⁻2, ⁻4)	Z (2,2)

II



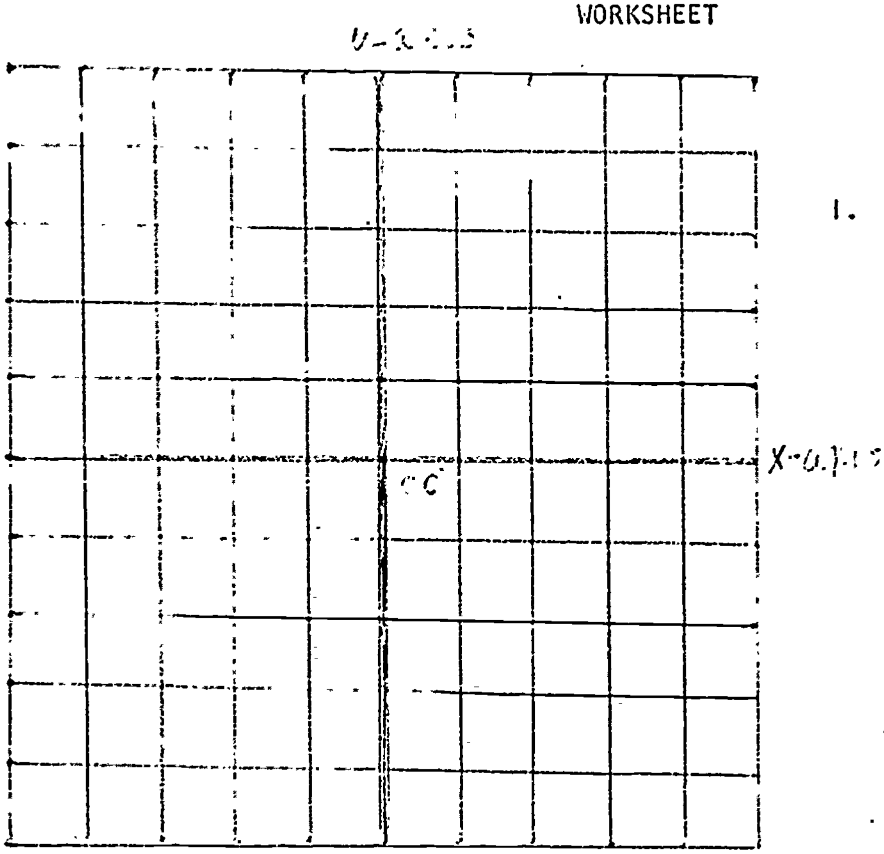
II. Write the coordinate of the following points.

A (___,___)	G (___,___)
B (___,___)	H (___,___)
C (___,___)	I (___,___)
D (___,___)	J (___,___)
E (___,___)	K (___,___)
F (___,___)	L (___,___)

21-16

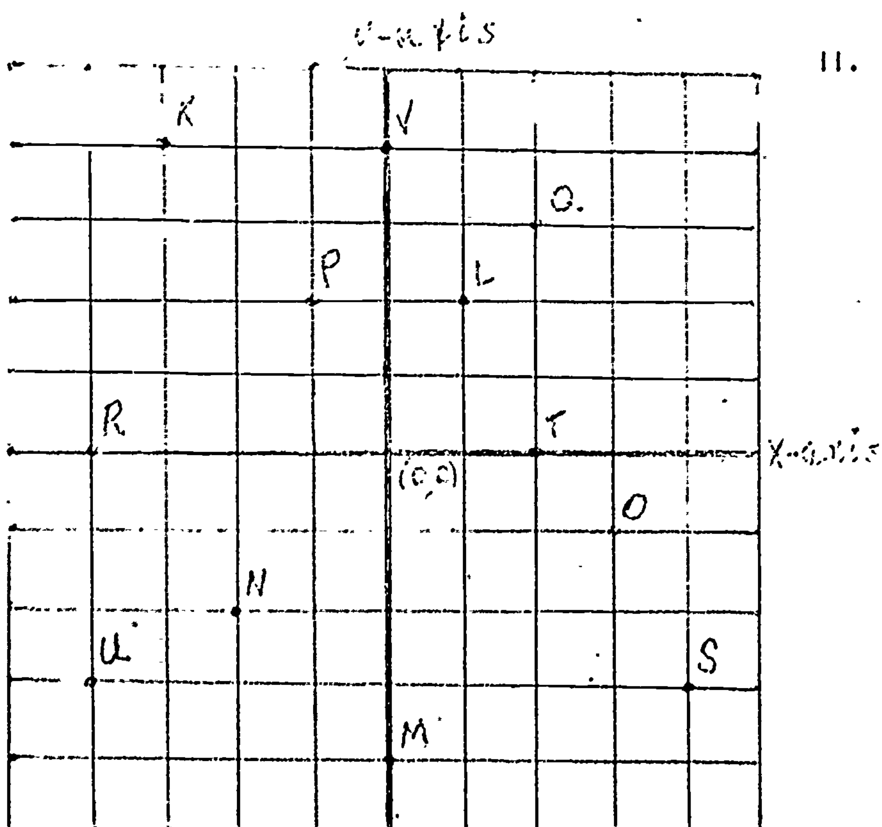
Level: 21 Step: B Concept: Graphs

WORKSHEET



I. Graph the ordered pair on the number plane. Use the letter to label the point.

A ($^-4$,1)	F ($^-4$, $^-4$)
B (0,3)	G ($^-3$, $^-1$)
C (5,3)	H (0, $^-2$)
D (4, $^-2$)	I (2, $^-3$)
E (3, $^-4$)	J ($^-1$, $^-1$)



II. Write the coordinate of the following points.

K (___,___)	Q (___,___)
L (___,___)	R (___,___)
M (___,___)	S (___,___)
N (___,___)	T (___,___)
O (___,___)	U (___,___)
P (___,___)	V (___,___)

Level: 21 Step: B Concept: Geometry

I. Concept:

Geometry: Finding the area of a trapezoid.

II. Behavioral Objectives:

The student given the length of the parallel sides and the altitude of a trapezoid will be able to find the area.

III. Mathematical Ideas:

Area of a trapezoid: $A = \frac{1}{2} \times h \times (b + b')$ where b and b' are the lengths of the parallel sides (the bases) and h is the altitude.

IV. Vocabulary To Stress:

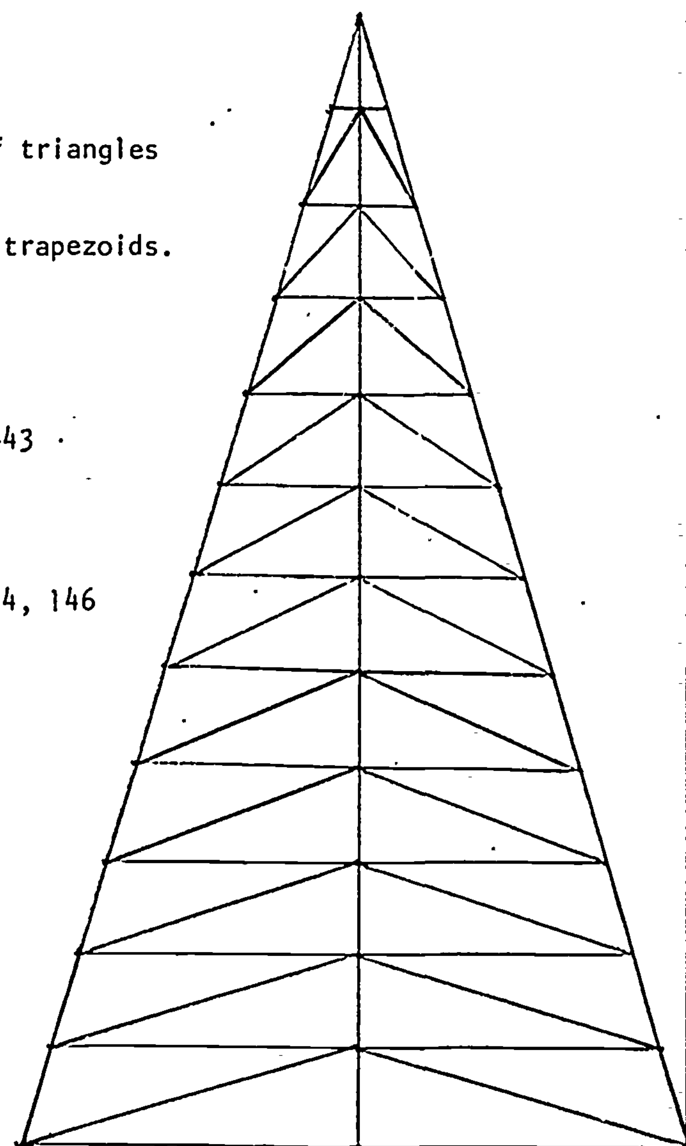
bases altitude isosceles trapezoid trapezoid

V. Activities:

Triangles and Trapezoids:

Have the students determine the number of triangles and trapezoids in the figure.

Answer: There are 113 triangles and 220 trapezoids.



Text References:

Book: 7

Houghton Mifflin (1967, 1970) pp. 434, 441-443
Houghton Mifflin (1972) p. 298

Book: 8

Houghton Mifflin (1967, 1970) pp. 141-142, 144, 146
Houghton Mifflin (1972) p. 351
Addison-Wesley (1971) pp. 306-307

21-18

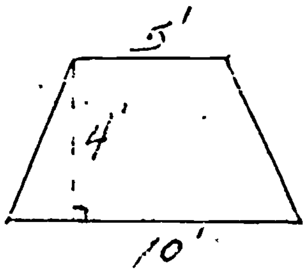
Level: 21 Step: B Concept: Geometry

WORKSHEET

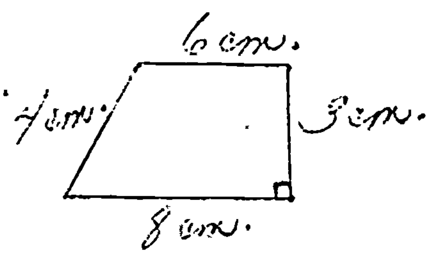
The area of a trapezoid = $\frac{1}{2}$ x _____ x (the _________ of the 2 bases).

Find the area of the following trapezoids. Show your work.

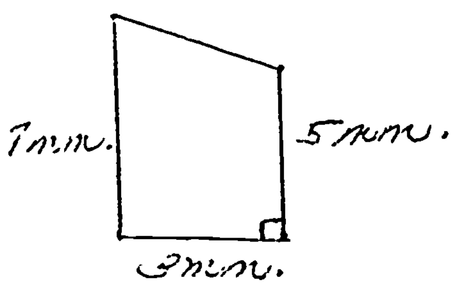
ANSWERS

1.


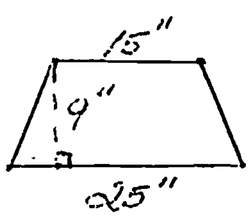
1. ______________

2.


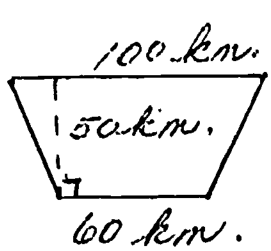
2. ______________

3.


3. ______________

4.


4. ______________

5.


5. ______________

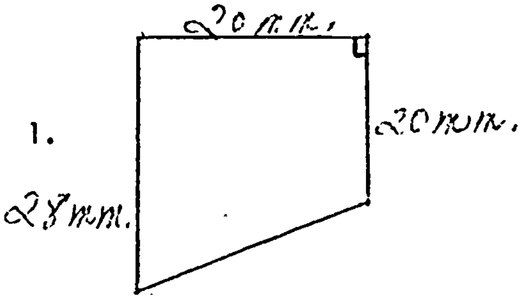
Level: 21 Step: B Concept: Geometry

WORKSHEET

Find the area of the following trapezoids. <u>Show</u> your work.

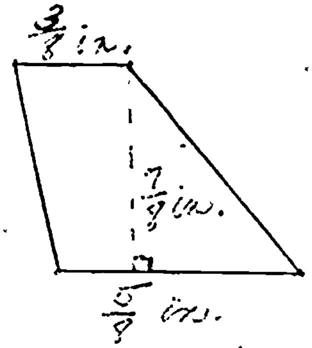
ANSWERS

1.



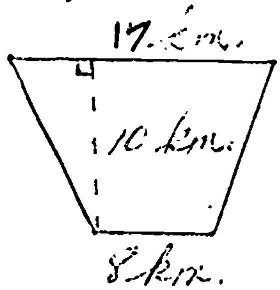
1. ______________

2.



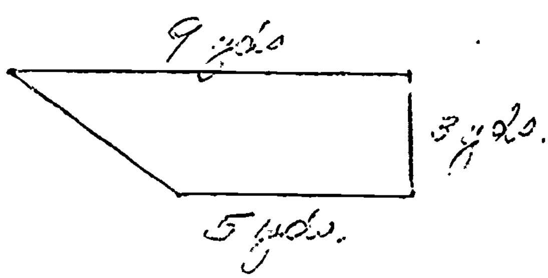
2. ______________

3.



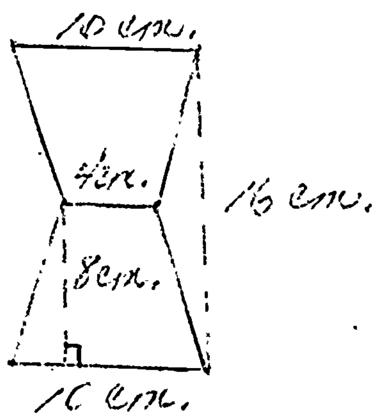
3. ______________

4.



4. ______________

5.



5. ______________

Level: 21 Step: B Concept: Number Sentences

I. Concept:

Number sentences: Solving word problems involving commission, interest and discount.

II. Behavioral Objective:

*The student given a word problem involving percent, will be able to find the commission, interest, discount or rate.

III. Mathematical Ideas:

A. Commission may be found by writing a proportion or by multiplying the amount of sales by the rate of commission.

B. The rate of commission may be found by writing a proportion or by dividing the commission by the total sales.

C. The rate of discount is equal to the amount of discount divided by the original price.

D. The basic relationship used in finding interest is: Interest = Principal x Rate x Time.

IV. Vocabulary To Stress:

commission	principal	discount
rate of commission	rate of interest	rate of discount
interest	time	sale price

V. Activities:

A. Clip sale advertisements from newspapers or catalogues to check prices or find the rate of discount.

B. Compare bank interest rates from newspaper ads.

Text References:

Book: 7

Houghton Mifflin (1967, 1970) pp. 476-481, (75)
Houghton Mifflin (1972) pp. 255, 336, 426 (65)
Addison-Wesley (1971) pp. 365-369, 373, (75, 76)

Book: 8

Houghton Mifflin (1972) pp. 194-195, 198, 219
Addison-Wesley (1971) pp. 284-285, 288 (65,66)

Other References:

Imperial Tapes (Intermediate) Tape #3

Level: 21 Step: B Concept: Number sentences

WORKSHEET.

Interest:

1. Mr. Jacobs needed $5500 to buy a business. If he borrowed this amount for 4 years at 4% per year, how much interest did he pay?

____

2. A bank loaned $400 to Mrs. Miller to buy a refrigerator. How much interest at 7% per year would she pay if she needed the money for 6 months?

____

3. Jerry needed $700 to buy a motorcycle. He had saved $300. The balance he borrowed at 6% per year from the bank for a period of 2 years. How much interest did he pay?

____

Commission:

4. Sally sold $84 worth of Christmas cards. If her commission was 20% of her sales, how much did she earn?

____

5. Mr. Rogers sold a house for $25,000. If his commission amounted to $750, what was the rate of commission?

____

6. A salesman sold 50 dozen shirts at $40 a dozen. If he received a 5% commission, how much did he earn?

____

Discount:

7. Luanne's Dress Shop advertised a 40% discount on all coats. If the coat Jane bought was originally makred $80, how much was the discount?

____

8. What is the rate of discount if a book originally priced at $5 was sold for $3?

____

9. A $75 camera was sold at a discount of $33\frac{1}{3}$%. What was the selling price?

____

10. What would be the selling price of the following items at a "50% off" sale?

(a) $8 ______ (b) $24.50 ______ (c) $75.00 ______ (d) $2.36 ______

Level: 21 Step: Z Concept: Numeral

I. Concept:

Numeral: Roman Numerals

II. Behavioral Objective:

The student given any numeral up through a million will be able to change from Hindu-Arabic to Roman or Roman to Hindu-Arabic numerals.

III. Mathematical Ideas:

Roman numerals involve th[illegible] [illegible]inciple of addition as well as the principles of multiplicati[illegible] subtraction, and economy.

Text References:

Book: 7

Houghton Mifflin (1967, 1970)	pp. 107-111
Houghton Mifflin (1972)	p. 28
Addison-Wesley (1971)	pp. 28, 30

Level: 21 Step: Z Concept: Numeral

WORKSHEET

I. Write the Hindu-Arabic number for:	II. Write the Roman numeral for:
1. XXI = ______	1. 8 = ______
2. IX = ______	2. 18 = ______
3. XV = ______	3. 19 = ______
4. XXV = ______	4. 14 = ______
5. $\overline{X}$C = ______	5. 29 = ______
6. $\overline{XX}$C = ______	6. 49 = ______
7. MXL = ______	7. 999 = ______
8. CCLI = ______	8. 449 = ______
9. CD = ______	9. 1969 = ______
10. MC = ______	10. 1357 = ______
11. MCD = ______	11. 64 = ______
12. MCM = ______	12. 3900 = ______
13. III = ______	13. 124 = ______
14. VII = ______	14. 1607 = ______
15. XIV = ______	15. 421 = ______
16. XXVI = ______	16. 2447 = ______
17. XLIV = ______	17. 110,000 = ______
18. XLIX = ______	18. 550,000 = ______
19. $\overline{X}$CI = ______	19. 1,010 = ______
20. $\overline{XXX}$DCII = ______	20. 1482 = ______
21. MCMLXVI = ______	21. 2808 = ______
22. MCDXCII = ______	22. 903 = ______
23. DCCIX = ______	23. 2002 = ______
24. CXC = ______	24. 98 = ______
25. MDCCLXXVI = ______	25. 194 = ______

Level: 21 Step: Z Concept: Numeral

ENRICHMENT EVALUATION

I. Write these Roman numerals as Hindu-Arabic numerals.

1. $\overline{XIV}$ = ____________________

2. CX = ____________________

3. $\overline{DL}$ = ____________________

4. MXXI = ____________________

5. MMCDXLVII = ____________________

II. Write these Hindu-Arabic numerals as Roman numerals.

6. 1,000,000 = ____________________

7. 1462 = ____________________

8. 1009 = ____________________

9. 20,459 = ____________________

10. 10,101 = ____________________

Level: 21 Step: Z Concept: Division

I. Concept:

Division: Euclidean Algorithm.

II. Behavioral Objective:

The student given any two numbers will be able to find the greatest common factor without factoring either of the numbers.

III. Mathematical Ideas:

The Euclidean Algorithm uses the division algorithm to find the greatest common factor without factoring either of the numbers.

IV. Vocabulary To Stress:

Euclidean Algorithm

V. Activities:

A. Use the eight problems on p. 241 (H.M.) for an assignemnt.

B. See article "The Euclidean Algorithm as a Means of Simplifying Fractions", in The Arithmetic Teacher, December 1970, pp. 657-660.

Text References:

Book: 7

Houghton Mifflin (1967, 1970) pp. 240-241

Book: 8

Addison-Wesley (1971) pp. 104,105

Level: 21 Step: Z Concept: Division

ENRICHMENT EVALUATION

Use the Euclidean Algorithm to find the GCF (greatest common factor) of the following numbers:

		Answers
1. 132; 680	1.	____________
2. 273; 378	2.	____________
3. 2,145; 7,007	3.	____________
4. 520; 728	4.	____________
5. 120; 328	5.	____________

Level: 21 Step: Z Concept: Geometry

I. Concept:

Geometry: Completing vector equation.

II. Behavioral Objective:

The student given points on a plane will be able to complete vector equations.

III. Mathematical Ideas:

A. Vectors are directed line segments used to show a move between two points. They are represented by arrows on a number plane.

B. The sum of two vectors may be written as a single vector.

IV. Vocabulary To Stress:

vector symbol for vector $\overrightarrow{AB}$

Text References:

Book: 7

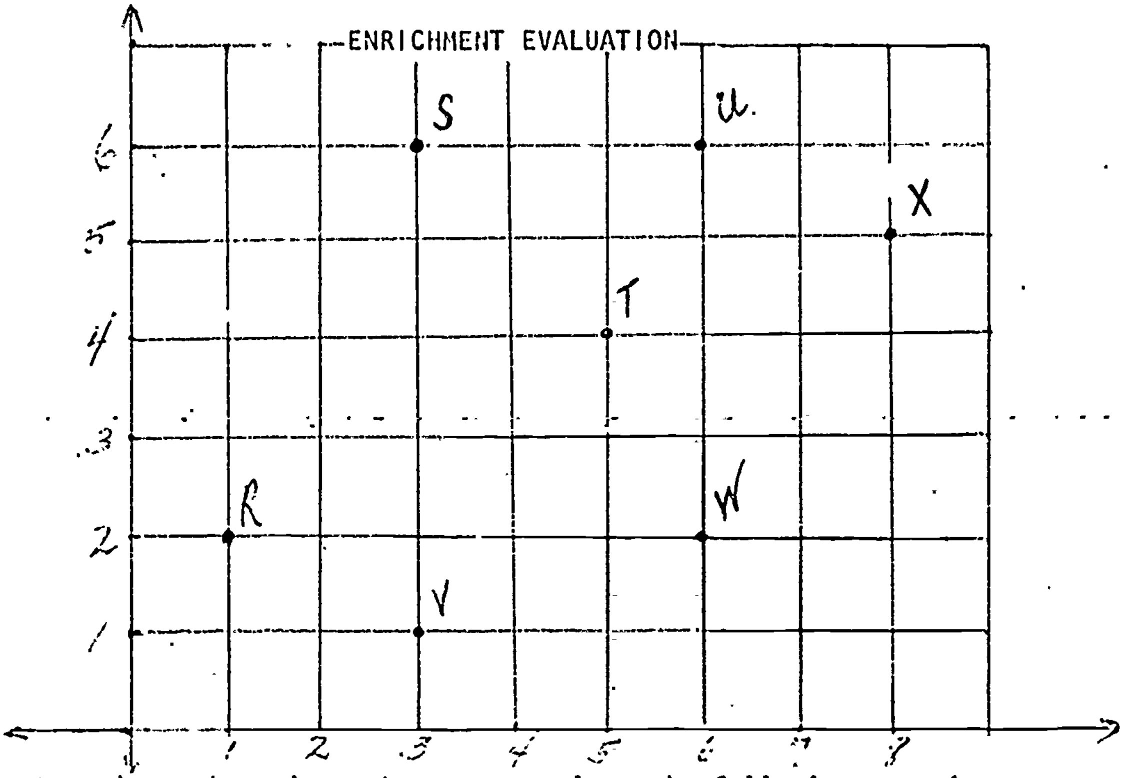
Houghton Mifflin (1972) pp. 98-99, 282-285 (20)

Book: 8

Houghton Mifflin (1972) pp. 106-107, 294-295 (21, 51).

Level: 21 Step: Z Concept: Geometry

ENRICHMENT EVALUATION



Use the number plane above to complete the following equations.

1. $\overrightarrow{TU} + \overrightarrow{UX} =$ __________

2. $[(3,1) \text{ to } (5,4)] + [(5,4) \text{ to } (6,2)] = [(\_\_,\_\_) \text{ to } (\_\_,\_\_)]$

3. $\overrightarrow{WR} +$ ______ $= \overrightarrow{WT}$

4. $[(\_\_,\_\_) \text{ to } (\_\_,\_\_)] + [(3,1) + (8,5)] = [(3,6) + (8,5)]$

5. ________ $+ \overrightarrow{RW} = \overrightarrow{TW}$

Level: 21 Step: Z Concept: Measurement

I. Concept:

<u>Measurement:</u> Frequency Distributions.

II. Behavioral Objective:

The student given a histogram, frequency table or frequency polygon will be able to determine the mean, median, and mode.

III. Mathematical Ideas:

A. The mean is the sum of all the items in a set of data divided by the number of the items.

B. The median is the middle item when the set of data is arranged in increasing or decreasing order.

C. The mode is the item in a set of data appearing with the greatest frequency.

D. The mean, median, and mode are used to summarize statistical relationships and can be presented in frequency tables, histograms, and frequency polygons.

IV. Vocabulary To Stress:

frequency distribution	mean
frequency table	median
histogram	mode
frequency polygon	

Text References:

Book: 7

Houghton Mifflin (1967, 1970) pp. 490-494, (77)

Book: 8

Houghton Mifflin (1972) p. 270 (48)

Level: 21 Step: Z Concept: Measurement

WORKSHEET (1)

From the following figures, determine the mean, median, and mode.

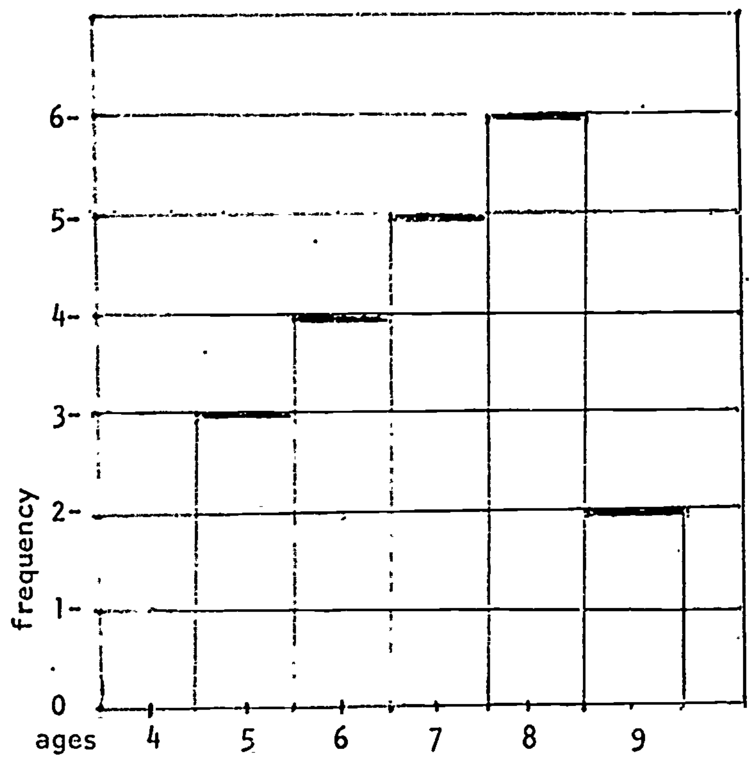


Figure A

Figure A shows the ages of 20 girls in the Junior Athletic Club. From the data shown in this figure determine the mean, median, and mode.

Answer

1. mean (a) 6 (b) 7 (c) 8 ____________

2. median (a) 7 (b) 8 (c) 9 ____________

3. mode (a) 6 (b) 7 (c) 8 ____________

Level: 21 Step: Z Concept: Measurement

WORKSHEET (cont. 2)

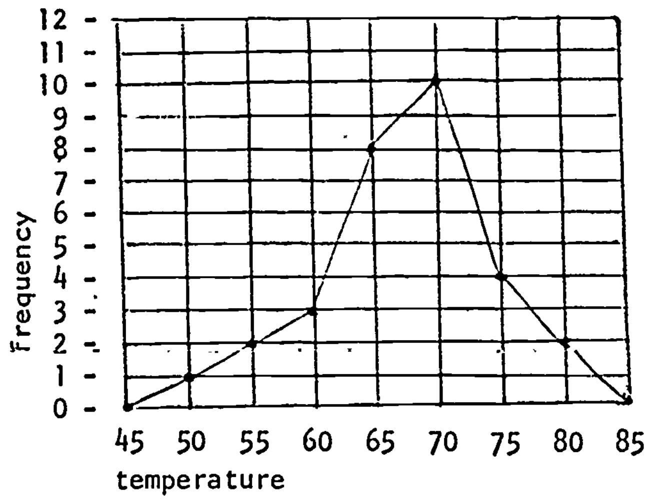


Figure B

Figure B shows the average temperatures for 30 days in June in New York City. From the data shown in this figure determine the mean, median, and mode.

Answer

4. mean (a) 70 (b) 65 (c) $67\frac{1}{3}$ ________

5. median (a) 70 (b) 65 (c) 60 ________

6. mode (a) 70 (b) 55 (c) 80 ________

Scores	Frequency	s x f
95	1	95
90	4	360
85	10	850
80	12	960
75	8	600
70	5	350
Total	40	3215

Figure C

Figure C shows the scores of 40 junior high students in a mathematics test. From this data determine the mean, median, and mode.

7. mean (a) $85\frac{1}{2}$ (b) $80\frac{3}{8}$ (c) 80 ________
8. median (a) 75 (b) 85 (c) 80 ________
9. mode (a) 75 (b) 85 (c) 80 ________

10. Which of the above figures is a histogram?

(a) Figure A (b) Figure B (c) Figure C ________

Level: 21 Step: Z Concept: Measurement

WORKSHEET (1)

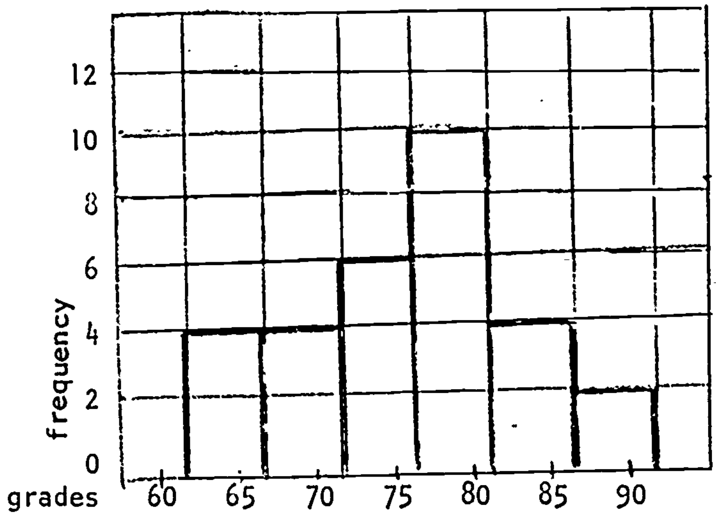


Figure A

Figure A shows the grades of 30 students in a science class. From this figure determine the mean, median and mode. Circle the correct answer.

1. mean (a) 80 (b) 77 (c) 75

2. median (a) 80 (b) 75 (c) 75 $\frac{1}{2}$

3. mode (a) 65, 70 & 85 (b) 75 (c) 80

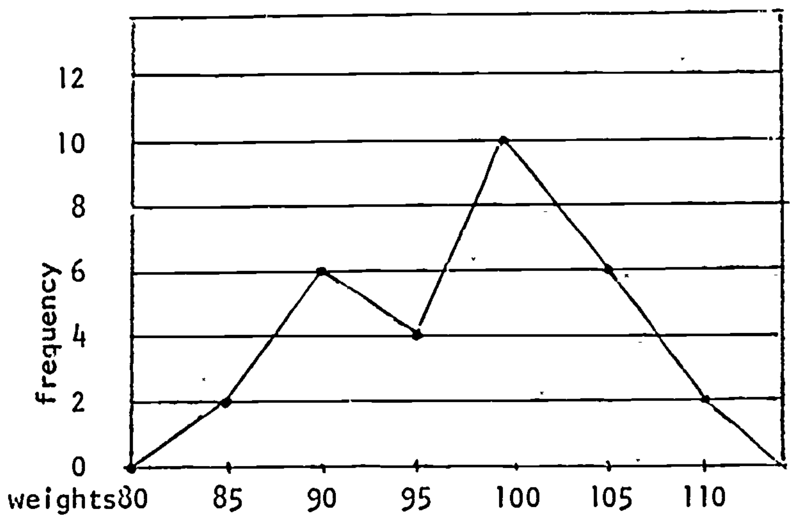


Figure B

Figure B shows the weights to the nearest 5 lbs of 30 boys in the Intermediate Swim Club. From this figure determine the mean, median, and mode. Circle the correct answer.

4. mean (a) 95 (b) 98 (c) 100
5. median (a) 95 (b) 100 (c) 105
6. mode (a) 90 & 105 (b) 95 $\frac{1}{2}$ (c) 100

Level: 21 Step: Z Concept: Measurement

WORKSHEET (cont. 2)

Salaries	Frequency	s x f
$6,000	4	24,000
$7,000	3	21,000
$8,000	3	24,000
$9,000	6	54,000
$10,000	4	40,000
$11,000	5	55,000
Total	25	218,000

Figure C

Figure C shows the salaries of a group of 25 salesmen. From this figure, determine the mean, median, and mode. Circle the correct answer.

7. mean (a) $9,000 (b) $8,720 (c) $8,500

8. median (a) $9,000 (b) $8,750 (c) $8,000

9. mode (a) $9,000 (b) $6,000 & $11,000 (c) none of these

10. Which of the above figures is a frequency polygon?

    (a) Figure A (b) Figure B (c) Figure C

Level: 21 Step: Z Concept: Measurement

ENRICHMENT EVALUATION (1)

From the following figures determine the mean, median, and mode.

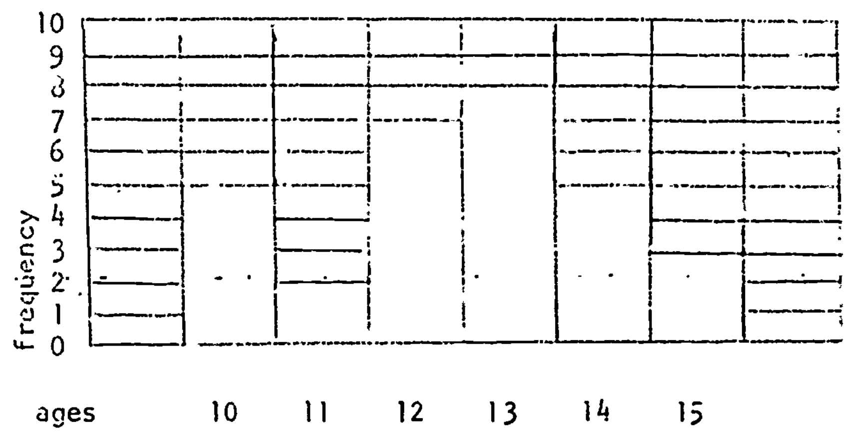


Figure A

Figure A shows the ages of 30 boys on the Intermediate Swim Team. From the data shown in the figure determine the mean, median, and mode. Circle the correct answer.

1. mean (a) 12 (b) 12.5 (c) 13
2. median (a) 12 (b) 12.5 (c) 13
3. mode (a) 12 (b) 12.5 (c) 13

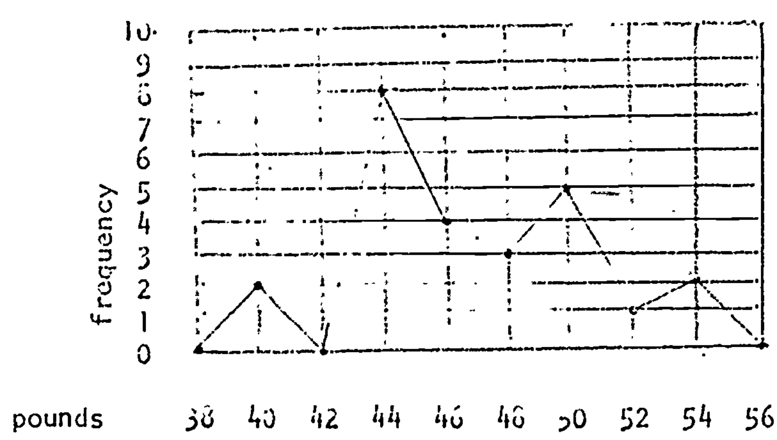


Figure B

Figure B shows the weights in pounds of 25 kindergarten children. From the data shown, determine the mean, median and mode. Circle the correct answer.

4. mean (a) $46\frac{4}{5}$ (b) $44\frac{2}{3}$ (c) 46
5. median (a) 45 (b) $44\frac{2}{3}$ (c) 46
6. mode (a) 44 (b) 46 (c) 48

Level: 21 Step: Z Concept: Measurement

ENRICHMENT EVALUATION (cont. 2)

Score	Frequency	s x f
100	1	100
95	3	285
90	3	270
85	9	765
80	8	640
75	6	450
Total	30	2510

Figure C

Figure C shows the test scores of 30 students. From the data shown determine the mean, median, and mode. Circle the correct answer.

7. mean (a) $84\frac{1}{3}$ (b) $83\frac{2}{3}$ (c) 85

8. median (a) 85 (b) $83\frac{2}{3}$ (c) 80

9. mode (a) 95 (b) 90 (c) 85

10. Which of the above three figures is a frequency polygon?

(a) A (b) B (c) C

Level: 21 Step: Z Concept: Number sentences & Phrases

I. Concept:

Number sentences and Phrases: Solving consumer problems (compound interest and installment buying.)

II. Behavioral Objective:

The student given a problem on compound interest or installment buying will be able to solve it.

III. Mathematical Ideas:

The basic relationship used in interest problems is:

Interest = Principal x Rate x Time.

IV. Vocabulary To Stress:

compound interest	balance
principal	installments
rate	carrying charge
time	semiannually
down payment	quarterly

Text References:

Book: 7

Houghton Mifflin (1967, 1970) pp. 477-481

Level: 21 Step: Z Concept: Number sentences & Phrases

WORKSHEET

1. Jean opened a savings account with a deposit of \$70. The bank pays $3\frac{1}{2}$% interest, compounded semiannually. What was the amount of Jean's account at the end of 2 years?

____

2. Find the compound int· r t on \$300 at 6% per year compounded semiannually for 3 years.

____

3. Find the compound interest on \$400 at 8% per year compounded quarterly for 2 years.

____

4. The cash price of a TV set is \$144.40. The installment price is \$24.50 down and \$7 a week for 20 weeks. What is the total installment price? How much more is the installment price than the cash price?

______________ ______________

5. The cash price of a suit is \$75. The installment price is \$15 down and \$6 a week for 12 weeks. Find the total of the installment payments. Find the carrying charge.

______________ ______________

6. Mrs. Thompson bought a washing machine that sold for \$238.00. She had to make a down payment of 10% of this price. What was the amount of the down payment? What was the balance due?

______________ ______________

Level: 21 | Step: Z | Concept: Number sentences & Phrases

ENRICHMENT EVALUATION

1. Find the compound interest on $800 at 9% compounded semiannually for 2 years.

____

2. At 5% per year compounded semiannually, how much will $2000 amount to in one year?

____

Find the installment price and carrying charge in Problems 3-5.

	Cash Price	Down Payment	Regular Payments	Time
3.	$60	$10	$6 a month	9 months
4.	$32	$5	$2 a week	15 weeks
5.	$150	$20	$20 a month	7 months

ANSWERS

3. ______________

4. ______________

5. ______________

Level: 21 ANSWERS TO WORKSHEETS

21-6

1. -3, -2, -1, 0, 1
2. -1, 0, 1, 2
3. -2, -1, 0, 1, 2, 3
4. -2, -1, 0, 1, 2
5. -5, -4, -3, -2
6. -4, -3, -2, -1
7. -3, -2, -1, 0, 1, 2, 3
8. -1, 0, 1, 2, 3, 4, 5
9. -4, -3, -2, -1, 0
10. -1, 0

21-8

1. (a) obtuse
   (b) isosceles
2. (a) acute
   (b) scalene
3. (a) right
   (b) isosceles
4. (a) acute
   (b) isosceles
5. (a) acute
   (b) equilateral
6. (a) obtuse
   (b) scalene
7. (a) right
   (b) scalene
8. (a) obtuse
   (b) isosceles

21-9

1. G
2. F
3. C
4. A
5. none
6. B
7. E
8. D

21-12

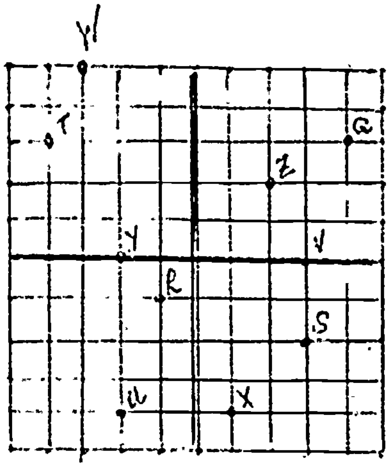
1. 8
2. 0
3. 21
4. 1
5. -5
6. 21
7. 20
8. -18
9. 4
10. 1
11. 16
12. 30
13. -2
14. -21
15. 2
16. -3
17. -25
18. 16
19. -2
20. 1

21-13

1. 16
2. 42
3. 2
4. 24
5. 8
6. 26
7. 15
8. -81
9. 7
10. 2
11. 2
12. -15
13. -24
14. -2
15. 0
16. 1
17. -12
18. -19
19. 16
20. -3

Level: 21 ANSWERS TO WORKSHEETS

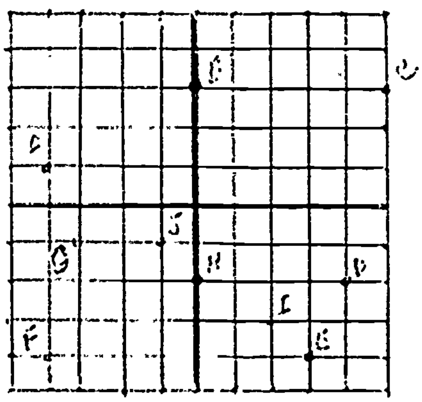
21-15



II.

A (-1, 3)	G (-2,2)
B (4,0)	H (0,-2)
C (4,-3)	I (1, 1)
D (-1, -4)	J (-2, 0)
E (-3, -1)	K (-4, 2)
F (4,4)	L (-4, -2)

21-16



II.

K (-3, 4)	Q (2,3)
L (1,2)	R (-4,0)
M (0,-4)	S (4,-3)
N (-2, -2)	T (2,0)
O (3, -1)	U (-4, -3)
P (-1,2)	V (0, 4)

21-18

1. 30 sq. ft.
2. 21 sq. cm.
3. 18 sq. mm.
4. 180 sq. in.
5. 4000 sq. km.

21-19

1. 480 sq. mm.
2. $\frac{7}{8}$ sq. in.
3. 125 sq. km.
4. 21. sq. yds.
5. 112 sq. cm.

Level: 21 ANSWERS TO WORKSHEETS

21-21

1. $880
2. $14
3. $48
4. $16.80
5. 3%
6. $100
7. $32, $48
8. 40%
9. $50
10. $4, $12.25, $37.50, $1.18

ANSWERS TO ENRICHMENT

WORKSHEETS

21-23

I.

1. 21	6. 20,100	11. 1490	16. 26	21. 1966
2. 9	7. 1,040	12. 1900	17. 44	22. 1492
3. 15	8. 251	13. 3	18. 49	23. 709
4. 25	9. 400	14. 7	19. 10,101	24. 190
5. 10,100	10. 1,100	15. 14	20. 30,602	25. 1776

II.

1. VIII	6. XLIX	11. LXIV	16. MMCDXLVII	21. MMDCCCVIII
2. XVIII	7. CMXCIX	12. MMMCM	17. $\overline{CX}$	22. CMIII
3. XIX	8. CDXLIX	13. CXXIV	18. $\overline{DL}$	23. MMII
4. XIV	9. MCMLXIX	14. MDCVII	19. MX	24. XCVIII
5. XXIX	10. MCCCLVII	15. CDXXI	20. MCDLXXXII	25. CXCIV

21-30 and 21-31

1. b
2. a
3. c
4. c
5. a
6. a
7. b
8. c
9. c
10. a

21-32 and 21-33

1. b
2. a
3. c
4. b
5. b
6. c
7. b
8. a
9. a
10. b

21-37

1. $75.04
2. $58.21
3. $63.66
4. $164.50, $20
5. $87, $12
6. $23.80, $214.20

Level: 21 EVALUATION

21-24

1. 14,000
2. 110
3. 550,000
4. 1.021
5. 2,447
6. $\overline{M}$
7. MCDLXII
8. MIX
9. $\overline{XX}$CDLIX
10. $\overline{M}$CI

21-26

1. 4
2. 21
3. 143
4. 104
5. 8

21-28

1. $\overrightarrow{TX}$
2. (3,1) to (6,2)
3. $\overline{RT}$
4. (3,6) to (3,1)
5. $\overline{TR}$

21-34 and 21-35

1. b
2. c
3. c
4. a
5. c
6. a
7. b
8. a
9. c
10. b

21-38

1. \$154.01
2. \$2101.25
3. \$64, \$4
4. \$35, \$3
5. \$160, \$10

Level: 21 ANSWERS - Post Test I

Step A

Addition & Subtraction

1. -12
2. 80
3. -46
4. -15
5. -22

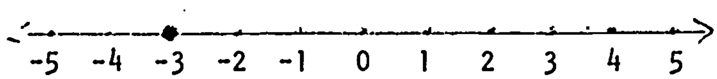
Multiplication

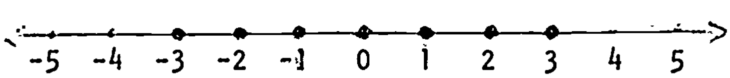
6. -36
7. 96
8. 42
9. -608
10. -48

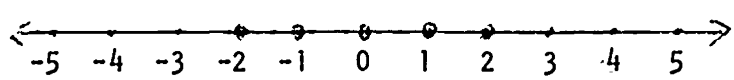
Division

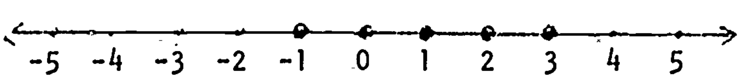
11. -12
12. -16
13. 14
14. -18
15. 8

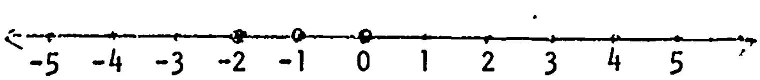
Graphs

16. 

17. 

18. 

19. 

20. 

Geometry

21. b
22. a
23. b
24. c
25. a
26. c
27. c
28. a
29. a
30. b

Measurement

31. 108 lbs.
32. 43°
33. \$2.40
34. 3
35. 5

Number Sentences

36. -2
37. 23
38. 2
39. 3
40. -2

Step B

Graphs

41-50. see graph answer sheet

Geometry

51. 55
52. $6\frac{1}{2}$
53. 50
54. 280
55. 12

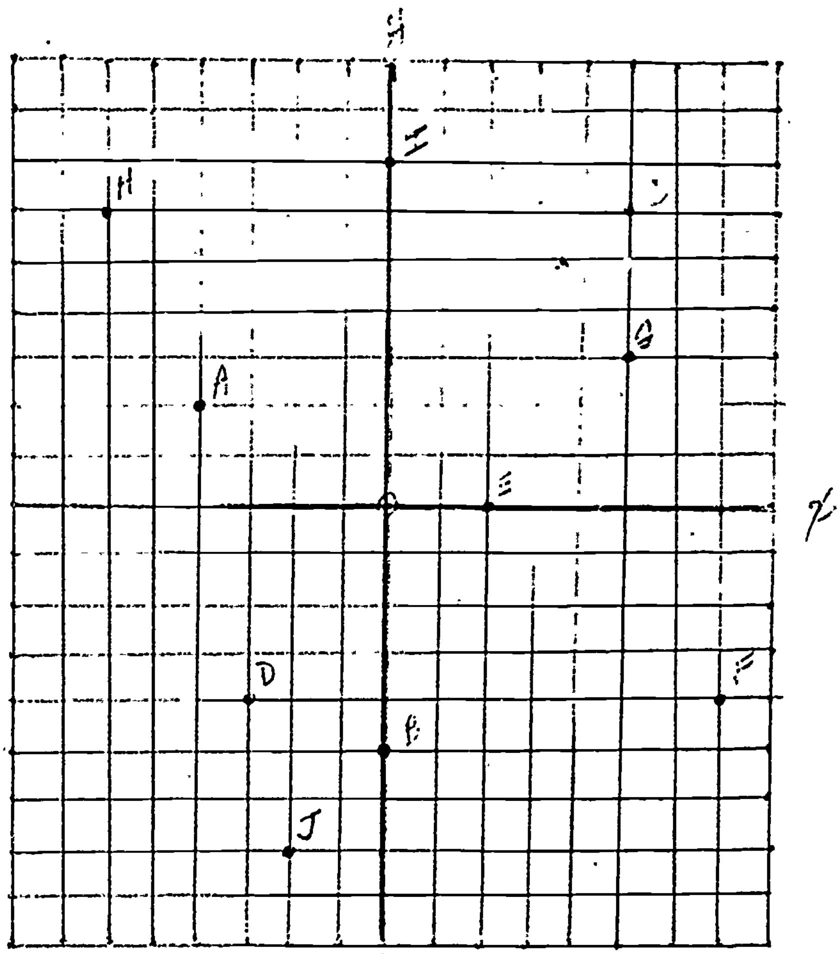
Number Sentences

56. \$36
57. 25%
58. \$175
59. \$7.20
60. \$50

Level: 21 ANSWERS - Post Test I

Step B

Graphs



Level: 21 ANSWERS - Post Test II

Step A

Addition & Subtraction

1. -7
2. 75
3. 1
4. -22
5. -58

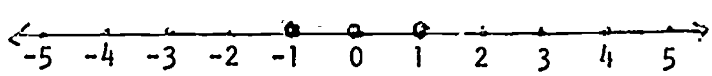
Multiplication

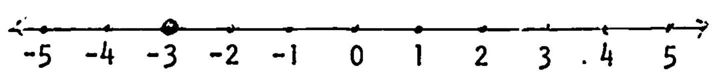
6. -231
7. 125
8. -132
9. 52
10. -690

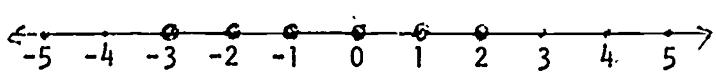
Division

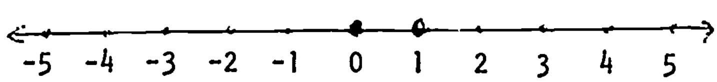
11. -25
12. 7
13. 11
14. -14
15. 16

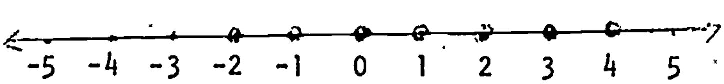
Graphs

16. 

17. 

18. 

19. 

20. 

Geometry

21. a
22. c
23. b
24. a
25. c
26. b
27. a
28. a
29. c
30. c

Measurement

31. 26
32. 16
33. $3.80
34. 4
35. 6

Number Sentences

36. 96
37. -4
38. 0
39. -9
40. 4

Step B

Graphs

41 - 50 see graph answer sheet

Geometry

51. 54
52. 84
53. 80
54. 15
55. 16

Number Sentences

56. $12
57. $501
58. 20%
59. $64
60. $75

21-46

Level: 21 ANSWERS - Post Test II

Step B

Graphs

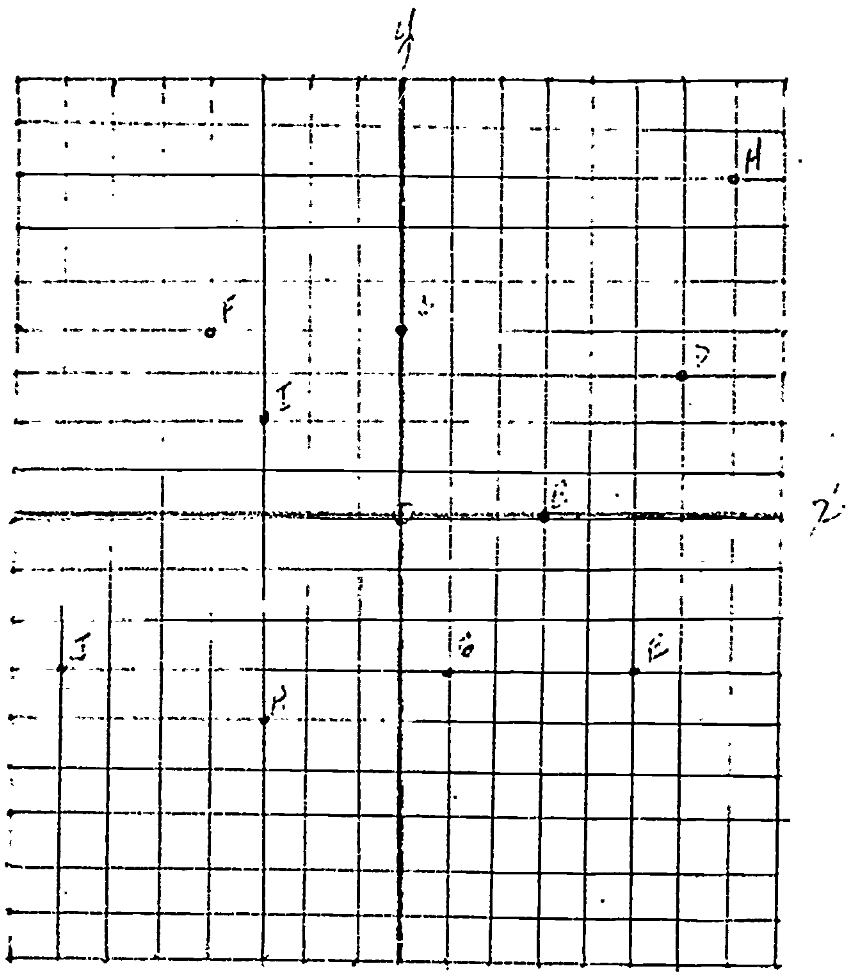


TABLE OF CONTENTS

Level 22

Step A

Step B

Step C

Step D

TABLE OF CONTENTS - Continued

Step Z

Level: 22 Step: A Concept: Numeral

I. Concept:

Numeral: Extending the set of nonnegative rational numbers to include negative rationals.

II. Behavioral Objective:

The student given either the graph of any rational number or its coordinate will be able to write the one not given.

III. Mathematical Ideas:

Negative rational numbers are rational numbers assigned to points to the left of the origin on the number line.

IV. Vocabulary To Stress:

negative rational number
directed number (sign indicates direction, numeral indicates distance)

V. Activities:

A. Assign Houghton Mifflin (1967, 1970) pp. 5-6: 15-32
or Addison-Wesley (1971) pp. 129: 1

B. See worksheet on following page.

Text References:

Book: 8

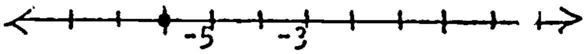
Houghton Mifflin (1967, 1970) pp. 2-6, (1, 2, 4: Sec. 1-3)
Addison-Wesley (1971) pp. 128-129

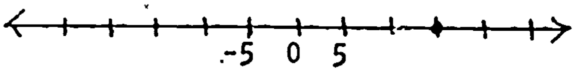
22-2

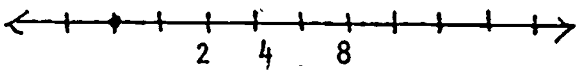
Level: 22 Step: A Concept: Numeral

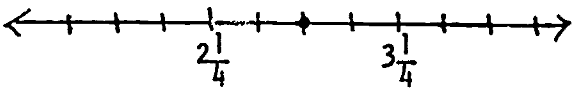
WORKSHEET

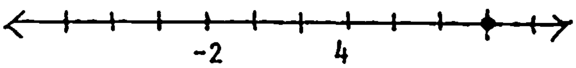
Name the number whose graph is shown.

1. 

2. 

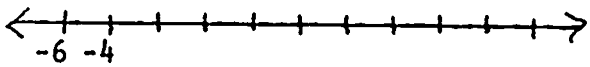
3. 

4. 

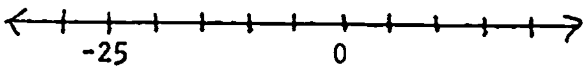
5. 

Graph each number on the given number line.

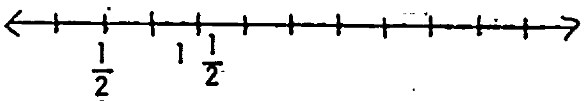
6. 4



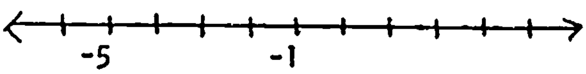
7. -15



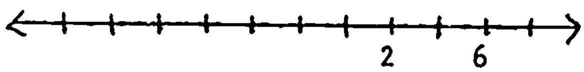
8. 3



9. -2



10. -4



Level: 22 Step: A Concept: Order

I. Concept:

Order: Comparing rational numbers.

II. Behavioral Objective:

The student given any 2 rational numbers will be able to write a true statement comparing them by using the symbols <, = or >.

III. Mathematical Ideas:

Comparison property: If M, N $\in$ R, then one and only one of the following statements is true: $M < N$, $M = N$, $M > N$.

IV. Vocabulary To Stress:

rational numbers

V. Activities:

Assign Houghton Mifflin (1967, 1970) Book 8 p. 20: 1-8

Extend understanding by using Houghton Mifflin (1967, 1970) Book 8, p. 20: 9-18

Text References:

Book: 8

Houghton Mifflin (1967, 1970) pp. 17-21, (4: Sec. 10, 11)

Level: 22 Step: A Concept: Geometry

I. Concept:

Geometry: applying the properties of corresponding, vertical, alternate exterior, alternate interior, and supplementary angles.

II. Behavioral Objective:

The student given two parallel lines cut by a transversal and the degree measure of one of the eight resulting angles will be able to write the degree measure of each of the other seven angles.

III. Mathematical Ideas:

When a transversal intersects two parallel lines, eight angles are formed:

A. pairs of corresponding angles, vertical angles, alternate exterior angles, and alternate interior angles have equal measure.

B. two adjacent angles which form a straight angle are supplementary (the sum of their measures in degrees is 180).

IV. Vocabulary To Stress:

supplementary angles (see Houghton Mifflin (1967, 1970) Book 7, p. 425)
transversal
corresponding angles
vertical angles
straight angle (see Houghton Mifflin (1967, 1970) Book 7, p. 268)
alternate exterior angles
alternate interior angles

V. Activities:

See worksheet on the following page.

Text References:

Book: 7

Addison-Wesley (1971) pp. 380-381

Book: 8

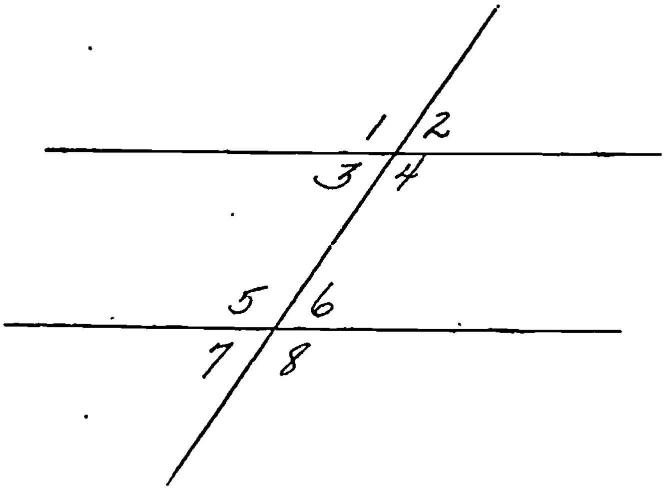
Houghton Mifflin (1967, 1970) pp. 87-89, (15: Sec. 1-9 only)
Houghton Mifflin (1972) pp. 284
Addison-Wesley (1971) pp. 292-293

Level: 22 Step: A Concept: Geometry

WORKSHEET

Refer to the figures and complete the statements.

I



m° ∠ 1 = 120

m° ∠ 2 = ______

m° ∠ 3 = ______

m° ∠ 4 = ______

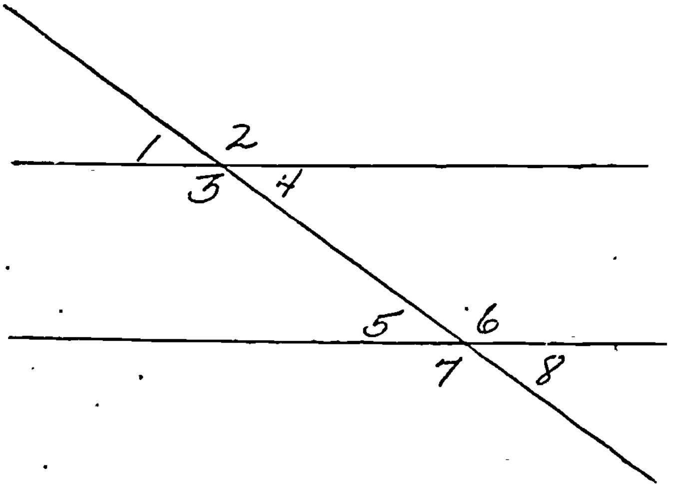
m° ∠ 5 = ______

m° ∠ 6 = ______

m° ∠ 7 = ______

m° ∠ 8 = ______

II



m° ∠ 1 = ______

m° ∠ 2 = ______

m° ∠ 3 = ______

m° ∠ 4 = 30

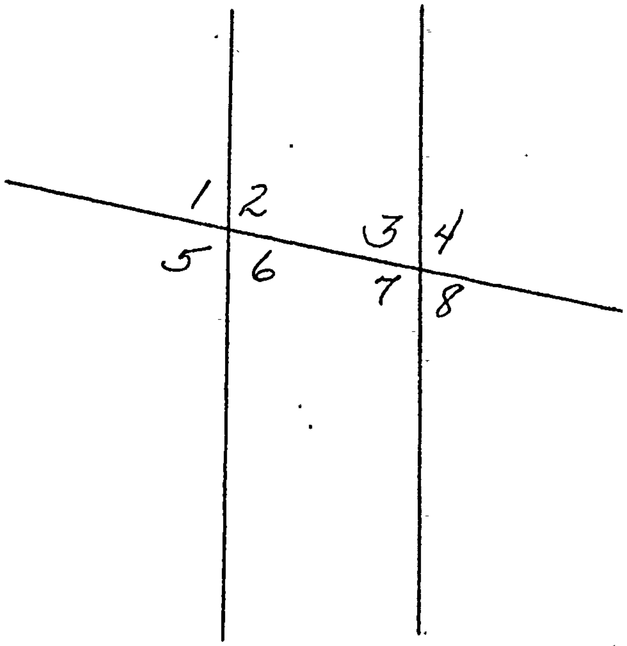
m° ∠ 5 = ______

m° ∠ 6 = ______

m° ∠ 7 = ______

m° ∠ 8 = ______

III



m° ∠ 1 = ______

m° ∠ 2 = ______

m° ∠ 3 = ______

m° ∠ 4 = ______

m° ∠ 5 = ______

m° ∠ 6 = ______

m° ∠ 7 = 100

m° ∠ 8 = ______

Level: 22 Step: A Concept: Number sentences

I. Concept:

Number sentences: translating word phrases into number phrases.

II. Behavioral Objective:

The student given a word phrase will be able to write the corresponding open number phrase.

III. Mathematical Ideas:

A number phrase is a phrase that names a number. An open number phrase is a number phrase containing a variable.

IV. Vocabulary To Stress:

number phrase	variable	replacement set or
expression	open number phrase	reference set

V. Activities:

A. Assign Houghton Mifflin (1967, 1970) Book 8, pp. 261-2: 1-12 and 25-36

B. See worksheet on following page.

Text References:

Book: 7

Houghton Mifflin (1972) pp. 8, 43

Book: 8

Houghton Mifflin (1967, 1970) pp. 258-262, (41)
Houghton Mifflin (1972) p. 8
Addison-Wesley (1971) pp. 40-42 (10)

Level: 22 Step: A Concept: Number sentences

WORKSHEET

Write an open number phrase for each word phrase.

1. the product of x and 4 ____________________

2. 6 less than y ____________________

3. the product of 5 and the sum of 3 and x ____________________

4. the quotient when the product of x and 4 is divided by 9 ____________________

5. half the sum of 7 and y ____________________

6. the sum of 5 times x and 7 ____________________

7. 4 less than twice y ____________________

8. the square of the sum of x and y ____________________

9. the sum of the square of x and y ____________________

10. the quotient when the sum of 5 and x is divided by the square of x ____________________

Level: 22 Step: B Concept: Numeral

I. Concept:

Numeral: naming opposites of rational numbers (additive inverses).

II. Behavioral Objective:

The student given any rational number will be able to write its opposite (additive inverse).

III. Mathematical Ideas:

A. Opposites are pairs of numbers whose graphs on a number line are the same distance from zero but on opposite sides of it.

B. Opposites are pairs of numbers whose sum is zero.

IV. Vocabulary To Stress:

opposites additive inverse
the negative of a number = the opposite of a number

V. Activities:

Note: Stress the difference between " a negative number" and "the negative of" or "opposite of" a number.

Text References:

Book: 8

Houghton Mifflin (1967, 1970) pp. 14-16, 47, 237, (3)
Addison-Wesley (1971) pp. 128-129

Level: 22 Step: B Concept: Order

I. Concept:

<u>Order</u>: Simplifying inequalities involving the symbols $\nless$, $\ngtr$, $\nleq$, $\ngeq$.

II. Behavioral Objective:

The student given an inequality of the form $a \nleq b$, $a \ngeq b$, $a \nless b$, $a \ngtr b$, when a and b are rational numbers, will be able to write an equivalent statement without the negation symbol.

III. Mathematical Ideas:

a and b are rational numbers. If $a \nleq b$, then $a > b$. If $a \ngeq b$, then $a < b$. If $a \nless b$, then $a \geq b$. If $a \ngtr b$, then $a \leq b$.

IV. Vocabulary To Stress:

$\leq$	nonpositive
$\geq$	negation symbol (/)

V. Activities:

A. Assign Houghton Mifflin (1967, 1970) Book 8, p. 21: 19-26.

B. See worksheet on following page.

Text References:

Book: 8

Houghton Mifflin (1967, 1970) pp. 17-21

Level: 22 Step: B Concept: Order

WORKSHEET

Fill in the blank with one of the symbols , , , to make an equivalent inequality.

1. $x \nless y$ x ________ y

2. $y \ngeq z$ y ________ z

3. $a \nleq b$ a ________ b

4. $c \ngtr d$ c ________ d

5. $e \nleq f$ e ________ f

6. $g \ngeq h$ g ________ h

7. $m \nless n$ m ________ n

8. $p \ngtr q$ p ________ q

9. $r \nleq s$ r ________ s

10. $t \nless y$ t ________ y

Level: 22 Step: B Concept: Geometry

I. Concept:

Geometry: constructing with a compass and ruler.

II. Behavioral Objective:

The student given only a compass and ruler will be able to:

A. bisect a given line segment

B. copy a given angle

C. bisect a given angle

D. construct a perpendicular to a given line from a point on the line

E. construct a perpendicular to a given line from a point not on the line

III. Mathematical Ideas:

A. A compass is used to draw a circle or an arc with a given radius.

B. A compass is also used to mark off a given length on a line.

C. See Addison-Wesley (1971), Book 7, p. 264 (TE)

IV. Vocabulary To Stress:

compass bisect perpendicular

V. Activities:

See Houghton Mifflin (1972)

A. Book 7: p. 106
Book 8: p. 105

B. Book 7: p. 292
Book 8: p. 289

C. Book 7: p. 107
Book 8: p. 105

D. Book 8: p. 105

E. Book 8: p. 105

Houghton Mifflin (1967, 1970)
Book: 8

A. p. 94

B. p. 95

C. p. 96

D. p. 126

E. p. 127

Level: 22 Step: B Concept: Geometry

Text References:

Book: 7

Houghton Mifflin (1972) pp. 106-107, 292
Addison-Wesley (1971) pp. 264-266 (51, 52, 54)

Book: 8

Houghton Mifflin (1972) pp. 105, 289 (20, 50)
Addison-Wesley (1971) pp. 183-184, (41)
Houghton Mifflin (1967, 1970) pp. 93-96, 126-127

Level: 22 Step: B Concept: Number sentences

I. Concept:

Number sentences: evaluating open number phrases.

II. Behavioral Objective:

The student given an open number phrase and a value for the variable will be able to evaluate the phrase.

III. Mathematical Ideas:

Order of operations: perform the operation within the grouping symbols before performing other operations. In the absence of grouping symbols: (1) multiply and divide together in order from left to right, (2) then add and subtract in order from left to right.

IV. Vocabulary To Stress:

value evaluate a number phrase replacement set

Text References:

Book: 8

Houghton Mifflin (1967, 1970) pp. 262-265, (42)
Houghton Mifflin (1972) (2)

Level: 22 Step: C Concept: Numeral

I. Concept:

Numeral: determining absolute value

II. Behavioral Objective:

The student given any rational number will be able to write its absolute value.

III. Matnematical Ideas:

$$\left(\begin{array}{l} \text{If } a \geq 0, \quad |a| = a \\ \text{If } a < 0, \quad |a| = -a \end{array}\right)$$

The absolute value of a rational number is either that number or its opposite, whichever is nonnegative.

IV. Vocabulary To Stress:

absolute value = distance from zero on the number line
$|a|$ = the absolute value of a

V. Activities:

A. If you introduce absolute value as "distance from zero" the student will find it easier to understand. (We have already described a directed number as one whose sign indicates direction, and whose numeral indicates distance.) Since distance is always nonnegative, absolute value is always nonnegative.

B. Discussion question: Can you walk a negative distance? (If they say "yes", have them show how. Then show them that they were walking a positive distance in a negative direction.)

Text References:

Book: 8

Houghton Mifflin (1967, 1970) pp. 21-25, (4, Sec. 4, 5, 6)
Houghton Mifflin (1972) p. 227
Addison-Wesley (1971) pp. 71-73, (18)

Level: 22 Step: C Concept: Addition & Subtraction

I. Concept:

<u>Addition & Subtraction</u>: adding and subtracting rational numbers.

II. Behavioral Objective:

The student given two or more rational numbers will be able to compute their sum or difference.

III. Mathematical Ideas:

A. Addition on the number line.

B. The sum of any two negative rational numbers is always a negative rational number.

C. If a and b represent negative rational numbers, then $|a| + |b| = |a + b|$.

D. To find the sum of two rational numbers a and b when one is positive and the other is negative and $|a| \neq |b|$, you subtract the lesser of $|a|$ and $|b|$ from the greater, and prefix the sign of the number having the greater absolute value to the numeral for the difference.

E. Closure, commutative, associative, additive identity, additive inverse properties of addition.

F. The sum of any two positive rational numbers is always positive.

G. To add two fractions first write them with their LCD, then add the numerators and write this sum over the LCD.

H. Definition of subtraction: If a, b, c are rational numbers and $a + b = c$, then $c - a = b$ and $c - b = a$.

I. If a and b are rational numbers, then $a - b = a + (-b)$.

J. To subtract a rational number, add its opposite.

K. <u>Note</u>: It might help to emphasize that we change a subtraction problem into the associated addition problem in order to use the addition properties.

Text References:

Book: 8

Houghton Mifflin (1967, 1970)	pp. 32-55, 108, 220, (5, 6, 7, 8)
Houghton Mifflin (1972)	pp. 246, 386, 388-389, (43: 1-18, 72, 73)
Addison-Wesley (1971)	pp. 130-135, (26, 27, 28, 29)

Level: 12 Step: C Concept: Multiplication

I. Concept:

Multiplication: multiplying rational numbers.

II. Behavioral Objective:

The student given two or more rational numbers will be able to compute their product.

III. Mathematical Ideas:

A. If two rational numbers have like signs, their product is positive.

B. If two rational numbers have unlike signs, their product is negative.

C. Closure, commutative, associative, multiplicative identity, multiplicative inverse, and distributive properties of multiplication.

D. Procedure for multiplication of fractions and decimals.

IV. Activities:

A. Introduce " · " symbol for multiplication and expressing the multiplication of two letters or one letter and one numeral by writing them together. Multiplication is also expressed by using parentheses.

Example:
2 · 3 instead of 2 x 3
3a instead of 3 x a
ab instead of a x b
a(b) instead of a x b
(a) b instead of a x b

B. A good review of multiplying positive fractions and decimals would be helpful in beginning this lesson. Houghton Mifflin (1972) Book 8, has a good variety of these problems in the "Extra Practice" pages at the back of the book.

Text References:

Book: 8

Houghton Mifflin (1967, 1970) pp. 62-72, 108, 220, (10, 11, 12)
Houghton Mifflin (1972) pp. 247, 249, 250, 390-391 (43: 19-27, 73)
Addison-Wesley (1971) pp. 136-139, 258-260 (30)

Level: 22 Step: C Concept: Division

I. Concept:

<u>Division</u>: dividing rational numbers.

II. Behavioral Objective:

The student given two rational numbers will be able to compute their quotient.

III. Mathematical Ideas:

A. If a, b, n are rational numbers, and $b \neq 0$, then $a \div b = n$ if and only if $nb = a$.

B. To divide by a rational number, multiply by its reciprocal (multiplicative inverse).

C. Since division has been re-written as multiplication, the laws of signs for multiplication hold.

IV. Activities:

<u>Note</u>: Students can usually follow this line of reasoning very easily:

$$a \div b = \frac{a}{b} = a\left(\frac{1}{b}\right)$$

Text References:

Book: 8

Houghton Mifflin (1967, 1970)	pp. 73-77, 108, 220, (13, 14 +, -, x, ÷)
Houghton Mifflin (1972)	pp. 247, 249, 250 (43: 28-36)
Addison-Wesley (1971)	pp. 144-145, 147-148, (32)

Level: 22 Step: C Concept: Number sentences

I. Concept:

Number sentences: using the replacement set to find the solution set.

II. Behavioral Objective:

The student given an open sentence with a finite replacement set will be able to write the solution set.

III. Mathematical Ideas:

Solution set $\subset$ replacement set

IV. Vocabulary To Stress:

number sentence	equality	root
open number sentence	statement	solution set
equation	solution	replacement set

V. Activities:

Note: Since the vocabulary in this section is extremely important, it would be a good idea to cover this section verbally in H. M.(1967, 1970), Book 8, pp. 266-269 and pp. 270-271: 1-10 and 17-30, stressing the vocabulary above. Ditto #43 goes along with this. Then assign pp. 270-271: 11-16 and 31-34.

Text References:

Book: 8

Houghton Mifflin(1967, 1970)	pp. 266—271, (43)
Houghton Mifflin(1972)	pp. 384-385
Addison-Wesley (1971)	pp. 29-30

Level: 22 Step: D Concept: Sets

I. Concept:

Sets: determining whether a given set is closed for a specified operation.

II. Behavioral Objective:

The student given a set and an operation will be able to state whether the set is closed for that operation.

III. Mathematical Ideas:

A. A set is said to be closed under a particular operation when the result of performing the operation on members of the set is also a member of the set.

B. The set of rational numbers has the closure property for addition and multiplication because the sum or product of any two rational numbers is a rational number.

IV. Vocabulary To Stress:

closure

V. Activities:

A. See Houghton Mifflin (1972) Book 7, p. 382: 1-12
Book 8, p. 385: 19-22

Note: The chart on p. 385 in Book 8 has an error. Under Subtraction {whole numbers} and {even numbers} the answer is NO.

B. See worksheet on following page.

Text References:

Book: 7

Houghton Mifflin (1972) p. 382
Addison-Wesley (1971) p. 170: 3

Book: 8

Houghton Mifflin (1972) pp. 223, 384-385, 404

Level: 22 Step: D Concept: Sets

WORKSHEET

In the blank, write "yes" or "no" to tell whether each set is closed for the given operation.

1. {1, 0}, multiplication ______________

2. {multiples of 3}, addition ______________

3. {whole numbers}, subtraction ______________

4. {rational numbers}, division ______________

5. {odd numbers}, addition ______________

6. {1, 0}, addition ______________

7. {negative rational numbers}, multiplication ______________

8. {negative integers}, addition ______________

9. {1, 2, 4}, multiplication ______________

10. {1, 0}, subtraction ______________

11. {integers}, division ______________

12. $\{\frac{1}{2}, 1, 1\frac{1}{2}\}$, addition ______________

13. {rational numbers}, subtraction ______________

14. {1}, division ______________

15. {integers}, multiplication ______________

Level: 22 Step: D Concept: Number sentences

I. Concept:

Number sentences: solving equations by inspection.

II. Behavioral Objective:

The student given a simple linear equation in one variable will be able to solve it over the set of rational numbers by inspection.

III. Mathematical Ideas:

Solving an equation by inspection means finding the solution set by looking at it, reading it for meaning, and using common sense.

IV. Vocabulary To Stress:

solving over a given set　　　　solving by inspection

Text References:

Book: 8

Houghton Mifflin (1967, 1970)	pp. 271-275, (44)
Houghton Mifflin (1972)	pp. 10-11
Addison-Wesley (1971)	pp. 34-35 (5, 6)

Level: 22 Step: Z Concepts: Sets

I. Concept:

Sets: Specifying sets using set-builder notation.

II. Behavioral Objective:

The student given a set specified in set-builder notation will be able to write it out in words.

III. Mathematical Ideas:

A set S may be specified by showing in braces a variable x, then a colon (:) meaning "such that", and finally a description of the variable.

Example: $S = \{x: x + 2 = 7\}$ (Read as "the set of all x such that x + 2 = 7".)

Sometimes a vertical slash (|) is used instead of a colon.

Example: $S = \{x \mid x + 2 = 7\}$

IV. Vocabulary To Stress:

See Houghton Mifflin (1967, 1970) Book 8, pp. 9-11.

N = {natural numbers}
W = {whole numbers}
J = {integers}
R = {rational numbers}

V. Activities:

See worksheets on the following pages.

Text References:

Book: 8

Houghton Mifflin (1967, 1970) pp. 9-11

Other References:

Christian, Robert R., Introduction to Logic and Sets, pp. 50-55, Blaisdell Publishing Company.

Level: 22 Step: Z Concept: Sets

WORKSHEET

Specify each set by roster (list):

Example:

$\{a : a > 1, a \in N\} = \{2, 3, 4, \ldots\}$

1. $\{b : b < 4, b \in W\} =$

2. $\{c : c + 5 = 5, c \in W\} =$

3. $\{d : d > 8, d < 12, d \in W\} =$

4. $\{j : 4j = 10, j \in N\} =$

5. $\{f : f \in W\} =$

6. $\{x : x + 3 = 9, x \in N\} =$

7. $\{y : 2y - 2 = 2, y \in N\} =$

8. $\{z : 3z = 1, z \in$ [illegible]$\} =$

Level: 22 Step: Z Concept: Sets

WORKSHEET

Write out in words:

Example: $\{g : g < 7, g \in W\}$

The set of all g such that g is a whole number less than 7 or the set of g such that g is less than 7 and g is a whole number.

1. $\{b : b + 4 = 18, b \in R\}$

2. $\{x : 3x = 12, x \in R\}$

3. $\{y : 2y - 1 = 5, y \in J\}$

4. $\{c : 1 < c < 4, c \in R\}$

5. $\{d : d > 100, d \notin N\}$

Level: 22 Step: Z Concept: Sets

ENRICHMENT EVALUATION

Write out in words in the space provided.

1. $\{x: -2 < x < 5, x \in N\}$

2. $\{x: x < 11, x \in W, x \neq 0\}$

3. $\{y: 4y + 7 = 19, y \in N\}$

4. $\{a: a \in J, a < 0\}$

5. $\{b: 3b = 1, b \in R\}$

Level: 22 Step: Z Concept: Sets

I. Concept:

Sets: Thinking independently -- "Just For Fun".

II. Activities:

See Houghton Mifflin(1967, 1970), Book 8:

A. p. 29

B. p. 219

C. p. 309

D. pp. 404-405

Note: There is no separate evaluation for this unit; however, the answers to these problems are in the back of the teacher's edition.

Text References:

Book: 8

Houghton Mifflin(1967, 1970) pp. 29, 219, 309, 404-405.

Level: 22 Step: Z Concept: Sets

I. Concept:

Sets: Thinking logically -- beginning logic.

II. Behavioral Objectives:

The student given:

A. a sentence containing the word "is", will be able to state whether or not the "is" can meaningfully be replaced by " = ".

B. a proposition (p) will be able to state its truth value ($\mathcal{T}(p)$).

C. two propositions, will be able to state truth values using the operations of conjunction, disjunction, or negation.

III. Mathematical Ideas:

A. The word "is" has many meanings. One of these is that of identity. In logic, " = " and "is" are both used to mean "is identical with".

B. A proposition has the truth value T if it is true and truth value F if it is false.

$\mathcal{T}(p)$ means "the truth value of proposition p."

C. When 2 statements are combined by the word and, the result is their conjunction. The symbol "$\wedge$" is used for the operation of conjunction.

D. When 2 statements are combined by the word or, the result is their disjunction. The symbol "V" is used for the operation of disjunction.

E. The negation of a proposition is the expression "It is false that" followed by the proposition itself. The symbol "$\sim$" is used for the operation of negation.

IV. Vocabulary To Stress:

identity	equal	substitution
transitivity	symmetry	propositions
truth values	conjunction	disjunction
negation	symbolic form	"$\mathcal{T}(x)$"
"$\wedge$"	"V"	"$\sim$"

Level: 22 Step: Z Concept: Sets

V. Activities:

Christian, pp. 3-11, (Introduction, equality, propositions and truth values, basic operations)

References:

Robert R. Christian, Introduction to Logic and Sets, second edition, Blaisdell Publishing Company, 1965.

Level: 22 Step: Z Concpet: Sets

ENRICHMENT EVALUATION

I. Place a check mark (✓) by any of the following sentences in which the word "is" can meaningfully be replaced by the equality sign (=).

1. My name is John. ________________
2. John is in the 8th grade . ________________
3. Every mother is a women. ______________
4. The capital of Missouri is Jefferson City. ____________
5. Math is fun. ________________

II. Find $\mathcal{T}(x)$.

1. x = "7 is an integer."

   $\mathcal{T}(x)$ = ____________

2. x = "3 x 6 + 5 x 3 = 5 x 6 + 3 x 0"

   $\mathcal{T}(x)$ = ____________

3. x = "2y = 8 when y = 4"

   $\mathcal{T}(x)$ = ____________

4. x = "2 + 2 = 2 x 2"

   $\mathcal{T}(x)$ = ____________

5. x = "-3 < 2"

   $\mathcal{T}(x)$ = ____________

III. Let p be "2 x 3 = 6" and let q be "-4 + 6 = -10". Find $\mathcal{T}$(each of the following).

1. $\mathcal{T}(p \wedge \sim q)$ = ____________
2. $\mathcal{T}(\sim p \vee q)$ = ____________
3. $\mathcal{T}[\sim(\sim p \wedge \sim q)]$ = ____________
4. $\mathcal{T}[(\sim p \wedge q) \vee (p \wedge \sim q)]$ = ____________
5. $\mathcal{T}[\sim(p \vee q)]$ = ____________

22-30

Level: 22 Step: Z Concept: Numeral

I. Concept:

Numeral: Using greatest integer notation.

II. Behavioral Objective:

The student given a rational number written in "greatest integer" notation will be able to write the integer named.

III. Mathematical Ideas:

A. If n is a rational number, $[n] \leq n$.

B. If n is an integer, $[n] = n$

C. $\left[3\frac{1}{2}\right]$ means "the greatest integer not greater than $3\frac{1}{2}$"

D. $\left[3\frac{1}{2}\right] = 3$.

IV. Activities:

See H.M. (1967, 1970), Book 8, p. 81

Text References:

Book: 8

Houghton Mifflin (1967, 1970), p. 81

Level: 22 Step: Z Concept: Numeral

ENRICHMENT EVALUATION

State the integer named by each symbol.

1. $\left[7\frac{1}{3}\right]$ ________

2. $\left[5\frac{2}{3}\right]$ ________

3. $\left[-\frac{1}{3}\right]$ ________

4. $\left[-3\frac{1}{3}\right]$ ________

5. $-\left[-3\right]$ ________

Level: 22 Step: Z Concept: Function

I. Concept:

Function: Finding the probability with simple experiments.

II. Behavioral Objective:

The student given a simple experiment will be able to find the probability that a given event will occur.

III. Mathematical Ideas:

A. If an experiment can result in n different equally likely outcomes, then the probability that any given one of these will occur is $\frac{1}{n}$.

B. P(a favorable outcome) = $\frac{\text{no. of favorable outcomes}}{\text{no. of possible outcomes}}$

C. p(not E) = 1 - P(E).

D. P(E) means the probability that event E will occur.

Text References:

Book: 8

Houghton Mifflin (1967, 1970) pp. 470-477, (76)
Houghton Mifflin (1972) pp. 202-205
Addison-Wesley (1971) pp. 207-213, (47)

Other References:

Imperial Tapes (Intermediate), #40

Willerding, Probability: The Science of Chance. (Franklin Math Series), pp. 5-40.

Level: 22 Step: Z Concept: Function

ENRICHMENT EVALUATION

I. A deck of cards contains 52 cards. An experiment consists of shuffling the cards, shutting your eyes, and drawing a card from the deck. Find the probability that the card drawn is:

1. an ace ______________
2. a spade ______________
3. not a ten ______________
4. the ace of hearts ______________
5. a 10, jack, queen, or king ______________

II. An urn contains 4 red, 3 blue, and 5 green marbles. If one marble is drawn at random, find the probability that it is:

1. red ______________
2. blue ______________
3. green ______________
4. red or green ______________
5. not red ______________

III. If an ordinary die is rolled, find the probability that the number of dots on its top face is:

1. 3 ______________
2. an odd number ______________
3. greater than 4 ______________
4. not 6 ______________
5. 10 ______________

Level: 22 Step: Z Concept: Geometry

I. Concept:

<u>Geometry</u>: Tracing networks.

II. Behavioral Objectives:

The student given:

(A) a drawing of a network will be able to tell whether or not it is traceable.

(B) the number of twists in a moebius strip will be able to tell how many sides the strip has or how many loops will result from a given cut.

III. Mathematical Ideas:

A. A network is a series of connected curves called arcs.

B. Traceable networks are networks that can be traced without lifting the pencil from the paper or retracing an arc.

C. A moebius strip is a one-sided surface formed by taking a strip of paper, twisting one end, and pasting the ends together.

IV. Activities:

A. See Horne, <u>Patterns and Puzzles in Mathematics</u>, pp. 71-80.

B. See Bruyer, <u>Geometrical Models and Demonstrations</u>, pp. 100-106.

C. See H.M. (1967, 1970), Book 8, pp. 146-149

D. See Bell Series booklet #8 -- <u>Networks</u>.

Text References:

Book: 8

Houghton Mifflin (1967, 1970)	pp. 146-149
Houghton Mifflin (1972)	pp. 276-277, 303
Addison-Wesley (1971)	pp. 55, 143. (TE)

Other References:

Sylvia Horne, <u>Patterns and Puzzles in Mathematics</u>, Franklin Publications, 1970. (The Franklin Mathematics Series)

Donald L. Bruyer, <u>Geometrical Models and Demonstrations</u>, J. Weston Walch, 1963.

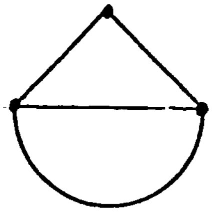
Stuart E. Bell, Mathematics in the Making, Booklet #8 -- <u>Networks</u>

Level: 22 Step: Z Concept: Geometry

ENRICHMENT EVALUATION

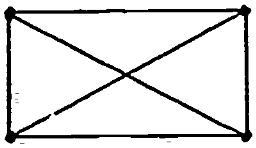
I. Write "yes" in the blank if the network is traceable; write "no" if it is not.

1.



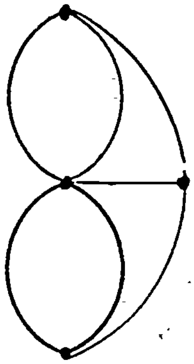
____

2.



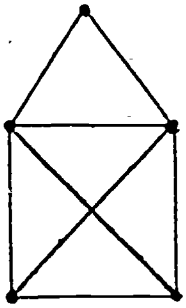
____

3.



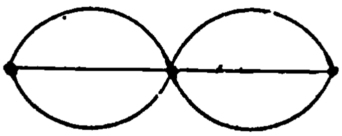
____

4.



____

5.



____

Level: 22 Step: Z Concept: Geometry

EVALUATION: continued

II. Fill in the blanks with the proper number.

1. A moebius strip with one twist has ______________ sides.

2. A moebius strip with 2 twists has ______________ sides.

3. If a moebius strip with one twist is cut down the middle, how many loops will result?

   ____

4. If a moebius strip with one twist is cut $\frac{1}{3}$ of the way from one edge, how many loops will result?

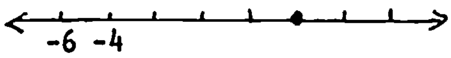
   ____

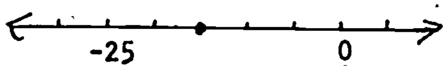
5. Two identical strips of paper are placed one on top of the other, given a single twist, and the ends of each strip are taped together. If they are opened out, how many bands will there be?

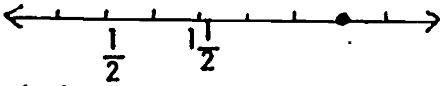
   ____

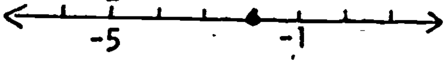
Level: 22 ANSWERS TO WORKSHEETS

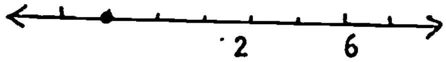
22-2

1. -6
2. 15
3. -2
4. $2\frac{3}{4}$
5. 10
6. 

7. 

8. 

9. 

10. 

22-5

I. $\angle 2 = 60$
$\angle 3 = 60$
$\angle 4 = 120$
$\angle 5 = 120$
$\angle 6 = 60$
$\angle 7 = 60$
$\angle 8 = 120$

II. $\angle 1 = 30$
$\angle 2 = 150$
$\angle 3 = 150$
$\angle 5 = 30$
$\angle 6 = 150$
$\angle 7 = 150$
$\angle 8 = 30$

III. $\angle 1 = 80$
$\angle 2 = 100$
$\angle 3 = 80$
$\angle 4 = 100$
$\angle 5 = 100$
$\angle 6 = 80$
$\angle 8 = 80$

22-7

1. $4x$
2. $y - 6$
3. $5(3 + x)$
4. $\frac{4x}{9}$ or $4x \div 9$
5. $\frac{1}{2}(7 + y)$ or $\frac{7 + y}{2}$
6. $5x + 7$
7. $2y - 4$
8. $(x + y)^2$
9. $x^2 + y$
10. $\frac{5 + x}{x^2}$ or $(5 + x) \div x^2$

22-10

1. $\geq$
2. $\leq$
3. $>$
4. $\leq$
5. $>$
6. $<$
7. $\geq$
8. $\leq$
9. $>$
10. $\geq$

22-20

1. yes
2. yes
3. no
4. yes
5. no
6. no
7. no
8. yes
9. no
10. no
11. no
12. no
13. yes
14. yes
15. yes

Level: 22

ANSWERS TO ENRICHMENT (STEP Z)

WORKSHEETS

22-23

1. $\{0, 1, 2, 3\}$
2. $\{0\}$
3. $\{9, 10, 11\}$
4. Φ
5. $\{0, 1, 2, \ldots\}$
6. $\{6\}$
7. $\{2\}$
8. $\{\frac{1}{3}\}$

22-24

1. The set of all b such that $b + 4 = 18$, and b is a rational number.
2. The set of all x such that $3x = 12$, and x is a rational number.
3. The set of all y such that $2y - 1 = 5$, and y is an integer.
4. The set of all c such that c is a rational number between 1 and 4, or the set of all c such that $c > 1$, $c < 4$, and c is a rational number.
5. The set of all d such that d is greater than 100, and d is not a natural number.

Level: 22 ANSWERS TO ENRICHMENT (STEP Z) (continued)

EVALUATION

22-25

1. The set of all x such that x is a natural number greater than -2 and less than 5.

2. The set of all x such that x is a whole number less than 11, but not zero.

3. The set of all y such that $4y + 7 = 19$, and y is a natural number.

4. The set of all a such that a is a negative integer.

5. The set of all b such that $3b = 1$, and b is a rational number.

NOTE: The above answers (page 25) may be expressed in ways other than those given.

22-29

I.	II.	III.
1. ✓	1. T	1. T
2. ___	2. F	2. F
3. ___	3. T	3. T
4. ✓	4. T	4. T
5. ___	5. T	5. F

22-31

1. 7
2. 5
3. -1
4. -4
5. 3

22-33

I.	II.	III.
1. $\frac{1}{13}$	1. $\frac{1}{3}$	1. $\frac{1}{6}$
2. $\frac{1}{4}$	2. $\frac{1}{4}$	2. $\frac{1}{2}$
3. $\frac{12}{13}$	3. $\frac{5}{12}$	3. $\frac{1}{3}$
4. $\frac{1}{52}$	4. $\frac{3}{4}$	4. $\frac{5}{6}$
5. $\frac{4}{13}$	5. $\frac{2}{3}$	5. 0

22-35

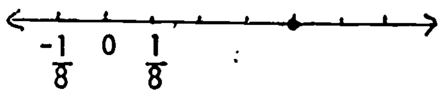
I.	II.
1. yes	1. 1
2. no	2. 1
3. no	3. 1
4. yes	4. 2
5. yes	5. 1

Level: 22 Answers - Post Test I

Step A

Numeral

1. -1
2. 18
3. 

4. 

5. 

Order

6. >
7. >
8. =
9. >
10. <

Geometry

11. 110
12. 120
13. 70
14. 90
15. 120

Number Sentences

16. x + 3 or 3 + x
17. x + 1
18. y - 4
19. $\frac{1}{2}(5 + a)$ or $\frac{5 + a}{2}$
20. 6(2 + y)

Step B

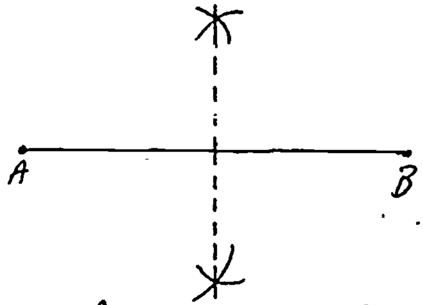
Numeral

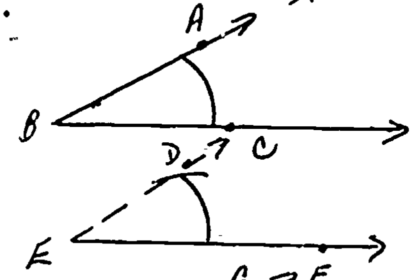
21. 3
22. 0
23. $-\frac{3}{4}$
24. 2.3^4
25. -1

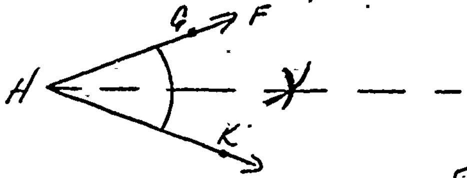
Order

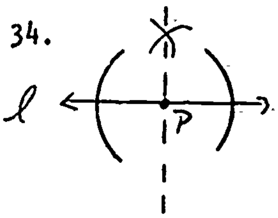
26. >
27. <
28. >
29. ≤
30. ≥

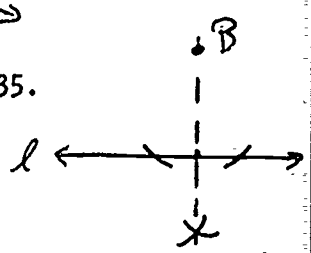
Geometry

31. 

32. 

33. 

34. 

35. 

Number Sentences

36. 0
37. 8
38. 16
39. 0
40. $\frac{1}{4}$

Step C

Numeral

41. 4.7
42. 5
43. 4
44. 0
45. $8\frac{1}{3}$

Addition & Subtraction

46. $-2\frac{3}{4}$ or $-\frac{11}{4}$
47. $1\frac{1}{7}$ or $\frac{8}{7}$
48. -.35 or $-\frac{7}{20}$
49. $-1\frac{1}{2}$ or $-\frac{3}{2}$
50. 9.79

Level: 22 Answers - Post Test I

Step C

Multiplication

51. 4
52. -20.345
53. -14
54. $2\frac{1}{5}$ or $\frac{11}{5}$
55. -30

Division

56. $-\frac{4}{3}$ or $-1\frac{1}{3}$
57. -2.4
58. $\frac{18}{11}$ or $1\frac{7}{11}$
59. $-\frac{45}{8}$ or $-5\frac{5}{8}$
60. .0486

Number Sentences

61. {-2}
62. {0, 1, 2, 3}
63. {-2, -1, 0, 1}
64. Φ
65. {-1, 1}

Step D

Sets

66. yes no yes no
67. yes no yes no
68. no no yes no
69. yes yes [illegible] [illegible]
70. yes yes [illegible] yes

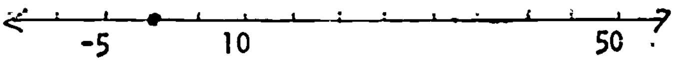
Number Sentences

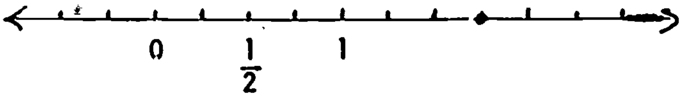
71. {21}
72. {6}
73. {36}
74. {-1}
75. {-5}

Level: 22 Answers - Post Test II

Step A

Numeral

1. 1
2. -4
3. 

4. 

5. 

Order

26. <
27. ≥
28. ≤
29. ≤
30. ≥

Order

6. >
7. <
8. =
9. >
10. >

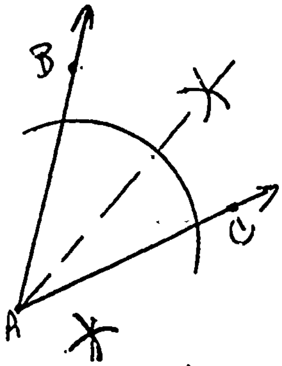
Geometry

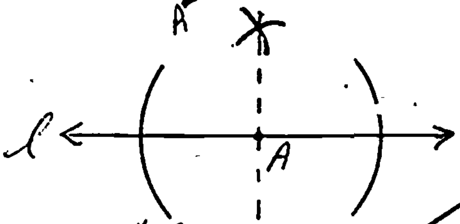
11. 145
12. 90
13. 80
14. 135
15. 50

Number Sentences

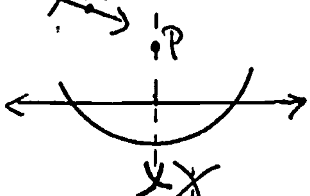
16. y + 5 or 5 + y
17. b - 6
18. y + 2
19. $\frac{1}{4}(6 + x)$ or $\frac{6 + x}{4}$
20. 4(3 + x)

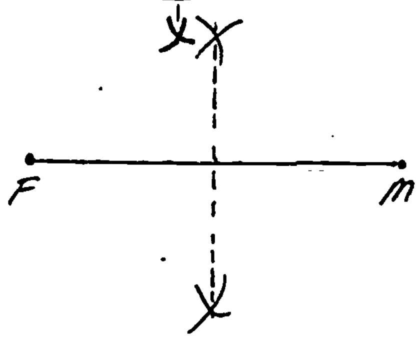
Geometry

31. 

32. 

33. 

34. 

35. 

Step B

Numeral

21. 0
22. 7
23. $-\frac{7}{8}$
24. -5.2
25. -3

Number Sentences

36. 0
37. 4
38. 4
39. 4
40. $\frac{3}{-2}$ or $-\frac{3}{2}$ or $-1\frac{1}{2}$

Level: 22 Answers - Post Test II

Step C

Numeral

41. 0
42. 6
43. 2
44. 8.78
45. 10

Addition & Subtraction

46. $-5\frac{7}{8}$ or $-\frac{47}{8}$
47. $-\frac{11}{14}$
48. -2.35 or $-2\frac{7}{20}$
49. 1.53
50. 1

Multiplication

51. 6
52. -30.149
53. -18
54. 3
55. -48

Division

56. $-\frac{5}{3}$ or $-1\frac{2}{3}$
57. -6.23
58. $\frac{2}{3}$
59. $-\frac{11}{32}$
60. .478

Number Sentences

61. {-3}
62. {-1, 0, 1, 2, 3}
63. {-3, -2, -1, 0, 1, 2}
64. Φ
65. {-3, 3}

Step D

Sets

66. yes yes yes no
67. yes no yes yes
68. yes yes yes yes
69. yes no yes no
70. yes no yes no

Number Sentences

71. {21}
72. {-3}
73. {-9}
74. {-2}
75. {3}

Other references used in levels 22-25:

Bruyer, Donald. L, Geometrical Models and Demontrations, J. Weston Walch, 1963

Christian, Robert R. Introduction to Logic and Sets (Second Edition), Blaisdell Publishing Co., New York, 1965.

Laycock, Mary, Straw Polyhedra, Creative Publications, Inc., 1970

TABLE OF CONTENTS

Level 23

Step A

Step B

Step C

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Level 23

Step D

Step Z

Level: 23 Step: A Concept: Multiplication

I. Concept:

Multiplication: Multiplying powers of 10 with positive integral exponents.

II. Behavioral Objective:

The student given two powers of 10 with positive integral exponents will be able to write their product as a single power of 10 in exponential form.

III. Mathematical Ideas:

$x^a \cdot x^b = x^{a+b}$ (x is rational, a and b are natural numbers)

IV. Vocabulary To Stress:

base exponent--An exponent is a "counter". It counts the number of times the base is used as a factor.

V. Activities:

NOTE: Do not say an exponent tells how many times a number is multiplied by itself. This is not true. If you multiply 5 by itself once you have 5 x 5, or 25. If you multiply 5 by itself twice you could have (5 x 5) x 5 = 125, or you could have 2(5 x 5) = 50.

Text References:

Book: 8

Houghton Mifflin(1967, 1970) pp. 154-158, 170
Houghton Mifflin(1972) p. 234
Addison Wesley (1971) p. 13, (9a-9f, 10a)

Level: 23 Step: A Concept: Division

I. Concept:

Division: Dividing powers of 10 with positive integral exponents.

II. Behavioral Objective:

The student given two powers of 10 with positive integral exponents will be able to write their quotient as a single power of 10 or as a fraction with 1 as numerator and a power of 10 as denominator.

III. Mathematical Ideas:

If x is rational, a and b are natural numbers, and $x \neq 0$:

A. $\frac{x^a}{x^b} = x^{a-b}$ if $a > b$ or

B. $\frac{x^a}{x^b} = \frac{1}{x^{b-a}}$ if $b > a$.

Text References:

Book: 8

Houghton Mifflin(1967, 1970) pp. 155-158
Addison-Wesley(1971) p. 13, (9g -91, 10b)

Level: 23 Step: A Concept: Geometry

I. Concept:

Geometry: Proving triangles congruent by ASA, SSS, SAS.

II. Behavioral Objective:

The student given drawings of 2 or more triangles with congruent sides labeled with strokes and congruent angles labeled with arcs will be able to name the congruent triangles and state why they are congruent. (ASA, SSS, SAS)

III. Mathematical Ideas:

A. Corresponding parts of congruent triangles are congruent.

B. Two triangles are congruent if 2 angles and the included side of one triangle are congruent respectively to 2 angles and the included side of the other.

C. Two triangles are congruent if 2 sides and the included angle of one triangle are congruent respectively to two sides and the included angle of the other.

D. Two triangles are congruent if the three sides of one triangle are congruent repectively to the three sides of the other.

IV. Vocabulary To Stress:

congruent	included angle	$\cong$	SSS
included side	$\longleftrightarrow$	ASA	SAS

Text References:

Book: 8

Houghton Mifflin(1967, 1970)	pp. 93, 118-131, (20,21)
Houghton Mifflin(1972)	pp. 280-281
Addison-Wesley(1971)	pp. 184-188, (42)

Level: 23 Step: A Concept: Number sentences

I. Concept:

Number sentences: Solving equations using the addition property of equality.

II. Behavioral Objective:

The student given a simple linear equation in one variable (no coefficient) will be able to solve it using successive transformations.

III. Mathematical Ideas:

A. If a, b, c are rational numbers and $a = b$, then $a + c = b + c$. (Addition property of equality).

B. An equation can be solved by successively transforming it into simpler equivalent equations until the solution set is obvious.

IV. Vocabulary To Stress:

equivalent equations transformations addition property of equality

Text References:

Book: 8

Houghton Mifflin(1967, 1970) pp. 282-288, (46)

Level: 23 Step: B Concept: Numeral

I. Concept:

Numeral: Extending the powers of 10 to include those with negative or zero integral exponents.

II. Behavioral Objective:

The student given a power of 10 written either in decimal notation or with an integral exponent will be able to write the numeral in the form not given.

III. Mathematical Ideas:

A. $a^0 = 1 \quad (a \neq 0)$

B. $a^{-b} = \frac{1}{a^b}$ where a is rational b is an integer, $a \neq 0$.

C. Specifically:

$10^0 = 1$ and $10^{-b} = \frac{1}{10^b}$ where b is an integer.

IV. Activities:

Assign Houghton Mifflin (1967, 1970), Book 8, p. 161: 1-12.

Text References:

Book: 8

Houghton Mifflin (1967, 1970)	pp. 158-161, (24, sec. 1-13)
Houghton Mifflin (1972)	pp. 234-235
Addison-Wesley (1971)	pp. 247-248

Level: 23 Step: B Concept: Multiplication

I. Concept:

Multiplication: Multiplying powers of 10 with integral exponents.

II. Behavioral Objective:

The student given two powers of 10 with integral exponents will be able to compute their product.

III. Mathematical Ideas:

$x^a \cdot x^b = x^{a+b}$ (x is rational, a and b are integers, $x \neq 0$).

Text References:

Houghton Mifflin (1967, 1970)	pp. 160-161, 170, (24)
Houghton Mifflin (1972)	p. 383
Addison-Wesley (1971)	p. 249

Level: 23 Step: B Concept: Division

I. Concept:

Division: Dividing powers of 10 with integral exponents.

II. Behavioral Objective:

The student given two powers of 10 with integral exponents will be able to compute their quotient.

III. Mathematical Ideas:

$$\frac{x^a}{x^b} = x^{a-b} \qquad (x \text{ is rational, } a \text{ and } b \text{ are integers, } x \neq 0)$$

Text References:

Book: 8

Houghton Mifflin(1967, 1970)	pp. 160-161, (24)
Houghton Mifflin(1972)	p. 383
Addison-Wesley(1971)	p. 249

Level: 23 Step: B Concept: Geometry

I. Concept:

Geometry: Identifying the 5 regular polyhedrons.

II. Behavioral Objective:

The student given drawings or models of the 5 regular polyhedrons will be able to name them.

III. Mathematical Ideas:

There are only 5 regular polyhedrons.

IV. Vocabulary To Stress:

polyhedron	hexahedron (cube)	dodecahedron
tetrahedron	octahedron	icosahedron

V. Activities:

A. Make each figure using patterns. See Franklin Mathematics Series, Sylvia Horne, Patterns and Puzzles in Mathematics, pp. 49-54, for directions on how to make a mobile with these figures with either closed or open surfaces.

B. Make polyhedra out of straws. See Straw Polyhedra, by Mary Laycock, Creative Publications, Inc., 1970, pp. 30-35, 40.

C. See Bruyr (listed in references). This is an excellent presentation of this topic.

D. See worksheet on the following page.

Text References:

Book: 8

Addison-Wesley (1971) pp. 299-300 .

Other References:

Bruyr, Donald L., Geometrical Models & Demonstrations, J. Weston Walch, 1963, pp. 123-126, 130.

Horne, Sylvia, Patterns and Puzzles in Mathematics, Franklin Mathematics Series, 1970.

Laycock, Mary, Straw Polyhedra, Creative Publications, Inc., 1970.

Level: 23 Step: B Concept: Geometry

WORKSHEET

POLYHEDRONS

Number of faces	Name of Polyhedron	On a <u>regular</u> polyhedron with this many faces, each face is a polygon called a ......
4		
6		This regular polyhedron is also called a ____________.
8		
	dodecahedron	
	icosahedron	

Level: 23 Step: B Concept: Number sentences

I. Concept:

Number sentences: Solving equations using the multiplication property of equality.

II. Behavioral Objective:

The student given a simple linear equation in one variable will be able to solve it using successive transformations.

III. Mathematical Ideas:

A. If a, b, x are rational numbers then $ax + bx = (a + b)x$. (Distributive Property).

B. If the variable appears on both sides of the equation, one of these may be eliminated by adding its opposite to both sides.

1. additive property of equality
2. additive inverses
3. additive identity

C. Multiplication property of equality:

If a, b, c are rational numbers, and $a = b$, then $ac = bc$.

1. Multiplicative inverses
2. Multiplicative identity

IV. Vocabulary To Stress:

$-x$ is read "the opposite of x"

Text References:

Book: 8

Houghton Mifflin(1967, 1970) pp. 288-294, (47)

Level: 23 Step: C Concept: Numeral

I. Concept:

Numeral: Expressing rational numbers in scientific notation.

II. Behavioral Objective:

The student given a rational number in decimal form or in scientific notation will be able to write it in the form not given.

III. Mathematical Ideas:

A. Multiplying and dividing a decimal numeral by powers of 10 can be done by changing the place values.

B. A number is expressed in scientific notation if it is represented in the form

$$a \times 10^n$$

where a is a number with absolute value between 1 and 10 and n is an integer.

1. If the number is a power of 10, then the first factor is 1 and need not be written.

2. If n = 0, then the second factor is 1 and need not be written.

IV. Vocabulary To Stress:

scientific notation standard position for the decimal point.

Text References:

Book: 8

Houghton Mifflin(1967, 1970) pp. 161-165, 169, 170, (25)
Houghton Mifflin(1972) pp. 24, 237 (7)
Addison-Wesley (1971) pp. 265-267, (59)

Level: 23 Step: C Concept: Multiplication

I. Concept:

Multiplication: Using scientific notation to simplify products.

II. Behavioral Objective:

The student given rational numbers in scientific notation or in decimal form will be able to use scientific notation to simplify their product.

III. Mathematical Ideas:

A. Expressing decimal numerals in scientific notation.

B. Laws of exponents for multiplication.

C. Associative and commutative properties of multiplication.

Text References:

Book: 8

Houghton Mifflin (1967, 1970)	pp. 166-168, 170, (26)
Addison-Wesley (1971)	pp. 271-272, 267, (59, sec. 3)

Level: 23 Step: C Concept: Division

I. Concept:

Division: Using scientific notation to simplify division.

II. Behavioral Objective:

The student given rational numbers in scientific notation or in decimal form will be able to use scientific notation to simplify their quotient.

III. Mathematical Ideas:

A. Expressing decimal numerals in scientific notation.

B. Laws of exponents for multiplication and division.

C. Associative and commutative properties of multiplication.

Text References:

Book: 8

Houghton Mifflin (1967, 1970) pp. 166-168, (26)
Addison-Wesley (1971) pp. 271-272

Level: 23 Step: C Concept: Geometry

I. Concept:

Geometry: Classifying prisms according to their bases.

II. Behavioral Objective:

The student given a drawing or model of a prism (base having 3, 4, 5, 6, or 8 sides) will be able to classify it according to its base.

III. Mathematical Ideas:

Prisms are classified according to the kind of congruent polygonal regions that form their bases.

IV. Vocabulary To Stress:

prism	triangular prism	pentagonal prism	octagonal prism
parallelepiped	rectangular prism	hexagonal prism	cube

V. Activities:

Make models. See Houghton Mifflin(1967, 1970), Book 8, p. 389, and Addison-Wesley, Book 8, p. 298.

Text References:

Book: 8

Houghton Mifflin(1967, 1970) pp. 385-389, (62, sec. 1-6, rest for review)
Addison-Wesley (1971) p. 297

Level: 23 Step: C Concept: Measurement

I. Concept:

Measurement: Extending conversions within the metric system.

II. Behavioral Objective:

The student given a measurement using any metric unit of length will be able to write it in scientific notation using any other metric unit of length.

III. Mathematical Ideas:

A. Multiplying and dividing by powers of 10.

B. Each metric unit is 10 times the next smaller unit.

IV. Vocabulary To Stress:

prefixes	μ	hecto	mm.	m.	km.
milli	deci	kilo	cm.	dkm.	M.
centi	deca	mega	dm.	hm.	micron

V. Activities:

An easier way to teach this section is to first show how to write 1 cm. in terms of any other unit (and 1 mm. in terms of any other unit, and 1 dkm. etc.).

For example:

1 dm. = _____ dkm.

answer: 1 dm. = 10^{-2} dkm. (1 dkm. ÷ 100)

Now assign the attached worksheets.

After the student can change easily from one unit to the other, then introduce problems involving more than 1 cm. (or any other unit).

For example:

27 dm. = __?__ dkm.

1 dm. = 10^{-2} dkm.

so 27 dm. = 27×10^{-2} dkm. = 2.7×10^{-1} dkm.

Now assign Houghton Mifflin(1967, 1970) Book 8, pp. 178-179.

Text References:

Book: 8

Houghton Mifflin(1967, 1970)	pp. 174-179, 196, 197, (28, sec. 1-11)
Houghton Mifflin(1972)	pp. 20-21
Addison-Wesley (1971)	pp. 256-257

Level: 23 Step: C Concept: Measurement

WORKSHEET

Complete each statement.

Example 1.

1 dkm. = ________ mm.

1 dkm. = 1 mm. x 10^4

so 1 dkm. = 10^4 mm.

Example 2.

1 micron = ________ km.

1 micron = 1 km. ÷ 10^9

so 1 micron = 10^{-9} km.

---

1. 1 cm. = ________ dkm.

2. 1 M = ________ dm.

3. 1 mm. = ________ cm.

4. 1 hm. = ________ km.

5. 1 dm. = ________ mm.

6. 1 hm. = ________ m.

Level: 23 Step: C Concept: Measurement

WORKSHEET

Complete each statement.

1. 1 micron = ________ mm.

2. 1 m. = ________ cm.

3. 1 km. = ________ dm.

4. 1 dm. = ________ m.

5. 1 hm. = ________ cm.

6. 1 mm. = ________ dkm.

7. 1 cm. = ________ microns

8. 1 dkm. = ________ km.

9. 1 M = ________ hm.

10. 1 cm. = ________ M

Level: 23 Step: C Concept: Number sentences

I. Concept:

Number sentences: Translating word sentences into number sentences.

II. Behavioral Objective:

The student given a simple word problem will be able to write an equation which expresses the number facts.

III. Mathematical Ideas:

Translating a word sentence into a number sentence is making a mathematical model of a real-life situation.

IV. Activities:

A. NOTE: (See H. M. (1967, 1970), Book 8, p. 266): A number sentence consists of two number phrases connected by some "verb symbol". In a word problem these symbols ($=, <, >, \leq, \geq$) usually replace some form of the verb "to be". With this in mind it is relatively easy to translate a word sentence into a number sentence by directly substituting symbols for the words. Emphasize H. M. (1967, 1970), Book 8, p. 312: "Scrambled X".

B. NOTE: The emphasis here is on writing equations, not solving them.

Text References:

Grade: 8

Houghton Mifflin(1967, 1970)	pp. 312-315, 327, 328, (50)
Addison-Wesley.(1971)	pp. 40-42, (10,11)

Level: 23 Step: D Concept: Numeral

I. Concept:

Numeral: Identifying significant digits.

II. Behavioral Objective:

The student given a number reporting a measurement will be able to name the number of significant digits.

III. Mathematical Ideas:

A. In a numeral reporting a measurement, each digit that serves to indicate the number of units of measure contained in the measurement is a significant digit.

B. When a number is written in scientific notation the first factor is written in such a way that all its digits are significant.

IV. Activities:

A. Assign Book 8

1. Houghton Mifflin (1972) p. 23: 10-19
2. Addison-Wesley (1971) p. 269: 1
3. Houghton Mifflin (1967, 1970) pp. 210-211 (Oral Exer.): 1-10, 31-42
   pp. 211-212 (Written Exer.): 1-8, 35-42

B. Discuss in Houghton Mifflin (1967, 1970), Book 8, pp. 208-209: Examples 1-3

C. See worksheet on the following page.

Text References:

Book: 8

Houghton Mifflin (1967, 1970) pp. 208-212
Houghton Mifflin (1972) p. 23
Addison-Wesley (1971) pp. 268-269

23-20

Level: 23 Step: D Concept: Numeral

WORKSHEET

State the number of significant digits for each measurement.

1. 23.20 m. ________________

2. 0.003 m. ________________

3. 3.6×10^6 m. ________________

4. 6.8420×10^2 m. ________________

5. 4.7 m. ________________

6. 2.72×10^{-5} m. ________________

7. 421.02 m. ________________

8. 4.2102×10^4 m. ________________

9. 0.006 m. ________________

10. 6.0×10^{-3} m. ________________

11. 75 m. ________________

12. 75.0 m. ________________

13. 75.00 m. ________________

14. .075 m. ________________

15. .0075 m. ________________

Level: 23 Step: D Concept: Geometry

I. Concept:

Geometry: Computing surface area and volume of prisms and pyramids.

II. Behavioral Objective:

The student given a drawing of a prism or pyramid with necessary dimensions given will be able to compute the surface area and volume.

III. Mathematical Ideas:

A. To find the surface area of a polyhedron, add the areas of all its faces.

B. To find the volume of any prism, multiply the area of the base by the height. ($V = Bh$)

C. To find the volume of any pyramid, multiply $\frac{1}{3}$ times the area of the base times the height. ($V = \frac{1}{3}Bh$)

IV. Activities:

A. Make models of a rectangular prism and a rectangular pyramid with the same base and height. See Addison-Wesley p. 300: 5 and p. 314: 3. Fill the pyramid with rice and pour into the prism. (It should take the contents of 3 pyramids to fill the prism.)

B. See worksheet on the following page.

Text References:

Book: 8

Houghton Mifflin (1967, 1970)	pp. 390-399, (63, 64)
Houghton Mifflin (1972)	pp. 305-307, (53)
Addison-Wesley (1971)	pp. 306, 313-315

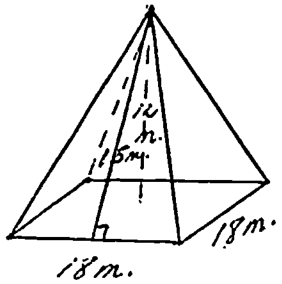
23-22

Level: 23 Step: D Concept: Geometry

WORKSHEET

Find the surface area and volume of each polyhedron:

1.

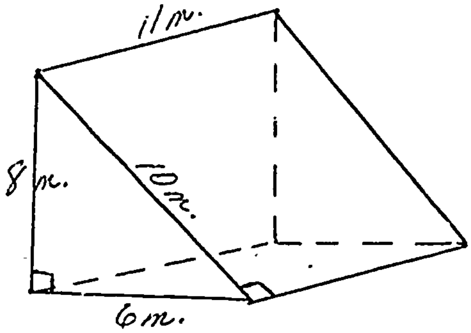


regular square pyramid

________________ surface area

________________ volume

2.

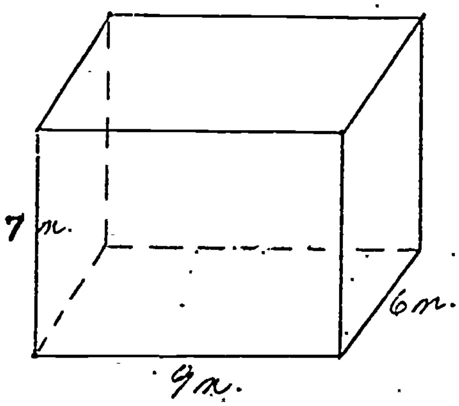


right triangular prism

________________ surface area

________________ volume

3.



rectangular prism

________________ surface area

________________ volume

Level: 23 Step: D Concept: Measurement

I. Concept:

Measurement: Converting linear English units to metric.

II. Behavioral Objective:

The student given a linear measure in the English system will be able to convert it to a specified linear unit in the metric system by using a table of equivalents.

III. Mathematical Ideas:

Refer to Table 3, p. 181 in Houghton Mifflin (1967, 1970), Book 8.

IV. Vocabulary To Stress:

convert conversion factor

V. Activities:

See worksheet on following page.

Text References:

Book: 8

Houghton Mifflin (1967, 1970) pp. 180-183, (28, Sec. 12-20)

Level: 23 Step: D Concept: Measurement

WORKSHEET

Using the table on p. 181 - Houghton Mifflin (1967, 1970), Book 8, convert the given measure to the indicated unit in the metric system.

1. 7 in. = ______ cm.

2. 5 ft. = ______ m.

3. 12 yd. = ______ m.

4. 4 mi. = ______ km.

5. 600 ft. = ______ km.

6. 3 in. = ______ mm.

7. 6 ft. = ______ cm.

8. 30 mi. per hr. = ______ km. per hr.

9. 530 yd. = ______ km.

10. 50 in. = ______ m.

11. 7.2 ft. = ______ m.

12. 120 in. = ______ m.

13. 50 mi. = ______ km.

14. 200 mi. = ______ km.

15. 634 ft. = ______ m.

Level: 23 Step: D Concept: Number sentences

I. Concept:

Number sentences: Using equations in one variable to solve word problems.

II. Behavioral Objective:

The student given a simple word problem will be able to solve it by writing and solving an equation in one variable.

III. Mathematical Ideas:

Six-step procedure:

1. What is asked for?
2. What number phrases can be written?
3. Will a sketch help?
4. Write a number sentence
5. Solve the number sentence.
6. Check with the facts given in the original problem.

Text References:

Book: 8

Houghton Mifflin (1967, 1970) pp. 316-321, (51)
Addison-Wesley (1971) pp. 40-42, (10, 11)

Level: 23 Step: Z Concepts: Sets

I. Concept:

Sets: Using truth tables.

II. Behavioral Objective:

The student given two propositions p and q, will be able to complete a truth table involving conjunctions, disjunctions, and negations.

III. Mathematical Ideas:

A. A truth table is a chart showing all possible truth values of a proposition, given the possible truth values of its components.

B. Two propositions are equivalent if they have the same truth value. The symbol "$\Longleftrightarrow$" is used to show that 2 propositions are equivalent.

IV. Vocabulary To Stress:

truth table equivalent propositions $\Longleftrightarrow$

V. Activities:

A. See Christian, pp. 12-20

References:

Robert R. Christian, Introduction to Logic and Sets, Blaisdell Publishing Company.

Level: 23 Step: Z Concept: Sets

ENRICHMENT EVALUATION

I. Complete the following truth table:

p	q	$p \vee q$	$\sim p$	$\sim q$	$\sim(p \vee q)$	$\sim p \wedge q$
T	T					
T	F					
F	T					
F	F					

II. Show by means of a truth table that

$$\sim(p \wedge q) \Longleftrightarrow \sim p \vee \sim q$$

Level: 23 Step: Z Concept: Numeral

I. Concept:

Numeral: Using unit fractions as exponents.

II. Behavioral Objective:

The student given a rational number in exponential form (the exponent a unit fraction) will be able to re-write it as a single numeral.

III. Mathematical Ideas:

A. $(a^{\frac{1}{2}})^2 = a$

B. $\sqrt{a} = a^{\frac{1}{2}}$

C. $\sqrt[n]{a} = a^{\frac{1}{n}}$

where $a \geqslant 0$, a is rational, n is a natural number

IV. Activities:

See Houghton Mifflin (1967, 1970) Book 8, pp. 252-253.

Text References:

Book: 8

Houghton Mifflin(1967, 1970) pp. 252-253.

Level: 23 Step: Z Concept: Numeral

ENRICHMENT EVALUATION

Re-write as a single numeral:

1. $64^{\frac{1}{2}}$ ______________

2. $400^{\frac{1}{2}}$ ______________

3. $64^{\frac{1}{3}}$ ______________

4. $81^{\frac{1}{4}}$ ______________

5. $32^{\frac{1}{5}}$ ______________

Level: 23 Step: Z Concept: Addition & Subtraction

I. Concept:

Addition and Subtraction: Adding and subtracting exponential expressions.

II. Behavioral Objective:

The student given an equation showing a possible sum, difference, product, or quotient of 2 rational numbers in exponential form, will be able to tell whether the statement is true or false, and show written proof.

III. Mathematical Ideas:

A. Although $10^2 \cdot 10^b = 10^{a+b}$, $(10^a + 10^b)$ cannot be simplified unless $a = b$.

B. The laws of exponents involve the operations of multiplication and division, not addition or subtraction.

C. Raising to a power is a shortened way of multiplying just like multiplying is a shortened way of adding. So adding $\longleftrightarrow$ multiplying $\longleftrightarrow$ raising to a power, but this relationship is not transitive.

IV. Activities:

A. Expand the laws of exponents to include bases of any integer, not just 10.

$$5^2 \cdot 5^4 = 5^6$$

$$6^3 \cdot 6^{-3} = 6^0 = 1$$

B. Explore examples like $a^x + a^y$ where a, x, y are integers. Example:

$$6^2 + 6^3 =$$

$$36 + 216 = \underline{252}$$

Therefore, $6^2 + 6^3 \neq 6^5$ because $6^5 = 7776$.

But $6^2 \cdot 6^3 = 6^5$ because $36 \cdot 216 = 7776$.

Conclusion: The laws of exponents involve only multiplication and division. There are no laws of exponents for addition or subtraction.

Level: 23 Step: Z Concept: Addition & Subtraction

WORKSHEET

Indicate whether each statement is true or false, and show your reason with written proof.

1. $3^2 + 3^4 = 3^6$ FALSE $9 + 81 = 729$

$90 \neq 729$

2. $2^3 \cdot 2^3 = 2^9$

3. $5^3 \cdot 5^2 = 5^6$

4. $3^4 \cdot 2^3 = 6^7$

5. $4^6 \div 4^2 = 4^3$

6. $3^8 \div 3^4 = 3^4$

7. $3^6 - 3^4 = 3^2$

8. $a^3 + a^4 = a^7$

9. $b^3 \cdot b^5 = b^8$

10. $x^4 \cdot x^2 = x^8$

Level: 23 | Step: Z | Concept: Addition & Subtraction

ENRICHMENT EVALUATION

Indicate whether each statement is true or false, and show your reason with written proof.

STATEMENT	TRUE OR FALSE	PROOF
1. $4^2 + 4^3 = 4^5$		
2. $3^2 : 3^4 = 3^8$		
3. $2^2 \cdot 2^3 = 2^5$		
4. $2^3 \cdot 3^2 = 6^5$		
5. $2^8 \div 2^4 = 2^2$		
6. $2^6 \div 2^3 = 2^3$		
7. $4^3 - 4^2 = 4^1$		
8. $x^2 + x^3 = x^5$		
9. $x^2 \cdot x^3 = x^5$		
10. $x^2 \cdot x^3 = x^6$		

Level: 23 Step: Z Concept: Function

I. Concept:

Function: Finding the probability of combined events.

II. Behavioral Objective:

The student given a sample space, two events, and their intersection will be able to find the probability of each event, their intersection, and their union.

III. Mathematical Ideas:

A. Mutually exclusive events are events which cannot both occur in one performance of an experiment.

B. $P(E \cup F) = P(E) + P(F)$ if E and F are mutually exclusive events.

C. $P(E \cup F) = P(E) + P(F) - P(E \cap F)$ if E and F are not mutually exclusive events.

IV. Activities:

A. See Houghton Mifflin (1967, 1970), Book 8, pp. 478-484.

B. Willerding pp. 41-65.

C. See Imperial Tapes (Intermediate), Teachers' Manual pp. 82-83

D. Addison-Wesley (1971) pp.214-217.

Text References:

Book: 8

Houghton Mifflin (1967, 1970) pp. 478, 484, (77)
Addison-Wesley (1971) pp. 214-217, (48)

Other References:

Willerding, Probability: The Science of Chance, (Franklin Math Series).

Imperial Tapes (Intermediate), Teachers' Manual "A Step Further", pp. 82-83.

Level: 23 Step: Z Concept: Function

ENRICHMENT EVALUATION

I. Fill in the ⌞ nks:

	NUMBER OF ELEMENTS IN							
	Sample Space	E	F	$E \cap F$	P(E)	P(F)	$P(E \cap F)$	$P(E \cup F)$
1.	12	4	3	1				
2.	15	3	5	0				
3.	20	4	5	3				
4.	18	6	3	2				
5.	8	2	4	2				

II. In Mr. Johnson's math class of 36 students, 12 have green eyes, and 20 have black hair. Six students have both green eyes and black hair. Find the probability that a student has:

1. green eyes
2. black hair
3. green eyes and black hair
4. green eyes or black hair
5. eyes any color but green.

Level: 23 Step: Z Concept: Geometry

I. Concept:

Geometry: Using Euler's formula.

II. Behavioral Objective:

The student given the name of a polyhedron and the number of vertices it has will be able to use Euler's formula to compute the number of edges.

III. Mathematical Ideas:

Euler's formula: $\boxed{F + V - E = 2}$

where F is the number of faces,

V is the number of vertices, and

E is the number of edges in a polyhedron.

IV. Activities:

A. See Roper, Paper and Pencil Geometry for a build-up to this concept, pp. 85-90.

B. See Laycock, Straw Polyhedra, pp. 31-38, especially pp. 37, 38.

C. See Houghton Mifflin (1967, 1970) Book 8, pp. 403, 404.

D. See Houghton Mifflin (1972) Book 8, p. 303, (5, 6, 7, 8)

Text References:

Book: 8

Houghton Mifflin (1967, 1970) pp. 403-404
Houghton Mifflin (1972) p. 303

Other References:

Mary Laycock, Straw Polyhedra, Creative Publications, Inc., 1970.

Susan Roper, Paper and Pencil Geometry, The Franklin Mathematics Series

Level: 23 Step: Z Concept: Geometry

ENRICHMENT EVALUATION

Suppose you were going to make a polyhedron out of straws. In order to know how many straws you will need for each figure, you need to know how many edges it has. Use Euler's formula to compute the number of edges in a:

	WORK SPACE	Number of edges
1. cube (8 vertices)		________
2. octahedron (6 vertices)		________
3. tetrahedron (4 vertices)		________
4. icosahedron (12 vertices)		________
5. dodecahedron (20 vertices)		________

Level: 23 Step: Z Concept: Number sentences

I. Concept:

Number sentences: Solving equations involving absolute value.

II. Behavioral Objective:

The student given a simple linear equation containing an absolute value, will be able to solve it over the set of rational numbers.

III. Mathematical Ideas

$$\{a: |a| = 2\} = \{-2, 2\}$$

Text References:

Book: 8

Houghton Mifflin (1967, 1970) pp. 271: 30; 257: 33, 34; 294: 32, 34
Addison-Wesley (1971) pp. 74, 81

Level: 23 Step: Z Concept: Number sentences

ENRICHMENT EVALUATION

Specify the solution set of each equation.

1. $|x| = 12$

2. $|x + 2| = 7$

3. $|2x + 3| = 11$

4. $|x| + 4 = -2$

5. $|x| - 4 = -2$

Level: 23 ANSWERS TO WORKSHEETS

23-9

4	tetrahedron	triangle
6	hexahedron	square (cube)
8	octahedron	triangle
12	dodecahedron	pentagon
20	icosahedron	triangle

23-16

1. 10^{-3}
2. 10
3. 10^{-1}
4. 10^{-1}
5. 10^{2}
6. 10^{2}

23-17

1. 10^{-3}
2. 10^{2}
3. 10^{4}
4. 10^{-1}
5. 10^{4}
6. 10^{-4}
7. 10^{4}
8. 10^{-2}
9. 10^{4}
10. 10^{-8}

23-20

1. 4
2. 1
3. 2
4. 5
5. 2
6. 3
7. 5
8. 5
9. 1
10. 2
11. 2
12. 3
13. 4
14. 2
15. 2

23-22

1. 864 sq. m.; 1296 cu. m.
2. 312 sq. m.; 264 cu. m.
3. 318 sq. m.; 378 cu. m.

23-24

1. 17.78
2. 1.50
3. 10.92
4. 6.44
5. .18
6. 76.2
7. 180
8. 48.3
9. .4823
10. 1.27
11. 2.16
12. 3.048
13. 80.5
14. 322
15. 190.2

Level: 23

ANSWERS TO ENRICHMENT (STEP Z)

WORKSHEETS

23-31

2. False — $64 \neq 512$
3. False — $3125 \neq 15{,}625$
4. False — $648 \neq 279{,}936$
5. False — $256 \neq 64$
6. True — $6561 \div 81 = 81$
7. False — $729 - 81 \neq 9$
8. False
9. True
10. False

(8–10) Replace the variable with any number and prove true or false.

EVALUATION

23-27

I.

$p \vee q$	$\sim p$	$\sim q$	$\sim(p \vee q)$	$\sim p \wedge q$
T	F	F	F	F
T	F	T	F	T
T	T	F	F	T
F	T	T	T	F

II.

p	q	$p \wedge q$	$\sim(p \wedge q)$	$\sim p$	$\sim q$	$\sim p \vee \sim q$
T	T	T	F	F	F	F
T	F	F	T	F	T	T
F	T	F	T	T	F	T
F	F	F	T	T	T	T

23-29

1. 8 2. 20 3. 4 4. 3 5. 2

23-32

1. False
$$4^2 + 4^3 = 4^5$$
$$16 + 64 = 1024$$
$$80 \neq 1024$$

2. False
$$3^2 \cdot 3^4 = 3^8$$
$$9 \cdot 81 = 6561$$
$$729 \neq 6561$$

3. True
$$2^2 \cdot 2^3 = 2^5$$
$$4 \cdot 8 = 32$$
$$32 = 32$$

4. False
$$2^3 \cdot 3^2 = 2^5$$
$$8 \cdot 9 = 7776$$
$$72 \neq 7776$$

Level: 23

ANSWERS TO ENRICHMENT

EVALUATION (CONTINUED)

23-32 (cont.)

5. False $2^8 \div 2^4 = 2^2$
$256 \div 16 = 4$
$16 \neq 4$

6. True $2^6 \div 2^3 = 2^3$
$64 \div 8 = 8$
$8 = 8$

7. False $4^3 - 4^2 = 4^1$
$64 - 16 = 4$
$48 \neq 4$

8. False $x^2 + x^3 = x^5$

9. True $x^2 \cdot x^3 = x^5$

10. False $x^2 \cdot x^3 = x^6$

Replace x with a constant and prove true or false.

23-34

1.

P(E)	P(F)	P(E∩F)	P(E∪F)
$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{12}$	$\frac{1}{2}$
$\frac{1}{5}$	$\frac{1}{3}$	0	$\frac{8}{15}$
$\frac{1}{5}$	$\frac{1}{4}$	$\frac{3}{20}$	$\frac{6}{20}$
$\frac{1}{3}$	$\frac{1}{6}$	$\frac{1}{9}$	$\frac{7}{18}$
$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{2}$

II.

1. $\frac{1}{3}$
2. $\frac{5}{9}$
3. $\frac{1}{6}$
4. $\frac{13}{18}$
5. $\frac{2}{3}$

23-36

1. 12
2. 12
3. 6
4. 30
5. 30

23-38

1. 12, -12
2. 5, -9
3. 4, -7
4. Ø
5. 2, -2

Level: 23 Answers - Post Test I

Step A

Multiplication

1. 10^{11}
2. 10^5
3. 10^9
4. 10^{11}
5. 10^5

Division

6. 10^4
7. $\frac{1}{10^6}$
8. 10^5
9. 10^2
10. $\frac{1}{10^3}$

Geometry

11. ABC $\cong$ DEF by SAS
12. ABD $\cong$ BCD by SSS
13. ACD $\cong$ BCE by ASA or ABF $\cong$ FDE by SAS
14. ABC $\cong$ DCE by SAS
15. DAB $\cong$ DBC by ASA

Number Sentences

16. $\{-11\}$
17. $\{-7\}$
18. $\{3\}$
19. $\{2\}$
20. $\{12\}$

Step B

Numeral

21. 10^5
22. 10^{-7} or $\frac{1}{10^7}$
23. 10^0
24. 0.00001
25. .001

Multiplication

26. 10^{-2}
27. 10^{-12}
28. 1
29. 10^6
30. 1

Division

31. 10^8
32. 10^{-7}
33. 10^{-4}
34. 10^{-5}
35. 10^{-7}

Geometry

36. dodecahedron
37. tetrahedron
38. icosahedron
39. cube or hexahedron
40. octahedron

Number Sentences

41. $\{-7\}$
42. $\{4\}$
43. $\{6\}$
44. $\{112\}$
45. $\{8\}$

Step C

Numeral

46. 1.776×10^3
47. 1.776×10^{-4}
48. 3.456×10
49. .000345
50. 34,500

Multiplication

51. 6×10^7
52. 4×10^8
53. 3.5×10^2
54. 4.08×10^7
55. 4.83×10

Division

56. 2×10^4
57. 2×10^3
58. 4×10^9
59. 5×10^8
60. 7×10^7

Level: 23 Answers - Post Test I

Step C

Geometry

61. pentagonal
62. rectangular
63. octagonal
64. triangular
65. hexagonal

Measurement

66. 10^3
67. 10^{-5}
68. 5×10^{-1}
69. 3.0×10^4
70. 2.3×10

Number sentences

71. $7x + x = 48$
72. $2(3w + w) = 56$ or
$2(3w) + 2w = 56$
73. $x + x + 1 = 87$
74. $.55h = 357.5$
75. $B + (B + 2) = 16$ or
$T + (T - 2) = 16$

Step D

Numeral

76. 1
77. 2
78. 4
79. 3
80. 2

Geometry

81. 144 sq. m.
82. 64 cu. m.
83. 60 cu. m.
84. 480 cu. m.
85. 376 sq. m.

Measurement

86. 30.48 cm.
87. 9.1 m.
88. 50.8 mm.
89. 9.66 km.
90. 1.5 km.

Number Sentences

91. 7
92. 11 cm.
93. 12
94. 35
95. 9 yrs.

Level: 23 Answers - Post Test II

Step A

<u>Multiplication</u>

1. 10^{11}
2. 10^{6}
3. 10^{12}
4. 10^{7}
5. 10^{7}

<u>Division</u>

6. 10^{5}
7. $\frac{1}{10^{4}}$
8. 10^{8}
9. $\frac{1}{10^{3}}$
10. 10^{4}

<u>Geometry</u>

11. ABC $\cong$ FDE by SAS
12. DAB $\cong$ DBC by SSS
13. ABD $\cong$ BCE by SAS
14. ABC $\cong$ CED by ASA
15. ABD $\cong$ BCD by SSS

<u>Number Sentences</u>

16. $\{-11\}$
17. $\{5\}$
18. $\{-1\}$
19. $\{4\}$
20. $\{2\}$

Step B

<u>Numeral</u>

21. 10^{6}
22. 10^{-6}
23. .001
24. 1.0
25. 10,000

<u>Multiplication</u>

26. 10^{2}
27. 10^{-11}
28. 10
29. 1
30. 10^{4}

<u>Division</u>

31. 10^{7}
32. 10^{-8}
33. 10^{-2}
34. 10^{12}
35. 10^{-3}

<u>Geometry</u>

36. icosahedron
37. tetrahedron
38. cube or hexahedron
39. dodecahedron
40. octahedron

<u>Number Sentences</u>

41. $\{3\}$
42. $\{-6\}$
43. $\{2\}$
44. $\{-9\}$
45. $\{12.1\}$

Step C

<u>Numeral</u>

46. 1.492×10^{3}
47. 1.492×10^{-4}
48. 1.923×10
49. .0192
50. 192

<u>Multiplication</u>

51. 9×10^{8}
52. 9×10^{-6}
53. 3.6×10^{2}
54. 5.12×10^{7}
55. 2.28×10^{-1}

<u>Division</u>

56. 3×10^{6}
57. 2×10^{3}
58. 4×10^{11}
59. 4×10^{7}
60. 3×10^{6}

Level: 23 Answers - Post Test II

Step C

Geometry

61. hexagonal
62. triangular
63. octagonal
64. rectangular
65. pentagonal

Measurement

66. 10^4
67. 10^{-6}
68. 4×10^{-1}
69. 2.5×10^4
70. 2.3

Number Sentences

71. $5x - x = 24$
72. $2(w + w + 3) = 46$
73. $x + x + 1 = 151$
74. $8\frac{1}{2}R = 510$
75. $A + 2A = 15$

Step D

Numeral

76. 3
77. 1
78. 4
79. 2
80. 3

Geometry

81. 576 cu. m.
82. 832 sq. m.
83. 512 cu. m.
84. 576 sq. m.
85. 1536 cu. m.

Measurement

86. 15.24 cm.
87. 7.28 m.
88. 101.6 mm.
89. 32.2 km.
90. 1.365 km.

Number Sentences

91. 7
92. 18m.
93. 24
94. 29
95. 19 yrs.

TABLE OF CONTENTS

Level 24

Step A

Step B

Step C

Step Z

Level: 24 Step: A Concept: Sets

I. Concept:

Sets: developing the set of real numbers

II. Behavioral Objectives:

The student given a finite set of real numbers (less than 10 elements) will be able to specify by roster (listing) the subsets consisting of: (1) integers, (2) rational numbers, (3) irrational numbers.

III. Mathematical Ideas:

A. A rational number in decimal form will either terminate or repeat.

B. An irrational number in decimal form will neither terminate nor repeat.

IV. Vocabulary To Stress:

real numbers	N = {natural numbers}	$\mathcal{R}$ = {real numbers}
irrational numbers	W = {whole numbers}	R = {rational numbers}
rational numbers	J = {integers}	

V. Activities:

A. Use the attached explanation and charts to develop the set of real numbers from the set of natural numbers. (pp. 24-3, 24-4, 24-5).

B. Then compare the set of rationals with the irrationals in terms of decimals and ratios:

RATIONAL	IRRATIONAL
1. Terminating or repeating decimal	1. Non-terminating and non-repeating decimal
2. Can be written as a ratio of two integers	2. Cannot be written as a ratio of two integers.

(Notice first five letters in word "rational.")

C. Assign worksheet #24-6.

Text References:

Book: 8

Houghton Mifflin (1967, 1970) pp. 234, 235, 239, (37) (38, Sec. 1-9, 17, 18)
Houghton Mifflin (1972) pp. 400, 403, (76)
Addison-Wesley (1971) p. 339

Level: 24 Step: A Concept: Sets

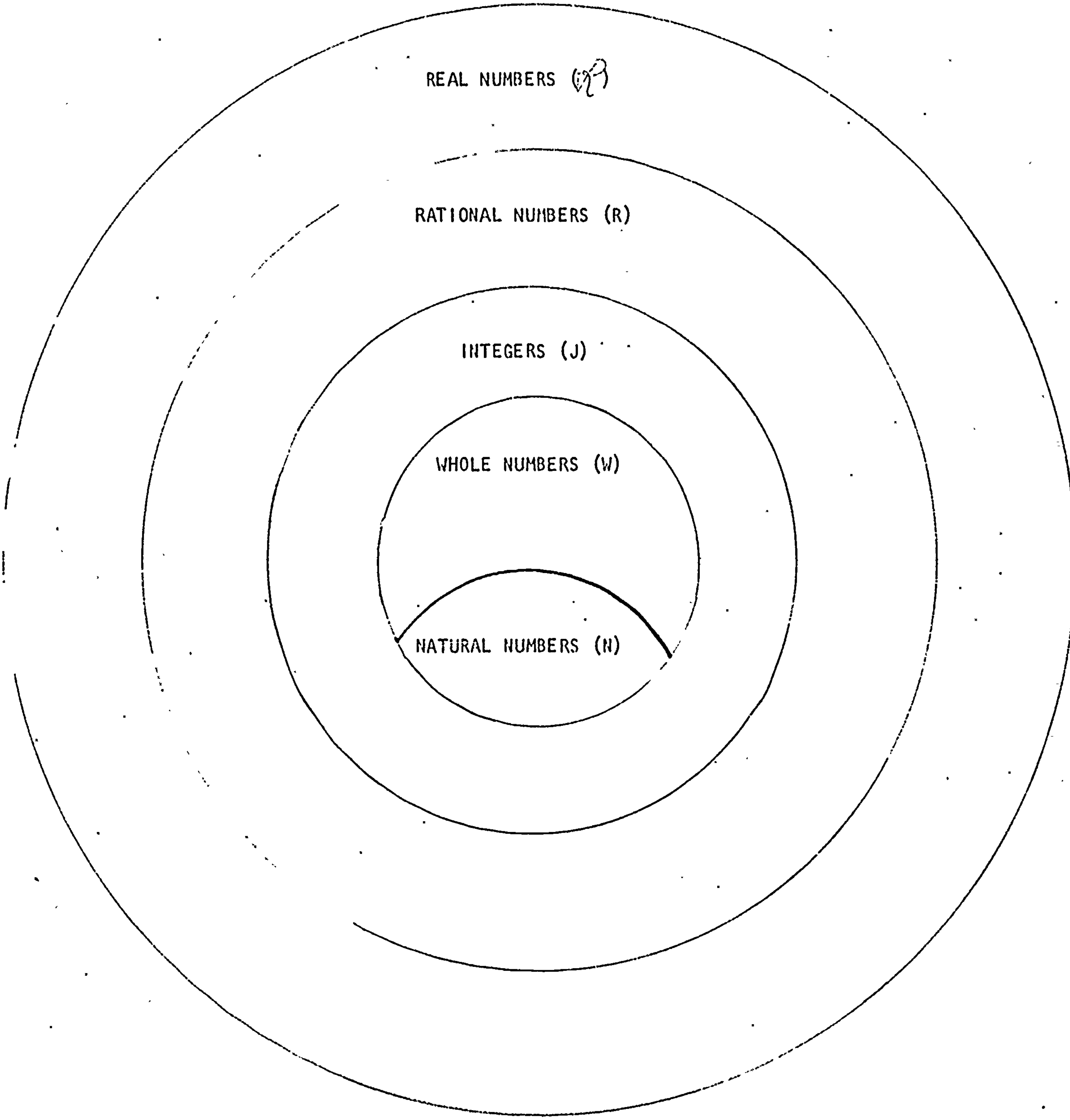
Development of the Set of Real Numbers

(To the teacher: Begin at the "bullseye" of the chart on the following page and work out to the larger sets.)

In the beginning, man needed numbers for counting things, so the set of Natural Numbers was created. Man could add or multiply these numbers, or he could subtract a smaller number from a larger number, but he could not subtract a number from itself; so he created "zero". When he added zero to his set he formed the set of whole numbers. For a long time the set of whole numbers was adequate for man's needs, but eventually the bookkeepers needed numbers to indicate that state of being "in the hole", so negative numbers were created. When the negative numbers were added to the whole numbers, the set of integers was formed. This is a very satisfactory set of numbers. It is closed for addition, subtraction, multiplication, and some division. But man soon found that division doesn't always come out even, so he created a new kind of number, a fraction. When the set of fractions was added to the set of integers, the set of rational numbers was formed. The set of rational numbers is closed for addition, subtraction, multiplication, and division. For this reason, these operations are often called the "rational operations". For most of us this set is sufficient, but scientists and engineers need numbers not in this set. Π or $\sqrt{2}$, for instance, cannot be expressed as a rational number, so for some people the set of rationals needs to be supplemented by the set of irrationals. Together these two sets form the set of real numbers. We will let the set of real numbers be our universal set. For our purposes this set is sufficient. (We might, however, mention that there is a larger set called the set of complex numbers. The complex numbers include the real numbers and the imaginary numbers. The imaginary numbers include numbers like $\sqrt{-2}$.)

Level: 24 Step: A Concept: Sets

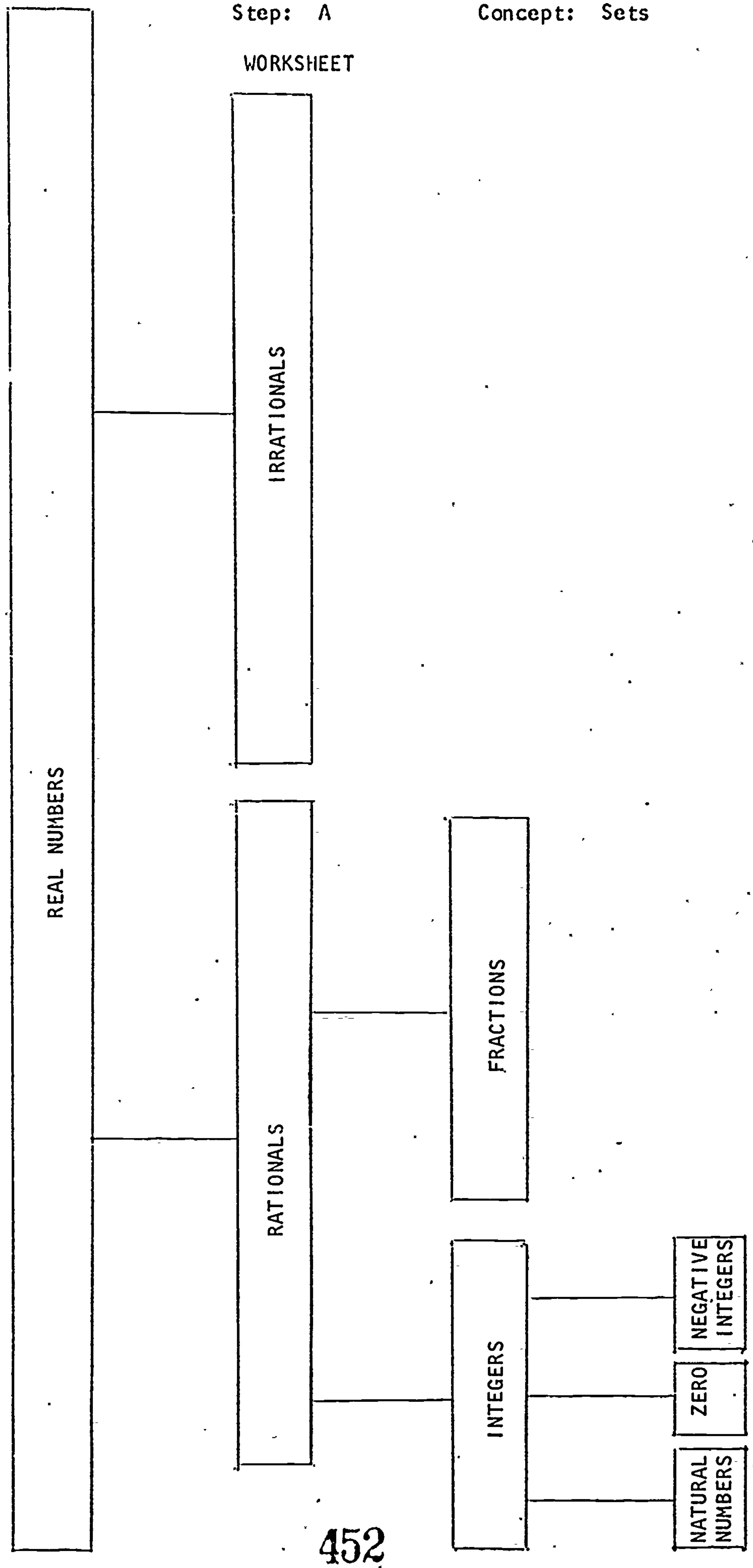
WORKSHEET



Step: A Concept: Sets

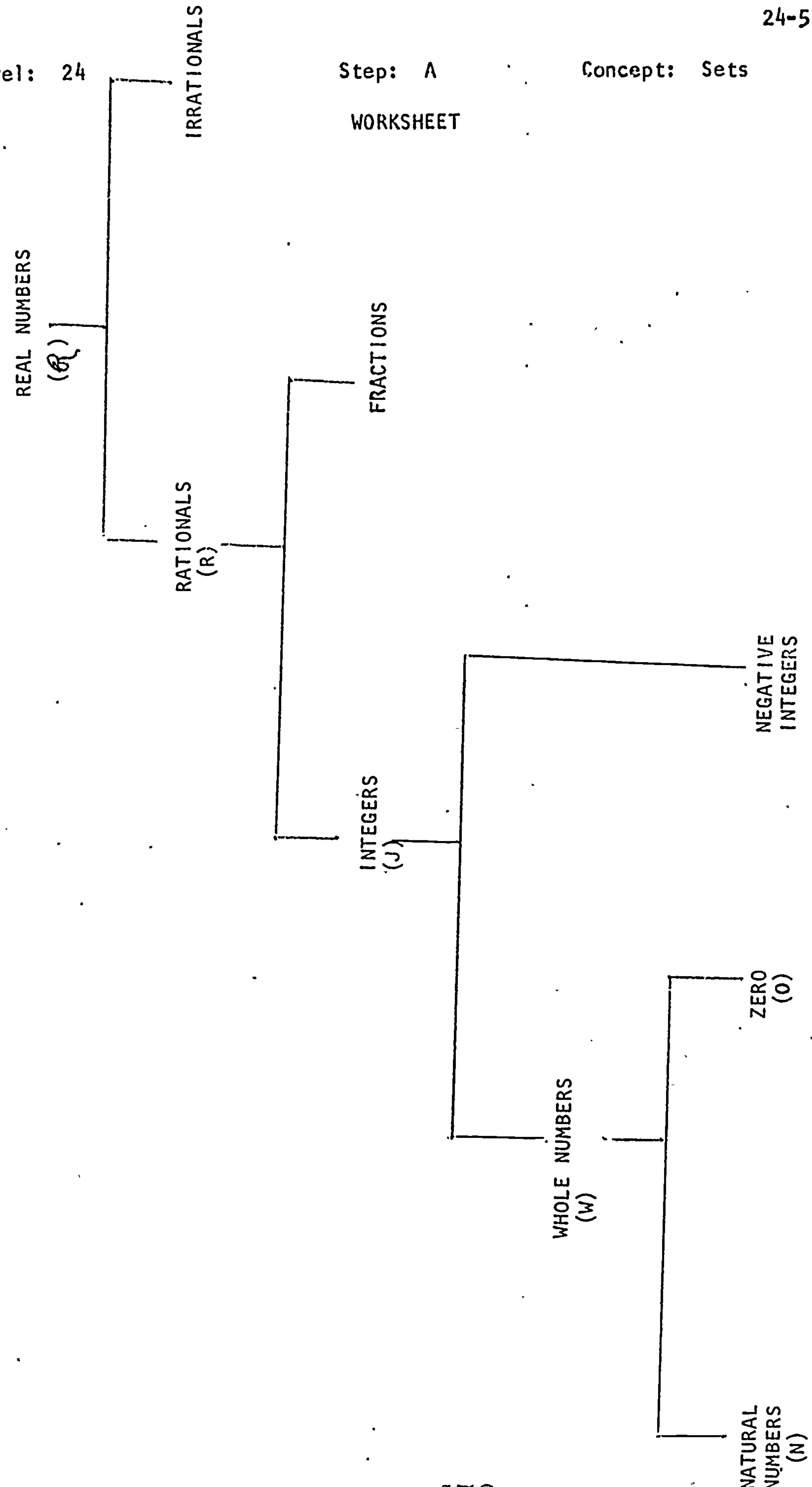
WORKSHEET

THE REAL NUMBER SYSTEM



Level: 24 Step: A Concept: Sets

WORKSHEET



Level: 24 Step: A Concept: Sets

WORKSHEET

$S = \{0, \frac{1}{2}, -7, -\frac{4}{5}, 6, 0.721, \pi, 2.12122...\}$

List these subsets of S:

A = {integers in S} = ______________________

B = {rational numbers in S} = ______________________

C = {irrational numbers in S} = ______________________

D = {natural numbers in S} = ______________________

E = {negative real numbers in S} = ______________________

F = {positive real numbers in S} = ______________________

G = {real numbers in S that are not in E and not in F}=

____

Level: 24 Step: A Concept: Numeral

I. Concept:

Numeral: changing a repeating decimal to a fraction.

II. Behavioral Objective:

The student given a repeating decimal numeral will be able to write it as a fraction in lowest terms.

III. Mathematical Ideas:

A. Every repeating decimal numeral names some rational number.

B. Multiplication property of equality.

C. Addition property of equality.

IV. Vocabulary To Stress:

repetend

V. Activities:

Note: Examples 2 and 3 p. 231 (H.M.) Book 8 (1967, 1970) give the procedure simply and concisely.

Text References:

Book: 8

Houghton Mifflin (1967, 1970) pp. 229-233, 250, 251, (38: sec. 1-9)
Houghton Mifflin (1972) pp. 261, 400
Addison-Wesley (1971) pp. 279-280 (exercises 7, 8), 336-338

Level: 24 Step: A Concept: Order

I. Concept:

Order: comparing real numbers.

II. Behavioral Objective:

The student given two real numbers will be able to state which is the greater.

III. Mathematical Ideas:

A. Comparison property of real numbers: If a and b are real numbers, then exactly one of these statements is true: $a < b$, $a = b$, $a > b$.

B. If two real numbers are represented by decimal numerals neither of which has the repetend 9, then the numbers are compared by comparing the digits in the decimal place where the numerals first differ.

C. Note: Avoid the repetend 9 when comparing real numbers. (See H.M. Book 8 (1967, 1970) p. 237.) $.\overline{9} = 1$

IV. Activities:

See worksheet on the following page.

Text References:

Book: 8

Houghton Mifflin (1967, 1970) pp. 237-241, (38, Sec. 12-16)
Houghton Mifflin (1972) (17: 29-31)

Level: 24 Step: A Concept: Order

WORKSHEET

State which of the given pair of numbers is the greater. If they are equal, write "equal".

1. $5.\overline{727}$ or $5.\overline{72}$ ________________

2. $-6.2\overline{78}$ or 6.278 ________________

3. $-0.53\overline{4}$ or -0.534 ________________

4. $-6.\overline{456}$ or $-6.4\overline{56}$ ________________

5. $-2.\overline{9}$ or -3.0 ________________

6. 34.55 or $34.5\overline{4}$ ________________

7. -7.6124 or -7.6 ________________

8. $6.4\overline{74}$ or $6.\overline{47}$ ________________

9. $-214.\overline{4}$ or -214.5 ________________

10. 12.642 or $12.64\overline{21}$ ________________

Level: 24 Step: A Concept: Addition & Subtraction

I. Concept:

Addition-subtraction: adding real numbers using rational approximations.

II. Behavioral Objective:

The student given rational approximations of any two real numbers will be able to compute their sum and tell whether it represents a rational or an irrational number.

III. Mathematical Ideas:

A. A rational number in decimal form either terminates or repeats.

B. An irrational number in decimal form neither terminates nor repeats.

C. Addition properties of real numbers: associative, commutative distributive, identity, inverse

IV. Vocabulary To Stress:

rational approximations

V. Activities:

A. This would seem to be the appropriate time to discuss all of the properties of real numbers for addition and multiplication. (See H.M. Book 8 (1967, 1970) p. 237.

B. Challenge the students to add .585585558... and .858858885...

Is the sum rational or irrational?
Does the 3 really exist?

Text References:

Book: 8

Houghton Mifflin (1967, 1970) pp. 236, 237, 240, (38, Sec. 10, 11)
Addison-Wesley (1971) p. 342 (exercises 10, 13)

Level: 24 Step: A Concept: Geometry

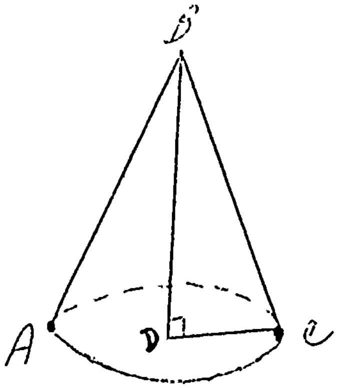
I. Concept:

Geometry: identifying parts of cones, cylinders, and spheres.

II. Behavioral Objective:

The student given a drawing of a cone, cylinder, or sphere will be able to identify: (a) base, vertex, altitude, height, slant height, or lateral surface of a cone; or (b) base, altitude, height, or lateral surface of a cylinder; or (c) center, radius, diametral chord, diameter, antipodal points, great circle, or small circle of a sphere.

III. Mathematical Ideas:



CONE

base - circular region D

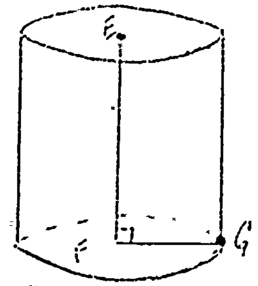
vertex - B

altitude - $\overline{BD}$

height - length of $\overline{BD}$

slant height - $\overline{BC}$

lateral surface - all of the cone except circular region D



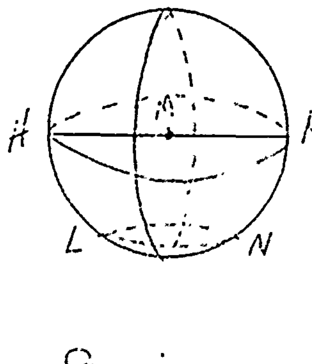
CYLINDER

bases - circular regions E and F

altitude - $\overline{EF}$

height - length of $\overline{EF}$

lateral surface - all of the cylinder except circular regions E and F



SPHERE

center - M

radius - $\overline{MK}$

diametral chord - $\overline{HK}$

diameter - $\overline{HK}$

antipodal points - H and K

great circle - the circle containing H and K

small circle - the circle containing L and N

Level: 24 Step: A Concept: Geometry

IV. Vocabulary To Stress:

(see objective)

V. Activities:

A. Make the models p. 412 (H.M.) Book 8 (1967, 1970) or page 399 in Addison-Wesley (1971), Book 8.

B. Use worksheet #24-13.

C. See Roper, Paper and Pencil Geometry, pp. 93-96.

Text References:

Book: 8

Houghton Mifflin (1967, 1970) pp. 408-412 (68)
Addison-Wesley (1971) pp. 395-397, 399

Other References:

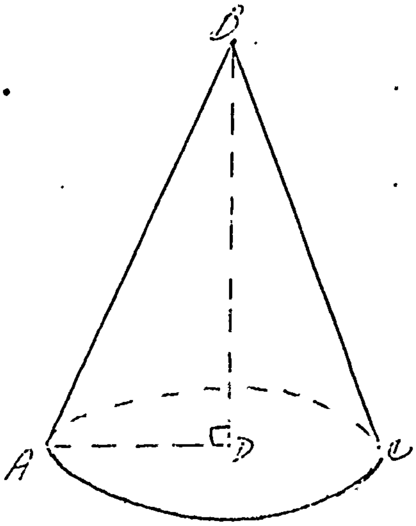
Susan Roper, Paper and Pencil Geometry, Franklin Mathematics Series, 1970

Level: 24 Step: A Concept: Geometry

WORKSHEET

IDENTIFY THE PARTS OF EACH FIGURE:

I.



The figure is called a ____________

The length of AB is called ____________

BD is called ____________

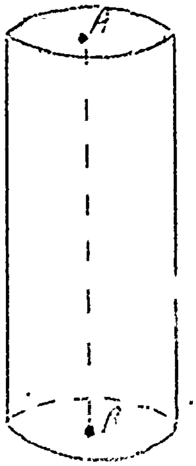
B is the ____________

Circular region D ____________

Length of $\overline{BD}$ ____________

All of the cone except circular region D is called ____________

II.



The figure is called a ____________

Circular region A ____________

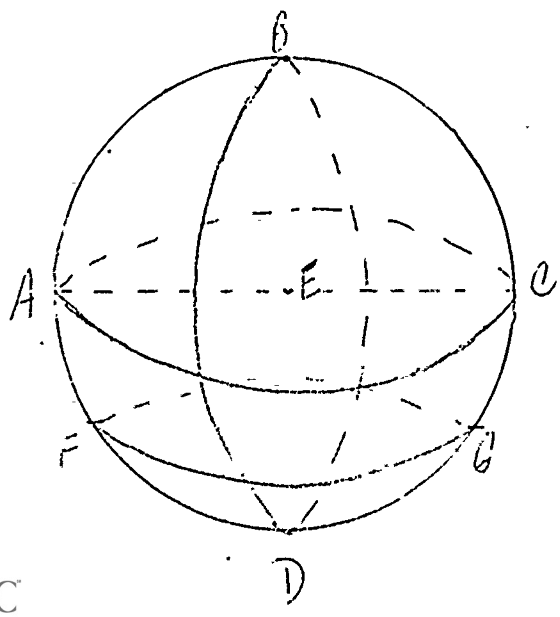
Circular region B ____________

$\overline{AB}$ is called ____________

The length of $\overline{AB}$ is called ____________

All of the cylinder except circular regions A and B ____________

III.



The figure is called a ____________

E ____________

$\overline{EC}$ ____________

$\overline{AC}$ ____________

A and C ____________

The circle containing A and C ____________

The circle containing F and G ____________

Level: 24 Step: A Concept: Measurement

I. Concept:

Measurement: converting to metric units of area.

II. Behavioral Objective:

The student given an area measure in either the English or the metric system will be able to convert it to another unit of area in the metric system by using two tables of equivalents. (See below.)

III. Mathematical ideas:

A. See Table 4 (approximately equivalent units of area) in H.M. p. 185, Book 8 (1967, 1970)

B. and the table on p. 186 in H.M., Book 8 (1967, 1970).

V. Activities:

A. Have pupils complete and use the table on p. 186 in H.M. Book 8 (1967, 1970).

B. Review metric prefixes and unit names. p. 177 in H.M. Book 8 (1967, 1970).

C. Assign in Houghton Mifflin, Book 8 (1967, 1970): pp. 185-186: oral exercises 1-4, 13-17, and 19-22.

p. 187 (written exercises 7-14, 23-24, and 27-29)

Text References:

Book: 8

Houghton Mifflin (1967, 1970) pp. 184-187, 197 (29)

Level: 24 Step: B Concept: Order

I. Concept:

Order: developing the property of density of the real numbers.

II. Behavioral Objective:

The student given any two real numbers will be able to find a rational and an irrational number between them.

III. Mathematical Ideas:

A. The set of points representing real numbers is dense on the number line.

B. Between any two real numbers there is a rational number and an irrational number.

IV. Vocabulary To Stress:

dense density average

Text References:

Book: 8

Houghton Mifflin (1967, 1970) pp. 241-245, 250, 251 (39)

Level: 24 Step: B . Concept: Function and graphs

I. Concept:

Function and Graphs: graphing intervals of real numbers.

II. Behavioral Objective:

The student given an interval of real numbers, specified by either a graph or an inequality, will be able to specify it by the form not given.

III. Mathematical Ideas:

A. Property of completeness: There is one-to-one correspondence between the set of real numbers and the set of points on the number line.

B. If a and b are real numbers and the graph of a lies to the left of the graph of b on the number line, then $a < b$.

C. If the graph of an interval ends in shaded circles, then the endpoints are included in the graph.

D. If the graph of an interval ends in unshaded circles (open circles) then the endpoints are not included in the graph.

IV. Vocabulary To Stress:

interval one-to-one correspondence

V. Activities:

A. Use Denholm, Making and Using Graphs and Nomographs, to lead up to and introduce this concept.

B. See worksheet on the following page.

Text References:

Book: 8

Houghton Mifflin (1967, 1970) pp. 245-248, (40, also includes work on density)

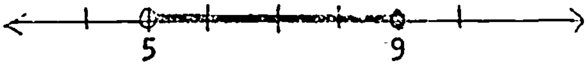
Other References:

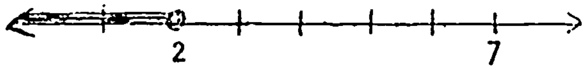
Richard A. Denholm, Making and Using Graphs and Nomographs, Franklin Mathematics Series, 1970.

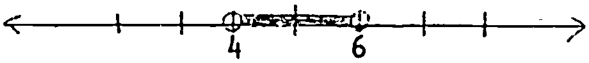
Level: 24 Step: B Concept: Function & Graphs

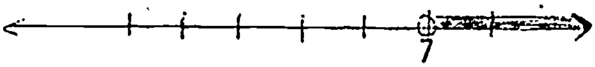
WORKSHEET

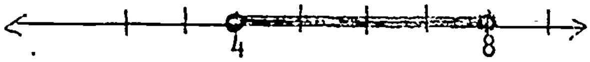
Write the inequality which describes the set of real numbers whose graph is shown.

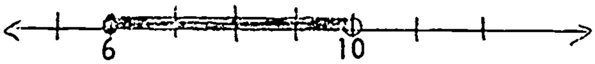
1.  ______________

2.  ______________

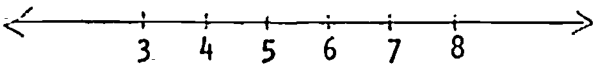
3.  ______________

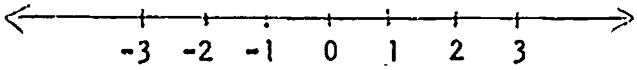
4.  ______________

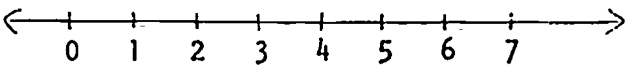
5.  ______________

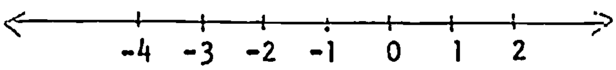
6.  ______________

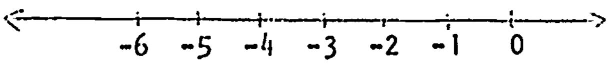
Graph each inequality over the set of real numbers.

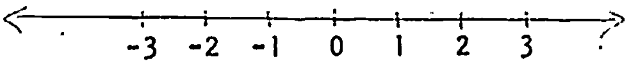
7. $3 < x \leq 7$ 

8. $x \leq 0$ 

9. $6 > x > 4$ 

10. $-4 \leq x \leq 0$ 

11. $x > -6$ 

12. $-3 \leq x < 3$ 

Level: 24 Step: B Concept: Geometry

I. Concept:

Geometry: Computing the surface area of cones, cylinders, and spheres.

II. Behavioral Objective:

The student given a drawing of a cone, cylinder, or sphere with necessary dimensions will be able to compute its total surface area, leaving the answer with π as a factor.

III. Mathematical Ideas:

	Lateral surface area	Area of base (s)	Total surface area
Cylinder	$2\pi Rh$	$2(\pi R^2)$	$2\pi R(h + R)$
Cone	πRs	πR^2	$\pi R(s + R)$
Sphere	———	———	$4\pi R^2$

IV. Vocabulary to Stress:

lateral surface
base
total surface

V. Activities:

A. Use the models they made for cone and cylinder. Use an orange or a rubber ball for the sphere.

B. To show the formula for surface area of a sphere: Cut a solid ball in half. Put a nail in the center of the flat side, and wind a cord around the nail until it covers the entire flat surface. Unwind and measure the length of the cord needed. Now turn over the hemisphere and insert the nail in the center of the curved side. Wind the cord around the nail until it covers the entire curved surface. Unwind and measure the length of the cord used.

Compare the two lengths of cord. (The area of the curved surface should be twice the area of the flat surface. If the area of the flat surface is πR^2, then the area of the curved surface is $2\pi R^2$. Since this is a hemisphere, the surface area of the entire sphere would be $2(2\pi R^2)$ or $4\pi R^2$.)

Level: 24 Step: B Concept: Geometry

Text References:

Book: 8

Houghton-Mifflin (1967, 1970)
pp. 417-422, (69)

Addison-Wesley (1971)
pp. 414-418

Houghton-Mifflin (1972)
pp. 305, 306, 309

Level: 24 Step: B Concept: Measurement

I. Concept:

Measurement: converting to metric units of volume

II. Behavioral Objective:

The student given a volume measure in either the English or the metric system will be able to convert it to another unit of volume in the metric system by using two tables of equivalents. (See below.)

III. Mathematical Ideas:

A. See Table 5 (approximately equivalent units of volume) in H.M. p. 189. Book: 8 (1967, 1970)

B. and the table on p. 191 in H. M. Book 8 (1967, 1970)

C. Property of powers:

If a and b are integers, then $(10^a)^b = 10^{ab}$

IV. Vocabulary To Stress:

metric unit names (see p. 177 in H. M. Book 8 (1967, 1970).

V. Activities:

A. Have pupils complete and use the table on p. 191 in H. M. Book 8 (1967, 1970)

B. Assign in H.M. - Book 8 (1967, 1970)
p. 191 (Oral exercises) 1-4, 14, 15, 17, 19-22
pp. 191-192 (Written exercises) 1-17, 19, 20, 23, 27, 28

Text References:

Book: 8

Houghton-Mifflin (1967, 1970)
pp. 188-192, 196, 197, 198 (30: Sec. 1-6, 9-12)

Level: 24 Step: C Concept: Function and Graphs

I. Concept:

Function and Graphs: specifying the domain and range of a relation and deciding whether a given relation is a function.

II. Behavioral Objective:

The student given a set of ordered pairs will be able to specify the domain and range and state whether the given relation is a function.

III. Mathematical Ideas:

A. A relation is a set of ordered pairs. The first coordinates of these ordered pairs form a set called the domain of the relation. The second coordinates are elements of the range of the relation.

B. A function is a relation in which the domain is restricted. In a function each element of the domain can be paired with only one element in the range. Each element of the domain can appear in only one ordered pair. Another way to test whether a relation is a function is to look at its graph. (See H.M. Book 8 (1967, 1970) --top of p. 446). (If two ordered pairs have the same first coordinate, then they must lie on the same vertical line.)

IV. Vocabulary to Stress:

domain	relation
range	rule of a relation
function	graph of a relation

Text References:

Book: 8

Houghton Mifflin (1967, 1970)
pp. 443-447, 461, 462 (73)

Addison-Wesley (1971)
p. 162

Level: 24 | Step: C | Concept: Geometry

I. Concept:

Geometry: finding the volume of cones, cylinders, and spheres.

II. Behavioral Objectives:

The student given a drawing of a cone, cylinder, or sphere with necessary dimensions will be able to compute its volume, leaving the answer with Π as a factor.

III. Mathematical Ideas:

	Area of Base	Volume
Cylinder	ΠR^2	$\Pi R^2 h$
Cone	ΠR^2	$\frac{1}{3}\Pi R^2 h$
Sphere	----------	$\frac{4}{3}\Pi R^3$

IV. Activities:

A. Use the models they made for cone and cylinder. Use an orange or an "Nerf" ball for the sphere.

B. Demonstrate the relationship in size of a sphere, cone, and cylinder by the use of water. (Can use rice instead).

See Donald L. Bruyr, Geometrical Models and Demonstrations, J. Weston Walch, 1963, pp. 131-132.

C. Or to compare the volume of the cylinder with the sphere, use a hollow rubber ball and a cylinder whose base and height equal the diameter of the ball.

Cut a small hole in the ball and fill with water (or sand or salt). Now pour the water into the cylinder.

(It should be $\frac{2}{3}$ full. A glass jar would do well in this experiment if you could find one with the same base and height as the diameter of a ball.)

So the volume of the sphere is $\frac{2}{3}$ the volume of the cylinder or $\frac{2}{3}(\Pi R^2 h)$, and h = 2R, so we have $\frac{2}{3}(\Pi R^2 2R) = \frac{4}{3}\Pi R^3$.

Level: 24 Step: C Concept: Geometry

Text References:

Book: 8

Houghton Mifflin (1967, 1970) pp. 422-427, (70)
Houghton Mifflin (1972) pp. 305, 307, 309
Addison-Wesley (1971) pp. 413-418

Other References:

Donald L. Bruyr, Geometrical Models and Demonstrations, J. Weston Walch, 1963.

Level: 24 Step: C Concept: Number Sentences

I. Concept:

Number Sentences: solving inequalities in one variable.

II. Behavioral Objective:

The student given a linear inequality in one variable will be able to specify the solution set over the set of real numbers by a graph or by a simpler equivalent inequality.

III. Mathematical Ideas:

A. If the replacement set is the set of real numbers, then the solution set for an inequality must be specified by a graph or by a description (usually using set-builder notation.)

B. Set-builder notation (See Level 22, Step Z, Sets).

C. Addition property of inequalities.

D. $a > b$ if and only if $b < a$.

E. To solve an inequality over a given set means to describe its solution set by using an equivalent inequality in which one member consists of the variable and the other member contains no variable.

F. See H.M. Book 8 (1967, 1970), T. M. p. 29:11-3 (3) for the multiplication property of inequalities. This has been avoided because multiplying both sides of an inequality by a negative number reverses the sense of the inequality. The teacher should be aware of this property, however, in order to answer any questions the students may ask.

G. Solving an inequality is just like solving an equation except that:

1. the solution set of an inequality over the set of reals must be in the form of a graph or an equivalent inequality

2. if an inequality is in the form $-x < a$, in order to solve for x, the sense of the inequality must be reversed. (see F under Mathematical Ideas.)

Text References:

Book: 8

Houghton Mifflin (1967, 1970) pp. 294-300, (48)
Houghton Mifflin (1972) p. 366

Level: 24 Step: Z Concept: Sets

I. Concept:

Sets: Extending operations on sets.

II. Behavioral Objective:

The student given three sets in Venn diagram or roster form will be able to specify their intersections, unions, or complements.

III. Msthematical Ideas:

A. Union and intersection of sets.

B. The complement of a subset A of a universal set $\mathcal{U}$ is the set of all elements of $\mathcal{U}$ which are not in A.

IV. Vocabulary to Stress:

The complement of A = A'

V. Activities:

See Christian pp. 50-61

Text References:

Book: 7

Houghton Mifflin (1967, 1970) pp. 19-27

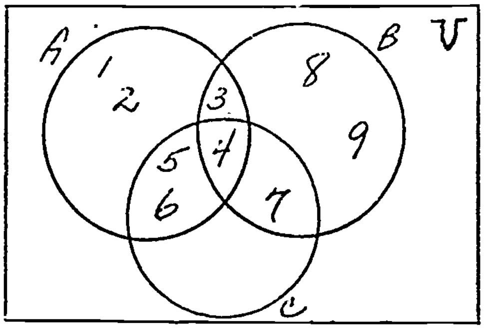
Other References:

Robert R. Christian, Introduction to Logic and Sets, Blaisdell Publishing Company.

Level: 24 Step: Z Concept: Sets

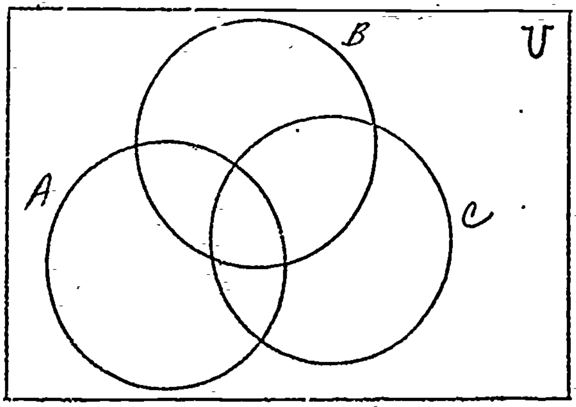
ENRICHMENT EVALUATION

I. Using the Venn diagram below, specify by roster (list) the following subsets of $\mathcal{U}$:



1. $A \cap B$ ____________________
2. $A \cap C$ ____________________
3. $A \cap (B \cap C)$ ____________________
4. $B \cup C$ ____________________
5. B' ____________________
6. A ____________________
7. $A \cap (B \cup C)$ ____________________
8. $B' \cap C$ ____________________
9. $A \cup (B' \cap C)$ ____________________
10. $B \cap (A \cup C')$ ____________________

II. Let U = {1, 2, 3, 4, 5, 6}, A = {1, 2, 3}, B = {3, 5}, C = {', 6}
Draw a Venn diagram to illustrate how these sets are related.

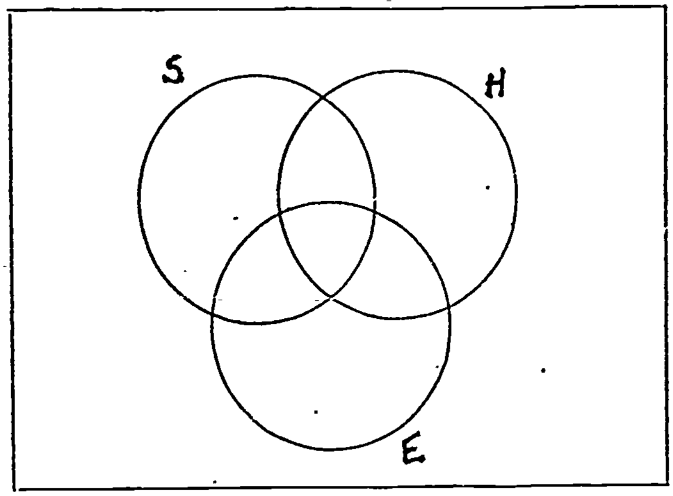


Level: 24 Step: Z Concept: Sets

ENRICHMENT: continued

III. In a certain math club, a survey revealed the following information:

25 liked Science
25 liked English
22 liked History
16 liked Science and History
15 liked Science and English
14 liked History and English
12 liked all three



Use the above Venn diagram to obtain your answers for the following.

1. If all members of the club responded to the survey, how many were in the club? __________

2. How many liked only Science? __________

3. How many liked Science or History but not English? __________

4. How many liked Science and English but not History? __________

5. How many liked only History? __________

24-28

Level: 24 Step: Z Concept: Function

I. Concept:

Function: Relating probability to statistics--empirical probability.

II. Behavioral Objective:

The student given a random sample of outcomes will be able to estimate the probability of an event.

III. Mathematical Ideas:

A. When the likelihood of an event is estimated by observing a sample of outcomes, it is called an empirical probability. Empirical probability is used when the sample space is too large to be tested in its entirety, or when such testing would destroy the samples.

B. A random sample is used in empirical probability because it provides a representative portion of the sample space. Care must be taken to make sure that the random sample includes enough occurrences to make the probability as accurate as possible.

IV. Vocabulary To Stress:

random sample

V. Activities:

A. Willerding, pp. 66-87

B. H.M. Book 8 (1967, 1970) pp. 485-487.

Text References:

Book: 8

Houghton Mifflin (1967, 1970) pp. 485-487
Houghton Mifflin (1972) pp. 212-213

Other References:

Willerding, Probability: The Science of Chance, (Franklin Math Series).

Level: 24 Step: Z Concept: Function

ENRICHMENT EVALUATION

I. An expreiment consists of drawing a marble from an urn. Assume that you know nothing about the number or color of marbles in the urn. The following chart shows the results of the first 50 drawings.

Red	~~////~~	~~////~~	~~////~~	//
White	~~////~~	~~////~~	////	
Blue	~~////~~	~~////~~	~~////~~	////

What is the probability that the next marble drawn will be:

1. red ________________
2. red or white ________________
3. green ________________
4. blue or green ________________
5. red, white, or blue ________________

II. Using the portion of a mortality table given below, answer the questions under the table.

AGE	NUMBER LIVING	DEATHS EACH YEAR
0	10,000,000	70,800
10	9,805,870	11,865
20	9,664,994	17,300
40	9,241,359	32,622

What is the probability that:

1. A person 40 years old will die within a year? ________________
2. A person 20 years old will die within a year? ________________
3. A person age 20 will live to be 40? ________________
4. A person will live to be 10? ________________
5. A person age 10 will live to be 40? ________________

Level: 24 Step: Z Concept: Geometry

I. Concept:

Geometry: finding any existing point of symmetry.

II. Behavioral Objective:

The student given a plane figure will be able to label any existing point of symmetry or to state that no such point exists.

III. Mathematical Ideas:

A geometric figure is said to be symmetrical with respect to a point 0 if and only if for each point A on the figure there is a point B on the figure for which 0 is the bisector of $\overline{AB}$.

IV. Vocabulary To Stress:

symmetry symmetrical center of symmetry
symmetrical with respect to a point

V. Activities:

A. In Houghton Mifflin Book 8 (1967, 1970), explore the points of symmetry of an ellipse (p. 102), a square p. 102, a parallelogram (p. 107), a rhombus (p. 107), a rectangle (p. 107), a circle (p. 101) and a line (p. 103: 17).

B. Assign Houghton Mifflin Book 8 (1967, 1970) pp. 103-104: 1-11

Text References:

Book: 8

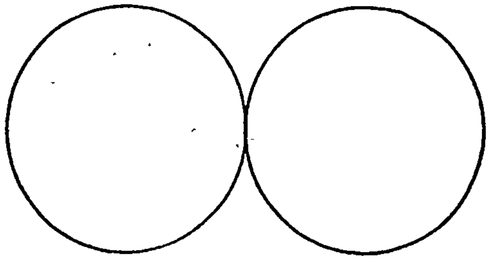
Houghton Mifflin (1967, 1970) pp. 101-104, (17, Sec. 6, 7, 8)

Level: 24 Step: Z Concept: Geometry

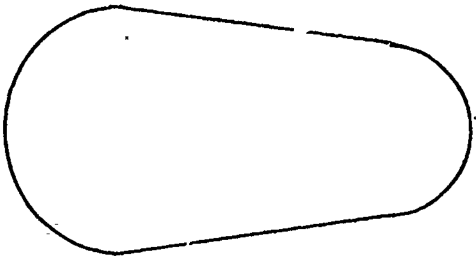
ENRICHMENT EVALUATION

If the figure has a point of symmetry, put a dot where it would be and label it "P". If the figure has no point of symmetry, write "none" on the line beside the figure.

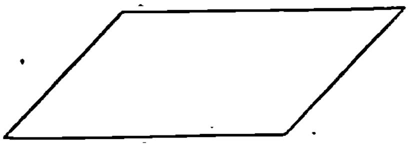
1.



2.

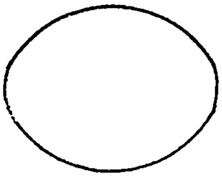


3.



4.

5.



Level: 24 Step: Z Concept: Number Sentences

I. Concept

Number Sentences: Graphing compound inequalities.

II. Behavioral Objective:

The student given a compound inequality will be able to graph it.

III. Mathematical Ideas:

A. The solution set of a compound sentence using "or" is the union of the solution sets of the two component sentences.

B. The solution set of a compound sentence using "and" is the intersection of the solution sets of the two component sentences.

IV. Vocabulary To Stress:

compound open sentence

V. Activities:

See H.M. Book 8 (1967, 1970) pp. 300-305

Text References:

Houghton Mifflin (1967, 1970) pp. 300-305, (49)

Level: 24 Step: Z Concept: Number Sentences

ENRICHMENT EVALUATION

Graph each of the following sets over the set of real numbers.

1. $\{x: x > -2 \text{ and } x < 4\}$

2. $\{x: x < -2 \text{ or } x > 4\}$

3. $\{x: x \leq -2\} \cup \{x: x \geq 4\}$

4. $\{x: x < -2\} \cap \{x: x > 4\}$

5. $\{x: x > -2\} \cap \{x: x \leq 4\}$

6. $\{x: x \geq -2\} \cup \{x: x < 4\}$

Level: 24 ANSWERS TO WORKSHEETS

24-6

$A = \{0, -7, 6\}$

$B = \{0, \frac{1}{2}, -7, -\frac{4}{5}, 6, .721\}$

$C = \{\pi, 2.12122...\}$

$D = \{6\}$

$E = \{-7, -\frac{4}{5}\}$

$F = \{\frac{1}{2}, 6, .721, \pi, 2.12122...\}$

$G = \{0\}$

24-9

1. 5.727
2. 6.278
3. -0.534
4. $-6.\overline{456}$
5. equal
6. 34.55
7. -7.6
8. equal
9. $-214.\overline{4}$
10. $12.64\overline{21}$

24-13

I. cone
   slant height
   altitude
   vertex
   base
   height
   lateral surface

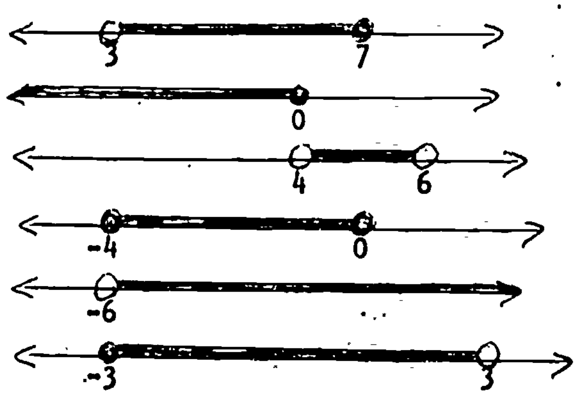
II. cylinder
   base
   base
   altitude
   height
   lateral surface

III. sphere
   center
   radius
   diameter
   antipodal points
   great circle
   small circle

24-17

1. $5 < x \leq 9$
2. $x < 2$
3. $4 \leq \;.\; 6$
4. $x > 7$
5. $4 \leq x \leq 8$
6. $6 \leq x \leq 10$

7\. 8. 9. 10. 11. 12.



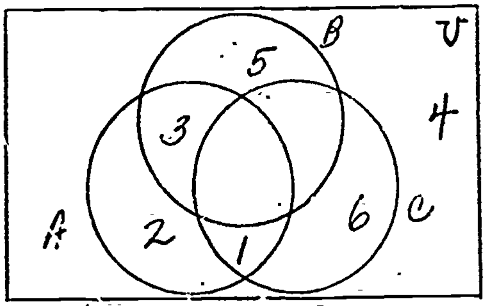
Level: 24 ANSWERS TO ENRICHMENT (STEP Z)

EVALUATION

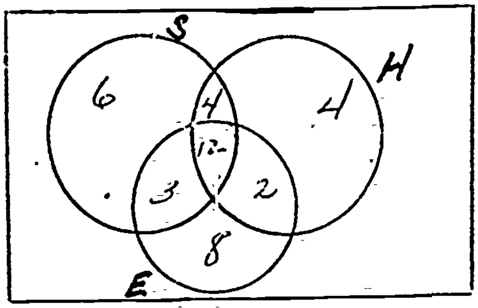
24-26

I.
1. {3, 4}
2. {4, 5, 6}
3. {4}
4. {3, 4, 5, 6, 7, 8, 9}
5. {1, 2, 5, 6}
6. { 1, 2, 3, 4, 5, 6}
7. { 3, 4, 5, 6}
8. {5, 6}
9. {1, 2, 5, 6}
10. {3, 8, 9}

II.



III.



1. 39
2. 6
3. 14
4. 3
5. 4

24-29

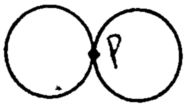
I.
1. $\frac{17}{50}$
2. $\frac{31}{50}$
3. 0
4. $\frac{19}{50}$
5. 1

II.
1. $\frac{32622}{9241359}$
2. $\frac{17300}{9664994}$
3. $\frac{9241359}{9664994}$
4. $\frac{9805870}{10,000,000}$
5. $\frac{9241359}{9805870}$

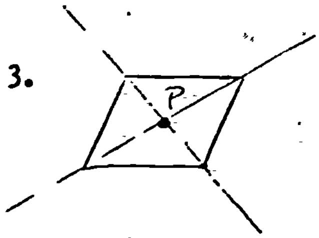
Level: 24

ANSWERS TO ENRICHMENT
(Continued)

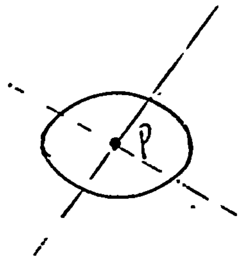
24-31

1. 

2. none

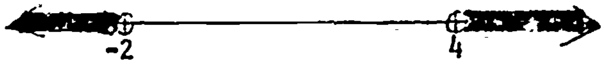
3. 

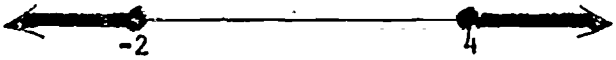
4. none

5. 

24-33

1. 

2. 

3. 

4. Ø (no graph)

5. 

6. 

(the entire number line

Level: 24 — Answers - Post Test I

Step A

Sets

1. $\{.777, -.\overline{7}, 0, -11, \frac{2}{3}, -4.3\}$
2. $\{0, -11\}$
3. $\{-\sqrt{3}, \pi, .161161116. . . \}$
4. $\{-\sqrt{3}, -.\overline{7}, -11, -4.3\}$
5. $\{0\}$

Numeral

6. $\frac{41}{90}$
7. $\frac{5}{11}$
8. $\frac{50}{11}$
9. $-\frac{454}{999}$
10. $\frac{4541}{999}$

Order

11. $3.4\overline{5}$
12. $-.44$
13. $72.\overline{56}$
14. $.00\overline{7}$
15. $-4.\overline{575}$

Addition & Subtraction

16. .656656665. . ., Irrational
17. .464464446. . ., Irrational
18. $.\overline{5}$, Rational
19. $7.\overline{96}$, Rational
20. $.\overline{9}$, Rational

Geometry

21. slant height
22. base
23. lateral surface
24. great circle
25. radius

Measurement

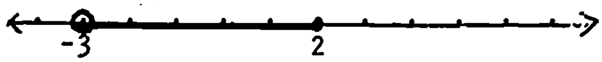
26. 5×10^{-4}
27. 7×10^{8}
28. 9×10^{-3}
29. 1.554×10^{2}
30. 3.225×10^{-8}

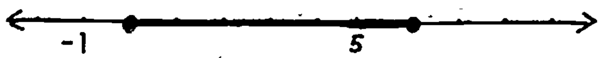
Step B

Order

31. B
32. C
33. A
34. B
35. C

Function and Graphs

36. $-4 \leq x < 2$
37. $-9 \leq x \leq 0$
38. 

39. 

40. 

Geometry

41. 48π sq. m.
42. 68π sq. m.
43. 144π sq. m.
44. 108π sq. m.
45. 33π sq. m.

Measurement

46. 5×10^{9}
47. 4×10^{-15}
48. 1.3112×10^{2}
49. 5.88
50. 4.917×10^{1}

Step C

Function and Graphs

	Domain	Range	Function
51.	{3,4,5}	{2,3,4}	yes
52.	{-4,4}	{-2,2}	no
53.	{5,4,3}	{4,5}	yes
54.	{4,5}	{3,4,5}	no
55.	{-2,-1,3,5}	{-7,2,4,6}	no

Level: 24 Answers - Post Test I

Step C

Geometry

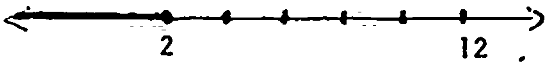
56. 108π cu. m.
57. 288π cu. m.
58. 64π cu. m.
59. 40π cu. m.
60. 21π cu. m.

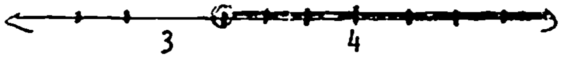
Number Sentences

61. $\{x: x < \frac{1}{6}\}$

62. $\{x: x \le 11\}$

63. $\{x: x \ge 2\}$

64. 

65. 

Level: 24 Answers - Post Test II

Step A

Sets

1. $\{\pi, \sqrt{5}, .121121112...\}$
2. $\{-16.3, -2.\overline{3}, 0, -71\}$
3. $\{-16.3, -2.\overline{3}, 43, \frac{1}{4}, 0, -71\}$
4. $\{43, 0, -71\}$
5. $\{43, 0\}$

Numeral

6. $\frac{8}{11}$ 8. $-\frac{80}{11}$ 10. $\frac{7265}{999}$

7. $\frac{13}{18}$ 9. $\frac{727}{999}$

Order

11. $-.66$
12. $8.\overline{78}$
13. $19.\overline{21}$
14. $.00\overline{5}$
15. $-3.\overline{454}$

Addition & Subtraction

16. $.\overline{5}$, Rational
17. $9.\overline{74}$, Rational
18. .494994999. . ., Irrational
19. .686686668. . ., Irrational
20. $.\overline{9}$, Rational

Geometry

21. altitude
22. antipodal points
23. slant height
24. lateral surface
25. sphere

Measurement

26. 4×10^{-4}
27. 10^{9}
28. 1.29×10^{4}
29. 1.036×10^{2}
30. 6.45×10^{-8}

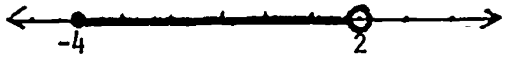
Step B

Order

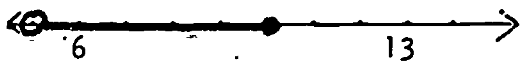
31. A
32. C
33. C
34. D
35. C

Function & Graphs

36. $-5 \leq x \leq 1$
37. $-5 < x \leq -1$

38. 

39. 

40. 

Geometry

41. 100π sq.m.
42. 90π sq. m.
43. 176π sq.m.
44. 90π sq. m.
45. 96π sq. m.

Measurement

46. 4×10^{9}
47. 6×10^{-15}
48. 3.278×10^{2}
49. 8.4×10^{-1}
50. 9.834

Step C

Function & Graphs

	Domain	Range	Function
51.	$\{7,6,5\}$	$\{8,7\}$	yes
52.	$\{8,7\}$	$\{5,6,7\}$	no
53.	$\{-2,3,2\}$	$\{3,-2,-3\}$	no
54.	$\{4,3,7,2,-2,5\}$	$\{6,12,9,-8,7\}$	yes
55.	$\{-1,1,0\}$	$\{2,3\}$	yes

Step C

Geometry

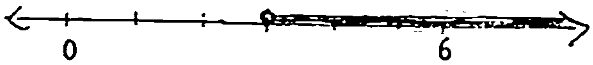
56. 36 π cu. m.
57. 192 π cu. m.
58. 100 π cu. m.
59. 108 π cu. m.
60. 96 π cu. m.

Number Sentences

61. $\{x: \ x > 11\}$

62. $\{x: \ x \leq \frac{1}{12}\}$

63. $\{x: \ x \geq -1\}$

64. 

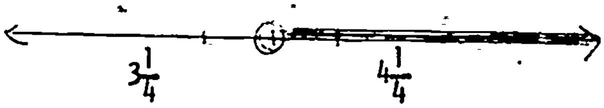
65. 

TABLE OF CONTENTS

Level 25

Step A

Step B

Step C

Step D

Step Z

Level: 25 Step: A Concept: Numeral

I. Concept:

Numeral: Reading radical expressions.

II. Behavioral Objective:

The student given an integer written as a radical expression (square root only) will be able to write the integer.

III. Mathematical Ideas:

A. If x and y are real numbers and $x^2 = y$, then x is a square root of y.

B. $\sqrt{a}$ is used to denote the positive square root of a ($a \geq 0$).

C. $-\sqrt{a}$ is used to denote the negative square root of a ($a \geq 0$).

IV. Vocabulary To Stress:

$\sqrt{a}$ = positive square root of a $-\sqrt{a}$ = negative square root of a

radical sign ($\sqrt{}$)

V. Activities:

A. Assign H.M. Book 8 (1967, 1970) p. 338: 1-8 and oral exercises 1-20 (omit 9, 10).

B. Important: Be sure to include the explanation of examples like $\sqrt{(-3)^2} = 3$ to insure the understanding of positive and negative square roots.

Text References:

Book: 8

Houghton Mifflin(1967, 1970) pp. 336-338, (55, sec. 1-25)
Addison-Wesley (1971) pp. 324

Level: 25 Step: A Concept: Order

I. Concept:

Order: Using order to estimate square roots.

II. Behavioral Objectives:

The student given an expression representing a real number in radical (square root) form will be able to name either the interger the expression represents or two consecutive integers between which the given number lies.

III. Mathematical Ideas:

A. If n is a perfect square, then $\sqrt{n}$ is an integer.

B. If a, b, x are positive integers and $a^2 < x < b^2$, then $a < \sqrt{x} < b$, and $-b < -\sqrt{x} < -a$

IV. Activities:

Assign Houghton Mifflin, Book 8 (1967, 1970) p. 339: 9-19
Addison-Wesley p. 329: 2,4

Text References:

Book: 8

Houghton Mifflin (1967, 1970) pp. 338, 339, (55, sec. 26-34)
Addison-Wesley (1971) pp. 329, (75)

Level: 25 Step: A Concept: Geometry

I. Concept:

Geometry: Solving right triangles.

II. Behavioral Objective:

The student given the measures of 2 sides of a right triangle will be able to find the measure of the third side, leaving any irrational numbers in simplified radical form.

III. Mathematical Ideas:

A. Pythagorean Property: If a, b, and c are the measures of the sides of a right triangle, and c is the hypotenuse, then $a^2 + b^2 = c^2$.

B. Multiplication property of square roots: If x and y are positive real numbers, then $\sqrt{xy} = \sqrt{x} \cdot \sqrt{y}$

C. When a number is in simplified radical form, (square root only), there are no perfect squares under the radical sign.

Example: $\sqrt{4a^2} = \sqrt{4} \cdot \sqrt{a^2} = 2a$

Example: $\sqrt{3a^2} = \sqrt{3} \cdot \sqrt{a^2} = (\sqrt{3})\,(a) = a\sqrt{3}$

D. Irrational numbers should be expressed in simplified radical form.

Example: $\sqrt{5x^2} = \sqrt{5} \cdot \sqrt{x^2} = (\sqrt{5})\,(x) = x\sqrt{5}$

Text References:

Book: 8

Houghton Mifflin (1967, 1970) pp. 339-343, (56)
Houghton Mifflin (1972) pp. 279, 402
Addison-Wesley (1971) pp. 310, 325, 326

Level: 25 Step: A Concept: Number sentences

I. Concept:

Number sentences: Solving equations in two variables.

II. Behavioral Objective:

The student given a simple linear equation in 2 variables and a finite replacement set of ordered pairs will be able to write the solution set as a set of ordered pairs.

III. Mathematical Ideas:

A. The solution set of an open number sentence in 2 variables consists of the set of ordered pairs that make the open sentence into a true statement.

B. In each ordered pair, the x value is the first component and the corresponding y value is the second component.

IV. Vocabulary To Stress:

ordered pairs first component second component

V. Activities:

See H.M. Book 8 (1967, 1970) pp. 433 (oral exercises 1-7) and (written exercises 1-12)

Text References:

Book: 8

Houghton Mifflin(1967, 1970) pp. 432-437, 460, 461, (71: sec. 1-13)

Level: 25 Step: B Concept: Numeral

I. Concept:

Numeral: Calculating square roots by the "divide and average" method.

II. Behavioral Objective:

The student given a positive rational number in decimal form will be able to approximate its positive square root to the nearest tenth using the "divide and average" method.

III. Mathematical Ideas:

A. Density of real numbers.

B. If a, b, x are positive real numbers and $a^2 < x < b^2$, then $a < \sqrt{x} < b$.

C. See Houghton Mifflin Book 8 (1967, 1970) p. 344, second method or Addison-Wesley Book 8 p. 330.

D. Note: The "divide and average" method is also called the "iterative" method, the "2nd method" in Houghton Mifflin, p. 344, and the Newton Method in the 6th grade Houghton Mifflin Teacher's Manual, p. 124.

Text References:

Book: 8

Houghton Mifflin (1967, 1970) pp. 344-346, (57)
Houghton Mifflin (1972) pp. 171, 260
Addison-Wesley (1971) pp. 330-331

Level: 25 | Step: B | Concept: Function & Graphs

I. Concept:

Functions and Graphs: Graphing linear equations in two variables.

II. Behavioral Objective:

The student given a simple linear equation in 2 variables will be able to graph its solution set over the set of real numbers.

III. Mathematical Ideas:

A. A linear equation in two variables is an equation that can be written in the form $Ax + By + C = 0$, where a, b, c are integers.

B. The solution set of an linear equation is a set of ordered pairs.

C. The graph of a linear equation is a straight line.

IV. Vocabulary To Stress:

linear equation | linear relation | linear function

V. Activities:

A. The conventional method is to first solve the equation for y in terms of x. Then make a chart and pick 3 x-values and find the corresponding y-values. Then plot these three points and draw the line.

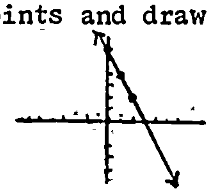
Example: $2x + y = 6$

1) Solve for y: $y = -2x + 6$

2) Make a chart, pick 3 x-values, and find the corresponding y-values:

x	y
0	6
1	4
2	2

3) Plot these 3 points and draw the line.



B. Do exercises 15-17 on p. 447, H.M. book 8 (1967, 1970).

C. Assign H.M. Book 8 (1967, 1970) p. 451 (written exercises)

Text References:

Book: 8

Houghton Mifflin(1967, 1970) pp. 447-451
Addison-Wesley (1971) pp. 164-165, (37)

Level: 25 Step: B Concept: Measurement

I. Concept:

Measurement: Converting into metric units of mass and capacity.

II. Behavioral Objective:

The student given a measure of mass or capacity in the English or the metric system will be able to convert it to another unit of mass or capacity in the metric system by using a table of equivalents. (See below)

III. Mathematical Ideas:

A. See Table 7 (approximately equivalent units of capacity) in H.M. Book 8 (1967, 1970) p. 194.

B. Also see the table made below (Activity A).

IV. Vocabulary To Stress:

capacity liter mass gram

V. Activities:

A. Make a table like the one on p. 177, H.M. Book 8 (1967, 1970) using units of capacity instead of length.

Example:

unit	Multiple of a liter
ml.	10^{-3}
cl.	10^{-2}
dl.	10^{-1}
l.	1
dkl.	10^{1}
hl.	10^{2}
kl.	10^{3}

B. The table above may also be used for units of mass--just replace liter with gram.

C. Relate volume, capacity, mass, and weight of water. See H.M. Book 8 (1967, 1970) pp. 193, 199 and A.W. Book 7 p. 327.

D. Assign in H.M. Book 8 (1967, 1970) p. 194 (oral exercises) and p. 195 (written exercises 1-6 and 8-17).

Level: 25 Step: B Concept: Measurement

Text References:

Book: 8

Houghton Mifflin(1967, 1970) pp. 192-195, 199, (30)
Addison-Wesley (1971) p. 257

Grade: 7

Addison-Wesley (1971) p. 327

Level: 25 Step: B Concept: Number sentences

I. Concept:

Number sentences: Solving inequalities in two variables.

II. Behavioral Objective:

The student given a simple linear inequality in 2 variables and a finite replacement set of ordered pairs will be able to write the solution set as a set of ordered pairs.

III. Mathematical Ideas:

The solution set of an open sentence in 2 variables is a set of ordered pairs. If the replacement set is finite, these may be specified by roster. If the replacement set is infinite, these may be specified by graph or rule (set-builder notation).

IV. Activities:

A. See H.M. Book 8 (1967, 1970) p. 437 (exercises 21-26)

B. See worksheet on the following page.

Text References:

Book: 8

Houghton Mifflin(1967, 1970) pp. 436-437, (71, sec. 14, 15)

Level: 25 Step: B Concept: Number sentences

WORKSHEET

Given the set $S = \{(-2, -1)\ (-1, 2)\ (1, -2)\ (-1, -2)\}$, write the set of ordered pairs in the set S that satisfy each of the following inequalities:

1. $x + y \leq 0$ ____________________

2. $y - x \geq 0$ ____________________

3. $2x + y > -2$ ____________________

4. $y - 2x < 2$ ____________________

5. $x + 2y < 1$ ____________________

Level: 25 Step: C Concept: Numeral

I. Concept:

Numeral: Calculating square roots by the algorithm method.

II. Behavioral Objective:

The student given a positive decimal numeral will be able to approximate its positive square root to the nearest tenth, using the algorithm method.

III. Mathematical Ideas:

A. $(a + b)^2 = a^2 + 2ab + b^2$ where a, b are real numbers.

B. $\sqrt{a^2 + 2ab + b^2} = a + b$ where a, b are real numbers.

C. See the two pages entitled "Algorithm Method for Calculating a Square Root"

IV. Vocabulary to Stress:

algorithm = a systematic procedure used in a computation (Houghton Mifflin Book 7 (1967, 1970)

V. Activities:

A. Note: This is also called the "Doubling Method".

B. For the teacher only: Read the 5 attached pages entitled "Explanation of Algorithm Method for Calculating Square Root". This explanation is too advanced for students at this level. Do not give it to your students.

C. Give students the 2 pages entitled "Algorithm Method for Calculating a Square Root" and the worksheet following it.

Level: 25 Step: C Concept: Numeral

FOR THE TEACHER ONLY:

EXPLANATION OF ALGORITHM METHOD FOR CALCULATING SQUARE ROOT

Any decimal numeral with 2 digits can be written in the form 10a + b where a is the first digit and b is the second.

$$23 = 20 + 3 = (10 \cdot 2) + 3$$

So when you square 10a + b, you get:

$$(10a + b)^2 = (10a + b)(10a + b) = 100a^2 + 10ab + 10ab + b^2 = 100a^2 + 20ab + b^2$$

Now let's try 23:

$$23 = 10 \cdot 2 + 3 = 20 + 3$$

Square:

$$\begin{array}{r} 23 \\ \underline{23} \\ 69 \\ \underline{46} \\ 529 \end{array} \qquad (20 + 3)(20 + 3) = 400 + 60 + 60 + 9 = 400 + 120 + 9 = 529$$

Compare:

$$\begin{array}{l} 100a^2 + 20ab + b^2 \\ 100 \cdot 4 + 20 \cdot 2 \cdot 3 + 9 \\ 400 + 120 + 9 \end{array} \qquad (a = 2,\ b = 3)$$

Now find the square root of $100a^2 + 20ab + b^2$

$$\sqrt{100a^2 + 20ab + b^2}$$

We know the square root of a trinomial is a binomial, so our root will be in the form x + y. Find x. We know that our first term ($100a^2$) came from squaring the first term in 10a + b.

So x must be 10a.

We now have:

$$\begin{array}{l} \phantom{\sqrt{}}10a \\ \sqrt{100a^2 + 20ab + b^2} \end{array}$$

Square the 10a and subtract.

$$\begin{array}{l} \phantom{\sqrt{}}10a \\ \sqrt{100a^2 + 20ab + b^2} \\ \phantom{\sqrt{}}\underline{100a^2} \end{array}$$

Bring down the rest of the terms:

$$\begin{array}{l} \quad 10a \\ \sqrt{100a^2 + 20ab + b^2} \\ \underline{100a^2} \\ \qquad\qquad 20ab + b^2 \end{array}$$

Where will the second term in the root come from?

$$20ab + b^2 = b(20a + b)$$

so divide:

$$20a + b \;\overset{\displaystyle b}{\overline{\big)\, 20ab + b^2}}$$

Our divisor is 20a + b.

Where did this come from?

20a is just 10a doubled, and the second term in the quotient is the same as the second term in the divisor. (Because the b^2 came from squaring b in the first place.)

Now let's return to our original problem.

$$\begin{array}{l} \quad 10a \\ \sqrt{100a^2 + 20ab + b^2} \\ \underline{100a^2} \\ \qquad\qquad 20ab + b^2 \end{array}$$

Double the quotient (10a) and use this for the trial divisor:

$$\begin{array}{rl} & \quad 10a \\ & \sqrt{100a^2 + 20ab + b^2} \\ & \underline{100a^2} \\ 20a\,\big) & \qquad\qquad 20ab + b^2 \end{array}$$

Divide 20ab by 20a and put this quotient above the dividend as the second term in the quotient.

$$\begin{array}{rl} & \quad 10a + b \\ & \sqrt{100a^2 + 20ab + b^2} \\ & \underline{100a^2} \\ 20a\,\big) & \qquad\qquad 20ab + b^2 \end{array}$$

b is also the second term in the new divisor: (See explanation above.)

$$\begin{array}{rl}
 & \quad 10a \quad + \quad b \\
 & \sqrt{100a^2 + 20ab + b^2} \\
 & \quad \underline{100a^2} \\
20a + b\;) & \qquad\qquad\; 20ab + b^2
\end{array}$$

Multiply:

$$\begin{array}{rl}
 & \quad 10a \quad + \quad b \\
 & \sqrt{100a^2 + 20ab + b^2} \\
 & \quad \underline{100a^2} \\
20a + b\;) & \qquad\qquad\; 20ab + b^2 \\
 & \qquad\qquad\; \underline{20ab + b^2}
\end{array}$$

Now apply this to 23^2:

$$23^2 = (20 + 3)^2 = 400 + 120 + 9$$

$$\begin{array}{rl}
 & \quad 20 \\
 & \sqrt{400 + 120 + 9} \\
 & \quad \underline{400} \\
 & \qquad\quad 120 + 9
\end{array}$$

Double 20 to get 40 for the trial divisor:

$$\begin{array}{rl}
 & \quad 20 \\
 & \sqrt{400 + 120 + 9} \\
 & \quad \underline{400} \\
40\;) & \qquad\quad 120 + 9
\end{array}$$

$120 \div 40 = 3$, so 3 is the second term in the quotient and also in the divisor.

$$\begin{array}{rl}
 & \quad 20 + 3 \\
 & \sqrt{400 + 120 + 9} \\
 & \quad \underline{400} \\
40 + 3\;) & \qquad\quad 120 + 9
\end{array}$$

Multiply:

$$\begin{array}{rl}
 & \quad 20 + 3 \\
 & \sqrt{400 + 120 + 9} \\
 & \quad \underline{400} \\
40 + 3\;) & \qquad\quad 120 + 9
\end{array}$$

Now let's write this as: $\sqrt{529}$ and find the square root.

First find the largest multiple of 10 whose square is less than 529. It is 400. The square root of 400 is 20.

$$\begin{array}{r|l} \sqrt{529} & 20 \\ \underline{400} & \\ 129 & \end{array}$$

Double 20 and divide into 129. $129 \div 40 = 3$. Put the 3 in the quotient and add it to the divisor. Multiply $43 \cdot 3$.

$$\begin{array}{rr|r} & \sqrt{529} & 20 \\ & \underline{400} & \\ 40 + 3 & 129 & 3 \\ 43 & 129 & \\ \hline & & 23 \end{array}$$

So $\sqrt{529} = 23$.

Now let's find a shorter method: $\sqrt{529}$

First divide the dividend into groups of 2 digits, starting at the decimal point. Why? (Because $10^2 = 100$, and for every 100 we have in the dividend, we will have a 10 in the quotient.)

$$\sqrt{5\,29.}$$

Now find the largest perfect square that is less than or equal to 5.

$$\begin{array}{r} 2 \\ \sqrt{5\,29.} \\ \underline{4} \end{array}$$

IMPORTANT: We have written a 2, but we should realize that this is really only the first digit in 20.

Now continue. Subtract and bring down.

$$\begin{array}{r} 2 \\ \sqrt{5\,29.} \\ \underline{4} \\ 1\,29 \end{array}$$

(We will always bring down 2 digits at a time because for every 2 digits in the dividend there will be one digit in the quotient. $10^2 = 100$.)

Find the new divisor by doubling the quotient.

IMPORTANT: Remember that the quotient here is 20, not 2. So we must do something about this. After you double the digits in the quotient, always multiply by 10.

```
      2
    √529
     4
40 ) 129
```

Now divide and place the quotient above the dividend and add it to the divisor.

```
          2 3
        √529
         4
40 + 3 ) 129
43
```

Multiply the last digit in the quotient by the divisor.

```
          2 3
        √529
         4
40 + 3 ) 129
    43   129
```

Now we are ready to learn the algorithm.

ALGORITHM METHOD FOR CALCULATING A SQUARE ROOT

Find the square root correct to the nearest tenth: $\sqrt{123.4}$

1. Group the dividend into pairs of digits beginning at the decimal point and going in both directions. (If necessary, annex a zero to get an even number of decimal places to the right of the decimal point. If there is an odd number of decimal places to the left of the decimal point, let the extra digit be the one farthest to the left.)

$$\sqrt{\underbrace{1}\underbrace{23}.\underbrace{40}}$$

2. Find the largest perfect square less than or equal to the number in the first group. Write it under the first group and put its square root above the first group.

```
   1
 √ 1 23. 40
   1
```

3. Subtract the perfect square from the first group, and bring down the second group. (Remember to always bring down two digits at a time.)

```
   1
 √ 1 23. 40
   1
    23
```

4. The number you brought down is now the dividend. To get a new divisor, double the entire number in the quotient and multiply by 10. This is now the trial divisor. Place it in front of the dividend.

```
        1
      √ 1 23. 40
        1
  20 )   23
```

5. Divide the new dividend by the trial divisor and put the quotient in two places: (1) write it above the group being used as the dividend, and (2) add it to the trial divisor. Re-write the trial divisor as one numeral.

```
            1  1.
          √ 1 23. 40
            1
  20 + 1 )   23
    21
```

6. Multiply the new digit in the quotient by this trial divisor, place this product under the dividend, and subtract.

```
                 1 1.
              \/ 1 23.40
                 1
      20 + 1  ) 23
          21  ) 21
                 2
```

7. Bring down the next group and repeat Steps 4 through 6 as many times as necessary. (See Step 8).

8. Place the decimal point in the quotient directly above the decimal point in the dividend. For every group (usually two digits) in the dividend there will be one digit in the quotient. Carry out the quotient to one more decimal point than is needed; then round off.

```
                 1 1. 1 0    = 11.1
              \/ 1 23.4000
                 1
      20 + 1  ) 23
          21  ) 21
     220 + 1  ) 240
         221  ) 221
    2220 + 0  ) 1900
        2220  )    0
                1900
```

WORKSHEET

1. $\sqrt{21.4}$

6. $\sqrt{2}$

2. $\sqrt{52.7}$

7. $\sqrt{3}$

3. $\sqrt{43.2}$

8. $\sqrt{5}$

4. $\sqrt{414.4}$

9. $\sqrt{32.3}$

5. $\sqrt{1024}$

10. $\sqrt{10}$

Level: 25 Step: C Concept: Function & Graphs

I. Concept:

Function & Graphs: Graphing linear inequalities in two variables.

II. Behavioral Objective:

The student given a simple linear inequality in two variables will be able to graph its solution set over the set of real numbers.

III. Mathematical Ideas:

A. A linear inequality in two variables is an inequality for which the associated equation is a linear equation in two variables.

B. The graph of a linear inequality in two variables is an open ($<$, $>$) or closed ($\leq$, $\geq$) half-plane on one side of the graph of the associated equation.

IV. Vocabulary To Stress:

open half-plane closed half-plane

V. Activities:

Graph the associated equation, then check whether the line should be dotted or solid, then shade the proper side.

If the inequality is solved for y first, it makes the shading much easier. If $y < mx + b$, the half-plane below the line is shaded; and if $y > mx + b$, the half-plane above the line is shaded.

You might also include problem #18 on p. 447, H.M. Book 8 (1967, 1970).

Text References:

Book: 8

Houghton Mifflin(1967, 1970) pp. 452-454, (74)
Houghton Mifflin(1972) pp. 367, 370-371. (67)

Level: 25 Step: D Concept: Numeral

I. Concept:

Numeral: Using tables to find rational approximations of positive square roots.

II. Behavioral Objective:

The student given any natural number < 101 or any perfect square $\leq$ 10,000 will be able to write its positive square root correct to the nearest tenth using tables.

III. Mathematical Ideas:

A. Tables:

Addison-Wesley (1971) Book 8, p. 333.

B. Rounding decimals to the nearest tenth.

IV. Vocabulary to Stress:

" $\doteq$ " and "$\approx$" mean "approximately"

V. Activities:

A. Using the table in Addison-Wesley (1971) p. 333, the square roots of the natural numbers less than 101 are listed in the column headed " $\sqrt{n}$ ". The perfect squares $\leq$ 10,000 are listed in the column headed " n^2 ". To find $\sqrt{7921}$ using the table in Addison-Wesley p. 333, look for 7921 under "n^2" and find its square root under "n". To find $\sqrt{31}$, look for 31 under "n" and find its square root under "$\sqrt{n}$ ".

Note: To be consistent with the other methods of finding square root, round off to tenths.

B. See worksheet on the following page.

Text References:

Book: 8

Houghton Mifflin (1967, 1970) pp. 345-346 501, (57, Sec. 5, 6, 7)
Addison-Wesley (1971) pp. 332-334

Level: 25 Step: D Concept: Numeral

WORKSHEET

Using the table in Addison-Wesley Book 8, p. 333, find the positive square root of each number correct to the nearest tenth.

1. $\sqrt{361}$	________	11. $\sqrt{841}$	________
2. $\sqrt{98}$	________	12. $\sqrt{3844}$	________
3. $\sqrt{2601}$	________	13. $\sqrt{84}$	________
4. $\sqrt{24}$	________	14. $\sqrt{52}$	________
5. $\sqrt{8464}$	________	15. $\sqrt{13}$	________
6. $\sqrt{196}$	________	16. $\sqrt{6724}$	________
7. $\sqrt{31}$	________	17. $\sqrt{7}$	________
8. $\sqrt{5041}$	________	18. $\sqrt{73}$	________
9. $\sqrt{65}$	________	19. $\sqrt{7744}$	________
10. $\sqrt{2401}$	________	20. $\sqrt{43}$	________

Level: 25 Step: D Concept: Function & Graphs

I. Concept:

Function and Graphs: Graphing systems of linear equations in two variables.

II. Behavioral Objective:

The student given a system of two linear equations in two variables will be able to solve the system by graphing.

III. Mathematical Ideas:

A. To solve a system of equations over $\mathcal{R}$ means to find the set of all ordered pairs that satisfy both equations.

B. The solution set of a system of linear equations in two variables consists of the point(s) where their graphs intersect.

IV. Vocabulary To Stress:

inconsistent dependent system of equations

V. Activities:

Assign in Houghton Mifflin (1972) pp. 368-369. This explanation is simpler than in the (1967, 1970) edition.

Text References:

Houghton Mifflin(1967, 1970) pp. 454-458, (75)
Houghton Mifflin(1972) pp. 368-369
Addison-Wesley (1971) p. 166

Level: 25 .Step: Z Concept: Sets

I. Concept:

<u>Sets</u>: Using quantifiers.

II. Behavioral Objective:

The student, given a set and a proposition with a quantifier describing that set, will be able to determine the truth value of the proposition.

III. Mathematical Ideas:

Expressions involving the idea of "how many" are called <u>QUANTIFIERS</u>.

IV. Vocabulary To Stress:

$\forall$ = for all, for every $\exists$ = for some, there exists

V. Activities:

See Christian, *Logic and Sets*, pp. 66-76.

References:

Robert R. Christian, *Logic and Sets*, Blaisdell Publishing Company, Second Edition, 1965.

Level: 25   Step: Z

ENRICHMENT EVALUATION

Let $B = \{0, 1, 2\}$. Determine the truth value of each of the following propositions, and defend your answer.

1. $(\forall x \in B)\ (x + 1 = 2)$ ____________________

2. $(\exists x \in B)\ (x + 1 = 2)$ ____________________

3. $(\exists x \in B)\ (x^2 + x + 1 = 0)$ ____________________

4. $(\forall x \in B)\ [x + (2 + 7) = (7 + x) + 2]$ ____________________

5. $(\forall x \in B)\ [x - (2 - 7) = (7 - x) - 2)]$ ____________________

6. $(\exists x \in B)\ [x - (2 - 7) = (7 - x) - 2]$ ____________________

Level: 25 Step: Z Concept: Numeral

I. Concept:

Numeral: Calculating square roots by the "sum of odd numbers" method.

II. Behavioral Objective:

The student given a perfect square will be able to calculate its positive square root, using the "sum of odd numbers" method.

III. Mathematical Ideas:

The sum of the first n odd natural numbers is a perfect square, and its positive square root is n.

IV. Activities:

A. Students can discover the above mathematical idea for themselves by filling in the blanks in the following two worksheets.

B. Use the pages entitled, "Calculating Square Roots by the 'Sum of Odd Numbers' Method".

C. Assign Worksheet on p. 25-32.

Level: 25 Step: Z Concept: Numeral

WORKSHEET

The Sum of Odd Numbers

Fill in the blanks:

Number of Odd Addends		Sum
1	1 =	1
2	1 + 3 =	4
	1 + 3 + 5 =	
	1 + 3 + 5 + 7 =	
	1 + 3 + 5 + 7 + 9 =	25
	1 + 3 + 5 + 7 + 9 + 11 =	
7	1 + 3 + 5 + 7 + 9 + 11 + 13 =	
	1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 =	
	1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 =	
	1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19 =	
11		
12		

Level: 25 Step: Z Concept: Numeral

WORKSHEET

The Sum of Odd Numbers

Compare the number of odd addends with the sum on each line.

1. If there are 4 addends, the sum is ______. $4^2 =$ ______. $\sqrt{} = 4$.

2. If the sum is 81, there are ______ addends. $\sqrt{81} =$ ______.

3. If there are 12 addends, the sum is ______. $\sqrt{} = 12$.

4. If the sum is 169, there are ______ addends. $\sqrt{169} =$ ______.

Now read the page entitled "Calculating Square Roots by the 'Sum of Odd Numbers' Method".

Level: 25 Step: Z Concept: Numeral

Calculating Square Roots by the "Sum of Odd Numbers" Method

1. Group the dividend into pairs of digits beginning at the decimal point and going in both directions.

$$\sqrt{\underbrace{3\ 8}\underbrace{4\ 4}.}$$

2. Subtract 1 from the first group.

$$\begin{array}{r}\sqrt{\underbrace{3\ 8}\underbrace{4\ 4}.}\\ \underline{-\ 1}\\ 3\ 7\end{array}$$

3. Subtract each successive odd number until there is not enough left to subtract the next odd number.

$$\begin{array}{r}\sqrt{\underbrace{3\ 8}\underbrace{4\ 4}.}\\ \underline{-\ 1}\\ 3\ 7\\ \underline{-\ 3}\\ 3\ 4\\ \underline{-\ 5}\\ 2\ 9\\ \underline{-\ 7}\\ 2\ 2\\ \underline{-\ 9}\\ 1\ 3\\ \underline{-\ 1\ 1}\\ 2\end{array}$$

Level: 25 Step: Z Concept: Numeral

Calculating Square Roots by the "Sum of Odd Numbers" Method -- Continued

4. Count the number of subtractions and place this number above the group.

$$\begin{array}{rl}
\overset{6}{} & \\
\sqrt{\underbrace{38}44.} & \text{(We subtracted 6 times.)} \\
\underline{-\ 1}\ ✓ & \\
37 & \\
\underline{-\ 3}\ ✓ & \\
34 & \\
\underline{-\ 5}\ ✓ & \\
29 & \\
\underline{-\ 7}\ ✓ & \\
22 & \\
\underline{-\ 9}\ ✓ & \\
13 & \\
\underline{-\ 11}\ ✓ & \\
2 &
\end{array}$$

5. Double the entire quotient and annex a 1.

$6 \times 2 = 12$

annex a 1 $\rightarrow$ 121

6. Bring down the next group and subtract the number found in Step 5.

$$\begin{array}{r}
\sqrt{3844.} \\
\underline{-\ 1} \\
37 \\
\underline{-\ 3} \\
34 \\
\underline{-\ 5} \\
29 \\
\underline{-\ 7} \\
22 \\
\underline{-\ 9} \\
13 \\
\underline{-\ 11} \\
244 \\
\underline{-\ 121} \\
123
\end{array}$$

Calculating Square Roots by the "Sum of Odd Numbers" Method -- Continued

7. Repeat Steps 3 - 6 as many times as necessary.

```
     6  2
  √3 8,4 4
  - 1
  3 7
  - 3
  3 4
  - 5
  2 9
  - 7
  2 2
  - 9
  1 3
 - 1 1
    2 4 4
  - 1 2 1
    1 2 3
  - 1 2 3
```

Now do the following worksheet.

Level: 25 Step: Z Concept: Numeral

WORKSHEET

Find the positive square root by the "sum of odd numbers" method.

1. $\sqrt{1\ \ 6\ \ 9}$

3. $\sqrt{1\ \ 7\ \ 6\ \ 4}$

2. $\sqrt{6\ \ 7\ \ 6}$

4. $\sqrt{4\ \ 6\ \ 2\ \ 4}$

Level: 25 Step: Z Concept: Numeral

WORKSHEET -- Continued

5. $\sqrt{1\ 1\ 5\ 6}$

8. $\sqrt{2\ 8\ 0\ 9}$

6. $\sqrt{5\ 2\ 9}$

9. $\sqrt{3\ 7\ 2\ 1}$

7. $\sqrt{5\ 1\ 8\ 4}$

10. $\sqrt{1\ 4\ 4\ 4}$

Level: 25 Step: Z Concept: Numeral

ENRICHMENT EVALUATION

Find the following square roots using the "sum of odd numbers" method:

1. $\sqrt{1024}$

4. $\sqrt{3136}$

2. $\sqrt{576}$

5. $\sqrt{2025}$

3. $\sqrt{1849}$

Level: 25 Step: Z Concept: Function & Graphs

I. Concept:

Function and Graphs: Using arrangements, permutations, and combinations.

II. Behavioral Objective:

The student given a set of objects will be able to determine how many ways those objects can be arranged, with or without regard to order.

III. Mathematical Ideas:

A. $n! = n \times (n - 1) \times (n - 2) \times \ldots \times 3 \times 2 \times 1$ (n factorial)

B. $4! = 4 \times 3 \times 2 \times 1 = 24$ (4 factorial)

C. A permutation is an arrangement of all or part of a set of objects in a certain order.

1. A permutation of n objects taken all at a time is an ordered arrangement of all objects in the set.

   If $P(n,n)$ represents the number of possible permutations of a set of n objects, then $P(n,n) = n!$

   Example:

   $\{a,b,c\}$ has 3 elements. There are 6 permutations of this set if all 3 elements are taken at a time.

   $P(3,3) = 3! = 3 \times 2 \times 1 = 6$

   The 6 permutations are: abc acb bac bca cab cba

2. A permutation of n objects taken r at a time is an ordered arrangement of only r objects in the set.

   If $P(n,r)$ represents the number of possible permutations of a set of n objects taken r at a time, then
   $P(n,r) = n \times (n-1) \times (n-2) \times \ldots \times$(to r factors)

   Example:

   $\{a,b,c\}$ has 3 elements. There are 6 pemutations of this set if 2 elements are taken at a time.

   $P(3,2) = 3 \times 2 = 6$

   The 6 permutations are: ab ac ba bc ca cb

Level: 25 Step: Z Concept: Function & Graphs

III. Mathematical Ideas: (Continued)

D. A combination is an arrangement of all or part of a set of objects without regard to order.

If $C(n,r)$ represents the number of possible combinations of a set of n objects taken r at a time, then

$$C(n,r) = \frac{P(n,r)}{P(r,r)}$$

Example:

$\{a,b,c\}$ has 3 elements. There are 3 combinations of this set if 2 elements are taken at a time.

$$C(3,2) = \frac{P(3,2)}{P(2,2)} = \frac{3 \times 2}{2 \times 1} = 3$$

The 3 combinations are: ab ac bc

E. $P(\text{a favorable outcome}) = \dfrac{\text{number of favorable outcomes}}{\text{number of possible outcomes}}$

IV. Vocabulary to Stress: permutation combination

V. Activities:

A. Addison-Wesley Book 8, pp. 221-228 (An excellent coverage of this subject!)

B. Relate Binomial Theorem to Pascal's Triangle (Addison-Wesley p. 227)

C. Willerding, p. 20 (tree diagrams)

D. Houghton Mifflin Book 8 (1967, 1970) pp. 493-500

Text References:

Book: 8

Houghton Mifflin (1967, 1970) pp. 493-500
Addison-Wesley (1971) pp. 221-228

Other References:

Willerding, Probability, The Science of Chance, Franklin Math Series.

Level: 25 Step: Z

ENRICHMENT EVALUATION

I. Permutations

A. How many different ways can 5 books be arranged on a shelf?__________

B. How many different 3-digit numerals can be formed from the digits in {1,2,3,4,5}, if no digit appears more than once in each numeral?__________

C. How many different 3-digit numerals can be formed from the set given in problem 2 if each digit may be repeated any number of times? __________

II. Combinations

A. How many committees of 4 persons can be appointed in a club with 10 members? __________

B. A bowl contains 4 red marbles and 6 white marbles. In how many ways can you select a set consisting of:

a) 2 red marbles? __________

b) 4 white marbles?__________

c) 4 red marbles? __________

d) 2 white marbles?__________

III. A bag contains 10 red marbles and 4 white marbles. If 3 marbles are drawn at random, what is the probability that:

A. all are red __________

B. all are white__________

C. 1 is red and 2 are white__________

D. 2 are red and 1 is white__________

Level: 25 Step: Z Concept: Geometry

I. Concept:

Geometry: Introducing trigonometric ratios.

II. Behavioral Objective:

The student given a drawing of a right triangle showing the length of each side will be able to write the sine, cosine, and tangent ratios of either acute angle.

III. Mathematical Ideas:

A. $\tan A = \frac{\text{Length of side opposite } A}{\text{Length of side adjacent } A}$ (when A is an acute $\angle$ in a right $\triangle$)

B. $\sin A = \frac{\text{Length of side opposite } A}{\text{Length of hypotenuse}}$

C. $\cos A = \frac{\text{Length of side adjacent to } A}{\text{Length of hypotenuse}}$

IV. Vocabulary To Stress:

tangent sine cosine

V. Activities:

Encourage students to cover as much as they can. Although the behavioral objective only includes finding ratios, the 3 texts listed under references all present excellent units covering more than this one concept.

Text References:

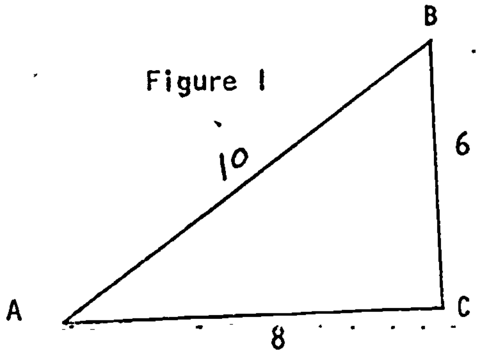
Book: 8

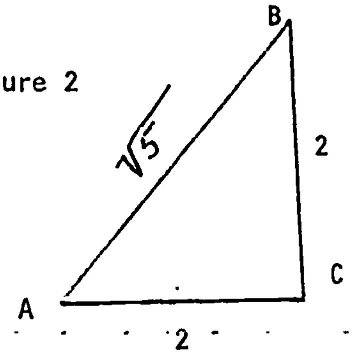
Houghton Mifflin(1967, 1970) pp. 353-358, (59)
Houghton Mifflin(1972) pp. 334-343, 348 (59,60,61)
Addison-Wesley (1971) pp. 363-366

Level: 25 Step: Z

ENRICHMENT EVALUATION

Complete the following statements:





In figure I above:

1. Sin A = ____________
2. Cos A = ____________
3. Tan A = ____________
4. Sin B = ____________
5. Cos B = ____________
6. Tan B = ____________

In figure II above:

7. $m^\circ\angle A$ = ____________
8. Sin A = ____________
9. Sin B = ____________
10. Tan A = ____________

Level: 25 Step: Z Concept: Number Sentences

I. Concept:

Number sentences: Using implication and proofs.

II. Behavioral Objective:

The student given a logic problem written as a series of hypotheses will be able to find the "best" conclusion by using relationships of sets.

III. Mathematical Ideas:

A. Equivalent conditions are conditions which specify the same truth set.

B. Condition p implies condition q if the truth set of p is a subset of the truth set of q. (See Christian p. 79).

In symbols, $p \Rightarrow q$ means

1. p implies q or
2. if p, then q or
3. p only if q or
4. q is a necessary condition for p or
5. p is a sufficient condition for q

C. Some important properties of implication:

1. If $p \Rightarrow q$ and $q \Rightarrow r$, then $p \Rightarrow r$
2. If $p \Rightarrow q$, then $\sim q \Rightarrow \sim p$
3. If $\sim q \Rightarrow \sim p$, then $p \Rightarrow q$

IV. Activities:

A. See Christian, pp. 76-90

B. See Christian, pp. 90-94 for application of logic to problems. Assign the odd-numbered problems for practice, and use the even-numbered ones for test purposes. (There is no written evaluation of this unit other than this.)

References:

Robert R. Christian, Introduction to Logic and Sets, Blaisdell Publishing Company, Second Edition, 1965.

ANSWERS TO WORKSHEETS

Level 25

25-10

1. $\{(-2, -1)\ (1, -2)\ (-1, -2)\}$
2. $\{(-2, -1)\ (-1, 2)\}$
3. $\{(-1, 2)\ (1, -2)\}$
4. $\{(1, -2)\ (-1, -2)\}$
5. $\{(-2, -1)\ (1, -2)\ (-1, -2)\}$

25-19

1. 4.6	6. 1.4
2. 7.3	7. 1.7
3. 6.6	8. 2.2
4. 20.4	9. 5.7
5. 32	10. 3.2

25-22

1. 19	6. 14	11. 29	16. 82
2. 9.9	7. 5.6	12. 62	17. 2.6
3. 51	8. 71	13. 9.2	18. 8.5
4. 4.9	9. 8.1	14. 7.2	19. 88
5. 92	10. 49	15. 3.6	20. 6.6

ANSWERS TO ENRICHMENT (Step Z) WORKSHEETS

25-27

3	9
4	16
5	
6	36
	49
8	64
9	81
10	100

1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19 + 21 = 121

1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19 + 21 + 23 = 144

25-28

1. 16, 16, 16	3. 144, 144
2. 9, 9	4. 13, 13

25-32

1. 13	3. 42	5. 34	7. 72	9. 61
2. 26	4. 68	6. 23	8. 53	10. 38

ANSWERS TO ENRICHMENT (Continued)

EVALUATION

25-25

1. F, $0 + 1 \neq 2$
2. T, $1 + 1 = 2$
3. F, $0^2 + 0 + 1 \neq 0$
   $1^2 + 1 + 1 \neq 0$
   $2^2 + 2 + 1 \neq 0$
4. T, commutative and associative properties of addition
5. F, $1 - (2 - 7) \neq (7 - 1) - 2$
6. T, $0 - (2 - 7) = (7 - 0) - 2$

25-34

1. 32
2. 24
3. 43
4. 56
5. 45

25-37

I
- A. 120
- B. 60
- C. 125

II.
- A. 210
- B. a) 6
  - b) 15
  - c) 1
  - d) 15

III.
- A. $\frac{c(10,3)}{c(14,3)} = \frac{30}{91}$
- B. $\frac{c(4,3)}{c(14,3)} = \frac{1}{91}$
- C. $\frac{c(10,1) \times c(4,2)}{c(14,3)} = \frac{15}{91}$
- D. $\frac{c(4,1) \times c(10,2)}{c(14,3)} = \frac{45}{91}$

25-39

1. $\frac{6}{10}$ or $\frac{3}{5}$
2. $\frac{8}{10}$ or $\frac{4}{5}$
3. $\frac{6}{8}$ or $\frac{3}{4}$
4. $\frac{8}{10}$ or $\frac{4}{5}$
5. $\frac{6}{10}$ or $\frac{3}{5}$
6. $\frac{8}{6}$ or $\frac{4}{3}$
7. 45
8. $\frac{2}{\sqrt{5}}$
9. $\frac{2}{\sqrt{5}}$
10. $\frac{2}{2}$ or 1

Level: 25

ANSWERS - Post Test 1

Step A

Numeral

1. 21
2. 7
3. 5
4. -4
5. 11

Order

6. $5<\sqrt{32}<6$
7. $-4<-\sqrt{12}<-3$
8. 9
9. $6<\sqrt{45}<7$
10. -2

Geometry

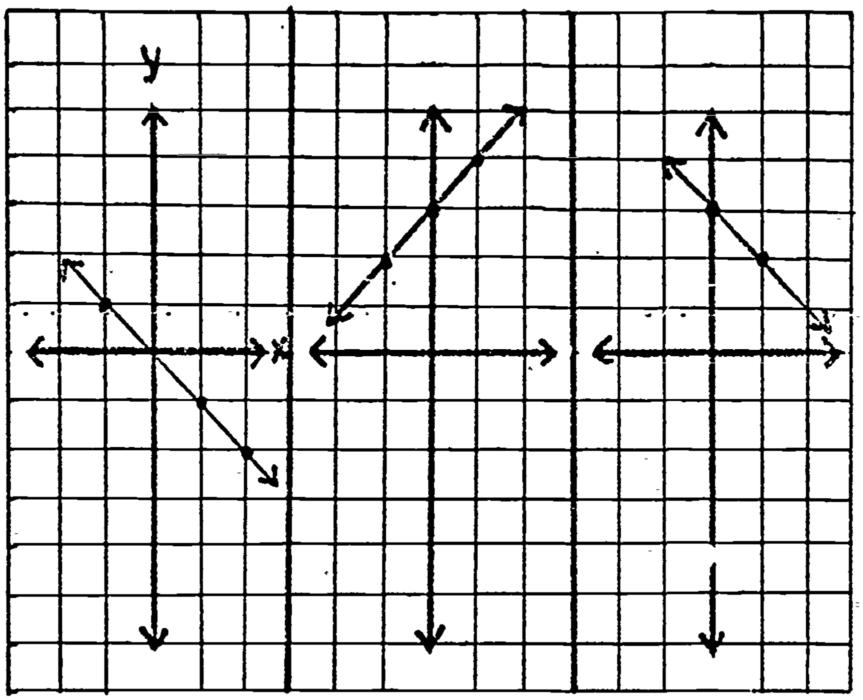
11. $x\sqrt{2}$
12. 12
13. 8a
14. $\sqrt{61}$
15. .4

Number Sentences

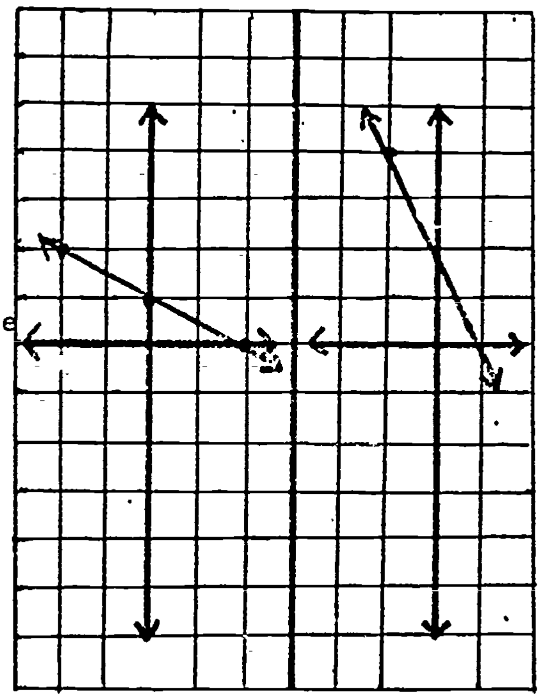
16. $\{(2,4), (4,3)\}$
17. $\{(3,4), (4,3)\}$
18. $\{(2,4), (3,2)\}$
19. $\{(3,2), (4,3)\}$
20. $\{(2,4), (4,3)\}$

Function & Graphs

26. 27. 28.



29. 30.



Step B

Numeral

21. 5.7
22. 6.4
23. 10.6
24. 4.4
25. 2.6

(The student's work should be shown on the back of the test page. Check to make sure the "divide and average" method was used).

Level: 25 Answers - Post Test I

Step B

Measurement

31. 1.1355×10^{1}
32. 5×10^{2}
33. 1.8925
34. 2.0×10^{5}
35. 1.00×10^{-2}

Number Sentences

36. {(4,5), (5,4)}
37. {(3,5)}
38. {(5,3), (4,3), (5,4)}
39. Ø
40. {(5,3), (4,3), (5,4)}

Step C

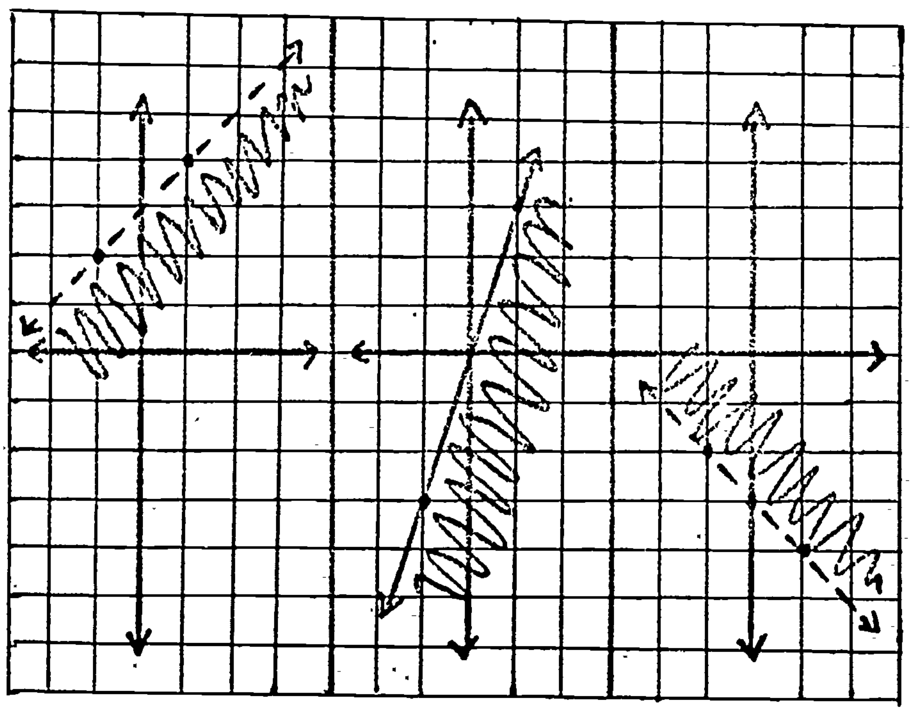
Numeral

41. 1.6
42. 6.9
43. 2.4
44. 5.1
45. 1.0

(The student's work should be shown on the back of the test page. Make sure the algorithm method was used.)

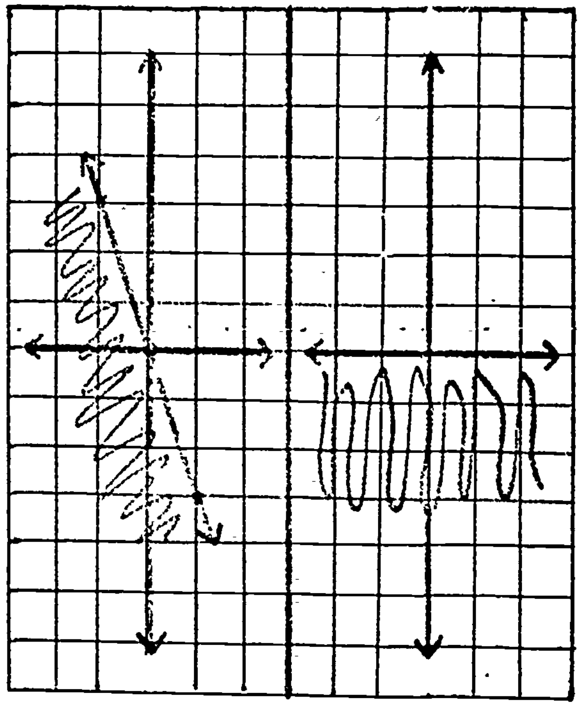
Function & Graphs

46. 47. 48.



Function & Graphs (continued)

49. 50.



Level: 25 Answers - Post Test I

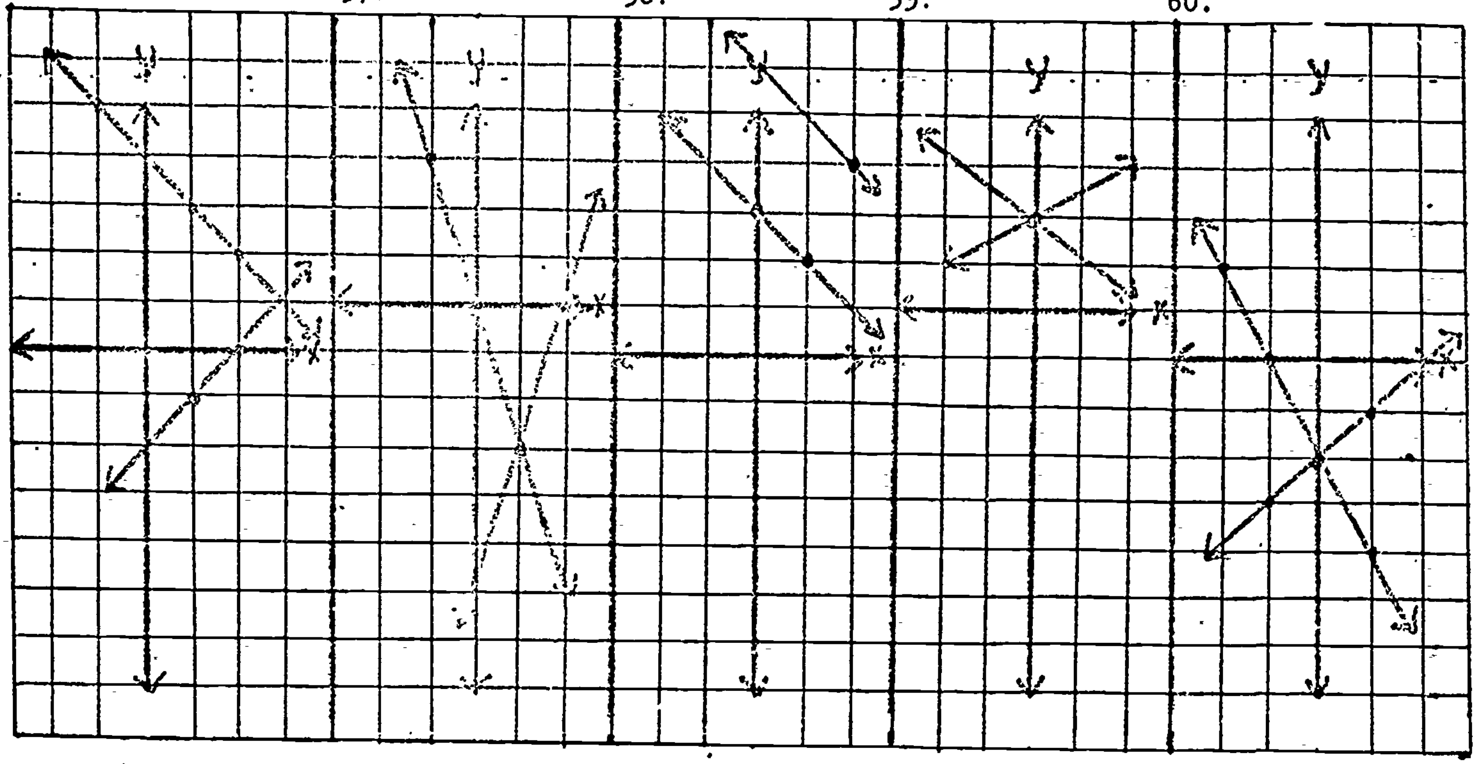
Step D

Numeral

51. 8.2
52. 73 or 73.0
53. 59 or 59.0
54. 8.7
55. 7.9

Function & Graphs

56. 57. 58. 59. 60.



{(3,1)} {(1,-3)} ∅ {(0,2)} {(0,-2)}

Level: 25 Answers - Post Test II

Step A

Numeral

1. 6
2. -3
3. 8
4. 4
5. 24

Order

6 $8 < \sqrt{72} < 9$
7. $-3 < -\sqrt{8} < -2$
8. 8
9. $5 < \sqrt{34} < 6$
10. -2

Geometry

11. 4a
12. $\sqrt{41}$
13. $a\sqrt{2}$
14. 2
15. 9

Number Sentences

16. {(3,4), (5,3)}
17. {(4,3), (5,4)}
18. {(4,3)}
19. {(4,5), (5,4)}
20. {(4,5)}

Step B

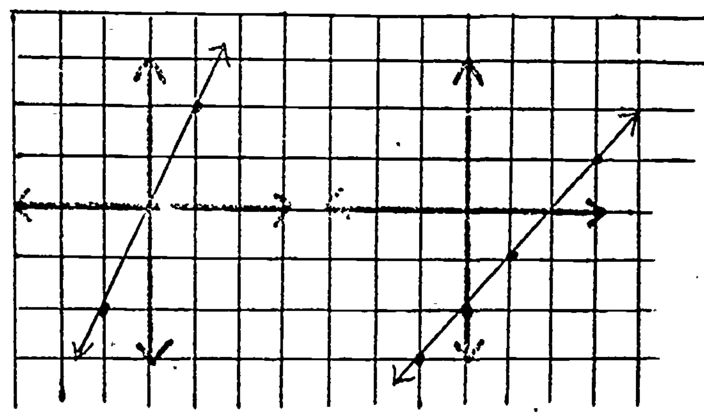
Numeral

21. 4.8
22. 7.2
23. 11.3
24. 4.1
25. 2.2

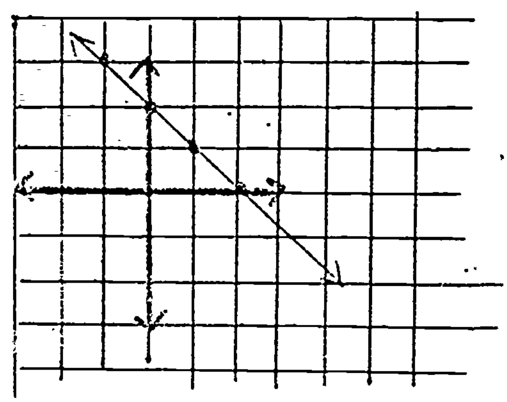
(The student's work should be shown on the back of the test page. Check to make sure the "divide and average" method was used.)

Function & Graphs

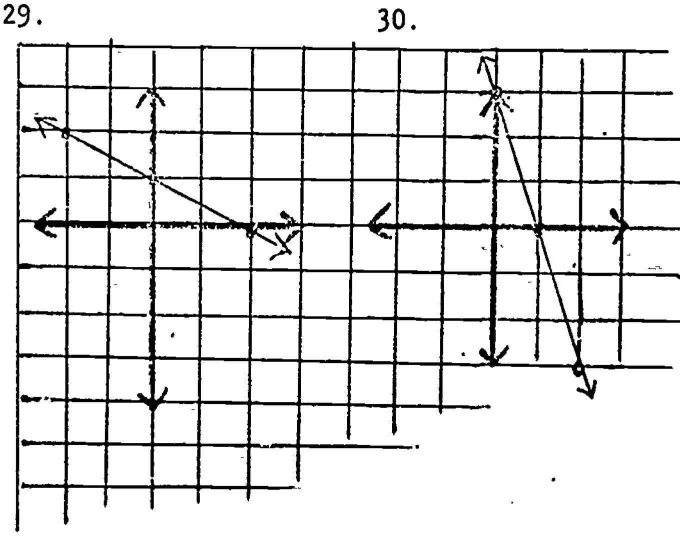
26. 27.



28.



29. 30.



Level: 25 Answers - Post Test II

Step B

Measurement

31. 1.514×10^{1}
32. 4×10^{3}
33. 7.57×10^{-1}
34. 5×10^{2}
35. 5.0×10^{-1}

Number Sentences

36. $\{(4,5), (5,4)\}$
37. $\{(4,5)\ (4,6), (5,6)\}$
38. $\{(4,6)\}$
39. $\emptyset$
40. $\{(4,5), (4,6), (5,6)\}$

Step C

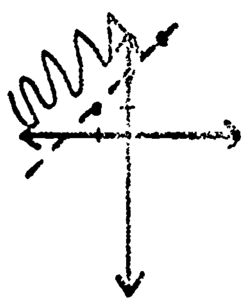
Numeral

41. 1.8
42. 6.2
43. 2.8
44. 1.0
45. 1.3

(The student's work should be shown on the back of the test page. Check to be sure the algorithm method was used).

Function & Graphs

46.



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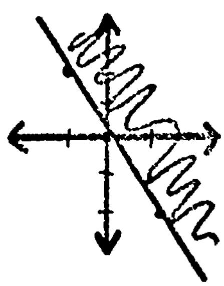

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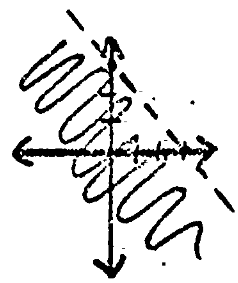

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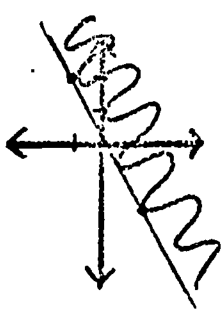
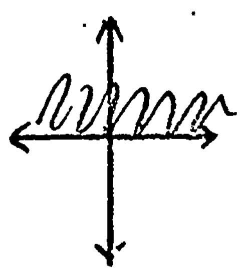


50.



Step D

Numeral

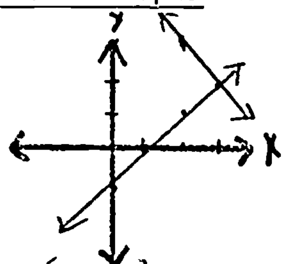
51. 8.1
52. 72 or 72.0
53. 69 or 69.0
54. 8.6
55. 7.9

Level: 25 Answers - Post Test II

Step D

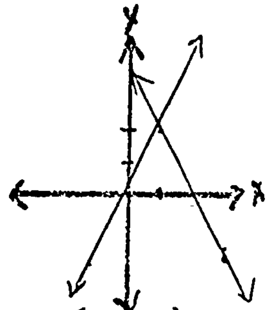
Function & Graphs

56.



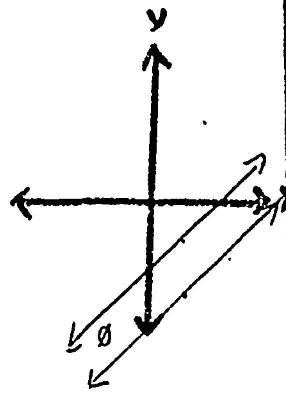
{(3,2)}

57.

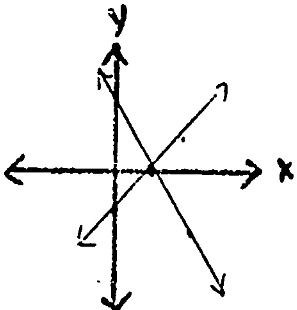


{(1,2)}

58.

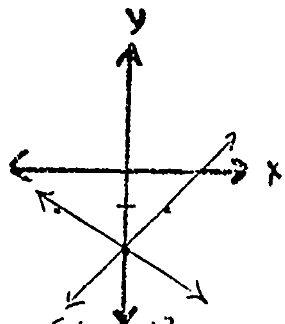


59.



{(1,0)}

60.



{(0,-2)}