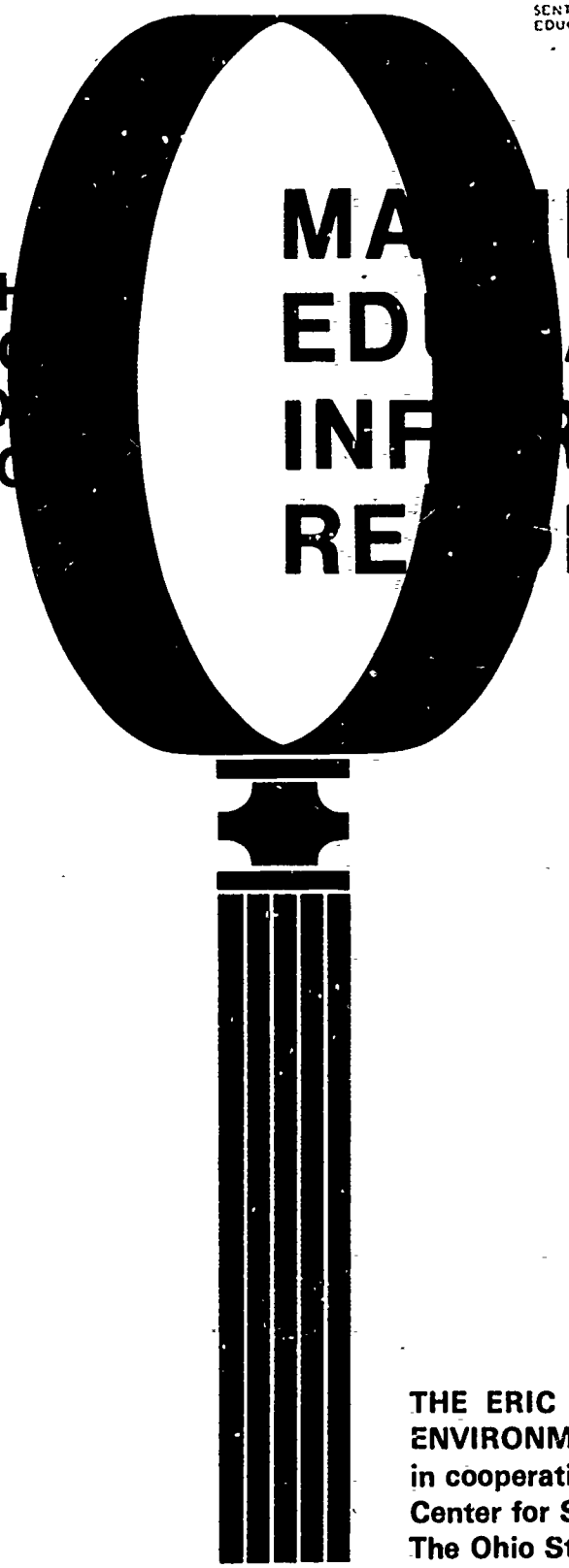
DOCUMENT RESUME

ED 104 720 SE 019 007

AUTHOR	Higgins, Jon L.; Sachs, Larry A.
TITLE	Mathematics Laboratories: 150 Activities and Games for Elementary Schools.
INSTITUTION	ERIC Information Analysis Center for Science, Mathematics, and Environmental Education, Columbus, Ohio.
SPONS AGENCY	National Inst. of Education (DHEW), Washington, D.C.
PUB DATE	Dec 74
NOTE	207p.
AVAILABLE FROM	Ohio State University, Center for Science and Mathematics Education, 244 Arps Hall, Columbus, OH 43210 ($3.50)
EDRS PRICE	MF-$0.76 HC-$10.78 PLUS POSTAGE
DESCRIPTORS	*Activity Learning; Basic Skills; Elementary Education; *Elementary School Mathematics; Games; Geometric Concepts; *Instruction; Instructional Materials; *Laboratories; *Manipulative Materials; Measurement; Number Concepts

ABSTRACT

This volume presents a collection of activities and games for use in elementary school mathematics laboratories. The activities and games included were submitted by classroom teachers and were selected for their use of manipulative materials or their reliance on student interactions. Several of the activities included have been described in "The Arithmetic Teacher," but many are first published in this volume. Activities described are in eight subject matter categories: (1) number concepts, (2) addition and subtraction, (3) multiplication and division, (4) number skills review, (5) measurement, (6) fractions, (7) graphs and functions, and (8) geometric concepts. The goals of activities range from increasing speed and power in computational situations to concept learning, development of strategies, and discovery. Each activity description is in outline form and includes statements of goals and purposes materials needed, procedures involved, and activity source. Many descriptions include diagrams and instructions for making any necessary materials. (SD)



MATHEMATICS EDUCATION INFORMATION REPORT

THE ERIC SCIENCE, MATHEMATICS AND ENVIRONMENTAL EDUCATION CLEARINGHOUSE
in cooperation with
Center for Science and Mathematics Education
The Ohio State University

MATHEMATICS EDUCATION REPORTS

Jon L. Higgins

and

Larry A. Sachs

MATHEMATICS LABORATORIES: 150 ACTIVITIES AND GAMES FOR ELEMENTARY SCHOOLS

ERIC Information Analysis Center for
Science, Mathematics, and Environmental Education
The Ohio State University
1800 Cannon Drive
400 Lincoln Tower
Columbus, Ohio 43210

December, 1974

Mathematics Education Reports

Mathematics Education Reports are being developed to disseminate information concerning mathematics education documents analyzed at the ERIC Information Analysis Center for Science, Mathematics, and Environmental Education. These reports fall into three broad categories. Research reviews summarize and analyze recent research in specific areas of mathematics education. Resource guides identify and analyze materials and references for use by mathematics teachers at all levels. Special bibliographies announce the availability of documents and review the literature in selected interest areas of mathematics education. Reports in each of these categories may also be targeted for specific sub-populations of the mathematics education community. Priorities for the development of future Mathematics Education Reports are established by the advisory board of the Center, in cooperation with the National Council of Teachers of Mathematics, the Special Interest Group for Research in Mathematics Education, the Conference Board of the Mathematical Sciences, and other professional groups in mathematics education. Individual comments on past Reports and suggestions for future Reports are always welcomed by the editor.

This publication was prepared pursuant to a contract with the National Institute of Education, U.S. Department of Health, Education and Welfare. Contractors undertaking such projects under Government sponsorship are encouraged to express freely their judgment in professional and technical matters. Points of view or opinions do not, therefore, necessarily represent official National Institute of Education position or policy.

This publication is the second in a series of mathematics laboratory publications produced by the ERIC Clearinghouse for Science, Mathematics, and Environmental Education. Some of the best teaching materials are those produced by teachers, and accordingly, when we began to plan a book of exemplary mathematics laboratory games and activities it was decided to solicit activities from teachers and school systems. The result of that solicitation forms the majority of this book. We recognized, however, that many teachers had also submitted descriptions of their activities to The Arithmetic Teacher, so back issues of that journal were also searched for suitable activities.

The key to our search and selection could be placed in one word: activity. We looked for experiences that either involved manipulation of materials, or student interactions (as in game situations). Accordingly, we discarded many excellent worksheets and individual study programs, since they did not fit within our activity restriction.

The result is what we hope to be a very practical collection of classroom activities in mathematics. Every teacher will recognize several old familiar activities. In addition, most teachers will find exciting new ideas that have never been published. Although we gave some consideration to balance, we make no claim as to thoroughness or completeness. Mathematics laboratories are continually growing entities that challenge both students and teachers. We hope that this collection of activities will both challenge and stimulate teachers to create additional activities. The possibilities for mathematics laboratories are endless, and the first-hand experiences they provide students are vital to meaningful mathematics learning.

We wish to thank Dr. Charles Thompson and Dr. Lee Erker for their assistance in condensing activity ideas from The Arithmetic Teacher. Illustrations from The Arithmetic Teacher are used with the permission of The National Council of Teachers of Mathematics.

Jon L. Higgins
Larry A. Sachs
Editors

TABLE OF CONTENTS

SECTION ONE:

NUMBER CONCEPTS

Section 1: Number Concepts

Number is an abstract idea. This does not mean that it must be taught abstractly, however. The foundations of number lie in processes of classifying, ordering, and grouping. Including activities which emphasize these processes helps build a foundation of experiences from which the idea of number can be extracted. The first activity in this section is a rather sophisticated classifying activity that is suitable for intermediate as well as primary students. Meaning and understanding often grow gradually. Classifying, ordering and grouping activities should not be confined to pre-school and kindergarten classrooms, but should be included in all the early grade levels.

Activities related to place value are also included in this section. Place value is often not recognized as a higher-order number concept. Place value is a grouping of numbers to facilitate record keeping. Yet number itself may be a grouping process. Procedures for the "grouping of groups" essentially require a child to make abstractions about abstractions. If forming abstractions about concrete objects requires a variety of experiences, place value must require many more experiences. The activities included in this section suggest some beginning experiences and will hopefully stimulate ideas for further activity.

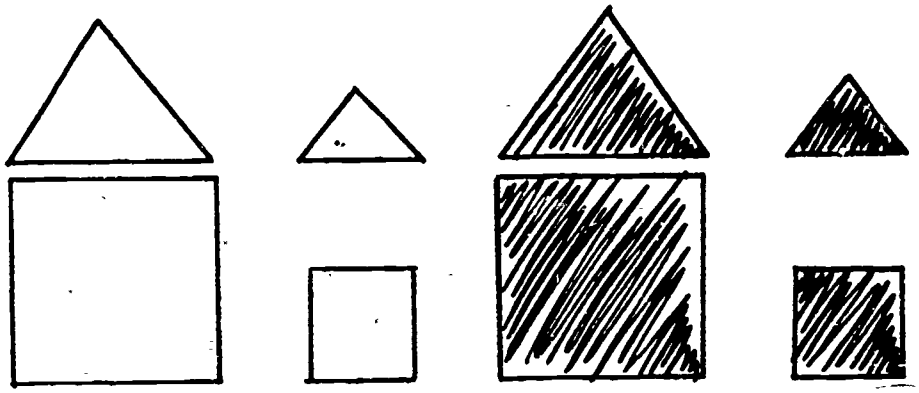
A GEOMETRIC MAGIC SQUARE

Goals and Purposes

Recognition of identifying characteristics.

Materials

Sixteen cardboard squares and a 4 x 4 chart on which to place the squares. The set of 16 squares should be made as follows:



The above should be drawn in red and a second set in blue for the total of 16. The shaded areas indicate these figures are colored solid. The triangles, of course, should be similar.

Procedure

The 16 pieces are to be arranged on the chart so that there are two of each characteristic (color, size, shape, and solid or not solid) in each row, column, and diagonal.

Source

Materials developed for U.S. Army Dependents Schools; submitted by Clyde Finnell, Mathematics Curriculum Coordinator for European Area.

ORDERING

Goals and Purposes

Ordering of geometric shapes by size.

Materials

Ten or more pieces of cardboard. Select a geometric shape; a triangle is good. Draw a similar triangle on each card but of different sizes.

Procedure

Shuffle the cards and place the top one in the chalk ledge. Have pupils draw successive cards and place them so that the cards are ordered according to size.

Source

Adapted from materials submitted by East Syracuse - Minoa Central Schools, East Syracuse, New York.

MUSICAL NUMBER CHAIRS

Goals and Purposes

To practice numeral recognition and naming.

Materials

One piece of tagboard (approximately 5" x 8") for each child. Write a numeral on each piece.

Procedure

Place chairs in a circle, and place a number card face down on each child's chair. The children march around the room and, when the teacher claps her hands, stop at the desk they are nearest. Each child picks up the card on that chair and sees what the number is. Each child must show and tell what number he has. The children who are correct sit down; the others must march again. The last child is "it" and takes the role of the teacher for the next game.

Source

Adapted from, "Math Ideas '+' for Grades 1,2,3," University City Public Schools, University City, Missouri.

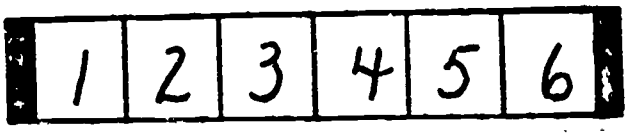
ALL COVERED

Goals and Purposes

Association of the numerals 1-6 with the corresponding quantities.

Materials

One die, 6 markers and a number strip (as shown below) for each player.



Procedure

Each player in turn tosses the die. He may then cover the numeral on his strip to match the set on the die. If this numeral has already been covered, he misses his turn. First player with all the numerals on his strip covered, wins the game!

Source

Material by Mary Wilkinson and Bonnie Tach of Waterloo County Board of Education; submitted through Ministry of Education in Ontario, Canada.

CONCENTRATION

Goals and Purposes

Matching the numerals 1 through 10 with the quantities for which they stand.

Materials

20 - 3 x 5 index cards. On ten of the cards write the numerals one through ten. On the other ten cards, make one card for each quantity one through ten. For example, the quantity card corresponding to three might look like this:

Use different symbols for each card.

Procedure

The 20 cards are shuffled and placed face down on the desk. The first player turns over two cards. If they match (i.e., the card with the 3 and the card with the three flowers shown above), he "books" these cards and takes another turn. If they do not match, he turns the cards back over and play passes to the next player. Game continues until all pairs have been made. The player with the most books wins.

Source

Adapted from material in "Guide and Resource Materials for Teaching," from Minneapolis Public Schools in Minneapolis, Minnesota.

A NUMBER GAME FOR KINDERGARTEN

Goals and Purposes

To provide learning experiences for counting and number recognition.

Materials

Some cards on which numbers have been written.

Procedure

Each child receives a number which he keeps secret. He then comes to the group he is playing with and taps his number of times on the door. He asks the group "Who am I?" The children reply "Come in three (or whatever his number is)."

Source

Beard, Virginia. "Mathematics in Kindergarten." The Arithmetic Teacher, 9: 22-25; January, 1962.

ZINGO

Goals and Purposes

Association of numerals with the corresponding quantities.

Materials

A set of Zingo cards, flash cards and markers.

Example: Zingo cards

2	6
0	1
4	8

3	2
5	0
4	6

7	2
1	8
3	4

6	5
2	1
4	8

Flashcards:  etc.

Procedure

1. Each player takes one large Zingo card.
2. A set of flashcards is placed face down on the table. (No answers on the back of these cards.)
3. The first player takes a flashcard. If the answer matches a numeral on his zingo card, he places the flashcard in that space. (A marker may be used to cover the numeral.)
4. Each player continues in turn.
5. The first player to fill his card calls out "Zingo."
6. The players can now change cards and play a new game!

Source

Material by Mary Wilkinson and Bonnie Tach of Waterloo County Board of Education; submitted through Ministry of Education in Ontario, Canada.

CROSS THE RIVER

Goals and Purposes

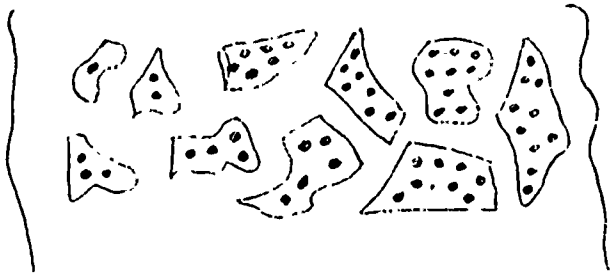
Recognition of quantities corresponding to the first ten natural numbers.

Materials

Ten irregularly cut pieces of cardboard -- approximate size of 8" by 10". Paint spots on the cardboard "rocks"; the number of spots corresponding to the first ten natural numbers.

Procedure

"River banks" are chalked on the floor and the rocks spread from bank to bank. Students then cross the river by stepping on the rocks in proper sequential order.



Source

Submitted by East Syracuse - Minoa Central Schools, East Syracuse, New York.

GET THE MESSAGE

Goals and Purposes

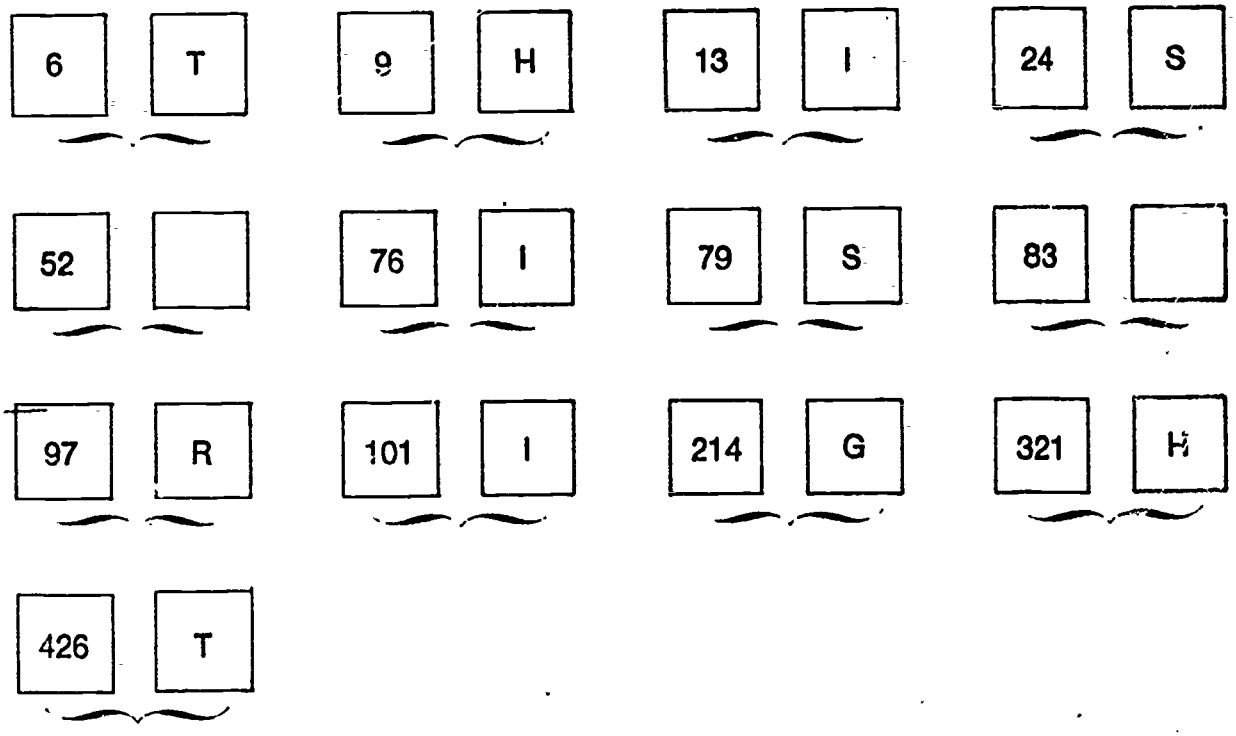
Practice in ordering a sequence of natural numbers.

Materials

Cardboard squares with numbers written on one side and letters on the other.

Procedure

This activity is best explained by an example:



One card has a 6 on one side and a T on the reverse side. The remaining cards are made the same way. The cards are shuffled and the student is to place the cards in increasing numerical order. Then turn the cards over and if the order arrangement is correct, the message will read:

GET THE MESSAGE (cont.)

Procedure (cont.)

Sets can be formed of any length and, of course, with any message desired.

Source

Adapted from Helpful Hints for Teachers of Basic Mathematics, Arkansas Department of Education, Little Rock, Arkansas.

THE LOWLY NUMBER LINE

Goals and Purposes

Introduction of many number concepts such as odd, even and negative integers to early elementary school children.

Materials

Masking tape, felt marking pens.

Procedure

Several children are asked to construct a large number line on the floor using masking tape. Other children can be asked to study the line and record the moves of the "one-jump cricket" and his friends (e.g., "three-jump cricket").

Discussions will soon occur on topics such as odd numbers, infinity, or whether a "two-jump backward cricket" can exist. Students will soon want to construct a new number line with numerals written in red on one side of zero and in green on the other side.

Source

Adapted from Rapp, Lois. "Arithmetic is Fun!" The Arithmetic Teacher, 10: 256-258; May, 1963.

LEARNING NUMBER CONCEPTS VIA A DECK OF CARDS

Goals and Purposes

To provide practice in ordering numbers, equal, greater-than and less-than relationships.

Materials

A deck of cards for each group of four students. Remove all the jacks, queens, kings and jokers from each deck. Mark the aces with the numeral "1."

Procedure

Divide the deck into its four distinct suits, give each of the four children a shuffled suit. They are to lay their cards in front of them in order of size (1 to 10) as quickly as possible.

For an advanced game, procede as before, but once the cards have been placed on the table in order of size, turn them face down. One child starts by pointing to a card of the child on his left; that child must then predict what numeral is under the card by secretly counting.

A game of concentration can also be played with these cards. A deck (all suits) is shuffled and laid out face down in a 5 x 8 array. One child starts the game by turning over any two cards. If they are equal, he may keep them and continue with another turn. If they are not equal the cards are turned face down again and play passes to the next child. Play continues until all cards have been removed; the child holding the greatest number of cards wins.

These cards may also be used for the game "War." For two players, each child gets half of a shuffled deck. Each player turns over one card from his deck at the same time. The player whose card has the greatest numerical value wins both cards which he places on the bottom of his deck. To emphasize "less-than" relationships the rules for winning could be reversed.

Source

Adapted from Gurau, Peter K. "A Deck of Cards, a Bunch of Kids, and Thou." The Arithmetic Teacher, 16: 115-117; February, 1969.

THE COUNTING BOARD AND THE UNIT BOX

Goals and Purposes

To provide experiences with concrete materials for learning basic concepts of the numbers 1 through 10.

Materials

A counting board (fig. 1) and a unit box (fig. 2) made of cardboard; Cuisenaire rods.

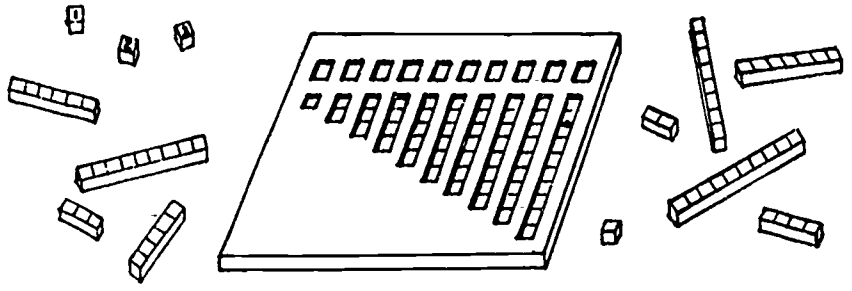


FIG. 1. The Counting Board.

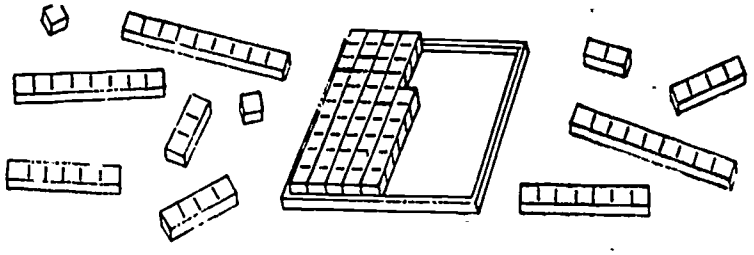


FIG. 2. The Unit Box.

Procedure

With the counting board the child fits the blocks into the corresponding grooves. This leads to counting the number of units in each block. He also learns the ordering of the numbers 1 through 10. At a later date the numbers 1 through 10 may be filled in across the top of the board so that the child can associate the number of units in each block with the correct numeral.

THE COUNTING BOARD AND THE UNIT BOX (continued)

With the unit box, 10 x 10 units, children pick any block and find one that will finish making 10 by fitting them into the box. Children learn the addition factors of 10 with this device. Boxes may also be made to teach the factors of 7 or 5, for example.

Source

Stern, Catherine. "The Concrete Devices of Structural Arithmetic." The Arithmetic Teacher, 5: 119-130; April, 1958.

"NON-RECTANGULAR" NUMBERS

Goals and Purposes

Determination as to whether a natural number is prime or composite.

Materials

Any suitable set of objects to be used as markers.

Procedure

To convince themselves that a number is composite the students should be able to arrange the corresponding number of markers in a rectangular array.

Example for 12:

On the other hand, 13 is prime because it will not yield any rectangular arrangement.

Source

Adapted from material submitted by Boyd Henry, College of Idaho, Caldwell, Idaho.

MANCOLA - AN EGYPTIAN GAME

Goals and Purposes

Practice in counting, determination of odd and even numbers, development of critical thinking.

Materials

12 cups or similar-sized containers and 72 marbles.

Procedure

a) Idea of the game.

At your turn, take all the balls from any of the 12 circular cups and place one each, into the next trays, moving clock- or counter-clockwise. Choose the cup which you empty carefully so that the last cup into which you drop a ball will then contain 2, 4, or 6 balls. These are yours. If this last cup contains an uneven number or more than six, you win nothing. The player who possesses more balls than any other at the end of the game is the winner.

b) Playing the game.

Place 6 balls into each of the 12 cups. It is usually wise to supply extra cups for the players to save their winnings.

At your turn you empty any of the 12 cups and drop one ball in succession into each of the next trays. You may move clockwise or counter-clockwise as you choose, but you must not change direction during one turn. When only 12 or fewer balls are left, moves must be made clockwise only.

After a few turns, there are fewer than 6 or no balls in some cups and more than 6 in others. You try to choose a cup to empty which will enable you to drop the last ball into a cup which will then contain 2, 4, or 6 balls. These balls you remove as your winnings. You may then also take the balls in the two preceding cups into which you have placed a ball during this move provided they contain 2, 4, or 6.

You must make a move at your turn. Passing is not permitted.

When all the balls have been picked up the game comes to an end. The winner is the player who possesses the largest number. He will start the next game.

When only 4 or fewer balls are left, it sometimes happens that all players move in such a way that no more balls can be won. This is a stalemate and play stops and the winner is judged as above.

Source

Adapted from material submitted by the Ministry of Education, Ontario, Canada.

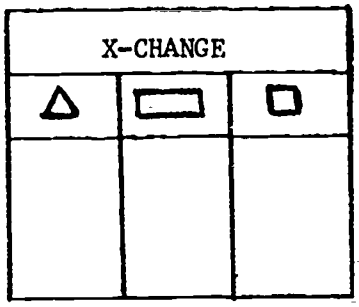
X-CHANGE

Goals and Purposes

Development of the place-value concept.

Materials

Make two playing boards from 9" x 12" tag:



Cut from construction paper: 30 green squares, 30 blue rectangles, 10 black triangles. Also needed for game ... 2 dice.

Procedure

For two players. First player rolls dice and puts as many green squares as the count indicated into the proper column on his board. Second player does the same on his turn. Whenever there are 10 or more squares in the squares column, X-CHANGE 10 squares for 1 blue rectangle. Put the rectangle in the proper column on the board. Alternate turns. Whenever there are 10 or more rectangles, X-CHANGE 10 rectangles for 1 black triangle.

Winner is the first player to achieve 5 triangles on his board.

Possible variations for the game would be:

1. Start with 5 triangles on the board, roll dice and remove that number of squares each time. Obviously a lot of exchanging will have to take place. First player to clear his board wins.

2. Play original rules but change the grouping base; i.e., base 3, base 5, base 8, etc.

Source

Materials from "Guide and Resource Materials for Teachers" from Minneapolis Public Schools in Minneapolis, Minnesota.

A PLACE-VALUE GAME

Goals and Purposes

Provides review situations for place value concepts.

Materials

Some pegs (as in fig. 1) and some sheets of cardboard cut into donut-like shapes. This game is based on the familiar ring-toss game.

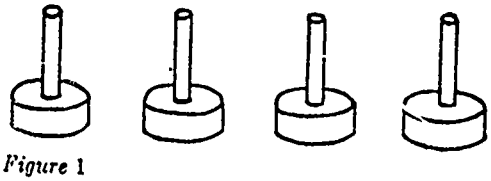


Figure 1

Procedure

The scoring in this game is different. The peg on the right counts 1, the next 10, the next 100, and the next 1000. A change in the game is that instead of tossing the rings and adding up the score, the score is given ahead of time and the task is to put the rings on in the right places. To win, the fewest number of rings possible for a given score must be used. The game may be played by several children with one child naming the score and the others placing the rings on their own pegs. The winner may be based on accuracy and speed.

This game provides a good situation in which to review place-value concepts. It can also be easily adapted to bases other than 10.

Source

Adapted from Shurlow, Harold J. "The Game of Five." The Arithmetic Teacher, 10: 290-291; May, 1963.

TEACHING PLACE VALUE VIA THE ABACUS AND OTHER MANIPULATIVE AIDS

Goals and Purposes

To provide concrete experiences for learning place value.

Materials

Spike or loop-type abacus, bundles of sticks, tin cans, and patch pockets which may be sewn or stapled to cardboard or cloth.

Procedure

Bundles of sticks, perhaps, would be the first device to use in learning about place value. Toothpicks, straws, or popsicle sticks can be bundled together with rubber bands in groups of tens, hundreds, and thousands.

After sufficient practice with bundles, children can move on to more abstract representations of numbers. A first step might be three tin cans labeled hundreds, tens, and ones with different colored sticks in each one. Patch pockets stapled to cardboard would embody the same idea. Two sets of pockets, one stapled directly above the other, work effectively to teach the addition and subtraction algorithms. Two types of abaci are also used for place value experiences. The loop-type abacus (fig. 1) has the advantage that tens may be exchanged for ones, for example, easily and conveniently.

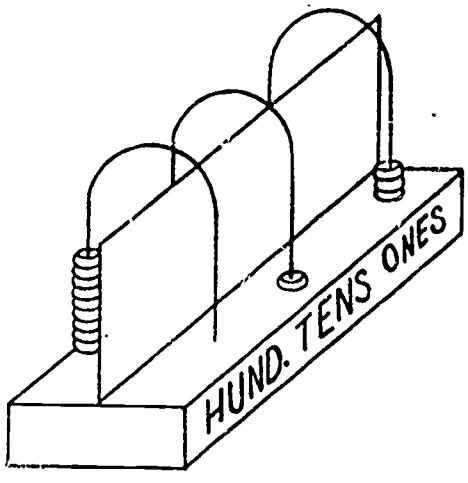


Figure 1

A disadvantage is that there will need to be more than 10 rings in any one category - something which differs from our place-value notation. The spike abacus (fig. 2) offers the same advantages of borrowing and yet provides a closer parellel to the way we think about place value.

TEACHING PLACE VALUE VIA THE ABACUS ... (continued)

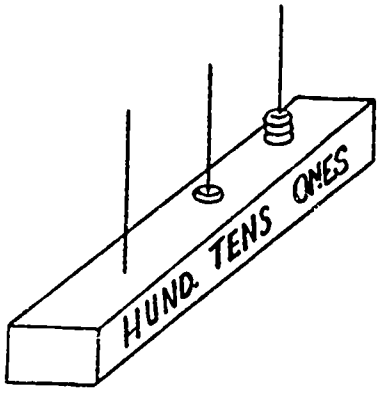


Figure 2

Ideally, each spike should hold only 10 markers.

Source

Adapted from Hamilton, E. W. "Manipulative Devices." The Arithmetic Teacher, 13: 461-467; October, 1966.

Hausdoerffer, William H. "Introducing Our Numbering System in the Primary Grades." The Arithmetic Teacher, 14: 61-63; March, 1967.

Mayer, Louise A. "The Scarbacus or Scarsdale Abacus." The Arithmetic Teacher, Vol. 2, #5, p. 159.

EGG CARTON HOLDER

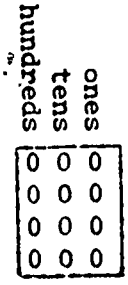
Goals and Purposes

Grouping to develop concept of place value.

Materials

Toothpicks and rubber bands, egg cartons.

Obtain egg cartons in the 3 by 4 shape. Remove cover. Label carton for either 3 or 4 place numerals.



Procedure

Use toothpicks for counters. Bundle tens and hundreds with rubber bands. Picks will fit into cups of carton. This device can be used for addition and subtraction problems requiring regrouping.

A variation of the above can be done using a muffin tin. Label places, but use beans as your counters. With this device the exchange from ones to tens would involve substituting a single bean, abacus fashion, rather than doing bundling.

Source

Materials from "Guide and Resource Materials for Teachers" from Minneapolis Public Schools in Minneapolis, Minnesota.

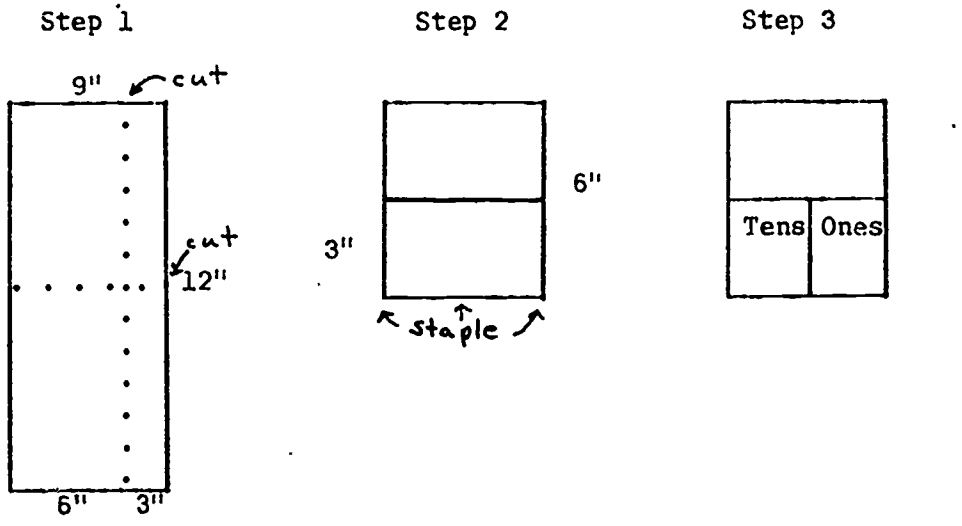
POCKET CHART

Goals and Purposes

A manipulative aid for place value.

Materials

Construct from tag. Cut 1 piece 9" x 12" as shown to create two square boards, 6" x 6". Use the cut off 3" strips across bottom of each square, staple. Cut strips of construction paper or tag to use as counters.



Procedure

Count the accumulation to ten and regroup. Bundle ten counters with a rubber band or paper clip. Use bundles of 10 strips in the tens pocket.

Take bundles apart to rename numbers. Use to illustrate regrouping in addition and subtraction.

To increase flexibility of use, mount a piece of clear contact paper on the lower half of the pocket chart. Numerals may be written with crayon or grease pencil on this material and erased with tissue for reuse.

When the bundling of ten ones for one ten is clearly understood, it is possible to move to the next level in development of place value representation; i.e., a single counter in tens place instead of the bundle of 10. Color coding and/or counters may be desirable and helpful for the above transition.

Source

Materials from "Guide and Resource Materials for Teachers" from Minneapolis Public Schools in Minneapolis, Minnesota.

AN AID FOR PLACE VALUE

Goals and Purposes

To provide practice with place value.

Materials

A number of sheets of paper that have been divided into columns representing ones, tens, hundreds, and so forth.

Procedure

This project may be carried out at various times during a school year. It has been used successfully at third grade but seems easily adaptable to other levels. Each child has a dittoed sheet. Initially the teacher asks the students to record a 1 in the units place. Then she may ask another student to name a number. If the student names 5, for example, the class records five more in the units column for a total of 6. This procedure continues and the students work with their record sheets much like they would with an abacus, continuing to add new numbers onto the cumulative total. When ten marks are recorded in any one column, they are "x-ed" out and one mark is recorded in the column to the immediate left. Later on larger numbers, such as 100, may be added. The result is that students become more aware of place value, the size of numbers, and addition. By the end of the year the total may be in the ten thousands. This activity may be performed with other counting aids also.

Source

Adapted from Rinker, Ethel. "Eight-ring Circus: A Variation in the Teaching of Counting and Place-Value." The Arithmetic Teacher, 19: 209-216; March 1972.

TEE OFF

Goals and Purposes

Stresses logical thinking.

Materials

A piece of pegboard with 18 holes and 18 golf tees.

Procedure

Three players are needed for the game. Put a tee into each hole and determine who is to move first. In turn, each player removes one, two, or three tees from the board. Player who takes out the last tee wins.

Source

Adapted from material submitted by Jane Riley, Community Consolidated School District 15, Palatine, Illinois.

BUZZ

Goals and Purposes

Recognition, in a speed situation, of certain number concepts.

Materials

None

Procedure

This game may be played in or out of doors, and any number of children may play. The children sit in a circle. One player starts by saying, "One." The child sitting next to him follows with, "Two," and so it goes until the number seven is reached. In this game any number containing seven or any multiple of seven must not be given. Instead, the word "buzz" is substituted for all numbers containing seven. Seven should be "buzz," likewise 14, 28, 35, 42, etc. But seventeen should be "one-buzz;" 27 should be "two-buzz;" 37 should be "three-buzz" and so on. And 71 is "buzz-one;" 72 is "buzz-two" and 77 is "buzz-buzz." When a player makes a mistake, he drops out of the game. In the intermediate grades the game may be made progressively more difficult by having the children count by twos, threes, fours, fractions, etc. The one remaining the longest in the game wins.

Source

Adapted from "Math Ideas '+' for Grades 1, 2, 3," University City Public Schools, University City, Missouri.

SET MEMBERSHIP

Goals and Purposes

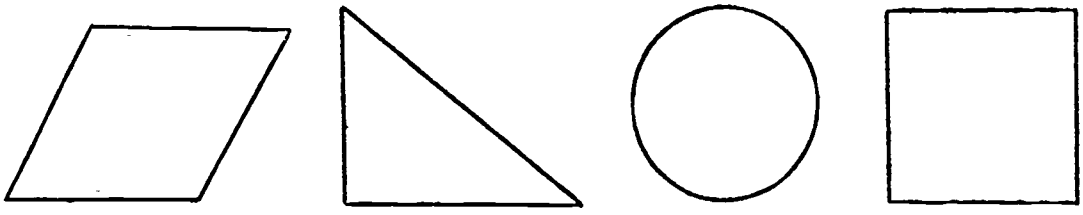
Determining to which set(s) various elements belong.

Materials

Four different colors of construction paper, scissors.

Procedure

Cut out of each of the four colors of paper a small and a large card of each of these shapes.



Make the small pieces about 1 inch square, the large pieces about 2 inches square.

Draw three large overlapping circles on a plain piece of paper. Label the three circles "blue," "polygons," and "large."

Place all the cards where they belong.

Did all the cards fit into a circle?

Were there any cards left over? Why?

Source

Adapted from "Math Lab, Middle School" submitted by Pittsburgh Public Schools, Pittsburgh, Pennsylvania.

SECTION TWO:

ADDITION AND SUBTRACTION

Section 2: Addition and Subtraction

Addition and subtraction need to be seen in a number of different contexts and in a wide variety of applications. The activities and games contained in this section suggest different contexts and varieties of experience.

The interaction of place value with addition and subtraction (carrying and borrowing) often presents difficulties. Two models for illustrating these concepts are included in this section. An additional place-value model is provided by number blocks or multibase arithmetic blocks. These commercially available sets consist of four basic pieces: a unit cube; a "long" stick which matches 10 cubes placed end to end; a "flat" which matches 10 longs placed side by side; and a "block" which matches 10 flats stacked on top of each other. Each piece represents a place value (1, 10, 100 and 1000) and the equivalence is built into the structure of the blocks. Thus they may be traded (carried or borrowed) in a natural manner without arbitrary rules of equivalence. Although cumbersome, a sample set of such place-value blocks may be constructed by gluing sugar cubes together with generous amounts of white glue.

A NUMBER LINE GAME

Goals and Purposes

Provides initial experiences for addition or subtraction using the number line.

Materials

A number line can be made on the floor of the classroom by using masking tape and a felt marker. A number line drawn on a sheet of tablet paper will also work. Poster board for making 3" x 5" game cards is also needed.

Procedure

On each card draw a group of circles. Use as many as you like. Below are two examples:

Fig. 1

You can choose the number of circles to draw on the cards according to the student's level of progress. After the deck of cards is made, it is shuffled and placed upside down. Students play the game in pairs. They take turns picking cards and moving a marker, from zero, the same number of spaces as there are circles on the cards that are chosen by each pupil. The winner is the first person to reach a pre-set point on the number line.

For subtraction practice, the game can be played by beginning at a pre-set number and moving to the left. The winner would be the first to reach zero, or some other pre-set number. Other games can be created by the students themselves.

Source

Adapted from, Deans, Edwina. "Remainders in Division and a Floor Number Line." The Arithmetic Teacher 8: 131-134; March 1961.

ADDITION AND SUBTRACTION SLIDE RULE

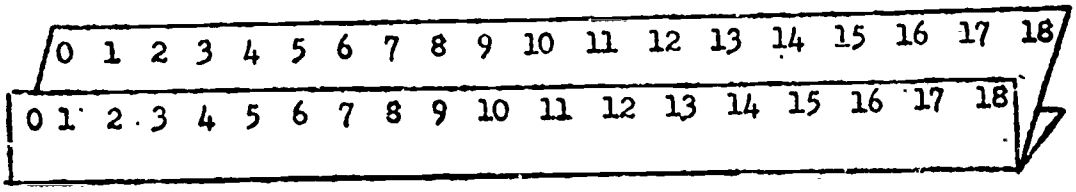
Goals and Purposes

Manipulative device for addition and subtraction facts up to 18.

Materials

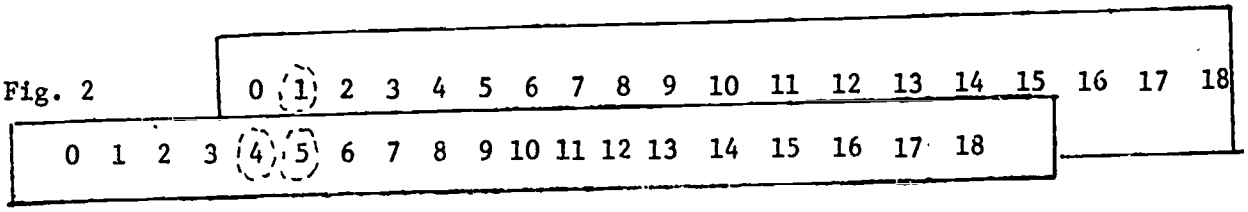
Use 4½" by 20" tagboard. Write numeral 0 through 18 at intervals of one inch, one-eighth inch down from the top. Fold in half lengthwise to make the body. Use another piece of tagboard 3½" by 20" and write the numerals 0 through 18, two and one-half inches from the bottom. This is called the slide.

Fig. 1



Procedure

Fig. 2



Slide rule shows 4 + 1 = 5, 4 + 2 = 6 etc.

Place the slide in the body. Move slide to the right until the zero is above one addend. Look along the slide for the second addend. The sum is below the second addend.

To subtract, locate the minuend on the body and move the slide so that the subtrahend on the slide stands over the minuend. The zero on the slide then stands over the remainder (on the body). In Figure 2, 10 - 6 = 4, etc.

Source

Adapted from "Math Ideas '+' for Grades 1,2,3," University City Public Schools, University City, Missouri.

BEAN BAG TOSS

Goals and Purposes

Work on beginning addition skills.

Materials

Three bean bags.

Procedure

Chalk off a section on the floor and number similar to the example. Each player has three throws. Each player records and adds his throws to determine the winner.

1	2	1
3	4	3
1	2	1

Player 1	Player 2
1	2
4	-
3	4
8	6

If a bean bag lands outside of the numbered section, no score is recorded (as in the second throw for player 2). Size of the squares and distance for throwing will depend on the age of the players.

Source

Adapted from "Math Ideas '+' for Grades 1,2,3," University City Public Schools, University City, Missouri.

AN AID FOR GROUPING CONCEPTS

Goals and Purposes

To provide a mainpulative aid for experiences with addition, subtraction, and grouping concepts.

Materials

Fifty-five one inch long dowels with small holes drilled lengthwise and at least 110" of string cut in lengths of 2", 4", 6", ..., 20". One rod is strung on the 2" string, 2 rods on the 4" string, and so forth as illustrated in figure 1.

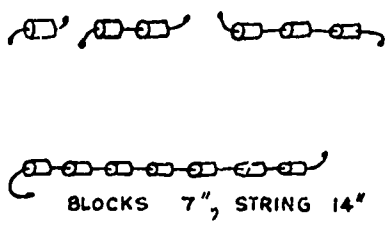


fig. 1

Procedure

These strings of rods may be used to discover the number of different combinations that may be formed from 10 rods, or 7 rods, etc. They may be used to help understanding of subtraction by removing a group of 4 from 7, for example. They may be used to build up the addition facts when the sums are less than 11. A big advantage of this aid is that rods do not become scattered.

Source

Adapted from Bridgers, Raymond B., Jr. "Easily Made Arithmetic Aids." The Arithmetic Teacher, 10: 507-508; December 1963.

AMAZING MACHINE

Goals and Purposes

Grouping with regard to place value in performing addition and subtraction.

Materials

Prepare the following materials cut from graph paper (½"):

10 squares of 100 graph cells (10 x 10)
18 strips of 10 graph cells (1 x 10)
20 single squares of 1 graph cell
3 signs for "ONES", "TENS", HUNDREDS" (on 9" x 12" tag)

Procedure

Mount one sign on each of 3 desks in the front of the room, with a pupil seated at each. They are the Machine Parts.

Select 3 or more pupils to be Banker, Programmer, Recorder. The rest of the class or group serve as Repairmen. The Banker holds all the graph paper pieces. The Programmer hands out pieces according to problem direction. The Recorder writes a problem on the board to be fed to the Machine. The 3 parts (pupils) of the Machine can accept only the pieces of graph paper corresponding to the name on the front of their desk.

A 3 digit numeral is written by the Recorder. The Programmer selects pieces from the Banker and distributes to the Machine Parts. The Machine Parts tell the Recorder what they have. This should check with the original numeral (now in expanded form).

For an addition problem each number is fed to the Machine separately; if no regrouping is needed, the Parts give back simple total to the Recorder. If regrouping is required (a ground rule is that no Machine Part may have more than 9 pieces of graph paper of its special kind), the Part exchanges his ten pieces for a single piece of the next size with the Banker and gives the single piece to the next Part who accepts this size piece.

Subtraction is done in a similar manner.

Source

Materials adapted from an idea in the MINNEMAST Unit 24; submitted through "Guide and Resource Materials for Teachers" from Minneapolis Public Schools in Minneapolis, Minnesota.

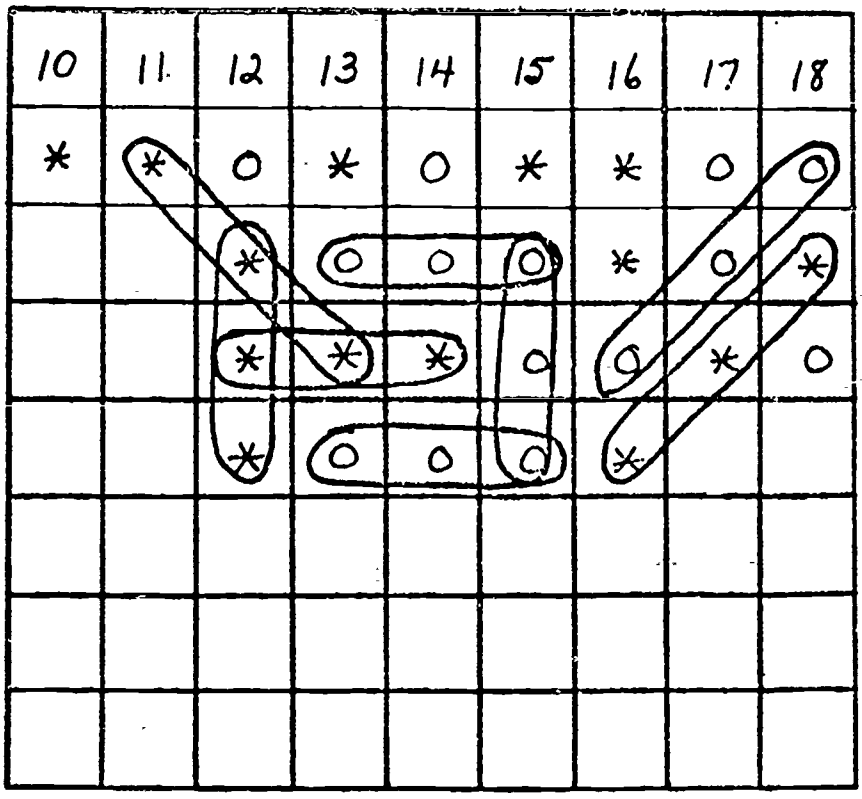
LOOP THREE

Goals and Purposes

Practice in using the digits 4-9 in addition facts.

Materials

Two cubes; one marked 4,5,6,7,8,9 and the other marked 6,7,7,8,8,9. A chart similar to one shown in example below:



Procedure

This is a game for two players. The first player rolls the dice and computes the addition fact. He places his mark (or initial) in the first empty square under the correct column. Play alternates. Any time three marks are in a row (horizontally, vertically or diagonally) they are circled. Play continues until it is impossible to form any more loops of three. Player with the most loops wins.

Source

Material by Mary Wilkinson and Bonnie Tack of Waterloo County Board of Education; submitted through Ministry of Education in Ontario, Canada.

NUMBER CHECKERS

Goals and Purposes

Practice in the addition facts of ten.

Materials

Two playing boards each lined into an array of 4 x 5 spaces. Two sets of 20 playing pieces each. The pieces should be about 1½" square for easy handling and each set should contain two pieces corresponding to each of the digits 1-9 except that there should be four pieces numbered five.

Procedure

The 20 pieces are shuffled and randomly placed on the 20 sections of the playing board. A move may be made upward, downward, sideways, or diagonally, and only one square at a time. The purpose is to place one numeral on another so that the numbers they represent total the agreed upon sum, e.g. 10. The cards with numerals representing numbers that total 10 are removed from the board. If a numeral cannot be placed on another to represent a total of 10, it may be moved to an adjoining blank space in preparation for a later move. The first person getting ten totals of 10 is the winner.

Example 1 shows several possible first moves that a player might make. A variation is, let the players position the pieces on their opponent's board for the start of the game. It should be noted that it is possible to position the pieces (as in example 2) such that no beginning move can be made.

5	8	6	2	6
3	7	5	9	3
9	4	8	7	5
1	2	5	1	4

Example 1

5	7	1	6	1
8	4	8	5	3
3	9	7	9	4
5	6	2	2	5

Example 2

Source

Adapted from "Math Ideas '+' for Grades 1,2,3," University City Public Schools, University City, Missouri.

SEVEN-UP

Goals and Purposes

Counting, recognition of numerals, and simple addition.

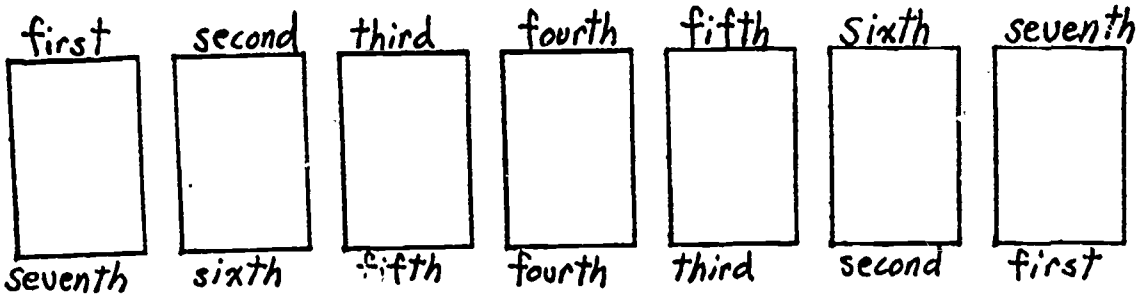
Materials

Seven cards numbered one through seven.

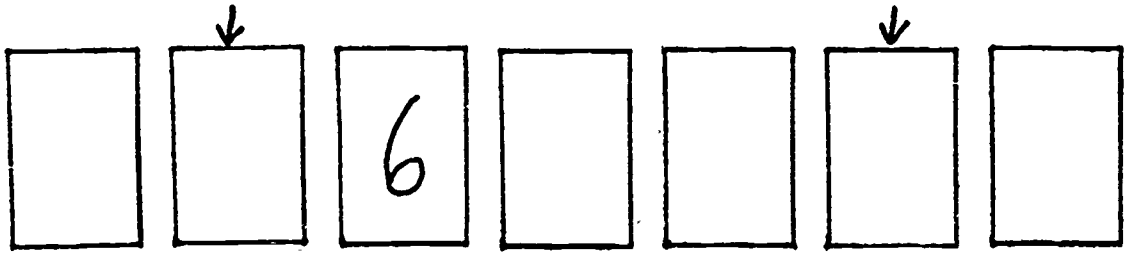
Procedure

This is a game for any number of players. Each player takes turns turning up cards. The game uses 7 cards.

The cards are schuffled and placed face down in front of the player. The cards are ordered in both directions.



First the player turns up any card. The number on this card tells him which card to turn up next. For example, if he turns up a "6", he then turns up the sixth card. Since two cards may be considered the sixth card, the player has two choices.



The game continues until either all cards are turned up or until no further moves are possible. The players score will be the sum of the number on the cards not turned up. For example, if cards 1, 2, and 7 are left, the player's score for that turn will be 10. The player who ends up with the lowest score will be the winner.

Source

Materials developed for U.S. Army Dependents Schools: submitted by Clyde Finnell, Mathematics Curriculum Coordinator for European Area.

ADDITION FACTS

Goals and Purposes

Provide realization that different addition combinations may give the same total.

Materials

Pair of dice and a score sheet ruled into a 6 by 6 grid with the co-ordinate axes numbered from one through six.

Procedure

Two players for the game; one selected to go first. He rolls the pair of dice, sums the two numbers, and marks his initial in any vacant square that would provide the same sum as that on his dice. For example, if the sum on the dice is 6, he could then initial any one of the following squares: (0,6), (6,0), (1,5), (5,1), (2,4), (4,2), or (3,3). Doubles give another turn and the person with the greatest number of initialed squares is the winner.

Source

Adapted from material submitted by the Halton County Board of Education, Burlington, Ontario, Canada.

ZYVE

Goals and Purposes

Recognizing specific two-number sums.

Materials

A deck of 56 cards, four cards for each of the numbers 1 through 14. (A commercial game "Rook" has such a deck of cards.)

Procedure

Three or four players works best. The cards are dealt face down to the players. The first player turns over his top card and puts it in front of his pack. The remaining players do similarly in turn. Whenever a player sees that the sum of any two exposed cards is 15, calls out "zyve" and takes the two cards. Game continues until all cards are played. Player having most cards wins. The sum or number of cards used could be changed to add variety to the game.

Source

Adapted from "Math Ideas '+' for Grades 1,2,3," University City Public Schools, University City, Missouri.

FIFTEEN

Goals and Purposes

Practice skills in basic addition facts and logical thinking.

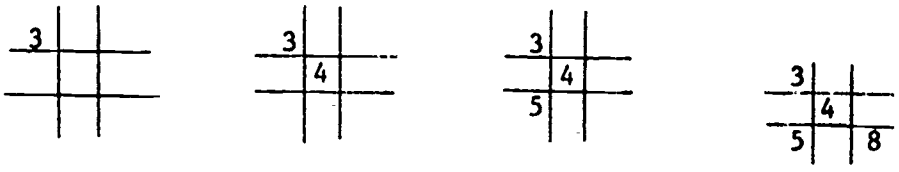
Materials

Pencil and paper.

Procedure

Playing board is like a tic-tac-toe diagram but the object is to get a row, column, or diagonal to sum to 15. Two players; first player uses only (1,3,5,7,9) and second player uses only (2,4,6,8). Each number may be used only once.

Example



Even wins

Source

Material submitted by Halton County Board of Education, Burlington, Ontario, Canada.

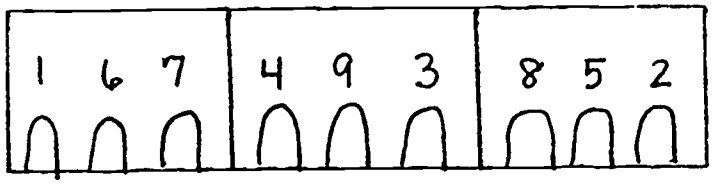
ADDITION BOWL

Goals and Purposes

Review of addition facts.

Materials

Three shoe boxes and five marbles. Cut three holes in one side of each shoe box and join together as shown below. The boxes may be painted or decorated with contact paper and the holes numbered.



Procedure

Each player takes a turn at rolling the five marbles. For each marble that goes through a hole, he receives the corresponding number of points. Each player totals his own score and the highest is the winner.

Source

Adapted from materials submitted by Florence Kostyc of No. Nine School, Perth Amboy, New Jersey.

ADD-FOUR-ROW

Goals and Purposes

To review addition facts.

Materials

Five dice, 15 markers of each of four colors, and a game board labeled as below:

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
1	8	9	16	17	24	25	2	3	10
2	7	10	15	18	23	26	1	4	9
3	6	11	14	19	22	27	30	5	8
4	5	12	13	20	21	28	29	6	7
30	29	28	27	26	25	24	23	22	21
20	19	18	17	16	15	14	13	12	11
10	9	8	7	6	5	4	3	2	1

Procedure

For each turn, the player may choose to roll one, two, three, four, or five dice. The sum of the numbers rolled indicates the number that may be covered on the board. The aim is to cover four adjacent numbers in a row, column or diagonal.

If a player rolls a total that has already been covered everywhere on the board, the player may roll one additional die and add this to his total to obtain a different number.

Source

Adapted from material of the Waterloo County Board of Education; submitted through the Ministry of Education in Ontario, Canada.

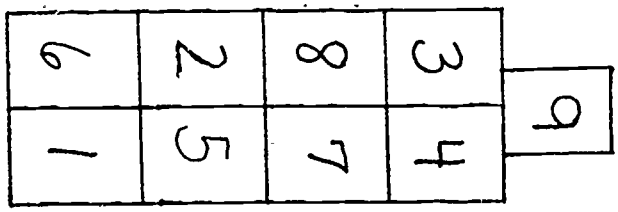
BEAN BAG TOSS

Goals and Purposes

Practice in addition skills.

Materials

Masking tape and a bean bag. With the masking tape, make a playing area on the floor similar to one shown below:



Procedure

Taking turns, each player throws the bean bag and records the result. Play continues for a determined time; when players total results, the highest correct score wins. As proficiency increases, variations may be incorporated. For example, every third score could be subtracted. Or additional blocks could be added to the layout, such as "lose a turn" or "double your score."

Source

Adapted from material submitted by Nancy Czech, J.J. Flynn School, Perth Amboy, New Jersey.

FIFTY

Goals and Purposes

Practice in addition skills.

Materials

None

Procedure

Two players; determine who is to start. First player calls out an integer from one to six. Second player chooses an integer from one to six, adds this to the previous total, and calls out the new sum. Play alternates in this manner with each player attempting to be the one to call out 50 and be declared the winner.

Source

Adapted from material submitted by Halton County Board of Education, Burlington, Ontario, Canada.

GRAND TOTAL

Goals and Purposes

Review of addition skills.

Materials

Container with 200 squares [illegible] numbered from 1 through 200.

Procedure

Each player in turn draws a slip from the container and adds that number to his total of previously drawn slips. At the end of the playing time, the person with the highest grand total is the winner. Several variations are possible: one would be to use fractions or decimals on the numbered squares; another would be to alternate between addition and subtraction following the drawing of squares (this would involve the use of signed numbers).

Source

Adapted from material submitted by Jane Riley, Community Consolidated School District 15, Palatine, Illinois.

PIG

Goals and Purposes

Practice in addition skills.

Materials

Two dice

Procedure

1. You need 2 or more friends to play.
2. Roll 2 dice - add the 2 numerals.
3. Player may choose to give the dice to next player or he may roll again as many times as he chooses, adding each score to the previous total. If he rolls a one he does not get the total for that turn and dice must be given to next player.
4. Record score if any.
5. Each person plays in turn trying to accumulate a total of 100 points.
6. If two 1's are rolled, the player loses all points accumulated in this and previous turns.
7. After one person reaches 100 or more, each has one more turn to try and beat him.

Source

Materials developed for U.S. Army Dependents Schools; submitted by Clyde Finnell, Mathematics Curriculum Coordinator for European Area.

ONE HUNDRED

Goals and Purposes

Addition skill and logical reasoning.

Materials

Pencil and paper.

Procedure

The class should be divided into groups of four students. There should be two students on a team, but one student can make a team if there is an odd number of students. The teams keep rotating with each other, playing one game with each team until all the teams have played each other exactly once. A team gets five points for a win and two points for a loss. The teacher or an extra student should keep score. Then the two highest scoring will play each other for the championship in front of the whole class. Using the numbers 1, 3, 5, 7, 9 each team will keep adding to the total until one team reaches 100 exactly. The team reaching 100 exactly is the winner. Example:

```
          total 82
"A" team selected 9
     new total 91
"B" team selected 9
   exact total 100
game over, "B" team wins
```

Students should think about the numbers used and try to cut-manipulate the other team. Teams should write the total down after each selection so there will be no arguments afterwards.

The team members are to work together to arrive at a selection.

Source

Adapted from "Math Lab, Middle School," submitted by Pittsburgh Public Schools, Pittsburgh, Pennsylvania.

BOW WOW, WHERE ARE MY BONES?

Goals and Purposes

Practice in subtraction from ten.

Materials

Cardboard, cut into the shape for ten bones.

Procedure

Children form a circle and sit on the floor if there is enough room space. One person is selected to go to the center of the circle. This person, given ten cardboard bones, will place them behind him. He then closes his eyes and a child is elected leader of the big circle. The leader points to a person in the circle who will quietly slip up behind the "dog" and take some of his bones. This person returns to his place in the circle and hides the bones behind him. Then the whole circle of children call out, "Bow Wow" and the doggie looks to see how many bones are gone. He then says, "Bow Wow, where are my _____ bones?" depending on how many are gone. If he subtracted correctly, he gets a number of chances, according to the number of bones missing, to guess who took the bones. If he did not subtract correctly, he has to start all over again. If he guesses who took the bones he then leaves the center of the circle and is replaced by the person who took the bones. The game then continues with the new "dog."

Other subtraction facts could be practiced by beginning with a different number of bones.

Source

Adapted from "Math Ideas '+' for Grades 1,2,3," University City Public Schools, University City, Missouri.

SUBTRACTION WAR

Goals and Purposes

To provide practice in subtraction combinations for numbers less than or equal to ten.

Materials

A deck of cards for each two children. Remove all the jacks, queens, kings, and jokers from each deck. Mark the aces with the numeral "1".

Procedure

This game proceeds according to the rules for "War." Each child receives half of a shuffled deck. Both players hold their cards face down and simultaneously turn up one card. The player with the card of higher value gets as many cards from the loser as are represented by the numerical difference between the two cards. Thus if a 7 and a 3 are turned up, the holder of the 7 card receives four cards from the loser. Chips could be substituted and the rules varied to shorten or lengthen the game.

Source

Adapted from Gurau, Peter K. "A Deck of Cards, A Bunch of Kids, and Thou." The Arithmetic Teacher, 16: 115-117; February 1969.

DIFFY

Goals and Purposes

Practice in subtraction skills.

Materials

A game sheet similar to one shown on following page.

Game sheets can be constructed with more or less sets of cells.

Procedure

The game is started by placing numbers in the four outermost corner circles. The difference between any two adjacent corners is recorded in the middle circle between them. The numbers in these positions now become corners of a new square. There differences are recorded in the middle circle between them. The game is continued until only zeros appear.

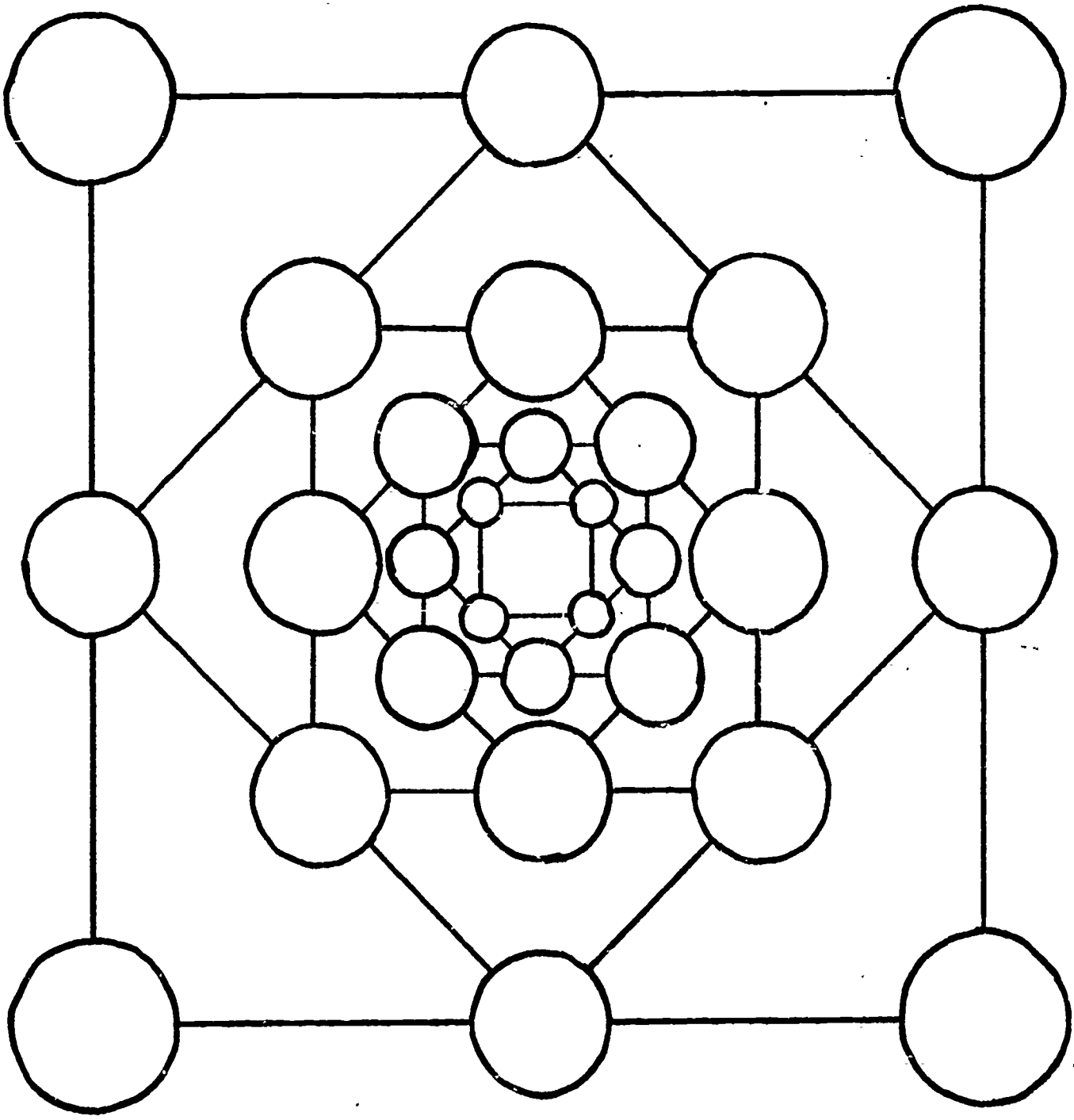
The only rule in DIFFY is that the smaller numbers be subtracted from larger ones.

A goal might be to reach the innermost set of cells without writing zero in each of them.

A variation would be to create a game of DIVVY. This time divide the numbers. Instead of ending with zero in the last cells, now you will end with ones. In playing DIVVY be sure not to enter any zeros (division by zero is not allowed in mathematics).

Source

Material devised by Herbert Wills (see The Arithmetic Teacher, October 1971, pp. 402-405); submitted through "Guide and Resource Materials for Teachers" by Minneapolis Public Schools in Minneapolis, Minnesota.



DIFFY

ADDITION AND SUBTRACTION SNAP

Goals and Purposes

To review and practice addition and subtraction facts.

Materials

Forty-four cards, 4 marked with each number from 0 to 10, inclusive.

Procedure

1. For two players: deal all cards between the two players. Each player turns up one card. The player calling out the sum first collects both cards. The player with the most cards when time is up wins.

2. For more than two players: same as #1, but each player turns up a card, and the winner must shout out the sum of all the cards showing.

3. Same as #2, but the sum of the two greatest numbers showing is called out.

4. Same as #2, but the sum of the largest and the smallest numbers showing is called out.

5. Same as #1, but the difference of the two numbers is called out.

6. Same as #4, but the difference of largest and smallest is called out.

Source:

Adapted from material of the Waterloo County Board of Education; submitted through the Ministry of Education, Ontario, Canada.

NINETY-NINE

Goals and Purposes

Practice in addition skills.

Materials

44 cards
4 of each number 0 to 10
(Mark 10's in a different color)

Procedure

1. Shuffle all of the cards.
2. Deal 3 cards face down to each player.
3. Place the remainder face down in the middle of the table.
4. The first player turns one of his cards face up and announces the number aloud.
5. This player then draws a replacement card.
6. The next player turns up one card, and announces the sum of the two cards turned up.
7. He draws one card.
8. Each player does the same as 6 & 7 above.
9. The player who has to play a card that takes the sum over 99 loses the game.

Note: The 10 cards may be used to add or subtract to the total. For example, if the total is 99 before my turn, if I play a 10 card, the total reduces to 89 and I don't lose the game on this turn.

Source

Adapted from material of the Waterloo County Board of Education; submitted through Ministry of Education, Ontario, Canada.

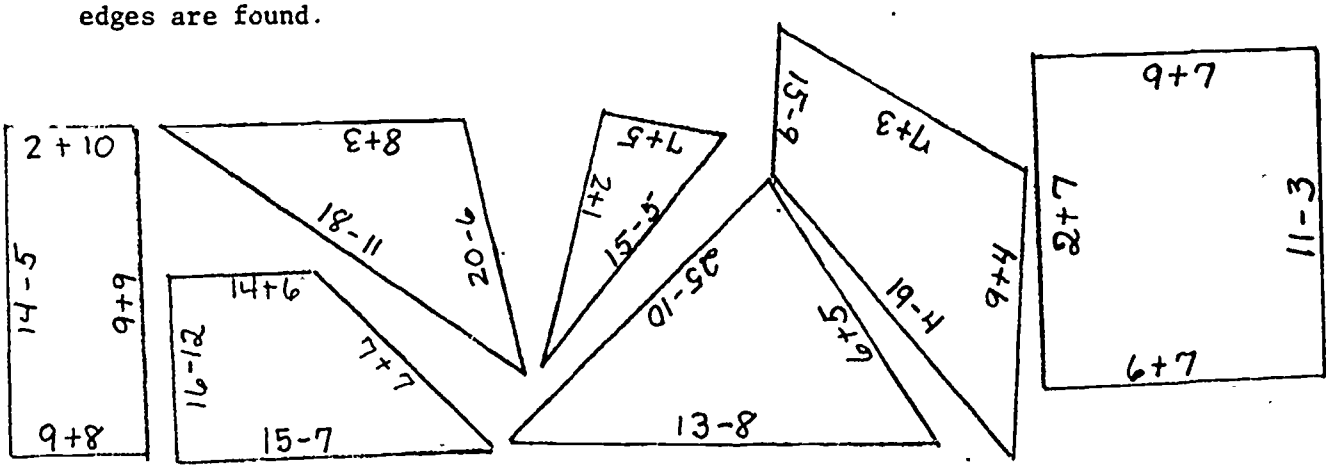
ADDITION AND SUBTRACTION FACTS

Goals and Purposes

Provide review and practice of basic addition and subtraction combinations.

Materials

Cut a geometric shape from tagborad, then cut the shape into puzzle pieces. Label each edge with an addition or subtraction fact, making sure that matching edges have facts that yield the same number as sums or differences. (Be sure the non-matching edges are labeled with facts giving different sums or differences.) The following puzzle pieces yield a square when matching edges are found.



Procedure

Find the value of each edge of each puzzle piece. (Values are not related to the lengths of the edges.) Put edges with the same value next to each other. Name the geometric shape you form.

Source

Adapted from "Math Lab, Middle School," submitted by Pittsburgh Public Schools, Pittsburgh, Pennsylvania.

PLUS AND MINUS

Goals and Purposes

Practice addition and subtraction facts.

Materials

Scissors, 3 x 5 cards cut in half.

Make a pack of cards. Make 40 cards of the same size. Write "one" on four cards then "two" on four cards and so on until you have four cards of every number from one through ten.

Procedure

Each player is dealt half of the pack. He places his cards face down in front of himself.

The players will turn up (at the same time) one of their cards. The player having the card with the smaller number should subtract that number from the number on his opponent's card. If he gives the correct answer, he scores a point. If he gives a wrong answer, his opponent scores a point. If both players turn cards having the same number, the first one who calls "zero", wins the point. The play continues until all cards have been turned.

The cards are shuffled for the next round of Plus.

Each player turns one of his cards and the one having the card with the smaller number starts the play. But now he should add the numbers on the cards. Two rounds make a game.

The player with the most points wins.

Source

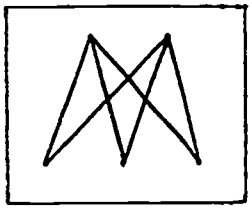
Material from "Math Lab, Middle School," submitted by Pittsburgh Public Schools, Pittsburgh, Pennsylvania.

SECTION THREE:

MULTIPLICATION AND DIVISION

Section 3: Multiplication and Division

Children need to see a variety of models for multiplication and division. The first activity in this section is an example of a good model. Another model may be easily constructed by driving two rows of nails into a small board. Rubber bands are then stretched from the first row to the second, connecting one nail in the first row to one nail in the second in as many different ways as possible. The total number of rubber bands used is the product of the number of nails in the first row and the number of nails in the second row. The product 2 x 3 is illustrated below:



2 x 3 = 6

Multiplication is usually applied to area. In fact, area can just as well serve as a model for multiplication. Paper rectangles and small square tiles can be used to illustrate each multiplication fact. A paper rectangle two tiles wide and three tiles long can be covered with exactly six tiles, illustrating 2 x 3 = 6. Another activity in this section extends this model in a useful way to illustrate the place-value algorithm for multiplying two-digit factors.

Multiplication can also be modeled as repeated addition, but in general it should be seen in many other ways as well.

A MULTIPLICATION AID

Goals and Purposes

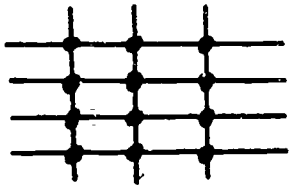
Determination of basic multiplication facts.

Materials

Strips of felt or pieces of yarn -- or anything similar that will allow manipulation of these representatives of line segments.

Procedure

To illustrate the product of 3 and 4 have student place 3 vertical and 4 horizontal lines so that they intersect. The total number of intersections is the product.



By making 4 vertical and 3 horizontal lines, the commutative property can be demonstrated.

Source

Submitted by Boyd Henry, College of Idaho, Caldwell, Idaho.

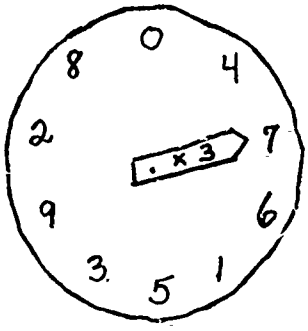
MULTIPLICATION WHEEL

Goals and Purposes

Review basic multiplication facts.

Materials

Cut a circle (approx 12" diameter or larger) from heavy cardboard. Write the numberals 0 through 9 around the edge. Cut a strip of tag board to a suitable length and write an indicated multiplier on it (a whole set could be made). Attach the strip to the center with a paper fastener. Answers may be written on the back side of the circle.



Example: 7 x 3 = 21

Procedure

Divide into teams and move the strip around the wheel asking alternating team members to give correct products. Score points to determine the winning team.

Source

Adapted from Helpful Hints for Teachers of Basic Mathematics, Arkansas Department of Education, Little Rock, Arkansas.

MULTIPLICATION GRAPHS

Goals and Purposes

Use of a graph to illustrate multiplication facts and concepts.

Materials

Graph paper with ¼" grid.

Procedure

1. Place a sheet of ¼" graph paper horizontally in front of you. Start in the lower left corner and number the squares up starting with 1. Do the same across.
2. Across the one row, place a dot in each square.
3. Across the two row, place a dot in every second square. Continue as shown in the example until you have used all of your graph paper.
4. On the following chart you can see how to multiply 3 x 2. Go up 3 and count 2 dots over - read 6 at bottom.
5. Do the following multiplication problems on the graph:

   5 x 7 = _____ 4 x 10 x 2 = _____

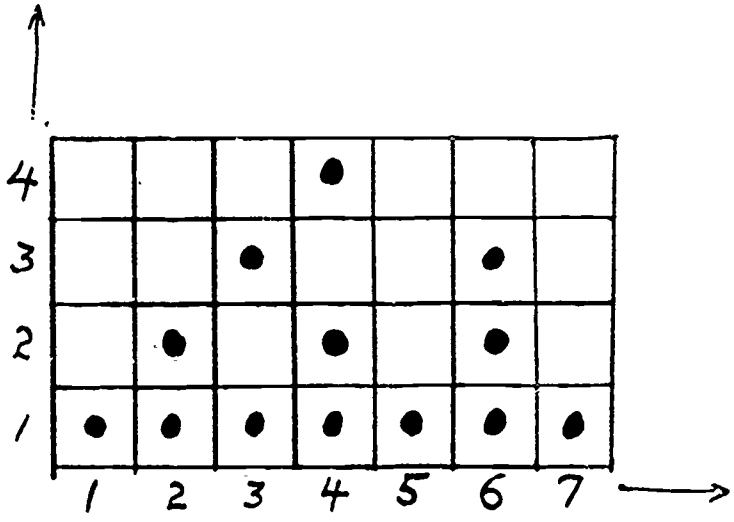
   3 x 2 = _____ 5 x 3 x 3 = _____

   4 x 10 = _____

6. Can you see why the least common multiple (LCM) of 5 and 4 is 20?
7. Make up some problems on your graph and try them on your classmates.
8. What kinds of numbers have only two dots above them?

MULTIPLICATION GRAPHS (continued)

Sample of how the bottom, left-hand corner of graph should look:



Source

Materials developed for U.S. Army Dependents Schools; submitted by Clyde Finnell, Mathematics Curriculum Coordinator for European Area.

MULTIPLICATION BOARD

Goals and Purposes

This board shows relationships between multiplication facts where the product is greater than 10.

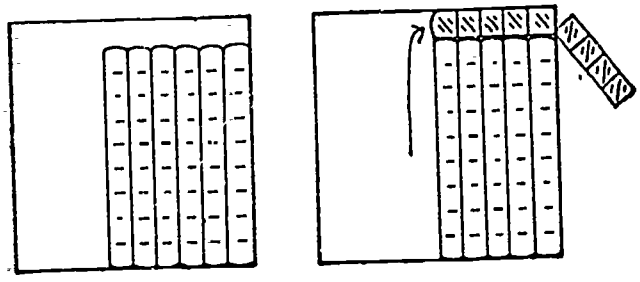
Materials

100 counting cubes. (These may be sugar cubes or wooden cubes cut from lumber. One-inch wooden cubes are available commercially from several school supply houses.)

1 frame whose dimensions are 10¼ by 10¼ cubes.

Procedure

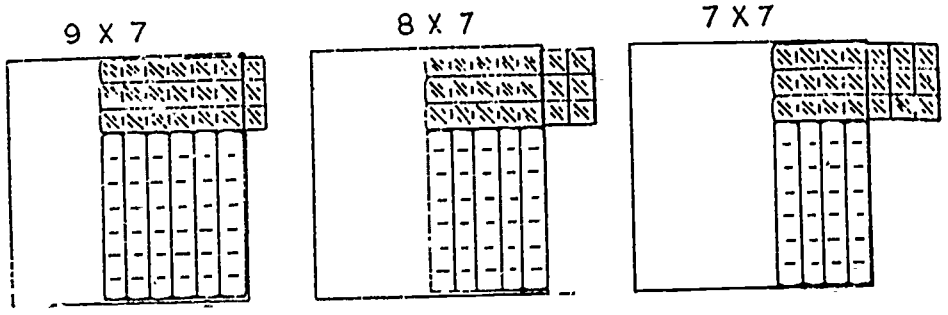
To illustrate 6 x 9, place six columns of 9 cubes each in the frame. Then use one column of the cubes to make tens-columns with units left over. This procedure is illustrated below:



THE TEN COMPARTMENT WITH 6 NINES

6 NINES = 5 TENS + 4 UNITS

Relationships between multiplication facts become apparent using this board. The sequence illustrated below shows units digits increasing by three's for the facts 9 x 7, 8 x 7, and 7 x 7.



NINE 7's = SIX 10's + (1X3)UNITS • EIGHT 7's = FIVE 10's + (2x3)UNITS • SEVEN 7's = FOUR 10's + (3x3)UNITS

MULTIPLICATION BOARD (cont.)

For Further Information

Stern, Catherine, "New Experiments with Multiplication," The Arithmetic Teacher, 7: 381-388; December, 1960.

NAIL BOARD

Goals and Purposes

Provides learning and review opportunities for multiplication and division facts when the factors are 1 through 10.

Materials

A square board with 100 nails arranged in a grid consisting of 10 rows and 10 columns; and 100 cardboard disks with holes punched so they can be hung on the nails. Nails should be spaced approximately 2 inches apart; cardboard disks can then be 1 and 3/4 inches in diameter.

Procedure

Write the digits 1 through 10 in order, beside each row, along the left-hand side of the grid arrangement. Do the same for the columns across the top of the grid. On the cardboard discs write the 100 products of all possible pairs of numbers from 1 through 10 and hang the discs on the appropriate nails on the board. Children can do this themselves as they are learning their multiplication facts. A good review exercise is for students to remove the discs, mix them up, and then try to replace them correctly on the nail board. This provides practice for multiplication, division, and factoring.

Source

Hertz, Pauline. "Manipulative Devices in Lower Grades." The Arithmetic Teacher, 4: 214-216; November, 1957.

MULTIPLICATION SNAP

Goals and Purposes

To review multiplication facts.

Materials

44 cards, 4 marked with each number from 0 to 10, inclusive.

Procedure

All cards are dealt equally to two players. Each player turns up one card. The player calling out the product first collects the two cards. The player with the most cards when the time for the game is used up, wins the game.

Source

Adapted from material of the Waterloo County Board of Education; submitted through the Ministry of Education, Ontario, Canada.

MULTIPLICATION SCRAMBLE

Goals and Purposes

Review of multiplication and addition facts.

Materials

One shoe box; 100 slips of heavy paper with a statement of one different basic multiplication problem on each.

Example:

7 x 9

Procedure

Each player chooses a slip, performs the multiplication, records his answer, and passes the box to the next player in turn. After five slips have been drawn by each player his answers are totaled and the player with the highest score wins. A variation would be to play to a certain sum, say 300. More addition would be involved this way as players would add each time as they got closer to 300. Similar games are possible for subtraction or division.

Source

Adapted from materials submitted by Florence Kostyc, No. Nine School, Perth Amboy, New Jersey.

MULTIPLICATION TOUCH-DOWN

Goals and Purposes

Drill and practice of selected multiplication combinations.

Materials

Playing board such as the one below.

A set of tickets with the answers for each space on the board.

X	0	1	2	3	4	5	6	7	8	9
1										
2										
3										
4										
5										

Procedure

Each player gets ten tickets. One ticket is placed somewhere on the board (in the correct answer space). Each player then has a turn in playing one of his tickets, but this ticket must in some way touch another ticket on the board.

Example:

X	X	X
X	10	X
X	X	X

Next player may play in one of the "X" spaces.

If he cannot play in one of these spaces, he must draw a ticket from the extra pile. First player with no tickets left is the winner. Cut the size of the game board down to suit the facts you want to drill.

Example:

X	1	2	3
0			
1			

HUNDRED'S BOARD

Goals and Purposes

Provides an interesting method for review of mathematical facts and concepts from any selected area.

Materials

A square board or heavy cardboard (approximately 20" by 20") should be ruled into 100 squares and numbered from 1 through 100. Playing cards slightly smaller than the ruled squares should be made and also numbered from 1 through 100.

Procedure

Deal the entire stack of cards as evenly as possible among the players-four players is probably best. The object is to be the first to get rid of all of your cards. Each player in turn defines a concept that will benefit him most; that is a concept that will allow him to play many of his cards but not allow his opponents to play very many. For example, if one player declares the concept "prime number", then all players having any cards corresponding to prime numbers remove these cards from their stacks and place them on the board. Play continues until one player has placed all of his cards on the board and is declared the winner.

Source

Adapted from material submitted by Jane Riley, Community Consolidated School District 15, Palatine, Illinois.

PRODUCTO

Goals and Purposes

Practice on basic multiplication skills.

Materials

Playing board ruled into 36 squares and numbered from 1 through 36. Cards numbered from 1 through 36 and 2 dice.

Procedure

All cards are dealt out to the players. First player throws the dice; multiplies the numbers that turn up. If he holds the card corresponding to this product, he plays it on the board. Play passes in turn until one player has played all of his cards and is thus the winner.

Source

Material submitted by Halton County Board of Education, Burlington, Ontario, Canada.

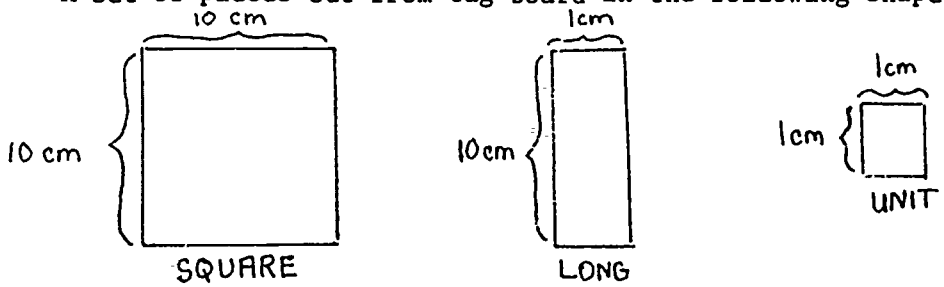
"RECTANGLES"

Goals and Purposes

A manipulative aid for determining the product of two 2-digit numbers.

Materials

A set of pieces cut from tag board in the following shapes:

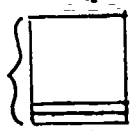


The number of each type needed will depend on the particular problems to be solved. Sixteen squares, forty longs and twenty-five units will handle problems with tens digits of four or less and units digits of five or less. Although the method can be generalized to larger problems, they do become cumbersome to handle.

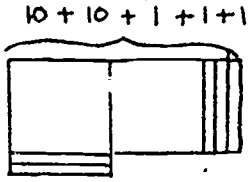
Procedure

The product of two 2-digit numbers is found by building a rectangle whose length is one of the two-digit numbers and whose width is the other number.

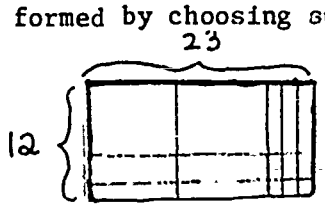
The length of one side of a "square" corresponds to 10 and the shorter side of a "long" to 1. Thus in the problem 12 x 23, 12 would be represented by

10 + 1 + 1 

Combining this with the representation for 23 gives



The product is formed by choosing suitable pieces to complete the rectangle.



"RECTANGLES" (cont.)

The rectangle is composed of 2 squares, 7 longs, and 6 units. Thus the product of 12 and 23 is 276.

Source

Adapted from material submitted by James K. Bidwell of Central Michigan University.

NAPIER'S RODS

Goals and Purposes

Development of multiplication skills through use of a manipulative device.

Materials

Tongue depressors or tag board. A complete set of rods consists of an index rod and a rod for each of the numbers 0 to 9. The index rod and rods for 1, 2, 4, and 9 are shown in figures 1 and 2 below.

Procedure

To multiply 49 by 4, use the index, the four, and the nine bones as shown in Figure 2. Read the numerals on the line to the right of 4 on the index bone . . . 196. The 6 and 3 are added since each indicates the number of 10's.

Fig. 1

Index	1	2
1	0/1	0/2
2	0/2	0/4
3	0/3	0/6
4	0/4	0/8
5	0/5	1/0
6	0/6	1/2
7	0/7	1/4
8	0/8	1/6
9	0/9	1/8

Fig. 2

Index	4	9
1	0/4	0/9
2	0/8	1/8
3	1/2	2/7
4	1/6	3/6
5	2/0	4/5
6	2/4	5/4
7	2/8	6/3
8	3/2	7/2
9	3/6	8/1

= 196

When the multiplier has more than two digits, then the partial products are found from the bones and written in the usual manner.

Source

Adapted from "Ideas Plus for Math, Grades 4,5,6," University City Public Schools, University City, Missouri.

MULTIPLICATION TIC-TAC-TOE

Goals and Purposes

Review and practice of basic multiplication facts.

Materials

Copies of the standard 10 x 10 multiplication table array with all entries blank except for the first row and the first column.

Procedure

Two players use the array to play a modified tic-tac-toe game. To place an "X" or an "O" in a particular square, the player must give the multiplication fact that belongs in the square. If he gives an incorrect fact, his opponent may place his mark in the square. Four X's or O's in a row, column or diagonal are required to win.

Different student abilities may be accounted for by placing certain rows and columns "off limits" for certain games. For example, forbidding play on the second, third and fourth columns and rows forces players to practice multiplication facts with larger numbers.

Source

Adapted from "Math Lab, Middle School" submitted by Pittsburgh Public Schools, Pittsburgh, Pennsylvania.

DIV-FOUR-ROW

Goals and Purposes

Practice in determining divisibility by the integers 1-6.

Materials

One 10" x 10" board ruled into 1" squares with squares numbered consecutively from 1 through 100. Fifteen markers of each of four colors and one die.

Procedure

1. Each player rolls the die. Highest numbers go first.
2. The first player rolls the die. He may cover one number on the board that is divisible by the number rolled.
3. Each player follows as in step #2.
4. The aim is to cover 4 numbers in a row horizontally, vertically or diagonally.
5. If a player rolls a 6, he may do as in 2 above or he may roll again and replace an opponent's marker with his own if the number he rolls divides a number on the board already covered by the opponent's marker.
6. When all 15 of each player's markers have been placed, the players move one of their markers to a new space on each turn until the game is won.

Source

Adapted from material of the Waterloo County Board of Education; submitted through Ministry of Education in Ontario, Canada.

SHORT WAY HOME

Goals and Purposes

Practice in short division with remainders.

Materials

A piece of cardboard or similar material made into a playing board as shown below. The numbers written in the 32 small squares can be any 3- or 4-digit numbers. From the square marked start, draw arrows directing the way to "home." Make a pack of 24 small cards; on each write a number 2 through 9, using each number three times.

3001 →	659	484	853	309	4021 ↓
813	1236 →	715	151	449 ↓	362
5162	418			2501	214
999	789	HOME		← 569	675
422	989 ↑	620	815	781	← 1216
↑ 2847	974	297	651	452	516 START ←

Procedure

Two or three pupils can play the game. Each pupil should have an identifying marker. To begin the game, a player puts his marker on the space labeled "Start." He turns over the top card of the pack of one-figure numerals. Using short division, he is to divide the number named by the numeral on the card he has turned up. (He may use scratch paper.) If there is a remainder, he can move his marker as many spaces as the remainder indicates. Otherwise, his marker must remain where it is for that round. Thus, in the first play, if the numeral on the card is 7, he divides 516 (the first number named on the sample playing board) by 7 and gets 73 and 5 remainder. He advances his marker 5 squares to the numeral 2847. The other players check his work. Then the next player takes his turn. The first player to reach "Home" is the winner.

Source

Adapted from "Ideas Plus for Math Grades 4,5,6," University City Public Schools, University City, Missouri.

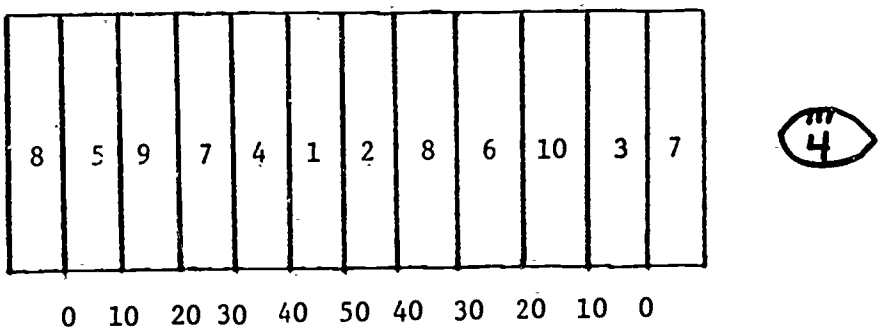
MULTIPLICATION AND DIVISION GAME

Goals and Purposes

Practice in multiplication skills.

Materials

Picture of a football field drawn on an overhead transparency or on chalkboard.



Procedure

Divide into two teams and select a number to put on the football. A player "carries the ball" by multiplying correctly the number on it by the numbers on the field. If he carries the ball into the opponents end zone, he scores a touchdown for his team. A mistake is considered a "fumble" and a player from the opposing team starts there and proceeds in the same manner - but opposite direction.

Source

Submitted by "Games and Challenges" from Livonia Public Schools in Livonia, Michigan.

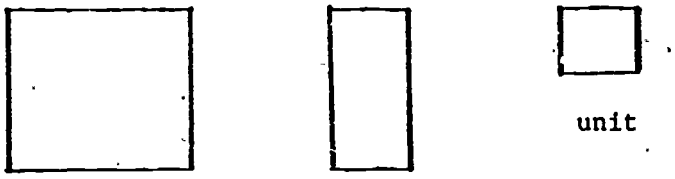
FACTORING QUADRATICS

Goals and Purposes

A manipulative model for factoring quadratic polynomials.

Materials

A set of pieces cut from tagboard in the following shapes:



square long

They can be made any size so long as the length of the square is not a multiple of the length of the unit. One-half inch for the length of the unit and 3¼ inches for the length of the square are convenient dimensions. An adequate "kit" for a student would have two sets of different colors; each set consisting of 4 squares, 15 longs, and 20 units.

Procedure

The square pieces represent $x \cdot x$ or x^2, the longs represent $1 \cdot x$ or x, and the units 1. At first, use only one color. Have students attempt to form rectangles using various combinations of units, longs, and squares. For example, given the pieces shown in figure 1, the square shown in figure 2 can be made.

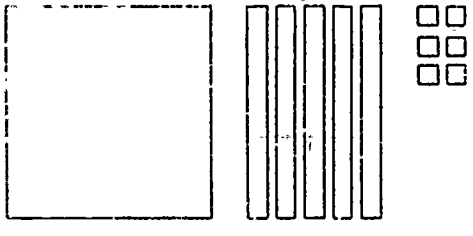


figure 1

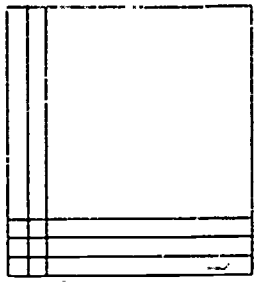


figure 2

This example corresponds to factoring $x^2 + 5x + 6$ into $(x + 2)(x + 3)$. Harder problems, such as $2x^2 + 13x + 15$, will lead to a variety of solutions corresponding to $(x + 5)(2x + 3)$. The combinations of pieces with which it is impossible to form a rectangle such as $x^2 + x + 1$, are representative of expressions that are not factorable over the rational numbers.

FACTORING QUADRATICS (continued)

If expressions involving negative numbers are to be considered, then both colored sets should be used. Suppose the two colors are white (for "positive area") and red (for "negative area").

White pieces are put down first adjoining each other and red pieces put on top of white pieces. A white and a red piece of the same size are equivalent to zero. Pieces may be added to a given collection in red-white pairs of the same size. As before, the attempt is to form a white rectangle. For example, consider the combination of one white square, one white long and two red units shown in figure 3.

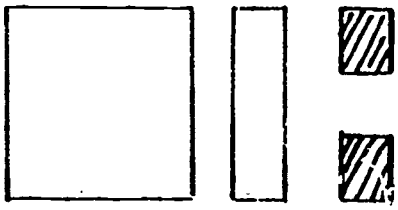


Figure 3

As such, a rectangle cannot be formed. But by adding a white-red pair of longs and positioning the whites as shown in figure 4, the reds can then be placed on top to form the rectangle as shown in figure 5.

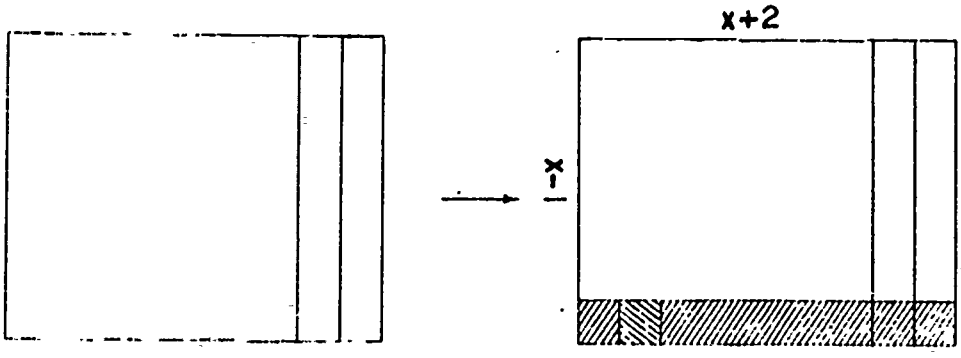


Figure 4

Figure 5

This example corresponds to rewriting $x^2 + x - 2$ as $x^2 + 2x - 1x - 2$ and factoring it into $(x + 2)(x - 1)$.

Source

Adapted from material submitted by James K. Bidwell of Central Michigan University. For further information, see Mathematics Teacher, Bidwell, James K. "A Physical Model for Factoring Quadratic Polynomials." The Mathematics Teacher, 65: 201-202; March 1972.

SECTION FOUR:

NUMBER SKILLS REVIEW

Section 4: Number Skills Review

The mathematics that is used is the mathematics that is remembered. This section includes games and activities that require the use of combinations of the four basic operations. They provide practice without drill, but they do not necessarily make mathematics fun! Remember that success is basic to having fun. A game that a student cannot win is a tedious bore. (So, for that matter, is a game that a student cannot lose.)

The key to successful use of mathematics games in the classroom is for the teacher to know the ability of students. Games must be carefully chosen to allow successful challenges.

PEGS AND PEGBOARDS IN KINDERGARTEN AND FIRST GRADE

Goals and Purposes

Provides concrete experiences with counting, ordering of numbers, and certain geometric concepts.

Materials

A ten-inch square pegboard containing 100 holes spaced one-inch apart and 20-30 different-colored pegs per child.

Procedure

Here are some representative activities which may be carried out with young children. Many more can be devised by the teacher and the students.

1. Count out three pegs: one red, one blue, and one yellow. Arrange them in a straight line with the yellow one in the middle. Make a triangle using the three pegs.

2. Match each peg with one of the same color, placing them in pairs. Arrange them in pairs each having two colors. Make two triangles. Make one triangle using all the pegs. Arrange all the pegs so that they form a rectangle. Arrange the six pegs in a broken line. How many squares can be made with six pegs? How many more pegs are needed to complete the second square?

3. Pick up as many pegs as possible in one grab. Count to see who has the most pegs, the least number of pegs, and the same number of pegs as someone else.

4. Make one row at the top of the pegboard, using all of the holes. Make one row at the bottom, using all of the holes. Make one row at the right side, using all of the holes. Make one row at the left side, using all of the holes.

5. Find row 1. Put one peg in the first hole. Find row 2. Put pegs in the first and the second holes. Continue with the remaining rows.

6. Find row 7. Put a peg in the next to the last hole.

7. Find row 4. Put pegs in half the holes.

8. Put pegs in every hole in row 4. Take out five pegs. How many pegs are left?

PEGS AND PEGBOARDS IN KINDERGARTEN AND FIRST GRADE (continued)

9. Put pegs in four holes so that you can put a rubber band around them to make a rectangle. Repeat, using three pegs, to make a triangle. Can you make a square?

Source

For further information: Brong, Tedi. "Fun With Pegs and Pegboards." The Arithmetic Teacher, 18: 234-235; April, 1971.

NUMBER CONCEPTS VIA EGG CARTONS

Goals and Purposes

Provides concrete materials for learning about division, addition, subtraction, and multiplication.

Materials

Egg cartons, some with groups of 6 painted different colors, some with groups of 4 painted differently, and so forth.

Procedure

Division can be readily illustrated. $12 \div 2$ can be rephrased as "How many groups of 2 are there in 12?" In fact, parts of one carton may be cut off and fitted into another one. Addition could be demonstrated by filling up an egg carton with 7 red beads and 5 blue beads, to see that $12 = 7 + 5$. By taking part of one carton away, subtraction may be emphasized. For example, if a group of 3 is taken from the carton, 9 remains. That is, $12 - 3 = 9$. The egg cartons may work best for multiplication. Students easily recognize that it takes 4 groups of 3 to make a dozen; that is, $4 \times 3 = 12$.

Source

Baumgartner, Margery. "What Can You Do With An Egg Carton?" The Arithmetic Teacher, 15: 456-458; May, 1968.

ENGINEER'S DICE

Goals and Purposes

Developing skill in basic operations in conjunction with logical thinking.

Materials

Five dice.

Procedure

1. Using five dice, the purpose of this activity is to explore the four operations of addition, subtraction, multiplication, and division.

2. Roll all five dice. Choose two and do one of the operations. Record your answers. Those two dice and that operation cannot be used again.

3. Roll the remaining three dice again. Choose one die and do another operation using the number on that die and the first score.

4. Roll the remaining two dice again. Choose one and follow the same rules as in number 3.

5. Roll the last die and do the operation which remains.

6. What is your total score?

7. Can your friend beat your score?

8. Try repeating the game; this time low score wins.

Source

Materials developed for U.S. Army Dependents Schools; submitted by Clyde Finnell, Mathematics Curriculum Coordinator for European Area.

TREASURE HUNT

Goals and Purposes

Using basic operations skills.

Materials

3 x 5 cards, small "treasures."

Procedure

Direct children to previously hidden "treasures" by writing mathematical clues on 3 x 5 note cards. Some examples might be:

1. Look in the (3 x 2) - 4 book from the left on the (9 ÷ 3) shelf.

2. Look under the (4 + 1) x 2 desk in the (8 ÷ 2) ÷ 4 row from the science activity center.

The treasure may be candy or other small treats. While the students are out of the room you can set this one up nicely.

Source

Materials developed for U.S. Army Dependents Schools; submitted by Clyde Finnell, Mathematics Curriculum Coordinator for European Area.

MAIL ORDER CATALOG

Goals and Purposes

Skill practice in addition and multiplication.

Materials

Mail order catalogs.

Procedure

Select five or more articles that you would like to buy from the catalog. The total price of your order must be between $95 and $100. (Taxes, or taxes and postage could also be required to be figured in if you desire a more realistic problem.) List the item, page, its price, and then the total for all items combined.

Source

Submitted by East Syracuse-Minoa Central Schools in East Syracuse, New York.

COVER THAT NUMBER

Goals and Purposes

Computation with the four basic operations and exponentiation.

Materials

Three dice, some colored markers, and a number grid similar to the one shown.

Procedure

Game is best played with 2 to 4 players. First player rolls the dice and uses the digits with the operations of addition, subtraction, multiplication and exponentiation to compute a number. If the roll resulted in a 3, 4, and 5, he could, for example, compute:

$3 + 4 + 5 = 12$
$(3 + 4) \cdot 5 = 35$
$4^3 \cdot 5 = 69$
$5 + 4 - 3 = 6$ or many other possibilities.

He covers the resulting number on the grid and play rotates to the next player. A player receives one point each time he covers a number that is adjacent horizontally, vertically, or diagonally to another covered number. When all players have experienced three successive failures to produce a coverable number, the game ends and player with highest score wins. For variation, the grid can be changed and four or five dice used.

Grid on following page.

Source

Materials developed for U.S. Army Dependents Schools; submitted by Clyde Finnell, Mathematics Curriculum Coordinator for European Area.

COVER THAT NUMBER (continued)

Grid

1	2	3	4	5	6	7	8
9	10	11	12	13	14	15	16
17	18	19	20	21	22	23	24
25	26	27	28	29	30	31	32
33	34	35	36	37	38	39	40
41	42	43	44	45	48	50	54
55	56	60	64	69	72	80	90
96	100	105	108	125	144	150	216

MATCH THE NUMBER

Goals and Purposes

Practice in the basic operations and use of logical thinking.

Materials

A deck of 60 cards, four each from 1 to 15. (Flinch cards, a commercial game by Parker Bros., can be used. It contains enough for 2½ sets.) If fewer than three players, use only half a deck.

Procedure

1. Deal five cards face up to each player. Make sure cards are shuffled well before dealing.

2. After all cards are dealt, turn up the next card and place it in the middle of the playing surface.

3. The objective of the game is to use all five cards and any mathematical operations or symbols to obtain the number in the middle.

4. You may not move the cards while you are trying to solve the puzzle.

5. There is a time-limit from 30 seconds to 5 minutes. If you haven't solved it by then, start over again with 5 new cards and a new number in the middle.

6. You must use all five cards and may not use a card more than once. For example: Say you have been dealt the following cards:

and the middle card is

Two possible combinations are:

$$2 + 11 \div 7 - 10 + 4 = 14$$

$$(2 \times 4) + 7 + 10 - 11 = 14.$$

Are there any other possibilities?

MATCH THE NUMBER (continued)

7. Scoring - At the end of each turn, if you have found a solution, add up the sum of the numbers on the cards. In the example above, you would have earned 2 + 11 + 7 + 10 + 4 = 34 points for finding the solution. A game continues until someone has reached 100 points in which he is declared the winner.

Source

Materials developed for U.S. Army Dependents Schools; submitted by Clyde Finnell, Mathematics Curriculum Coordinator for European Area.

MULTIPLES

Goals and Purposes

Using basic operations and the digits 1-9 to form numbers that are multiples of a given number.

Materials

Lengths of 1 x 1 lumber cut and sanded to form cubes. Six cubes are needed as described:

1 cube with +, +, -, -, x, ÷ on the sides.
3 cubes numbered so that the digits 1 - 9 are each used twice.
1 "goal" cube of a different color with 2, 3, 4, 5, 6, 7 on the sides.

Procedure

The first player rolls the goal cube. He then rolls the four remaining cubes at the same time. He tries to arrange these cubes so that the resulting number is a multiple of the number on the goal cube. (See examples.) He scores one point for each successful arrangement. The remaining players, in turn, have a chance to score points by displaying any correct arrangement the original player might have missed. Play proceeds to the next player who again starts by rolling the goal cube. Game can be determinated at a specified value, any number of rolls, or at the end of a certain time period. Player with highest total wins.

The goal is [3]

You roll [3] [7] [2] [+]

Possible arrangements are:

27 + 3 (30)
32 + 7 (39)
72 + 3 (75)

You roll [6] [8] [2] [+]

No successful arrangement is possible, so you get a zero for the round.

You roll [2] [7] [5] [x]

Possible arrangements are:

27 x 5 (135)
57 x 2 (114)

You roll [8] [1] [4] [-]

Possible arrangements are:

41 - 8 (33)
14 - 8 (6)

Source

Adapted from Classroom Proven Motivational Mathematics Games by Michigan Council of Teachers of Mathematics.

CONCENTRATION

Goals and Purposes

To review basic arithmetic operations, symbols, and equivalences.

Materials

16 cards: eight with basic facts or symbols and eight with matching equivalent values or expressions.

Examples:

2+3	5
5-3	2
2x3	6
6÷2	3

0+4	4
2x4	8
½	2/4
¼	.25

Procedure

Shuffle the cards and deal them face down into a 4 by 4 array. The first player turns up two cards. If they are equivalent the player keeps these cards and turns over two more. He continues until the two cards turned up do not match. The two non-matching cards are returned, face down, to their original positions. The next player then takes a turn. The player with the most cards when the game is over, wins.

Source

Adapted from material of the Waterloo County Board of Education; submitted by the Ministry of Education in Ontario, Canada.

BOTTLE CAP SCRAMBLE

Goals and Purposes

Practice in forming true mathematical sentences.

Materials

Two coffee cups and 54 bottle caps with the bottle caps labeled as follows:

On each bottle cap, a number from 0 to 10 is placed using adhesive tape and a small slip of paper:

4 caps with 0	4 caps with 6
4 caps with 1	4 caps with 7
4 caps with 2	4 caps with 8
4 caps with 3	4 caps with 9
4 caps with 4	4 caps with 10
4 caps with 5	

To the other 10 caps add the following signs:

2 caps with =	2 caps with >
2 caps with -	2 caps with <
2 caps with +	

Procedure

Each player shakes the can and draws out 1 number cap. The same player then chooses 1 cap with a sign on it. If he can make a true number sentence on his next draw from the number can, he keeps the caps. If he cannot make a true number sentence, he gets two more turns to see if he can make a true number sentence. If he cannot, then he must return his caps to the correct cans. The winner has the greatest number of caps.

Examples:

1st two draws: 5 >
third draw: 5 > 3
(player keeps all three caps)
1st two draws: 6 +
third draw: 6 + 4
fourth draw: 6 + 4 =
fifth draw: 6 + 4 = 3
(there is no true sentence using these symbols, therefore player returns all caps to corresponding coffee cups)

Source

Adapted from materials submitted by Florence Kostyc, No. Nine School, Perth Amboy, New Jersey.

MATH RACE TRACK

Goals and Purposes

Drill in basic operations.

Materials

Piece of heavy cardboard about 3' square. Set of miniature cars to use as game pieces.

Procedure

Draw a race track on the board and divide it into at least 100 spaces. Make a set of game cards containing basic operations questions - for example; 2 x 5, 9 + 7, 8 - 3, etc. Each player places his car at the starting line and in turn draws a card. If he answers correctly, he moves his car the corresponding number of spaces. First person to complete a predetermined number of laps is the winner. As an option, certain squares could be marked with special instructions such as "out of gas - lose a turn".

Source

Adapted from material submitted by Halton County Board of Education, Burlington, Ontario, Canada.

MATCH UP

Goals and Purposes

Review of mathematics vocabulary.

Materials

One set of desired vocabulary cards and one matching set of example cards such as

Vocabulary Card	Example Card
mixed number	3 1/2
greater than	>
prime number	7

Procedure

Divide into two teams and distribute the vocabulary cards to team #1 and the example cards to team #2. A student from team #1 displays a vocabulary card and a student from team #2 is to correctly match up with the example card. Score is kept until all cards are used. Team #1 then gets their chance at matching.

Source

Adapted from materials submitted by Jane Riley, Community Consolidated School District 15,Palatine, Illinois.

MATHEMATICAL SENTENCES

Goals and Purposes

Developing skill in forming mathematical sentences.

Materials

Nine wooden cubes for use as dice labeled as follows:

one die labeled	=	≠	=	>	<	=
two dice labeled	+	-	x	÷	+	-
five dice labeled	1	2	3	4	5	6
one die labeled	7	8	9	10	11	12

It is a good idea to have the numbered dice a different color than the dice with symbols on them.

Procedure

Any number of players may participate; one should be designated to be the time-keeper. A player rolls all nine dice and then attempts to make as many true mathematical sentences as possible within a certain time limit (3 minutes is good). Dice can be used over again in making different sentences. Another player writes down the sentences as they are made and scores one point for each sentence. After a certain number of turns for each player, the one with the most points wins.

Source

Submitted by Halton County Board of Education, Burlington, Ontario, Canada.

SECTION FIVE:

MEASUREMENT

Section 5: Measurement

Measurement is critical to mathematics, for it is the way that numbers are generated in the real world. The measurement process is basically a counting process, where the sizes or units to be counted are agreed upon in advance. Thus it is often best to begin by measuring with arbitrary units -- the width of a hand, the length of a pace. Discrepancies that arise can best be resolved by standardizing the unit, which is an important concept for children to understand.

The geometric concept of congruence is also important in measurement. We measure lengths, areas and volumes by constructing congruent (matching) lengths, areas or volumes from the smaller units; then count the number of units used to construct the congruent object. This is a relatively simple process if the unit chosen happens to be a multiple of the object measured. When this is not the case we construct an object as close as possible to the one being measured. One way to improve upon this "closeness" is to choose smaller units. Another way is to retain the original unit but to subdivide it into smaller subunits. The subunits are used only to construct the last partial unit, which saves effort in the counting process.

The metric system gains its advantage from the fact that subunits and units are related by powers of ten. Thus the system corresponds closely to our place value numeration system. All the activities of this section were developed before the present emphasis on teaching the metric system. Most of them have been converted to the metric system, but a few of the activities which use materials manufactured in the English measurement system have also been included. Despite the importance of the metric system of measurement students will live for many years in a world where both systems are present. Thus teaching activities in both systems will still be useful.

GUESS-TI-MATION

Goals and Purposes

Practice in estimating and measuring size and distance.

Materials

Pencil, ruler, oak-tag paper and small objects, e.g., books, boxes, small paper, etc.

Procedure

Pupils develop the concept of measurement by using one familiar object to measure another familiar object, e.g., "How many times will this pencil fit across this shelf?" It's important that children estimate the number of times one object is contained in another.

When a ruler is presented it is better if it is unmarked and perhaps made of oak-tag or cardboard. After a few trial measurements, the children will discover the need for a smaller unit of measure - such as the centimeter.

This same approach might also be used to introduce children to size (area) of simple objects, e.g., "How many sheets of this paper are needed to cover this desk?"

Source

Adapted from Thomson, Alice P. "Evaluation by Observation - Grade 3." The Arithmetic Teacher, 3: 104-108; April, 1956.

INDOOR FIELD MEET

Goals and Purposes

Use of measurement skills.

Materials

Straws, paper plates, paper cups, ping-pong balls, tape measure.

Procedure

Conduct a track and field meet with these four events: javelin throw (straw), discus (paper plate), shot put (ping-pong ball), and hammer throw (paper cup with string tied through the bottom). Students rotate through a turn at each event and measure other students results. Greatest distance in each event could receive a "gold medal" and the four distances for each student could be added to determine the "overall champion."

Source

Materials developed for U.S. Army Dependents Schools; submitted by Clyde A. Finnell, Mathematics Curriculum Coordinator for European Area.

TEAM MEASUREMENT

Goals and Purposes

To introduce and practice measurement of lengths, to introduce and use the terms "length and width."

Materials

Ten packs of rectangles cut from cardboard. Each pack (envelope) should contain three identical rectangles, and each pack should be different from every other pack. Rulers (specify the use of centimeters or inches).

Procedure

Divide the class into teams, with no more than three students per team. Each team is given one pack of rectangles and must use the rulers to agree on an answer for the length and width of the rectangles in the pack. The teacher checks answers, giving each team 5 points for each correct answer.

If student ability necessitates, each team may be given a second try if the first answers are wrong. Correct answers on the second try earn 3 points. Exchange packs until each team has measured all ten packs. Total the scores to find the winner.

Source

Adapted from "Math Lab, Middle School," submitted by Pittsburgh Public Schools, Pittsburgh, Pennsylvania.

STRAW LENGTHS

Goals and Purposes

To practice measurement of lengths with a ruler, and to relate lengths to fractions.

Materials

Soda straws, rulers (plastic straws are preferable to paper straws). For lower ability children, cut the straws in advance to whole numbers of centimeters or inches. For an easy activity, use 12cm lengths. For slightly more advanced students use lengths of 18cm or 15cm.

Procedure

1. Measure the length of the straw (specify cm or inches).
2. Fold the straw in half. Measure the length of half of the straw.
3. Fold the straw in fourths. Measure the length of one-fourth of the straw.
4. Unfold the straw. By trial and error, fold the straw into thirds (as closely as you can). Measure the length of one-third of the straw.
5. Examine the numbers you obtained as answers to questions 1,2,3 and 4. How would you compute one-half, one-fourth, or one-third of a number?

Source

Adapted from "Math Lab, Middle School," submitted by Pittsburgh Public Schools, Pittsburgh, Pennsylvania.

RULER RACE

Goals and Purposes

Practice in using the ruler for measurement.

Materials

Rulers and a set of the cards with the following fractions written one on the face of each card:

1/16, 3/16, 5/16, 7/16, 9/16, 11/16, 13/16, 15/16, 1/8, 3/8, 5/8, 7/8, 1/4, 3/4, 1/2

To have more cards, duplicates could be used or improper or unreduced fractions could be used.

Procedure

Players should rule off equal line segments on their playing papers, say 10" long. Players in turn draw a card from the deck which has been shuffled and placed face down. Starting from one end of their line segment, they rule off the fractional distance indicated on the drawn card. Play continues until one player reaches or passes the opposite end of his line segment. For better students, it could be required that the last distance must come exactly at the end of the segment and that they must pass any turn that would carry them too far.

Source

Adapted from material submitted by Jane Riley, Community Consolidated School District 15, Palatine, Illinois.

MAKING A LINEAR MEASUREMENT DEVICE

Goals and Purposes

Constructing a useful tool for linear measurement.

Materials

Clothes pin (non-spring type), straight pin, small thin wheel (can be made from cardboard) approximately 1-1½" in diameter.

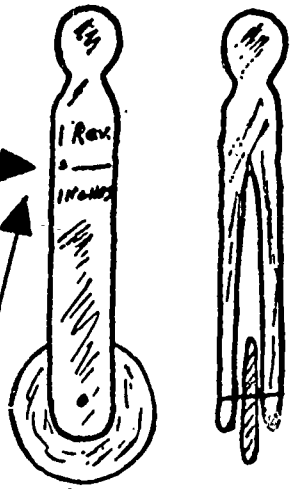
Procedure

Assemble as indicated.

Along a ruler measure the distance of one revolution of the wheel, write it here.

Make a line on the wheel so you can count rotations as it rolls.

To find distance or perimeter of figures, roll the wheel over the line you want to measure. Be careful on corners and curves. Count the number of times the wheel turns.



Multiply this by the scale on top of clothes pin. This tells you the total distance the wheel has rolled.

Source

Materials developed for U.S. Army Dependents Schools, submitted by Clyde Finnell, Mathematics Curriculum Coordinator for European Area.

PAPER AIRPLANE CONTEST

Goals and Purposes

Use of measuring instruments.

Materials

Measuring tape or yardstick, stopwatch.

Procedure

Planes should be made only from paper. Each student should make and fly his own plane. Measure the distance and duration of each flight. Awards can be given to the best in each category. If desired, the speed of each airplane could be determined by use of the "distance formula."

Source

Materials developed for U.S. Army Dependents Schools; submitted by Clyde Finnell, Mathematics Curriculum Coordinator for European Area.

HUMAN PROPORTIONS

Goals and Purposes

Practice in measurement and resulting work in multiplication and division.

Materials

Tape measures or yardsticks.

Procedure

Group students into threes. Determine the desired accuracy and have the students measure each others height, arm span, and length of foot. Write the ratios length of foot to height and arm span to height. Compare these ratios, first among members of the same sex, then all together. Predict someone's arm span when only his height is known.

Source

Adapted from Activities with Ratio and Proportion, submitted by Oakland County Mathematics Project, Pontiac, Michigan.

CAVEMAN MEASUREMENT

Goals and Purposes

Introduce students to older units of measure and give practice in using measuring instruments.

Materials

Rulers, yardsticks, and/or tape measures. Individual copies of the information sheet showing the various units of measure that would be helpful.

Procedure

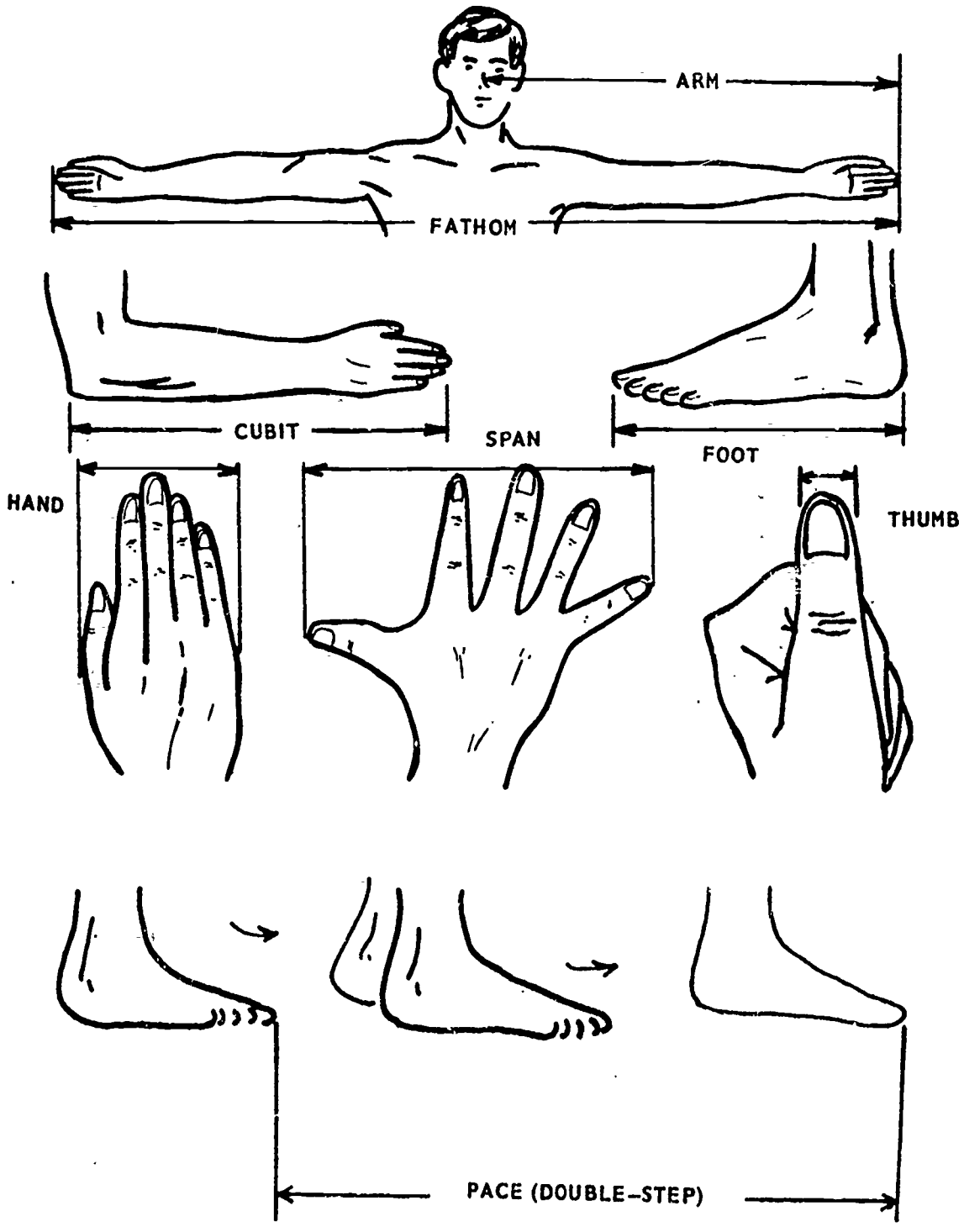
Assign students to work in pairs to complete the information required in Table 2-1. Class results could be tabulated to help in filling out Table 2-2 and averages could be determined if desired. Differences in measurements should lead to very good discussion questions.

See included information sheet and class activity sheet.

Source

Adapted from Exploring Linear Measure, submitted by Oakland County Mathematics Project, Pontiac, Michigan.

CAVEMAN MEASUREMENT
INFORMATION SHEET



CAVEMAN MEASUREMENT
CLASS ACTIVITY SHEET

With the aid of a partner, obtain these measurements for yourself to the nearest $\frac{1}{2}$ inch. Record your results in TABLE 2-1.

TABLE 2-1

DESCRIPTION OF UNIT	LENGTH TO NEAREST $\frac{1}{2}$ INCH
a) Width of THUMB	
b) Distance from first joint to end of FOREFINGER	
c) HAND	
d) SPAN	
e) CUBIT	
f) Length of FOOT	
g) Distance from nose to thumb of outstretched arms (ARM)	
h) Distance across outstretched arms (FATHOM)	
i) PACE	

COMPARING SIZES . . .

Compare your measurements in TABLE 2-1 with those of your classmates. Are they the same? Why or why not?

Complete TABLE 2-2. Use the longest and shortest lengths in your class for the units listed.

TABLE 2-2

UNIT	LONGEST LENGTH	SHORTEST LENGTH	DIFFERENCE
Width of thumb			
Hand			
Span			
Cubit			
Pace			

RECTANGULAR AREAS

Goals and Purposes

To measure rectangular areas, and relate these areas to the length and width of the rectangle.

Materials

Pencils, graph paper (uniformly ruled, no subdivisions).

Procedure

Each square on the graph paper will represent one unit of area. The edge of each square will represent one unit of length.

Draw rectangles on the graph paper with the following (length) dimensions: 7 by 5, 8 by 9, 6 by 11, 2 by 3, and 12 by 15.

Count the area in each rectangle.

How does each area relate to the dimensions of the rectangle?

Source

Adapted from, "Math Lab, Middle School," submitted by Pittsburgh Public Schools, Pittsburgh, Pennsylvania.

ESTIMATING AREA

Goals and Purposes

Skill in estimating areas by means of a grid.

Materials

Pieces of clear cellophane with grids of different sizes ruled on them. A supply of maps or other figures for areas to be measured.

Procedure

The grid should be placed on top of the figure whose area is being determined. First count the number of squares completely contained within the figure; then add the fractional parts to obtain an estimation of the area. The same figure's area can be computed using several different grid sizes and compared based on the relative size of the squares on each grid.

Source

Adapted from material submitted by Halton County Board of Education, Burlington, Ontario, Canada.

UNITS OF SQUARE MEASURE

Goals and Purposes

Comparison of units of square measure.

Materials

Construction paper, rulers, scissors.

Procedure

Instructions for students ---

1) Cut a square 1cm by 1cm; what is it called?
2) Cut a square 10cm by 10cm; what is it called?
3) How many of the smaller squares will fit in the larger square without overlapping?
4) If you have a square 1 meter by 1 meter, what would you call it?
5) How many of the 10cm by 10cm squares would fit in this still larger square?

Source

Adapted from material submitted by Halton County Board of Education, Burlington, Ontario, Canada.

AREA OF A CIRCLE

Goals and Purposes

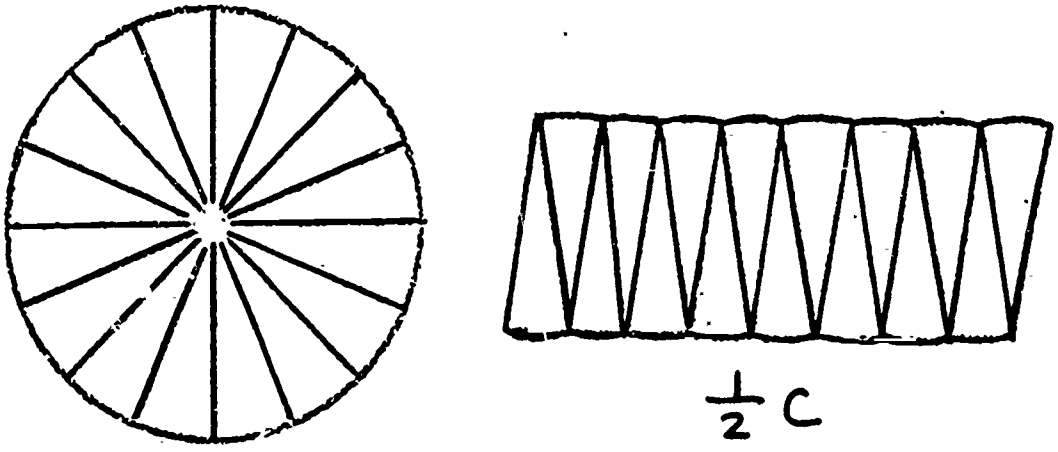
"Justification" of a formula for area of a circle.

Materials

Compass, ruler, scissors.

Procedure

Draw a circle, cut into 16 equal sectors, and reassemble the sectors as shown in the illustration below.



The resulting region is very similar to the region bounded by a parallelogram. Measurement, (or thought), will convince one that the base is roughly ½C and height is roughly r. Thus the area of this region, and hence of the circle is about:

$$A = \tfrac{1}{2}C \cdot r$$

Previously the student should have established that $C = 2\pi r$ and thus

$$A = \pi r^2$$

Source

Submitted by Boyd Henry, College of Idaho, Caldwell, Idaho.

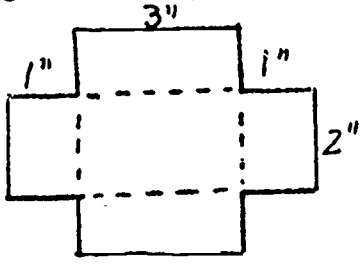
INTRODUCTION TO VOLUME

Goals and Purposes

To introduce the concept of volume of rectangular parallelapipeds as measured by unit cubes, and to relate this volume to length, width and height measurements.

Materials

Use tagboard to make a series of boxes whose dimensions are 1"x2"x3", 2"x2"x3", 2"x3"x3", 1"x2"x4", 2"x2"x4", 1"x3"x4", 2"x3"x4", 1"x4"x4", 2"x4"x4", etc. A simple cross-pattern may be used, folding on dotted lines and taping the corners together. Be generous with all dimensions to allow for stacking cubes.



Sample box pattern

Use 1" wooden cubes, or sugar cubes (which are ½" cubes). Each team needs 32 1" cubes or 128 (1 box) sugar cubes.

Procedure

Fill each box completely with cubes. Count the cubes as they are removed -- this measure of the size of the box is its volume. Measure the length, width, and depth of each box. Can you relate these three numbers to the volume of the box?

Source

Higgins, Jon L. "Sugar Cube Mathematics," The Arithmetic Teacher, 16: 427-431 October 1969 .

CAPACITY COMPARISONS

Goals and Purposes

Practice in estimation and comparison of volumes.

Materials

Common containers such as teacups, jars, vases, pails, teaspoons, cups, pints, etc.; rice, beans.

Procedure

Start with containers with which the children are familiar. Let the children estimate how many cups of water, or beans, it takes to fill the pail. After many experiences with non-standard units of measure, present the standard unit of measure, i.e., cup and liter.

Comparison exercises are also very helpful in improving the children's concepts of measure and volume. "Which of these boxes will hold more rice?"

To challenge in greater depth the abilities of children to estimate which of two containers could hold more of a given material, the teacher can construct sets of hollow containers of 1/16" acrylic-plastic cement in three geometric shapes: cube, rectangular prism, and tetrahedron.

Source

Adapted from Clintic, Joan. "Capacity Comparisons by Children." The Arithmetic Teacher, 17: 19-25; January, 1970.

Thomson, Alice P. "Evaluation by Observation - Grade 3." The Arithmetic Teacher, 3: 104-106; April, 1956.

HOW MANY BEANS?

Goals and Purposes

Estimating volume measurements.

Materials

Jar filled with beans and a small paper cup.

Procedure

Have students guess the number of beans in the jar. Fill the small cup with beans and actually count the number of beans in the cup. Use this to estimate the number of beans in the jar by seeing how many cups of water it would take to fill the jar.

Source

Materials developed for U.S. Army Dependents Schools; submitted by Clyde Finnell, Mathematics Curriculum Coordinator for European Area.

CONSTRUCTING SQUARE ROOTS

Goals and Purposes

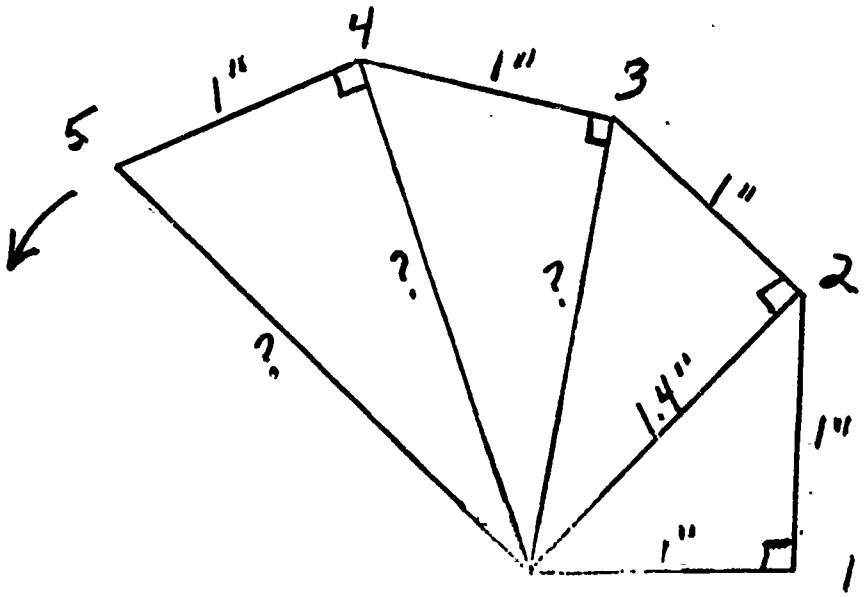
Determination of square roots of integers through construction and measurement.

Materials

Ruler, object for making right angles (protractor, square, etc.).

Procedure

Construct the following figure:



Multiply 1.4 x 1.4 and see how close your answer approximates 2.

Take the measure of the other three lines and see how close they approximate their squares.

Draw two more sections and see how close they approximate their squares. Can you see why this line approximates the square root of the number?

Source

Adapted from, "Math Lab, Middle School," submitted by Pittsburgh Public Schools, Pittsburgh, Pennsylvania.

APPROXIMATING PI

Goals and Purposes

Practice in measurement and an experimental determination of pi.

Materials

Tin cans, coins, cups, wastebaskets, and other "round" objects.

Procedure

Using a non-stretchable but flexible tape measure, measure the circumference and diameter of each object. Compute the ratio C:D and compare with the ratios determined by the other class members.

Source

Adapted from Equa.Formu.Alities, submitted by Oakland County Mathematics Project, Pontiac, Michigan.

BUFFON'S NEEDLE PROBLEM

Goals and Purposes

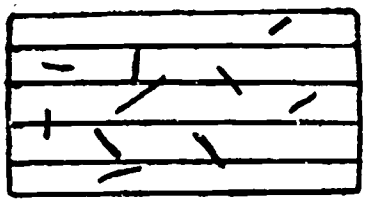
Determine an approximation for π.

Materials

Large piece of paper or cardboard, wire or thin plastic tubing for cutting into small segments.

Procedure

1. Draw parallel lines one inch apart on a large sheet of paper.
2. Cut a piece of wire into half-inch segments.
3. Place the paper with the parallel lines on the floor and drop the wire segments on the paper.
4. Count the number of times a wire fell on a line and call it "D".
5. Count the number of wires dropped and call it "C".
6. Now divide "C" by "D".



Source

Materials developed for U.S. Army Dependents Schools; submitted by Clyde Finnell, Mathematics Curriculum Coordinator for European Area.

PLANNING A ROUTE

Goals and Purposes

Skill in using measurement in a practical experience.

Materials

A large supply and variety of road maps.

Procedure

Below are suggestions as to the type of questions students could be asked to consider when planning a trip.

1) Get out a road map and trace a route to a vacation area.
2) Use the scale factor given on the map to calculate the distance from your home to the vacation area.
3) How accurate is your measurement?
4) If you average 50 miles per hour on the trip, how long would the trip take?
5) Find out how many miles your car will travel on one gallon of gasoline; estimate the cost of gasoline for your trip.
6) Draw a straight line from your home to the vacation area and calculate the distance 'as the crow flies'.
7) How much shorter is the direct route?
8) How much time would be saved at 50 miles per hour?
9) How much money would be saved on buying gasoline?

Source

Adapted from materials submitted by Halton County Board of Education, Burlington, Ontario, Canada.

PER CENT CALCULATOR

Goals and Purposes

Make students more familar with the concept of percent and have them try estimates of whole number percents of line segments.

Materials

One half rubber band per student -- a good size is #31, 1/8" by 2 1/2".

Procedure

Pair students off and follow directions given on the accompanying information page. A sample exercise page is also included.

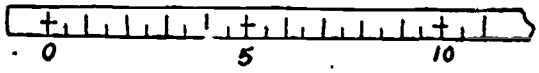
Source

Adapted from Activities with Ratio and Proportion, submitted by Oakland County Mathematics Project, Pontiac, Michigan.

INSTRUCTIONS FOR MAKING AND USING YOUR PERCENT CALCULATOR

1. To make the calculator you will need a piece of rubber band.

2. Choose a partner. Stretch the rubber band the length of the ruler which is printed along the edge of this page. Then have your partner put a pencil mark on the rubber band every half inch.



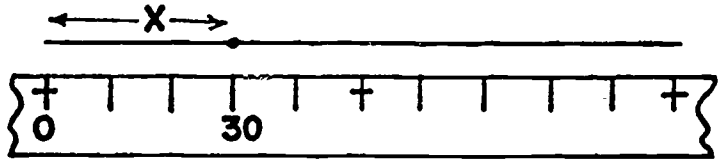
3. When you let the rubber band relax, there is a set of evenly spaced marks. They will stay evenly spaced for any length the rubber band is stretched.

   Let's try it. Mark off 30% of the segment drawn below.

Here's how.

a. Stretch the rubber band next to the segment until it is divided into 10 units of length. (Each unit represents 10%, and there will be some marks on the rubber band that will not be used.)

b. Place a dot on the segment after the third unit which will be the fourth mark on the rubber band.



c. The x on the segment indicates that the left hand part of the segment is 30% of the entire length.

1. Place a mark on each segment so that the left hand part represents the given per cent.

   a. 50% ———————————————————

   b. 40% ——————————————

   c. 60% ————————————————————

   d. 25% ———————————————

   e. 75% ——————————————————————————

   f. 85% ————————————————————

   g. 90% ———————————————

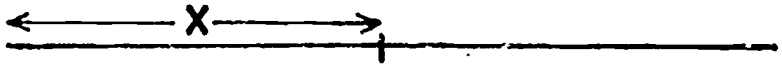
   h. 100% ———————————————

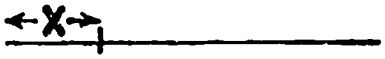
   i. 125% ———————————————

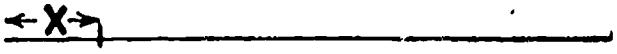
2. (I) Estimate the per cent represented by the part of the segment indicated by |←——x——→|.

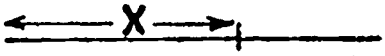
   (II) Check your estimate using the rubber band.

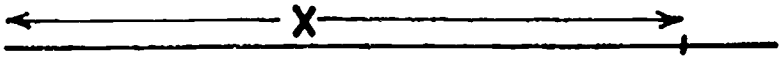
	A	B	C	D	E	F	G
I ESTIMATE							
II CHECK							

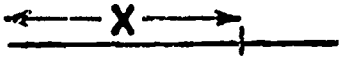
A 

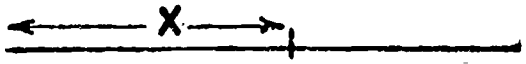
B 

C 

D 

E 

F 

G 

PULSE RATE

Goals and Purposes

Learning how to use a stopwatch to measure time and practicing how to make a graph to organize data.

Materials

Stopwatches -- at least three or four. Wristwatches with second hands or egg timers could be substituted if stopwatches are not available.

Procedure

Divide students into convenient groups of 8 to 10 persons. Demonstrate how to take a pulse count. Each student should get to time and count pulses at least once. After the pulse rate per minute of each member of the group has been determined, the students should each make a chart showing the pulse rates for the members of his group and then present this information in terms of a bar or line graph.

Source

Adapted from materials submitted by Halton County Board of Education, Burlington, Ontario, Canada.

PROBABILITY

Goals and Purposes

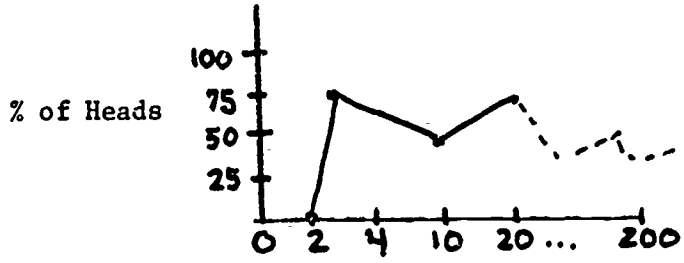
Introduction to probability and use of skills in computation of percentages and construction of graphs.

Materials

Coins.

Procedure

Toss a coin approximately 200 times. After the following tosses: 2, 4, 10, 20, 40, 60, 80, 100, 125, 150, 175, 200, compute the percentage of "head results" to total tosses and enter on the graph. Finished graph might look like this:



Number of Tosses

What has happened as the number of tosses increased?

Source

Adapted from material submitted by East Syracuse-Minoa Central Schools in East Syracuse, New York.

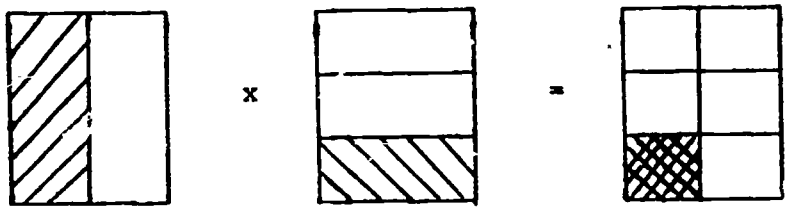
SECTION SIX:

FRACTIONS

Section 6: Fractions

Fractions represent an entirely new number system -- the system of rational numbers. Parts of this new system can be related to the familiar system of counting numbers, but much of the system cannot. Thus several unsettling things occur. Multiplication is no longer simply repeated addition. Products may be smaller than either of the factors; quotients may be larger. It has traditionally been assumed that relating rational numbers to the counting numbers at every possible occasion will build familiarity and confidence. This may not be the case. There are so many exceptions where the correspondence cannot be made that over-anxious attempts to relate the two systems may generate confusion instead of confidence. Perhaps the treatment of fractions as a separate system is an alternative to be considered.

Learning psychologists believe that learning is facilitated if we can relate new facts to things we already know. Would the treatment of fractions as a separate system violate that principle? Not if they could be related to something else already known. Fortunately, an alternative relationship is easily at hand. The subdivision of measurement units generates fractions in a familiar way. Thus fractions can be related to the measurement process. Addition and subtraction of fractions can be developed through the addition of appropriate lengths, as the first two activities in this section illustrate. Multiplication and division can be illustrated by fractional areas in the following manner:



1/2 x 1/3 = 1/6

(A set of fraction squares can be made from overhead transparencies and overlapped to show multiplication.)

It is worth noting that the selection of lengths (fraction strips or bars) and areas (fraction squares) as models is crucial. The old subdivided circle (pieces of a pie) does not lend itself to manipulation and activity. A unit square area may be easily related to unit lengths. But a unit of circular area is not easily related to the radius of the circle. For these reasons, pieces of circles are <u>not</u> useful in illustrating fractions and should almost never be used in mathematics laboratories.

ADDING UNLIKE FRACTIONS

Goals and Purposes

To provide concrete experiences with the addition of unlike fractions and a rationale for the addition of fractions algorithm.

Materials

An 8" x 11" acetate parjectual transparency and corresponding fractional parts (fig. 1). The fractional parts are made from a second transparency of another color. The second acetate is cut into pieces that represent fractions.

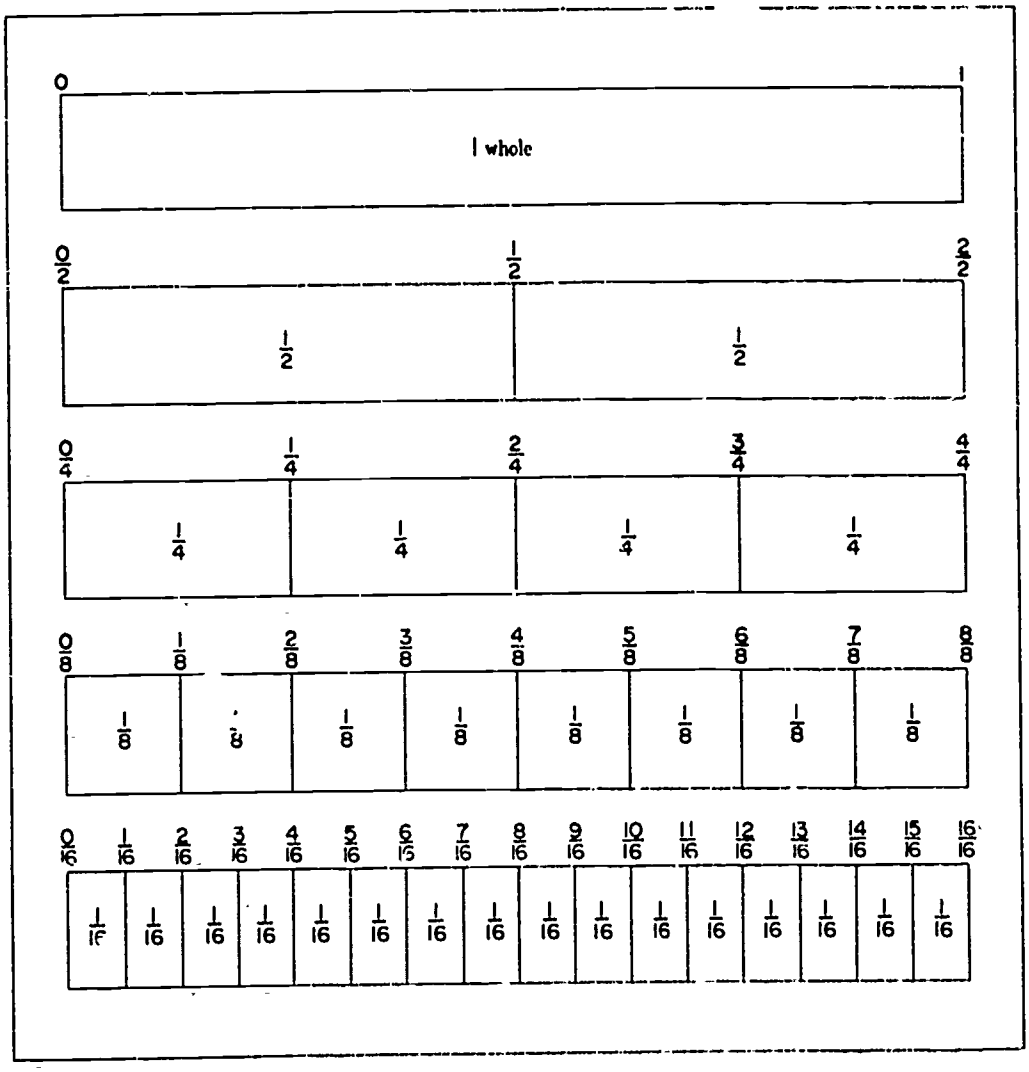


Fig. 1

ADDING UNLIKE FRACTIONS (continued)

1. Place the transparency (fig. 1) on the desk.

2. Gather the pieces that represent the fractional numbers to be added, for example, 1/2 + 2/8 (fig. 2).

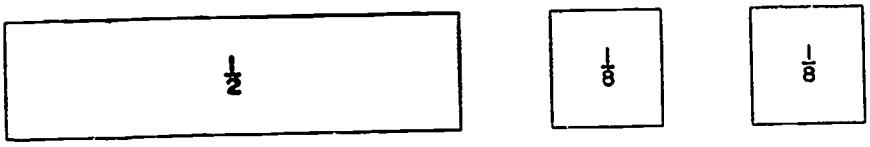


Fig. 2

3. Join the pieces and place them on the portion of the acetate marked "1 whole."

4. Slide the pieces down until the right end meets a line -- for this example, 3/4 (fig. 3).

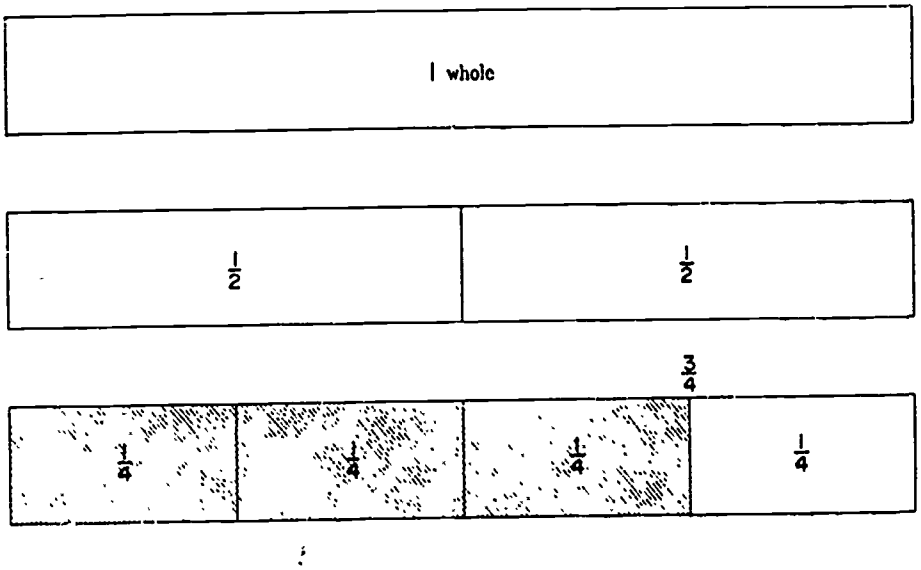


Fig. 3

5. Look above the line to the name of the point. This represents the sum of the two fractional numbers.

After several examples of this type the children will be in a position to discover the algorithm for the addition of unlike fractions.

Source

Adapted from Jacobson, Ruth S. "Fun With Fractions for Special Education." The Arithmetic Teacher, 15: 221-223; March 1968.

PAPER-STRIP FRACTIONS

Goals and Purposes

To provide concrete experiences with the addition of halves and thirds and a rationale for the appropriate algorithm.

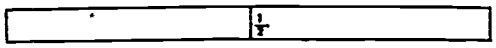
Materials

Paper cut into 4" x ½" strips.

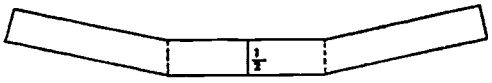
Procedure

Fold the strip of paper in half. The unit strip is now divided into two equivalent sections.

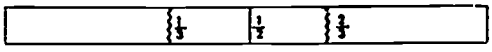
Note the fold and mark it $\frac{1}{2}$.



Take the same strip of paper and fold it into three equivalent sections.

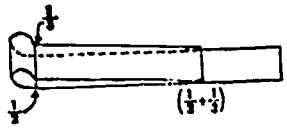


Mark the new folds $\frac{1}{3}$.

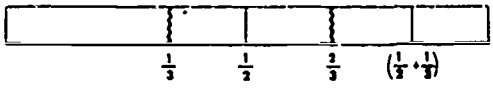


We can now relate the halves and the thirds on the paper strip. Can we determine $\frac{1}{2} + \frac{1}{3}$?

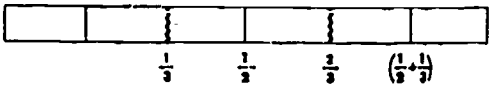
We can now mark off to the right of the $\frac{1}{2}$, a $\frac{1}{3}$ by folding the paper strip from left to right by setting the $\frac{1}{3}$ mark on the $\frac{1}{2}$ mark.



The strip of paper will now have the following markings:

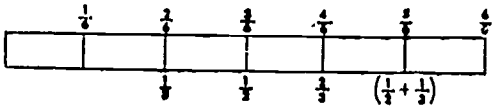


Suppose we fold the $\frac{1}{3}$ section on the left of the paper strip into two equivalent sections. The paper strip will now have the following folds:



Will each of the folds be of equivalent size? How many equivalent sections is the unit strip of paper divided into?

Can we see that $\frac{1}{2} + \frac{1}{3} = \frac{5}{6}$?



We note that $\frac{1}{3}$ is equivalent to $\frac{2}{6}$ and $\frac{1}{2}$ is equivalent to $\frac{3}{6}$. Is $\frac{3}{6} + \frac{2}{6} = \frac{5}{6}$?

Source

Adapted from Adachi, Mitsuo. "Addition of Unlike Fractions." The Arithmetic Teacher, 15: 221-223; March 1968.

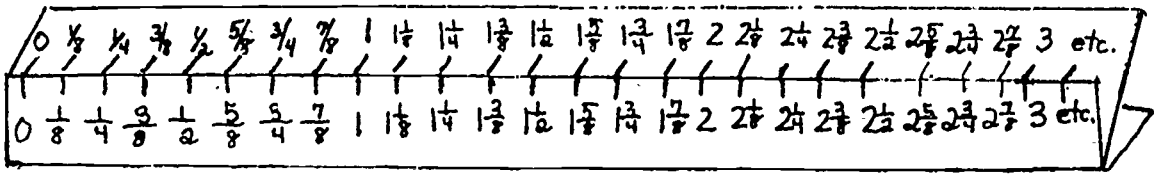
ADDITION AND SUBTRACTION SLIDE RULE

Goals and Purposes

Manipulative device for adding and subtracting fractions.

Materials

Use 4½" by 20" tagboard. Write numbers at intervals of one-half inch, one-eighth inch down from the top as shown. Fold in half lengthwise to make the body. Use another piece of tagborad 3½" by 20" and write a similar set of numbers two and one-half inches from the bottom. This piece is called the slide.



Procedure

Place slide in body. Move the slide to the right until the zero is above one addend. Look along the slide for the second addend. The sum is below the second addend (on the body).

To subtract, locate the minuend on the body and move the slide so that the subtrahend on the slide stands over the minuend. The zero on the slide then stands over the remainder (on the body).

Source

Adapted from "Math Ideas '+' for Grades 1, 2, 3," University City Public Schools, University City, Missouri.

ADDITION OF FRACTIONS WITH UNLIKE DENOMINATORS

Goals and Purposes

Provides practice in forming common denominators and adding unlike fractions.

Materials

A sheet of poster board marked into 9 square regions like those for tic-tac-toe, and 10 cardboard squares, each of which will fit into the regions marked on the poster board.

Procedure

On five of the cardboard squares write the fractions 0/15, 2/15, 4/15, 2/5, 8/15; on the others write 1/15, 1/5, 1/3, 7/15, 3/5. The game is played by two players, one having the first set of squares and the other having the second set. To play, the students alternately place one of their fractions on the poster board. The winner is the first player who makes a play so that a row, column, or diagonal of fractions add to 1. Sometimes there will be no winner. Students benefit by playing with both sets of fractions.

Source

Hammond, Robert C. "A Device for Practice with Common Denominators and Addition of Unlike Fractions." The Arithmetic Teacher, 8: 373; November 1961.

FRACTION STICKS

Goals and Purposes

Improving skills with equivalent fractions, and using a computational aid for addition and subtraction of fractions.

Materials

Ten tongue depressors (or wooden popsicle sticks). Mark each stick into ten sections. Number the sections of the first stick 1-10, the second stick multiples of 2-20, the third stick multiples of 3-30, etc. The last stick should be labeled in multiples of 10-100. (See figures 1 and 2.)

Procedure

Equivalence of fractions can be shown with two sticks as shown in figure 1. The sticks can be used to find common denominators for addition and subtraction as shown in figure 2.

Figure 1. Show equivalent fractions 1/5 = 2/10 = 3/15

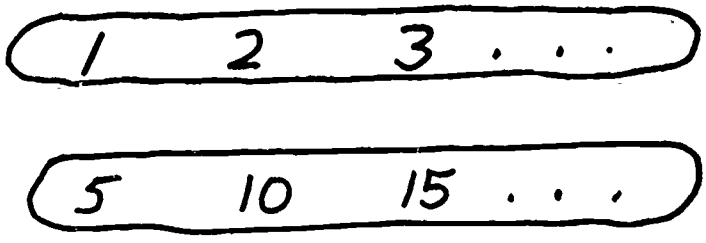
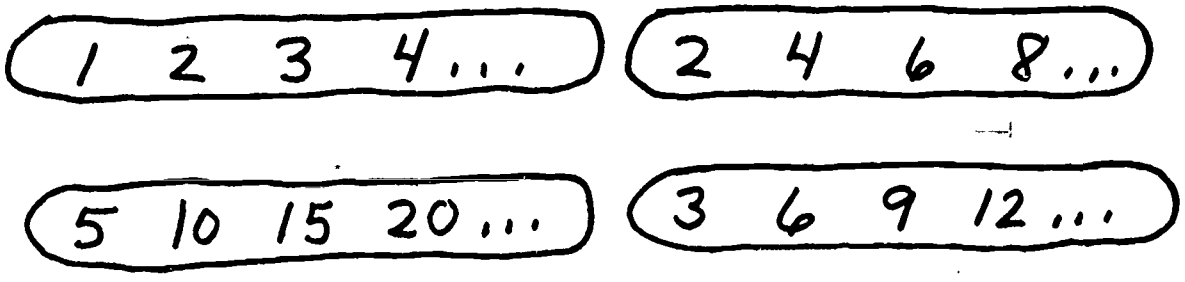


Figure 2. add fractions 1/5 + 2/3 = 3/15 + 10/15 = 13/15.



Source

Adapted from material submitted by Jane Riley, Community Consolidated School District 15, Palatine, Illinois.

COMPARING SIZE OF FRACTIONS

Goals and Purposes

Pictorial method for determining relative size of fractions.

Materials

Graph paper.

Procedure

1. Draw five rectangles containing two blocks, three blocks, four blocks, five blocks, and six blocks respectively.
2. Shade in one block of each of the rectangles.
3. Below each rectangle list what fractional part of the total is shaded.

Questions:

1. Are any of the above fractions identical?
2. If none of the fractions are identical, what is the major difference between them?
3. Which fraction above is the largest?
4. Which fraction above is the smallest?
5. Can you answer questions three and four by looking at the rectangle?
6. Write the fractions for each rectangle if you shade two parts instead of one.

Source

Adapted from "Math Lab, Middle School," submitted by Pittsburgh Public Schools in Pittsburgh, Pennsylvania.

FRIO
(Fractions in Order)

Goals and Purposes

Practice in determining relative size of common fractions.

Materials

60 cards. On these write the numerals that represent 1/2, 2/2, 1/4, 2/4, 3/4, 4/4, 1/8, 2/8, 3/8, 4/8, 5/8, 6/8, 7/8, 8/8, 1/16, 2/16, 3/16, 4/16, 5/16, 6/16, 7/16, 8/16, 9/16, 10/16, 11/16, 12/16, 13/16, 14/16, 15/16, 16/16. Make two copies of each card to make the 60 card deck. (Use other fractions to make the game more challenging, as required.)

Procedure

Three or four players works best.

One player shuffles the deck and deals five cards to each player. The cards are placed face up in front of the player in the order in which they are dealt to him. Each player in turn draws from the pile that is left face down, or he may take the last card to be discarded. If this new card will help him to get his cards in order from low value to high, he exchanges it for one of his; if not, he discards it. The object of the game is to be the first one to arrange his cards in order beginning with the smallest value.

The following example may help to clarify the rules.

Suppose a player is dealt the following cards:

$\boxed{\frac{1}{4}}$ $\boxed{\frac{1}{8}}$ $\boxed{\frac{9}{16}}$ $\boxed{\frac{1}{2}}$ $\boxed{\frac{3}{4}}$

On his first turn he draws $\boxed{\frac{7}{16}}$ and replaces the $\boxed{\frac{1}{8}}$ with it.

When his next turn comes, there is a $\boxed{\frac{1}{8}}$ on the discard pile, so he decides to draw. He draws a $\boxed{\frac{2}{8}}$ and discards it. His cards are now arranged $\boxed{\frac{1}{4}}$ $\boxed{\frac{7}{16}}$ $\boxed{\frac{9}{16}}$ $\boxed{\frac{1}{2}}$ $\boxed{\frac{3}{4}}$. All he needs is a card to replace the $\boxed{\frac{1}{2}}$ that is between $\boxed{\frac{9}{16}}$ and $\boxed{\frac{3}{4}}$. As soon as he has done this, he has won.

Source

Adapted from material submitted by the Ministry of Education, Ontario, Canada.

FRACTION RALLY

Goals and Purposes

Practice the addition of simple fractions.

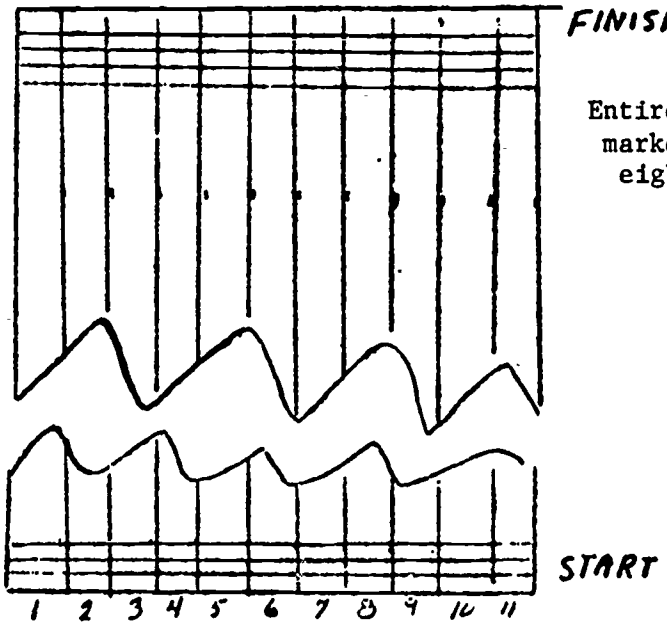
Materials

Three wooden blocks, or unmarked dice, marked as follows:

(1st)	1/2	3/3	1/4	3/6	1/8	6/12
(2nd)	2/8	3/4	6/8	5/8	6/12	3/12
(3rd)	3/8	5/8	2/4	2/2	7/8	4/8

or other suitable markings.

Game board made of tagborad and marked as shown in the figure:



Entire board marked in eights.

Procedure

Each player chooses a lane marked on the gameboard. Each person rolls the dice and adds the fractions that turn up. He moves his marker the correct number of eights and then awaits his next turn. The first person to reach the finish line is the winner of the rally.

MAKE A WHOLE

Goals and Purposes

Addition of fractions 1/2, 1/3, 1/4, 1/6 and 1/8 to make a whole unit.

Materials

From different colored 6" tagboard disks cut 8, 1/2 sections; 12, 1/3 sections; 16, 1/4 sections; 24, 1/6 sections; and 32, 1/8 sections. Use several different colors for the sections which may or may not be identified.

From 4, 8" x 8" tagboard squares prepare a frame. Cut a 6" circular disk from the center of three of the squares and then glue the squares to the complete square so that a frame is formed.

Cut a cube (a die) from a block, approximately a 2" cube. Paint the fractions 1/2, 1/3, 1/4, 1/6, 1/8 (one to a face), and a star on faces of the cube. Paint the cube with clear varnish to prevent smudging.

Procedure

Place the fractional sections in a box, from which they are to be selected by the players. Individuals or teams may compete. Two to eight students may play the game. The player rolling the star, or the fraction with greatest value, goes first. He rolls the die and selects the section represented by the fraction showing on top of the die. If he can use the section, he places it in the frame. If the player cannot use the section, he passes. Should he decide to take a section that cannot be used, or should he pass a section that he could have used, he loses his next turn. The star is a bonus; when it is rolled, the player may choose any section that he can use. If a section taken completes the "whole", the player wins the game. If it does not complete the "whole", the die is passed to the next player.

Source

Adapted from Helpful Hints for Teachers of Basic Mathematics, Arkansas Department of Education, Little Rock, Arkansas.

SPEEDWAY

Goals and Purposes

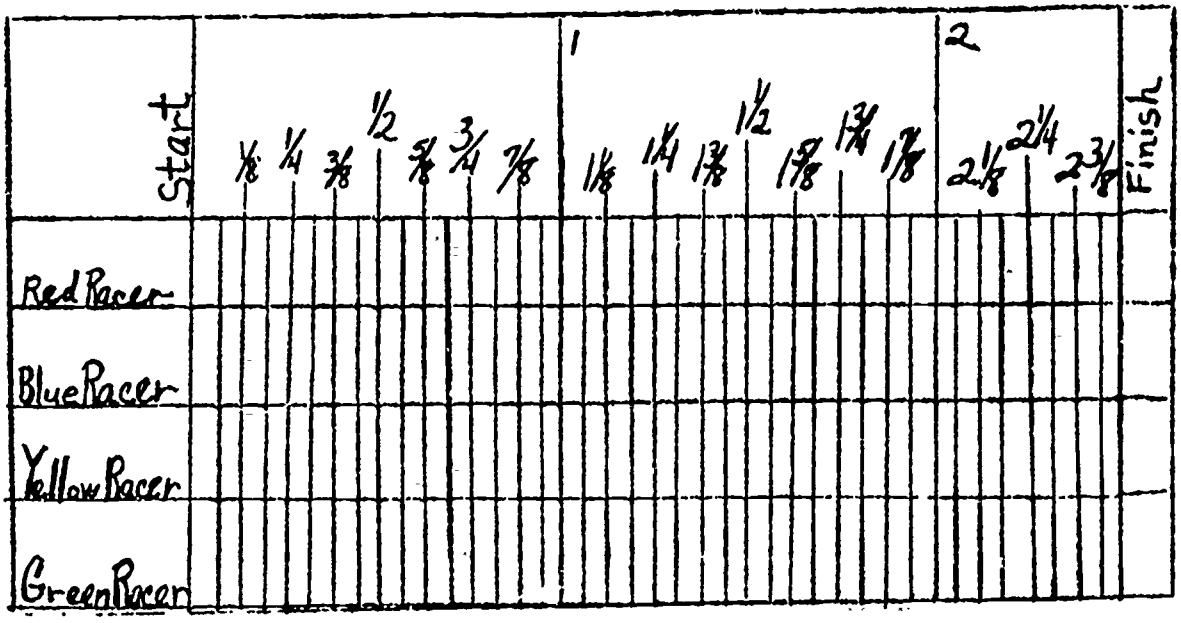
Addition practice with the ... e common fractions.

Materials

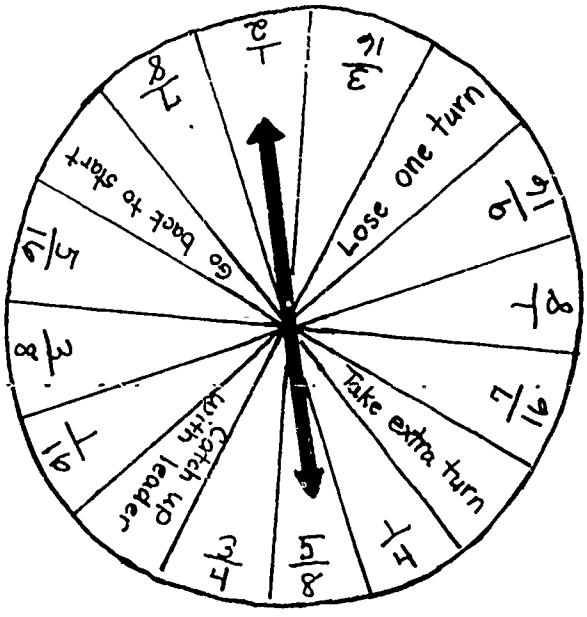
Cardboard for making game board and spinner as shown below. Small objects to be used as game pieces.

Procedure

Students should take turns with the spinner and move their playing pieces as indicated after each spin.



SPEEDWAY (cont.)



Source

Adapted from "Ideas Plus for Math Grades 4, 5, 6," University City Public Schools, University City, Missouri.

FRACTION SPIN

Goals and Purposes

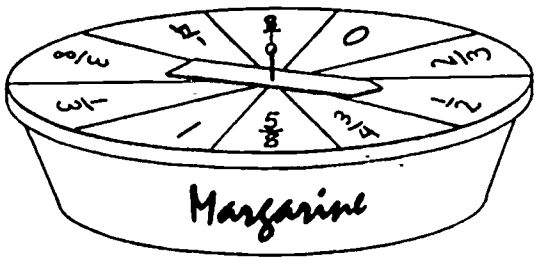
Using fractions to complete mathematical statements.

Materials

Margarine tub, heavy paper, straight pin.

Procedure

Divide the lid of a margarine tub into ten equal sectors and label as shown. Make an arrow for the indicator and attach to the center of the lid with the straight pin. Players alternate turns, writing the outcome in one of the squares on the game sheet each has. (A sample game sheet is shown.) The first player to complete 10 true statements is the winner.



SAMPLE GAMESHEET

$\square < \frac{1}{2}$ $\square + \frac{2}{3} = 1$ $\square + \frac{1}{2} = 1$

$\square > \frac{1}{2}$ $\square + \frac{1}{4} = 1$ $\square + \frac{5}{8} = 1$

$\square + \square = \frac{1}{2}$ $\square + \square = \frac{1}{4}$ $\square + \square = \frac{2}{3}$

Source

Materials developed for U.S. Army Dependents Schools; submitted by Clyde Finnell, Mathematics Curriculum Coordinator for European Area.

DIVISION OF FRACTIONS

Goals and Purposes

To provide concrete learning experiences involving division of fractions.

Materials

A number of egg cartons may be colored as in figure 1.

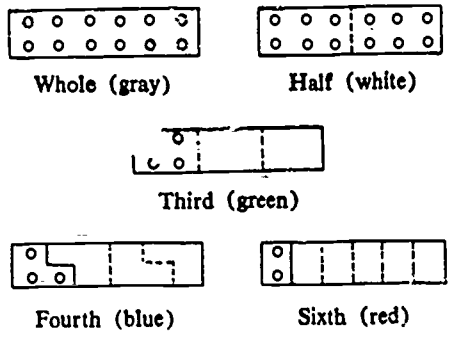
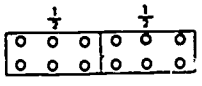


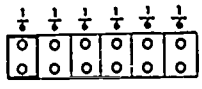
FIGURE 1

Procedure

Division problems involving fractions may be posed as "How many times will one fraction fit into another fraction?" For example: 1/2 ÷ 1/6 means "How many 1/6's are there in 1/2?" Egg cartons can be used to clarify this.

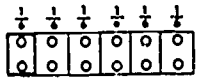


How many 1/6's are there in ½? There are 3 1/6's in ½.

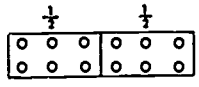


thus- $1/2 \div 1/6 = 3$ or $\frac{1/2}{1/6} = 3.$

Similarly, to illustrate 1/6 ÷ 1/2 we use egg cartons as follows:



How many ½'s are there in 1/6? There is ⅓ of ½ in 1/6.



thus - $1/6 \div 1/2 = 1/3$ or $\frac{1/6}{1/2} = 1/3.$

Source

Adapted from Hyde, David and Nelson, Marvin N. "Save Those Egg Cartons!" The Arithmetic Teacher, 14: 578-579; November 1967.

NOMOGRAPH FOR DECIMALS

Goals and Purposes

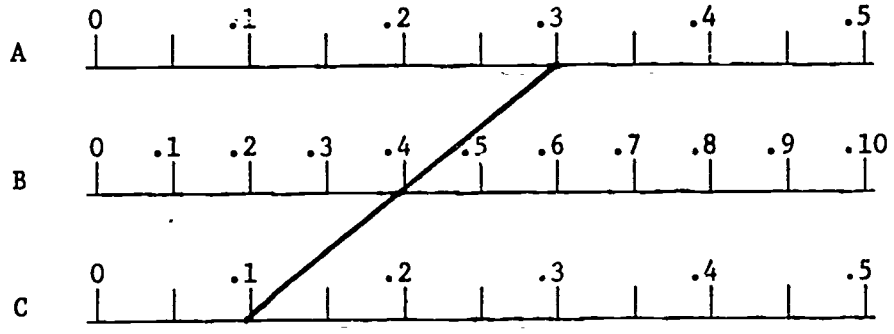
Manipulative device for addition of decimals.

Materials

Graph paper and string.

Procedure

Make 3 number lines similar to ones shown below:



Stretch the string across the three number lines, starting with number line C at point .1. Crossing number line B at point .4. Up to number line A at point .3.

Do you see how you can use this to do the addition problem, .3 + .1 = .4?

Source

Adapted from "Math Lab, Middle School," submitted by Pittsburgh Public Schools, Pittsburgh, Pennsylvania.

FRACTION - DECIMAL DOMINOES

Goals and Purposes

Recognition of equivalent fractions and decimals.

Materials

Cardboard cut into pieces approximately 2" x ".

Procedure

To make the dominoes, divide each card in half and write a fraction on one end and a decimal on the other.

Example:

3/10	.25

For best results, each fraction or decimal used should be placed on four different cards and between 32 and 48 cards work best depending on the number of players. To play, the stack of cards is placed face down and each player draws four cards. The first player selects one of his cards and places it in the center. Succeeding players may match the decimal or the fraction with its equivalent. If a person cannot play, he draws a card from the stack and play passes on. The first player to use all his dominoes is the winner.

Source

Adapted from materials submitted by Halton County Board of Education in Burlington, Ontario, Canada.

HOT SHOT

Goals and Purposes

Practice in converting from fraction to decimal equivalent.

Materials

Wastebasket, paper ball, score sheet, small box with numbers 2, 4, 5, 6, 8 inside, paper and pencil.

Procedure

Each team's score sheet should list player's name, number of shots taken, number of shots made, ratio of shots made to shots taken, and percent of shots made.

Each team should consist of five players.

Each player on one team will pick a number from the box; after the whole team has picked a number, that team will replace the numbers and another team will pick a number. Each player should stand nine feet from the wastebasket and take as many shots as his number indicated. Each team should make sure their scores are marked correctly. Each player should figure his own percentage of shots made.

Source

Adapted from "Math Lab, Middle School" submitted from Pittsburgh Public Schools, Pittsburgh, Pennsylvania.

SECTION SEVEN:

GRAPHS AND FUNCTIONS

Section 7: Graphs and Functions

The concept of function is one of the most fundamental ideas of mathematics. Many functions can be pictured and analyzed through simple graphing techniques. This section includes activities which can be used to introduce these ideas in the elementary classrooms. The suggestions contained here can be easily modified and extended to be appropriate to a wide variety of situations.

FISHING

Goals and Purposes

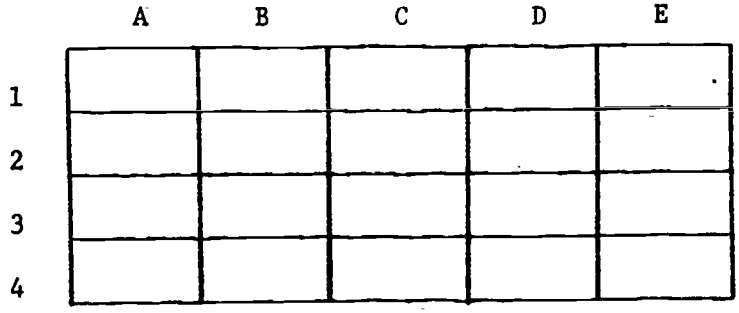
Use of a coordinate system for plotting locations.

Materials

Aquarium with at least two different kinds of fish. The aquarium could be anywhere in the building for daily or hourly observations.

Procedure

Make a grid the same size of the aquarium.



Place it behind the aquarium so when you look through the front glass it covers the back.

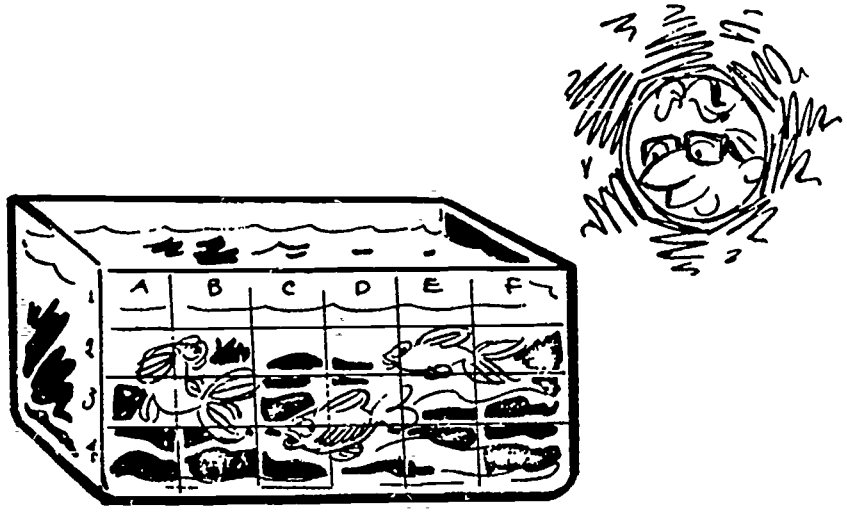
Using two or more fish (example: angel fish and guppies) plot their position in the tank at a certain time each day, such as A3, E4, etc. Do this for one week and complete the following chart. You may wish to do it hourly on a certain day. The positions shown are examples.

	MON	TUE	WED	THUR	FRI
GUPPY	A3				
ANGEL FISH	E4				

1. Determine from your data which of the fish swims highest in the water.

FISHING (continued)

2. Another interesting discovery is to place the aquarium so the sun shines in one end like so:



Do the same data collecting as before and try to determine if one fish favors the sun more than the other one.

Source

Materials developed for U.S. Army Dependents Schools; submitted by Clyde Finnell, Mathematics Curriculum Coordinator for European Area.

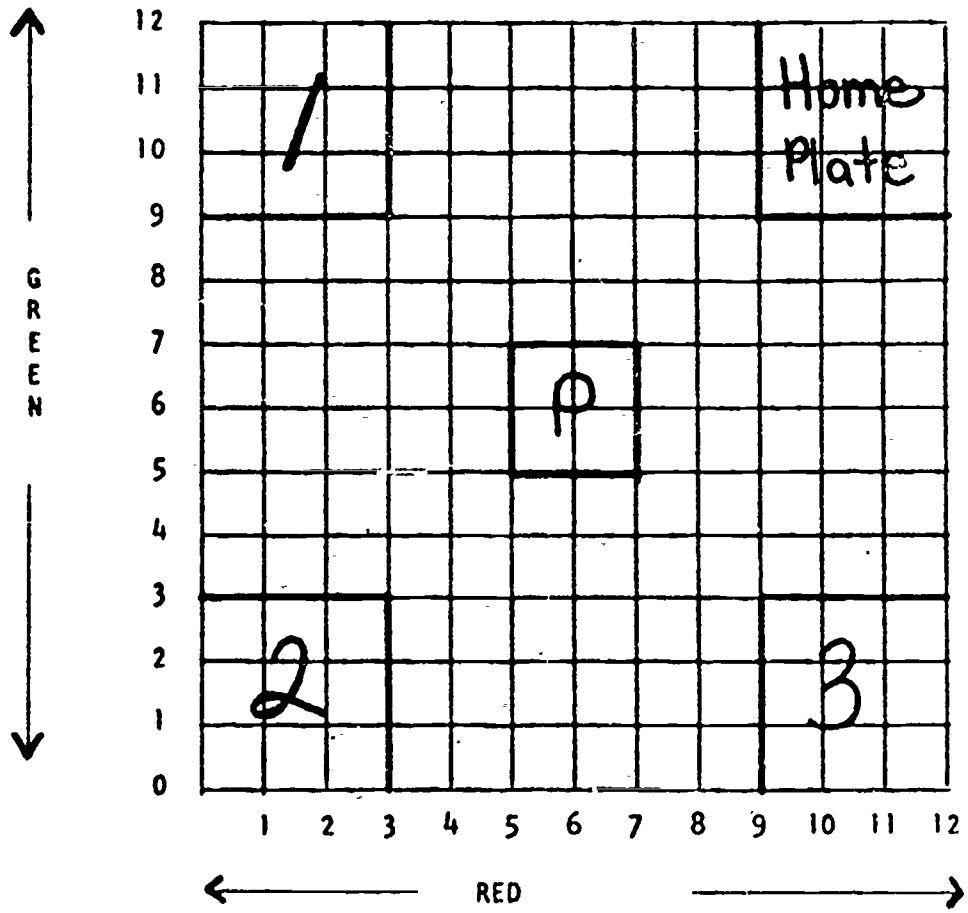
ORDERED-PAIR BASEBALL

Goals and Purposes

Introduction to a finite coordinate system and the concept of an ordered pair.

Materials

Two dice of different colors (e.g., red and green) and the following game board:



ORDERED-PAIR BASEBALL (continued)

Procedure

Roll the red dice twice and sum to find the number across. Roll the green dice twice and sum to find the number up. Plot your position and find your score according to the following table:

Pitcher's mound	- scores 3
Home plate	- scores 10
Third base	- scores 5
Second base	- scores 4
First base	- scores 3
Field (any other location)	- scores 1

For a variation which practices simple number facts and emphasizes strategy, roll a dice four times to generate four numbers. Then combine any two of the numbers by addition, subtraction or multiplication to get the first element of the number pair. The remaining two numbers must be combined to get the other element of the number pair.

Source

Adapted from material submitted by Halton County Board of Education, Burlington, Ontario, Canada.

RED AND BLUE NUMBERS

Goals and Purposes

Manipulative aid for addition of signed numbers.

Materials

Colored counters, such as red and blue poker chips.

Procedure

Let blue chips represent positive numbers and red chips represent negative numbers. To add (+3) and (+4) the students combine 4 blue chips and 3 blue chips for a total of 7 blue chips or (+7). Similarly when both are negative. When adding (+3) and (-3), students seem intuitively to know to match red and blue, one for one, so long as supply of both lasts. Remaining chips, 2 reds in this case, indicates answer of (-2).

Source

Submitted by Boyd Henry of College of Idaho, Caldwell, Idaho.

FUNCTION BOARD

Goals and Purposes

Provides a manipulative aid for graphing ordered pairs.

Materials

Large piece of pegboard with a co-ordinate axis painted on or put on with chalk or masking tape so that the position and/or scale may be changed. Package of golf tees.

Procedure

The golf tees are used to locate desired ordered pairs. Large rubber bands or string can be used to designate line segments. The board can also be used for graphing inequalities and systems of equations and may even be used as a large geo-board.

Source

Adapted from material submitted by Jane Riley, Community Consolidated School District 15, Palatine, Illinois.

LOCATING ORDERED PAIRS

Goals and Purposes

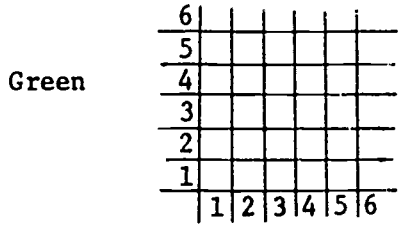
Introduction to a finite co-ordinate system and the concept of an ordered pair.

Materials

Two dice of different colors, two different colored pencils, and paper for the co-ordinate system.

Procedure

Pair students off to start the game. Make and label a 6 x 6 co-ordinate system as shown below:



Determine which color die is to correspond with which axis; for example, red with the x-axis and green with the y-axis. Have players roll the dice and the one with the highest total goes first. Dice are rolled and the results announced in ordered pair form. Player colors in the square corresponding to his roll. Turn passes to the other player who does the same. Rolling doubles allows another turn; if square is already colored, turn is passed. After all squares are colored, player with the greatest number of squares in his color is the winner.

Source

Adapted from material submitted by Halton County Board of Education, Burlington, Ontario, Canada.

SECTION EIGHT:

GEOMETRIC CONCEPTS

Section 8: Geometric Concepts

Not all mathematics concepts need be reduced to numbers. Concepts of space and orientation form perhaps the most practical mathematics possible. Thus it is important to introduce geometric activities to the mathematics laboratory early and continuously. The activities and games in this section illustrate imaginative ways to do this.

PUTTING SAM TOGETHER

Goals and Purposes

Recognition of and familiarity with geometric figures.

Materials

A "bag-o-shapes" consisting of three or four sizes each of the following geometric shapes -- rectangles, triangles, circles, semi-circles, trapezoids, parallelograms, and sectors. Dimensions should be between 1" and 3" for the best results. The shapes can be cut from heavy cardboard or masonite or 1/8" plywood. Scissors and construction paper are necessary for the student part of the work. Instruction sheet.

Procedure

Students are asked to follow the instructions given on the following pages to form a person.

Source

Adapted from material submitted by Halton County Board of Education, Burlington, Ontario, Canada.

INSTRUCTIONS

1. From the "Bag-O-Shapes", find a large 

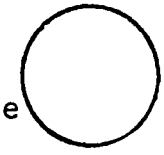
rectangle

This is Sam's body. Trace out the shape on a piece of paper. Cut out the shape and paste it in the middle of the large paper.

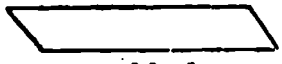
2. Find a 

triangle

This is Sam's neck and shoulders. Trace and cut out this shape. Paster the shape on Sam's body.

3. Find the large circle 

This is Sam's head. Trace and cut out this shape. Paste Sam's head in place.

4. Find a 

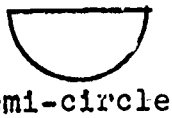
parallelogram

This is Sam's arm. Trace and cut out TWO of these shapes. Paste Sam's arms on his body.

5. Find a large 

triangle

This is Sam's leg. Trace and cut out TWO of these shapes. Paste Sam's legs on his body.

6. Find a 

semi-circle

This is Sam's foot. Trace and cut out TWO of these shapes. Paste Sam's feet onto Sam's legs.

7. Find a 
sector

This is Sam's hand. Trace and cut out TWO of these shapes Paste Sam's hands onto his arms.

8. Find a 
trapezium

This is Sam's hat. Trace and cut out this shape. Paste Sam's hat on his head.

9. Draw  Sam's eyes.

Draw  Sam's nose.

Draw  Sam's mouth.

10. Draw clothes on Sam with your crayons.

11. What is Sam doing? Draw a picture around Sam to show what he is doing.

12. Write a story about Sam.

FINDING GEOMETRIC SHAPES IN THE SURROUNDINGS

Goals and Purposes

Encourage students to find examples of uses of geometric shapes in their everyday surroundings.

Materials

Construction paper, magic markers or crayons, scissors. (As an option, catalogs may also be used.)

Procedure

Have the student select one geometric shape and cut it out of the construction paper in a fairly large size. Have him list on it places in the classroom where he sees objects of that shape. Example:

I see a rectangular shape on
1. my desk
.
.
.
10. the window

Another possibility would be to have them select items from a catalog and similarly list them according to shape.

Source

Adapted from material submitted by Halton County Board of Education, Burlington, Ontario, Canada.

GEOMETRIC BEAN BAG TOSS

Goals and Purposes

Review of the characteristics of different geometric shapes.

Materials

A large piece of heavy cardboard or masonite or pegboard. Several bean bags.

Procedure

Cut a circle, triangle, rectangle, and square in the board each large enough to throw a bean bag through. Prop the board up securely and have the students toss a bean bag through the hole made by one of the geometric shapes. He receives one point for each correct characteristic he can tell about the geometric shape corresponding to the hole his bean bag went through.

Source

Adapted from material submitted by Halton County Board of Education, Burlington, Ontario, Canada.

BUILDING SKELETONS

Goals and Purposes

To study the characteristics of solids through examination of skeletons and to construct models of useful objects in our environment.

Materials

Straws, pipe cleaners, and scissors.

Procedure

Permit the class to experiment and discover what skeletons can be built. Challenge them to build skeletons of certain objects in the room. Perhaps you could add constraints such as the number of straws or size of straws that would be allowed.

Source

Adapted from material submitted by Halton County Board of Education, Burlington, Ontario, Canada.

SPROUTS

Goals and Purposes

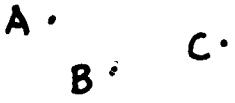
Introduction to the geometric concept of a network.

Materials

Pencil and paper.

Procedure

The game of SPROUTS is a game for 2 players. It begins with a network of 3 dots.



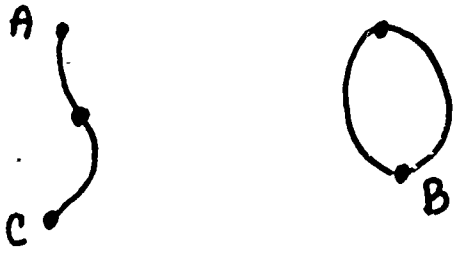
Players take turns drawing lines from one dot to another.

The object of the game is to draw the last possible line joining two dots.

The rules for drawing lines from one dot to another are as follows:

a. A line may not cross itself.
b. A line may not cross another line.
c. A line may not pass through a dot.
d. A dot that has 3 lines can not be used again as the end of a line.
e. When a line is drawn, another dot is marked on it. The winner is the player who draws the last line.

For example: Start with three points A, B, and C. Two possible moves are shown in the figure below. A to C and B to itself. At this moment A and C each have one line entering it but B has two lines. Also each new point has already two lines entering it.



Source

Materials developed for U.S. Army Dependents Schools; submitted by Clyde Finnell, Mathematics Curriculum Coordinator for European Area.

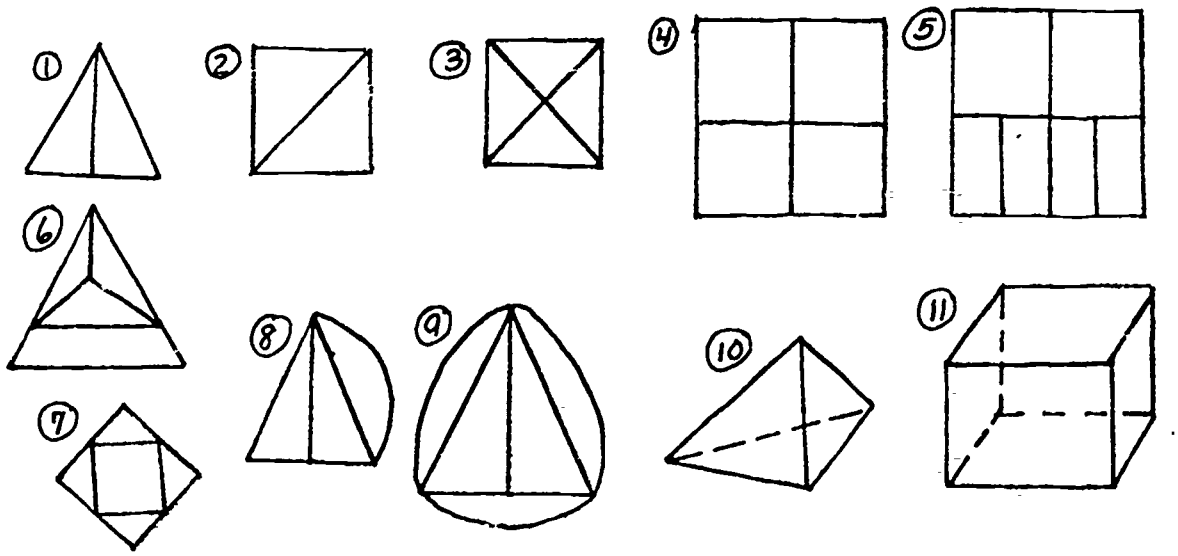
NETWORKS

Goals and Purposes

Introduction to the topological concept of networks.

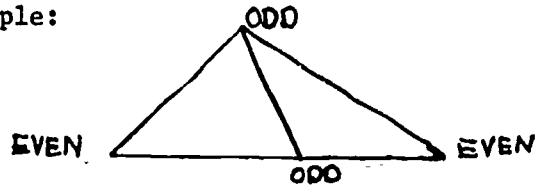
Materials

Ditto sheet containing a variety of geometric shapes (some sample ones are shown below).



Procedure

Try to draw each of the figures by starting at a point, not lifting the pencil, and not going over any line more than once. Now make a chart -- recording the number of "even" and "odd" vertices for each figure. "Even" and "odd" are determined by counting the number of line segments leaving a vertex as demonstrated in this example:



Compare the entries in the chart with whether or not the corresponding figure could be traced. Can you determine a rule that will explain when any given figure can be traced?

Source

Adapted from "Math Lab, Middle School," submitted by Pittsburgh Public Schools, Pittsburgh, Pennsylvania.

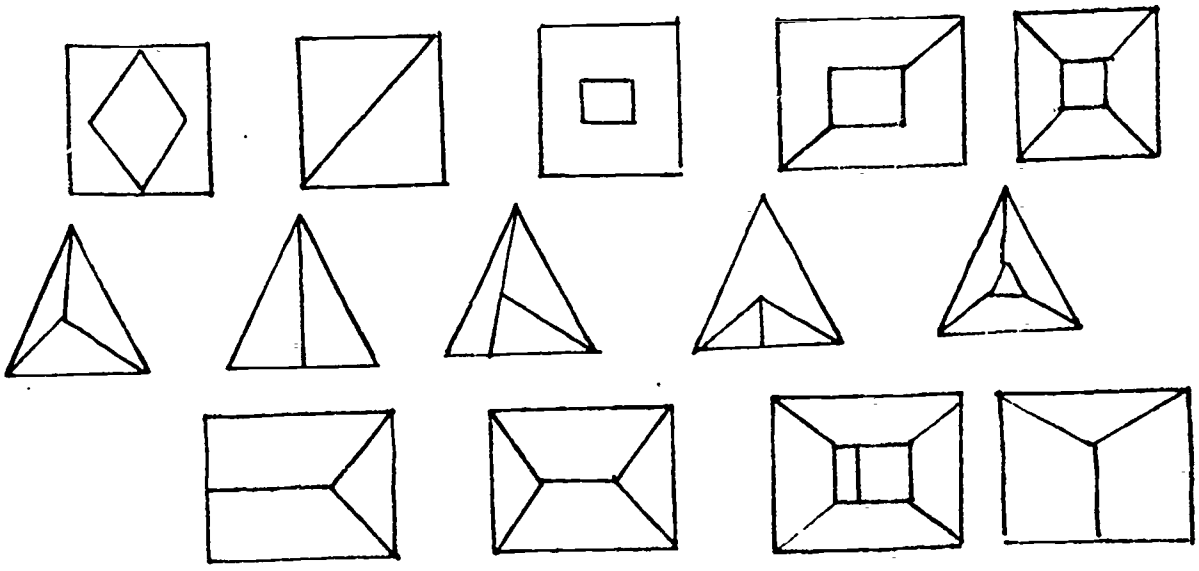
MAP COLORING

Goals and Purposes

An intuitive attempt to discover the minimum number of colors necessary to color any 2-dimensional map.

Materials

Crayons or colored pencils, a ditto sheet of various figures (samples shown below). Actual maps, such as those showing states in U.S. or counties in a state, are very useful after initial practice on the ditto.



Procedure

Color each section of the above "maps." Use as few colors as possible, but sections with a common edge should not be colored the same. What seems to be the most colors ever needed? The proof of this is still an unsolved mathematical problem.

Source

Adapted from "Math Lab, Middle School," submitted by Pittsburgh Public Schools, Pittsburgh, Pennsylvania.

A MOEBIUS STRIP

Goals and Purposes

Acquaintence with an interesting topological property.

Materials

Roll of adding machine or cash register tape, scissors, paste (glue, or tape).

Procedure

1. Cut a strip in the shape of a long, thin rectangle.
2. Now bend the strip to connect the two ends.
3. Paste or tape the ends together.
4. Make a pencil line down the middle of the loop.
5. Cut along the line. How many loops?
6. Use another strip of paper and twist it once before taping.
7. Cut along the line. How many loops?
8. Try giving 2, 3, and 4 twists before cutting.
9. Record your results.

Source

Materials developed for U.S. Army Dependents Schools; submitted by Clyde A. Finnell, Mathematics Curriculum Coordinator for European Area.

SYMMETRY IN LETTERS

Goals and Purposes

Recognition of lines of symmetry in letters of the alphabet.

Materials

Paper and pencil.

Procedure

Print all the letters of the alphabet on a sheet of paper. Place the line of symmetry through each letter. Which letters have at least one line of symmetry? Could any letters have more than one line of symmetry? If so, which letters? Are there any letters without a line of symmetry? Arrange the letters in the word MATH so that you have a common line of symmetry. List four words in which the letters have a common line of symmetry.

Source

Adapted from "Math Lab, Middle School," submitted by Pittsburgh Public Schools, Pittsburgh, Pennsylvania.

A GAME WITH ATTRIBUTE BLOCKS

Goals and Purposes

To provide experiences for learning the various attributes of three-dimensional shapes via logical reasoning.

Materials

A set of 48 attribute blocks consisting of circles, squares, oblongs, and triangles in red, yellow, green and blue with three different thicknesses. Blocks may be made from colored poster board, with several layers pasted together to vary thickness, or they may be purchased commercially. No two blocks must be alike in all 4 of the attributes size, thickness, color, and shape.

Procedure

To encourage children to become conscious of such differences and similarities, the difference games are introduced. One child puts down a piece out of the set. The next child puts down a piece which is different from the first piece in only one attribute. The next player (or the first, if there are only two playing), puts down a third piece which differs from the second piece in only one attribute. The game goes on in this way until all, or nearly all, the pieces in the set have been placed in a row. Each player has the right to challenge the previous player. If the challenge is justified, he gains a point. If he wrongly challenges a previous player, he loses a point. Each successful, unchallenged placement of a piece gains a point. So, points can be gained by playing a piece according to the rules and by discovering that the opponent has broken the rules. The child with the largest number of points wins the game. The fact that the players are allowed to challenge each other encourages them to concentrate not only on their own moves but also on the moves of other players. Elaborations of the one-difference game would be a two-, three-, and even four-difference game. Children will want to set up their own rules after some experiences have been developed.

Source

Rogers, Lloyd V. "Australian Demonstration in California: Dr. Dienes and Mathematics Learning." The Arithmetic Teacher, 11: 359-360; May, 1964.

COMPLETE THE SQUARES

Goals and Purposes

Cooperative work in solving geometric puzzles.

Materials

6" x 6" cardboard squares, large envelopes.

Procedure

Prepare several sets of squares so that you have enough for one set for each group of five students. The set consists of five envelopes containing pieces of cardboard cut into patterns that will form five 6" x 6" squares as shown in the diagrams following. Several ways exist to make individual squares but only one way exists to make all five squares.

Cut each square into parts A through J and lightly pencil in the letters. Then label the five envelopes I, II, III, IV, V and distribute the pieces like so:

Envelope #	Pieces
I	I, H, E
II	A, A, A, C
III	A, J
IV	D, F
V	G, B, F, C

Now erase the lightly penciled letters and write instead the number of the envelope to which the piece belongs so it is easy to place them in the envelopes after the exercise.

Divide the class into groups of five and seat each group around a table. Pass out the envelopes. Read the following instructions to the group:

1. Your task is for each person to make a 6" x 6" square. Your group is not finished until every member has the square completed.

2. You MUST NOT TALK or leave your seat during the exercise.

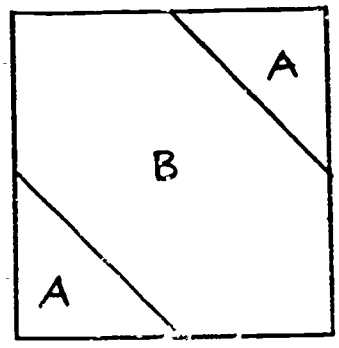
3. You may take pieces from anyone else if you think the piece will help you. You may NOT have more than four pieces at any one time.

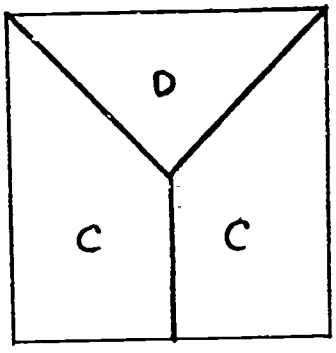
4. Your group will be timed.

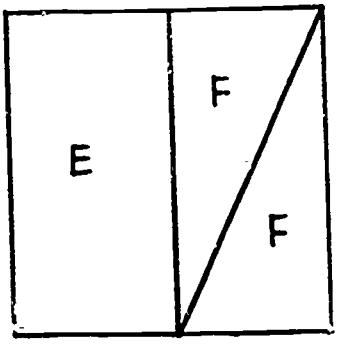
COMPLETE THE SQUARES (continued)

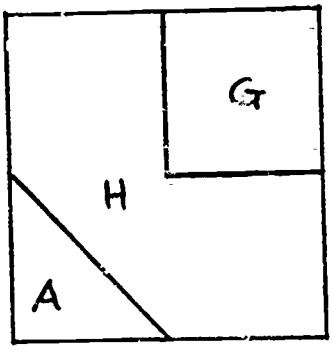
The same rules apply to this game except the following:

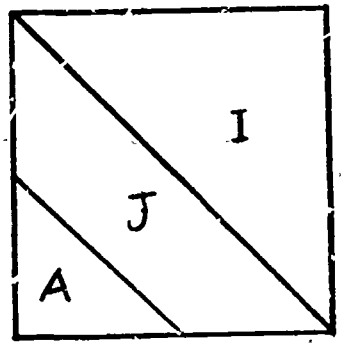
You may GIVE pieces to other people if you think they can use them. You must have no less than two pieces at all times.











Source

Materials developed for U.S. Army Dependents Schools; submitted by Clyde Finnell, Mathematics Curriculum Coordinator for European Area.

RIGID FIGURES

Goals and Purposes

Formation of geometric plane figures and study of their rigidity.

Materials

Cardboard strips and paper fasteners.

Procedure

Make a skeleton model of a square using the cardboard strips and paper fasteners. Add one or more cardboard strips as necessary to make it rigid. Do the same for a triangle, rectangle, pentagon, hexagon, and octagon. What is the strongest shape? What do you discover about the number of sides of a plane figure and the number of triangles formed when it is made rigid?

Source

Submitted by Halton County Board of Education, Burlington, Ontario, Canada.

SUMMING TO 180°

Goals and Purposes

An intuitive demonstration of the sum of the measures of the angles of a triangle being 180 degrees.

Materials

Piece of wood, 2 nails, hammer, large rubber band, wire bent to form a hook.

Procedure

Hammer two nails into the wood at points A and B as illustrated in figure 1. Stretch the rubber band around the nails. At the midpoint, pull up on one side of the rubber band so that an isoceles triangle is formed. Examine the sum of the measures of the angles of various triangles formed as in figure 2. At one point an equilateral triangle will be formed and its additional properties may be discussed. Pulling up at a point other than the midpoint will allow scalene triangles to be formed and these triangles can be examined similarly. A good discussion could be based on what happens as the measures of two of the angles approach either 0° or 90°.

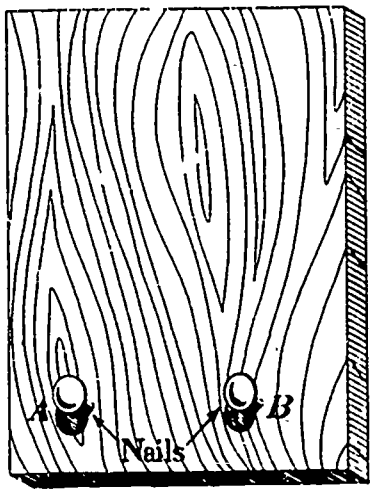


Fig. 1

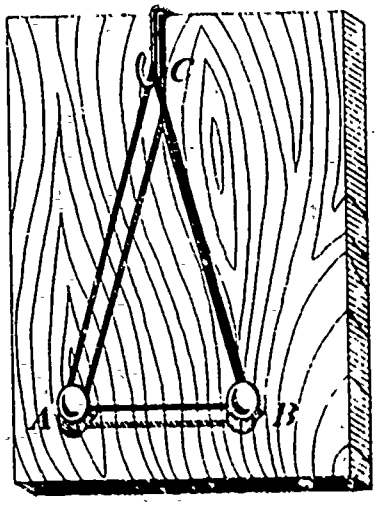


Fig. 2

For Further Reference

Bruni, James V. "A 'Limited' Approach to the Sum of the Angles of a Triangle," The Arithmetic Teacher, 19: 85-87 February 1972 .

SUM OF THE ANGLES OF A TRIANGLE

Goals and Purposes

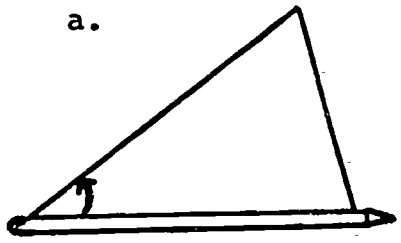
Lead the students to make an intuitive guess as to the sum of the measures of the angles of any triangle.

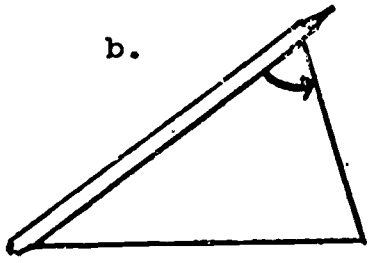
Materials

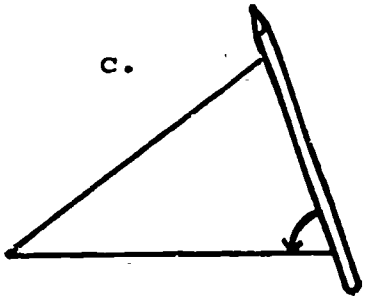
Pencil and paper.

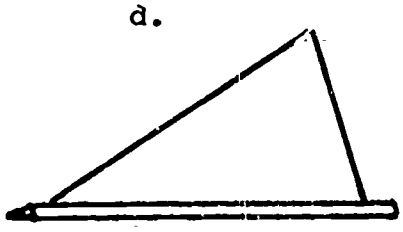
Procedure

Draw any triangle. Place a pencil along any side of the triangle and turn it about each angle successively as shown in figures a - d. What do you notice about the final position of the pencil as compared with its first position? What would you then guess about the sum of the measures of the angles of any triangle?









The procedure may be repeated for quadrilaterals, pentagons, and hexagons.

Source

Adapted from material submitted by Halton County Board of Education, Burlington, Ontario, Canada.

CONSTRUCTION OF A RIGHT ANGLE

Goals and Purposes

Discovery of a method for construction a right angle, and the relationships between diameters and chords of a circle.

Materials

A piece of heavy cardboard about 2' square, with a circle of 10" radius drawn on it and the center labeled. Thumbtacks, a yardstick, and two pieces of yarn each 30" long (preferably of two different colors), protractor.

Procedure

Position the yardstick on the cardboard so that the center of the circle is on one edge of the yardstick. On this same edge of the yardstick, put a thumbtack at each of the two places where the yardstick crosses the circle. Connect the two thumbtacks with a piece of yarn, stretched to form a diameter. Pick any other point on the circle and place a thumbtack there. Connect the three thumbtacks with the second piece of yarn so that the ends of the yarn are at the ends of the diameter. Measure the angle formed by the second piece of yarn with a protractor.

Source

Adapted from material submitted by East Syracuse-Minoa Central Schools, East Syracuse, New York.

GETTING TO THE CENTER OF THINGS

Goals and Purposes

Determining the center of a circle.

Materials

Compass and scissors.

Procedure

Use the compass to draw a circle and cut it out. Fold the circle to make the edges meet. Open it back up and then fold it another way. Repeat several more times. What do you notice?

Source

Materials developed for U.S. Army Dependents Schools; submitted by Clyde Finnell, Mathematics Curriculum Coordinator for European Area.

INSCRIBED REGULAR HEXAGON

Goals and Purposes

Use of measurement as an aid in discovering a method for construction of a regular hexagon.

Materials

A regular hexagon inscribed in a circle on individual dittoed paper or a larger model constructed on cardboard. For best results the radius should be at least 3''. Rulers.

Procedure

Have students measure the length of each side of the hexagon and the radius of the circle. Compare these lengths and see if they can use this information to discover a method for constructing a regular hexagon. Have them construct one of a certain size.

Source

Adapted from material submitted by Halton County Board of Education, Burlington, Ontario, Canada.

CENTER OF GRAVITY

Goals and Purposes

Use of a method for determining the center of gravity for any plane figure.

Materials

Cardboard, scissors, paper clips, rubber band or string.

Procedure

With the cardboard cut out a circle and square at least 6 inches across. Then cut out a very irregular shaped object.

You are now ready to find the center of gravity of each of these pieces of cardboard.

Straighten one end of a paper clip and punch three holes anywhere in the circle.

Stick the end of the paper clip through one of the holes, holding clip in back, let the cardboard figure swing freely and come to rest. Then place the rubber band or string on the paper clip so that the weight at the other end hangs straight down.

Now draw a line along the string or rubber band. Repeat the process for another hold. Where the lines cross would be the center of gravity. You may use the third hole to check yourself. Make a hole where the lines cross and see if the circle spins freely in an upright position on the paper clip.

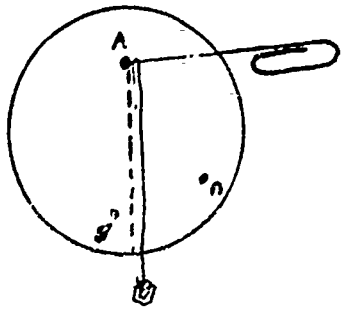


Fig I

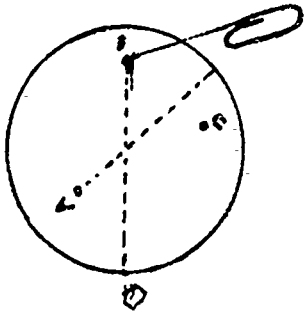


Fig II

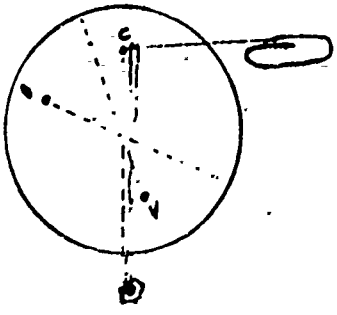


Fig III

CENTER OF GRAVITY (continued)

Do the same for the square and the irregular shaped object.

Using an outline map of the United States, a single state, Germany, or England, place the map on heavy cardboard with glue or paste. Using the same procedure find the geographical center of each map. How far do you live from the center?

Source

Materials developed for U.S. Army Dependents Schools; submitted by Clyde Finnell, Mathematics Curriculum Coordinator for European Area.

DOT GAME

Goals and Purposes

Encourage logical thinking; introduction to square regions.

Materials

A piece of poster board should have four rows of five dots each evenly spaced on it. The board should be covered with contact paper so that the crayons or felt-tip markers used for drawing can be wiped off.

Procedure

Start anywhere and connect dots by straight lines trying to form as many squares as possible. The marker may not be lifted from the paper and no line may be crossed or retraced. When you finally end in a blind alley, score as follows: complete square - 1 point, 3 sides - 3/4 point, 2 sides - 1/2 point, and 1 side - 1/4 point.

Source

Adapted from material submitted by Jane Riley, Community Consolidated School District 15, Palatine, Illinois.

LARGEST POSSIBLE AREA

Goals and Purposes

Formation of geometric figures and determination of their areas.

Materials

String, rulers, and graph paper.

Procedure

Take a length of string about 2' long. Lay it on top of a piece of graph paper and form various geometric figures with it such as triangles, rectangles, circles, squares, etc. Determine the area of each figure by counting the number of squares enclosed. For a given perimeter, what geometric figure has the largest area.

Source

Adapted from material submitted by East Syracuse-Minoa Central Schools in East Syracuse, New York.

AREA OF A TRIANGLE

Goals and Purposes

Discovery of a formula for the area of a triangle.

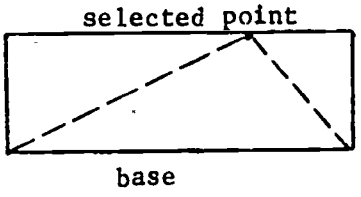
Materials

Paper, pencil and sissors.

Procedure

Cut a rectangle from a sheet of paper. Call any side the base and select any point on the opposite side. Connect this point with the endpoints of the base.

Example



Cut along these lines making three triangles. Rearrange the two smaller triangles to cover as much of the larger triangle as possible. Compare the area of the larger triangle to the area of the original rectangle. Can you guess a formula for the area of any triangle?

Source

Material submitted by Halton County Board of Education, Burlington, Ontario, Canada.

NON-RECTANGULAR AREAS

Goals and Purposes

To develop formulas for area of parallelograms, triangles, trapezoids and circles if the formula for computing the area of a rectangle is known.

Materials

Scissors, several copies of parallelograms, triangles, trapezoids and circles cut from tagboard. A ruler and protractor will increase accuracy but are not necessary.

Procedure

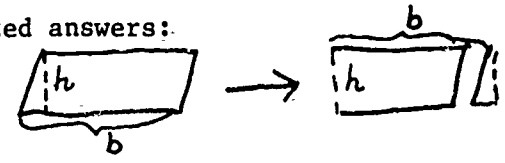
1. Cut a parellelogram into two pieces so that the two pieces can be fit together to form a rectangle. How would you compute the area of the rectangle? What measurement of the parallelogram would you need to know?

2. Find two identical triangles, and use them to form a parallelogram. Can you always do this with any two identical triangles? If you know how to compute the area of a parallelogram, how could you compute the area of <u>one</u> triangle?

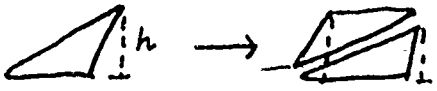
3. Find two identical trapezoids and use them to form a parallelogram. If you know how to compute the area of the parallelogram, how could you compute the area of the trapezoid?

4. Cut the circle into pieces that can be rearranged into a shape <u>close</u> to a rectangle. (Hint: don't "destroy" the radius.)

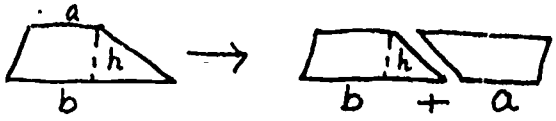
Suggested answers:

1. 

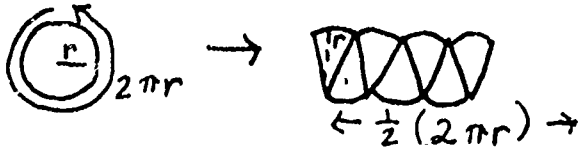
$V = h \times b$

2. 

$V = \frac{1}{2}h \times b$

3. 

$V = \frac{1}{2}(a + b)h$

4. 

$V = (\pi r)r = \pi r^2$

Source

Adapted from "Math Lab, Middle School," submitted by Pittsburgh Public Schools, Pittsburgh, Pennsylvania.

IDENTIFYING SIMILAR FIGURES

Goals and Purposes

Increasing the ability to identify similar figures.

Materials

Geometric figures you can cut out for comparison, such as those shown on the following two pages.

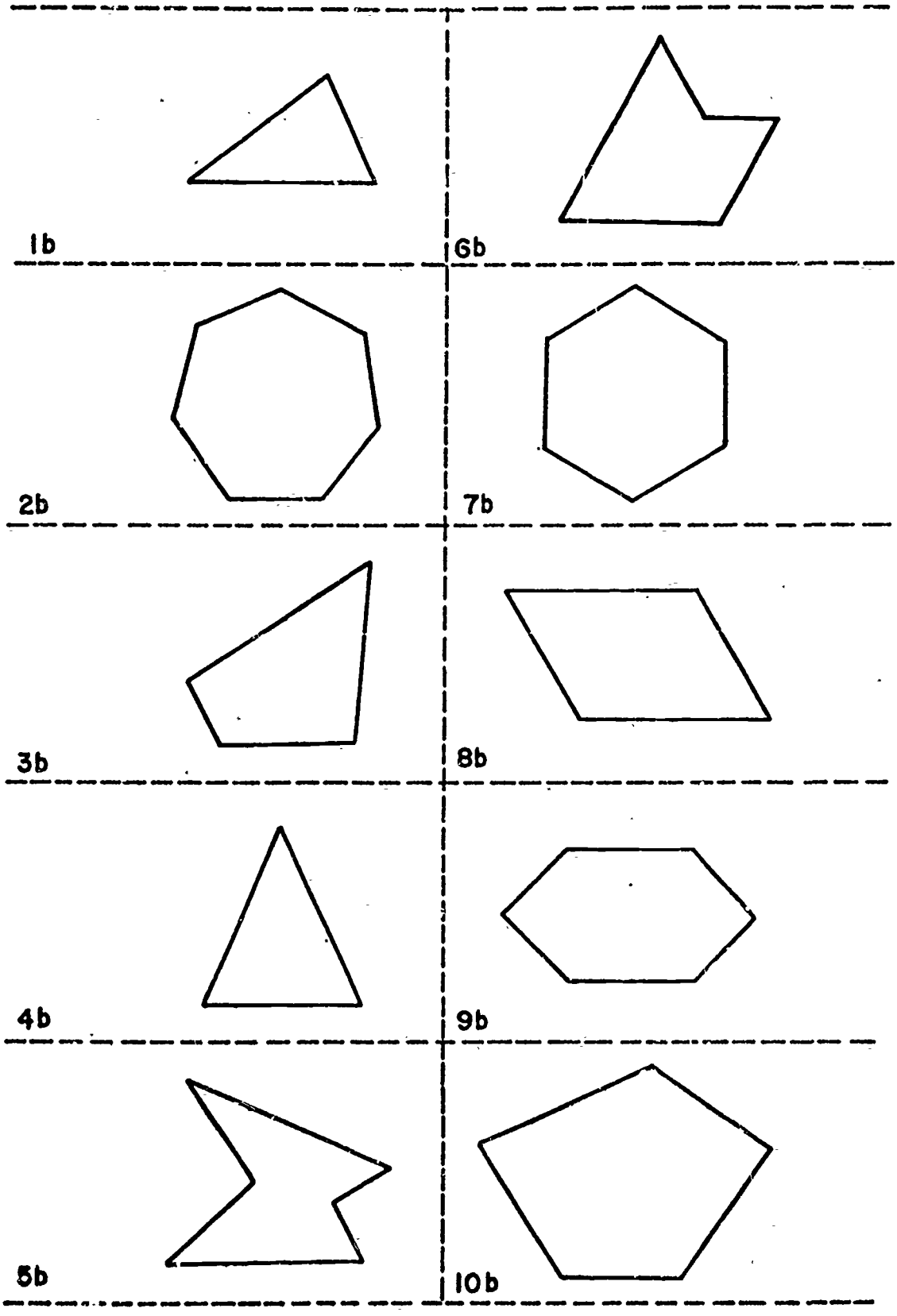
Procedure

Cut out figures 1a and 1b. Try placing one over the other and holding them up to the light. Move them around, flip one over, compare the angles; continue until you see the similarity or are satisfied that they cannot possibly be similar. Do the same with the remaining pairs of figures.

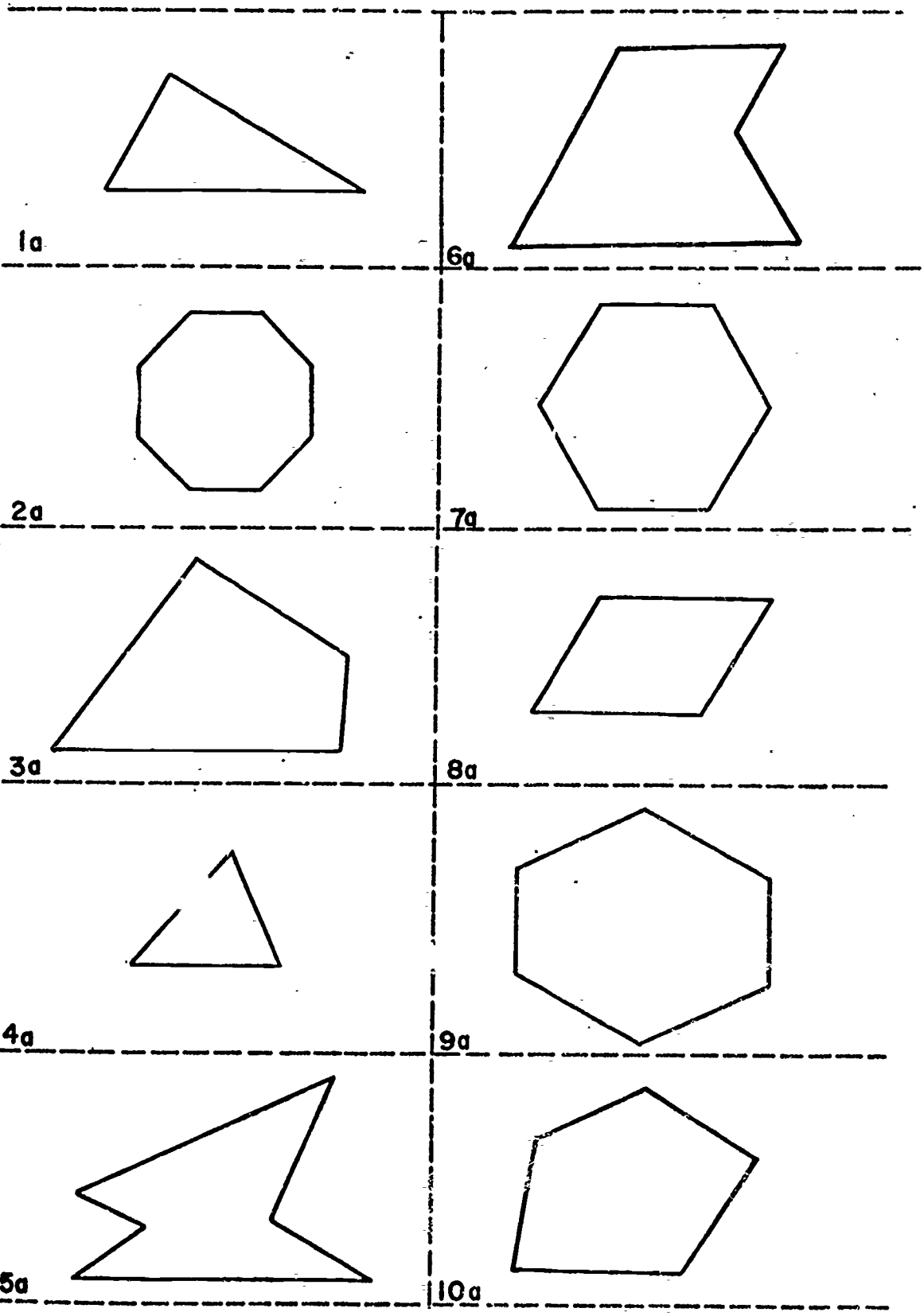
Source

Adapted from Similarity and Congruence, submitted by Oakland County Mathematics Project, Pontiac, Michigan.

Identifying Similar Figures
Worksheet



Identifying Similar Figures
Worksheet



FORMING SIMILAR FIGURES

Goals and Purposes

Combining two or more geometric figures to make another figure which is similar to a third given figure.

Materials

Worksheet with geometric figures to be cut out. Worksheet containing instructions and questions.

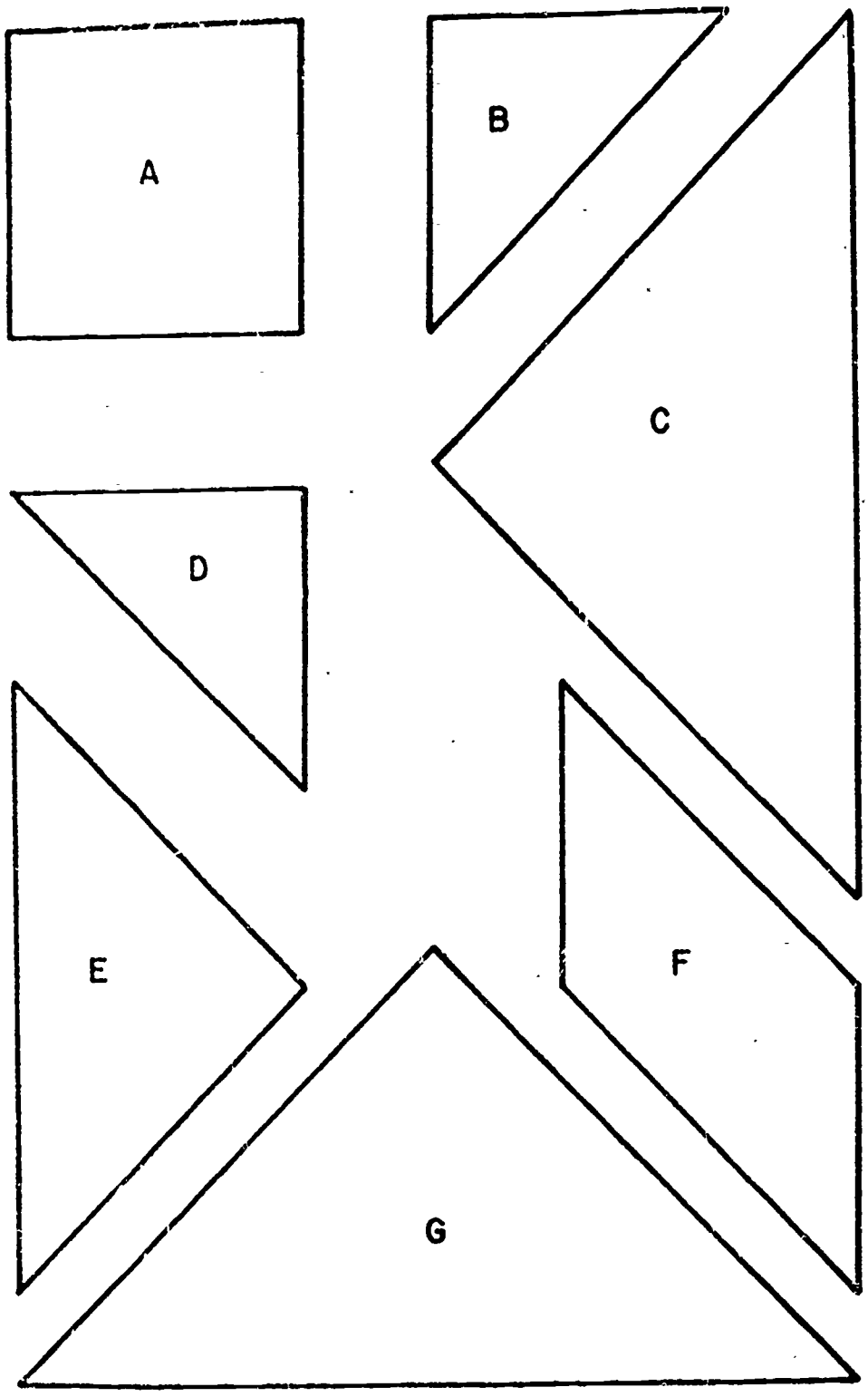
Procedure

After cutting out the figures, have students answer the questions on the second worksheet. Have them make sketches of their solutions to compare with others. (A solution sheet is included in case some escape you -- some have more than one solution.)

Source

Adapted from Similarity and Congruence, submitted by Oakland County Mathematics Project, Pontiac, Michigan.

WORKSHEET OF GEOMETRIC FIGURES



WORKSHEET OF INSTRUCTIONS AND QUESTIONS

Cut out insert A and carefully cut out each figure. Which figures are similar to each other? _____ Show how figures B and D can be combined to form a figure the same size and shape as figure A, E or F. Name some combinations which will form a figure the same size and shape as figure C.

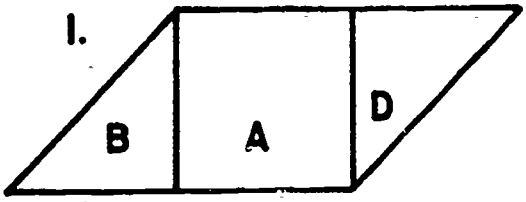
Are the statements true or false?

1. Figures A, B and D will form a figure similar to figure F.
2. Figures B, D and E will form a figure similar to figure A.
3. Figures A and B will form a figure similar to figure E.
4. Figures C and G will form a figure similar to figure F.
5. Figures B, D and F will form a figure similar to figure A.
6. Figures D, E and F will form a figure similar to figure A.
7. Figures G and C will form a figure similar to figure E.
8. Figures A, B, D, E and F will form a figure similar to figure C.
9. Figures B, C, D and E will form a figure similar to figure F.
10. Figures D, B, E and G will form a figure similar to figure F.
11. All the figures will form a figure similar to figure A.
12. All the figures will form a figure similar to figure C.

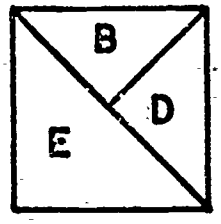
SOLUTIONS

A solution is given for each statement that is true. Several statements have more than one solution. Use the solutions only as a last resort. Students enjoy seeing the teacher "stumped" on occasion.

1.

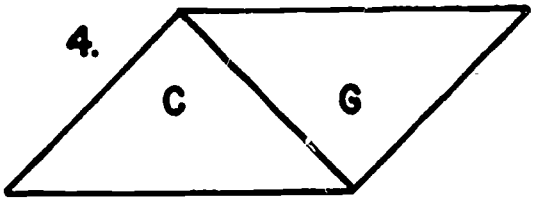


2.



3. FALSE

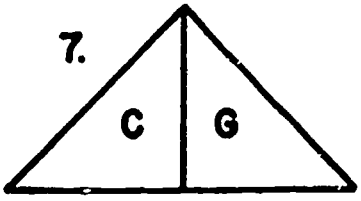
4.



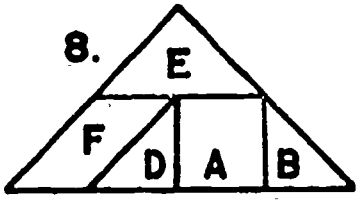
5. FALSE

6. FALSE

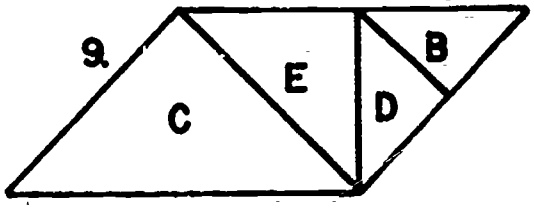
7.



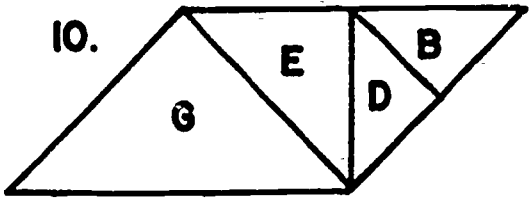
8.



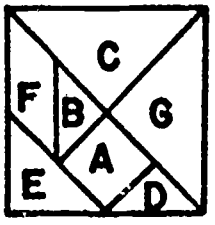
9.



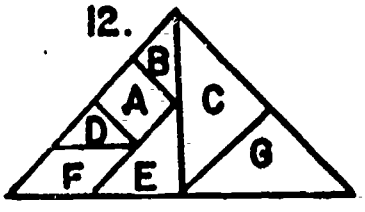
10.



11.



12.



SUGAR CUBE

Goals and Purposes

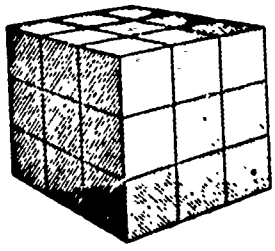
Development of spatial reasoning.

Materials

Sugar cubes and food coloring.

Procedure

Stack the sugar cubes to form a larger cube like the one shown. Color the outside of the large cube with food coloring.



Questions

1. How many sugar cubes were used? _____

2. How many sugar cubes are painted:
   a. on four sides? _____
   b. on three sides? _____
   c. on two sides? _____
   d. on one side? _____
   e. on zero sides? _____

3. Imagine if you stacked 64 sugar cubes in the same way and painted the outside. Answer questions 2a through 2e for this cube and write your answers below:
   a. _____
   b. _____
   c. _____
   d. _____
   e. _____

Source

Materials developed for U.S. Army Dependents Schools; submitted by Clyde Finnell, Mathematics Curriculum Coordinator for European Area.

A PERFECT MATCH

Goals and Purposes

Introduction to the concept of congruence.

Materials

Data sheet (as suggested on the accompanying page) showing the keys and asking several questions. Scissors.

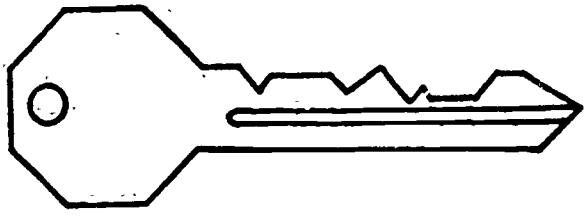
Procedure

After students have visually selected their answers, pass out scissors so that the top key may be cut out and their answers either verified or revised. A collection of real keys could be made and the matching process applied to them.

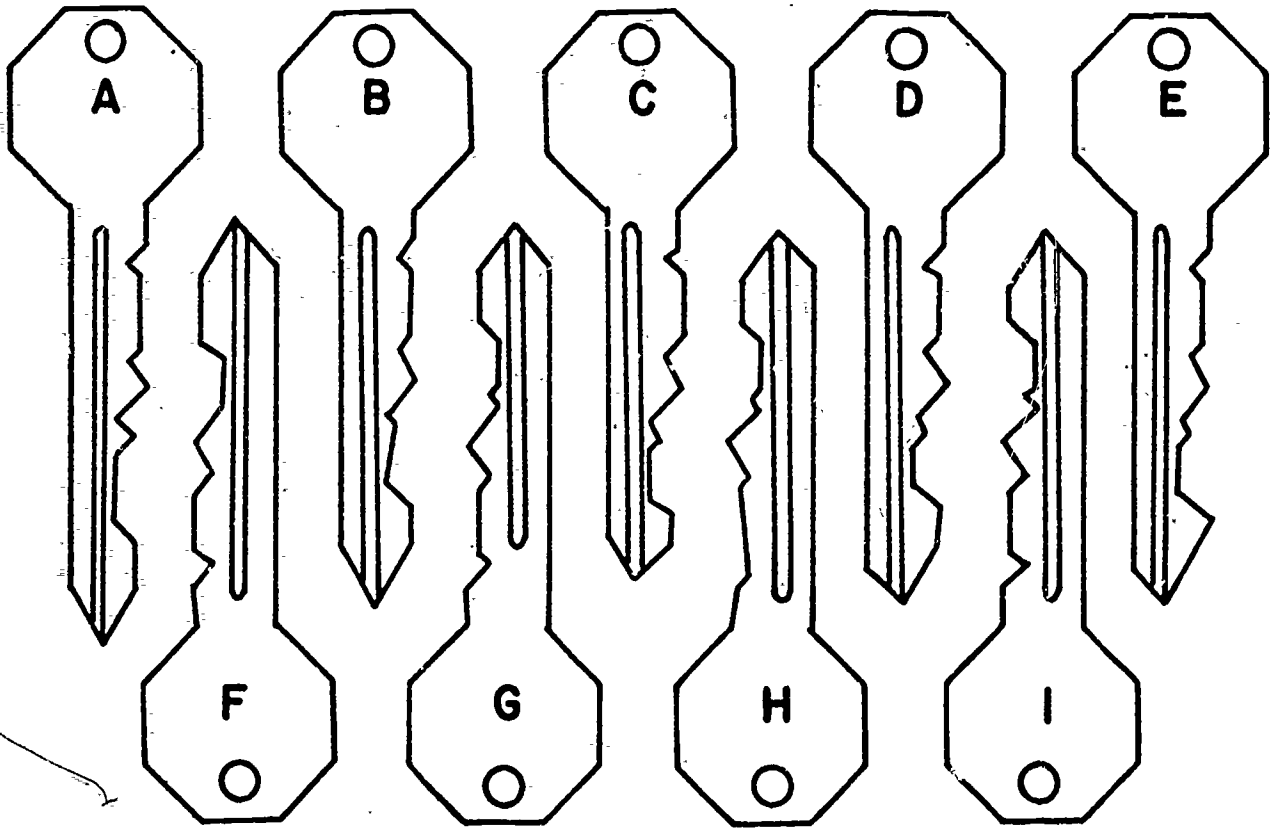
Source

Adapted from Similarity and Congruence, submitted by Oakland County Mathematics Project, Pontiac, Michigan.

DATA SHEET



One of the keys below will unlock the same door as the key above. Can you find it? Place a circle around your suggestion.



DISCUSSION QUESTIONS

1. Why do you think the key you selected will unlock the door?
2. Can you think of a way to prove your selection is correct?
3. Explain your reasons for "ruling out" each of the other keys.

MAKING CUBES

Goals and Purposes

Manipulation of solid geometric figures and development of space perception.

Materials

A sheet of heavy paper ruled into squares. Scissors.

Procedure

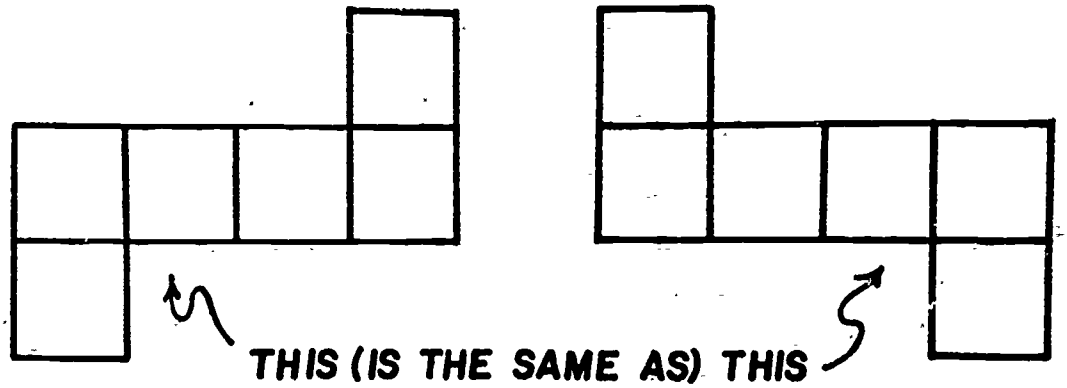
Show the students one pattern (such as the one shown on the accompanying sheet) that can be folded into a cube. Ask them to find as many different patterns as possible that will also fold into a cube. The scissors may be used to cut out and try any pattern that the student may not be able to visualize. A solution sheet is also included.

Source

Adapted from Geometric Excursions, submitted by Oakland County Mathematics Project, Pontiac, Michigan.

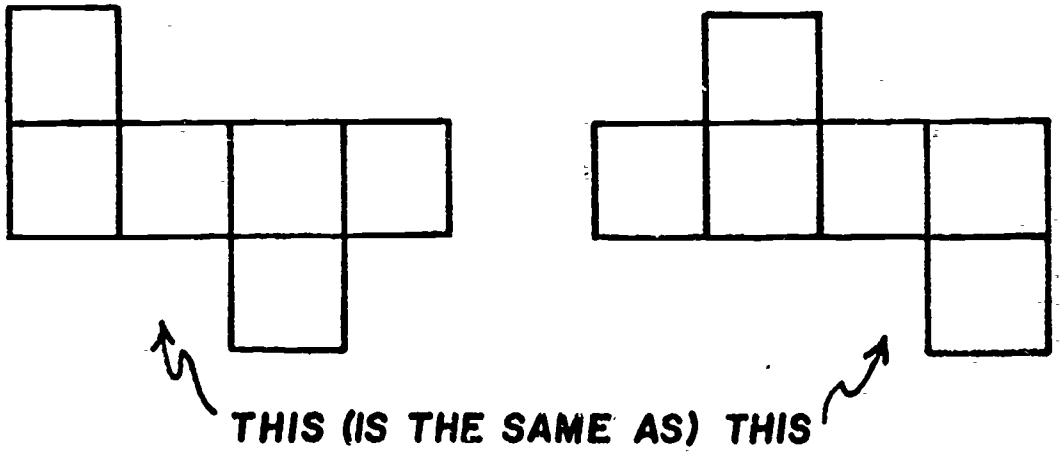
CAUTION:

Make sure each pattern you record is different.



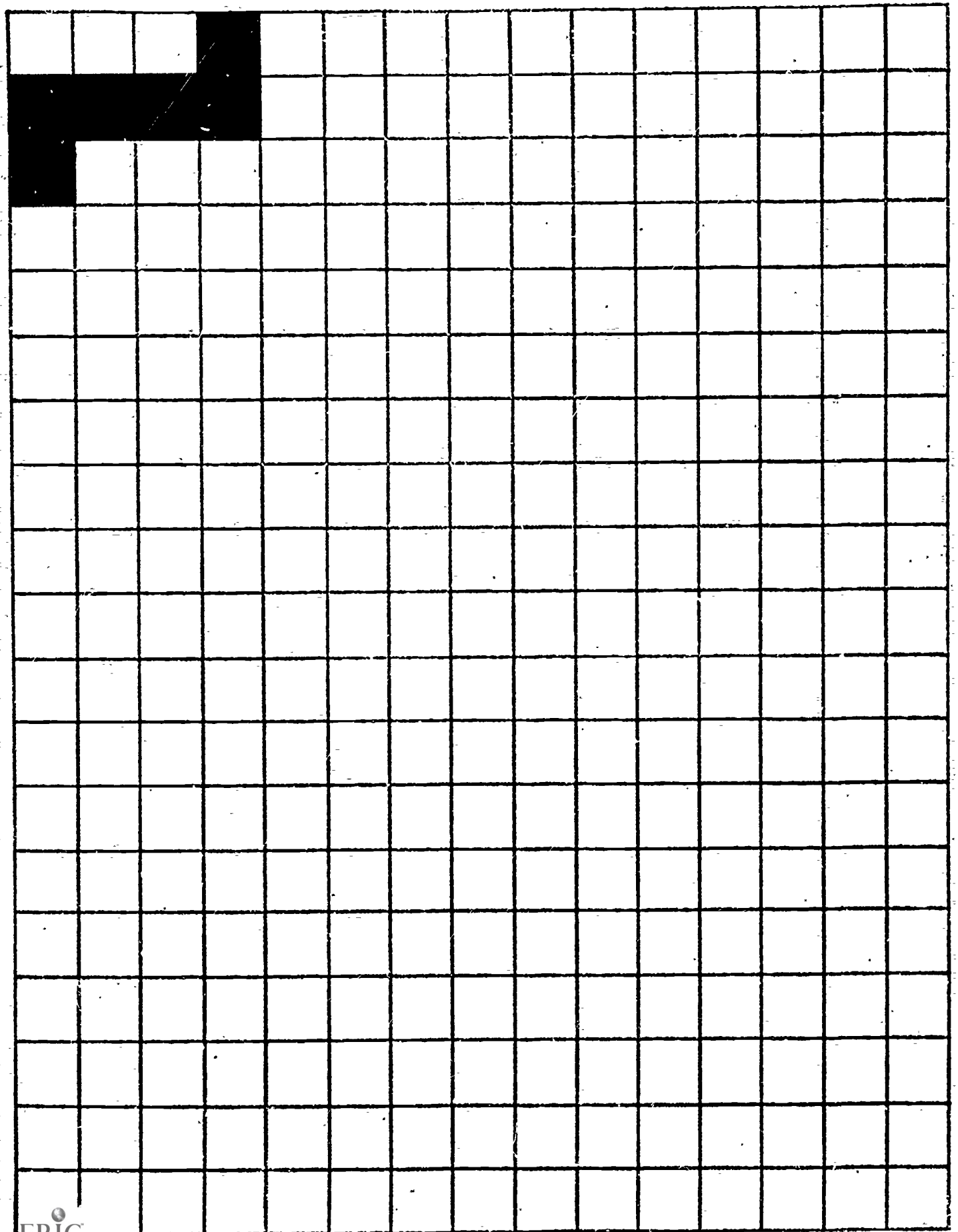
These two patterns are the same. The first can be flipped to get the second.

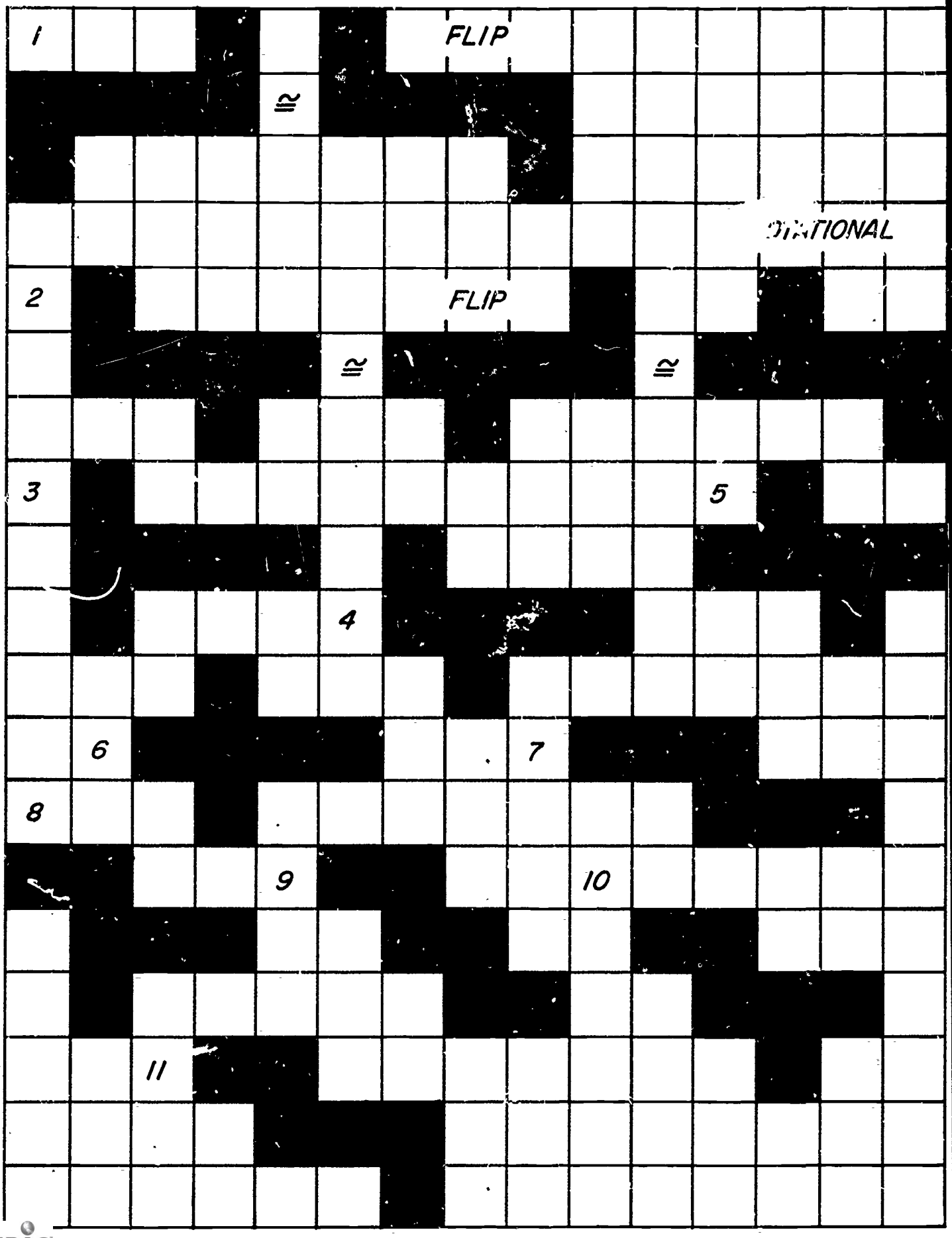
These two are also the same:



The first has been rotated to get the second.

REMEMBER – no flips or rotations.




1
FLIP
DIAGONAL
2
FLIP
3
5
4
6
7
8
9
10
11

CHARACTERISTICS OF SOLIDS

Goals and Purposes

To guide the student through handling of the solids in making discoveries concerning the characteristics of solids.

Materials

A set of solids (preferably wooden or strong plastic) consisting of a sphere, cube, cone, cylinder, rectangular prism, and possibly other prisms and pyramids.

Procedure

As they examine the solids, have each student make a list of the characteristics that are common to all of the solids (eg. all are 3-dimensional) and a list of characteristics that are common to some of the solids (eg. some have edges, some roll, etc.) Given a set of solids, ask each student to group them into subsets on the basis of some characteristics and explain the basis for this particular grouping.

Source

Adapted from material submitted by Halton County Board of Education, Burlington, Ontario, Canada.

TREASURE HUNT

Goals and Purposes

Combining computational skills, reading of a compass, and determination of directions in a discovery adventure.

Materials

Compasses, 3 x 5 cards prepared with directions similar to example:

Starting Point No. 4

*Direction	Feet
325°	40
140°	50
90°	20

*All directions indicated are directions from North

Procedure

First, have students compute the number of feet in their walking pace. This can be done by dividing a measured distance, say 100 feet, by the number of steps it takes them to walk the distance. Previously the teacher should have selected five or more starting places on the school grounds and mapped out a compass route for each starting point on a 3 x 5 card. The routes should all end at the same landmark (or treasure if you desire). The students are divided into groups and given the compass route card. Object is for the whole group to get to the final landmark before the other team.