

DOCUMENT RESUME

ED 103 095

PS 007 507

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TITLE Structures-of-the-Whole: Is There Any Glue to Hold  
the Concrete-Operational "Stage" Together?  
PUB DATE Jun 74  
NOTE 18p.; Paper presented at the Canadian Psychological  
Association Symposium (Windsor, Ontario, June  
1974)

EDRS PRICE MF-\$0.76 HC-\$1.58 PLUS POSTAGE  
DESCRIPTORS \*Abstraction Levels; \*Children; Classification;  
Cognitive Processes; \*Concept Formation; Conservation  
(Concept); \*Developmental Psychology; \*Logical  
Thinking; Models; Number Concepts; Serial Learning;  
Skill Development  
IDENTIFIERS \*Piaget (Jean)

ABSTRACT

Studies concerned with the synchronous emergence prediction of Piaget's structures-of-the-whole principle are discussed in conjunction with three groups of concrete-operational skills: (1) transitivity/conservation/class inclusion; (2) double classification/double seriation; and (3) ordinal, cardinal, and natural number concepts. Findings show that contrary to the principle, asynchronous emergence of stage-related skills appears to be the rule rather than the exception. Children appear to grasp the relevant relational skill before they grasp the corresponding classificatory skill: transitivity is understood before class inclusion; double seriation is understood before double classification; ordinal number before cardinal number. It is suggested that the specific asynchronies considered are not isolated idiosyncratic phenomena but instead are part of some underlying pattern in the growth of human logic that is not yet fully understood. It may be that the ontogenesis of logical thought mirrors the axiomatic development of logic itself. (Author/CS)

ED103095

PS 002507

U.S. DEPARTMENT OF HEALTH  
EDUCATION & WELFARE  
NATIONAL INSTITUTE OF  
EDUCATION

AUG. 28 1974

STRUCTURES-OF-THE-WHOLE

IS THERE ANY GLUE TO HOLD THE CONCRETE-OPERATIONAL "STAGE" TOGETHER?

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This paper was presented as part of the symposium "Quantity and Logical Concept Development: Alternatives and Extensions of the Piagetian Model." Canadian Psychological Association, Windsor, Ontario, June, 1974.

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The notion of structures-of-the-whole is one of the major defining attributes of Piaget's stages of mental growth. Unlike the other key properties commonly associated with these stages (hierarchization, integration, coordination, equilibration), which are basically teleological meta-principles of little or no predictive consequence, structures-of-the-whole entails some widely recognized predictions which are subject to direct test. These predictions are concerned with the order in which the prototypic reasoning skills of each stage will be observed to emerge in children's thinking.

According to the structures-of-the-whole principle, each of the reasoning skills that characterize a given Piagetian stage presupposes one or more members of a set of abstract cognitive structures. In Piagetian theory, the term "presupposes" connotes far more than the assumption that a certain finite set of structures is a formal process model for a large, presumably infinite, set of reasoning behaviors. Piaget's structures also are supposed to be "really there" controlling thought, in the same sense that the moon is "really there" controlling the tides or a program is "really there" controlling how a computer processes an input. [At this point, the philosophically inclined listener no doubt will be anticipating another chapter in the old realism-nominalism and mind-body controversies. However, we shall attempt to avoid these metaphysical male-storms.] During each Piagetian stage of mental development, the specific reasoning skills associated with the stage presumably are generated (or "deduced") from the set of structures which define the stage.

Turning to the empirical implications of structures-of-the-whole,

the principle entails patterns of synchronous emergence of the key reasoning skills of each stage. This widely recognized prediction (e.g., cf. Pinard & Laurendeau, 1969; Wohlwill, 1963; Flavell, 1963, 1971) follows from an assumption that Piaget makes about the emergence of the underlying cognitive structures. He assumes that the structures which define a stage emerge in tight synchrony, rather than in some invariant order. An important caveat must be appended to this assumption. The assumption does not entail that, in every child, the structures associated with given stages emerge suddenly and abruptly. Although Piaget's writings are by no means clear on this point, it seems that stage-defining structures may very well emerge in some fixed order in any given child. However, the order is always idiosyncratic to that child. If the child acquires some stage-defining structure X before some other stage-defining structure Y, then some other child will do the reverse. More generally, the members of the set of structures defining any given Piagetian stage do not emerge in an order that is constant for all or most children.

From the structural synchrony assumption, it follows that the reasoning skills generated by these structures also cannot emerge in an order that is constant for all or most children. If this prediction is violated, then one of the following must be true: (a) the structures themselves emerge in some constant order; (b) the reasoning skills associated with Piaget's stages do not presuppose the structures that the theory says they presuppose; (c) we have not fairly tested the prediction (i.e., we have committed gross measurement and/or sampling errors). There

is an important exception to the general rule that stage-defining reasoning skills must emerge synchronously: the ad hoc principle of horizontal decalage. After the fact, Piaget has decided that synchronous emergence of underlying structures does not necessarily exclude asynchronous emergence of a given reasoning skill in different content areas. The classic illustration of this point involves the conservation concept of middle-childhood. Children are known to conserve in some content areas (e.g., number, length) long before they conserve in others (e.g., area, volume). The explanation offered for this phenomenon is that, somehow, certain content areas tend to resist application of the relevant structures more than other content areas. Unfortunately, this ad hominem argument offers no basis for making directional predictions about two or more content areas.

The sort of within-stage asynchrony that cannot be accommodated (or, at least, has not been to date) by either ad hoc principles or ad hominem arguments is of the following type. We are given two different reasoning skills A and B which the theory says are both generated by the same stage-defining set of cognitive structures. We examine the order in which A and B emerge in several content areas and they are observed to emerge in the same constant order in each area. The concept of conservation, together with the concept transitivity, provide a case in point. The theory tells us that the so-called groupement structures of the concrete-operational stage (Piaget, 1942, 1949) generate both of these concepts. Suppose we assess both conservation and transitivity in several content areas--e.g., length, weight, height--and we observe that transitivity always appears before conservation in individual areas. It is this sort of finding that structures-of-the-whole predicts will not be observed.

Since the early years of the preceding decade, it has been strongly suspected that developmental evidence would fail to confirm this prediction (cf. Wohlwill, 1963). Although the prediction was the subject of more than one investigation during the 1960s (e.g. Lovell & Ogilvie, 1960; Kofsky, 1966, McManis, 1969), measurement issues such as test sensitivity and response criteria often precluded interpretation of the findings (cf. Braine, 1962, 1964; Brainerd, 1973a, 1973b, 1974a, 1974b; Gruen, 1966; Smedslund, 1963, 1969). It is only within the present decade that these issues have been resolved to a sufficient degree that we can be fairly certain that the developmental data do indeed fail to support the prediction.

In the remainder of this paper, I propose briefly to discuss some findings that illustrate the asynchronous emergence of different stage-related reasoning skills in individual content areas. Rather than try to deal with a large number of such skills, I shall focus narrowly on three select groups. The members of each group normally are associated with Piaget's concrete-operational stage. Hence, each skill presumably is a by-product of the eight groupement structures used to define this stage. The criteria employed in selecting these illustrative groups was that each skill should play a central role in Piaget's seminal discussions of concrete operations and that the empirical evidence for the asynchronous emergence of the members of each group should be as unequivocal as possible. The three groups of skills are: (a) transitivity/conservation/class inclusion; (b) double classification/double seriation; (c) ordinal, cardinal, and natural number concepts.

Turning to group a, most listeners no doubt are aware that transitivity, conservation, and class inclusion are the three skills which Piaget most

frequently resorts to in general discussions of the concrete-operational stage. Hence, evidence pertaining to the synchronous or asynchronous emergence of these particular skills is especially interesting. Although the developmental relationship between class inclusion and the other two skills rarely has been studied, the order of emergence of transitivity and conservation was the subject of several investigations conducted during the late 1950s and the 1960s (e.g., Kooistra, 1963; Lovell & Ogilvie, 1961; McManis, 1969; Smedslund, 1961, 1963). Conservation and transitivity were observed to emerge synchronously, in concept areas such as length and weight, in only one of these studies. In the other studies, conservation was observed to emerge before transitivity. Unfortunately, all of these earlier studies are subject to some important measurement criticisms which I have discussed at length in the literature (Brainerd, 1973a, 1973b, 1974a, 1974b). This fact led me to conduct a new series of investigations in which class inclusion also was included. As a result of this particular series of studies, plus two follow-up experiments on class inclusion, it now seems reasonably clear that, during late preschool and middle-childhood, transitivity, conservation and class inclusion emerge in the following constant order in most concept areas: transitivity first; conservation second; class inclusion third.

The first three studies in this series (Brainerd, 1973a) were normative and cross-sectional in design. Three different groups of children, whose ages fell within the broad ranges that Genevans report for transitivity, conservation, and class inclusion, were administered tests of all three skills. The tests were new ones devised to eliminate the measurement objections alluded to earlier. It was observed that



transitivity appeared to precede both conservation and class inclusion in children's thinking: large proportions of subjects passed the transitivity tests but failed both the conservation and class inclusion tests. It also was observed that conservation appeared to precede class inclusion: large proportions of subjects passed the conservation tests but failed the class inclusion tests. To examine this order of emergence under more carefully controlled conditions, a training experiment was conducted (Brainerd, 1974c). A large group of preschoolers was pretested for transitivity, conservation, and class inclusion. Subjects who failed all three tests were then assigned to one of three training conditions or one of three control conditions. For the three training conditions, a simple verbal feedback procedure was used to induce the three skills. A comparative analysis of the three training conditions indicated that, on a number of learning variables, the findings were consistent with the order of emergence observed in the earlier developmental studies: transitivity proved much easier to induce experimentally than either conservation or class inclusion; conservation proved much easier to induce experimentally than class inclusion.

Perhaps the most intriguing and unexpected finding of our studies of transitivity/conservation/class inclusion was the enormous developmental gap between class inclusion, on the one hand, and transitivity-conservation, on the other. The oldest subjects in these studies had been 8-year-olds. The findings indicated that, by age 8, virtually all the subjects understood transitivity (length and weight) and slightly more than half of them understood conservation (length and weight). However, to our surprise and contrary to Genevan norms for this skill, there was almost no evidence



of class inclusion by age 8. This prompted two follow-up experiments of class inclusion with subjects up to and including 11-years-old (Brainerd & Kaszor, 1974). These experiments were designed to examine the possibility that the virtual absence of class inclusion in 8-year-olds was an artifact of procedure. In the literature, there is a tendency to regard the inordinate difficulty of the class inclusion problem as something of an epiphenomenon (cf. Ahr & Youniss, 1970; Wohlwill, 1968). According to this view, certain features of the stimulus materials normally employed in these problems inhibit class inclusion reasoning. If these features are eliminated, the argument continues, then class inclusion tests will be passed at a much younger age. These features were controlled in our follow-up experiments; however, the results confirmed our earlier findings. There was little evidence of class inclusion before about age 10. Moreover, even in 10- and 11-year-olds, only about half the subjects really understood class inclusion. Since the completion of these experiments, our discussant, Dr. Hooper, has uncovered two findings which tend to support my own findings on the late emergence of class inclusion. First, he has observed (cf. Hooper et al., 1974) that the solution rate on class inclusion problems continues to accelerate gradually throughout the adolescent years--i.e., it continues to accelerate long after the solution rate on corresponding transitivity and conservation problems have reached asymptote. Second, he also has observed substantial deficits in class inclusion reasoning among college males.

To sum up the best available evidence on transitivity/conservation/ class inclusion, there seems to be consistent support for the constant

order of emergence I mentioned earlier. More over, when my data on these three skills are combined with those of Dr. Hooper on class inclusion and with Trabasso's recent data on transitivity (Bryant & Trabasso, 1971; Riley & Trabasso, 1974), the following normative statements seem reasonable: Transitivity is worked out in most content areas during the preschool and very early middle-childhood years; conservation is worked out in most content areas during the middle-childhood years; class inclusion is worked out in most content areas during the juvenile and adolescent years. Further, class inclusion is never grasped at a level comparable to transitivity and conservation.

Let us turn now to our second group of skills: double classification and double seriation. Double classification and double seriation are quite interesting theoretically because they are generally believed to provide fairly direct estimates of two different concrete-operational structures (Groupement IV and Groupement VII, respectively). Although these skills are becoming more common in the neoPiagetian literature, they are not nearly as common as transitivity/conservation/class inclusion and, hence, they may be unfamiliar to some listeners. For "double classification," envisage a 3 x 3 matrix in which the columns are defined by the intensions "red," "blue," "green," and the rows are defined by the intensions "circle," "square," "triangle." For "double seriation," envisage another 3 x 3 matrix in which the three columns are defined by the relation "height" and the three rows are defined by the relation "diameter." In each cell of the double seriation matrix, there is a cylinder. As we go across the matrix, the cylinders increase in height. As we go down the matrix, the cylinders increase in diameter. With both the double classification matrix and the

double seriation matrix, the child's main task is to fill in missing cells. In order to do this reliably, he must have both intensional properties in mind, in the case of the classification matrix, and he must have both relations in mind, in the case of the seriation matrix.

About 2 1/2 years ago, our discussant, Dr. Hooper, noted in a letter to me that he recently had gathered some data which indicated that children understood and could solve double seriation problems before they could solve double classification. Subsequently he was able to replicate this finding in a methodologically sophisticated large-scale study of the development of children's classification abilities. During the past year, I included double classification and double seriation tasks, which were slightly different than those employed by Dr. Hooper, in a number development study I was conducting. I observed that same sequence as Dr. Hooper; my subjects were more successful with double seriation than they were with double classification. Shortly after the first of this year, a third replication was obtained, quite independently of Hooper's and my studies, by Mrs. Bernice Wong who is a doctoral candidate at the University of British Columbia. During a recent visit to Edmonton, Mrs. Wong informed me of this finding and was surprised to learn that Dr. Hooper and I had already obtained the same datum. Mrs. Wong's tasks were different from both Hooper's and mine. Finally, Dr. Hooper, in conjunction with Ann Burke-Merkel, has conducted a training experiment that involved instruction in both double seriation and double classification. His major finding, that seriation instruction was more successful than either classification training or combined seriation/classification training, is consistent with

the developmental findings we have just considered.

Thus, there seems to be fairly consistent support for the developmental priority double seriation over double classification. However, I should like to caution the listener about the magnitude of this difference. Although the findings I have just mentioned consistently support the priority of double seriation, the data also indicate that the gap between double seriation and double classification is fairly small--probably six months to a year in the average child. From a psychometric standpoint, this means that the double seriation/double classification is not nearly as robust as the transitivity/conservation/class inclusion sequence discussed earlier. Thus, while one must commit fairly crude measurement errors to mask the latter sequence, small methodological perturbations will suffice to mask the former.

Finally, let us turn to the development of number concepts. As most Piaget observers are aware, number development has been an abiding theme throughout Piaget's long research career. As things now stand, the development of basic number skills is associated with the concrete-operational stage in Piagetian theory. Number and number concepts mean different things to different investigators. Piaget's work has been restricted to three broad categories of numerical competence: ordinal number (numbers as representations of the terms in any ordered progression); natural number (arithmetic computation and the fundamental laws of arithmetic); cardinal number (numbers as representations of the manyness of any collection of terms). According to the theory, all three of these generic forms of number are the province of concrete operations and, of course, emerge

in strict synchrony during this stage.

About three years ago, I initiated a series of investigations of ordinal, cardinal, and natural number concepts. I began these studies as an empirical off-shoot of some mathematical interests of mine and with no thought of providing a definitive test of the structures-of-the-whole principle as it applies to number concepts. Hence, the major implications of this research are for mathematical epistemology and not for Piagetian theory per se. Nevertheless, certain of the findings are relevant and I should like to note them.

The studies initially were focused on the early elementary school years (Brainerd, 1973c, 1973d, Brainerd & Fraser, 1975). Though some number ideas are known to emerge earlier, these are the years when natural number concepts first appear. Because I was interested primarily in the relative contributions of ordinal number and cardinal number to children's natural number skills, this is where I began. The initial phase of the investigation consisted of two large scale developmental studies. These two studies indicated that the three generic forms of number emerge in the following order: ordinal number first; natural number third. It appeared that most children enter elementary school with a reasonably good grasp of ordinal number, then proceed to work out the basic components of natural number, and finally, by the late middle-childhood years, begin making substantial progress with cardinal number. These findings subsequently were replicated in a study conducted with Michelle Fraser (Brainerd & Fraser, 1975). A training study (Brainerd, 1973d) then was conducted with children of this age level. The study was designed to determine: (a) the

relative susceptibility of ordinal number and cardinal number to training experiences and (b) the extent to which ordinal number training and cardinal number training enhance natural number skills. It was observed that ordinal number was far easier to train than cardinal number and that there was substantial transfer of ordinal number training to natural number. Cardinal number training, on the other hand, was not observed to enhance natural number skills. Another training study was conducted with preschoolers (Brainerd, 1974d). This particular study focused on training preschoolers to use numerical symbols either as representations of terms in an ordered progression (ordinal number) or as representations of the manyness of collections of terms (cardinal number). It turned to be much easier to train the former sort of representations than the latter.

From the perspective of the structures-of-the-whole principle, of course, the important aspect these findings on number development is the indication of a constant order of emergence for ordinal, natural, and cardinal number. Since these findings have been available in the literature for only a short period, there has not yet been time for comprehensive replicatory evidence to appear from other laboratories. However, some supportive independent evidence is beginning to appear. Our chairman, Dr. Siegel, recently conducted an investigation of ordering and correspondence operations in preschoolers (Siegel, 1974). She concluded that certain aspects of her data were consistent with my finding that ordinal number skills generally appear before cardinal number skills. Mrs. Bernice Hong, whom I mentioned earlier with regard to research on classification/seriation, examined the relationship between the correspondence operation and

a measure of natural number. Her data appear to be consistent with my general findings about the developmental relation between natural number and cardinal number.

To sum up very briefly, it is obvious that the synchronous emergence prediction of the structures-of-the-whole principle has not fared well in conjunction with the three groups of concrete-operational skills which we have considered. Contrary to the principle, asynchronous emergence of stage-related skills appears to be the rule rather than the exception. Some listeners may also have noticed that there is a discernible pattern running through specific sequences observed for the three groups of skills. In each case, children appear to grasp the relevant relational skill before they grasp the corresponding classificatory skill: transitivity is understood before class inclusion; double seriation is understood before double classification; ordinal number is understood before cardinal number. This suggests that the specific asynchronies we have considered are not isolated idiosyncratic phenomena but, rather, they are part of some underlying pattern in the growth of human logic that we do not yet fully understand. The best hunch about what this underlying pattern it seems to be that the ontogenesis of logical thought mirrors the axiomatic development of logic itself--in which the abstract logic of classes is generated from the abstract logic of relations.



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