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ABSTRACT

This document is a collection of course outlines, syllabi, and test materials designed for several high school level and lower division mathematics courses taught in an auto-tutorial learning laboratory at Skagit Valley College (Washington). The courses included are: Pre-Algebra, Basic Algebra, Plan Geometry, Intermediate Algebra, Probability and Statistics, Functions and Relations, Periodic Functions, Analytic Geometry, Differential and Integral Calculus. To determine his entering level, each student solves increasingly more difficult problem on the Student Decision Placement Test, which is included; his level of ease determines his proper program entry level. Students attend one schedule conference each week and may study in the learning laboratory at other times. Most of the work is completed in programmed textbooks. Only "A" and "B" grades are given. Each course outline contains performance objectives, course goals, average student completion time, and the number of credits allotted, as well as a list of suggested student materials and texts. Each course is presented with two approaches (tracks)--one for those who are prepared for, but unfamiliar with, the course material, and one for review and in-depth study. (DC)

Oleanna Math
Program

OLEANNA MATH PROGRAM MATERIALS
Walter A. Coole, Skagit Valley College

This is a collection of course outlines, syllabi, and test materials for several high-school level and lower-division courses intended for open-classroom use. Other ancillary materials will be presented in a separate publication.

Using institutions are authorized to reproduce these materials, modifying them as necessary for their own programs.

I wish to thank the publishers of textbooks used in the Oleanna Math Program for their permission to reproduce passages of text and handbook content.

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I'd welcome comments and suggestions...

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Oleanna Math Program

This STUDENT-DECISION PLACEMENT TEST is designed to take the guesswork out of your course registration. It's not foolproof. You don't absolutely _have_ to follow the advice we give you.

You'll need scratch paper and a pencil or pen.

Please do not write in this booklet.

I. Work the following problems on a piece of scratch paper.

1. 378 + 123 = ?
2. 1247 - 347 = ?
3. 278 X 476 = ?
4. 647 ÷ 27 = ?
5. Is .33 the same as one-third?
6. How do you find the area of a square?

Please turn to the next page.

Now, check your answers.

1. 501
2. 900
3. 132,328
4. 23 with a remainder of 26 or
   23 26/27 or
   23.9637 — you needn't have carried it out any further
5. No. It's almost the same, but not quite.
6. To find the area of a square, multiply the length of one side by itself.

Did you get five of them right? Did you find the problems easy? If the answer to either of these questions is no, you should ask for help in fundamental arithmetic of a special kind. You will not do well, entering the Oleanna Math Program at this time.

....

If the answer to both of these questions is yes, then it's time to tell you a little about the Oleanna Math Program.

In the Oleanna Math Program, you will not attend daily lectures. You will only be required to attend a scheduled conference once a week (two times a week during summer terms).

Most of your work will be done in "programmed textbooks" and you will be encouraged to skip topics you have already mastered thoroughly. You may proceed as quickly through your course of study as you can—and go on to the next course as soon as you want to.

Between scheduled conference periods, you may use the learning lab to study, if you wish. Student coaches and your instructor will be available to assist you most of the time.

We award only two grades: A and B. If you do not complete a particular course by the end of a grade-reporting period, we turn in a "no-credit" report. You may continue in the course by signing up for it again the following term.

....

II. Now, let's try some harder problems....

1. .15 + 12.85 = ?

2. 43/37 + 12/67 = ?

3. .83 - 6.75 = ?

4. 1 23/27 - 2/3 = ?

5. 1.4 X .028 = ?

6. 6 3/5 X 15/22 = ?

7. 1.45 X 100 = ?

8. -.27 X -.12 = ?

9. .112 ÷ .04 = ?

10. Express 1/8 as a percentage.

Please turn to the next page.

The correct answers for the problems on page 3 are:

1. 13
2. 3325/2479
3. -5.92
4. 1 5/27
5. .0392
6. 4 1/2
7. 145.
8. +.0324
9. 2.8
10. 12 1/2%

Did you get at least 9 of theses problems right?
Did you feel comfortable working the problems?
Were you fairly sure of your answers before checking them?

If the answer to <u>any</u> of these questions was <u>no</u>, you should enter the Oleanna Math Program by enrolling in Pre-Algebra.

This is a high-school level course, designed to prepare you for the study of algebra. You probably won't have to study every topic in the course to complete it; but you'll find it easier in the long run to begin with this course. Please return this pamphlet and register for the course.

If you're beginning at the first of the term, be sure you attend one of the initial meetings on the first day of class. See the class schedule for meeting times.

....

If the answer to <u>all</u> of these questions were <u>yes</u>, you are probably prepared to take more advanced work. Let's just see how advanced:

Please turn to the next page.

III. Please work the following problems....

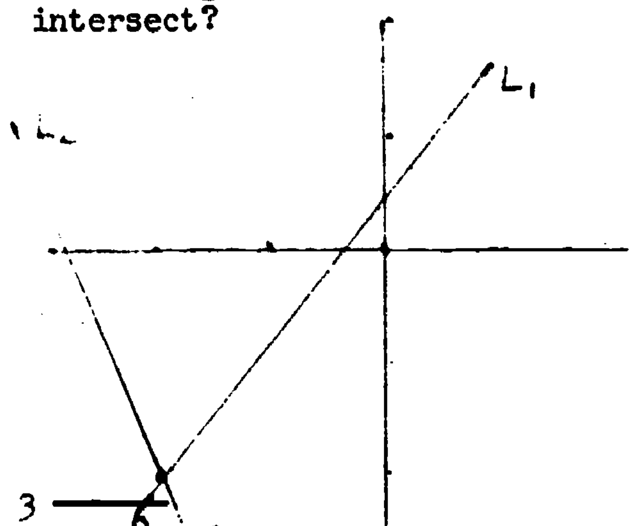
1. Solve: $4x + 2 = 1$

2. $(-6) + (+3) = ?$

3. What is the coefficient of A in the expression 5Ax?

4. $X^2 / 2X^3 = ?$

5. In the figure shown below, give the point at which L_1 and L_2 intersect?



6. $\sqrt[3]{27x^6} = ?$

7. Factor completely: $12X^2 - 2X - 2$

8. $\frac{8A^3B^2}{5} \div \frac{12A^2}{25B} = ?$

9. If $X - 1 \geq -12$, $X \geq$?

10. Which of these describes the number π ?
Real? Rational? Positive? Integer?

Please check your answers.

1. $X = -1/4$
2. -3
3. $5x$
4. $1/2X$
5. $(-2,-2)$
6. $3X^2$
7. $2(2X-1)(3X+1)$
8. $10/3\ AB^3$
9. -11
10. Real, Positive

Did you get at least 9 of these problems r ght?
Did you feel comfortable working these problems?
Were you fairly sure of your answers before checking them?

If the answer to <u>any</u> of these questions was <u>no</u>, you should enter the Oleanna Math Program by enrolling in Basic Algebra, Part I.

This is the first of two 3-credit, high school level courses which are equivalent to the first year of algebra. You probably won't have to study every topic in the two-course sequence to complete it; but you'll find it easier to begin with basic algebra, rather than undertaking more advanced work without a proper foundation.

If you complete Part I before the end of the term, you'll be allowed to begin Part II, immediately.

Please see the current Class Schedule for instructions about meeting times.

Please return this pamphlet.

....

If you answered all of the questions above <u>yes</u>.....
Have you taken Plane Geometry in high school? If not, you may wish to take Geometry, a 5-credit course. This high-school level course is useful, but not essential for entry at a more advanced level.

Otherwise, please turn to the next page.

Please work the following problems....

1. $\sqrt[3]{\text{[illegible]}} = ?$

2. Solve for X: $X^2 + X - 6 = 0$

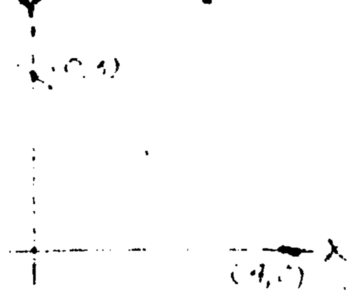
3. $\sqrt{12} - \sqrt[4]{9} = ?$

4. $25^{1/2} - 12^0 + 4^{-1/2} = ?$

5. What is the slope of the line whose equation is $4y-5x = 9$?

6. If $\log_{10} X=3$, then $X = ?$

7. Give the equation of the line graphed below in slope-intercept form.



8. $(-i)(-2i) = ?$

Please turn to the next page.

Please check your answers.

1. -4
2. $X = +2$ and $X = -3$
3. $\sqrt{3}$
4. 1 1/2
5. 5/4
6. 1000
7. $y = -3/4x + 1$
8. -2

Did you get at least 7 of these right?
Did you feel comfortable working these problems?
Were you fairly sure of your answers before checking them?

If the answer to <u>any</u> of these questions was <u>no</u>, you should enter the Oleanna Math Program by enrolling in Intermediate Algebra.

Intermediate Algebra is a 5-credit college course. Perhaps you'll be able to skip some topics you've already learned thoroughly; but you'll find it easier to begin with a good mastery of algebra before undertaking more advanced work.

....

If you answered all of the questions above yes...

You are ready to enroll in Functions and Relations (4 credits), the first pre-calculus course. If you prefer, you can enroll in Probability and Statustics (5 credits); you're well qualified for that course, also.

....

Please see the current Class Schedule for instructions about meeting times.

Please return this pamphlet.

Oleanna Math Program

PRE-ALGEBRA. Course Outline by Walter A. Coole, Skagit Valley College.

Skagit Valley College Course Number: Mathematics 1

Quarter credits: 3 Semester credits: 2

Average student completion time: 100 hours

Goal: The student should master thoroughly, all mathematical pre-algebraic operations needed to perform arithmetic calculations required in higher mathematical studies.

At the end of this course, the student will be able to perform the four fundamental operations on rational numbers; and compute decimal fractions and percentages. His terminal examination will establish his mastery at the 90th percentile for urban high school students.

The two-track approach.

A. The standard path assumes that the student has had little real mastery of elementary arithmetic, but allows for some skipping of already-mastered materials, based upon pre-testing of lessons.

The standard path's performance objectives, lesson-by-lesson are:

1. interpret standard numerals and exponentiated notation;
2. add and subtract whole numbers;
3. multiply and divide whole numbers;
4. factor whole numbers to prime factors;
5. multiply and divide rational numbers and state reciprocals;
6. add and subtract rational numbers;
7. add and subtract decimal numerals;
8. multiply and divide decimal numerals
9. compute percentages;
10. express quantities in terms of both metric and English conventions;
11. express ratios and proportions mathematically

12. compute averages, medians and squares; approximate square roots.

B. The review path assumes that the student is familiar with basic arithmetic and wants a bit of novelty in his study. The sequence's first two units are recommended as first projects for the course. Their performance objectives are...

1. COMBINATIONS. To perform basic combinatory computations on whole numbers.

2. COUNTING. To count in various number-bases.

The remaining units of the review path may be taken in any order the student desires.

3. FIBONACCI NUMBERS. To discover numeric patterns which develop according to the Fibonacci and Lucas numbers sequences.

4. THE GEOBOARD. To find areas of both simple and complicated regions without having to resort to formulae and create his own unique solutions to geometric problems.

5. THE GOLDEN MEAN. To determine the Golden Ratio and extract square roots.

6. GOOGOLS AND GOOGOLPLEXES. To name large numbers and express them in scientific notation.

7. THE GREAT CHASE. To perform computations of the kind enabled by the Pythagorean Theorem.

8. PALINDROMIC NUMBERS. To read, write, and add whole numbers in various base-notations.

9. PHYLOTAXIS. To be able to count and record observations of natural phinomena.

10. PRIME NUMBERS. To find prime numbers and use them with fractions.

11. PROBABILITY. To compute probabilities, using fractional calculations.

12. RANDOM DIGITS. To be able to explain what is meant by randomness in his own words and recognize random situations.

13. THE SCHIZOPHRENIC RABBIT. To convert between common and decimal fractions.

14. SHORT LONG DIVISION. To estimate quotients and calculate them by long and short division.

15. THE SQUIGGLE. To graph and find areas of simple closed curves; to convert between fractional and percentage notation.

Entry. In addition to basic numerical familiarity, the entering student should be able to:

i. read and follow simple written instructions
ii. state his educational objectives in simple, coherent terms
iii. study systematically and diligently

Student materials.

Testing form: Automata Student-response card (1-50)
Pencil, paper, protractor

Standard Path:

Keedy & Bittinger: *Arithmetic, A Modern Approach.* Reading, Mass.: Addison-Wesley Publishing Co. 1971.

Coole, Walter A.: *Pre-Algebra Arithmetic--Syllabus for Standard Path*

Review Path:

Curl, James C.: *Developmental Arithmetic.* New York: McGraw-Hill Book Company. 1973

Teacher preparation:

Study instructor's manuals and testing materials provided by the publishers.

Other materials required:

Oleanna Math Program: *The Student-Decision Placement Test.*

Cooperative Testing Service: *Cooperative Math Test--Arithmetic* (Forms A, B, and C and users' manual.) Palo Alto, CA: Cooperative Testing Service. 1969.

Oleanna Math Program: *Smorgasbord.*

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Oleanna Math Program

Syllabus for PRE-ALGEBRA (Standard Path)
by Walter A. Coole, Skagit Valley College

Your goal for this course is to master all of the arithmetic fundamentals necessary to do well in a basic algebra course. Along with this mastery, you'll learn a number of useful tricks in applying your mathematical skills to "real world" problems.

This course is divided into four "units", each of which will require about 25 hours' work. By following directions in this syllabus, you'll be able to avoid spending time unnecessarily on information you've already mastered. The units of the course are:

Unit	Lesson	Completion Date	
I	Pre-test	__________	
	1	__________	
	2	__________	
	3	__________	
	4	__________	*
II	5	__________	
	6	__________	
	7	__________	*
III	8	__________	
	9	__________	
	10	__________	*
IV	11	__________	
	12	__________	*
	Final	__________	

Your completion date for the pre-test should be the day of your earliest scheduled conference.

Completion dates for each unit (Marked by asterisks *) should be filled in from the schedule provided. If you're beginning at the opening of a school term, your schedule will be posted on the bulletin board; otherwise, your teacher will work out a special schedule for you.

For this course, you'll need paper, pencil, and the following textbook:

Keedy & Bittinger: *Arithmetic--A Modern Approach*

DO ALL OF YOUR WORK IN PENCIL!!!

Pre-test

At the very front of the textbook, you'll find a 'PRETEST". Write your answers to the pre-test on a sheet of notebook paper.

Score your results from the answers given in the back of the book.

Note the "Pretest Analysis" which tells you which lessons you may skip. If you wrote as many as 75 correct answers on the pre-test, you should then skip to the "Final Examination" at the back of the text. If you can write 100 correct answers from this test, you should contact the instructor for the "official" course-completion test.

How to Study Each Lesson

Each chapter in the textbook corresponds to a lesson in this course. By using your pretest results, you should be able to decide which lessons to omit.

Each chapter of the text is divided into several sections. Begin each section by reading the objectives (what you should learn) and then the explanation.

Write the answers to problems as you are directed in the text.

As you complete each section's "Margin Exercises", check your answers in the back of the book. If you have difficulty, see your instructor or a math coach as soon as possible.

Next, complete the odd-numbered exercises in all exercise sets at the end of the chapter. Then score your results, using the answers given in the back of the book.

To complete the lesson, take the test at the end of the chapter. Score your results, using the answers at the back of the book and follow the directions given in the test analysis.

When you've scored satisfactory results, remove the chapter test from the book and turn it in. If the test uses more than one sheet, staple them together at the upper left-hand corner. The tests will be returned as soon as they are recorded.

Completing the Course

After you've mastered all of the chapters of the textbook — either by scoring perfect on the pre-test or by achieving a satisfactory grade on the end-of-chapter test—complete the final examination provided at the back of the book. Score your results against the answers in the back of the book and follow directions given in the analysis.

When you've scored 80 or better on the final examination, you are ready to take the "official" course-completion test.

You may take this test at any scheduled conference or by appointment. You'll need paper, pencil, and 50-entry student response card (on sale at the bookstore). You may use your textbook and notes during the test. Average completion time for the end-of-course test is 40 minutes, but you may take longer if you wish.

Grading

When you've completed the end-of-course test, you may close off the course with a grade of "B". If you wish to improve your grade to an "A"; you may act as a coach or undertake optional projects from the "Smorgasbord". This may be done during the following term and your "B" will be changed to an "A".

Oleanna Math Program

BASIC ALGEBRA. Course Outlines by Walter A. Coole, Skagit Valley College.

Skagit Valley College Course Numbers: Part I: Mathematics 2
Part II: Mathematics 3

Quarter credits for each part: 3 Semester credits for each part: 2

Average student completion time for each part: 100 hours

Goal. This course is equivalent to a first-year course in high school algebra. In it, the student will...

1. gain mathematical proficiency by learning and using algebra as an extention of the number system--to the irrational numbers; and how to use this proficiency in solving verbal problems and working with formulas of a moderately difficult nature

2. develop an understanding of the properties and structure of the number system

3. prepare for future work in mathematics, science, and related fields by:
   a. developing competence in the use of algebraic language and symbols
   b. learning to use signed numbers, formulas, and equations
   c. exploring how mathematics has contributed to human betterment
   d. mastering the skills of graphing to express in a precise way, how events relate to one another

The two-track approach.

A. The standard path assumes that the student has had little real mastery of basic algebra, but allows for some skipping of already-mastered materials, based upon pre-testing of lessons.

The standard path's performance objectives, lesson-by-lesson are as follows.

Part I

1. solve simple equations involving rational numbers;

2. solve more complex equations involving rational numbers

3. solve difficult equations involving rational numbers

4. perform complex operations on polynomials;

5. graph linear equations;

Part II

6. solve equation-pairs by substitution and addition;

7. represent inequalities graphically;

8. solve higher-degree equations whose notation includes polynomials with several variables;

9. solve equations involving fractional expressions;

10. manipulate expressions with radical notation;

11. solve second-degree equations, using the quadratic formula;

* * * * *

b. The review path assumes that the student is familiar with basic algebra, but needs extensive review and consolidation of his knowledge. It consists in five modules of eight lessons each. The lessons' constituent topics are as follows...

Part I

Module I: Operations on Numbers

1. prime numbers
2. multiplication and division of fractions
3. addition of fractions
4. zero and the signed numbers
5. multiplication and division of signed numbers
6. addition of signed fractions
7. subtraction of signed numbers
8. order of operations

Module II: Operations on Polynomials

1. operations on signed numbers
2. algebraic notation and addition of polynomials
3. subtraction of polynomials and multiplication monomials
4. multiplication of polynomials
5. factoring
6. exponents
7. division of monomials by monomials
8. division polynomials by monomials

Module III: Linear Equations and Lines

1. linear equations in one variable
2. advanced linear equations in one variable
3. linear equations in two variables
4. advanced linear equations in two variables
5. slope
6. graphing by the intercept-slope method
7. simultaeous solutions of linear equations by graphing
8. simultaneous solutions of linear equations by algebra

Part II

Module IV: Factoring and Operations on Algebraic Fractions

1. factoring and simplifying algebraic fractions
2. foil multiplication and factoring trinomials
3. more factorin
4. still more factoring
5. You guessed it! More factoring.
6. multi-lication and division of algebraic fractions
7. addition and subtraction of algebraic fractions
8. advanced addition and subtraction of algebraic fractions

Module V: Quadratic Equations and Curves

1. number sets
2. quadratic equations
3. the quadratic formula
4. graphing quadratics
5. imaginary numbers
6. graphing and algebraic solutions of quadratics
7. simultaneous solutions of equations by graphing
8. simultaneous solutions of equations by algebra

Upon completing the Basic Algebra Review Path, the student should have mastered all of the performance objectives of the Standard Path and be prepared to complete the Oleanna Math Program Intermediate Algebra Review Path at a somewhat accelerated pace. He may, however, choose to complete Intermediate Algebra by the Standard Path.

Entry.

The student entering either path of Basic Algebra should be able to perform with ease, all four fundamental operations on rational numbers.

In addition, he/she should be able to:

i. read and follow simple written instructions
ii. state his educational objectives in simple, coherent terms
iii. study systematically and diligently

Student materials.

Testing form: Automata Student Response Card (1-50)
Pencil, paper

Standard Path

Keedy & Bittinger: *Introductory Algebra--A Modern Approach.* Reaeing, Mass.: Addison-Wesley. 1971

Coole, Walter A.: *Basic Algebra Syllabus For Standard Path.* Parts I & II.

Review Path

Ablon, Leon J., Blackman, Sherry, Giangrasso, Anthony P., & Siner, Helen: *Series in Mathematics Modules.* Menlo Park, CA: Cummings Publishing Co. 1973. Modules I-V

Coole, Walter A.: *Basic Algebra Syllabus to Accompany Series in Mathematics Modules.* Sedro-Woolley, WA: The Courier-Times. 1974. Parts I & II.

Teacher preparation.

Study instructor's manuals, testing materials, and texts.

Other materials required.

Oleanna Math Program: *The Student-Decision Placement T st.*

Cooperative Testing Service: *Cooperative Math Test--Algebra I* (Forms A and B and user's manual.) Palo Alto, CA: Cooperative Testing Service. 1969.

Oleanna Math Program: *Smorgasbord.*

Syllabus for BASIC ALGEBRA—
Parts I and II—(Standard Path)
by Walter A. Coole, Skagit Valley
College

Your *goal* for this course is to master all of the basic concepts of algebra necessary to do well in further mathematical studies. Along with this master, you'll learn a number of practical applications of algebra of "real world" problems.

Basic algebra is divided into two parts, each of which requires separate registration. If you finish Part I before the end to the current term, you may begin working on Part II immediately.

Basic algebra is divided into four "units", each of which will require about 40 hours' work. By following directions in this syllabus, you'll be able to avoid spending time unnessarily on information you've already mastered.

Part	Unit	Lesson	Completion date
I	I	Pre-test	________
		1	________
		2	________*
	II	3	________
		4	________
		5	________*
II	III	6	________
		7	________
		8	________*
	IV	9	________
		10	________
		11	________
		Final	________

Your completion date for the pre-test should be the day of your earliest scheduled conference.

As you begin Part I, fill in completion dates for each unit indicated by an asterisk (*). If you're beginning at the opening of the school term, your schedule will be posted on the bulletin board; otherwise, your teacher will work out a special schedule for you.

For this course, you'll need paper, pencil, and the following textbook:

Keedy & Bittinger: *Introductory Algebra--A Modern Approach*

DO ALL OF YOUR WORK IN PENCIL!!

Pre-test

At the very front of the textbook, you'll find the 'PRETEST'. Write your answers to the pre-test on a sheet of notebook paper.

Score your results from the answers given in the back of the book.

Note the "Pretest Analysis" which tells you which lessons you may skip. If you wrote as many as 30 correct answers on the pre-test, you should then skip to the "Final Examination" at the back of the text. If you can write 70 correct answers from this test, you should contact the instructor for the "official" course-completion test (or Basic Algebra, Part II.)

How to Study Each Lesson

Each chapter in the textbook corresponds to a lesson in this course. By using your pretest results, you should be able to decide which lessons to omit.

Each chapter of the text is divided into several sections. Begin each section by reading the objectives (what you should learn) and then the explanation.

Write the answers to problems as you are directed in the text.

As you complete each section's "Margin Exercises", check your answers in the back of the book. If you have difficulty, see your instructor or a math coach as soon as possible.

Next, complete the odd-numbered exercises in all exercise sets at the end of the chapter. Then score your results, using the answers given in the back of the book.

To complete the lesson, take the test at the end of the chapter. Score your results, using the answers at the back of the book and follow the directions given in the test analysis.

When you've scored satisfactory results, remove the chapter test from the book and turn it in. If the test uses more than one sheet, staple them together at the upper left-hand corner. The tests will be returned as soon as they are recorded.

Completing Part I

When you have turned in the end-of-chapter test for lesson 5, you will be given a grade of "B" for Basic Algebra, Part I. If you wish to achieve a grade of "A", see the instructor.

Completing Part II

After You've mastered all of the chapters of the textbook--either by scoring perfect on the pre-test or by achieving a satisfactory grade on the end-of-chapter test--complete the final examination provided at the back of the book. Score your results against the answers in the back of the book and follow directions given in the analysis.

When you've scored 56 or better on the final examination, you are ready to take the "official" course-completion test.

You may take this test at any scheduled conference or by appointment. You'll need paper, pencil, and a 50-entry student response card (on sale at the bookstore). You may use your textbook and notes during the test. Average completion time for the end-of-course test is 40 minutes, but you may take longer if you wish.

Grading

When you've completed the end-of-course test, you may close off the course with a grade of "B". If you wish to improve your grade to an "A", you may act as a coach or undertake optional projects from the "smorgasbord". This may be done during the following term and your "B" will be changed to an "A".

Oleanna Math Program

BASIC ALGEBRA (Part I) -- Syllabus for the Review Path, to accompany SIMM: Series in Mathematics Modules, I-III. by Walter A. Coole, Skagit Valley College--with the assistance of Uncle Thorbald

This is the first of two short courses which make up a first course in algebra. In order to recieve credit for a full basic-algebra course, BOTH courses must be completed.

Your OBJECTIVES for the two short courses will be to...

1. gain mathematical proficiency by learning and using algebra as an extention of the number system you've already mastered--an extension to the rational number system and to learn how to use this proficiency in solving verbal problems and in working with formulas of a moderately difficult nature

2. develop an understanding of the properties and structure of the number system

3. prepare for future work in mathematics, science, and related fields by:

   a. developing competence in the understanding of the use of algebraic language and symbols

   b. learning to use signs, numbers, formulae, and equations

   c. exploring how mathematics has contributed to human betterment

   d. mastering the skills of graphing to express in a precise way, how events relate to one another

And while your grade won't depend on it, we hope to show you that widespread use of mathematical language is essential to achieving a better society.

....

Contrary to a widespread myth that mathematically ignorant people take seriously, mathematics is a human language, developed for the purpose of communication among people and for the solution of human problems.

Its advantages over the "natural" languages, such as English, French, German, or Coptic, is that mathematics can express quantity ideas more clearly and conveniently. It is particularly useful in planning for human cooperation.

The algebraic language you will learn in this course is essential for the citizen to exercise his civic rights and responsibilities intelligently.

The widespread requirement of basic algebra for jobs is an indication that many employers feel that the work of their business or service emphasizes numeric abilities.

....

To give you an example of the use of numbers in planning human effort--a very simple one--we're going to ask you to plan your progress through the course.

This little project will not predict when you will finish the course, but it will allow you to plan your progress and to provide your making revisions as the situation progresses.

If you're beginning this course at the first of a term, you can check your answers against the posted schedule of unit completions.

Please use a pencil in working out your schedule.

Pick a date at which you intend to complete the course. Write that date here: __________

Write your planned examination date here: __________

Using a calendar, count the number of days you have planned to allow for the course: __________

and divide that number by 3: __________

Starting from today's date, then, you should now be able to identify "target dates" for each unit of the course.

UNIT I: ____________________

UNIT II: ____________________

UNIT III: ____________________

You should plan to spend about 33 hours of study on each unit or "module" of work in this course. If you can't make the plan fit, erase your entries and try again. If you run into trouble in this process, see the instructor or a coach.

Let's see how much of the course can be skipped. Circle the correct answer among the choices given.

PART A. This part contains ten questions.

1. 709×864

(a) 612,576
(b) 602,576
(c) 611,576
(d) 612,566
(e) NG*

*NG = "Not given". Use this choice if all of the other answers are incorrect.

2. $86\overline{)8342}$

(a) 96
(b) $96\frac{76}{86}$
(c) $97\frac{6}{86}$
(d) 97
(e) NG

3.
$$\begin{array}{r} 141606 \\ -\ 94679 \\ \hline \end{array}$$

(a) 46,837
(b) 46,927
(c) 46,937
(d) 47,027
(e) NG

4. $\frac{2}{3} + \frac{5}{6}$

(a) $\frac{1}{6}$
(b) $\frac{1}{2}$
(c) $1\frac{1}{2}$
(d) $1\frac{1}{3}$
(e) NG

5. $3\frac{1}{2} \div \frac{9}{10}$

(a) $\frac{1}{4}$
(b) $2\frac{1}{2}$
(c) $2\frac{3}{6}$
(d) $3\frac{3}{6}$
(e) NG

6. $2\frac{1}{4} - 1\frac{3}{4}$

(a) $1\frac{1}{2}$
(b) $\frac{1}{2}$
(c) $1\frac{1}{4}$
(d) $\frac{3}{4}$
(e) NG

7. Simplify

$\frac{36}{45}$

(a) $\frac{1}{2}$
(b) $\frac{4}{5}$
(c) $\frac{4}{9}$
(d) $\frac{3}{5}$
(e) NG

8. $\left(+\frac{3}{7}\right) \cdot \left(-\frac{4}{5}\right)$

(a) $-\frac{1}{2}$
(b) $\frac{13}{35}$
(c) $-\frac{12}{35}$
(d) 1
(e) NG

---

9 Add -15 and +35

(a) -50
(b) +20
(c) -20
(d) +10
(e) NG

---

10. Simplify
$\frac{(+2) + (-6)}{(+2)}$

(a) -6
(b) +6
(c) -2
(d) +2
(e) NG

PART B. This part contains ten questions.

1. Add $3x^2 - 5x + 2$ and $2x^2 + 8x - 4$

(a) $6x^4 - 40x^2 - 8$
(b) $6x^2 + 13x - 8$
(c) $5x^2 + 3x - 2$
(d) $x^2 - 1$[illegible]$x + 6$
(e) NG

---

2. Subtract -10y from -12y

(a) -2y
(b) -22y
(c) -120y
(d) +2y
(e) NG

---

3. Multiply $-3x^2y$ by $-5xy^2$

(a) $8x^2y^2$
(b) $15 + x^3 + y^3$
(c) $-8xy$
(d) $15x^3y^3$
(e) NG

---

4. Simplify $3 - 2(x - 5)$

(a) $x - 5$
(b) $-2x - 2$
(c) $x - 7$
(d) $-2x + 7$
(e) NG

5. Multiply $2x + 4$ by $x - 3$

(a) $2x^2 - 12$
(b) $2x^2 - 2x - 12$
(c) $3x + 1$
(d) $x + 7$
(e) NG

---

6. Factor completely
$3x^3 + 9x^2 + 3x$

(a) $3x(x^2 + 3x)$
(b) $3(x^3 + 3x^2 + x)$
(c) $x(3x^2 + 9x + 3)$
(d) $3x(x^2 + 3x + 1)$
(e) NG

---

7. What number does the following expression stand for when x is -2?
$3x^2$

(a) 12
(b) 36
(c) -36
(d) -12
(e) NG

---

8. Divide $18a^3$ by $-6a$

(a) $-12a^2$
(b) $-3a^2$
(c) $\frac{1}{-3a^2}$
(d) $24a^4$
(e) NG

---

9. Simplify:
$\frac{15x^2y^3 - 10x^4y}{5xy}$

(a) $3xy^2 - 2x^3$
(b) x^5y^3
(c) $20x^3y^4 - 15x^5y^2$
(d) $10xy^2 - 5x^3y$
(e) NG

---

10. What number does the following expression stand for when x is 5 and y is 2?
$3x - 4y + 17$

(a) 16
(b) -24
(c) 24
(d) 40
(e) NG

PART C. This part contains ten questions.

1. What is the solution of the following equation?

$$2x + 3 = -5$$

(a) $x = 0$
(b) $x = 4$
(c) $x = -4$
(d) $x = -1$
(e) NG

2. What is the solution of the following equation?

$$x + x = x$$

(a) $x = -1$
(b) $x = 0$
(c) $x = 1$
(d) It has no solution
(e) NG

3. What is the y-intercept of the line whose equation is $3x + 2y = 6$?

(a) -6
(b) 2
(c) 6
(d) 3
(e) NG

4. What is the solution of the following equation?

$$2(x + 1) = 3 - 2(x - 1)$$

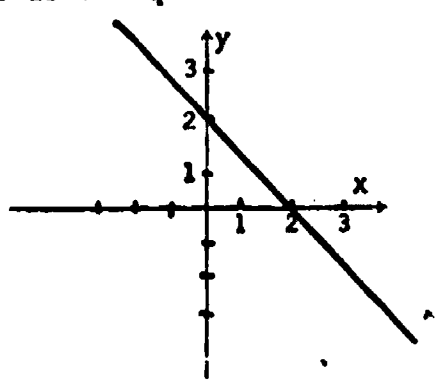
(a) $x = -\frac{3}{4}$
(b) $x = \frac{3}{4}$
(c) $x = -2$
(d) $x = \frac{1}{4}$
(e) NG

5. What is the solution of the following equation?

$$\frac{x}{2} + \frac{x}{3} = 5$$

(a) $x = 1$
(b) $x = -1$
(c) $x = 6$
(d) $x = 25$
(e) NG

6. What is an equation of the line below?



(a) $x + y = -2$
(b) $y = x$
(c) $x + y = 2$
(d) $x - y = 2$
(e) NG

7. What is the solution of the following equation?

$\frac{3x}{4} = 12$

(a) 16
(b) 12
(c) 8
(d) 9
(e) NG

8. What is the slope of the line whose equation is $y = 2x + 5$?

(a) $\frac{2}{5}$
(b) -5
(c) +5
(d) +2
(e) NG

9. Which point lies on the graph of the equation $y = 3x - 5$?

(a) (0, 0)
(b) (3, -5)
(c) (-3, 5)
(d) (-7, -26)
(e) NG

10. What is the simultaneous solution of the following two equations?

$x + y = 10$

$x - y = 4$

(a) x = 5 and y = 5
(b) x = 5 and y = 1
(c) x = 6 and y = 4
(d) x = 7 and y = 3
(e) NG

PART D. This part contains ten questions.

1. $(4y - 7x)(2y - 5x) =$

(a) $8y^2 - 37xy + 35x^2$
(b) $8y^2 + 35x^2$
(c) $8y^2 - 6xy - 35x^2$
(d) $8y^2 - 37xy - 35x^2$
(e) NG

2. $\frac{3x - 1}{1 - 3x} =$

(a) 1
(b) -1
(c) 0
(d) $\frac{1 + 3x}{1 - 3x}$
(e) NG

3. $\frac{3x}{x^2 - 4} + \frac{2}{x + 2} =$

(a) $\frac{5x - 4}{x^2 - 4}$

(b) $\frac{3x + 2}{x^2 - 4}$

(c) $\frac{3}{x - 4} + \frac{1}{x}$

(d) $\frac{5x + 2}{x^2 - 4}$

(e) NG

---

4. $\frac{(x + 3)^2}{6} \div \frac{3x + 3}{x + 3} =$

(a) $\frac{(x + 3)^2}{2}$

(b) $\frac{x + 3}{6}$

(c) $\frac{x + 1}{2}$

(d) $\frac{x + 1}{6}$

(e) NG

---

5. $\frac{3}{x} + \frac{2}{y} =$

(a) $\frac{5}{x + y}$

(b) $\frac{5}{xy}$

(c) $\frac{3y + 2x}{xy}$

(d) $3y + 2x$

(e) NG

---

6. Factor completely $a^3 - a$

(a) $a(a + 1)(a - 1)$

(b) $a^2(a - 1)$

(c) $a(a - 1)^2$

(d) $(a^2 - 1)(a + 1)$

(e) NG

---

7. $\frac{x}{y} - \frac{x - 1}{y} =$

(a) $-\frac{1}{y}$

(b) $\frac{1}{y}$

(c) 1

(d) -1

(e) NG

---

8. Factor completely $5x^2 + 11x + 2$

(a) $(5x - 1)(x - 2)$
(b) $(5x + 1)(x + 2)$
(c) $(5x + 1)(x + 1)$
(d) $(5x - 2)(x - 1)$
(e) NG

---

9. $\frac{a - b}{a} \div \frac{a^2 - b^2}{a^2} =$

(a) $\frac{a + b}{a^2}$
(b) $\frac{a^2 - b^2}{a^2}$
(c) $\frac{a - b}{a + b}$
(d) $\frac{a}{a + b}$
(e) NG

---

10. $\frac{x^2 + 4x + 4}{2x + 4} =$

(a) $\frac{x + 4}{2}$
(b) $\frac{x + 2}{2}$
(c) $x^2 + 2x + 1$
(d) $x^2 + 2$
(e) NG

END OF DIAGNOSTIC PLACEMENT EXAMINATION*

Now, please score your placement examination according to the following answers...

PART A		PART B		PART C		PART D	
1.	a	1.	c	1.	c	1.	e
2.	d	2.	a	2.	d	2.	b
3.	b	3.	d	3.	d	3.	a
4.	c	4.	e	4.	b	4.	e
5.	e	5.	b	5.	c	5.	c
6.	b	6.	d	6.	c	6.	a
7.	b	7.	a	7.	a	7.	b
8.	c	8.	b	8.	d	8.	b
9.	b	9.	a	9.	d	9.	d
10.	c	10.	c	10.	d	10.	b

*Reproduced from SIMM: Program Rationale and Tests. New York: Cummings Publishing Co. 1973.

--If you scored less than 7 on Part A, begin your work with Unit I.

--If you scored 7 or better on Part A, but less than 7 on Part B, you may begin with Unit II.

--If you scored 7 or better on Parts A and B, but less than 7 on Part C, you may begin with Unit III.

--If you scored 7 or better on Parts A, B, and C, you may skip all assignments in this course and procede directly to the final examination. Read the passage below in this syllabus for directions concerning the final examination.

HOW TO STUDY EACH UNIT

There are three units of eight lessons in this course. In each unit, you'll study one module of the text. Begin the unit by reading page iv: "TO THE STUDENT". Complete the unit's work as directed.

When you've completed the module, take the unit post test. (If you've forgotten where the post tests are, refer to your initial meeting sheet.) Score your post tests by the answers provided. You should make 90% correct or else review your study. Then, procede to the next unit.

Keep a record of your post test scores and dates of completion.

UNIT	TEST SCORE	COMPLETION DATE
I	________	________
II	________	________
III	________	________

COMPLETING THE COURSE

In order to achieve a "pass" grade, ask your instructor for the final examination for Basic Algebra, Part I. This test will take about 90 minutes. You'll need paper and pencil.

KEEP THIS SYLLABUS FOR USE IN PART II.

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Oleanna Math Program

BASIC ALGEBRA (Part II) -- Syllabus for Review Path, to accompany SIMM: Series in Mathematics Modules IV-V. By Walter A. Coole -- with the assistance of Uncle Thorbald

This short course is a continuation of the basic algebra course.

Your final examination will be a 40-item multiple-choice examination. To extend your grade for the course, you may complete an additional 10 hours' work from the Smorgasbord. If you're interested, ask your instructor or a coach about it. If you don't have sufficient time to complete the A-project before the end of the term, your "B" will be recorded initially, and the grade will be revised upward when the project is completed.

We shall continue the unit and module numbering we began in Basic Algebra, Part I. Please fill out the following plan...

Date to complete the course ____________________

Date to take the final exam ____________________
(Should be at least 3 days before completing the course.)

UNIT IV ____________________

UNIT V ____________________

If you're beginning the course at the first of the term, check the bulletin board for required schedules of completion.

IF YOU COMPLETED PART I BY CHALLENGE...

and you scored 7 or better on Part D of the placement examination, you may skip to Unit V. Note the syllabus for Part I for instructions on how to study each unit.

Keep a record of your post test scores and dates of completion.

UNIT	TEST SCORE	COMPLETION DATE
IV	________	________________
V	________	________________

Having completed Unit V, you are now ready for the final examination.

Make an appointment with your instructor or see him during a conference period for your final exam. You'll need a 50-entry student response form--available at the Bookstore.

GOOD LUCK!
****
*****
***
*

When you've passed the final exam (which entitles you to a grade of "B") you may wish to go on to a grade of "A". If you don't have time to complete your "A" project during the term, a grade of "B" will be reported for you immediately; it will be revised on the record when your project is completed.

ASK YOUR INSTRUCTOR OR A COACH ABOUT THE SMORGASBORD.

Another way to raise your grade from "B" to "A" is to turn in answers to all the additional exercises in Modules I through V.

Having completed both parts of Basic Algebra, you may procede to Intermediate Algebra. You needn't wait for the beginning of the next term, you may start now. See the instructor or a coach.

Oleanna Math Program

ELEMENTS OF PLANE GEOMETRY. Course Outline
by Walter A. Coole, Skagit Valley College

Skagit Valley College course number: Mathematics 8

Quarter credits: 3 Semester credits: $1\frac{1}{2}$

Average student completion time: 100 hours

Goal: to provide a bridge between elementary algebra and more advanced mathematics, in which the properties of planar space become important topics. This course differs from the standard high-school course in plane geometry in that it omits certain topics of the classical Euclidean geometry in favor of the salient results of that schedule.*

Performance objectives. This course of study consists of five units whose objectives are...

I. to be able to describe, classify, and measure angles; to compute complementary, supplementary, and vertical angles;

II. to be able to decide when polygons are similar and/or congruent; to compute the number of degrees in both exterior and interior angles, given sufficient data; to recall and apply the Pythagorean theorem;

III. to recon the perimeter and area of certain kinds of polygons;

IV. to describe and compute certain measures of circles and associated figures (eg. radii, apothems, and recall an approximation for the number π;

V. to identify the sine, cosine, and tangent of acute angles; use a simple table to derive approximations of these functions; to apply trigonometric practice to the computation of unmeasurable physical dimensions.

Entry. The student entering this course should have completed basic algebra with a grade of "B". In addition, he/she should be able to:

- i. read and follow simple written instructions
- ii. state his educational objectives in simple, coherent terms
- iii. study systematically and diligently

*The Oleanna Math Program contains a study of the Euclidean schedule in its Smörgåsbord of optional modules.

Student materials

Zlot, Graber, & Rausch: *Elementary Geometry*
Paper, pencil, protractor, ruler, compass
Coole: *Syllabus to accompany Elementary Geometry*

Other materials required

Coole: *Final examination for Elementary Geometry*

Teacher preparation

Study text, syllabus, and test.

Syllabus to accompany *Elementary Geometry*
by Walter A. Coole, Skagit Valley College

The *purpose* of this course is to provide a bridge between elementary algebra and more advanced mathematics, without dwelling on the more antiquarian interest of Euclid's *Geometry*. Thus, it will deal with the *results* of plane geometry as applies to advanced algebra and leaves off with some of the unnecessary material.

Materials:

Zlot, Graber & Rausch: *Elementary Geometry*
Paper, pencil, protractor, ruler, compass

Timing. This course will require about a hundred hours' work. It is divided into five units of approximately equal difficulty. Please set target dates for the five units so that you will be sure of completing the course by the required time...

Unit I ______________________

Unit II ______________________

Unit III ______________________

Unit IV ______________________

Unit V ______________________

Final Examination ______________________

....

UNIT I: Your *objective* for this unit will be to be able to describe, classify, and measure angles; also to compute complementary, supplementary, and vertical angles.

Study chapter I of the text and solve all problems. Check your answers with those given in the back of the book. (You should have almost all of them right.)

If you encounter difficulty, see your instructor or a coach as soon as possible.

UNIT II: Your *objective* for this unit will be to be able to decide when polygons are similiar and/or congruent; to compute the number of degrees in both exterior and interior angles, given sufficient data; and to recall and apply the Pythagorean theorem.

Study Chapter II of the text and solve all problems. Check your answers with those given in the back of the book (you should have almost all of them right.)

If you encounter difficulty, see your instructor or a coach as soon as possible.

UNIT III: Your *objective* for this unit will be to be able to reckon the perimeter and area of certain kinds of polygons.

Study Chapter III of the text and solve all problems. Check your answers with those given in the back of the book.

UNIT IV: Your *objective* for this unit will be to be able to describe and compute certain measure of circles and associated figures (eg. radii, apothems) and to recall an approximation for the number π.

Study Chapter IV of the text and solve all problems. Check your answers with those given in the back of the book.

UNIT V: Your *objective* for this unit will be to be able to identify the sine, cosine, and tangent of acute angles; use a simple table to derive approximations of these functions; and to apply trigonometric practice to the computation of unmeasurable physical dimensions.

Study Chapter V of the text and solve all problems. Check your answers with those given in the back of the book.

Your final examination will require a 50-entry answer form, the text, paper, pencil, protractor, compass, and ruler.

Oleanna Math Program

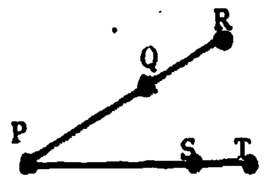
Final Exam for *Elementary Geometry*. Form A. by Walter A. Coole, Skagit Valley College

DO NOT WRITE IN THE EXAMINATION BOOKLET!

Use a 50-entry student response form for your answers. You may use the text and notes if you wish. There is no time limit for this exam.

1. T-F. The word *geometry* means *earth measure*.

2. Which point is the vertex of the angle formed?



A. - P

B. - A

C. - R

D. - S

E. - T

3. Which of the following correctly names the angle shown in problem 2?

A. < RQT

B. < RPT

C. < AST

D. Both B & C

E. None of the above

4. How many degrees are in this angle?

A. 150°

B. 60°

C. 120°

D. 30°

E. None of the above

5. T-F. A 90° angle is acute.

6. The supplement of a 34° angle is _______?

A. 146°

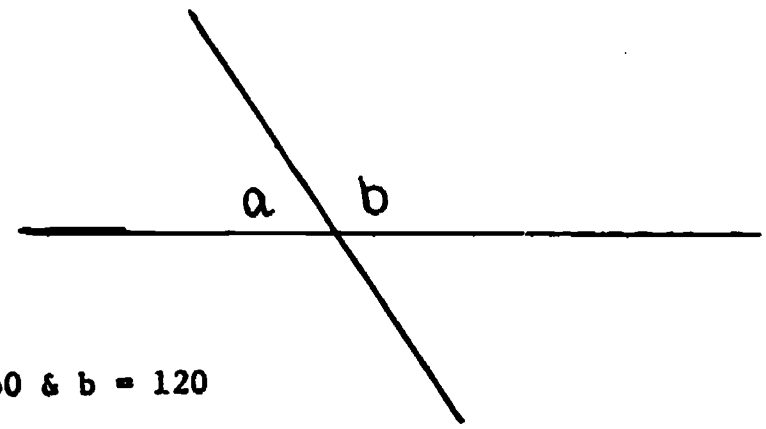
B. 154°

C. 156°

D. 56°

E. None of these

7. Find the number of degrees in <a and ∢b.



A. a = 60 & b = 120

B. a = 120 & b = 60

C. a = 40 & b = 140

D. a = 140 & b = 40

E. None of these

8. T-F. All plane figures can be divided into assemblies of triangles.

9. How many degrees are in the sum of the angles of a triangle?

A. 90°

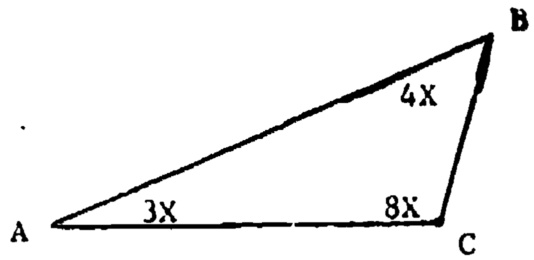
B. 120°

C. 270°

D. 360°

E. None of these

10. How many degrees are there in <C?



A. 35°

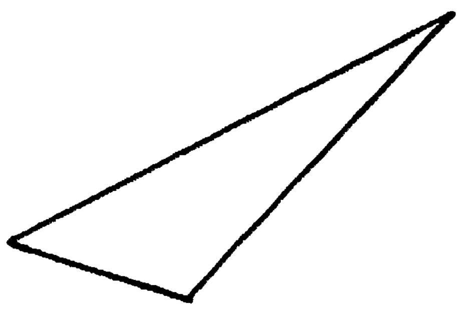
B. $47\frac{1}{2}$°

C. 60°

D. 105°

E. None of these

11. Classify this:



A. Acute scalene triangle

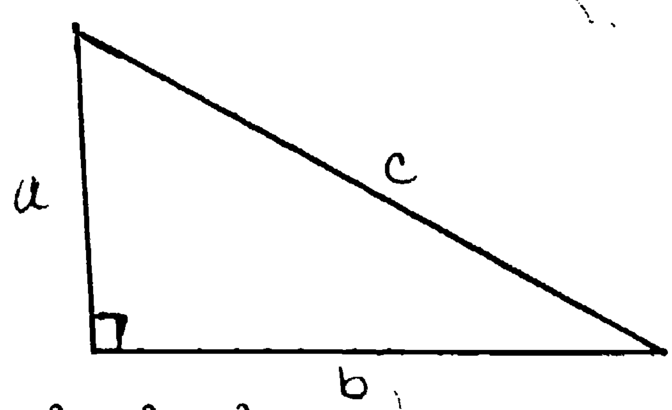
B. Acute isosceles triangle

C. Obtuse scalene triangle

D. Obtuse isoceles triangle

E. None of these

12. Which of these formulas applies?



A. $a^2 + b^2 = c^2$

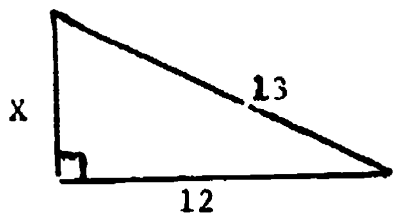
B. $a^2 + c^2 = b^2$

C. $b^2 + c^2 = a^2$

D. All of these

E. None of the above

13. Find the length of side X.



A. 3

B. 4

C. 5

D. 6

E. 7

14. Find the length of side Q.

A. 3

B. 4

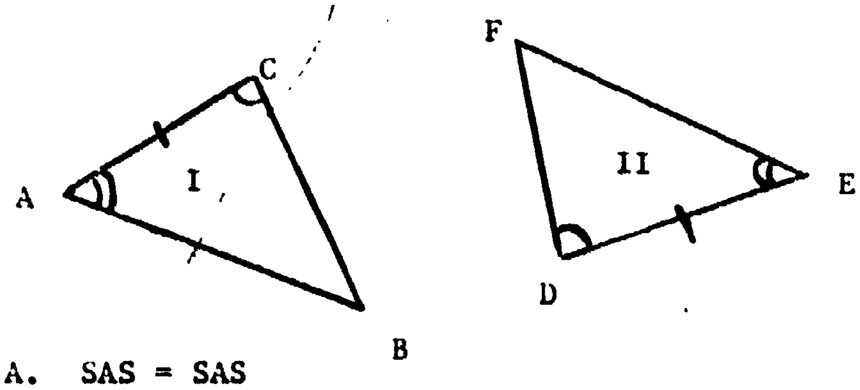
C. 5

D. 6

E. 7

15. T-F. Two objects are congruent when all parts correspond with a common proportion.

16. Which of these rules confirm that ΔI = ΔII?



A. SAS = SAS

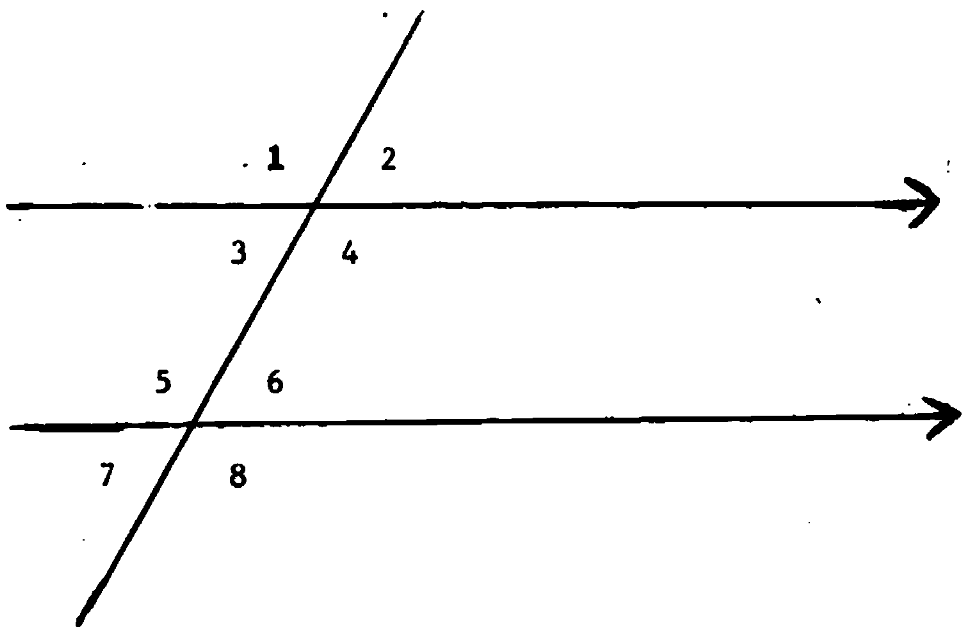
B. ASA = ASA

C. SSS = SSS

D. AAA = AAA

E. None of these

17. In this sketch...



Which of these choices are *corresponding angles?*

A. <1 & <2

B. B. <1 & <3

C. <1 & <5

D. <3 & <6

E. None of these

18. Which are alternate interior angles.

A. <1 & <2

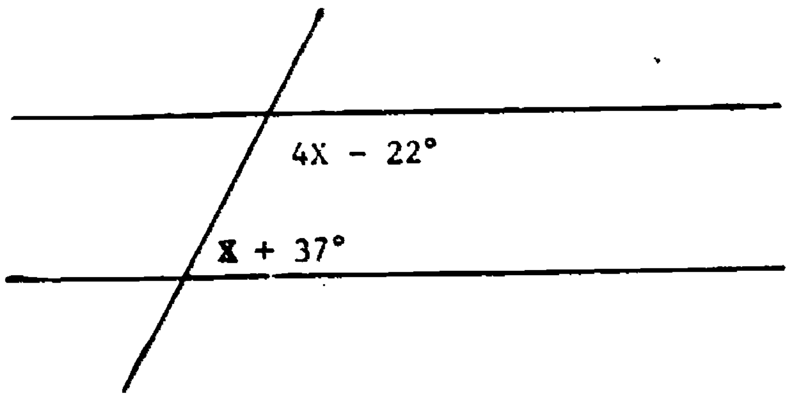
B. <1 & <3

C. <1 & <5

D. <3 & <6

E. None of these

19. Find X.



A. 18°

B. 24°

C. 27

D. $42\frac{1}{3}°$

E. None of these

20. A boy 1.7m. tall casts a 2.6m. shadow. In the same location and at the same time, a flagpole casts a shadow 19.5m. long. Find the height of the flagpole.

A. 10.25m.

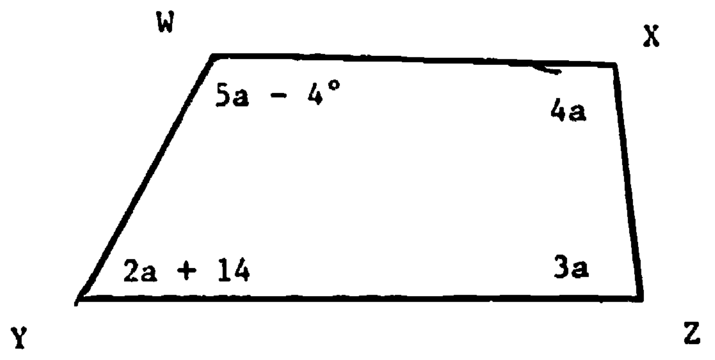
B. 12.75m.

C. 25.5m.

D. 39.5m.

E. None of these

21. Consider the following quadrilateral...



A. W = 121°

B. Y = 64°

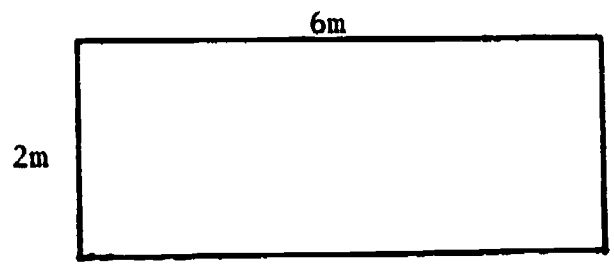
C. X = 100°

D. Both A & B

E. A, B, & C above

22. T-F All squares are rhombuses.

23. What is the area of this rectangle?



A. 8m

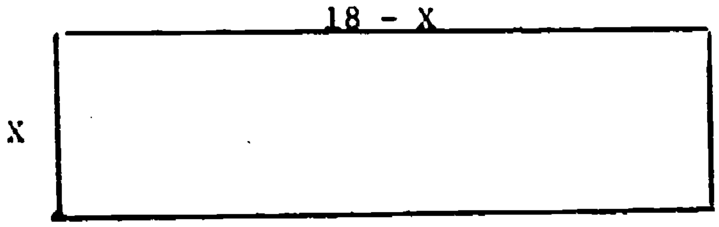
B. 12m

C. 10m

D. 26m

E. None of the above

24. What is the area of this rectangle?



A. X^2

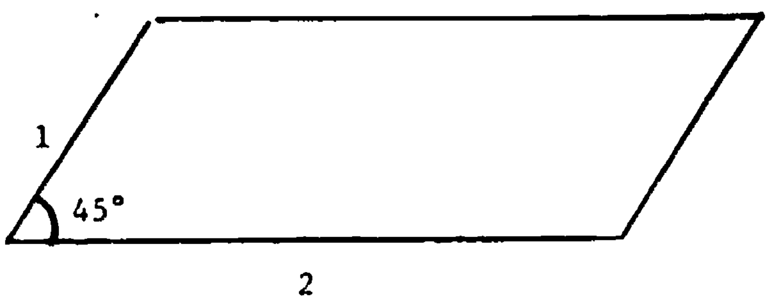
B. 18

C. $18X - X^2$

D. $18 - X^2$

E. None of these

25. What is the area of this parallelogram?



A. 1

B. $\sqrt{2}$

C. $\sqrt{3}$

D. 2

E. None of these

26. T-F. The approximate value of π is 3.1416.

27. T-F. A *tangent* to a circle touches the circle at 2 points.

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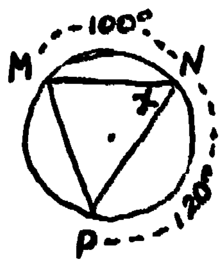
28. How many degrees in <X?



A. 35°

B. 52°

C. 68°

D. 70°

E. None of these

29. The tangent of 45° is....

A. .1

B. .5

C. 1.0

D. 2

E. None of these

30. T-F. Cos A = 1/sin A

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Oleanna Math Program

INTERMEDIATE ALGEBRA. Course Outline by Walter A. Coole, Skagit Valley College

Skagit Valley College Course Number: Mathematics 101

Quarter credits: 5 Semester credits: 3

Average student completion time: 165 hours

Goal.

This course is equivalent to a second-year high school algebra course. Its primary goal is to fully-equip the student for advanced mathematical studies; much of the technique, however has practical applications.

The two-track approach.

A. The standard path assumes that the student has mastered, thoroughly, the content of a basic algebra course, but has no knowledge of the content of more advanced subject-matter.

The standard path's performance objectives, lesson-by-lesson, are as follows:

1. perform the four fundamental operations on real numbers;
2. solve linear equations and interpret linear inequalities;
3. solve systems of equations, using determinants;
4. multiply and divide polynomials;
5. solve equations involving fractional polynomials;
6. interpret and manipulate numbers expressed in exponential and root notation;
7. solve quadratic equations;
8. graph second-degree equations;
9. use the theory of logarithms to compute various values.

* * * *

B. The review path assumes that the student is familiar with the content of an intermediate algebra course, but requires extensive review before proceeding to the next course.

Its performance objectives, lesson-by-lesson are:

Chapter 1 Sets

1. Be able to identify sets and their elements.
2. Learn to use and read the symbols $\in$, $\notin$, $\{\}$, ϕ, $\cap$, $\cup$, $|$, $\leq$, $\geq$, $\subseteq$, $\supseteq$, (a,b), AxB.
3. Learn the definitions of the terms well defined, subset, intersection, union, ordered pair, cross product
4. Be able to describe sets using set-builder notation.

Chapter 2 From N to R

1. Define the sets N, N*, I, and R.
2. Work with the fundamental laws of numbers.
   (a) Commutative lows
      1. of addition
      2. of multiplication
   (b) Associative laws
      1. of addition
      2. of multiplication
   (c) Distributive laws
3. State the properties of closure of each set with respect to addition, subtraction, multiplication, and division.
4. Define subtraction and division in each set in terms of addition and multiplication.
5. Give the inclusion relations between the sets N, N*, I, and R and the logical development of each as an extension of the included sets.
6. Add, multiply subtract, and divide (when defined) the elements in each set.

Chapter 3 Other Number Fields

1. Recognize elements of R* and C.
2. Define a for a R*.
3. Add, subtract, multiply, and divide real numbers.
4. Add, subtract, multiply, and divide complex numbers.

Chapter 4 Sets of Polynomials

1. Define a polynomial over I, R, R*, or C.
2. Identify elements of sets of polynomials.
3. Define and identify coefficients, indeterminate, leading coefficient, constant term, and degree of a polynomial.
4. Add, subtract, and multiply polynomials.

Chapter 5 Factorization of Polynomials

1. Factor numbers in N, I, R, R*, and C.
2. Factor polynomials over I, R, R*, and C.
3. Multiply certain special products by inspection.
4. Factor quadratic polynomials by completing the square.

Chapter 6 Rational Algebraic Expressions

1. Simplify complex fractions.
2. Reduce algebraic fractions to lowest terms.
3. Find the LCM (or LCD) of a set of polynomials (or a set of fractions).

Chapter 7 Solution Sets of Equations in One Variable

1. Solve linear equations.
2. Solve quadratic equations.
3. Solve some equations of higher degree.
4. Solve equations involving rational algebraic expressions.
5. Solve equations involving radicals.

Chapter 8 Solution Sets of Inequalities

1. Solve linear inequalities.
2. Solve quadratic inequalities.
3. Solve inequalities involving rational expressions.
4. Solve inequalities involving absolute values.

Chapter 9 Solution Sets of Systems of Linear Equations

Solve directly or by using matrices:

1. Two equations in two unknowns.
2. Three equations in three unknowns.
3. Systems with more equations than unknowns.
4. Systems with more unknowns than equations.
5. Homogeneous systems.

Chapter 10 Solution of Linear Systems of Determinants

1. Solve any linear system of two equations in two unknowns by using determinants.

Chapter 11 Proofs and Mathematical Induction

1. Understand what constitutes direct and indirect proofs.
2. Be able to put conditional statements together in a logical sequence to make up a proof by either direct or indirect proof.

Chapter 12 Synthetic Division and the Remainder Theorem

1. Divide polynomials to find a quotient and a remainder.
2. Divide by synthetic division.
3. Given $P_{(x)}$ find $P_{(a)}$ by synthetic division.

Chapter 13 Some Theory of Equations

1. Find upper and lower bounds for roots of a polynomial equation over R*.
2. Find all rational roots, and in some cases all roots, of polynomial equations over R.
3. Factor polynomials over R into linear factors.

Entry.

The student entering either path of Intermediate Algebra should be able to perform with ease, all four fundamental operations on rational (algebraic) expressions and solve linear equations of considerable difficulty. In addition, he/she should be able to:

i. read and follow simple written instructions

ii. state his educational objectives in simple, coherent terms

iii. study systematically and diligently

Student materials.

Testing form: Automata Student Response Card (1-50)
Paper and pencil

Standard Path

Keedy & Bittinger: *Intermediate Algebra--A Modern Approach.* Reading, Mass. Addison-Wesley. 1971.

Coole: *Syllabus for Intermediate Algebra* (Standard Path)

Review Path

Howes: *Pre-Calculus Mathematics* Book I: Algebra. NY: John Wiley & Sons. 1967

Coole: *Syllabus for Intermediate Algebra* (Review Path)

Teacher preparation.

Study instructor's manuals, testing materials, and texts.

Other materials required.

Oleanna Math Program: *The Student-Decision Placement Test.*

Cooperative Testing Service: *Cooperative Math Test--Intermediate Algebra* (Forms A and B and user's manual.) Palo Alto, CA: Cooperative Testing Service. 1969.

Oleanna Math Program: Smorgasbord.

Syllabus for INTERMEDIATE ALGEBRA (Standard Path) by Walter A. Coole, Skagit Valley College

Your *goal* for this course is to master all of the principles of the intermediate stage of algebraic studies. Such mastery will enable you to do well in more advanced studies. Along with this mastery, you'll learn a number of useful ways to solve "real world" problems with algebraic methods.

This course is divided into four "units", each of which will require about 40 hours'work. By following directions in this syllabus, you'll be able to avoid spending time unnecessarily on information you've already mastered. The units of the course are:

Unit	Lesson	Completion date
I	Pre-test	________________
	1	________________
	2	________________*
II	3	________________
	4	________________
	5	________________*
III	6	________________
	7	________________
IV	8	________________
	9	________________*
	Final	________________*

Your completion date for the pre-test should be the day of you earliest scheduled conference.

Completion dates for each unit (marked by asterisks*) should be filled in from the scheduled provided. If you're beginning at the opening of a school term, your schedule will be posted on the bulletin board; otherwise, your teacher will work out a special schedule for you.

For this course, you'll need paper, pencil, and the following textbook:

Deedy & Bittinger: Intermediate Algebra--A Modern Approach

DO ALL OF YOUR WORK IN PENCIL!!

Pre-test

At the very front of the textbook, you'll find a 'PRETEST'. Write your answers to the pre-test on a sheet of notebook paper.

Score your results from the answers given in the back of the book.

Note the "Pretest Analysis" which tells you which lessons you may skip. If you wrote as many as 55 correct answers on the pre-test, you should then skip to the "Final Examination" at the back of the text. If you can write 50 correct answers from this test, you should contact the instructor for the "official" course-completion test.

How to Study Each Lesson

Each chapter in the textbook corresponds to a lesson in this course. By using your pretest results, you should be able to decide which lessons to omit.

Each chapter of the text is divided into several sections. Begin each section by reading the objectives (what you should learn) and then the explanation.

Write the answers to problems as you are directed in the text.

As you complete each section's "Margin Exercises", check your answers in the back of the book. If you have difficulty, see your instructor or a math coach as soon as possible.

Next, complete the odd-numbered exercises in all exercise sets at the end of the chapter. Then score your results, using the answers given in the back of the book.

To complete the lesson, take the test at the end of the chapter. Score your results, using the answers at the back of the book and follow the directions given in the test analysis.

When you've scored satisfactory results, remove the chapter test from the book and turn it in. If the test uses more than one sheet, staple them together at the upper left-hand corner. The tests will be returned as soon as they are recorded.

Completing the Course

After you've mastered all of the chapters of the textbook—either by scoring perfect on the pre-test or by achieving a satisfactory grade on the end-of-chapter test—complete the final examination provided at the back of the book. Score your results against the answers in the back of the book and follow directions given in the analysis.

When you've scored 40 or better on the final examination, you are ready to take the "official" course completion test.
You may take this test at any scheduled conference or by appointment. You'll need paper, pencil, and a 50-entry student response card (on sale at the bookstore). You may use your textbook and notes during the test. Average completion time for the end-of-course test is 40 minutes, but may take longer if you wish.

Grading

When you've completed the end-of-course test, you may close off the course with a grade of "B". If you wish to improve your grade to an "A", you may act as a coach or undertake optional projects from the "Smorgasbord". This may be done during the following term and your "B" will be changed to an "A".

INTERMEDIATE ALGEBRA--Syllabus (Review Path)

Walter A. Coole, Skagit Valley College

In this course of study, you will extend your algebraic competence by understanding mathematics as a systematic study of quantity, treated abstractly.

This syllabus is to be the basic "map" by which you will progress through your course of study.

Rationale. Contrary to a widespread myth that mathematically ignorant people take seriously, mathematics is a purely human language; it was developed to make human problems easier to solve and to be used by people to talk about *quantities* of things important to them.

The advantage that mathematics has over the "natural" languages (ie. English, French, Coptic) is that mathematics is deliberately contrived to deal with quantitative expressions more easily. When it is necessary for a society with many individuals to cooperate, then quantities of things must enter significantly into the dialogue.

Your objectives in this course will be to:

(1) understand algebra as a systematic study of quantitative language, particularly as it applies to the system of real numbers;

(2) master all computational skills to enter advanced mathematics and begin studies in the sciences, natural and behavioral.

Pre-requisite testing: This process should take about 3 hours[1] and should be completed within one day after beginning the course. Check off each item as you have completed it in the space provided.

( )Read page xi of your textbook, Howes: *Pre-calculus Mathematics, Book I: Algebra*

( )Write out your answers for the "Prerequisite Test for Book I." Score your results, using the answers in the textbook. *If you get more than one wrong, see your instructor. You may be in over your head!*

( )Write out your answers for the "Post Test for Book I." Score your results by using the answers given in the text. If you didn't get them all right, you should be satisfied that you're in the right course--if you did get them all right, you may complete this course, simply by <u>taking the final examination.</u>

1. Time-estimates given in this syllabus are based on an 11-week term. If your schedule is different, you should make adjustments accordingly.

Sample lesson. (Approximately 10 hours.)

( ) Keep track of the time you spend on this lesson; write down each hour of study you invest into the course. This will allow you to estimate how much more or less to spend on each subsequent lesson.

( ) Read the lesson's objectives and rationale on page 1 of the text.

( ) Complete the lesson assignment, beginning on page 2 of the textbook and continuing to the end of Chapter 1. If you have difficulty, see your instructor or a math coach in the learning laboratory.

( ) Review the chapter by referring back to page 1. Examine each objective and satisfy yourself that you can perform as required by the text.

( ) Test your mastery of Chapter 1 by writing out the answers to the problems at the end of the chapter. Score your answers.

--If you do not achieve 90%, repeat the lesson.
--If you do score over 90%, prcede to the next lesson...

( ) Write the number of hours spent on this Sample Lesson: ____________. This will give you a rough basis for estimating the number of hours to allow for the remaining lessons in this course.

Your instructor will want to see this section of the syllabus completed by the time you attend your first scheduled conference.

....

BEFORE WE GO ANY FURTHUR

Let's plan your way through the course.

If you're beginning this course at the first of the term, you'll find a completion-schedule posted in the learning laboratory. Use the dates given there to fill in the unit completion schedule below. If not...

Pick a date at which you intend to complete the course. Write that date

here:_________________________. Now, back up three days; write that date

here: ___________________________. This will be your examination date.

Count the number of days between today's date and the examination date.

____________. Divide the number of available days by 4: __________. This is the number of days you should spend on each of the four units of study. Please note that each unit consists of three lessons. Enter the date for completing each unit of study below. (If you plan to undertake special projects to make a grade of "A", allow for about 30 hours' work.)

Unit I:______________________ Unit III:________________

Unit II:_____________________ Unit IV: ________________

One of the course requirements is to maintain this completion-schedule.

You should plan to attend every scheduled conference in your program.

Remember that your instructor and fellow-students who are serving as coaches will be available between conferences to help you with the rough spots. The best way to ask for help is to be able to point out a specific part of the textbook that's causing you trouble. BRING YOUR TEXTBOOK AND SYLLABUS FOR COACHING!

UNIT I

Unit I consists of Lessons 1, 2, and 3 (corresponding to textbook chapters 2, 3, and 4) and should be studied in the same way that you did Chapter 1 in the foregoing Sample Lesson.

As you complete each lesson, write the time and date. Record your test score, simply by noting the number of problems correctly solved.

Lesson Number	Read Objectives	Complete Programmed Work	Review Objectives	Test Score
1				
2				
3				

UNIT II

Unit II consists of Lessons 4, 5, and 6 (Chapters 5, 6, and 7).

Lesson Number	Read Objectives	Complete Programmed Work	Review Objectives	Test Score
4				
5				
6				

AT THIS POINT, YOU'RE HALFWAY DONE!

UNIT III

Unit III's lessons cover Chapters 8, 9, and 10.

Lesson Number	Read Objectives	Complete Programmed Work	Review Objectives	Test Score
7				
8				
9				

UNIT IV

In this unit, you'll polish off the remaining three chapters.

Lesson Number	Read Objectives	Complete Programmed Work	Review Objectives	Test Score
10				
11				
12				

When you've completed Lesson 12 (Chapter 13), you're ready for the final examination. You may take it at your scheduled conference hour or you may make an appointment to take it at another time.

Oleanna Math Program

PROBABILITY AND STATISTICS. Course outline
by Walter A. Coole, Skagit Valley College

Skagit Valley College Course Number: Mathematics 108

Quarter credits: 5 Semester credits: 3

verage student completion time: 165 hours

Goal. Upon completion of this course, the student should be able to retrieve and apply the basic principles of probability theory and statistical techniques to solving problems of empirical research; he should also be able to read standard scientific-mathematical treatises critically.

Performance objectives. There are four units (17 lessons) in this course. Lesson objectives are given thus...the student should be able to...

1. express relative frequency in terms of sets and set-counts
2. describe sample spaces mathematically
3. calculate complementary events from event-data given
4. compute conditional probabilities
5. recall and apply principles for calculating the probabilities of dependent and independent events from given data
6. recall and apply principles by which unions of events may be computed
7. apply the principle of binomial distributions to calculating probabilities in appropriate situations
8. compute permutations and combinations, given the necessary data
9. determine whether a set of data represents a sample or a population; give reasons why samples rather than populations are often studied; draw a random sample, using a table of random numbers; select appropriate sampling procedures for specific situations; ask appropriate questions in order to determine the source and nature of data being presented; identify the primary difference between *descriptive statistics* and *statistical inference* as fields of study in statistics
10. classify properly the variables within the following categories: (a) ordered-unordered (b) scaled-unscaled (c) continuous-discrete... categorize variables into a general descriptive model, when given information about variables; identify the real and score limits of any specified score
11. arrange a set of raw data into cumulative frequency and cumulative percentage tables; determine, given the cumulative frequency or cumulative percentage the opposite; determine the percentage of the distribution between two specified percentiles, quartiles, or deciles; and the percentage of the distribution below or above a specified percentile, quartile, or decile

12. locate and designate any point on a braph by means of the rectangular system; group raw data into intervals (including the discrimination between the real limits and score limints of any interval and determination of the class midpoint of an interval and the number of intervals to be used in any specific distribution); construct a histogram (making proper decisions about the selection of the number and size of the intervals, based upon the significance of changing the shape of the graph); construct and obtain information from a frequency polygon;

13. compute the various measures of central tendency for a given set of grouped or ungrouped data (mean, mode, median); explain the effect of arithmetic operations with constants on the mean of raw score distributions

14. apply to grouped and ungrouped data various computational formulas for the variance and standard deviation; express Z-scores and determine when Z transformations are indicated

15. describe the normal curve in terms of (a) central tendency and (b) the relationship of the mean and standard deviation to the area under the curve; compute the percentile rank for scores given means and standard deviations; compute T scores

16. analyze correlation coefficients in terms of direction and magnitude; estimate magnitude and direction of r from a scatter diagram; define test-reliability and test-validity, giving examples of each

17. write general formulas and plot graphs for approximate statistical correlations, identifying dependent and independent variables correctly; plot the means for conditional distributions; partition variations around regression lines into explained and unexplained portions; identify $\Sigma(Y - \tilde{Y})^{-2}$, $\Sigma(Y - \tilde{Y})^2$, and $\Sigma(\tilde{Y} - \overline{Y})^2$ in terms of concepts of the total variation.

Enrichment

The student who has special academic programs or "real world" applications in mind will find a number of projects--varying in length, difficulty, and depth-of-contemplation-in the Olaanna Math Program's Smörgåsbord appropriate to such plans.

Entry

The student entering this course should be recall easily and apply, all of the principles and techniques treated in high school mathematics sequences through intermediate algebra. In addition, he/she should be able to:

i. read and follow moderately difficult instructions
ii. state his/her educational objectives in simple, coherent terms
iii study systematically and diligently

Student materials

Earl, Boyd: *Introduction to Probability*. N. Y. McGraw-Hill. 1969.
Gotkin & Goldstein: *Descriptive Statistics: a Programmed Textbook*. (In 2 vols.) N. Y. John Wiley & Sons. 1967.
Coole: *Syllabus for Elementary Probability & Statistics*.

Other materials required

Earl, Boyd: *Instructor's Manual for Introduction to Probability*. N. Y. McGraw-Hill, 1969.
Gotkin & Goldstein: *Teacher's Manual for Descriptive Statistics: A Programmed Textbook*. N. Y. John Wiley & Sons. 1967.
Coole: *Tests for Elementary Probability and Statistics*.
Oleanna Math Program: *Smörgåsbord*

Teacher preparation

Study student materials, instructor's manual, and texts. The teacher should be easily familiar with many applications of statistical techniques.

Oleanna Math Program

SYLLABUS FOR ELEMENTARY PROBABILITY & STATISTICS by Walter A. Coole, Skagit Valley College

GOAL. When you complete this course of study, you should be capable of retrieving and applying basic principles of probability theory and statistical techniques to solving problems of empirical research; also, you are expected to be able to read standard scientific-mathematical treatises critically.

The course is separated into four units of study. Units I and II will deal specifically with probability; Units III and IV with statistics.

While we will concentrate on your lrearning about practical applications, principles will not be neglected: understanding principles will help you in applying probability and statistical theory to problems not dealt with within this course.

Planning your way through the course

Using the completion-schedule provided, enter unit completion dates where indicated by a star (*). Then, use a calendar and your own plans to determine target dates for each lesson.

UNIT	LESSON	PLANNED COMPLETION	ACTUAL COMPLETION
I	1		
	2		
*	3		
II	4		
	5		
	6		
	7		
*	8		

UNIT	LESSON	PLANNED COMPLETION	ACTUAL COMPLETION
III	9		
	10		
	11		
	12		
	13		
	14		
*	15		
IV	16		
*	17		

Your *objectives* for the first two units of study will be to be able to: calculate the "ideal" relative frequency of events in an intuitively reasonable way.

RATIONALE

Events, particularly those involving human affairs, cannot be predicted precisely. Thus, when basing actions on predictions, one is "gambling" in the sense that he is playing the most-likely odds.

Not to "gamble" on human affairs is simply not to act. So we we are forced, in many cases, to act with incomplete knowledge of the consequences of our decisions. But does that mean that we are acting "in the dark?"

Certainly not. We plan and act for the *most likely outcome*. And that's where probability theory comes in...

Your textbook for Units I and II will be...

Earl, Boyd: *Introduction to Probability*

( ) Read pp. *v*–*x*.

UNIT I

Lesson 1

Your *objective* for this lesson will be to be able to express relative frequency in terms of sets and set-counts.

PRETEST: Self-Test I, p. 17 (Answers on p. 251) Score:__________

(If you feel a bit hazy on your set-theory, see Appendix A, pp. 245-249.)

ASSIGNMENT: Frames 1-75, pp. 1-16.

POST-TEST: Self-Test I, p. 17. Score:__________

Lesson 2

Your *objective* for this lesson will be to be able to describe sample spaces mathematically.

PRE-TEST: Self-Test II, p. 39. Score:__________

ASSIGNMENT: Frames 76-150, pp. 19-39.

POST-TEST: Self-Test II, p. 39. Score:__________

Lesson 3

Your *objective* for this lesson will be to be able to calculate complementary events from event data.

PRE-TEST: Self-Test III, pp. 61-62. Score:__________

ASSIGNMENT: Frames 151-235, pp. 41-61.

POST-TEST: Self-Test III, pp. 61-62. Score:__________

UNIT II

Lesson 4

Your *objective* for this lesson will be to be able to calculate conditional probabilities.

PRE-TEST: Self-Test IV, pp. 101-102. Score:__________

ASSIGNMENT: Frames 236-408, pp. 63-101.

POST-TEST: Self-Test IV, pp. 101-102. Score:__________

Lesson 5

Your *objective* for this lesson will be to be able to recall and apply principles for calculating probabilities of dependent and independent events from sufficient data given.

PRE-TEST: Self-Test V, pp. 140-141. Score:__________

ASSIGNMENT: Frames 409-583, pp. 103-140.

POST-TEST: Self-Test V, pp. 140-141. Score:__________

Lesson 6

Your *objective*: to be able to recall and to apply principles by which event-unions are computed.

PRE-TEST: Self-Test VI, pp. 158-159. Score:__________

ASSIGNMENT: Frames 584-660, pp. 141-158.

POST-TEST: Self-Test VI, pp. 158-159. Score:__________

Lesson 7

Your *objective* for this lesson will be to be able to apply the principle of binomial distributions to calculating probabilities in appropriate situations.

PRE-TEST: Self-Test VII, pp. 191-192. Score:__________

ASSIGNMENT: Frames 661-810, pp. 161-191.

POST-TEST: Self-Test VII, pp. 191-192. Score:__________

Lesson 8

Your *objective* for this lesson will be to master computations for permutations and combinations.

PRE-TEST: Self-Test VIII, pp. 226-227. Score:__________

ASSIGNMENT: Frames 811-950, pp. 192-226.

POST-TEST: Self-Test VIII, pp. 226-227. Score:__________

YOU ARE NOW ALMOST READY FOR THE MID-COURSE TEST. For review, complete Frames 951-1019, pp. 229-244.

When you've completed the review, ask your instructor for the mid-course test.

When you've passed that examination, record your completion of Unit II.

Your textbooks for Units III and IV will be:

Gotkin, Lassar G. & Goldstein, Leo S.: *Descriptive Statistics--A Programmed Textbook* (In two volumes).

( ) Read Volume 1, pp. iii-xi.

UNIT III

This unit will correspond to Volume 1 of the set. Lessons 9-15 will correspond to Units I-VI of the text. Each lesson's objectives are stated at the beginning of the relevant portion of the text.

There will be no lesson pre-tests or post-tests.

When you've completed Unit III, please report it on the chart.

UNIT IV

This unit will correspond to Volume 2 of the set. Lessons 16-17 will correspond to Units VII-IX of the text. Each lesson's objectives are stated at the beginning of the relevant portion of the text.

There will be no lesson pre-tests or post-tests.

When you've completed Unit IV, please report it on the chart.

For your final examination, ask your instructor for the final examination on statistics.

This examination is a "take home" examination. You have three days between the time it is picked up and the time you must turn it in to the instructor.

Your work should be typed or written neatly in pencil.

Plan to meet with your instructor after he has evaluated your examination results--*make a written appointment!*

Oleanna Math Program

FUNCTIONS AND RELATIONS. Course Outline by Walter A. Coole, Skagit Valley College

Skagit Valley College Course Number: Mathematics 111

Quarter Credits: 4 Semester Credits: 3

Average student completion time: 120 hours

Goal. This course begins a three-course sequence of "precalculus mathematics"; it is roughly equivalent to "advanced algebra".

The two-track approach.

A. The standard path assumes that the student has mastered, thoroughly, the content of an intermediate algebra course, but is not all familiar with its constitutent topics.

The standard path's performance objectives, lesson-by-lesson, are as follows:

1. perform all operations treated in elementary and intermediate algebra courses;
2. solve quadratic equations;
3. express relations, functions, and transformation in set-theoretic language;
4. express inequalities and quadratic functions in set-theoretic language;
5. use determinants to solve linear equations;
6. apply logarithmic principles to perform complex computations;
7. write imaginary numbers as complexes;
8. solve equations involving polynomials;
9. graph conic sections;
10. compute terms of series and sequences;
11. calculate permutations, combinations, and probabilities;

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B. The review path assumes that the student is somewhat familiar with the concepts of the course, but wishes a more figorous preparation.

Its performance objectives, lesson-by-lesson, are:

0. recognise, read, and use the symbols $\in$, $\subseteq$, $\emptyset$, N, I, R, R*, $\cap$, $\cup$, $\{x \mid x \in N \text{ and } 2x = 5\}$; identify elements of sets which are defined either by listing elements or in set-builder notation;

   identify elements of sets which are defined either by listing elements or in set-builder notation; find and graph solution sets of equations and inequalities in one variable.

1. give the Cartesian product of two sets; define ordered pairs, Caresaian products, relations, functions, inverse functions, domain and range; define a relation or a function as a set, either in set-builder notation or by listing the elements of the set; determine the domain and range of a given relation or function; determine the inverse of a given relation or function.

2. find the largest possible domain of a function defined in A X B when the domain is not specified; find the domain and range of a function defined by a formula, $y = f(x)$, in R* X R*; find the image of a number or other algebraic expression under a given function; add, subtract, multiply, and divide functions; find the composite, $f(g(x))$, of two functions; find an expression for $\frac{f(x + b) - f(x)}{b}$ for a function defined by $y = f(X)$. (A problem from calculus)

3. graph finite Cartesian product sets and their subsets; graph infinite Cartesian product sets; graph relations and functions on the Cartesian plane; determine the domain and range of a relation from a graph of the relation; determine whether a relation is a function from its graph; determine whether a function is one-to-one from its graph; graph the inverse of a relation given a graph of the relation.

4. recognize and graph: (a) constant functions, (b) linear functions, (c) quadratic functions. (d) polynomial functions of higher degree; graph a parabola by finding its vertex.

5. use synthetic division to find upper and lower integral bounds for the zeros of a polynomial; use synthetic division to compute functional values of polynomial functions; use the graph of a polynomial function to help locate the real zeros; use linear interpolation to approzimate irrational zeros to any number of decimal places.

6. graph rational functions (many are discontinuous); find where rational functions are discontinuous; graph absolute value functions (most graphs have sharp angles); graph functions defined by radicals (their domains are frequently restricted); find an equation for the inverse of functions defined by radicals.

7. find any term of a sequence defined by a formula; give arithmetic and geometric sequences by recursion formulas; define an arithmetic or geometric sequence by formula given its first term and common difference or common ratio; find the numerical value of (finite) series; find the numerical value of an arithmetic or geometric serries by formula; evalu te such expression as:

$$\sum_{n=1} (n^2 + n), \sum_{k=1} (3 + (k - 1)5), \sum 5 (\tfrac{1}{2})k \cdot 1$$

8. define a^n, a^o, a^{-k}, $a^{\frac{p}{q}}$; evaluate or simplify expressions as: $x^{-1} + y^{-2}$, $x^{\frac{5}{4}}$, $x^{-\frac{2}{3}}$, $\sqrt[3]{\sqrt{5}}$; graph exponential functions.

9. define the symbol $\log_b x$; graph logarithmic functions; derive the identities (a) $y = \log_b b^y$; (b) $x = b^{\log_b x}$; derive and use the the properties (a) $\log_b MN = \log_b M + \log_b N$ (b) $\log_b \frac{M}{N} = \log_b M - \log_b N$ (c) $\log_b M^r = 4 \log_b M$; prove the statement:

$$\log_a x = (\log_b x)(\log_a b).$$

10. use a table of common logarithms for finding the logarithm of any positive number or antilogarithm of any number; use logarithm s to approximate such expressions as:

$$\frac{(25.6)(.0035)}{4.752}, \frac{\sqrt[3]{374.5}}{.157}$$

solve exponential and logarithmic equations.

11. graph relations defined by linear equations in two variables; graph relations defined by certain quadratic equations in two variables; graph relations defined by linear and quadratic inequalities in two variables.

12. solve systems of linear equations; solve systems of equations where one equation is linear; solve some systems of quadratic equations; illustrate solutions of systems of equations and inequalities graphically.

Entry.

The student entering either path of Functions and Relations should have mastered thoroughly, the content of a standard intermediate algebra program. (Ie. as evidenced by a grade of B or achieving the 90th percentile on the Cooperative Math Test for Algebra II.)

In addition, he/she should be able to:

i. read and follow simple written instructions

ii. state his educational objectives in simple, coherent terms

iii. study systematically and diligently

Student materials.

Testing form Automata Student Response Card (1-50)
Paper and pencil

Standard path

Keedy & Bittinger: *College Algebra--A Functions Approach.* Reading, Mass. Addison-Wesley. 1974.

Coole: *Syllabus for Functions & Relations* (standard path)

Review path

Howes: *Pre-Calculus Mathematics Book II: Functions and Relations* NY: John Wiley & Sons. 1967

Coole: *Syllabus for Functions & Relations* (review path)

Teacher preparation.

Study instructor's manuals, testing materials and texts.

Other materials required.

Cooperative Testing Service: *Cooperative Math Test--Algebra III.* Forms A & B. Also, user's manual. Palo Alto, CA. Cooperative Testing Service. 1969.

Oleanna Math Program: Smorgasbord.

Oleanna Math Program

Syllabus for FUNCTIONS & RELATIONS (Standard Path) By Walter A. Coole, Skagit Valley College

Your *goal* for this course will be to master the notion of a *function* as it is used in the study of mathematics. Sets, their relations and functions are basic to understanding any mathematical subject matter beyound intermediate algebra.

This course is divided into four "units," each of which will require about 30 hours' work. By following directions in this syllabus, you'll be able to avoid spending time unnecessarily on information you've already mastered. The units of the course are:

Unit	Lesson	Completion date
I	1	________________
	2	________________
	3	________________*
II	4	________________
	5	________________
	6	________________*
III	7	________________
	8	________________
	9	________________*
IV	10	________________
	11	________________*
	Final	________________

You should attempt to complete Lesson 1 by your first scheduled conference if at all possible.

Completion dates for each unit (marked by asterisks*) should be filled in from the schedule provided. If you're beginning at the opening of a school term, your schedule will be posted on the bulletin board; otherwise, your teacher will work out a special schedule for you.

For this course, you'll need paper, pencil, and the following textbook:

Keedy & Bittinger: *College Algebra--A Functions Approach*

How to Study Each Lesson

Corresponding to each lesson, there is a chapter of the textbook. Each lesson begins with a 'PRETEST'. The pretest serves two p rposes: (1) it tells you what kind of problems you will learn about in the chapter and (2) it allows you to skip over chapters whose content you already know.

Begin the chapter's study by taking the pretest. Then score you results by checking against these given in this syllabus. If you score 90% or better on the pre-test, review the chapter briefly and procede to the chapter test at the end of the chapter. (See below).

Should you score less than 90% on the pretest, study the chapter thoroughly.

Each chapter in this text contains a number of sections. Each section has--

objectives given in the beginning of the margin

explanation with sample problems--you should work the sample problems and check the answers in the back of the book as you go

exercise set which should be worked through--answers to the
od odd numbered problems are in the back of the text

To complete a chapter, you should work the chapter test at the end. Turn the test in to be recorded; when your instructor's had a chance to inspect your work and record it, the test will be returned.

Completing the Course

After you've mastered all of the chapters of the textbook--either by scoring 90% on the pretest or by studying the chapter--you are ready for the final examination. This may be taken during any scheduled conference or by appointment.

You'll need paper, pencil, and a 50-entry student response card (On sale at the bookstore). You may use your textbook and notes during the test. Average completion for the end-of-course test is 40 minutes, but you may take longer if you wish.

Grading

When you've completed the end-of-course test, you may close off the course with a grade of "B". If you wish to improve your grade to an "A", you may act as a coach or undertake optional projects from the "smorgasbord". This may be done during the following term and your "B" will be changed to an "A".

Chapter 1
Pretest - KEY
pp. 3, 4

ANSWERS

1. -4
2. 0
3. -36
4. -6
5. 30
6. $3,261,000$
7. $.041$
8. 6.321×10^{1}
9. 1.432×10^{-2}
10. $-14a^{-2}b^{6}$
11. $6x^{9}y^{-6}z^{6}$
12. 4
13. 3
14. $\frac{xy}{x^2-y^2}$
15. -4
16. $y^3\sqrt[4]{y}$
17. $\sqrt[5]{b^7}$
18. $\frac{r^2s^3}{2}$
19. $-5, 3$
20. $y > -2$
21. $7, 3$
22. $(r-s)^2$
23. $\frac{2}{x-y}$
24. $\frac{y-\sqrt{xy}-2x}{x-y}$
25. $2000\ \frac{yd.}{min.}$

Chapter 1
Test - KEY
pp. 47, 48

ANSWERS

1. 14
2. -8
3. 63
4. 3
5. 24
6. $97,240$
7. $.000321$
8. 4.321×10^{4}
9. 2×10^{-5}
10. $6x^{-1}y^{5}$
11. $\frac{2}{3}x^{3}yz^{-7}$
12. 3
13. -2
14. $\frac{x+y}{xy}$
15. 2
16. $\sqrt[3]{(a+b)^2}$
17. $\sqrt[5]{a^3}$
18. $\frac{m^4n^2}{3}$
19. $6, -3$
20. $y < -3$
21. $-2, 1$
22. 3
23. $\frac{2a}{a^2-1}$
24. $\frac{x-2\sqrt{xy}+y}{x-y}$
25. $880\ \frac{ft}{min}$

Chapter 2
Pretest - KEY
pp. 51, 52

ANSWERS

1. $\{3\}$
2. $\{4, -\frac{5}{2}, -6\}$
3. $\{-\frac{3}{2}, 4\}$
4. $\{\frac{27}{7}\}$
5. 94%
6. $1\frac{1}{2}$ hr
7. 60
8. $\{8, -2\}$
9. Two real number Solutions
10. $\frac{3 \pm \sqrt{57}}{6}$
11. $S = \frac{4}{3}$, $P = \frac{2}{3}$
12. $x^2 - \frac{5}{2}x + 1 = 0$, or $2x^2 - 5x + 2 = 0$
13. $t = \frac{R - R_0}{R_0 a}$
14. 40 mph
15. $\{5\}$
16. $\{1\}$
17. $T = \frac{1}{180} \cdot \frac{x^2}{P}$
18. $533\frac{1}{3}$ cc

Chapter 2
Test - KEY
pp. 89, 90

ANSWERS

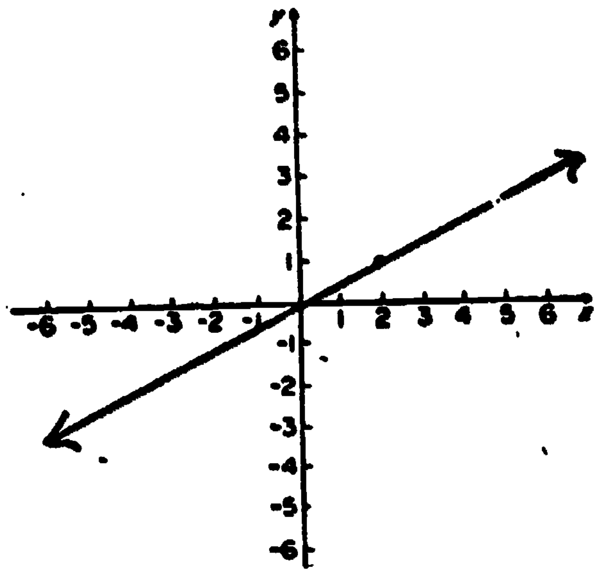
1. $\{-1\}$
2. $\{3, -\frac{2}{3}, -2\}$
3. $\{\frac{4}{3}, -2\}$
4. $\{\frac{1}{2}\}$
5. 19, 20, 22
6. $1\frac{1}{3}$ hr
7. 4.5
8. $-1 \pm \sqrt{17}$
9. Two real number Solutions
10. $\frac{-3 \pm \sqrt{13}}{2}$
11. $S = \frac{4}{5}$, $P = \frac{2}{5}$
12. $x^2 + \frac{7}{2}x + \frac{3}{2} = 0$, or $2x^2 + 7x + 3 = 0$
13. $h = \frac{v^2}{2g}$
14. 250 mph, 600 mph
15. $\pm\sqrt{\frac{3 \pm \sqrt{5}}{2}}$
16. $\{0, 3\}$
17. $Y = \frac{.5}{x^2}$
18. $s = 16t^2$, $7\frac{1}{2}$ sec

Chapter 3
Pretest - KEY
pp. 93-96

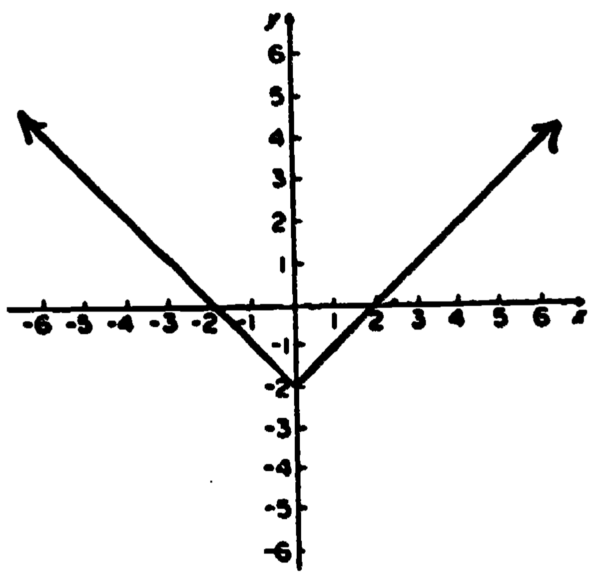
(a,2), (a,3), (a,a),
(b,2), (b,3), (b,a)

1. (1,2), (1,3), (1,a)

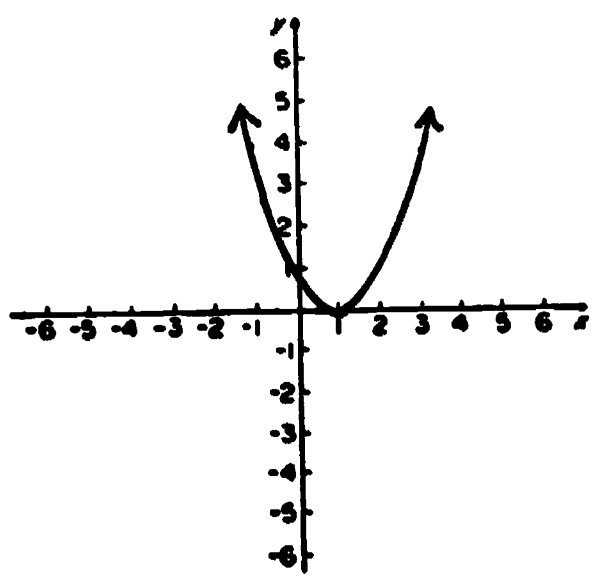
2.



3.



4.



5. a, c, e

6. b, c

7. $x = \sqrt{y+2}$

8. b, c

9. d

10. -3

11. 9

12. $a^2 - a - 3$

13. $g^{-1}(x) = \sqrt{x-2}$

14. a

15. See graphs →

16. a, b, c

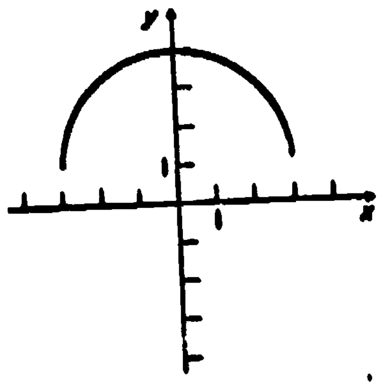
17. e, f

18. d

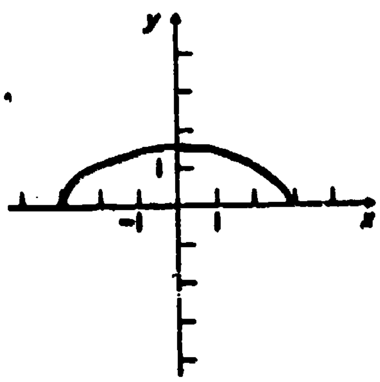
19. a, c, d

Chapter 3
Protest - KEY
pp. 93-96

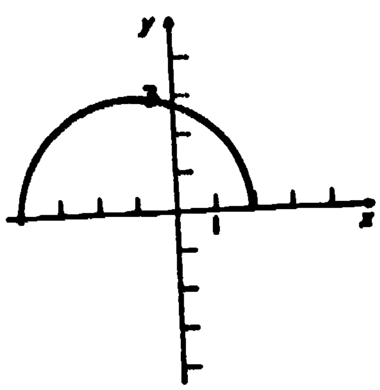
a) $y = 1 + f(x)$



b) $y = \frac{1}{2} f(x)$



c) $y = f(x + 1)$



20. 4

a. no
b. yes
21. ______

22. [0,7]

23. (-4,3)

24. b

25. d

26. a, c

27. -1

28. $y+1 = 3(x+2)$, or $y = 3x+5$

29. $y+4 = \frac{6}{5}(x+3)$, or $y = \frac{6}{5}x - \frac{2}{5}$

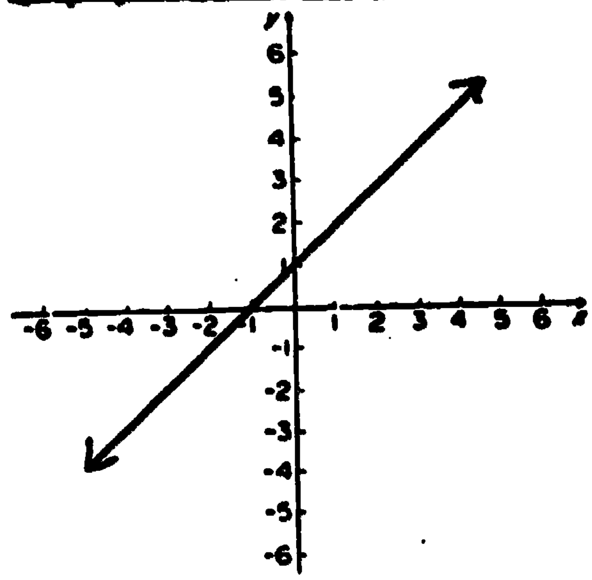
30. $\sqrt{130}$

31. $\left(\frac{5}{2}, \frac{7}{2}\right)$

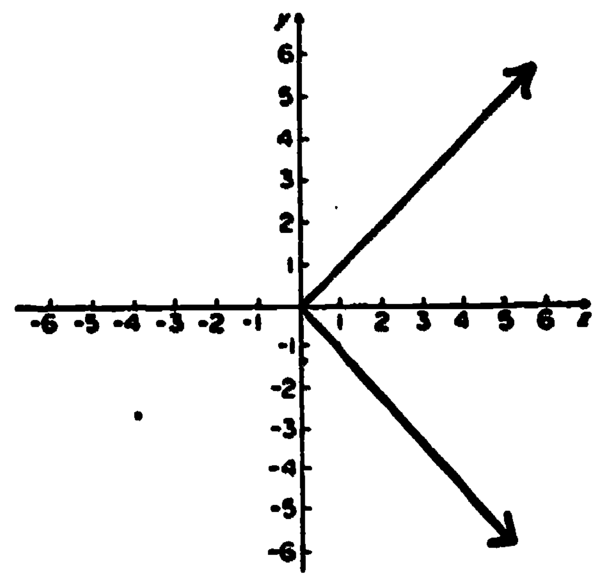
Chapter 3
Test - KEY
pp. 149-153

1. (b,a) (b,1) (b,3) (1,a) (1,1) (1,3) (3,a) (3,1) (3,3)

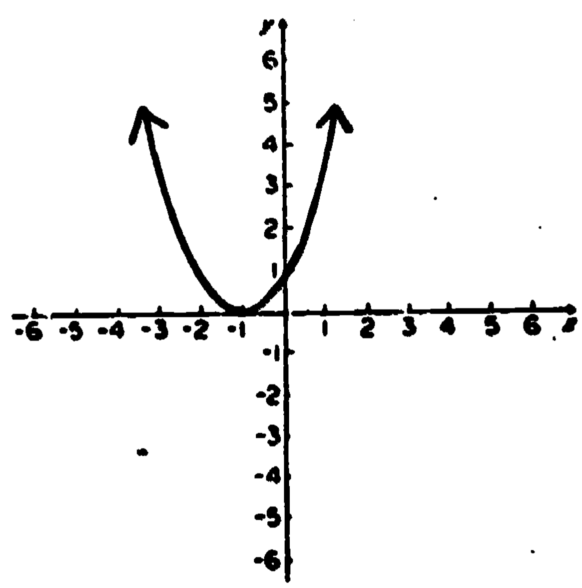
2.



3.



4.



Chapter 3
Test-KEY
pp. 149-153

5. b, d, f

6. b, c, d

7. $x = 3y^2 + 2y - 1$

8. a, b, d

9. a, b, d

10. 0

11. 4

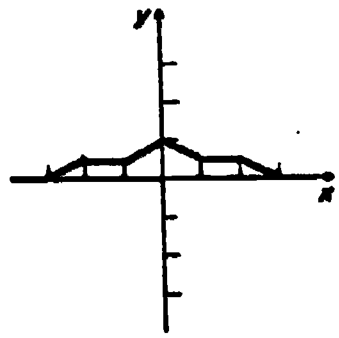
12. $2\sqrt{a+1}$

13. $g^{-1}(x) = (2x-4)^2$

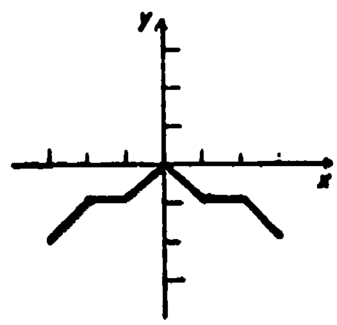
14. S

15. See graphs.

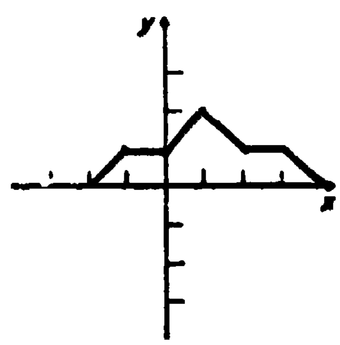
15-a)



15-b)



15-c)



16. a, c

17. b, f

18. d, e

19. b, c

20. 2

Chapter 3
Test-KEY
pp. 149-153

a. yes
21. b. no

22. $[-\pi, 2\pi]$

23. $(0, 1]$

24. c, d

25. a

26. b

27. -2

28. $y-1 = -\frac{1}{2}x$

29. $y-1 = \frac{1}{3}(x-4)$, or $y = \frac{1}{3}x - \frac{1}{3}$

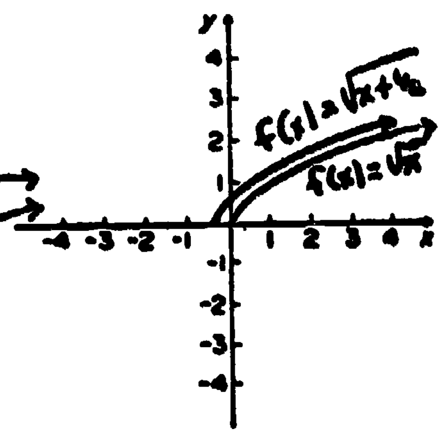
30. $\sqrt{34}$

31. $(\frac{1}{2}, \frac{11}{2})$

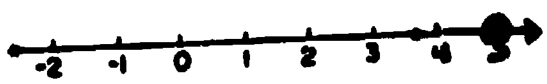
Chapter 4
Pretest-KEY
pp. 157, 158

ANSWERS

1. a) $f(x) = \frac{1}{2}(x-2)^2 - 1$
   b) $(2, -1)$
   c) $x = 2$
   d) -1, Min
2. a) $f(x) = -2(x-\frac{3}{4})^2 + \frac{1}{8}$
   b) $(\frac{3}{4}, \frac{1}{8})$
   c) $x = \frac{3}{4}$
   d) Max
3. $(\frac{1}{2}, 0)$, $(-3, 0)$
4. a) See graph.
   b) See graph.



5. 4
6. $6\frac{1}{4}$ sec, $156\frac{1}{4}$ ft
7. {p, o, r, k}
8. {o, k}
9. See graph. Whole real line
10. See graph.



11. $\{x \mid 2 < x \le 4\}$

12. $\{c \mid \$360 < c < \$450\}$

13. $\{x \mid 1 < x < 11\}$

14. $\{x \mid x > 0$ or $x < -\frac{4}{3}\}$

15. $\{x \mid -20 \le x \le 4\}$

16. $\{2, -7\}$

17. $\{x \mid -2 < x < 2\}$

18. $\{x \mid x > 2$ or $x < -\frac{1}{2}\}$

19. $\{x \mid x < -\frac{14}{3}$ or $\dots -3\}$

Chapter 4
Test - KEY
pp. 189, 190

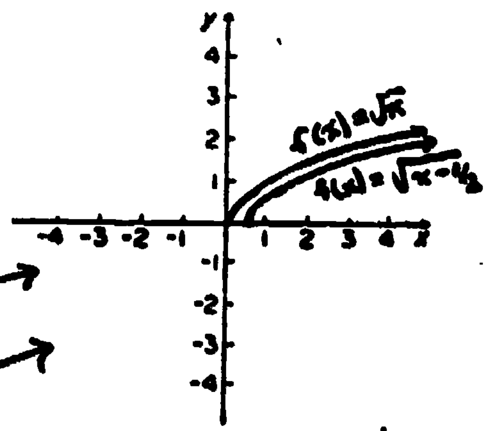
ANSWERS

1. a) $f(x) = 3(x+1)^2 + 5$
   b) $(-1, 5)$
   c) $x = -1$
   d) 5, Min

2. a) $f(x) = -2\left(x + \frac{3}{4}\right)^2 + \frac{57}{8}$
   b) $\left(-\frac{3}{4}, \frac{57}{8}\right)$
   c) $x = -\frac{3}{4}$
   d) Max

3. Do not exist

4. a) See graph.
   b) See graph.



5. 15, 15

6. 3 sec., 144 ft

7. $\{0, 2, 3, 7, 9, 1, 11\}$

8. $\{8, 32\}$

9. See graph.

10. See graph.

11. $\{x \mid 4 < x \leq 4\frac{1}{2}\}$

12. $\{b \mid 10 < b < 18\}$

13. $\{x \mid -5 < x < 9\}$

14. $\{x \mid -\frac{9}{2} \leq x \leq \frac{15}{2}\}$

15. $\{x \mid x \geq 41$ or $x \leq -17\}$

16. $\{3, -4\}$

17. $\{x \mid x > 1$ or $x < 1\}$

18. $\{x \mid -2 < x < \frac{1}{8}\}$

19. $\{x \mid x > -\frac{1}{2}$ or $x < -\frac{6}{5}\}$

Chapter 5
Pretest - KEY
pp. 193, 194

ANSWERS

1. (1, -2)

2. (3, -2)

3. \$1500 @ 4%
\$3500 @ 5%

4. (-3, -1)

5. (-1, -2, 5)

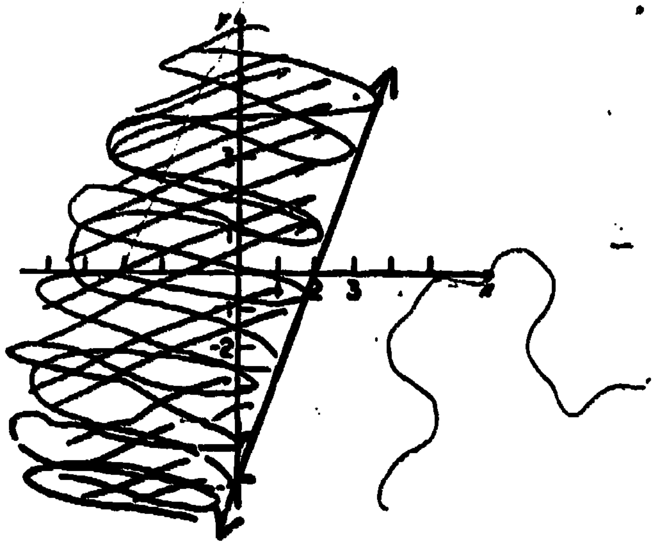
6. $y = x^2 - 3x + 4$

7. {3, -3}

8. 8

9. $\begin{vmatrix} x & y & 1 \\ 7 & 2 & 1 \\ -2 & 4 & 1 \end{vmatrix} = 0$

10. See graph →



11. Min = 12 at (2, 0)
Max = 60 at (10, 0)

12. Type A: 0
Type B: 10
Max. Score = 120 pts

13. $\begin{bmatrix} 1 & -1 \\ 3 & 5 \end{bmatrix}$

14. $\begin{bmatrix} 2 & 1 \\ 3 & -4 \end{bmatrix}$

15. $\frac{1}{11}\begin{bmatrix} 4 & 1 \\ -3 & 2 \end{bmatrix}$

Chapter 5
Test - KEY
pp. 245, 246

ANSWERS

1. $(-2, -2)$

2. $(-5, 4)$

3. 31 nickels, 44 dimes

4. $(1, 2)$

5. $(-3, 4, -2)$

6. $y = -x^2 - 2x + 3$

7. $\{4, -4\}$

8. 9

9. $\begin{vmatrix} x & y & 1 \\ -3 & -4 & 1 \\ 3 & 2 & 1 \end{vmatrix} = 0$

10. See graph.

11. Min = 20 at (0,2)
Max = 56 at (6,2)

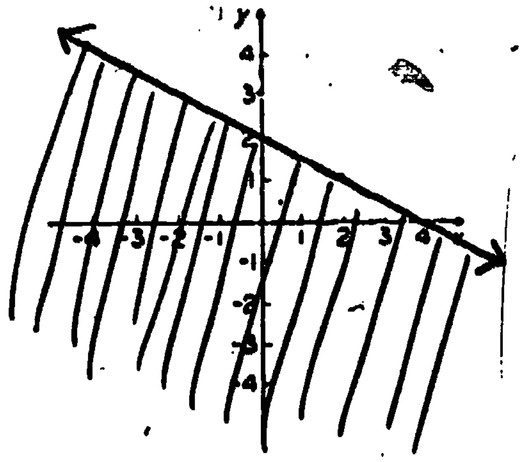
12. Type A: 5
Type B: 6
Max Score = 68

13. $\begin{bmatrix} -3 & 2 \\ 4 & 3 \end{bmatrix}$

14. $\begin{bmatrix} -3 & 3 \\ -1 & 1 \end{bmatrix}$

15. $-\frac{1}{11}\begin{bmatrix} 4 & -1 \\ -3 & -2 \end{bmatrix}$

10. Graph $x + 2y < 4$.



Chapter 6
Pretest - KEY
pp. 249, 250

1. See graph.

2. See graph.

3. $8^{-\frac{2}{3}} = \frac{1}{4}$

4. $\{4\}$

5. $\left\{\frac{1}{2}\right\}$

6. $\log_b \frac{a^{\frac{1}{2}} c^{\frac{3}{2}}}{d^4}$

7. $\{3, -2\}$

8. 1.255

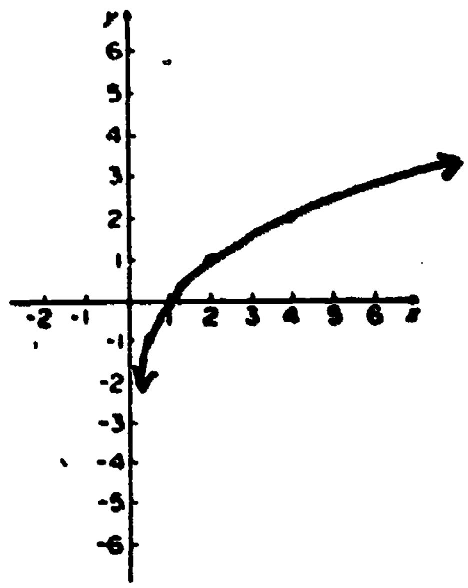
9. .544

10. −.602

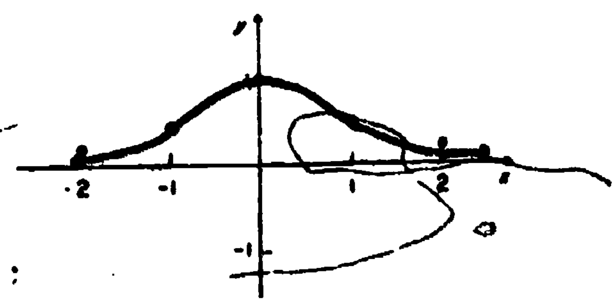
11. .2385

12. $\frac{2}{3}\log_a M - \frac{1}{3}\log_a N$

1. $y = \log_2 x$



2. $y = 2^{-x^2}$



13. 1.4200

14. 7.9034 − 10

15. 73.9

16. 3.5

17. .0276

18. .0006934

19. $\frac{1}{5}$

20. −9

21. a) $x \leq 0$
b) $0 < x < 1$

22. 14.2 yr

Chapter 6
Test - KEY
pp. 283, 284

1. See graph.

2. See graph.

3. $3^2 = 9$

4. $\{5\}$

5. $\left\{\frac{1}{2}\right\}$

6. $\log_a \frac{b^2\sqrt{d}}{c^3}$

7. $\left\{1, -\frac{1}{2}\right\}$

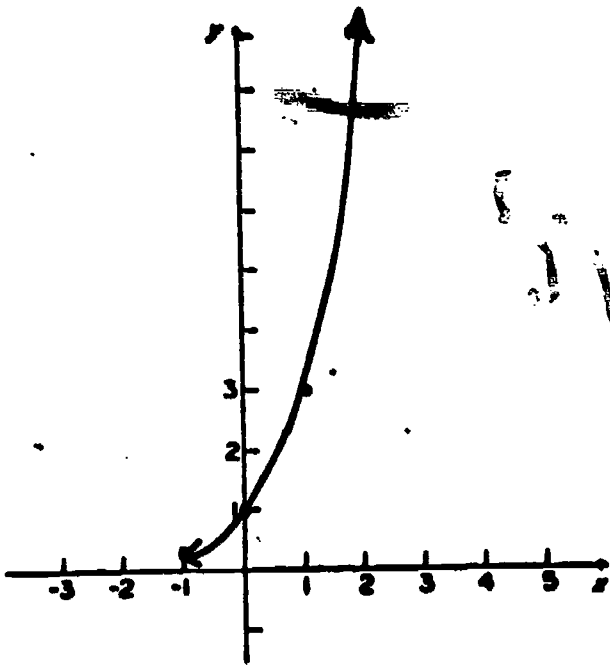
8. .902

9. 1.075

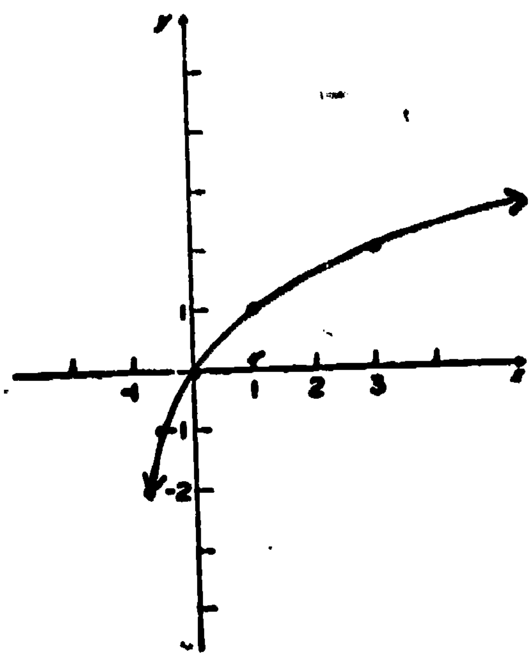
10. .059

11. .255

1. $y = 3^x$



2. $y = \log_3(x+1)$



12. $\frac{1}{3}\log_a x - \frac{2}{3}\log_a y$

13. 1.1553

14. 7.5113−10

15. .00342

16. 4370

17. .36

18. 3.483×10^9

19. 6

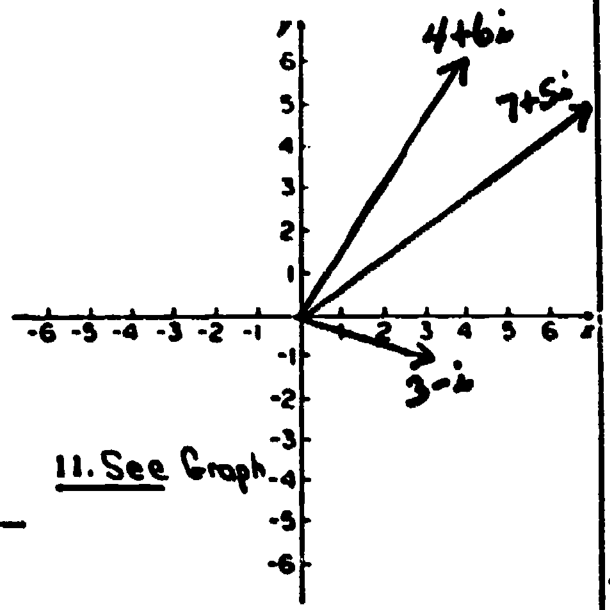
20. 5

21. a) $x > 1$

b) 1

22. 12.5 g

Chapter 7
Pretest - KEY
pp. 287, 288



1. $5 - i$

2. $6 - 6i$

3. $-5i$

4. $1 + 2i$

5. $x = -\frac{4}{3}$, $y = 1$

6. $2\bar{z}^6 - 4\bar{z}^3 + \bar{z} + 1$

7. $x^2 - 4x + 5 = 0$

8. $-\frac{3}{2} \pm \frac{\sqrt{3}}{2} i$

9. $\frac{-(2+i) \pm \sqrt{-4i - 1}}{2}$

10. $1 + i$, $-1 - i$

11. See Graph

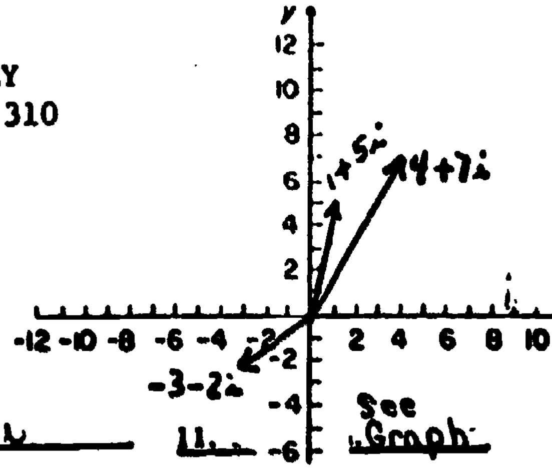
12. $1 + \sqrt{3}\, i$

13. $2 \text{ cis } 30°$

14. $12 \text{ cis } 70°$

15. $-\frac{\sqrt{3}}{2} + \frac{1}{2} i$, $\frac{\sqrt{3}}{2} + \frac{1}{2} i$, $-i$

Chapter 7
Test - KEY
pp. 309, 310



1. $14 + 2i$

2. $1 - 4i$

3. $2 - i$

4. $\frac{11}{10} + \frac{3}{10} i$

5. $x = 2$, $y = -4$

6. $3\bar{z}^3 + \bar{z} - 7$

7. $x^2 - 2x + 5 = 0$

8. $\frac{2}{5} \pm \frac{1}{5} i$

9. $\frac{(-3 \pm \sqrt{5})\, i}{2}$

10. $\sqrt{2} + \sqrt{2}\, i$, $-\sqrt{2} - \sqrt{2}\, i$

11. See Graph

12. $-\sqrt{2} + \sqrt{2}\, i$

13. $\sqrt{2} \text{ cis } 45°$

14. $70 \text{ cis } 50°$

15. $\sqrt[6]{2} \text{ cis } 15°$, $\sqrt[6]{2} \text{ cis } 135°$, $\sqrt[6]{2} \text{ cis } 255°$

Chapter 8
Pretest - KEY
pp. 313, 314

1. $Q = 5x^5 - 5x^4 - x^3 + x^2 - x + 1$, $R = 0$

2. -7069

3. $x^4 - 1$

4. $\frac{5+\sqrt{15}}{10}$, $\frac{5-\sqrt{15}}{10}$, $\frac{1}{2}$

5. $x^3 - x^2 - x + 1$

6. $(x+2i)(x-2i) \cdot (x+1)(x-2)$

7. 73

8. 4

9. $-3-4i$, $2+\sqrt{5}$

10. Yes

11. 2.1

Chapter 8
Test - KEY
pp. 339, 340

1. 0

2. $Q = 2x^3 - 10x^2 + 27x - 59$, $R = 119$

3. $x^3 - 3x^2 + 2x$

4. $(x^2-1)(x-2)^2(x+3)^3$

5. $(x-1)(x+\frac{1}{2}+i\frac{\sqrt{3}}{2})(x+\frac{1}{2}-i\frac{\sqrt{3}}{2})$

6. 3, 2

7. 88

8. 5

9. No

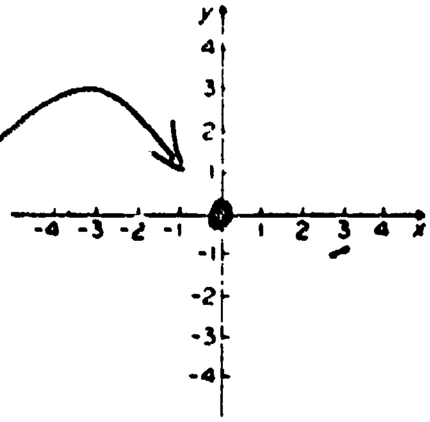
10. $-8+7i$, $10-\sqrt{5}$

11. -0.9, 1.3, 2.5

Chapter 9
Pretest - KEY
pp. 341, 342

ANSWERS

1. See graph



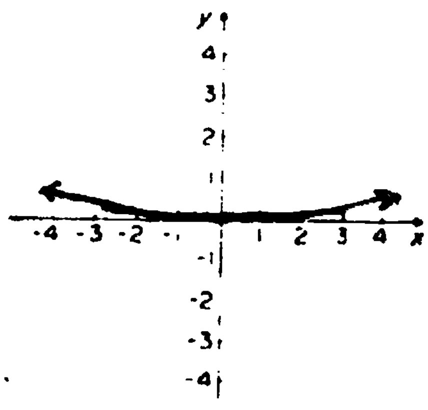
2. $(x-4)^2+(x+1)^2=25$

3.

Center $C(4,-6)$

Radius $r=\sqrt{3}$

4. See graph. $x^2=28y$



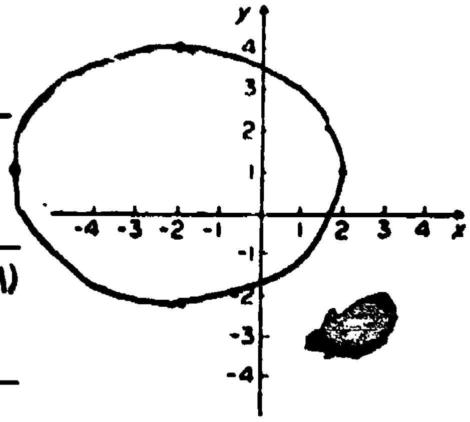
5.

Focus $F(-\frac{1}{4},0)$

Directrix $x=\frac{1}{4}$

Vertex $V(0,0)$

6. See graph.



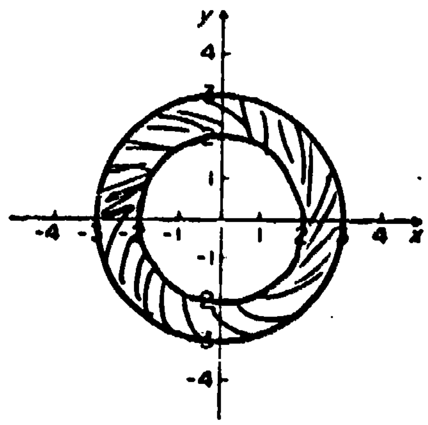
Center $C(-2,1)$;

Vertices $V(2,1),(-6,1),(-2,4)$ $(-2,-2)$

Foci $F(-2+\sqrt{7},1)$, $(-2-\sqrt{7},1)$

7. $\frac{x^2}{4}-\frac{y^2}{12}=1$

8. See graph.



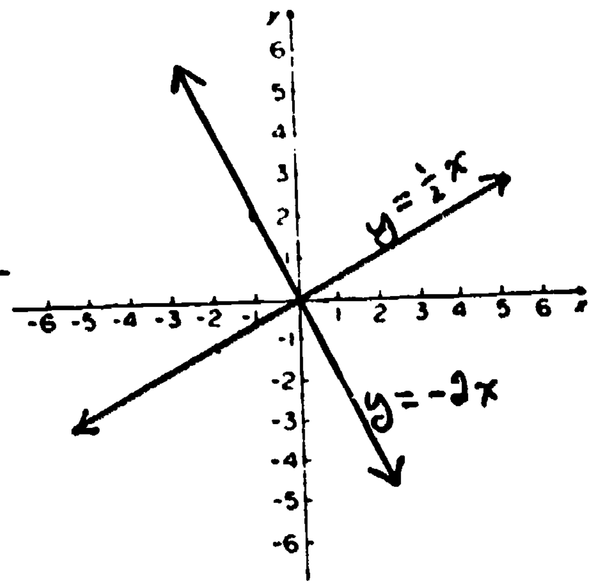
9. $(2,4),(2,-4),$ $(-4,2),(-4,-2)$

10. $(4,3),(4,-3),$ $(-4,3),(-4,-3)$

11. 24 in x 10 in

87

Chapter 9
Test - KEY
pp. 385, 386



ANSWERS

1. See graph.

$(x+2)^2+(y-6)^2=13$

2.

3.
Center $C\left(\frac{3}{4}, \frac{5}{4}\right)$

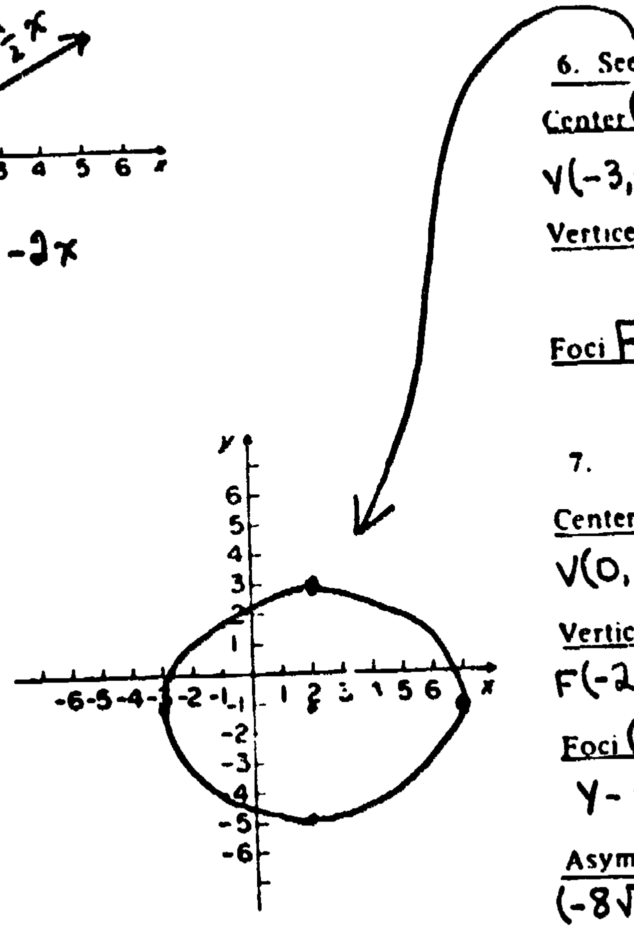
Radius $r = \frac{\sqrt{10}}{4}$

4. $y^2 = -6x$

5.
Focus $F(-3,0)$

Vertex $V(0,0)$

Directrix $x = 3$



6. See graph.

Center $C(2,-1)$

$V(-3,-1), (7,-1)$
Vertices $(2,3), (2,-5)$

Foci $F(-1,-1), (5,-1)$

7.
Center $C\left(-2, \frac{1}{4}\right)$

$V\left(0, \frac{1}{4}\right), \left(-4, \frac{1}{4}\right)$
Vertices

$F\left(-2+\sqrt{6}, \frac{1}{4}\right)$,
Foci $\left(-2-\sqrt{6}, \frac{1}{4}\right)$

$y - \frac{1}{4} = \pm \frac{\sqrt{2}}{2}(x+2)$
Asymptotes

$(-8\sqrt{2}, 8)$,
8. $(8\sqrt{2}, 8)$

$\left(3, \frac{\sqrt{29}}{2}\right), \left(-3, \frac{\sqrt{29}}{2}\right)$
$\left(3, -\frac{\sqrt{29}}{2}\right), \left(-3, -\frac{\sqrt{29}}{2}\right)$
9.

10. 7, 4

$\frac{11\sqrt{6}}{7} \approx 5.6$
11.

Chapter 10
Pretest - KEY
pp. 389, 390

1. $a+4b$

2. 531

3. -4

4. $n=6$, $S_n = -126$

5. $\frac{1}{2}, -\frac{1}{6}, \frac{1}{18}$

6. $\frac{123}{99}$

7. \$163, \$4498

8. $\approx$ 50 ft

9. See work.

9. Use mathematical induction. Prove:

$1+3+3^2+\cdots+3^{n-1}=\frac{3^n-1}{2}.$

$S_n: 1+3+3^2+\cdots+3^{n-1} = \frac{3^n-1}{2}$

$S_1: 1 = \frac{3-1}{2}$

$S_k: 1+3+3^2+\cdots+3^{k-1} = \frac{3^k-1}{2}$

$S_{k+1}: 1+3+3^2+\cdots+3^{k-1}+3^k = \frac{3^{k+1}-1}{2}$

1) Basis Step. $1 = \frac{2}{2} = \frac{3^1-1}{2}$ is true

2) Induction Step. Assume S_k. Prove S_{k+1}

$\underbrace{1+3+3^2+\cdots+3^{k-1}}+3^k$

$= \frac{3^k-1}{2} + 3^k$, by S_k

$= \frac{3^k-1}{2} + \frac{2\cdot 3^k}{2}$

$= \frac{3^k-1+2\cdot 3^k}{2}$

$= \frac{3\cdot 3^k-1}{2}$

$= \frac{3^{k+1}-1}{2}$

Chapter 10
Test - KEY
pp. 419, 420

1. $3\frac{3}{4}$

2. 465

3. 11

4. $a_1 = 8$, $a_5 = \frac{1}{2}$

5. .27, .0027, .000027

6. $\frac{192}{90}$, or $\frac{64}{30}$, or $\frac{32}{15}$

7. \$7.38, \$1365.10

8. 7,680,000

9. See work.

9. Use mathematical induction. Prove: For all natural numbers n,

$$1 + 4 + 7 + \ldots + (3n - 2) = \frac{n(3n-1)}{2}$$

$$S_n : 1+4+7+\ldots+(3n-2) = \frac{n(3n-1)}{2}$$

$$S_1 : 1 = \frac{1(3-1)}{2}$$

$$S_K : 1+4+7+\ldots+(3K-2) = \frac{K(3K-1)}{2}$$

$$S_{K+1} : 1+4+7+\ldots+[3(K+1)-2] =$$
$$1+4+7+\ldots+(3K-2)+(3K+1) = \frac{(K+1)(3K+2)}{2}$$

1) Basis Step. $1 = \frac{2}{2} = \frac{1(3-1)}{2}$ is true

2) Induction Step. Assume S_K. Prove S_{K+1}.

$$\underbrace{1+4+7+\ldots+(3K-2)}+(3K+1)$$

$$= \frac{K(3K-1)}{2} + (3K+1)$$

$$= \frac{K(3K-1)}{2} + \frac{2(3K+1)}{2}$$

$$= \frac{3K^2-K+6K+2}{2}$$

$$= \frac{3K^2+5K+2}{2}$$

$$= \frac{(K+1)(3K+2)}{2}$$

90

Chapter 11
Pretest - KEY
pp. 423, 424

1. $6!$ or 720

2. $\binom{15}{8}$, or 6435

3. $n = 14$

4. $\frac{13}{52}$, or $\frac{1}{4}$

5. $\frac{{}_4C_2 \cdot {}_4C_1}{{}_{52}C_3}$, or $\frac{6}{5525}$

6. $x^8 + 12x^6y + 54x^4y^2 + 108x^2y^3 + 81y^4$

7. $220a^9x^3$

8. 0.817

9. 36

10. $5!$, or 120

11. $\frac{9}{35}$

Chapter 11
Test - KEY
pp. 451, 452

1. $9 \cdot 8 \cdot 7 \cdot 6$, or 3024

2. $24 \cdot 23 \cdot 22$, or 12,144

3. 36

4. $\frac{1}{2}$

5. $\frac{1}{12}$, 0

6. $\binom{18}{11} a^7 x^{11}$, or $\frac{18!}{11!\,7!} a^7 x^{11}$

7. $m^7 + 7m^6n + 21m^5n^2 + 35m^4n^3 + 35m^3n^4 + 21m^2n^5 + 7mn^6 + n^7$

8. 1.159

9. $\frac{9!}{1!\,4!\,2!\,2!}$, or 3780

10. $6!$ or 720

11. $\frac{5}{24}$

Syllabus: PRE-CALCULUS: FUNCTIONS AND RELATIONS (Review Path) Walter A. Coole, Skagit Valley College

Your objectives for this course will be to learn the principles of mathematical functions and how to apply these principles to numerical problems.

Rationale. Modern mathematics uses the idea of a FUNCTIONAL RELATIONSHIP to express ideas about numbers. This course should develop your familiarity with the language of functions and the concepts behind its symbols.

Pre-requisite testing. (This process should take about 3 hours and should be completed within one day after beginning the course.[1]) Check off each item as you have completed it in the space provided.

( ) Read page xi of your textbook, Howes: *Pre-Calculus Mathematics, Book II: Functions and Relations*

( ) Write out your answers for the "*Pre-requisite Test for Book II.*" Score your results, using the answers given in the text.

( ) If you didn't get them *all* right, see your instructor immediately.

( ) Write out your answers for the "Post Test for Book II." Score your results by using the answers given in the textbook.

( ) If you didn't get them all right, you can be fairly sure you're in the right course.

( ) If you *did* get them all right, you may want to skip this course and take the final examination. Turn to the last page of this syllabus for instructions about the final exam.

Sample lesson. (This lesson should take about 10 hours and should be completed within three days after beginning the course.[1])

1. Time-estimates given in this syllabus are based on an eleven-week term. If your schedule is different, you should make adjustments accordingly. In general, each of the thirteen lessons of this course (including the sample lesson) take most students ten hours' study.

( ) Keep track of the time you spend on this lesson; write down each hour of study you invest into the course. This will allow you to estimate how much more or less to spend on each subsequent lesson.

( ) Read the lesson's objectives and rationale on page 1 of your textbook.

( ) Complete the lesson assignment, biginning on page 2 of the textbook and continuing to the end of Chapter 1. If you have difficulty, see your instructor or a math coach in the learning laboratory.

( ) Review the chapter by referring back to page 1. Examine each objective and satisfy yourself that you can perform as required by the text.

( ) Test your mastery of Chapter 1 by writing out the answers to the problems at the end of the chapter. Score your answers.

--If you don't achieve 90%, repeat the lesson.
__If you score over 90%, proceed to the next lesson...

( ) Write the number of hours spent on this Sample Lesson ____________.
This will give you a rough basis for extimating the number of hours to allow for the remaining lessons in this course.

Your instructor will want to see this section of the syllabus completed during your scheduled conference.

....

BEFORE WE GO ANY FARTHUR

Let's plan your way through the course.

If you're beginning this course at the first of the term, you'll find a completion-schedule posted in the learning laboratory. Use the dates given there to fill in the Unit completion Schedule below. If not...

Pick a date at which you intend to complete the course. Write that date Here____________. Now, back up three days; write that date here...

EXAMINATION DATE: __________ ___ __________

Count the number of days between today's date and the examination date.___

Divide the number of available days by 4: ______________. This is the number of days you should plan to spend on each of the four units of study. Please note that each unit consists of three lessons. Enter the date for completing each unit of study below. (If you plan to undertake special projects to make a grade of "A" allow for about 30 hours work.)

UNIT COMPLETION SCHEDULE

Unit I: ______________________________
Unit II: ______________________________
Unit III: ______________________________
Unit IV: ______________________________

You must maintain this schedule of completions to continue in the course, unless you have made special arrangements with the instructor.

You should plan to attend every scheduled conference you have selected in the learning laboratory. Bring your textbook and syllabus. You should be prepared to use the time spent waiting for the instructor studying.

Remember that your instructor and fellow-students who .e serving as coaches will be available between conferences to help you with the rough spots. The best way to ask for help is to be able to point out a specific part of the textbook that's causing you trouble. BRING YOUR TEXTBOOK AND SYLLABUS FOR COACHING!

UNIT I

Unit I consists of Lessons 1, 2, and 3 (corresponging to textbook chapters 2, 3, and 4) and should be studied in the same way that you did Chapter 1 in the foregoing Sample Lesson.

As you complete each lesson, write the time and date. Record your test score, simply by noting the number of problems correctly solved.

Lesson Number	Read Objectives	Complete Programmed Work	Review Objectives	Test Score
1				
2				
3				

When you have completed Unit I, report your progress on the sign-in sheet during your next visit to the learning laboratory.

UNIT II

Unit II consists of Lessons 4, 5, and 6 (Chapters 5, 6, and 7).

Lesson Number	Read Objectives	Complete Programmed Work	Review Objectives	Test Score
4				
5				
6				

Please record your completion of Unit II.

At this point, you're halfway done!

UNIT III

Unit III's lessons cover Chapters 8, 9, and 10.

Lesson Number	Read Objectives	Complete Programmed Work	Review Objectives	Test Score
7				
8				
9				

Please record your completion of Unit III.

UNIT IV

In this unit, you'll polish off the remaining three chapters.

Lesson Number	Read Objectives	Complete Programmed Work	Review Objectives	Test Score
10				
11				
12				

Now, finally, you can record the completion of the last unit of study.

You are now ready for your final examination. You may take it at your scheduled conference period OR you may make an appointment to take it at another time.

PERIODIC FUNCTIONS. Course Outline by
Walter A. Coole, Skagit Valley College

Skagit Valley College Course Number: Mathematics 112

Quarter credits: 4 Semester credits: 3

Average student completion time: 120 hours

Goal. In this course, the student should master theoretical trigonometry organized on the basis of set-theoretical and functional notions.

The two-track approach.

A. the standard path's performance objectives, lesson-by-lesson, are as follows:

1. express functions and relations as ordered pairs and graph them;

2. relate circular functions algebraically;

3. perform standard computations, using logarithms;

4. interpret tables of trigonometric functions;

5. prove identities and describe inverse trigonometric functions;

6. solve problems involving right triangles and vectors;

7. express imaginary numbers in complex notation;

B. the review path's performance objectives, lesson-by-lesson, are as follows:

S. Upon finishing this lesson the student should be able to:

use a ruler to locate (approximately) points on the real line, corresponding to real numbers; locate points in the Cartesian plane corresponding to ordered pairs of real numbers; recognize a function, its domain, and its range; graph functions; determine and graph the inverse of a function.

1. define a unit circle; given a real number, locate the approximate corresponding point on a unit circle; express any given real number as an "equivalent" number between 0 and 2π; give the approzimate rectangular coordinates of any point on a unit circle; construct the points correspoiding to, and compute the exact rectangular coordinates of

$$0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}, \frac{2\pi}{3}, \frac{3\pi}{4}, \frac{5\pi}{6}, \pi, \frac{7\pi}{6}, \frac{5\pi}{4}, \frac{4\pi}{3}, \frac{3\pi}{2}, \frac{5\pi}{3}, \frac{7\pi}{4}, \frac{11\pi}{6}.$$

2. give exactly the sine and cosine of $0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}$, etc., $-\frac{\pi}{6}, -\frac{\pi}{4}, -\frac{\pi}{3}, -\frac{\pi}{2}$, etc., $\frac{10\pi}{3}, 3\pi, -21\pi, \frac{61\pi}{2}, -\frac{53\pi}{6}$, etc., approximate to four decimals the sine and cosine of any real number.

3. develop the graph of the function defined by $f(x) = \sin x$; plot enough points to determine the general shape of the sine curve; work with the general equation of a sine curve or sine wave $f(x) = a \sin (bx+c)$; investigate the effect on the curve of different values of a, b, and c; analyze the graph of the sine function and examine the graph of the cosine function defined by $f(x) = \sin x + \frac{1}{2} \sin 3x$.

4. find the exact values of tan x, ctn x, sec x, and csc x for

$$x = 0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}, \frac{3\pi}{3}, \frac{3\pi}{4}, \text{etc.},$$

use Table I to approximate values of tax x, ctn x, sec x, and csc x for x and real number given to two or three decimals; graph the tangent, cotangent, secant, and cosecant functions.

5. give eight fundamental trigonometric identities; use the fundamental identities to make other changes such as

(a) $\sin x \sec x = \tan x$
(b) $\cos^2 x - \sin^2 x = 1 - 2 \sin^2 x$
(c) $\frac{1 + \cos x}{\sin x} + \frac{\sin x}{\cos x} = \frac{1 + \cos x}{\sin x \cos x}$

give the tribonometric functions of such expressions as

$-x, \pi - x, 2\pi - x, \pi + x. \frac{\pi}{2} - x, x - \frac{\pi}{2}, \frac{3\pi}{2} + x$ in terms of trigonometric functions of x.

6. derive the addition formulas; derive the double and half-number formulas; prove other trigonometric identities in two unknowns; compute exact values for functions of some rational multiples of π other than those already studied (such as $\frac{\pi}{8}, \frac{\pi}{12}$, etc.,).

7. solve equations of the type TF (x) = a; solve equations of the type TF[f(x)] = a; solve equations involving more than one trigonometric function; solve factorable equations of higher degree.

8. determine whether a function is one-to-one; define the inverse of a one-to-one function; restrict the domain, if necessary, to obtain a one-to-one function; define and graph

(a) y = Arcsin x
(b) y = Arcos x
(c) y = Arctan x

evaluate such expressions as

(a) Arcos $\frac{1}{2}$
(b) Arcsin (-0.4213)
(c) Sin (Arctan 1)

9. measure angles in revolutions, radians, and degrees; change from one unit of measure to another; define the six trigonometric functions of angles; use Table II to approximate the trigonometric functions of angles measured in degrees; find an angle given one of its trigonometric functions.

10. give the trigonometric functions of an angle in terms of x, y, and r; given any two values for x, y, and/or r, evaluate the trigonometric functions of 0; given one function of an angle, find the other five; define the six right triangle functions; solve a right triangle given one side and one angle; solve a right triangle given two sides; use right triangle fundtions in applications.

11. solve the following triangles;

    (a) given two angles and any side

    (b) given two sides and an angle opposite one of them

    (c) given two sides and the included angle

    (d) given three sides

    apply the law of sines and the law of cosines to solve problems.

12. represent complex numbers

    (a) as ordered pairs of real numbers

    (b) in rectangular form

    (c) in trigonometric form

    plot complex numbers on the complex plane; add, subtract, multiply, and divide complex numbers; multiply and divide easily in trigonometric form; find powers and roots of complex numbers by DeMoivre's Theorem.

Entry.

The student entering either path of this course should have mastered thoroughly, the content of the course "Functions & Relations". In addition, he/she should be able to:

i. read and follow simple written instructions

ii. state his educational objectives in simple, coherent terms

iii. study systematically and diligently

Student materials.

Testing form: Automata Student Response Card (1-50)
paper and pencil

Standard Path

Keedy & Bittinger: *Trigonometry--A Functions Approach.* Reading, Mass. Addison-Wesley. 1974

Coole: *Syllabus for Periodic Functions* (Standard Path)

Review Path

Howes: *Pre-Calculus Mathematics* Book III: Analytic Trigonometry. NY: John Wiley & Sons. 1967

Coole: *Syllabus for Periodic Functions* (Review Path)

Teacher preparation-Study instructor's manuals, testing materials and texts.

Other materials required.

Cooperative Testing Service: *Cooperative Math Test--Trigonometry* (Forms A & B and user's manual). Palo Alto, CA: Cooperative Testing Service. 1969.

Oleanna Math Program: *Smorgasbord.*

Oleanna Math Program

Syllabus for PERIODIC FUNCTIONS (Standard Path) By Walter A. Coole, Skagit Valley College

Your *goal* for this course will be to master the *theory of periodicity*. Since trigonometry is the basic concept involved, you'll also learn how trigonometry relates to periodicity.

This course is divided into four "units", each of which will require about 30 hours' work. By following directions in this syllabus, you'll be able to avoid spending time unnecessarily on information you've already mastered. The units of the course are:

Unit	Lesson	Completion date
I	1	____________
	2	____________ *
II	3	____________
	4	____________ *
III	5	____________
	6	____________ *
IV	7	____________ *
	Final	____________

You should attempt to complete Lesson 1 by your first scheduled conference if at all possible.

Completion dates for each unit (marked by asterisks*) should be filled in from the schedule provided. If you're beginning at the opening of a school term, your schedule will be posted on the bulletin board; otherwise, your teacher will work out a special schedule for you.

For this course, you'll need paper, pencil, and the following textbook:

Keedy & Bittinger: *Trigonometry--A Functions Approach*

How to Study Each Lesson

Corresponding to each lesson, there is a chapter of the textbook. Each lesson begins with a "PRETEST". The pretest serves two purposes: (1) it tells you what kind of problems you will learn about in the chapter and (2) it allows you to skip over chapters whose content you already know.

Begin the chapter's study by taking the pretest. Then score your results by checking against those given in this syllabus. If you score 90% or better on the pre-test, review the chapter briefly and procede to the ch chapter test at the end of the chapter. (See below).

Should you score less than 90% on the pretest, study the chapter thoroughly.

Each chapter in this text contains a number of sections. Each section has--

> *objectives*, given in the beginning of the margin
>
> *explanation* with sample problems--you should work the sample problems and check the answers in the back of the book as you go
>
> *exercise set* which should be worked through--answers to the odd numbered problems are in the back of the text

To complete a chapter, you should work the chapter test at the end. Turn the test in to be recorded; when your instructor's had a chance to inspect your work and record it, the test will be returned.

Completing the Course

After you've mastered all of the chapters of the textbook--either by scoring 90% on the pretest or by studying the chapter--you are ready for the final examination. This may be taken during a scheduled conference or by appointment.

You'll need paper, pencil, and a 50-entry student response card (on sale at the bookstore). You may use your textbook and notes during the test. Average completion for the end-of-course test is 40 minutes, but you may take longer if you wish.

Grading

When you've completed the end-of-course test, you may close off the course with a grade of "B". If you wish to improve your grade to an "A", you may act as a coach or undertake optional projects from the "Smorgasbord". This may be done during the following term and your "B" will be changed to an "A".

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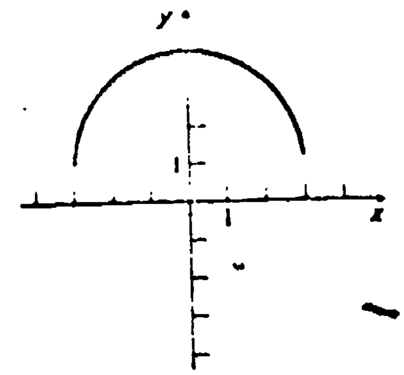
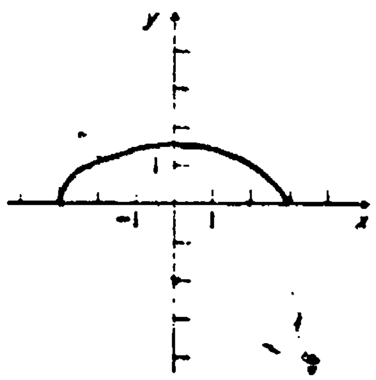
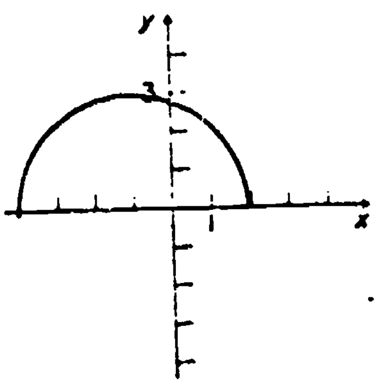
a)



b)



c) y = f(x + 1)



20. 4

21. a. no
b. yes

22. [0,7]

23. (-4,3)

24. b

25. d

26. a, c

27. −1

28. $y+1 = 3(x+2)$

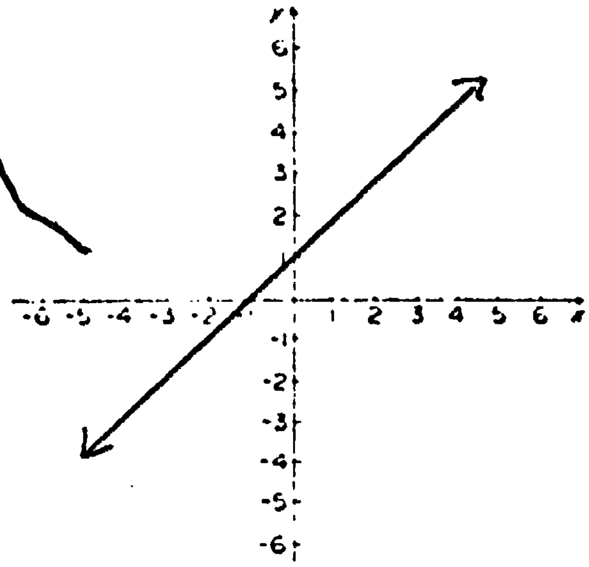
29. $y+4 = \frac{6}{5}(x+3)$

30. $\sqrt{130}$

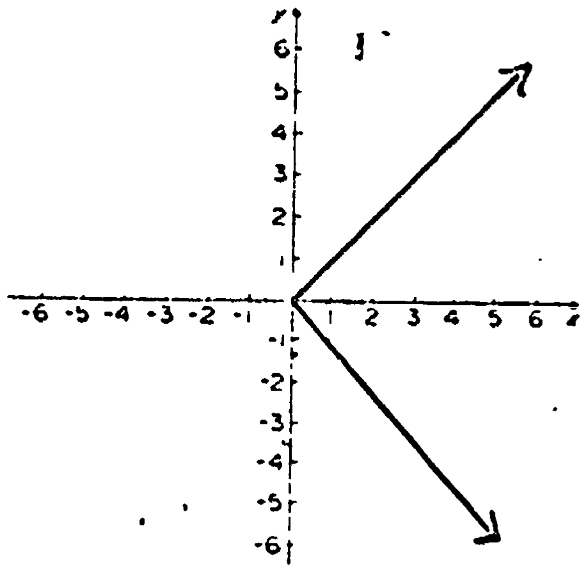
31. $\left(\frac{5}{2}, \frac{7}{2}\right)$

1. (b,a) (b,1) (b,3)
(1,a) (1,1) (1,3)
(3,a) (3,1) (3,3)

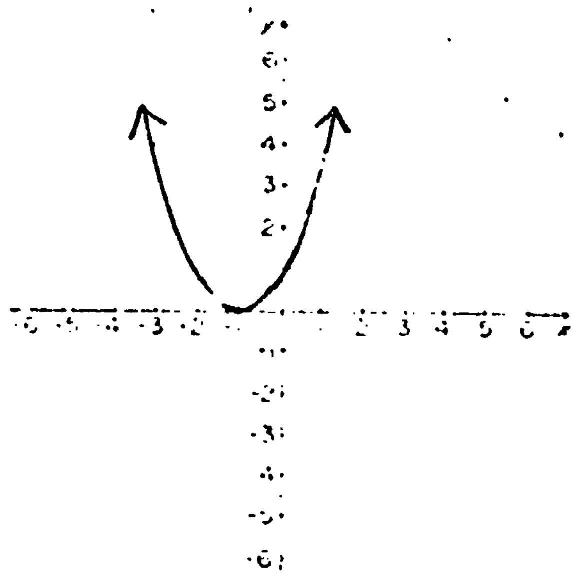
2.



3.



4.



5. b, d, f

6. b, c, d

7. $x = 3y^2 + 2y$ 1

8. a, b, d

9. a, b, d

10. 0

11. 4

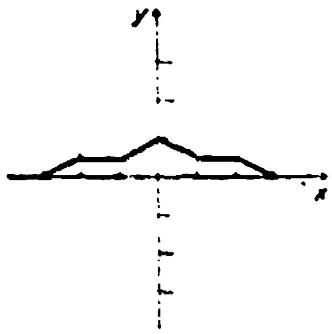
12. $2\sqrt{a+1}$

13. $g^{-1}(x) = (2x-4)^2$

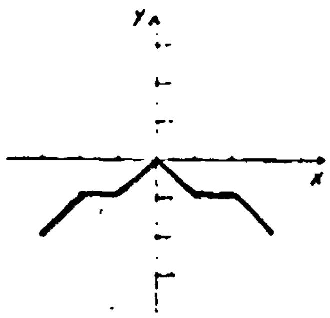
14. 5

15. See graphs.

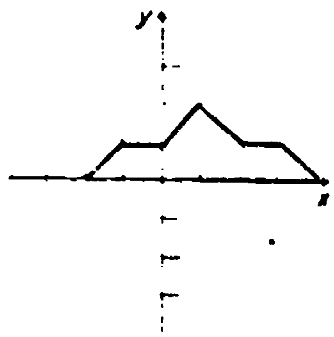
15-a)



15-b)



15-c)



16. a, c

17. b, f

18. d, e

19. b, c

20. 2

21. a. yes
b. no

22. $[-\pi, 2\pi]$

23. $(0, 1]$

24. c, d

25. a

26. b

27. -2

28. $y - 1 = -\frac{1}{2}$

29. $y - 1 = \frac{1}{3}(x - 4)$

30. $\sqrt{34}$

31. $\left(\frac{1}{2}, \frac{11}{2}\right)$

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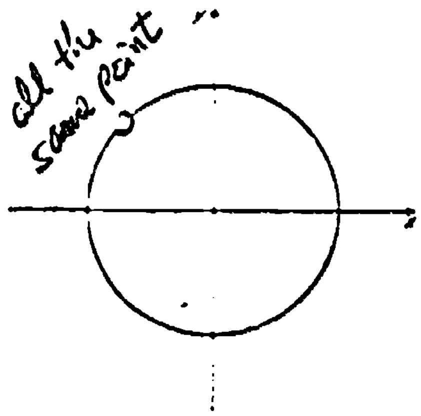
The point $(-3, -4)$ is on a circle centered at the origin. Find the coordinates of its reflection across

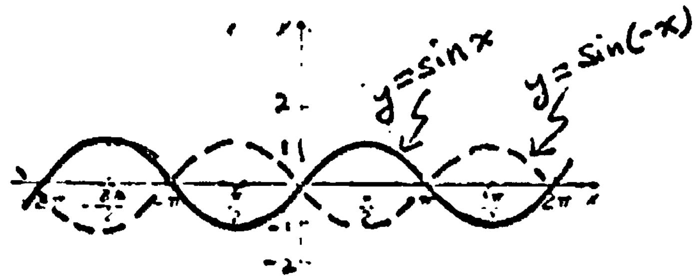
1. the x-axis
2. the y-axis
3. the origin.

On a unit circle, mark the points determined by the following numbers.



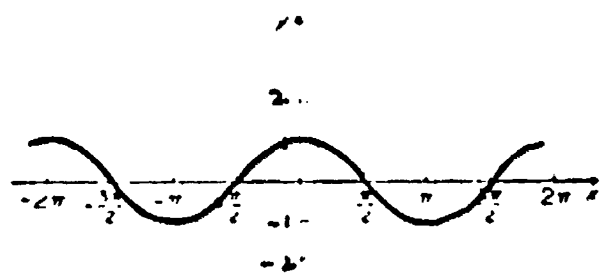
4. $\frac{3\pi}{4}$
5. $\frac{11\pi}{4}$
6. $-\frac{5\pi}{4}$
7. $\frac{19\pi}{4}$

8. Sketch a graph of $y = \sin x$.



9. What is the period of the sine function.

10. Sketch a graph of $y = \cos x$.



11. What is the range of the cosine function?

12. Sketch a graph of $y = \sin(-x)$. Use the axes of exercise 8.

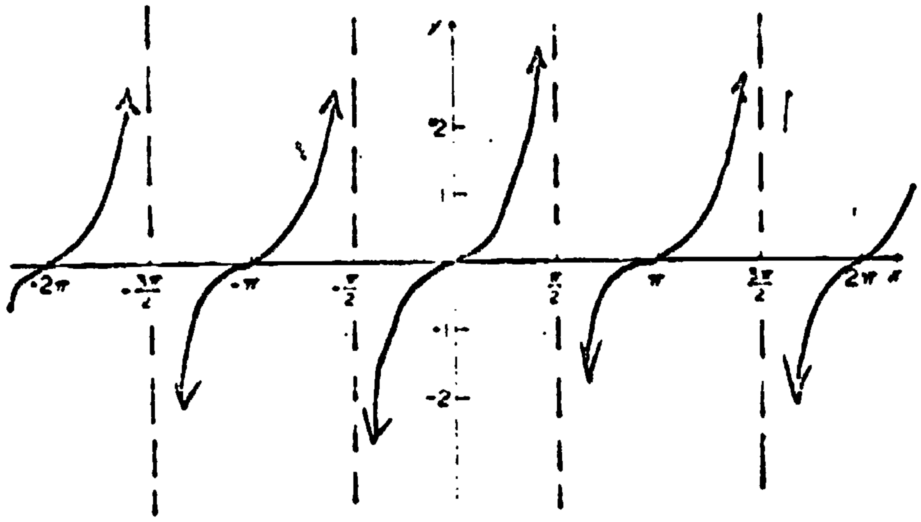
ANSWERS

1. (-3, 4)
2. (3, -4)
3. (3, 4)
4. See graph
5. See graph
6. See graph
7. See graph
8. See graph
9. 2π
10. See graph
11. [-1, 1]
12. See graph for exercise 8

13. Complete the following table.

x	$\sin x$	$\cos x$
$\frac{3\pi}{4}$	$\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{2}}{2}$
π	0	-1
$\frac{5\pi}{4}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{2}}{2}$
$\frac{3\pi}{2}$	-1	0
$\frac{7\pi}{4}$	$-\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$

14. Sketch a graph of $y = \tan x$.

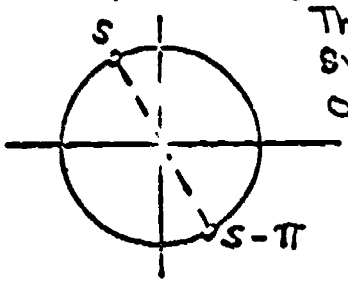


15. What is the period of the tangent function?

16. What is the domain of the tangent function?

17. In which quadrants are the signs of the sine and cosine the same?

18. Verify the following identity. $\sin(\pi - s) \equiv \sin s$



The points for s and s-π are symmetric with respect to the origin, so $\sin(s-\pi) \equiv -\sin s$

Since $\sin(-s) \equiv -\sin s$,
$\sin(\pi - s) \equiv \sin[-(s-\pi)] \equiv -\sin(s-\pi)$
So $\sin(s-\pi) \equiv \sin s$

Complete these Pythagorean identities.

19. $\sin^2 x + \cos^2 x \equiv$ ______.   20. $1 + \tan^2 x \equiv$ ______.

Complete these cofunction identities.

21. $\sin\left(x + \frac{\pi}{2}\right) \equiv$ ______.   22. $\sin\left(\frac{\pi}{2} - x\right) \equiv$ ______.   23. $\cos\left(x - \frac{\pi}{2}\right) \equiv$ ______.

13. See table
14. See graph
15. π
16. all real numbers except odd multiples of $\frac{\pi}{2}$
17. I + III
18. See work.
19. 1
20. $\sec^2 x$
21. $\cos x$
22. $\cos x$
23. $\sin x$

24. $\pm\sqrt{\sec^2 x - 1}$

25. See graph.

26. 2

27. 2π

28. See graph.

29. $\frac{1}{\sin x}$ or $\sin x + \frac{\cos^2 x}{\sin x}$

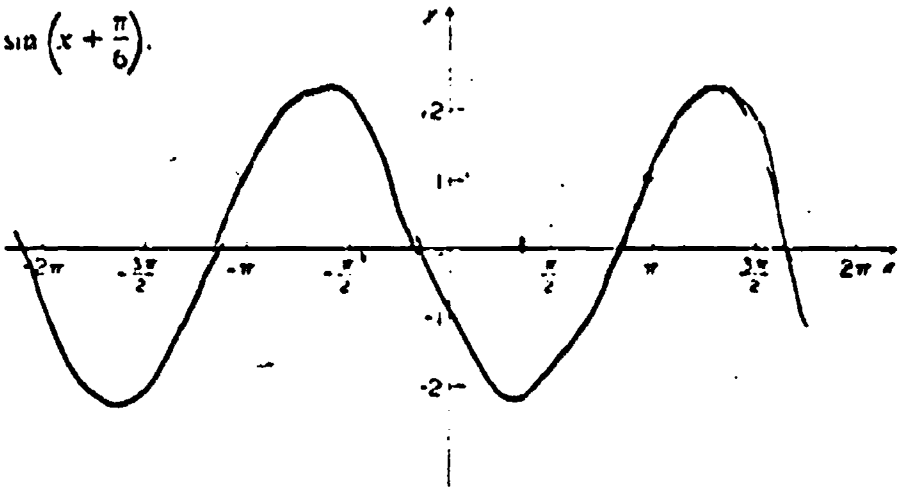
30. 1

31. $\frac{\sqrt{\sin x \tan x}}{|\sin x|}$

32. $\frac{-1 \pm \sqrt{10}}{3}$

24. Express $\cot x$ in terms of $\sec x$.

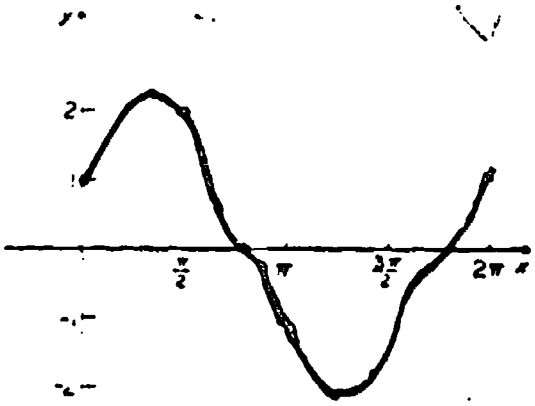
25. Sketch a graph of $y = -2\sin\left(x + \frac{\pi}{6}\right)$.



26. What is the amplitude of the function in exercise 25?

27. What is the period of the function in exercise 25?

28. Sketch a graph of $y = 2\sin x + \cos x$, for values of x between 0 and 2π.



Simplify.

29. $\frac{1}{\sec x}(\tan x + \cot x)$

30. $\frac{\tan^2 x \csc^2 x - 1}{\csc x \tan^2 x \sin x}$

31. Rationalize the denominator. $\sqrt{\frac{\tan x}{\sin x}}$

32. Solve for $\sin x$. $3\sin^2 x + 2\sin x = 3$

Chapter [illegible]
Test - [illegible]
pp. [illegible]

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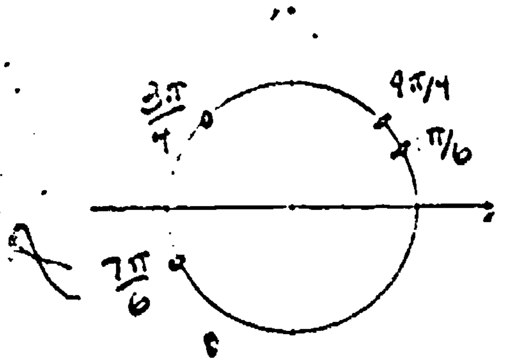
The point (3, −2) is on a circle centered at the origin. Find the coordinates of its reflections across

1. the y-axis. 2. the origin. 3. the x-axis.

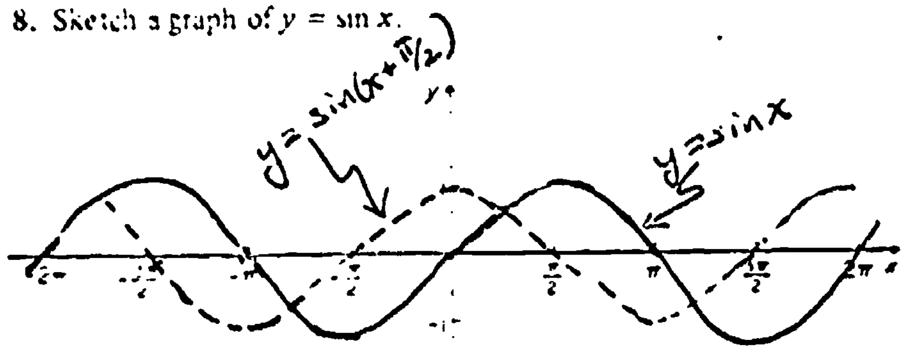
On a unit circle, mark the points determined by the following numbers.



4. $\frac{7\pi}{6}$ 5. $\frac{3\pi}{4}$

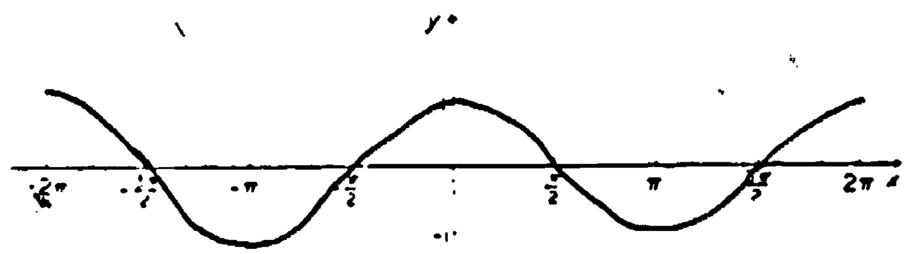
6. $\frac{\pi}{6}$ 7. $\frac{9\pi}{4}$

8. Sketch a graph of $y = \sin x$.



9. What is the domain of the sine function?

10. Sketch a graph of $y = \cos x$.



11. What is the period of the cosine function?

12. Sketch a graph of $y = \sin\left(x + \frac{\pi}{2}\right)$.
Use the axes of exercise 8.

ANSWERS

1. (−3, −2)

2. (−3, 2)

3. (3, 2)

4. See graph.

5. See graph

6. See graph

7. See graph.

8. See graph.

9. set of all real numbers

10. See graph.

11. 2π

12. See graph for exercise 8

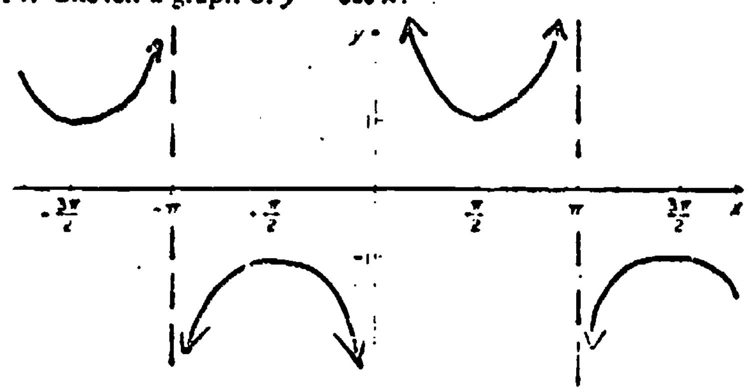
Chapter 2
Test - KEY
pp. 105-107

13. See table
14. See graph
15. 2π
16. $x \geq 1$ and $x \leq -1$
17. I, IV
18. See work.
19. 1
20. $\csc^2 x$
21. $-\sin x$
22. $\sin x$
23. $-\cos x$
24. $\pm\sqrt{\sec^2 x - 1}$

13. Complete the following table.

x	$\sin x$	$\cos x$
$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$
$\frac{\pi}{2}$	1	0
$\frac{3\pi}{4}$	$\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{2}}{2}$
π	0	-1
$\frac{5\pi}{4}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{2}}{2}$

14. Sketch a graph of $y = \csc x$.

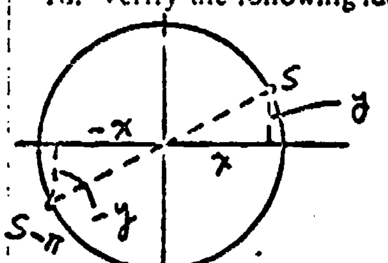


15. What is the period of the cosecant function?

16. What is the range of the cosecant function?

17. In which quadrants are the signs of the sine and tangent the same?

18. Verify the following identity. $\cot(x - \pi) \equiv \cot x$.



The points for s and s−π are symmetric with respect to the origin.
$\cot s = \frac{x}{y}$ and $\cot(s-\pi) = \frac{-x}{-y} = \frac{x}{y}$
so $\cot s = \cot(s-\pi)$

Complete these Pythagorean identities.

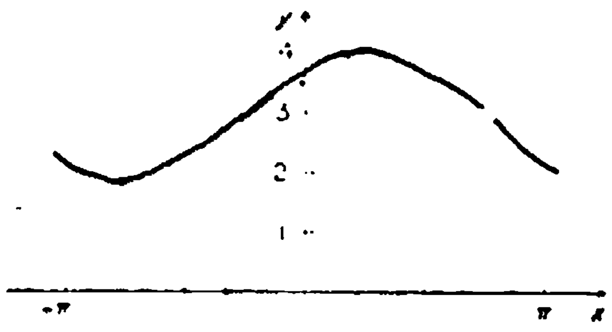
19. $\sin^2 x + \cos^2 x \equiv$ ______ 20. $1 + \cot^2 x \equiv$ ______

Complete these cofunction identities.

21. $\cos\left(x + \frac{\pi}{2}\right) \equiv$ ______ 22. $\cos\left(\frac{\pi}{2} - x\right) \equiv$ ______ 23. $\sin\left(x - \frac{\pi}{2}\right) \equiv$ ______

24. Express $\tan x$ in terms of $\sec x$.

25. Sketch a graph of $y = 3 + \cos\left(x - \frac{\pi}{4}\right)$.



25. See graph.

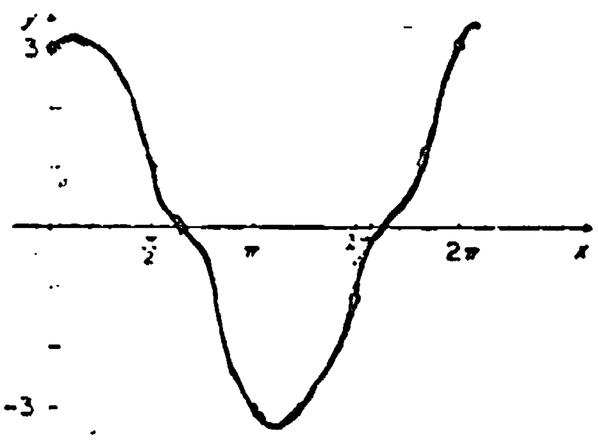
26. What is the phase shift of the function in exercise 25?

26. $-\frac{\pi}{4}$

27. What is the period of the function in exercise 25?

27. 2π

28. Sketch a graph of $y = 3\cos x + \sin x$, for values of x between 0 and 2π.



28. See graph.

Simplify.

29. $\cos x(\tan x + \cot x)$

29. $\frac{1}{\sin x}$

30. $\dfrac{\csc x(\sin^2 x + \cos^2 x \tan x)}{\sin x + \cos x}$

30. 1

31. Rationalize the denominator. $\sqrt{\dfrac{\tan x}{\sec x}}$

31. $\dfrac{\sqrt{\sec x \tan x}}{|\sec x|}$

32. Solve for $\tan x$. $3\tan^2 x - 2\tan x - 2 = 0$

32. $\dfrac{1 \pm \sqrt{7}}{3}$

1. See graph

2. See graph

3. $8^{-\frac{2}{3}} = \frac{1}{4}$

4. $\{4\}$

5. $\left\{\frac{1}{2}\right\}$

6. $\log_b \frac{a^{\frac{1}{2}} c^{\frac{3}{2}}}{d^4}$

7. $\{3, -2\}$

8. 1.255

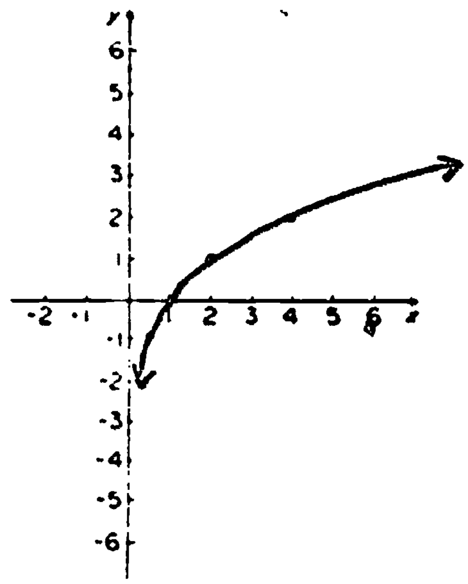
9. .544

10. −.602

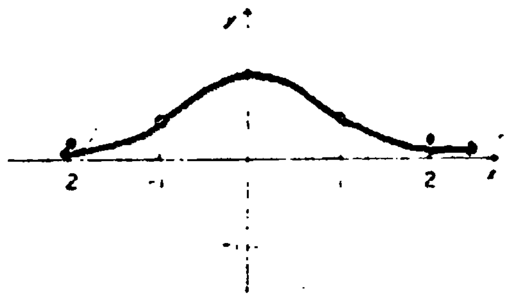
11. .2385

12. $\frac{2}{3}\log_a M - \frac{1}{3}\log_a N$

1. $y = \log_2 x$



2. $y = 2^{-x^2}$



13. 1.4200

14. 7.9034−10

15. 73.9

16. 3.5

17. .0276

18. .0006934

19. $\frac{1}{5}$

20. 9

21. a) $x \leq 0$

b) $0 < x < 1$

22. 14.2 yr

Chapter 3
Test - KEY
pp. [illegible]

1. See graph.

2. See graph.

3. $3^2 = 9$

4. $\{5\}$

5. $\{\frac{1}{2}\}$

6. $\log_a \frac{b^2\sqrt{d}}{c^3}$

7. $\{1, -\frac{1}{2}\}$

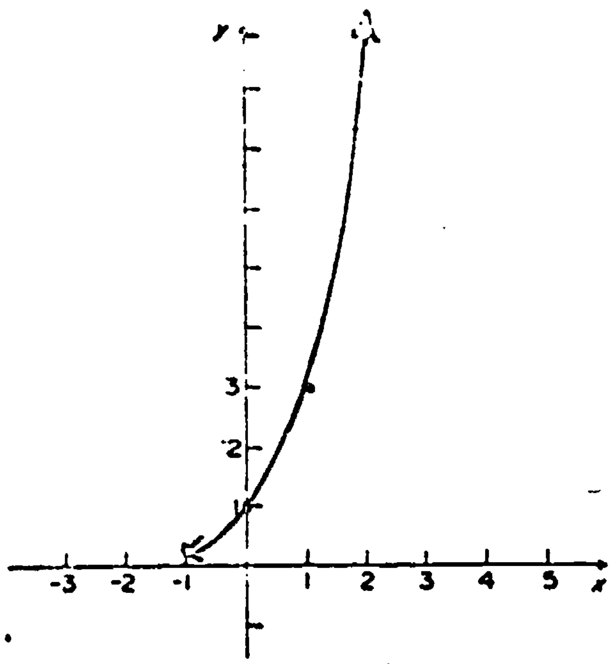
8. .902

9. 1.075

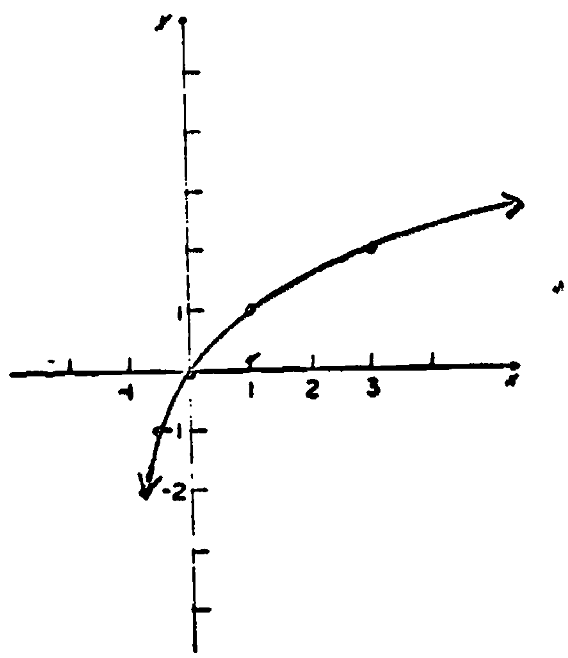
10. .059

11. .255

1. $y = 3^x$



2. $y = \log_2(x + 1)$



12. $\frac{1}{3}\log_a x - \frac{2}{3}\log_a y$

13. 1.1553

14. 7.5113−10

15. .00342

16. 4370

17. .36

18. 3.483×10^9

19. 6

20. 5

21. a) $x > 1$

b) 1

22. 12.5 g

1. I
.15π
.471

2. III
-.89π
-3.79

3. IV
1.75π
5.495

4. I
.167π
.524

5. 172°

6. 1080°

7. $\frac{5\pi}{3}$ or 5.23 in

8. $2,115^\circ$

9. 1884 $\frac{ft}{min}$

10. 6 $\frac{rad}{sec}$

11. $\sin\theta = \frac{1}{2}$
$\cos\theta = \frac{\sqrt{3}}{2}$
$\tan\theta = \frac{1}{\sqrt{3}}$
$\cot\theta = \sqrt{3}$
$\sec\theta = \frac{2}{\sqrt{3}}$
$\csc\theta = 2$

12. See table.

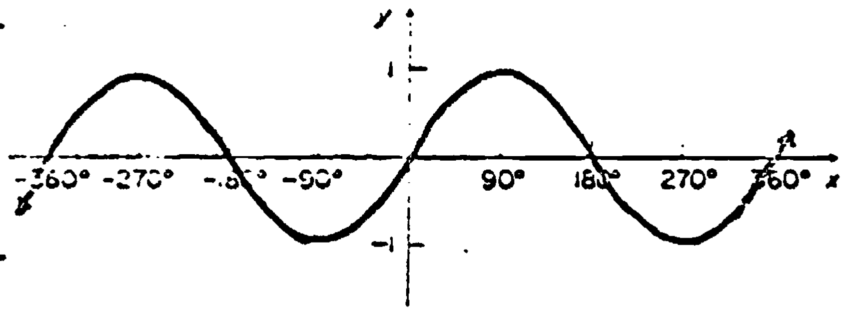
θ	0°	30°	45°	60°	90°	180°
$\sin\theta$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	0
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	-1
$\tan\theta$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	X	0
$\cot\theta$	X	$\sqrt{3}$	1	$\frac{\sqrt{3}}{3}$	0	X
$\sec\theta$	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	X	-1
$\csc\theta$	X	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1	X

13. $-\frac{\sqrt{2}}{2}$

14. $-\frac{\sqrt{3}}{2}$

15. -1

16. See graph.



17. $-\frac{\sqrt{5}}{3}$

18. $-\frac{2}{\sqrt{5}}$ or $-\frac{2\sqrt{5}}{5}$

19. $-\frac{\sqrt{5}}{2}$

20. $-\frac{3}{\sqrt{5}}$ or $-\frac{3\sqrt{5}}{5}$

21. $\frac{3}{2}$

22. .8746

23. .4848

24. 1.804

25. .5543

26. 2.063

27. 1.143

28. 44° 15'

29. 33.4°

30. .1074

31. .9320

32. 1.123

33. .7050

34. 6° 20'

9. 1130 $\frac{in}{min}$

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1. I
.483π
1.52

2. II
.806π
2.53

10. 146,000 $\frac{rad}{hr}$

3. I
.167π
.524

4. IV
-.167π
-.524

11. $\sin\theta = \frac{3}{\sqrt{13}}$
$\cos\theta = -\frac{2}{\sqrt{13}}$
$\tan\theta = -\frac{3}{2}$
$\cot\theta = -2/3$
$\sec\theta = -\sqrt{13}/2$
$\csc\theta = \sqrt{13}/3$

5. 270°

6. 720°

12. See table

θ	0°	30°	45°	60°	90°	270°
sin θ	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	-1
cos θ	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	0
tan θ	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	X	X
cot θ	X	$\sqrt{3}$	1	$\frac{\sqrt{3}}{3}$	0	0
sec θ	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	X	X
csc θ	X	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1	-1

7. $\frac{7}{4}\pi$ or 5.5 ft.

2.25,

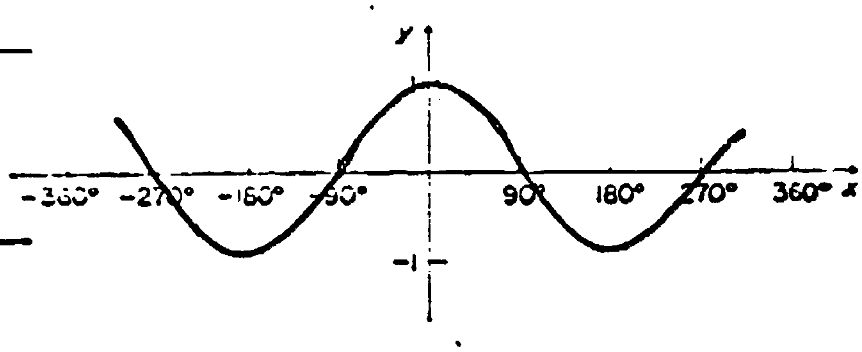
8. 129°

13. $\frac{\sqrt{3}}{2}$

14. -1

15. $\sqrt{3}$

16. See graph.



17. $-\frac{2}{3}$

18. $-\frac{\sqrt{5}}{3}$

19. $\frac{\sqrt{5}}{2}$

20. $-\frac{3}{\sqrt{5}}$ or $-\frac{3\sqrt{5}}{5}$

21. $-\frac{3}{2}$

22. .7314

23. .6820

24. 1.0724

25. .9325

26. 1.4663

27. 1.3673

28. 22° 12'

29. 47.55°

30. .9894

31. .5147

32. 1.402

33. 16° 10'

34. 39°

1. $\frac{\tan C - \tan D}{1 + \tan C \tan D}$

2. $\sin 60^\circ \cos 45^\circ + \cos 60^\circ \sin 45^\circ$

3. $\sin(76^\circ - 34^\circ)$ or $\sin 42^\circ$

4. 0

5. $\frac{1}{2}\sqrt{2 + \sqrt{2}}$

6. $1 - \tan^2\theta$ or $2 - \frac{1}{\cos^2\theta}$

7. See proof.

8. $\frac{\pi}{6} + k \cdot \pi$

9. $27^\circ + k \cdot 360^\circ$, $153^\circ + k \cdot 360^\circ$

10. [illegible] or $\frac{5\pi}{6}$

11. $\frac{\sqrt{3}}{2}$

12. $81^\circ 50'$

13. Neither

14. Perpendicular

15. Parallel

16. $y - 3 = -\frac{1}{2}(x + 2)$

17. $\frac{\pi}{2}$, $\frac{3\pi}{2}$

18. $5 \sin\left(2x + \operatorname{Arcsin}\frac{4}{5}\right)$

7. $\cot 2\theta = \frac{\cot^2\theta - 1}{2\cot\theta}$

$\frac{\cos 2\theta}{\sin 2\theta}$	$\frac{\frac{\cos^2\theta}{\sin^2\theta} - \frac{\sin^2\theta}{\sin^2\theta}}{2\frac{\cos\theta}{\sin\theta}}$
$\frac{\cos^2\theta - \sin^2\theta}{2\sin\theta\cos\theta}$	$\frac{\cos^2\theta - \sin^2\theta}{2\sin^2\theta} \cdot \frac{\sin\theta}{\cos\theta}$
	$\frac{\cos^2\theta - \sin^2\theta}{2\sin\theta\cos\theta}$

1. $\cos X \cos Y - \sin X \sin Y$

2. $\dfrac{\tan 45^\circ - \tan 30^\circ}{1 + \tan 45^\circ \tan 30^\circ}$

3. $\cos(27^\circ - 16^\circ)$ or $\cos 11^\circ$

4. $2 - \sqrt{3}$

5. $\frac{1}{2}\sqrt{2-\sqrt{2}}$

6. $2 \cot \theta$

7. See proof.

8. $\frac{\pi}{6} + 2k\pi$, $\frac{5}{6}\pi + 2k\pi$

9. $81^\circ + k \cdot 180^\circ$

10. -45° or $-\frac{\pi}{4}$

11. $\frac{7}{8}$

12. $18^\circ 30'$

13. Parallel

14. Neither

15. Perpendicular

16. $y - 4 = -1 \cdot (x - 1)$

17. $0, \pi$

18. $\sqrt{40} \sin\left(3x + \text{Arcsin} \frac{\sqrt{10}}{10}\right)$

7. $\tan 2\theta = \dfrac{2 \tan \theta}{1 - \tan^2 \theta}$.

$\dfrac{\sin 2\theta}{\cos 2\theta}$	$\dfrac{2 \frac{\sin\theta}{\cos\theta}}{\frac{\cos^2\theta}{\cos^2\theta} - \frac{\sin^2\theta}{\cos^2\theta}}$
$\dfrac{2\sin\theta\cos\theta}{\cos^2\theta - \sin^2\theta}$	$\dfrac{2\sin\theta}{\cos\theta} \cdot \dfrac{\cos^2\theta}{\cos^2\theta - \sin^2\theta}$
	$\dfrac{2\sin\theta\cos\theta}{\cos^2\theta - \sin^2\theta}$

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Chapter 6
Pretest - KEY
pp. 229, 230

ANSWERS

1. a = 40.4
A = 43° 10'
B = 46° 50'

2. B = 41°
c = 50.4
b = 33.0

3. 35° 10'

4. C = 68° 20'
b = .603
a = .443

5. 37.2 cm
46.1 cm

6. 25

7. 2.24 tons
34°

8. vert. 494 lb
horiz. 310 lb

9. (-6.88, 9.83)

10. 722 lb
in each

Chapter 6
Test - KEY
pp. 265, 266

ANSWERS

1. A = 58° 10'
B = 31° 50'
c = 4.53

2. A = 38° 50'
b = 37.9
c = 48.6

3. 1748 ft

4. A = 34°
a = .619
c = .514

5. 420 ft.

6. 8.0

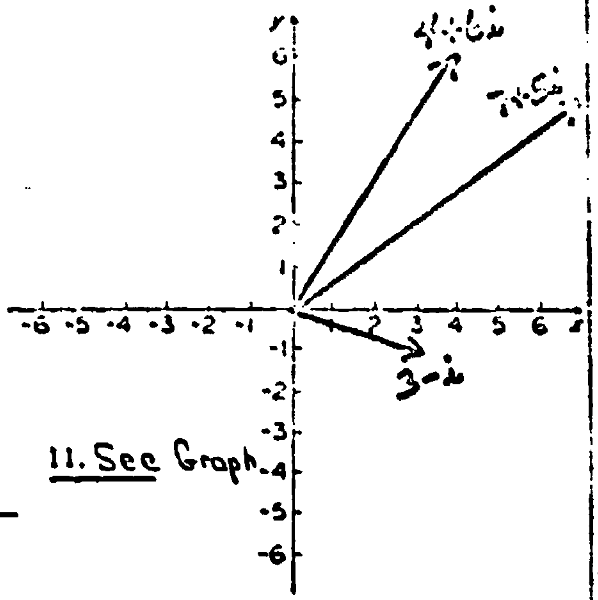
7. 75.8 mph
N 20° 10' E

8. Parallel: 106 lb
⊥ : 106 lb

9. ($\sqrt{13}$, 123° 40')

10. 577 lb

Chapter 7
Pretest - KEY
pp. 269, 270



1. $5 - i$

2. $6 - 6i$

3. $-5i$

4. $1 + 2i$

5. $x = -\frac{7}{3}$, $y = 1$

6. $2z^6 - 4z^3 + z + 1$

7. $x^2 - 4x + 5 = 0$

8. $-\frac{3}{2} \pm \frac{\sqrt{3}}{2} i$

9. $\frac{-(2+i) \pm \sqrt{-4i - 1}}{2}$

10. $1 + i$, $-1 - i$

11. See Graph

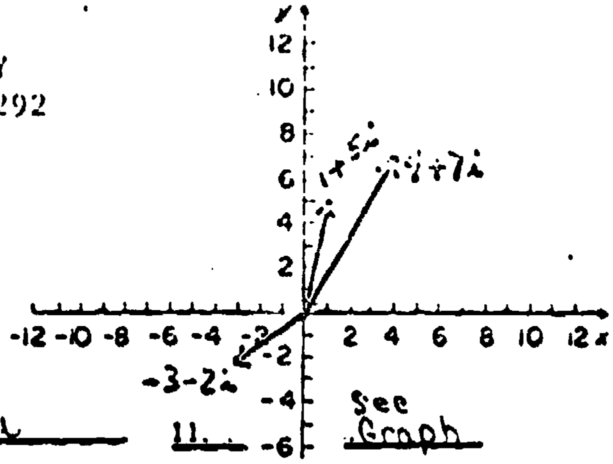
12. $1 + \sqrt{3}\, i$

13. $2 \text{ cis } 30^\circ$

14. $12 \text{ cis } 70^\circ$

15. $-\frac{\sqrt{3}}{2} + \frac{1}{2} i$, $\frac{\sqrt{3}}{2} + \frac{1}{2} i$, $-i$

Chapter 7
Test - KEY
pp. 291, 292



1. $14 + 2i$

2. $1 - 4i$

3. $2 - i$

4. $\frac{11}{10} + \frac{3}{10} i$

5. $x = 2$, $y = -4$

6. $3z^3 + z - 7$

7. $x^2 - 2x + 5 = 0$

8. $\frac{2}{5} \pm \frac{1}{5} i$

9. $\frac{(-3 \pm \sqrt{5})i}{2}$

10. $\sqrt{2} + \sqrt{2}\, i$, $-\sqrt{2} - \sqrt{2}\, i$

11. See Graph

12. $-\sqrt{2} + \sqrt{2}\, i$

13. $\sqrt{2} \text{ cis } 45^\circ$

14. $70 \text{ cis } 50^\circ$

15. $\sqrt[6]{2} \text{ cis } 15^\circ$, $\sqrt[6]{2} \text{ cis } 135^\circ$, $\sqrt[6]{2} \text{ cis } 255^\circ$

Oleanna Math Program

Syllabus: PRECALCULUS MATHEMATICS: PERIODIC (TRIGONOMETRIC) FUNCTIONS (Review Path)

Walter A. Coole, Skagit Valley College

Your OBJECTIVES for this course will be to learn the principles of periodic functions and how to apply these principles to both practical and theoretical problems.

This syllabus will be your basic map through the course. Always refer to it when you have completed each instruction so that you will know what to do next.

. . . .

Rationale. Recurring phenomena is the stuff of our lives. Without simple recurrences, we'd not know what to do from day to day, except by learning our way through each moment of time.

And yet, when recurring phenomena from different sources overlap, we sometimes have difficulty perceiving the regularity that exist in nature.

In order to account for complexities that occur as the result of an interplay of periodic events, we need sophisticated conceptual apparatus; and such an apparatus has been developed by men of the past with the study of trigonometry.

. . . .

PRE-REQUISITE TESTING: (Should take 3 hours and be completed within one day form beginning the course.)

( ) Read p. xi of your text.

( ) Write out your answers for the "PRE-REQUISITE TEST FOR BOOK III." Score your answers. If you didn't get them all right, see your instructor immediately.

( ) Write out your answers for the "POST TEST FOR BOOK III". Score your results by using the answers given in the text. If you didn't get them all right, you should be satisfied that you're in the right course. If you *did* get them all right, you may wish to complete this course simply by taking the final examination. Turn to the last page of this syllabus for instructions about the final exam.

Sample lesson. (This lesson should take about 10 hours and should be completed within three days of beginning the course.[1])

1. Time estimates given in this syllabus are based on an 11-week term. If your schedule is different, you should make adjustments accordingly.

( ) Keep track of the time you spend on this lesson; write down each ho hour of study you invest into this course. This will allow you to estimate how much more or less to spend on each subsequent lesson.

( ) Read the lesson's objectives and rationale on page 1 of your textbook.

( ) Complete the lesson assignment, beginning on page 2 of the textbook. and continuing to the end of Chapterl. If you have difficulyt, see your instructor or a math coach in the learning laboratory.

( ) Review the chapter by referring back to page 1. Examine each objective and satisfy yourself that you can perform as required by the text.

( ) Test your mastery of Chapter 1 by writing out answers to the problems at the end of the chapter. Score your answers.

--If you don't achieve 90%, repeat the lesson.
--If you score over 90%, procede to the next lesson...

( ) Write the number of hours spent on this Sample Lesson ____________. This will give you a rough basis for estimating the number of hours to allow for the remaining lessons in this course.

Your instructor will want to see this section of the syllabus completed during your scheduled conference.

....

BEFORE WE GO ANY FARTHUR

Let's plan your way through the course.

If you're beginning this course at the first of the term, you'll find a completion-schedule posted in the learning laboratory. Use the dates given there to fill in the Unit Completion Schedule below. If not...

Pick a date at which you intend to complete the course. Write that date here________________. Now, back up three days; write that date here...

EXAMINATION DATE:____________________________

Count the number of days between today's date and the examination date._____

Divide the number of available days by 4:..______________. This is the number of days you should plan to spend on each of the four units of study. Please note that each unit consists of three lessons. Enter the date for completing each unit of study below. (If you plan to undertake special projects to make a grade of "A", allow for about 30 hours' work.)

UNIT COMPLETION SCHEDULE

Unit I:____________________________________
Unit II:___________________________________
Unit III:__________________________________
Unit IV:___________________________________

You must maintain this schedule of completions to continue in the course, unless you have made special arrangements with the instructor.

You should plan to attend every scheduled conference you have selected in the learning laboratory. Bring your textbook and syllabus. You should be prepared to use the time spent waiting for the instructor studying.

Remember that your instructor and fellow-students who are serving as coaches will be available between conferences to help you with the rough spots. The best way to ask for help is to be able to point out a specific part of the textbook that's causing you trouble. BRING YOUR TEXTBOOK AND SYLLABUS FOR COACHING!

UNIT I

Unit I consists of Lessons 1, 2, and 3 (corresponging to textbook chapters 2, 3, and 4) and should be studied in the same way that you did Chapter 1 in the foregoing Sample Lesson.

As you complete each lesson, write the time and date. Record your test score, simply by noting the number of problems correctly solved.

Lesson Number	Read Objectives	Complete Programmed Work	Review Objectives	Test Score
1				
2				
3				

When you have completed Unit I, report your progress on the sign-in sheet during your next visit to the learning laboratory.

UNIT II

Unit II consists of Lessons 4, 5, and 6 (Chapters 5, 6, and 7).

Lesson Number	Read Objectives	Complete Programmed Work	Review Objectives	Test Score
4				
5				
6				

Please record your completion of Unit II.

At this point, you're halfway done!

UNIT III

Unit III's lessons cover Chapters 8, 9, and 10.

Lesson Number	Read Objectives	Complete Programmed Work	Review Objectives	Test Score
7				
8				
9				

Please record your completion of Unit III.

UNIT IV

In this unit, you'll polish off the remaining three chapters.

Lesson Number	Read Objectives	Complete Programmed Work	Review Objectives	Test Score
10				
11				
12				

Now, finally, you can record the completion of the last unit of study.

You are now ready for your final examination. You may take it at your scheduled conference period OR you may make an appointment to take it at another time.

Oleanna Math Program

ANALYTIC GEOMETRY. Course Outline by Walter A. Coole, Skagit Valley College.

Skagit Valley College Course Number: Mathematics 120

Quarter credits: 4 Semester credits: 3

Average student completion time: 120 hours

Goal. In this course, the student should learn how to describe plane figures algebraically--including "conic sections"; and conversely, how to depict algebraic formulae in conventional ways. In doing so, the student should be able to locate mathematically and scientifically significant regions relating to the "conic sections."

Performance objectives. Lesson-by-lesson, the student is required to demonstrate the following abilities: to...

1. locate points and line-segments on one-dimensional coordinate systems

2. translate statements of inequality, involving one variable, into graphic representations, using the one-dimensional coordinate system

3. translate statements of absolute value, involving one variable into graphic representations, using the one-dimensional coordinate system

4. translate one-variable formulae involving both absolute value and inequality into graphic representations

5. account for signed numbers in terms of directed distances

6. construct cartesian coordinate systems; locate points in cartesian space; explain in his/her own words, conventional quadrant-assignments

7. compute distances between two points located in cartesian space

8. given two cartesian points, compute a midpoint between; locate points on joining-lines in any proportional distance

9. given any two cartesian points, compute the slope of a line which includes them

calculate the slope of lines parallel to and perpendicular to a line, given the location of any two included points

11. produce the tangent of the angle between two lines, given points which determine them

12. given an equation in two variables, represent it as a line in planar cartesian space

13. calculate x- and y-intercepts of a line corresponding to an equation; describe figures in terms of mathematical symmetry; describe asymptotes of curved figures in algebraic language

14. describe lines in terms of four conventional "standard" forms: point-slope, two-point, slope-intercept, and intercept

15. specify the degree of an equation

16. give the standard form of a circle's equation; given a circle's equation, compute its center and radius

17. give the standard form of a parabola's equation; given a parabola's equation, compute: directrix, focus, vertex, axis, latus rectum

18. give the standard form of an elipse's equation; given an elipse's equation, compute: foci, constant distance, axes of symmetry (major and minor), center, latera recta

19. give the standard equation of an hyperbola; given an hyperbola's equation, compute its foci, axes (conjugate and transverse), center, vertices, asymptotes, latus rectum

20. "move" figures from one coordinate system to another without dropping, bending, or breaking them, given their formulae in the original system and some important clues as to where the other system could possibly be

21. figure out just how eccentric a conic could be, given its formula; and what sort of goofy things it will do as a result of that eccentricity

---

Entry.

The student entering this course should have mastered thoroughly, the content of "Periodic (Trigonometric) Functions". In addition, he/she should be able to:

i. read and follow simple written instructions

ii. state his educational objectives in simple, coherent terms

iii. study systematically and diligently

Student materials.

Paper, pencil, graph paper, straight edge

Davis, Thomas A. : *Analytic Geometry--A Programmed Text.* NY. McGraw-Hill Book Company. 1967.

Coole: *Analytic Geometry--Syllabus.*

Teacher preparation: study instructor's manual, testing materials and text

Other materials required.

Cooperative Testing Service: *Cooperative Math Test--Analytic Geometry* (Forms A and B and user's manual). Palto Alto, CA: Cooperative Testing Service. 1969.

Oleanna Math Program: *Smorgasbord*

Oleanna Math Program

ANALYTIC GEOMETRY. Syllabus by Walter A. Coole, Skagit Valley College

Your *goal* for this course will be to learn how to describe plane figures algebraically--including "conic sections"; and conversely, how to depict algebraic formulae in conventional ways. In doing so, you should be able to locate mathematically and scientifically significant regions relating to the "conic sections."

This course is divided into two equal "units", each of which will require about 60 hours' work. By following directions in this syllabus, you'll be able to avoid spending time unnecessarily on information you've already mastered. The units of the course are:

Unit	Lesson	Completion date
I	1	____________
	2	____________
	3	____________
	4	____________
	5	____________
	6	____________
	7	____________
	8	____________
	9	____________
	10	____________
	11	____________
	12	____________
	13	____________
	14	____________

Unit	Lesson	Completion date
II	15	
	16	
	17	
	18	
	19	
	20	
	21	*

The lesson-numbers correspond to the chapter of the textbook. You should attempt to complete Lesson 1 by your first scheduled conference if at all possible.

Completion dates for each unit (marked by asterisks *) should be filled in from the schedule provided. If you're beginning at the opening of a school term, your schedule will be posted on the bulletin board; otherwise, your teacher will work out a special schedule for you. Intervening lesson-completion targets must be filled in by your first conference period--but you are free to set up any realistic schedule you feel will work for you.

For this course, you'll need:

Paper, pencil, graph paper, straight edge

Davis, Thomas A.: *Analytic Geometry--A Programmed Text*

UNIT I

Pretest

Circle the correct answer.

I. Find the values of x such that $4 < |2 - \frac{1}{3}x| < 7$.

1. Which of the following is the correct answer to problem I above?

(A) $-15 < x < -6$
(B) $-15 < x < -6$ and $18 < x < 27$
(C) $x < -6$ and $x > 18$
(D) $-15 < x < 27$
(E) No values of x

II. Find the distance and the <u>directed distance</u> (if it exists) from the point P_1 to the point P_2 whose coordinates are given below:

(i) $P_1(-b, 0)$, $P_2(n, 0)$

2. The directed distance is
(A) $-b - n$
(B) $n + b$
(C) $n - b$
(D) $-b + n$
(E) None

3. The distance is
(A) $n + b$
(B) $|n - b|$
(C) $|n + b|$
(D) $n - b$
(E) None

(ii) $P_1(0, 3)$, $P_2(0, -5)$

4. The directed distance is
(A) 8
(B) −8
(C) 2
(D) −2
(E) None

5. The distance is
(A) 8
(B) −8
(C) 2
(D) −2
(E) None

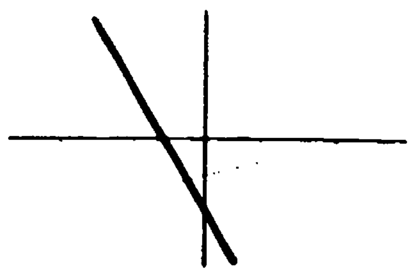
(iii) $P_1(-1, 4)$, $P_2(3, -2)$

6. The directed distance is
(A) $\sqrt{52}$
(B) $-\sqrt{8}$
(C) $-\sqrt{20}$
(D) $\sqrt{10}$
(E) None

7. The distance is
(A) $\sqrt{52}$
(B) $\sqrt{8}$
(C) $\sqrt{20}$
(D) $\sqrt{10}$
(E) None

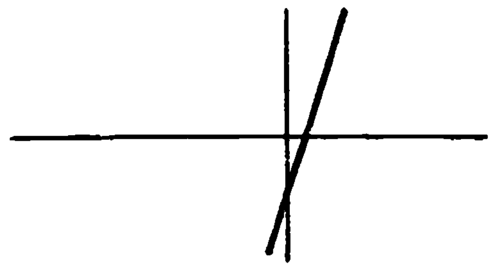
III. In problems 8 through 11, use your common sense to assign the appropriate slope to each line.

8. The slope is



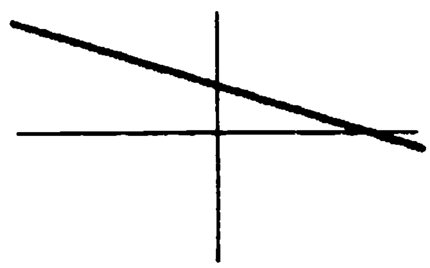
(A) 5
(B) $-\frac{1}{5}$
(C) −2
(D) 2
(E) $\frac{1}{2}$

9. The slope is



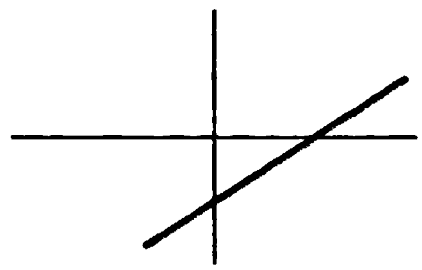
(A) 5
(B) $-\frac{1}{5}$
(C) −2
(D) 2
(E) $\frac{1}{2}$

10. The slope is



(A) 5
(B) $-\frac{1}{5}$
(C) −2
(D) 2
(E) $\frac{1}{2}$

11. The slope is



(A) 5
(B) $-\frac{1}{5}$
(C) −2
(D) 2
(E) $\frac{1}{2}$

IV. Given the lines L_1 through $(-4, -2)$ and $(1, 5)$, and L_2 through $(-7, 1)$ and $(9, -9)$, are they parallel, perpendicular, or neither?

12. In the problem above, the lines are:
(A) parallel
(B) perpendicular
(C) neither

V. Find the equation of the line through the points $(-4, 3)$ and $(-9, 5)$. Put your answer in the form $y = Ax + B$ and answer question 13 below.

13. The sum of the coefficients $A + B$ is
(A) 1
(B) 9
(C) $16\frac{1}{5}$
(D) -20
(E) $8\frac{1}{5}$

VI. Find the equation of the line through the point $(4, -6)$ parallel to the line $11y + 2x + 33 = 0$. Put your answer in the form $y = Ax + B$ and answer question 14 below.

14. The sum of the coefficient $A + B$ is
(A) $-5\frac{5}{11}$
(B) $-22\frac{1}{2}$
(C) $10\frac{1}{2}$
(D) $-6\frac{6}{11}$
(E) $2\frac{8}{11}$

VII. Find the coordinates of the point $M(x, y)$ which is $\frac{4}{7}$ of the way from the point $(a, -b)$ to the point (J, K).

15. The coefficients of the point are:
(A) $x = \frac{3a + 4J}{7}$ $y = \frac{3b - 4K}{7}$
(B) $x = \frac{3J - 4a}{7}$ $y = \frac{3K + 4b}{7}$
(C) $x = \frac{3a - 4J}{7}$ $y = \frac{3b + 4K}{7}$
(D) $x = \frac{3J + 4a}{7}$ $y = \frac{3K - 4b}{7}$
(E) $x = \frac{4J - 3a}{7}$ $y = \frac{4K + 3b}{7}$

VIII. Find the intercepts and asymptotes of the curve whose equation is $y(1 - x^2) = 5x$, and test for symmetry. Now answer questions 16 through 20 below.

16. The x-intercept(s) are:
(A) $+1, -1$
(B) 0
(C) $0, +1, -1$
(D) $0, +\sqrt{5}, -\sqrt{5}$
(E) there are none

17. The y-intercept(s) are
(A) $+1, -1$
(B) 0
(C) $0, +1, -1$
(D) $0, 5, -5$
(E) there are none

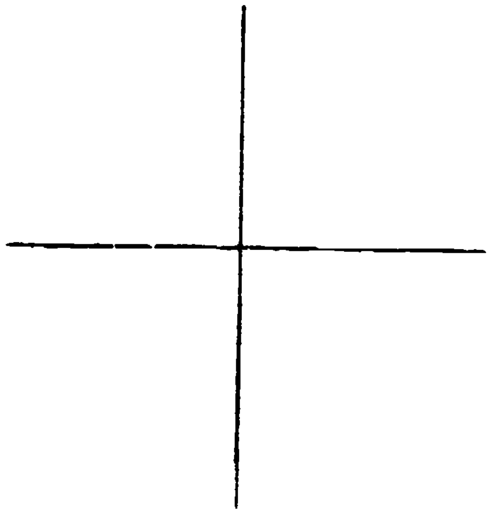
18. The horizontal asymptote(s) pass through
(A) $(0, 1)$
(B) $(0, -1)$
(C) $(0, 0)$
(D) $(0, 1)$ and $(0, -1)$
(E) there are none

19. The vertical asymptote(s) pass through
(A) $(1, 0)$
(B) $(-1, 0)$
(C) $(0, 0)$
(D) $(1, 0)$ and $(-1, 0)$
(E) there are none

20. The curve is symmetric with respect to
(A) x-axis only
(B) y-axis only
(C) origin only
(D) x and y axes and the origin
(E) none of these

IX. Use the information from problem VIII (questions 16 through 20) to <u>sketch</u> the curve whose equation is $y(1 - x^2) = 5x$.

Sketch



Score your pretest according to the answers below and total the points achieved.

Question Number	Correct Answer	
I. ($12\frac{1}{2}$ points)		
1	B	$-15 < x < -6$ and $18 \le x < 27$
II. (12 points)		
2	B	The directed distance from $P_1(-b, 0)$ to $P_2(n, 0)$ is $n + b$.
3	C	The distance from $P_1(-b, 0)$ to $P_2(n, 0)$ is $\lvert n + b \rvert$
4	B	The directed distance from $P_1(0, 3)$ to $P_2(0, -5)$ is -8.
5	A	The distance is 8.
6	E	There is no directed distance from $P_1(-1, 4)$ to $P_2(3, -2)$.
7	A	The distance is $\sqrt{52}$
III. (8 points)		
8	C	-2 is the slope.
9	A	5 is the slope.
10	B	$-\frac{1}{5}$ is the slope.
11	E	$\frac{1}{2}$ is the slope.
IV. ($12\frac{1}{2}$ points)		
12	C	The lines are neither parallel nor perpendicular.

V. ($12\frac{1}{2}$ points)

The equation in the form $y = Ax + B$ is $y = -\frac{3}{5}x + \frac{2}{5}$.

13	A	$1 = A + B$

VI. ($12\frac{1}{2}$ points)

The equation in the form $y = Ax + B$ is $y = -\frac{2}{11}x - \frac{58}{11}$.

14	A	$-5\frac{2}{11} = A + B$

VII. (10 points)

15	D	$x = \frac{3J + 4a}{7}$, $y = \frac{3K - 4b}{7}$ are the coordinates of the point which is $\frac{4}{7}$ of the way from $(a, -b)$ to (J, K).

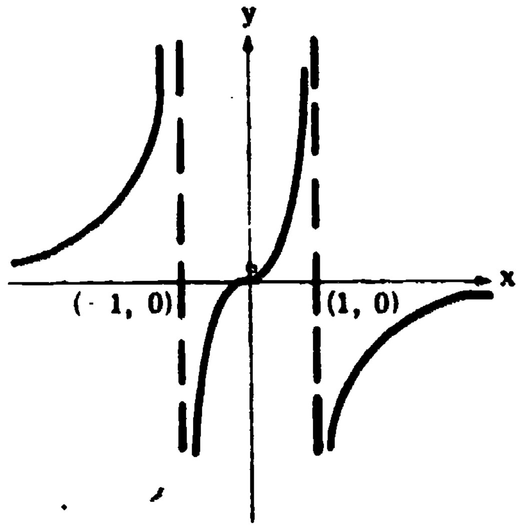
VIII. (10 points)

Given the curve whose equation is $y(1 - x^2) = 5x$.

16	B	0 is the x-intercept.
17	B	0 is the y-intercept.
18	C	The horizontal asymptote passes through (0, 0).
19	D	The vertical asymptotes pass through (1, 0) and (−1, 0).
20	C	The curve is symmetric with respect to the origin only.

IX. (10 points) $y(1 - x^2) = 5x$

Sketch



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If your score was 90 or more points, procede directly to Unit II.

If your score was less than 90 points, complete the work assigned for Unit I.

How to Study Each Lesson

First: read the objective for the lesson (given below).

Second: complete the programmed work in the text in the chapter corresponding to the lesson

Third: complete the "Supplementary Problems" at the end of the chapter; check your answer in the back of the book; if you get 90% right, procede to the next lesson--if you get less than 90% right, repeat the lesson (note: there are no supplementary problems for chapter 1).

LESSON OBJECTIVES

At the end of each lesson, you are expected to be able to do certain specific things. Lesson-by-lesson, they are to be able to...

1. locate points and line-segments on a one-dimensional coordinate system

2. translate statements of inequality, involving one variable, into graphic representations, using a one-dimensional coordinate system

3. translate statements of absolute value, involving one variable into graphic representations, using a one-dimensional coordinate system

4. translate one-variable formulae involving both absolute value and inequality into graphic representations

5. account for signed numbers in terms of directed distances

6. construct cartesian coordinate systems; locate points in cartesian space; explain in your own words, conventional quadrant-assignments

7. compute distances between two points located in cartesian space

8. given two cartesian points, compute a midpoint between; locate points on joining-lines in any proportional distance

9. given any two cartesian points, compute the slope of a line which includes them

10. calculate the slope of lines parallel to and perpendicular to a line, given the location of any two included points

11. produce the tangent of the angle between two lines, given points which determine them

12. given an equation in two variables, represent it as a line in planar cartesian space

13. calculate x- and y-intercepts of a line corresponding to an equation; describe figures in terms of mathematical symmetry; describe asymptotes of curved figures in algebraic language

14. describe lines in terms of four conventional "standard" forms: point-slope, two-point, slope-intercept, and intercept

---

When you have completed lesson (chapter) 14 and have scored 90% on the supplemental problems for chapters 2-14, you are ready for ...

Unit I Post-Test

Circle the correct answer.

I. Find the values of x such that $3 < |7 - 3x| \le 8$.

1. Which of the following is the correct answer to problem I above?
   (A) $x > \frac{10}{3}$ and $x < \frac{4}{3}$
   (B) $-\frac{1}{3} \le x \le 5$
   (C) $-\frac{1}{3} \le x < \frac{4}{3}$
   (D) $-\frac{1}{3} \le x < \frac{4}{3}$ and $\frac{10}{3} < x \le 5$
   (E) no values of x

II. Find the distance and the directed distance (if it exists) from the point P_1 to the point P_2 whose coordinates are given below:

(i) $P_1(0, k)$, $P_2(0, -a)$

2. The directed distance is
   (A) $k + a$
   (B) $a - k$
   (C) $k - a$
   (D) $-a - k$
   (E) None

3. The distance is
   (A) $k + a$
   (B) $a - k$
   (C) $|k - a|$
   (D) $|-a - k|$
   (E) None

(ii) $P_1(-2, 0)$, $P_2(4, 0)$

4. The directed distance is
   (A) 2
   (B) −2
   (C) 6
   (D) −6
   (E) None

5. The distance is
   (A) 2
   (B) −2
   (C) 6
   (D) −6
   (E) None

(iii) $P_1(3, 1)$, $P_2(-7, 5)$

6. The directed distance is
   (A) $-\sqrt{14}$
   (B) $\sqrt{116}$
   (C) $-\sqrt{32}$
   (D) $\sqrt{40}$
   (E) None

7. The distance is
   (A) $\sqrt{14}$
   (B) $\sqrt{116}$
   (C) $\sqrt{32}$
   (D) $\sqrt{40}$
   (E) None

III. In problems 8 through 11, use your common sense to assign the appropriate slope to each line. Mark your answers on the answer card.

8. The slope is

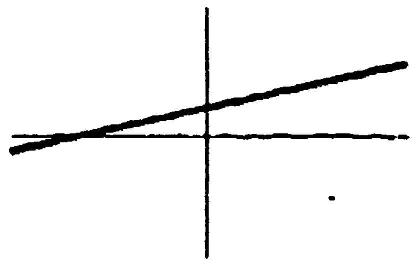
(A) 6

(B) $\frac{1}{6}$

(C) 3

(D) -3

(E) $-\frac{1}{3}$



9. The slope is

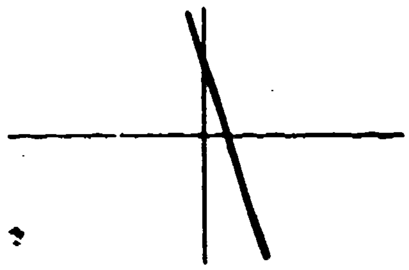
(A) 6

(B) $\frac{1}{6}$

(C) 3

(D) -3

(E) $-\frac{1}{3}$



10. The slope is

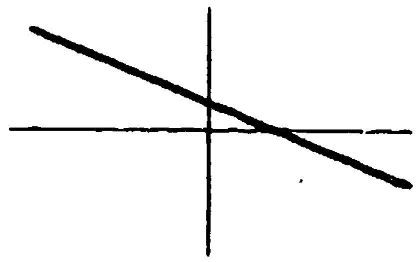
(A) 6

(B) $\frac{1}{6}$

(C) 3

(D) -3

(E) $-\frac{1}{3}$



11. The slope is

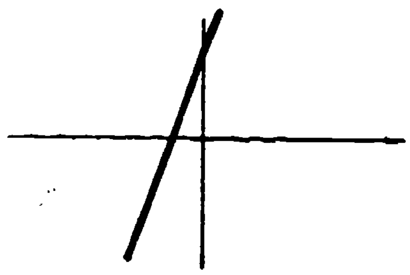
(A) 6

(B) $\frac{1}{6}$

(C) 3

(D) -3

(E) $-\frac{1}{3}$



IV. Given the lines L_1 through $(-3, 2)$ and $(4, -3)$, and L_2 through $(1, -15)$ and $(-13, -4)$, are they parallel, perpendicular, or neither?

12. In the problem above the lines are:
(A) parallel
(B) perpendicular
(C) neither

V. Find the equation of the line with x-intercept -4 and y-intercept -5. Put your answer in the form $y = Ax + B$ and answer question 13 below.

13. The sum of the coefficients $A + B$ is
(A) $6\frac{1}{4}$
(B) $-6\frac{1}{4}$
(C) $-11\frac{1}{4}$
(D) $-4\frac{1}{5}$
(E) $4\frac{1}{5}$

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VI. Find the equation of the line through the point $(-3, 5)$ perpendicular to the line $2x + 4y - 7 = 0$. Put your answer in the form $y = Ax + B$ and answer question 14 below.

14. The sum of the coefficients $A + B$ is
(A) -11
(B) -3
(C) 3
(D) 7
(E) 13

VII. Find the coordinates of the point $M(x, y)$ which is $\frac{2}{5}$ of the way from the point (m, n) to the point $(r, -s)$.

15. The coefficients of the point M are:
(A) $x = \frac{3m + 2r}{5}$ $\quad y = \frac{3n - 2s}{5}$
(B) $x = \frac{3m - 2r}{5}$ $\quad y = \frac{3n + 2s}{5}$
(C) $x = \frac{2r - 3m}{5}$ $\quad y = \frac{2s + 3n}{5}$
(D) $x = \frac{2m - 3r}{5}$ $\quad y = \frac{2n + 3s}{5}$
(E) $x = \frac{2m + 3r}{5}$ $\quad y = \frac{2n - 3s}{5}$

VIII. Find the intercepts and asymptotes, of the curve whose equation is $y(x^2 - 4) = -7x$ and test for symmetry. Now answer questions 16 through 20 below.

16. The x-intercept(s) are:
(A) $+2, -2$
(B) 0
(C) $0, +2, -2$
(D) $0, \sqrt{7}, -\sqrt{7}$
(E) there are none

17. The y-intercept(s) are
(A) $+2, -2$
(B) 0
(C) $0, +2, -2$
(D) $0, 7, -7$
(E) there are none

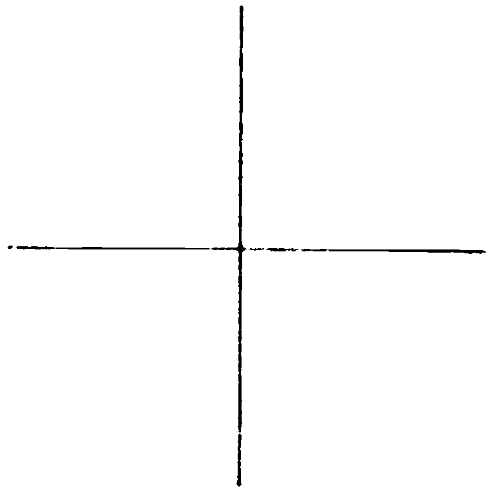
18. The horizontal asymptote(s) pass through
(A) $(0, 2)$
(B) $(0, -2)$
(C) $(0, 0)$
(D) $(0, 2)$ and $(0, -2)$
(E) there are none

19. The vertical asymptote(s) pass through
(A) $(2, 0)$
(B) $(-2, 0)$
(C) $(0, 0)$
(D) $(2, 0)$ and $(-2, 0)$
(E) there are none

20. The curve is symmetric with respect to
(A) x axis only
(B) y-axis only
(C) origin only
(D) x and y axes and the origin
(E) there are none

IX. Use the information from problem VIII (questions 16 through 20) to sketch the curve whose equation is $y(x^2 - 4) = -7x$.

sketch



Score your post-test according to the answers below and total the points achieved.

Question Number	Correct Answer	
I. ($12\frac{1}{2}$ point)		
1	D	$-\frac{1}{3} < x < \frac{4}{3}$ and $\frac{10}{3} < x \le 5$
II. (12 points)		
2	D	$-a - k$ is the directed distance from $P_1(0, k)$ to $P_2(0, -a)$.
3	D	$\lvert -a - k \rvert$ is the distance from $P_1(0, k)$ to $P_2(0, -a)$.
4	C	6 is the directed distance from $P_1(-2, 0)$ to $P_2(4, 0)$.
5	C	6 is the distance from $P_1(-2, 0)$ to $P_2(4, 0)$.
6	E	There is no directed distance from $P_1(3, 1)$ to $P_2(-7, 5)$.
7	B	$\sqrt{116}$ is the distance between $P_1(3, 1)$ and $P_2(-7, 5)$.
III. (8 points)		
8	B	$\frac{1}{6}$ is the slope.
9	D	-3 is the slope.
10	E	$-\frac{1}{3}$ is the slope.
11	C	3 is the slope.
IV. ($12\frac{1}{2}$ points)		
12	C	The lines are neither parallel nor perpendic... .

V. ($12\frac{1}{2}$ points)

The equation in the form $y = Ax + B$ is $y = -\frac{5}{4}x - 5$.

13 B $-6\frac{1}{4} = A + B$.

VI. ($12\frac{1}{2}$ points)

The equation in the form $y = Ax + b$ is $y = 2x + 11$.

14 E $13 = A + B$.

VII. (10 points)

15 E $x = \frac{2m + 3r}{5}$, $y = \frac{2n - 3s}{5}$ are the coordinates of the point which is $\frac{3}{5}$ of the way from (m, n) to (r, −s).

VIII. (10 points)

Given the curve whose equation is $y(x^2 - 4) = -7x$.

16 B 0 is the x-intercept.

17 B 0 is the y-intercept.

18 C The horizontal asymptote passes through (0, 0).

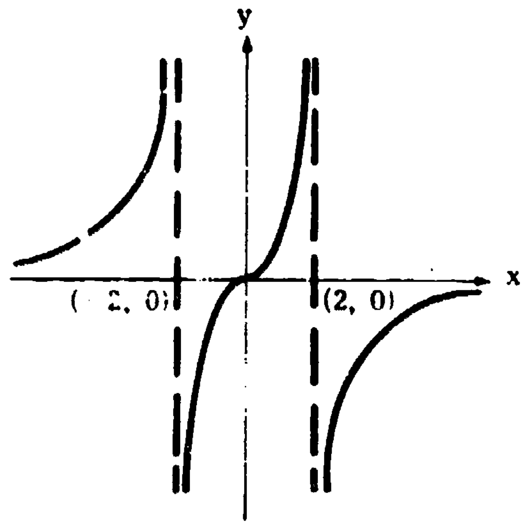
19 D The vertical asymptotes pass through (2, 0) and (−2, 0).

20 C The curve is symmetric with respect to the origin only.

IX. (10 points)

$y(x^2 - 4) = -7x$

Sketch



If you achieved 90 or more points, procede to Unit II. If not, repeat Unit I.

UNIT II

Pretest

Circle the correct answer.

1. Write the equation $5x^2 + 5y^2 + 30x - 10y + 14 = 0$ in standard form and identify the conic. Then answer the questions that follow.

1. The conic is
(A) a circle
(B) a parabola
(C) an ellipse
(D) a hyperbola
(E) degenerate

2. The focus is (or foci are) at
(A) $(1, -3)$
(B) $(-3, 1)$
(C) $(1 - \sqrt{5}, -3)$ and $(1 + \sqrt{5}, -3)$
(D) $(-3 - \sqrt{5}, 1)$ and $(-3 + \sqrt{5}, 1)$
(E) There are none

3. The eccentricity is
(A) 0
(B) 1
(C) $\sqrt{2}$
(D) $1/\sqrt{2}$
(E) There is none

4. The vertex is (or the vertices are) at
(A) $(1, -3)$
(B) $(-3, 1)$
(C) $(1 - \sqrt{5}, -3)$ and $(1 + \sqrt{5}, -3)$
(D) $(-3 - \sqrt{5}, 1)$ and $(-3 + \sqrt{5}, 1)$
(E) There are none

5. The center is at
(A) $(1, -3)$
(B) $(-1, 3)$
(C) $(3, -1)$
(D) $(-3, 1)$
(E) There is none

6. The radius is
(A) 1
(B) $6/\sqrt{5}$
(C) $\sqrt{5}$
(D) 6
(E) There is none

7. The directrix (or one of the directrices) is
(A) $y = 1 - 2\sqrt{5}$
(B) $x = -3 - 2\sqrt{5}$
(C) $y = 1 + 2\sqrt{5}$
(D) $x = -3 + 2\sqrt{5}$
(E) There is(are) none

8. The ends of the latus rectum (or one of the latera recta) are
(A) $(1 + \sqrt{10}, -3 \pm \sqrt{5})$
(B) $(1 + \sqrt{10}, -3 \pm 1/\sqrt{5})$
(C) $(1 - \sqrt{10}, -3 \pm \sqrt{5})$
(D) $(1 - \sqrt{10}, -3 \pm 1/\sqrt{5})$
(E) There are none

9. The axis of parabola is
(A) the x-axis
(B) the y-axis
(C) parallel to the x-axis
(D) parallel to the y-axis
(E) There is none

10. The parabola is concave
(A) right
(B) left
(C) up
(D) down
(E) It is not a parabola

11. The major axis is
(A) the x-axis
(B) the y-axis
(C) parallel to the x-axis
(D) parallel to the y-axis
(E) There is none

12. The minor axis is
(A) the x-axis
(B) the y-axis
(C) parallel to the x-axis
(D) parallel to the y-axis
(E) There is none

13. The transverse axis is
(A) the x-axis
(B) the y-axis
(C) parallel to the x-axis
(D) parallel to the y-axis
(E) There is none

14. The conjugate axis is
(A) the x-axis
(B) the y-axis
(C) parallel to the x-axis
(D) parallel to the y-axis
(E) There is none

15. The asymptotes are
(A) $y = x - 4$, and $y = -x - 2$
(B) $y = x + 4$, and $y = -x + 2$
(C) $y = x - 4$, and $y = -x + 2$
(D) $y = x + 4$, and $y = -x - 2$
(E) There are none

II. Write the equation $4x^2 + 9y^2 - 56x + 54y + 241 = 0$ in standard form and identify the conic. Then answer the questions that follow.

16. The conic is
(A) a circle
(B) a parabola
(C) an ellipse
(D) a hyperbola
(E) degenerate

17. The focus (or one of the foci) is at
(A) $(-2, 3)$
(B) $(-4, 3)$
(C) $(7 + \sqrt{5}, -3)$
(D) $(10, -3)$
(E) There are none

18. The eccentricity is
(A) 0
(B) 1
(C) $\sqrt{5}/3$
(D) $3/\sqrt{5}$
(E) There is none

19. The vertex (or one of the vertices) is at
(A) $(-5, 3)$
(B) $(9, -3)$
(C) $(-4, 3)$
(D) $(10, -3)$
(E) There are none

20. The center is at
(A) $(-3, 7)$
(B) $(7, -3)$
(C) $(3, -7)$
(D) $(-7, 3)$
(E) There is none

21. The radius is
(A) 1
(B) 2
(C) 3
(D) 6
(E) There is none

22. The directrix (or one of the directrices) is
(A) $x = 4\frac{1}{3}$
(B) $x = \frac{4\sqrt{5} - 35}{5}$
(C) $x = \frac{9\sqrt{5} + 35}{5}$
(D) $\frac{1}{3}$
(E) There are none

23. The ends of the latus rectum (or one of the latera recta) are
(A) $(7 + 3, -3 \pm \frac{2}{3})$
(B) $(-7 + \sqrt{5}, 3 \pm \frac{4}{3})$
(C) $(7 + \sqrt{5}, -3 \pm \frac{4}{3})$
(D) $(-7 + 3, 3 \pm \frac{2}{3})$
(E) There are none

24. The axis of parabola is
(A) the x-axis
(B) the y-axis
(C) parallel to the x-axis
(D) parallel to the y-axis
(E) There is none

25. The parabola is concave
(A) right
(B) left
(C) up
(D) down
(E) It is not a parabola

26. The major axis is
(A) the x-axis
(B) the y-axis
(C) parallel to the x-axis
(D) parallel to the y-axis
(E) There is none

27. The minor axis is
(A) the x-axis
(B) the y-axis
(C) parallel to the x-axis
(D) parallel to the y-axis
(E) There is none

28. The transverse axis is
(A) the x-axis
(B) the y-axis
(C) parallel to the x-axis
(D) parallel to the y-axis
(E) There is none

29. The conjugate axis is
(A) the x-axis
(B) the y-axis
(C) parallel to the x-axis
(D) parallel to the y-axis
(E) There is none

30. The asymptotes are
(A) $y = \pm\frac{2}{3}x$
(B) $y = \frac{3}{2}x - \frac{27}{2}$ and $y = -\frac{3}{2}x + \frac{15}{2}$
(C) $y = \pm\frac{3}{2}x$
(D) $y = \frac{2}{3}x - \frac{22}{3}$ and $y = -\frac{2}{3}x + \frac{2}{3}$
(E) There are none

III. Find the equation of the parabola with vertex at $(-3, 0)$ and focus at $(5, 0)$. Put your answer in the form $Ax^2 + Cy^2 + Dx + Ey + F = 0$ and answer questions 31 and 32 below.

31. In the equation above, the sum of the coefficients $A + D =$
(A) −16
(B) −20
(C) −32
(D) 16
(E) 1

32. In the equation above, $C + E + F =$
(A) 97
(B) −95
(C) −47
(D) 64
(E) −59

IV. Find the equation of the ellipse centered at the origin with eccentricity $\frac{3}{4}$ and major axis on the x-axis of length 8. Put your answer in the form $Ax^2 + Cy^2 + Dx + Ey + F = 0$ and answer questions 33 and 34 below.

33. In the equation, the sum of the coefficients $A + D$ is
(A) −16
(B) −7
(C) 7
(D) 9
(E) 16

34. In the equation, $C + E + F$ is
(A) −138
(B) −135
(C) −128
(D) −105
(E) −96

V. Find the equation of the parabola with vertex at $(4, -6)$ and focus at $(4, -1)$. Put your answer in the form $Ax^2 + Cy^2 + Dx + Ey + F = 0$ and answer questions 35 and 36 below.

35. In the equation, the sum of the coefficients $A + D$ is
(A) −20
(B) −10
(C) −7
(D) 9
(E) 13

36. In the equation, $C + E + F$ is
(A) −124
(B) −54
(C) 116
(D) 129
(E) 156

VI. Find the equation of the hyperbola with vertices at $(-2, 2)$ and $(4, 2)$ which passes through the point $(6, \frac{10}{3})$. Put your answer in the form $Ax^2 + Cy^2 + Dx + Ey + F = 0$ and answer questions 37 and 38 below.

37. In the equation, the sum of the coefficients $A + D$ is

(A) -9
(B) -3
(C) -1
(D) 1
(E) 3

38. In the equation, $C + E + F$ is

(A) -89
(B) -17
(C) -1
(D) 1
(E) 17

Score your pretest according to the answers below and total the points achieved.

Question Number	Correct Answer		Points
1. (25 points)		$5x^2 + 5y^2 + 30x - 10y + 14 = 0$ Standard form: $(x + 3)^2 + (y - 1)^2 = \frac{36}{5}$	
1	A	The conic is a circle.	3
2	E	There are no foci	3
3	E	There is no eccentricity.	3
4	E	There are no vertices.	3
5	D	The center is at $(-3, 1)$.	3
6	B	The radius is $6/\sqrt{5}$.	1
7	E	There is no directrix.	1
8	E	There are no latera recta.	1
9	E	There is no axis of parabola.	1
10	E	It is not a parabola.	1
11	E	There is no major axis.	1
12	E	There is no minor axis.	1
13	E	There is no transverse axis.	1
14	E	There is no conjugate axis.	1
15	E	There are no asymptotes.	1

Question Number	Correct Answer		Points
II. (25 points)		$4x^2 + 9y^2 - 56x + 54y + 241 = 0$	
		Standard form: $\frac{(x-7)^2}{9} + \frac{(y+3)^2}{4} = 1$	
16	C	The conic is an ellipse.	3
17	C	One focus is at $(7 + \sqrt{5}, -3)$.	3
18	C	The eccentricity is $\sqrt{5}/3$.	3
19	D	One of the vertices is at $(10, -3)$.	3
20	B	The center is at $(7, -3)$.	3
21	E	There is no radius.	1
22	C	One of the directrices is $x = \frac{9\sqrt{5} + 35}{5}$	1
23	C	The ends of one of the latera recta are $(7 + \sqrt{5}, -3 \pm \frac{4}{3})$.	1
24	E	There is no axis of parabola.	1
25	E	It is not a parabola.	1
26	C	The major axis is parallel to the x-axis.	1
27	D	The minor axis is parallel to the y-axis.	1
28	E	There is no transverse axis.	1
29	E	There is no conjugate axis.	1
30	E	There are no asymptotes.	1
III. ($12\frac{1}{2}$ points)		Parabola with vertex at $(-3, 0)$ and focus at $(5, 0)$.	
		Standard form: $y^2 = 32(x + 3)$	
		Form of $Ax^2 + Cy^2 + Dx + Ey + F = 0$: $y^2 - 32x - 96 = 0$	
31	C	$-32 = A + D$	
32	B	$-95 = C + E + F$	
IV. ($12\frac{1}{2}$ points)		Ellipse with center at the origin, eccentricity $\frac{3}{4}$, and major axis on the x-axis of length 8.	
		Standard form: $\frac{x^2}{16} + \frac{y^2}{7} = 1$	
		Form of $Ax^2 + Cy^2 + Dx + Ey + F = 0$: $7x^2 + 16y^2 - 112 = 0$	
33	C	$7 = A + D$	
34	E	$-96 = C + E + F$	
V. ($12\frac{1}{2}$ points)		Parabola with vertex at $(4, -6)$ and focus at $(4, -1)$.	
		Standard form: $(x - 4)^2 = 20(y + 6)$	
		Form of $Ax^2 + Cy^2 + Dx + Ey + F = 0$: $x^2 - 8x - 20y - 104 = 0$	
35	C	$-7 = A + D$	
36	A	$-124 = C + E + F$	
VI. ($12\frac{1}{2}$ points)		Hyperbola with vertices at $(-2, 2)$ and $(4, 2)$ which passes through the point $(6, \frac{10}{3})$.	
		Standard form: $\frac{(x-1)^2}{9} - \frac{(y-2)^2}{1} = 1$	
		Form of $Ax^2 + Cy^2 + Dx + Ey + F = 0$: $x^2 - 9y^2 - 2x + 36y - 44 = 0$	
37	C	$-1 = A + D$	
38	B	$-17 = C + E + F$	

If your score was 90 or more points, procede to the final examination.

If your score was less than 90 points, complete the work assigned for Unit II.

First, read "How to Study Each Lesson", on page 6. If your study-habits have deteriorated, get back into the prescribed pattern.

LESSON OBJECTIVES

Unit II's objectives, lesson-by-lesson are for you to be able to...

15. specify the degree of an equation

16. give the standard form of a circle's equation; given a circle's equation, compute its center and radius

17. give the standard form of a parabola's equation; given a parabola's equation, compute: directrix, focus, vertex axis, latus rectum

18. give the standard form of an elipse's equation; given an elipse's equation, compute: foci, constant distance, axes of symmetry (major and minor), center, latera recta

19. give the standard equation of an hyperbola; given an hyperbola's equation, compute its foci, axes (conjugate and transverse), center, vertices, asymptotes, latus rectum

20. "move" figures from one coordinate system to another without dropping, bending, or breaking them, given their formulae in the original system and some important clues as to where the other system might be

21. figure out just how eccentric a conic coulc be, given its formula; and what sort of goofy things it will do as a result of that eccentricity.

When you have completed lesson (chapter) 21 and have scored 90% on the supplemental problems for all chapters, you are ready for...

Unit II Post-Test

Circle the correct answer.

I. Write the equation $9x^2 - 6x + 18y + 91 = 0$ in standard form and identify the conic. Then answer the questions that follow.

1. The conic is
(A) a circle
(B) a parabola
(C) an ellipse
(D) a hyperbola
(E) degenerate

2. The focus is (or foci are) at
(A) $(\frac{1}{3}, -\frac{11}{2})$ and $(\frac{1}{3}, -\frac{9}{2})$
(B) $(-\frac{1}{6}, -5)$
(C) $(\frac{1}{3}, -\frac{11}{2})$
(D) $(-\frac{1}{6}, -5)$ and $(\frac{5}{6}, -5)$
(E) There are none.

3. The eccentricity is
(A) 0
(B) 1
(C) $\frac{1}{3}$
(D) 3
(E) There is none.

4. The vertex is (or the vertices are) at
(A) $(\frac{1}{3}, -\frac{13}{2})$ and $(\frac{1}{3}, -\frac{7}{2})$
(B) $(\frac{1}{3}, -5)$
(C) $(\frac{11}{6}, -5)$ and $(-\frac{7}{6}, -5)$
(D) $(\frac{1}{3}, -\frac{7}{2})$
(E) There are none

5. The center is at
(A) $(-5, \frac{1}{3})$
(B) $(-\frac{1}{3}, 5)$
(C) $(5, -\frac{1}{3})$
(D) $(\frac{1}{3}, -5)$
(E) There is none

6. The radius is
(A) $\frac{3}{2}$
(B) $\frac{1}{2}$
(C) 1
(D) $\sqrt{10}$
(E) There is none

7. The directrix (or one of the directrices) is
(A) $y = -\frac{9}{2}$
(B) $x = \frac{5}{6}$
(C) $x = -\frac{1}{6}$
(D) $y = -\frac{11}{2}$
(E) There are none.

8. The ends of the latus rectum (or one of the latera recta) are
(A) $(-\frac{2}{3}, -7)$ and $(-\frac{2}{3}, -3)$
(B) $(\frac{4}{3}, -\frac{9}{2})$ and $(-\frac{2}{3}, -\frac{9}{2})$
(C) $(\frac{4}{3}, -\frac{11}{2})$ and $(-\frac{2}{3}, -\frac{11}{2})$
(D) $(\frac{7}{3}, -7)$ and $(\frac{7}{3}, -3)$
(E) There are none

9. The axis of parabola is
(A) the x-axis
(B) the y-axis
(C) parallel to the x-axis
(D) parallel to the y-axis
(E) There is none

10. The parabola is concave
(A) right
(B) left
(C) up
(D) down
(E) It is not a parabola.

11. The major axis is
(A) the x-axis
(B) the y-axis
(C) parallel to the x-axis
(D) parallel to the y-axis
(E) There is none

12. The minor axis is
(A) the x-axis
(B) the y-axis
(C) parallel to the x-axis
(D) parallel to the y-axis
(E) There is none

13. The transverse axis is
(A) the x-axis
(B) the y-axis
(C) parallel to the x-axis
(D) parallel to the y-axis
(E) There is none.

14. The conjugate axis is
(A) the x-axis
(B) the y-axis
(C) parallel to the x-axis
(D) parallel to the y-axis
(E) There is none

15. The asymptotes are
(A) $y = \frac{1}{3}x - \frac{46}{9}$ and $y = -\frac{1}{3}x - \frac{44}{9}$
(B) $y = 3x + 6$ and $y = -3x + 4$
(C) $y = 3x - 6$ and $y = -3x - 4$
(D) $y = \frac{1}{3}x + \frac{46}{9}$ and $y = -\frac{1}{3}x + \frac{42}{9}$
(E) There are none

II. Write the equation $7y^2 - 9x^2 - 18x - 28y - 44 = 0$ in standard form and identify the conic. Then answer the questions that follow.

16. The conic is
(A) a circle
(B) a parabola
(C) an ellipse
(D) a hyperbola
(E) degenerate

17. The focus (or one of the foci) is at
(A) (- 1, 5)
(B) (2, 2)
(C) (−1, 6)
(D) (3, 2)
(E) There are none

18. The eccentricity is
(A) 0
(B) 1
(C) $\frac{4}{3}$
(D) $\frac{3}{4}$
(E) There is none

19. The vertex (or one of the vertices) is at
(A) $(-1 + \sqrt{7}, 2)$
(B) (−1, 5)
(C) (−1, 6)
(D) (−1, 2)
(E) There is none.

20. The center is at
(A) (−2, 1)
(B) (2, −1)
(C) (1, −2)
(D) (−1, 2)
(E) There is none

21. The radius is
(A) $\sqrt{7}$
(B) 3
(C) 4
(D) $\sqrt{63}$
(E) There is none.

22. The directrix (or one of the directrices) is
(A) $x = -\frac{13}{4}$
(B) $x = -\frac{13}{9}$
(C) $y = -\frac{1}{4}$
(D) $y = \frac{14}{9}$
(E) There is none

23. The ends of the latus rectum (or one of the latera recta) are
(A) $(-\frac{10}{3}, 6)$ and $(\frac{4}{3}, 6)$
(B) $(3, \frac{13}{4})$ and $(3, \frac{1}{4})$
(C) $(-\frac{11}{4}, 6)$ and $(\frac{1}{4}, 6)$
(D) $(3, \frac{13}{3})$ and $(3, -\frac{1}{3})$
(E) There are none.

24. The axis of parabola is
(A) the x-axis
(B) the y-axis
(C) parallel to the x-axis
(D) parallel to the y-axis
(E) There is none

25. The parabola is concave
(A) right
(B) left
(C) up
(D) down
(E) It is not a parabola

26. The major axis is
(A) the x-axis
(B) the y-axis
(C) parallel to the x-axis
(D) parallel to the y-axis
(E) There is none

27. The minor axis is
(A) the x-axis
(B) the y-axis
(C) parallel to the x-axis
(D) parallel to the y-axis
(E) There is none

28. The transverse axis is
(A) the x-axis
(B) the y-axis
(C) parallel to the x-axis
(D) parallel to the y-axis
(E) There is none

29. The conjugate axis is
(A) the x-axis
(B) the y-axis
(C) parallel to the x-axis
(D) parallel to the y-axis
(E) There is none

30. The asymptotes are
(A) $3y - \sqrt{7}x + (\sqrt{7} + 6) = 0$ and $3y + \sqrt{7}x - (\sqrt{7} - 6) = 0$
(B) $3y - \sqrt{7}x - (\sqrt{7} + 6) = 0$ and $3y + \sqrt{7}x + (\sqrt{7} - 6) = 0$
(C) $\sqrt{7}y - 3x - (3 + 2\sqrt{7}) = 0$ and $\sqrt{7}y + 3x + (3 - 2\sqrt{7}) = 0$
(D) $\sqrt{7}y - 3x + (3 + 2\sqrt{7}) - 0$ and $\sqrt{7}y - 3x - (3 - 2\sqrt{7}) = 0$
(E) There are none

III. Find the equation of the parabola whose directrix has the equation $y = 3$ and whose focus is the point $(0, -5)$. Put the answer in the form $Ax^2 + Cy^2 + Dx + Ey + F = 0$ and answer questions 31 and 32 below.

31. In the equation, the sum of the coefficients $A + D$ is
(A) -16
(B) -8
(C) 1
(D) 3
(E) 16

32. In the equation, $C + E + F$ is
(A) -32
(B) -15
(C) 0
(D) 16
(E) 32

IV. Find the equation of the ellipse with vertices at $(2, -3)$ and $(2, 5)$ and one focus at $(2, 4)$. Put your answer in the form $Ax^2 + Cy^2 + Dx + Ey + F = 0$ and answer questions 33 and 34 below.

33. In the equation, the sum of the coefficients $A + D$ is
(A) -48
(B) -21
(C) 21
(D) 48
(E) 80

34. In the equation, $C + E + F$ is
(A) -176
(B) -84
(C) -80
(D) -48
(E) -20

V. Find the equation of the hyperbola whose vertices are $(0, 4)$ and $(0, -4)$ and whose eccentricity is $\frac{3}{2}$. Put your answer in the form $Ax^2 + Cy^2 + Dx + Ey + F = 0$ and answer questions 35 and 36 below.

35. In the equation, the sum of the coefficients $A + D$ is
(A) -16
(B) -4
(C) 4
(D) 16
(E) 20

36. In the equation, $C + E + F$ is
(A) -340
(B) -336
(C) -300
(D) -60
(E) -15

VI. Find the equation of the ellipse with vertices $(7, -3)$ and $(7, 7)$ which passes through the point $(8\frac{3}{5}, 5)$. Put your answer in the form $Ax^2 + Cy^2 + Dx + Ey + F = 0$ and answer questions 37 and 38 below.

37. In the equation, the sum of the coefficients $A + D$ is
(A) -52
(B) -325
(C) -832
(D) $-8,125$
(E) $-24,375$

38. In the equation, $C + E + F$ is
(A) 121
(B) 1,129
(C) 1,229
(D) 2,161
(E) 49,675

Score your post-test according to the answers below and total the points achieved.

Question Number	Correct Answer		Points
I. (25 points)			
		$9x^2 - 6x + 18y + 91 = 0$ Standard form: $(x - \frac{1}{3})^2 = -2(y + 5)$	
1	B	The conic is a parabola.	3
2	C	The focus is at $(\frac{1}{3}, -\frac{11}{2})$.	3
3	B	The eccentricity is 1.	3
4	B	The vertex is at $(\frac{1}{3}, -5)$.	3
5	E	There is no center.	3
6	E	There is no radius.	1
7	A	The directrix is $y = -\frac{9}{2}$.	1
8	C	The ends of the latus rectum are $(\frac{4}{3}, -\frac{11}{2})$ and $(-\frac{2}{3}, -\frac{11}{2})$.	1
9	D	The axis of parabola is parallel to the y-axis.	1
10	D	The parabola is concave down.	1
11	E	There is no major axis.	1
12	E	There is no minor axis.	1
13	E	There is no transverse axis.	1
14	E	There is no conjugate axis.	1
15	E	There are no asymptotes.	1

Question Number	Correct Answer		Points
II. (25 points)			
		$7y^2 - 9x^2 - 18x - 28y - 44 = 0$	
		Standard form: $\frac{(y-2)^2}{9} - \frac{(x+1)^2}{7} = 1$	
16	D	The conic is a hyperbola.	3
17	C	One of the foci is at $(-1, 6)$.	3
18	C	The eccentricity is $\frac{4}{3}$.	3
19	B	One of the vertices is at $(-1, 5)$.	3
20	D	The center is at $(-1, 2)$.	3
21	E	There is no radius.	1
22	C	One of the directrices is $y = -\frac{1}{4}$.	1
23	C	The ends of one of the latera recta are $(-\frac{11}{4}, 6)$ and $(\frac{3}{4}, 6)$.	1
24	E	There is no axis of parabola.	1
25	E	It is not a parabola.	1
26	E	There is no major axis.	1
27	E	There is no minor axis.	1
28	D	The transverse axis is parallel to the y-axis.	1
29	C	The conjugate axis is parallel to the x-axis.	1
30	C	The asymptotes are $\sqrt{7}y - 3x - (3 + 2\sqrt{7}) = 0$ and $\sqrt{7}y + 3x + (3 - 2\sqrt{7}) = 0$.	1
II. (12½ points)			
		Parabola with directrix $y = 3$ and focus at $(0, -5)$.	
		Standard form: $x^2 = -16(y + 1)$.	
		Form of $Ax^2 + Cy^2 + Dx + Ey + F = 0$: $x^2 + 16y + 16 = 0$.	
31	C	$1 = A + D$	
32	E	$32 = C + E + F$	
IV. (12½ points)			
		Ellipse with vertices at $(2, -3)$ and $(2, 5)$ and one focus at $(2, 4)$.	
		Standard form: $\frac{(y-1)^2}{16} + \frac{(x-2)^2}{7} = 1$	
		Form of $Ax^2 + Cy^2 + Dx + Ey + F = 0$: $16x^2 + 7y^2 - 64x - 14y - 41 = 0$.	
33	A	$-48 = A + D$	
34	D	$-43 = C + E + F$	
V. (12½ points)			
		Hyperbola with vertices at $(0, 4)$ and $(0, -4)$ and eccentricity $\frac{3}{2}$.	
		Standard form: $\frac{y^2}{16} - \frac{x^2}{20} = 1$	
		Form of $Ax^2 + Cy^2 + Dx + Ey + F = 0$: $-16x^2 + 20y^2 - 320 = 0$	
35	A	$-16 = A + D$	
36	C	$-300 = C + E + F$	

Question Number	Correct Answer	
VI. ($12\frac{1}{2}$ points)		
		Ellipse with vertices at $(7, -3)$ and $(7, 7)$ which passes through the point $(8\frac{2}{5}, 5)$.
		Standard form: $\frac{(y-2)^2}{25} + \frac{(x-7)^2}{4} = 1$
		Form of $Ax^2 + Cy^2 + Dx + Ey + F = 0$: $25x^2 + 4y^2 - 350x - 16y + 1141 = 0$
37	B	$-325 = A + D$
38	B	$1{,}129 = C + E + F$

If you achieved 90 or more points, you are ready for the final examination. If not, you should repeat Unit II.

Completing the Course

If you've mastered the text and met the lesson objectives of the course--either by scoring 90% on pre-tests or by studying the lessons' content--you are ready for the final examination, a multiple-choice objective examination. You'll need paper and pencil. You may take the test during a scheduled conference period or by appointment.

Grading

When you've completed the end-of-course examination, you may close off the course with a grade of "B". If you wish to improve your grade to an "A", you may act as a coach or undertake optional projects from the "Smorgasbord". This may be done during the following term and your "B" will be changed to an "A".

Oleanna Math Program

THE CALCULUS OF DIFFERENTIALS AND INTEGRALS.
Course outlines for a series of three courses
by Walter A. Coole, Skagit Valley College

Skagit Valley College Course Numbers:
Part I. Techniques: Mathematics 121
Part II. Applications: Mathematics 122
Part III. Theory: Mathematics 123

Quarter credits: 6, 5, 5 Semester credits: 4, 4, 3

Goals:

This series, taken together with Analystic Geometry (Oleanna Math Program) deals effectively with the content of a standard "calculus with analytic geometry" sequence, presented conventionally. However, the order of presentation within the whole sequence has been totally changed for sound pedagogical reasons.

For this reason, it is almost impossible for a student ot successfully negotiate a change from conventional to "systems" courses in the middle of the sequence.

Performance objectives.

At the end of Part I, the student is expected apply formulas of the combined calculus to treat with the following kinds of mathematical problems:

1. locate functional limits
2. determine differentials of various functions
3. inferring antiderivatives
4. computing integrals

Upon completing Part II, (s)he should be capable of applying acquired mathematical skills to appropriate situations in physics and economics.

In the terminal portion, Part III, the student learns about the theoretical structure of the calculus as a deductive system; to demonstrate his competence, he reconstructs significant portions from memory.

Entry

These three courses must be taken in order.

The student entering the sequence must have mastered thouroughly: (i) functions and relations (ii) periodic functions and (iii) analytic geometry. The Oleanna Program provides these three subjects as courses of study, requiring a high degree of mastery of the subject matter. alternate evidence of entry-competence can be taken from the Cooperative Math Program tests:

Algebra II
Trigonometry
Analytic Geometry

in each case, scoring in the 90th percentile range or higher.

Some familiarity with the slide rule is useful, but not essential.

In addition he/she should be able to:

a. read and follow difficult instructional material
b. state educational goals succinctly and relate mathematical skills to them
c. study systematically and diligently
d. maintain a high degree of effort in his/her work

Student materials.

Paper and pencil
Slide rule (optional)
Pocket calculator (optional)

Merriell, David M.: Calculus: A Programmed Text. (Vols. I & II) Menlo Park, CA. W. A. Benjamin, Inc.

Burlington, R. S. Handbook of Mathematical Tables and Formulas. New York. McGraw-Hill, 1973

Coole: Syllabus for Calculus

Teacher preparation

Study instructor's manual, testing materials, texts.

Other materials required

Cooperative Testing Service: Cooperative Math Test--Calculus. Forms A & B. Also, user's manual. Palo Alto, CA. Cooperative Testing Service. 1969

Oleanna Math Program: Smorgasbord.

Teacher's manual for text.

Olcanna Math Program

Syllabus for THE CALCULUS OF DIFFERENTIALS AND INTEGRALS. By Walter A. Coole, Skagit Valley College.

This syllabus contains complete instructions to accompany the two-volume set of programmed texts, Merriell: *Calculus, A Programmed Text*.

The two-volume set and syllabus, together, provide you with basic instructions for three courses--or "parts"--which, taken in a series, treat the standard content of elementary calculus.

You should note that this sequencetakes up the subject of calculus in quite a different order from that used in "conventional" classroom courses. *Therefor, you will not be able to shift into or out of this sequence in the middle.* You must do it all one way or the other.

PART I: TECHNIQUES

Your *goal* in this course will be to apply formulae of the combined calculus to treat with the following kind of mathematical problems:

i. locate functional limits
ii. determine differentials of various functions
iii. inferring antiderivatives
iv. computing integrals

Approximately 200 hours of work is required to complete the five units of this course. Please set target dates for yourself so that you complete Unit V several days before the end of the term.

I. ____________________

II. ____________________

III. ____________________

IV. ____________________

V. ____________________

Now, read "Directions to the Student," Volume I, pp. vii-viii.

How to Study Each Unit

1. Read the objectives given for the unit in the syllabus.

2. Work your way through the assigned chapter, following "Directions to the Student."

3. Read the summary at the end of the chapter.

4. Work the odd-numbered review exercises at the end of the chapter, checking your results in the back of the book.

5. Review the objectives for the unit, checking off each objective you are sure of.

6. Report each unit's completion during your next scheduled conference.

Your Final Exam

The end-of-course test will consist of problems drawn from the even-numbered exercises in the course. This is not a timed test--you may take as long as you need.

You will need paper, pencil, and the *Handbook of Mathematical Tables and Formulas*. No notes or books may be used.

If you do not pass the test on the first try, you may re-take it later, using another form--after you have studied to correct your weak areas.

A-project (Optional)

Read the text and work the problems in each chapter indicated as you progress through the basic course unit-by-unit. Submit all written work before the end of the course (examination). The text is:

> Ayers, Frank: *Schaum's Outline of Theory and Problems of Differential and Integral Calculus*. New York. Mc-Graw-Hill Book Company. 1964.

UNIT	Chapters
I	-----
II	1, 50
III	2, 3
IV	4, 5, 6, 12, 13, 14, 15, 22, 29, 30, 53, 56
V	25, 26, 27, 28, 29, 30, 31, 32, 33, 55

UNIT OBJECTIVES

I

( )discuss the nature of calculus problems orally

( )interpret:

$$\lim_{x \to a} f(x)$$

( ) compute the slope of a line
( ) decide when a function is differentable
( ) discuss the process of repeated estimation to find an area
( ) explain the idea of computing distances from velocities
( ) relate functions to operations performed to gain them
( ) recite from memory, the Fundamental Theorem of Calculus and apply it in simple cases

ASSIGNMENT: Chapter I

UNIT OBJECTIVES

II

( ) use with fluency, the notation of sets and intervals
( ) recall important definitions and principles of inequalities and absolute values
( ) use with fluency, the language of summation ( ) and the principle of mathematical induction
( ) form such functionally-relate sets as domain and range; decide when functions are one-to-one
( ) tell whtt the term 'f^{-1}' means
( ) represent functions geometrically
( ) sketch curves
( ) identffy and describe conic sections
( ) perform fundamental operations and functions
( ) execute sophisticated computations involving periodic functions
( ) graph and interpete logarithmic and exponential functions

ASSIGNMENT: Chapter II

UNIT OBJECTIVES

III

( ) define 'limit'
( ) recall and apply the Basic Limit Theorem
( ) decide when a function is continuous
( ) compute limits of functions

ASSIGNMENT: Chapter III

UNIT OBJECTIVES

IV

( ) define precisely: derivative, right-hand derivative, differentiable functions
( ) differentate function sums, differences, and products
( ) relate continuity and differentiability
( ) differentiate transcendental functions
( ) differentiatie quotients
( ) differentiate composite functions
( ) recall and apply the Chain Rule
( ) differentiate certain functions through implication
( ) relate the derivative of an inverse function to that of the original function
( ) recall and apply L' Hopital's Rules

( ) construct Taylor's series for functions
( ) differentiatie functions with more than one independent variable

ASSIGNMENT: Chapter IV

UNIT OBJECTIVES

V

( ) define 'antiderivative' and 'integration'
( ) use with fluency, terminology and notation of antiderivatives
( ) perform simple integrations
( ) integrate by parts
( ) produce trigonometric integrals
( ) evaluate definite integrals
( ) perform numerical integration
( ) apply the nethod of integration by partial fractions
( ) use a table of integrals
( ) compute improper integrals
( ) find all functions satisfying a given differential equation

ASSIGNMENT: Chapter V

PART II: APPLICATIONS

Your *goal* in this course will be to apply acquired mathematical skills to appropriate situations in physics and the social sciences, as well as abstract geometry.

Approximately 165 hours of work is required to complete the three units of this course. Please set target dates for yourself so that you complete Unit VIII before the end of the term.

VI. ______________________

VII. ______________________

VIII. ______________________

Then, re-read "How to Study Each Unit" and "Your Final Exam" on page 2 of this syllabus.

A-project (Optional)

As in Part I, *one* way to make a grade of "A" is to do extra work in *Schaum's Outline* parallel to your basic course work.

UNIT	Chapters
VI	7, 9, 17, 34, 16, 20, 35, 36, 42
VII	10, 11, 40, 37, 38, 60, 61, 62
VIII	8, 21, 69, 70

UNIT OBJECTIVES

VI

to be able to...

- ( ) compute the tangent of a curved figure as a function of its independent variable
- ( ) use differentials to approximate 2 functions dependent variable
- ( ) locate and compute a functions minimal and maximal values
- ( ) apply calculus techniques to obtain information about a functions graphic shape
- ( ) compute areas bounded by curves, using definite integrals
- ( ) calculate curve-lengths, using parametric equations
- ( ) relate rectangular and polar coordinates and find both curve-length and bounded areas defined by polar coordinates
- ( ) sketch polar-coordinate curves
- ( ) compute volumes of solids generated when a curve is rotated around the x-axis
- ( ) compute areas of surfaces generated in the same way

UNIT OBJECTIVES

VII

- ( ) compute quantities related by an equation which determines between (hormonic) rate of change
- ( ) associate number with work done when bodies are displaced by forces acting along line of displacement
- ( ) calculate moment and center of gravity
- ( ) define vectors and perform combinitory operations on vectors
- ( ) relate derivative to volcities and accelerations of porticles moving in planar space

UNIT OBJECTIVES

VIII

- ( ) solve a variety of practical problems using maxima and minima
- ( ) determine a functions mean value
- ( ) make mathematical models of physical and social problems
- ( ) discuss functions with more than two variables

PART III: Theory

Your <u>goal</u> in the course will be to learn about the theoretical structure of the calculus as a deductive system.

approximately 165 hours work is required for this course's six units. Please set target dates so that you complete Unit XIV a few days before the ind of the term.

IX. ______________________

X. ______________________

XI. ______________________

XII. ______________________

XIII. ______________________

XIV. ______________________

A-project (Optional)

In Part III, you may earn an "A" by completing the following assignments parallel to your basic course work. Your text will be: Granville, Smith, and Longley: *Elements of the Differential and Integral Calculus*. Waltham, Mass. Blaisdell Publishing Co. 1962.

Submit all exercises from the assigned chapters.

UNIT	Chapter
IX	II
X	III, IV
XI	------
XII	XIV
XIII	VII
XIV	XV

UNIT OBJECTIVES

IX

( ) discuss theoretical mathematics as a system of deductions
( ) prove theorems about sets of real numbers
( ) define 'limit and apply this definition to inferences about neighborhodds
( ) prove limit theorems
( ) show that the inverse of a monotone function is continuous

UNIT OBJECTIVES

X

( ) relate, theoretically, differentiablilty and continuity
( ) derive rules about operations on functions
( ) prove the above rule
( ) prove the mean value theorem
( ) deduce various theorems about second derivatives

UNIT OBJECTIVES

XI

( ) recall the definition of a set-maximum and apply it
( ) define upper and lower bounds
( ) prove the intermediate value theorem
( ) show that continuous functions are bounded and certain related theorems

UNIT OBJECTIVES

XII

( ) relate lower sums to uppersums deductively
( ) prove theorems concerning integrability
( ) show mathematical existence of integ als
( ) demonstrate properties of definite integrals
( ) deduce the Fundamental Theorem of Calculus
( ) relate limits to definite integrals theoretically

UNIT OBJECTIVES

XIII

( ) define natural logarithms
( ) define the exponential function and deduce its properties
( ) prove basic theorems aoout der vatives and integrals of trigonimetric functions

UNIT OBJECTIVES

XIV

( ) explain convergence and divergence in mathematical ways
( ) prove theorems about convergent series
( ) apply techniques for determining convergence or divergence in most infinite series
( ) decide about differentiability of power series
( ) prove Taylor's Theorem

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Oleanna Math Program

FINAL EXAMINATION: Techniques of Calculus*

This is NOT a timed test. You may take as much time as you need. Do your work in pencil. Show your answers and work on a separate piece of paper.

DO NOT MARK THE TEST!

You should use Burlington's *Handbook of Mathematical Formulas and Tables*, but may not use your text or notes.

---

1. Let f be a function such that $f(x) = \dfrac{|x|}{x}$.

   a) Find $f(2)$, $f(-2)$, $f(1/100)$, $f(-1/100)$, $f(0)$.

   b. Does $\lim_{x\to 0} f(x)$ exist? Give the value of the limit or a reason for nonexistence.

2. Let h be the function defined by $h(t) = \dfrac{3}{t+1}$. Find $\lim_{t\to\infty} h(t)$, $\lim_{t\to-\infty} h(t)$, $\lim_{t\to 0} h(t)$, and $\lim_{t\to -1} h(t)$ if they exist.

3. The position function for a point moving along the x axis is $x(t) = \pi\, t^2$. Find the velocity when $t = 3$.

4. Find $f'(0)$ if $f(x) = \begin{cases} x \text{ if } x<0 \\ x \text{ if } x\geq 0 \end{cases}$ and sketch the graph of f.

5. Given that the derivative of $\tan x$ is $\sec^2 x$, find

$$\int_0^{\pi/2} \sec^2 t \; dt.$$

6. a) $e^{2\ln x}$ = ______________ b) $\ln e$= ____________

   c) $\ln 1$ = ____________ d) $\ln e^{\sin x}$ = ____________

   e) $\log_2 (1/16)$ = ____________ f) $\ln \sqrt[3]{e}$ = ____________

   g) $\ln 1/\sqrt{e}$ = ____________

7. Find $\ln x$ if:

   a) $x = 1/e$ b) $x = e$

   c) $x^2 = e^3$ and $x > 0$ d) $1/x = e^3$

*Adapted from Merriel, David: *Calculus--A Programmed Text, Vol. I.* Menlo Park, CA. W. A. Benjamin, Inc. 1974 by permission of the publisher.

8. Let $f(x) = 1/x^3$. Find

$\lim_{x\to 0^+} f(x)$, $\lim_{x\to 0^-} f(x)$, $\lim_{x\to\infty} f(x)$ and $\lim_{x\to -\infty} f(x)$.

9. For the function f such that $f(x) = \sqrt{(x-1)(x-2)}$, determine the values of x at which f is (a) continuous on the right but not on the left; (b) continuous on the left but not on the right.

10. Let $f(x) = \sec^2 x$ and $g(x) = \tan^2 x$. Find the limit as $x \to \pi/2$ of f, g, $f + g$, $f - g$, fg and f/g.

11. Find the derivative of: $\dfrac{1}{1 - 2\cos x}$

12. Find the derivative of: $\ln\left(\dfrac{e^x + \sqrt{e^{2x} - 4}}{2}\right)$

13. Find $Df^{-1}(x)$ if $f(x) = \ln|x|$, $x<0$.

14. Evaluate: $\lim_{x\to\pi/4} (1 - \tan x)\sec 2x$

15. Integrate: $\displaystyle\int \frac{dx}{1 + e^x}$

16. Integrate: $\int x \sec^2 2x\, dx$

17. Integrate: $\int \tan^4 x\, dx$

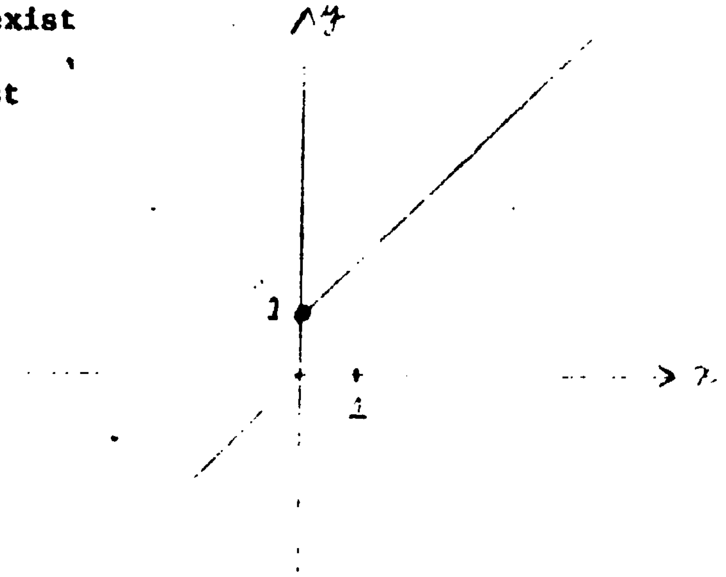
18. Integrate: $\displaystyle\int \frac{x^2\, dx}{\sqrt{8 + 2x - x^2}}$

19. Integrate: $\displaystyle\int \frac{\sec^2\theta\, d\theta}{2 + 5\tan^2\theta}$

20. Solve: $\cosh x\, dy + (y \sinh x + e^x)\, dx = 0$

ANSWERS TO FINAL EXAMINATION: Techniques of Calculus*

1. a) 1, -1, 1, -1, does not exist
   b) No. For positive x near 0, $f(x) = 1$; but for negative x near 0, $f(x) = -1$.
2. 0, 0, 3, does not exist
3. 6π
4. $f'(0)$ does not exist



5. 1

6. a) x^2 b) 1 c) 0 d) $\sin x$ e) -4 f) 1/3 g) 1/2

7. a) -1 b) 1/2 c) 3/2 d) -3

8. ∞, $-\infty$, 0, 0
9. a) $x = 2$ b) $x = 1$
10. $\lim f(x) = \infty$, $\lim g(x) = \infty$, $\lim [f+g](x) = \infty$, $\lim [f-g](x) = 1$, $\lim [f/g](x) = 1$, $\lim [fg](x) = \infty$.
11. $\dfrac{-2 \sin x}{(1 - 2 \cos x)^2}$

12. $\dfrac{e^x}{\sqrt{e^{2x} - 4}}$

13. $- e^x$

14. 1

15. $\ln \left| \dfrac{e^x}{1 + e^x} \right| + C$

*Adapted from Merriel, David: *Solutions Manual for Calculus: A Programmed Text*. Menlo Park, CA. W. A. Benjamin, Inc. 1974 by permission of the publisher.

16. $1/2\ x \tan 2x - 1/4 \ln|\sec 2x| + C$

17. $1/3 \tan^3 x - \tan x + x + C$

18. $11/2 \arcsin \frac{x - 1}{3} - 1/2\ (x + 3) \quad 8 + 2x - x^2 + C$

19. $\frac{1}{\sqrt{10}} \arctan\left(\sqrt{5/2} \tan \theta\right) + C$

20. $y \cosh x + e^x = C$

Oleanna Math Program

FINAL EXAMINATION: Applications of Calculus*

This is NOT a timed test. You may take as much time as you need. Do your work in pencil. Show your answers and work on a separate piece of paper.

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1. Find the differential dy: $y = \arctan x$

2. Find the extrema and tell whether maximum or minimum:

   $$f(x) = x - 2 \sin x \text{ on } [0, \pi]$$

3. Find the area of the region bounded by the given curve:

   $$y = \tan x,\ x = \pi/3,\ x\text{-axis}$$

4. Find an equation of a tangent to the curve $x = \ln(t - 2)$, $y = t/3$ where $t = 3$.

5. Find the length of the curve given in polar coordinates:
   $r = 3(1 + \cos\theta)$ for $\theta \in [0, 2\pi]$

6. Find the volume of the solid generated by revolving the region bounded by $y = \sqrt{x}$ and $y = x^3$ about the y axis.

7. Find the area of the surface of revolution generated by revolving

   $y = 3 \cosh x/3$, $x = -3$, $x = 3$, and $y = 0$ about the x axis.

8. The position of a particle moving on a line is $s(t) = 3t + \cos 3t$, where $0 < t \leq \pi/3$.
   a) When is the velocity 0? b) When is the acceleration 0?
   c) When does the velocity reach its maximum value?
   d) When does the acceleration reach its maximum value?

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*Adapted from Merriel, David: *Calculus--A Programmed Text, Vol. I.* Menlo Park, CA. W. A. Benjamin, Inc 1974 by permission of the publisher.

9. A bag of sand initially weighing 100 lb is lifted at a uniform rate. The sand leaks out of the bag at a rate of 1/2 lb/ft. Find the work done in lifting the bag 30 ft.

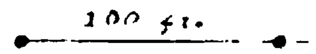
10. Find the centroid of the region bounded by $y = 6x - x^2$ and $y = x$.

11. If u and v are any vectors, show that $(u + v)\cdot(u + v) = |u|^2 + |v|^2 + 2u\cdot v$.

12. The position of a particle is $F(t) = e^{2t}i + e^{-t}j$. Find the unit tangent vector, the unit normal vector, and the curvature, at the point at which $t = 0$.

13. Let u, v, w be three vectors having different directions and containing arrows (O, A), (O, B), and (O, C), respectively. Let $u + 3v - 4w = 0$. Show that A, B, and C lie on a straight line.

14. A farmer has a fence 100 ft long along one side of his property. He wishes to make a rectangular enclosure by using 200 ft more fencing, using the original fence as part of the boundary, as shown below. What should the dimensions be in order to enclose the greatest possible area?



15. For the function $f(x) = \dfrac{x}{x^2 + x - 2}$, find $M_{\frac{1}{2}}(f)$.

16. Bacteria increase at a rate proportional to the number present. The original number doubles in two hours. In how many hours will it be 20 times as great?

17. The demand function for a commodity is $p = 8000 - 100x$, where p is the price and x is the number of units demanded. The total cost function is $C = 1500 + 400x + 100x^2$. How many units should a monopolist produce for maximum profit?

18. Show that the function $f(x,y) = xe^{x/y}$ is homogenous of degree 1, and verify that it satisfies Euler's theorem.

19. Find the values of x and y for which $f(x,y) = x^2 + y^2 - 2xy + 2x - 3y + 4$ has a relative maximum or minimum.

20. The production function for z units of a commodity is $z = 100\sqrt{xy}$, where x units of production factor A and y units of production factor B are used. If the respective prices of A and B are \$10 and \$25 per unit, how many units of each factor should be used to produce 200 units of the commodity at minimum cost? Use the method of Lagrange and express solutions to the nearest tenth.

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ANSWERS TO FINAL EXAMINATION: Applications of Calculus*

1. $\dfrac{dx}{1 + x^2}$

2. maximum of $e^{\pi/4}$ at $x = \pi$, mimimum of $\pi/3 - \sqrt{3}$ at $x = \pi/3$

3. ln 2

4. $y - 1 = 1/3\ x$

5. 24

6. $2\pi/5$

7. $9\pi(2 + \sinh 2)$

8. a) $t = 0, \pi/3$ b) $t = \pi/6$ c) $t = 0, \pi/3$ d) $t = \pi/3$

9. 2775 foot-pounds

10. (5/2, 5)

11. Let $\vec{u} = [u_1, u_2]$ and $\vec{v} = [v_1, v_2]$. Then $(\vec{u} + \vec{v}) \cdot (\vec{u} + \vec{v}) =$

$[u_1 + v_1, u_2 + v_2] \cdot [u_1 + v_1, u_2 + v_2] =$

$(u_1 + v_1)^2 + (u_2 + v_2) =$

$(u_1^2 + u_2^2) + (v_1^2 + v_2^2) + 2(u_1v_1 + u_2v_2) =$

$|\vec{u}|^2 + |\vec{v}|^2 + 2\vec{u}\cdot\vec{v}$.

12. $\dfrac{2\vec{i} - \vec{j}}{\sqrt{5}}$, $\dfrac{\vec{i} + 2\vec{j}}{\sqrt{5}}$, $\dfrac{6\sqrt{5}}{25}$

13. The line segment (A, B) belongs to the vector $\vec{v} - \vec{u}$ and the line segment (A, C) to the vector $\vec{w} - \vec{u}$. Now $4(\vec{w} - \vec{u}) = 4\vec{w} - 4\vec{u}$

$= (\vec{u} + 3\vec{v}) - 4\vec{u}$ (since $\vec{u} + 3\vec{v} - 4\vec{w} = 0$)

$= 3(\vec{v} - \vec{u})$.

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So the lines containing (A, B) and (A, C) have the same direction. Therefore they coincide since A is a common point.

*Adapted from Merriel, David: *Solutions Manual for Calculus: A Programmed Text*. Menlo Park CA. W. A. Benjamin, Inc. 1974 by permission of the publisher.

14. 100 feet by 50 feet

15. 1/6 ln 27/4

16. $\frac{2 \ln 20}{\ln 2} \doteq 8.6$ hours

17. 19

18. $f(tx, ty) = (tx)e^{(tx)/(ty)} = txe^{x/y} = t\, f(x, y)$;

$x f_x + y f_y = x[x(1/y)e^{x/y} + e^{x/y}] + y[x(-x/y^2)e^{x/y}]$

$= x\, e^{x/y} = f(x, y)$

19. relative minimum when x = 1, y = 5/2

20. Minimize C(x, y) = 10x + 25y subject to the constraint xy = 4.

The solutions are $x \doteq 3.2$, $y \doteq 1.3$.

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FINAL EXAMINATION: Theory of Calculus*

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1. Find positive numbers δ such that, for all x satisfying $|x - 1| < \delta$, it is true that

   a) $|x^2 - 1| < 1$
   b) $|x^2 - 1| < 0.01$
   c) $|x^2 - 1| < 0.0001$

2. Let f be the function with domain [0, 1] such that...

   $$f(x) = \begin{cases} 0 & \text{if } x \text{ is irrational} \\ 1/q & \text{if } x \text{ is rational and } x = p/q \text{ in lowest terms} \end{cases}$$

   where p and q are positive integers.

   a) Show that if a is rational and $a \in (0, 1)$, then f is discontinuous at a.

   b) Show that if a is irrational and $a \in (0, 1)$, then f is continuous at a.

3. Let f be a function with domain D. Suppose that there is a positive constant k such that for all s and t in D,

   $|f(s) - f(t)| \leq k|s - t|$.

   Prove that f is continuous on D.

4.a) Show that the function $f(x) = x - \tan x$ has only negative values in $(0, \pi/2)$.

b) Show that $(\sin x)/x$ has values between $2/\pi$ and 1 in the interval $(0, \pi/2)$.

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*Adapted from Merriel, David: *Calculus--A Programmed Text, Vol. II*. Menlo Park, CA. W. A. Benjamin, Inc. 1974 by permission of the publisher.

5. Let f and g be functions that are differentialbe on $[a, b]$ and such that $fg' - f'g$ never has the value 0 on $[a, b]$. Suppose that $f(a) = f(b) = 0$. Show that there must be a value c in (a, b) for which $g(c) = 0$.

6. Show that $\sqrt{x} > \ln x$ for all $x > 0$.

7. Show that if glb $S \in S$, then glb S = min S.

8. Let f be continuous on $[a, b]$ and let k be a real number such that $0<k<1$. Show that there is a real number $c \in [a, b]$ such that $kf(a) + (1 - k)f(b) = f(c)$.

9. Let k be a positive real number and n be a positive integer. Show that there exists one and only one positive real number satisfying the equation $x^n = k$.

10. Show that $0 \le \int_0^\pi \sin(x^2)\,dx \le \pi$.

11. Let f and g be continuous on $[a, b]$ and let g be non-negative. Prove that there is a number $c \in [a, b]$ such that..

$$\int_a^b (fg) = f(c)\int_a^b g.$$

12. Let f be integrable over $[a, b]$ and let $G(x) = \int_a^x f(t)\,dt$ for each $x \in [a, b]$. Show that G is continuous on $[a, b]$.

13. Express without using ln or e (or exp):

a) $\ln(e^{2x})$ b) $e^{\ln x^2}$ c) $e^{-\ln x}$

14. The function $f(x) = e^{-1/x^2}$ has a removable discontinuity at $x = 0$. Show that the function

$$g(x) = \begin{cases} e^{-1/x^2} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$$

is differentiable at $x = 0$.

15. Given the identity $\sin 2x = 2 \sin x \cos x$, differentiate both sides to find an identity for $\cos 2x$. Do the same for the identity $\sin(x + \alpha) = \sin x \cos \alpha + \cos x \sin \alpha$, where α is constant.

16. Show that $\tan x > x$ for $x \in (0, \pi/2)$.

17. Show that $\frac{n+1}{2n-1}$ converges to 1/2.

18. Test for convergence and determine whether it converges conditionally or absolutely...

$$\sum_{j=1}^{\infty} \frac{1}{3^j - 2}$$

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19. Find the interval of convergence for this power series.

$$\sum_{j=0}^{\infty} j!(x - 2)$$

20. Show that the power series

$$\sum_{j=0}^{\infty} a_j x^j \qquad \text{and} \qquad \sum_{j=0}^{\infty} (j + 1)(j + 2)\, a_{j+2} x^j$$

have the same radius of convergence.

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ANSWERS TO FINAL EXAMINATION: Theory of Calculus

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1. a) The largest δ is $\sqrt{2} - 1$.

   b) The largest δ is $\sqrt{1.01} - 1$.

   c) The largest δ is $\sqrt{1.0001} - 1$.

2. a) Let $a = p/q$ in lowest terms so that $f(a) = \frac{1}{q}$. Any neighborhood of a will contain irrational numbers and $f(x) = 0$ for those numbers. Hence, if we choose $\epsilon = 1/q$, then no matter how δ is chosen, there are values $x \in N_\delta(a)$ such that $f(x) \notin N_\epsilon(\frac{1}{q})$. Therefore f is discontinuous at a since it has no limit there.

   b) If a is irrational, $f(a) = 0$. Let ϵ be any positive number. If $\epsilon \geq 1$, any choice of δ will satisfy the definition for $\lim_{x \to a} f(x) = 0$. If $\epsilon < 1$, let $1/k$ be the largest rational of that form less than ϵ. In $(0, 1)$, there is a finite number of rationals p/q such that $q \leq k$. Therefore, we can choose a neighborhood $N_\delta(a)$ that excludes all these rationals. Hence any rational x in $N_\delta(a)$ will have a denominator greater than k and so $f(x) < \epsilon$. Therefore $\lim_{x \to a} f(x) = 0$.

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3. Let $a \in D$ and choose any $\epsilon > 0$. Let $\delta = \epsilon/k$. Then for any $s \in N_\delta(a)$, $|f(s) - f(a)| \le k|s - a| < k(\epsilon/k) = \epsilon$, so f is continuous at a.

4. a) $f'(x) = 1 - \sec^2 x$, which is negative in $(0, \pi/2)$. Hence f is decreasing. Since $f(0) = 0$, there can only be negative values for $f(x)$ in $(0, \pi/2)$.

   b) $f'(x) = \dfrac{x \cos x - \sin x}{x^2} = \dfrac{x - \tan x}{x^2 \cos x}$, which is negative in $(0, \pi/2)$ as a result of part (a). Hence f is decreasing. Since $\lim\limits_{x \to 0} \dfrac{\sin x}{x} = 1$ and $f(\pi/2) = 2/\pi$, the values of f are between 1 and $2/\pi$.

5. Suppose there is no such value c. Since $f(a)g'(a) - f'(a)g(a) = -f'(a)g(a) \neq 0$, $g(a)$ is not 0. Similarly, $g(b) \neq 0$. ~~As a result of this~~ and the continuity of f and g resulting from their differentiability, $h(x) = \dfrac{f(x)}{g(x)}$ is continuous on $[a, b]$ and differentiable on (a, b). Since $h(a) = h(b) = 0$, $h'(c) = 0$ for some $c \in (a, b)$. But $h'(c) = \dfrac{f'(c)g(c) - f(c)g'(c)}{[g(c)]^2}$ so $f'(c)g(c) - f(c)g'(c) = 0$, contrary to the hypothesis.

6. The function f has minimum value $2 - \ln 4$ when $x = 4$. Since this is positive, all other values of f are also positive.

7. Since glb $S \le x$ for all $x \in S$ and glb $S \in S$, S has a minimum and glb S is the minimum.

8. If $f(a) = f(b)$, one may take $c = a$ or $c = b$. If $f(a) < f(b)$, then $f(b) - f(a) > 0$. Since $0 < k < 1$, $0 < 1 - k < 1$ and $0 < (1 - k)[f(b) - f(a)] < f(b) - f(a)$ so $f(a) < f(a) + (1 - k)[f(b) - f(a)] < f(b)$. By the intermediate value theorem, there is a value $c \in (a, b)$ such that $f(c) = f(a) + (1 - k)[f(b) - f(a)] = kf(a) + (1 - k)f(b)$. If $f(a) > f(b)$, then $0 < k[f(a) - f(b)] < f(a) - f(b)$. Therefore there is a value $c \in (a, b)$ such that $f(c) = f(b) + k[f(a) - f(b)] = kf(a) + (1 - k)f(b)$.

9. If $f(x) = x^n$, then $f(0) = 0$ and $f(k + 1) = (k + 1)^n > k^n \geq k$. Since $0 < k < (k + 1)^n$, by the intermediate value theorem there is a value $c \in (0, k + 1)$ such that $f(c) = k$. Also, since $f'(x) = nx^{n-1}$, $f'(x) > 0$ on $(0, k + 1)$ so f is increasing. Consequently f has value k only when $x = c$.

10. The function $\sin(x^2)$ is continuous so it is integrable. Since $|\sin(x^2)| \leq 1$, $\int_0^{\pi} \sin(x^2)dx \leq \int_0^{\pi} dx = \pi$. To show that the integral is positive, consider the partition $P = \{0, \sqrt{\pi/6}, \sqrt{\pi/3}, \sqrt{2\pi/3}, \sqrt{5\pi/6}, \sqrt{\pi}, \sqrt{7\pi/6}, \sqrt{4\pi/3}, \sqrt{5\pi/3}, \sqrt{11\pi/6}, \sqrt{2\pi}, \sqrt{3\pi}, \pi\}$. On $(0, \sqrt{\pi})$ and $(\sqrt{2\pi}, \sqrt{3\pi})$, the function $\sin(x^2)$ is positive, whereas it is negative on $(\sqrt{\pi}, \sqrt{2\pi})$ and $(\sqrt{3\pi}, \pi]$. The lower sum for P is

$$s(P) = \sqrt{\pi}\Big[\tfrac{1}{2}\Big(\sqrt{\tfrac{1}{3}} - \sqrt{\tfrac{1}{6}}\Big) + \tfrac{1}{2}\sqrt{3}\Big(\sqrt{\tfrac{2}{3}} - \sqrt{\tfrac{1}{3}}\Big) + \tfrac{1}{2}\Big(\sqrt{\tfrac{5}{6}} - \sqrt{\tfrac{2}{3}}\Big)$$
$$- \tfrac{1}{2}\Big(\sqrt{\tfrac{7}{6}} - 1\Big) - \tfrac{1}{2}\sqrt{3}\Big(\sqrt{\tfrac{4}{3}} - \sqrt{\tfrac{7}{6}}\Big) - \Big(\sqrt{\tfrac{5}{3}} - \sqrt{\tfrac{4}{3}}\Big)$$
$$- \tfrac{1}{2}\sqrt{3}\Big(\sqrt{\tfrac{11}{6}} - \sqrt{\tfrac{5}{3}}\Big) - \tfrac{1}{2}\Big(\sqrt{2} - \sqrt{\tfrac{11}{6}}\Big) - \sin \pi^2(\sqrt{\pi} - \sqrt{3}),$$

which is greater than $0.03\sqrt{\pi}$. Since the integral is greater than or equal to any lower sum, it must be positive

11. If $\int_a^b g = 0$, then by Exercise 7, $g(x) = 0$ for all $x \in [a, b]$ so $\int_a^b (fg) = 0$ and $\int_a^b g = 0$. Suppose $\int_a^b g \neq 0$. Since f is continuous, it is bounded on $[a, b]$ and $m \leq f(x) \leq M$ where M and m are the maximum and minimum of f on $[a, b]$. Because g is nonnegative, $mg(x) \leq f(x)g(x) \leq Mg(x)$ for all $x \in [a, b]$. Hence $m\int_a^b g \leq \int_a^b (fg) \leq M\int_a^b g$. Therefore $m \leq [\int_a^b (fg)]/[\int_a^b g] \leq M$. By the intermediate value theorem, there is a value $c \in [a, b]$ such that $f(c) = [\int_a^b (fg)]/[\int_a^b g]$. Consequently, $\int_a^b (fg) = f(c)\int_a^b g$.

12. Let x_0 be any number in [a, b] and choose h so that $x_0 + h \in [a, b]$. Then

$$G(x_0 + h) = \int_a^{x_0+h} f = \int_a^{x_0} f + \int_{x_0}^{x_0+h} f.$$

Since f is integrable, it is bounded on [a, b] and $lh \le \int_{x_0}^{x_0+h} f \le uh$ by Theorem 12.5.22, where l and u are lower and upper bounds for f on [a, b].

As $h \to 0$, $lh \to 0$ and $uh \to 0$, so by the Pinching Theorem, $\lim_{h \to 0} \int_{x_0}^{x_0+h} f = 0$. Consequently

$\lim_{h \to 0} G(x_0 + h) = \int_a^{x_0} f = G(x_0)$ so G is continuous at x_0.

13. a) $2x$ b) x^2, where $x \neq 0$

14. $$g'(0) = \lim_{h \to 0} \frac{e^{-1/h^2}}{h} = \lim_{h \to 0} \frac{1/h}{e^{1/h^2}} = \lim_{h \to 0} \frac{-1/h^2}{e^{1/h^2}(-2/h^3)}$$

$$= \lim_{h \to 0} \left(\frac{h}{2} e^{-1/h^2}\right) = 0$$

15. $D(\sin 2x) = 2 \cos 2x$, $D(2 \sin x \cos x) = 2 \cos^2 x - 2 \sin^2 x$, so $\cos 2x = \cos^2 x - \sin^2 x$. $D[\sin(x + \alpha)] = \cos(x + \alpha)$, $D(\sin x \cos \alpha + \cos x \sin \alpha) = \cos x \cos \alpha - \sin x \sin \alpha$, so $\cos(x + \alpha) = \cos x \cos \alpha - \sin x \sin \alpha$.

16. Let $f(x) = \tan x - x$. Then $f'(x) = \sec^2 x - 1 > 0$ for $x \in (0, \pi/2)$. Therefore f is increasing. Since $f(0) = 0$, $f(x) > 0$ for $x \in (0, \pi/2)$. Hence $\tan x > x$ in this interval.

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17. The terms of the sequence are all positive. Also $\frac{n+1}{2n-1} - \frac{1}{2} = \frac{3}{2(2n-1)}$ which is less than ϵ if and only if $n > \frac{3+2\epsilon}{4\epsilon}$. Let ϵ be any positive real number and choose a positive integer $N > \frac{3+2\epsilon}{4\epsilon}$. Then for all integers $n \geq N$, $|a_n - \frac{1}{2}| < \epsilon$ so $\lim_{n \to \infty} a_n = \frac{1}{2}$.

18. Absolutely convergent (comparison test)

19. 2}

20. The series $\sum_{j=0}^{\infty} a_j x^j$ has the same radius of convergence as its derivative series $\sum_{j=1}^{\infty} j a_j x^{j-1}$, which in turn has the same radius of convergence as $\sum_{j=2}^{\infty} j(j-1) a_j x^{j-2}$. The last series can be written in the form $\sum_{j=0}^{\infty} (j+2)(j+1) a_{j+2} x^j$.