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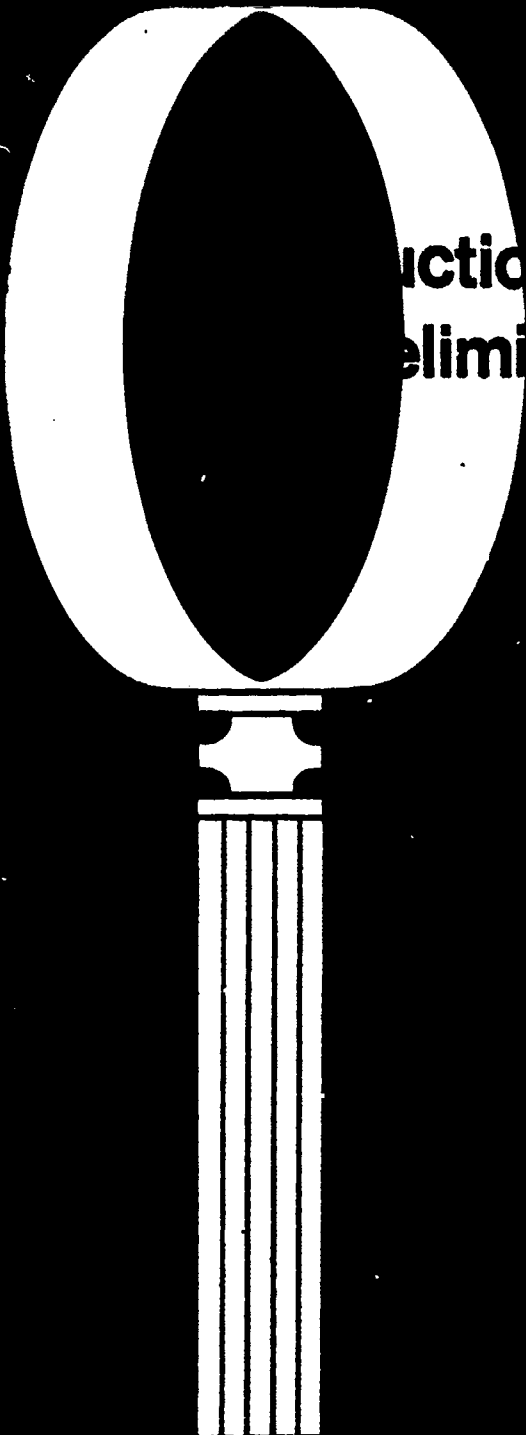
ABSTRACT

This paper proposes that instruction consists of four relatively independent facets: learner aptitudes, content structure, delivery systems, and instructional strategies. The purpose of this paper is to develop a taxonomic vocabulary and a model for portraying instructional strategies. Instructional strategies are defined as sequences of two or more instructional displays. To describe individual displays, eight variables are identified: content type, content mode, content representation, mathemagenic prompting, response conditions, response mode, response representations and mathemagenic feedback. Various parameters are suggested for each. To describe the relationship between displays, quantitative and sequence specifications and a class of qualitative interdisplay relationships are suggested. Manipulation of the qualitative relationships is considered to be the factor that affects instructional effectiveness and efficiency. The proposed theory and accompanying flow chart conventions should have value in any discipline for the development of instructional theory, the synthesis and interpretation of research, the analysis of existing strategies, and the design of materials and systems. (CR)

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MATHEMATICS EDUCATION REPORT

Instructional Strategies: A Preliminary Taxonomy

by

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Mathematics Education Reports

Mathematics Education Reports are being developed to disseminate information concerning mathematics education documents analyzed at the ERIC Information Analysis Center for Science, Mathematics, and Environmental Education. These reports fall into three broad categories. Research reviews summarize and analyze recent research in specific areas of mathematics education. Resource guides identify and analyze materials and references for use by mathematics teachers at all levels. Special bibliographies announce the availability of documents and review the literature in selected interest areas of mathematics education. Reports in each of these categories may also be targeted for specific sub-populations of the mathematics education community. Priorities for the development of future Mathematics Education Reports are established by the advisory board of the Center, in cooperation with the National Council of Teacher of Mathematics, the Special Interest Group for Research in Mathematics Education, the Conference Board of the Mathematical Sciences, and other professional groups in mathematics education. Individual comments on past Reports and suggestions for future Reports are always welcomed by the editor.

For the past several years, ERIC/SMEAC has sponsored a presentation in cooperation with the Special Interest Group for Research in Mathematics Education at the annual meeting of the American Educational Research Association. This publication is based on the presentation made by the senior author on April 16, 1974 at the AERA meeting in Chicago.

The focus of the paper is apparent: strategies for instruction. The authors propose a taxonomic vocabulary with which instructional strategies can be described. They suggest a taxonomic organization for relating the variables involved in instruction, and symbols for representing these variables and their relationships. The premise is that, through the application of such a taxonomy to research on instruction, the development of a theory-based approach to instruction will be facilitated.

There is much detail in the descriptions and many illustrations to clarify specific points. However, as the authors point out, all of the variables are not specified: the paper represents a stage in a continuing analysis of research on instruction. While some examples are specific to mathematics instruction, it is readily apparent that the taxonomy is not limited to any one field.

The applicability of the taxonomy should provide the basis for much discussion among educators. Hopefully, it will also lead to some consideration of how this (or some other) taxonomy might be used so that research results from various studies might be related more readily.

ERIC/SMEAC is pleased to make this publication available not only to mathematics educators, but to all educators interested in instruction.

Marilyn N. Suydam
Editor

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Instructional Strategies: A Preliminary Taxonomy

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Is there empirical evidence for a theory based approach to instruction? The answer is yes. There is much carefully conceived, controlled, and executed research on instruction. However, synthesizing, interpreting, and developing theorems from this data base is difficult because we lack a common vocabulary for the variables investigated. Furthermore, we lack a taxometric formulation relating these variables to one another in a meaningful way.

In a thoughtful paper on the development of theory, Snow (1973) suggested that one necessary step toward an axiomatic theory is the specification of a taxometric vocabulary which can unambiguously describe and relate the variables comprising a given system.

The purpose of this paper is to suggest a carefully selected vocabulary for describing instructional strategies, to suggest a taxometric organization for relating these variables in a way which facilitates the statement of instructional theorems, and to suggest a set of symbolic flow chart conventions for representing these variables and their relationships in existing and yet to be conceived strategies. In the same spirit as the excellent work of Heimer and Lottes (1973) we agree that what is needed are theorems

". . . that would specify relations between particular courses of action and their consequences in terms of achievement of specified ends." The ultimate aim of this work is the review of existing research in terms of the variables suggested in the hope of formulating some such theorems and to identify questions which have not yet been asked but which are needed if we are to have a theory based approach to instruction. Limitations of time and space necessitate restricting the scope of this paper to part of the taxometric specifications as a step toward a theory based approach to instruction.

FACETS OF INSTRUCTION

The primary thrust of this paper is instructional strategies. In this section we have attempted to put instructional strategies into a broader context and to indicate how research on instructional strategies is related to other types of instructionally relevant research.

For purposes of discussion, instruction can be conceptualized as consisting of four facets: (1) learner aptitudes, (2) subject matter content, (3) instructional strategies, and (4) instructional delivery systems.

Instructional Facets Defined

Aptitude. "'Aptitude' is . . . any characteristic of the person that forecasts his probability of success under a given (instructional) treatment." (Cronbach and Snow, 1973, pp. 1-12.)

Two major classes of aptitudes can be identified. Those most frequently investigated by differential psychologists are trait

aptitudes, those pervasive characteristics of the individual which are relatively stable over relatively long periods of time. A second class of aptitudes are state aptitudes, those dynamic characteristics of the learner which change from moment to moment.¹ It is the opinion of the authors that state aptitudes are likely to make a bigger difference on the effectiveness of particular instructional strategies than are the more pervasive trait aptitudes. This argument has been developed elsewhere (Merrill, 1974).

Content. The content facet is concerned with two classes of variables: content structure-- what are subject matter components and how are they related-- and content sequence-- which subject matter components should be taught in a given instructional setting and how should they be sequenced.

Several authors have suggested that subject matter can be represented by means of only a few types of content elements. Organized subject matter exists when these elements are related in some useful or meaningful way (Donio, 1967; Merrill, 1973; Macdonald-Ross, 1974; and Pask, 1974).

Perhaps the most difficult task in instructional development is the selection and sequencing of content components for presentation to students. Hierarchical arrangement has been strongly advocated during the last several years (Gagne, 1967, 1968, 1974; Merrill, 1973). However, recently serious questions have been raised about the adequacy of learning hierarchies as a prescription for sequence (Merrill and Gibbons, 1974).

¹Labeling dynamic moment-to-moment characteristics of learners as aptitudes may be contrary to the usual connotation of the word. However, it is consistent with the definition adopted from Cronbach and Snow and thus seemed appropriate for our use in this paper.

Strategy. An instructional strategy is composed of the type of displays (not media type), the sequence of displays, and the relationship among the displays that are presented to the student. The remaining sections of the paper will define and illustrate instructional strategies in detail; hence, further elaboration will be omitted at this point.

Delivery systems. A delivery system is the organic or mechanical device(s) used to provide the sensory input and to receive and record the response output from the learner. This definition is meant to include all types and combinations of media including various audio-visual devices, live teachers, and instructionally structured social settings.

Delivery systems are frequently cataloged by means of type, such as motion picture, video, printed materials, etc. The authors suspect that for purposes of the proposed model more abstract variables will prove more useful than merely the type of media employed.

A Geometric Metaphor-- Facets

In Figure 1 these four facets of instruction are represented as faces of a tetrahedron. The illustration shows the tetrahedron unfolded so that all four facets are easily visible. Each facet (surface) of the tetrahedron represents a set of variables which is involved in the instructional process. The metaphor of facets of a tetrahedron was adopted for several reasons.²

²Since this paper was originally prepared as an invited address to the Special Interest Group for Research in Mathematics Education it seemed that a mathematical figure was an appropriate metaphor for representing these ideas.

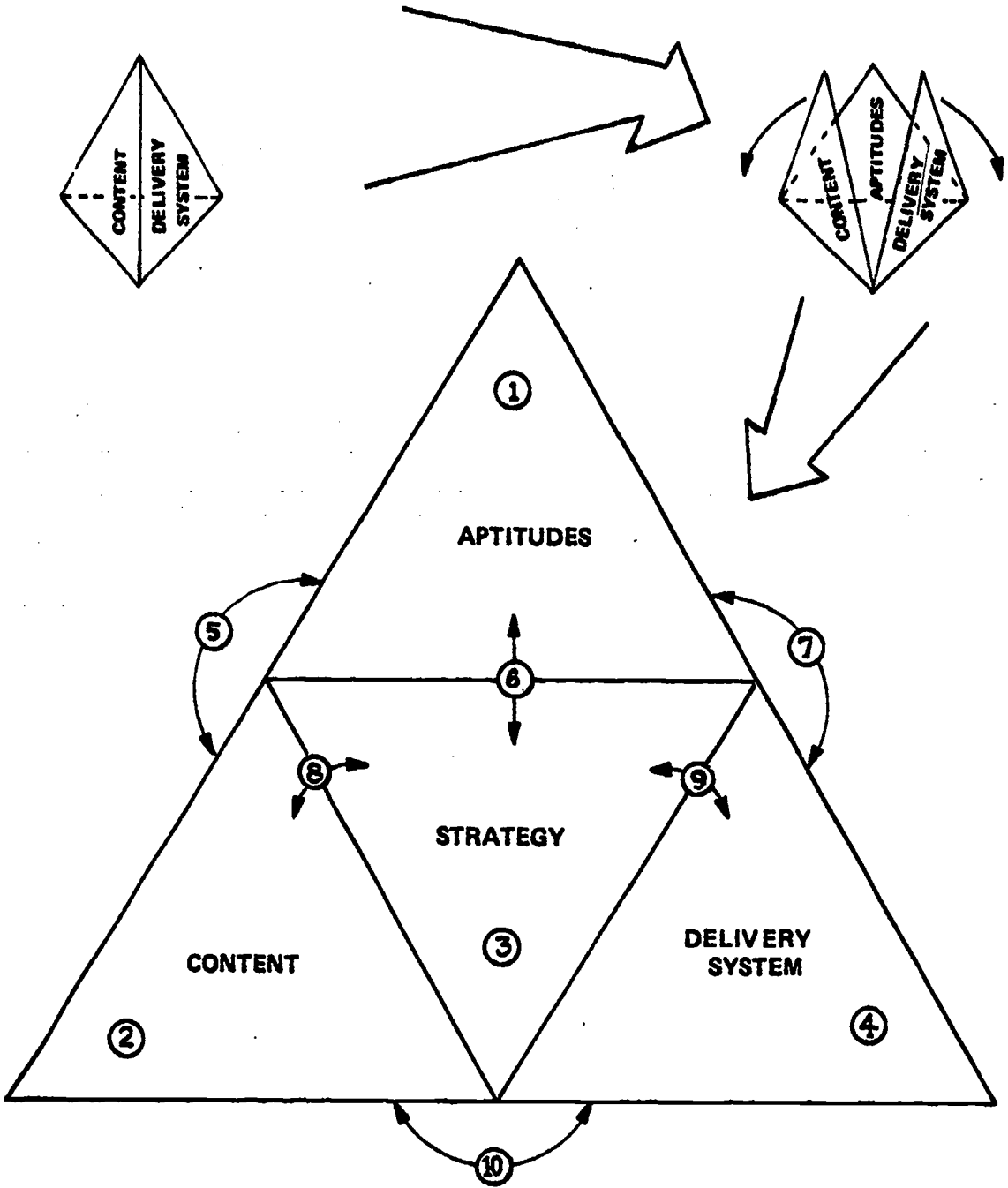


Figure 1. Facets of instruction.

One assumption of the proposed model is that the variables within each facet are relatively independent of the variables within another facet. That is, a given instructional strategy (facet 3) may be effective for a variety of aptitude configurations (facet 1), may be used for presenting different subject matter content (facet 2), and can be presented by means of a variety of different delivery systems (facet 4). This is not to say that there are not interactions, as described below, but merely to indicate that a given set of parameter values³ on the set of variables in one facet does not simultaneously fix parameter values on the variables in another facet.

Another assumption is that to be included in the model the variables to be identified within each facet must be present in every instructional situation. Thus for an instructional situation to be completely specified it is necessary that a particular value be specified for each parameter of each variable in each facet.

It follows from the above assumptions that the proposed model would consist of a parsimonious set of variable categories and associated parameters in each facet such that any instructional product or system can be unambiguously described by the appropriate identification of parameter values or changes in a parameter value on each variable in each facet.

³Parameters are those dimensions which characterize a particular variable. Some variables are unidimensional and are characterized by a single parameter. Other variables are multidimensional and are characterized by several parameters. Parameters may assume any of several quantitative characteristics, that is, a given parameter may assume either ordinal, interval, or ratio scale values.

For each of the facets identified there exists one or more bodies of research literature which reports investigations of variables within a given facet while ignoring or controlling variables in the other facets.

The field of differential psychology and psychological testing is concerned with the identification of learner aptitudes. Developmental psychology is also concerned with the identification of aptitudes that characterize individuals at various stages of development.

The curriculum revision projects of the past two decades are efforts related to the content facets. Other areas of activity in this facet include task analysis, the behavioral objective movement, and recent emphasis on competency based programs. Learning hierarchies (Gagne 1967, 1968, 1970, 1974) and attempts to specify content networks (Macdonald-Ross, 1974; Pask, 1974; and Merrill and Gibbons, 1974) represent additional activities in this domain.

The strategy facet has been investigated by research efforts in programmed instruction and computer assisted instruction. These efforts have recently expanded to include much of what is now called "instructional psychology" quite independent of P.I. or C.A.I. as media (see the following reviews: Anderson, 1967; Gagne and Rohwer, 1969; Glaser and Resnick, 1972; and Merrill and Boutwell, 1973).

New delivery systems are continually being invented and tested. In addition to this technical research there has been increasing interest in more effective utilization of various media. More

recently this investigation has taken on a multimedia emphasis rather than being limited to a single medium. (For a recent review of research on delivery systems see Jamison, Suppes, and Wells, 1974.)

In spite of the research activities briefly mentioned in the preceding paragraphs there does not yet exist an agreed-to set of variables within any of these facets. Furthermore, most of these areas of investigation have proceeded quite independent of one another so that the scholars in one area have only infrequently tried to conceptualize variables which are compatible with the variables investigated in other facets. A major purpose of this paper is to suggest a possible integrated set of variables for the strategy facet.⁴

A Geometric Metaphor-- Interaction of Facets

Each interface (edge) of the tetrahedron represents the interactions of particular parameter values of the variables in one facet with parameter values of the variables in another facet. Another reason for the choice of a tetrahedron metaphor was to choose a figure which conveys the idea that every facet interacts with every other facet. In other words, the effectiveness of a given instructional strategy will be affected by the aptitude of the students involved, the content presented and the delivery system used to make the presentation. The ease with which a given set of subject matter content is mastered is affected by the aptitude of the students, the strategy used, and the delivery system employed. Similar arguments follow for given aptitude configurations and given delivery systems.

⁴The authors are also working in other facets of the instructional model proposed. It is beyond the scope of this paper to describe this effort.

For each of the six areas of interaction there exists one or more bodies of research literature which investigate the interactions between the variables in the various facets. Often these studies ignore or control other interactions or effects between various parameter values within facets.

A major thrust of developmental psychology has been the investigation of what concepts (content) are most easily acquired at what stage of development (5) (see Figure 1). The work of Piaget-oriented researchers has been particularly involved with this question, especially in mathematics. To some extent curriculum research is concerned with the interaction of aptitude and content. The testing movement has also been concerned with predicting success in particular content areas by persons of particular aptitude.

The recent emphasis on aptitude-treatment interactions research (see Cronbach and Snow, 1973) has been largely concerned with the interaction of aptitude with treatment (6). To a lesser extent this same research effort has looked at aptitude delivery system interactions (7). The senior author has suggested that learner control provides an alternative approach to dealing with aptitude strategy interactions (Merrill, 1974).

The interactions of content with strategy (8), strategy with delivery system (9), and content with delivery system (10) have not received as much attention as the other areas, perhaps because content, strategy and delivery systems have not frequently been conceptualized as independent facets. One purpose of the proposed model is to stimulate systematic study of these interactions.

Instructional Strategies

An instructional strategy can be characterized as consisting of two or more instructional displays arranged in a specified sequence.

An instructional strategy can also be described as a series of instructional moves⁵ made by the teacher, in the case of live instruction, or by the instructional designed, in the case of mediated instruction. An instructional move consists of presenting an instructional display which bears a particular relationship to a previously presented display and to the move made or assumed to have made on the part of the student.

Instructional strategy is not the same as learner strategy. Learner strategy consists of a series of learning moves made by the student. A learner move consists of a particular overt or covert response to a particular instructional display or series of displays which enables the student to relate the ideas being presented to previously acquired ideas in such a way that he can remember and use them at a later time.

Chess provides a useful metaphor for describing the dynamics of instruction. In chess there are a limited number of pieces which can each be moved in a particular way. Instruction consists of a limited number of display characteristics which can be combined

⁵The authors are indebted to Thomas Cooney, Edward Davis, and Kenneth Henderson (1975) for the use of the term "instructional moves". Our definition of the term may not correspond completely with their intent but there is a close relationship between their description of the instructional process and the description provided in this paper.

and sequenced in a limited number of ways. A given display sequenced in a particular way consists of an instructional move. Unlike the chess game, where both players are allowed the same number of pieces with each piece moving in the same way, the moves available to the instructor do not completely correspond to the moves available to the learner. Also unlike chess the objective of the instructor is to get the student to win.⁶ Each instructional move is designed to get the learner to make an optimal learning move so that the learner eventually wins by having the idea being taught in check.

In the terms of the chess metaphor, the purpose of this paper is to describe the basic display characteristics (pieces) and the interdisplay relationships (moves) which are possible for the instructor. It is not a discussion of how an instructor can help the student win at instructional chess. This paper is not a discussion of effective strategies for beginning instructors.

Our definition of an instructional strategy suggests that we must be concerned with two major classes of variables: the characteristics of a given instructional display and the relationships between one display and another display. In the "Display Characteristics"

⁶Some instructors act as if their objectives were to make the student fail. These tactics are seen when an instructor grades on a curve by selecting test items which produce a wide distribution. Another "student-should-lose" strategy is to select test items on topics which have not been included in instruction. Many teachers, especially college professors, assume no instructional responsibility but see their only goal as making subject matter available and leaving its acquisition entirely to the student. These inappropriate strategies can still be described by instructional moves and the authors would like to believe that the instructors involved would subscribe to the "student-should-win" purpose of instruction even though their strategies may be inconsistent with this purpose.

section of the paper we have identified those variables that are thought necessary for describing an instructional display. The "Interdisplay Relationships" section provides an incomplete description of the relationship between instructional displays.

In chess, having mastered the basic moves for each piece, there are innumerable ways to combine these moves into effective strategies. So also in instruction; having acquired the ability to identify basic characteristics of displays there are almost innumerable ways to combine these displays into effective instructional strategies. In chess the strategy that is likely to be most effective depends on your opponent; in instruction the strategy that is likely to be most effective depends on the learner and the subject matter involved in a given situation. In chess there is usually more than one strategy that is likely to be effective for a given opponent; in instruction there is usually more than one strategy that is likely to be effective for a given learner. Furthermore, different strategies are likely to be most effective at different times even with the same learner.

Limitations on Variables

A series of instructional displays present a very complex stimulus situation. A description of this event could include innumerable variables. The following are some of the criteria which were used in selecting the variables which are described.

Parsimony. Any acceptable theoretical system should contain as few variables as possible.

Generality. The variables which are selected should subsume variables which have been investigated and reported in the experimental literature. For any variable which has been demonstrated to have made a difference in student learning, it should be possible to show that it is an instance of one of the variables included in the model.

Relevancy. The variables included should be instructionally relevant, i.e., changing a parameter value on the variable should make a difference in learning a particular type of subject matter content for students with a particular configuration of aptitudes.

Control. The parameters included must be susceptible to either instructor, system, or learner control. If, for a given piece of subject matter content, a particular variable or its parameters can not be altered or manipulated, then this content variable is an instructional constant for a given situation and can not be an instructional variable.

Universal. The variables selected must be present in every display regardless of the subject matter content, the configuration of learner aptitudes, or the type of delivery system involved. If, for a particular content, a given variable is not present, then it is a content variable rather than an instructional variable. A similar argument holds for aptitudes and delivery systems. Differences in displays are created by modifying the parameter values of the variables included rather than by changing variables themselves.

Flow Chart Conventions

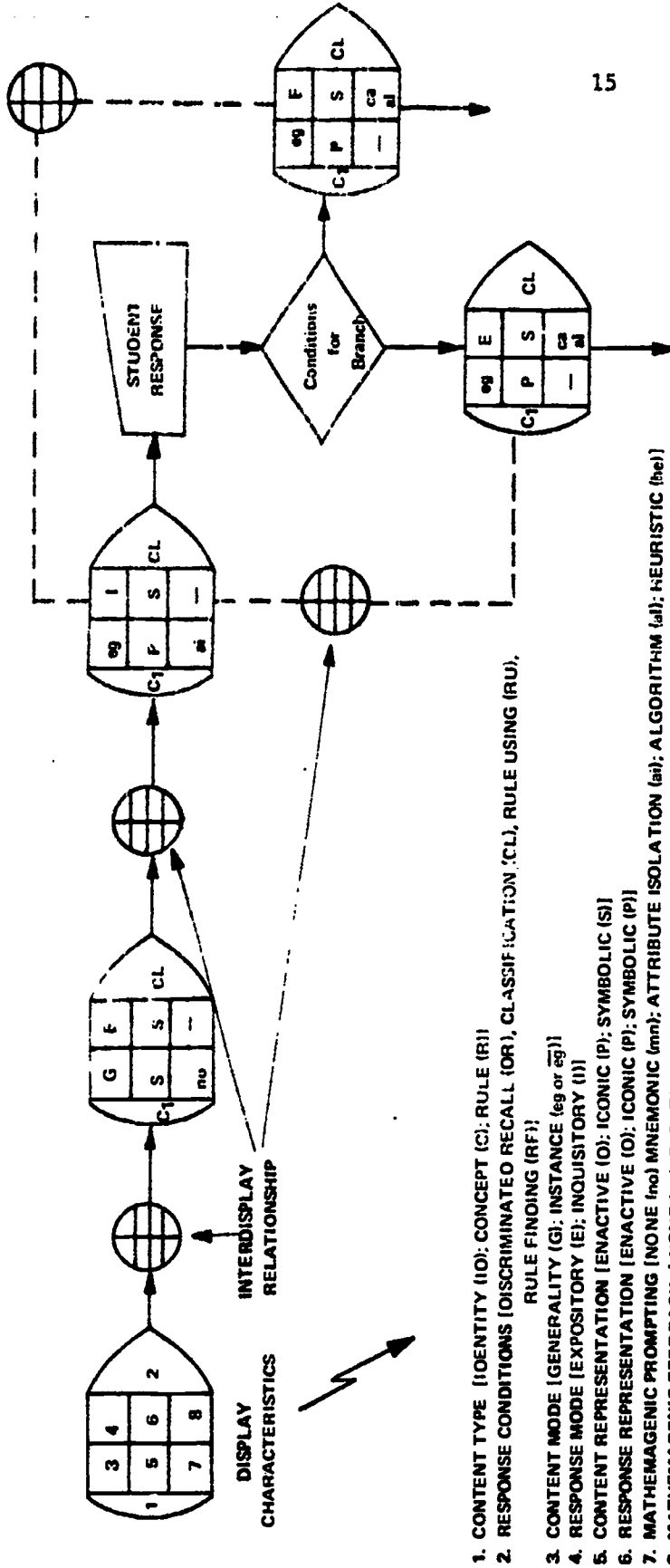
In addition to defining the display variables (pieces) and interdisplay relationships (moves) the authors would like to suggest a schematic representation for these variables which can provide considerable economy in describing instructional strategies.

Following is a brief description of the flow chart conventions which will be used throughout the remainder of the paper

Instructional strategies are described in terms of two sets of variables: the characteristics of individual displays and the relationship between two or more displays in sequence. It is proposed that a display be represented by the symbol indicated in Figure 2.

In the "Display Characteristics" section of the paper, eight display variables will be described. They are: content type, content mode, content representation, mathemagenic prompting, response mode, response representation, mathemagenic feedback, and response conditions. For each of these variables various parameters will be suggested. The display representation is sectioned as shown in Figure 2 to provide a location for recording the primary parameter value for each of these display variables. Symbols for the various parameter values and other conventions for recording this information will be introduced as the variables are described.

It is proposed that the relationship between two displays be represented as indicated in Figure 2.



1. CONTENT TYPE (IO); CONCEPT (C); RULE (RI)
2. RESPONSE CONDITIONS (DISCRIMINATED RECALL (OR), CLASSIFICATION (CL), RULE USING (RU), RULE FINDING (RF))
3. CONTENT MODE (GENERALITY (G); INSTANCE (eg or eg))
4. RESPONSE MODE (EXPOSITORY (E); INQUIRY (I))
5. CONTENT REPRESENTATION (ENACTIVE (O); ICONIC (P); SYMBOLIC (SI))
6. RESPONSE REPRESENTATION (ENACTIVE (O); ICONIC (P); SYMBOLIC (SI))
7. MATHEMAGENIC PROMPTING (NONE (no); MNEMONIC (mn); ATTRIBUTE ISOLATION (ai); ALGORITHM (al); HEURISTIC (he))
8. MATHEMAGENIC FEEDBACK (NONE (no); RIGHT/WRONG KNOWLEDGE OF RESULTS (r/w); CORRECT ANSWER K OF R (ca); MNEMONIC (mn); ATTRIBUTE ISOLATION (ai); ALGORITHM (al); HEURISTIC (he))

Figure 2. Instructional strategy flow chart symbols.

In the "Interdisplay Relationships" section of the paper, it is suggested that qualitative relationships can be identified between each of the display variables. Not all of these relationships are described nor are all of the parameter values identified. The connector symbol suggested is sectioned as shown in Figure 2 to provide a place for indicating each of the qualitative relationships involved. Details for recording this information will be described as the variables are introduced.

Sequence is indicated by direction of flow using the conventional arrows. Process descriptions or notes about the strategy are indicated in boxes. Branches in the instruction are indicated by diamonds with the conditions for the branch indicated in the diamond symbol as is conventionally done. Student moves are indicated by the keyboard symbol shown in Figure 2. Since this paper does not include a description of learner moves, use of this symbol will be undifferentiated. It is anticipated that a description of learner moves would enable us to partition this symbol and allow a more complete description of the learner moves similar to the partitioning used for instructional displays. Additional special conventions will be introduced as necessary in the remainder of this paper.

Display Characteristics

Because it is defined in terms of the variables yet to be described, this incomplete definition of a display will be more meaningful after the reader has completed this section.

An instructional display is the presentation of a concept or rule generality or an identity, concept, or rule instance, together with such mathemagenic information as may be included.

A display is differentiated from a frame in that a frame includes that which is presented simultaneously via the delivery system in use. Thus for printed materials a frame is synonymous with a page but a page is not synonymous with a display. A given frame or page may consist of several displays. For a tape/slide (or filmstrip) delivery system a frame consists of a single picture with its accompanying audio. A single frame may consist of one or several displays, or a single display may require more than one frame. For continuous type delivery systems such as audio tapes, video tapes or motion pictures, the definition of a frame is somewhat more problematic but the presentation can still be segmented into a sequence of instructional displays.

For purposes of analysis when one or more of the parameter values of the variables to be described have been changed the presentation is recorded as a new display. For some delivery systems many elements of a new display may be identical or even the same as elements of a previous display but if some parameter of the previous display has changed then it is recorded as a new display.

Prior to defining each of the variables and their associated parameters, some of the overall characteristics of the variables will be defined. In a previous section the interaction of strategy with content, learner aptitudes, and delivery systems was noted.

Strategy variables reflect these various interactions. Ignoring delivery systems for the time being, the variables can be divided into two categories: content oriented variables and learner (aptitude) oriented variables. The content oriented variables include content type, content mode, content representation and mathemagenic prompting. The learner oriented variables are response mode, response representation, mathemagenic feedback, and response conditions.

The schematic symbol for display characteristics (see Figure 2) was partitioned in such a way that content oriented variables appear on the left and response oriented variables appear on the right. From a learner system point of view, content variables can be referred to as input variables while response variables can be referred to as output variables (see Heimer and Lottes, 1973).

The model has a form of symmetry in that each content oriented variable has a particular relationship with a response oriented variable. Hence, the instructional objective for a particular instructional segment is determined by a combination of content type with response conditions. Content mode and response mode combine to form four basic presentation forms. Content representation is parallel to response representation and mathemagenic prompting is parallel to mathemagenic feedback. We have therefore, elected to describe these pairs of variables together rather than first describing all content oriented variables and then all response oriented variables.

Content Type

For purposes of instruction, subject matter content can be classed into three categories: identities, concepts, and rules. Each of these categories will be described and illustrated in the following paragraphs.⁸

The type of content (identity = ID, concept = C, rule = R) is represented in space 1 of the display flow chart symbol (see Figure 2). When a given instructional presentation consists of several identities, concepts, or rules, subscripts are used to indicate which is being presented in a given display.

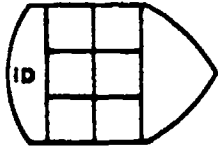
Identity (ID). An identity is a symbol, object, or event associated on a one-to-one basis with another symbol, object, or event.

Figure 3 illustrates sets of identities from several subject matters.

Concepts (C). A concept consists of a set of objects, symbols, or events (referents) which have been grouped together because they share some common characteristics (attributes). Concepts are usually referenced by some concept name.

Figure 4 illustrates concepts from several subject matters.

⁸The role of representation of referents will be described and parameters related to representational variables will be identified in a latter section of this paper.



MUSICAL NOTATION SYMBOLS

RESISTOR
CAPACITOR
COIL

SCHEMATIC SYMBOLS

Is Perpendicular to
Is Parallel to
Angle BOA
Triangle ABC

MATH SYMBOLS

Figure 3. Sample sets of identities.

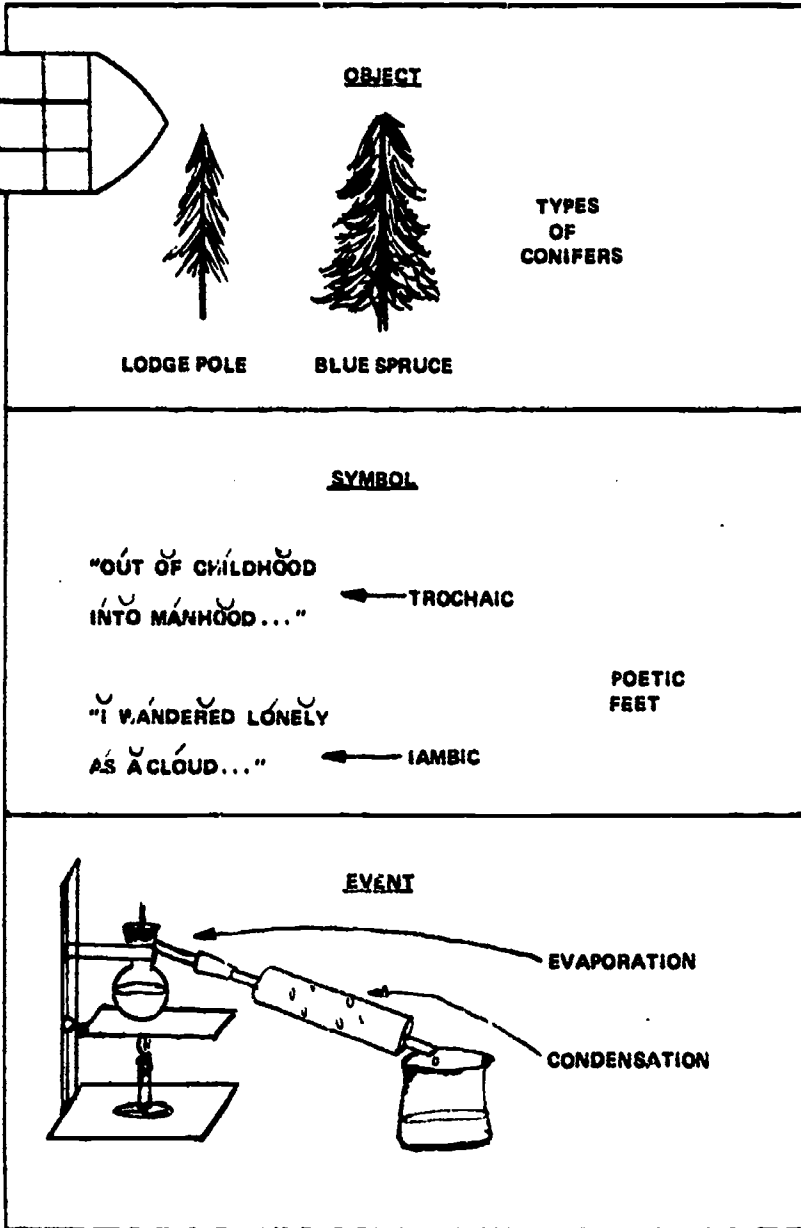


Figure 4. Samples of object, symbol, and event concepts.

Rule (R). A rule is an ordered relation consisting of a set of domain concepts, an operation, and a set of range concepts.

An operation is a procedure for describing one concept by means of other concepts (definition), a procedure for comparing one concept to other concepts, or a procedure for changing instances of one set of concepts into instances of another set of concepts. An object, symbol, or event of each domain concept is acted upon as per the operation resulting in the symbol(s), object(s), or event(s) of the domain concepts. This relationship is illustrated in Figure 5.

From the above definitions the following characteristics can be derived. First, all operations are concepts but not all concepts are operations. Operations are relational concepts for describing, ordering, or changing other concepts. Second, operations are not synonymous with rules. An operation is part of a rule but a rule also includes the domain and range concepts as well as the operation. Third, all concepts can be represented by a rule involving a descriptive operation. This means that the distinction between concepts and rules is sometimes confusing and the classification of existing subject matter is sometimes a little ambiguous. This confusion can be reduced by the following convention. A given piece of content is classed as a concept if its definition involves a descriptive operation, but as a rule if its definition involves a change operation. Figure 6 illustrates this convention. Ordering operations are identified as concepts when attributes of the instances are still present in the range instant(s), but as rules when they are not.

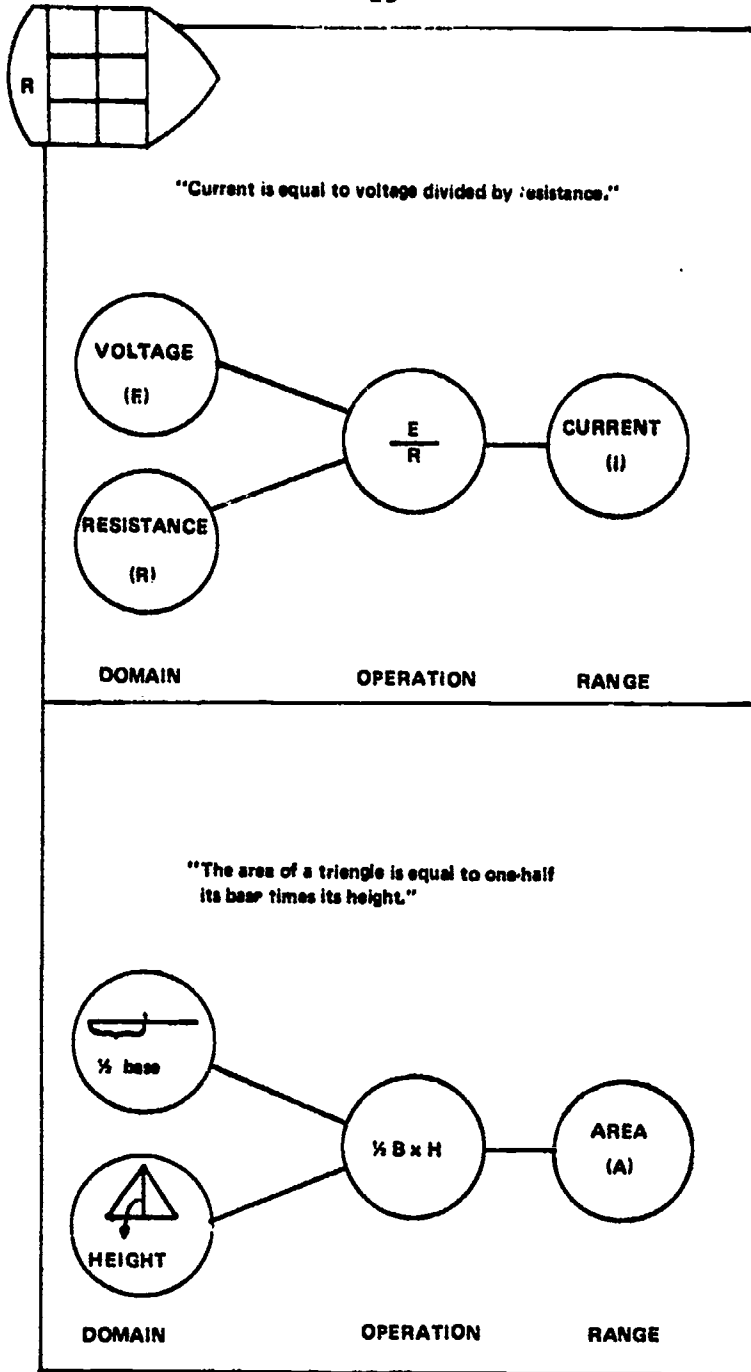


Figure 5. Sample rules expanded into domain, operation, and rule concepts.

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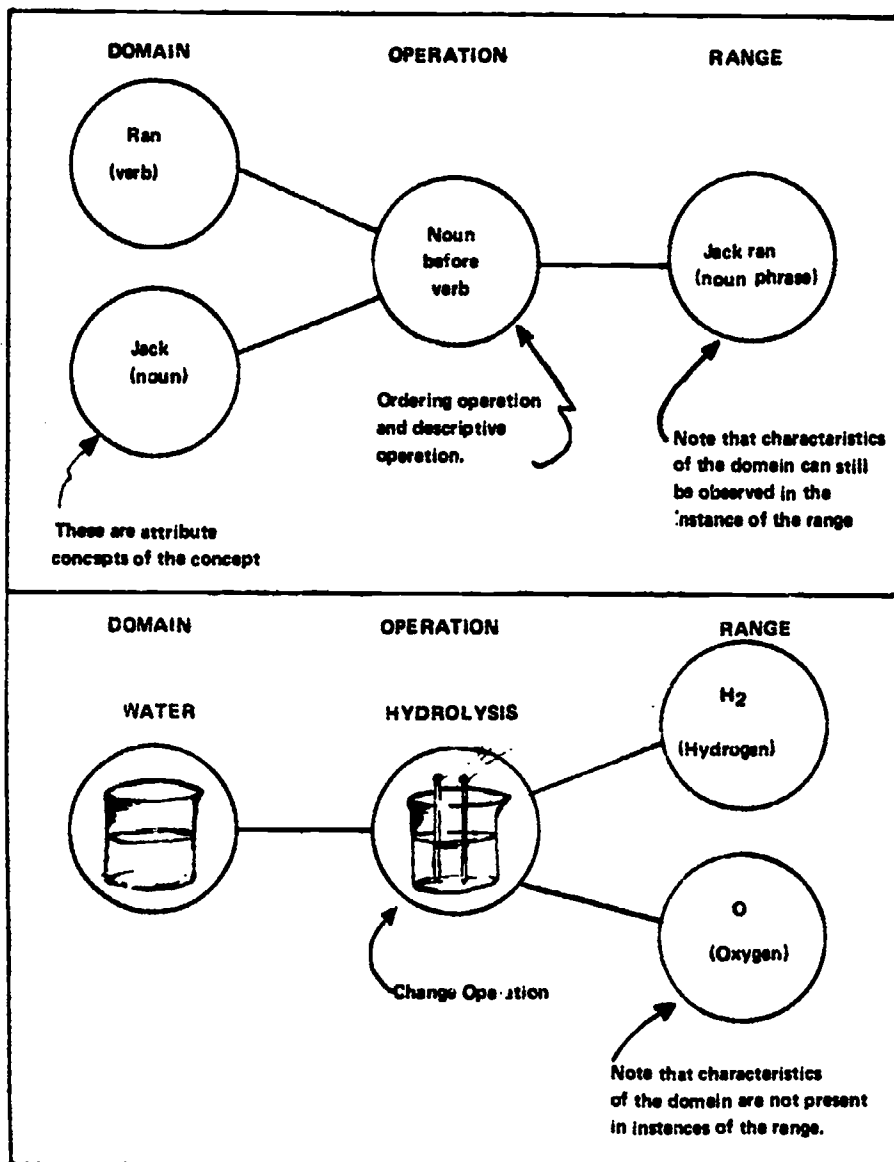


Figure 6. Descriptive (concept) rule contrasted with change (rule) rule.

Response Conditions

For purposes of instruction four levels of student response can be inferred, depending on the conditions surrounding the request to respond. This is not to say that the actual response as seen from inside the learner is at the inferred level but only that, based on the conditions surrounding the response, the inferred level is all that can be logically justified. For example, the conditions of the instructional display may allow inference of a "discriminated recall" response; the student may actually be employing some problem solving strategy rather than simple association. However, unless modified, the conditions surrounding the response would not allow an observer to infer other than discriminated recall.

The following paragraphs describe the necessary conditions for inferring discriminated recall (DR), classification (CL), rule using (RU), and rule finding (RF) levels of behavior. Note that the four levels of response include only those necessary for instruction in organized subject matter and do not include emotional (affective) or psychomotor levels of response. Both psychomotor and emotional behaviors are involved in all of the response levels described, but in this paper we have directed our attention only to the cognitive aspects of these behaviors.

The response level which can be inferred by the conditions surrounding the response is represented in space 2 of the display flow chart symbol.

Discriminated recall (DR). Discriminated recall can be inferred when given a set of identity pairs (symbol - symbol; symbol - object; symbol - event; etc.) the student is able to recall or recognize one given the other.

For purposes of strategy analysis we have not made a distinction between recall or recognition. Research on this question clearly shows that performance is better on recognition as compared with recall tasks. However, the authors feel that the same level of behavior is involved and that the constraints of the instructional situation will usually determine which form of response must be used. For example, if "real world" performance will require recall, the instructional situation should use recall.

Figure 7 illustrates some objectives and test questions from which discriminated recall can be inferred.

Classification (CL). Classification can be inferred when given an unencountered symbol, object, or event the student is able to correctly identify class membership.


Figure 8 illustrates some objectives and test questions from which classification can be inferred.


Rule using (RU). Rule using can be inferred when given unencountered instances of each of the domain concepts and the operation the student is able to produce or identify the resulting instance(s) of the range concept(s).


OBJECTIVE: Given a set of identity pairs (Symbol - Symbol, Symbol - Object, Symbol - Event) the student will recall or recognize one given the other.

	eg	I	
ID	S	S	DR
	no	no	


Write the name for these electronic symbols:

1.  _____

2.  _____

3.  _____

	eg	I	
ID	S	S	DR
	no	no	



1 2 3 4

Give the note names for each numbered note.

	eg	I	
ID	S	S	DR
	no	no	

Name the state capitols for each of these states:

CALIFORNIA _____

UTAH _____

FLORIDA _____

Figure 7. General objective and possible test items requiring discriminated recall of identities.

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OBJECTIVE:
Given an **UNENCOUNTERED** symbol, object or event the student will identify class membership

TEST ITEM:
"Circle the symbols that are REAL numbers."

1 π $7/8$ $\sqrt{13}$.101

TEST ITEM:
"Name the cloud shown in each picture."

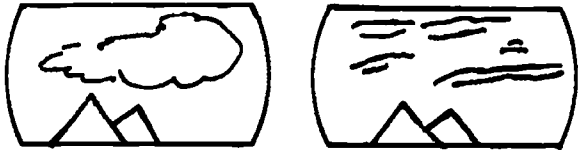


Figure 8. Objective and test item requiring classification.

Based on this definition classification is an instance of rule using behavior when the operation involved is a descriptive operation. In this situation the instances of the domain concept are the instances of the attributes as embodied in the instance of the concept being tested. The instance of the range concept is the concept name or class membership. This situation is illustrated in Figure 9. For purposes of instructional analysis whenever the operation involved is a descriptive operation the behavior will be categorized at the classification rather than the rule using level.

Figure 10 illustrates an example and a test question from which rule using with order or change operations can be inferred.

Rule finding (RF). Rule finding can be inferred when, given labels and unencountered instances of the domain concepts and range concept(s), the student is able to find or invent an operation that will complete the ordered relationship between the domain and range.

Figure 11 illustrates an objective and a potential test question from which rule finding can be inferred.

Rule finding as defined does not include all problem solving or creative behaviors. Several other possibilities exist. One type of behavior might be called domain finding. In this situation the student is given the label and unencountered instances of the range concept and the operation and asked to find domain concepts with instances such that the operation will indeed produce the instance of the range given.

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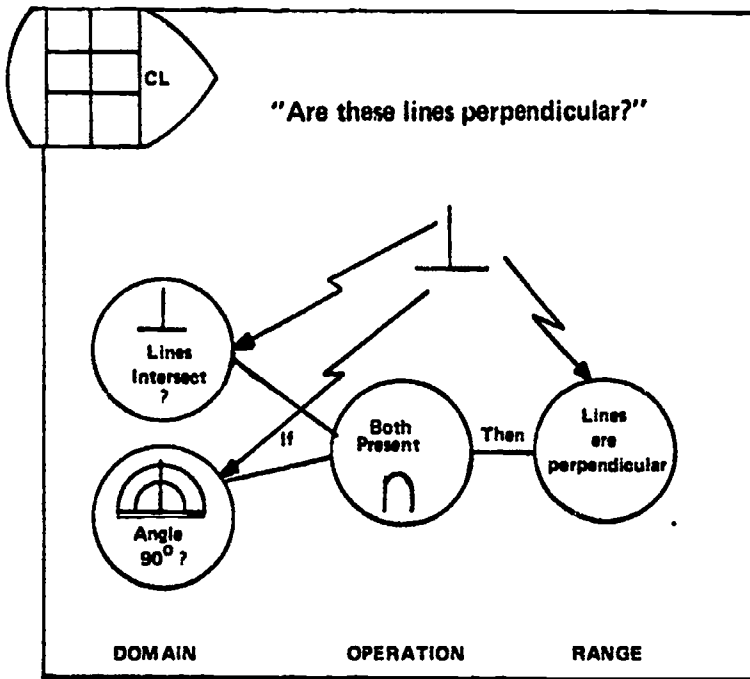


Figure 9. Classification as rule using.

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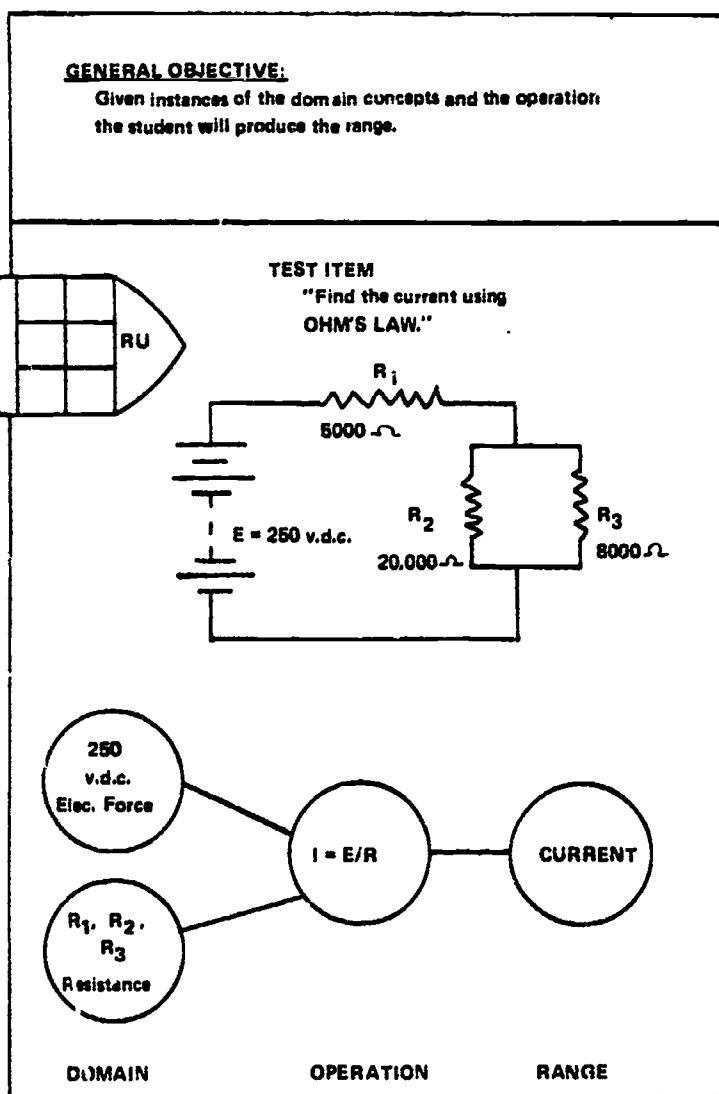


Figure 10. Objective and test item requiring rule using (change operation).

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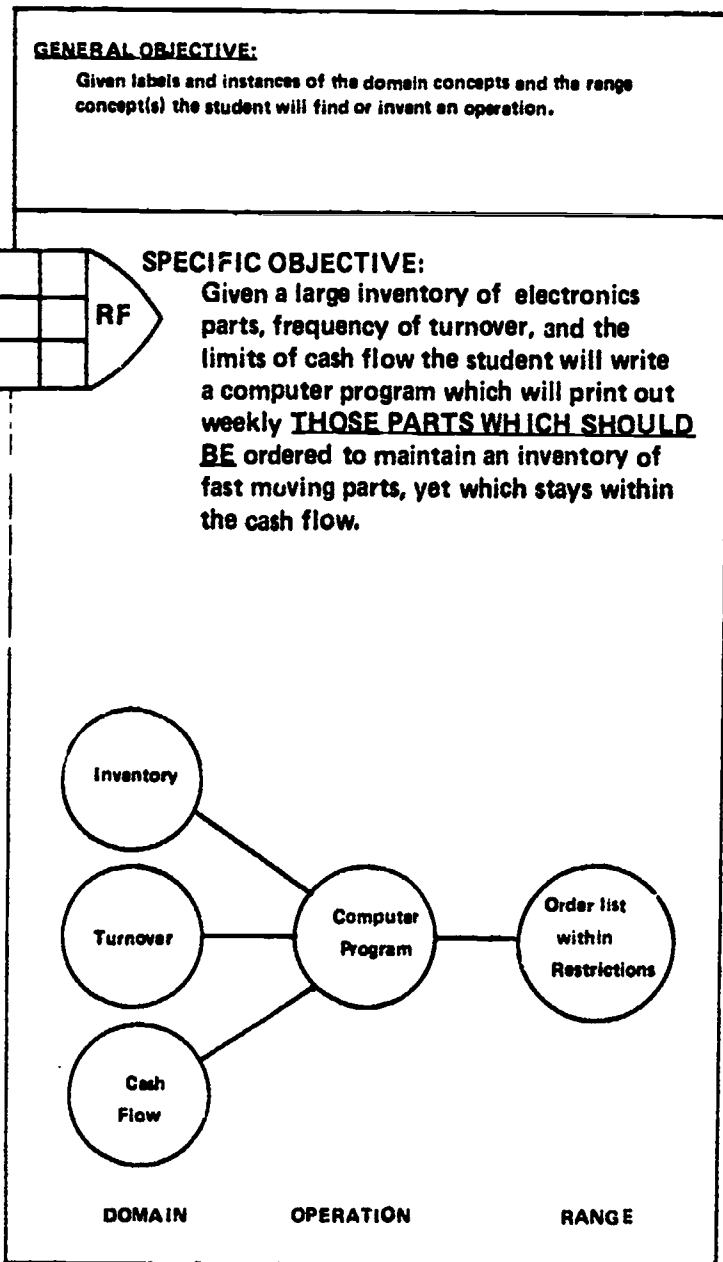


Figure 11. General objective and specific objective requiring rule finding behavior.

In another type of creative behavior the student is given labels or instances of the domain concepts and asked to find his own operation and to define his own range. An example could be an assignment in art where the student is told to paint something using water color. Similar creative expression can be found in other subject areas. Because it is much less defined such behavior is difficult to evaluate.

Another creative situation merely instructs the student to do something. In this case, the domain and operation are all left for student selection and he is given a range label. For example, write a short story, paint a picture, etc. These expressions of creativity are difficult to judge and may not really be included in the realm of instruction as defined in this paper.

One of the assumptions underlying this instructional model is that the type of subject matter content and the inferred level of student response to that content are somewhat independent phenomena (see Merrill and Boutwell, 1973; Merrill, 1973). This means that for a given type of content as embodied in a given display the conditions surrounding a request to respond may be adjusted to allow inference of any of the four response levels.

Figure 12 illustrates this relationship. Note that since classification is a special type of rule using there are only three behavioral levels indicated. For purposes of instructional analysis, however, classification is so fundamental that it makes sense to continue to record it as a separate category.

Rule Finding	Rule Using	Discriminated Recall	CONTENT TYPE
Find a Mnemonic	Find a descriptive or order operation	Find a change operation	Identity
Use a Mnemonic	Use a descriptive or order operation (Classification)	Use a change operation (Rule Using)	Concept
Remember a S-R Association	Remember a Specific descriptive or order operation	Remember a Specific change operation	Rule

Figure 12. Two-way classification of response conditions and content type.

It is proposed that all instructional objectives are instances of one or more cells of this matrix. Many objectives are simple objectives involving the content and behavioral level of a single cell. Other objectives are compound objectives and require combinations of cells. In another place the authors have suggested that this matrix can greatly facilitate the process of preparing instructional objectives (see Wood and Merrill, 1974). Figure 13 illustrates some instructional objectives and indicates the content type/response condition combinations involved in the objective.

Content Mode

All organized subject matter consists of referents which have been grouped into classes such that these classes are or can be related by propositional statements.

Referents are the actual objects, events, or symbols that exist or could exist in the "real world." Until such referents are cataloged they do not comprise subject matter. Hence, there are many referents which are not yet part of any organized subject matter. As previously defined, concepts are these sets of referents which have been grouped together because they share common attributes. Propositional statements are sentences which relate concept labels by means of relational concept labels.

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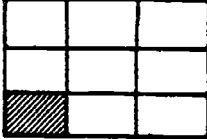
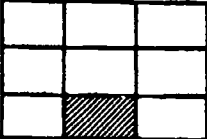

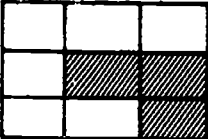
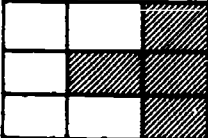
	<p>Given any one of the numerals 1, 2, 3, . . . , 31, the student will read it aloud.</p>
<p>(See figure 12)</p>	
	<p>The student will define from memory the following terms:</p> <ol style="list-style-type: none"> 1. parallel 2. perpendicular
	<p>Given two line segments the learner can tell whether they are parallel, perpendicular, or neither.</p>
	<p>Given the measure of an acute angle of a right triangle and a table of square roots, the student will find the sine, cosine, and tangent of the angle.</p>
	<p>Given unencountered descriptions and raw score data from several related experiments, and given the conclusions reached by the experimenters, the student will reinterpret the conclusions in light of his comparison.</p>

Figure 13. Illustration of various simple and compound objectives.

It follows from the above that concepts and rules can thus be represented in three ways: first, by a symbolic label; second, by a propositional statement relating domain and range concepts by means of relational concepts; third, by specific referents involved. For purposes of instructional strategy analysis, content mode is recorded either as a generality (G), a propositional statement, or an instance (eg) presentation of the referent or some high fidelity representation of the referent.⁸ The mere listing of a concept or rule label would not be considered as an instructional display. In most displays the generality or the instance is accompanied by the label or the student is asked to supply the label; hence, a separate symbol for a label alone is not necessary.

The content mode used in a given display is recorded in space 3 of the display flow chart symbols as illustrated in Figure 14.

Generality (G). For a concept, a generality is a list of the critical characteristics (attributes) of the concept; in other words, a rule statement involving a descriptive operation. For a rule, a generality is the identification by label of the domain concepts, the operation, and the range concepts. Figure 14 illustrates generalities from several subject matter areas.

Instance (eg). For a concept an instance is a particular object, event or symbol from the class or a high fidelity representation

⁸The role of representation of referents will be described and parameters related to representational variables will be identified in a latter section of this paper.

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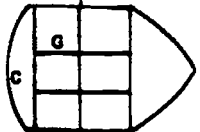
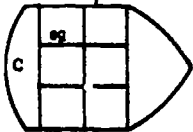
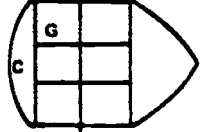

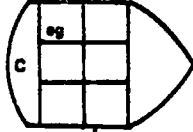
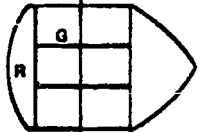
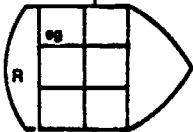
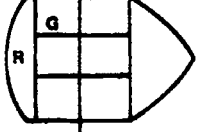
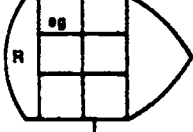
	<p>A declarative sentence is one which states or asserts a fact or a probability, or a possibility or an impossibility.</p>	<p>TUESDAY IS THE TENTH OF MAY.</p>	
	<p>Two lines are said to be parallel to one another if they have the same slope.</p>	<p>THESE LINES ARE PARALLEL.</p> 	
	<p>To make a compound sentence out of two simple sentences do the following:</p> <ol style="list-style-type: none"> 1. Change period of last sentence to linking word or semicolon. 2. Put a comma before the linking word, if necessary. 3. Un-capitalize the first word of the sentence (unless it is a name noun). 	<p>Edit the following sentences to form one compound sentence:</p> <p>"I WENT TO THE THEATER. THE PLAY, 'A MAN FOR ALL SEASONS' WAS THE PRODUCTION."</p>	
	<p>To find the sum of two numbers with unlike signs, find the difference between their absolute values and give the number having the greatest absolute value.</p>	<p>$(+17) + (-32) = ?$ Difference between absolute values. $32 - 17 = 15$ Give sign of greatest number. ANSWER = - 15</p>	

Figure 14. Displays showing generalities and related instances.

of a particular referent. Note that the word 'instance' includes both referents and their representations. In instructional situations it is frequently necessary to use representation of referents rather than referents themselves.

For a rule an instance is a referent or its representation from each of the domain concepts, the operation as applied or to be applied to these referents, and identification of the range concept(s) label. Figure 14 includes instances for each of the generalities illustrated.

Effective instructional strategies often make use of nonexamples or counterexamples in presentations. It is therefore useful to distinguish examples from nonexamples in our representation of instructional strategies. For a concept, a counterexample is an instance of a related concept which shares some of the attributes of the concept being taught, and hence, it is probable that the student might incorrectly identify it as an example. For a rule several types of counterexamples can be constructed. One type of counterexample is an inappropriate operation which is similar (shares attributes with) to the operation being taught and hence, a source of possible confusion to the student. Another type of rule counterexample consists of various combinations of counterexamples for one or more of the domain concepts which the student might incorrectly use in applying the operation, hence, incorrectly using the rule.

Nonexamples or counterexamples are also recorded in position 3 of the flow chart display symbol using the following symbol, $\overline{\text{eg}}$. Displays sometimes contrast an example and a counterexample. For convenience it is often easier to record both an example (eg) and a counterexample ($\overline{\text{eg}}$) in the same display symbol rather than indicating two separate displays. Figure 15 illustrates some examples and counterexamples.

By definition, an identity involves a one-to-one correspondence so there is not a propositional representation of the referents involved. In other words, the generality and the instance are identical. When the display involves an identity, we will therefore adopt the convention of indicating the content mode as an example and will use the eg symbol in box number 3 of the flow chart symbol.

Response Mode

Generalities or instances can be presented in either an expository (to tell) or inquisitory (to ask) mode. Response mode is recorded in space 4 of the display flow chart symbol as illustrated in Figures 16 and 17.

Expository (E). If the primary purpose of the display is to present either a generality or an instance to the student without soliciting an overt response, it is said to be an expository presentation. Figures 16 and 17 illustrate several expository displays for both generalities and instances.

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	<p>Any straight line from the vertex to the circle of a cone is an ELEMENT.</p>	
	<p>A SIMILE is a figure of speech which employs the use of the words "LIKE" or "AS" to make a comparison between similar objects.</p>	
	<p>THIS IS A SIMILE "Fresh as the first beam glittering on a sail . . ."</p>	<p>THIS IS NOT A SIMILE "The trees are roots . . ."</p>

Figure 15. Examples and counterexamples related to generality statements.

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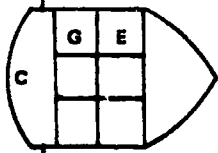
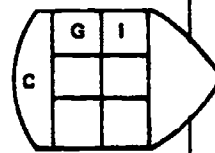
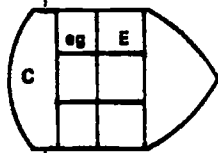

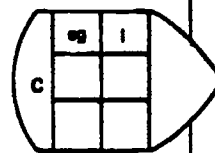

 <p data-bbox="282 564 526 645">A fault hanging wall is the wall on the block above an inclined fault.</p>	 <p data-bbox="638 564 842 614">Write the definition of a fault hanging wall.</p>
 <p data-bbox="361 866 559 897">A fault hanging wall.</p> 	 <p data-bbox="638 866 776 977">What kind of fault is represented in this drawing?</p> 

Figure 16. Expository and inquisitory displays involving a concept.

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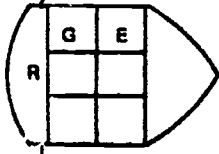
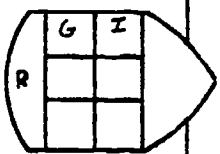
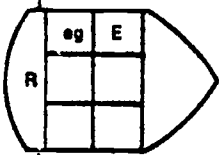
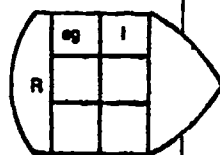
 <p>The product of a whole number and a fraction can be found as follows:</p> $m \times \frac{e}{d} = \frac{m \times e}{d} = \frac{me}{d}$	 <p>Explain how the product of a whole number and a fraction can be found.</p>
 $4 \times \frac{7}{17} = \frac{4 \times 7}{17} = \frac{28}{17}$	 <p>Find the product of the whole number and the fraction.</p> $4 \times \frac{7}{11} =$

Figure 17. Expository and inquisitory displays involving a rule.

An expository presentation of a concept generality would include the concept label (range) together with some descriptive operation and listing of the attribute (domain) concept labels. An expository presentation of a rule generality would include a stated proposition including domain concept labels, the operation, and range concept label(s). An expository presentation of a concept instance would include the concept label (range) together with an instance of the concept class. Note that this instance must contain instances of each of the attribute concepts. An expository presentation of a rule instance would include instances from each of the domain concepts, an application of the operation to each of these instances, and the resulting instance of the range concept(s).

In a previous section it was indicated that response conditions are part of every display. However, the definition of an expository display indicates that an overt response is not required of the student. To adequately process the input necessary, even in an expository display, the student should know how he will be required to respond to the information in the future. If this direction is not explicit then the student must infer how he will be asked to respond. Therefore, one potential part of an expository display is some indication to the student concerning how he will be asked to respond. In many instructional situations this direction is given prior to a series of displays (e.g., the student is given a behavioral objective) rather than on every display, thus enabling one to indicate the response conditions for expository displays.

When response conditions are not indicated to the student as part of an expository display or prior to a series of expository displays, it is sometimes possible to infer the response conditions that the student will assume. When response conditions are inferred rather than expressly stated in the instructional materials, the symbol in part 2 of the display symbol indicating response conditions should be put in quotation marks (e.g., "DR", "CL", etc.).

Inquisitory (I). If the primary purpose of the display is to solicit an overt response from the student, it is said to be an inquisitory presentation. Figures 16 and 17 illustrate several inquisitory displays for both generalities and instances.

An inquisitory presentation for a concept generality would present the attribute labels with the descriptive operation (the definition) and ask the student to supply the concept label. Or it would present the concept label and ask the student to provide the attribute labels and the descriptive operation (the definition). An inquisitory presentation for a rule generality would present a label for the rule and have the student state the proposition consisting of the domain concept labels, the operation, and the range label. Or it could present the proposition and have the student provide the identifying label. Or it could present various subsets of the domain labels, the operation, or the range label and have the student supply the missing elements. Whenever the student has previously been exposed to the generality under question, the conditions are such that the response level which can be inferred is discriminated recall. This is not the case when the student is asked to "discover" the rule after having been presented a series of instances.

The conditions for response should usually be indicated or apparent as part of a complete inquisitory display. Thus for most inquisitory displays the determination of response conditions is much less problematical than for many expository displays.

Figure 18 indicates that the combination of content mode and response mode makes possible four primary presentation forms. A sequence of these four forms is the skeleton of any instructional strategy. One definition of an instructional strategy is that an instructional strategy is a sequence of primary presentation forms... The dots indicate that this is not a complete definition. The following sections indicate that attached to these basic presentation forms are variables related to representation and the use of mathemagenic information. We have already indicated that a presentation form relates to a given identity, concept, or rule and implies a given level of response.

Content Representation

Bruner (1966) suggested that learners can internally represent the world in three distinct ways: enactive, iconic, and symbolic. Enactive refers to action or response. It is the means by which the student represents psychomotor skills in his cognitive storage. Iconic refers to images or pictorial representations. Iconic representation is governed by those relationships which have been observed for perception (Attneave, 1954). Symbolic representation refers to use of various languages and symbolic structures to represent the world.

BEST COPY AVAILABLE**HOW WAS IT PRESENTED?****EXPOSITORY INQUISITORY****GENERALITY****EG
"RULE"****IG
"RECALL"****WHAT IS PRESENTED?****INSTANCE****Eeg
"EXAMPLE"****Ieg
"PRACTICE"**

Figure 18. Primary presentation forms.

It is suggested by the authors that quite independent of the learner a given generality or instance as presented by a single display can be represented in a variety of ways that correspond to some degree to the meanings given these terms by Bruner. It should be remembered that while there may be some correspondence between the internal cognitive processes of the learner and the characteristics of the stimulus materials as presented in the display, our concern in this paper is with the latter.

The representation mode used for a given generality or instance is indicated in space 5 of the display flow chart symbol as illustrated in Figure 19.

Enactive (O). When the display presents the referent itself in such a way that the student can manipulate the attributes of the actual objects involved, the representation is said to be enactive. The flowchart symbol "O" was selected to stand for "object" and to avoid confusion with "E" which was used for an expository response mode. Figure 19 uses an iconic display to illustrate enactive displays.

When the referents are symbols, such is often the case in mathematics, it is possible and perhaps desirable to use an inverse form of representation. Objects are used to represent the symbols or abstract entities and students are allowed (required) to manipulate these objects which now represent symbolic referents. The use of Cuisenaire rods is an example of such inverse representation. Such enactive representation of abstract ideas is likely to result in considerable instructional improvement especially in instructing young children who have not yet acquired all of the formal processing strategies that characterize a mature learner. Figure 20 illustrates inverse enactive representation.

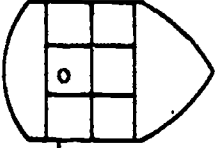
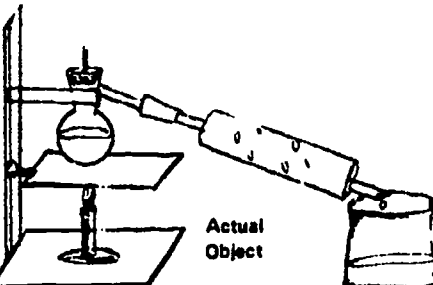
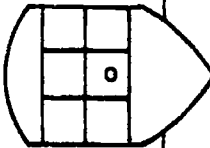
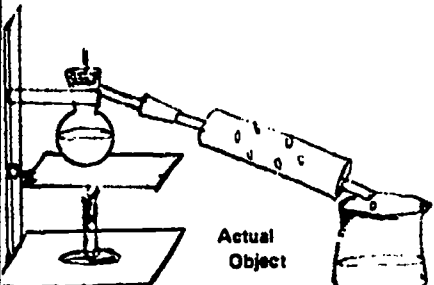
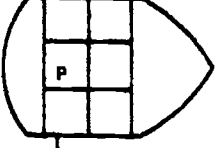

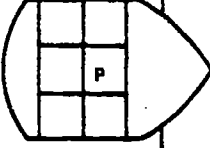
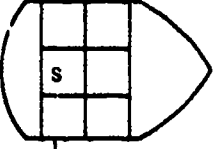
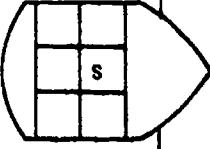
 <p>Here is an EVAPORATION- CONDENSATION APPARATUS.</p>  <p>Actual Object</p>	 <p>CONDENSE 50 ml. of liquid.</p>  <p>Actual Object</p>
 <p>This is an Inclined Fault.</p> 	 <p>Draw a general diagram of an inclined fault.</p>
 <p>Straight lines in the same plane which do not intersect or meet no matter how far they are extended in either direction are PARALLEL.</p>	 <p>Define the term "PARALLEL":</p>

Figure 19. Displays illustrating content and response representation.

IT WOULD TAKE
4 WHITE RODS TO
BE AS LONG AS THE
BLACK ROD.

WHICH NUMBER
CORRESPONDS
TO THE BLACK
ROD?

1 2 4 6

NOTICE THAT THE REPRESENTATION
IN EACH INQUISITORY EXAMPLE IS
REVERSED FROM THE EXPOSITORY
EXAMPLE.

THE RED ROD
IS 4 TIMES THE
WHITE ROD.

FIND THE ROD
WHICH IS FOUR
TIMES THE WHITE
ROD.

Figure 20. Displays involving inverse representation.

While instances are most easily presented via an enactive representation it is also possible to represent the generality enactively. In many cases, especially where the generality is a proposition for some abstract relationship, a form of inverse representation may also be used.

Iconic (P). When the referents for a particular concept or rule are represented by some graphic or symbolic representation such that the attributes can be viewed but not manipulated, then the representation is said to be iconic (pictorial). The flow chart symbol "P" was selected to stand for pictorial and to avoid confusion with "I" which was used for inquisitory response mode. Iconic representations are not limited merely to pictures, however, but include a wide range of possibilities such as models, diagrams, drawings, photographs, video tapes, or motion pictures. Figure 19 illustrates iconic representations.

Symbolic (S). When referents or the relationship between referents are represented via language or some other symbol system such that the attributes can be neither viewed nor manipulated, the representation is said to be symbolic. There is sometimes confusion as to whether a given representation is iconic (e.g., in the case of a diagram) or symbolic. If the main function of the illustration is to show the object or event as close to the referent as possible--that is, there is a degree of isomorphism between the representation and the real world referent--then the representation is classed as iconic. If, however, there is little or no isomorphism between the referent and the symbolic representation then the

representation is classed as symbolic. As with other display characteristics content representation represents a continuum rather than a set of discrete categories. Thus in particular cases there may be confusion as to which category is most appropriate. Figure 19 illustrates symbolic representations.

Response Representation

A given display not only presents content material that is represented in a particular way but it also requires (in inquisitory mode) or implies (in expository mode) a response representation on the part of the student. In other words, when the student answers the question or processes the information for later retrieval, he must know how he is to represent his response. Is he to make a symbolic response, prepare some diagram or picture to represent his response, or must he perform some physical manipulation of the referents involved? Response representation is recorded in space 6 of the display flow chart symbol as illustrated in Figures 19 and 20.

Enactive (O). A given display requires an enactive response representation if the student is requested to manipulate referents in response to an inquisitory display or if he anticipates that he will be asked to manipulate referents at some future time as a result of an expository display. Figure 20 illustrates several displays which request an enactive response representation.

Iconic (P). A given display requires an iconic response representation if the student is requested in some way to prepare a diagrammatic or pictorial representation of some referent or interaction of referents in response to an inquisitory display or anticipates that he will be asked to prepare such an iconic representation at some future time as a result of an expository display.

Symbolic (S). A given display requires a symbolic response representation if the student is requested to provide a symbol or set of symbols in response to an inquisitory display or anticipates that he will be asked to provide a symbol or set of symbols at some future time as a result of an expository display. Figure 20 also illustrates symbolic response representation.

As with response mode on expository displays, response representation is not always clearly indicated to the student. In many cases it may have been indicated earlier in the instructional sequence as part of an objective or other directions to the student regarding the expectations of the system and the instructional materials. When response representation is inferred rather than expressly stated in the instructional materials, the symbol in part 6 of the display flow chart symbol should be put in quotation marks (e.g., "O", "P", "S").

Mathemagenic Prompting

The display characteristics described to date have all had something to do with the content of the presentation or the student's response to that content. Rothkopf (1965) coined the phrase "mathemagenic behavior" as a label for those behaviors that give

rise to learning while a student is processing information in an instructional situation. The authors have borrowed the term **mathemagenic** and applied it to information which is added to an instructional display which is believed to facilitate or give rise to student processing behaviors resulting in more efficient or effective learning. Mathemagenic information, therefore, consists of information which is added to the content information for the express purpose of directly or indirectly facilitating a learner's processing of the content information. Each of the various types of mathemagenic information which is applied to the content information in either an expository or inquisitory display is coded in part 7 of the display flow chart symbol. /

Mathemagenic information can be provided either before or after a student responds to a given display. When mathemagenic information is provided to an expository display or when it is provided on an inquisitory display prior to a student's responding to the display it is called mathemagenic prompting. It is necessary to distinguish mathemagenic prompting from the type of prompting which has been frequently studied and reported in the research literature (see Anderson, 1967 for a review of some of this research). The prompting which has often been studied has involved primarily the learning of identities (often word pairs or nonsense syllable pairs), and the prompting has consisted of giving the learner the response prior to his responding. This might be called response prompting. In our variable system such a prompt would change the

display to an expository display. Mathemagenic prompting refers rather to stimulus prompting. That is, the emphasis is on calling the student's attention to salient features of the attributes rather than giving him the response.

None (NO). Unlike each of the other display variables described thus far which must take some value other than zero in every display, mathemagenic prompting may or may not be present. When no facilitating directions or information is provided with a given display this variable is coded "NO" in part 7 of the display flow chart symbol. Figure 21 illustrates an unprompted display.

Mnemonic (mn). A mnemonic is some form of memory aid thought to facilitate later recall of some specific information. It is believed that mnemonics have particular value for helping the student retrieve specific generalities or identities. Mnemonic mathemagenic prompting is illustrated in Figure 21.

Attribute isolation (ai). Attribute isolation is the use of attention focusing devices designed to call the student's attention to the attributes of a concept. It is proposed that attribute isolation is most relevant in clarifying instances of a concept. Figure 21 illustrates the use of attribute isolation.

Algorithm (al). An algorithm might be defined as a rule for using a rule. Usually an algorithm is a step-by-step analysis of the process involved in identifying instances of a given concept or in finding the instances of the range concept(s) in a given rule using task. Figure 22 illustrates the use of algorithms.

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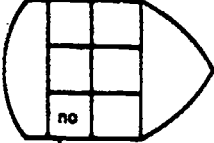
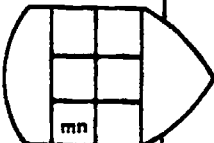
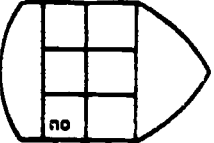
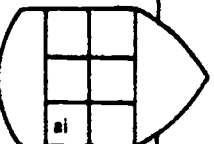
 <p>BOF XAJ MIB ZYQ</p> <p>(List of nonsense syllables)</p>	 <p>BOF "BOF... XAJ exaggerates his ... MIB misery because ... ZYQ he is not sick."</p> <p>(Mnemonic aid added) [Adopted from MILLER, GALANTER, and PRIBRAM (1960)]</p>
 <p>The following line is classified as trochaic meter:</p> <p>"Out of childhood into manhood ..."</p>	 <p>The following line is classified as trochaic meter:</p> <p>"OUT of CHILDhood INTO MANhood ..."</p> <p>(Stressed syllables are all in caps.)</p>

Figure 21. Displays illustrating mathemagenic prompting--mnemonic and attribute isolation.

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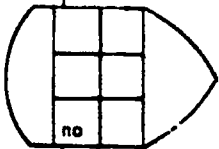
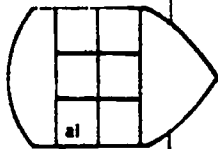
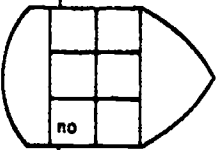
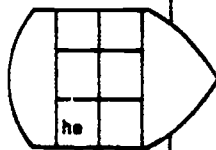
	<p>Since lines in the same plane which do not intersect are parallel, the distance between parallel lines is everywhere the same.</p>		<p>Since lines in the same plane which do not intersect are parallel, the distance between parallel lines is everywhere the same.</p>
	<p>Since lines in the same plane which do not intersect are parallel, the distance between parallel lines is everywhere the same.</p>		<ol style="list-style-type: none"> 1. Construct a perpendicular between two lines (A). 2. Construct another perpendicular at another point between the two lines (B). 3. Does $A = B$? \longrightarrow YES <p>NO \swarrow</p> <p>LINES ARE NOT PERPENDICULAR</p> <p>LINES ARE PERPENDICULAR</p>
			<p>Since lines in the same plane which do not intersect are parallel, the distance between parallel lines is everywhere the same.</p> <p>Is the distance between the two lines everywhere the same?</p>

Figure 22. Displays illustrating mathemagenic prompting--algorithm and heuristic.

Heuristic (he). A heuristic consists of an incomplete algorithm. In other words, giving the student hints for some useful procedures to try comprises a heuristic. Heuristics are most useful, it is proposed, when a student is involved in a rule finding task. Figure 22 illustrates the use of heuristics.

Using the definitions given for concepts, operations, and rules enables a more formal analysis of the relationship between attribute isolation, algorithms, and heuristics.⁹

Since a concept is defined by a rule involving a descriptive operation, then classification behavior involves using this rule. In most concept tasks, several attribute concepts are involved in the domain of the concept rule definition. In many concepts it is necessary to identify the instances of these domain concepts in a particular order if one is to determine whether or not an instance of the concept is present. This ordered procedure for applying rules is what is meant by an algorithm.

Perhaps the following incomplete algorithm for indicating whether or not a given passage of poetry is an instance of a given type of metric foot will illustrate the above. In this example

⁹The authors are indebted to their colleague, Rowland Blake, for this formal analysis of attribute isolation. Blake is presently preparing a lengthy monograph on attribute isolation and its formal analysis. Readers interested in further information on this work should contact him at the following address: Division of Instructional Resources, Central Michigan University, Mt. Pleasant, Michigan 48859. Blake has also worked out the complete algorithm for scanning poetry. Its presentation here would have unnecessarily lengthened this paper.

we have not defined the domain concepts and the operation involved in each step but have merely described the range and provided an example. Consider the following:

Out of childhood into manhood now has grown my Hiawatha. In order to scan this line and classify the metric foot involved, it is necessary to complete the following rule-using steps.

Step 1. Divide the line into separate syllables.

Range: Out·of·child·hood·in·to·man·hood...

Step 2. Determine those syllables which are stressed.

Range: OUT·of·CHILD·hood·IN·to·MAN·hood...

Step 3. Divide the line into poetic feet.

Range: OUT·of·/ CHILD hood·/ IN to·/ MAN·hood...

Step 4. Identify which stress pattern is used.

Range: Trochaic (A stressed syllable followed by an unstressed syllable).

This algorithm can be represented abstractly as shown in Figure 23.

Note that the range of one step becomes a significant part of the domain for the next step. If domain concepts and operations had been specified it could be observed that this ordering of steps is necessary in order for the learner to scan a given line of poetry. It should also be noted that the algorithm given is in its simplest form. Most poetry involves a number of exception steps which complicates the algorithm. A complete algorithm has been identified which can unambiguously determine meter for any given passage.¹⁰

¹⁰Ibid.

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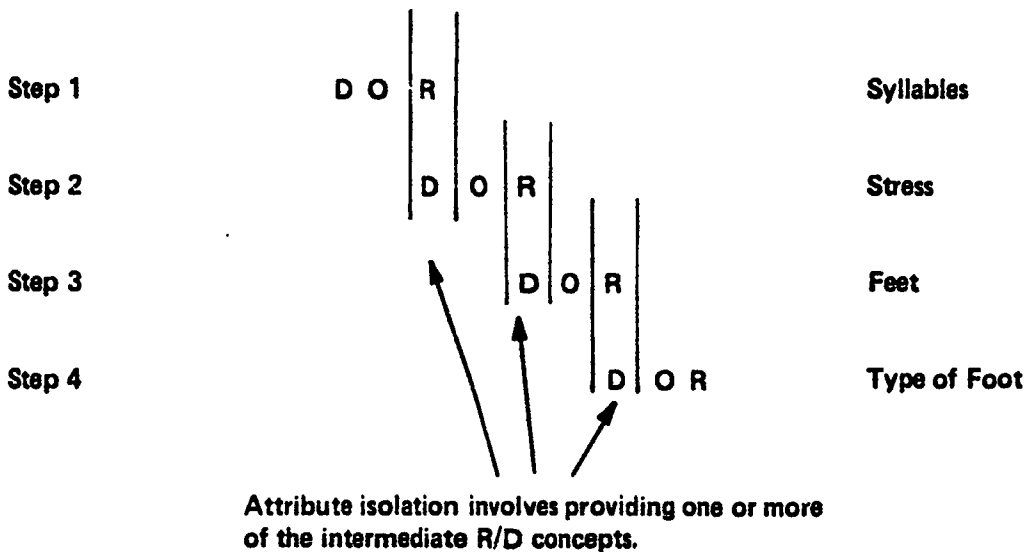


Figure 23. Algorithm for determining type of poetic meter.

Using this illustration it is possible to more formally define attribute isolation, algorithm, and heuristic. First, an algorithm is the presentation of the ordering operation for using a more general operation. Almost all rule using tasks (including classification) involve an algorithm. These algorithms are only rarely explicitly taught. When an algorithm of this type is explicitly taught it is classed as mathemagenic prompting.

Attribute isolation, in terms of this analysis, consists of providing the student the range value for one or more of the preceeding steps for a specific instance as indicated in Figure 23. In other words, giving the student any of the range instances of steps 1 through 3 of the above example constitute varying degrees of attribute isolation for this particular instance of poetry.

It should be obvious that providing the range of the fourth step makes the example an expository frame (an instance of response prompting). In another sense, however, all attribute isolation is response prompting. It consists of providing preceeding range values (responses) earlier in the algorithm.

When not all of the component operations or rules are known in a given task, it is not possible to provide an algorithm. By definition an algorithm should work (provide the range) in every case. Providing an incomplete algorithm is the formal definition of a heuristic. Space prevents a more detailed discussion of heuristics in this paper.

One final note is that this detailed analysis of mathemagenic prompting also has implications for sequencing concepts and rules. This paper has only vaguely touched on content sequence and the whole area of content analysis. But the reader should begin to see the close relationship between strategy analysis and content analysis.

This analysis is necessarily sketchy. An adequate treatment would require another lengthy paper. Such a paper is currently in preparation by Blake.¹¹

Mathemagenic Feedback

Prompting was defined as providing information to the learner prior to his responding; feedback is providing information to the learner following his response. After a student has responded several types of information can be provided. He can be told that he was right or wrong, he can be provided with the correct answer, or his attention focusing devices can be provided with his response to help him see why his response was correct or why it was incorrect. This information added to the student's response is called mathemagenic feedback. Like mathemagenic prompting it is information which has been added to the content information for the express purpose of facilitating the learning process.

None (no). Like mathemagenic prompting it is possible to provide no additional facilitating information. When no information is provided this is indicated by the word "no" in the eighth space of the display flow chart symbol.

¹¹ Ibid.

Right/wrong knowledge of results (r/w). This is merely the message that the student's response was right or wrong without additional information (see Figure 24).

Correct answer knowledge of results (ca). The problem is worked for the student showing him the correct answer. Correct answer knowledge of results may be provided with or without right/wrong knowledge of results. When both are used, both symbols should be placed in the eighth space of the flow chart symbol (see Figure 24).

Attribute isolation (ai). Attention focusing devices can be added to the student's responses to help him see why his answer was correct or incorrect and to prompt future responses to similar instances. Attribute isolation as feedback information is very similar to attribute isolation in prompting mode (see Figure 24).

Algorithm (al). In reference to the student's response, he is shown the algorithm for using the rule involved. This is very similar to providing an algorithm in prompting mode except that the student's response is used as the instance or part of the instance of the algorithm (see Figure 25).

Heuristic. Providing the student a heuristic using his response as an instance or part of an instance of the heuristic.

The various types of mathemagenic feedback can be used in combination with the two types of K of R (knowledge of results). When such combinations are used they should be indicated by the appropriate symbols in the flow chart symbol (see Figure 25).

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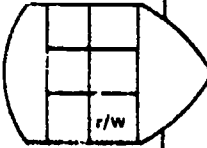
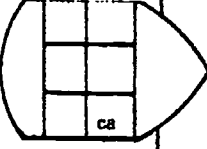
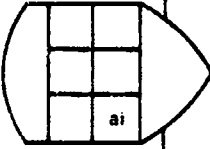
<p>STUDENT RESPONSE</p> $4 \times \frac{7}{17} = \frac{7}{4 \times 14}$ <p style="text-align: center;">↑ (student response)</p>	 $4 \times \frac{7}{17} = \frac{7}{4 \times 14}$ <p style="text-align: center;">"wrong"</p>
<p>STUDENT RESPONSE</p> $4 \times \frac{7}{17} = \frac{7}{4 \times 14}$ <p style="text-align: center;">↑ (student response)</p>	 $4 \times \frac{7}{17} = \frac{7}{4 \times 14}$ $= \frac{28}{17}$ <p style="text-align: right;">correct answer</p>
<p>STUDENT RESPONSE</p> $4 \times \frac{7}{17} = \frac{7}{4 \times 14}$ <p style="text-align: center;">↑ (student response)</p>	 <div style="border: 1px solid black; padding: 5px; display: inline-block;"> 4×7 <p style="margin: 0;">in numerator</p> $4 \times \frac{7}{17} = \frac{7}{4 \times 14}$ $= \frac{28}{17}$ </div>

Figure 24. Student responses and accompanying mathemagenic feedback displays.

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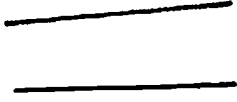
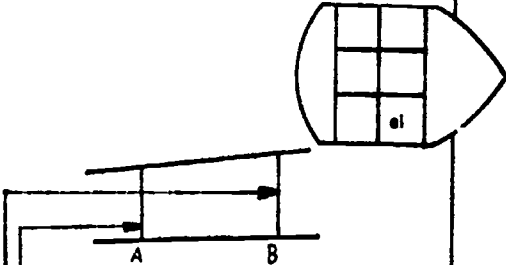

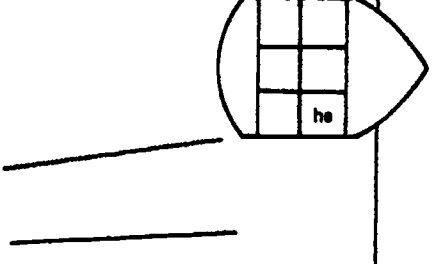
<p>STUDENT RESPONSE</p>  <p>(Student draws lines that are to be parallel.)</p>	 <ol style="list-style-type: none"> 1. Perpendicular at point A. 2. Perpendicular at point B. 3. $A < B$ 4. Lines are not parallel.
<p>STUDENT RESPONSE</p>  <p>(Student draws lines that are to be parallel.)</p>	 <p>Is distance between lines everywhere the same?</p>

Figure 25. Displays illustrating mathemagenic feedback--algorithm and heuristic.

Interdisplay Relationships

In the "Display Characteristics" section of this paper those variables and broad parameter values which characterize a given display were described. When a series of displays are arranged in sequence three additional classes of variables are introduced. These are quantity, sequence, and qualitative relationships between variables.

Quantity

Quantity is the easiest to describe in that this is a parameter which specifies how many, and which applies to each of the display characteristic variables previously described. Specification of quantity provides values for questions like the following: How many concepts or rules should be included in a given instructional session or in a given lesson? How many instances for a given generality? How many nonexamples or counterexamples for a given generality? Supplying values for this parameter does not usually change the nature of the characteristics involved in a given display.

Sequence

Sequence refers to order. This parameter also can be applied to any two or more instructional displays, but like quantity does not usually change the nature of the display characteristics. However, unlike quantity there are many sequence patterns that can be simultaneously or sequentially manipulated as part of an instructional strategy. Which concept or rule should be presented first, which second, etc., is a content structure sequence question. This

particular question is critical in the content structure facet of the instructional process. In fact, the authors would hypothesize that this may be the most crucial question related to instructional effectiveness. It may be that an inadequately sequenced content structure provides a limit on the learning outcome of a given instructional strategy regardless of what is done in manipulating the other strategy variables described in this paper.

Other sequence questions within a given content structure deal with the sequence of basic presentation forms (expository generalities, expository instances, inquisitory generalities, inquisitory instances). Manipulating this sequence parameter produces the types of treatments which have been referred to by such labels as discovery learning, expository presentations, guided discovery, and inquiry training. It is suggested by the authors that if the investigators or advocates of these various types of strategies would describe their particular strategy in terms of a specific sequence of presentation forms, there would be much less confusion in the outcome of research comparing these strategies.

Other sequence questions deal with the presentation of instances. The simultaneous versus sequential presentation of generalities with instances or instances with instances fall in this category. Questions concerned with the ratio of examples to non-examples are a combination of sequence and quantity parameters.

Other sequence questions involve the use of mathemagenic prompting versus mathemagenic feedback; the sequencing of changes in

representation for given generalities or sets of instances; the sequence of response level conditions--i.e., should a student first recall a definition and then use the definition to classify unencountered instances, or find the domain instance in rule tasks?

Qualitative Relationships

Quantity and sequence constitute parameters which can be applied to the display characteristic variables previously identified. Interdisplay relationships, however, include another class of variables which are not merely manipulations in the quantity and sequence of display characteristic variables, but which qualitatively change the nature of these display characteristic variables. In our previous analogy with a chess game, display characteristics represented the pieces and interdisplay relationships represented the moves.

This paper is already lengthy and a complete specification of these interdisplay relationships would require a considerable number of additional pages. Furthermore, the authors have only just begun to identify the variables and parameters involved, so a more complete report would be premature. What follows is to be considered illustrative of some possible variables involved rather than to be considered comprehensive. In a sense our work is like a primitive version of the periodic table in chemistry. We haven't identified all of the elements but our analysis to date convinces us that there are some variables which exist which will have certain specified characteristics when they are identified.

It is suggested that there are qualitative relationships between each of the display characteristic variables which have been identified. In some cases there may be several such variables. It is manipulation of these interdisplay variables which gives a particular instructional strategy its power and its unique character. It is proposed that the manipulation of these interdisplay variables are those factors which affect instructional effectiveness and efficiency for a given learner in a given instructional situation.

In outline form some of the interdisplay variables yet to be adequately identified include the following:

Content structure variables. These are qualitative relationships between concepts and rules in a given content structure. For example, a concept which is part of the domain of a given rule is qualitatively different from a concept which is one of several parallel concepts. A scaffolding rule designed to help a student understand some higher order rule is qualitatively different from the higher order rule or from a series of equal status rules. The challenge is to identify the nature of these relationships and the values which their principal parameters can assume.

Response conditions variables. These are qualitative relationships between various levels of response. For example, a series of independent discriminated recall tasks are qualitatively different from a discriminated recall task which is prerequisite to some classification task. How should this difference be characterized? What are the parameter values involved?

Content mode relationships. This relationship has been chosen for more detailed analysis and is discussed in detail below.

Response mode variables. These are qualitative relationships between inquisitory and expository frames. For example, an inquisitory display which builds on information present in a previous display is qualitatively different from a relatively independent inquisitory frame. What is the nature of this qualitative difference? How can it be characterized? What are the parameters involved?

Representation variables. These are qualitative relationships between various enactive, iconic, and symbolic displays. For example, an iconic presentation of content which has been presented symbolically in a previous display is qualitatively different from an independent display. What is this difference? What are the parameters involved?

Mathemagenic information. The same mathemagenic prompting display is quite different if in one case it is one of a series of displays which are slowly eliminating the facilitating information as compared to the same information presented only once. What is the nature of this difference? Can the relevant parameters be identified?

Interactive relationships. In addition to the simple relationships briefly outlined there are also possibilities for many interactive relationships. For example, is an iconic representation following an inquisitory frame qualitatively different from the same display following an expository frame? There are a large number of other possible combinations.

The authors believe that a simple variable system can be identified which will reduce these many qualitative relationships to a manageable number of variables and parameters. It is also believed that the identification of these variables will enable instructional designers and instructional researchers to more adequately describe their instructional strategies and to more adequately test empirical relationships in ways that are far less ambiguous than the current loosely-formulated descriptions of instructional treatments.

Content mode relationships -- an illustration. There are several possible relationships between content mode parameter values of generality and instance. These include: the relationship of a generality to a generality, the relationship of a generality to an instance, and the relationship of one instance to another instance.

One generality-to-generality variable is generality scope. One of the parameters of this variable can take the values of restricted (R) and parallel (P). One generality is more restricted than another when the set of objects, symbols, or events referenced are a subset of the set referenced by the more general generality. Two generalities are parallel when both are at the same level of generality, neither being a subset or partial subset of the other. As with many of the other parameters identified in this paper, there is a continuum of values involved rather than the simple dichotomy indicated. The symbolic representation for generality scope is indicated in the third space of the interrelationship flow chart symbol as illustrated in Figure 26.

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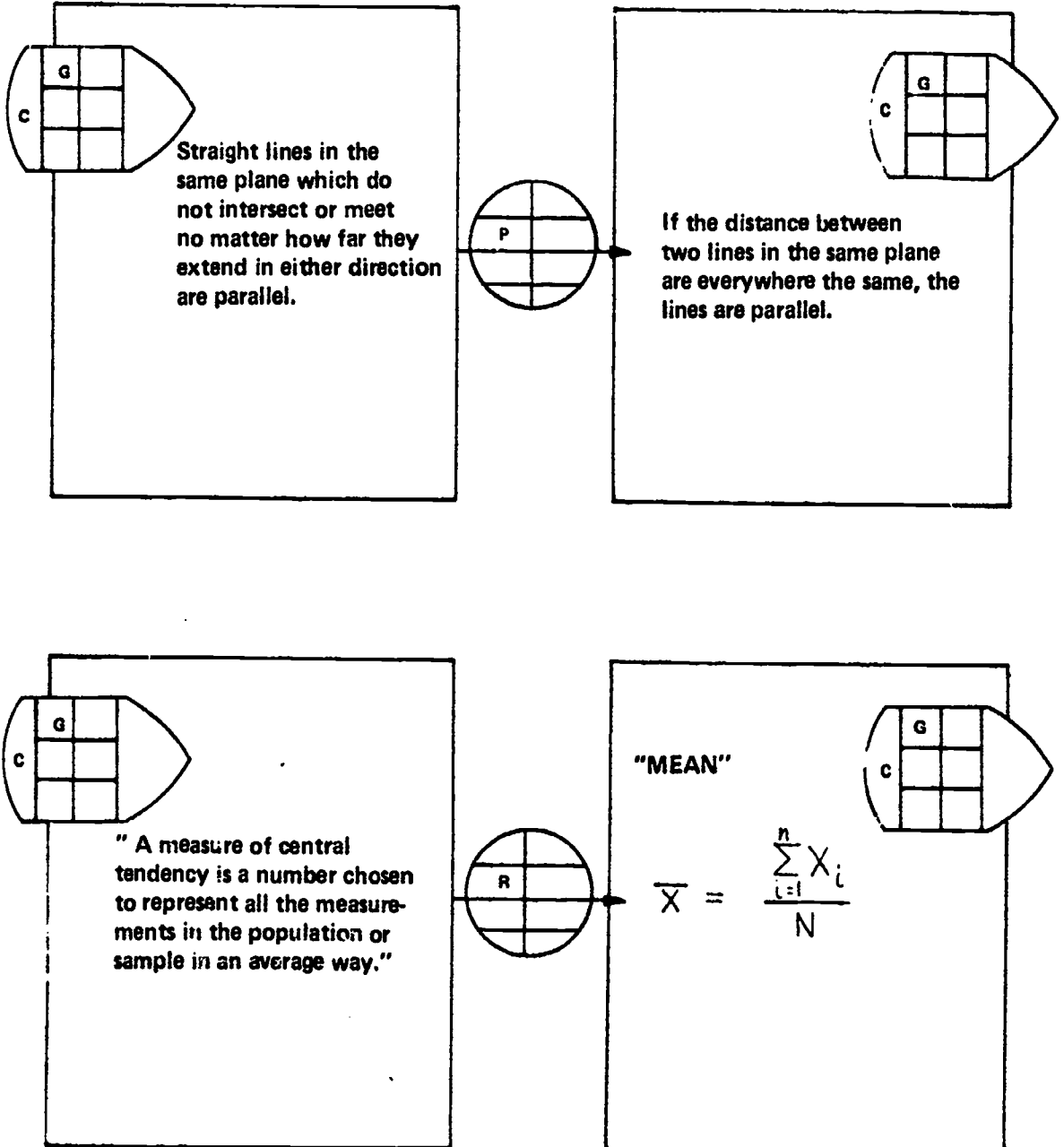


Figure 26. Displays illustrating generality scope.

One of the generality-to-instance variables is instance scope. One of the parameters of this variable can take the values within scope (w) and extra scope (x). A within scope instance is one which possesses the attributes identified by the stated generality. An extra scope instance possesses attributes of a more general generality (less restricted) but may not possess attributes as specified by the restricted generality. Instance scope and the flow chart symbolic representation are illustrated in Figure 27.

One of the instance-to-instance variables is attribute matching. This variable involves three subvariables: the matching of an example to an example of an example to a nonexample and of a nonexample to a nonexample. For most situations only the first two are of interest. One parameter of this attribute matching variable can take the following values: Matched (m) -- the irrelevant attributes of the two instances are as similar as possible. For two examples this would mean that the two examples were of the same type and resembled each other. For an example and a nonexample, this would mean that the nonexample was as similar to the example as possible without sharing the critical attributes. Divergent (d) -- the irrelevant attributes are as different as possible. In this case, two examples would be as different as possible and still belong to the class under consideration. Random (r) -- instances are selected either haphazardly or based on random occurrence rather than systematically arranged. The matching relationships are unspecified in a random attribute matching situation.

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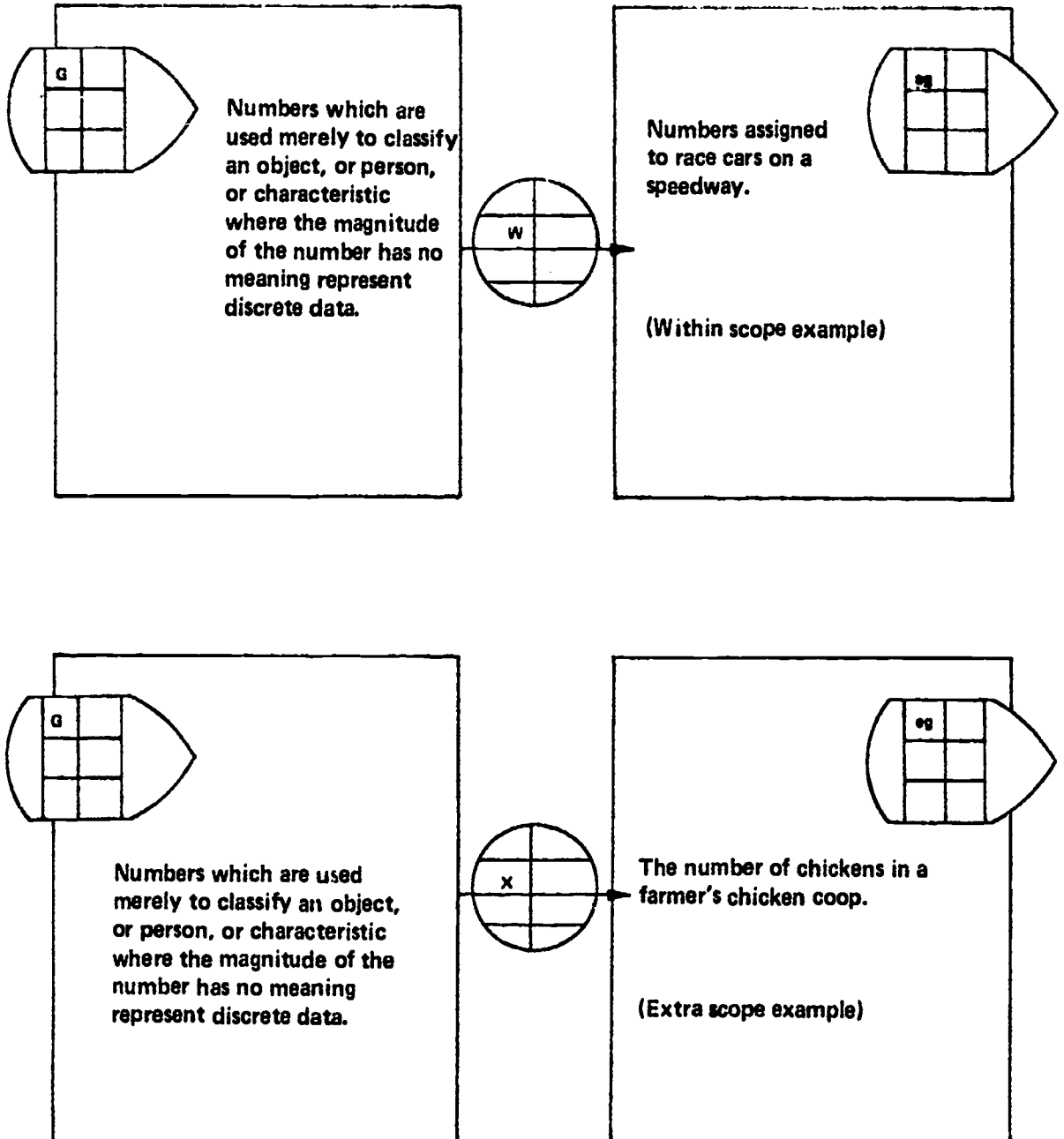


Figure 27. Displays illustrating instance scope.

Research by the senior author and his associates has demonstrated that for teaching concepts (descriptive operations), instructional strategies which use matched example - nonexample pairs and divergent example - example pairs are more effective than any of the other possible combinations on these variables (see Tennyson, Merrill, and Woolley, 1972; Tennyson, 1973). Figure 28 illustrates matched example - nonexample pairs and divergent example - example pairs.

The reader should be able to detect that much more could be said about interdisplay relationships. Hopefully these few paragraphs have indicated the nature of this area of investigation and has, in a rough way, indicated the boundaries of such an investigation. The authors are currently working on further specification of interdisplay relationships so that the instructional strategy taxonomy promised in the opening paragraphs of this paper might someday be realized. We invite critical comments of all who read and from others working in the same field of labor.

Directional Displays

As the authors have attempted to apply the above variables in the description of existing instructional materials they have become aware that a complete description of an instructional strategy requires the identification of another class of display not subsumed by the previous variables. These displays serve at least three distinct functions: providing directions to the student which are designed to direct his cognitive processing of the information presented in the remaining displays; providing the student with

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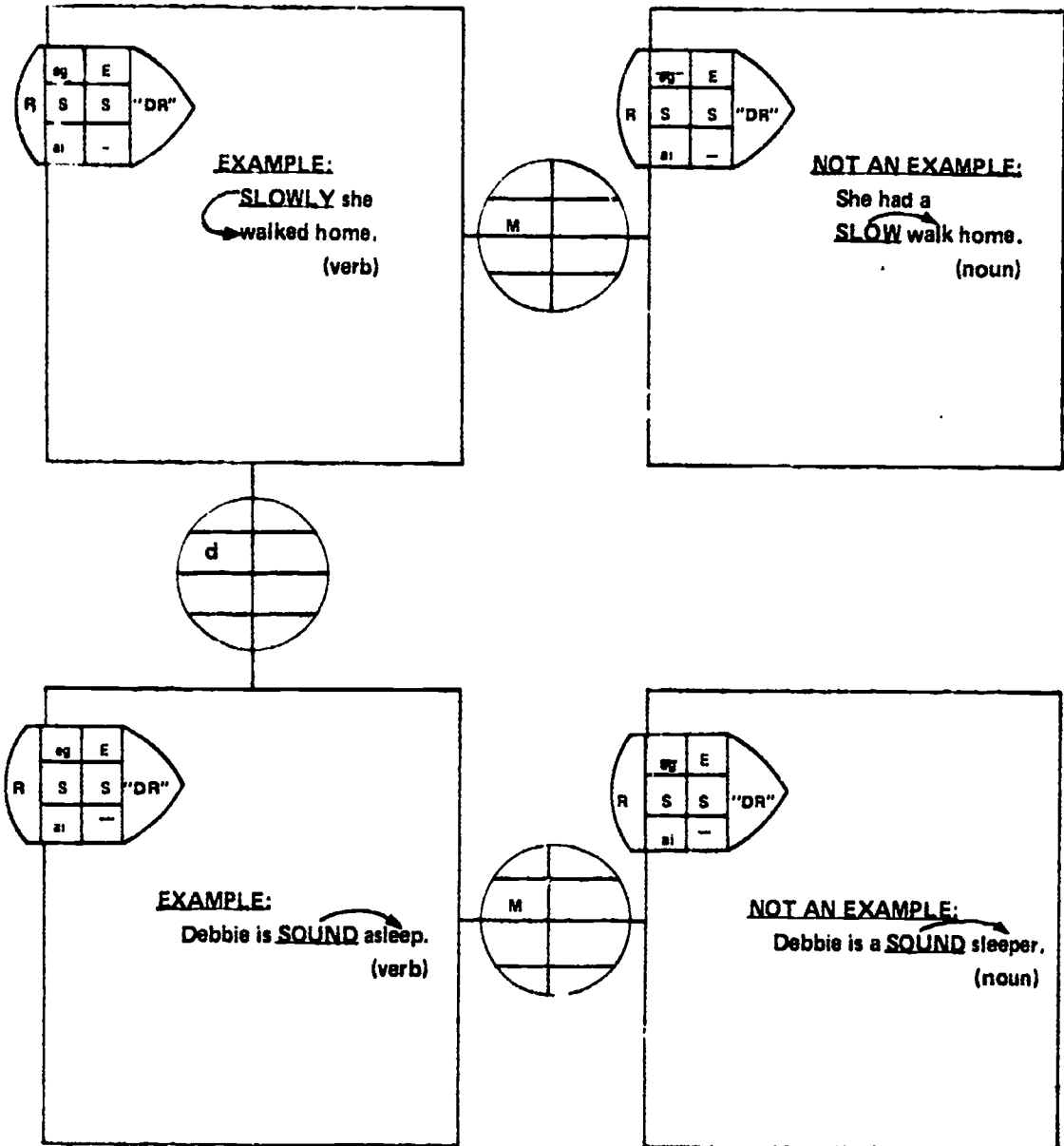


Figure 28. Displays illustrating matched example - nonexample and divergent example - example pairs.

directions for manipulating the apparatus constituting the particular instructional system on which he is working; and displays which contain little or no content information and little or no processing or procedural directions, but provide for smooth transitions from one part of the instructional sequence to another.

The authors, while having identified these three types of directional displays, are not yet prepared to suggest definitive variables which can be used to characterize such displays. In our analysis of existing materials, directional displays are indicated with a flow chart display symbol undifferentiated as to variables and parameter values, but with one of the following words enclosed: "Process" -- for those displays directing the cognitive processes of the student; "Procedure" -- for those displays telling the student how to run the system; and "Glue" -- for those displays which seem to have only a transitional function to facilitate moving from one part of a strategy to another. Figure 29 illustrates these directional displays.

APPLICATION

In the final paragraphs of this paper we would like to suggest some of the reasons we feel that the development of a definitive category system for instructional strategy variables is desirable. In addition, we would like to provide a couple of sample analyses of instructional strategies so that the reader can observe the application of the flow chart system.

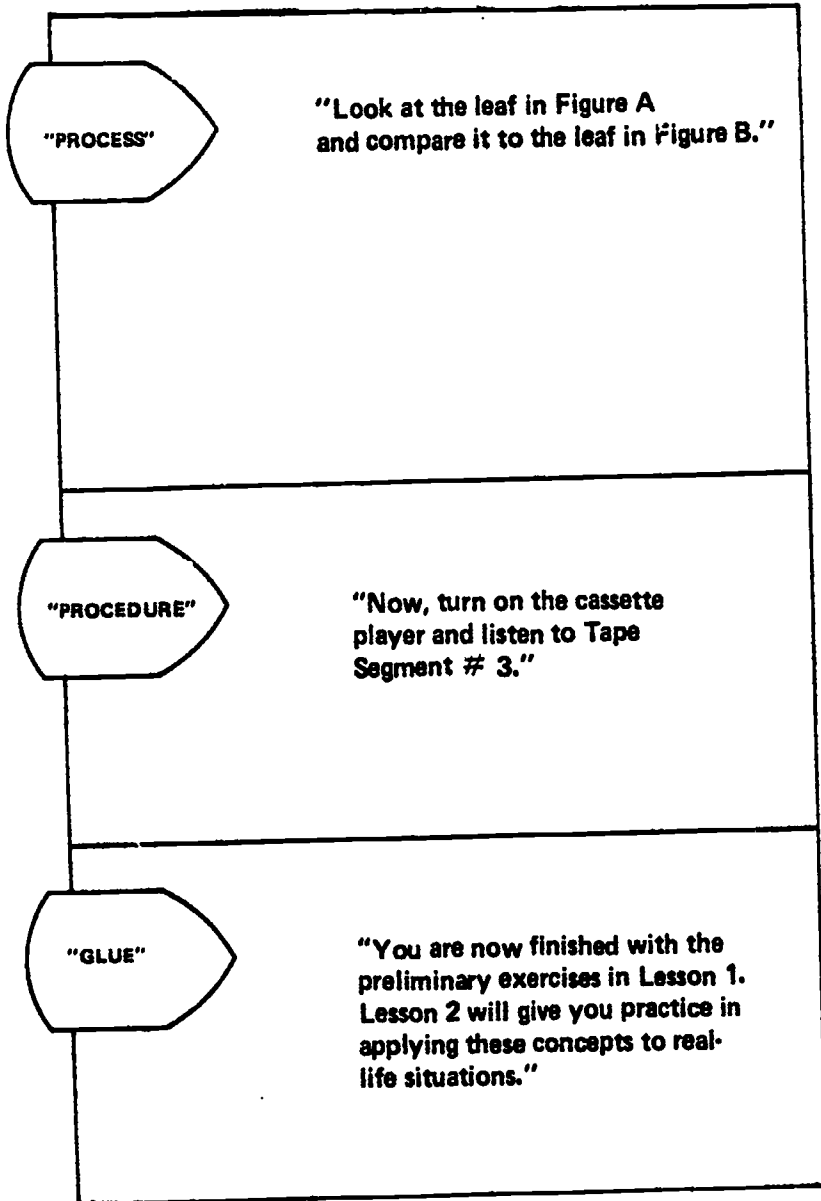
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Figure 29. Illustrative directional displays.

Theory Construction

As suggested in the opening paragraphs a carefully conceived taxonomy is a necessary first step toward the construction of an adequate instructional theory. The intent of the authors is to work toward the development of such a theory which can guide the development of instructional materials and the design of instructional systems in such a way that the materials and systems so designed are more effective and efficient than strategies based on folklore, artistic creativity, and empirical validation alone.

The taxonomy can have an initial value in the construction of such a theory by enabling us to have a vocabulary with which to describe the treatments involved in experimental investigations of the instructional process (with much less ambiguity than has hitherto been the case). One of the difficulties in reviewing existing research literature, is the fact that when complex instructional strategies are involved it is almost impossible to determine how the strategy was constructed. If investigators would flow chart their strategies using the variables and flow chart conventions suggested, it would be much easier to determine the type of strategy involved and to observe the treatment differences.

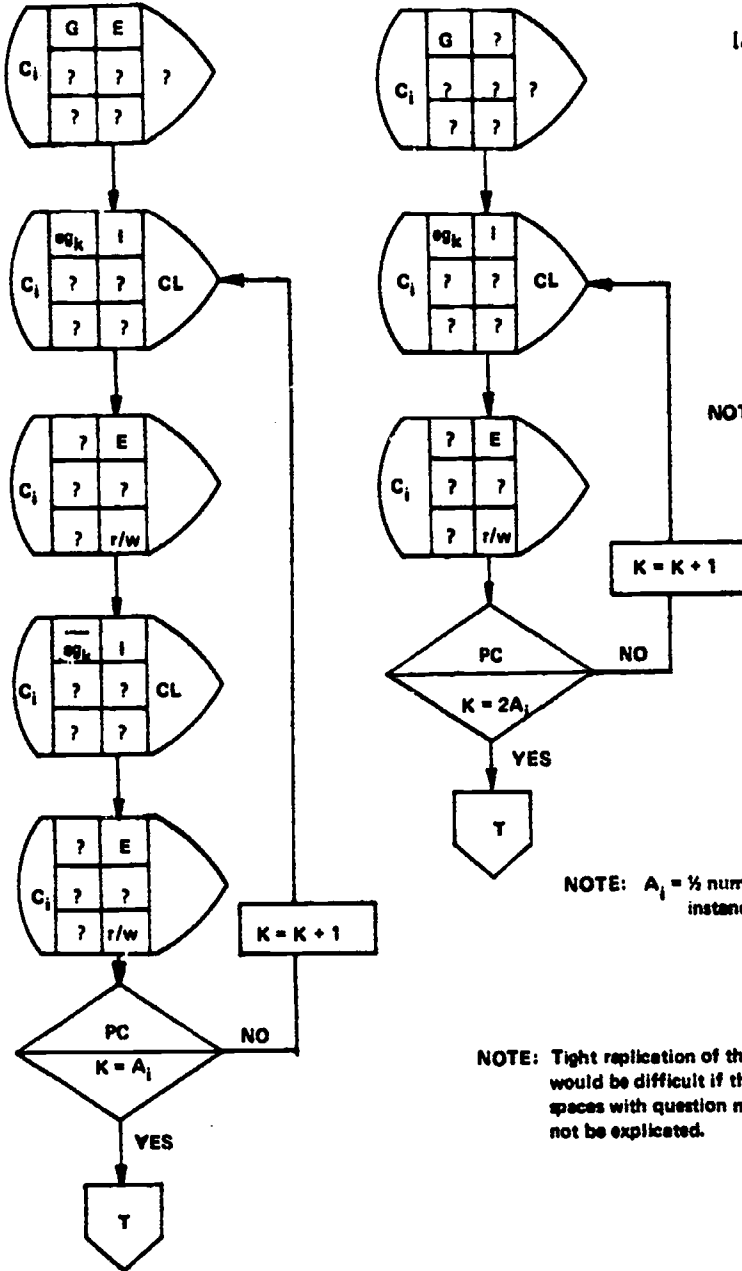
To illustrate this point, two different experimental studies involving strategies for teaching mathematics have been diagrammed using the flow chart symbols described in this paper (see Figures 30 and 31). The variable differences are indicated in bold face

EXPERIMENT A

EXPERIMENTAL GROUP

CONTROL GROUP

[Adapted from Shumway (1971).]



NOTE: Differences in treatments pertained to negative and positive examples.

NOTE: $A_i = \frac{1}{2}$ number of instances for C_i

NOTE: Tight replication of this study would be difficult if the display spaces with question marks could not be explicated.

Figure 30. An instructional strategy flow chart description of the Shumway mathematics education research study.

[Adapted from Dossey and Henderson (1974)]

TREATMENT CONDITIONS 1, 2, 7, 8

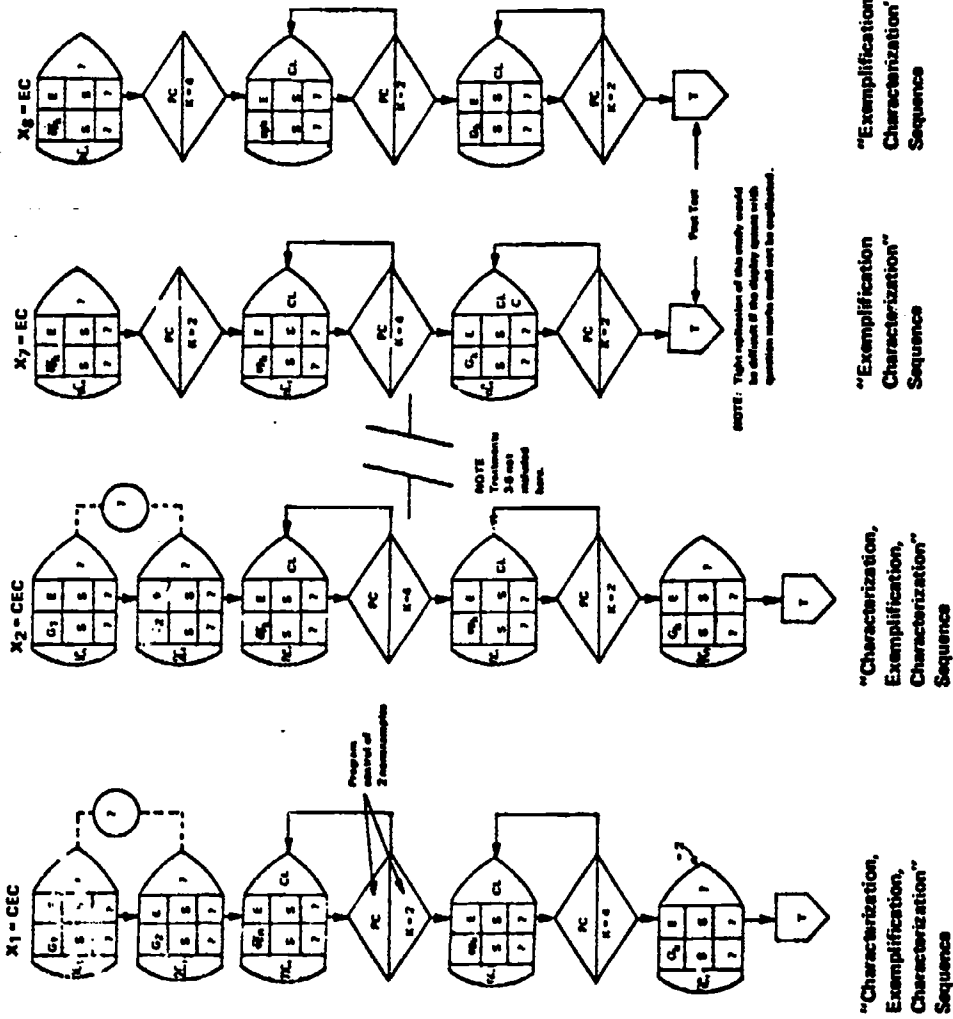


Figure 31. An instructional strategy flow chart description of the Dossey and Henderson mathematics education research study.

and the descriptions provided in the experimental reports have been included for comparison. Both of these studies are very well articulated in their existing reports, but the reader will readily observe the additional information which flow charting the strategy provides, as well as the ease with which one can compare the treatment differences.

Strategy Analysis

A second value of the proposed system is to provide a means whereby existing instructional materials can be unambiguously described and compared with other instructional materials. As an example, a sample segment from a mathematics textbook is analyzed in Figure 32.

By comparing the flow charts of empirically validated, effective strategies with less effective strategies, we should be able to develop some guidelines which would allow one to analyze a strategy and then, by comparing certain characteristics of the strategy to the guidelines, know where the strategy can be improved and what improvements may be required. The unior author of this paper is currently working on a diagnostic system for use in evaluating the effectiveness of public school curriculum materials.

Strategy Design

Use of the proposed variable system should greatly facilitate the development of instructional materials. By a careful specification of an instructional strategy using the symbols suggested, an instructional designer has provided a recipe or blueprint for the construction of instructional materials. It should be much easier for media designers, writers, and other professionals to work with instructional designers using such a tool. Much current instructional

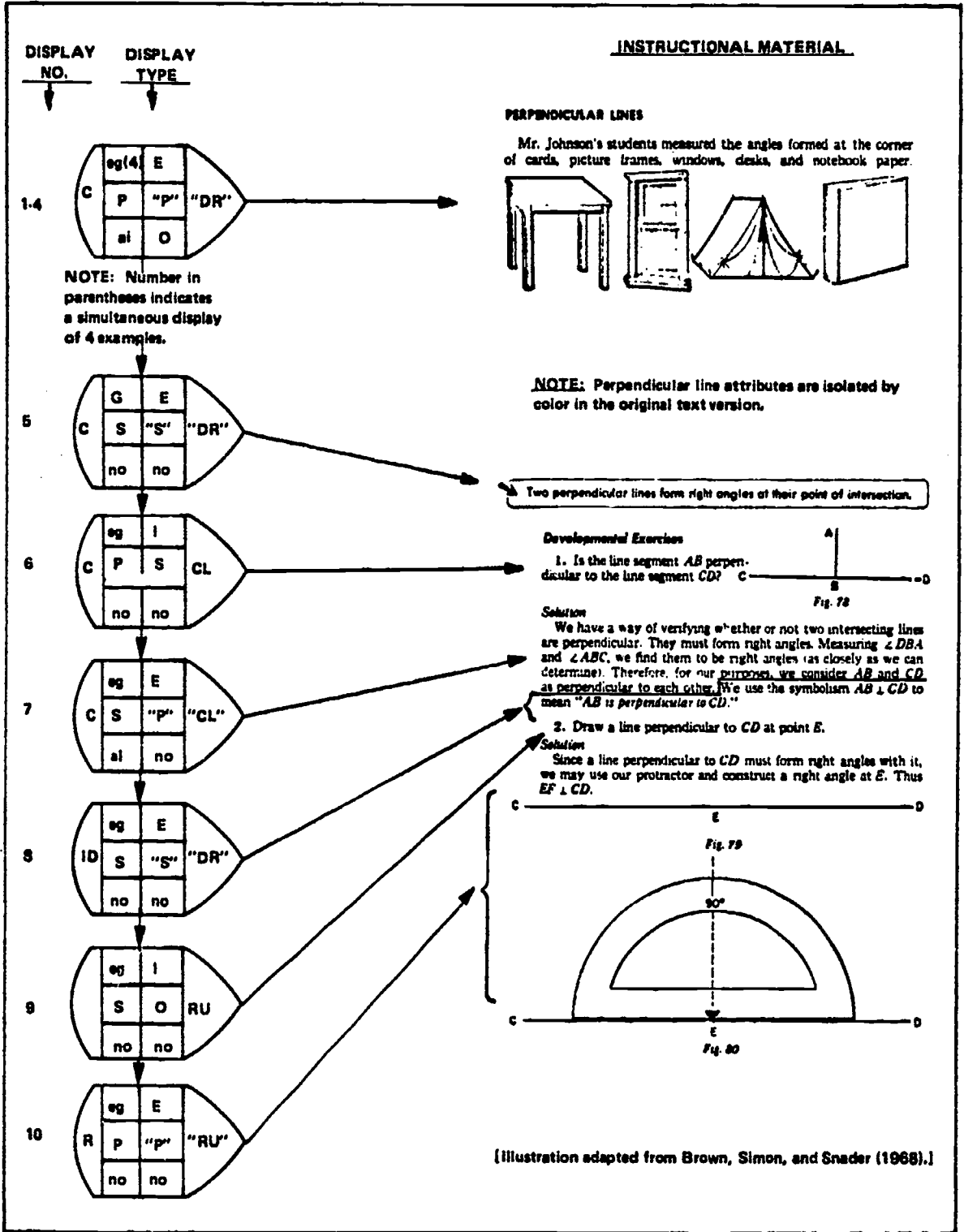


Figure 32. Strategy flow chart describing an instructional segment from a mathematics textbook.

development is labor intensive because there is no convenient way for the designers to communicate with the producers of the instructional materials. Hence, we see much current development operating in "shoe shop" mode where a single highly trained person does every aspect of the development himself. An adequate strategy variable specification would free instructional psychologists to do strategy design while leaving the actual preparation of the materials to professional writers, media specialists, subject matter experts and other specialized personnel. The senior author has been involved in several instructional development projects where this type of team approach has been tried and has proven to be effective. A more complete taxonomy system should improve the effectiveness of such team-based approaches even more.

SUMMARY

The authors have proposed that instruction consists of four relatively independent facets: learner aptitudes, content structure, delivery systems, and instructional strategies. Instructional strategies consist of sequences of informational displays. Eight variables were proposed which characterize a given informational display as follows: content type, content mode, content representation, mathemagenic prompting, response conditions, response mode, response representation, and mathemagenic feedback. For each variable several principal parameter values were suggested. It was proposed that a strategy also involved quantity and sequence parameters and a class of qualitative interdisplay relationships. All of the interdisplay relationship variables were not specified. It was suggested that strategies also consist of directional displays which differ from informational displays. The instructional strategy system described and its accompanying flow chart conventions should have value in the construction of instructional theory, the review of instructional research, the analysis of existing instructional strategies, and the design of instructional materials and systems.

References

- Anderson, R. C. Educational psychology. Annual Review of Psychology, 1967, 18, 129-164.
- Attneave, F. Some informational aspects of visual perception. Psychological Review, 1954, 61, 183-193
- Brown, K., Simon, L., and Snader, D. W. General Mathematics: Book Two. River Forest, Illinois: Laidlaw Brothers Publishers, 1968.
- Bruner, J. S. Toward a Theory of Instruction. New York: W. W. Norton and Company, 1966.
- Cooney, T., Davis, E., and Henderson, K. Dynamics of Teaching Secondary School Mathematics. Accepted for publication by Houghton-Mifflin Company, 1975.
- Cronbach, L. J., and Snow, R. E. Aptitudes and Instructional Methods. Preliminary edition, Stanford University, 1973.
- Donio, J. Personal communication. Paris, France: Institute de Recherche d'Informatique et d'Automatique, 1972.
- Dossey, J. A., and Henderson, K. B. The relative effectiveness of four strategies for teaching disjunctive concepts in mathematics. Journal for Research in Mathematics Education, 1974, 5, No. 1, 6-19.
- Gagne, R. M. Task analysis: Its relation to content analysis. Educational Psychologist, 1974, 11, 11-18.
- Gagne, R. M. Curriculum research and the promotion of learning. In R. Tyler, et al. (Eds.), Perspectives of Curriculum Evaluation. Chicago: Rand McNally, 1967.
- Gagne, R. M. Learning hierarchies. Educational Psychologist, 1968, 6, No. 1, 1-6.
- Gagne, R. M. The conditions of learning. New York: Holt, Rinehart and Winston, 1970.
- Gagne, R. M., and Rohwer, W. D., Jr. Instructional psychology. Annual Review of Psychology, 1969, 20, 381-418.
- Glaser, R., and Resnick, L. B. Instructional psychology. Annual Review of Psychology, 1972, 23, 207-276.

- Heimer, R. T., and Lottes, J. J. Toward a theory of sequencing: An integrated program of research. Journal for Research in Mathematics Education, 1973, 4, 85-93.
- Jamison, D., Suppes, P., and Wells, S. The effectiveness of alternative instructional media: A survey. Review of Educational Research, 1974, 44, No. 1, 1-67.
- Macdonald-Ross, M. Glass beads and geometric monsters: the problems of representing knowledge. Paper presented at the meeting of the Association for Educational Communications and Technology, Atlantic City, March, 1974.
- Merrill, M. D. Learner control: Beyond aptitude treatment interactions. Unpublished manuscript, Brigham Young University, 1974.
- Merrill, M. D. Content and instructional analysis for cognitive transfer tasks. Audio Visual Communication Review, 1973, 21, No. 1, 109-125.
- Merrill, M. D., and Boutwell, R. C. Instructional development: Methodology and research. In F. N. Kerlinger (Ed.), Review of research in education (Vol. 1). Itasca, Illinois, Folio Publishers, 1973.
- Merrill, M. D., and Gibbons, A. S. Heterarchies and their relationship to behavioral hierarchies for sequencing content in instruction. Paper presented at the American Educational Research Association convention, Chicago, April, 1974.
- Miller, G. A., Galanter, E., and Pribram, K. Plans and the structure of behavior. New York: Holt, Rinehart and Winston, 1960.
- Pask, G. An outline of conversational domains and their structure. A paper presented at the Conference of Structural Learning, University of Pennsylvania, 1974.
- Rothkopf, E. Z. Some theoretical and experimental approaches to problems in written instruction. In J. D. Krumboltz (Ed.), Learning and the educational process. Chicago: Rand McNally, 1965.
- Shumway, R. J. Negative instances and mathematical concept formation: A preliminary study. Journal for Research in Mathematics Education, 1971, 2, No. 3, 218-227.
- Snow, R. E. Theory construction for research on teaching. In R. M. W. Travers (Ed.) Second handbook of research on teaching. Chicago: Rand McNally, 1973.

Tennyson, R. D. Effect of negative instances in concept acquisition using a verbal learning task. Journal of Educational Psychology, 1973, 64, 247-260.

Tennyson, R. D., Woolley, F. R., and Merrill, M. D. Exemplar and nonexemplar variables which produce concept classification behavior and specified classification errors. Journal of Educational Psychology, 1972, 63, 144-152.

Wood, N. D., and Merrill, M. D. Cognitive objectives - 3 x 3 = 0: The taxonomy revisited. A paper presented at the Competency Based Teacher Education convention at Wayne State University, Detroit, May, 1974.