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ABSTRACT

In this volume, the results of a study of concept learning are presented. (The background for the study, and descriptions of the treatments--rote reception, rote discovery, conceptual reception, and conceptual discovery--are provided in Part 1.) Several hypotheses concerning the effectiveness of these treatments in learning concepts and on the ability to use those concepts were analyzed using multivariate analysis of covariance. The results of these analyses indicated some differences for the reception and discovery treatments, and for the rote and conceptual treatments; no interaction between these dimensions was discovered. The data are shown to fit well with the Bower model for paired-associate learning. (SD)

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Technical Report No. 307 (Part 2 of 2 Parts)

A STUDY OF THE ROTE-CONCEPTUAL AND RECEPTION-DISCOVERY DIMENSIONS OF LEARNING MATHEMATICAL CONCEPTS

Report from the Projection Conditions of School Learning and Instructional Strategies

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STATEMENT OF FOCUS

Individually Guided Education (IGE) is a new comprehensive system of elementary education. The following components of the IGE system are in varying stages of development and implementation: a new organization for instruction and related administrative arrangements; a model of instructional programing for the individual student; and curriculum components in prereading, reading, mathematics, motivation, and environmental education. The development of other curriculum components, of a system for managing instruction by computer, and of instructional strategies is needed to complete the system. Continuing programmatic research is required to provide a sound knowledge base for the components under development and for improved second generation components. Finally, systematic implementation is essential so that the products will function properly in the IGE schools.

The Center plans and carries out the research, development, and implementation components of its IGE program in this sequence: (1) identify the needs and delimit the component problem area; (2) assess the possible constraints--financial resources and availability of staff; (3) formulate general plans and specific procedures for solving the problems; (4) secure and allocate human and material resources to carry out the plans; (5) provide for effective communication among personnel and efficient management of activities and resources; and (6) evaluate the effectiveness of each activity and its contribution to the total program and correct any difficulties through feedback mechanisms and appropriate management techniques.

A self-renewing system of elementary education is projected in each participating elementary school, i.e., one which is less dependent on external sources for direction and is more responsive to the needs of the children attending each particular school. In the IGE schools, Center-developed and other curriculum products compatible with the Center's instructional programing model will lead to higher morale and job satisfaction among educational personnel. Each developmental product makes its unique contribution to IGE as it is implemented in the schools. The various research components add to the knowledge of Center practitioners, developers, and theorists.

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ABSTRACT

The study reported in this thesis concerns the learning and use of mathematical concepts and the learning of relations described by a hierarchy of mathematical concepts.

The rote-conceptual and reception-discovery dimensions of learning were studied. The study was designed not only to allow study of the hypothesized main effects and interactions between these dimensions but also to determine the feasibility of studying the learning of relations between concepts.

Three types of relations that exist between mathematical concepts are designated which are used to define a hierarchy of mathematical concepts. The procedure used was to designate a hierarchy of mathematical concepts. Then, instructional units were prepared and taught to students in fifteen sections of college algebra by rote reception, rote discovery, conceptual reception, or conceptual discovery methods. The rote treatments allowed, but did not require, rote learning of single facts. In the conceptual treatments students were never given the same items more than once so that only conceptual learning was demonstrated. The procedure in the reception treatments was to give the S the correct definitions and examples of concepts; whereas, in the discovery treatements, the S had to discover the correct rule.

The results of the study indicated differences between rote and conceptual learning as well as between reception and discovery learning. No evidence of an interaction between the rote-conceptual and reception-discovery dimensions were found. An excellent fit for data from the conceptual reception treatments to theoretical values from Bower's model of paired-associate learning was found.

Conclusions drawn from the study are: (1) there are differences between rote and conceptual learning and between reception and discovery learning that can be studied using hierarchies of mathematical concepts as the content to be learned and by fitting observed data to different models of learning noting variation in parameters and fit, (2) rote learning does not hinder conceptual learning, and (3) if there is an interaction between rote-conceptual and rote-discovery learning, either in learning concepts or in learning relations between concepts, more refined methods are needed to analyze them.

Chapter V

THE STUDY

With only minor changes, the study was conducted as described in Chapter IV. The students and teaching assistants were most cooperative. Many students seemed to like the change from the regular classroom routine afforded by participation in the study; this leads to the feeling that there may be some effect caused by the novelty of the experience. However, since students were randomly assigned to treatments it is assumed that whatever the effect of participation, it was uniform across treatments.

The Population

The study was conducted at the University of Wisconsin - Madison. The subjects were students enrolled in College Algebra (Mathematics 112). During the semester in which the study was conducted, there were 30 sections of this course taught by 16 teaching assistants (TA) with the supervision of a faculty member. The textbook was Groza and Shelly (1969) together with a suggested syllabus. Permission had been granted by the professor for one of the TAs involved in the study to write his own syllabus for Groza and Shelly and to give his own examination.

The professor in charge had also granted permission to five TAs to individualize their ten sections so as to permit students to progress at their own rate; these sections used Spiegel (1956) and a great deal of supplementary material written by the five TAs.

<u>Initial Contact with Teaching Assistants</u>

The TAs for the individualized sections had indicated an interest in participating in the study. The materials were designed with these sections in mind. The TAs reviewed the materials before use with their students. It was determined that the individualized sections would not provide enough subjects so that it was necessary to contact other TAs. The unit and study were then described to three TAs teaching regular sections at least a week before the material was to be taught. Each TA agreed to use the material.

Due to the different instructional methods used by the TAs, it was desirable to consider effects of TA section on data collected in the experiment. It was determined that students in the ten individualized sections moved freely from one section to another and that team teaching was occurring in these sections so that distinction between sections was considered to be unimportant, what was important was whether students were in an individualized section or not. Three covariates, SECTV1, SECTV2, and SECTV3 were used in the analyses of data. These titles refer to sections of Mathematics 112 taught by three different TAs and were designed to contrast the performance of <u>S</u>s in sections taught by each of the TAs not teaching individualized sections with performance of <u>S</u>s in individualized sections. This was done by assigning value one (for SECTV1) to each <u>S</u> in sections taught by TA number one (designations were arbitrary), zero to Ss taught by TA Two and Three, and -1 to all Ss in individualized sections. Values for SECTV2 and SECTV3 were determined similarly with Ss taught by TA Two being assigned value for one for SECTV2 and Ss for TA Three assigned value one for SECTV3, with other Ss for TAs being assigned value 0, and Ss from individualized sections always being assigned value -1.

Other covariates were SEX, CQT, and ALG. The values for CQT were the scores of Ss on the College Qualification Test, numerical portion. The covariate ALG was the scores of Ss on the algebra part of the University of Wisconsin-Madison mathematics placement test.

Timetable for the Study

Students from the individualized sections were allowed to work on the unit from April 25, 1973 to May 2, 1973. Data obtained after May 2 were not included in the study.

Three regularly scheduled class periods of fifty minutes each were used for the regular sections. The periods were on April 25, 27, and 30, 1973, with make-up periods for students from April 26 through May 2. These three days were suggested for teaching sequences and series in the syllabus.

Summary of Treatments

Subjects from individualized sections. Students from the individualized sections were required to complete the unit to be eligible for a grade of A or B in the course. The unit had been designated as an optional unit at the beginning of the semester and grading procedures set before the study was conceived. When a student from an individualized section was ready to study the unit he was instructed by his TA to come to the experimenter's office; there, he was randomly assigned to one of the eight treatments. If the treatment included a pretest it was given at once. When the pretest was finished or if no pretest was administered the subject was given a sheet of instructions. (Figure 4.1, p. 49 shows the instructions for one treatment in Part One. Figure 5.1 shows the instructions all subjects received for Part Two. The instructions for the other treatments are in Appendix A).

Part 2, Sequence and Series

This part of the unit is designed to teach you to use formulas associated with sequences. For each sequence, you are to find the 20th term, the nth term, the sum of the first 20 terms, the sum of the first n terms and the sum of the terms of the infinite sequence (if it exists). After you have found each of these expressions for a particular example write each one in the place indicated in your unit. If a requested value does not exist place an X where the answer should be. With these exceptions the directions are the same as for the first part of the unit.

Some of the answers will involve arithmetic with large numbers. It is necessary only to indicate what should be done; $3 + 2^{17}$ is a perfectly good answer, it is not necessary to do the computation. DO NOT GET TIED UP IN ARITHMETIC PROBLEMS.

Figure 5.1. Instructions all subjects received for Part Two

If the subject had questions about what he was to do he answered orally. At this point the subject proceeded as described in Chapter IV for his particular treatment.

If the subject did not finish both parts of the unit he was instructed to return at his next opportunity. Upon completion of the unit and both posttests the subject was given a sheet with definitions of all terms used and derivations of all formulas regardless of whether he had been in the reception or discovery treatments. This material is shown in Figures 5.2 and 5.3.

Subjects from non-individualized sections. The class roster from the five regular sections were obtained from the TAs and the students were randomly assigned to the treatments before the first day of the experiment. At the first class meeting subjects were given a short oral description of the experiment stating that the content covered was a part of the regular course, that they were to work alone, and that they were not to change responses after correct responses were given (since these responses had no effect on their grades). At this point the roll was called, and each student was given a letter W, X, Y, or Z according to the treatment to which he was assigned regardless of whether or not a pretest would be given. Students with the same letter were asked to sit in the same section of the classroom. Next pretests were given to subjects assigned to pretest treatments; then the other subjects were given materials according to their treatment assignment as described in Chapter IV.

The experimenter and TA worked together in grading grid lists, checking booklet lists when subjects indicated no mistakes (if a subject indicated mistakes on a booklet list he was given another of the same form) and observing the progress of the class. As lists and

Conclusion: Sequences and Series

Be sure to get copies of the definitions and formulas before you leave. The definitions should require no comments, except that sequences are sometimes called progressions. The term "sequence" is used consistently in calculus and higher mathematics.

The nth term of an arithmetic sequence is found by counting the number of times the common difference is added to the first term. For instance if a is the first term and d is the common difference then:

1st term = a

2nd term = a + d

3rd term = a + 2d

...

10th term = a + 9d

...

nth term = a + (n-1)d

The same ideas apply to the terms of a geometric sequence. If a is the first term and r is the common ratio then:

1st term = a

2nd term = ar

3rd term = ar^2

...

10th term = ar^9

...

nth term = ar^{n-1}

The idea behind the formula for the sum of the first n terms of an arithmetic sequence can be seen in finding the sum of the even numbers from 2 to and including 100. (This is the arithmetic sequence 2, 4, 6, 8, ..., 94, 96, 98, 100).

Figure 5.2. Handout given to Ss upon completion of Part Two posttest.

$$25 \begin{cases} 2 + 100 = 102 \\ 4 + \ \ 98 = 102 \\ 6 + \ \ 96 = 102 \\ 8 + \ \ 94 = 102 \\ \ldots \\ 50 + 52 = 102 \end{cases}$$

It is only necessary to multiply 102 by one half the number of terms in the sequence (50) so the sum is $25 \cdot 102$.

To find the sum of the first n terms of an arithmetic sequence add the first term, a, and the nth term, $a + (n-1)d$, and multiply by the number of pairs that have sum $a + [a + (n-1)d]$, which is $\frac{n}{2}$:

sum of the 1st n terms $= \frac{n}{2}[2a + (n-1)d]$

The sum of 1st n terms of a geometric sequence can be indicated by: (a is the 1st term and r is the common ratio)

$$S_n = a + ar + ar^2 + \ldots + ar^{n-1}$$

Multiply both members of this equation by r:

$$rS_n = ar + ar^2 + \ldots + ar^{n-1} + ar^n$$

subtract

$$Sn - rS_n = a + ar + ar^2 + \ldots + ar^{n-1} - ar - ar^2 - \ldots - ar^n$$
$$= a - ar^n$$

therefore $(1-r)Sn = a(1-r^n)$

$$Sn = \frac{a(1-r^n)}{1-r}$$

This gives the formula for the sum of n terms of a geometric sequence. As n increases, if $|r| < 1$, r^n gets closer to 0. This is stated $\lim_{n \to \infty} r^n = 0$. So that $\lim_{n \to \infty} \frac{a(1-r^n)}{1-r} = \frac{a}{1-r}$. Thus, if $|r|<1$ the sum of an infinite geometric sequence is $\frac{a}{1-r}$.

These same ideas are discussed in your text and you should be sure to read the relevant sections. If you feel you do not understand the material after you have read the discussion in your text I will be happy to make an appointment to work out the problems. My office is 707 Van Vleck.

Figure 5.3. Handout given to Ss upon completion of Part Two Posttest.

tests were completed they were collected in a box to be sorted later. If a student was unable to complete a list at the end of the period he was given that list to complete at the first of the next class meeting.

Before the next class meeting each list was checked to verify that responses were clearly marked and the correct lists were being used. A list of subjects with the starting point for the next class period was prepared. If a subject completed all lists and tests before the last day of the experiment he was excused and asked to return to class the class session following the completion of the experiment at which time he would be given the definitions and formulas used in the study. Subjects that took the three class periods to complete the material were given the definitions and formulas before leaving on the last day of the experiment.

Descriptive Statistics for Pretests and Posttests

The pretest indicated students entered the experiment not knowing the concepts to be taught. Of the thirty-three subjects taking the pretest, 25 gave no correct responses at all. This may help account for the low Hoyt reliability indicated in Table 5.1 for the pretest. On the other hand, approximately half the subjects made no mistakes on the posttests (31 out of 63 on Part One and 23 out of 46 on Part Two) which is a possible explanation for low Hoyt reliability coefficients for these tests. The situation where pretest scores are low and posttest scores are high was described by Hambleton and Novick (1973):

> ...criterion-referenced tests are usually administered either immediately before or after small units of instruction. Thus, it is not surprising that we frequently observe homogeneous distributions of test scores on the pre- and posttests, but centered at the low and high ends of the achievement scales, respectively. It is well known from the study of classical test theory (Lord and Novick, 1968) that when the variance of test scores is restricted, correlational estimates of reliability and validity will be low (p. 167).

Ebel (1968) pointed out that measures of internal consistency give only estimates of reliability. He stated that reliability is:

> ...the ratio of true score variance to obtained score variance and operationally, the correlation between measurements of the same characteristic obtained from equivalent but independent operations. Reliability of this kind should never be <u>defined</u> as internal consistency, though it may often be estimated by a measure of internal consistency. Theoretically, a test that is perfectly reliable in the variance ratio sense or in the correlation of equivalent measurements sense may have zero internal consistency (p. 72).

Popham and Husek (1969) stated:

> If a criterion-referenced test has a high average inter-item correlation, this is fine. If the test has a high test-retest correlation, that is also fine. The point is <u>not</u> that these indices cannot be used to support the consistency of the test. The point is that a criterion-referenced test could be highly consistent, either internally or temporarily, and yet indices dependent on variability might not reflect that consistency (pp. 5-6).

Table 5.1 shows the mean, standard deviation, Hoyt reliability coefficients, and standard error for the tests.

<u>Descriptive Statistics for the Lists</u>

Table 5.2 shows the mean and standard deviations for the number of lists used and number of different forms of the lists used in each treat-

Table 5.1

DESCRIPTIVE STATISTICS FOR PRETESTS AND POSTTESTS

Observation	No. of Items	No. of Subjects	Range	Mean	Standard Deviation	Hoyt Reliability	Standard Error
Pretest	6	33	3	0.424	0.830	0.529	0.510
Posttests:							
Part I Total	12	63	4	11.238	0.946	0.290	0.763
Part I Old	6	63	3	5.476	0.759	0.299	0.580
Part I New	6	63	2	5.762	0.465	-0.040	0.433
Part II Total	20	46	6	18.826	1.568	0.614	0.949
Part II Old	10	46	5	9.239	1.303	0.690	0.688
Part II New	10	46	2	9.587	0.686	0.210	0.578

Table 5.2

DESCRIPTIVE STATISTICS FOR LISTS

	Number of Lists Used		Number of Forms of Lists	
Treatment	Mean	Standard Deviation	Mean	Standard Deviation
PR RO RE	4.000	1.927	2.375	.744
PR RO DI	6.111	1.965	2.778	.667
PR CO RE	3.222	1.302	3.222	1.302
PR CO DI	4.286	.951	4.286	.951
NP RO RE	4.222	1.641	2.222	.441
NP RO DI	7.714	3.200	3.429	1.272
NP CO RE	3.143	1.345	3.143	1.345
NP CO DI	4.000	1.291	4.000	1.291
RO RE	4.118	1.781	2.294	.603
RO DI	6.812	2.579	3.062	.979
CO RE	3.188	1.321	3.188	1.321
CO DI	4.143	1.134	4.143	1.134

ment. The pretest (PR), no pretest (NP), rote (RO), conceptual (CO), reception (RE), and discovery (DI) designations for treatments are also used in Chapter VI.

The study has been described and the method of data collection indicated. The statistical analyses of the data are reported in Chapter VI and Chapter VII contains the conclusions reached based on these results.

Chapter VI

RESULTS OF THE DATA ANALYSIS

Data were gathered as indicated in Chapter V to test the hypotheses stated in Chapter II. The analyses of these data are presented here. Observed and predicted values for the models from Chapter II are also included.

Each hypothesis is discussed within the section dealing with the appropriate set of dependent measures. The hypotheses were tested using either a 2 x 2 x 2 multivariate analysis of covariance (MANCOVA) for Part One (concept identification) or a 2 x 2 MANCOVA for Part Two (concept use). The covariates used were SECTV1, SECTV2, and SECTV3, which contrasted the performance of Ss in sections taught by each of the three TAs not teaching individualized sections with performance of Ss in individualized sections; CQT, ALG, and SEX, which were scores on the numerical portion of the College Qualification Test, scores on the algebra part of the University of Wisconsin-Madison mathematics placement test, and sex of the S, respectively. The significance level was set at .05. Since the study is exploratory in nature, weak results are also considered. A significance level of .10 was used to indicate marginal results and to suggest possible research hypotheses for future study.

Another type of analysis is required for the Bower model. The estimated parameters for each treatment for the Bower model are given in the last section of this chapter. The D^2 values for goodness of fit are indicated but due to problems of calculating degrees of freedom, knowing whether or not observations are independent, and small cell size (seven to nine), care must be exercised in viewing D^2 as a Chi-square statistic with the usual "number of observations minus one minus number of parameters estimated" degrees of freedom.

Part One - Concept Identification

Relation learning. Three of the research hypotheses stated at the end of Chapter II concerned Ss learning relations between concepts. These research hypotheses are:

Hypothesis 1a. There is a significant difference between reception and discovery learning on recognition of relations between concepts.

Hypothesis 1b. There is a significant difference between learning that includes rote learning and learning that is conceptual on recognition of relations between concepts.

Hypothesis 1c. There is a significant interaction between reception-discovery learning and rote-conceptual learning on recognition of relations between concepts.

Briefly stated the relations were: an item was to be classified as either finite or infinite but not both, an item was to be classified as either a sequence or a series but not both, and an item was not to be classified as both arithmetic and geometric but possible was neither. The dependent variables related to these classifications are FIV, SSV, and AGV, respectively, and are defined as follows:

FIV the number of mistakes on the lists that indicate the subject does not recognize an item cannot be both finite and infinite.

SSV the number of mistakes on the lists that indicate the subject does not recognize an item cannot be both a sequence and a series.

AGV the number of mistakes on the lists that indicate the subject does not recognize an item cannot be both arithmetic and geometric.

The following null hypotheses correspond to Research Hypotheses 1a, 1b, and 1c:

H.1a: There is no significant difference in cell means for observed values of FIV, SSV, and AGV between reception (RE) and discovery (DI) treatments.

H.1b: There is no significant difference in cell means for observed values of FIV, SSV, and AGV between rote (RO) and conceptual (CO) treatments.

H.1c: There is no significant interaction between the RE/DI and the RO/CO dimensions as measured by observed values of FIV, SSV, and AGV.

Of interest also are hypotheses concerning the effects of the pretest (PR) and no pretest (NP) treatments. These null hypotheses are:

H.1d: There is no significant difference in cell means for observed values of FIV, SSV, and AGV between PR and NP treatments.

H.1e: There is no significant interaction between the PR/NP and the RE/DI dimensions as measured by observed values of FIV, SSV, and AGV.

H.1f: There is no significant interaction between the PR/NP and the RO/CO dimensions as measured by observed values of FIV, SSV, and AGV.

H.1g: There is no significant differences among cell means for observed values of FIV, SSV, and AGV.

No support was found for Research Hypothesis 1a or Research Hypothesis 1c since tests of null hypotheses H.1a and H.1c were not significant. Also, null hypotheses H.1c, H.1f, and H.1g cannot be rejected for the same reason.

Null hypothesis H.1b is rejected ($p < .002$) giving support to Research Hypothesis 1b. There is a significant difference between the rote (RO) treatments and the conceptual (CO) treatments. The means for FIV, SSV, and AGV for RO treatments are 1.09, .79, and .12, respectively. The means for the same variables but for the CO treatments are 4.87, 1.90, and .50, respectively. Thus Ss in the RO treatments made significantly fewer errors. Univariate analyses disclosed significant differences between means for variable FIV and SSV for RO and CO treatments ($p < .003$ and $p < .017$, respectively) but the values for AGV were not significantly different (see Table 6.1).

Null hypothesis H.1d is rejected ($p < .0354$). The means for FIV, SSV, and AGV for pretest (PR) treatments are 4.52, 1.36, and .15, respectively, and for the same variables when a pretest was not given (NP) are 1.10, 1.27, and .47, respectively. The exact nature of this main effect is not clear. Univariate analyses indicate a significant difference on FIV scores ($p < .02$) but not for SSV nor AGV. The FIV and SSV mean scores favor the NP treatments but the AGV mean scores do not (see Table 6.2).

Table 6.1

MANCOVA FOR RO/CO MAIN EFFECT ON LEARNING RELATIONS BETWEEN CONCEPTS FOR VARIABLES FIV, SSV, AND AGV

F Ratio for Multivariate Test of Equality of Mean Vectors = 5.8213, df = 3 and 47, $p < .0019$

Variable	MS	Univariate F	p <
FIV	126.3541	10.1882	.0025
SSV	25.0236	6.2056	.0162
AGV	1.119	1.2026	.2782

Table 6.2

MANCOVA FOR PR/NP MAIN EFFECT ON LEARNING RELATIONS BETWEEN CONCEPTS FOR VARIABLES FIV, SSV, AND AGV

F Ratio for Multivariate Test of Equality of Mean Vectors = 3.1043, df = 3 and 47, $p < .0354$

Variable	MS	Univariate F	p <
FIV	71.4091	5.7579	.0203
SSV	.5324	.1320	.7180
AGV	2.3961	2.5916	.1139

<u>Concept identification</u>. Three research hypotheses concerned with <u>S</u>s' recognition of examples and non-examples of concepts are:

> <u>Hypothesis 2a</u>. There is a significant difference between reception learning and discovery learning on recognition of examples of concepts learned.
>
> <u>Hypothesis 2b</u>. There is a significant difference between learning that includes rote learning and learning that is conceptual on recognition of examples of concepts learned.
>
> <u>Hypothesis 2c</u>. There is a significant interaction between reception-discovery learning and rote-conceptual learning on recognition of examples of concepts learned.

Four different sets of null hypotheses corresponding to these research hypotheses were tested. The null hypotheses will be stated after the appropriate dependent variables are described.

The first set of dependent measures used to study Research Hypotheses 2a, 2b, and 2c is related to items that were included on list A. These items are distinguished as "old" items. These dependent variables are:

- SS OLD the number of mistakes on the posttest in classifying items from the previously learned items as either a sequence or a series.
- A OLD the number of mistakes on the posttest in classifying items from the previously learned items as either arithmetic or not.
- G OLD the number of mistakes on the posttest in classifying items from the previously learned items as either geometric or not.
- TOTOLD the total number of mistakes in classifying items from previously learned items.

The null hypotheses that were tested are:

H.2a: There is no significant difference in cell means for observed values of SS OLD, G OLD, and TOTOLD between reception (RE) and discovery (DI) treatments.

H.2b: There is no significant difference in cell means for observed values of SS OLD, G OLD, and TOTOLD between rote (RO) and conceptual (CO) treatments.

H.2c: There is no significant interaction between the RE/DI and the RO/CO dimensions as measured by observed values of SS OLD, G OLD, and TOTOLD.

H.2d: There is no significant difference in cell means for observed values of SS OLD, G OLD, and TOTOLD between PR and NP treatments.

H.2e: There is no significant interaction between the PR/NP and the RE/DI dimensions as measured by observed values of SS OLD, G OLD, and TOTOLD.

H.2f: There is no significant interaction between the PR/NP and the RO/CO dimensions as measured by observed values of SS OLD, G OLD, and TOTOLD.

H.2g: There is no significant difference among cell means for observed values of SS OLD, G OLD, and TOTOLD.

No multivariate tests were statistically significant (each test is reported in Appendix B). The null hypotheses H.2a, H.2b, and H.2c correspond to Research Hypotheses 2a, 2b, and 2c, respectively. Hence, no support was found for these research hypotheses.

The second set of dependent measures used to study Research Hypotheses 2a, 2b, and 2c is related to "new" items, items not previously seen by the Ss. These dependent variables are:

SS NEW the number of mistakes on the posttest in classifying items from the new items as either a sequence or a series.

A NEW the number of mistakes on the posttest in classifying items from the new items as either arithmetic or not.

G NEW the number of mistakes on the posttest in classifying items from the new items as either geometric or not.

TOTNEW the total number of mistakes in classifying items from new items.

The null hypotheses that were tested are:

H.3a: There is no significant difference in cell means for observed values of SS NEW, A NEW, G NEW, and TOTNEW between RE and DI treatments.

H.3b: There is no significant difference in cell means for observed values of SS NEW, A NEW, G NEW, and TOTNEW between RO and CO treatments.

H.3c: There is no significant interaction between the RE/DI and the RO/CO dimension as measured by observed values of SS NEW, A NEW, G NEW, and TOTNEW.

H.3d: There is no significant difference in cell means for observed values of SS NEW, A NEW, G NEW, and TOTNEW between PR and NP treatments.

H.3e: There is no significant interaction between the PR/NP and the RE/DI dimensions as measured by observed values of SS NEW, A NEW, G NEW, and TOTNEW.

H.3f: There is no significant interaction between the PR/NP and the RO/CO dimensions as measured by observed values of SS NEW, A NEW, G NEW, and TOTNEW.

H.3g: There is no significant difference among cell means for observed values of SS NEW, A NEW, G NEW, and TOTNEW.

No multivariate tests were statistically significant (Appendix B reports the analyses). Thus, Research Hypotheses 2a, 2b, and 2c were not supported.

The third set of dependent variables related to Research Hypotheses 2a, 2b, and 2c pooled the old and new items. These dependent variables are SS SUM, A SUM, G SUM, and TOTSUM which are the sums of SS OLD and SS NEW, A OLD and A NEW, G OLD and G NEW, and TOTOLD and TOTNEW, respectively.

The null hypotheses that were tested are:

H.4a: There is no significant difference in cell means for observed values of SS SUM, A SUM, G SUM, and TOTSUM between RE and DI treatments.

H.4b: There is no significant difference in cell means for observed values of SS SUM, A SUM, G SUM, and TOTSUM between RO and CO treatments.

H.4c: There is no significant interaction between the RE/DI and the RO/CO dimensions as measured by observed values of SS SUM, A SUM, G SUM, and TOTSUM.

H.4d: There is no significant difference in cell means for observed values of SS SUM, A SUM, G SUM, and TOTSUM between PR and NP treatments.

H.4e: There is no significant interaction between the PR/NP and the RE/DI dimensions as measured by observed values of SS SUM, A SUM, G SUM, and TOTSUM.

H.4f: There is no significant interaction between the PR/NP and the RO/CO dimensions as measured by observed values of SS SUM, A SUM, G SUM, and TOTSUM.

H.4g: There is no significant difference among cell means for observed values of SS SUM, A SUM, G SUM, and TOTSUM.

No multivariate tests were statistically significant (Appendix B contains the analyses). Research Hypotheses 2a, 2b, and 2c were not supported.

The fourth set of dependent measures used to study Research Hypotheses 2a, 2b, and 2c was used to determine if differences existed between scores on old items and scores on new items. These dependent variables are SS DIF, A DIF, G DIF, TOTDIF which are SS OLD minus SS NEW, A OLD minus A NEW, G OLD minus G NEW, and TOTOLD minus TOTNEW, respectively.

The null hypotheses that were tested are:

H.5a: There is no significant difference in cell means for observed values of SS DIF, A DIF, G DIF, and TOTDIF between RE and DI treatments.

H.5b: There is no significant difference in cell means for observed values of SS DIF, A DIF, G DIF, and TOTDIF between RO and CO treatments.

H.5c: There is no significant interaction between the RE/DI and the RO/CO dimensions as measured by observed values of SS DIF, A DIF, G DIF, and TOTDIF.

H.5d: There is no significant difference in cell means for observed values of SS DIF, A DIF, G DIF, and TOTDIF between PR and NP treatments.

H.5e: There is no significant interaction between the PR/NP and the RE/DI dimensions as measured by observed values of SS DIF, A DIF, G DIF, and TOTDIF.

H.5f: There is no significant interaction between the PR/NP and the RO/CO dimensions as measured by observed values of SS DIF, A DIF, G DIF, and TOTDIF.

H.5g: There is no significant difference among cell means for observed values of SS DIF, A DIF, G DIF, and TOTDIF.

The MANCOVA indicated that null hypothesis H.5g should be rejected ($p < .0204$). The corresponding univariate analyses indicated that probably the significant difference is due to the differences between values for G DIF. The univariate F ratios for SS DIF, A DIF, G DIF, and TOTDIF are .0865, .0004, 12.4878, and 7.5721, respectively. The correlation between G DIF and TOTDIF is high (.816) which is expected since TOTDIF was the sum of the other three variables (see Table 6.3).

Table 6.3

MANCOVA FOR DIFFERENCE BETWEEN CELL MEANS
FOR VARIABLES SS DIF, A DIF, G DIF, AND TOTDIF

F Ratio for Multivariate Test of Equality
of Mean Vectors = 3.2288, df 4 and 46, $p < .0204$

Variable	MS	Univariate F	p <
SS DIF	.0121	.0865	.7700
A DIF	.0000	.0004	.9842
G DIF	3.4230	12.4878	.0010
TOTDIF	4.1815	7.5721	.0083

Consideration of the cell means shows that most are positive or zero which indicates that more new items were missed than old. In fact rote treatments, which required correct responses on all old items before the posttest was given, have fewer negative mean scores than the conceptual treatments (see Table 6.4). The indication is strong that the old items had been forgotten and responses made on the basis of a rule rather than on the basis of a remembered isolated fact.

Table 6.4

CELL MEANS FOR VARIABLES SS DIF, A DIF, G DIF, AND TOTDIF

Cell	Number of Subjects	SS DIF	A DIF	G DIF	TOTDIF
PR RO RE	8	.000	.000	.375	.375
PR RO DI	9	.111	.000	.111	.222
PR CO RE	9	.111	.222	.667	.889
PR CO DI	7	.000	.000	.286	.286
NP RO RE	9	.000	-.111	.000	-.111
NP RO DI	7	.000	.000	.571	.571
NP CO RE	7	-.143	.000	.571	.429
NP CO DI	7	-.143	-.143	-.143	-.429
Standard Deviation		.369	.245	.580	.784

No evidence was found to support Research Hypotheses 2a, 2b, or 2c for any of the four sets of dependent variables. A significant difference was found between cell means for dependent variables SS DIF, A DIF, G DIF, and TOTDIF.

Part Two - Concept Use

Three research hypotheses from Chapter II concerning Part Two of the study are:

> Hypothesis 3a. There is a significant difference between reception and discovery learning on the ability to do computations related to sequences.
>
> Hypothesis 3b. There is a significant difference between learning that includes rote learning and learning that is conceptual on the ability to do computations related to sequences.
>
> Hypothesis 3c. There is a significant interaction between reception-discovery and rote-conceptual learning on the ability to do computations related to sequences.

Fourteen sets of null hypotheses were tested using MANCOVA for different sets of dependent measures. Only those that were statistically significant are reported here; the others may be found in Appendix B. Four MANCOVA indicate support for Research Hypothesis 3a and a fifth provides marginal support for that hypothesis.

The first set of dependent variables is related to the mean number of mistakes made on each kind of calculation required:

- CL1MN the mean number of incorrect responses in finding the twentieth term of a given sequence.
- CL2MN the mean number of incorrect responses in finding the nth term of a given sequence.
- CL3MN the mean number of incorrect responses in finding the sum of the first twenty terms of a given sequence.
- CL4MN the mean number of incorrect responses in finding the sum of the first n terms of a given sequence.

CL5MN the mean number of incorrect responses in finding the sum of the terms in an infinite sequence if it exists.

There was no variance between CL1MN and CL2MN so that CL2MN was deleted from the set of variables before the MANCOVA was performed. The null hypothesis tested is:

H.7a: There is no significant difference in cell means for observed values of CL1MN, CL3MN, CL4MN, and CL5MN between reception (RE) and discovery (DI).

This hypothesis is tentatively rejected ($p < .0649$). Thus there is weak support for the hypothesis that there is a difference between the reception (RE) treatments and the discovery (DI) treatments. The univariate F ratios do not provide any particular aid in interpretation of the result (see Table 6.5).

Table 6.5

MANCOVA FOR RE/DI MAIN EFFECT FOR VARIABLES CL1MN, CL3MN, CL4MN, AND CL5MN

F Ratio for Multivariate Test of Equality of Mean Vectors = 2.4573, df = 4 and 33, $p < .0649$

Variable	MS	Univariate F	p <
CL1MN	.0644	2.9083	.0968
CL3MN	.2708	3.9524	.0545
CL4MN	.4491	9.0610	.0048
CL5MN	.2266	1.0294	.3171

When the cell means are considered, it may be observed that fewer mistakes were made by Ss in discovery treatments than by Ss in reception treatments as measured by variables CL1MN, CL3MN, and CL4MN, but Ss in discovery treatments made more mistakes with respect to CL5MN (see Table 6.6).

Table 6.6

CELL MEANS FOR VARIABLES CL1MN, CL3MN, CL4MN, AND CL5MN

Cell	Number of Subjects	CL1MN	CL3MN	CL4MN	CL5MN
RO RE	16	.063	.188	.188	.250
RO DI	11	.000	.000	.000	.409
CO RE	12	.083	.125	.208	.125
CO DI	7	.000	.071	.000	.357
Standard Deviation		.143	.254	.227	.471

The elements in the second set of dependent variables providing supporting evidence for Research Hypothesis 3a are used to measure differences in computing the sum of the first twenty terms of arithmetic and geometric sequences. The variables are:

Cl3ON the number of mistakes on old items in CL3 minus the number of mistakes on new items in CL3.

CL3AG the number of mistakes on arithmetic items in CL3 minus the number of mistakes on geometric items in CL3.

CL3IT the number of mistakes on old geometric and new arithmetic items in CL3 minus the number of mistakes on old arithmetic and new geometric items in CL3.

The null hypothesis tested is:

H.8a: There is no significant difference in cell means for observed values of CL3ON, CL3AG, and CL3IT between RE and DI treatments.

The null hypothesis is rejected ($p < .0238$). Strong support is given to Research Hypothesis 3a. The univariate tests for CL3ON and CL3AG both indicate significant differences (see Table 6.7). The nature of these differences will be discussed below.

Table 6.7

MANCOVA FOR RE/DI MAIN EFFECT
FOR VARIABLES CL3ON, CL3AG, AND CL3IT

F Ratio for Multivariate Test of Equality of Mean Vectors = 3.5782, df 3 and 34, $p < .0238$

Variable	MS	Univariate F	p <
CL3ON	.2472	9.9605	.0033
CL3AG	.5242	7.9752	.0077
CL3IT	.0862	3.3402	.0760

The only cell mean that is negative for either CL3ON or CL3AG was for the conceptual discovery treatment. Thus only Ss in this treatment missed more new items than old and more arithmetic items than geometric. Subjects in both rote treatments missed at least as many old items as new (Table 6.8 reflects these observations).

Table 6.8

CELL MEANS FOR VARIABLES CL3ON, CL3AG, AND CL3IT

Cell	Number of Subjects	CL3ON	CL3AG	CL3IT
RO RE	16	.063	.188	.063
RO DI	11	.000	.000	.000
CO RE	12	.125	.125	.125
CO DI	7	-.071	-.071	.071
Standard Deviation		.170	.254	.170

The third set of dependent variables related to Research Hypothesis 3a is:

CL4ON the number of mistakes on old items in CL4 minus the number of mistakes on new items in CL4.

CL4AG the number of mistakes on arithmetic items in CL4 minus the number of mistakes on geometric items in CL4.

CL4IT the number of mistakes on old geometric and new arithmetic items in CL4 minus the number of mistakes on old arithmetic and new geometric items in CL4.

The null hypothesis tested is:

H.9a: There is no significant difference in cell means for observed values of CL4ON, CL4AG, and CL4IT between RE and DI treatments.

The null hypothesis is rejected ($p < .0449$). This supports Research Hypothesis 3a. The univariate tests indicated significant differences between mean scores for CL4AG and CL4IT but not for CL4ON (see Table 6.9).

Table 6.9

MANCOVA FOR RE/DI MAIN EFFECT
FOR VARIABLES CL4ON, CL4AG, AND CL4IT

F Ratio for Multivariate Test of Equality of Mean Vectors = 2.9836, df 3 and 34, $p < .0449$

Variable	MS	Univariate F	p <
CL4ON	.1414	2.8159	.1020
CL4AG	.3502	5.5051	.0246
CL4IT	.2064	4.5599	.0397

The fourth set of dependent variables related to Research Hypothesis 3a is:

CL5ON the number of mistakes on old items in CL5 minus the number of mistakes on new items in CL5.

CL5AG the number of mistakes on arithmetic items in CL5 minus the number of mistakes on geometric items in CL5.

CL5IT the number of mistakes on old geometric and new arithmetic items in CL5 minus the number of mistakes on old arithmetic and new geometric items in CL5.

The null hypothesis is:

H.10a: There is no significant difference in cell means for observed values of CL5ON, CL5AG, and CL5IT between RE and DI treatments.

This null hypothesis is rejected ($p < .0457$). The only univariate F ratio that indicated significance was for CL5ON (see Table 6.10).

Table 6.10

MANCOVA FOR RE/DI MAIN EFFECT
FOR VARIABLES CL5ON, CL5AG, AND CL5IT

F Ratio for Multivariate Test of Equality of Mean Vectors = 2.9671, df 3 and 34, $p < .0457$

Variable	MS	Univariate F	p <
CL5ON	.2923	6.0060	.0193
CL4AG	.1301	.8856	.3530
CL5IT	.0674	1.2707	.2671

Examination of the cell means indicate that Ss in reception treatments made more errors on old items but Ss in discovery treatments made more errors on new items. This result, as well as other results reported in this section, provide strong evidence that there is a main effect on the reception-discovery dimension. One more significant result indicating the same thing is given next.

The fifth set of dependent variables related to Research Hypothesis 3a is 34XON, 145XON, and 124XON. The variable 34XON is defined as the sum of the mistakes made in computing the sum of the first twenty terms of old items and the sum of the first n terms for the new items minus the errors in computing the sum of the first n terms for old and the first twenty terms for new items. The variables 145XON and 124XON are defined analogous to 34XON.

The null hypothesis is:

H.11a: There is no significant difference in cell means for observed values of 145XON, 124XON, and 34XON between RE and DI treatments.

This hypothesis is rejected ($p < .0260$). The only significant univariate test is for 124XON (see Table 6.11) That there is a difference between the reception and the discovery treatments is once again demonstrated but that one is better than the other is not. Even though the results are significant it is clear that more research is required before the nature of the difference is understood.

Table 6.11

MANCOVA FOR RE/DI MAIN EFFECT
FOR VARIABLES 145XON, 124XON, AND 34XON

F Ratio for Multivariate Test of Equality of Mean Vectors = 3.4946, df 3 and 34, $p < .0260$

Variable	MS	Univariate F	p <
145XON	.0525	1.5050	.2279
124XON	.0925	4.1582	.0489
34XON	.0073	.3350	.5664

Strong support for Research Hypothesis 3a was reported in this section. It should be noted that the precise nature of the differences between the reception treatments and the discovery treatments is not clear and it would be a mistake to conclude that one method is superior to another. No support was found for either Research Hypothesis 2b or Research Hypothesis 2c.

The Learning Models

Bower's Model. The estimate of $\frac{1}{N} = g$ (where g is the probability of a correct guess before an item is learned) in Bower's model was found to be incorrect. The value $\frac{1}{N}$ is $(\frac{1}{2})^8$ which is much too small for the actual data. Also, the other parameter c (the probability of transition from state C_0, the unlearned state, to state C_1, the learned state) is not specified by the model so that it was necessary to estimate both c and g from the data. The method used was to minimize $D^2 = \Sigma\frac{(\text{observed} - \text{theoretical})^2}{\text{theoretical}}$. Where the observed and theoretical values are for $P(X_n = 1)$, the probability of an error on trial n, and $P(Te = t)$, the probability of a total of t errors for a subject-item sequence. Under the assumption of independent observations (which should be questioned for these models since the learning of one item should effect the learning of another) D^2 is asympotically distributed as Chi-square so that large values of D^2 would indicate a poor fit between the data and the model. Regardless of the statistical validity, the method does give one method of estimating the parameters.

In this section the following designations are used: PR for pretest given, NP for no pretest given, RO for rote, CO for conceptual, RE for reception and DI for discovery.

For the PR RO RE treatment g was estimated to be .804 and c to be .611 with a resulting value of D^2 equal to 10.967 which indicates a poor fit between theory and data (Table 6.12). Figure 6.1 gives a graphical representation of this poor fit.

Table 6.12

OBSERVED AND THEORETICAL VALUES OF THE PROBABILITY OF AN ERROR ON TRIAL n AND THE PROBABILITY OF t ERRORS BEFORE LEARNING FOR TREATMENT PR RO RE

	n =		1	2	3
$P(X_n=1)$	observed		.208	.083	0
	theoretical		.196	.076	.030
	t =	0	1	2	3
P(Te=t)	observed	.771	.167	.062	0
	theoretical	.715	.254	.028	.003

g = .804 c = .611 D^2 = 10.967

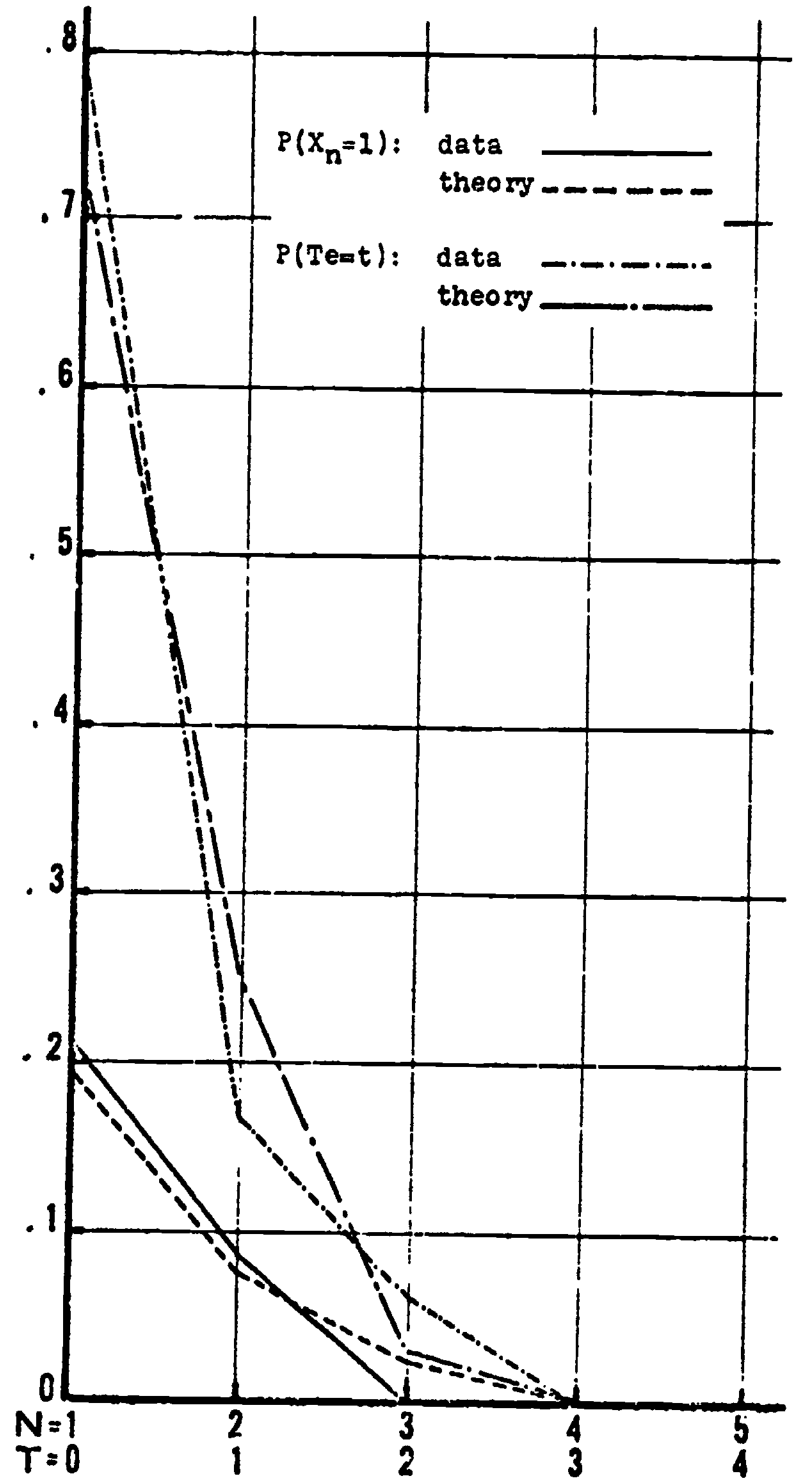

Figure 6.1. Observed and Theoretical Values of $P(X_n=1)$ and $P(Te=t)$ for Treatment PR RO RE.

Estimates of g and c for the PR RO DI treatment are .573 and .601, respectively, with D^2 equal to 1.355 (Table 6.13) which is a much better fit (Figure 6.2 shows this also).

Table 6.13

OBSERVED AND THEORETICAL VALUES OF THE PROBABILITY OF AN ERROR ON TRIAL n AND THE PROBABILITY OF t ERRORS BEFORE LEARNING FOR TREATMENT PR RO DI

	n =		1	2	3	4	5
$P(X_n=1)$	observed		.426	.194	.074	.018	0
	theoretical		.427	.170	.068	.027	.01
	t =	0	1	2	3	4	5
P(Te=t)	observed	.463	.398	.120	.009	.009	0
	theoretical	.447	.431	.095	.021	.005	.001

g = .573 c = .601 D^2 = 1.355

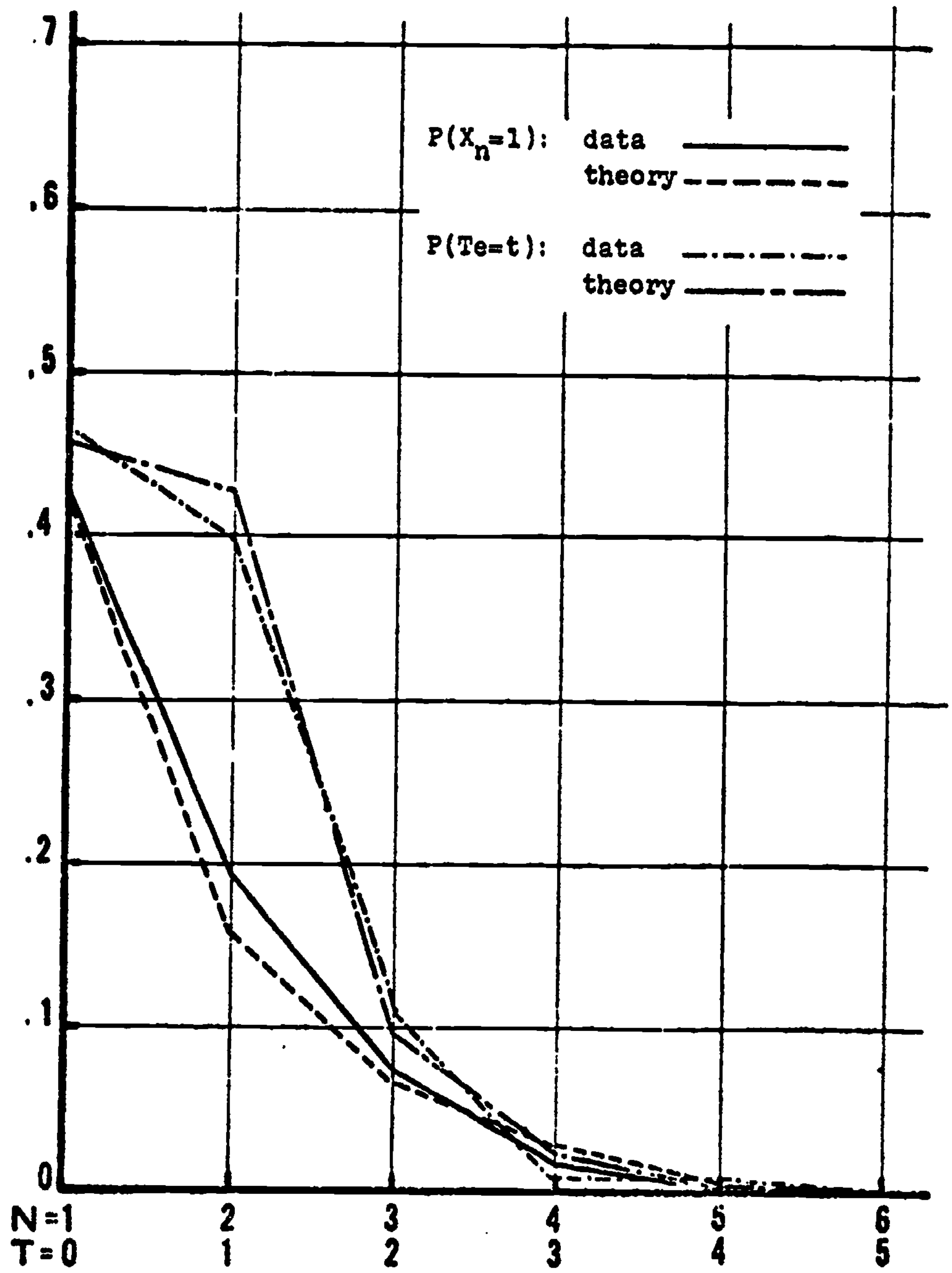

Figure 6.2. Observed and Theoretical Values of $P(X_n=1)$ and $P(Te=t)$ for Treatment PR RO DI.

A moderately good fit was found between data and theory with g equal to .646 and c equal to .638 for the NP RO RE treatment (D^2 = 4.344, Table 6.14 and Figure 6.3).

Table 6.14

OBSERVED AND THEORETICAL VALUES OF THE PROBABILITY OF AN ERROR ON TRIAL n AND THE PROBABILITY OF t ERRORS BEFORE LEARNING FOR TREATMENT NP RO RE

	n =		1	2	3	4
$P(X_n=1)$	observed		.352	.157	.037	0
	theoretical		.354	.128	.046	.017
	t =	0	1	2	3	4
P(Te=t)	observed	.565	.333	.093	.009	0
	theoretical	.538	.385	.064	.011	.002

g = .646 c = .638 D^2 = 4.344

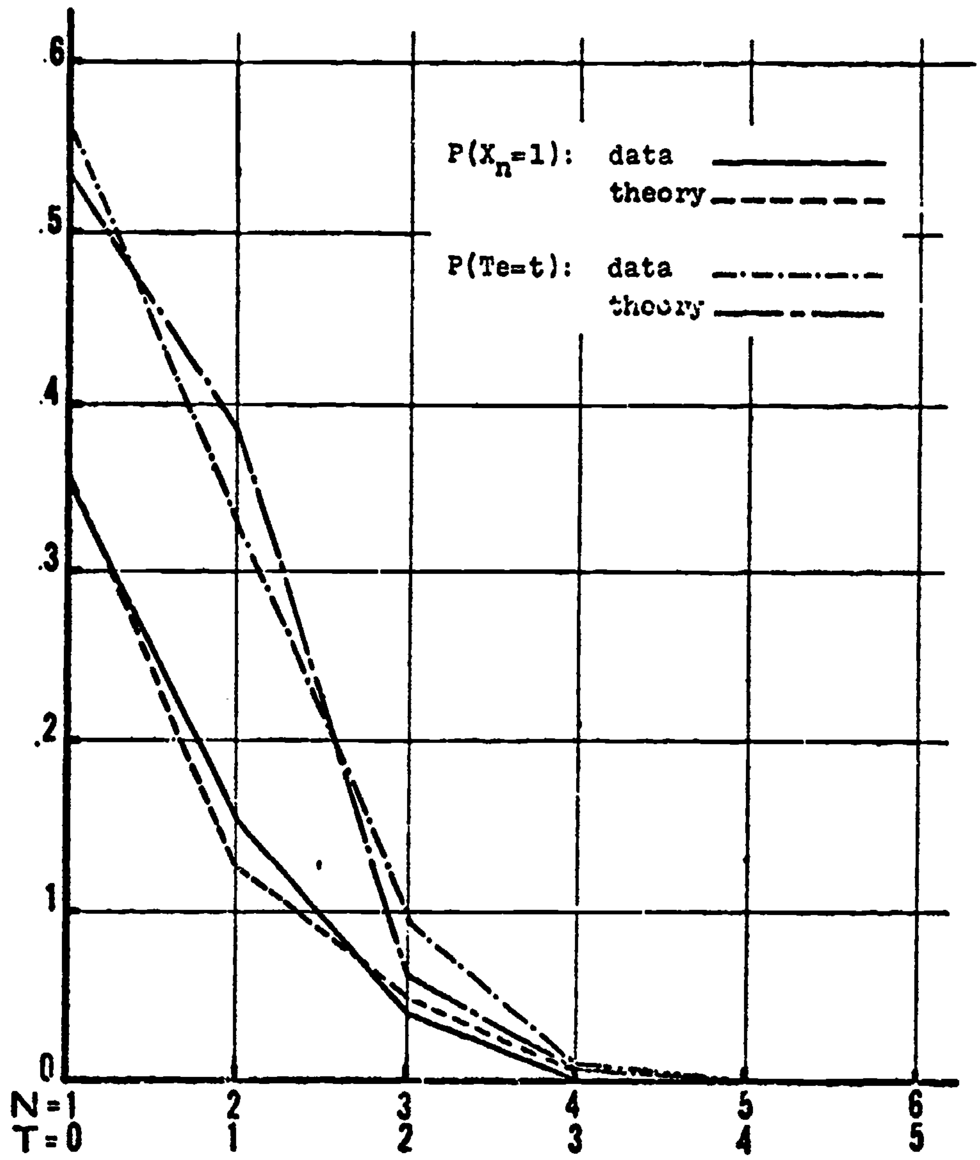

Figure 6.3. Observed and Theoretical Values of $P(X_n=1)$ and $P(Te=t)$ for Treatment NP RO RE.

A poor fit, D^2 equal to 8.090, for g equal to .460 and c equal to .554 is illustrated in Figure 6.4 and is reflected in values in Table 6.15 for the NP RO DI treatment.

Table 6.15

OBSERVED AND THEORETICAL VALUES OF THE PROBABILITY OF AN ERROR ON TRIAL n AND THE PROBABILITY OF t ERRORS BEFORE LEARNING FOR TREATMENT NP RO DI

	n =		1	2	3	4	5
$P(X_n=1)$	observed		.560	.286	.071	.048	0
	theoretical		.540	.241	.107	.048	.021
	t =	0	1	2	3	4	5
P(Te=t)	observed	.369	.357	.226	.036	.012	0
	theoretical	.320	.474	.144	.044	.013	.004

g = .460 c = .554 D^2 = 8.090

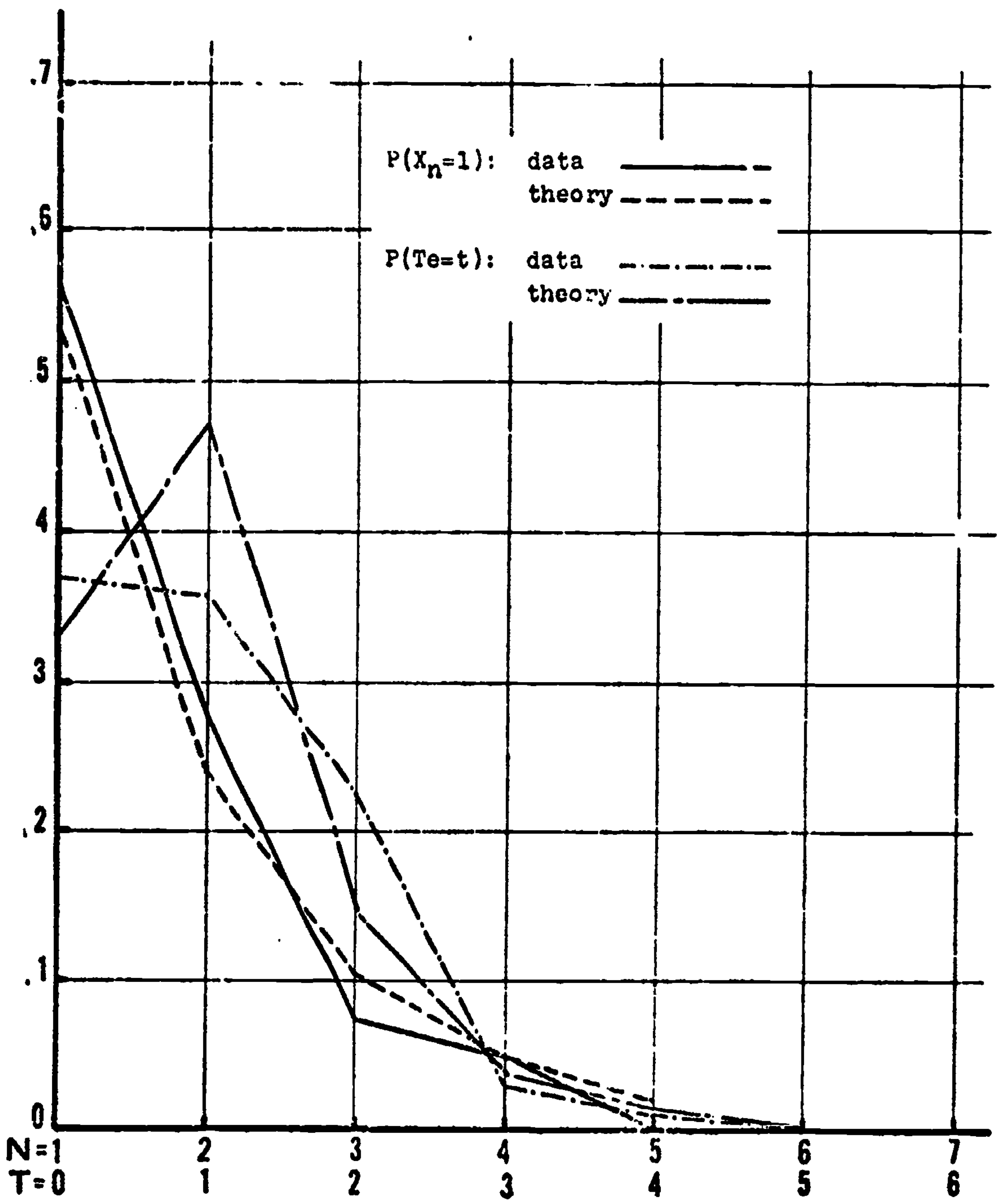

Figure 6.4. Observed and Theoretical Values of $P(X_n=1)$ and $P(Te=t)$ for Treatment NP RO DI.

The data from the PR CO RE treatment, with values of .364 and .709 for g and c, respectively, resulted in a value for D^2 of .854, as reported in Table 6.16, fit the Bower model very well, as is evident in viewing Figure 6.5.

Table 6.16

OBSERVED AND THEORETICAL VALUES OF THE PROBABILITY OF AN ERROR ON TRIAL n AND THE PROBABILITY OF t ERRORS BEFORE LEARNING FOR TREATMENT PR CO RE

	n =		1	2	3	4	5	
$P(X_n=1)$	observed		.685	.167	.056	.028	0	
	theoretical		.636	.185	.054	.016	.005	
	t =	0	1	2	3	4	5	6
P(Te=t)	observed	.306	.546	.093	.028	.019	.009	0
	theoretical	.290	.564	.117	.024	.005	.001	0

g = .364 c = .709 D^2 = .854

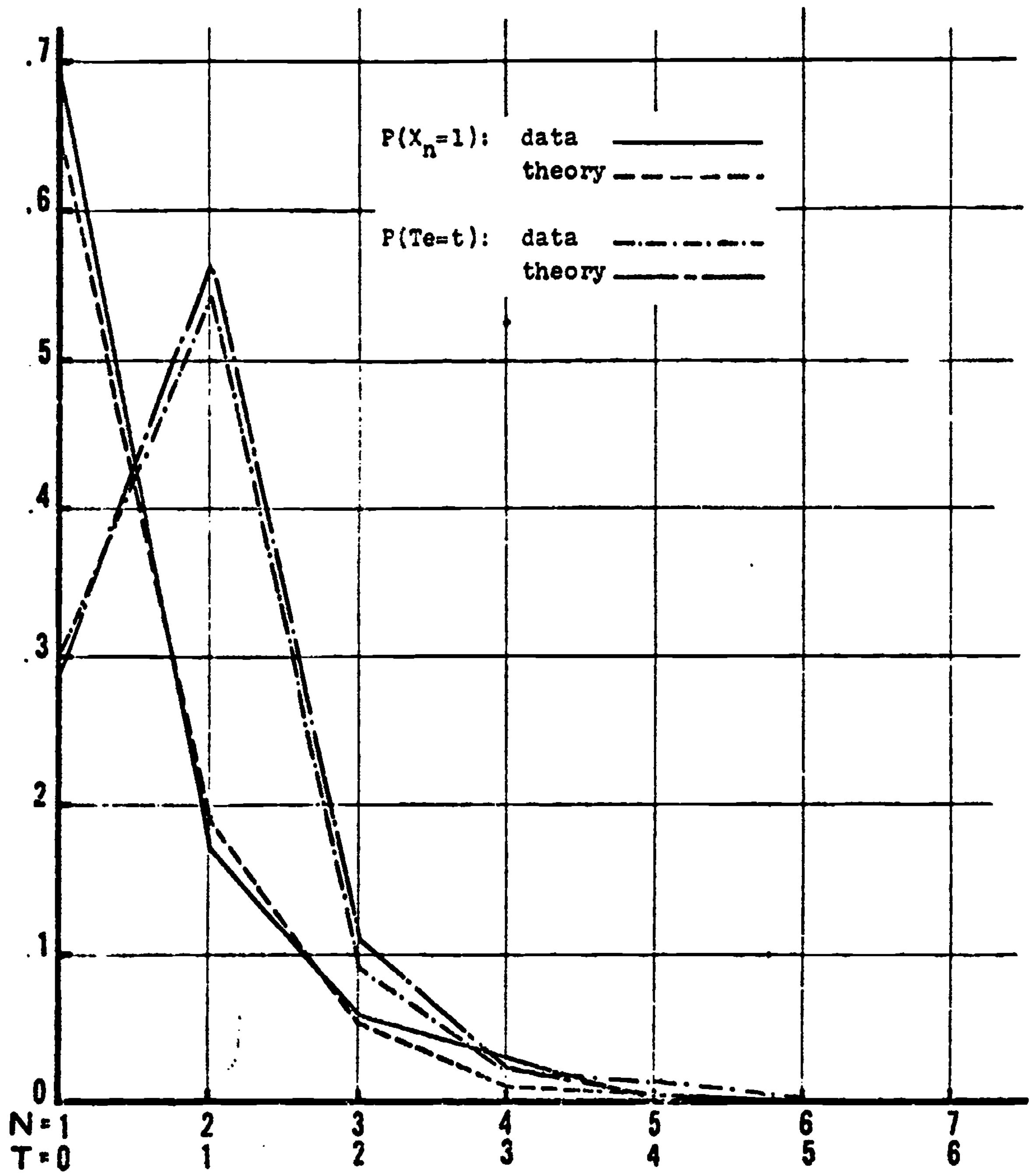

Figure 6.5. Observed and Theoretical Values of $P(X_n=1)$ and $P(Te=t)$ for Treatment PR CO RE.

If the RE part of the above treatment is changed to DI so that the treatment is PR CO DI the fit is not good as indicated by D^2 equal to 8.172 for values of g and c ot .057 and .528, respectively. Table 6.17 indicate these values and Figure 6.6 graphically illustrates the poor fit.

Table 6.17

OBSERVED AND THEORETICAL VALUES OF THE PROBABILITY OF AN ERROR ON TRIAL n AND THE PROBABILITY OF t ERRORS BEFORE LEARNING FOR TREATMENT PR CO DI

	n =		1	2	3	4	5
$P(X_n=1)$	observed		.964	.452	.203	.095	0
	theoretical		.943	.445	.210	.099	.047

	t =	0	1	2	3	4
P(Te=1)	observed	.036	.393	.417	.155	0
	theoretical	.030	.526	.241	.110	.050

g = .057 c = .528 D^2 = 8.172

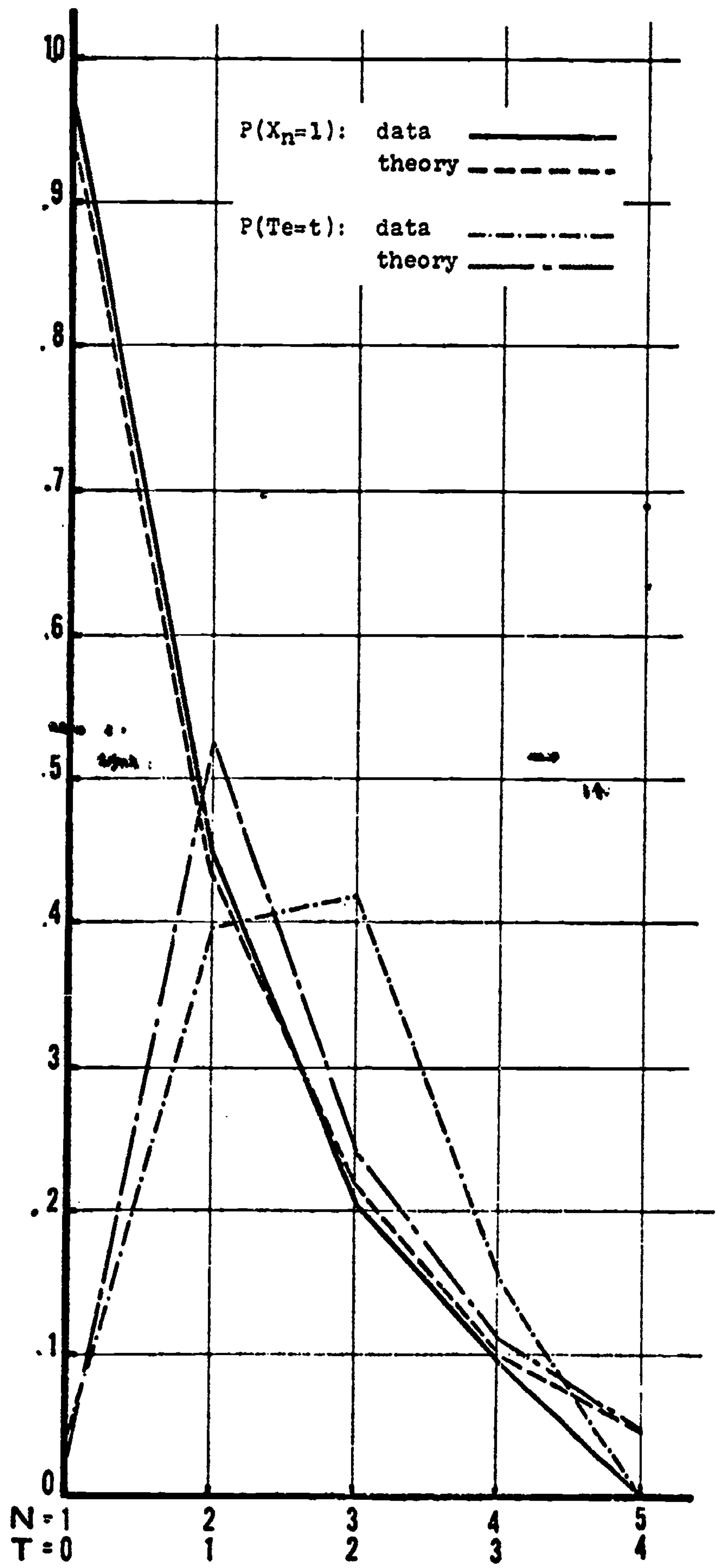

Figure 6.6. Observed and Theoretical Values of $P(X_n=1)$ and $P(Te=t)$ for Treatment PR CO DI.

The NP CO RE treatment has the smallest value of D^2 for any group, .141, indicating a very good fit. This value of D^2 is achieved with values of g and c equal to .476 and .684, respectively. Table 6.18 and Figure 6.7 reflect these observations.

Table 6.18

OBSERVED AND THEORETICAL VALUES OF THE PROBABILITY OF AN ERROR ON TRIAL n AND THE PROBABILITY OF t ERRORS BEFORE LEARNING FOR TREATMENT NP CO RE

	n =		1	2	3	4	5
$P(X_n=1)$	observed		.512	.179	.060	.012	0
	theoretical		.524	.166	.052	.017	.005
	t =	0	1	2	3	4	
P(Te=t)	observed	.381	.500	.107	.012	0	
	theoretical	.383	.500	.100	.019	.003	

g = .476 c = .684 D^2 = .141

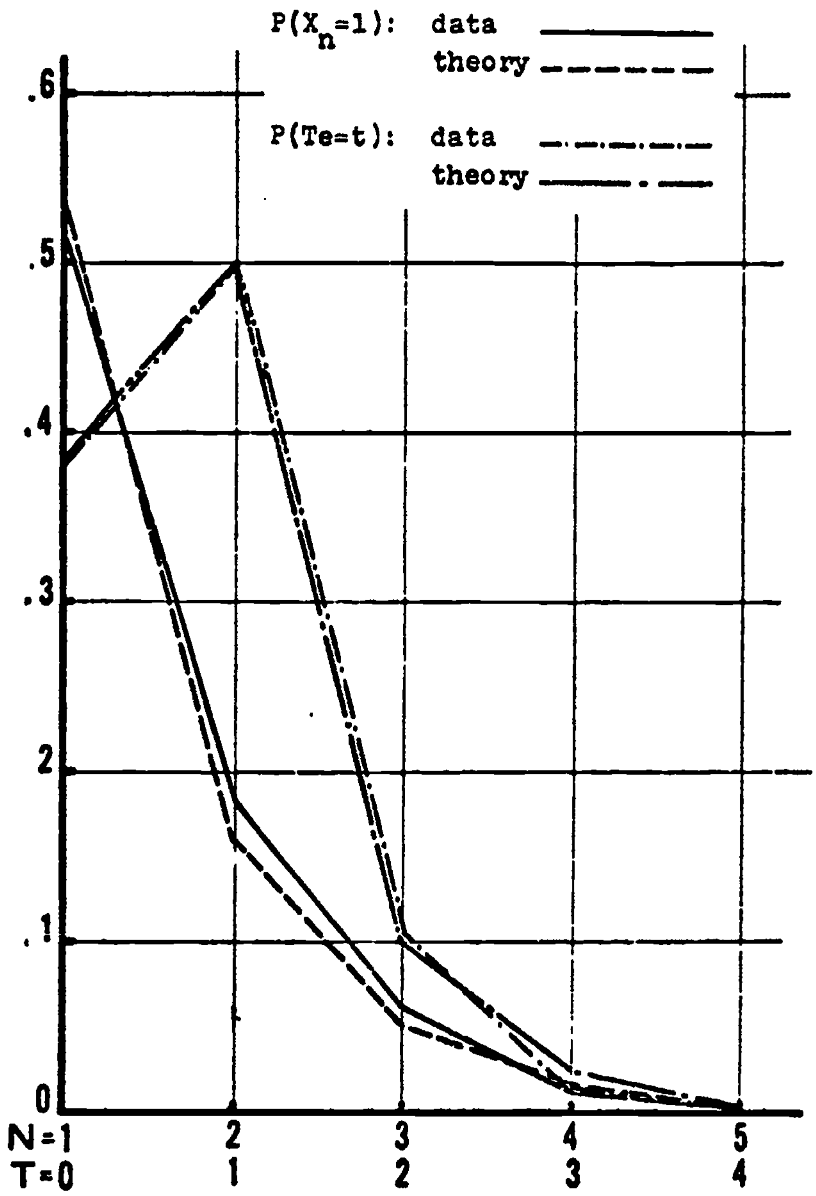

Figure 6.7. Observed and Theoretical Values of $P(X_n=1)$ and P(Te=t) for Treatment NP CO RE.

On the other hand, the NP CO DI treatment has the largest value of D^2 which is 14.485. The values of g and c corresponding to this value of D^2 are .058 and .483, respectively. Table 6.19 and Figure 6.8 reflect these observations.

Table 6.19

OBSERVED AND THEORETICAL VALUES OF THE PROBABILITY OF AN ERROR ON TRIAL n AND THE PROBABILITY OF t ERRORS BEFORE LEARNING FOR TREATMENT NP CO DI

	n =		1	2	3	4	5	6
$P(X_n=1)$	observed		.917	.607	.262	.083	.012	0
	theoretical		.942	.488	.252	.130	.067	.035

	t =	0	1	2	3	4	5
P(Te=1)	observed	.036	.333	.405	.167	.060	0
	theoretical	.029	.483	.243	.122	.061	.030

g = .058 c = .483 D^2 = 14.485

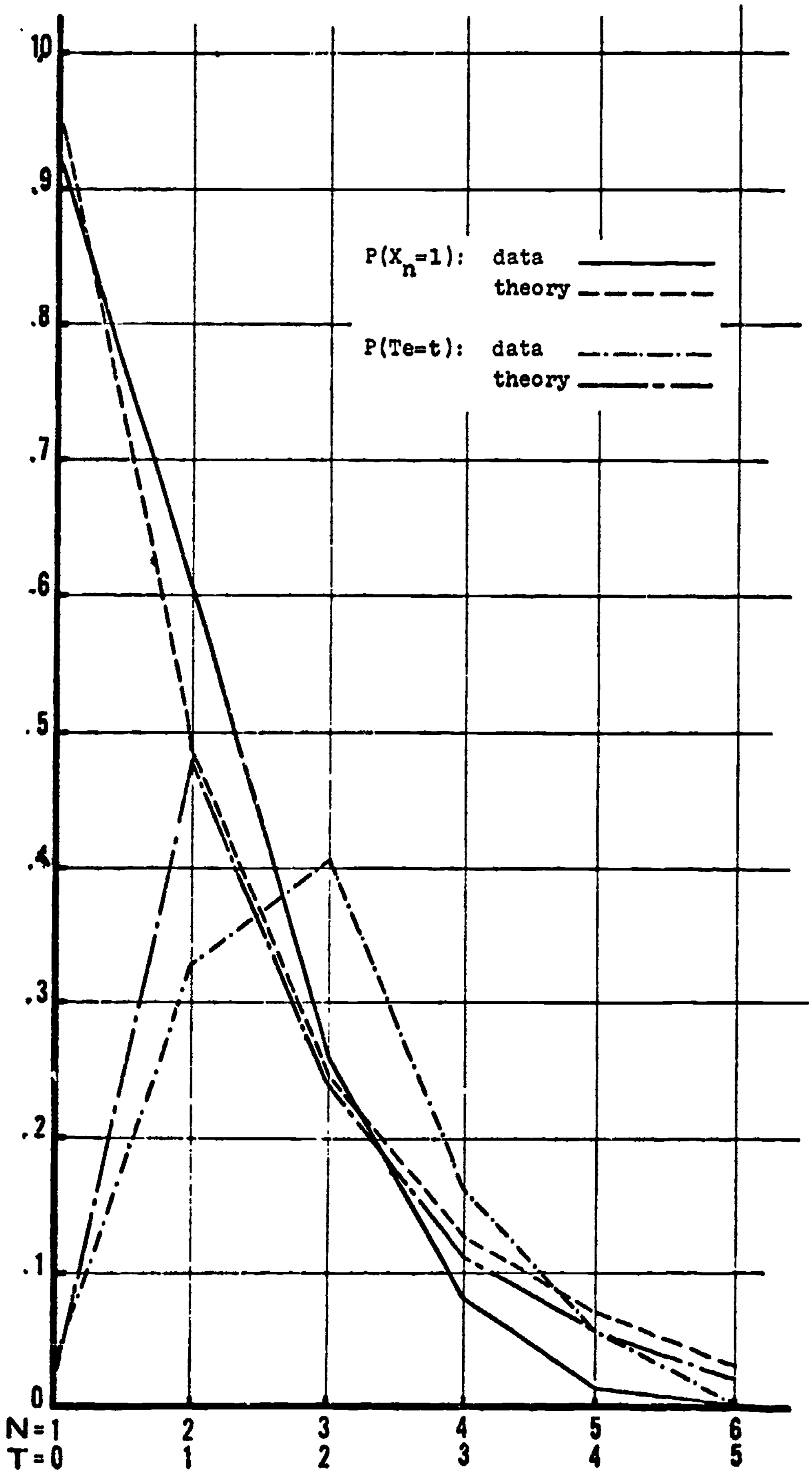

Figure 6.8. Observed and Theoretical Values of $P(X_n=1)$ and $P(Te=t)$ for Treatment NP CO DI.

Model II and Model III. One purpose for considering Models II and III was to facilitate parameter estimation, however, the method of minimizing D^2 gave much better estimates. The transition matricies for the different treatments are given in Appendix C.

Summary

The results of the data analysis is reported in this chapter. Significant differences were found between rote and conceptual treatments and between pretest given and pretest not given treatments for learning relations between concepts. Significant differences were found among cell means for the differences between errors on old items and errors on new items. Significant differences were also found between reception and discovery treatments in Part Two of the study. Data from conceptual reception treatments were found to agree well with theoretical values from Bower's model.

Conclusions based on these analyses will be discussed in Chapter VII with recommendations for future research and some limitations of the study.

Chapter VII

CONCLUSION

This concluding chapter contains a discussion of the evidence gathered to answer the questions stated in Chapter II. Romberg (1969) stated, "The educational research process involves five sequential activities: asking good questions, gathering relevant evidence to answer the questions, ruling out alternative hypotheses, extrapolating minimally beyond the data and recylcing (p. 1)." Except for recycling this chapter completes the report of the five sequential activities not already done.

The questions in Chapters I and II could be briefly summarized as:

1. Could evidence of an interaction between rote-conceptual and reception-discovery dimensions of learning be found?

2. Could a method of studying learning relations between concepts be developed?

3. To what extent would models like Bower's fit data about learning mathematical concepts?

The ideas discussed in Chapters I and II and the research cited in Chapter III led to the formulation and execution of the study as reported in Chapters IV and V. The data were analyzed as reported in Chapter VI. To reiterate, this chapter will report some conclusions. However, some limitations of the study must be considered before drawing these conclusions.

Limitations

The external validity of the study must be questioned since the teaching assistants that participated in the experiment were not randomly selected. Also the students knew they were participating in an experiment, and this is another source of possible invalidity. Campbell and Stanley (1966) pointed out that when subjects know they are participating in an experiment ". . . a higher-order problem-solving task is generated, in which the procedures and experimental treatment are reacted to not only for their simple stimulus values, but also for their role as clues in divining the experimenter's intent (p. 20)." The study was designed to provide internal validity by randomly assigning the subjects to treatments.

The problem of the low reliabilities of the pre- and posttests was pointed out in Chapter V. The position taken is that conclusions drawn from this test data provides hypotheses for future studies. The tests are not believed to be unreliable but rather that appropriate methods for assessing the reliability of criterion-referenced tests have not been discovered. Perhaps reliability should not be considered at all.

The content validity of the pre- and posttests is assumed to be high. Recall that the tests were constructed from the same item pool used in instruction. Anastasi (1970) said,

> ...content validity provides an adequate technique for evaluating achievement tests. It permits us to answer two questions that are basic to the validity of an achievement test: (1) Does the test cover a representative sample of curricular content? (2) Is test performance reasonably free from the influence of irrelevant variables (p. 102)?

It seems reasonable to answer "yes" to both questions.

In spite of careful proof reading, a mistake in writing the posttest for Part One was found after the data were collected. The item 9/7 + 9/14 + 3/7 + 9/28 + . . . on list A was written as 9/7 + 9/14 + 3/7 + 9/8 + . . . on the posttest so that what was intended to be an "old" item on the posttest was actually new to the Ss. Both items are examples of an infinite series that is neither arithmetic nor geometric. A conclusion cannot be drawn on the basis of this being an "old" item.

With these limitations in mind, some conclusions are discussed.

Conclusions

The three research hypotheses about learning relations between concepts are:

> Hypothesis 1a. There is a significant difference between reception and discovery learning on recognition of relations between concepts.

> Hypothesis 1b. There is a significant difference between learning that includes rote learning and learning that is conceptual on recognition of relations between concepts.

> Hypothesis 1c. There is a significant interaction between reception-discovery learning and rote-conceptual learning on recognition of relations between concepts.

No evidence was found to support Research Hypothesis 1a. It may be that no differences exist for learning hierarchies of concepts in the different ways discussed or it may be that more difficult hierarchies of mathematical concepts are required to detect the differences. More discussion will be provided in the next section.

There is statistical evidence for the existence of a main effect on the rote (RO) - conceptual (CO) dimension. However, the data may not warrant inferring Research Hypothesis 1b is true. Note correct responses, whether made because a concept is learned or because an item is learned by rote, do not give evidence that relations between concepts have been learned. The only evidence about learning relations is of a negative sort and shows at times a relation is not learned but never that a relation is learned. The conclusion is that Ss in CO treatments make more mistakes (than Ss in RO treatments) of a kind that indicate relations between concepts are not learned. Thus, either the RO treatment increases the probability of learning relations between concepts or it increases the probability of guessing correctly before learning. Since the experiment does not distinquish between these two possibilities experiments would be required to determine which alternative is the better interpretation.

There is no evidence to support Research Hypothesis 1c. Comments made about Research Hypothesis 1a are appropriate.

The ability to test these three hypotheses leads to the following important conclusion: It is possible to study the learning of relations between concepts in a hierarchy of mathematical concepts by observing Ss' responses while learning.

Three research hypotheses about concept recognition are:

Hypothesis 2a. There is a significant difference between reception learning and discovery learning on recognition of examples of concepts learned.

Hypothesis 2b. There is a significant difference between learning that includes rote learning and learning that is conceptual on recognition of examples of concepts learned.

Hypothesis 2c. There is a significant interaction between reception-discovery learning and rote-conceptual learning on recognition of examples of concepts learned.

No evidence was found to support these hypotheses.

There is no evidence that rote learning will hamper relation learning or concept learning. In fact, no variables related to new items on the posttest had observed mean values that indicated better performance for subjects in conceptual treatments than for subjects in rote treatments. This fact is certainly not consistent with the view that rote learning is not meaningful in the sense that to encourage rote learning is to discourage conceptual learning. However, it is consistent with the view that there are hierarchies of learning starting with the learning of basic facts and proceeding to higher levels.

The three research hypotheses concerned with Part Two of the study are:

Hypothesis 3a. There is a significant difference between reception and discovery learning on the ability to do computations related to sequences.

Hypothesis 3b. There is a significant difference between learning that includes rote learning and learning that is conceptual on the ability to do computations related to sequences.

Hypothesis 3c. There is a significant interaction between reception-discovery and rote-conceptual learning on the ability to do computations related to sequences.

No evidence was found to support either Research Hypothesis 3b or Research Hypothesis 3C. Research Hypothesis 3a was strongly supported by the data. That one method of learning is better than another is not clear since statistically significant differences were found that in some cases favored reception learning and in other cases favored discovery learning. In addition, some situations revealed differences that were difficult to interpret as favoring either. It may be a mistake to try to prove, in general, that reception learning is better or worse than discovery learning. There is evidence that, in some situations, there are differences, but the nature of these differences needs to be understood more clearly.

One part of the study attempted to detect differences while learning was occurring. Data collected while *Ss* were learning agree rather well with the theoretical values from the Bower model for conceptual reception treatments when the estimate that g (the probability of a correct guess) was the reciprocal of the number of possible guesses was discarded and replaced by an empirical estimate of g. The data does not agree as well for other treatments; this leads to the conclusion that there are differences during learning which can be detected using mathematical models.

It is most interesting that of all the treatments in the experiment the conceptual reception treatments resemble most closely what would be expected in classroom activities designed to teach students the concepts of the study. In most classrooms, before being asked to recognize examples, the students see both the definitions and the examples of the concepts either in a reading assignment or in a lecture. Then the students are given items to classify after which they are given feedback by having their homework graded or perhaps by checking an answer book. If it is determined that the concepts are not learned, the student would most likely be directed to reread the definitions and examples and then try some more problems. If a way of estimating the parameters could be found that did not depend upon empirical data, great progress toward estimation of the difficulty of acquiring specific concepts in the classroom could be developed.

No values of parameters g and c in the Bower model will give a larger probability of learning a concept on the third trial than on the second trial yet that is what occurred for both conceptual discovery treatments. Just what this means in terms of learning concepts is not clear but it does mean the Bower model is not appropriate for this kind of learning.

Recommendations for Future Research

Is it possible that problems in understanding the differences between reception and discovery learning in the past have been caused by looking for main effects when the whole question is really one of interacting variables? The evidence is at least strong enough to prompt further research. This research may need to include reconsideration of the treatments termed as rote and conceptual. But the method of specifying from a mathematical discourse a hierarchy of concepts and then proceeding to study different methods of teaching and learning the relations with respect to interacting variables seems quite promising. It is reiterated that the structure of mathematics is one aspect that sets it apart from other subjects taught in schools. It may be possible to learn a great deal about mathematics learning without considering the formal logical relations between concepts but surely much more could be gained by detailing the role of the structure in the learning situation.

The lack of evidence found to support Research Hypotheses 1a, 1c, 2a, 2b, or 2c indicate that if there are differences caused by the rote-conceptual and reception-discovery treatments more subtle tests must be devised. It would be interesting to find a way to measure differences in relations learned. To do this, much more difficult relations would be required. Since correct responses provide no information about learning relations between concepts, more mistakes are needed to study this kind of learning. The possibility exists that a relation has not been learned even though the S has no errors on a posttest. In the study no errors were made on any posttest that indicated a S did not know the relations between the concepts studied.

It had been hoped that evidence as to whether or not learning the relations between concepts fit the Bower model would be found. Too few errors were made to draw conclusions one way or the other. It still seems possible that Bower's model may be appropriate for sets of more difficult relations but for a small number of easy to learn relations it may not be very accurate. Three research hypotheses are:

> The Bower model is adequate for describing learning of relations in a hierarchy of mathematical concepts.
>
> The Bower model is adequate for describing learning in the conceptual reception treatments.
>
> The Bower model is not adequate for learning in the conceptual discovery treatments or for learning in any of the rote treatments.

Summary

This chapter contains a discussion of the conclusions that there are significant differences between rote and conceptual learning and significant differences between reception and discovery learning. It was also concluded that the method of specifying a hierarchy of mathematical concepts and then proceeding to study learning the relations between the concepts was promising. A very good fit between data and theoretical values for the Bower model for some treatments and not for others seems to indicate a method that could be developed to study differences in learning.

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APPENDIX A

INSTRUCTIONAL MATERIALS

DIRECTIONS FOR PART ONE RECEIVED BY Ss IN RO RE TREATMENTS

A Unit on Sequences and Series

You will be given a set of definitions. You are to learn to identify the different kinds of sequences and series that are defined there.

DIRECTIONS:

1. After you finish reading the directions, spend a few minutes reading the definitions. It is not necessary to memorize each one since you will find as you use them that you will learn them easily.
2. After you have read the definitions, return them to the instructor and get a booklet containing examples from him/her. In the booklet place an X in front of those terms that describe the example. For instance:

   Phydeau (a local dog)

   X animal   X dog

   plant

3. The correct responses are marked on the back of the page. Your answers are not used to determine your grade. PLEASE DO NOT CHANGE ANY OF YOUR ANSWERS!
4. When you have finished your booklet be sure your name is on it and give it to your instructor.
5. Get a list of definitions from the instructor and reread any you wish, then return the definitions and get a new booklet.
6. Repeat this fun process until you can use the definitions correctly. Don't spend more than 20 minutes per table since a certain amount of speed is necessary.

PLEASE WORK INDIVIDUALLY.

DIRECTIONS FOR PART ONE RECEIVED BY Ss IN RO DI TREATMENTS

A Unit on Sequences and Series

You will be given a booklet containing examples and terms. You are asked to identify the terms that describe each example.

DIRECTIONS:

1. After you finish reading the directions, get a copy of a booklet of examples from the instructor. In the booklet place an X in front of those terms that describe the example. For instance:

   Phydeau (a local dog)

   X Animal

   Plant

   X Dog

2. The correct responses are marked on the back of the page. Your answers are not used to determine your grade. PLEASE DO NOT CHANGE ANY OF YOUR ANSWERS!
3. When you have finished your booklet be sure your name is on it and give it to your instructor.
4. Get another booklet from the instructor and repeat this fun process until you can recognize examples of each term. Don't spend more than 20 minutes per booklet since a certain amount of speed is necessary.

PLEASE WORK INDIVIDUALLY.

DIRECTIONS FOR PART ONE RECEIVED BY Ss IN CO RE TREATMENTS

A Unit on Sequences and Series

You will be given a set of definitions. You are to learn to identify the different kinds of sequences and series that are defined there.

DIRECTIONS:

1. After you finish reading the directions, spend a few minutes reading the definitons. It is not necessary to memorize each one since you will find as you use them that you will learn them easily.
2. After you have read the definitions, return them to the instructor and get a table containing examples from him/her. In the table, place an X below those terms that describe the example. For instance:

	Animal	Plant	Dog
Phydeau (a local dog)	X		X

3. After you have completed the table an instructor will give you the correct answers (<u>your answers are not used to determine your grade</u>). PLEASE DO NOT CHANGE ANY OF YOUR ANSWERS!
4. Study the correct responses a few minutes then turn in your table (with your name on it) and get the list of definitions. Reread any of these you wish, then return the definitions and get a new table from the instructor.
5. Repeat this fun process until you can use the definitions correctly. Don't spend more than 20 minutes per table since a certain amount of speed is necessary.

PLEASE WORK INDIVIDUALLY.

DIRECTIONS FOR PART ONE RECEIVED BY Ss IN CO DI TREATMENTS

A Unit on Sequences and Series

You will be given a table containing examples and terms. You are asked to identify the terms that describe each example.

DIRECTIONS:

1. After you finish reading the directions, get a copy of a table of examples from the instructor. In the table place an X below those terms that describe the example. For instance:

	Animal	Plant	Dog
Phydeau (a local dog)	X		X

2. After you have completed the table the instructor will give you the correct answers (your answers are not used to determine your grade). PLEASE DO NOT CHANGE ANY OF YOUR ANSWERS!

3. Study the correct responses a few minutes then turn in your table (with your name on it) and get another one.

4. Repeat this fun process until you can recognize examples of each term. Don't spend more than 20 minutes per table since a certain amount of speed is necessary.

PLEASE WORK INDIVIDUALLY.

A.	Infinite Sequence	Infinite Series	Arithmetic Sequence	Geometric Sequence	Finite Sequence	Geometric Series	Finite Series	Arithmetic Series
8, 11, 14, 17								
1 + 7 + 14 + 22								
7 + 2 + (-3) + (-8)								
9, 18, 28, 39								
$6, -\frac{21}{4}, \frac{147}{32}$								
9, 18, 27, 36								
$\frac{1}{7} + \frac{1}{14} + \frac{[illegible]}{7} + \frac{1}{28} + \cdots$								
$1, \frac{1}{6}, \frac{1}{9}, \frac{1}{13}, \ldots$								
8 + (-4) + 2 + (-1)								
$9 + (-\frac{27}{8}) + \frac{81}{64} + (-\frac{243}{512}) + \cdots$								
8 + 0 + (-8) + (-16)								
$5, \frac{35}{9}, \frac{245}{81}, \ldots$								

PART ONE - LIST B

B.

	Infinite Series	Arithmetic Series	Finite Sequence	Geometric Series	Infinite Sequence	Geometric Sequence	Finite Series	Arithmetic Sequence
$1 + (-2) + (-5) + (-8)$								
$\frac{1}{4}, \frac{1}{4}, \frac{1}{8}, \frac{1}{12}, \ldots$								
$7 + \frac{7}{9} + \frac{7}{81} + \frac{7}{729} + \ldots$								
$4, 7, 10, 13$								
9, [illegible], 17, [illegible]								
$6 + 13 + 20 + 27$								
$2 + 1 + \frac{2}{3} + \frac{1}{2} + \ldots$								
$2, \frac{12}{7}, \frac{72}{49}, \frac{432}{343}, \ldots$								
$1 + 8 + 16 + 25$								
$1, \frac{1}{4}, \frac{1}{16}, \frac{1}{64}$								
$8 + 6 + \frac{9}{2} + \frac{27}{8}$								
$7, 16$, [illegible], [illegible]								

PART ONE - LIST C

C.

	Infinite Sequence	Arithmetic Series	Geometric Sequence	Geometric Series	Finite Sequence	Finite Series	Arithmetic Sequence	Infinite Series
$\frac{2}{9} + \frac{1}{9} + \frac{2}{27} + \frac{1}{18} + \cdots$								
$\frac{1}{3}, \frac{1}{4}, \frac{1}{7}, \frac{1}{11}, \ldots$								
$3 + (-7) + \frac{49}{5} + (-\frac{343}{64}) + \cdots$								
$3, 10, 17, 24$								
$5 + 11 + 17 + 23$								
$1, 3, 6, 10$								
1 + 10 + 20 + [illegible]								
$2 + (-\frac{1}{4}) + \frac{1}{32} + (-\frac{1}{256})$								
$3 + 7 + 13 + 17$								
$6, 4, 2, 0$								
$4, 1, \frac{1}{4}, \frac{1}{16}$								
$2, -\frac{1}{2}, \frac{1}{8}, -\frac{1}{32}, \ldots$								

D	Arithmetic Series	Geometric Sequence	Infinite Sequence	Arithmetic Sequence	Finite Series	Infinite Series	Geometric Series	Finite Sequence
$5 + 14 + 24 + 35$								
$7, \frac{7}{4}, \frac{7}{16}, \frac{7}{64}$								
$1 + (-\frac{3}{8}) + \frac{9}{64} + (-\frac{27}{512})$								
$3, -2, \frac{4}{3}, -\frac{8}{9}, \ldots$								
$4 + (-\frac{8}{3}) + \frac{16}{9} + (-\frac{32}{27}) + \ldots$								
$6 + 12 + 18 + 24$								
$7, 5, 3, 1$								
$5, 11, 15, 21$								
$\frac{1}{9} + \frac{1}{18} + \frac{1}{27} + \frac{1}{36} + \ldots$								
$7 + 6 + 5 + 4$								
$4, 12, 20, 28$								
$\frac{1}{5}, \frac{1}{7}, \frac{1}{12}, \frac{1}{19}, \ldots$								

PART ONE - LIST E

	Infinite Series	Geometric Sequence	Arithmetic Series	Finite Series	Finite Sequence	Arithmetic Sequence	Infinite Sequence	Geometric Series
$3 + 8 + 13 + 18$								
$5, \frac{30}{7}, \frac{180}{49}$								
$\frac{1}{3}, \frac{1}{9}, \frac{1}{27}, \frac{1}{81}, \ldots$								
$12 + 21 + 31$								
$, -2, -7$								
$, 12, 15$								
$12 + 16 + 20$								
$4 + (-\frac{8}{3}) + \frac{16}{9} + (-\frac{32}{27})$								
$\frac{1}{4} + \frac{1}{8} + \frac{1}{12} + \frac{1}{16} + \cdots$								
$5, 8, 12, 11$								
$5 + \frac{25}{7} + \frac{125}{49} + \frac{625}{343} + \cdots$								
$2, -\frac{10}{9}, \frac{50}{81}, -\frac{250}{729}, \ldots$								

PART ONE - LIST F

FORM F	Arithmetic Sequence	Arithmetic Series	Geometric Series	Geometric Sequence	Infinite Sequence	Finite Series	Finite Sequence	Infinite Series
$3, -2, \frac{4}{3}, -\frac{8}{9}$								
$1 + \frac{1}{5} + \frac{1}{25} + \frac{1}{125} + \cdots$								
$\frac{3}{2} + \frac{3}{4} + \frac{1}{2} + \frac{3}{8} + \cdots$								
$2 + (-6) + (-14) + (-22)$								
$3 + (-\frac{5}{3}) + \frac{25}{27} + (-\frac{125}{243})$								
$2, -4, -10, -16$								
$7 + 1 + (-5) + (-11)$								
$6, 5, \frac{25}{6}, \frac{125}{36}, \cdots$								
$\frac{1}{2}, 1, \frac{1}{3}, \frac{1}{4}, \cdots$								
$3 + 11 + 20 + 30$								
$8, 10, 13, 17$								
$2, -1, -4, -7$								

PART ONE - LIST G

FORM G

	Finite Series	Geometric Sequence	Arithmetic Series	Infinite Series	Finite Sequence	Geometric Series	Arithmetic Sequence	Infinite Sequence
$1, -\frac{5}{6}, \frac{25}{36}, -\frac{125}{216}$								
$9 + 15 + 21 + 27$								
$7 + (-1) + (-9) + (-17)$								
$4 + 8 + 13 + 19$								
$3, \frac{2}{3}, \frac{4}{27}, \frac{8}{243}, \ldots$								
$8 + \frac{4}{3} + \frac{4}{18} + \frac{4}{108} + \cdots$								
$5, 4, 3, 2$								
$\frac{1}{5}, 1, \frac{1}{6}, \frac{1}{7}, \ldots$								
$5, 0, -5, -10$								
$+ (-\frac{5}{2}) + \frac{5}{4} + (-\frac{5}{8})$								
$\frac{1}{8} + \frac{7}{16} + \frac{7}{24} + \frac{7}{32} + \cdots$								
[illegible], 11, 18, 26								

FORM H	Aritinmetic Series	Arithmetic Sequence	Infinite Series	Geometric Series	Finite Series	Infinite Sequence	Geometric Sequence	Finite Sequence
$4, -3, -10, -17$								
$\frac{1}{4}, \frac{1}{7}, \frac{1}{11}, \frac{1}{18}, \ldots$								
$5 + 4 + 3 + 2$								
$8, 4, 2, 1$								
$2 + 4 + 7 + 11$								
$8, 12, 17, 23$								
$6, -\frac{21}{4}, \frac{147}{32}, \ldots$								
$1 + 3 + 5 + 7$								
$\frac{4}{3} + \frac{2}{3} + \frac{4}{9} + \frac{1}{3} + \ldots$								
$5 + \frac{5}{9} + \frac{5}{81} + \frac{5}{729} + \ldots$								
$8 + (-\frac{24}{5}) + \frac{72}{25} + (-\frac{216}{125})$								
$9, 11, 13, 15$								

PART ONE - LIST I

FORM I

	Infinite Sequence	Infinite Series	Arithmetic Series	Finite Series	Arithmetic Sequence	Finite Sequence	Geometric Sequence	Geometric Series
$6, 8, 11, 15$								
$2 + 9 + 16 + 23$								
$1 + 6 + 12 + 19$								
$6 + \frac{21}{4} + \frac{147}{32} + \cdots$								
$\frac{8}{9} + \frac{8}{18} + \frac{8}{27} + \frac{2}{9} + \cdots$								
$\frac{1}{3}, 1, \frac{1}{4}, \frac{1}{5}, \ldots$								
$6 + 4 + \frac{8}{3} + \frac{16}{9}$								
$2 + (-4) + (-10) + (-16)$								
$9, 17, 25, 33$								
$2, \frac{8}{5}, \frac{32}{25}, \frac{128}{125}$								
$8, -\frac{8}{3}, \frac{8}{9}, -\frac{8}{27}$								
$8, -1, -10, -19$								

PART ONE - LIST J

FORM J	Arithmetic Series	Geometric Series	Finite Sequence	Infinite Series	Geometric Sequence	Finite Series	Infinite Sequence	Arithmetic Sequence
$5, 14, 23, 32$								
$6, 14, 23, 33$								
$3 + 2 + \frac{4}{3} + \frac{8}{9} + \cdots$								
$4, \frac{8}{7}, \frac{16}{49}, \frac{32}{343}, \ldots$								
$7 + 13 + 19 + 25$								
$8 + 14 + 20 + 26$								
$9, -\frac{63}{8}, \frac{441}{64}$								
$\frac{1}{7}, \frac{1}{3}, \frac{1}{10}, \frac{1}{13}, \ldots$								
$6 + 11 + 17 + 24$								
$\frac{1}{4} + \frac{1}{8} + \frac{1}{12} + \frac{1}{16} + \cdots$								
$2 + \frac{8}{5} + \frac{32}{25} + \frac{128}{125}$								
$6, 8, 10, 12$								

PART ONE - POSTTEST

Name:

Section:

	Arithmetic Series	Geometric Sequence	Infinite Series	Finite Series	Arithmetic Sequence	Infinite Sequence	Finite Sequence	Geometric Series
$1+7+14+22$								
$6, -\frac{21}{4}, \frac{147}{32}$								
$9, 18, 27, 36$								
$\frac{9}{7}+\frac{9}{14}+\frac{3}{7}+\frac{9}{8}+\cdots$								
$8+(-4)+2+(-1)$								
$8+0+(-8)+(-16)$								
$7, 9, 11, 13$								
$1+8+15+22$								
$5, 7, 10, 14$								
$\frac{1}{6}, \frac{1}{2}, \frac{1}{8}, \frac{1}{10}, \ldots$								
$4+2+1+\frac{1}{2}+\cdots$								
$4, \frac{4}{5}, \frac{4}{25}, \frac{4}{125}, \ldots$								

PART TWO - LIST A

Name: ____________

A.	20th term	nth term	sum of 1st 20 terms	sum of 1st n terms	sum of the terms of the infinite sequence
$6, -\frac{21}{4}, \frac{147}{32}, \ldots$					
$5, \frac{35}{9}, \frac{245}{81}, \ldots$					
$9, 18, 27, 36, \ldots$					
$8, 11, 14, 17, \ldots$					

PART TWO - LIST B

Name: ______ B.	20th term	nth term	sum of 1st 20 terms	sum of 1st n terms	sum of the terms of the infinite sequence
$4, 7, 10, 13, \ldots$					
$1, \frac{1}{4}, \frac{1}{16}, \frac{1}{64}, \ldots$					
$7, 12, 17, 22, \ldots$					
$2, \frac{12}{7}, \frac{72}{49}, \frac{432}{343}, \ldots$					

PART TWO - LIST C

Name: C.	20th term	nth term	sum of 1st 20 terms	sum of 1st n terms	sum of the terms of the infinite sequence
$2, -\frac{1}{2}, \frac{1}{8}, -\frac{1}{32}, \ldots$					
$3, 10, 17, 24, \ldots$					
$4, 1, \frac{1}{4}, \frac{1}{16}, \ldots$					
$6, 4, 2, 0, \ldots$					

PART TWO - LIST D

Name: ______ D.	20th term	nth term	sum of 1st 20 terms	sum of 1st n terms	sum of the terms of the infinite sequence
$9, \frac{9}{4}, \frac{9}{16}, \frac{9}{64}, \ldots$					
$3, -2, \frac{4}{3}, -\frac{8}{9}, \ldots$					
$7, 5, 3, 1, \ldots$					
$4, 12, 20, 28, \ldots$					

PART TWO - LIST E

Name: E.	20th term	nth term	sum of 1st 20 terms	sum of 1st n terms	sum of the terms of the infinite sequence
$8, 3, -2, -7, \ldots$					
$2, -\frac{10}{9}, \frac{50}{81}, -\frac{250}{729}, \ldots$					
$5, \frac{30}{7}, \frac{180}{49}, \ldots$					
$6, 9, 12, 15, \ldots$					

PART TWO - LIST F

Name: ______ F.	20th term	nth term	sum of 1st 20 terms	sum of 1st n terms	sum of the terms of the infinite sequence
$6, 5, \frac{25}{6}, \frac{125}{36}, \ldots$					
$2, -1, -4, -7, \ldots$					
$3, -2, \frac{4}{3}, -\frac{8}{9}, \ldots$					
$2, -4, -10, -16, \ldots$					

PART TWO - LIST G

Name: ____ G.	20th term	nth term	sum of 1st 20 terms	sum of 1st n terms	sum of the terms of the infinite sequence
$5, 0, -5, -10, \ldots$					
$5, 4, 3, 2, \ldots$					
$1, -\frac{5}{6}, \frac{25}{36}, -\frac{125}{216}, \ldots$					
$3, \frac{2}{3}, \frac{4}{27}, \frac{8}{243}, \ldots$					

PART TWO - LIST H

Name: H.	20th term	nth term	sum of 1st 20 terms	sum of 1st n terms	sum of the terms of the infinite sequence
4, -3, -10, -17, ...					
8, 4, 2, 1, ...					
9, 11, 13, 15, ...					
$6, -\frac{21}{4}, \frac{147}{32}, \ldots$					

PART TWO - LIST I

Name: ____________

I.	20th term	nth term	sum of 1st 20 terms	sum of 1st n terms	sum of the terms of the infinite sequence
$8, -1, -10, -19, \ldots$					
$8, -\frac{8}{3}, \frac{8}{9}, -\frac{8}{27}, \ldots$					
$9, 17, 25, 33, \ldots$					
$2, \frac{8}{5}, \frac{32}{25}, \frac{128}{125}, \ldots$					

LIST J

Name: J.	20th term	nth term	sum of 1st 20 terms	sum of 1st n terms	sum of the terms of the infinite sequence
$5, 14, 23, 32, \ldots$					
$4, \frac{8}{7}, \frac{16}{49}, \frac{32}{343}, \ldots$					
$9, -\frac{63}{8}, -\frac{441}{64}, \ldots$					
$6, 8, 10, 12, \ldots$					

PART TWO - POSTTEST

Name: Section:	20th term	nth term	sum of 1st 20 terms	sum of 1st n terms	sum of the terms of the infinite sequence
$6, -\frac{21}{4}, \frac{147}{32}, \ldots$					
$8, 11, 14, 17, \ldots$					
$4, 2, 1, \frac{1}{2}, \ldots$					
$5, 7, 9, 11, \ldots$					

APPENDIX B

MANCOVAS AND CELL MEANS

The Following Symbols Are Used:

RE = Reception

DI = Discovery

RO = Rote

CO = Conceptual

PR = Pretest Given

NP = Pretest Not Given

MANCOVA FOR LEARNING RELATIONS BETWEEN CONCEPTS

Source	Dependent Variables FIV, SSV, and AGV F-Ratio	p <
RE/DI	.6969	.5587
RO/CO	5.8213	.0019
PR/NP	3.1043	.0354
RE/DI x RO/CO	1.0832	.3655
RE/DI x PR/NP	1.6245	.1964
RO/CO x PR/NP	1.7970	.1607
RE/DI x RO/CO x PR/NP	2.1451	.1072

Degrees of freedom = 3 and 47

MANCOVA FOR PART ONE POSTTEST

	Dependent Variables			
	SS OLD, A OLD, G OLD, and TOTOLD		SS NEW, A NEW, G NEW, and TOTNEW	
Source	F-Ratio	p <	F-Ratio	p <
RE/DI	.4277	.7879	.4981	.7372
RO/CO	.7944	.5350	.4140	.7977
PR/NP	1.3152	.2785	1.6124	.1873
RE/DI x RO/CO	.6499	.6299	.9631	.4369
RE/DI x PR/NP	.1606	.9572	.5422	.7056
RO/CO x PR/NP	.7342	.5734	1.7975	.1456
RE/DI x RO/CO x PR/NP	.7871	.5396	.1966	.9389

Degrees of freedom = 4 and 46

	Dependent Variables			
	SS SUM, A SUM, G SUM, and TOTSUM		SS DIF, A DIF, G DIF, and TOTDIF	
Source	F-Ratio	p <	F-Ratio	p <
RE/DI	.3286	.8574	.6107	.6571
RO/CO	.6215	.6495	.5623	.6912
PR/NP	1.4062	.2469	1.4856	.2221
RE/DI x RO/CO	.0926	.9844	1.9005	.1265
RE/DI x PR/NP	.3417	.8484	.3968	.8099
RO/CO x PR/NP	1.5671	.1991	.4308	.7856
RE/DI x RO/CO x PR/NP	.3707	.8283	1.2415	.3067

Degrees of freedom = 4 and 46

MANCOVA FOR PART TWO POSTTEST

	Dependent Variables			
	OVSN, AVSG, and ONXAG		145XAG, 124XAG, and 34XAG	
Source	F-Ratio	p <	F-Ratio	p <
MEAN	1.7932	.1671	1.1533	.3418
RE/DI	2.1789	.1086	2.1490	.1122
RO/CO	1.0035	.4032	1.2828	.2960
RE/DI x RO/CO	1.1888	.3286	.2111	.8881

Degrees of freedom = 3 and 34

	Dependent Variables			
	145XON, 124XON, and 34XON		1-4VS5, 12VS34, and 3VS4	
Source	F-Ratio	p <	F-Ratio	p <
MEAN	.3356	.7997	.5965	.6217
RE/DI	3.4946	.0260	1.9313	.1431
RO/CO	.8374	.4829	.4835	.6960
RE/DI x RO/CO	.3861	.7638	.5154	.6745

Degrees of freedom = 3 and 34

	Dependent Variables			
	145XIT, 124XIT, and 34XIT		CL3ON, CL3AG, and CL3IT	
Source	F-Ratio	p <	F-Ratio	p <
MEAN	1.4093	.2570	1.7859	.1685
RE/DI	1.0852	.3686	3.5782	.0238
RO/CO	.4104	.7466	.9847	.4116
RE/DI x RO/CO	.3675	.7770	1.8227	.1617

Degrees of freedom = 3 and 34

MANCOVA FOR PART TWO POSTTEST

Source	CL4ON, CL4AG, and CL4IT F-Ratio	CL4ON, CL4AG, and CL4IT p <	CL5ON, CL5AG, and CL5IT F-Ratio	CL5ON, CL5AG, and CL5IT p <
MEAN	.2701	.8465	.5760	.6348
RE/DI	2.9836	.0449	2.9671	.0457
RO/CO	.0282	.9935	.3546	.7861
RE/DI x RO/CO	.5348	.6616	.3322	.8021

Degrees of freedom = 3 and 34

Source	CL1IT F-Ratio	CL1IT p <	CL2IT F-Ratio	CL2IT p <	MEAN F-Ratio	MEAN p <
MEAN	.4413	.5108	.4413	.5108		
RE/DI	.7880	.3806	.7880	.3806	2.2052	.1463
RO/CO	1.4146	.2421	1.4146	.2421	.7399	.3954
RE/DI x RO/CO	.6535	.4242	.6535	.4242	.0052	.9431

Degrees of freedom = 1 and 36

Source	CL1MN, CL3MN, CL4MN and CL5MN F-Ratio	CL1MN, CL3MN, CL4MN and CL5MN p <	CL1ON and CL1AG F-Patio	CL1ON and CL1AG p <
RE/DI	2.4573	.0649	1.4144	.2567
RO/CO	.3614	.8343	1.0094	.3748
RE/DI x RO/CO	.4401	.7787	.3593	.7008
Degrees of freedom =	4 and 33		2 and 35	

PART ONE OBSERVED CELL MEANS

CELL	SEX	COT	ALG	SECTV1	SECTV2	SECTV3	SS OLD
PR RO RE	1.625	34.500	10.875	-.125	-.500	-.375	.000
PR RO DI	1.222	41.000	15.222	-.111	.000	-.222	.222
PR CO RE	1.111	33.444	11.444	.333	.222	.000	.111
PR CO DI	1.286	30.429	9.429	-.571	-.571	-.143	.000
NP RO RE	1.778	34.778	12.222	-.222	-.556	-.444	.000
NP RO DI	1.571	33.429	14.143	-.143	-.571	-.571	.000
NP CO RE	1.429	35.429	14.143	-.143	-.286	-.286	.000
NP CO DI	1.286	30.143	13.429	-.143	-.286	.286	.000

CELL	N INST	N ADMN	FIV	SSV	AGV	FI OLD	A OLD
PR RO RE	2.375	4.000	2.750	1.000	.000	.000	.000
PR RO DI	2.778	6.111	.776	.889	.111	.000	.000
PR CO RE	3.222	3.222	5.556	2.111	.444	.000	.222
PR CO DI	4.286	4.286	10.000	1.429	.000	.000	.143
NP RO RE	2.222	4.222	.444	.444	.111	.000	.000
NP RO DI	3.429	7.714	.429	.857	.286	.000	.000
NP CO RE	3.142	3.143	1.571	.714	.143	.000	.143
NP CO DI	4.000	4.000	2.143	3.286	1.429	.000	.000

PART ONE OBSERVED CELL MEANS

CELL	G OLD	TOTOLD	FI NEW	SS NEW	A NEW	G NEW	TOTNEW
PR RO RE	.375	.375	.000	.000	.000	.000	.000
PR RO DI	.111	.333	.000	.111	.000	.000	.111
PR CO RE	.667	.889	.000	.000	.000	.000	.000
PR CO DI	.714	.714	.000	.000	.143	.429	.429
NP RO RE	.333	.333	.000	.000	.111	.333	.444
NP RO DI	.714	.714	.000	.000	.000	.143	.143
NP CO RE	.571	.714	.000	.143	.143	.000	.286
NP CO DI	.143	.143	.000	.143	.143	.286	.571

CELL	FI SUM	SS SUM	A SUM	G SUM	TOTSUM	FI DIF
PR RO RE	.000	.000	.000	.375	.375	.000
PR RO DI	.000	.333	.000	.111	.444	.000
PR CO RE	.000	.111	.222	.667	.889	.000
PR CO DI	.000	.000	.286	1.143	1.143	.000
NP RO RE	.000	.000	.111	.667	.778	.000
NP RO DI	.000	.000	.000	.857	.857	.000
NP CO RE	.000	.143	.286	.571	1.000	.000
NP CO DI	.000	.143	.143	.429	.714	.000

PART ONE OBSERVED CELL STANDARD DEVIATIONS

CELL	SEX	CQT	ALG	SECTV1	SECTV2	SECTV3
PR RO RE	.518	4.810	3.643	.991	.535	.744
PR RO DI	.441	3.808	1.716	.782	.866	.667
PR CO RE	.333	4.216	3.539	.707	.667	.500
PR CO DI	.488	6.399	4.577	.535	.535	1.069
NP RO RE	.441	6.037	4.055	.972	.527	.726
NP RO DI	.535	6.241	3.532	1.069	.535	.535
NP CO RE	.535	5.255	5.242	.900	.756	.756
NP CO DI	.488	14.064	2.992	.690	.488	.951

CELL	N INST	N ADMN	FIV	SSV	AGV	FI OLD
PR RO RE	.744	1.927	2.252	2.070	.000	.000
PR RO DI	.667	1.965	1.394	1.965	.333	.000
PR CO RE	1.301	1.301	4.391	3.060	1.014	.000
PR CO DI	.951	.951	8.287	1.618	.000	.000
NP RO RE	.441	1.641	1.014	.726	.333	.000
NP RO DI	1.272	3.200	.787	.690	.488	.000
NP CO RE	1.345	1.345	2.507	.756	.378	.000
NP CO DI	1.291	1.291	1.952	3.147	2.573	.000

PART ONE OBSERVED CELL STANDARD DEVIATIONS

CELL	SS OLD	A OLD	G OLD	TOTOLD	FI NEW	SS NEW
PR RO RE	.000	.000	.744	.744	.000	.000
PR RO DI	.667	.000	.333	1.000	.000	.333
PR CO RE	.333	.441	.500	.601	.000	.000
PR CO DI	.000	.378	.488	.488	.000	.000
NP RO RE	.000	.000	.707	.707	.000	.000
NP RO DI	.000	.000	.756	.756	.000	.000
NP CO RE	.000	.378	.787	1.113	.000	.378
NP CO DI	.000	.000	.378	.378	.000	.378

CELL	A NEW	G NEW	TOTNEW	FI SUM	SS SUM	A SUM	G SUM
PR RO RE	.000	.000	.000	.000	.000	.000	.744
PR RO DI	.000	.000	.333	.000	.707	.000	.333
PR CO RE	.000	.000	.000	.000	.333	.441	.500
PR CO DI	.378	.535	.535	.000	.000	.756	.900
NP RO RE	.333	.500	.527	.000	.000	.333	1.118
NP RO DI	.000	.378	.378	.000	.000	.000	1.069
NP CO RE	.378	.000	.488	.000	.378	.756	.787
NP CO DI	.378	.488	.787	.000	.378	.378	.535

PART ONE OBSERVED CELL STANDARD DEVIATIONS

CELL	TOTSUM	FI DIF	SS DIF	A DIF	G DIF	TOTDIF
PR RO RE	.744	.000	.000	.000	.744	.744
PR RO DI	1.014	.000	.782	.000	.333	1.093
PR CO RE	.601	.000	.333	.441	.500	.601
PR CO DI	.900	.000	.000	.000	.488	.488
NP RO RE	1.093	.000	.000	.333	.500	.601
NP RO DI	1.069	.000	.000	.000	.535	.535
NP CO RE	1.414	.000	.378	.000	.787	.976
NP CO DI	.756	.000	.378	.378	.690	.976

PART TWO OBSERVED CELL MEANS AND STANDARD DEVIATIONS

CELL	SEX	CQT	ALG	SECTV1	SECTV2	SECTV3
RO RE	1.625	35.125	12.188	.000	-.250	-.250
RO DI	1.364	37.636	13.545	-.182	-.272	-.363
CO RE	1.167	35.417	13.500	.167	-.083	-.083
CO DI	1.143	32.000	12.000	-.714	-.714	-.429
Standard Deviation	.458	6.186	3.959	.827	.682	.727

CELL	CL1ON	CL1AG	CL1IT	CL2ON	CL2AG	CL2IT	CL4ON
RO RE	.000	.063	.000	.000	.063	.000	.063
RO DI	.000	.000	.000	.000	.000	.000	.000
CO RE	.083	.083	.083	.083	.083	.083	.125
CO DI	.000	.000	.000	.000	.000	.000	.000
Standard Deviation	.148	.143	.148	.148	.143	.148	.218

CELL	CL4AG	CL4IT	CL5ON	CL5AG	CL5IT	MEAN	1-4VS5
RO RE	.125	.125	.063	.125	.063	.335	-.112
RO DI	.000	.000	-.045	.227	-.045	.183	-.366
CO RE	.208	.125	.125	.125	.125	.280	.000
CO DI	.000	.000	-.071	.214	.071	.192	-.303
Standard Deviation	.243	.208	.215	.376	.215	.357	.434

PART TWO OBSERVED CELL MEANS AND STANDARD DEVIATIONS

CELL	12VS34	1VS2	3VS4	OVSN	AVSG	145XON	124XON
RO RE	-.125	.000	.000	.084	.252	-.028	-.063
RO DI	.000	.000	.000	-.020	.102	.041	.000
CO RE	-.083	.000	-.059	.242	.280	-.019	-.042
CO DI	-.036	.000	.051	-.064	.064	.048	.036
Standard Deviation	.210	.000	.139	.295	.345	.182	.162

CELL	12XON	34XON	145XAG	124XAG	12XAG	34XAG	ONXAG
RO RE	.000	.000	-.014	-.094	.000	.044	.112
RO DI	.000	.000	-.203	.000	.000	.000	-.020
CO RE	.000	.000	.000	-.083	.000	-.059	.242
CO DI	.000	-.051	-.208	.036	.000	-.051	.064
Standard Deviation	.000	.143	.348	.216	.000	.137	.291

CELL	145XIT	124XIT	12XIT	34XIT
RO RO	-.014	-.094	.000	-.044
RO DI	.041	.000	.000	.000
CO RE	-.019	-.042	.000	.000
CO DI	-.048	-.036	.000	.051
Standard Deviation	.182	.156	.000	.140

APPENDIX C

TRANSITION MATRICIES FOR MODEL II AND MODEL III

TRANSITION MATRIX FOR MODEL II
TREATMENT NP RO RE

		State on trial n + 1		
		L	U	C
State on trial n	L	1	0	0
	U	.797	.186	.017
	C	0	.833	.167

TRANSITION MATRIX FOR MODEL II
TREATMENT NP RO DI

		State on trial n + 1		
		L	U	C
State on trial n	L	1	0	0
	U	.654	.296	.050
	C	0	.833	.167

TRANSITION MATRIX FOR MODEL II
TREATMENT PR RO RE

		State on trial n + 1		
		L	U	C
State on trial n	L	1	0	0
	U	.786	.214	0
	C	0	1	0

TRANSITION MATRIX FOR MODEL II
TREATMENT PR RO DI

		State on trial n + 1		
		L	U	C
State on	L	1	0	0
trial n	U	.753	.234	.013
	C	0	.722	.278

TRANSITION MATRIX FOR MODEL II
TREATMENT NP CO RE

		State on trial n + 1		
		L	U	C
State on	L	1	0	0
trial n	U	.812	.172	.016
	C	0	.714	.286

TRANSITION MATRIX FOR MODEL II
TREATMENT FOR NP CO DI

		State on trial n + 1		
		L	U	C
State on	L	1	0	0
trial n	U	.513	.443	.044
	C	0	1	0

TRANSITION MATRIX FOR MODEL II
TREATMENT PR CO RE

		State on trial n + 1		
		L	U	C
State on	L	1	0	0
trial n	U	.745	.255	0
	C	0	1	0

TRANSITION MATRIX FOR MODEL II
TREATMENT PR CO DI

		State on trial n + 1		
		L	U	C
State on	L	1	0	0
trial n	U	.569	.333	.097
	C	0	.882	.118

TRANSITION MATRIX FOR MODEL III
TREATMENT NP RO RE

		State on trial n + 1			
		R	P	U	C
State on	R	1	0	0	0
trial n	P	1	0	0	0
	U	.305	.492	.186	.017
	C	.593	0	.370	.074

TRANSITION MATRIX FOR MODEL III
TREATMENT NP RO DI

		State on trial n + 1			
		R	P	U	C
State on	R	1	0	0	0
trial n	P	1	0	0	0
	U	.111	.543	.296	.049
	C	.571	0	.357	.071

TRANSITION MATRIX FOR MODEL III
TREATMENT PR RO RE

		State on trial n + 1			
		R	P	U	C
State on	R	1	0	0	0
trial n	P	1	0	0	0
	U	.101	.121	.707	.070
	C	.592	0	.407	0

TRANSITION MATRIX FOR MODEL III
TREATMENT PR RO DI

		State on trial n + 1			
		R	P	U	C
State on	R	1	0	0	0
trial n	P	1	0	0	0
	U	.156	.597	.234	.013
	C	.581	0	.302	.116

APPENDIX D

STUDENT RESPONSES

The Following Symbols are Used:

RE = Reception

DI = Discovery

RO = Rote

CO = Conceptual

PR = Pretest Given

NP = Pretest Not Given

PART ONE POSTTEST

CORRECT RESPONSES IN GRID FORMAT

PR RO RE

8	8	8	8	8	8	8	8
8	7	8	8	8	8	8	8
8	8	8	8	8	8	8	8
8	8	8	8	8	8	8	7
8	8	8	8	8	8	8	7
8	8	8	8	8	8	8	8
8	8	8	8	8	8	8	8
8	8	8	8	8	8	8	8
8	8	8	8	8	8	8	8
8	8	8	8	8	8	8	8
8	8	8	8	8	8	8	8
8	8	8	8	8	8	8	8

PR CO RE

9	9	9	9	9	9	9	9
9	6	9	9	9	9	9	9
9	9	9	9	8	9	9	9
9	9	9	9	9	9	9	8
8	8	9	9	9	9	9	6
9	9	9	9	9	9	9	9
9	9	9	9	9	9	9	9
9	9	9	9	9	9	9	9
9	9	9	9	9	9	9	9
9	9	9	9	9	9	9	9
9	9	9	9	9	9	9	9
9	9	9	9	9	9	9	9

PR RO DI

9	9	9	9	9	9	9	9
9	8	9	9	9	9	9	9
9	9	9	9	9	9	9	9
9	9	9	9	9	9	9	9
9	8	9	8	9	9	8	8
8	9	9	8	8	9	8	9
8	9	9	9	8	9	9	9
9	9	9	9	9	9	9	9
9	9	9	9	9	9	9	9
9	9	9	9	9	9	9	9
9	9	9	9	9	9	9	9
9	9	9	9	9	9	9	9

NP RO RE

9	9	9	9	9	9	9	9
9	8	9	9	9	9	9	9
9	9	9	9	9	9	9	9
9	9	9	9	9	9	9	8
9	9	9	9	9	9	9	8
9	9	9	9	9	9	9	9
9	9	9	9	8	9	9	9
9	9	9	9	9	9	9	9
9	9	9	9	9	9	9	9
9	9	9	9	9	9	9	9
9	9	9	9	9	9	9	7
9	8	9	9	9	9	9	9

PART ONE POSTTEST

CORRECT RESPONSES IN GRID FORMAT

PR CO DI

7	7	7	7	7	7	7	7
7	3	7	7	7	7	7	7
7	7	7	7	7	7	7	7
7	7	7	7	7	7	7	7
6	7	7	7	7	7	7	6
7	7	7	7	7	7	7	7
7	7	7	7	7	7	7	7
7	7	7	7	7	7	7	7
7	7	7	7	7	7	7	7
7	7	7	7	7	7	7	7
6	7	7	7	7	7	7	4
7	7	7	7	7	7	7	7

NP CO RE

6	7	7	7	7	7	7	7
7	4	7	7	7	7	7	7
7	7	7	7	7	7	7	7
7	7	7	7	7	7	7	6
7	7	7	7	7	7	7	7
7	7	7	7	7	7	7	7
7	7	7	7	7	7	7	7
7	7	7	7	7	7	7	7
7	7	7	7	6	7	7	7
7	7	7	7	7	7	7	7
7	7	7	7	7	7	7	7
7	6	7	7	7	7	7	6

NP CO DI

7	7	7	7	7	7	7	7
7	6	7	7	7	7	7	7
7	7	7	7	7	7	7	7
7	7	7	7	7	7	7	7
7	7	7	7	7	7	7	7
7	7	7	7	7	7	7	7
7	7	7	7	7	7	7	7
6	7	7	7	7	7	7	7
7	7	7	7	7	7	7	7
7	7	7	7	7	7	7	7
7	7	7	7	7	7	7	6
7	5	7	7	7	7	7	6

NP RO DI

7	7	7	7	7	7	7	7
7	4	7	7	7	7	7	7
7	7	7	7	7	7	7	7
7	7	7	7	7	7	7	7
7	7	7	7	7	7	7	5
7	7	7	7	7	7	7	7
7	7	7	7	7	7	7	7
7	7	7	7	7	7	7	7
7	7	7	7	7	7	7	7
7	7	7	7	7	7	7	7
7	7	7	7	7	7	7	6
7	7	7	7	7	7	7	7

PART TWO POSTTEST

(Correct Responses Indicated by 0, Format in Booklet Order)

CO RE

11111000000000000000
00000000000001000000
00010000000000000000
00000000000000000000
00010000000000000000
00000000000000000000
00010000000000000000
00100000000000000000
00000000000000000000
00000000000000000000
00000000000000000000
11111000000000000000

RO DE

00000000000000000000
00001000000000100000
00001000010000100000
00000000000000000000
00001000000000100001
00000000000000000000
00000000000000000000
00000000000000000000
00000000000000000000
00000000000000100000
00000000000000000000

CO DI

00000000000000000000
00001000000000100001
00000000000000000000
00000000000000000000
00000000000000000000
00000000000000000000
00001000000000100100

RO RE

00000000000000000000
00000000010000000001
00001000000000100000
00110000000000000000
00001000000000000000
00110000000001100000
00000000000000000010
00000000001100000000
00000000000000000000
00001000000000100000
00000000000000000000
00000000000000000000
00000000000000000000
00000000000000000000
11111000000001000000
00110000000000000000

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