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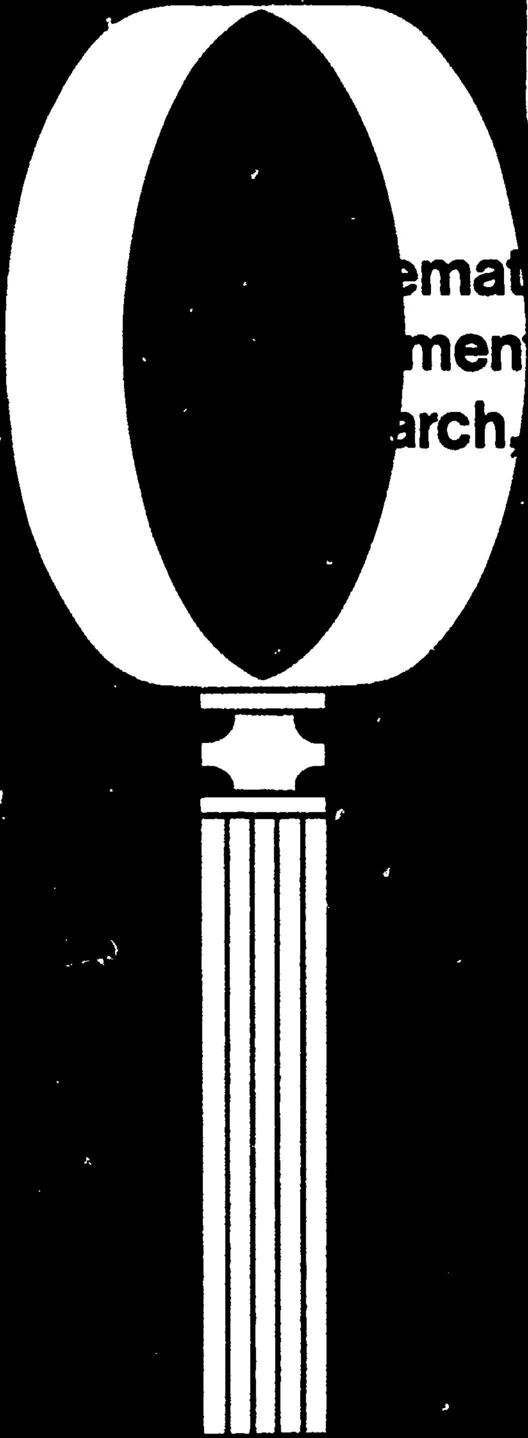
ABSTRACT

This publication reviews the mathematical laboratory from three perspectives: a practical view of laboratories in operation, a review of related research, and a view of current laboratory evaluation procedures. After a discussion of definitions, types, and purposes of math labs, the first paper concentrates on their historical development in Philadelphia, Pennsylvania, from their inception in 1967 through 1972. Despite continued interest throughout this period, actual numbers declined rapidly after 1970. Several reasons for this decline are cited. Several projects are reviewed; specifically discussed are problems, solutions, and results. Although generally pessimistic regarding the future of math labs, the author mentions several innovative movements as possible reversing trends. The second paper reviews research on math labs and activity learning, describes ways to employ math labs most effectively, and discusses some apparent effects on student achievement and attitudes (specifically, achievement gains in less able elementary children, with no effect on attitudes). This paper is followed by an extensive bibliography. The third paper critically analyzes seven specific project evaluations--their data collection techniques, types of analyses, and results. Following a discussion of difficulties encountered, the author makes some suggestions, based on the need for individually tailored evaluations. (CR)



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MATHEMATICS EDUCATION REPORTS

Mathematics Laboratories: Implementation, Research, and Evaluation

Edited by

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and
Jon L. Higgins

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Science, Mathematics, and Environmental Education
The Ohio State University
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November, 1974

Mathematics Education Reports

Mathematics Education Reports are being developed to disseminate information concerning mathematics education documents analyzed at the ERIC Information Analysis Center for Science, Mathematics, and Environmental Education. These reports fall into three broad categories. Research reviews summarize and analyze recent research in specific areas of mathematics education. Resource guides identify and analyze materials and references for use by mathematics teachers at all levels. Special bibliographies announce the availability of documents and review the literature in selected interest areas of mathematics education. Reports in each of these categories may also be targeted for specific sub-populations of the mathematics education community. Priorities for the development of future Mathematics Education Reports are established by the advisory board of the Center, in cooperation with the National Council of Teachers of Mathematics, the Special Interest Group for Research in Mathematics Education, the Conference Board of the Mathematical Sciences, and other professional groups in mathematics education. Individual comments on past Reports and suggestions for future Reports are always welcomed by the editor.

Jon L. Higgins
Editor

This publication was prepared pursuant to a contract with the National Institute of Education, U.S. Department of Health, Education and Welfare. Contractors undertaking such projects under Government sponsorship are encouraged to express freely their judgment in professional and technical matters. Points of view or opinions do not, therefore, necessarily represent official National Institute of Education position or policy.

Papers in this publication developed out of a symposium on research pertaining to mathematics laboratories for the 1972 Annual Meeting of the National Council of Teachers of Mathematics. I was pleased to have the opportunity to organize that symposium. The concepts underlying mathematics laboratories are so pervasive today that careful assessment, research and evaluation is becoming very important.

To provide balance in the presentations, we requested three quite different papers: one to present a schoolman's practical view of laboratories, one to review related research, and one to review evaluation procedures.

Alan Barson was appropriate to write the schoolman's paper for, as the Madison Project coordinator in Philadelphia for several years, he was in a position to witness a wide variety of successes and failures.

Professor Jack Wilkenson was asked to write the second paper because he had recently completed his doctoral thesis and was still in possession of his sensitive antenna which one develops during such a period.

The final paper was written by Professor Donald Kerr who, with his colleague Professor John LeBlanc, was just embarking on the development of a new teacher training program. In preparation for that effort, they visited many laboratory projects and were particularly prepared to write on this subject.

These papers offer a significant contribution to the literature of mathematics education. When we compare what we write today with the message of E. H. Moore in 1902, we can see an occasional advancement. After all, E. H. Moore didn't know what a Cuisenaire rod was.

William M. Fitzgerald
Editor

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by

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A SCHOOLMAN'S VIEW OF THE HISTORICAL DEVELOPMENT AND CHANGE OF THE MATHEMATICS LABORATORY IN A LARGE URBAN SCHOOL SYSTEM

Before we discuss the genesis, metamorphosis, and paralysis of the mathematics laboratory movement in Philadelphia, some definitions need to be clarified at the beginning:

I. Characteristics of a Math Lab -- in general, a mathematics laboratory is activity-centered; the child is placed in a problem-solving situation and through self-exploration and discovery provides a solution based on his experiences, needs, and interests. Below are listed some common bonds of mathematics laboratories.

   a. The room is organized with stations of activities where the children (individually, in small groups, or as an entire class) may work simultaneously on different materials or on the same materials at different rates.

   b. The room is rich in materials, making use of commercial, teacher-made, and pupil-made devices.

   c. The teacher works with small groups, with individuals, or with the entire class in a child-centered rather than a teacher-dominated atmosphere.

   d. The activities are usually open-ended to enable the students to extend their discoveries as far as they wish.

   e. The organization of the laboratory work is flexible so that a child can move from one activity to another, depending on his interests and needs.

   f. There is a multimedia or multisensory approach to learning, using tapes, films, concrete objects, records, listening centers, and so forth.

g. Textbooks and pamphlets are used as reference materials. They are mostly ungraded and include a large variety of topics.

Basically there are four types of mathematics laboratories:

1. Decentralized, or classroom laboratories
    a. All classrooms containing laboratories
    b. Some classrooms containing laboratories
2. Centralized laboratory
3. Team-room laboratory
4. Roving, or movable, laboratory

Following is a brief description of each type:

1. A decentralized laboratory provides a good situation in which the teacher has a permanent mathematics laboratory in his room. This is ideal because the teacher is responsible for all areas of the curriculum and for a major portion of the time the child is in school. This organization facilitates movement and scheduling (which is a problem with the other types of laboratories). It also allows the children and the teacher to use the material whenever the need arises. A variation can be adopted when it is too expensive to supply all teachers with the materials, or when it is desirable to make use of any special talents of teachers. In this situation, several teachers join together and cycle the classes for mathematics instruction into one room. The teacher for that room, which is equipped with the laboratory, is then responsible for the mathematics instruction for all the children in the cycle.

2. Centralized mathematics laboratories are used in many schools where children from all the grades, or from certain grades, share the facilities of the laboratory for part of their mathematics instruction. The ideal program would have a specially trained mathematics teacher in the room full-time to instruct the children with the aid of the regular teacher. In this situation the mathematics laboratory teacher could do all the scheduling, take care of the materials, create activities, and use his special talents to help all the children. It also provides an excellent opportunity for the development of the other teachers.

   If it is necessary to have a central laboratory without a special laboratory teacher, each teacher is then responsible for the use of the laboratory and its maintenance. Often this creates a difficult situation because scheduling becomes cumbersome, materials are lost, and the activities become rigid and standardized.

   The basic problem with either situation is that the teacher can send the class into the laboratory for only a few periods a week and at predetermined times, not necessarily when its service is needed.

3. Team-room laboratories were created as schools became equipped for team teaching. The most distinguishing feature of this type of lab is that it is in constant use by the children, and only a small number of them attend the laboratory at a time. Usually there is no adult in the laboratory because the children are in sight of a

teaching team member in an adjacent room. Children work in the laboratory on mathematics concepts of current interest or on any mathematics topic that the team considers useful to the children. The arrangement provides easy movement in the team situation and allows great flexibility -- a child might spend all day in the laboratory to finish something in which he is interested.

4. The roving, or movable, laboratory is useful when a school cannot afford to buy many materials and does not have a room for a central laboratory. The materials are carefully itemized and placed in containers for easy assembling by the teachers. The container for a particular topic has all the necessary materials for the child to investigate the related problems freely. All the objects are then put on a large movable cart, and a schedule is created to allow access. The teacher who gets the laboratory during a certain period of time can quickly lay out the containers and produce an instant mathematics laboratory in the room. In this situation the teacher must take the entire cart during the mathematics period and return all the materials afterward. There are disadvantages to this type of laboratory: for example, access is not immediate when the need arises; the laboratory does not grow in increasing variety; maintenance is difficult; and unless staff development is provided, the activities lose their mathematical significance. However, if this is the only type of laboratory that is possible in the school, it is better than no system at all.

The purposes of a mathematical laboratory must also be considered since they determine the type of laboratory needed and the characteristics of the activities. Children can derive many benefits from a lab period which can be translated into objectives and summarized under these four major purposes:

a. motivation

b. enrichment

c. articulation with the regular mathematics program

d. review, reinforcement, and remediation.

Generally a mathematics laboratory is designed to satisfy the needs of all the children and will provide activities consistent with the four purposes of a laboratory listed above. Even though a school might think of a laboratory as having a special purpose, and as a specific type of room, the laboratory can serve the needs of all the children in the school. While to one child an activity is motivating, to another it might be review and reinforcement or even enrichment. Still another child might be working with a game that is motivating and suddenly find it has meaning for the concept he is presently learning in class.

THE GENESIS IN PHILADELPHIA

The movement toward math laboratories began in the summer of 1967 with a concentrated course in the use of concrete materials given by the Madison Project for teachers and administrators. A portion of the program was devoted to observing children at work in a math laboratory setting. Strong emphasis was placed on manipulation, discovery, freedom of movement and choice, and a large variety of materials and projects.

This was most certainly the first time many of the teachers and principals had witnessed this type of environment and excitement was evident throughout the weeks of the course. While the teachers saw "MANNA," some administrators saw "MADNESS." At the end of the course, the principals were still at grips with these questions:

What is the curriculum for a Math Lab?

How does it fit in with our present graded Mathematics curriculum?

Where will the money come from to purchase the needed materials to properly stock the Math Lab?

Who will train the Math Lab teachers?

Where will I find the room to house the Math Lab?

How do I get a teacher released full-time to organize, run, and in-service the other members of the staff in the Math Lab?

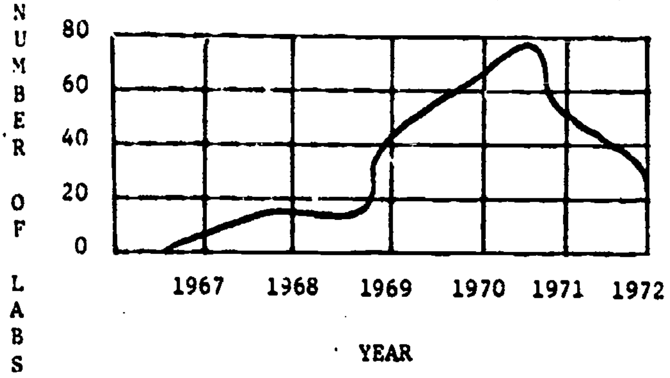
How do I get the parents and other teachers excited enough to see the significance of the Math Lab?

Do I want this type of noisy experience that allows freedom of movement throughout the room and the school, when I stress discipline so strongly in my school?

Each principal obviously left the course with a different view of what the math lab should be for his school situation, tempered with the reality of the above questions.

If a graph were to be drawn of the number of math laboratories created after that initiation, a graph of the intensity of enthusiasm for organizing activity-centered classrooms, it would look like this:

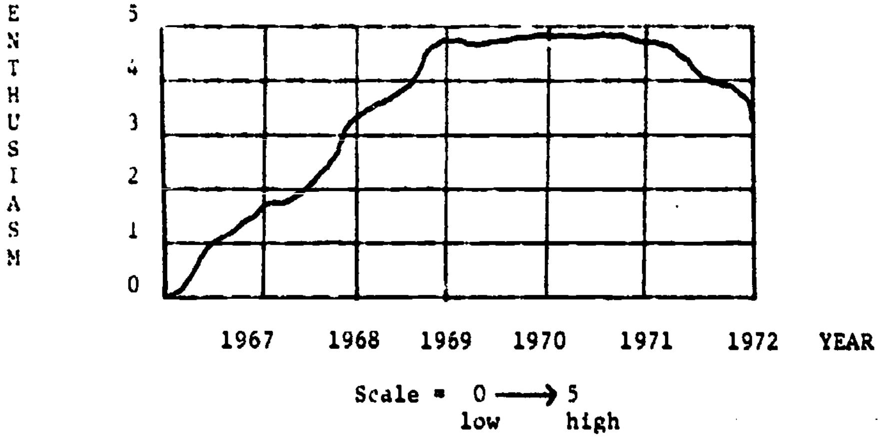
NUMBER OF SCHOOLS WITH MATH LAB ORGANIZATIONS



This graph clearly shows that the number of labs is declining rapidly since 1970. At the high point, 75 schools out of the 266 in Philadelphia were participating in the math lab concept. While there were other schools trying pieces of the activity-centered approach, only these 75 schools succeeded in organizing a program that comprehensively embodied the principles recognized as a full activity-centered program. It is also significant that the breakdown of the different types of labs showed a wide variety of alternatives, each possessing admirable qualities with both staff and parent approval. There were 27 schools with separate lab rooms coordinated by a full-time released teacher, 32 schools with individual classrooms organized as math lab rooms providing leadership to the other members of the staff, 9 schools with team-room math labs necessitated by the organizational pattern of team teaching being tried by the new schools and some old converted ones, 3 centers were set up to act as both demonstration schools on a limited basis and as teacher centers to help in proselytizing, and 4 schools had portable math labs that travelled around the school in a most effective way.

The next graph will show how the principals' enthusiasm varied from the beginning to the present:

ENTHUSIASM FOR THE MATH LABS BY ADMINISTRATORS
(subjective analysis by the Madison Math Coordinator)



Why is it that the demand for the math labs increased over the years, while the actual units decreased sharply? The answer lies in the various current urban thrusts in education, and society at large, that produced the fervor for labs, facilitated their growth, but unfortunately produced debilitating side effects that aborted many of the attempts. Such trends as:

1. A concentrated effort in early childhood education which accelerated the demand in concrete materials for the very young and sought alternatives to the traditional methods of teaching. The federal government supplied the funds for many experimental programs in this area such as Follow-Through models (one being fashioned after the British Infant Schools), Get Set Centers, learning centers, day care centers, etc.
2. The movement toward the national testing program which brought to realization how ineffective our present methods of reaching children were and that new methods had to be employed.
3. Decentralization of the large city school systems to allow more freedom for the local principals to produce their own budgets and spend their money in consonancy with their philosophy of educating children.

4. The movement toward unionization of teachers which clearly provided them with a share in the decision-making policies of management.
5. Parental disgust with the present school system and their involvement in striving for something different.
6. The influence of many national movements in curriculum that continually stressed the activity-centered approach to learning. The accepted works of Bruner and Piaget vividly changed the materials and instructional aids being produced by the companies, and the current research appearing in the literature noted the effectiveness of the Infant Schools, open classrooms, Madison Project, and learning centers.
7. The proliferation of federal funds for alternative means of educating the children in deprived areas.
8. In Philadelphia, a new responsive administration, responsive to change and innovation. Stress was placed on providing alternative means to reach the children. New experimental schools were started, such as the Parkway Project (school without a building), the Intensive Learning Center, the Pennsylvania Advancement School, the Rosmuessan Learning Centers, and many others that were so closely allied to the math lab concept.
9. A more responsive university program geared to the training of teachers for urban realities. Included in many of the programs were courses devoted to the math lab concept, along with practica in organizing and operating them (Temple University has a number of such courses in Philadelphia).
10. The daring new designs in school construction facilitate a flexible

transition from the traditional to the open space concept of teaching. Most new schools have rooms for math labs (these are self-contained) or large rotunda type units designated as learning centers. Practically all the interior classrooms provide space for small group or large group experiences with ante-rooms set aside for activity centers in math and science.

Why then, with all those movements clearly snowballing the momentum toward math labs, did the trend reverse itself so abruptly? The answer lies in the saddening realities that eventually cripple many programs prematurely, when they reach the stage of practical application.

The six following conditions may explain part of the problem:

1. Although federal funds increased, the money available for school operating funds from state and local governments became insufficient. Most urban school systems had to cut back on services requiring larger pupil-teacher ratios and less supportive services. With these severe cuts, the continuation of any innovative program becomes suspect. The boards of education in their zeal to balance the budget, cut past the marrow and reflected kindly on the basic skill programs rather than any project termed "experimental."
2. The high turnover of teachers in the urban areas meant constant in-service training just to maintain the present level of competence, let alone improve it! There is just not enough money available to provide the type of continuous, concentrated in-service needed in an activity-centered approach.
3. A communication gap appeared between the parents and their schools. Any program not specifically geared to training children in memorization of the basic number facts became a frill and was not warmly received by some members of the community.

4. Discipline problems had become so numerous in the past several years that a full swing toward the more traditional, formal, isolated learning experiences became expedient.
5. A long awaited, concentrated effort to improve reading occurred, but unfortunately at the cost of providing funds and services for the subject areas. Any program not clearly stamped "Reading" got a lower priority.
6. And the most dangerous condition of all that usually occurs with innovative attempts to improve learning, the bandwagon syndrome. Some principals, realizing that the current thrust is toward active learning classrooms, proceeded to initiate programs without the slow methodical research and communication necessary to gain acceptance and success with this type of radical change.

Although some of the above points may seem incongruous with the present trends mentioned before, remember that this often happens when drawing board thoughts meet the day to day existing conditions.

THE METAMORPHOSIS

When administrators and teachers face a formidable barrier such as previously described, what happens? Some creatively find alternative routes around the obstacle, some set their sights higher and hurdle it, others barrel through and suffer setbacks, the less creative souls lower their sights and burrow under, while some relent--turn around--and retreat. Certainly it would be beneficial to examine what adjustments and solutions were tried during the last four years.

Case #1 -- A middle-sized elementary school, housing grades K-6, realized a serious problem in its mathematics instruction. According to their scores from the Iowa Test of Basic Skills for November of 1969, the majority of students were two years to two-and-a-half years behind in their level of mathematics competence. The teachers, parents, and principal decided to start a central mathematics laboratory to help solve the problem, primarily designed for remediation, manned by a full-time laboratory teacher who would also provide an intensified training program in "Modern Math" for the staff. Since this school was due for a new replacement building in September of 1970, the principal included in the building plans the construction of a large room (1600 sq. ft.) as a mathematics lab. Materials and equipment were also ordered for over $11,000.00, including a small computer. Fortunately, a new school receives an extremely liberal allowance for equipment and supplies during its initial year. However, after the first year, money is equalized with the other elementary schools in the city, which makes maintaining existing experimental programs difficult. In spite of that fact, the laboratory was built. The laboratory teacher, of course, was not included in the staff budget, since money is not liberalized for manpower as it is for materials. The principal talked the faculty into splitting up one class among the other teachers in order to release someone full-time to man the laboratory. The released teacher agreed to take special courses during the summer in the Madison Project and in the Learning Centers Project to prepare himself properly. With all good intentions, the laboratory opened along with the new school in September of 1970. There were thirty classroom units in the school and each teacher demanded equal time in the laboratory. The laboratory teacher wanted to focus on the lower grades and only work with those children whose scores on the Iowa were below the 16-percentile (the non-functioning educational level) and give them concentrated sessions throughout the week. The teachers demanded that complete classes work in the lab since they felt that all children would benefit from the laboratory. They also felt since they had increased their class size voluntarily, the sessions should be considered free time for the classroom teachers. With these restrictions, naturally the laboratory was doomed! The principal agreed to all the conditions because he felt so strongly that the laboratory was needed and he knew the lab teacher had prepared himself well for the chore.

In order for each of the thirty teachers to get a laboratory period, the lab teacher had to give up his lunch periods and any preparation time needed between the classes. Certainly, no time was left for in-service training and since the teachers refused to remain in the laboratory during the sessions, no articulation took place in the classrooms. No matter how hard the lab teacher worked during each thirty-five minute lab period per week, none of the classes truly benefited. How much individualization can be done with over 950 students and one lab teacher shackled without time for preparation?

The Iowa scores for the following year remained the same and in some cases decreased. By the end of the year, the Board of Education cut its budget, laid off teachers, reduced supportive services to the schools, and increased class size. The teachers at the school felt the lab really did not accomplish its goals, and discipline problems had increased alarmingly because of the large classes, so the lab was closed and the teacher went back to the classroom as part of a three-man team teaching unit (his responsibility--Language Arts!).

Although this is the story of one school, it reflects the conditions existing in other schools. No matter how hard the principal tried to get support for the program, he was thwarted in his every attempt. The laboratory remained empty, the materials crated up and inaccesible, and teachers did not move away from the traditional lecture-textbook method of mediocrity.

Case #2 -- A large, over-crowded, "vintage" junior high school took a survey and found that the average ninth grade student in their school had a fifth grade reading level and a beginning sixth grade level of competence in mathematics. Based on this survey, a creative, resourceful math teacher wrote a proposal for a federal grant to set up a motivational type Mathematics Laboratory for remedial eighth grade students, designed to rekindle an interest in mathematics. The proposal was funded for three years and consisted of a salaried position for organizing and operating the laboratory, plus $3,500 for materials and equipment. The principal found an unused stockroom to house the operation and the laboratory opened in September, 1968. All the remedial mathematics classes in eighth grade were scheduled into the laboratory for two of their mathematics periods a week, with open slots for independent study. The homeroom teachers had the option of attending the sessions: it was not made mandatory! At first the laboratory was suspect. Most of the students felt it was childish, a playroom, demeaning their level of sophistication, comparable to the "Dick and Jane" readers they were forced to enjoy in their condescending, remedial reading classes. The lab teacher had stocked the room with games, calculators, computers, Cuisenaire rods, geoboards, geoblocks, pattern blocks, attribute games, mirrors, tangrams, etc. After a few months of recalcitrance, the laboratory teacher realized that he had to remold the image of the activities and make them relevant to eighth grade students. He invited "star" academic classes to the laboratory a few periods a week as an enrichment center and taught new subjects previously reserved for high school, using the same materials! When the remedial classes came into the laboratory, they also wanted to be part of the enrichment program. New topics were tried, such as: transformations, symmetry, tessellations, Boolean Algebra, calculus, computer programming, etc. The approach worked! The students flocked to the laboratory with enthusiasm, not even realizing they were working with the same materials. They were learning new areas, but also cementing the primary concepts that had before caused such rebellion. They tackled new and interesting topics, without nausea, because they had not previously developed that pernicious failure syndrome so closely allied with repeated learnings.

By the eighth month, almost all the homeroom teachers stayed during the laboratory periods and assiduously helped the students with their projects. Sometimes the atmosphere electrified the teachers more than the students and the laboratory teacher found it harder to remove the homeroom instructors! Eventually, the ideas found their way into the regular classes and a system of borrowing materials during the off laboratory periods developed naturally.

A follow-up study the next year showed a substantial increase in both mathematics and reading scores for over 65% of the laboratory students. In the September 1969-70 term, success became even more pronounced as the seventh grade remedial classes were added. The teacher was sedulous enough even to visit the "feeder" elementary schools and observe the instructional program the students were experiencing in the sixth grade, so he could provide a facile articulation. He even ran in-service sessions for the elementary school teachers so they could better prepare their students for secondary schools.

Unfortunately, as is usually the case in junior high schools, the laboratory teacher transferred out of the school. Knowing he was leaving, he trained someone else as a replacement for the laboratory, but the enthusiasm regressed. The new teacher returned to the remedial activities, with which he felt more confident, and rapidly evoked a violent visceral reaction among the students. As conditions in the school became more overcrowded, and the cut in services occurred once again, the laboratory teacher feared for his tenure. The laboratory closed. The remaining year of the proposal was forfeited because it became impossible to fulfill its provisions, and once again the children returned to the soporific, lecture-textbook method.

Continuity in junior high school mathematics is an encroaching problem. The highest turnover rate among teachers (30%) occurs at this level, and so do the most serious discipline cases, which is concomitant with the self-realization and self-seeking stages of development that all teenagers experience.

Case #3 -- Let's now look at a new middle school that was built and organized for team teaching. A cluster of three open-spaced rooms, an enclosed planning room, and another small anteroom, comprised each cloistered team model. The principal and the teachers wanted a non-graded organization (of course, they really never had the planning time before the school opened to develop a smooth transitional change from the self-contained to the team situation) based on vertical teaming within the subject areas. Three men teachers voluntered to teach all the mathematics to one hundred students at a time, in forty-five minute class periods with mixed age groups from grades five through eight. In some cases, double periods were scheduled.

The small anteroom was generously stocked and labeled Mathematics Laboratory and the students were allowed complete access to the room for varying lengths of time. The teachers worked well together and spent all their free time at lunch each day planning the various activities.

Two interesting problems occurred. The non-graded organization, which actually was the principal's preference, was too difficult for the team to handle (especially since it was a new experience for them). No matter how they grouped the children, it eventually fell upon grade lines since fifth graders and eighth graders had too wide an experiential background to work together on similar concepts. The peer group relationships also produced a tense situation. The team felt it could do a better job of individualization by operating within grade lines and taking one grade level at a time during the mathematics periods because the social adjustments played havoc with the lessons throughout the day. However, the principal remained adamant in continuing the present organization.

Secondly, the diminutive effects of the team teaching situation must also be considered. Because of the professionalism, experience, and dedication of the three teachers, the five ubiquitous problems that eventually decimate most team teaching experiments seemed to be abated. The problem of a lack of team planning time was solved by having daily "lunch-ins" (although an unfair method). The unfortunate situation of "turn-teaching" that usually develops in the vacillating interrelationships of adults when one member decides to lessen his load and take "small breaks" was resolved by the exemplary manner in which all the members pulled their share. The difficult problem created by a lack of paraprofessionals which so severely limit the amount of individual or small group instruction that is concomitant with team teaching was alleviated by building a favorable attitude of peer assistance between the upper grades and the lower grades. The teacher absence problem that literally destroys a team never became an issue because the teachers were extremely responsible in that respect. Lastly, the problem of a lack of materials and activities that propagate individual instruction resolved itself by drawing upon the fertile store of knowledge of the experienced teachers and the strong support of the principal who gave them more than their fair share of the school's budget. It was truly a successful program, but a most difficult one to administer. The unfair load shared by the teachers was soon to become a bane, thus ending this happy troika. The second year of the program one of the team members left for a promotional job, which necessitated training a new teacher to fill the abyss. Another member soon left in the middle of the third year and that was the final coup! The remaining teacher, already overburdened with providing in-service sessions for the two new members, sealed off his middle room and taught a self-contained classroom for the rest of the year. The mathematics laboratory became a stock room and the unique program a forgotten dream.

One painful lesson educators can learn from this case is that the master teachers (especially men) look toward promotions after a certain degree of tenure is realized. It always strikes me as odd that we reward the good teachers by removing them from the classrooms. Why not provide merit scale pay to keep them where they are most effective? Certainly the "Peter Principle" pervades the school system: time and time again, whenever you find a good team of teachers operating a program, the odds are that they will not be together very long.

Case #4 -- By now it seems that all the mathematics laboratory situations have become aborted after a meteoric attempt at success. Let's now examine a successful laboratory in an elementary school, located in an inner-city area. The principal, impressed by the pervasive excitement and apparent success of a laboratory he visited in a neighboring school, dedicated himself to organizing a similar program in his building. He realized it would be a difficult task, because he had a 35% turnover of staff each year, with an average teacher tenure of $2^1/_2$ years. The principal in his wisdom, however, decided he could carefully avoid the pitfalls ahead if he planned for them! He knew ingenious solutions had to be found for the six persistent problems associated with previously existing laboratories that he had seen end prematurely:

1. Finding perennial funds outside the realm of the school system that would insure program continuity.
2. Finding a teacher willing to assume the arduous task and insure her continual tenure.
3. Finding the planning time necessary for the laboratory teacher to prepare materially, emotionally, and mentally for the position.
4. Gaining the acceptance of the staff and parents to expand the program, therefore assuring its existence beyond the present team of teachers.
5. Finding the room to house the laboratory in an already overcrowded school.
6. Reorganizing the school, if necessary, to free the laboratory teacher full-time and be reasonably certain that future cuts would not endanger her position.

For one full year, the principal carefully planned his moves with administrative acumen. He decided that a nearby university would be ideal for stability, and arranged to have a student teacher center for the training of undergraduates in mathematics laboratory techniques initiated in his school. The undergraduates provided four cogent services for his program:

1. The student teachers could provide released time for the laboratory teacher to be used for planning or intra-demonstrating.
2. They could act as paraprofessionals, lowering the pupil-adult ratio and aiding in individualizing activities.
3. Since it has been established that almost 75% of undergraduates request appointments in the school that they did their student teaching in, the program would then actually be a pre-service training for future staff members, thereby insuring the active learning program's continuance.
4. With special arrangements, the principal was able to secure a substantial sum of money for the laboratory teacher in consideration for the time she would be spending in-servicing the undergraduates. The university was quite pleased with this on-the-job training plan and this added incentive cemented the lab teacher's positive feelings toward her new role.

Several professors from the university offered on-site staff development programs for the school before the program began. The principal watched with apprehension, hoping to find one member on his staff willing to assume the new role. With certain alacrity, one teacher became extremely excited with the activities and volunteered her services as the laboratory teacher. This teacher formed a team with three others in the same grade, each one agreeing to cycle their classes into her room for mathematics, while they divided up the remaining areas of the curriculum. This meant that all morning the laboratory teacher had four different classes rotating into her room for all their mathematics instruction. The principal, an adroit scavenger, found all the unused mathematics materials in the school (and there were plenty) and syphoned them into the laboratory. The teacher was given one full year of grace before beginning the program, during which time she would be experimenting and learning from many of the consultants available from the central mathematics office. Her in-service training was carefully planned by the principal to insure a well-rounded program. The laboratory teacher took courses at the university, offered by a professor who worked with Dienes, which exposed her to one model for active learning. In July, she spent two weeks at Syracuse University working with Dr. Robert Davis and the Madison Project; the remaining two weeks of July were spent in an infant school project in Philadelphia, operated by Sybil Marshall for the Committee on Independent Schools. In August, two weeks were spent in New York working in The Schools for the Future instilling Gategno's philosophical beliefs. In the last two weeks of August, she visited the learning centers already operating, enabling her to spend many hours discussing with the successful laboratory teachers in the city.

Assured and anxious when the new term began, the program was launched with anticipatory glee.

Every Tuesday the laboratory teacher was required to give in-service sessions for the student teachers after school. The school staff was also invited and the adult community surrounding the school, in the hopes of proselytizing. Eventually, the volunteer trainees totaled twenty-six by the fifth month of the term. Three other teachers were so impressed with the effectiveness of the laboratory program that they "bargained" with the principal to allow them to organize similar cycles in their grades. The principal readily agreed, only if they were willing to devote the required time that the laboratory teacher did in pre-service training. They consented and by the beginning of the next year, the school had four operating cycles in grades 3, 4, 5, and 6. This meant that sixteen classes were now receiving all their mathematics instruction from well-trained, enthusiastic teachers. When the ax finally fell, tolling severe cuts in services and manpower, the program remained unaffected. The principal had done his job well; the funding, extra personnel, and lab teachers were sacrosanct because they were clothed from extra-territorial sources.

Case #5 -- A high school on the fringes of a well integrated neighborhood was faced with an unusual situation; their main building was so overcrowded that the placement of any new students defied solution. The school district decided to decentralize the program and experiment with the "House" concept by placing all the tenth grade students in one annex, several miles away, for a cloistered learning experience. The usual passive tenth graders presented a unique, pleasurable challenge, and the teachers assigned to the annex decided to capitalize on it. They were given an opportunity for wide leverage in their planning because of the newness of the program and the small size of the student body. The school district also felt that a number of innovative programs could be tried by the staff without complete chaos because of the diminutive discipline problems presented by the freshmen classes. The Mathematics Department chose their own chairman, separate from the main building chairman, and decided to center their program around a mathematics laboratory.

The building was an abandoned warehouse that the school district rented and planned to renovate to relieve the overcrowding. As a precursor for future policy, the district allowed the teachers to be in on the initial phase of the building construction. Their suggestions were not only listened to--but followed!

The mathematics department designed their own classrooms and the central laboratory; and coordinated with the roster chairman to schedule each class into the laboratory for two of their five mathematics periods a week. The laboratory that was envisaged by the mathematics teachers was significantly different than others previously tried in our city. Two cogent reasons accounted for this dissimilarity. First, high school teachers are specifically trained in their disciplines, which is not usually the case in elementary, middle, or junior high schools. This factor leads to a more sophisticated, academic type laboratory, with activities carefully selected and sequenced to reach specific goals. Teachers lower on the hierarchical grade scale do not possess this single-disciplined background; their laboratories tend to be less integrated and provide fewer alternatives for the children. A mathematician can always find several routes to travel in reaching his goal, while "generalists" have to search assiduously just to find one alternative approach. The second difference is related to the length and depth of in-service training needed by the laboratory teachers. Certainly with the intense course requirements mathematics teachers need for certification, it would require only a few short lessons in the active learning approach for their edification and stimulation. They are quite quick to grasp the mathematical significance of concrete activities and can readily extrapolate numerous innovative ideas from just one suggested lesson.

The responsibility for the operation of the laboratory was cheerfully accepted by the new department head, who is usually only rostered for two periods a day; the remaining time reserved for administrative details. Instead, the department was so small that they felt the non-teaching duties were minimal, so it was natural for the chairman to assume the leadership for the laboratory. The chairman, along with the department, decided that the center should be an "Articulation" laboratory, utilizing concrete materials and active learning projects based on the same concepts that the classroom teachers were simultaneously teaching. In this way the students would be getting both the concrete applications of mathematical ideas and their theory in abstract form.

All the components for a successful program were there:

1. The money for materials and supplies was generally requisitioned.
2. The team of mathematics teachers was highly competent in their respective areas.
3. No new personnel were needed to operate the program.
4. There was clear evidence of coordination between the laboratory and the classroom lessons.
5. The laboratory teacher was the mathematics department head, and was highly respected by the staff and provided the key leadership in curriculum.
6. Cooperative team planning preceeded the laboratory's initiation, which insured motivational impetus (the halo syndrome).

Obviously, the program did work and is still working in the high school annex. Other schools (even junior high and middle schools) have since adopted the house plan with the central core of activity being the learning center. Some adaptations had to be made for the lower grade teachers because of the nature of the learner and the nature of the curriculum. The learning centers are usually not concentrated in the mathematics area alone, instead they are integrated activity centers with all the areas of the curriculum given an equal status. This movement, necessitated by the new school constructions of gargantuan proportions, is a reflection of the ambiguity of pedagogy. The teachers are seeking controlled situations that allow freedom for individualization and identification of learning problems, while the architects are promoting mausoleums of mass education, which change the noticed to the nameless.

Parenthetically, there are reversing trends that hopefully will have a salutary effect on the thinking of the designers of tomorrow. Two interesting developments in Philadelphia are manifestations of these innovative movements. The Parkway Program, which is called the "school without walls," resides along a tree-lined avenue densely populated with cultural, esthetic, industrial, and administrative organizations vital to our city's operation. The students from grades nine to twelve (however, the school is non-graded) are rostered to these multiple sites according to their interests. They choose the curriculum and avail themselves of the expertise abounding in the Parkway area. Participants in the program include the Franklin Institute and all its scientific resources, many insurance companies with their computerized actuary departments, the central office for police administration, a television studio, five different museums (including our world-famous art museum), the main public library of Philadelphia, the Philadelphia school administration building with its curriculum experts and reference library, along with many other buildings and individuals who volunteer their services to the children. The students are continually mobile and content, studying relevant concepts offered by field experts. The cost of the operation is lower than the average per pupil cost experienced by the school district, the savings being realized from the absent cost of a central building and the use of outside, volunteer teachers. Presently, another unit is operating as a middle school in a different part of the city and plans are being formulated for starting an elementary school age model.

The "scattered" middle school concept is another fascinating innovation in the constant search for providing mini-self-contained schools within behemoth organizations. Developed by one of our school districts, it plans for small family type units scattered throughout the community. Although collectively the student population will be large, it will be so geographically spaced as to allow the individualizing of learning styles which is only present when teachers truly know the learners.

These three approaches, the Parkway Program, the Scattered Middle School and the House Plan, transcend the abyss and provide a sanguine sign for future planners.

THE PARALYSIS

At one time our Director of Mathematics developed a program of concentrated in-service training, specifically designed to produce two key teachers in each elementary school as grass-roots leaders in mathematics. Realizing the job of training every teacher would be too costly and overwhelming, it seemed pedagogically sound to just locate two leaders in each school -- one from the primary and one from the intermediate grades-- and saturate them with the current trends and activities useful for the elementary grades. The supposition being they would be the storehouse and transmitter to the staff in the area of mathematics, eventually elevating their status to a respected leadership position in their schools. It would then be quite easy to effect changes in curriculum and management dealing with only a small number of consistently motivated professionals.

Clearly the area of reading is far ahead in this respect. Each elementary school in Philadelphia has a lead reading teacher released full-time to provide the necessary leadership in staff development· but mathematics has never enjoyed this salutary position. Being second-best, we have to try harder!

Unfortunately, the plan formulated by the Director of Mathematics never materialized. No matter how assiduously we trained the submitted candidates, we found the program was self-stultifying for five reasons:

1. The key teachers were usually selected for other criteria than their competence in mathematics or their ability to command the respect of the staff. The principal forcibly chose the key teachers from seniority status or even more regrettably, "closeness."
2. The turnover of the teachers was too devastating to enable the mathematics department to provide any continuity of service to the schools. Each succeeding workshop produced more and more new faces, until the gaps between the tenured and novice became too wide to handle effectively.
3. The principals could not adequately accommodate the released-time, staff development sessions that the key teachers required, with only in-house coverage. Without any outside assistance for substitutes' pay or stipends for after school training, it was untenable to believe any consistent service could be provided.
4. The elementary school teachers' background and perception of mathematics education were so traditionally oriented that the applicability gap between present practice and modern theory seemed unbridgeable. This compounded the insecurity of the key teachers in their leadership roles since they soon realized their inadequacies were causing withdrawal symptoms, inhibiting them from assuming their responsibility for directing others. They had so many adjustments to make themselves, that to mold

others was incomprehensible.

5. The assumed extra burden of leadership carried with it no compensatory time or compensation for the key teachers who still had the ubiquitous task of full-time classroom teaching. Obviously, little transmitting could be done.

Failing that scheme to build in local expertise that would eventually promote the mathematics laboratory concept in the schools, another operational plan was proposed. It entailed utilizing the district-wide mathematics collaborators (each of the eight districts had one resource person in mathematics instruction for an average of twenty-five elementary schools) in producing up-to-date, easy-to-follow mathematics guides and activity booklets that would sensibly lead the teachers into mathematics laboratory experiences. Many arduous hours of dedication were spent in molding a comprehensive elementary mathematics program that was truly a vanguard in education. Even the most uninitiated could successfully provide a healthy learning environment for his children by following the suggested program.

The proliferation of materials included:

1. A new intermediate guide (affectionately called the "Jolly Green Giant" because of its enormity) which grouped the mathematics concepts into 18 levels, enabling the teachers to transgress sensibly into the realm of non-gradeness. The guide included not only the sequential concepts, but background and methodology.
2. Levels tests and summary forms that pinpointed the weaknesses and strengths of the students. The tests were specifically designed to be used with the guide.
3. Activity booklets for each level demonstrating the use of concrete and semi-abstract materials in concept formation.

4. Teacher Resource Centers in each district, opened after school and during school hours when needed, as an immediate source of help in giving teachers advice, workshop instruction, materials, and space to construct instructional aids.
5. A large quantity and variety of task cards that teachers could request from the central mathematics department in sufficient numbers to help them get started in their laboratory.
6. A series of creative workshops for interested teachers based on current topics of need. This was done to continue the stimulation previously given the key teachers.

Secure in the feeling that the back-up materials and supportive services were sufficient to create the transition from the insecure traditionalists to motivated mathematicians, all watched with anticipated success. But, plaintively, we once more experienced the inevitability of frustration; well-written guides and booklets -- no matter how "teacher-proof" -- still presented serious debilities. (You can lead a horse to water--but you can't make him drink.) The educational moral being, "Indirect stimulation through the written word is a far cry from getting ideas put into action."

Simultaneously, another approach was tried which involved the initiation of the Madison Mathematics Program in 1967. The program supplied a coordinator for the city and a generous budget to begin the project. The Madison coordinator was well advised by the Mathematics Director, collaborators, and supervisors as to where the best prospects existed for implementing his program. This time the candidates to be trained were chosen by outside experts who knew, objectively, the abilities and motivation of the teachers in the city. During the following three years, the coordinator slowly and meticulously trained laboratory teachers and helped organize activity centers in many schools. With his own separate budget and non-affiliation with the administration of the schools, he was able to work independently and avoid the usual red tape problems associated with large bureaucracies.

The Madison mathematics program was pervasive. Working only with the most promising teachers, a strong following developed that soon built a dynasty of mathematics laboratories across the city. Eventually, the momentum declined in 1970, related to the reasons outlined previously in the first section of this paper.

Regrettably, an unusual situation existed in our mathematics program. The three-pronged attack at improving the teaching of mathematics, including the key teacher program, teacher-proof materials, and the Madison Math Project, failed to build that elusive bridge between antiquated practice and currently accepted practice.

The most recent strategy being tried by the Division of Mathematics is to apply various pressures on the undergraduate institutions to update their current teacher-training programs and provide practical experiences in the mathematics laboratory techniques. Temple University (which supplies over half of our new teachers) accepted the challenge willingly and created many innovative programs in their Math Education Department. The new courses provide the prospective teachers with active materials-oriented curriculum, along with practica of various lengths (some as soon as the sophomore term) as student teacher laboratory assistants to successful mathematics laboratory teachers in the elementary schools. It appeared logical that providing students with the properly accepted techniques of teaching math to children and giving them the necessary background to put those ideas into action, would eventually prove fruitful. This could alleviate the sterile machinations that quickly overtake some teachers later on as they face the everyday crisis in the classrooms.

But once more the paralysis occurred. The Board of Education had to cut funds to survive, which meant that the hiring of any new teachers for the year 1971-72 was at a standstill. The freshly graduated seniors had no place to go -- the system was closed!

This then is the urban dilemma. The mathematics laboratories are declining, for obvious reasons, and no matter what means are applied to stabilize or reverse the trend, paralysis soon sets in!

A REVIEW OF RESEARCH
REGARDING
MATHEMATICS LABORATORIES

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A REVIEW OF RESEARCH REGARDING MATHEMATICS LABORATORIES

This paper is written in three parts. Part one presents a description of what a mathematics laboratory might or might not be. Part two contains a review of research dealing with math labs and activity learning. Part three presents a summary and a charge.

I

Over the past ten years, "mathematics laboratory" has become an "in" term in mathematics education. In the world of the classroom, it means many things to many teachers. In the world of research, the findings regarding math labs are mixed. The current research concerning math labs is clouded in the same way that discovery-learning research is clouded; namely, the definition of the term presents a problem.

While many interpretations try to coexist, there appear to be two different basic interpretations of what constitutes a mathematics laboratory. One interpretation focuses on the physical nature of the classroom and the presence of hardware. There is a place which is referred to as the mathematics laboratory. A second interpretation focuses on the mathematics laboratory as a teaching strategy--a way of reshaping the role of the student and the teacher in the classroom. In the judgment of this writer, a mathematics laboratory should be a synthesis of the physical interpretation and the teaching-learning interpretation. In a mathematics laboratory, students process information. Students use concrete, physical materials to help them arrive at generalizations or solutions. They are working individually, in small groups, or as a class. The teacher's role is consultative--not answer giving or expository.

Kieren (1971) refers to a macro-instructional versus a micro-instructional role for activity learning. I would like to apply this dichotomy to the mathematics laboratory. The macro-instructional use of mathematics laboratory will be interpreted to be applicable over the several content objectives at a given grade level. The micro-instructional use of mathematics laboratory applies when teaching a single idea or concept. Although it might be argued that a macro-instructional use of math labs necessarily implies that there is a facility and lots of hardware, the atmosphere of the classroom is what makes it a mathematics laboratory--be it micro or macro. The micro-instructional use of math labs fits with the teaching-learning strategy interpretation.

It seems fairly obvious to this writer that there are ideas in mathematics programs at any grade level which lend themselves well to interpretations in the physical world. It also seems reasonable to me that some abstract ideas should be treated as just that. The decision to use math labs in a classroom should be made on the basis of the daily content objective.

If math labs are thought of as a place, then the physical world and the mathematical world must be compatible before any great success will be realized. If math labs are thought of as a teaching strategy, then any situation where it is expected that students will process information and make judgments could be incorporated into the laboratory. The atmosphere of the classroom is of primary importance--not the hardware. It is what you do with the materials rather than the fact that they exist.

II

The research reported in this paper will consist entirely of citations that were not given in the October, 1969 Review of Educational Research, Kieren (1969) or the December, 1971 Arithmetic Teacher, Vance and Kieren (1971).

First, consider some older research dealing with manipulative materials. Some of the early research dealing with manipulative materials attempted to get at the effectiveness of specific materials. For example, the research dealing with rods had mixed results. Hollis (1965) and Lucow (1964) reported more learning when rods were used. Haynes (1964) reported no significant difference, while Passy (1963) found that the use of rods caused significantly lower scores in computational skill and reasoning.

In the older research, it is rather common to find studies which dealt with learning aids. If these studies were done in the '70's, they would likely be math lab studies. Sole (1957) reported a study using manipulative materials to teach arithmetic. He concluded that the effectiveness of the learning of arithmetic depended more on the teacher than the materials used. He warned that educators must be careful not to confuse the manipulation of materials with the learning of arithmetic. Sole further stated that arithmetic is a set of ideas and not a system of concrete materials. Eidson (1956) made a similar observation in citing that instructional aids themselves seldom teach arithmetic. The role of the teacher in their use is paramount. Swick (1959) cited data which gave strong support to the desirability of using multisensory aids in teaching arithmetic computation and reasoning. He did not find any unusual interaction between ability of pupils and the use of multisensory aids. He did note an improvement in the attitude toward arithmetic in those second- and third-grade pupils in the experimental sections. Ebeid (1964) reported on a study at the junior high level whereby students were allowed to self-select activity-oriented materials. He found no significant difference between experimental and control groups on achievement or attitudes. Cohen (1959) in a study dealing with solid geometry in the twelfth grade concluded that there is no justification for the claim that construction of models by students during the study of solid geometry will further growth in ability to visualize. It is the judgment of this writer that the inclusion of some examples of research prior to 1965 may well help to set a stage for the more recent research which will be reported in the remainder of this paper.

This next section deals with research which I have classified as being macro-instructional in nature. This means that the mathematics laboratory was used to teach several different content objectives and for a period of twenty or more consecutive school days. Wynroth (1970) reported a study involving the use of game activities in kindergarten and first grade. On the basis of achievement tests, the results were significantly higher in favor of the experimental groups. Game type activities, whether they be pencil and paper or involve some manipulative materials, are often classified as being part of a mathematics laboratory. On that basis, I have included this study.

Weber (1969) in a study using first-grade subjects examined the relative effectiveness of two treatment groups whereby treatment group one consisted of reinforcement of mathematical concepts through the use of paper and pencil follow-up activities, and treatment two consisted of reinforcement of mathematical concepts by the use of manipulative and concrete materials. The study was conducted for thirty consecutive school days, and the results indicate that there was no significant difference between methods as measured by the instrument used, although a definite trend favored manipulative materials. Children from the manipulative materials group scored significantly higher on the understanding instrument. The statistical design provided for an examination of interaction effects, and while the interaction effect was not significant, the trend favored manipulative materials with low socio-economic status children.

Howard (1969) reported on a study involving culturally deprived academically retarded rural children in a mathematics laboratory setting. There were only twelve subjects in this study. On the basis of her findings, one conclusion that she mentioned deals with the older child who has missed the experiences of sorting, counting, classifying, and partitioning sets. Howard feels that this child needs experience with concrete objects. She also made the observation that the laboratory setting can accelerate the usually retarded communication development of the culturally deprived and academically retarded child.

Dienes (1971) reported on a project which includes 60 experimental elementary classes. The project is investigating the nature of complex learning in mathematics. Two additional goals are to: (1) construct a mathematics curriculum, grades one through seven, which is based on the personal interaction of each child with a rich, concrete, mathematical environment, and (2) develop a degree in Elementary Education which has about one-third of the time spent on learning mathematics. The mathematical learning would be based on laboratory work, concrete manipulation, and inductive principles.

Research findings in the above project identified six stages in the learning of mathematical abstractions: Stage 1, initial interaction with environment, random behavior; Stage 2, awareness of constraints and self-imposition of constraints (games); Stage 3, comparison of activities, search for isomorphisms or inclusion or exclusion relationships between games learned; Stage 4, representation, usually spatially, of the abstractions achieved; Stage 5, description of properties of representation (symbolization process); Stage 6, formalization (axioms, theorems, and the like).

Nickel (1971) designed a study to complement Piaget's developmental theory. Nickel designed materials which would test three pedagogical methods attributed to Piaget. These methods were verbal, intuitive, and active. The verbal method relied on use of abstract materials: the spoken and written word. The intuitive method utilized representational materials: pictures, diagrams, and the like. The active method utilized concrete materials: things which can be seen, manipulated, and transformed. Three different treatment methods were assigned to fifteen different groups of fourth-grade children. Treatment one was the control method, treatment two was a strictly verbal approach, and treatment three was a multi-experience approach using the active, intuitive, and verbal methods. One conclusion deals with the multi-experience approach to verbal problem solving. This treatment was demonstrated to be significantly more effective than the strictly verbal approach. Generally the multi-experience approach proved to be a viable means of implementing Piaget's developmental theory in terms of organized classroom instruction for the purposes of improving verbal problem solving.

Toney (1968) conducted another fourth-grade study. Students were randomly assigned to one of two treatment groups. Treatment one called for the students to individually manipulate the instructional materials. Treatment two called for the students to be spectators while the teacher demonstrated the same materials as were used in treatment one. An analysis of the data for this study suggests the following: First, the data indicate a trend toward greater achievement by the group using individually manipulated materials, although this was not significant at the .05 level. A second conclusion stated that the individually manipulated materials seemed to be somewhat more effective in building understanding than the control treatment which consisted of teacher demonstration. A third conclusion dealt with the general mathematics achievement of the two groups. Teacher demonstration and individual manipulation seemed to be equally effective.

Reyes (1972) reported on a study involving second graders and sixth graders. The treatment group had one period per week devoted to the mathematics laboratory while the control group had no mathematics laboratory experience at all. The data indicated that no statistically significant changes took place, although treatment group means exceeded control group means on virtually all the criteria used. The criteria consisted of attitudes toward math, ability in problems similar to those encountered in the math lab, and achievement on the standardized arithmetic test. Even though those students in the experimental group received one less period of arithmetic per week, this did not result in lowering their scores on the standardized arithmetic test.

Wilkinson (1970) conducted a study which used three treatments to teach geometry to nine classes of sixth-grade students. Students taught by the laboratory method did as well as those taught by the expository method. The third treatment group used cassette tapes for instructions. In the case of the low I.Q. male, significant attitude changes in favor of the laboratory treatment were reported. Wilkinson found a tendency for brighter students to show a negative attitude toward laboratory materials. The findings were not significant at the .05 level. Wilkinson expressed the opinion that brighter students may have their thinking slowed when forced to use the physical world. The more able student functioned quite well in the verbal world, and the use of physical materials seemed to create a negative attitude toward mathematics.

Schwartz (1971) reported on the COLAMDA project for low achievers in the Denver area. COLAMDA used the math lab as an approach for teaching the low achiever. A study investigated the achievement growth with seventh and eighth graders. Two treatments were staffed with COLAMDA teachers. The third (T3) was taught by a teacher who had not participated in the project. The COLAMDA teachers used the project materials in treatment one (T1), but did not use project materials in T2. The following conclusions were reported: (1) In the seventh grade, the use of COLAMDA materials did not prove to be significant in affecting achievement. In the eighth grade, the students who were not required to use project materials attained a significantly higher level of achievement, (T2 compared with T3). (2) In a comparison of T1 and T3, there was no significant difference between classes which used the materials and classes which did not use the materials. In the eighth grade, the classes with the teachers who did not use the material attained a significantly higher level of achievement than the classes taught by teachers who did use the material.

The next study, Cohen (1970), deals with a group of boys in a middle school mathematics program and the comparative effects of laboratory and conventional mathematics teaching upon achievement and attitude toward mathematics. While the study did cover thirty-four school days, the content centered entirely about fraction concepts and computation. Comparison of the difference in achievement between the groups revealed a statistically significant increase in improvement on the part of students taught by the conventional approach. The laboratory approach was found to require considerably more class time than did the conventional treatment. There also was a marked difference in favor of the control group with respect to computational ability. There was no clear-cut pattern regarding attitude on the part of control versus experimental group.

Johnson (1970) randomly assigned seventh graders to one of six classes. There were three treatments which were randomly assigned. Treatment A consisted of the textbook only, treatment B consisted of exclusive use of instructional modes other than textbook and was titled "activity." Treatment C, which was called enriched, was a synthesis of the textbook mode and the activity mode. Achievement and attitude tests were used to measure the outcomes. On the basis of results from three achievement tests, Johnson suggested the following conclusions: (1) From the data obtained one cannot conclude that the exculsive continuous use of activity oriented lesson in seventh-grade mathematics classes will result in improved achievement over exclusive textbook based or activity enriched instruction. When the activity is the dominant feature of the instruction and occupies the great portion of class time, then the performance of students on achievement tests appears to be inferior to the performance of students who have little or no activity based learning experience. (2) No differences were detected in achievement between activity enriched and textbook based instruction. Activity enriched was the synthesis of the activity lab oriented treatment and the textbook treatment. (3) A third conclusion dealt with low and middle-ability students; these students were probably aided in the learning of some concepts in seventh-grade mathematics by the use of activity oriented lessons. Johnson went on to suggest that laboratory lessons in the study of geometry and measurements might be appropriate for low and middle-ability students. Finally, the use of activity oriented lessons failed to produce any significant differential effect between treatment on attitude measures.

Chandler (1971) reported on a study which examined Experiences in Mathematical Ideas, NCTM (1970). The study measured changes in the learning environment as perceived by students, and changes in teaching practices as determined by teachers' opinions, as a result of introducing a set of curriculum materials designed for low achievers. Chandler cites the following conclusions: (1) Girls felt the rate of presentation of ideas in EMI was matched with their individual characteristics. (2) The students were not as satisfied with EMI as with their former mathematics program, (3) The students felt that EMI decreased the confusion in the class and helped them see the goals of the class. (4) Teacher opinion of their teaching practices were not changed by the use of EMI. (5) Teacher and supervisor attitudes toward EMI were positive.

Shoecraft (1971) reported a study using three treatments. The treatments were: T1 - low imagery, T2 - high imagery using materials, and T3 - high imagery using drawings. The treatments were used to teach problem solving in seventh- and ninth-grade algebra classes. The materials treatment (T2) classes scored the best on transfer instruments. For seventh-grade students categorized to upper, middle, and lower arithmetic levels, the diagram treatment (T3) was associated with the lowest performance among the middle one-third, and the materials treatment (T2) was associated with highest performance among the lower one-third. For ninth-grade students, T1 was favored among the upper one-third and T2 was favored among the lowest one-third. Shoecraft concluded that low achievers seem to derive particular benefit from representing problems using materials. The other two achievement levels seemed to derive no benefit from materials.

In an extensive project of teaching of mathematics through science by SMSG, more than 12,000 (seventh, eighth, and ninth) grade children studied special materials written by a team of mathematicians, scientists, and teachers. Higgins (1969) conducted a study of the students of 29 eighth-grade mathematics teachers from junior high schools in Santa Clara County, California, as they taught a unit entitled "Graphing, Equations, and Linear Functions."

There were 853 students in the experiment. At the conclusion of the experiment, the students were grouped in eight "natural" attitude groups such that all of the children in a given group had similar attitudes toward mathematics. It was found that differences in attitude patterns among groups were not reflected in significant differences in either ability or achievement. It was concluded that attitude change clusterings were not a major consideration if one is concerned with mathematics achievement during a unit taught via physical approaches. The study found about six percent of the children developing rather strong cohesive unfavorable attitudes toward the content. At the other end, he found eight percent of the children developing attitude shifts favorable toward mathematics, but in general, most students changed attitudes very little. Some liked the unit; some liked it, but found it harder; others found it easier, but less interesting; and a few disliked it quite strongly.

In addition to the above studies, other studies combined films, filmstrips, computer assisted instruction, programmed instruction, and the like with other manipulative laboratory oriented materials. It is difficult enough to design a study to manipulate a.single variable. It is almost impossible to tease out the significant effects when a synthesis of many different teaching strategies and materials takes place within what is identified as a single treatment.

The next section of this paper contains a report of research that deals with a micro-instructional use of the mathematics laboratory. For the purposes of this paper, micro-instructional will apply when there is a single mathematical content idea and the period of instruction is fewer than twenty days.

Moody, Abell, and Bausell (1971) reported on a study of ninety third-grade students. The majority of the subjects came from lower middle socio-economic families. These ninety subjects were randomly assigned to four treatments. One treatment consisted of an activity oriented setting in which the subjects manipulated instructional materials. A second treatment consisted of a rote treatment where the emphasis was placed upon memorizing basic multiplication facts and algorithms. A third treatment consisted of a rote word problem treatment where the instruction was the same as in the rote treatment. The third treatment also included practice in solving multiplication worded problems. The control group received instruction in addition. The results indicate that activity oriented instruction did not result in superior original learning at the .05 level when compared to the other two treatments. It was also found that the additional instruction in the solving of worded problems did not significantly affect computational performance involving basic facts. Difference scores between a pre-test and a post-test indicated that instruction in the solving of worded problems did not result in superior worded problem performance and also that activity oriented instruction did not result in superior transfer compared to the other instructional methods. In the matter of retention, there was no significant difference between the activity oriented instruction and the other two treatments on the retention of multiplication facts. In general, the authors pointed out that all treatments failed to affect transfer of learned computational skill to worded problems involving identical computational requirements. Subjects who were specifically taught worded problems did not manifest a trend towards superior performance immediately following instruction. On the basis of this study, it was concluded that the activity oriented instruction did not result in higher original learning, higher transfer, or higher retention of the instructional content.

Trueblood (1967) assigned subjects, aged nine to eleven, to two different treatment groups. Treatment one involved the actual manipulation of visual tactile aids while treatment two called for the subjects to observe and tell the teacher how to manipulate such devices. The result of the statistical analysis indicated that pupils taught by the demonstration method scored higher on the immediate post-test than pupils who manipulated the visual tactile aids. The significance was at the .10 level. Both T1 and T2 had a high degree of retention.

A second demonstration-comparison type study dealt with fifth graders, Bisio (1970). This study concerned itself with the teaching of addition and subtraction of like fractions to fifth-grade pupils. In treatment one, neither the teacher nor the students used manipulative materials while in treatment two, the teacher used manipulative materials as a demonstration. In treatment three, both teachers and pupils used the manipulative materials. An analysis of co-variance design was used and the following conclusions were cited by the author. Children taught to add and subtract like fractions with manipulative materials, whether they were demonstrated or actually used by the child, were at least equal in measures to children taught by a method not involving manipulative materials. A second general observation seems to be that the demonstration of manipulative materials on the part of the teacher appeared to be as effective as the actual experience using manipulative materials on the part of the student, and both of these were better than the absence of the manipulative materials.

Green (1969) combined two interpretations of fractional number with two different kinds of instructional materials. The instructional materials were diagrams and cardboard strips labeled "materials." It was concluded that diagrams and "materials" were equally effective in learning multiplication of fractional numbers. A second observation dealt with the fact that the students using the diagram techniques liked those techniques significantly better than the students who were using the manipulative materials.

Carmody (1970) reported on a study which investigated the role of concrete and semi-concrete materials in the teaching of elementary school mathematics. Three sixth-grade classes spent eleven class periods in this experiment. Three treatment groups were set up--one used concrete aids, a second treatment group used semi-concrete aids, and a third group used no aids at all. In general, the results favored the semi-concrete and concrete groups over the symbolic group. There was no significant difference between the concrete and semi-concrete groups. The experiment supported the use of concrete or semi-concrete materials if the goal is transfer. The study emphasized the importance of having specific behavioral objectives for the use of concrete aids and of recognizing the many factors that must be considered in deciding on their usage. The necessity of distinguishing between the physical situation to which a mathematical concept is related and the media used for establishing the relationships was also indicated by the study.

The next section deals with mathematics laboratories as an integral part of content or methods courses for prospective elementary school teachers. Smith (1970) reported on an experiment which was to determine whether laboratory experiences improved the performance of undergraduate students in an introductory course in abstract modern mathematics. There were four treatment groups--a control group and three experimental groups. The control group class had four lecture sessions per week, while the experimental groups had one, two, or three laboratory periods per week in addition to lecture sessions for a maximum of four class meetings per week. It was concluded that the laboratory experience of the three experimental groups did facilitate learning of concepts and improved the retention of those concepts significantly. Smith also noted that even though college students are in the formal stage of operations, it may still be the case that they learn a discipline better when they move from concrete to abstract by means of physical materials and models.

Postman (1971) also examined the effect of a mathematics laboratory on the teaching behavior of preservice elementary school teachers. Four components of the laboratory approach were identified: (1) active use of concrete materials by students, (2) the guided discovery approach, (3) students working independently as individuals or in small groups, and (4) the teacher directing her comments to individuals or small groups. Teachers were video taped while teaching a topic. Next, the experimental group of teachers was sectioned off into a six-week laboratory experience as a part of the methods course. The control group did not receive laboratory experiences in the methods course. Because the sample was small, a statistical design was not reported. There was a tendency for students who had received the laboratory activities instruction, as a part of the methods class, to talk less and use materials more in the post-test observation. It appears that the mere involvement in laboratory experiences at the preservice level is not sufficient to cause teachers to use the laboratory approach. Postman does point to the potential that the laboratory approach may have in the changing of teacher behavior.

Boonstra (1970) conducted a study at Michigan State with the purpose of studying, recording, and analyzing the classroom behavior of student teachers who had been given two mathematics laboratory experiences. The study answered two basic questions: (1) Do student teachers who have been taught a concept in a mathematics laboratory use manipulative materials as they teach the same concept? (2) Do student teachers who have experienced a student centered learning situation in a mathematics laboratory employ a student centered teaching approach as they teach? On the basis of this study, it was concluded that two preservice classes incorporating laboratory experiences are not sufficient to affect the teaching behavior of student teachers. The laboratory experiences did not result in the use of manipulative materials and did not affect the dominant role of the teacher in the classrooms observed.

Hendrickson (1970) reported a study dealing with the preservice education of elementary teachers. Three treatments were used in the course. One treatment used a math lab, a second treatment used an enrichment approach, while the third treatment was a conventional approach. There were no significant differences between the three groups in achievement. The conventional group showed a significant attitude gain.

David Fitzgerald (1971) reported on a study conducted at the University of Houston which investigated the effect of a mathematics laboratory upon the performance of prospective elementary teachers enrolled in a mathematics class for elementary teachers. The investigation consisted of the comparison of the effect of a mathematics laboratory teaching technique to the effect of a traditional lecture-discussion teaching technique. Statistically there was no difference between the groups which had participated in laboratory activities and the groups which had participated in lecture only in terms of the achievement in the course. However, those in the treatment group had received only one-half of the lecture time as those in the control group; therefore, it can be concluded that the laboratory experiences can be included in the preservice experience without causing poor performance or poor attitudes on the part of the prospective teachers. In every instance the performance and attitude of the teachers who had the laboratory experiences were slightly better though not statistically significant. Wilkinson found similar results in an unpublished study at the University of Northern Iowa.

In the judgment of this writer, it would appear that our best hope would be to teach several of the courses in the preservice mathematical experience with a laboratory strategy as an integral part of the variety of strategies that we as collegiate level people use to teach content and methods courses. We should model the strategy which we want to impart to the prospective teachers.

Douthitt (1971) reported on a study whose purpose was to design an effective laboratory in college freshman mathematics. One hundred fifty "high risk" students in an analytic geometry course at the University of Houston were the subjects. The following conclusions seem noteworthy:

1. A mathematics laboratory can produce:

   a) higher achievement
   b) more positive attitude scores
   c) a lower failure-withdrawal rate

2. The expository method does not seem to result in the coverage of more course content.

Bluman (1971) reported on the mathematics laboratory as a part of the community college mathematics program and the results of his study indicate that there were no differences in student achievement as a result of mathematics laboratory but that there were more favorable attitudes toward mathematics on the part of those students that used the laboratory method.

As part of the Specialized Teacher Project in California, which is one of the Miller Mathematics Improvement Projects, Frank (1970) published a report which summarizes the special training sessions for in-service teachers held during the summer. These sessions stressed individualized learning experiences and the techniques employed in using mathematical possibilities in the environment, as well as physical materials in a mathematics laboratory setting. Results of the pre- and post-testing of pupils showed that the teacher training had a very strong effect on pupil mathematical achievement in grade two, particularly on those pupils from low socio-economic areas.

In grade five, the effect was more limited with achievement gains significant in certain new topics introduced in the summer training. In a subsequent summer, 1969, an analysis of the results of the pre- and post-testing during 1969-1970 revealed that pupils whose teachers received in-service training scored significantly higher on measure of comprehension and computation. Again, the project was particularly effective with pupils from low socio-economic areas. Both the second- and fifth-grade pupils made significant gains on nine of the ten skills measured.

In all of the above research which used analysis of variance or analysis of covariance design, the F statistic was the only measure of variance comparison reported. The writer suggests that the "multiple R^2" is more easily understood by the average reader and just as meaningful to the sophisticated researcher. The search for significance at the .05 level is greatly enhanced by having a large sample in the study. Often results are reported as significant at the .05 level when by looking at the multiple R^2, we find 4 or 5% of the variance attributable to that effect. The multiple R^2 is defined to be (SS due to regression) + (Total SS corrected). The R^2 measures the proportion of total variation about the mean explained by the source. It is easily computed and would be meaningful if reported with the standard ANOVA tables.

III

In the judgment of this writer, the research on math labs and activity learning does reveal some trends. They are:

a) Math labs seem to produce favorable achievement gains in primary age children. The usual comparison treatments are pencil and paper or expository. The favorable results are not generally present in studies dealing with older children.

b) Math labs seem to produce favorable achievement gains in the less able elementary age child. The favorable results are not generally present in groups of average or above-average ability.

c) Math labs do not seem to have any differential effect on attitudes of elementary or junior high students.

d) Demonstration treatments using activity oriented materials seem to be as effective as "hands-on" treatments.

e) Viewing math labs as a teaching strategy and selecting that strategy when it seems more appropriate than others in a teacher's "repertoire" seems to hold more promise than adopting a lab approach to a large body of content without regard for the compatibility of content and teaching strategy.

f) Math labs show promise as a strategy to teach pre-service classes in mathematics methods.

I would be presumptuous and naive to assert that the above bear any great resemblances to fact. As a community of mathematics educators, we need to identify what it is that we know to be facts, what we know to be opinions, and what are our fantasies. No single researcher, institution, or group can perform this task. It will take cooperation, insight, and hard work. We must get a narrow focus on the issues and provide many replications. In this way, we may more clearly identify the facts, opinions, and fantasies dealing with mathematical learning and mathematics laboratories.

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MATHEMATICS LABORATORY EVALUATION

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MATHEMATICS LABORATORY EVALUATION

Introduction

Mathematics laboratories in one form or another at one level or another are currently a major interest of mathematics educators throughout the world. A natural consequence of this interest is a call for evaluation. This paper is concerned with the evaluation of mathematics laboratory projects. It will report on several project evaluations; it will discuss the difficulties in evaluating laboratory projects; and it will outline some possible directions for the future of mathematics laboratory evaluations.

Interest in Mathematics Laboratories

No one who has attended any recent meeting dealing with mathematics instruction will demand additional evidence of the interest in mathematics laboratories. The programs swell with talks and workshops that are at least tangentially related to someone's definition of a mathematics laboratory. The commercial exhibits have blocks, cubes, rods and mirrors, and publishing houses are looking for manuscripts dealing with laboratory instruction.

There seem to be several forces contributing to this push for mathematics laboratories. Some innovators believe that the research of Piaget and his Geneva Group has provided a rationale for an instructional strategy that involves concrete embodiments of abstract concepts. Moreover, men like Bruner and Dienes have lent support through their writings and research to the notion that the laboratory may be an appropriate strategy for mathematical learning. In acknowledged response to some of these forces, the Nuffield Foundation in England has made possible the development of one extensive model of mathematics laboratory instruction. The creative and attractive materials developed by the Nuffield Foundation and glowing reports of their effectiveness have, themselves, supplied a force for further laboratory activity.

In the United States the realization that the mathematics curriculum reform of the 60's has not had the impact on child learning that was hoped for has inspired a search for a new pedagogy. Some of the searchers have found the new that they seek among the old. The mathematics laboratory has a long history in both writing and action in this country. In his oft-quoted and now seemingly prophetic presidential address to the American Mathematical Society in 1902, E. H. Moore asked "... Would it not be possible for the children in the grades to be trained in the power of observation, experimentation, reflection, and deduction so that always their mathematics would be directly connected with matters of thoroughly concrete character?..." The seventy years since Professor Moore's talk have been spotted with considerable writing about laboratories and many laboratory projects. But it does seem that interest in laboratories has achieved at least a local maximum at this time.

Need for Evaluation

Despite all of the forces fostering mathematics laboratory instruction, the average mathematics class at every level is conducted in 1972 pretty much as it has been for 70 years. There may be a few more materials on the shelves of some elementary classrooms. There may even be a weekly game period which exhibits a non-directedness which tends to be associated with laboratories. But there are very few classrooms at any level that are being conducted according to a laboratory strategy. It would certainly be naive to lay all of the blame for this situation on the lack of laboratory evaluation. There is, however, a posture of skepticism and a demand for accountability that prevails in this country. Innovators, in particular those who advocate mathematics laboratories, are pressed to support their positions with hard data. Unfortunately, such innovations tend to be regarded as threats to existing programs rather than as alternatives, the best of which should be assimilated.

The fact that a mathematics laboratory strategy generally implies a higher degree of classroom organization, that it suggests additional expenditures for equipment, and that it questions existing mathematics priorities puts the advocate of mathematics laboratories under additional pressure to support his position with evaluative data. Furthermore, the serious educator wants objective answers to questions that have been raised concerning the laboratory strategy of instruction.

In an attempt to determine just what is being done toward evaluating the laboratory strategy of teaching, the authors have identified certain laboratory projects throughout the world. In the following pages we will put forward a definition of mathematics laboratory; we will report on selected projects and their evaluation; and we will make some comments concerning problems and possible directions for laboratory evaluation.

Procedures for This Paper

In order to learn of some mathematics laboratory projects and their evaluation the authors devised a questionnaire. This was sent to a list of people which was compiled from

1. AAAS 7th Clearinghouse Report, 1970
2. Personal acquaintance
3. Suggestions from respondents to the questionnaire.

In mailing the questionnaire we were not very restrictive in our definition of a laboratory. We included any project or individual that came to our attention as being involved in something that would satisfy someone's definition of a mathematics laboratory.

However, in deciding which projects to report on in this paper it was necessary to agree on some meaningful, yet not too restrictive, definition of a mathematics laboratory. Many of our questionnaire respondents had given considerable thought to a definition and our thought was colored by theirs.

The mathematics laboratory has to do with mathematics and its implications in and relationships to the real world. It also has to do with learning by doing rather than by being told. There is not general agreement as to exactly what kind of "doing" should go on in a mathematics laboratory. There seems to be a continuum on which most definitions fit. On one end is a highly structured learning strategy involving planned experiences with concrete embodiments of certain mathematical concepts. On the other end is a totally unstructured open-ended involvement in loosely defined real-world situations. Reason seems to suggest that some ingredients from each extreme should be employed in devising a mathematics learning strategy.

For the purposes of this paper it seems desirable to be as inclusive as possible in defining laboratory strategy of instruction. Hence the following:

Definition

The mathematics laboratory is a strategy of instruction in which the learner himself interacts with mathematics and its real-world applications. The techniques used in a laboratory strategy may be varied; they may include discussion, discovery activities, model construction or even some directed teaching. Likewise the interaction of the learner with mathematics and its applications may vary. But the laboratory strategy focuses the learner's attention and activities on the relationship between mathematics and its real-world applications.

The projects discussed below were selected according to the following criteria:

1. the authors' awareness of the project
2. the above definition of mathematics laboratory
3. the presence of a serious attempt at evaluation*
4. the apparent quality and importance of the project
5. the desire for diversity in projects mentioned.

Criterion 1 was mentioned because of the conviction of the authors that they have probably overlooked several very significant projects. There is no claim for completeness in this survey, and the authors are anxious to have other projects brought to their attention. In the process of selection, certain important pioneering projects have been omitted because of their lack of evaluation. This lack is easily understood in such ventures because they are usually under-financed and directed by committed and involved individuals who lack either the time or the training for systematic evaluation. Also some projects are at too early a stage for summative evaluation. Such projects include the laboratory teacher training projects of Professor Dora Skypeck at Emory University and Professor Robert Reys at the University of Missouri, the laboratories of Professor William Fitzgerald at Michigan State University and in the East Lansing schools, the learning center mathematics program in the Winnetka Schools directed by Dr. Lola May, the multilingual mathematics programs of the Southwest Regional Laboratory directed by Kelly Hamby in consultation with Professor Glenadine Gibb, and the Mathematics Specialist Internship Program at Teachers College of Columbia University under the direction of

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* This paper will concentrate on the summative evaluation of projects rather than on their formative evaluation.

Professors Rosskopf and Kaplan, and the Comprehensive School Mathematics Program of CEMREL under the direction of Burt Kaufman.

Seven Mathematics Laboratory Projects and Their Evaluation

I. COLAMDA PROJECT
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COLAMDA (Committee on Low Achievers in Mathematics - Denver Area) is a teacher training project for teachers of low-achieving junior high and senior high students. This Title III funded project has a small pre-service component, but its main energies seem to be directed toward implementing a detailed in-service training model. This model is designed to modify teacher attitude toward slow learners and to introduce the teacher to a wealth of laboratory materials that can be used with slow learners. A snowballing effect is achieved by having teachers who have been trained in the project train new teachers.

Among the projects we have surveyed COLAMDA has given unusual attention to evaluation. The project has an evaluator, Mr. Dan Colvin and an evaluation consultant, Dr. Doug Smogren. The most extensive evaluation reported occurred in the year 1970-1971. This evaluation will be reported here.

Principal foci of this evaluation were student attitude and achievement, teacher attitude, and project implementation. Student achievement was measured by the Stanford Achievement Test in a pre-test - post-test design in which regression to the mean was controlled. The results indicated that there were gains that could not be accounted for by maturation and which were not typical of the population. On these tests as well as on a test designed for topics specifically taught by the project teachers there was a serious problem with getting complete data from properly administered instruments. A semantic differential scale was used to measure student and teacher attitudes toward ideas and things relevant to the program. It was found that pre- and post-test scores were high for both groups on most items. A questionnaire was used to determine the extent to which specific aspects of the COLAMDA materials were implemented in the classrooms of teachers who were trained by the project. Also, questionnaires were used to determine the impact of workshops on teachers and informational luncheons on principals.

The COLAMDA evaluation does reflect gains. It is worth noting, however, that this evaluation was effected by trained evaluators who devoted a great deal of time and money to the evaluation. Even under these conditions the evaluators reported some problems in getting complete and accurate data.

II. Specialized Teacher Project
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The Specialized Teacher Project, one of California's Miller Mathematics Improvement Programs provides volunteer elementary teachers with a two week summer workshop which is designed on a laboratory format. In addition to the training, each teacher is given an allowance for the purchase of some laboratory materials to take back to her classroom. The director feels that this allowance greatly increases the probability of implementation of new techniques learned. The participating teachers are also urged to trade off with another teacher so that they will be teaching mathematics to two elementary classes upon return. This project has trained more than 3,000 teachers during the last four years and has effected extensive evaluation.

As with the COLAMDA project the focus of the testing was on the pupils of the teachers rather than on the teachers themselves. Pupils were tested at grade levels two and five. In grade two the test battery consisted of basic tests of mathematical comprehension and computational ability. These tests were modified by the addition of more difficult items for the post-testing. For the fifth graders selected scales from the National Longitudinal Study of Mathematics Achievement were used for pre- and post-tests. "These tests covered understanding as well as computational ability in the areas of whole number operations, operations with fractions, and informal geometric ideas. A special achievement test covering graphs, functions, and probability was administered in the spring. Those particular areas had been emphasized in the in-service training and in the instructional program for the fifth grade. In grade 5 a selected set of attitude scales was also administered..."

The results on the achievement tests showed significant achievement gains in the experimental group over the control group for both grade levels tested. The gain for the second grade was, however, greater than that for the fifth grade. As is frequently the case there was not a significant attitude gain except for those children whose classes were in disadvantaged areas.

The training for the teachers in this program had, of course, many goals which did not get measured. Nor was any measure taken (or at least reported) on a change of teaching technique by the participating teachers. This evaluation, under the direction of Robert Dilworth, does seem to have been thorough and careful and seems to have demonstrated the effectiveness of the program.

III. USMES
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The Unified Science and Mathematics for Elementary Schools (USMES) project has developed some laboratory materials that appear to be near the open-ended, undirected end of the strategy continuum proposed earlier. As an outgrowth of the Cambridge Conference on the Correlation of Mathematics and Science in the Schools (1967), the USMES project was funded by the

National Science Foundation. The project materials are built around a series of challenges to the student, for example:

-- How can you make practical fair dice for different numbers of players?

-- How can you design and build at reasonable cost a burglar alarm which would give adequate warning?

-- What is the best description to help in picking a person out of a crowd at the airport or bus station or to help in finding an amnesia victim or other missing person?

-- How would you improve the safety and convenience of a pedestrian crossing?

-- How would you improve your cafeteria service?

The USMES materials include the challenges, skill cards to help a student with technical problems that he encounters in trying to answer his challenge, the plans for a design laboratory for the student to work in, and materials to help a teacher organize and facilitate student participation in the program. The materials are now being tried with elementary pupils at various locations throughout the country, and they are being used in pre-service teacher training at the California State College at Bakersfield and at Michigan State University.

The arithmetic and reading comprehension subtests of the California Test Battery are being applied to 50 USMES classes and 50 control classes this year on a pre-test - mid-test - post-test basis. The project is also collecting observational data from teachers and outside observers. Most interesting, though, is the problem test which is being applied to a small random sample from each of the 100 classes. The student is asked to help the school to make a decision on which of three exercise books (notebooks) the school should buy for the students to use in science and mathematics classes. The student is told that he has 30 minutes and that he will be given any information that he asks for. He is asked to record in writing or verbally the steps that he goes through to make a decision. The observer is asked to note such things as the number of measurements, tools used, accuracy of measurements, number of comparative calculations, data used in calculations, questions and statements of students, number of times instructions need to be repeated, elapsed time to selection of first of three books, elapsed time to final selection.

This latter technique of evaluation has obvious implications for the evaluation of mathematics laboratories. It is tailored specifically for an objective that is near to the heart of many laboratory advocates, namely, improving the abilities of students in solving real, open-ended problems. The evaluation design of the USMES project is still in the developmental stage under the direction of Professor Bernard Shapiro of Boston University. It bears watching by those who are interested in mathematics laboratory evaluation.

IV. The Nuffield Mathematics Project
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London, England

The Nuffield Mathematics Project in England was formally begun in 1964 and has since been developing a series of materials for teaching mathematics in an open-ended inquiry setting with a great deal of focus on one's environment and one's interaction with it. This most famous of mathematics laboratory projects has been the source of a great deal of interest and controversy in this country. Its dedication to process goals and lack of controlled evaluation of those goals have added fuel to the fire.

Of interest to this paper is the fact that the Nuffield Mathematics Project has appointed Mr. Murray Ward of the University of Reading to direct an evaluation of the project. He has not as yet formulated how he will carry out his evaluation. He is visiting schools, teacher centers, and projects that have developed materials to determine the nature and scope of the evaluation program. The results of this evaluation seem several years away but will be of the greatest interest.

V. WYMOLAMP Project
David Flory, Director
Riverton Schools
Riverton, Wyoming

The WYMOLAMP Project is related to the COLAMDA Project in origins and objectives. It has as its goals attitude change and skill improvement in mathematics for K-12 (pre-algebra) disadvantaged students. The project has developed activity packets with manipulative materials for skill development in younger (K-6) children, and it has developed occupational mathematics packets which are designed to increase skills which might be of use in particular occupations. As with the COLAMDA project, WYMOLAMP devotes considerable energy to developing teacher attitudes and skills for handling the problems of low-achievers in mathematics.

Project evaluation includes attitude questionnaires and some interesting individually administered tests. In these tests the child was given an opportunity to respond to a problem at several levels of abstraction. It was found that many of the children could solve certain problems by manipulating a physical referent but could not solve an equivalent problem symbolically. The interesting feature of these manipulative tests is that they were particularly tailored to the skill development strategy of the project.

VI. Institute for Research in Psychomathematics
Zoltan P. Dienes, Director
University of Sherbrooke
Sherbrooke, Quebec

Another laboratory project which, like the Nuffield Project, has had a great impact on the mathematics laboratory movement is that of Professor

Zoltan Dienes at the University of Sherbrook in Quebec province. Professor Dienes has the full scope of mathematics education activities in progress. Built around a theory which states, in part, that children learn abstract concepts by discovering isomorphisms between multiple embodiments of those concepts, the institute has a program of research in child learning, a materials development program, a pre-service teacher training program and an in-service teacher training program. Children are taught in several local schools via the multiple embodiment principle.

Professor Dienes has had at least some influence in the development of projects throughout the world including the Canary Islands, Australia, England, several European countries, San Diego and Philadelphia.

Statistical evaluation has not been a major thrust of the Dienes projects. In Sherbrooke, 9 year olds in the Dienes program were compared with those in regular classes on achievement tests which are tailored to school objectives. The Dienes children were reported to have done equally well on computational skills and better on problem solving than the control students. Dienes himself feels that one needs but walk into one of his classes, and one will be convinced that the children are engaged in profitable mathematical activity.

VII. TTT Mathematics Systems Laboratory
Viggo P. Hansen, Director
San Fernando Valley State College
Northridge, California

The TTT Mathematics Systems Laboratory at San Fernando Valley State College is unique in several respects. The most relevant uniqueness is that it has had a high degree of success with an outside evaluator. For the fairly nominal fee of $5,000 the Research Bureau of the California Teachers Association developed questionnaire and interview instruments and administered them to the student teachers trained by the project, their supervising teachers, the college personnel involved, public school administrators, and various people from the community. Psychologists were hired to do the interviewing. The evaluators found that the respondents to the questionnaires and interviews were very enthusiastic about the project.

In this TTT project the prospective junior high school teachers receive their methods and practice teaching experiences at the same time in the schools. The mathematics component is handled entirely in a laboratory setting, with lessons involving manipulative devices and open-ended activities. Another unique feature of this program is its use of the computer as a tool in record keeping, diagnosis and prescription. The added cost that a computer introduces makes the project directors particularly sensitive to the need for evaluation to provide accountability for the additional expenditures.

In addition to the funded evaluation activities the fifth-grade children who were taught by project students were pre- and post-tested on the 1956 basic skills component of the Stanford Achievement Test. Significant gains were recorded. The project hopes to develop further evaluative instruments including such unobtrusive measures as time-on-task.

Summary of Listed Project Evaluations

The projects reported had differing goals, different levels of funding, different local problems, and each came up with a different approach to its evaluation. One commonality among those projects that reported some satisfaction with their evaluation is that their evaluation design was tailored specifically to the goals of the project, and their evaluative instruments tended to be sensitive to those goals.

The Specialized Teacher Project recognized that, while it was training teachers, its principal goal was improved achievement of pupils. So the evaluators went to the pupils to measure the effectiveness of the program.

The evaluators of the TTT Program at San Fernando Valley State College felt that existing paper-and-pencil and observational instruments were going to be insensitive to the goals of that project. They were also faced with the serious time-lag and geographic distribution problems of measuring the impact of pre-service teacher preparation on the subsequent performance of graduates. In this case the evaluators chose to sacrifice "cleanness" of evaluation design for the sensitivity of questionnaires and interviews. If well thought out, this design seems to make sense for early evaluation of a project since it can yield considerable formative information. But it does not so easily produce those all-convincing means and correlation coefficienta.

The COLAMDA and WYMOLAMP projects both went to their ultimate goal, the pupils, for their evaluation, despite the fact that the treatment group consisted of teachers. WYMOLAMP actually designed test instruments to measure the pupil's performance on the concrete materials with which he was trained.

It is not difficult to appreciate the concern that led the USMES evaluators to devise an instrument that was very specific to their project. The goals of flexibility, open-ended problem solving techniques, and analytic skills would certainly not be reflected in scores on any of the standardized achievement tests. The USMES problem situation instrument seems to show some promise of providing a new direction in the development of instruments for mathematics laboratory evaluation.

One message, then, is that individually tailored evaluations are needed which employ instruments that are sensitive to project goals. We now consider the implications of this message for the design of mathematics laboratory evaluation.

Articulation of Goals

One implication is that a clear definition of goals must precede effective evaluation. However, as the cliche so aptly puts it, "this is easier said than done." Most laboratory projects share some of the traditional achievement goals with other strategies of instruction. These goals have been carefully thought out and many of them have been articulated in behavioral terms. The evaluation of these goals should present few problems.

However, most mathematics laboratory projects have other goals. These may include confidence, flexibility, positive attitude, and problem solving process skills, all to be exhibited in the context of mathematics. Clearly, such goals are most difficult to express in behavioral terms. And yet, it is exactly these goals that motivate the development of most mathematics laboratories. So they must be articulated and they must be evaluated.

Sensitive Instruments

Again the traditional goals, at least for children, present fewer problems. Standardized achievement tests abound. Unfortunately, many evaluations that employ these instruments in a comparative design seem to find no significant difference between experimental group and control group. There is some question as to whether the usual paper-and-pencil test puts a laboratory-trained child at a disadvantage. It may be that the traditional goals could be measured by instruments that are more compatible with the laboratory strategy of instruction. Possibly, for some laboratories an instrument that would measure these goals in an activity setting would be more appropriate. However, like it or not, one of the pressures for the evaluation of mathematics laboratories is to demonstrate to education decision makers that laboratories do not sacrifice traditional skills in their pursuit of other goals. There is probably nothing that could be as effective to that end as demonstrating sizeable gains on recognized standardized tests.

It seems, however, that the evaluation of the set of process and affective goals is going to require some very creative instrument construction. Fortunately, considerable energy is already being expended by evaluators on the development of alternatives to paper-and-pencil tests. Many such sensitive instruments as the USMES problem solving test must be developed. Observational techniques, self reports, interviews, interaction with computers in simulated problem solving situations and other, as yet unknown, techniques may prove to be helpful.

Defining the Audience*

An obvious and very important first step for evaluation which some projects overlook is that of deciding for whom the evaluation is being done. That is, the evaluators and the project director must decide to whom they are going to demonstrate the project's effectiveness. Is it taxpayers, parents, scientists, National Science Foundation administrators, or is it just the project staff and local school personnel that are to comprise the audience for the evaluation? For some, testimonial would be the most convincing argument for or against a project's effectiveness. For others, clean statistical data collected under controlled conditions is the least that will be accepted. One must enter into the epistemological consideration of what constitutes "knowing" for people with various points of view. Agreement between project director and evaluator on the audience and the audience's criteria for knowing could provide a clear focus for evaluation. However, such evaluatee-evaluator

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* The authors are indebted to Dr. Richard Turner of Indiana University for his comments on this section.

agreement is not always easily obtained.

Evaluator-Evaluatee Agreement

One common problem between director and evaluator is reflected in a conversation that one of the authors recently had with the director of a laboratory project. This individual spoke animatedly about the evaluation that his funding agency had imposed on him. He felt that the evaluators forced him to admit to objectives that he did not really have in order to effect a clean evaluation. Furthermore, he felt that one of his project staff members could tell him much more about the effectiveness of his materials after an hour with a class than could any amount of formal evaluation.

One cause of such incompatibility between development and evaluation may be the fact that the evaluative criteria and even the evaluator were imposed on the project by its funding agency. Furthermore, some evaluators tend to lack flexibility in their approach. They tend to impose an established evaluation design onto a project rather than tailoring the design to the project and the audience. And frequently the project director has no interest at all in formal evaluation. His interest and energies are directed toward development, and his contact with the project materials completely satisfies his criteria for knowing that they are successful. For such a person the hours required to develop an effective evaluation are not going to be gladly given.

It does not seem likely that a simple cure will be forthcoming for the problems mentioned above. One can do little more than call for consultation between funding agency and project director on evaluative criteria, flexibility and ingenuity on the part of the evaluator, and commitment to formal evaluation on the part of the project. There is, however, another problem which plagues evaluators and which can, under some conditions be dealt with.

Money

Generally, formal evaluation is expensive. Evaluators, assistants, tests, and computers all cost money and many projects, even if they have sufficient money, hesitate to spend it on evaluation. A partial practical solution to this money problem is the in-house development of an evaluator. Those projects that work within the framework of a university can employ a mathematics education graduate student who has an interest in evaluation. He can be involved in the development of the project; he can be trained by the evaluators on campus as part of his doctoral work; and he can effect the evaluation of the project with the consultation of one of the professional evaluators (possibly his thesis director). This approach cuts costs, and seems to increase the likelihood of sensitive and open-minded evaluation without sacrificing validation by a professional evaluator.

Need for Communication and Cooperation

No one project is likely to afford clear articulation of affective and process goals, the development of sensitive instruments, and the creation of an acceptable design for evaluation. Projects must share their experiences and their instruments, and they must cooperate in the development of better evaluation programs.

Summary

There are problems with mathematics laboratory evaluation, and the elimination of most of these problems seems a long way off. When one considers these problems and recalls that a certain number of individuals are mystically or emotionally opposed to all evaluation, one may be disappointed but should not be terribly surprised that the list of mathematics laboratory evaluation projects is fairly short.

We have reported here on some laboratory evaluation projects that have been effective or that have introduced interesting techniques. We have discussed the need for clear articulation of goals, the need for the development of sensitive instruments, and the need for flexible leadership from professional evaluators. And we have hinted at some possible steps toward these ends.

Hopefully, the cooperation of many mathematics laboratory projects on these matters will improve the art of their evaluation.