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ABSTRACT

Described are two different statistical methods (component and discriminant analysis) used to compare electroencephalographic patterns of normal and three types of mentally retarded persons ages 7- to 41-years-old. (CL)

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Application of Multivariate Analysis to Quantitative Classification of EEG Patterns of the Mentally Retarded

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APPLICATION OF MULTIVARIATE ANALYSIS TO QUANTITATIVE CLASSIFICATION OF EEG PATTERNS OF THE MENTALLY RETARDED

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Multivariate analysis is that branch of statistics which is devoted to summarizing the relationships of sets of dependent variables. It includes, for example, principal component analysis or discriminant analysis, where the problem is developed in the statistical context of determining a linear combination of a given set of variables that has a larger variance than any other linear combination, or that best differentiates among several groups. The detailed theory and trends of extensive use particularly in behavioral science research will be introduced by such books as those by Cooley and Lohnes [1971], Rulon [1967] and Tatsuoka [1971].

Those techniques will be useful in the realm of EEG pattern analysis, since the individual patterns may be assumed to be single points located in a multidimensional space. Hence, several multivariate techniques have already been taken up recently in this field: step-wise discriminant analysis [SWDA] to average evoked potentials [Donchin et al., 1970], component analysis to spectra, and the like. Particularly, it is with the hope that application of multivariate analysis will facilitate an dealing with taxonomic or classification problem of EEG.

The purpose of this report is to describe sections of our studies in making use of the multivariate mathematical models to quantitative discrimination of EEG patterns among the groups of normal and several types of mentall retarded. We outline the preliminary approach in applying the techniques of principal component analysis and discriminant analysis.

1. MATERIAL AND DATA PROCESSING

Thirty-eight EEG pattern samples of 10 second epoch during resting condition were extracted from three groups of mentally retarded, namely, the predisposed [Group P], the exogenous [Group E] and the Down's syndrome [Group D], who ranged in age from 7 to 41 years. Those typical samples of each group, 11, 13 and 14 respectively, were selected from more than four hundred cases of the mentally retarded that were clinically examined. The subjects with evidence of any epileptic and other neurological signs were excluded in selection.

As controls, 32 normal samples ranging from 4 days to 20 years of age were also used [Group N], which consisted of the younger 17 samples ranged up to 6 years [Group N_1] and the other 15 samples matched in age to the retarded [Group N_2]. These were extracted from the Gibbs' Atlas [1951].

Data processing was performed in three steps. In the beginning, EEG patterns were digitized by A/D conversion of sample waves on the magnetic tape and were punched automatically in 8 bit paper tape for computer processing, using TEAC R-400/ ATAC 501-10/ TH-800 system at the laboratory.

The binary coded decimal outputs thus obtained were then used to computer processing on multiple variables, listed in Table 1, to gain overall infoimations of each pattern. Figure 1 shows the flow of data processing except that in auto/cross correlation and spectrum analysis, and its computer program is also given in the Appendix.

The final step is to apply the component analysis and the discriminant analysis to sample values of variables obtained. Since the variables to use should be limited in number at these procedures, due to the limit of computer memory, 26 variables were selected. Those are marked by X_1 to X_{26}.

2. COMPONENT ANALYSIS

Suppose the random p-dimensional vector $X' = [x_1, x_2, \ldots, x_p]$ has the variance and covariance matrix Σ. We shall assume that the mean vector is O and x's have the unit variance. The object of component analysis is to economize in the number of variates, and for that, is to seek for a linear combination of type $Z = a'X$ which maximizes variance.

Let a be a p-dimensional column vector such that $a'a = 1$. Then the variance of Z is

$$E[a'X]^2 = E[a'XX'a] = a'\Sigma a = a'Ra \qquad [1]$$

where R is the correlation matrix.

To determine the normalized linear combination $a'X$ with maximum variance, we must find a vector a satisfying $a'a = 1$ which maximizes [1]. In order to get a solution, we should seek for a satisfying

$$[R - \lambda I]a = O \qquad [2]$$

where λ is a Lagrange multiplier. If a satisfies [2] and $a'a = 1$, then the variance of $a'X$ is λ. Thus for the maximum variance we should use in [2] the largest λ, namely, λ_1.

Let a_1 be a normalized solution of

$$[R - \lambda_1 I]a = 0 . \qquad [3]$$

Then $Z_1 = a_1'X$ is a linear combination with maximum variance, and is called the first component. Furthermore, we may find another vector a_2 corresponding to the second largest root λ_2 of [2], such that $Z_2 = a_2'X$ has maximum variance of all linear combinations uncorrelated with Z_1. Z_2 is called the second component. This procedure is carried on, and we may thus transform to new variates Z_1, Z_2,.., Z_p which are uncorrelated and have variances λ_1, λ_2,.., λ_p in decreasing order.[1]

The results are shown in Table 2. It describes the coefficients of first seven linear combinations, which we obtained when applying the component analysis to 70 EEG samples simul'' .eously. The leading four extracted components, Z_1, Z_2, Z_3 and Z_4 account respectively for 23.0, 16.6, 13.3 and 8.9 per cent of the total variance, and evidently 75 per cent are accounted for by the seven components given in Table 2.

Multiplying each coefficient by $\sqrt{\lambda_j}$ we have the correlation coefficient r_{ij} of ith variable and jth component; therefore, the signs of the coefficients and their relative magnitudes are useful to examine the nature of components.[2] In the present results, it may be observed that Z_1 sums up information on "general development of EEG". Likewise, Z_2, Z_3, Z_4,.. may be named

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[1] To get a solution of [2] with $a'a = 1$ we must have $R - \lambda I$ singular; in other words, λ must satisfy $|R - \lambda I| = 0$. The function $|R - \lambda I|$ is a polynomial in λ of degree p. Therefore, the equation $|R - \lambda I| = 0$ has p roots; let these be $\lambda_1 \geq \lambda_2 \geq \ldots \geq \lambda_p$.

respectively "frequency versatility", "amount of irregular slowing", "amount of beta-activities at the occipital region", and so on [See Figure 2].

Figure 2 depicts the scatter diagrams representing relations between component scores and ages for Groups P, E, D and N. The lines and broken lines are showing the regression of the scatter diagrams for Group N. What is evident from these diagrams is that Groups E and D are retarded in regard to Z_2 as compared with Group N. Furthermore, it is clear that Group D keeps high amount of occipital beta-waves, contrary to the result of decreasing of those waves after 15 years in the other groups. However, definite tendencies cannot be observed in Group P; that will be one of topics for further discussion.

Thus, owing to reduction in the dimensions, the classification of EEG patterns may be discussed economically in terms of a set of fewer new variables, namely, components. But we think it is satisfactory to consider that the discriminant analysis will be more effective to the classification problem, which will be described in the next section.

3. DISCRIMINANT ANALYSIS

Suppose we have the the vector of p-dimensional measurements $X' = [x_1, x_2, \ldots, x_p]$ on an individual. We shall now consider the assignment of that

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2 The covariance of Z_j and X is $E[(a_j'X)X'] = a_j'E[XX'] = a_j'R$. The variance of Z_j is λ_j, then the correlation coefficient $R_j' = [r_{1j}, \ldots, r_{pj}]$ should be

$$R_j' = \frac{a_j'R}{\sqrt{\lambda_j}} = \frac{a_j'\lambda_j}{\sqrt{\lambda_j}} = a_j'\sqrt{\lambda_j} ,$$

since $Ra_j = \lambda_j a_j$ is derived from $|R - \lambda_j I| a_j = 0$.

individual into one of two normal populations, namely, $\pi_1 : N[\mu_1, \Sigma]$ and $\pi_2 : N[\mu_2, \Sigma]$, where $\mu_i = [\mu_1^i, \ldots, \mu_p^i]$ is the vector of means of the ith population $[i = 1, 2]$ and Σ is the matrix of variances and covariances of each population.

In this case, if the observation X is actually from π_i, the linear combination $Z = a'X$ should be distributed according to one-dimensional normal distribution $N[a'\mu_i, a'\Sigma a]$. Then the problem is to classify into either π_1 or π_2 to minimize the distance $|a'X - a'\mu_i|$, between $|a'X - a'\mu_1|$ and $|a'X - a'\mu_2|$.

Since the probability of misclassification, in this case, is the monotonic decreasing function of the Mahalanobis' distance between π_1 and π_2

$$\Delta^2 = \frac{[a'\mu_1 - a'\mu_2]^2}{a'\Sigma a} \qquad [4]$$

the most appropriate weight $a' = [a_1, \ldots, a_p]$ may be obtained by seeking for a so as to maximize under the restriction of

$$a'\Sigma a = 1 \; . \qquad [5]$$

Thus we find

$$a = K\Sigma^{-1}[\mu_1 - \mu_2] \qquad [6]$$

where K is the constant. The linear function $Z = a'X$, thus obtained, is the well-known discriminant function, that is to differentiate best the observations from two populations, π_1 and π_2.

The same result may be reached by a different route. The ith normal density function is

$$P_i[X] = \frac{1}{[2\pi]^{\frac{p}{2}} \, |\Sigma|^{\frac{1}{2}}} \exp[-\tfrac{1}{2}[X - \mu_i]'\Sigma^{-1}[X - \mu_i]] \qquad [7]$$

Then the ratio of densities is

$$\frac{P_1[X]}{P_2[X]} = \frac{\exp[-\frac{1}{2}[X - \mu_1]'\Sigma^{-1}[X - \mu_1]]}{\exp[-\frac{1}{2}[X - \mu_2]'\Sigma^{-1}[X - \mu_2]]} \qquad [8]$$

$$= \exp[X'\Sigma^{-1}[\mu_1 - \mu_2] - \frac{1}{2}[\mu_1 + \mu_2]'\Sigma^{-1}[\mu_1 - \mu_2]].$$

The region of classification into π_1, which we denote by R_1, is the set of X's for which [8] is $\geq$ k [for k suitably chosen]. Since the logarithmic function is monotonic increasing, the inequality can be written in terms of the logarithm of [8] as

$$X'\Sigma^{-1}[\mu_1 - \mu_2] - \frac{1}{2}[\mu_1 + \mu_2]\ \Sigma^{-1}[\mu_1 - \mu_2] \geq \log k\ . \qquad [9]$$

If we denote the left-hand side as U, and if π_i has the density [7] [i = 1, 2], the best regions of classification are given by

$$\begin{aligned} R_1&:\ U \geq \log k \\ R_2&:\ U < \log k\ . \end{aligned} \qquad [10]$$

If a priori probabilities q_1 and q_2 are known, then k is given by

$$k = \frac{q_2\ C[1/2]}{q_1\ C[2/1]}\ , \qquad [11]$$

where C[1/2] is the cost of misclassifying an individual from π_2 as from π_1, and C[2/1] is that in the opposite direction.

In the particular case of the two populations being equally likely and the costs being equal, k = 1 and log k = 0. Then the regions of classification into π_1 and π_2 are respectively

$$\begin{aligned} R_1&:\ U \geq 0 \\ R_2&:\ U < 0\ . \end{aligned} \qquad [12]$$

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If the first term of U will be denoted as a, namely,

$$a = \Sigma^{-1}[\mu_1 - \mu_2] , \quad [13]$$

then we obtain

$$U = X'a - \frac{1}{2}[\mu_1 + \mu_2]'a . \quad [14]$$

The first term is the discriminant function previously introduced.

Transforming U slightly, we get

$$U = [X'\Sigma^{-1}\mu_1 - \frac{1}{2}\mu_1'\Sigma^{-1}\mu_1] - [X'\Sigma^{-1}\mu_2 - \frac{1}{2}\mu_2'\Sigma^{-1}\mu_2]. \quad [15]$$

Let us now define

$$f_i = X'\Sigma^{-1}\mu_i - \frac{1}{2}\mu_i'\Sigma^{-1}\mu_i , \quad [16]$$

then U may be stated as

$$U = f_1 - f_2 . \quad [17]^3$$

We may then rewrite the classification procedure as

$$\begin{aligned} R_1&: f_1 \geq f_2 \\ R_2&: f_1 < f_2 . \end{aligned} \quad [18]$$

With using f_i's, we may proceed to the classification among m populations [$m \geq 3$]. The population π_k corresponding to the greatest f_i, say f_k, among m f_i's will be the most appropriate population, to which X should be allotted. In such a case, the probability of correct classification into π_i is

$$P_i = \frac{\exp[f_i - \max f_i]}{\sum_{i=1}^{m} \exp[f_i - \max f_i]} . \quad [19]$$

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[3] The Mahalanobis' distance between X and the centroid of π_i, if we denote this by $D^2{}_i$, should be $D^2{}_i = [X - \mu_i]'\Sigma^{-1}[X - \mu_i] = X'\Sigma^{-1}X - 2f_i$. Hence, we may also write $f_i = \frac{1}{2}[X'\Sigma^{-1}X - D^2{}_i]$ and $U = f_1 - f_2 = \frac{1}{2}[D^2{}_2 - D^2{}_1]$.

Things to be investigated will be divided into two cases: the one is allotting an individual EEG sample to one of two populations, namely, the normal and the mentally retarded in general, and the another is allotting to one of four populations: P, E, D and N. These are schematically illustrated in Figure 3.

The purpose of the analysis is to seek for *p* linear functions of the variables, f_i, i equals one to p, so that a ~~sample observation~~ can be allotted to appropriate one of p populations, according to which of the f's is the greatest when the sample values are substituted. Therefore, f_i might be called a measure of proximity to population π_i. As is evident from the upper diagram, in case of two populations, where to allocate a sample will be decided according to the value of the function U which we have by taking f_2 from f_1. If the value of U is positive, the sample should be allotted to the mentally retarded, and if negative, to the normal.

The table in Figure 4 shows the weights and the constant term of U, which we found by computation in case of two populations, using the sample data. At tha , Group N_2 alone were used for the normal, for the sake of matching in age to Groups P, E and D. The values given to the whole samples by the discriminant function Z obtained in this way are also distributed in Figure 4. It is evident that two distributions for the mentally retarded samples and for the normal are clearly separated. Therefore, the probability of misclassification seems to be estimated as extremely low.

When we proceed to discrimination among four populations, however, results are more complicated. Table 3 shows the coefficients and the constant terms of the four linear functions f_1, f_2, f_3 and f_4, which we obtained by computation. From the values given to the samples by four functions, probabilities of assignment to each one of four populations may be computed for each sample. These probabilities are tabulated in Table 4. The samples having a some amount of probabilities, large and small, to be assigned tc the other groups are added by dashed lines. Probabilities of assignment to the normal are 1 for all normal EEG samples, but it can be seen that two samples of Group P and Group D, namely, P-7 and D-14, are misclassified to the popupation E with the probabilities of 0.618 and 0.945 respectively. Besides, complicated problem on classification may be pointed out for the sample E-6. The EEG patterns of those complicated samples classified to E are shown in Figure 6, comparing with E-8 that is typical of Group E.

Thus, it is concluded that members o: any groups of the mentally retarded were not misclassified to Group N at least in this study. However, the general veracity of this conclusion is doubtful because such a result can be drawn merely from sampling bias, which should be the subject for a future study. We assume that the sampling bias of the normal EEG was the primary factor affecting the result.

Figure 5 gives the two-dimensional chart for f_1 through f_3 with respect to which the individuals of the mentally retarded can be classified into three regions such that

$$\begin{aligned} R_p&: U_{13} > 0;\ U_{12} > 0 \\ R_e&: U_{12} < 0;\ U_{23} > 0 \\ R_d&: U_{13} < 0;\ U_{23} < 0\ , \end{aligned} \qquad [20]$$

where $U_{12} = f_1 - f_2$, $U_{13} = f_1 - f_3$ and $U_{23} = f_2 - f_3$. The space is divided by three boundary lines, $U_{12} = 0$, $U_{13} = 0$, $U_{23} = U_{13} - U_{12} = 0$, intersecting at a single point.

The dots, squares and circles represent members of Groups P, E and D, respectively, and those added by sample numbers are the practically or probably misclassified ones given in Figure 6. What is evident from Figure 5 is that Group E lies in close proximity to Group D, when compared with the relations of P to E and P to D. It is consistent with the results of the component analysis shown in Figure 2, and such a result can be expected on pathological and empirical grounds [Hirai and Izawa, 1964].

4. DISCUSSIONS

The following points are left as future problems: the one is what sort of multidimensional variables should be introduced to identify and differentiate an individual EEG pattern exactly, and the unbiased sampling also should carefully be considered; that is another point.

The results of multivariate statistical analysis may be said to depend finally on those two points. It may be true that the variables we introduced are mere preliminary ones; for that reason, further strict discussion, from physiological as well as statistical point, will be required on selecting appropriate variables. As to the sampling, as well, it becomes a serious problem that we used the normal EEG samples of the Gibbs' Atlas; those samples seem to be biased to fewer amount of fast waves and versatility, which will act in favor of discrimination from samples of the mentally retarded.

The discriminant analysis may also be accomplished by finding the weights a that maximize the discriminant criterion λ, defined by the ratio of the between-groups to within-groups sums-of-squares of a linear combination $a'X$. The criterion λ should be

$$\frac{SS_b}{SS_w} = \frac{a'Ba}{a'Wa} \equiv \lambda \qquad [21]$$

where B and W are the between-groups and within-groups SSCP matrices, respectively. The necessary condition for maximizing λ reduces to

$$[B - \lambda W]a = 0 \qquad [22]$$

which is equivalent to

$$[W^{-1}B - \lambda I]a = 0 \qquad [23]$$

provided, as will generally be true, that W is non-singular.

Thus, when this equation is solved, we get non-zero eigenvalues, which will be denoted as $\lambda_1, \lambda_2, .., \lambda_r$ in descending order of magnitude, and r associated eigenvectors $a_1, a_2, .., a_r$. The elements of those eigenvectors may be used as combining weights to form r uncorrelated discriminant functions, the entire set of which constitutes the discriminant space [Rulon, 1967; Tatsuoka, 1971].

For this attempt, the severe restrictions of normality and identical dispersion matrix in each group are not required; in this respect, this method exceeds that we used, apart from the discussions of discriminatory power. It seems to be a worthwhile subject to seek relationship between two methods in applying to EEG patterns.

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Table 1.
Variables employed for data processing.

A: Intra-area Variables [LP & LO]					
		LP			**LO**
1	X_1	mean wave frequency	18	X_{14}	mean wave frequency
2	X_2	SD of wave frequency	19	X_{15}	SD of wave frequency
3	X_3	mean wave amplitude	20	X_{16}	mean wave amplitude
4	X_4	SD of wave amplitude	21	X_{17}	SD of wave amplitude
5		weighted mean of frequency[1]	22		weighted mean of frequency
6		weighted mean of amplitude[2]	23		weighted mean of amplitude
7		sum of D•A[3]	24		sum of D•A
8		mode of frequency [1][4]	25		mode of frequency [1]
9		mode of frequency [2]	26		mode of frequency [2]
10	X_5	sum of θ amplitude	27	X_{18}	sum of θ amplitude
11	X_6	sum of α amplitude	28	X_{19}	sum of α amplitude
12	X_7	sum of β_1 amplitude	29	X_{20}	sum of β_1 amplitude
13		total amount of θ waves[5]	30		total amount of θ waves
14		total amount of α waves	31		total amount of α waves
15		total amount of β_1 waves	32		total amount of β_1 waves
16		auto-correlation	33		auto-correlation
17		auto-spectrum	34		auto-spectrum
	X_8	power of 4-7 Hz		X_{21}	power of 4-7 Hz
	X_9	power of 8 Hz		X_{22}	power of 8 Hz
	X_{10}	power of 9-10 Hz		X_{23}	power of 9-10 Hz
	X_{11}	power of 11-12 Hz		X_{24}	power of 11-12 Hz
	X_{12}	power of 13-19 Hz		X_{25}	power of 13-19 Hz
	X_{13}	peak frequency		X_{26}	peak frequency
B: Inter-area Variables [LP-LO]					
35		cross-correlation	36		cross-spectrum

1 using amplitude as weight
2 using wave duration as weight
3 D•A: wave duration multiplied by amplitude
4 [1] frequency which shows the highest peak of sum of amplitude
[2] frequency which shows the highest peak of wave numbers
5 band-width: θ[4-8 Hz], α[8-13 Hz] & β_1[13-20 Hz]

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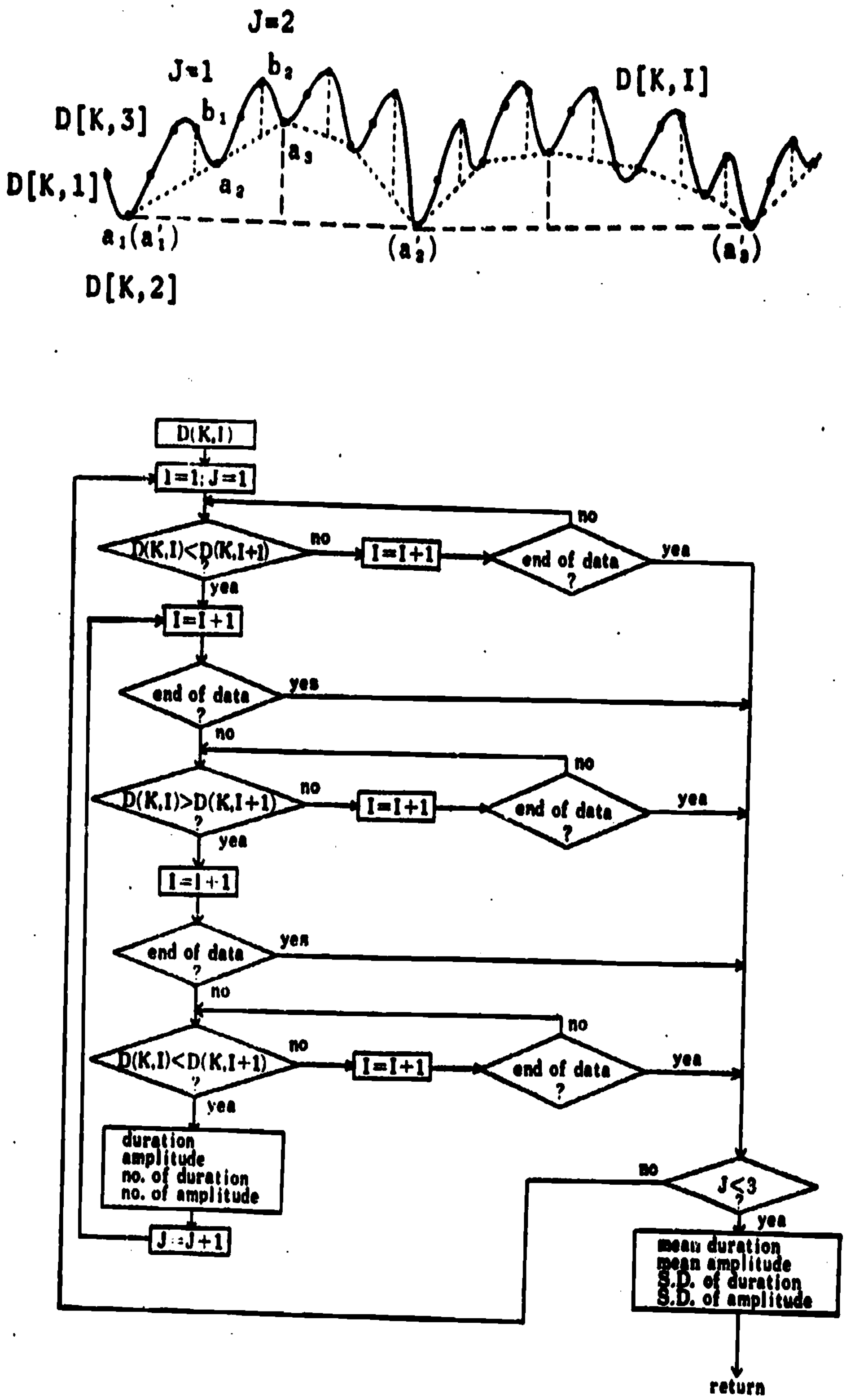

Figure 1. Data processing of waves and its flow-diagram

Table 2.
Results of component analysis applied to EEG samples.

	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6	Z_7
1	-.164	-.265	-.214	.165	.168	-.150	.188
2	-.109	-.333	-.168	-.135	.145	-.219	.101
3	-.065	.213	-.367	-.275	-.176	.104	-.054
4	-.037	.022	-.233	-.438	-.203	-.116	.050
5	.141	.228	-.332	-.205	-.105	.088	-.101
6	-.350	.045	-.022	-.080	-.171	.032	-.030
7	-.123	-.168	-.193	-.352	-.024	-.228	.019
8	.099	.254	-.112	.079	-.063	-.078	.157
9	-.132	.136	.102	.189	-.238	-.277	.441
10	-.278	.134	.113	.051	-.112	.243	.151
11	-.315	.068	.082	.027	-.024	.306	.109
12	-.247	.050	-.007	-.372	.040	.123	.326
13	-.294	.117	.080	-.020	.145	-.189	-.242
14	-.196	-.274	.087	-.157	.166	-.095	-.039
15	-.037	-.392	-.180	.041	-.225	.092	.111
16	-.082	.190	-.389	.269	-.024	-.012	-.206
17	-.001	.044	-.352	.223	.428	-.007	.137
18	.144	.299	-.239	.048	.236	.003	.088
19	-.288	.106	-.088	.072	-.008	-.256	-.376
20	-.093	-.280	-.307	.196	.007	.078	-.030
21	.000	-.188	-.205	.301	-.510	.259	.017
22	-.108	.240	-.002	.187	-.165	-.373	.214
23	-.269	.109	.073	.123	-.117	-.192	.048
24	-.234	.045	.067	.070	.103	.378	-.192
25	-.226	.110	-.102	-.011	.330	.279	.328
26	-.308	.013	-.011	.058	-.003	-.034	-.311
λ_j	5.980	4.335	3.476	2.335	1.292	1.231	1.121
$\lambda_j/26$	.230	.166	.133	.089	.049	.047	.043
$\Sigma\lambda_j/26$		.396	.530	.620	.669	.717	.760

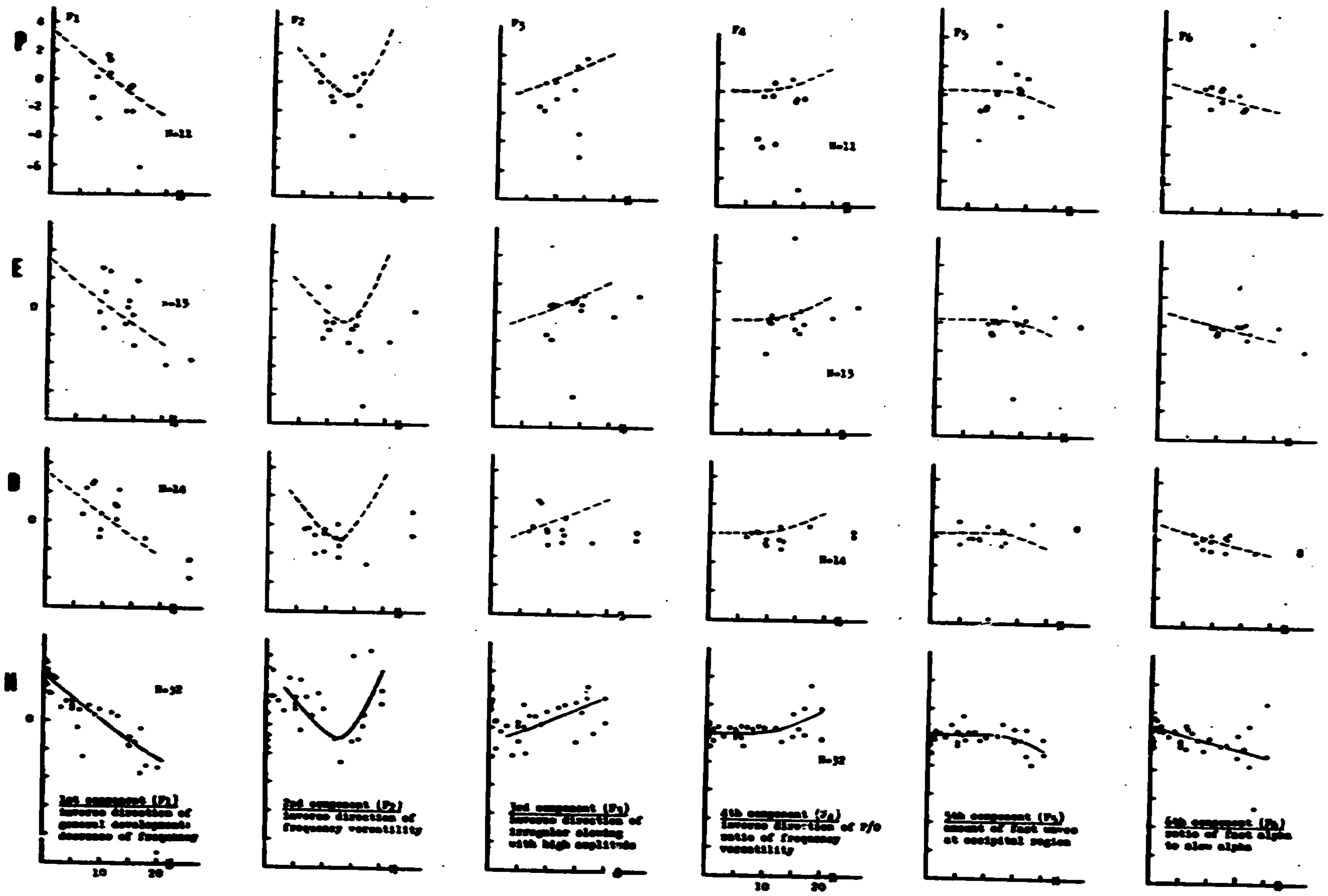

Figure 2. Scatter diagrams representing relations between components and ages.

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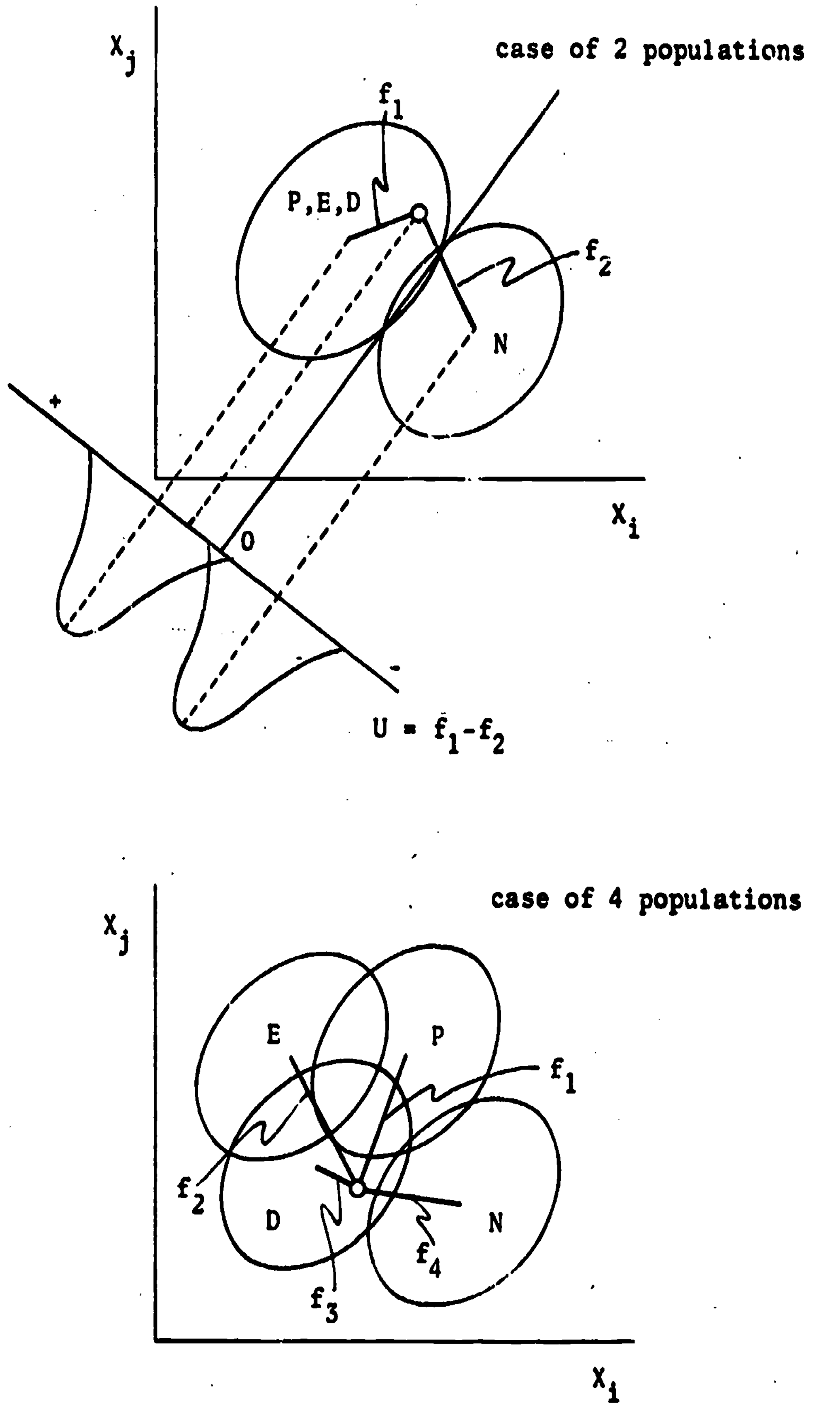
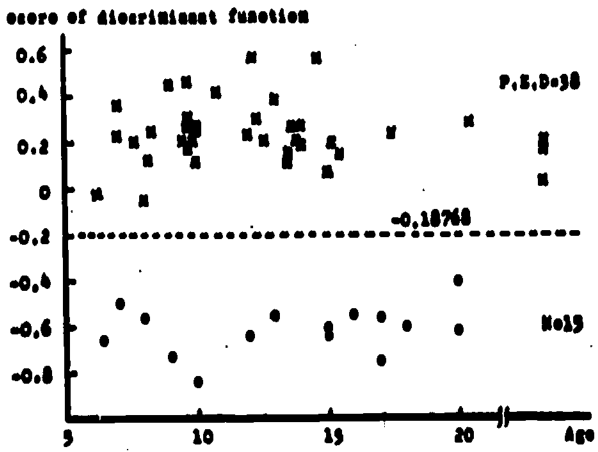


Figure 3. Schematic diagrams of the discriminant analysis

	mean vector μ_1 PED	mean vector μ_2 N_2	weight a in U
1	0.05248	-0.13295	-0.24660
2	0.47658	-1.20733	0.31724
3	0.11252	-0.28506	0.28110
4	0.10907	-0.27633	-0.18618
5	0.10331	-0.26169	-0.17325
6	-0.05692	0.14419	-0.05455
7	0.17885	-0.45310	-0.00081
8	-0.17876	0.45286	-0.06425
9	-0.26807	0.67913	-0.01910
10	-0.21218	0.53752	-0.11437
11	-0.21247	0.53827	-0.11831
12	-0.06747	0.17091	0.15645
13	-0.18181	0.46056	0.08145
14	0.09548	-0.24187	-0.05142
15	0.33489	-0.84836	0.11653
16	-0.02507	0.06351	0.17912
17	0.07079	-0.17935	-0.04060
18	-0.09239	0.23402	-0.04485
19	-0.07713	0.19539	-0.01324
20	0.20712	-0.52471	0.05940
21	0.05433	-0.13765	-0.07203
22	-0.26977	0.68342	-0.04468
23	-0.11415	0.28916	0.00912
24	-0.01476	0.03741	0.10985
25	-0.07969	0.20190	-0.07484
26	-0.13377	0.33889	-0.10360
	constant term of U		-0.18768
	Mahalanobis' distance		44.11321
	prob. of misclassification		0.00000



$$Z = -0.24660\,X_1 + 0.31724\,X_2 + 0.28110\,X_3 - 0.18618\,X_4 - 0.17325\,X_5 - 0.05455\,X_6 - 0.00081\,X_7 - 0.06425\,X_8 - 0.01910\,X_9 - 0.11437\,X_{10} - 0.11831\,X_{11} + 0.15645\,X_{12} + 0.08145\,X_{13} - 0.05142\,X_{14} + 0.11653\,X_{15} + 0.17912\,X_{16} - 0.04060\,X_{17} - 0.04485\,X_{18} - 0.01324\,X_{19} + 0.05940\,X_{20} - 0.07203\,X_{21} - 0.04468\,X_{22} + 0.00912\,X_{23} + 0.10985\,X_{24} - 0.07484\,X_{25} - 0.10360\,X_{26}$$

Figure 4. Results of discriminant analysis in assignment into two populations

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Table 3.
Coefficients of four linear functions of the variates f_i when a sample will be allotted to one of four populations.

	f_1	f_2	f_3	f_4
1	-3.21223	-1.70721	-4.73777	8.25725
2	5.51442	1.50793	5.75684	-10.72541
3	3.15795	3.56681	4.56734	-9.66987
4	1.87781	-5.30259	-3.32030	6.31740
5	-1.88480	-0.36555	-4.13029	5.55404
6	-1.16677	-1.09835	-0.27543	2.06444
7	-1.34622	-0.55439	1.15727	0.38750
8	-1.08231	-2.64588	0.44676	2.66974
9	0.48790	-0.28345	-0.69710	0.53852
10	-0.31535	-2.60035	-1.62966	4.00583
11	-1.34608	-1.40560	-1.97901	4.05245
12	0.69760	3.79400	1.90643	-5.57898
13	-0.32422	2.90533	0.74993	-2.98014
14	1.45954	-2.00025	-1.09966	1.68953
15	1.32677	8.80853	-3.23178	-5.59030
16	0.20775	1.60024	4.44757	-5.68913
17	1.35211	-1.14910	-1.28686	1.20499
18	2.31651	-0.27492	-2.60211	0.96760
19	1.86774	2.47589	-3.25302	-0.47992
20	-0.15522	0.44112	1.66944	-1.82678
21	-0.08244	-3.60310	0.27595	2.92482
22	-1.46977	0.65849	-1.00890	1.44881
23	-0.82224	1.83958	-0.50786	-0.51727
24	3.50552	1.92631	0.07033	-4.30573
25	-1.09734	-2.54092	0.04611	2.96390
26	0.45734	-4.19572	-0.63146	3.39018
C[1]	-5.22886	-3.81538	-2.53466	-11.07762

[1] The second term of function $f_i = X'\Sigma^{-1}\mu_i - \frac{1}{2}\mu_i'\Sigma^{-1}\mu_i$

Table 4.
Probabilities of classification when samples will be allotted to each one of four populations.

		P_1	P_2	P_3	P_4
P	1	0.99993	0.00001	0.00007	0.00000
	2	0.99939	0.00042	0.00019	0.00000
	3	1.00000	0.00000	0.00000	0.00000
	4	0.99824	0.00025	0.00151	0.00000
	5	0.99918	0.00023	0.00059	0.00000
	6	0.97467	0.00082	0.02451	0.00000
	7 →E[1]	0.00876	0.61791	0.37250	0.00083
	8	1.00000	0.00000	0.00000	0.00000
	9	0.80300	0.01264	0.18436	0.00000
	10	0.83053	0.01064	0.15883	0.00000
	11	0.99989	0.00002	0.00011	0.00000
E	1	0.00001	0.99966	0.00034	0.00000
	2	0.00115	0.96988	0.02897	0.00000
	3	0.00065	0.99599	0.00336	0.00000
	4	0.16471	0.82509	0.01020	0.00000
	5	0.00008	0.99881	0.00112	0.00000
	6	0.37708	0.41097	0.21195	0.00000
	7	0.00014	0.79351	0.20636	0.00000
	8	0.00004	0.93902	0.06094	0.00000
	9	0.00007	0.96553	0.03440	0.00000
	10	0.00001	0.97994	0.02005	0.00000
	11	0.00006	0.99872	0.00121	0.00000
	12	0.00005	0.93809	0.06186	0.00000
	13	0.00090	0.99364	0.00546	0.00000
D	1	0.29930	0.00114	0.69956	0.00000
	2	0.00526	0.00026	0.99448	0.00000
	3	0.00033	0.00056	0.99911	0.00000
	4	0.14899	0.03791	0.81310	0.00000
	5	0.00034	0.02543	0.97423	0.00000
	6	0.00010	0.03347	0.96643	0.00000
	7	0.00019	0.18173	0.81808	0.00000
	8	0.00202	0.00227	0.99571	0.00000
	9	0.00034	0.28947	0.71019	0.00000
	10	0.01906	0.00422	0.97652	0.00021
	11	0.00000	0.00026	0.99974	0.00000
	12	0.00081	0.00575	0.99344	0.00001
	13	0.00023	0.07799	0.92179	0.00000
	14 →E	0.00027	0.94497	0.05476	0.00000

[1] misclassification

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Table 4.
Probabilities of classification when samples will be allotted to each one of four populations.
[continued]

		P_1	P_2	P_3	P_4
N_2	1	0.00000	0.00000	0.00000	1.00000
	2	0.00000	0.00000	0.00000	1.00000
	3	0.00000	0.00000	0.00000	1.00000
	4	0.00000	0.00000	0.00000	1.00000
	5	0.00000	0.00000	0.00000	1.00000
	6	0.00000	0.00000	0.00000	1.00000
	7	0.00000	0.00000	0.00000	1.00000
	8	0.00000	0.00000	0.00000	1.00000
	9	0.00000	0.00000	0.00000	1.00000
	10	0.00000	0.00000	0.00000	1.00000
	11	0.00000	0.00000	0.00000	1.00000
	12	0.00000	0.00000	0.00000	1.00000
	13	0.00000	0.00019	0.00000	0.99981
	14	0.00000	0.00000	0.00000	1.00000
	15	0.00000	0.00000	0.00000	1.00000

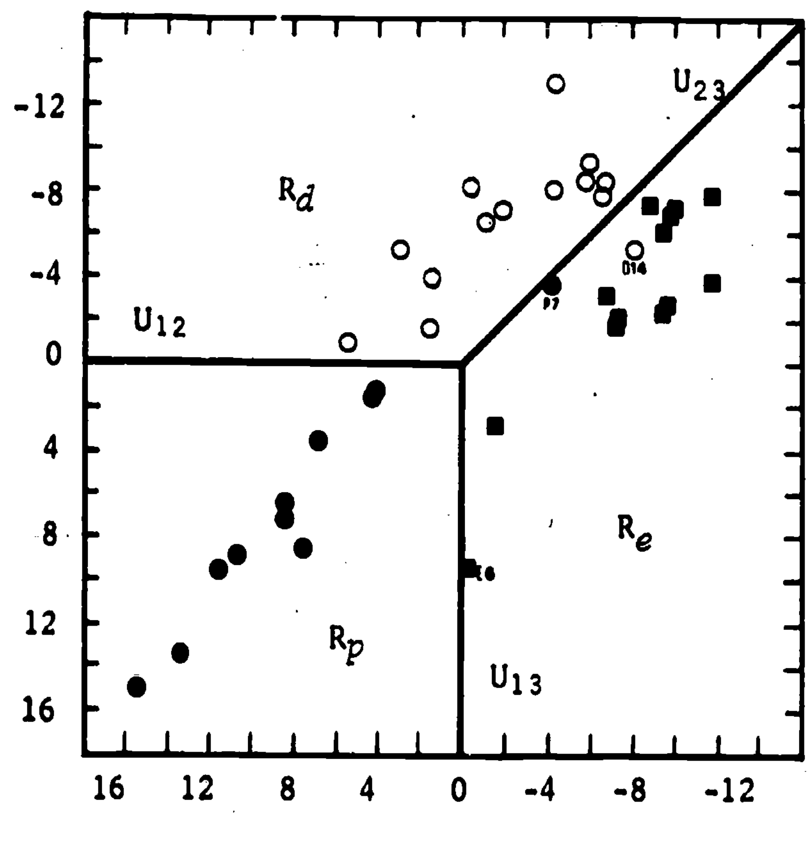

Figure 5. The regions separating the three groups of the mentally retarded

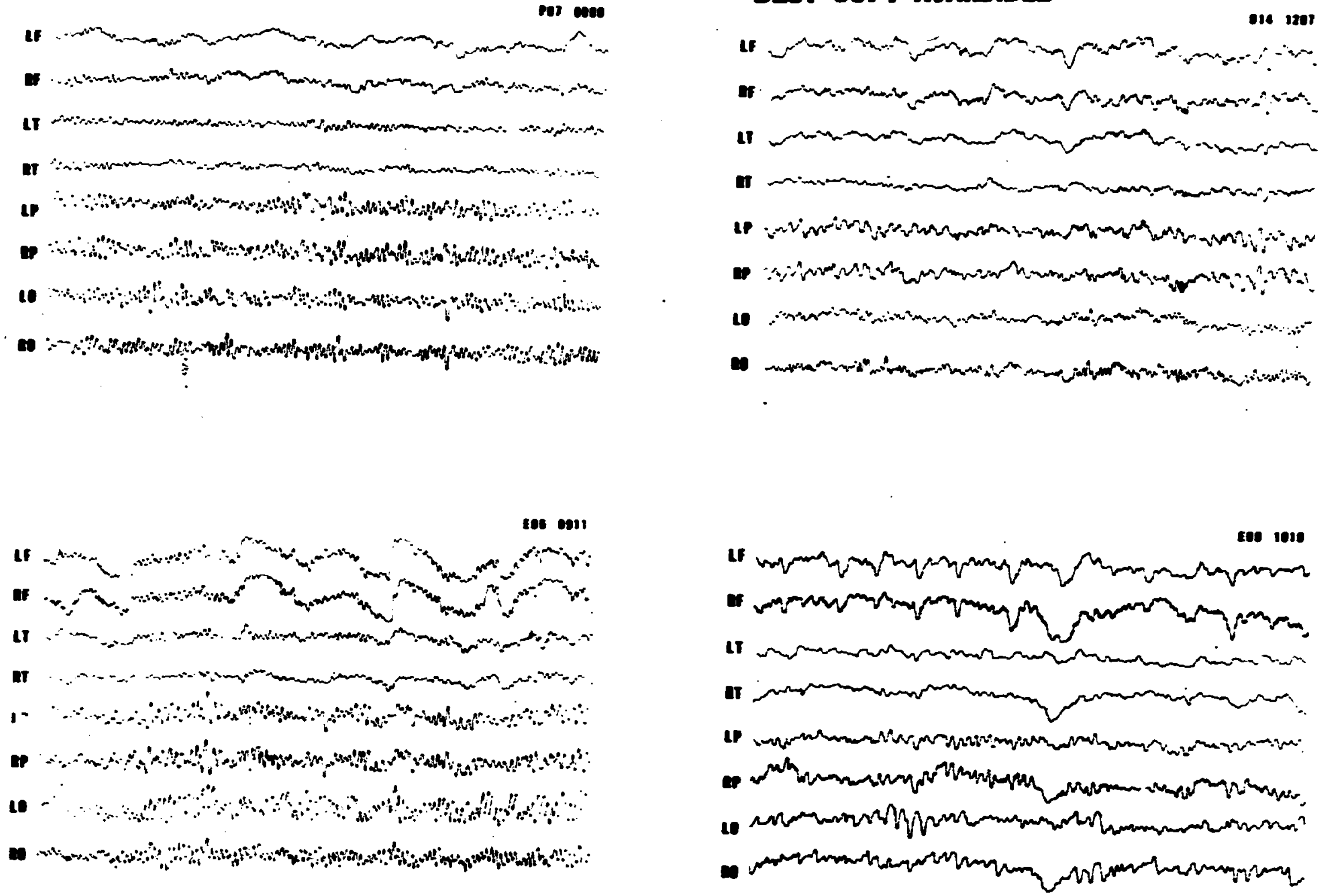

Figure 6. EEG patterns misclassified to Group E [P-7 and D-14] as compared with two E-patterns, one the "typical"[E-8] and the other the "complicated"[E-6]

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C     EEG ANALYSIS ( HISTOGRAM METHOD )
      DIMENSION FNM(10,3),D(16),X(700,2),R(3),AV(2),SD(2),ASP(30,2),FREQ
     1(2),AMP(2),SF(2),SA(2),TH(2),BE(2),
     2N(2),NUM(700),NSP(30,2),NAV(2),NSD(2),NTH(2),NAL(2),NBE(2)
      DO 10 J=1,3
      READ(2,101) (FNM(I,J),I=1,10)
  101 FORMAT(10A8)
   10 CONTINUE
      L=1
      J=1
   16 I1=0
   14 READ(2,102) (D(I),I=1,16)
  102 FORMAT(16F5.0)
      DO 12 I=1,16
      IF(D(I).EQ.0.01) GO TO 13
      I1=I1+1
   12 X(I1,J)=D(I)
      GO TO 14
   13 I2=I+1
      N(J)=I1
      IF(J.EQ.2) GO TO 15
      J=2
      IF(I.EQ.16) GO TO 16
      I1=0
      DO 17 I=I2,16
      I1=I1+1
   17 X(I1,J)=D(I)
      GO TO 14
   15 IF(N(1).LE.N(2)) NS=N(1)
      IF(N(1).GT.N(2)) NS=N(2)
      COV=0.0
      DO 18 J=1,2
      AV(J)=0.0
      SD(J)=0.0
      DO 18 I=1,NS
      AV(J)=AV(J)+X(I,J)
   18 SD(J)=SD(J)+X(I,J)**2
      DO 19 I=1,NS
   19 COV=COV+X(I,1)*X(I,2)
      SN=NS
      R(1)=(COV*SN-AV(1)*AV(2))/SQRT((SD(1)*SN-AV(1)**2)*
     1(SD(2)*SN-AV(2)**2))
      DO 20 J=1,2
      NO=0
      NS=N(J)
      FREQ(J)=0.0
      AMP(J)=0.0
      SF(J)=0.0
      SA(J)=0.0
      NTH(J)=0
      NAL(J)=0
      NBE(J)=0
      TH(J)=0.0
      AL(J)=0.0
```

```
      BE(J)=0.0
      DO 21 I=1,30
      ASP(I,J)=0.0
   21 NSP(I,J)=0
      DO 22 I=1,700
   22 NUM(I)=I
   32 I=1
      K=1
   25 I1=I+1
      IF(X(I,J).LT.X(I1,J)) GO TO 23
      I=I+1
      IF(I.EQ.NS) GO TO 24
      GO TO 25
   23 X(K,J)=X(I,J)
      XMIN1=X(I,J)
      KS=NUM(I)
   27 I=I+1
      IF(I.EQ.NS) GO TO 24
      I1=I+1
      IF(X(I,J).GT.X(I1,J)) GO TO 26
      GO TO 27
   26 XMAX=X(I,J)
      K1=NUM(I)
   29 I=I+1
      IF(I.EQ.NS) GO TO 24
      I1=I+1
      IF(X(I,J).LT.X(I1,J)) GO TO 28
      GO TO 29
   28 XMIN2=X(I,J)
      K2=NUM(I)
      G1=FLOAT(K1-KS)
      G2=FLOAT(K2-KS)
      A=XMAX-XMIN1-G1*(XMIN2-XMIN1)/G2
      G1=60.0/G2
      MF=G1
      FREQ(J)=FREQ(J)+G1
      AMP(J)=AMP(J)+A
      SF(J)=SF(J)+G1**2
      SA(J)=SA(J)+A**2
      NO=NO+1
      IF(MF.LT.1) GO TO 30
      IF(MF.GT.30) GO TO 30
      ASP(MF,J)=ASP(MF,J)+A
      NSP(MF,J)=NSP(MF,J)+1
   30 NUM(K)=KS
      K=K+1
      NUM(K)=K2
      XMIN1=XMIN2
      X(K,J)=XMIN2
      KS=K2
      GO TO 27
   24 IF(K.LT.3) GO TO 31
      NS=K
      GO TO 32
   31 BN=NO
      FREQ(J)=FREQ(J)/BN
```

```
      AMP(J)=AMP(J)/BN
      SF(J)=SF(J)/BN-FREQ(J)**2
      SA(J)=SA(J)/BN-AMP(J)**2
      DO 33 I=4,7
      TH(J)=TH(J)+ASP(I,J)
   33 NTH(J)=NTH(J)+NSP(I,J)
      DO 34 I=8,12
      AL(J)=AL(J)+ASP(I,J)
   34 NAL(J)=NAL(J)+NSP(I,J)
      DO 35 I=13,19
      BE(J)=BE(J)+ASP(I,J)
   35 NBE(J)=NBE(J)+NSP(I,J)
   20 CONTINUE
      DO 36 J=1,2
      AV(J)=0.0
      SD(J)=0.0
      NAV(J)=0
      NSD(J)=0
      DO 36 I=1,30
      AV(J)=AV(J)+ASP(I,J)
      SD(J)=SD(J)+ASP(I,J)**2
      NAV(J)=NAV(J)+NSP(I,J)
   36 NSD(J)=NSD(J)+NSP(I,J)**2
      COV=0.0
      NO=0
      DO 37 I=1,30
      COV=COV+ASP(I,1)*ASP(I,2)
   37 NO=NO+NSP(I,1)*NSP(I,2)
      R(2)=(COV*30.0-AV(1)*AV(2))/SQRT((SD(1)*30.0-AV(1)**2)
     1*(SD(2)*30.0-AV(2)**2))
      COV=NO
      DO 9 I=1,2
      AV(I)=NAV(I)
    9 SD(I)=NSD(I)
      R(3)=(COV*30.0-AV(1)*AV(2))/SQRT((SD(1)*30.0-AV(1)**2)
     1*(SD(2)*30.0-AV(2)**2))
      WRITE(3,300) (FNM(I,L),I=1,10)
  300 FORMAT(1H7,10A8/1H2,2X,10HMEAN FREQ.,10X,2HSD,3X,9HMEAN AMP.,
     110X,2HSD,2(7X,5HTHETA,7X,5HALPHA,8X,4HBETA))
      DO 38 J=1,2
      WRITE(3,301) FREQ(J),SF(J),AMP(J),SA(J),TH(J),
     1AL(J),BE(J),NTH(J),NAL(J),NBE(J)
  301 FORMAT(1H2,7F12.4,3I12)
   38 CONTINUE
      WRITE(3,302) (R(I),I=1,3)
  302 FORMAT(1H3,1X,11HCORRELATION,3F12.4)
      IF(L.EQ.3) GO TO 11
      L=L+1
      J=1
      IF(I2.EQ.17) GO TO 16
      I1=0
      DO 39 I=I2,16
      I1=I1+1
   39 X(I1,J)=D(I)
      GO TO 14
   11 STOP
      END
```