DOCUMENT RESUME

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ABSTRACT

Using this guide and the four popular books noted in it, a student, working independently, will learn about some of the classical ideas and problems of topology: the Moebius strip and Klein bottle, the four color problem, genus of a surface, networks, Euler's formula, and the Jordan Curve Theorem. The unit culminates in a project of the students' choosing; recommended projects are designed to be shared with others and could be placed in a mathematics laboratory. Worksheets, answers to exercises, and a review test are provided. (SD)

INDEPENDENT STUDY PROJECT

TOPIC: TOPOLOGY

Notre Dame High School
Easton, Pa. 18042

1974

TO THE STUDENT:

Using a finite set of books
(see listing below)
as directed
over a finite period of time -
you will emerge
with finite understanding
of the branch of mathematics known as

TOPOLOGY.

Beyond this, you will possess as immeasurable supply of intuition and curiosity which will hopefully move you beyond the confines of this Independent Study Project.

The next page of this booklet provides a flowchart to guide you through the project. Answers to all exercises can be found on the last three pages of the booklet.

Special emphasis is placed on the Response Project (see p. 17). As the name implies, the Response Project offers you an opportunity to crystallize the basic ideas of topology and then to share these ideas in some creative way. It is highly possible that a major percentage of the time spent on the entire booklet would be used in preparing your Response.

In order to begin this ISP you will need some of the books listed below. So, right now, go to your research center, or math lab, or library and get a copy of each of the books marked below with an asterisk.

Then begin. Good luck
and enjoy your journey through the world of distortion!

Barr, Stephen. Experiments in Topology. Thomas Y. Crowell Company, 1964.

* Bergamini, David. Mathematics Life Science Library. Time, Inc. 1963.

Clark, Frank. Cortemporary Math. Franklin Watts, Inc. 1964.

* Fadiman, Clifton, ed. Fantasia Mathematica. Simon and Schuster, 1958.

* Glenn, William H. and Johnson, Donovan. Exploring Math on Your Own. Doubleday and Company, 1960.

Kasner, Edward and Newman, James. Mathematics and the Imagination. "Rubber Sheet Geometry" Simon and Schuster, 1940.

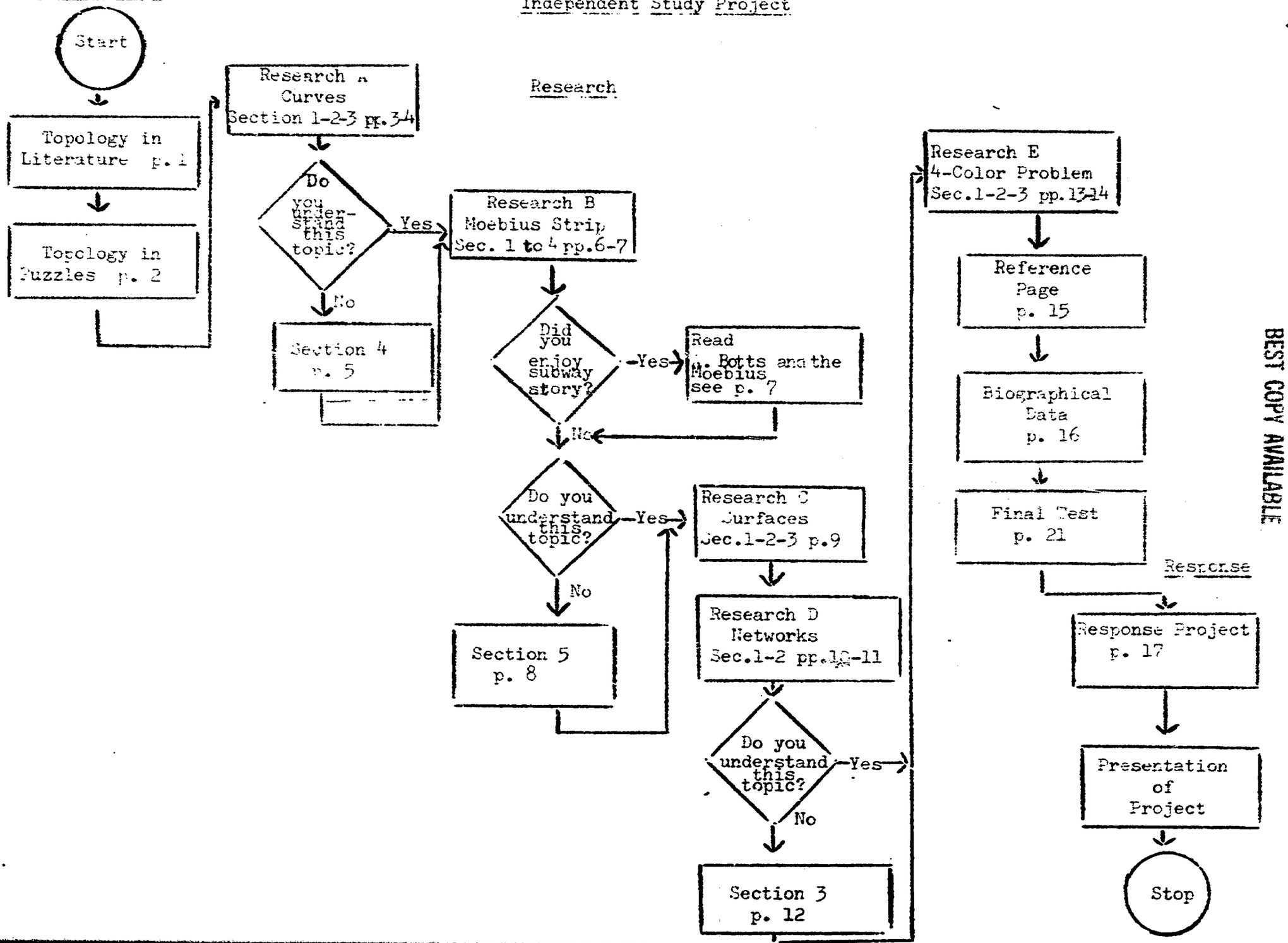
* Jacobs, Harold R. Mathematics A Human Endeavor. W.H. Freeman and Company, 1970.

Reid, Constance. A Long Way from Euclid. "Where Is In and Where Is Out?" Thomas Y. Crowell Company, 1963.

Introduction

Independent Study Project

Research



INTRODUCTION PART A TOPOLOGY IN LITERATURE

Read "A Subway Named Moebius" A.J. Deutsch. Fantasia Mathematica Clifton Fadiman, ed.

Write the first ten words that come to your mind after reading this selection.

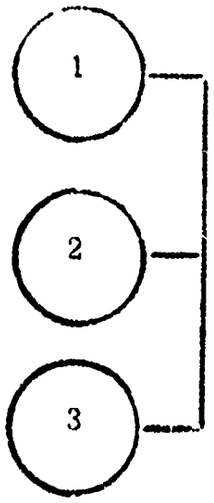
Would you classify this as science fiction or nonfiction. Discuss.

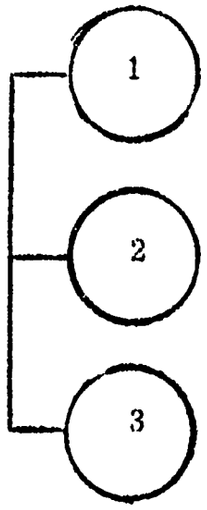
INTRODUCTION PART B TOPOLOGY AND PUZZLES

Solve the following puzzle. Adher strictly to the time limit.

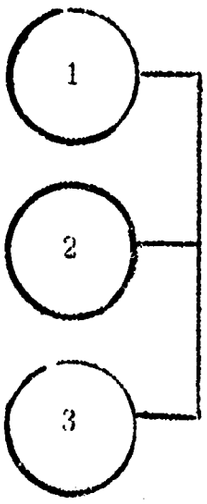
Connect like numbers by lines that do not cross each other and do not cross any given lines.

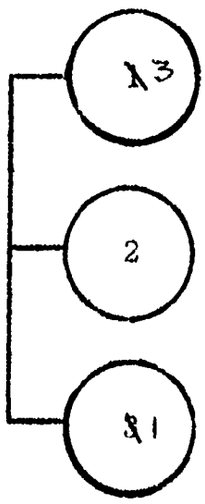
I. time limit: 3 seconds.





II. time limit: 3 hours





RESEARCH PART A CURVES

1. Mathematics A Human Endeavor. Chapter 10 Lesson 1 pp.448-456

read pp.448-449

Do Exercises Set I, II, III. Record answers here. Check your responses with correct answers found at the end of this booklet.

Set I

1.

2. 3. 4.

5. 6. 7. 8.

9. 10.

11. 12. 13.

Set II

1.

2. 3. 4.

5. 6. a) _____ b) _____ c) _____

7.

8.

9. 10.

Set III

Follow directions as given in text

2. Mathematics Life Science Library "Topology: The Mathematics of Distortion."

Read pp. 186-187

3. In view of what you learned in assignments 1 and 2 of this section, explain why you could not do the second part of the puzzle on page 2 of this booklet.

Before proceeding:

1) Check reference page: Have you recorded as many entries as possible?

2) If you feel at ease with the material presented so far, proceed to next topic found on page 6. If you feel you need further explanation of this topic, proceed to assignment 4 on the next page.

4. Exploring Math on your Own Part V, "Strange World of Topology"

Read pp. 198-205

Do Exercise Set 1 - p.205. Record answers here. Check solutions with correct answers at the end of this booklet.

Set I

1.

2.

3.

RESEARCH PART B MÖEBIUS STRIP AND KLEIN BOTTLE

1. Mathematics A Human Endeavor. Chapter 10 Lesson 5 pp. 478-484.

Read pp. 478-479

Do Exercises Sets I, II, III. Record all observation here. Check your results with the correct solutions at the end of this project.

Set 1

1.

2.

3.

4.

5.

6.

7.

Set II

1.

2.

3.

4.

5. 6.

7.

8.

9.

10.

11.

12.

Set III

1.

2.

2. "Paul Bunyan versus the Conveyor Belt" William Hazlett Upson
Exploring Math on Your Own.

Read this short story on pp. 208-209.

3. Mathematics Life Science Library "A Twisted World of One-sided Surfaces".

Read pp. 182-183

4. Take your watch and slide it along a Moebius strip.

Time on watch before you slide it _____

Time on watch after you slide it _____

Before proceeding:

1) Check reference page: Have you made as many new entries as possible?

2) If you enjoyed the story about the subway, you might also enjoy reading "A. Botts and the Moebius." It can be found on page 155 of Fantasia Mathematica.

3) If you feel pretty sure of the behavior of the Moebius strip, go on to the next page, found on page 9. If not, you will find further explanation in the assignment on the next page.

5. Exploring Math on your Own. "Fun with the Moebius Strip"

Read pp. 206-207

Complete the following table:

Number of half-twists	Number of sides and edges	Kind of cut	Result *
0		center	
1		center	
1		1/3	
2		center	
2		1/3	
3		center	
3		1/3	

*number of sides and edges, length and width, number of loops, twists, and knots

Check your solution with the correct answers at the end of this booklet.

RESEARCH PART 3 TOPOLOGICAL SURFACES

1. Mathematics Life Science Library "A Topsy-Turvy World of Topological Changes"

   Read pp. 178-181

2. Exploring Math on your Own "Classifying Topological Figures"

   Read pp. 219-221

   Do question 1 from exercise set 7 (p.221)

   a) _____ b) _____

   c) _____ d) _____

3. Answer the following:

   1) How can you determine the genus of an object?

   2) Give three examples of common objects to genus 0:

   Give three examples of common objects of genus 1:

   Give three examples of common objects of genus 2:

Compare your answers to question 1 with the picture at the bottom of p. 178 in Life Science Library.

Compare your examples in question 2 with the examples on p. 181 of Life Science Library.

RESEARCH PART D NETWORKS

1. Exploring Math on your Own. "Topology Solves Some Interesting Problems"

Read pp. 210-211

Do exercise set 3. Use the following table for your answers:

Network	Even verticles	Odd verticles	Can it be traveled?
1			
2			
3			
4			
5			
6			
7			
8			
9			
10			
11			
12			

Read pp. 213-215

Do exercise set 5. Use the following table for for answers:

Network	Vertices	Arcs	Regions
1			
2			
3			
4			
5			
6			

Read top of page 217

State Euler's formula _____

2. Mathematics Life Science Library "The Ancient Bridges That Led to Modern Network Theory".

Read pp. 188-189

Before proceeding:

1) Check reference page: Is it up to date as far as possible?

2) If you are satisfied with your understanding of networks and Euler's formula, then proceed to the final research topic found on page 13. If you feel you want another explanation of network theory, go on to the next page.

3. Mathematics A Human Endeavor Lesson 2

Read pp. 457-459

Do exercises Sets I, II, III

Set I (tracing paper included at the back of the booklet)

Set II (enter your answers here)

1. network 2 _____ _____ _____ _____

network 3 _____ _____ _____ _____

network 4 _____ _____ _____ _____

network 5 _____ _____ _____ _____

network 6 _____ _____ _____ _____

network 7 _____ _____ _____ _____

network 8 _____ _____ _____ _____

network 9 _____ _____ _____ _____

network 10 _____ _____ _____ _____

network 11 _____ _____ _____ _____

network 12 _____ _____ _____ _____

network 13 _____ _____ _____ _____

2. 3. 4.

5. 6. 7.

8. 9.

10.

Set III

Write the results of your games of "Sprouts".

RESEARCH PART E FOUR COLOR PROBLEM

1. Mathematics Life Science Library "Map Coloring: The Topologists' Enigma"

   Read pp. 184-185

2. Exploring Math on Your Own "The Four-Color Map Problem"

   Read pp. 217-218

   Do exercise set 6. #2-4. Write your answers here and then check with correct solutions at the end of the booklet.

   2.

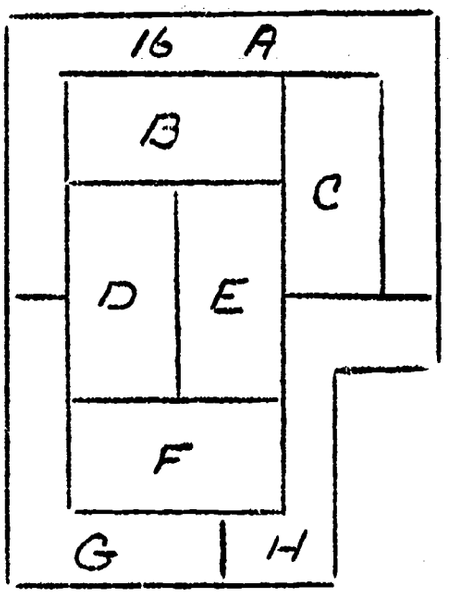
   3.

   4.

No. of lines	No. of regions
1	
2	
3	
4	
5	

3. Solve this puzzle:

You are required to color the map below. Each region has an area of 8 square inches except the top one, which has 16. You have the following colors: exactly enough red for 24 square inches, enough yellow for 24 square inches, enough green for 16 square inches, and enough blue for 8 square inches. The rules are: no touching areas may be colored the same. Watch out for unicorns!!
(answer at the end of the booklet)



REFERENCE PAGE

Briefly identify each of the following:

1) topology

2) topological equivalence

3) Klein bottle

4) Moebius strip

5) Jordan Curve Theorem

6) four color problem

7) network

8) Euler formula

9) genus

10) torus

11) simple closed surface

BIOGRAPHICAL DATA

Give a brief biographical sketch of the following contributors to the mathematics of topology:

Euler

Klein

Moebius

Jordan

N.B. It might be necessary to use references beyond the ones mentioned in this project. For instance, a History of Mathematics books might be helpful.

RESPONSE PROJECT

Choose any one of the following projects. Discuss with your teacher the mode of presentation - e.g. a seminar presentation to your class, an actual lesson presentation, an audio-visual presentation...

1) Prepare a demonstration of topological puzzles. See Exploring Math on Your Own. pp. 225-230.

2) Using ideas from topology write a fantasy or a science fiction short story. Tape this story for use on a cassette player.

3) Prepare a classroom lesson of the genus of everyday objects. See Life Science Library, p. 181.

4) Make a filmstrip and an audio tape to accompany it on any one or two ideas of topology.

5) Define your own project. Consult your teacher for approval.

ANSWERS TO EXCERISES

booklet p. 3

Set I 1. Because it can be twisted and stretched into the same shape.
2. L 3. J 4. E and I 5. F 6. N 7. O
8. Yes, M 9. C, J, L, M, N, S, U, V, W, Z 10. D
11. F, T, Y 12. X 13. R

Set II 1. No. It is not a simple closed curve because it crosses itself.
2. Yes. This is a simple closed curve. 3. Outside, even
4. Inside, odd 5. Outside -- inside 6. a) inside
b) outside c) inside 9. Yes, Y 10. No

booklet p. 5

Set I 1. a, c, 2. a, b, 3. a-2, b-3, c-0, d-5

booklet p. 3 & p. 7

Set I 1. When the line has been drawn once around the band, it appears on the "other" side.
2. The result is a twisted band twisted band twice as long and half as wide.
3. One is twice as long as the other.
4. Their widths are the same.
5. Two interlocking loops are produced, one of which is twice as long as the other.
6. The shorter loop is twice as wide.
7. Two interlocking loops, one twice as long as the other. The shorter loop is 3 times as wide.

Set II 1. 2 sides 2. interlocking loops of the same length and width.
3. Two interlocking loops of the same length. One is twice as wide as the other.
4. Two interlocking loops of the same length. One would be three times as wide as the other. 5. one side.
6. A single loop with a know in it is produced.
7. Two interlocking loops of the same width. One is twice as long as the other. The longer loop has a know in it and the shorter loop passes through the knot..
8. The results is the same as the previous one, except that the shorter loop is twice as wide. 9. two sides.
10. A band with an odd number of half-twists has 2 sides.
11. Two interlocking loops of the same length and width.
12. Two interlocking loops of the same length and width.

Set III 1. A chain of 3 equal interlocking loops.
2. One long loop.

booklet p. 7 The time after you slide your watch along a Moebius strip should be the "Upside down and backwards" time of what it was before you slide it.

ANSWERS TO EXCERISES

booklet p. 8

Number of half-twists	Number of sides and edges	Results
0	2	2 separate loops
1	1	1 loop, 2 twists
1	1	2 loops interlocked
2	2	2 loops interlocked
2	2	2 loops interlocked
3	1	1 loop, 1 knot
3	1	2 loops interlocked, 1 knot

booklet p. 9

exercise set 7 1. a-2, b-3, c-3, d-0

booklet p. 10

exercise set 3

Figure	Even vertices	Odd vertices	traveled
1	2	2	yes
2	0	6	no
3	4	0	yes
4	1	4	no
5	101	0	yes
6	9	4	no
7	4	8	no
8	10	0	yes
9	2	2	yes
10	8	0	yes
11	2	6	no
12	14	2	yes

exercise set 5

	V	A	R
1	2	1	1
2	2	2	2
3	2	3	3
4	4	6	4
5	5	7	5
6	4	8	5

Euler's formula: V - A + R = 2

ANSWERS TO EXERCISE

booklet p. 12

exercise set I

To draw network 3 it is necessary to start at either A or B; network 4 are either E or F; and network 7 at either C or F.

exercise set II

1.	network 2	4	0	4	no
	3	6	4	2	yes
	4	6	4	2	yes
	5	10	10	0	yes
	6	8	0	8	no
	7	7	5	2	yes
	8	8	4	4	no
	9	7	1	6	no
	10	3	1	2	yes
	11	10	6	4	no
	12	9	9	0	yes
	13	16	16	0	yes

2. no - network 11 has more even corners than odd corners and cannot be traveled.
3. no - network 10 has more odd corners than even corners and can be traveled.
4. yes 5. yes 6. no 7. no
8. the simplest case is a triangle.
9. such a network must have exactly two odd corners.
10. impossible

exercise set III

"Sprouts" A game starting with 2 points cannot last longer than 5 moves. The limits on games with 3 and 4 points are 8 and 11 moves respectively. In general, a game starting with no points must end in 3 - 1 moves or less.

booklet p. 13

exercise set 6

4. No. of regions: 2, 4, 7, 11, 16 Differences: 2, 3, 4, 5,
   With 6 lines: 22 regions.

booklet p. 14

Puzzle: If you really were mindful of the unicorns, you would have thought to mix 1/3 of the red with all of the blue getting enough purple for 16 square feet. Colors, then, by region are: A and F: yellow; B and H: purple; C and D: red; E and G: Green.

TOPOLOGY REVIEW TEST

Refer to Exploring Math on Your Own pp. 231-232. Answer each of the test questions in the space below. The correct answers can be found at the bottom of this page.

1. a)

   b)

   c)

   d)

   e)

2.

3.

4.

5.

6.

7. a)

   b)

   c)

   d)

   e)

ANSWERS:

1. a) figure c b) figure b c) figure a d) figure a e) figure b
2. a, b, c 3. a, b, c 4. one 5. and odd vertex
6. two 7. V - A + R = 2 a) 4 - 6 + 4 = 2
b) 8 - 12 + 6 = 2 c) 9 - 16 + 9 = 2
d) 12 - 18 + 8 = 2 e) 12 - 12 + 2 = 2