DOCUMENT RESUME

ED 100 103	95	EC 070 973

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TITLE	Teaching Gifted Children Mathematics in Grades Four Through Six.
INSTITUTION	California State Dept. of Education, Sacramento. Div. of Special Education.
SPONS AGENCY	Bureau of Elementary and Secondary Education (DHEW/OE), Washington, D.C.
PUB DATE	74
NOTE	44p.; For additional information see ED 082 415
EDRS PRICE	MF-$0.75 HC-$1.85 PLUS POSTAGE
DESCRIPTORS	*Class Activities; Concept Formation; Creative Development; *Curriculum Design; Exceptional Child Education; *Gifted; Instructional Materials; Intermediate Grades; *Mathematics; Resource Teachers; Sequential Learning; *Teaching Guides; Teaching Methods
IDENTIFIERS	Elementary Secondary Education Act Title V; ESEA Title V

ABSTRACT

Intended for teachers of gifted students in grades 4-6, the guide emphasizes the need for specialized instruction in mathematics, suggests methods for teaching mathematical facts and concepts, describes approaches and materials to develop students' understanding of mathematical principles, and explores ways to build skills and creativity. Stressed is the resource role of the mathematics specialist in diagnosing individual student needs and in planning a program to build sequential understandings and skills. Listed are mathematical facts and concepts (for sets and subsets, numbers and numeration, operations, mathematical sentences, measurement, graphs, and geometric figures) followed by suggested teaching activities such as using graph paper to diagram multiplication facts and using both a yardstick and a meter stick to measure student height. Suggested are games and experiences to help children discover and test mathematical generalizations. Recommended instructional approaches include using magic squares to develop computational skills, adapting the seminar teaching/learning style to encourage higher intellectual skills, and the discovery of alternate problem-solving methods to develop creativity. Noted is the relationship of mathematics to other subjects such as science, geography, and music and the need for coordination between mathematics specialists and teachers of gifted children at the elementary and junior high school levels. (LH)

Teaching Gifted Children Mathematics in Grades Four Through Six

Prepared for the

Special Education Support Unit
California State Department of Education

by

Juliana T. Gensley

This publication, which was funded under provisions of Title V of the Elementary and Secondary Education Act, was edited and prepared for photo-offset production by the Bureau of Publications, California State Department of Education, and was published by the Department, 721 Capitol Mall, Sacramento, CA 95814.

Printed by the Office of State Printing
and distributed under the provisions
of the Library Distribution Act

1974

FOREWORD

Today's elementary pupils are far more sophisticated in the field of mathematics than were their predecessors. And this is particularly true for gifted children—those taught by the educators to whom this publication is directed.

Teaching Gifted Children Mathematics in Grades Four Through Six emphasizes the need for the specialized instruction that will help the gifted elementary pupil "discover the beauty, fascination, and recreational possibilities of the spectrum of mathematics," according to the author. Instead of teaching children to memorize without question the generalizations handed down from another generation, the author suggests that the mathematics teacher or specialist encourage them to test generalizations that are commonly accepted. She says, "To emphasize the importance of being right without also emphasizing the importance of the mathematical process is to destroy interest and motivation." The *Mathematics Framework for California Public Schools* also emphasizes this approach, calling for the establishment of a climate that is "pupil-oriented, self-directed, and nonauthoritarian."

The development of higher intellectual abilities and creativity is stressed in *Teaching Gifted Children Mathematics in Grades Four Through Six*. These qualities, as well as the important skills that are measured in test scores on standardized instruments must be included in our educational objectives for gifted pupils

Mathematics permeates almost every area of life and almost every area of the curriculum. With the introduction of the metric system of measurement in our schools and national life, the application of mathematics to everyday life is taking on an even keener significance. The next few years promise to be exciting and challenging ones to study mathematics; this publication will help make them so for gifted children in grades four through six.

Superintendent of Public Instruction

PREFACE

This publication is one of the products of an education project authorized and funded under provisions of the Elementary and Secondary Education Act, Title V. It is intended for use by the teachers of pupils whose mental ability is such that they are classified as mentally gifted. It is also recommended for use by administrators, consultants, and other professional personnel involved in helping gifted children.

Teaching Gifted Children Mathematics in Grades Four Through Six is one of a group of curriculum materials designed for use by teachers of the mentally gifted in grades one through three, four through six, seven through nine, and ten through twelve. These materials were prepared under the direction of Mary N. Meeker, Associate Professor of Education, and James Magary, Associate Professor of Educational Psychology, both of the University of Southern California.

Also developed as part of the education project is a series of curriculum guides for use in the teaching of mentally gifted minors in elementary and secondary schools. The guides, which contain practical suggestions that teachers can use to advantage in particular subject areas, were prepared under the direction of John C. Gowan, Professor of Education, and Joyce Sonntag, Assistant Professor of Education, both of California State University, Northridge.

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Chapter 1

Gifted Children and Mathematics

Gifted children form a diverse group. They have an infinite range of talents and, viewed realistically, a wide range of weaknesses. A child who has been identified as gifted by tests is not necessarily gifted in mathematics. By the same token, some children who have not reached the cutoff score which defines the gifted child in California can become mathematical experts.

The understanding of mathematics develops sequentially. A teacher often discovers that a gifted child in the intermediate grades has missed one step in the sequence and therefore is not only limited in what he can achieve but also performs less well at grade level. Thus the first step in planning a mathematics program for a gifted child is that of diagnosis. Questions should be asked about how far the child has progressed in his understanding of mathematics and whether there are steps he has not grasped which hinder his achievement in this sequentially developed understanding.

Need for Specialized Instruction

A mathematics specialist is an essential resource person for planning individual programs for gifted children since the elementary teacher in the regular classroom is usually limited in mathematics training. When a child asks why there are 180 degrees in the three angles of every triangle, he needs someone who can help him discover the answer. A mathematics specialist is the person who can give such help. However, the regular classroom teacher can help a child realize that mathematics is involved in all parts of his environment. The following are some examples:

History has a time line that is a number line.

Geography has distances, areas, altitudes, populations, time zones, latitude, and longitude.

Physical education involves scorekeeping, batting averages, stopwatches, and dimensions of courts, diamonds, football fields, and circles.

Music is interrelated with mathematics in many ways.

The daily school schedule and many extracurricular activities require the use of mathematics—the collection of milk money and money for Parent-Teacher Association drives, the plotting of graphs to show progress or relationships, and the keeping of attendance records.

The teacher who shows pupils the mathematics of daily living at school and who suggests similar inquiry projects at home is providing an enrichment program that is practical and educational and lays the foundation for the kind of careful observation which is basic to scientific thinking.

Essential to the mathematics program for gifted children are (1) a mathematics specialist in charge of the program; (2) analysis of the needs of the individual child (diagnosis); (3) interdisciplinary use of mathematical principles; and (4) opportunities for mental relaxation through the use of mathematical games.

The mathematics specialist should be able to convey to teachers and children the beauty, fascination, and recreational possibilities of the spectrum of mathematics. The specialist should also be able to remove a problem that affects an appreciation of mathematics on the part of the gifted and other children; that is, anxiety from the timing of seatwork and tests. This obsolete approach is contrary to the method of the mathematician. To emphasize the importance of being right without also emphasizing the importance of the mathematical process is to destroy interest and motivation.

Recent Trends in Mathematics Instruction

In 1959 Dinkel first proposed exploring the possibility of introducing algebra as early as the seventh or eighth grades.[1] Since then limited amounts of algebra and geometry have been introduced as early as kindergarten. The initial task for the teacher of gifted children in grades four through six, therefore, is to discover how much each child in the program already knows.

Each strand of the mathematics program in California is developed in its own sequence in kindergarten and grades one through eight. Therefore, the gifted child comes to the fourth grade with some working knowledge about sets, numbers, operations, mathematical sentences, measurement, and geometry.

In the late 1950s and 1960s, enrichment programs for gifted children introduced many concepts of the new mathematics such as

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[1] Robert L. Dinkel, "Prognosis for Studying Algebra," *The Arithmetic Teacher*, VI (December, 1959), 317-19.

bases other than ten, the mathematical sentence, associative and commutative properties, set theory, and other principles of the new mathematics. Such a background has caused the child in the elementary school today to be far more sophisticated in the field of mathematics than was his predecessor. Enrichment for the gifted child in the intermediate grades can be based upon a foundation from which the new mathematics has been growing for a number of years.

Children's Knowledge of Diagnostic Facts

The teacher should be aware of the knowledge that the children have of diagnostic facts.

Sets

Before progressing to new material, the teacher should be sure that the children in the group can do the following:

Recognize equal sets.
Recognize equivalent sets through one-to-one correspondence.
Recognize nonequivalent sets and use the signs $\neq$, $>$, and $<$ to express their nonequivalence.
Identify disjoint sets.
Make diagrams to show the overlapping relationship of sets.
Make diagrams to show the inclusion relationship of sets.

Numbers

The extent of children's knowledge of number facts should also be explored. The teacher should determine whether or not the children understand the following:

Difference between number, numeral, and systems of enumeration
Use of cardinal and ordinal numbers
Use of more than one set of numerals
Use of more than one set of number names
Use of place-value system for base ten
Use of bases other than ten
Use of system of measures used in the United States as everyday uses of bases other than ten
Foot/yard = base 3
Inch/foot = base 12
Inch/yard = base 36
Day/week = base 7
Month/year = base 12
Cup/quart/gallon = base 4
Ounce/pound = base 16 (avoirdupois)

Pound/ton = base 2,000 (U.S.A.)
Pound/ton = base 2,240 (Great Britain)
Quart/peck/bushel = base 4 (Note that gallon should come between quart and peck to make this base 4.)
Dozen/gross = base 12

The children should also know (1) that base 2 is used in computers; (2) that base 60 is used for angles and for time related to angles (the clock being a protractor); (3) that base 8 is used by stockbrokers; (4) that natural numbers are abstract concepts; and (5) how to use a number line to explain mathematical relationships.

Operations

The teacher should be aware of how far the children have progressed in the use of mathematical operations and how able the children are to do the following:

Define addition as set union.
Define addition on a number line.
Compare sets and express nonequivalence as subtraction.
Define subtraction on a number line.
Remove a subset and express the result as subtraction.
Find the missing addend as a result of subtraction.
Recognize the sign $\cup$ (cup) to express union of sets.
Recognize the sign $\cap$ (cap) to express intersection of sets.
Recognize the many signs for multiplication.
Recognize the many signs for division.

Mathematical Sentences

The teacher needs to find out whether the children can express their mathematical ideas in the form of mathematical sentences and whether they can recognize a true mathematical sentence, a false mathematical sentence, and an open mathematical sentence. The teacher also must determine whether the children can change a false mathematical sentence to a true mathematical sentence and an open mathematical sentence to a true mathematical sentence. It is also important to know whether the children have a complete vocabulary of symbols that enables them to express their ideas as mathematical sentences.

Measurements

The teacher should find out whether the children know that all measurement is based on points of reference to show relationships and that each unit of measure was arbitrarily invented to fill a particular need The children need to know the following:

The United States uses a form of linear measure that in part evolved from body measurements.

Almost all the rest of the world uses the metric system, which is related to a fraction of a meridian line of the earth.

If the children enter a scientific field in high school or college, they will be using the metric system.

The measurements now used in the United States have several bases with no unifying relationship.

By the mid-1980s the metric system will probably be predominant the United States.

The metric system, which is used in almost every part of the world, is coordinated in terms of base ten; this base is used for linear measure, volume, mass (weight), and temperature, and the units are so definitive that they can measure very small or very large things.

Most money systems in the world use base ten, just as the United States does with cents, dimes, and dollars.

The teacher needs to find out which of the following units of measure the children are familiar with: inches, feet, yards, fathoms, miles (land and nautical), millimeters, centimeters, decimeters, meters, kilometers, degrees of latitude, teaspoons, tablespoons, cups, pints, quarts, gallons, cubic centimeters, liters (1,000 cubic centimeters), ounces, pounds, tons, grams, kilograms, metric ton (1,000 kilograms), seconds, minutes, hours, days, years, centuries, and millenia.

It is particularly important that children be familiar with the metric system of measurement. By the fall of 1976, all new state mathematics and science textbooks in California will present most measurement instruction in metric units.

Geometry

Children absorb some geometrical principles through experience whether they have had systematic lessons in geometry or not. The diagnostic task of the teacher, therefore, is to discover whether the gifted have the vocabulary to express their concepts and to what extent each child has recognized geometrical relationships. All of the children will find the words *point, line, plane, solid, space, curve,* and *circle* very familiar. The task of the mathematics teacher is to build upon the concepts that the children know and to help these children discover new mathematical relationships.

When the level of learning reached by each child in the group has been explored, the program can be planned. Flexibility is essential. For this reason this publication is not arranged according to grade levels but rather as a continuum and contains suggestions for activities to encourage the growth of logical thinking.

Traditionally, the arithmetic program for children in the fourth grade has devoted much time to the memorization of multiplication tables. The child who is gifted in mathematics should be devising his own tables, thus developing a sense of the logical relationships of facts that he has discovered. When he sees mathematics as a pattern and understands arithmetical and geometrical progression, he will be prepared to make creative responses in the realm of mathematics.

Chapter 2

Facts and Concepts: Suggested Teaching Activities

This chapter is devoted to a discussion of mathematical facts and concepts. Suggested teaching activities are included for each of the following areas of instruction: sets and subsets, numbers and numeration, operations, mathematical sentences, measurement, and geometry.

Mathematical Facts

The mathematical facts listed under each of the six categories are followed by suggestions for teaching activities.

Sets and Subsets

Tables are the arrangement of facts into sets for easy reference.

Use graph paper to diagram addition facts. Make a simple reference table of the discovered facts.

Use graph paper to diagram multiplication facts. Make a simple reference table of the discovered facts.

Use these tables in reverse as reference for subtraction facts and division facts.

Make a simple reference table of the squares of whole numbers. (A study of the multiplication table will reveal a pattern related to the squares.)

Reverse the table of squares to make a table of square roots.

Become acquainted with a table of square roots of the numbers from 1 to 1,000 and discuss how this table is used by statisticians, engineers, and other scientists.

Make a cube of wood and score it into equal units. Make a simple reference table of cubes.

Reverse the table of cubes to make a table of cube roots.

Use the exponents ^{2} and ^{3} to indicate square and cube. Explore the use and meaning of other exponents.

Help the children read timetables for bus lines, trains, and airlines.

Correlate the social sciences program in geography with mathematics by making tables showing time in California, Japan, and Africa (to be used in the fourth grade) or showing time in New York, Chicago, Denver, Sacramento, Honolulu, and Juneau (to be used in the fifth grade).

Numbers and Numeration

The concept of number has been designated in different civilizations by different systems of numeration and different sets of number names.

Do research to find out about systems of numeration used in ancient civilizations.

Make charts to show the symbols for numbers used by the Hindu-Arabic, Egyptian (hieroglyphic), Mayan, Greek, Roman, and Chinese cultures. Discuss the strengths and weaknesses of each in terms of ease in performing the operations of addition, subtraction, multiplication., and division.

Find out the names of numbers from one to ten in English, French, Spanish, Japanese, German, and any other language that may be spoken by members of the community.

Use the whole width of the chalkboard to write numbers which have meaning in the place-value system even though they may be so large that we do not know all their names. Some gifted children enjoy learning the names billion, trillion, quadrillion, quintillion, sextillion, septillion, octillion, nonillion, decillion, unidecillion, and duodecillion. These prefixes from Latin are also used to define geometrical figures and solids (such as duodecahedron). However, geometry also uses Greek prefixes as well.

The number line extends both to the right and left of zero. It also extends above and below vertically.

Using sea level as zero. have the children read maps to discover altitudes above sea level. Secure a tide book and have the children look up high tides and minus tides.

Natural numbers are the numbers used for counting.

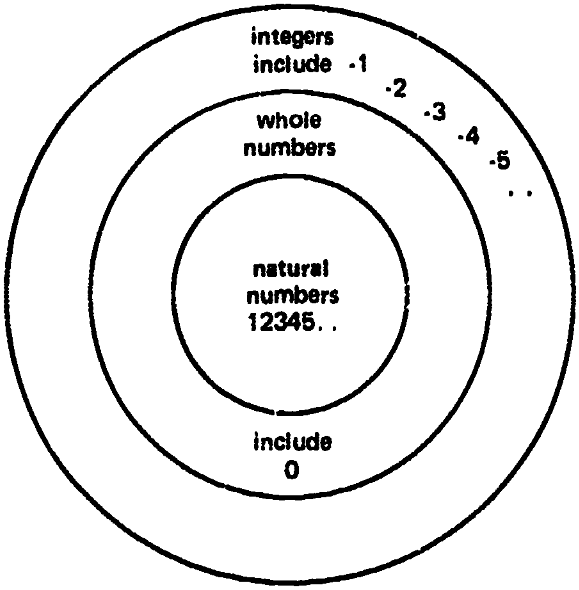
Make a Venn diagram with the natural or counting numbers in the inner subset.

Whole numbers are all the natural numbers and the additional element zero.

In the Venn diagram show that the subset of whole numbers includes all the natural numbers, but that it also includes zero.

Integers include all the natural numbers and their opposites.

Diagram, as shown, the set of integers including both of the subsets described and also the set of negative numbers.



A rational number is any number which can be expressed as the ratio of two integers.

Make a number line. Express rational numbers on the number line.

Use a yardstick for a number line. Show that $\frac{1}{3}$ and $\frac{12}{36}$ express the same ratio. Show other ratios that are equivalent.

The product of a rational number and its reciprocal is always one.

Make a table of rational numbers and their reciprocals. Demonstrate the use of reciprocals in the division of fractions.

An irrational number is any number that cannot be expressed as a ratio of two integers.

Check the table of square roots from 1 to 1,000 to find out which are irrational numbers.

Operations

Addition is the union of sets. (The answer is called the *sum.*)

Two rulers can be used to form a slide rule for addition. Place them so that the end with the numeral 12 on the first ruler is next to the end with the numeral one on the second ruler. Every numeral is now beside the numeral on the other ruler, which will complete the sum of 12. To find the sum of five and three, slide the rulers so that the numeral five on one touches the numeral three on the other. The end of each ruler should touch the numeral eight on the other ruler, and all touching numerals should complete the sum of eight. If the other edge of each ruler has a scale for centimeters, a similar method can be used to demonstrate sums up to 30.

Subtraction can be defined as removing a subset (the answer is called the *remainder*) or comparison (the answer is called the *difference*) or as finding the missing addend (the answer is called the *missing addend*).

Two rulers may be used to form a slide rule for subtraction. Place them so that both ends having the numeral one are at the left. To find the difference between 24 and 19, place the 19 on the centimeter scale of one ruler directly under the 24 on the centimeter scale of the other ruler. The left end of the lower ruler should be directly beneath the five on the upper ruler, and all of the touching numerals should have a difference of five.

Multiplication is also defined as the union of sets. However, it refers to the union of several equivalent sets. (The answer is called the *product.*)

Have the children discover which tables are related to each other. For instance, the products in the 4 table are the same numbers as the products of the 2 table with even numbers, as shown in the following list:

2 X 2 = 4	4 X 1 = 4
2 X 4 = 8	4 X 2 = 8
2 X 6 = 12	4 X 3 = 12

Note that the 2 table, 4 table, 8 table, and 16 table share some products in common. The 3 table and the 6 table are related to each other; the 3 table and the 9 table are also related to each other.

Some numbers do not appear as products in any of the tables. The name for these numbers is *prime* numbers. Have the children try to discover which are prime numbers.

Check products by using a standard slide rule, and then check products by making an array.

Discover the use of Cartesian products to estimate all possible combinations for matching sets.

Use a number line to demonstrate multiplication.

Construct a set of Napier's bones and use them to check products.

Division is the inverse operation of multiplication. (The answer is called the *quotient.*)

Use division to locate prime numbers and factors.

Use division to determine average temperature for the week or average daily attendance for the week.

Create number puzzles, riddles, and games, using division as one operation. For example, think of a number. Double it. Multiply the product by ten. Divide the product by the number you first thought of. The answer is 20.

Check division with a slide rule.

Have the children learn how to read square root on a slide rule.

Mathematical Sentences

Mathematical ideas can be expressed in mathematical sentences. Mathematical sentences may be true or false. Inequalities are indicated by such symbols as the following:

$<$: is less than
$>$: is greater than
$\nless$: is not less than
$\ngtr$: is not greater than
$\neq$: is not equal to

Equality is indicated by =. Equations (equalities) can be expressed for addition, subtraction, multiplication, and division. Associative and commutative properties can be expressed as mathematical sentences.

Have the children give each other pairs of numbers to change into a mathematical sentence by using symbols that indicate inequality or equality. Inequalities may be used for estimating trial quotients.

Measurement

Most of the world uses the metric system for measurement.

Have several meter sticks available for the children to use. Have a scale that indicates both grams and kilograms, as well as ounces and pounds. Have at least one thermometer that has the centigrade scale on one side and the Fahrenheit scale on the other. (The height of the mercury at any point can be read either in degrees Celsius [centigrade] or degrees Fahrenheit.)

Ask the children to check the odometers of cars in which they ride in order to discover the actual distance of a mile. They should then check the odometer to discover $\frac{6}{10}$ of a mile, which is the approximate equivalent of a kilometer.

Obtain a liquid measure which is graduated to show cubic centimeters. Obtain a liter measure.

Measure the height of the children in centimeters. Record their heights both in inches and in centimeters.

Weigh books and other objects and record their weights both in grams and ounces, or in kilograms and pounds. Do not confuse the children by having them use a formula to convert from one system to the other.

Most of the currencies of the world use base ten.

Have the children pretend they are shopping in Tokyo and have 10,000 yen. Have them make a list of items purchased and the cost of each in yen and determine how much change they would have left.

Have the children pretend they are shopping in Mexico City with 100 pesos to spend. Have them make a list of items purchased and determine how many pesos and centavos they would receive in change.

Extend this project to other countries. France and Italy would be good prospects. Since Great Britain has a very complicated monetary system, only a very highly motivated child should attempt to go on a shopping tour in London.

When the children are working on these problems, do not have them try to convert the money into dollars since the rate of exchange varies from day to day. The children may become so interested that they will do some research on the rate of exchange.

Units of measure are established to fill specific needs.

Have the children do research on modern units of measure, such as the following:

Ohms measure electrical resistance
Amperes electrical volume
Volts electrical pressure
Watts volts times amperes
Calories measure heat
Ergs measure energy
Angstroms measure color wavelength

Statistics are used to organize measurement data.

Make a bar graph to show the enrollment in each class in the school.

Make a circle to show the proportion of boys to girls in the classroom.

Graph two variables on squared paper, using the domain of real numbers.

Teach a simple computer language.

Geometry

Geometry is the mathematics of space. The word geometry is derived from two Greek words meaning earth measurement. Much earth measurement is related to the measurement of angles. The various instruments with which angles can be measured include a protractor, a sextant, a pelorus, a compass, a quadrant, an astrolabe, the face and hands of a clock, and a cord knotted at 3 inches, 4 inches, and 5 inches (or another unit may be substituted for inch) to measure a 90-degree angle.

Time is frequently measured by angles, such as the hands of a clock, the angles on a sundial, the angle between the sun and the horizon, the angle between the moon and the horizon, and the angle between a star and the horizon.

Use a *National Geographic* 12-inch globe with the clear plastic "thinking cap" to compute the areas of geographic regions. (The "thinking cap" has a grid with a scale for four million square kilometers, another grid for one million square miles, an embossed meridian scaled to kilometers, and another meridian indicating degrees.) Any distance on the globe can be read in kilometers, miles, or degrees of latitude simply by placing the desired scale on the "thinking cap" over it.

Have the pupils use various instruments to measure angles. They can make a pelorus to measure horizontal angles by fastening two protractors to a cardboard disc to form a 30-degree circle. The telescope can be made from a jumbo plastic straw pivoted on a pin which runs through the diameter of the straw to the center of the circle. The angle between two horizontal objects is measured by aligning one object with the line extending from zero to 180 degrees and then sighting the other object through the "telescope." The degrees between the zero and the line of the "telescope" can be read to find the angle.

Read the story of Eratosthenes and how he measured the circumference of the earth.

Help the pupils create devices for reckoning time by reading the angles made by shadows.

Mathematical Concepts

Mathematical concepts are learned in context while children are working with mathematical facts. As the patterning of mathematics develops, children learn or discover for themselves various mathematical relationships.

In their studies of nongifted children, Inhelder and Piaget documented the development of concepts through initial concrete experiences.[2] They observed the ways in which children of different age levels explained tne phenomena in each experiment. All of the children were given the same experiences, but their concepts varied according to their age levels.

The younger children, to seven or eight years of age, were unable to comprehend conservation of space or volume. That is, their judgments were based exclusively on and bound by their perceptions, and they could neither foresee displacements nor conserve classes and classification. This concrete, perception-bound thinking is the first level in thinking and is considered to be the preoperations level.

Children who were somewhat older and at the second stage were able to internalize their experiences in a manner analogous to abstract thinking. During this stage the facts integrated earlier become the bases for conceptual thinking.

When children reach the third or "formal" stage, they begin to comprehend or discover laws and principles governing the observed phenomena. Children in the Inhelder-Piaget studies were at least twelve years old before they reached the formal stage of thinking.

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[2]Bärbel Inhelder and Jean Piaget. *The Growth of Logical Thinking from Childhood to Adolescence: An Essay on the Construction of Formal Operational Structures.* Translated by A. Parsons and S. Milgram. New York: Basic Books, 1958

In working with gifted children ten, eleven, and twelve years old, the mathematics specialist may find that, regardless of age, some of them will already have developed formal thinking and logic. Others will not have progressed to that stage and will still be developing concepts.

The activities suggested in the section on mathematical facts are not all-inclusive. Each activity, however, is an example of an open-ended learning experience which can be valuable for children who need more experiences at the concrete operations level.

Sets

Activities relating to tables give gifted children the opportunity to explore the relationships and patterns of various sets. As children develop concepts and patterns in mathematics, they discover methods for arranging the patterns in a logical order for easy reference. Planned programs guide children into discovering what laws or principles govern these patterns. Thus the concept of sets as a collection, which the children developed in the primary grades, can be extrapolated to organization of set patterns.

Various approaches to mathematical understanding are needed for the particular interests and orientation of gifted children. The gifted child whose major interest is mathematics finds it much easier to remember number facts when he understands the logical relationship of those facts. The gifted child who prefers memorizing rules and following them may not understand the "why" of the relationships although he can solve the problem. The gifted child whose strength is primarily semantic can learn mathematics best if he is led into the understandings through a conceptual, verbal approach as used in the School Mathematics Study Group (SMSG) program.[3] Negro and Mexican children who are outstanding in mathematics but whose semantic abilities may not be at the gifted level can be taught as the gifted are.

The teacher who finds gifted children disliking mathematics and becoming bored or making careless computation errors may well turn to a conceptual approach. On the other hand, some gifted children do not handle numerical (symbolic) thought well at all. Two reference books that can provide stimulation for these children are *Temporal Learning: Dimensions in Early Learning Series*,[4] and

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[3]See Mary N. Meeker's *The Structure of Intellect: Its Interpretation and Uses*. Columbus, Ohio: Charles E. Merrill Publishing Co., 1969.

[4]Barbara D. Bateman, *Temporal Learning: Dimensions in Early Learning Series*. San Rafael, Calif.: Dimensions Publishing Co., 1968.

Creative Teaching of Mathematics in the Elementary School.[5] Both books are highly recommended for their exciting ideas and new ways to teach concepts when children have difficulty in learning.

Numbers

The unifying concepts of number become more vital to gifted children when they discover that the ideas of number have been basic to every civilization. Number concept is international even though names of numbers and symbols for numbers vary. The number line can be used in infinite variations to develop concepts. After children have had numerous experiences in extending the number line both to the right and left of zero, they have a clear concept of negative numbers. A further refinement of this concept is shown in Figure 1.

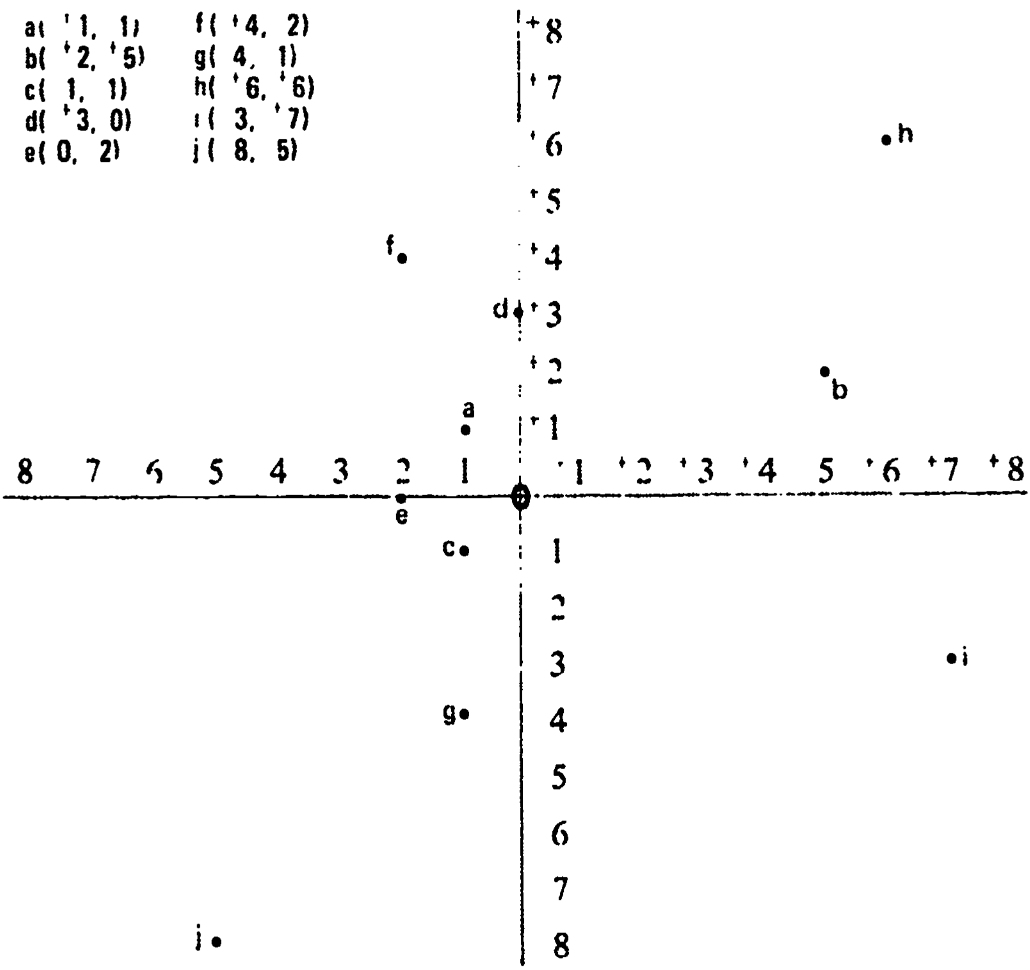

Figure 1. Plotting points in four quadrants

[5] James A. Smith and Alvin M. Westcott, *Creative Teaching of Mathematics in the Elementary School.* Boston: Allyn & Bacon, 1967.

Operations

The program in the regular classroom presents operations in a carefully graded sequence so that concepts of the operations of addition, subtraction, multiplication, and division are developed throughout the mathematical experiences of every child. These concepts can be enlarged in programs for gifted children if the interrrelationships between these operations are made explicit. Examples of such interrelationships are as follows:

Addition and subtraction are inverse to each other.

Multiplication and division are inverse to each other.

Addition and multiplication are related because they are two ways of combining sets.

Subtraction and division are related because they are two ways of removing subsets.

Each operation can be checked by means of its inverse operation. Moreover, each operation can be used to manipulate whole numbers, fractions, negative numbers, and problems without numbers.

Since the commutative properties of addition and multiplication are presented in the regular program, gifted children are, by the time they enter the fourth grade, probably well acquainted with all the possibilities.

The associative property, however, has many variations and applies to all four operations an understanding which leads to an introduction of basic algebraic concepts. The associative property can also provide opportunities for creativity on the part of children who can make up equations for each other.

The concept of the distributive property is equally challenging for gifted children when they understand distributive properties because then they are moving into the realm of abstract symbolic thought.

Mathematical Sentences

Concurrently with other learning in mathematics, pupils learn to use mathematical sentences to communicate. Skill in using the language of mathematics must be developed in a way very similar to that used to develop skills in other languages. Through frequent use of mathematical sentences to express their problems or findings, children become aware of the range, versatility, and precision of this language. A few of the concepts they develop through these experiences are the following:

Mathematical sentences can express inequality or equality.

Mathematical sentences can be simple or complex.

Mathematics sentences can direct one to perform any or all of the operations.

Mathematical sentences can demonstrate the associative property.
Mathematical sentences can demonstrate the commutative property.
Mathematical sentences can demonstrate the distributive property.
Mathematical sentences can be stated without using numbers.
Formulas are mathematical sentences which can be used as a pattern for solving or finding the answers to many similar problems.

Measurement

As they use different measuring systems, children begin to develop concepts about ratios between different number systems. Formulas which show the relationship of different units of measure are the outgrowth of this experience. Many American children have been completely baffled by the metric system because they were taught the formula for changing meters into inches without ever having measured anything with a meter stick.

Reversing this process, children's initial experiences should include recording two readings for every measurement—one in the metric system and the other in the American system. On many canned goods dual readings are printed: Net weight 1 lb. (453 g.) or Net weight 12 fl. oz. (354 ml.). When dual reading becomes a familiar process, children can make educated guesses and estimates.

Children should measure with various instruments: meter sticks, the metric side of the ruler, Celsius thermometers, scales which give readings in grams and kilograms, and barometers which can be read in millibars as well as in inches of mercury. Concepts emerge as the children become familiar with these tools.

The ability to make educated guesses and estimates gives evidence of concept formation. These answers must be checked and proved by the guesser. Guesses become more accurate as the concepts are clarified.

Graphs and Geometric Figures

In the California-adopted textbook *Modern Arithmetic Through Discovery*,[6] considerable space is devoted to reading and using graphs. Activities for the gifted beyond those mentioned in the state textbook are suggested here in this publication. Summarizing data by means of graphs is an activity which communicates information and is instrumental in concept development. The communication aspects

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[6]Robert Lee Morton and Others. *Modern Arithmetic Through Discovery*. Vol. IV. Morristown, N.J.: Silver Burdett Co., 1964.

need further exploration by the gifted. Graphs provide a practical use of geometrical figures for interpreting data. Organizing data so that it can be graphed by using geometric figures is very effective in clarifying concepts.

Creative experiences are possible when children are freed from rote work in order to develop graphs on data that they find interesting. These tasks are especially recommended for gifted children who have difficulty in mathematics or for the gifted who have strong figural and motor skills.

Chapter 3

Generalizations, Understandings, and Principles

In the new approach to mathematics, concept development is emphasized before students are asked to make generalizations. The development of understandings and principles follows.

Generalizations

In dealing with nongifted children, Inhelder and Piaget did not find any ten-, eleven-, or twelve-year-old children who had yet progressed to the formal stage of thinking. They found that the intellectual capacities which enable children to make generalizations first become evident in adolescence. Regarding intellectual growth, Inhelder and Piaget state that "the analyzable facts of the growth of experimental reasoning are interesting because they show us that a number of new operations and concepts emerge in close linkage with the establishment of propositional logic. . . ."[7]

In working with a group of gifted children, we can assume that some of them have progressed to advanced intellectual stages and may already be making generalizations when they are in grades four, five, and six. This progress is particularly evident if they show skills in transformational and implication thinking. But it would be dangerous to generalize this assumption for all gifted children who show strengths in other intellectual dimensions. The ability to generalize often has to be developed in steps.

Mathematical games such as *What Are My Rules?* encourage even younger children to search for generalizations. This game has unlimited possibilities for creative approaches to mathematics. Although the *Math Workshop* has devised sets of cards for playing

[7]Bärbel Inhelder and Jean Piaget. *The Growth of Logical Thinking from Childhood to Adolescence: An Essay on the Construction of Formal Operational Structures.* New York: Basic Books, 1958, p. 105.

What Are My Rules?[8], children enjoy arranging their own sets of facts to present to their friends.

Instead of teaching gifted children to memorize without question the generalizations handed down from another generation or pressuring them to come up with their own generalizations, the mathematics teacher can encourage them to test generalizations that are commonly accepted. As they check for discrepancies, they can begin to frame valid generalizations such as the following: "You can add only things that are alike. You can add apples to apples or oranges to oranges, but you cannot add apples to oranges." Is this a valid generalization? How can you test it? Using the same facts, what words can you use to state a generalization that would be valid?

This generalization can be tested very simply. Have the children place apples and oranges in a bowl. They have added apples to oranges. What statement can be made to explain both the generalization that the "old math" teacher made and the facts that the children have discovered? Statements can include the following:

If the desired result is a larger set of apples, then a set of apples should be joined to another set of apples.

When a set of apples is joined to a set of oranges, the result is a set of fruit.

This exercise also teaches classification.

Another generalization that was memorized as one of the 100 facts of addition in the "old math" ($2 + 2 = 4$) can be tested by the following questions:

Can 2 plus 2 ever equal 1? If you add two cups and two cups, the result is one quart.

Could 2 plus 2 ever equal 11? In base 3, 2 plus 2 equal 11.

If two feet and two feet are added together, the result is one yard and one foot.

In base 4, 2 and 2 make 10.

Let the children frame statements that express these facts as a valid generalization.

The mathematics teacher will never lack generalizations to test. After students are thoroughly familiar with the "new math," they can explore the generalizations so carefully memorized by older generations. They may have fun testing "old math" generalizations, hopefully in such a manner as not to threaten their parents, who feel uncomfortable enough with new math and quite helpless to assist their children with difficult homework assignments.

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[8]Robert W. Wirtz and Others, *Math Workshop: Level C; Teacher's Guide.* Chicago: Encyclopedia Britannica Press, 1964, pp. 34-40.

Understandings

After testing and reframing generalizations and discovering some generalizations of their own, gifted children can see more clearly the overall pattern of the field of mathematics. Will this understanding take place when they are ten, eleven, or twelve years old? With gifted children anything can happen.

Understanding involves much more than being able to stand up and say, "The square of the hypotenuse is equal to the sum of the squares of the other two sides." To understand the theorem of Pythagoras, a child needs to know and understand what a right angle is, which part of a right triangle is the hypotenuse, what properties a square has, what operation produces a sum, and how to recognize the "other two sides." He needs to know the vocabulary of mathematics.

It can be useful to know that ancient builders used to knot a cord at three units, four units, and five units. By placing a stake at each knot and uniting the end of the cord with the last knot, they were able to produce without fail a right (90-degree) angle. Exemplified in mathematical symbols, the Pythagorean theorem can be stated as follows:

$3 \times 3 = 9$
$4 \times 4 = \underline{16}$
$5 \times 5 = 25$, which is the square of the hypotenuse or the long side of the triangle.

This statement can be varied by the use of 6, 8, and 10; or 9, 12, and 15; or 12, 16, and 20. Questions that can be raised for children to consider are the following: How useful is this information? How important was the right triangle to the ancient builders? Is the 90-degree angle important to us? Why? By what process do modern builders determine a 90-degree angle?

Source books of ideas are also available to develop the understandings of all pupils. Robert B. Davis has written extensively on the Madison Project. He recommends such criteria as the following for choosing experiences:

Adequate readiness
Close relationship to fundamental ideas
Active role for students
Interesting patterns under the surface of every task
Worthwhile experiences[9]

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[9]Robert B. Davis, *The Madison Project: A Brief Introduction to Materials and Activities* Reading, Mass.: Addison-Wesley Publishing Co., 1964.

Whether or not the particular ten-, eleven-, and twelve-year-olds in a gifted program have reached Piaget's third stage of formal thinking in intellectual development is unimportant because they soon will reach it. Experiences in the program must be worthwhile. This worthwhile quality should contribute to understandings even if the beginning of understanding occurs a year or two later.

To realize the value of worthwhile experiences when understanding occurs later, some interesting questions can be raised for gifted children to consider about the immediate understanding of such a mathematical genius as Galileo, for instance. How many times, we might ask, did Galileo watch the swaying chandelier in the cathedral before he began to wonder about it? How long did he wonder before he developed a hypothesis? How long did the hypothesis nag at his consciousness before he thought of the ingenious method of testing it mathematically by using his own pulse as the timekeeper? These technicalities of his great contribution to science have not been recorded. Galileo was a youth before his understanding of the phenomenon had developed to the point that he could formulate the law of the pendulum. There is something for educators to learn from Galileo's story. He had time to observe. It is vital that we guarantee some of this free time to our children.

Davis has many suggestions for teachers to help them set the stage for the discovery that leads to understandings. The materials and activities used in the Madison Project are explained in his book, *The Madison Project: A Brief Introduction to Materials and Activities.*[10]

The *Math Workshop* contains unifying ideas or strands (sets, number and numeration, operations, functions and relations, measurement, and geometry) that are developed sequentially with approaches especially appealing to gifted children.[11] Each step in the sequence is planned for the development of concepts and understandings.

Two volumes from the series *Exploring Modern Mathematics*[12] have been used as textbooks for rapid learners in the seventh and eighth grades. All three authors of this series participated in the University of Maryland Mathematics Project and in the School Mathematics Study Group (SMSG).

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[10] *Ibid.*

[11] Robert W. Wirtz and Others, *Math Workshop: Level C: Teacher's Guide.* Chicago: Encyclopedia Britannica Press, 1964, pp. 34-40.

[12] Mervin L. Keedy, Richard E. Jameson, and Patricia L. Johnson, *Exploring Modern Mathematics*, Books 1 and 2 (Third edition). New York: Holt, Rinehart and Winston, 1964.

A wealth of materials and suggested activities are currently available to help the mathematics specialist or the classroom teacher broaden experiences for the gifted group and set the stage for the development of understandings basic to mathematics.

The spiral curriculum, which incorporates each of the strands at every grade level in situations appropriate to the development of the child, can be recognized in the mathematics projects mentioned. It is recommended that the multiple textbook approach, so familiar in the realm of the social sciences, also be used in mathematics programs for gifted children. The children should also be allowed to compare textbooks and to evaluate them.

In *The Wonderful World of Mathematics*, Hogben leads children to understand the scope and interrelationships of mathematics.[13] He presents the field of mathematics as the keystone of civilization. Long before Hogben started writing for children, he wrote a book for adults entitled *Mathematics for the Millions*, which enabled readers to understand what mathematics has done for civilization. It became a best seller.

Hogben expressed his understandings of mathematics in chapter headings such as the following:

First steps in measurement, or mathematics in prehistory
The grammar of size, order, and number, or translating number language
Euclid without tears, or what you can do with geometry
From crisis to crossword puzzle, or the beginnings of arithmetic
The size of the world, or what we can do with trigonometry
The dawn of nothing, or how algebra began
The world encompassed, or spherical triangles
The reformation geometry, or what are graphs
The collectivism of arithmetic, or how logarithms began
The arithmetic of growth and shape, or what the calculus is about
Statistics, or the arithmetic of human welfare[14]

In his book *The Giant Golden Book of Mathematics*, Irving Adler suggests many activities which increase children's understanding of mathematics.[15] He presents a good explanation of the metric

[13] Lancelot Hogben. *The Wonderful World of Mathematics.* Garden City, N.Y.: Garden City Books, 1955.

[14] Lancelot Hogben. *Mathematics for the Millions.* New York: W. W. Norton & Co., 1943, p. ix.

[15] Irving Adler, *The Giant Golden Book of Mathematics: Exploring the World of Numbers and Space.* New York: Golden Press, 1960.

system. His mathematics puzzles are recreational, but they also help the young reader to understand the patterns of mathematics.

Principles

As understandings become clearer, underlying principles emerge. Again, it must not be assumed that children who have been identified as gifted are ready for the formal stage in logical thinking. The mathematics teacher should encourage the child who is ready to discover principles. Some of the principles which can be discovered are the following:

Sets represent a way of thinking of collections of things.

Sets may be equal, equivalent, nonequivalent, disjoint, or overlapping; or a set may include one or more subsets.

The number of elements in a set may be any amount from zero to infinity.

Number concepts permeate every aspect of our civilization.

Number systems have been invented by many civilizations that have used various methods to show increasing value such as additive (the Egyptian system); additive-subtractive (the Roman system); multiplicative (the Chinese system); and place value (Hindu, Arabic, and the modern system).

Base ten is most commonly used for the positional or place-value system, but any base can be used.

Different positions indicate different powers of the base.

Exponents can also indicate the powers of a number and are helpful in scientific notation to indicate very large or very small numbers.

Natural numbers, sometimes called counting numbers, were the earliest numbers devised. Number theory is the study of natural numbers.

All natural numbers are either prime or composite numbers. A composite number has factors other than itself or one. A prime number has no factors other than itself or one.

Every composite number can be expressed as a product of prime factors (e.g., $20 = 2 \times 2 \times 5$).

Zero and other whole numbers are all natural numbers.

Zero is the additive identity because it can be added to any integer without changing the value of that integer.

One is the multiplicative identity because any number can be multiplied by one without changing its value.

A rational number can be expressed as a ratio.

An irrational number cannot be expressed as a ratio.

All the rational numbers and all the irrational numbers form a set called the set of real numbers.

All the real numbers are ordered and belong somewhere on the sequence of the number line.

The operations of addition, subtraction, multiplication, and division can be used to manipulate any combination of real numbers.

Rational numbers can be used to express fractions and division.

Mathematical sentences show number, operation, and relationship.

No matter how efficient computer programs become, people still have to compose the mathematical sentence that tells the computer what its work must be. This operation is known as programming.

A mathematical sentence can express equality or inequality, or it can open with a placeholder to indicate where the missing element should be.

Measurement is one of the most ancient ways to show mathematical relationships.

Units of measure are developed to meet immediate needs in problem solving and are still being invented. The most primitive measurements were based on units which corresponded to parts of the human body, such as a foot; hand; fathom (both arms outstretched); ell (midpoint of body to fingertip of one hand); cubit (elbow to fingertip); stride; inch (second joint of forefinger); and palm.

An estimate is an educated guess.

The principles of statistics and probability are the mathematics which help predict the future.

Geometry helps us to measure the world or parts of it through the study of points, lines, planes, space, angles, and solids.

Some of the principles that the gifted child may discover if he has reached the formal stage of intellectual development have been discussed. Of course, his knowledge of mathematical vocabulary is almost indispensable to his growth toward deriving principles. If the child is a collector, as many gifted children are, he may build the requisite vocabulary into his own book, box, or chart.

Chapter 4

Skills and Creativity

A comprehensive mathematics program for gifted children emphasizes the strengthening of subject-area skills such as computation; the development of higher intellectual skills such as evaluation, cognition, and divergent production; and the encouragement of such creative responses as questioning, experimenting, devising new approaches, and testing results.

Subject-Area Skills

When one thinks of mathematics, the first essential that comes to mind is ability in computational skills. Can the child manipulate numbers with ease? Does he feel at home with addition, subtraction, multiplication, and division? Can he use these operations to show the relationship of whole numbers, rational numbers, and irrational numbers? Can he use them to find solutions to problems?

As with the skills in any other field, a comfortable familiarity with computational skills comes about through practice. The teacher who works with gifted children should use a creative approach in providing such practice. Gifted children learn quickly; thus rote work should be avoided because it can be painful, disinteresting, and disillusioning.

Mathematical games and puzzles are effective means for providing practice in a recreational setting. The *Scientific American* features each month "Mathematical Games," which provides a rich source of recreational mathematics. While many of these games are quite advanced and difficult, some are enjoyed by children. One of the "Mathematical Games" articles describes games entitled *Jam, Hot,* and an unnamed game which are all related to tic-tac-toe and end in a draw when players are equally skillful.[16]

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[16]Martin Gardner, "Mathematical Games," *Scientific American*, CCXVI (February, 1967), 116-20.

The Giant Golden Book of Mathematics contains a section entitled "Mathematics for Fun!"[17] Many children's games are based on the principle of the number line, with directions to "go forward four" or "go back six."

Many games involve the collection of sets. Almost all card games fall into this category. Occasionally, the periodical *Highlights for Children* features exciting mathematics problems or articles.

If the gifted group needs special help in multiplication of three-place numbers or long division, these skills must not be neglected. Nevertheless, there are creative ways of teaching these skills to imprint them vividly on the pupils' memories. Often children invent games and activities to replace drill work on paper. Sometimes children enjoy covering the chalkboard with problems and solutions (much as a college mathematics instructor might), a practice that makes discussion and correction easier. The mathematics teacher must diagnose and recommend the kind of work needed for the proper development of skills.

Much of the literature concerning mathematics programs for gifted children pertains to the development of subject-area skills. Schwartz suggests that the gifted child learn shorter methods of computation.[18] For example, there are several short ways in which the child can add a column of two-place numbers such as the following:

21
36
27
43

The child can add 57, 84, and 127; or he can zigzag down with 27, 57, 64, 84, 87, and 127.

Schwartz has also explained shorter methods of computing the products of two-digit numbers. These methods can be used by ten-, eleven-, and twelve-year-old gifted children. The author has demonstrated how the solution can be checked.

Working with and developing magic squares is a recreation which also helps a gifted child develop skills in arithmetic with ease and flexibility. Using all the operations both horizontally and vertically in many situations often reinforces skill learning and encourages flexibility. Traditionally, the checking process for column addition

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[17]Irving Adler, *The Giant Golden Book of Mathematics: Exploring the World of Numbers and Space*. New York: Golden Press, 1960, pp. 81-82.

[18]A. N. Schwartz, "Challenging the Rapid Learner," *The Arithmetic Teacher*, VI (December, 1959), 311-13.

has been to add the column from the bottom upward. Directions can be reversed for all the other computational operations. For instance, the pupil can start with a product and discover appropriate combinations of multipliers and multiplicands that can be used to produce that result. Or the pupil can start with a quotient and fill in the divisor and the dividend. The teacher can ask how many right answers are possible.

The three cases of percent can be developed creatively with the use of the familiar mathematical sentence and the placeholder. In fact, the whole idea of percent becomes much clearer when it is approached as a statement in mathematical terms or, in other words, a formula.

Spelling scores, for instance, can be recorded as percents. The teacher can explain that since different spelling groups were assigned word lists of different lengths, an easy comparison can be made if the ratios of words spelled correctly to words spelled incorrectly were all expressed in ratios of 100. All words spelled correctly can be expressed as 100 percent no matter how long the list. If half of any list were correct, the ratio would be expressed as 50 percent. At this point the teacher can show the similarity to the ratio of dollars and cents and suggest that a table for converting frequently used ratios to percentages would be helpful.

A meter stick makes a convenient number line for checking the percentage table since it is scaled in 100 centimeters and 1,000 millimeters. When the percentage table is complete, the children can answer questions by reading the table as follows:

24 correct out of 25 = ___% (first case of percentage)
___ correct out of 25 = 96% (second case of percentage)
24 correct out of ___ = 96% (third case of percentage)

Plenty of graph paper should be available at all times to be used in numerous creative ways to develop skills in mathematics. Demonstrated earlier in this publication was the plotting of points in four quadrants. This particular skill is immediately applicable to locating points on a globe if the latitude and longitude are known. Graph paper is useful for creating mathematical games which, in the course of play, involve the practice of mathematical skills.

Higher Intellectual Skills

Historically, mathematics, like Latin, was considered to be the subject which was "good" for students and which aided in the development of higher intellectual skills. Dutton points out that today the schools do not evaluate the development of these skills, and standardized tests scarcely probe the surface of the pupil's intellect.[19] Yet so great is the concern of teachers and principals for the scores on these tests that they tend to ignore any objectives which do not produce scores on a standardized instrument.

Dutton's study further shows that the pressure for test scores is not limited to California or to the United States; the French-speaking schools of Europe, as well as English and Scottish schools, also evaluate largely in terms of testing, with emphasis on memory and convergent production.[20] If gifted children are going to develop higher intellectual skills, then children, teachers, and administrators must value other parts of intellectual abilities such as evaluation, cognition, and divergent production.

Dutton suggests that the evaluative process should be expanded beyond cognition and memory to include the following:

A comprehensive range of objectives rather than just subject-matter achievement

A variety of techniques for securing data on pupil progress

A report of the Southern Regional Project for Education of the Gifted suggests that the seminar style of teaching and learning, once reserved for students in graduate school, is appropriate at the elementary level for gifted children.[21] At both levels the use of the seminar has led to the development and successful use of the higher intellectual skills.

In such a seminar for gifted children who are ten, eleven, and twelve years old, pupils can do independent research in the field of mathematics; bring their own written reports (convergent thinking); brainstorm, starting with the ideas presented in their reports (divergent thinking); extrapolate and explore possibilities (divergent thinking): and make evaluations together. They can invent games and try them out with each other. They can identify (cognition) mathematical needs around the school (e.g., a map of the school and playground to scale) and work toward meeting these needs through a group project.

A seminar approach to teaching stimulates the development and use of higher intellectual skills and sets the climate for exchange and development of ideas. The "think factories" in industry have found discussion groups to be highly productive. Similar groups in which gifted children have participated have been equally successful.

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[19] Wilbur H. Dutton, *Evaluating Pupils' Understanding of Arithmetic*, Englewood Cliffs, N.J.: Prentice-Hall 1964.

[20] *Ibid.*, pp. 56-57.

[21] "The Gifted Student: A Manual for Program Improvement." A report to the Southern Regional Project for Education of the Gifted. New York: Southern Regional Education Board, 1962.

Gifted children in the fourth, fifth, and sixth grades may be at any of the learning stages. Some of these children may show evidence of using higher intellectual skills, of generalizing, looking for underlying principles, framing hypotheses, and testing them. An opportunity for communication and discussion, guided by a mathematics teacher who neither pressures nor prods, can help pupils clarify their thinking, recognize significant factors, relate these factors to each other, and search out the underlying principles.

Creativity

Creative mathematicians of the past designed the vehicles for many of the steps in man's progress toward controlling his environment. For example, Copernicus computed many astrological formulas that changed basic ideas in mathematics. Mathematicians subsequently measured the earth and advised many explorers. They were often mistaken about the size of the land mass of Asia or the area of the Pacific and Atlantic oceans, but their confidence led others to seek new lands. Today mathematicians, in command of computers, anticipate and control every move of the Apollo spacecraft, the Mariner space probes, and subterranean investigations.

The whole focus of the new mathematics encourages the bright child to look at every possibility, to question, experiment, discover new approaches, and to test results. Children who are taught simple computer language today will become the computer mathematicians of tomorrow.

Realistically. however, today's child can be limited in his growth by a teacher who feels insecure in the field of mathematics or by a teacher who clutches at the security of an answer book or the "old mathematics" simply because they are familiar. For instance, some creative pupils in the fifth grade were asked to write 99 in Roman numerals. Knowing that this system of numeration was additive-subtractive, they immediately wrote IC. How should a teacher respond? Should he clutch his answer book and say, "No, that's wrong. The right answer is XCIX"? Or should he say, "That's a good idea, and it's much more efficient than the conventional method which subtracts twice and adds once"?

If a teacher values creativity in mathematics, children are likely to try creative approaches. Why shouldn't the teacher paraphrase some of Torrance's creative activities?[22] Instead of the teacher asking, "How many ways can you use a brick?" he should try asking, "How

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[22] E. Paul Torrance, *Educational Achievement of the Highly Intelligent and the Highly Creative*. Research Memorandum 60-18. Minneapolis: Bureau of Educational Research. University of Minnesota. 1960.

many ways can you express the number idea 146?" Recognition can be given for the greatest number of ways in which the problem is attacked as well as for the most ingenious way and the manner of approach.

Torrance suggests that classroom activities emphasizing fluency, flexibility, originality, and curiosity be used to foster creativity. Fluency in number language can be encouraged by suggestions such as, "How many mathematical sentences can you think of to show various relationships between three and five?" (3 is greater than -5.)

Multiple textbooks give children opportunities to evaluate different approaches to similar problems. Experience with multiple textbooks gives them a philosophical set for trying a variety of methods to reach solutions. A question that can stimulate flexibility is, "Can you think of other ways we can solve the problem?" In a seminar situation some time should be devoted to original problems or discoveries presented in turn by each member of the group and thought about by the rest.

In the realm of curiosity, mathematics offers a key to many ideas, beginning with, "I wonder . . ." The child who wonders must be given time to follow up his trend of thought. One characteristic typifies all creative people, particularly mathematicians and scientists: they spend hours involved with their work without regard to time. The system of formal schooling is such that the interested child is usually cut off by the scheduling of activities. The teacher who wants to encourage creative endeavor must have flexibility in his schedule to permit the growth of dedication to an interest area.

Creative expressions are the ultimate proof of a good mathematics program for gifted children. They occur when a child approaches his mathematics teacher on the playground or in the hall and says, "I have just thought of a good way to find out which are the prime numbers," or "I figured out the 25 tables. You just count by quarters."

A teacher demonstrates that he values creative responses by showing interest in and acceptance of every answer given by a child—even incorrect answers! He can say, "Show us how you reached that conclusion." The child may discover his own mistake as he explains, or he may reveal a weakness in his mathematical background. The teacher's next step is to provide the pupil with experiences to strengthen his background and build his concepts. Respect must always be shown for the child's ideas by the teacher and by his peers who adopt the teacher's attitude.

When this respect permeates the atmosphere of a classroom, creativity can occur.

A study of how creative mathematicians lived and how their divergent thinking ushered in new epochs of civilization can also help children to value creativity. The story of Galileo timing the swings of the chandelier in the cathedral by counting the beating of his own pulse is a story that every gifted child should know. Galileo's was a creative approach to measurement. After hearing Galileo's story, children can be encouraged to devise creative methods of measurement and to look for laws or principles. What dramatic changes in the world were made when the law of the pendulum, discovered so ingeniously by Galileo, was applied to the invention of the first clocks! Kepler, Newton, and Einstein were all creative mathematicians with fascinating lives.

Chapter 5

The Role of Mathematics

In the development of mathematics programs for gifted children, it is important to recognize the role which mathematics plays in the development of human potential, its relationship with other subject areas in the curriculum, and the need for an orderly progression of mathematics instruction through the various grade levels.

Development of Human Potential

As we approach the twenty-first century, the role of mathematics in the future of this universe is of increasing importance. We cannot afford to have the masses of mathematically inadequate pupils who just "got by" in past generations and felt rather frightened and bewildered by the mysteries of calculations. Studies of prospective elementary school teachers and actual elementary school teachers indicate that both groups have difficulties with some phases of the arithmetic they are supposed to teach children.

When the gifted children of this generation become adults, almost every field they might wish to enter will actively involve them in mathematics. The social sciences are becoming increasingly concerned with statistics as well as with other mathematical fields. All sciences depend upon the kind of mathematics relevant to their disciplines. Business, government, and labor are all dependent upon finance, which is a branch of mathematics.

Within the past few years legislation was passed in California which made a college course in mathematics mandatory for elementary schoolteachers. Previously many teachers came into the field of education without having had a course in mathematics since the eighth grade.

If gifted children are to be given the opportunity to develop their potential, then they must have a mathematics specialist who is available for consultation, for class work. and for planning. Because gifted children have the ability to see relationships, to do abstract thinking, and to extrapolate, they need all the experiences they can get in the field of mathematics.

By the time the gifted children of this generation become adults, the "new mathematics" may be replaced by newer and more effective systems. Therefore, subject-matter content alone cannot be relied upon to develop potential in gifted children. For these children to develop their potential to the fullest, they need a creative approach to all aspects of mathematics as it now exists; they also need open and alert minds that are ready for future revelations in this ever-expanding realm.

In ancient times the secrets of mathematics remained with the priestly caste and were handed down only to those of the inner circle. These were the mysteries by which men were able to predict the phases of the moon, the changing of the seasons, the procession of the stars, and the times for planting and harvesting. Today every schoolchild has access to those early secrets, and mathematics still holds the key to the secrets of the universe. As the world faces the beginning of space exploration, mathematicians are needed more than ever before. Computers control space flights, but gifted mathematicians are needed to program the computers.

The teacher who works with gifted children must be more than a classroom teacher. He cannot be a high priest, divulging the secrets by which others are controlled through oracles and predictions, but he should give his gifted pupils insights into future possibilities in the development of their potential through mathematics.

Relationship with Other Areas in the Curriculum

To develop a rich mathematics program for gifted children, the classroom teacher should help pupils see the direct relationship between mathematics and other subject areas. An understanding of the history of mathematics as well can give children a clearer perspective of the breadth of this subject.

Astronomy is a subject clearly tied to mathematics because of its units of measure and points of reference. Historical reference can also be used. For example, in 150 B.C. Hipparchus developed the idea of latitude and longitude to create a celestial globe. This invention enabled Ptolemy to map the visible stars. Latitude and longitude also proved to be a convenient method for mapping the earth. The location of these imaginary lines on earth has been established by sighting the stars. For instance, Polaris, the North Star, is 90 degrees above the horizon (or at the zenith) at the North Pole. The North Pole is located at 90 degrees of latitude. The equator is zero degrees of latitude, and the North Star seems to rest on the horizon (zero latitude). Every point on earth has a latitude and a longitude and is located at an intersection of these imaginary lines.

Much of the study of mathematics deals with the measurements of the earth's surface.

The *National Geographic* globe, which has a clear plastic "thinking cap," should be included in every program for gifted children in the fields of geography and mathematics. The embossed scales on the thinking cap help pupils to figure areas in square miles or square kilometers or to compute distances in miles, kilometers, or degrees. A plastic ring, embossed with numerals representing 24 hours, makes it possible to compare time differences between any two locations on earth.

The social science program for pupils in the fourth grade includes studies of California and its neighbors. Japan is a neighbor often chosen for study, possibly because it lies at approximately the same latitude as California. These two regions suggest many opportunities for geographical mathematics or mathematical geography. The westward movement, a typical unit of study for pupils in the fifth grade, suggests problems in time, distance, latitude, longitude, area, and, of course, the historical number line known as a time line.

Pupils in the sixth grade study the Western Hemisphere (a mathematical reference). Canada, Mexico, Central America, and South America can all be described in terms of latitude and longitude, areas, distance, and time. In addition, a study of the currencies of these countries provides information which can lead to more interest in the mathematics of economics. Another related area is population statistics, which can introduce the mathematics of sociology.

Music and mathematics have always been closely related. Octaves, intervals, and scales are terms which are both musical and mathematical. Notes are designated as whole, half, quarter, sixteenth, and thirty-second. In harmony the third, fifth, and seventh tones play a significant part.

Even literature has its own share of mathematical puzzles and games. Longfellow's puzzle of the bee is one example. Lewis Carroll was the pseudonym of mathematician Charles Lutwidge Dodgson, who packed *Alice's Adventures in Wonderland* and *Through the Looking Glass* full of mathematical jokes.

Physical education programs are filled with mathematics. The dimensions of playing fields, the distances in track events, the time recorded on a stopwatch, the scores of the teams playing, and the height and weight of the players are all part of the mathematics of physical education.

Once the interrelationships between mathematics and other subject areas have been introduced and studied to some extent, it is very important that these interrelationships be maintained in the curriculum throughout the entire school program.

Curriculum Continuity and Articulation

A concern for a broad view of mathematics and its integration with other subjects must be complemented by a continuity of the curriculum so that children experience meaningful changes and deeper understandings as they progress through their school years. School districts should make every effort to facilitate meetings between teachers of gifted children and mathematics specialists in the elementary schools and those in the junior high schools. Many gifted children have experienced a vital, creative special program in the elementary school only to reach the junior high school and find a restrictive program that gives little or no challenge to them.

The larger enrollment in the junior high school makes homogeneous grouping practicable. More opportunities are available for mathematics specialists at this grade level. But if children in the gifted program are then graded on the curve, some very able children are branded as "C" students because they are only average in the gifted class. Another problem occurs when children who are highly gifted and have been taught in elementary school by a gifted and creative mathematics teacher reach junior high school and find that they are expected to mark time doing the same work over again because there has been no continuity between programs.

Efforts must be made by individual school districts to coordinate their programs. Each district plans its own program for gifted children; it should therefore take the responsibility for making sure that children have appropriate opportunities at every level.

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