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ABSTRACT

Groups of students were required to respond to the same eight two-person games, two variants each of four qualitatively different separable games including Prisoner's Dilemma. The games were generated by varying a payoff parameter which altered the potential maximum per trial difference between payoffs for the two players. The groups differed systematically in terms of the types of numerical representations chosen for each game. For each pair of games, the subjects were asked to indicate which strategy for each game they would choose to play against a hypothetical opponent and for which of the two games they would most prefer to play the dominant strategy. The paired-comparison data for each subject was entered into a dominance matrix and processed by triangular analysis. The data from 33 of the 35 subjects yielded satisfactory unidimensional scales. It was concluded that it is possible to scale qualitatively different games along the same dimension. It was further concluded that most game players prefer to select strategies which will maximize the difference between their score and their opponent's score even when those strategies are not dominant. (Author/RC)

UNIDIMENSIONAL SCALING OF TWO PERSON NON-ZERO SUM GAMES

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This paper focuses on an old interpretation of the dynamics of the game "prisoner's dilemma" and on a new analysis technique which might generate a new taxonomy of 2 x 2 games based on response measures. Several theorists (Rapoport & Guyer, 1966; Harris, 1972; and Hamburger, 1974) have proposed rational taxonomies of 2 x 2 games based on both ordinal and interval relationships among payoff parameters. For example, games could be classified on the basis of whether neither, only one, or both players have a dominant strategy. Alternatively, games could be treated as stimuli like weights, hues, or sound frequencies, and then be scaled using any one of a variety of standard scaling techniques. In this fashion the structure of the stimulus space would be revealed by the manner in which subjects responded to the games. Games could then be classified according to their coordinates in this space.

The practical problem of determining what and how many games should be used in a first attempt to scale games was solved by choosing a set of games which could be related to one and another by a very simple mathematical function. Hamburger (1969) has shown that all symmetric separable games can be generated by varying a single parameter if payoffs are assumed to be given on an interval scale. There are only four different games in the class of symmetric separable games if only ordinal relationships between parameters are taken into consideration. We chose to scale two variants each of the four different games and hypothesized (wished) to ourselves that the resultant scale would be unidimensional and would be correlated with the parameter used to generate the games. A second reason for choosing separable games was that it is very easy to explain to college sophomores how to play separable games. One problem with separable games is that any given separable game in normal form can be represented to subjects in an infinite number of decomposed forms which are logically but perhaps not psychologically equivalent. Indeed, Messick & McClintock (1968), and Pruitt (1967, 1970) have shown that different decomposed forms of the same game can produce different choice behaviors. If this were the case in the present study, then it would be doubtful that any derived scale would have transituational applicability. In order to control for this potential criticism, each game was presented in four highly variant decomposed forms.

This study bears on an interpretation of the dynamics of prisoner's dilemma because some prisoner's dilemma games are symmetric and separable and because the parameter used to generate the games can be cloaked with psychological significance. The maximum per trial difference between the payoffs of the two players in this study is the value of the parameter, x, plus eight. A number of theorists (Rapoport & Chammah, 1965; Messick & Thorngate, 1967; Shubik, 1971; Griesinger & Livingston, 1973; and Brew, 1973) have directly or indirectly indicated that the desire to maximize relative gain may be the strongest factor determining choice behavior in 2 x 2 non-zero sum games. According to this view, subjects do not cooperate when playing prisoner's dilemma because that is the only strategy which can produce a win, i.e., scoring more points than the other player. Evidence of a unidimensional scale correlated with the x parameter from this study would support the maximizing relative gain point of view and extend it to a broader class of games.

Method

Thirty-five introductory psychology students were divided into four groups, 10 Ss in Group I, 8 in Group II, 9 in Group III, and 8 in Group IV. Each group was required to respond to the same eight two-person games, two variants each of four qualitatively different separable games. The games were presented in decomposed form. Figure 1 shows the form of the decomposed game used to generate all of the games used in this study. Figure 2 shows the same game in normal form. Each subject was told that he was going to play a series of games with an hypothetical partner. For each game, both players were to independently and simultaneously chose one of the two available strategies, X or Y. A choice of either strategy would guarantee the award of a fixed number of points to both players. The "real" subjects' points are listed in the "yours" column and the hypothetical subjects' points are listed in the "others" column. Thus, the total number of points won on any given play of a game would consist of those points guaranteed to oneself plus those received from the hypothetical partner. For example, if both players chose X, then each player would receive 8 points (8 = (8 + a) - a). Each player was told that his objective was to win as many points for himself as possible.

The eight different games were generated by varying the x parameter from 14 to -14 in 4 point steps. According to the Rapoport and Guyer (1966) taxonomy, this parameter variation generated variants of Games 12, 9, 3, and 6. These games are shown in normal form in Figure 4. In each of these games, X is the dominant strategy for both players. Game 12, prisoner's dilemma, has a strongly stable deficient equilibrium. Game 9 has a strongly stable equilibrium and Games 3 and 6 are no-conflict games. The four experimental groups differed in the manner in which the a parameter was varied. In Group I, a was set at one for all values of x. In Group II, a equalled -.25x + 4.5. In Group III, a equalled .25x - 4.5, and in Group IV, a varied randomly with x with $-10 \leq a \leq 10$.

The games were presented in paired-comparison form and each subject was required to respond to each of the 28 possible pairs. A sample page of a test booklet is presented in Figure 3. Each subject was asked to circle the strategy he would play against an hypothetical partner for each game and to check that game for which X appeared to be the better strategy relative to the other game. In order to clarify this task, the subjects were asked to imagine that they were required to play the X strategy but could check which of the two games they would rather play.

The pages of the test booklets were randomized within groups. The game with higher value of the x parameter appeared in the top position of each test page 14 times. The subjects were given no feedback on what play the hypothetical partner might have made and received no compensation for participating in the experiment other than fulfilling an introductory psychology course requirement.

Results

The paired-comparison data for each subject was entered into a dominance matrix. A sample matrix appears in Figure 5. The row and column labels refer to the value of the x parameter in the decomposed games. A number one in the ijth cell of the matrix indicates that the column stimulus was prefered over the row stimulus in the sense that when the subject was forced to state a preference for playing the X strategy for one of the two games having the jth column and ith row value for the x parameter, respectively, the subject checked the first game. The data was then processed using a form of triangular analysis (Coombs, 1964). For each <u>S</u>, the number of intransitivities were computed and the number of deviations from an "ideal" dominance matrix generated by using the ordinal values of the x parameter to define a unidimensional scale. Analysis of variance indicated that the four experimental groups did not differ in number of intransitivities, $F(3,31) = .62$, or in number of deviations, $F(3,31) = .05$.

Strickly speaking, if the data is presumed to be errorless, then the unidimensional hypothesis can be rejected for any given subject if that subject produces a single intransitivity. Twenty-one of 35 Ss produced at least one intransitivity out of a total of 20 possible intransitivities. However, 26 of the 35 Ss produced 6 or fewer instransitivities.

Another method of assessing unidimensionality is to assume that Ss are responding randomly and to determine whether there are more correct responses than would be expected by chance. In this context, a correct response is defined as one consistent with the hypothesized unidimensional scale. Equivalently, a correct response may be considered as a number one above the diagonal in Figure 5. Using the binomial distribution and a .05 significance level, the null hypothesis was rejected for 28 of the 35 Ss. Table I lists the number of rejections of the null as a function of number of intransitivities.

To determine whether Ss were responding differentially to the games, the probability of playing the X strategy was computed for each S for each game. These results are summarized in Table II. The probabilities of playing the X strategy were analyzed by an 8 x 4 (Games x Experimental Groups) mixed analysis of variance design. Significant effects of games $F(7,217) = 53.8$, $p < .001$, and the games x experimental groups interaction, $F(21,217) = 1.62$, $p < .05$, were obtained. The games effect is clearly attributable to the relatively low probability across groups of playing the X strategy for games with the x parameter set at -10 and -14.

Discussion

The four experimental groups were differentiated in the manner in which the _a_ parameter covaried with the x parameter. As can be seen by comparing Figures 1 and 2, variations the _a_ parameter did not affect the payoff structure of the games. However, changes in _a_ did affect the manner in which equivalent games were presented to the subjects. Contrary to the results of Messick & McClintock (1968) and Pruitt (1967, 1970), varying the manner in which games were decomposed produced no substantial behavioral differences in this study. This result could be attributed to a number of procedural differences between the four studies. For example, Messick & McClintock and Pruitt in both studies ran their subjects against real opponents for a substantial number of trials.

The relatively few number of intransitivities produced by subjects performing what we consider to be a very difficult judgemental task strongly supports the proposition that the eight games may be scaled along the same continuum. This conclusion is buttressed by the analysis of correct responses in which the null hypothesis was rejected in 28 out of 35 cases. Although these rejections strongly support the hypothesis that the unidimensional scale may be defined in terms of the x parameter, they do not preclude other possibilities. We have not found another way of ordering games along a single continuum which produces a superior result. Thus it appears that the eight games used in this study may be ordered along a dimension which might be called "relative gain". Subjects are more inclined to play the X strategy, the dominant strategy, when playing that strategy has the potential of producing a large positive difference between their payoff and their opponent's payoff. The protocols of three Ss do not support the last generalization. In each case, these Ss produced both a small number of intransitivities and a small number of correct responses. In other words, they were responding consistently but to a different dimension. The best fitting scale for all three of these Ss was related to the absolute value of the x parameter. They apparently were more disposed to play the X strategy when playing that strategy had the potential of minimizing the absolute difference between their scores and their opponent's scores.

The most surprising results of this study appear in Table II. For games with $\cdot$ = 14, 10, 6, 2, -2, -6, Ss strongly preferred the X strategy. However, for games with x = -10, -14, Ss overall preferred the Y strategy. The implications of this result can be better visualized by referring to Figure 6 which shows the two most extreme games, the endpoints of the scale, in normal form. The game on the left (x = 14) is prisoner's dilemma, and the game on the right (x = -14) is a so-called "no-conflict" game. In the no-conflict game, most Ss chose not to play the dominant strategy thereby risking a loss of 14 points apparently in an effort to realize an actual outcome of 0 points and a relative gain of 6 points. Although we would not expect this result if Ss were playing for something of material value we believe this is the best evidence available in support of the maximize relative gain theory. We wonder whether prisoner's dilemma poses any dilemma at all. The real dilemma facing game theorists may be to rationalize deviant behavior in no-conflict games.

It may be argued that the results of this study are atypical because Ss played each game only once. We would counter this argument by agreeing with Guyer and Rapoport (1972) that games should be viewed as independent variables or stimuli and that it is seldom the case in the "real world" that people repeatedly play the same game.

In conclusion, we believe it is possible and useful to treat games as stimuli and to scale these stimuli.

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TABLE I

		Number of Rejections	Number of Ss Having Same Number of Intransitivites
Number of Intransitivites	0	12	14
	1	1	1
	2	5	5
	3	2	2
	4	1	1
	5	1	1
	6	1	1
	7	0	0
	8	1	1
	9	2	3
	≥10	2	6
Totals		28	35

TABLE II

Probability of Choosing the
X Strategy as a Function of the
x Parameter and Experimental Group

x parameter		Group I	Group II	Group III	Group IV
	14	.90	1.00	.94	.98
	10	.90	1.00	.87	.95
	6	.91	1.00	.90	.93
	2	.91	1.00	.92	1.00
	-2	.94	1.00	.83	.96
	-6	.91	.95	.83	.89
	-10	.54	.14	.27	.52
	-14	.50	.13	.16	.32

Figure 1

	Yours	Others
X	8+a	-a
Y	a	x-a

Figure 2

		Person II	
		X	Y
Person 1	X	8, 8	8+x, 0
	Y	0, 8+x	x, x

Figure 3

	Yours	Others	Most likely to choose X
X	10	-10	
Y	10	-8	________

	Yours	Others	
X	-2	10	
Y	-10	0	________

Figure 4

Game 12

2, 2	4, 1
1, 4	3, 3

Game 9

3, 3	4, 1
1, 4	2, 2

Game 3

4, 4	3, 2
2, 3	1, 1

Game 6

4, 4	2, 3
3, 2	1, 1

$4 > 3 > 2 > 1$

Figure 5

	-14	-10	-6	-2	2	6	10	14
-14				1	1		1	1
-10	1		1		1	1	1	1
-6	1			1	1	1	1	1
-2		1					1	1
2				1		1	1	1
6	1			1			1	
10								1
14						1		

Figure 6

x = 14

	X	Y
X	8, 8	22, 0
Y	0, 22	14, 14

x = -14

	X	Y
X	8, 8	-6, 0
Y	0, -6	-14, -14